A device to extract energy from water waves

A. J. HERMANS AND E. VAN SABBEN

Delft University of Technology, Dept. of Applied Mathematics, Julianalaan 132, Delft, The Netherlands

J. A. PINKSTER

*MARIN, Haagsteeg 2, Wageningen, The Netherlands**

1. INTRODUCTION

In recent years attention has been focussed on means of extracting energy from ocean waves. As a result, a number of systems have been devised by means of which wave energy, which consists of potential and kinetic fluid energy, can be captured and transformed into either mechanical, hydraulic or electrical energy. The majority of the systems transform the wave motion into a reciprocating motion of a mechanical system which in turn, drives some other form of energy transformation system finally yielding electric or hydraulic energy. The selection of a reciprocating system to transform wave motion into mechanical motion is an obvious choice when the waves are interpreted in terms of being a free-surface phenomenon. It can be shown however, that it is possible to transform wave energy directly into rotational mechanical energy based on a different viewpoint of wave motion. In this paper, the fundamental characteristics of such a device will be discussed and a mathematical model developed at the Delft University of Technology describing the properties of the device will be verified by means of results of model tests carried out at the Maritime Research Institute Netherlands.

The device has been conceived based on knowledge of the fluid kinematics in a regular long-crested wave travelling in deep water. Under these conditions the fluid velocity in any fixed point in the fluid domain takes the form of a vector with constant magnitude which rotates with constant angular velocity equal to the wave frequency (see Fig. 1).

We now consider a submerged horizontal shaft aligned with the crest of the regular waves to which a foil is attached as shown in Fig. 2. At time zero, we let the fluid velocity vector at the location of the shaft be directed at the foil. As time progresses the velocity vector of the fluid rotates and the angle of attack of the foil increases. This results in an increasing lift force on the foil which is directed so that if the shaft is allowed to rotate it will tend to reduce the foil's angle of attack. This system will in fact rotate with wave frequency as long as the load on the shaft is exceeded by the torque exerted by the foil thus constituting a 'synchronous' motor. The concept was verified by preliminary concept tests carried out at MARIN. During the tests which were carried out in regular waves, weights were lifted by using the rotating shaft as a winch. The test set-up is shown in Fig. 3.

The feasibility of the system being verified, the next step is to analyse the hydrodynamic aspects of the system with a view to optimization. To this end, a mathematical model based on two-dimensional linear potential theory has been set up. Results obtained based on this model have been compared with results of model tests with a rotating foil in deep water.

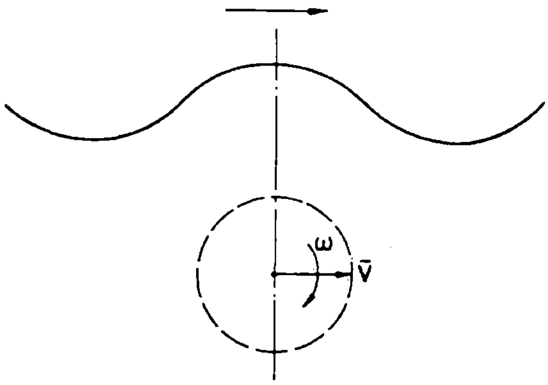

Fig. 1. Wave motion

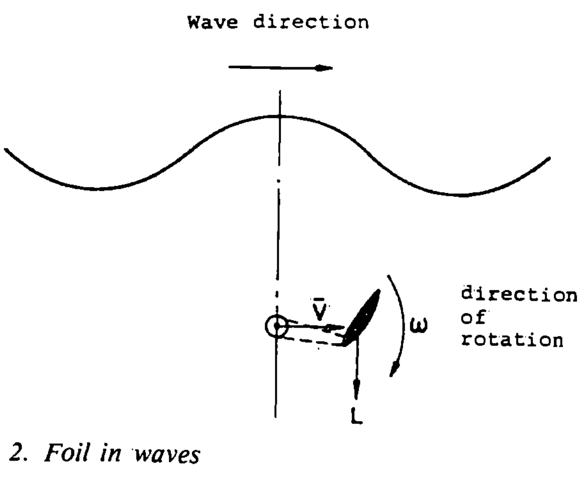

Fig. 2. Foil in waves

*Present address: Delft University of Technology, Ship Hydromechanics Lab., Mekelveg 2, Delft, The Netherlands.

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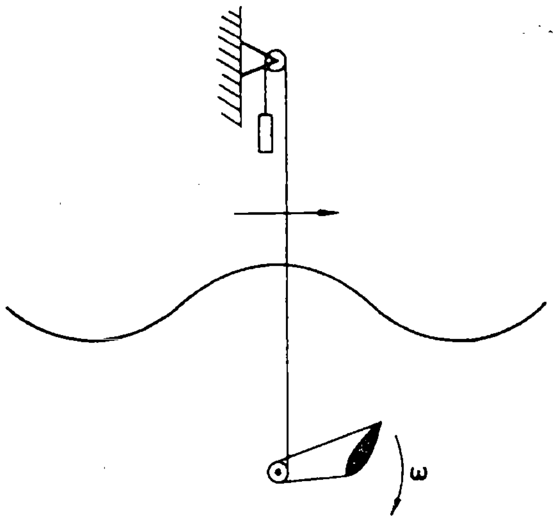


Fig. 3. Test set-up lift experiment

2. MATHEMATICAL FORMULATION

In this section we discuss some properties of a rotating device to extract energy from a travelling plane wave at deep water. The device consists of a profile rotating around an axis. The fluid velocity is at an angle with respect to the profile. Hence, a lift force is exerted at the profile. This driving force generates the rotation of the device at the wave frequency. The energy dissipation of the device determines the angle of incidence of the flow at the profile. If this angle exceeds the stall angle, the lift force diminishes and the rotation stops. Therefore, we know beforehand that the amount of energy extracted from the waves is limited by the hydrodynamics properties of the device. The efficiency of the device is determined by the energy loss of the waves. Therefore, we study the influence of the rotating profile on the amplitude of the travelling wave and the amplitude of higher harmonics generated.

We assume that the lift force can be determined by means of the balance of angular momentum of the device. With the total fluid velocity known, the angle of attack can be computed under the assumption that linearized theory holds. The rotating profile is then replaced by a rotating lifting line. In other words, the wave field is influenced by a rotating vortex if we consider the problem two-dimensional. To obtain more accurate results we also consider the two-dimensional profile completely, together with a simplified description of the shed vortices due to the fluctuating incident velocity vector. It is assumed that the fields due to the incoming wave and the rotating vortex may be superimposed due to the linearity of the problem. Therefore, we study the effect of the rotating vortex or the profile at a small angle in still water.

As mentioned before, we restrict ourselves to the two-dimensional case. First we consider the rotor and assume that at time t the relative fluid velocity $\underline{V}(t)$ at the profile makes a small angle $\alpha(t)$ with respect to the profile. This results in an approximate lift force

$$L = \rho \,|\, \underline{V}(t) \,|\, \Gamma(t) \tag{1}$$

where

$$\Gamma(t) = \pi \,|\, \underline{V}(t) \,|\, \ell \; tg\; \alpha(t) \tag{2}$$

The balance of angular moments gives

$$J \frac{d^2\theta}{dt^2} = L(t) - M \tag{3}$$

where J is the moment of inertia and M the moment exerted externally at the axis of the device. For instance damping may be incorporated in M. In the case of linear damping we write

$$M = \tilde{M} + \beta\dot{\theta} \tag{4}$$

The fluid velocity is described by means of the velocity potential $\phi(\underline{x}, t)$ with $\underline{V}(\underline{x}, t) = \text{grad } \phi$. In the fluid domain D we have

$$\Delta\phi = 0 \tag{5}$$

while at the free surface we assume that the dynamic and the static condition may be linearized; we have

$$\phi_{tt} + g\phi_y = 0 \qquad \text{at } y = 0 \tag{6}$$

The free surface elevation $\eta(x, t)$ is given by

$$\eta(x, t) = -\frac{1}{g}\phi_t(x, 0, t) \tag{7}$$

The incident wave is defined as

$$\tilde{\eta}(x, t) = A \cos[k(x - x_c) - \omega t] \tag{8}$$

hence

$$\tilde{\phi}(\underline{x}, t) = Re\left\{\frac{igA}{\omega}\exp[ky - ik(x - x_c) + i\omega t]\right\} \tag{9}$$

From Fig. 4 point ℓ on the profile is situated at the vectorial position

$$\underline{x}_p = \underline{x}_c + R(\cos\theta - \sin\theta) \tag{10}$$

where we assume R to be small with respect to the wave length ($Rk \ll 1$).

We take as the local fluid velocity at the position of the profile, the fluid velocity at the position of the axis ($\underline{x}_c$).

The relative fluid velocity $\underline{V}(t)$ becomes

$$\begin{aligned} \underline{V}(t) &= \nabla\tilde{\phi}(\underline{x}_c, t) + R\dot{\theta})(\sin\theta, \cos\theta) \\ &= \omega A e^{ky_c}(\cos(kx_c - \omega t),\ \sin(kx_c - \omega t)) \\ &\quad + R\dot{\theta}(\sin\theta, \cos\theta) \end{aligned} \tag{11}$$

In the ideal case of $M(t) = 0$ stationary rotation is possible with $\theta = \omega t + \gamma$ while $V(t) = V(\cos\theta, -\sin\theta)$ which means that the angle of attack $\alpha(t)$ is zero. The values

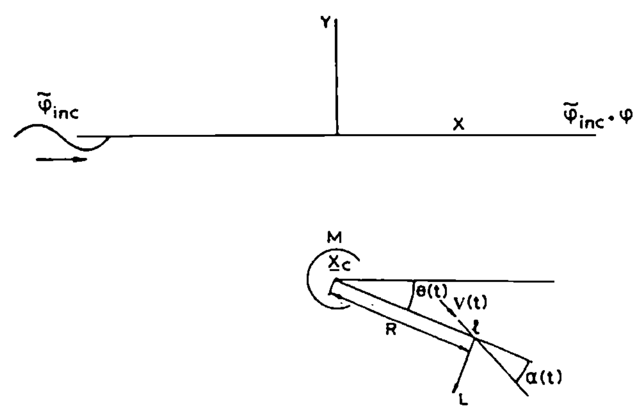


Fig. 4. Notation

of V and γ are determined by equation (11)

$$V = \{(\omega A e^{ky_c} \cos kx_c)^2 + (\omega A e^{ky_c} \sin k\underline{x}_c + R\omega)^2\}$$

$$\gamma = \text{arc } tg\left\{\frac{-(\omega A e^{ky_c} \sin kx_c + R\omega)}{\omega A e^{ky_c} \cos kx_c}\right\} \qquad (12)$$

In the case $M(t) = M(\text{const})$, we may take $\alpha(t) = \alpha(\text{constant})$ for small value of kR. If $M(t)$ is a general function of time or the assumption $kR \ll 1$ does not hold, we should consider $\alpha(t)$ as a function of time.

In this study we assume the lift force to be a constant therefore the profile is replaced by a lifting line rotating at constant speed around x_c. In two dimensions we may use the model of a vortex with constant strength in the moving point

$$\underline{x} = \underline{x}_c + R[\cos(\omega t + \gamma), \; -\sin(\omega t + \gamma)]$$

in the presence of the free surface. The moving vortex generates waves. The energy of these waves equals the energy extracted from the incident wave. The field of a moving vortex can be derived by means of an asymptotic evaluation for large values of t of the field of a vortex started at $t = 0$.

We define

$$\underline{c}(t) := \underline{x}_c + R[\cos(\omega t + \gamma), \quad -\sin(\omega t + \gamma)] = \underline{V}(t) \qquad \text{for } t \geqslant 0$$

$$\underline{c}(t) := 0 \qquad \text{for } t < 0.$$

The complex potential $f(z, t) = \phi + i\psi$, $(z = x + iy)$ can be derived by means of complex function theory or may be found in Wehausen and Laitone[1].

It becomes for $t \geqslant 0$

$$f(z,t) = \frac{\Gamma}{2\pi i} \log \frac{z - c(t)}{z - \bar{c}(t)} + \frac{g\Gamma}{\pi i} \int_0^t \int_0^\infty \frac{1}{\sqrt{gk}} e^{-ik(z - \bar{c}(\tau))} \sin[\sqrt{gk}(t - \tau)] \, dk \, d\tau \qquad (13)$$

where $c(t) = c_1 + ic_2$ which $\bar{c}(t)$ is its complex conjugate.

We study the integral in equation (13) for a large value of t in order to obtain the wave contribution. First we study the integration with respect to τ

$$I = \frac{g\Gamma}{\pi i} \int_0^\infty \frac{e^{-ik(z - \bar{z}_c)}}{\sqrt{gk}} \times \left[\int_0^t \exp\{ikRe^{i(\omega\tau + \gamma)}\} \sin\{\sqrt{gk}(t - \tau)\} \, d\tau\right] dk \qquad (14)$$

partial integration with respect to τ leads to

$$I = \frac{\Gamma}{\pi i} \int_0^\infty \frac{e^{-ik(z - \bar{z}_c)}}{k} \left[\exp(ikRe^{i(\omega\tau + \gamma)} \cos\sqrt{gk}(t - \tau)\right]_{\tau = 0}^{t}$$

$$+ \frac{R\omega\Gamma}{\pi i} \int_0^\infty e^{-ik(z - \bar{z}_c)} \left[\int_0^t e^{i(\omega\tau + \gamma)} \times \exp[ikR \, e^{i(\omega\tau + \gamma)}) \cos\sqrt{gk}(t - \tau) \, d\tau\right] dk \qquad (15)$$

the stock term for $\tau = 0$ leads, after partial integration with respect to k, to terms that tend to zero for $t \to \infty$.

Further partial integration leads to

$$I = \sum_{n=0}^{\infty} \frac{\Gamma}{2\pi i} \left(\frac{R\omega}{i}\right)^n \int_0^\infty e^{-ik(z - \bar{z}_c) + ikRe^{i(\omega t + \gamma)}} \times k^{n-1} \left\{\frac{1}{\Pi_{m=1}^{n} (m\omega - \sqrt{gk})} + \frac{1}{\Pi_{m=1}^{n} (m\omega + \sqrt{gk})}\right\} e^{in\omega t} \, dk \qquad (16)$$

(where we define $\Pi_{m=1}^{0} \equiv 1$).

This expression is meaningful if the series expansion is convergent. We are interested in waves at the free surface $y = 0$ while y_c is negative, hence the integrals with respect to k exist and the rate of convergence of the series depends on the magnitude of $R\omega$. The integral has first order singularities for $gk = m^2\omega^2$ $m = 1, 2, 3, \ldots$. The integral with respect to k originates from a Fourier integral, therefore the integration contour passes the singularities in the lower or upper complex k-plane. We require a radiation condition and assume a radiated wave travelling in the same direction as the incoming wave and hence and no disturbance far upstream. This leads to a unique choice of the contour.

For $x > x_c$ the contour may be closed in the lower quarter plane. The contribution of the poles are the waves travelling to the right hand side while the integral along the negative imaginary axis tends to zero as $x - x_c \to \infty$.

For $x < x_c$ the contour may be closed in the upper quarter plane. There is no contribution of poles, hence no waves, while the integral along the positive imaginary axis tends to zero as $x - x_c \to -\infty$.

For a rotating vortex this means that for clockwise rotation, $\omega > 0$, waves are generated travelling to the right only. For counter clockwise rotation, $\omega < 0$ (corresponding to waves travelling from the right) the poles have to be passed in the lower plane, waves are generated travelling to the left.

For clockwise rotation we compute the wave components. To do so, we restrict ourselves to the situation of R being small compared to the wave length. In this case the wave spectrum becomes as follows:

$$\phi = \alpha_0 - 2\Gamma \sum_{\substack{n=2 \\ \text{even}}}^{\infty} \frac{(i\omega^2 n^2 R/g)^n)}{n!} \exp\left\{\frac{\omega^2 n^2}{g}(y + y_c)\right\} \times \cos\left[n(\omega t + \gamma) - \frac{\omega^2 n^2}{g}(x - x_c)\right] + -2\Gamma \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \times i^{n+1} \frac{(\omega^2 n^2 R/g)^n}{n!} \exp\left\{\frac{\omega^2 n^2}{g}(y + y_c)\right\} \times \sin\left[n(\omega t + \gamma) - \frac{\omega^2 n^2}{g}(x - x_c)\right] \qquad (17)$$

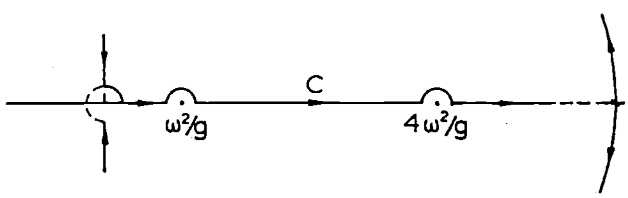


Fig. 5. Contour in the complex k-plane

The constant α_0 may be computed; it results from the singularity at the origin. However, it leads to a zero contribution to the wave height.

We write

$$\phi = \frac{\omega}{g}\left\{A_0 + \sum_{n=1}^{\infty} A_n \times \cos\left[\omega nt - \frac{\omega^2 n^2}{g}(x - x_c) + n\gamma - \delta_n \frac{\pi}{2}\right]\right. \quad (18)$$

when

$$\delta_n = \begin{cases} 0, & n \text{ even} \\ 1, & n \text{ odd} \end{cases}$$

The amplitudes A_n are defined according to equation (17). The mean energy of the incident wave is

$$\bar{E}_{inc} = \tfrac{1}{2}\rho g A^2 \quad (19)$$

while the wave at the right-hand side has an energy of

$$\bar{E}_{trans} = \frac{1}{2}\rho g\left[A^2 - 2AA_1 \cos\gamma + \sum_{n=1}^{\infty} A_n^2\right] \quad (20)$$

We keep in mind that A_i is proportional to A because the vortex strength is proportional to the velocity of the incident field (2). The energy, averaged over one period, absorbed by the device becomes

$$\Delta E = \frac{1}{2}\rho g\left[-2AA_1 \cos\gamma + \sum_{n=1}^{\infty} A_n^2\right] \quad (21)$$

As mentioned before, these results hold for a rotating profile at a constant angle with respect to the local fluid velocity. A more elaborate model has been used in the case that the angle varies due to the velocity profile of the incident wave. We used the thin airfoil theory where the potential function is described by means of a vortex distribution on the foil while vortices are shed in the wake due to the variations in vortex strength. We employ a simple model to displace these vortices and assume that the velocity field is a small perturbation with respect to the incident field. Hence, the free vortices are displaced by the local unperturbed fluid velocities. This means that each vortex makes a circular motion. It turns out that after a few cycles the pattern becomes stationary and the waves can be computed. It turns out that the amplitude of the wave components in equation (17) have changed 10% at most, depending on the configuration. Therefore we restrict ourselves here to the case of constant angle of attack.

3. EXPERIMENTS AND COMPUTATION

To check the theory, a long profile is rotated around a fixed axis

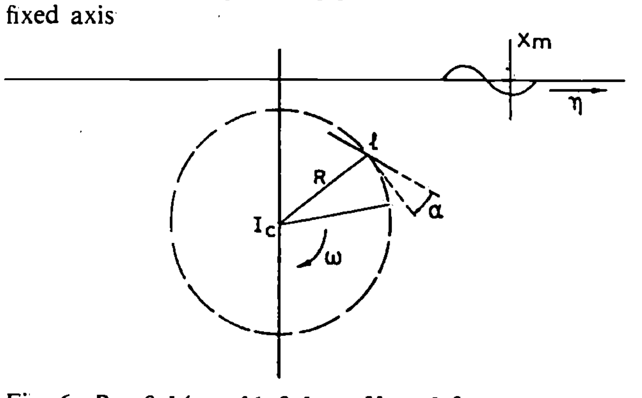


Fig. 6. R = 0.14 m; ℓ = 0.1 m; X_m = 1.8 m

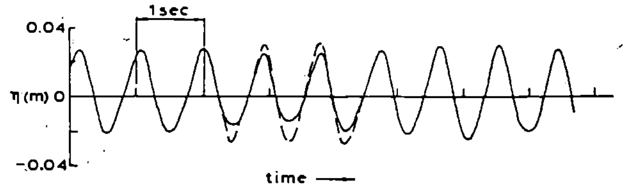


Fig. 7. $y_0 = -0.271$ m, $\omega = 6.91$ rad/s, $\alpha = 0.384$ rad — measured, ---- computed

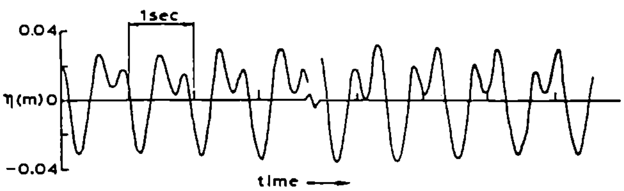


Fig. 8. $y_0 = -0.271$ m, $\omega = 6.91$ rad/s, $\alpha = 0.576$ rad

The radius R and the length of the profile ℓ are fixed but the position of the center y_c, the angle of attack and angular velocity ω may be varied. Tests were carried out in the deep water basin of MARIN. The span of the profile was 1.5 m, long enough to observe a two-dimensional wave profile at the point x_m when the wave height was measured. We have chosen $x_m = 1.8$ m.

The wave height is computed by evaluating equation (13) in (7) numerically. Some results of the experiments are shown in Figs 7–10.

If the linearized free surface holds, Fig. 8 must follow from Fig. 7 by means of amplification of $tg(0.576)/tg(0.384)$ because the angle only plays a role in the determination of $\Gamma(2)$. The change in shape is due to higher harmonics generated by the nonlinearity at the free surface. The computation predicts the wave reasonably well. A closer look at (17) shows that higher harmonics due to the rotating device play a minor role in this case. A closer study of Figs 9 and 10 reveals that it is not possible to make a proper distinction between higher order harmonics generated by the second order effects at the free surface and the higher order harmonics generated by the device. The theory leads to

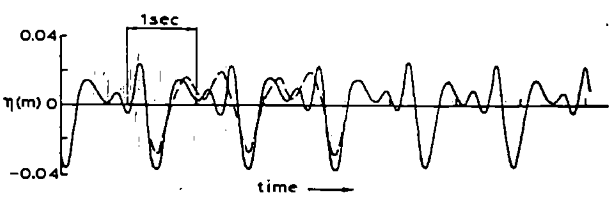


Fig. 9. $y_0 = -0.231$ m, $\omega = 4.6$ rad/s, $\alpha = 0.384$ rad — measured, ---- computed

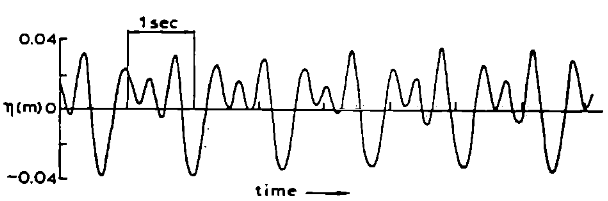
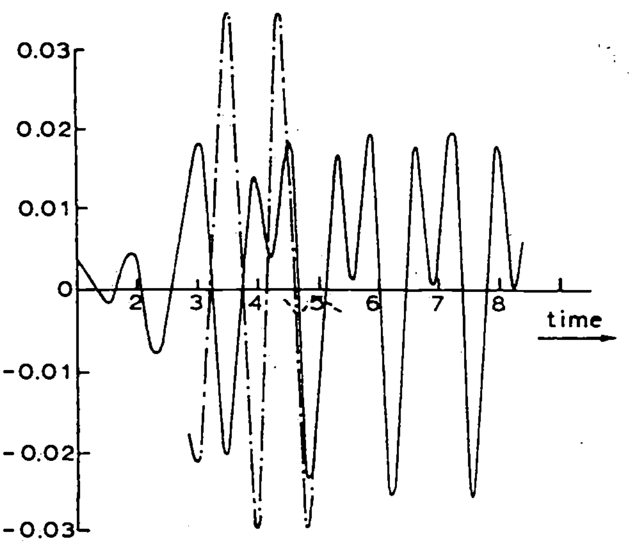


Fig. 10. $y_0 = -0.231$ m, $\omega = 4.6$ rad/s, $\alpha = 0.576$ rad



	y_0	ω	α	x_m
—	−0.231	4.6	0.384	1.8 m
----	−0.231	4.6	0.384	−1.8 m
-·-·-	−0.271	6.91	0.384	1.8 m

Fig. 11. Computed results

a good description of the wave profile in general. In Fig. 11 some computed results are compared. It shows that for decreasing values of ω and y_c higher order harmonics are more relevant. This is explained by the exponential factor in equation (17).

A result is also shown in front of the profile, where according to the asymptotic derivation of equation (17) the disturbance dies out. This is confirmed by the computations. The solid line in Fig. 11 suggests that the transient may be neglected at $t \approx 4$ sec. The computations show that at $x_m = -1.8$ at $t = 5$ sec. the transient is about 10% of the signal at $x_m = 1.8$ m.

Some experiments are carried out to check the energy generator in regular waves. It is shown that the device rotates at the wave frequency. However, the influence at the wave height could not be measured because of the small values at α. The radiated wave height is about one percent of the incident wave height.

4. CONCLUSIONS

The waves radiated by the rotating device may be calculated by means of the theory developed here. The non-linear effects of the free surface may be neglected because in the realistic case the vortex strength is less than the one generated in the rotating experiments. An important conclusion is that waves are generated "down stream", only. Higher order harmonics become significant if the center is situated closer to the free surface, which is the same region where the device becomes more efficient. They are also of importance in the case of long waves. Finally equation (17) can be used in equation (7) to compute the wave height for large values of time, at model scale after 10 sec. The energy absorption of the rotating device in regular waves is given in equation (21). Future efforts will be directed towards more experimental verification of the theoretical results.

REFERENCE

1 Wehausen, J. V. and Laitone, E. V., Surface waves, *Handbook of Physics*, Vol. 9, 1960