Replacement optimisation for public infrastructure assets

Quantitative optimisation modelling taking typical public infrastructure related features into account

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DOI
10.4233/uuid:3cef9da8-d432-4d6a-8805-4c094440bd56

Publication date
2020

Document Version
Final published version

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Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft
op gezag van de Rector Magnificus prof. dr. ir. T.H.J.J. van der Hagen;
voorzitter van het college voor Promoties,
in het openbaar te verdedigen op
woensdag 15 juli, 2020 om 15:00
door

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Keywords: Replacement, optimisation, public infrastructure, uncertainty, reliability, real options, life cycle costs

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Omslagontwerp Promotie In Zicht, www.promotie-inzicht.nl
Drukker Ipskamp Printing B.V., Enschede
ISBN 978-94-028-1965-6
Financiering Dit onderzoek is tot stand gekomen op basis van eigen financiering

Replacement optimisation for public infrastructure assets

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Dissertation

for the purpose of obtaining the degree of doctor
at Delft University of Technology
by the authority of the Rector Magnificus prof. dr. ir. T.H.J.J. van der Hagen;
Chair of the Board for Doctorates,
to be defended publicly on
Wednesday 15 July 2020 at 15:00 o'clock
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Cover design Promotie In Zicht, www.promotie-inzicht.nl
Printed by Ipskamp Printing B.V., Enschede
ISBN 978-94-028-1965-6
Funding This research was self-funded

Preface

A PhD journey is life in miniature. Looking back, I describe it as a lifetime experience which goes way beyond the expectations I had when embarking this incredible journey. True growth is anchored in those places, way out of my comfort zone where I felt scattered, insecure and lost but managed to bounce back.

On this journey I met wonderful people which I would like to thank deeply. First my two promotors, Hans Bakker and Zoran Kapelan who guided me smoothly to the final beacon. Their experience, professionality, balanced approach and confidence were uplifting. My doctoral committee members were carefully selected based on their deep knowledge and experience. Their time and commitment are highly appreciated.

I would like to express special thanks to Pablo Ballesteros-Pérez, an associate editor of the Journal of Construction Management and Economics. Pablo provided more than once valuable feedback on my writing which has been beneficial for my entire dissertation and will be for my future path.

My coauthors on my articles in my dissertation deserve my gratitude. Rob Schoenmaker, Yue Shang and Georgios Leontaris provided feedback on clarity, structure and line of reasoning. Moreover, Yue Shang scrutinized my formulae and asked excellent questions. I am proud of having her beside me as my paranymph. Matthijs Spaan and Pieter van den Berg assisted with my mathematical formulations. Rogier Wolfert provided a platform for discussions and a mirror in which I could sharpen my visions.

Furthermore, I thank Sandra Schuchmann-Hagman, our secretary, for her caring support in all aspects. Students are fond of her and I know why. I would like to thank the Graduate School, especially Ilse Oonk, Wilma ter Hark and Giovanni Bertotti whose mission is to support PhD students. My PhD peers and graduation students were wonderful. Knowing we're on the same journey, with the same struggles and victories, showing interest in each other's work and approaches felt encouraging.

My deepest gratitude goes to my husband, my family and my family in law who provided unconditional support and advice. Without this strong life-long foundation I would never have reached this milestone.

Let's end with Faysal, my very special Tibetan Terrier who talks. For that reason, he cannot join my defense. He would take the floor and lacks an off switch. He kept me company, took me out for walks and made me laugh in all those hours of studying and writing.

Summary

Infrastructure assets are vital for a sustainable and economically strong society. Many infrastructure assets were built between 1950 – 1970 and reach the end of the life they were originally designed for. Increased utilisation accelerates the ageing of infrastructure. Moreover, climate change, transition to new sources of energy and changing societal demands contribute to infrastructure replacement challenges.

The Netherlands Court of Audit warns in 2019 for an unprecedented backwash of infrastructure replacements in the coming decennia because of ageing and current underinvestment. Financing these replacements is an issue of great concern. The Dutch Ministry of Infrastructure and Water Management presently (2019) investigates the magnitude of the financing need which likely outperforms all previous estimates from the Dutch Economic Institute for Construction in 2016 and the ministry itself.

After the period of large-scale construction, focus has shifted to operation and maintenance of infrastructure assets. Asset management as a dedicated life cycle management strategy for infrastructure assets has emerged from 2005 onwards. Asset management intends to maximise value from assets by balancing performances, risks and life cycle costs. As such, asset management directly contributes to levelling the back wash of future replacements as it addresses the optimal timing and costs.

However, concrete methods for balancing infrastructure assets' performances, risks and life cycle costs are still absent in the scientific literature and in practice. Especially optimising life cycle cost in decision making is immature in its development. This observation led to the following research question:

What life cycle cost modelling approaches should be applied for public infrastructure replacement optimisation taking their relevant features into account?

The research commenced with an analysis of several current life cycle costing calculations in public sector organisations in the Netherlands. This analysis revealed common misunderstandings in the application of classical economic present value comparison for infrastructure investment and replacement decisions. Moreover, it resulted in the observation that typical infrastructure related features make classical net present value comparison unsuitable in its application for optimising replacements. Especially the low discount rate of public sector organisations and price increases contribute to this phenomenon in which the application of classical

net present value comparison leads to suboptimal timing and costs. Moreover, the classical net present value comparison does not account for uncertainty nor the flexibility to act upon uncertainty as more information becomes available.

The conclusion that classical net present value comparison is often misunderstood in its application, together with its inherent methodological unsuitability for replacement optimisation, has led to the development of six replacement models for common types of infrastructure replacement challenges. In doing so, theory of different scientific domains has been combined such as Operation Research, Real Options Analyses, Markov Decision Processes and Portfolio Theory. The models are based on case studies and as such serve as blueprints for similar types of infrastructure replacement challenges.

The six models are primarily classified in three types of replacement challenges as depicted in Table S1, which provides a description of each model. The models can be adapted to case specific situations. Guidelines for selecting a model or method are presented in Chapter 8 and encapsulated in three core questions:

1. What is the sequence of intervention strategies?
2. Are the cash flows of the intervention strategies repetitive?
3. Is the future certain or uncertain and to what extent?

For the case studies, comparison of the advanced optimisation models with the inherently wrong application of the classical net present value approach results in deviations of 2% to 44% in total discounted costs. Moreover, abnormalities in optimal replacement times occur. The main contributors to the deviations are identified as neglecting price increases, its uncertainty and wrong method selection.

The primary conclusions of the current research are:

1. Infrastructure related features determine the life cycle costing method. The commonly applied classical net present value comparison leads to significant errors in results and consequently sub-optimisation in timing and discounted life cycle costs.

2. Price increases and its uncertainty influence optimal replacement times but are generally neglected in practice. Historic price indices are provided by the Dutch Bureau for Statistics and CROW. These historic prices can be used to forecast future expected prices and a cone of uncertainty around these expectations. Because of the low discount rate applied by public sector organisations, each substantiated price forecast is better than no forecast.

3. The current research demonstrates the influence of price uncertainty on short, mid and long-term replacement decisions. However, the inclusion of uncertainty complicates the interpretation of long-term results as the number of uncertainty states to be evaluated increases exponentially further in time. The complex approaches to replacement optimisation that include uncertainty are very appropriate for short and mid-term decision making. However, if the interest is establishing a long-term asset planning, the current research advises to use a model which includes price forecasts but excludes its uncertainty. Hereafter, a sensitivity analysis can support a decision maker to gain some insight in the impact of price uncertainty.

4. If the public sector wants to benefit from the methods developed for optimising infrastructure replacements, the current approach to life cycle costing analysis in practice needs to be lifted to a higher maturity level. This requires training of professionals in replacement optimisation modelling. Moreover, knowledge institutes can assist in developing modular software to support these trained professionals. The current research does not support one software model applicable to all types of replacement decisions.

5. Infrastructure replacement decisions are based on multiple criteria among which the economical optimisation as presented in the current research. Therefore, the results of the current research are supportive to a wider decision-making framework which embraces both qualitative and quantitative criteria, acknowledges the interconnectivity between infrastructure systems and the impact of current and future trends such as climate change, energy transition and circularity on the life cycle management of infrastructure assets. Such wider decision-making framework does not yet exist.

The current research is part of solving a large puzzle aimed at levelling the approaching backwash of infrastructure replacements. As outlook for future research, forecasting of price increases and price uncertainty emerge as important contributors. Moreover, the current research recommends as a future research direction to develop integrative decision methods combining quantitative and qualitative replacement criteria. Finally, the current research recommends translating trends, such as climate change, energy transition and circular construction into quantitative cash flow scenarios and to investigate their impact on optimal replacement times with the models provided by the current research.

Table S1 Description of developed replacement optimisation models

<table>
<tr><th colspan="2">Optimise a like-for-like replacement</th></tr>
<tr><td>Chapter 3: Age replacement model
Calculates the optimal preventive replacement interval for i.e. (the conservation of) a lock gate under a strategy in which (the conservation of) the gate is correctively replaced upon failure or preventively at the optimal interval whichever comes first. The preventive interval can be optimised based on least life cycle costs or a reliability threshold. This model does not account for inflation.</td><td>Chapter 3: Interval replacement model
Calculates the optimal preventive replacement interval for i.e. streetlights under a strategy where an individual light is replaced upon failure and the entire group of lights preventively at the optimal interval. This model does not account for inflation.</td></tr>
<tr><th colspan="2">Replace an old asset with a like-for-like replacement</th></tr>
<tr><td>Chapter 4: Inflation adjusted capitalised equivalent model
Calculates the optimal preventive replacement time of i.e. an old bridge to be replaced with a new bridge while accounting for increasing costs as a consequence of ageing and inflation.</td><td>Chapter 5: Simple decision tree and real options analysis
Calculates the optimal preventive replacement time of i.e. an old bridge to be replaced by one out of two scenarios for a new bridge while accounting for failure costs, price uncertainty and political uncertainty</td></tr>
<tr><th colspan="2">Optimise multiple sequential intervention strategies</th></tr>
<tr><td>Chapter 6: Network optimisation model
Calculates the optimal duration of sequential intervention strategies, for example when dealing with an old pumping station and the options to maintain, renovate and replace. The model accounts for increasing costs caused by ageing and inflation.</td><td>Chapter 7: Compound decision tree and real options analysis
Calculates the optimal duration of sequential intervention strategies for example when dealing with an old pumping station and the options to maintain, renovate and replace. The model accounts for ageing, structural failure costs and price uncertainty.</td></tr>
</table>

Samenvatting

Infrastructuur is vitaal voor een leefbare, duurzame en economisch sterke samenleving. Veel infrastructuur is aangelegd in de periode 1950 – 1970 en nadert haar ontwerplevensduur. Toenemend gebruik en mobiliteitsdruk versnelt de veroudering. Ook klimaatverandering, energietransitie en veranderende maatschappelijke eisen dragen bij aan het vervangingsvraagstuk.

De Algemene Rekenkamer waarschuwt in 2019 dat verouderende infrastructuur en achterstallige vervangingen in de komende decennia tot een boeggolf aan vervangingsinvesteringen leiden. Financiering van deze opgave staat onder druk. Het Ministerie van Infrastructuur en Waterstaat onderzoekt momenteel (2019) de omvang van de investeringsbehoefte die naar alle waarschijnlijkheid fors hoger ligt dan in het verleden door het Economisch Instituut voor de Bouw in 2016 en het ministerie is ingeschat.

Na een periode van grootschalige aanleg, laten de afgelopen decennia een accentverschuiving naar beheer en onderhoud van infrastructuur zien. Assetmanagement als beheersstrategie voor infrastructuur is rond 2005 geïntroduceerd. Assetmanagement beoogt om te sturen op een gezonde balans tussen prestaties, risico's en levensduurkosten van infrastructuur. Assetmanagement als zodanig draagt bij aan het nivelleren van de boeggolf aan vervangingsinvestering. Echter, concrete methoden om deze balans aan te brengen ontbreken in de literatuur en de praktijk. Met name het optimaliseren op basis van levensduurkosten door een goede timing van vervangingsinvesteringen is onderbelicht. Deze observatie leidde tot de volgende onderzoeksvraag:

Welke modellen zijn nodig voor het optimaliseren van het tijdstip van vervanging van infrastructuur, rekening houdend met de specifieke eigenschappen en context van deze infrastructuur?

Het onderzoek ving aan met een analyse van levensduurkostenberekeningen en variantenanalyses voor vervangingsbeslissingen bij een aantal publieke organisaties. Dit resulteerde in de vaststelling dat de gebruikte klassieke netto contante waarde (NCW) vergelijking vaak niet goed wordt toegepast. Bovendien maken de specifieke eigenschappen en context van infrastructuur deze klassieke methode ongeschikt voor gebruik voor optimalisatievraagstukken. Met name de lage discontovoet van infrastructuurbeheerders en prijsstijgingen blijken grote invloed te hebben. Het toepassen van klassieke netto contante waarde vergelijking leidt tot suboptimalisatie van het vervangingstijdstip en als zodanig tot hogere kosten of hogere risico's. De klassieke aanpak houdt bovendien geen rekening met prijsonzekerheid, andere onzekerheden en de flexibiliteit hierop te acteren als meer informatie beschikbaar komt. Prijsonzekerheid duidt op de spreiding rond de verwachtingswaarde van prijsstijgingen.

De constateringen dat klassieke NCW-methoden vaak niet goed worden toepast maar ook niet toereikend zijn voor de optimalisatievraagstukken voor vervanging van infrastructuur vormde de basis voor de ontwikkeling van zes optimalisatiemodellen voor veel voorkomende situaties. Hierbij zijn theorieën uit verschillende wetenschapsdomeinen zoals Operations Research, Reële Optie Analyses, Markov Decision Processes en Portfolio Theory gecombineerd. De modellen zijn gebaseerd op casestudies en fungeren als een blauwdruk voor soortgelijke vraagstukken. De modellen zijn primair ingedeeld in drie type vraagstukken zoals weergegeven in Tabel S2. Deze tabel geeft tevens een beschrijving van de modellen die als blauwdruk dienen voor gelijksoortige vraagstukken. Generieke richtlijnen om tot een model- of methodeselectie te komen zijn ontwikkeld in Hoofdstuk 8. De drie kernvragen voor deze selectie zijn:

1. Wat is het type vervangingsvraagstuk?
2. Zijn de levensduurkasstromen van de interventie- of vervangingsoptie repetitief?
3. Is de toekomst zeker of onzeker en in welke mate?

Vergelijking van de geavanceerde modellen met toepassing van de inherent verkeerde toepassing van klassieke methoden resulteert voor de casestudies in afwijkingen van 2 % tot 44 % in totale gedisconteerde kosten. Ook treden verschillen in optimale vervangingstijdstippen op. De verschillen blijken voor infrastructuur voornamelijk veroorzaakt te worden door het niet meenemen van prijsstijgingen en verkeerde methodeselectie. De hoofdconclusies uit het onderzoek zijn:

1. De typische infrastructuur gerelateerde eigenschappen en het type optimalisatievraagstuk bepalen de optimalisatiemethode. De brede toepassing van traditionele netto contante waarde vergelijking leidt tot fouten die significant zijn en tot suboptimalisatie leiden.

2. Prijsstijgingen en prijsonzekerheid blijken een grote invloed te hebben op optimale vervangingstijdstippen. Het voorliggende onderzoek heeft geen praktijkcasussen gevonden waarbij prijsstijging of prijsonzekerheid zijn meegenomen in variantenanalyses. Historische prijzen voor constructie,

onderhoud en beheer zijn beschikbaar bij bijvoorbeeld het CBS en CROW. Op basis van historische prijzen kunnen onderbouwde schattingen voor toekomstige prijzen worden gemaakt. Vanwege de lage discontovoet die publieke organisaties hanteren mogen prijsstijgingen niet zomaar achterwege gelaten worden.

3. Het onderzoek laat zien dat prijsonzekerheid invloed heeft op besluitvorming. Dit speelt voor de korte, middellange en lange termijn. Echter, onzekerheid meenemen in modellen compliceert de interpretatie van de uitkomsten op de lange termijn omdat het aantal onzekerheidstoestanden exponentieel toeneemt in de tijd. De complexere modellen die onzekerheid meenemen zijn uitermate geschikt voor de korte- en middellange termijn besluitvorming. Voor een lange termijn assetplanning adviseert dit onderzoek om wel prijsstijging mee te nemen, maar geen prijsonzekerheid. Om toch enig inzicht te krijgen in de invloed van prijsonzekerheid kan vervolgens een eenvoudig toe te passen gevoeligheidsanalyse worden ingezet.

4. Als de sector serieus werk wil maken van vervangingsoptimalisatie van infrastructuur zal de huidige levensduurkosten aanpak naar een hoger niveau getild moeten worden. Dit betekent dat professionals getraind moeten worden in het toepassen van vervangingsoptimalisatiemethoden. De kennisinstituten kunnen werk maken van het ontwikkelen van modulaire software om vervangingsoptimalisatie te ondersteunen. Een generiek model dat alle vraagstukken aankan, ondersteunt het voorliggende onderzoek niet. Daarvoor spelen te veel factoren mee. Modelleren blijft maatwerk en vakmanschap.

5. Er zijn meer criteria die een rol spelen bij de vervanging van infrastructuur dan de criteria benoemd in het huidige onderzoek. Daarom moeten de resultaten van dit onderzoek gezien worden als een onderdeel van een breder besluitvormingsraamwerk. Dit raamwerk bestaat uit kwalitatieve en kwantitatieve criteria, houdt rekening met relaties tussen infrastructuur-systemen en vertaalt de impact van klimaatverandering, energietransitie en circulariteit naar levensduurkosten van infrastructuur. Een dergelijk raamwerk bestaat nog niet.

Het voorliggende onderzoek is onderdeel van het oplossen van een grotere puzzel met als doel het nivelleren van de naderende boeggolf aan vervangings-investeringen. Als toekomstig onderzoek springen het voorspellen van prijsont-wikkelingen en prijsonzekerheid in het oog en het meenemen hiervan in vervangingsoptimalisatie. Andere aanbevelingen voor verder onderzoek zijn gericht op het ontwikkelen van instrumenten of software om professionals te ondersteunen bij de modellering en de interpretatie van resultaten. Verder beveelt dit onderzoek aan om te onderzoeken hoe kwantitatieve vervangingscriteria samen met kwalitatieve criteria de besluitvorming kunnen ondersteunen. Tot slot is het nodig om trends zoals klimaatverandering, energietransitie en circulair bouwen te vertalen naar concrete varianten voor het levensduurmanagement van infrastructuur, die in de modellen van het voorliggende onderzoek kunnen worden opgenomen.

Tabel S2 Omschrijving van de ontwikkelde vervangingsoptimalisatiemodellen

Levensduuroptimalisatie van nieuw aan te schaffen infrastructuur	
Hoofdstuk 3: Leeftijdsvervanging Berekent het optimale preventieve vervangingsinterval voor bijvoorbeeld een sluisdeur onder een strategie waarbij de sluisdeur correctief vervangen wordt als de sluisdeur eerder aan vervanging toe is. Het model optimaliseert op basis van levensduurkosten of een betrouwbaarheidseis. Het model neemt geen inflatie mee.	Hoofdstuk 3: Intervalvervanging Berekent het optimale preventieve vervangingsinterval van bijvoorbeeld straatverlichting onder een strategie waarbij een individuele lamp vervangen wordt als deze kapot gaat, en alle lampen als groep vervangen worden op het preventieve tijdstip. Het model neemt geen inflatie mee.
Vervangingsoptimalisatie van verouderende infrastructuur door nieuwe infrastructuur	
Hoofdstuk 4: Voor inflatie gecorrigeerde capitalised equivalent methode Berekent het optimale vervangingstijdstip van bijvoorbeeld een oude brug die wordt vervangen door een nieuwe brug waarbij rekening gehouden wordt met toename van kosten door veroudering en inflatie.	Hoofdstuk 5: Eenvoudige beslisboom en reële optie analyse Berekent het optimale vervangingstijdstip van bijvoorbeeld een oude brug door twee varianten van een nieuwe brug waarbij rekening gehouden wordt met faalkosten, prijsonzekerheid en politieke onzekerheid.
Optimalisatie van de levensduren van opeenvolgende interventie-strategieën	
Hoofdstuk 6: Netwerkoptimalisatie Berekent de optimale levensduren van bijvoorbeeld de opeenvolgende interventiestrategieën voor in stand houden, renoveren en vervangen voor een gemaal waarbij rekening gehouden wordt met toename van kosten door veroudering en inflatie.	Hoofdstuk 7: Samengestelde beslisboom en reële optie analyse Berekent de optimale levensduren van bijvoorbeeld de opeenvolgende interventiestrategieën in stand houden, renoveren en vervangen voor een gemaal waarbij rekening gehouden wordt met toename van kosten door veroudering, meerdere prijsonzekerheden en faalkosten.

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1 Introduction

1.1 Unprecedented backwash of infrastructure replacements

Society relies on infrastructure assets such as bridges, pumping stations, locks, dikes, transport mains, water treatment facilities, rails and roads. Infrastructure assets are characterised by long service lives, huge investment costs and predominantly owned by (semi-)public sector organisations as these assets serve public functions.

In the Netherlands, the first infrastructure assets were built before 1900. Accelerated expansion is observed during the years 1950 - 1970. In the coming decennia many infrastructure assets reach their design lives and need to be rebuilt. In addition, increased utilisation adds to accelerated ageing. Moreover, changing societal demands, climate change, energy transition and technology development contribute to the complexity of keeping infrastructure safe in usage and fit for purpose.

A major challenge faced by infrastructure owners is the funding of increasing maintenance, reconstructions and investments. The Dutch Economic Institute for Construction estimated a required funding of € 80 billion for infrastructure reconstructions and replacements for the period 2015 – 2030. In addition, € 96 billion was reserved for maintenance and € 24 billion for expansions (Groot, Saitua, & Visser, 2016). However, available budgets lag behind and proved to fall short at each update (Groot, 2019). Underspending in the past decennium adds a backlog to the increasing maintenance and replacement needs. The Netherlands Court of Audit warns for an unprecedented backwash of infrastructure replacements (Algemene Rekenkamer, 2019). The Minister and Secretary of State of Infrastructure and Water Management refer to the biggest replacement challenge in history and just recently gave order to investigate the full scale and impact of required maintenance and replacement needs to satisfy future performance levels (kst-35000-A-98, 2019). Interesting is the notion of increasing prices and their potential impact on future funding by the Ministry of Infrastructure and Water Management.

1.2 Infrastructure asset management and research gap

The ageing of infrastructure and the maturity in spatial planning portray a shift in focus from expansion towards preservation, reconstruction and replacements of infrastructure assets. In response, infrastructure asset management has emerged as a specified strategy in 2005 in the form of a British Standard, the PAS 55. The PAS 55 was replaced by the first European ISO 55000 standard series on infrastructure asset management in 2014 (ISO 55000:2014; ISO 55001:2014; ISO 55002:2014). The ISO 55000 defines asset management as “the coordinated activity of an organisation to realize value from assets”, where the realisation of value involves in short a balancing of performances, risks and life cycle costs (ISO 55000:2014). This definition identifies opportunities for the maintenance, reconstruction and replacement challenges as described in the previous paragraph as it introduces the aspect of optimising multiple objectives and as such introduces the aspect of timing. The ISO 55000 standard, however, does not prescribe how to balance performances, risks and life cycle costs.

From 2005 onwards, emphasis in infrastructure asset management has been put on performance management, risk management and reliability-based maintenance as is demonstrated in a vast amount of literature, standards and implementation practices which are addressed in subsequent chapters. However, the pillar life cycle costing has received little attention and the optimising of life cycle costs even less (Korpi & Ala-Risku, 2008; Van den Boomen, Schoenmaker, Verlaan, & Wolfert, 2016). Moreover, the commonly applied traditional life cycle cost analysis approaches are not equipped to handle such optimisation challenges. In addition, scientific literature does not offer dedicated replacement optimisation models for ageing infrastructures as is shown in subsequent chapters.

The research gap identified by the current research is the absence of dedicated methods for optimising maintenance, reconstruction and replacement decisions for ageing infrastructure assets from a life cycle costs perspective while accounting for performance requirements and uncertainty about asset integrity and prices. The optimal timing of these decisions contributes to levelling off the expected backwash of required investment expenditures. Moreover, these methods support organisations in short-, mid- and long-term planning of infrastructure maintenance, reconstruction and replacement expenditures.

1.3 Difficulties with traditional life cycle cost analyses

This research commenced with a practical evaluation of ten current life cycle costing analyses (LCCA’s) obtained from Dutch public sector organisations which resulted in the identification of some common misunderstandings in the application of traditional LCCA and led to guidelines how to avoid them (Van den Boomen et al., 2016). A core observation is that traditional discounted cash flow comparison is used for economic replacement optimisation in public sector organisations. Cash flows are forecasted on a finite timeline, often 50 to 100 years, discounted and compared (ISO 15686-5:2017). However, traditional LCCA does not properly account for typical features of infrastructure assets and public sector organisations who own these assets. The following key features are identified:

1. Public sector organisations use low discount rates

   Public sector organisations, who own infrastructure assets use low discount rates ranging from 2% to 5%. The consequence of low discount rates is that future cash flows have more impact on present values and current decisions. Consequently, low discount rates require a more careful estimation of cash flows in LCCA over longer time horizons than currently applied (Treiture et al., 2018).

2. Prices are subject to inflation

   In addition, prices for construction and maintenance are subject to distinct inflation rates (CBS Stateline, 2018; CROW, 2018). Such inflation is generally ignored in LCCA (Faghih Sayed Amir & Kashani, 2018; Van den Boomen et al., 2016). However, inflation further decreases the effective discount rate of public infrastructure organisations as is demonstrated in Treiture et al. (2018). Consequently, the presence of inflation reinforces the need for a careful estimation of future cash flows in LCCA.

3. Inflation is subject to uncertainty

   Inflation is the expected price increase over time. Nevertheless, prices will fluctuate around this expectation. This fluctuation is represented as a cone of uncertainty which widens further in time. Price uncertainty will influence future replacement decisions of infrastructure. Currently, price uncertainty is hardly accounted for in infrastructure replacement decisions (Ilbeigi, Castro-Lacouture, & Joukar, 2017; Van den Boomen, Spaan, Schoenmaker, & Wolfert, 2018). As an example, the Dutch standard for life cycle cost estimates in construction (CROW, 2019), pays attention to probabilistic cost estimates based on user defined confidence bounds and Monte Carlo Simulations but does not make a connection to the registered price indices and their volatilities.

4. Infrastructure assets have long service lives and are generally not for sale
   It is common in traditional LCCA to truncate cash flows at the end of a finite calculation horizon with a salvage value. This is real cash to be received when selling an asset (Brealey, Myers, & Allen, 2017). However, infrastructure assets generally are not for sale and more important, have public service lives extending beyond their design lives, requiring reinvestments in LCCA. In this situation traditional LCCA prescribes truncation of a finite calculation horizon with the expected future value of all cash flows beyond the calculation horizon, or equivalently, discounting cash flows over an infinite calculation horizon (Newnan, Lavelle, & Eschenbach, 2016; Prassas & Roess, 2012). This feature of infrastructure assets again requires a careful estimate of future cash flows over longer time horizons than currently applied.

5. Infrastructure assets are repairable and can also fail beyond repair
   Failure rates influence repair costs and structural failure or reaching a limit state requires rebuilds. Both are realistic features for infrastructure assets but hardly addressed in traditional LCCA. For example, the Dutch SSK manual for cost estimates in construction (CROW, 2019) does not provide guidelines how to incorporate ageing and infrastructure reliability in LCCA. Excluding these (risk) costs from LCCA may result in erroneous results as both can influence the optimal replacement time.

6. Ageing infrastructure assets are often challenged by a sequence of intervention strategies
   The optimal replacement time is not just balancing the costs of maintaining a current asset with the life cycle costs of a new asset. Often, multiple sequential intervention strategies are available such as life-time extension by maintenance, major overhauls and renovation before the actual replacement takes place. Sequential decision making is an optimisation challenge which traditional LCCA cannot handle properly as addressed in Chapters 6 and 7.

7. Managerial flexibility has value in infrastructure replacement optimisation
   Managerial flexibility is the option to choose the best strategy when the future becomes more certain. Capital intensive infrastructure assets generally have long design lives (Dawson et al., 2018; Newnan et al., 2016). Especially in an uncertain environment, postponement of capital expenditures generally will be beneficial. For example, one could follow price uncertainty, the outcome of a political decision or the development of demand and base future decisions on

these outcomes. Traditional LCCA does not take this type of managerial flexibility into account (Buyukyoran & Gundes, 2018; Herder, de Joode, Ligtvoet, Schenk, & Taneja, 2011; Martins, Marques, & Cruz, 2015). This type of managerial flexibility in infrastructure replacement optimisation has value and is addressed in Chapters 5 and 7.

Not all features mentioned above are typical for infrastructure assets. Private sector assets like buildings, airplanes, industrial installations and vehicles are also subject to inflation, price uncertainty, ageing, end-of-life failure and multiple intervention strategies, equally resulting in dedicated optimisation modelling requirements.

However, the most prominent distinction is that private sector organisations have high discount rates. High discount rates make the estimation of future cash flows less relevant. Even when traditional LCCA should not be applied from a mathematical perspective, the errors obtained with private sector assets are less severe because the future estimates have limited impact on current decisions. In contrast, estimating future cash flows for infrastructure assets which are discounted with low discount rates and subject to inflation, requires a careful consideration for which optimisation modelling is applied.

Summarising: traditional LCCA generally does not integrate reliability and is mathematically not equipped to handle price uncertainty, flexibility and optimising a sequence of intervention strategies. This observation is the starting point for the current research and led to the following research question and objective.

1.4 Research question and objective

The main research question to be answered is:

What life cycle cost modelling approaches should be applied for public infrastructure replacement optimisation taking their relevant features into account?

The following sub research questions contribute to answering the main research question:

a) *What LCC methods in general are available for replacement optimisation?*
b) *How are these LCC methods shaped into dedicated infrastructure replacement optimisation models taking the relevant infrastructure related features into account?*
c) *What optimisation models should be selected under what circumstances?*
d) *What is learned from the application of these models in case studies?*
e) *What are the current limitations of these dedicated LCC optimisation models?*

f) *What are the conclusions and directions for future research to reduce these limitations?*

The prime objective of the current research is development of dedicated modelling approaches for optimal replacements of infrastructure assets taking their relevant features into account. Underlying objectives are to learn from the application of these models on infrastructure case studies and to investigate the impact of infrastructure related features on model selection and optimisation method.

1.5 Research approach

This research started from the observation that appropriate LCC modelling approaches for infrastructure replacement optimisation are absent in practice and in the literature. Based on literature research covering the domains of Engineering Economy, Operations Research, Markov Decision Processes, Real Options Analysis and Portfolio Theory, quantitative LCC optimisation models have been developed for common infrastructure replacement challenges. Novel and dedicated approaches were generated by combining existing theory. Data on prices and failure rates were analysed to determine their magnitudes and impact. Case studies were used to demonstrate the application of the models and their added value in comparison to the current traditional LCC approaches.

1.6 Scope and thesis outline

The scope of the current research is quantitative economic optimisation modelling for infrastructure replacement decisions. These quantitative models support a wider decision-making context in which qualitative and quantitative decision criteria should be balanced.

The outline of this thesis follows the sub research questions. Chapter 2 deals with a general overview of LCC methods for replacement optimisation which are divided in classical and advanced methods. Sub research question b is covered in the Chapters 3 to 7. These chapters contain published articles with dedicated replacement optimisation models for infrastructure assets. These models are demonstrated on case studies to investigate the impact of infrastructure related features on method selection. The case studies are meant as blueprints for a wide range of similar challenges. A more in-depth structure for the Chapters 3 to 7 is provided in Paragraph 1.7. Chapter 8 deals with sub research questions c, d and e. In this chapter guidelines are developed for selecting a proper optimisation method based on a classification of the replacement challenge. Moreover, Chapter 8 encapsulates the overarching learning objectives based on the articles in the previous chapters and discusses the limitations of the current models and their application in the case studies. Chapter 9 presents the conclusions following from this research and provides prospects for further research to reduce current limitations. As such it addresses sub question f.

1.7 Classification of infrastructure replacement models

The chapters 3 to 7 contain distinct infrastructure replacement optimisation models. This research builds on selected generic case studies or typical replacement challenges obtained from public sector organisations. These challenges are classified based on the number of different intervention strategies and their main features as depicted in Figure 1.1 and Table 1.1. Taking relevant infrastructure related features into account, dedicated modelling approaches belonging to three common classes of infrastructure replacement optimisation challenges are classified as follows:

1. Optimise a like-for-like replacement (greenfield)

A like-for-like replacement is defined as a strategy where a new asset is bought and exploited over its economically optimised life cycle, assuming it will be replaced with an identical asset. This identical asset will again be exploited over the economically optimised life cycle and replaced with another identical asset, and so forth. This strategy is common to find optimal replacement intervals and supports long-term maintenance and replacement planning. The current research extends this like-for-like replacement with the time-variant probability of end-of-life failure and balances discounted preventive and corrective replacement costs to find optimal replacement intervals. Two models are developed (Van den Boomen, Schoenmaker, & Wolfert, 2018):

- Chapter 3: Discounted age replacement model
- Chapter 3: Discounted interval block-replacement model

2. Replace an old asset with a like-for-like replacement (brownfield)

This class of replacement challenges refers to an old asset in place which is challenged by a like-for-like replacement as described under 1. Instead of starting with a brand-new asset, this class starts with an existing asset with different life cycle cash flows than the replacement option. This approach is also known as a defender/challenger analysis, where the defender is the existing asset, and the challenger the like-for-like replacement. Such approach is common to find the economically optimised replacement time of the defender (existing asset). The like-for-like replacement provides a fair estimate of future cash flows of a replacement. Two dedicated models are developed for this class of replacement challenges. The first model adds inflation, the second model adds managerial flexibility and price uncertainty:

- Chapter 4: Inflation adjusted capitalised equivalent approach (Van den Boomen, Leontaris, & Wolfert, 2019)
- Chapter 5: Fundamental decision tree and real options approach (Van den Boomen, Spaan, et al., 2018)

3. Optimise a sequence of intervention strategies such as life-time extension for an ageing asset before implementing a like-for-like replacement strategy (brownfield)

   This class of replacement challenges optimises an entire chain of possible intervention strategies, such as maintain, overhaul and renovate, and truncates with a like-for-like replacement as an estimate for all future cash flows. Unsurprisingly, this is the most complex class of replacement challenges. Two models are developed. The first model optimises a chain of intervention strategies in a certain future, whereas the second model optimises a chain of intervention strategies under uncertainty. The first model is well equipped to support large-scale maintenance and replacement planning, whereas the second model supports more detailed short and mid-term decision making.

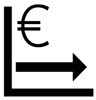
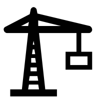
   - Chapter 6: Network optimisation using dynamic programming (Van den Boomen, Van den Berg, & Wolfert, 2019)
   - Chapter 7: Compound real options analysis using a Markov decision process (Van den Boomen, Spaan, Shang, & Wolfert, 2019)

Infrastructure related features which impact cash flow developments (prices, failure rates and structural failure) are labeled with symbols in Figure 1.1, whereas Table 1.1 explains the meaning of these symbols. The flexibility feature is incorporated in the class of optimisation challenge. The service life feature is reflected in the (approximated) infinite calculation horizons. The added value of these models compared to existing approaches is the inclusion of ageing, reliability, inflation, price uncertainty and flexibility.

The models are presented as cost models. Benefits (income) are assumed to be equal for alternatives to be compared as similar public services need to be delivered with a replacement option. Therefore, the benefits are assumed to be non-differential for the replacement timing question. In situations where this assumption is not valid, benefits can easily be included in the presented models.

In the current research, the word inflation is reserved for both price increases and decreases. Therefore, inflation can also be read as deflation. Moreover, the word inflation incorporates the notion of distinct inflation rates for different cost groups. In Chapter 4 and onwards, inflation will be further specified in total inflation, general inflation and differential inflation.

Table 1.1 Symbols used in Figure 1.1

Symbol	**Description**
	This symbol depicts repetitive life cycle cash flows of a replacement option. Cash flows may differ on a yearly basis; however, the cash flows of the full life cycles repeat themselves. This situation occurs for example in the absence of inflation (or negligible inflation).
	This symbol illustrates inflation. Different types of costs are subject to distinct inflation rates. The consequence is that life cycle cash flows of a replacement option are non-repetitive.
	This symbol indicates price uncertainty. The optimisation model accounts for inflation and a cone of uncertainty around this expected value.
	This symbol represents ageing of repairable infrastructure. Ageing is modelled with a failure rate and influences operational expenditures which are subject to inflation or price uncertainty.
	This symbol portrays infrastructure reaching a limit state which induces the need for a corrective replacement investment subject to inflation or price uncertainty. Reaching a limit state is modelled with a time variant probability obtained i.e. from a reliability function or inference theory (load-resistance).

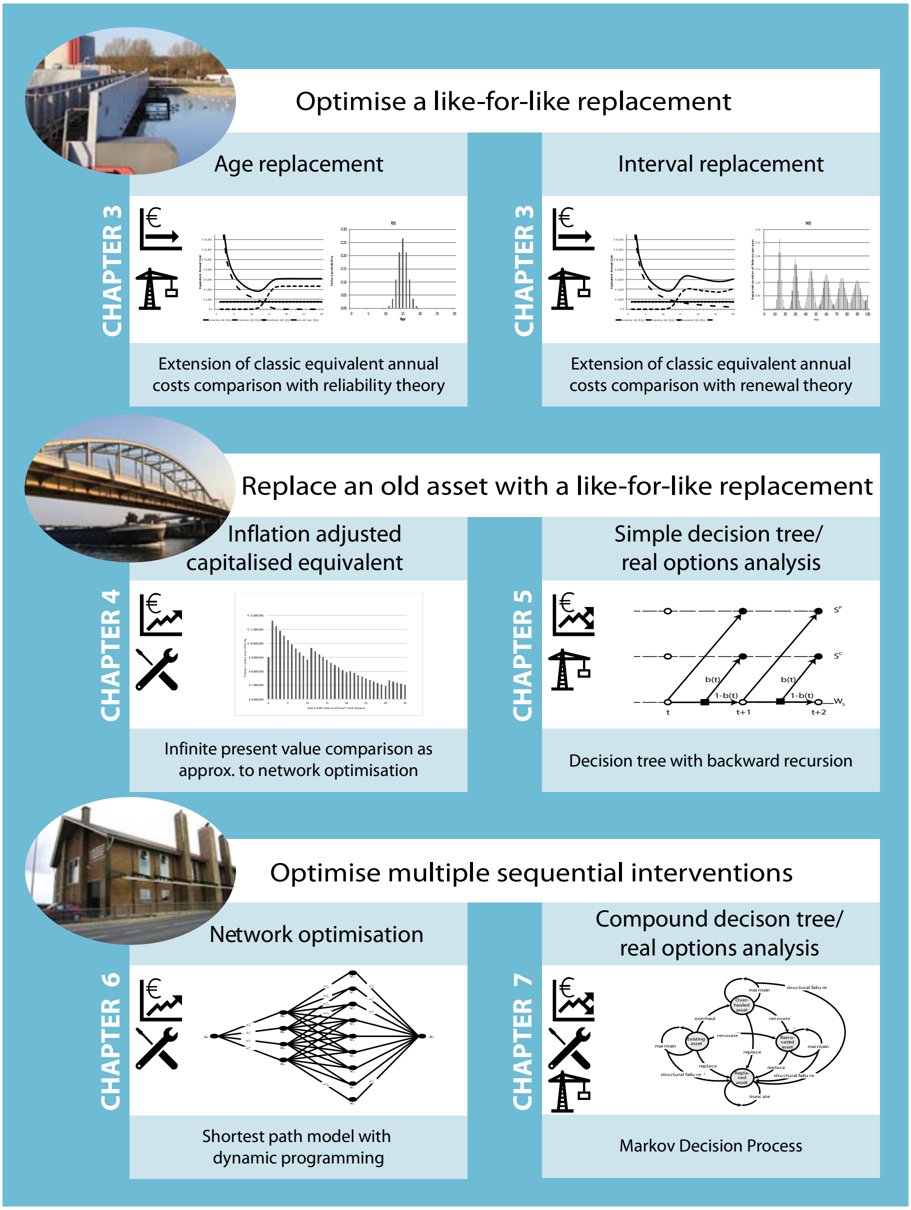


Figure 1.1 Scope of the current research for infrastructure assets

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2
Fundamental replacement optimisation methods

This chapter summarises the fundamental classical and advanced methods for replacement optimisation. It identifies the research gaps and establishes a foundation for the modelling of dedicated replacement optimisation models in the subsequent chapters and their comparison. The material in this chapter is a summary of the detailed literature reviews in the following chapters.

Classical approaches to maintenance and replacement optimisation are found in the domain of Engineering Economics (Blank & Tarquin, 2012; Newnan, Lavelle, & Eschenbach, 2016; Park, 2011; Sullivan, Wicks, & Koeling, 2012). Classical techniques designate an existing asset as a defender and a replacement option as a challenger.

These classical methods are straightforward in their application but have their limitations as these techniques assume that the life cycle cash flows of a challenger are repetitive. In other words, these classical methods are built on the assumption that the replacement option with its investment and operational expenditures will be repeated forever with identical cash flows.

Such a repeatability assumption can be very reasonable especially for public infrastructure assets with long design lives, even longer service lives, steady price developments and slow technology changes. Future cash flows have less weight because of the discounting process. Small changes in the far future will be insignificant.

However, there are also circumstances which refute a repeatability assumption of a replacement option, such as distinct inflation rates for cost components, technology change, the inclusion of flexibility or taking multiple sequential alternatives into account instead of one replacement option. When the assumption for identical renewals is invalid advanced replacement optimisation techniques are required. In the literature, such advanced replacement analysis approaches are labelled as network optimisation models, real options analyses and Markov Decision Processes. In essence all these approaches can be classified under Operations Research mathematics. The mathematics behind these advanced approaches will be elaborated on in subsequent chapters.

The current chapter provides a fundamental summary of four classical approaches and three advanced approaches to replacement optimisation which leads to the identification of research gaps and facilitates the understanding of their application and limitations under specific circumstances.

2.1 Classical approaches

The four underlying fundamental classical approaches to replacement analysis are the net present value comparison over a finite time horizon, the comparison of the equivalent annual costs at the economic life, the marginal analysis and the capitalised equivalent approach. The following explanations are built on and extend work presented in Van den Boomen, Leontaris, and Wolfert (2019).

Net present value comparison over a finite time horizon

The mainstream approach to replacement analysis in practice is a straightforward net present value comparison of different alternatives over a finite time horizon. First, there is a fundamental limitation with this approach when considering replacement optimisation of infrastructure assets which is truncation with expected future cash flows at the end of a calculation horizon. Treiture et al. (2018) demonstrated the impact of approximation errors as a consequence of a wrong or premature truncation when considering infrastructure replacements. Low discount rates of public organisations and inflation enhance the impact of these approximation errors.

The second limitation is that the time-variant estimate of a truncation value often incorporates one or more renewals which are not necessarily identical. In the latter case, advanced optimisation methods are required because the number of potential alternatives increases significantly and there are better mathematical approaches available to define these alternatives and to find the optimal one.

Comparison of equivalent annual cost at the economic life

The second classical technique first searches for and subsequently compares the minimum equivalent annual cost of a defender (EAC_D^*) with the minimal equivalent annual cost of a challenger (EAC_C^*) (Newnan et al., 2016; Park, 2011). The minimum EAC* is defined as:

$$EAC^* = \min_n \left[P \cdot \left(\frac{r(1+r)^n}{(1+r)^n - 1} \right) \right] \tag{2.1}$$

where P is the present value of the life cycle costs, n is the length of the life cycle generally expressed in years and r is the real discount rate.

The minimum EAC of the defender is found by defining scenarios: maintaining the defender for 1 year, maintaining the defender for 2 years, …, maintaining the defender for n years. The present value of each scenario is calculated and transformed to its EAC. The lowest EAC provides EAC* and n*. The minimum EAC of a challenger is calculated likewise based on the challenger's cash flows. In a cost model, a defender will be replaced immediately when the $EAC_D^* \geq EAC_C^*$ because that would indicate the challenger's costs to be equal or less than the defender. In the reverse situation where $EAC_D^* < EAC_C^*$, the subsequent two classical replacement techniques in Paragraphs 2.3 and 2.4 come into view.

The problem with the EAC* comparison is as follows: Mathematically the EAC calculated over one life cycle n* does not differ from the EAC calculated over an arbitrary compound number of these life cycles when the life cycle cash flows remain identical. Therefore, comparing the EAC_D^* and EAC_C^* implicitly assumes identical renewals over the n_C^* of the challenger. The problem with this assumption is that for example price increases would make the EAC_C^* over life cycle n_C^* an invalid assumption for a longer time horizon than n_C^*. Under price increases the future chain of challengers will have a higher EAC than the EAC of its first optimal life cycle. Comparing the EAC_C^* of the first cycle with the EAC_D^* is not a fair comparison any longer.

Therefore, price fluctuations or other factors that disrupt the challenger's repeatability assumption will make the EAC* comparison unsuitable for its application.

Marginal Analysis

The marginal analysis builds on the previous paragraph. If the EAC_D^* is lower than the EAC_C^*, the defender should at least be kept until n_D^* and possible some years beyond n_D^*. How long beyond n_D^* is answered by the marginal analysis (Park, 2011). The marginal analysis compares the year-by-year costs (marginal costs) of maintaining a defender with the challenger's EAC_C^*. (Newnan et al., 2016; Park, 2011) As soon as the marginal costs of the defender exceed the EAC_C^* of the challenger, the defender should be replaced.

Just as explained in the previous paragraph, the marginal analysis equally assumes identical repetitive life cycle cash flows of the replacement option. Another constraint is that the marginal analysis can only be applied when the cash flows of the defender increase gradually after n_D^* (Newnan et al., 2016). If the year-by-year costs of a defender go up and down again, it will not provide a fair basis for comparison. Major overhauls which are common for infrastructure assets, disrupt gradually increasing operational expenditures. The fourth classical replacement optimisation technique in the following paragraph does not need gradually increasing operating expenditures.

Capitalised equivalent approach

The capitalised equivalent approach takes the minimum EAC of the challenger (EAC_C^*), projects these equivalent annual costs over an infinite time horizon and calculates its present value. Hereafter this present value is used as a truncation value for the cash flows of keeping the defender for 1 year, 2 years, etc. before replacing it with the challenger (Park, 2011). The capitalised equivalent method is a present value analysis over an infinite time horizon. It uses the discounted value of an infinite repetitive chain of challengers with identical life cycle cash flows as truncation value. The present value of an infinite stream of EAC_C^* is calculated as:

$$P_C^*[0,\infty] = \frac{EAC_C^*}{r} \tag{2.2}$$

Equation 2.2 is derived from rearranging Equation 2.1 and letting n approach infinity. The capitalised equivalent approach now combines the present value of the cash flows of the defender until replacement time T with the present value of the infinite cyclic cash flows of the challenger from replacement time T onwards. This total present value is shown in Equation 2.3. Minimising P^* provides the optimal replacement time T.

$$P^* = \min_T \left(\sum_{i=0}^{T} \frac{F_{D,i}}{(1+r)^i} + \frac{P_C^*[0,\infty]}{(1+r)^T} \right) \tag{2.3}$$

where $F_{D,i}$ represents the cash flows of the defender in year i.

Although this technique provides a very elegant solution for dealing with fluctuating cash flows of a defender (a limitation of the marginal analysis), it is again built on the same repeatability assumption of the cash flows of the challenger. As said before, such assumption does not hold when price fluctuations or other disrupting factors are involved (Van den Boomen et al., 2019).

Classical life cycle cost optimisation methods are often used to support life cycle cost comparison in the literature and in practice because of their ease in application. For example, a recent application is provided by Farahani, Wallbaum, and Dalenbäck (2018). These authors use the EAC comparison for optimising maintenance intervals for buildings. Other practical applications in equipment maintenance and replacement are provided by Campbell, Jardine, and McGlynn (2011) and Jardine and Tsang (2017).

Classical discounted cash flow analysis can deal with uncertainty. Scope, Ilg, Muench, and Guenther (2016) and Ilg, Scope, Muench, and Guenther (2017) identified and classified approaches which incorporate uncertainty in discounted cash flow analysis. Common approaches are sensitivity analysis and Monte Carlo simulations on cost variables and discount rates.

Despite their aforementioned limitations, classical life cycle cost optimisation methods can be very powerful in many real-life applications. Chapter 3 addresses a research gap in this class of optimisation methods which adds to the work of Van Noortwijk (2003), Campbell et al. (2011) and Jardine and Tsang (2017). In this chapter a flexible approach to a reliability-based discounted age replacement model is developed and extended with a reliability-based interval block replacement model. These results were hereafter used and stretched by Shang, van den Boomen, de Man, and Wolfert (2019) in a railway application.

2.2 Advanced approaches

Advanced methods for replacement optimisation come into view when life cycle cash flows of a challenger or series of challengers are non-repetitive. Circumstances causing such non-repeatability are for example price increases, technology change, probabilities for certain events, but also the inclusion of flexibility to respond to uncertainty. Advanced approaches can roughly be divided in deterministic approaches and probabilistic approaches and are found in different domains of science. A deterministic approach yields one output for a set of input variables whereas a probabilistic approach accounts for a range of outcomes, each having their own probability of occurrence (Frangopol, Kallen, & Noortwijk, 2004). Within the deterministic range, network optimisation approaches are found. Two important probabilistic approaches are labelled as Real Options Analyses and Markov Decision Processes. These three advanced approaches are discussed in the following paragraphs.

Deterministic network optimisation approaches

The foundation for network optimisation is laid by Bellman (1955) who developed a functional equation for optimising a sequence of intervention strategies. Dynamic Programming (DP) or Linear Programming (LP) techniques are required to solve such functional equation. Multiple alternative strategies or a sequence of strategies are combined in a network or branched decision tree and LP or DP algorithms are applied to find the shortest path in such network or tree (Hillier & Lieberman, 2010; Wagner, 1975).

Numerous authors investigated deterministic DP and LP modelling for maintenance and replacement optimisation (Brekelmans, den Hertog, Roos, & Eijgenraam, 2012; Büyüktahtakın & Hartman, 2016; Dupuits, Schweckendiek, & Kok, 2017; Hartman & Tan, 2014; Zwaneveld & Verweij, 2014) but none of them integrated the infrastructure related features as depicted in Chapter 1. Most prominent is the absence of price fluctuations.

A study close to the interest of the current research is provided by Regnier, Sharp, and Tovey (2004). These authors developed an approach to a deterministic DP model for replacement optimisation under price increases and technology change. The authors' model is basically an elaboration of a fundamental regeneration model presented by Wagner (1975). The model optimises a sequence of replacements of a newly installed asset whereas the life cycle cash flows of successive assets are subject to inflation and technology change. However, the limitation of this model is the assumption that future cash flows are a geometric series based on the first-year asset's cash flows. Another limitation is that the model starts with a brand-new asset and not an ageing asset.

In Chapter 4 the current research addresses a research gap for a simplified case where only two sequential strategies, i.e. maintain and replace, need to be optimised under non-stationary cash flows caused by ageing and inflation. Normally this would require network optimisation with a DP-solution. Of interest here is the development of an approximated solution which builds on the aforementioned classical life cycle cost optimisation methods. The approximated solution is compared with the more complex DP solution and shown to be equally accurate as the DP-solution. The validity of this approximated solution is due to some typical infrastructure related features.

In Chapter 6 the current research addresses another research gap for a common situation where multiple sequential strategies, such as maintain, renovate and replace need to be optimised under non-stationary cash flows induced by ageing and inflation. Here a network with a nested DP-solution is presented.

Real Options Analyses

Real Options Analysis (ROA) originates from the financial domain. An option gives a holder the right but not the obligation to exercise this option at a future date. This right has value and ROA basically is a method to quantify this value. Typical real options are the right to invest, to defer, to expand and to abandon. As such ROA is of interest for replacement decisions as these are investment timing decisions.

Fundamental work for option pricing is provided by Black and Scholes (1973) and Merton (1973) who developed the famous Black-Scholes-Merton formula. Cox, Ross, and Rubinstein (1979) developed a discrete binominal approach as a more flexible alternative to the continuous Black-Scholes-Merton formula. This discrete binominal approach has become the foundation for real options analysis, where not options on financial securities are valued but options on real investments. The underlying binominal lattice is a discrete representation of future price uncertainty as it contains all expected future price paths.

It is important to realise that ROA originally is a method from the financial domain meant to value flexibility and as such traditional ROA methods are tied to market prices and market risks. Schwartz and Trigeorgis (2001) describe ROA as an economically corrected decision tree. The underlying mathematics, depending on the complexity of the ROA model, resemble those used in decision tree analysis. However, ROA additionally corrects for market risks.

In the literature numerous ROA applications are found. ROA is often used to value risk sharing between public and private partners and to value investment timing decisions under price uncertainty (Cheah & Liu, 2006; Liu, Gao, & Cheah Charles Yuen, 2017; Mathews, 2015; Pellegrino, Ranieri, Costantino, & Mummolo, 2011).

Other applications expand the ROA theory to address uncertainties in investment timing which are not necessarily related to financial markets such as climate change and fluctuating demand (De Neufville & Scholtes, 2011; Erfani, Pachos, & Harou, 2018; Kim, Ha, & Kim, 2017; M. Woodward, Gouldby, Kapelan, Khu, & Townend, 2011; Michelle Woodward, Kapelan, & Gouldby, 2014; Zambujal-Oliveira & Duque, 2011).

The difference between these two domains of applications is the economic valuation of risky cash flows. Cash flows subject to financial market risks are valued with a risk neutral probability approach or the equivalent replicating portfolio approach. Cash flows subject to other risks are valued with a project related risk adjusted discount rate. To distinguish between both types of ROAs, some authors refer to ROA when valuing market risks, decision tree analysis when valuing project risks and hybrid real options when combining both risks (Martins, Marques, & Cruz, 2015; Neely & De Neufville, 2001; Peters, 2016).

A prime observation of the current research is that real options applications obtained from the financial domain are shallow on maintenance and replacement and generally omit asset integrity. In contrast, real options applications obtained from the engineering domain omit price uncertainty and its proper financial valuation.

The research gap identified by the current research is the valuation of flexibility in both a simple and complex infrastructure replacement timing decision. The simple case considers two strategies: maintain or replace (a classic defender-challenger analysis) while accounting for structural integrity and uncertainty with respect to capacity planning. Moreover, two valuation methods are considered: without market risks (decision tree analysis) and with market risks caused by price uncertainty (traditional ROA). Both models are presented in Chapter 5,

The complex investment timing decision concerns the optimisation of multiple sequential intervention strategies under ageing, a probability for structural failure and price uncertainties for investments and operational expenditures. This model is presented in Chapter 7 and is labelled as a compound ROA with a Markov Decision Process as underlying mathematics.

Markov Decision Processes

A Markov Decision Process (MDP) is a probabilistic approach applicable to maintenance and replacement optimisation, which inherently addresses uncertainty. A MDP is a stochastic process which defines states, actions for each state and transition probabilities for transferal from one state to another when a certain action is taken. Algorithms such as policy iteration or value iteration determine the sequence of optimal actions under defined constraints (Frangopol et al., 2004; Puterman, 1994).

In the literature, MDPs are widely used to address the uncertainty of asset integrity in maintenance and replacement optimisation. Many applications are found in but not limited to road and gravity sewer maintenance (Baik, Jeong, & Abraham, 2006; Memarzadeh & Pozzi, 2016; Oliveira, Santos, Denysiuk, Moreira, & Matos, 2017). MDPs are popular because the asset integrity of these infrastructure assets can be expressed in condition scores ranging from 1 to 5 which facilitates the definition of transition probability matrices. The difficulty, however, lies in estimating the transition probabilities from one condition state to another when a certain improvement action (or none) is taken.

The current study observes that price uncertainty is generally omitted in MDPs found in the literature which is supported by several authors (Faghih Sayed Amir & Kashani, 2018; Ilbeigi, Castro-Lacouture, & Joukar, 2017; Mirzadeh, Butt, Toller, & Birgisson, 2014; Swei, Gregory, & Kirchain, 2017). Moreover, the current study observes that many infrastructure assets do not undergo visual inspections resulting in condition scores.

The research gap addressed in the current research is optimising a sequence of intervention strategies for infrastructure assets which age according to a failure rate and where investment and operational expenditures are subject to price uncertainty. This model is presented in Chapter 7.

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3

Discounted age and interval replacement optimisation

Like-for-like replacement
Repetitive cash flows
Certain future

This chapter presents a discounted age replacement model and a discounted interval replacement model developed by Van den Boomen, Schoenmaker, and Wolfert (2018). Both models are based on a like-for-like replacement strategy. Age and interval replacement models balance the cost of corrective replacement with the cost of preventive replacement given a reliability function and reliability threshold.

With age replacement, an asset is correctively replaced upon failure or preventively at a specified interval, whichever comes first. Such strategy is for example appropriate for the conservation of lock gates. The coating is preserved with a fixed interval or earlier if inspection reveals its necessity.

With block replacement, an asset is correctively replaced upon failure and preventively as part of a group at a designated interval. This type of strategy is appropriate for i.e. streetlights. A lamp is replaced upon failure but also at a designated interval when all lamps are replaced simultaneously.

The novelty in this work is the inclusion of discounting in age and interval replacement models by means of classical LCC techniques. Because infrastructure assets generally have long service lives, discounting should not be neglected. The literature only offers very few dedicated formulae for discounted age- and interval replacement models which were used to validate the results.

The added value of the approach using classical LCC techniques is flexibility. The models are easily expanded for other cost components such as operational expenditures, which has been demonstrated in Shang, Van den Boomen, De Man, and Wolfert (2019).

A Life Cycle Costing Approach for Discounting in Age and Interval Replacement Optimisation Models for Civil Infrastructure Assets

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Published online in Structure and Infrastructure Engineering at June 5, 2018
https://doi.org/10.1080/15732479.2017.1329843

Civil infrastructure assets, such as roads, locks, bridges, treatment plants and storm surge barriers, are often characterised by long service lives and corresponding technical life cycles. When life cycles are long, the time value of money plays a role in asset management decision making on capital investments and operation and maintenance expenditures. In this paper, a new life cycle costing (LCC) approach for discounting in two classes of maintenance optimisation models is developed. These models are the age replacement model and the interval replacement model. Three well-known life cycle costing (LCC) techniques, which are the present worth, the capital recovery and the capitalised equivalent worth, are combined and used to develop a stepwise methodology. This methodology is validated with the few case-specific mathematical equations that exist in the literature. The advantage of using this alternative LCC approach is its applicability and flexibility for reliability and maintenance engineers. The resulting LCC method builds on well-known LCC formula and enhances the understanding of the inclusion of discounting principles in reliability models. Understanding these principles makes the method flexible. Practitioners can extend or adapt the method to changing circumstances, such as additional cash flows and altering reliability modelling.

3.1 Introduction

The international standard on infrastructure asset management (ISO 55000:2014) and the British Institute of Asset Management (IAM, 2015) both stress the importance of life cycle cost optimisation at a desired service level. The application of infrastructure life cycle costing (LCC) in practice is supported worldwide by several standards and guidelines. Good examples hereof are provided by the U.S. National Highway Federation (FHWA, 2017), who presents a whole range of reports and case studies, including supporting software. Other examples are given by the U.S.

Department of Energy (DOE, 2014), the U.S. Transportation and Research Board (NCHRP, 2003), the World Road Association PIARC (PIARC, 2017) and the International Standards Organisation (ISO, 2008). The guidelines stress the importance of a probabilistic approach and dealing with uncertainty in LCC analyses, an area that is in development.

Probabilistic life cycle costing for maintenance optimisation is of importance for asset owners, asset managers and service providers. In general, fundamental probabilistic cost optimisation in maintenance strategies is widely covered in the literature on reliability engineering but often lacks discounting of costs. The cost of failure is set against the cost of preventive maintenance to find optimised preventive or corrective maintenance strategies. An overview of fundamental probabilistic maintenance and replacement costs optimisation models is provided by Jardine and Tsang (2013) and Campbell, Jardine, and McGlynn (2011). In our paper the focus is on two of these optimisation models: the age replacement and interval replacement models.

The fundamental probabilistic models provide a quick estimate for optimised preventive replacement (or major overhaul) intervals considering a trade-off between corrective and preventive replacement costs. The value of these generic optimisation models is their ease and broad applicability for practitioners to establish a long-term asset planning for similar types of assets, in addition to more case-specific and advanced probabilistic LCC-analyses. However, the fundamental probabilistic maintenance cost optimisation models hardly include discounting of costs. Discounting accounts for the time value of money. The time value of money gains in importance when maintenance or replacement intervals cover more than a few years, which is often the case for civil infrastructure assets. To allow for fair comparison of life cycle costs of different optimisation strategies, future costs are to be converted to their present values.

Although, LCC concepts are well-known, life cycle costing analyses are still far from satisfactory in many fields in practice. Korpi and Ala-Risku (2008) only found 55 international LCC cases studies suitable for analysis out of a total of 205 potential articles. The authors concluded an overall unsatisfactory level of the execution of LCC analyses and specifically addressed the deterministic nature of most LCC case studies. Similar conclusions were drawn in a small-scale study on the quality of LCC analyses in the Dutch public water sector (Van den Boomen, Schoenmaker, Verlaan, & Wolfert, 2016). Here only 10 suitable case studies for analysis were identified. The study primarily addressed common mistakes found in the execution of the investigated LCC case studies, which were all deterministic in nature.

Fundamental probabilistic maintenance optimisation models deal with uncertainty, however, hardly with the discounting of costs. The inclusion of the time value of money complicates the calculations. Mathematical solutions for discounting in specific fundamental maintenance optimisation models have been provided by only a few authors. Fox (1966) demonstrated a mathematical relationship for discounting in age replacement models. Chen and Savits (1988) established mathematical discounted cost relationships for both age and block replacement policies and the relation between them. Rackwitz (2001) incorporated discounting in a renewal model for structural failures with systematic reconstruction. Van Noortwijk (2003) derived a formula for calculating the present value over an unbounded time horizon in age replacement optimisation models as input for a condition-based lifetime extension model. Practical implications were shown in several papers, for example in an article by Van Noortwijk and Frangopol (2004). Van der Weide, Suyono, and van Noortwijk (2008) extended these results to other types of discounting such as hyperbolic and generalised hyperbolic discounting in renewal processes. Mazzuchi, van Noortwijk, and Kallen (2007) reviewed mathematical decision models to optimise time-based and condition-based maintenance intervals. The results were later extended to the derivation of formulas for calculating the discounted costs in combined condition-based and age-based optimisation models (Van der Weide, Pandey, & van Noortwijk, 2010).

These papers all have in common the derivation of mathematical formulas for discounting of costs for explicit and case-specific types of maintenance optimisation models. Other case-specific literature combines advanced probabilistic deterioration models with discounted life cycle costs for structure and infrastructure assets. E.g., Frangopol, Lin, and Estes (1997) developed an approach for optimising inspection and repair intervals based on discounted costs, related to the maximum allowable service life of a bridge. Furuta, Frangopol, and Nakatsu (2011) extended the work of Frangopol et al. (1997) to allow for the inclusion of more variables, like different combinations of inspection techniques, by developing multi objective mathematical algorithms to find the minimum discounted life cycle costs. Almeida, Teixeira, and Delgado (2015) developed degradation algorithms using Markov matrices for bridges and discounted the costs over medium and long-term finite time horizons. An extension to a discounted renewal approach is provided by Kumar and Gardoni (2014). These authors developed mathematical equations to calculate model variables such as repair time and age, as a function of the system's reliability and, discounted expenditures over a finite time horizon.

All these papers have a strong focus on developing case-specific probabilistic deterioration models. The total discounted costs of preventive and corrective measures over the allowable service life are hereafter minimised to arrive at optimised intervention intervals (inspection, preventive maintenance, repair, partial replacement).

The focus of our study is not on developing advanced probabilistic deterioration models for specific types of infrastructure assets to predict and optimise life cycle costs. Instead, focus is put on discounting life cycle costs in existing and fundamental maintenance optimisation models, using the concepts of equivalent annual costs and the capital equivalent worth, which will be explained in section 2. This alternative LCC approach for discounting in age and interval replacement models has not yet been elaborated on in the literature. From an engineering asset management point of view, there is an interest in a more generalised, rather quick and flexible approach that allows for discounting in different types of fundamental maintenance optimisation models. Instead of the derivation of a unique set of mathematical formulas for a specific maintenance optimisation problem, three LCC techniques are used in combination and in a specific order to arrive at the required results. This stepwise LCC approach is demonstrated in two fundamental maintenance optimisation models: the age replacement model and the interval (block) replacement model. In the age replacement model, an asset is replaced upon failure or at a preventive replacement interval, whichever comes first. In the interval replacement model, an asset is replaced upon failure and at a preventive replacement interval.

The reason for selecting the age and interval replacement models for developing this alternative LCC method is twofold. First, the existence of mathematical formulas for discounting in age and interval replacement models allows for validation of the alternative LCC approach. A second reason is their ease and quick applicability in practice for infrastructure assets with long life cycles and periodic major overhauls. An inventory of different maintenance policies over the last 50 years still denotes the popularity of these models (Asis, Subhash Chandra, & Bijan, 2011). The models are used in practice by organisations that own and/or maintain infrastructure assets with long life cycles, for instance for the interval estimation of the conservation of steel lock gates (age replacement), the block replacement of street lighting luminaires (interval replacement) and the revisions, major overhauls or replacements of hydraulic cylinders (both age and interval replacement).

One may argue that the age and interval replacement optimisation models are based on oversimplifications on the failure behaviour of assets and forecasts of future expenditures. An additional argument is that preventive age and interval replacement models ignore the benefits to be gained by measures directed at lifetime extension, risk reduction and postponement of the actual replacement. Periodic age and interval replacement optimisation is just one of the alternatives for optimised life cycle management of infrastructure assets. The value of age and interval replacement optimisation models is not the actual decision for a preventive or corrective replacement. These short-term and mid-term decisions are made based on actual condition monitoring and technical state combined with detailed LCC analyses which are commonly referred to as a defender (the existing asset) and challenger (the alternative option) analyses. In these types of LCC analyses is investigated whether the postponement of a replacement justifies the cost of measures like major overhauls or renovations to keep an asset some additional years in service. (Blank & Tarquin, 2012; Newnan, Lavelle, & Eschenbach, 2016; Park, 2011; Sullivan, Wicks, & Koeling, 2012).

The discounted age and interval replacement models are also not a substitute for the more advanced probabilistic life cycle optimisation models as referred to in the before mentioned literature. Within their field of application, the value of using age and interval replacement strategies is that the models provide quick and easy long-term costs and interval estimates as input for the overall long-term asset and maintenance planning. The models also support the decision where successive detailed probabilistic LCC analyses are most effective. Even with simplifications, the probabilistic generic age and interval replacement models provide accurate results for the objectives they are used and meant for.

In this paper, three areas of expertise are merged: reliability engineering, engineering economics (life cycle costing analysis) and infrastructure asset management. Terminology differs between and within these fields. The terminology used in this paper follows the best common denominator of the herein stated literature. The focus in this paper is on cost optimisation models. In the maintenance optimisation models covered in this paper, yearly benefits are considered to be non-differential for different scenarios and are therefore left out of the equations. Salvage values are also left out in the method development. The reason is that salvage values are cash flows that become available when an asset is sold at the end of its life (Brealey, Myers, & Allen, 2017, p. 131). In general, civil infrastructure assets have long service lives, even longer functionalities and are often replaced or renewed at the end of their service lives. Most infrastructure assets cannot be sold.

Occasionally, infrastructure assets may have some scrap or recycling values but these are frequently negligible compared to the renewal costs. Another common situation is that worn-out parts (without salvage values) are periodically renewed. In that case, an infrastructure asset will never be fully replaced. Infrastructure assets do have demolition costs which are often included in the renewal costs. Demolition costs are mostly not differential in a sequence of continuous renewals for age and interval replacement strategies. For these reasons, salvage values are left out in development of the alternative LCC approach and demolition costs are considered to be part of the new investment costs. However, the LCC approach developed in this paper is flexible, and allows for easy separate inclusion of salvage values or demolition costs.

The outline of this article is as follows: first, three generic LCC techniques, which are the present worth, the capital recovery and the capitalised equivalent worth, will be explained. This is followed by a stepwise approach on how to use these three LCC techniques in combination for discounting in age and interval replacement optimisation models. After this, the article is divided into two parts: one for age replacement modelling and one for interval (block) replacement modelling. For each part, the fundamental optimisation model without discounting will be shortly reviewed. Hereafter, the LCC techniques will be used to include discounting in the fundamental maintenance optimisation models. The results will be validated by using the mathematical discounted cost relationships found in the literature on an example. The paper ends with overall conclusions on using a LCC approach for discounting in fundamental maintenance optimisation models.

3.2 Life cycle costing techniques and method development

For the inclusion of the time value of money in both the age replacement model and the interval (block) replacement model, three life cycle costing techniques are of immediate interest: the so-called single payment present worth factor, the equal payment series capital recovery factor and the capitalised equivalent worth. These will be explained briefly. The terminology used follows the stated literature.

The single payment **present worth factor** $(\boldsymbol{P/F,r,t})$ transforms a future value F to its present value P and is given by (Park, 2011; Sullivan et al., 2012):

$$(P/F,r,t)=\frac{1}{(1+r)^t}\,, \tag{3.1}$$

where r is the real interest or discount rate [-] and t is the time of occurrence [time]. The functional notation $(P/F,r,t)$ reads as follows: find the present value P, given the future value F, the discount factor r and the time of occurrence t. Both t and r are generally expressed (but not necessarily), respectively, in years and discount rate per year.

The present worth factor is used for standard discrete discounting and is, for commonly used interest rates, comparable with and close to continuous exponential discounting. Continuous exponential discounting is frequently used in the literature that demonstrates mathematical derivations for the inclusion of the time value of money in maintenance optimisation models. In the latter case, a continuous discount function e^{-rt} is used instead of the present worth factor. General inflation is implicitly incorporated by using an inflation-free discount rate (real discount rate). Many considerations can be made on discount rate estimations and fluctuations in time. In general, the discount rate in cost models should at minimum cover the long-term weighted average cost of capital of an organisation. The methodology described in this paper allows for a flexible handling of discount rates if required.

The second factor of interest is the equal payment series **capital recovery factor or annuity factor** $(\boldsymbol{A/P, r, t})$**.** This factor transforms a present value into the equivalent annual costs (EAC) over a chosen number of time units t, generally years. The EAC is analogous to A. The capital recovery factor is given by (Park, 2011; Sullivan et al., 2012):

$$(A/P, r, t) = \frac{r(1+r)^t}{(1+r)^t - 1}. \tag{3.2}$$

Here, $(A/P, r, t)$ reads as: find A (analogous to EAC) given a present value P, a discount rate r and a number of time units t. An interesting, important and often forgotten feature of the EAC is that the equivalent annual costs of one life cycle equal the equivalent annual costs of any number of repeating life cycles assuming identical replacements and identical life cycle costs (Blank & Tarquin, 2012; Newnan et al., 2016). Therefore, the EAC of one life cycle is the same as the EAC of an infinite number of replacement cycles, under the given assumptions.

The third expression of interest is the **capitalised equivalent worth (CW)**. The capitalised equivalent worth equation converts the equivalent annual costs (EAC) of one life cycle to the present value of an infinite number of replacement cycles (Park, 2011; Sullivan et al., 2012):

$$CW = \frac{EAC}{r}. \tag{3.3}$$

The capitalised equivalent worth factor r^{-1} is found by letting t approach infinity in the recursive formula of the capital recovery factor (Park, 2011; Sullivan et al., 2012):

$$\lim_{t\to\infty}(P/A,r,t) = \lim_{t\to\infty}\frac{(1+r)^t-1}{i(1+r)^t} = \frac{1}{r}\,. \tag{3.4}$$

The framework in Figure 3.1 depicts how these three LCC techniques, are used in combination for the inclusion of the time value of money in age and interval replacement models. In sections 3.3 and 3.4, this approach will be expressed in formulas, demonstrated with a practical example and validated with the dedicated mathematical equations found in the literature. Step 3 considers the expected cycle length which will be explained in section 3.3 for the age replacement model and section 3.4 for the interval replacement model. Step 4 accounts for the initial investment which can be done in two alternative manners.

The total EAC of step 5 gives the basis for comparison of an age or interval replacement strategy with another age of interval replacement strategy. The optimum is found at the minimum total EAC. Following the same principles as shown in Figure 3.1, one can add time-dependent operation and maintenance expenditures without searching for another dedicated mathematical formula.

There is one important limitation that is hardly mentioned in the literature and textbooks. The age and interval replacement models, with or without discounting, assume a repeatability of the costs of a replacement cycle. If this repeatability assumption does not hold, neither approach can be used. The repeatability assumption will not hold if, for example, an asset is replaced by another alternative with a different cost and/or failure probability density profile. This may be the case when replacement options are prone to technology development.

These limitations do not automatically refute (discounted) age and interval replacement models for civil infrastructure assets. There are many situations where the age and interval replacement models provide good estimates for an initial investment decision and the long-term asset planning. Changing cash flow patterns of replacement cycles due to technology developments are often not that deviant for civil infrastructure assets with long life cycles. Furthermore, deviations frequently occur after decennia and the discounting process mutes the deviations. The argument here is that the applicability of discounted age and interval replacement models should be checked on the presence of an approximated repeatability assumption of replacement cycles. Something that is not well stated in the literature.

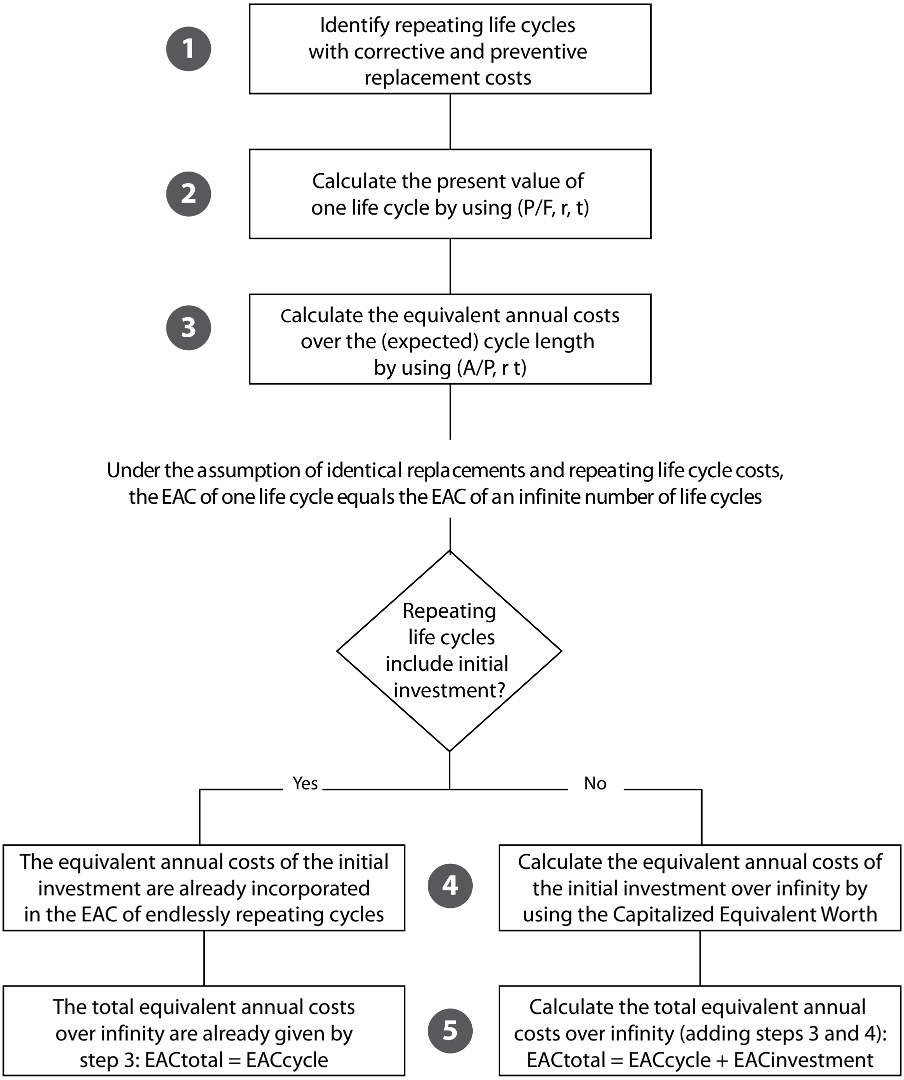

Figure 3.1 LCC approach to include the time value of money in the age and interval replacement models

3.3 Age replacement model

This section addresses the age replacement model. After a short review of the fundamental age replacement model without discounting, the LCC approach as described in Section 3.2 is demonstrated. Two situations are dealt with: ending and starting a cycle with a preventive replacement. Hereafter, a dedicated mathematical formula that includes discounting of costs over an infinite time horizon is presented. A practical example is used to compare the mathematical equation with the LCC techniques. After this, the results will be discussed, and conclusions formed.

Fundamental age replacement model without the time value of money

In an age replacement model, an asset is replaced correctively upon failure or preventively at a certain replacement interval, whichever comes first. As an example, Figure 3.2 depicts the cash flow development of an age replacement model with a preventive replacement interval of three years. Here, it is assumed that the initial investment I_0 equals preventive replacement costs C_p. The cost of a corrective replacement is given by C_f. The failure probability density function is designated with $f(t)$. The reliability function $R(t)$ is defined by $1 - \int_0^t f(t)dt$.

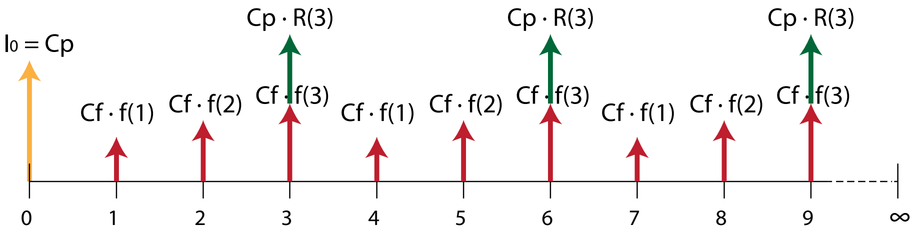

Figure 3.2 Cash flow diagram of an age replacement policy for a preventive replacement interval of three years. Three full cycles and an initial investment are shown

An age replacement model searches for the optimum of a preventive replacement interval, given a certain failure probability density function and corrective and preventive replacement costs. Age replacement models are well described in the literature, early by Barlow and Proschan (1965) and more recently, for example, by Jardine and Tsang (2013). In its basic form, the economic optimum is found by minimising the expected total costs per unit of time. In formula (Jardine & Tsang, 2013):

$$c(t) = \frac{C_f \cdot (1 - R(t)) + C_p \cdot R(t)}{M(t) \cdot (1 - R(t)) + t \cdot R(t)}, \tag{3.5}$$

where:

$c(t)$	expected total costs per unit of time for interval [0,t] [currency / unit of time]
t	time [unit of time]
C_f	corrective replacement costs or failure costs [currency]
$R(t)$	reliability [-]
C_p	preventive replacement costs [currency]
$M(t)$	the mean of the failure probability density function from t = [0,t] [unit of time]

The numerator of this equation expresses the expected total costs per cycle length, which is given by the probability of a failure multiplied by the corrective replacement costs and the probability of no failure multiplied by the preventive replacement costs. The denominator expresses the expected cycle length $E(L)$, which is a weighted average of the probability of a corrective cycle length in the case of failure and the probability of a preventive cycle length in the case of no failure:

$$E(L) = M(t) \cdot (1 - R(t)) + t \cdot R(t). \tag{3.6}$$

$M(t)$ is defined as (Jardine & Tsang, 2013):

$$M(t) = \int_0^t \frac{t \cdot f(t)dt}{1 - R(t)}, \tag{3.7}$$

Where $f(t)$ = failure probability density function [-]. For practical reasons, the failure probability density function is assumed to be 0 at $t = 0$.

Discounted age replacement optimisation model with use of the LCC techniques

In this section, the time value of money is included in the fundamental age replacement model (Equation 3.5). Hereby, the LCC techniques and approach described in Section 3.2 are used. Two situations are dealt with: ending a repeating cycle with a preventive replacement and starting a repeating cycle with a preventive replacement. The reason for doing so is that the mathematical equations in the literature all end a repeating cycle with a preventive replacement, while in practice a maintenance engineer would like to start a cycle with a preventive replacement.

Alternative 1: Ending a repeating cycle with a preventive replacement C_p

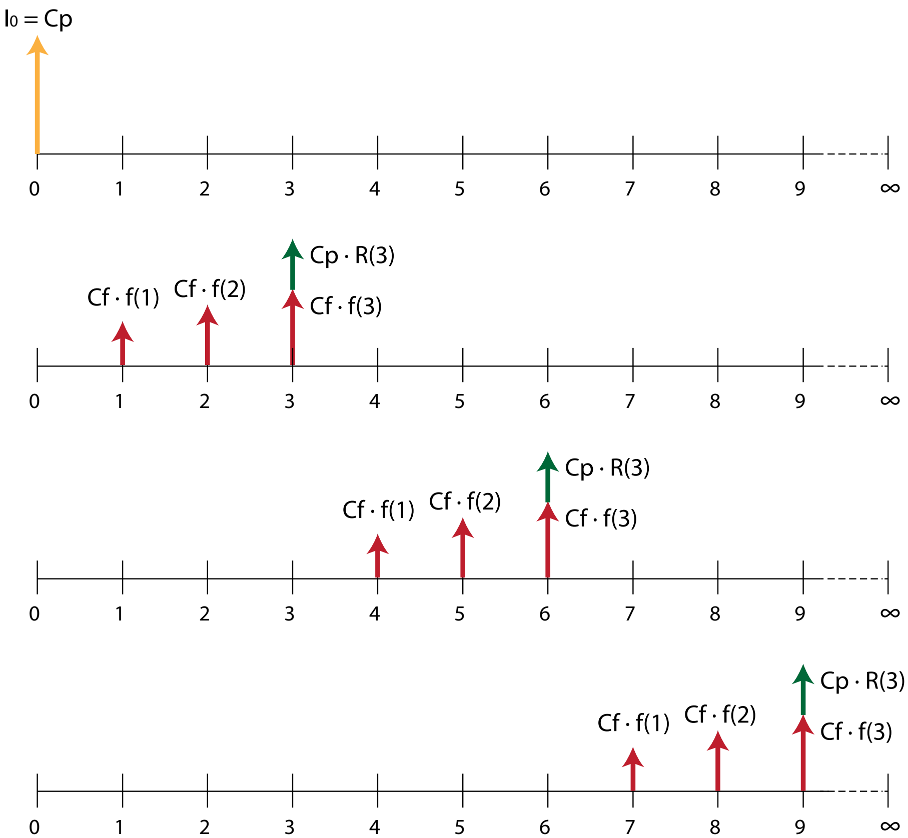

Figure 3.3 Cash flow diagram of an age replacement policy for a preventive replacement interval of three years, ending with a preventive replacement. Three full cycles are shown. The initial investment is fully excluded from the repeating life cycle costs

Step 1: Identify repeating life cycle costs.

A repeating pattern of cash flows is identified in Figure 3.2 by taking the initial investment $I_0 = C_p$ with probability 1 out of the cash flow development, as shown in Figure 3.3. If the asset fails in this example at t = 1, 2 or 3, it will be replaced correctively. If the asset has not failed at the end of t = 3, it will be replaced preventively. The total probability of a replacement cycle is 1. Because of the repeatability assumption, only the present value and equivalent annual costs of one cycle need to be calculated for derivation of the present value and EAC for repeating cycles up to infinity (Blank & Tarquin, 2012; Newnan et al., 2016).

Step 2: Calculate the present value of one life cycle.
Using the present worth factor $(P/F, r, t)$, the present value of the expected total replacement costs of the first cycle are given by:

$$P_{cycle} = C_f \cdot \sum_{t=1}^{T} (P/F, r, t) f(t) + C_p \cdot (P/F, r, T) \cdot \left(1 - \sum_{t=1}^{T} f(t) \right). \tag{3.8}$$

Equation 3.8 can also be written as:

$$P_{cycle} = \frac{C_f \cdot f(1)}{(1+r)^1} + \frac{C_f \cdot f(2)}{(1+r)^2} + \ldots + \frac{C_f \cdot f(T)}{(1+r)^T} + \frac{C_p \cdot R(T)}{(1+r)^T}. \tag{3.9}$$

Step 3: Calculate the equivalent annual costs (EAC) over the expected cycle length.
The expected cycle length $E(L)$ is calculated according to Equation 3.6. The equivalent annual costs (EAC) of a cycle are found by using the capital recovery factor $(A/P, r, t)$ (Equation 3.2) where t is equal to the expected cycle length $E(L)$:

$$EAC_{E(L)} = (A/P, r, E(L)) \cdot P_{cycle}, \tag{3.10}$$

Under the assumption of identical replacements and repeating life cycle costs, the EAC of one life cycle equals the EAC of an infinite number of life cycles.

Step 4: Calculate the EAC of the initial investment over infinity.
The initial investment costs $I_0 = C_p$ are equally distributed over an infinite time horizon by using the capitalised equivalent worth (Equation 3.3).

$$EAC_{I_0} = I_o \cdot r. \tag{3.11}$$

Step 5: Calculate the total EAC over infinity.

The total EAC of the age replacement strategy concerned is given by $EAC_{total} = EAC_{E(L)} + EAC_{I_0}$. The optimum is found by minimising the EAC_{total} of different age replacement strategies.

Alternative 2: Beginning a repeating cycle with a preventive replacement C_p

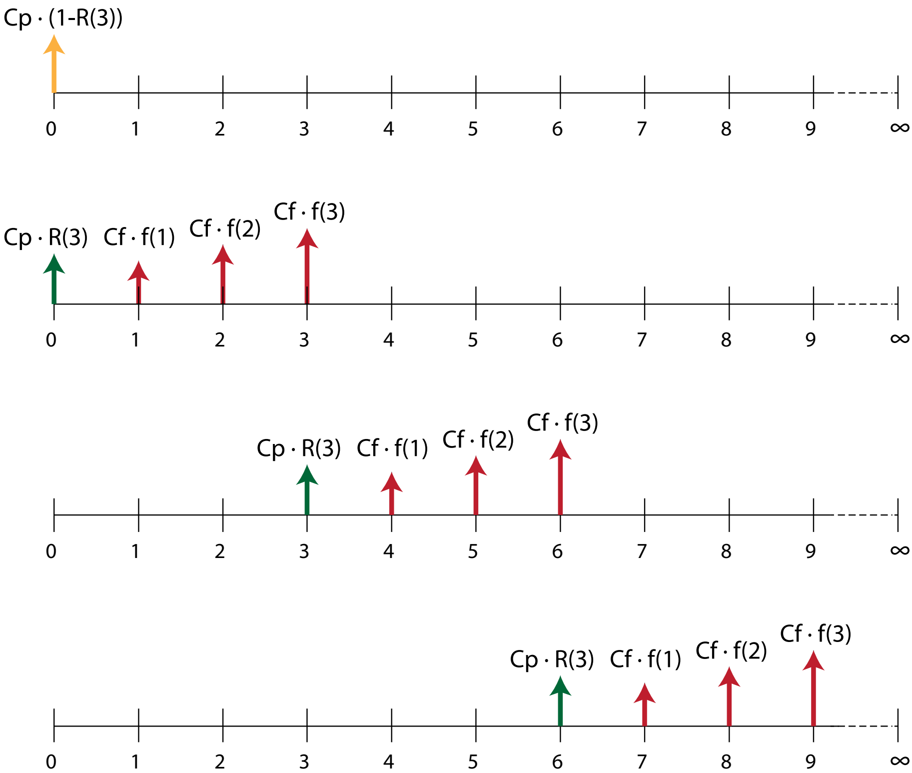

Figure 3.4 Cash flow diagram of an age replacement policy for a preventive replacement interval of three years, starting with a preventive replacement. Three full cycles are shown. The initial investment is partly excluded from the repeating life cycle costs

In the section dealing with Alternative 1, a repeating cycle of cash flows was identified after instalment of a new asset with investment costs I_0. The investment costs were converted to equivalent annual costs over an infinite time horizon and added to the equivalent annual costs of the cycles. In practice, a maintenance engineer would prefer to start an asset's life cycle with a preventive maintenance or initial investment. To show the deviations, the discounted age replacement model that starts with a preventive replacement will be presented.

Step 1: Identify repeating life cycle costs.
A repeating pattern that starts with a preventive replacement can also be derived from Figure 3.2 by dividing the initial investment (or preventive replacement) $I_0 = C_p$ with probability 1 into a part $C_p \cdot R(t)$ and a part $C_p \cdot (1 - R(T))$. This is shown in Figure 3.4.

Step 2: Calculate the present value of one life cycle.
The present value of the replacement costs of the first cycle are now given by:

$$P_{cycle} = C_f \cdot \sum_{t=1}^{T} (P/F,r,t) f(t) + C_p \cdot \left(1 - \sum_{t=1}^{T} f(t)\right). \tag{3.12}$$

There is no need for discounting C_p as in this approach C_p of the first cycle occurs at $t = 0$. Equation 3.12 can also be written as:

$$P_{cycle} = \frac{C_f \cdot f(1)}{(1+r)^1} + \frac{C_f \cdot f(2)}{(1+r)^2} + ... + \frac{C_f \cdot f(T)}{(1+r)^T} + C_p \cdot R(T). \tag{3.13}$$

Step 3: Calculate the equivalent annual costs (EAC) over the expected cycle length.
The expected cycle length is unchanged, as the probabilities and interval times of a preventive cycle and corrective cycle are unchanged. Again, the expected cycle length is calculated according to Equation 3.6. The equivalent annual costs (EAC) of a cycle are found by using the capital recovery factor $(A/P, r, t)$ (Equation 3.2) where t is equal to the expected cycle length $E(L)$:

$$EAC_{E(L)} = (A/P,r,E(L)) \cdot P_{cycle}, \tag{3.14}$$

Step 4: Calculate the EAC of the initial investment over infinity.
The rest of the term of the initial investment costs $C_p \cdot (1 - R(T))$ is equally distributed over an infinite time horizon by using the capitalised equivalent worth (Equation 3.3):

$$EAC_{rest\_term\_I_0} = C_p \cdot (1 - R(T)) \cdot i. \tag{3.15}$$

Step 5: Calculate the total EAC over infinity.

The total EAC of the age replacement strategy concerned is given by $EAC_{total} = EAC_{E(L)} + EAC_{rest\_term\_I_0}$. The optimum is found by minimising the EAC_{total} of different age replacement strategies.

Summarising: The differences between the LCC approaches with a preventive replacement at the end or beginning of a cycle are, respectively:

- Discounting or no discounting of C_p for the first cycle;
- Distributing the entire initial investment I_0 or the part $C_p \cdot (1 - R(T))$ over an infinite time horizon.

Both approaches will be demonstrated with an example after the following section.

Mathematical equation for discounted age replacement optimisation found in the literature

Chen and Savits (1988); Fox (1966) and Van Noortwijk (2003) established mathematical relationships for discounting in a fundamental age replacement model. Apart from differences in mathematical expressions, these relationships do not differ from each other. The expression of Van Noortwijk (2003) will be used in this article, slightly adapted for reasons of uniform notations.

The expected total discounted costs of an age replacement interval, assuming identical replacements over an infinite time horizon $E(K(n, \alpha))$, are written as (Van Noortwijk, 2003):

$$\lim_{n\to\infty} E(K(n,\alpha)) = \frac{C_f \cdot \left(\sum_{t=1}^{T} \alpha^t \cdot f(t)\right) + C_p \cdot \alpha^T \left(1 - \sum_{t=1}^{T} f(t)\right)}{1 - \left[\left(\sum_{t=1}^{T} \alpha^t \cdot f(t)\right) + \alpha^T \left(1 - \sum_{t=1}^{T} f(t)\right)\right]}, \quad (3.16)$$

where:

α	discount factor defined as $\frac{1}{(1+r)}$ with r as inflation free discount rate
t	time, often expressed in years [unit of time]
T	preventive replacement time [unit of time]
C_p	preventive replacement costs [currency]
C_f	corrective replacement costs or failure costs [currency]
$f(t)$	failure probability density function [-]

Note that α^t in Equation 3.16 is equal to the present worth factor in Equation 3.1. The numerator of Equation 3.16 expresses the discounted costs of one cycle length and is equal to Equation 3.8. It is further observed that the preventive replacement costs C_p are discounted with α^T at the end of a replacement interval and not at the beginning.

The denominator of Equation 3.16 transfers the discounted costs of one cycle length to the total discounted costs over an infinite time horizon by assuming continuous repeatability of the first cycle. The denominator of Equation 3.8 follows from a mathematical derivation where Van Noortwijk (2003) uses Feller (1950, chapter 13). Transformation to an infinite time horizon is practical for reasons of comparison. It is, for example, not justified to compare the total discounted costs of a cycle of 10 years with the total discounted costs of a cycle of 15 years. However, if both cycles are repeated to infinity, the same time basis of comparison is created. The expected total discounted costs over an infinite time horizon in Equation 3.16 excludes the initial investment costs. These can be added. In that case, Equation 3.16 is extended to:

$$\lim_{n\to\infty} E(K(n,\alpha)) = I_0 + \frac{C_f \cdot \left(\sum_{t=1}^{T} \alpha^t \cdot f(t)\right) + C_p \cdot \alpha^T \left(1 - \sum_{t=1}^{T} f(t)\right)}{1 - \left[\left(\sum_{t=1}^{T} \alpha^t \cdot f(t)\right) + \alpha^T \left(1 - \sum_{t=1}^{T} f(t)\right)\right]}, \quad (3.17)$$

where I_0 represents the initial investment costs at $t = 0$.

Practical example comparing discounted age replacement calculations

For comparison, a slightly adapted example of Van Noortwijk (2003) is used. It concerns the maintenance of a cylinder on an existing swing bridge. The cost of preventive and corrective replacements are, respectively, € 30,000 and € 100,000. The initial investment is equal to the cost of a preventive replacement. The failure of the cylinder is modelled with a normal probability distribution with a mean of 15 years and a standard deviation of 1.5 years. The inflation free discount rate is 5% per year.

A discrete approach on a yearly basis is used to perform the calculations. These were checked with more accurate discrete computations on a monthly basis. The differences were marginal from a practical point of view. The computations are made for the situations: (1) discounting with use of the LCC techniques and ending an interval with a preventive replacement, (2) discounting with use of the LCC techniques and starting an interval with a preventive replacement and (3) discounting with use of mathematical Equation 3.17. The results of the calculations on a yearly basis are given in Table 3.1.

The discrete failure probability density function is presented in Figure 3.5. The graphs that contain the age replacement interval calculations on a yearly basis are shown in Figure 3.6, Figure 3.7 and Figure 3.8. The equivalent annual cost of the mathematical equation of Van Noortwijk (2003) is obtained by using the capitalised equivalent approach (Equation 3.3) on Equation 3.17; that is, the EAC of the present value $E(K(\alpha, n))$ is obtained by multiplying this present value by the interest rate r.

Table 3.1 Discounted age replacement model: results calculated on a yearly basis

Approaches to discounted age replacement models	Optimum [y]	EAC [€]	CW=P∞ [€]	R(T)
1. Discounting with LCC techniques alternative 1	12	€ 3,586	€ 71,716	96%
2. Discounting with LCC techniques alternative 2	12	€ 3,587	€ 71,734	96%
3. Discounting with mathematical Equation 3.17	12	€ 3,586	€ 71,717	96%

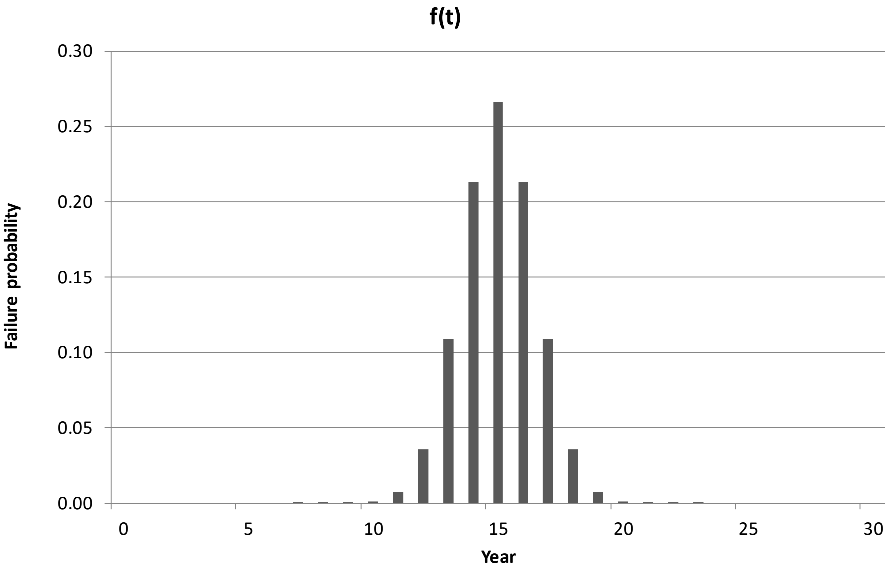

Figure 3.5 Failure probability density function f(t)

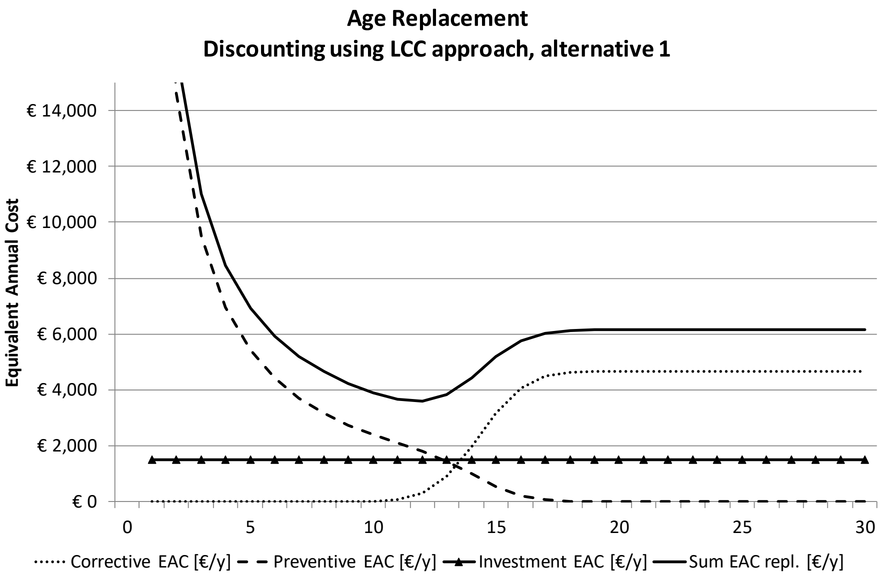

Figure 3.6 Age replacement with discounting using LCC techniques and ending an interval with a preventive replacement (alternative 1)

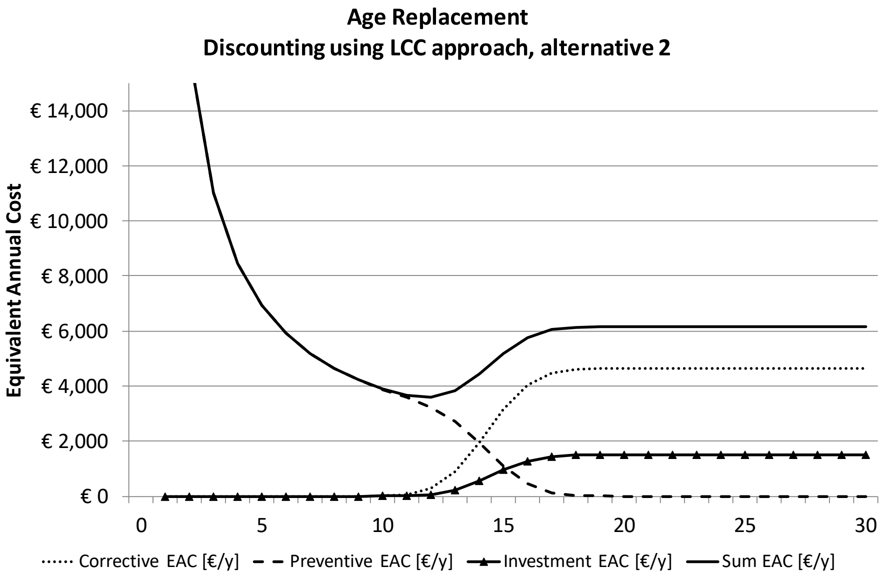

Figure 3.7 Age replacement with discounting using LCC techniques and beginning an interval with a preventive replacement (alternative 2)

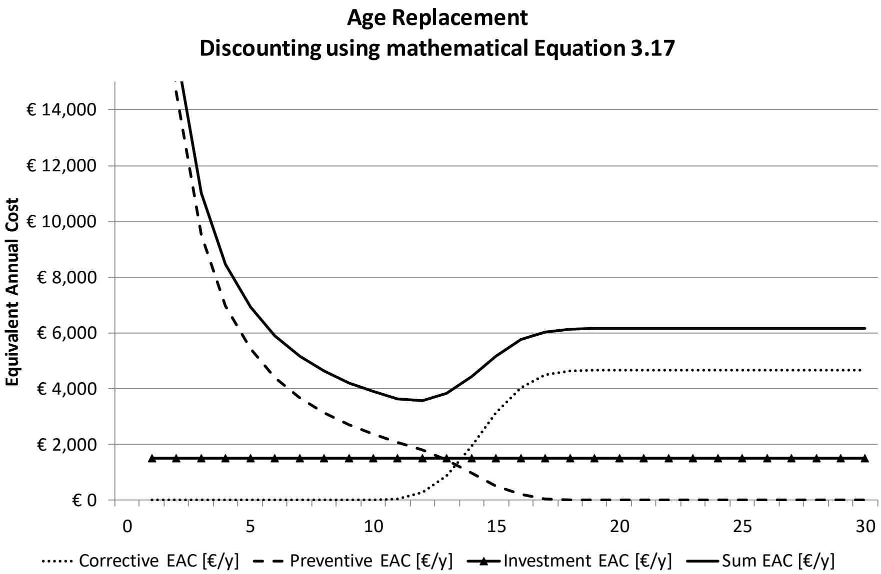

Figure 3.8 Age replacement with discounting using mathematical Equation 3.17 An interval ends with a preventive replacement. Transformation to EAC conform Equation 3.3

Discussion of results

The first observation is the marginal differences between the mathematical Equation 3.17 and the use of the LCC techniques. The three calculations give nearly identical outcomes. All calculations arrive at the same economic optimum for the preventive replacement interval. The equivalent annual costs only slightly differ. The slight difference between calculations of situation 1 and 3 (Figure 3.6 and Figure 3.8) is explained by the mathematical transform that is used in Equation 3.17.
The difference between LCC alternatives 1 and 2 (Figure 3.6 and Figure 3.7) is more difficult to explain. From a life cycle costing perspective, scenarios 1 and 2 should arrive at the same results for the total EAC because the total cash flows in Figure 3.3 and Figure 3.4 do not differ from each other. The difference is explained by the influence of the expected cycle length (the denominator of Equation 3.5) when calculating the EAC of a cycle. Instead of distributing the present value of one cycle over a preventive replacement interval, the present value is distributed over the expected cycle length, which is a weighted average of the probability of a preventive cycle length and the probability of a corrective cycle length.

The initial investment, however, is converted to equivalent annual costs over an infinite time horizon by using the capitalised equivalent worth. The capitalised equivalent worth does not consider expected cycle lengths. Thus, there is a distortion that disappears when the expected cycle length is replaced by the length of a preventive replacement cycle. From a reliability point of view, this would not be acceptable, as the expected cycle length is bounded by the failure probability function. The total probability of a replacement is always 1 for a cycle. There is no probability of survival after a certain time, which is the reason for a horizontal asymptote in Figure 3.5, Figure 3.6 and Figure 3.7. However, as the capital recovery factor $(A/P, r, E(L))$ for the time span considered is close to $(A/P, r, t)$, the distortion is small and from a practical point of view hardly significant. A sensitivity analysis supports this statement in the previous example. Increasing the standard deviation, decreasing and increasing the C_p/C_f -ratio and increasing the interest rate would not lead to differences in optimised preventive replacement intervals. Slight differences in equivalent annual costs may occur.

A second observation concerns the reliability at the economic optimum. In this example, the economic optimum is found at 12 years. The reliability at that point is approximately 96%. One could argue whether an organisation or maintenance department would accept 96% reliability, for example, for critical assets. Optimised replacement costs are not the only replacement criterion and should always be viewed in a broader context.

3.4 Interval or block replacement model

A second fundamental model in the field of maintenance optimisation is the interval (block) replacement model. In this case, an asset is correctively replaced upon failure and preventively at a certain interval. This type of maintenance optimisation is often found in combination with asset groups. The entire group (block) is preventively replaced at a certain interval. In between, corrective replacements of individual assets are carried out when assets fail. First, the interval (block) replacement model without discounting is reviewed. Then, the LCC techniques presented in Section 3.2 are used to include the time value of money into the interval (block) replacement model. Hereafter, two mathematical equations for discounting of costs in interval replacement models are shown. Finally, the LCC approach and mathematical approach are demonstrated with an example, compared and discussed.

Fundamental interval replacement model without the time value of money

The interval replacement model searches for the optimum preventive replacement interval given preventive replacement costs C_p, corrective replacement costs C_f and a renewal function $H(t)$. The renewal function expresses the total number of failures in an interval given a failure probability density function $f(t)$ and constant renewal at failure with identical assets. The interval replacement model is described by Barlow and Proschan (1965, p. 95). In this paper, the expression of Jardine and Tsang (2013, p. 41) is used to describe the model:

$$c(t) = \frac{C_p + C_f \cdot H(t)}{t}, \tag{3.18}$$

where:

$c(t)$	expected total costs in interval [0, t] / length of interval [currency/unit of time]
t	time [unit of time]
C_p	preventive replacement costs [currency]
C_f	corrective replacement costs or failure costs [currency]
$H(t)$	the expected number of failures between t = [0, t] [-]

For block replacement, the number of assets m is added:

$$c(t) = m \cdot \left[\frac{C_p + C_f \cdot H(t)}{t} \right]. \tag{3.19}$$

The number of assets is not relevant for the optimisation question. For practical reasons, m is assumed to be 1 in this paper. The difficulty in the interval replacement model is the determination of the renewal function $H(t)$ and its derivate $h(t)$, which expresses the expected number of failures per unit of time, often year. The renewal density function $h(t)$ is needed for discounting on a yearly (or other time unit) basis. The renewal density function $h(t)$ is given by (Barlow & Proschan, 1965, p. 50):

$$h(t) = \sum_{k=1}^{\infty} f^{(k)}(t), \qquad (3.20)$$

where $f^{(k)}(t)$ is the k-fold convolution of the probability density function $f(t)$ with itself. Suppose that an asset fails according to a certain probability density function $f(t)$. An asset can only fail once. A failure will lead to a full replacement by an identical asset with the same probability density function $f(t)$ that will start at the time of replacement. The probability functions move along the time axis and are combined to find the k-fold convolution. The expected number of failures in time [0, t] is given by:

$$H(t) = \int_0^t h(t)dt, \qquad (3.21)$$

Discounted interval replacement optimisation model with use of the LCC techniques

Including the time value of money in the fundamental interval replacement model is done with the LCC techniques explained in Section 3.2. Again, two alternatives will be demonstrated, ending a repeating cycle with a preventive replacement and beginning a repeating cycle with a preventive replacement. The reason for demonstrating two alternatives is that the mathematical equations found in the literature all end repeating cycles with a preventive replacement. For comparison between using the LCC techniques and mathematical equations, the same cash flow pattern should be used. However, from a practical point of view, a maintenance engineer would prefer to consider the first instalment as the start of a cycle. Therefore, two alternatives are demonstrated with the use of the stepwise LCC approach.

Alternative 1: Ending a repeating cycle with a preventive replacement C_p

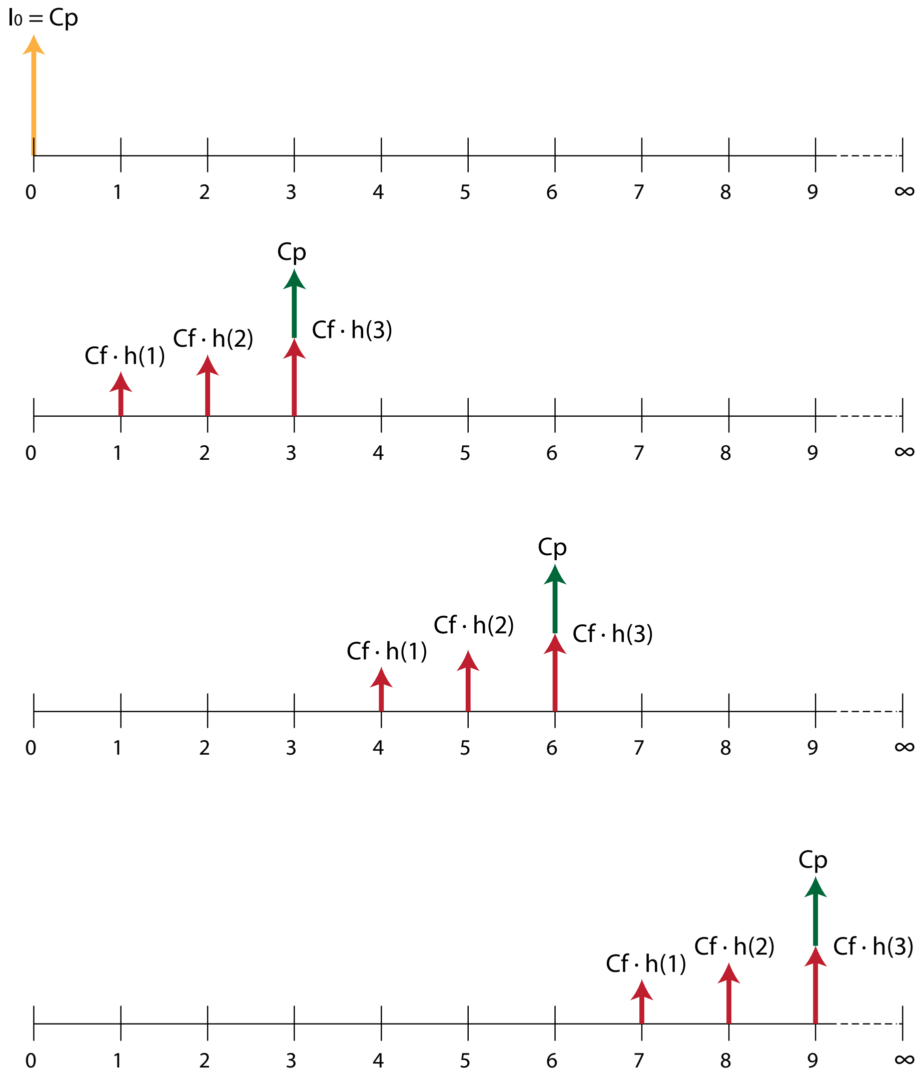

Figure 3.9 Cash flow diagram of an interval replacement policy for a preventive replacement interval of three years, ending with a preventive replacement. Three full cycles are shown. The initial investment is fully excluded from the repeating life cycle costs

Step 1: Identify repeating life cycle costs.

The initial investment and repeating cycles are presented in Figure 3.9. The renewal density function for the expected number of failures per year is represented as $h(t)$ (Equation 3.20).

Step 2: Calculate the present value of one life cycle.

The present value of the expected total costs of a cycle is calculated by using the present worth factor $(P/F, r, t)$ according to Equation 3.1.

$$P_{cycle} = C_f \cdot \sum_{t=1}^{T} (P/F, r, t) \cdot h(t) + C_p \cdot (P/F, r, T) . \tag{3.22}$$

This can also be written as:

$$P_{cycle} = \frac{C_f \cdot h(1)}{(1+r)^1} + \frac{C_f \cdot h(2)}{(1+r)^2} + \ldots + \frac{C_f \cdot h(T)}{(1+r)^T} + \frac{C_p}{(1+r)^T} . \tag{3.23}$$

Step 3: Calculate the equivalent annual costs (EAC) of one life cycle.

The present value of a cycle is now distributed over the cycle length $L = [0, T]$ by using the capital recovery factor $(A/P, r, t)$ with $t = T$:

$$EAC_L = (A/P, r, T) \cdot P_{cycle} . \tag{3.24}$$

Under the repeatability assumption, the EAC of a cycle is equal to the EAC of repeating cycles up to infinity.

Step 4: Calculate the EAC of the initial investment over infinity.

The initial investment costs $I_0 = C_p$ are equally distributed over an infinite time horizon by using the capitalised equivalent worth (Equation 3.3):

$$EAC_{I_0} = C_p \cdot r . \tag{3.25}$$

Step 5: Combining the EAC's of step 3 and 4 gives the total EAC of the strategy concerned.

The total EAC of the interval replacement strategy concerned is given by $EAC_{total} = EAC_L + EAC_{I_0}$. The optimum is found by minimising EAC_{total} of different interval replacement strategies.

Alternative 2: Beginning a repeating cycle with a preventive replacement C_p

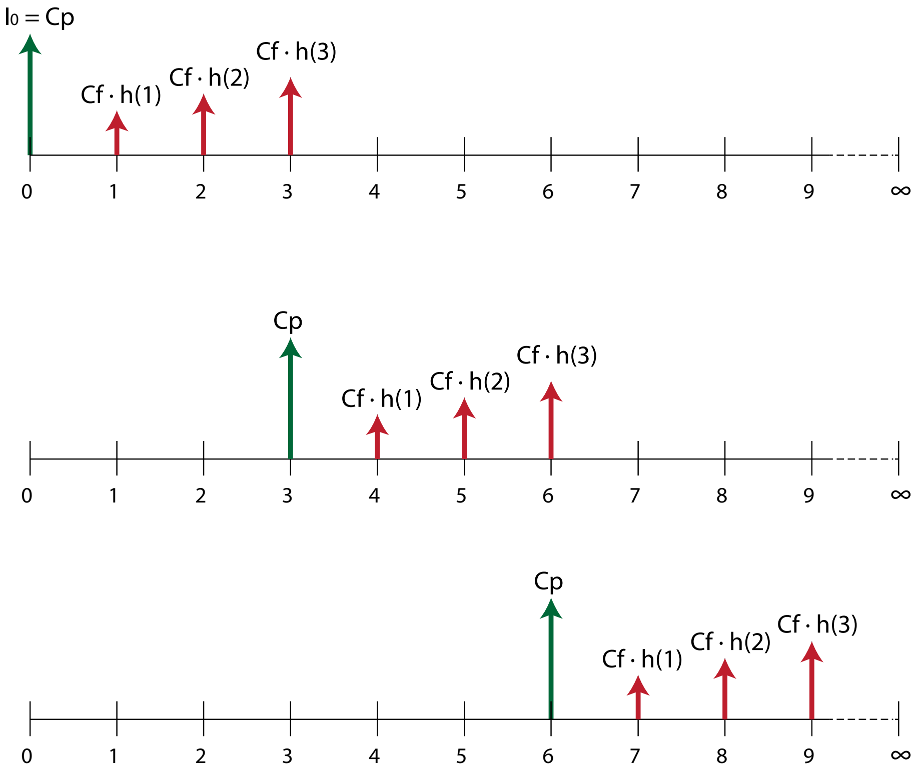

Figure 3.10 Cash flow diagram of an interval replacement policy for a preventive replacement interval of three years, beginning with a preventive replacement. Three full cycles are shown. The initial investment is fully included in the repeating life cycle costs

Instead of ending an interval with a preventive replacement (that is optimising a replacement interval after instalment of a brand-new asset), one could start an interval with a preventive replacement and include the initial investment costs directly. This is intuitively considered to be more realistic from a maintenance perspective.

Step 1: Identify repeating life cycle costs.
An even faster result is obtained by starting each repeating cycle with a preventive replacement C_p. In that case, there is no need to distribute the initial investment $I_0 = C_p$. The repeating cash flows are illustrated in Figure 3.10.

Step 2: Calculate the present value of one life cycle.
The present value of the expected total costs of a cycle is now calculated as:

$$P_{cycle} = C_p + C_f \cdot \sum_{t=1}^{T} (P/F, r, t) \cdot h(t) \,. \tag{3.26}$$

Step 3: Calculate the equivalent annual costs (EAC) of one life cycle.
The present value of a cycle is again distributed over the cycle length $L = [0, T]$ by using the capital recovery factor $(A/P, r, t)$ with $t = T$:

$$EAC_L = (A/P, r, T) \cdot P_{cycle} \,. \tag{3.27}$$

Because of the validity of the repeatability assumption, the EAC_L already provides a basis for comparison between various preventive replacement intervals and gives the required result. Steps 4 and 5 are redundant, as the initial investment $I_0 = C_p$ is already taken into account in the EAC of a cycle.

Mathematical equations for discounted interval replacement optimisation found in the literature

Chen and Savits (1988) developed a mathematical relationship for the expected total discounted costs over an infinite time horizon for an interval (block) replacement model that is given by:

$$J_B = \frac{\int_0^T e^{-rt} dQ(t)}{1 - e^{-rT}} \,, \tag{3.28}$$

where J_B represents the expected total discounted costs from $t = [0, \infty]$ for repeating cycles with a preventive replacement interval T. i is the discount rate. The integral represents the sum of the yearly discounted costs of one cycle. The factor e^{-rt} approximates the present worth factor $(P/F, r, t)$ and Q represents yearly costs. It is observed in the literature that Chen and Savits (1988) discount the preventive replacement costs C_p at T, the end of a preventive replacement interval. The initial investment is not taken into account in this mathematical model.

A nearly similar mathematical relationship is established by Mazzuchi et al. (2007):

$$\lim_{t\to\infty} E(K(T,r)) = \frac{C_f \cdot E\left(N(T,r)\right) + C_p \cdot e^{-rT}}{1-e^{-rT}}, \tag{3.29}$$

where $E(K(T,r))$ is the expected total discounted costs over an infinite time horizon of a continuous repeating cycle, $E(N(T,r))$ is the expected number of discounted failures in a preventive replacement interval $[0,T]$, T is the preventive replacement time and r is the discount rate. In the terminology used in this paper, $E(N(T,r))$ in the numerator of Equation 3.29 is explained as $\sum_{t=1}^{T} e^{-rt} \cdot h(t)$, where $h(t)$ is the renewal density function. From a life cycle costing perspective, it is not common to use the term discounted failures, as the term discounting is reserved for monetary values. However, from a mathematical perspective, there is no difference, as in this case $C_f \cdot E\big(N(T,r)\big) = C_f \cdot \sum_0^t e^{-rt} \cdot h(t) = \sum_0^t e^{-rt} \cdot C_f \cdot h(t)$.

It is again noticed that Mazzuchi et al. (2007) discount the preventive replacement costs at the end of a cycle. The initial investment is not included in this model. Chen and Savits (1988) and Mazzuchi et al. (2007) have in common that the numerators of Equations 3.28 and 3.29 calculate the present value of one cycle, and the denominator transforms this present value into the present value over an infinite time horizon. In the following subsection, the LCC approach is compared to the mathematical equations for interval replacement optimisation by means of an example.

Practical example comparing discounted interval replacement calculations

The previous example of the maintenance of a hydraulic cylinder is used. A practical application is the interval replacement optimisation of hydraulic cylinders at the Dutch Eastern Scheldt storm surge barrier. For the example and demonstration purposes, the number of assets (m) in Equation 3.19 is set at one as this will not influence the optimal replacement interval. The costs of preventive and corrective replacements are, respectively, € 30,000 and € 100,000. The initial investment is equal to the costs of a preventive replacement. The failure of the cylinder is modelled with a normal probability distribution with a mean of 15 years and standard deviation of 1.5. The inflation-free discount rate is 5% per year.

Discrete computations are made on a yearly basis (Table 3.2) and checked with more accurate discrete computations on a monthly basis. In this example, the results do not differ much from a practical point of view. Only slight differences in the order of magnitude of a few months were found.

The renewal density function $h(t)$ is calculated as the sum of the first to tenfold convolution of $f(t)$ and shown in Figure 3.11. The k-fold convolution of a normal probability density function is obtained by using mathematical rules (DasGupta, 2010, p. 203). For a probability density function $f(t)$ with a normal distribution having a mean μ_1 and a standard deviation σ_1, the twofold convolution $h^{(2)}(t)$ of $f(t)$ with itself is again a normal distribution with a mean $\mu_2 = \mu_1 + \mu_1$ and a standard deviation $\sigma_2 = \sqrt{\sigma_1{}^2 + \sigma_1{}^2}$. Even so, let $h^{(3)}(t) = h^{(2)}(t) \cdot f^{(1)}(t)$ then $h^{(3)}(t)$ is a normal distribution function with a mean $\mu_3 = \mu_2 + \mu_1$ and a standard deviation $\sigma_3 = \sqrt{\sigma_2{}^2 + \sigma_1{}^2}$.

The interval cost optimisation graphs on a yearly basis are presented in Figure 3.12, Figure 3.13 and Figure 3.14 for the approaches: (1) discounting with the use of LCC techniques and ending an interval with a preventive replacement, (2) discounting with use of the LCC techniques and beginning an interval with a preventive replacement and (3) discounting with the use of mathematical Equation 3.29 with a correction for the initial investment. The mathematical Equation 3.29 gives the present value over an infinite time horizon. This is transformed to EAC by using the capital equivalent worth approach (Equation 3.3).

Table 3.2 Discounted interval replacement model: results calculated on a yearly basis

Approaches to discounted interval replacement models	Optimum [y]	EAC [€]	CW=P∞ [€]	H(T)
1. Discounting with LCC techniques alternative 1	12	€ 3,669	€ 73,376	0.04
2. Discounting with LCC techniques alternative 2	12	€ 3,669	€ 73,376	0.04
3. Discounting with mathematical Equation 3.29	12	€ 3,660	€ 73,197	0.04

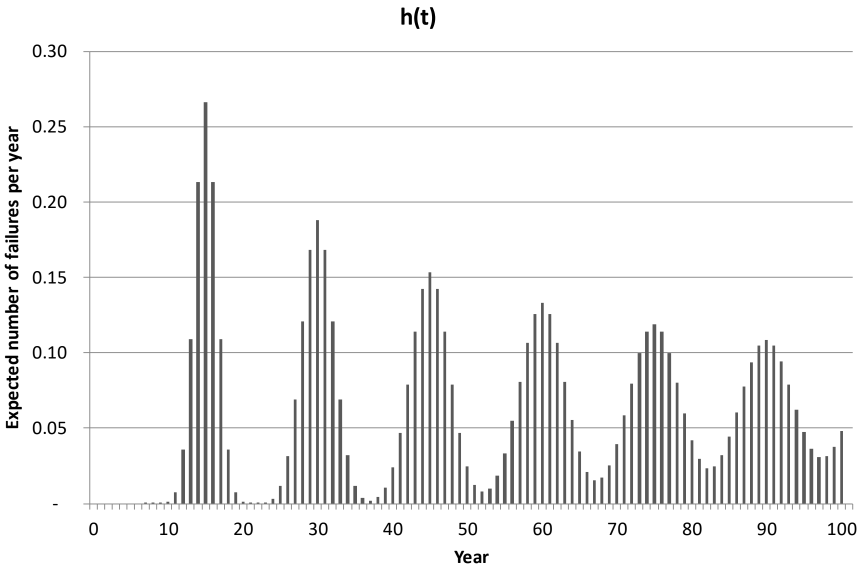

Figure 3.11 Renewal density function h(t): expected number of failures per year. The time axis is stretched to show the impact of the convolutions

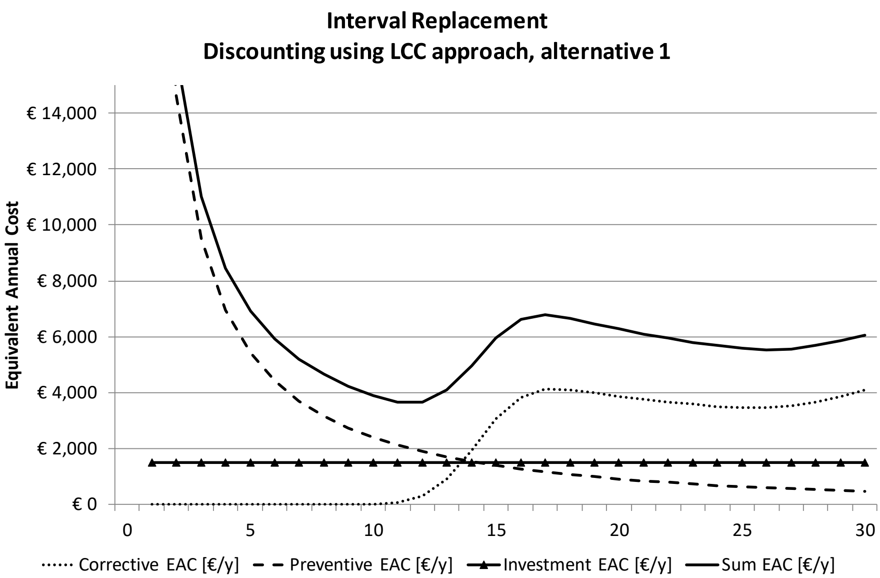

Figure 3.12 Interval replacement with discounting using LCC techniques and ending an interval with a preventive replacement (alternative 1)

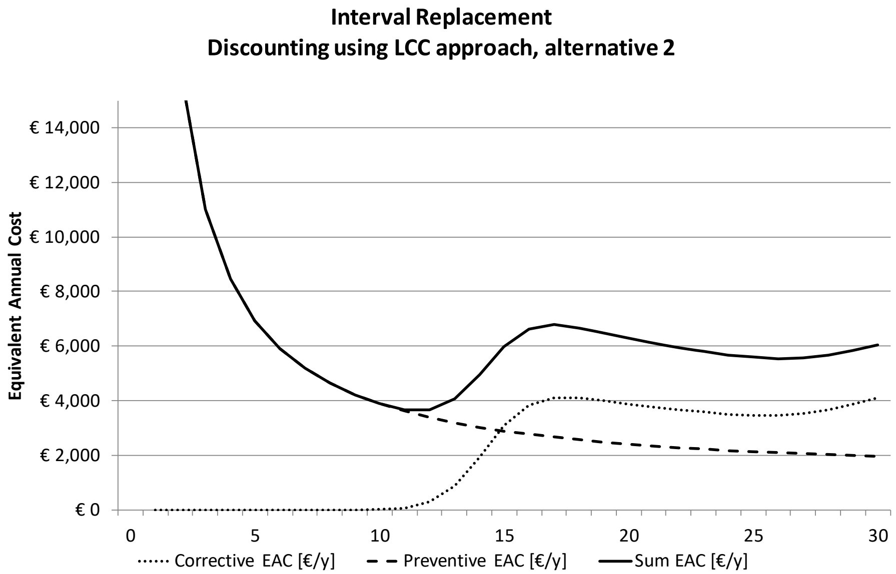

Figure 3.13 Interval replacement with discounting using LCC techniques and beginning an interval with a preventive replacement (alternative 2)

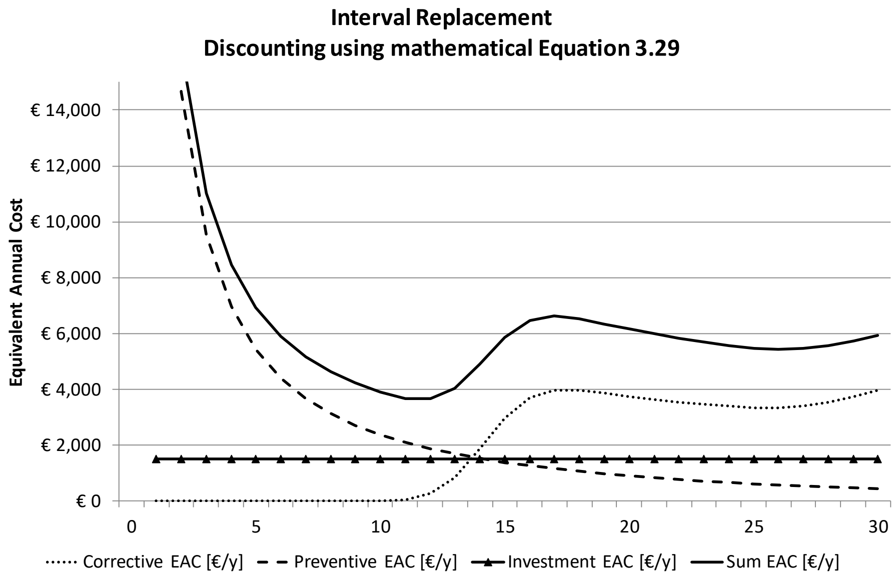

Figure 3.14 Interval replacement with discounting by use of mathematical Equation 3.29 (Mazzuchi et al., 2007) with a correction for the initial investment and transformation to EAC conform Equation 3.3

Discussion of results

A first observation is that the differences between discounting with mathematical Equation 3.29 and the LCC approach are marginal. There is no need for a sensitivity analysis here, as these findings are not surprising. The continuous discount function e^{-rt} used in Equation 3.29 is comparable to the discrete present worth factor $(P/F, r, t)$. For normally used interest rates, the following approximation is valid:

$$e^{-rt} \approx \frac{1}{(1+r)^t}. \tag{3.30}$$

The factor $\frac{1}{1-e^{rt}}$ in Equation 3.29 represents a continuous function for the transformation of the present value of one cycle to the present value of an endless stream of these cycles. Using the discrete LCC techniques, this transformation is achieved by combining the capital recovery factor $(A/P, r, t)$ and the capitalised equivalent worth (CW). The continuous transformation and the discrete transformation approximate each other:

$$\frac{1}{1-e^{-rt}} \approx \frac{(A/P,r,t)}{r} = \frac{r(1+r)^t}{(1+r)^t-1} \cdot \frac{1}{r} = \frac{(1+r)^t}{(1+r)^t-1}. \tag{3.31}$$

Therefore, there is not much difference in computations in using the mathematical formula (3.29) or the LCC techniques.

A second observation is that LCC alternatives 1 and 2 give the same results. In scenario 1, an interval ends with a preventive replacement, and the initial investment $I_0 = C_p$ is compensated for afterwards. In scenario 2, an interval begins with a preventive replacement, and there is no need to compensate for an initial investment, as it is already incorporated in the first cycle. The total cash flows, however, are identical, and there is no distortion due to an expected cycle length as was seen in the age replacement modelling in Section 3.3. Therefore, it is not surprising that the results of scenarios 1 and 2 are identical.

The last observation concerns $H(t)$ for the optimised preventive replacement interval. $H(t)$ approximates 0.04 failures per interval of 12 years. From an economical point of view, one should not accept more expected failures. This is explained by the relatively high corrective replacement costs and the characteristics of the probability density function $f(t)$. $H(t)$ is the cumulative density function of $h(t)$, which is shown in Figure 3.11. $h(t)$ is constructed by calculating the tenfold convolution of $f(t)$ with itself. It is observed that in the previous example, the second and higher convolutions of $f(t)$ do not influence the economic optimum, which suggests that situations exist where $h(t)$ can well be approximated by $f(t)$. However, a higher standard deviation of $f(t)$ would increase the influence of the renewal density function.

Compared to age replacement models, discounting in interval replacement models is not difficult because of the absence of an expected cycle length. In the case that the initial investment equals the cost of a preventive replacement, there is no need to distribute an initial investment over an infinite time horizon because one can start a calculation with a preventive replacement. The difficulty in the interval replacement model lies in the determination of the renewal density function $h(t)$ and/or the renewal function $H(t)$, irrespective of discounting.

3.5 Conclusions

The authors developed a stepwise and flexible life cycle costing approach for discounting in age and interval replacement models for civil infrastructure assets and validated the new approach by comparing the results with case-specific formulas. Age and interval replacement optimisation strategies support the long-term asset and maintenance planning of organisations that operate and maintain these infrastructure assets. Some typical examples for the application of these models are the conservation of steel lock gates, the replacements of streetlight luminaires and the major overhauls or replacements of hydraulic cylinders.

Life cycles of civil infrastructure assets are often long. Therefore, the time value of money should be incorporated. Discounting in fundamental probabilistic maintenance optimisation models is hardly covered in the literature on engineering economy and reliability engineering. For instance, just a few authors developed dedicated mathematical formulas for the fundamental and popular age and interval replacement models (Chen & Savits, 1988; Fox, 1966; Mazzuchi et al., 2007; Van Noortwijk, 2003). These mathematical formulas were used for validation of the developed LCC approach.

The LCC approach builds on well-known LCC techniques which are the are the present worth, the capital recovery and the capitalised equivalent worth. The LCC techniques are used in a specific order and combined with reliability formula. The advantage of this stepwise LCC approach is that it enhances the understanding of discounting principles, their constraints and their field of applicability, for reliability and maintenance engineers in practice. In addition, the stepwise LCC approach explicitly takes the initial investment into account and allows for easy adaptation and extension when conditions change, for instance changing cash flow patterns or reliability profiles.

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4

Inflation adjusted capitalised equivalent

Replace an old asset with a like-for-like replacement
Non-repetitive cash flows
Certain future

This chapter presents a replacement optimisation model for a situation where an old asset is challenged by a new asset with a different cost profile. All costs are subject to their own inflation rates.

The classical approach to such challenge is a marginal analysis or capital equivalent worth approach as described in Chapter 2. However, these classical approaches are built on a repeatability assumption of the challenger's cash flows. Inflation rejects such an assumption.

The model presented in this chapter is an adaptation of the classical capitalised equivalent approach by correcting the classical approach for inflation and ageing. This adaptation is an approximation method as the mathematically correct approach would require shortest path optimisation using dynamic programming algorithms as shown in Chapter 6. However, the special characteristics of infrastructure assets often allow for such approximation which is demonstrated by comparison of both methods.

Additionally, the presence and magnitude of inflation is investigated by comparing the inflation-adjusted capitalised equivalent approach with the classical approach, which demonstrates that application of the classical approach leads to errors. The added value of this model is a set of dedicated mathematical equations which reduces a complex optimisation challenge to a spreadsheet exercise.

Replacement optimisation of ageing infrastructure under differential inflation

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Published online in Construction Management and Economics at 12 March 2019
https://doi.org/10.1080/01446193.2019.1574977

Ageing public infrastructure assets necessitate economic replacement analysis. A common replacement problem concerns an existing asset challenged by a replacement option. Classical techniques obtained from the domain of engineering economics are the mainstream approach to replacement optimisation in practice. However, the validity of these classical techniques is built on the assumption that life cycle cash flows of a replacement option are repetitive. Differential inflation undermines this assumption and therefore more advanced replacement optimisation techniques are required under these circumstances. These techniques are found in the domain of operations research and require linear or dynamic programming (LP/DP). Since LP/DP techniques are complex and time-consuming, the current study develops an alternative model for replacement optimisations under differential inflation. This approach builds on the classical capitalised equivalent replacement technique. The alternative model is validated by comparison with a DP model showing to be equally accurate for a case with characteristics that apply to many infrastructure assets.

4.1 Introduction

Public sector organisations confront ageing infrastructure assets. Infrastructure assets, once built, often need to be replaced at the end of their economic or functional life, whichever comes first. The economic life is determined by a life cycle cost calculation. An asset is at the end of its economic life when it becomes less expensive to replace it with an equivalent alternative (Hartman & Tan, 2014; Park, 2011). In contrast, functional life is reached when an asset is not able to fulfil its original function (Lemer, 1996). In addition, the functional service life designates the time-period in which a certain function such as transportation, high water protection or water crossing should be provided. In this context, the functional service life is the life cycle of a users' system which incorporates multiple successive infrastructure assets' life cycles (Wasson, 2016).

Replacement decisions are time-variant optimisation challenges of such a system. The purpose of an economic replacement analysis is to assess whether postponing a replacement justifies the cost of lifetime extension, major overhauls and renovations, and for how long. This type of replacement problem is commonly designated a defender-challenger replacement analysis. A defender is the existing asset that can remain in service for a limited number of years with a major overhaul or renovation; a challenger is the replacement option.

Replacement analyses are a special class of problems in the field of engineering economics, and the literature offers an array of fundamental and advanced calculation approaches that are applicable under specific circumstances. However, for cost engineers, economic replacement analysis is challenging for such reasons as selecting the correct calculation technique in relation to the specific circumstances and understanding of inflationary effects (Korpi & Ala-Risku, 2008). Moreover, circumstances such as inflation require advanced replacement approaches with underlying linear programming (LP) or dynamic programming (DP) techniques. These techniques require case-specific modelling, are complex and time-consuming to apply and are often not known to practitioners (Hartman & Murphy, 2006). The current study evaluates the applicability of existing replacement analysis techniques for infrastructure assets.

Moreover, this study investigates the presence and impact of differential inflation (the difference between total inflation and general inflation) for public sector organisations and develops a pragmatic approach for inclusion in a common class of infrastructure replacement challenges, as an alternative for deployment of advanced LP or DP techniques.

The outline of this paper is as follows. First, the literature on classical and advanced replacement analysis techniques is reviewed. The different techniques are evaluated for their applicability on infrastructure assets under different circumstances. Hereafter, differential inflation is defined, supported with an illustrative quantitative analysis of long-term values for common infrastructure cost groups in the Netherlands. The subsequent section presents the method development and results into a set of practical equations.

The alternative method is demonstrated in a case study and compared with the full DP calculation. The current study is finalised with the conclusion that care should be taken when differential inflation is present, as it can lead to sub-optimal replacement times. The developed method offers a pragmatic solution for a common class of public infrastructure replacement problems, as opposed to complex dynamic programming modelling.

4.2 Literature review

Price developments affect decisions on infrastructure assets. For example, in establishing service payments contracts in long-term Design Build Finance Maintain and Operate (DBFMO) public-private partnerships. Another example is the long-term operational and capital expenditure planning of infrastructure owners. Mirzadeh, Butt, Toller, and Birgisson (2014) and Yu and Ive (2011) emphasise the significance of a proper assessment of price developments in Swedish road infrastructure and UK construction industry respectively.

The current study conforms these findings for the Dutch construction industry based on an analysis of historic price developments, which is provided after the literature review. Such price developments have consequences for replacement decisions and their underlying mathematics.

Replacement decisions are dealt with in the domains of engineering economics and operations research. Engineering economics offers classical techniques to compare the discounted time-variant life cycle cost of an existing asset (a defender) with a new asset (a challenger). These classical techniques are generally used in practice. However, these classical techniques are founded on a repeatability assumption of the cash flows of the challenger. When this repeatability assumption does not hold, i.e. because of price developments, advanced techniques like linear and dynamic programming are required which are found in the domain of operations research. The disadvantage of these advanced techniques is their complexity in practice.

The following literature review is structured along the two domains. The research gap identified by the current study is a replacement analysis technique that can handle price increases in a fundamental defender-challenger replacement problem without the need for dynamic programming. Based on the domains engineering economics and operations research, the current study categorises defender-challenger analyses in two classes:

- Engineering economics: classical defender-challenger replacement analysis with a **repetitive** challenger’s cash flows or a possibility for truncation of the challenger’s cash flows;
- Operations research: advanced defender-challenger replacement analysis with a **non-repetitive** challenger’s cash flows.

Classical defender-challenger replacement analysis

A classical defender-challenger replacement analysis is the mainstream approach to replacement problems. An existing asset is compared with a replacement option. The comparison answers the question: “what is the best time to replace the current asset with its challenger?” Several approaches are available for a classical defender-challenger analysis: present value analysis over a bounded time horizon, economic life comparison, marginal analysis and the capitalised equivalent analysis. The classical literature on engineering economics offers practical guidelines for their application. Essential work is provided by Blank and Tarquin (2012); Newnan, Lavelle, and Eschenbach (2016); Park (2011) and Sullivan, Wicks, and Koeling (2012). In the classical literature replacement decisions are treated as a special topic in engineering economics. A short mathematical review of classical approaches is provided in Chapter 2 and a descriptive review and evaluation is provided next.

The first classical method is a present value analysis over a bounded time horizon. This technique originates from the traditional investment analysis comparison where alternatives are compared based on least present value (in a cost model) or least equivalent annual costs (EAC) when alternatives have unequal lives (Blank & Tarquin, 2012). The EAC is a discounted annual average cost and is found by transforming the conventional present value to EAC over the service life of a scenario.

An application of the EAC comparison is provided by Farahani, Wallbaum, and Dalenbäck (2018) who compared maintenance and renovation scenarios of housing plans with unequal lives. Another application is provided by van den Boomen, Schoenmaker, and Wolfert (2017) who used EAC comparison to find optimal age and interval replacement intervals. Also, Safi, Sundquist, Karoumi, and Racutanu (2013) used this technique for comparison of life cycle scenarios of bridges with unequal lives. This technique, however, is not suitable for replacement optimisation affected by price increases or decreases as its underlying assumption is that EAC values of alternatives are comparable. EAC values are only comparable when they remain constant over time and this incorporates a repeatability assumption of life cycle cash flows (Newnan et al., 2016). Price developments undermine this assumption of a constant EAC as future life cycle cash flows will be affected by these price developments.

The second classical technique is the economic service life comparison which compares the minimum equivalent annual cost (EAC*) over the remaining economic life of a defender with the minimum EAC* over the economic life of a challenger. When the EAC* of a defender is less than the EAC* of a challenger (in a cost model), there is no reason to replace a defender immediately because it is cheaper to keep it. Instead, the defender should be kept in service for at least its remaining economic life. This approach also assumes that the EAC* of the challenger remains constant over infinity, irrespectively of its instalment year. Navon and Maor (1995) provide a clear implementation of an economic service life comparison for navel equipment.

The third classical technique, the marginal analysis, tells how long the defender should be kept beyond its economic life before replacing it by the challenger. The marginal analysis compares the year-by-year cost of keeping a defender in service with the EAC* of the challenger. The defender should be replaced as soon as the marginal (year) costs exceed the EAC* of the challenger.

Newnan, Eschenbach, and Lavelle (2009) clearly point to a constraint of the marginal analysis. This technique only works with gradually increasing operational expenditures of a defender. Major overhauls disrupt gradually increasing operational expenditures. Park (2011) provides a solution for this problem in replacement decisions and introduces the concept of the capitalised equivalent. The capitalised equivalent approach first transforms the EAC* of a challenger to a total present value over infinity and 'truncates' the time-variant cash flows of the defender with this challenger's present value. The capitalised equivalent approach is implicitly a traditional NPV analysis over an infinite time horizon.

The applicability of these four classical calculation approaches depends on the validity of the underlying assumptions. Two in the literature prominent characteristics that reject the repeatability assumption of the challenger's cash flows are technology change and differential inflation. Both lead to the following main class of defender-challenger replacement analysis.

Advanced defender-challenger replacement analysis

The second class of a defender-challenger replacement problem is a situation where a defender is challenged by a repetitive challenger or a chain of challengers with different life cycle cash flows. Both technology changes and differential inflation cause the non-repeatability of the future life cycle cash flows of a challenger. In this case, the optimal replacement time of a defender is influenced by all the optimal replacement times of future challengers and requires case-specific modelling and advanced techniques such as DP or LP.

These approaches for replacement decisions are described as a shortest path problem in domain of operations research. All possible defender-challenger replacement scenarios are visualised in a network in which nodes represent states and arcs between the nodes, the cost of transferring from one state to another. Backward induction (a DP-solution algorithm) or solving a set of linear equations with multiple unknows (LP) is applied to find the shortest path or least-cost route in such a network.

Numerous authors have studied this type of replacement problem. Bellman (1955) laid a foundation by developing a functional equation and suggested to solve this equation by successive approximations based upon an initial policy space approximation (a DP-approach). Wagner (1975) is one of the first authors who provided a pragmatic and accessible dynamic programming solution to calculate economic service lives of successive challengers. This approach is provided in Supplemental material, Appendix A in Van den Boomen, Leontaris, and Wolfert (2019) and can also be found in Hillier and Lieberman (2010).

Only few authors explicitly dealt with inflation. Karsak and Tolga (1998) handled inflation and developed a DP model to optimise maintain and replace strategies in a case study of an industrial plant. The authors stressed the importance of recognising the impact of inflation. Regnier, Sharp, and Tovey (2004) also included inflation and technology change in their case-specific DP model and concluded that applying classical replacement techniques under these circumstances will lead to errors. Mardin and Arai (2012) proposed a simplified approach for the model provided by Regnier et al. (2004) based on restricting the optimisation objective to minimising the EAC of two successive assets and used LP to solve the objective function. This slight simplification of DP modelling was originally introduced by Christer and Scarf (1994) and elaborated on by Scarf, Dwight, McCusker, and Chan (2007) who demonstrated that for their specific case studies, the cash flows of two challengers' optimal life cycles are sufficient for determining the optimal replacement time of a defender.

Other DP and LP replacement models incorporating technological change, variable utilisation and parallel asset replacements have been developed by Büyüktahtakın and Hartman (2016) and Hartman (2004). These authors again stress the importance of DP and LP modelling in replacement analysis opposed to classical engineering economics techniques as the latter will result in errors when future life cycle cash flows are non-stationary.

Brekelmans, den Hertog, Roos, and Eijgenraam (2012); Zwaneveld and Verweij (2014), and Dupuits, Schweckendiek, and Kok (2017) provided LP models for the optimisation of intervention strategies for coastal flood defence systems. Inflation was not taken into account but, in contrast to most other DP/LP studies, these authors dealt with a long calculation horizon of 300 years. As public discount rates are low, this is realistic for public infrastructure but also enlarges the modelling state space.

Van den Boomen, van den Berg, and Wolfert (2019) developed a nested DP model to optimise a sequence of multiple intervention strategies under differential inflation. However, such model has a large state space and needs more data and intermediate calculations. Moreover, it may be overqualified for a common generic case dealing with only two intervention strategies: maintain or replace.

DP techniques also underlie real options and decision tree analysis. In real options analysis the non-repeatability of future cash flows is caused by flexibility or the option to choose between future favourable and unfavourable developments. A case-specific replacement model incorporating real options based on DP techniques is provided by Van den Boomen, Spaan, Schoenmaker, and Wolfert (2018).

The literature shows that DP or LP techniques are required to optimise replacement decisions when the successive challengers' cash flows are non-repeatable. Various circumstances can cause non-repeatability such as price developments, technology change, changing demands and the option to choose.

Moreover, the studies demonstrate the assumptions underlying the estimation of future cash flows, such as the chain of challengers, cash flow growth or decline, selection of calculation horizons and cash flow truncation methods, require a case-specific type of DP or LP modelling. Each DP or LP model is different. The literature does not offer a generalisation for a common case. The closest is the DP-approach presented by Wagner (1975) to determine the optimal cost route for a new investment, to be replaced by itself, in a bounded time horizon (Supplemental material, Appendix A in Van den Boomen, Leontaris, et al. (2019)). The complexity of DP or LP modelling makes this optimisation approach difficult to apply in practice (Hartman & Murphy, 2006).

The available replacement analysis techniques are summarised in Table 4.1. The current study is interested in the extension of the classical defender-challenger replacement analysis with differential inflation and ageing without the need for applying complex DP modelling. This case is generic for many infrastructure assets with long economic or functional lives (whatever comes first) for which there is not a reason to incorporate technology change in the first life cycle. Once built, it is often extremely costly to replace infrastructure assets prematurely because better technology becomes available during its lifespan.

The following section shows that differential inflation significantly affects the present value calculations in organisations that use low discount rates. Therefore, the current study is particularly interested in determining a generic solution for a defender-challenger replacement problem: an approach to an inflation adjusted defender-challenger analysis for public infrastructure assets.

Table 4.1 Overview of classical and advanced replacement techniques.

	Underlying assumptions	Limitations
Classical replacement techniques		
EAC comparison when traditional NPV replacement scenarios have unequal lives	- Assumes that the EAC's of different replacement scenarios are comparable which means and underlying repeatability assumption of the combined cash flows of the defender and challenger.	- Cannot handle differential inflation, technology change, multiple different successive challengers.
Economic service life comparison	- Repeatability assumption of the challenger's life cycle cash flows.	- Cannot handle differential inflation, technology change, multiple different successive challengers.
Marginal analysis	- Repeatability assumption of the challenger's life cycle cash flows. - Gradually increasing operating marginal expenditures of the defender.	- Cannot handle differential inflation, technology change, multiple different successive challengers. - Cannot handle fluctuating operational marginal expenditures of a defender caused by e.g. major overhauls.
Capitalised equivalent approach	- Repeatability assumption of the challenger's life cycle cash flows.	- Cannot handle differential inflation, technology change, multiple different successive challengers.
Advanced replacement techniques		
Case-specific DP or LP modelling	- No mathematical assumptions. Can handle non-repeatable cash flows caused by whatever reason	- Complex and time consuming in its application.

4.3 Differential inflation

This section defines differential inflation and discusses its importance in infrastructure asset management based on a quantitative long-term analysis of producer price indices (PPI) of common engineering goods and services and consumer price indices (CPI). As an example, publicly available Dutch PPI and CPI data are analysed to demonstrate the magnitude and impact of differential inflation. Each government and related financial institutions publish PPI and CPI data.

Differential inflation (d) is the variation between general inflation (f) and total inflation (f_{tot}) on prices of specific goods and services (Sullivan et al., 2012). Certain goods and services have higher or lower price increases than general inflation. The relationship between total inflation, general inflation and differential inflation is defined by Equation 4.1.

$$d = \frac{f_{tot} - f}{1 + f} \tag{4.1}$$

In present value calculations, real cash flows are inflated with differential inflation and discounted with a real discount rate (r). An equal alternative is to inflate the cash flows with the total inflation and discount the cash flows with the nominal discount rate (r_{nom}), which is defined by Equation 4.2.

$$r_{nom} = r + f + r \cdot f \tag{4.2}$$

In the current study the first approach is systematically applied. Cash flows are expressed in real values, in the literature also referred to as constant currency, and discounted with a real discount rate. By definition (Equation 4.1 and Equation 4.2) differential inflation is included in real cash flows while total inflation is included in nominal cash flows. In the literature, nominal cash flows are also referred to as actual currency. To demonstrate the equivalence relationships, a small example is provided.

Assume a general inflation rate of 1.8% (economy wide) and a total inflation rate of 3% for a specific good or service, in this example an investment. The real interest rate is 6%. What is the present value of this investment when this investment is made five years from now? The same investment today would cost 1,000 (the current price level).

In a nominal expression (actual currency), the investment is inflated with total inflation and discounted with the nominal discount rate. To obtain the nominal discount rate Equation 4.2 is used: $r_{nom} = 6\% + 1.8\% + 6\% \cdot 1.8\% = 7.91\%$. The present value is now calculated as:

$$P = 1{,}000 * \left(\frac{1 + 3\%}{1 + (6\% + 1.8\% + 6\% \cdot 1.8\%)}\right)^5 = 792.35$$

Alternatively, in a real expression (constant currency), the investment is inflated with differential inflation only and discounted with a real discount rate. To obtain the differential inflation rate, Equation 4.1 is used: $d = \frac{3\% - 1{,}8\%}{1 + 1.8\%} = 1.18\%$. The present value now follows from:

$$P = 1{,}000 * \left(\frac{1 + \left(\frac{3\% - 1{,}8\%}{1 + 1.8\%}\right)}{1 + 6\%}\right)^5 = 792.35$$

As present value is literally a present value, both calculation approaches by definition lead to the same result as is illustrated with this example.

Total inflation rates can be obtained from PPI data. The general inflation rate is obtained from CPI data. The differential inflation follows from Equation 4.1. In the Netherlands, the Central Bureau for Statistics (CBS, a governmental organisation) publishes quarterly CPI data and aggregated PPI data for some cost groups relevant for civil engineering. CBS data are publicly available. In addition, private sector organisations collect and publish more specific quarterly PPI data on engineering cost groups and projects.

For public infrastructure assets with long life cycles, the interest lies in the long-term development of PPI and CPI data. In the Netherlands, official quarterly and yearly CPI data are published from 1996 onwards. A suitable PPI data set for construction works is published from 2000 onwards. Both data sets are obtained to investigate the magnitude and impact of differential inflation for organisations that use low discount rates. These data are presented in Figure 4.1. The results of the analysis are presented in Table 4.2.

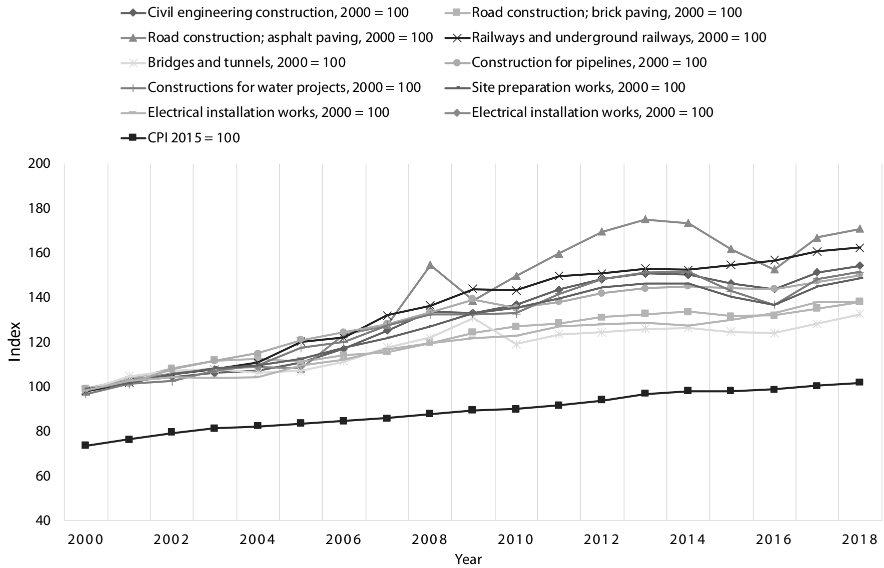

Figure 4.1 Typical engineering PPI and CPI developments in the Netherlands during 2000 – 2018

The top part of Table 4.2 shows the total inflation rates, general inflation rates and differential inflation rates for nine aggregated civil engineering cost components. The magnitude of the differential inflation rates ranges from -0.2% to 1.3% for these data set. In the Netherlands, public sector organisations use real discount rates between 3% and 5%.

The middle part of Table 4.2 shows the impact of the differential inflation rates on net discounting for an organisation A that uses a real discount rate of 5%. The impact of differential inflation on discounting follows the ratio: $(1+d)/(1+r) = 1/(1+r_{net})$, where r_{net} = net discount rate in real terms (opposed to nominal). For organisation A, differential inflation changes the effective real discount rate from 5% to a range of 3.7% to 5.2%. This results in deviations from the real discount rate from -3.4% to 26.9%. The deviation is calculated by $1 - r_{net}/r$.

The bottom part of Table 4.2 shows the same analysis for organisation B which uses a real discount rate of 3%. In this situation, the net discount rate ranges between 1.7% and 3.2% and the deviations from the real discount range between -5.5% and 44%.

Table 4.2 Long-term differential inflation rates and their impact on discounting

	1	2	3	4	5	6	7	8	9
Inflation									
Total	2.6%	1.9%	3.1%	2.8%	1.7%	2.3%	2.5%	2.3%	1.9%
General	1.8%	1.8%	1.8%	1.8%	1.8%	1.8%	1.8%	1.8%	1.8%
Differential	0.7%	0.0%	1.3%	1.0%	-0.2%	0.5%	0.7%	0.4%	0.1%
Discount rate for organisation A									
Real	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%	5.0%
Net	4.2%	5.0%	3.7%	4.0%	5.2%	4.5%	4.3%	4.5%	4.9%
Deviation	15.2%	0.7%	26.9%	20.6%	-3.4%	10.6%	14.2%	9.3%	1.5%
Discount rate for organisation B									
Real	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
Net	2.3%	3.0%	1.7%	2.0%	3.2%	2.5%	2.3%	2.5%	2.9%
Deviation	24.8%	1.1%	44.0%	33.7%	-5.5%	17.4%	23.3%	15.3%	2.5%

1. Civil engineering construction (weighted average)
2. Road construction; brick paving
3. Road construction; asphalt paving
4. Railways and underground railways
5. Bridges and tunnels
6. Construction of pipelines
7. Construction for water projects
8. Site preparation works
9. Electrical installation works

From the analysis, two conclusions are drawn. First, the differential inflation rates between cost components differ and can be positive or negative values. Second, for organisations that use low real discount rates, this differential inflation can significantly affect the net discounting in present value calculations. Present value calculations are known to be sensitive to changes in discount rates. Therefore, the presence of differential inflation should be carefully assessed.

The analysis is conducted with publicly available aggregated data that levels off values for specific subgroups and the impact of local circumstances. This type of aggregated data should be used with caution. Non-aggregated data fitting local circumstances are also available but not publicly published, because this information is often confidential or competitive.

Although not similar in objective, a UK study on the development of construction output prices, illustrates the significance of inflationary effects in UK construction industry (Yu & Ive, 2011). A study by Mirzadeh et al. (2014) emphasis a proper assessment of differential inflation rates reflecting specific circumstances. This study provides a model for such assessment for road infrastructure in Sweden.

4.4 Method development

The previous section demonstrates differential inflation and how it affects the net discounting in present value calculations. Classical replacement analysis techniques cannot handle differential inflation because differential inflation undermines the repeatability assumption of the challenger's cash flows. Therefore, the approach to infrastructure replacement analysis under differential inflation is based on the capitalised equivalent approach with adjustments to ageing and differential inflation.

The capitalised equivalent approach compares the present values of defender-challenger replacement scenarios over an infinite time horizon. For a cost model: let T be the time for replacing the defender in number of years starting from zero, then, the objective is to identify the value of T^* that minimises the total present value of the combined keep-replace scenarios according to Equation 4.3.

$$P^* = \min_T \left(\sum_{i=0}^{T} \frac{F_i}{(1+r)^i} + \sum_{k=T}^{\infty} \frac{F_k}{(1+r)^k} \right), \tag{4.3}$$

where P^* = minimum present value of a keep-replace scenario over an infinite time horizon; F_i = real cash flow of the defender in year i; F_k = real cash flow of the challenger(s) in year k, T = year of replacement of a defender relative to starting year zero and r = real interest rate.

The following sections derive generic mathematical relationships to perform the present value calculations for different types of life cycle activities subject to ageing and differential inflation for the defender and challenger. Combining these mathematical equations allows for a relatively compact spreadsheet calculation as opposed to complex DP-modelling, which will be used for comparison. The restrictions of this approach are twofold. First, this approach assumes the inflation adjusted perpetuity of the first challenger's cashflows is a suitable approximation of all future cash flows.

The second restriction is the assumption that the economic life of the challenger remains unchanged when evaluated in the future and is approximated by its forecasted functional life. Differential inflation can cause changes in economic lives when evaluated on future dates. However, because of high investment costs and relatively low operational expenditures combined with long functional lives (e.g., 100 years), the potential impact of changing economic lives is not differential for determining the optimal first replacement time, which is demonstrated in the case study.

Life cycle activities consist of investments, major overhauls and operation and maintenance (O&M) expenditures. For the capitalised equivalent approach, the cumulative cash flows of the defender are combined with the perpetuities of the challenger's cash flows. In the following paragraphs, formulae are derived including differential inflation and ageing for both the defender and challenger. For readability, differential inflation (d), ageing (g) and interval (n) are not indexed for specific cost components in the generic derivations. In the specific application of the formulae, different cost components will have different values for differential inflation, ageing and intervals.

Cumulative present values of the defender's cash flows subject to differential inflation and ageing

The cash flows of the defender generally consist of an initial renovation, intermediate major overhauls and yearly O&M costs. The defender is replaced by a challenger at a future time T relative to year zero. Potential salvage values, scrap and demolition values of a defender are considered part of the investment costs of the challenger.

Present values of the defender's major overhauls subject to differential inflation

The cumulative present values of the initial renovation and major overhauls of a defender follow traditional discounting. The cash flows of the renovation and major overhauls are inflated with differential inflation and discounted using a real discount rate. In general, major overhauls are not subject to ageing; however, ageing can be included in the cash flows if required.

For example, the present value of a defender's major overhaul with a current price level M_0, starting at time $t \leq T$, with an interval n, which is repeated z times within period $[t, T]$ with $t + zn \leq T$ is calculated as Equation 4.4.

$$P_D^M[0,T]_t = M_0\left(\frac{1+d}{1+r}\right)^t + M_0\left(\frac{1+d}{1+r}\right)^{t+n} + M_0\left(\frac{1+d}{1+r}\right)^{t+2n} + \ldots + M_0\left(\frac{1+d}{1+r}\right)^{t+zn} \tag{4.4}$$

Including ageing with a yearly percentage growth of g would require substituting $(1+d)$ with $(1+d)(1+g)$.

Present values of defender's annuities subject to differential inflation and ageing
More interesting is the calculation of the cumulative present values of the regular yearly O&M expenditures due to different possibilities of ageing and differential inflation. Three situations are noted: operation expenditures are an annuity subject to (1) differential inflation only, (2) ageing only and (3) ageing and differential inflation. The engineering economics toolbox provides the standard geometric gradient series formula for discrete compounding (Park, 2011), which is adapted to include differential inflation and ageing. This standard formula calculates the present value of an annuity starting at year 1, growing with x% per year, and is given by Equation 4.5.

$$P[0,T]_1 = A_0(1+x)\cdot\frac{1-\left(\frac{1+x}{1+r}\right)^T}{r-x} \qquad \forall\, x \neq r \quad , \tag{4.5}$$

with: $P[0,T]$= present value at $t = 0$ of an annuity starting at year 1 and ending in year T; A_0 = year zero price level of the annuity; x = the percentage of yearly growth; $A_0(1+x)$ = the first year cost of the annuity; and r = the real discount rate. The percentage of yearly growth x in Equation 4.5 can be substituted with differential inflation d or ageing g. However, including both differential inflation d and ageing g simultaneously requires substituting $(1+x)$ with $(1+d)(1+g)$ and consequently: $x = (1+d)(1+g) - 1$.

Performing these substitutions results in the following relationship (Equation 4.6) to calculate the present value of a defender's annuity over time $[0,T]$, subject to both differential inflation and ageing:

$$P_D^A[0,T]_1 = A_0(1+d)(1+g)\cdot\frac{1-\left(\frac{(1+d)(1+g)}{1+r}\right)^T}{r-(1+d)(1+g)+1}, \tag{4.6}$$

With: $P_D^A[0,T]_1$= present value of a defender's annuity starting at year 1 and ending in year T; A_0 = year zero price level of the annuity; d = differential inflation; g = yearly growth percentage for ageing; $A_0(1+d)(1+g)$ = first year cost of the annuity; and r = the real discount rate.

With generic Equations 4.4 and 4.6, the present values of the cash flows of a defender in time interval $[0,T]$ are calculated. The following section develops mathematical equations to calculate the present values of the challenger's cash flows occurring in interval $[T,\infty]$.

Present values of the challenger's perpetual cash flows subject to differential inflation and ageing

This section develops equations to calculate the present values ($t=0$) of the cash flows of the challenger $P_C\ [0,\infty]_T$ for cash flows occurring in interval $[T,\infty]$. The challenger is installed at time T and perpetually replaced over its life cycle N.

Present values of the challenger's perpetual investment and major overhauls costs

Activities such as the initial investment and major overhauls are perpetuities with intervals of N and n, respectively, and are generally subject to differential inflation only. In exceptional cases, major overhauls may also be subject to ageing. However, in these cases, it is easier to consider the ageing major overhauls as different activities with their own repeating intervals than to derive a mathematical equation that includes differential inflation and ageing.

The cash flows of a perpetuity of the initial investment I_0 (price level year 0) starting at time T relative to start year zero, with interval N and subject to differential inflation d, are depicted in Figure 4.2.

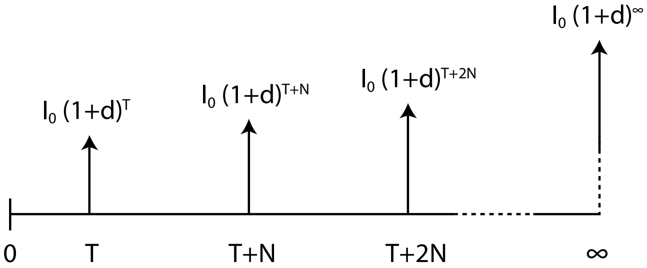

Figure 4.2 Cash flows of a perpetuity of investment I_0 with interval N, starting at time T and subject to differential inflation

The present value of this challenger's investment perpetuity is given by straightforward discounting as in Equation 4.7.

$$P_C^I[0,\infty]_T = \left(\frac{1}{1+r}\right)^T \cdot \left(I_0(1+d)^T + I_0(1+d)^T\left(\frac{1+d}{1+r}\right)^N + I_0(1+d)^T\left(\frac{1+d}{1+r}\right)^{2N} + \ldots \right) \tag{4.7}$$

where $P_C^I[0,\infty]_T$ = the present value at $t = 0$ of a perpetuity of the challenger's investment; I_0 is the cost of the investment at price level 0; T is the start time of the investment; N is the interval of the investment; d is the differential inflation specific for this investment; and r is the real interest rate.

The coefficients in Equation 4.7 are a geometric series. Therefore, Equation 4.7 can be rewritten as Equation 4.8.

$$P_C^I[0,\infty]_T = I_0 \cdot \frac{\left(\frac{1+d}{1+r}\right)^T}{1-\left(\frac{1+d}{1+r}\right)^N} \tag{4.8}$$

Let:

$$K = \left(\frac{1+d}{1+r} \right), \tag{4.9}$$

Then, Equation 4.8 simplifies to Equation 4.10.

$$P_C^I[0,\infty]_T = I_0 \cdot \frac{K^T}{1-K^N} \tag{4.10}$$

Summarising: Equation 4.10 calculates the present value at $t = 0$ of an investment starting at time T, repeating itself with an interval N and subject to differential inflation d.

For the present values of a perpetuity of a major overhaul, two situations are considered. The first and most common situation is: the interval n of the major overhaul is a common multiple in the functional life N of the challenger, for example $n = 20\,years$ and $N = 100\,years$. In that case, the present value of a major overhaul M_0 starting at year $t = T + n$, with an interval n is calculated similar to the perpetuity of the investment. Additionally, a perpetuity of this major overhaul with interval N starting at time $T + N$ (time of second investment) needs to be subtracted to prevent the simultaneous occurrence of a major overhaul with successive investments. Hence, a challenger's present value with major overhauls starting at $t = T + n$, with an interval n that is a common multiple in the challenger's functional life N, is computed by Equation 4.11.

$$P_C^M[0,\infty]_{T+n} = M_0 \cdot \left(\frac{K^{T+n}}{1-K^n} - \frac{K^{T+N}}{1-K^N} \right) \tag{4.11}$$

For the readers' convenience, K as defined in Equation 4.9 is not substituted with another symbol in Equation 4.11. However, in the application of the formula, the value of K depends on the specific differential inflation of a cost component.

The second situation, which is less common, encompasses major overhauls with an interval n that is not a common multiple in the challenger's functional life N. For example, $N = 100\,years$ and $n = 40\,years$. Interval n may not be stationary. The approach in this situation is to calculate the relative present value $P_C^M[T, T+N]_{T+n}$ at time T of the major overhauls occurring in the first challenger's life cycle (analogous to Equation 4.4) and treat this value as a perpetuity starting at time T with interval N, such as investment I_0, thus, analogous to Equation 4.10. Consequently, the challenger's present value with major overhauls with an interval n that **is not** a common multiple in the functional life N, is computed by Equation 4.12.

$$P_C^M[0,\infty]_{T+n} = P_C^M[T, T+N]_{T+n} \cdot \left(\frac{K^T}{1-K^N} \right) \tag{4.12}$$

Present values of perpetuities of the challenger's yearly operational expenditures
Operational expenditures of the challenger are modelled as yearly costs A_0 (price level year 0), starting at year $t = T + 1$, and grow each year with a factor. Again, three situations are considered: (1) differential inflation only, (2) ageing only or (3) differential inflation and ageing. In contrast to the approach followed for the defender, differential inflation and ageing cannot be treated similarly for the challenger's operational perpetuities. This finding is illustrated in the cash flow diagrams in Figure 4.3, Figure 4.4 and Figure 4.5, which depict the situations for differential inflation only, ageing only and differential inflation and ageing, respectively.

Considering differential inflation only
The calculation of the present value of a perpetuity of yearly expenditures subject to only differential inflation follows the same approach as Equation 4.10. Operational expenditures are a perpetuity with an interval of $n = 1$ and subject to differential inflation. The cash flows are depicted in Figure 4.3.

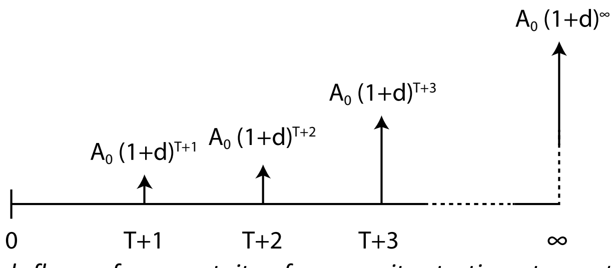

Figure 4.3 Cash flows of a perpetuity of an annuity starting at year $t = T + 1$ and subject to differential inflation d

The present value of a perpetuity of yearly operational expenditures starting at year $t = T + 1$ and subject to only differential inflation d follows from Equation 4.13.

$$P_C^{A,d\,only}[0,\infty]_{T+1} = A_0 \cdot \frac{K^{T+1}}{1-K} \tag{4.13}$$

Considering ageing only

The second case concerns an operational activity subject to only ageing (no differential inflation) due to increasing corrective maintenance. The calculation of the present value of the perpetuity follows a different approach. After replacement of the challenger, the corrective maintenance expenditures will start at their first-year level again. The cash flows are depicted in Figure 4.4. N is the life cycle of the challenger, installed at time T, and $t = T + 1$ are the first year of the annual costs.

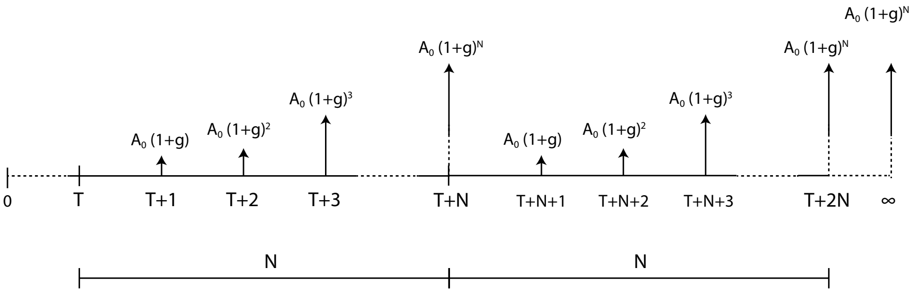

Figure 4.4 Cash flows of a perpetuity of an annuity starting at year t=T+1 and subject to ageing

The *relative* present value of yearly expenditures subject to ageing over one life cycle N of a challenger is independent of the challenger's start time T. The first-year costs for ageing will always be $A_0(1 + g)$. Therefore, the *relative* present value of a challenger's annuity subject to only ageing follows directly from Eq. (8), where x is substituted with the ageing growth factor g and is given by Equation 4.14.

$$P_C^{A,g\,only}[T, T+N]_{T+1} = A_0(1+g)\frac{1-\left(\frac{1+g}{1+r}\right)^N}{r-g} \tag{4.14}$$

$P_C^{A,g\,only}[T, T + N]_{T+1}$ is a *relative* present value at time T of annuities subject to ageing over life cycle N. This relative present value will repeat itself with an interval N (the challenger's life cycle). In accordance with the approach of Equation 4.8 with $d = 0$, the present value of its perpetuity for only ageing (without differential inflation) becomes Equation 4.15.

$$P_C^{A,g\,only}[0,\infty]_{T+1} = P_C^{A,g\,only}[T,T+N]_{T+1} \cdot \frac{\left(\frac{1}{1+r}\right)^T}{1-\left(\frac{1}{1+r}\right)^N} = A_0(1+g)\frac{\left(\frac{1}{1+r}\right)^T}{r-g}$$

(4.15)

Equation 4.15 calculates the present value at $t = 0$ of the challenger's perpetuity of yearly operational costs, starting at time $T + 1$, subject to only ageing.

Considering ageing and differential inflation

The third case concerns yearly operational expenditures subject to differential inflation and ageing. The cash flows are shown in Figure 4.5. The approach is to calculate the current present value of an annuity subject to differential inflation and ageing over one life cycle N, assuming an immediate instalment of the challenger. This present value is calculated with Equation 4.6. Second, the present value is considered an infinite repetitive activity with interval N, which can start at any time T and is subject to differential inflation, analogous to the calculation of the perpetuity of the investment in Equation 4.8.

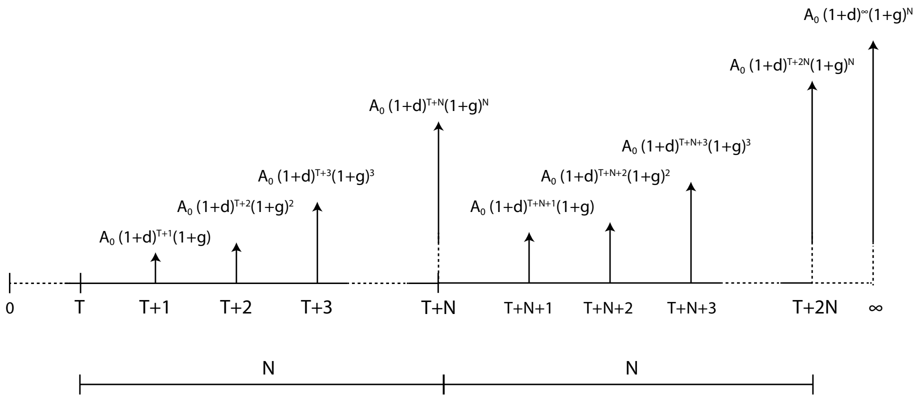

Figure 4.5 Cash flows of a perpetuity of an annuity starting at year t=T+1 and subject to differential inflation and ageing

The present value of the first life cycle of cash flows occurring over period N for an annuity subject to differential inflation and ageing is calculated as Equation 4.16 (analogous to Equation 4.6).

$$P_C^{A,d\,\&\,g}[0,N]_1 = A_0(1+d)(1+g)\cdot\left(\frac{1-\left(\frac{(1+d)(1+g)}{1+r}\right)^N}{r-(1+d)(1+g)+1}\right) \tag{4.16}$$

The present value (at $t = 0$) of its perpetuity with interval N, starting at an arbitrary time $T+1$ and subject to differential inflation, is calculated as Equation 4.17.

$$P_C^{A,d\,\&\,g}[0,\infty]_{T+1} = A_0(1+d)(1+g)\cdot\left(\frac{1-\left(\frac{(1+d)(1+g)}{1+r}\right)^N}{r-(1+d)(1+g)+1}\right)\cdot\left(\frac{\left(\frac{1+d}{1+r}\right)^T}{1-\left(\frac{1+d}{1+r}\right)^N}\right) \tag{4.17}$$

The last expression in Equation 4.17 represents the differential inflation when a challenger is installed at time T instead of time zero. Using the same notation, Equation 4.17 can be simplified to Equation 4.18.

$$P_C^{A,d\,\&\,g}[0,\infty]_{T+1} = P_C^{A,d\,\&\,g}[0,N]_1\cdot\frac{K^T}{1-K^N} \tag{4.18}$$

Equation 4.17 is generic for the three situations of differential inflation only, ageing only and differential and ageing. Setting d or g equal to zero and making proper substitutions results in Equation 4.13 and Equation 4.15, respectively.

The formulae for calculating the present values of the defender and challenger subject to ageing and differential inflation are combined in the capitalised equivalent approach. The proper substitutions are verified by discounting forecasted cash flows subject to differential inflation and age-related growth on a time horizon that approximates infinity. Forecasting cash flows subject to differential inflation and ageing for all combined keep-replace scenarios is much more time consuming and prone to mistakes.

4.5 Demonstration of method

For demonstration of the method, an existing case study in the Netherlands is used, a steel bridge owned by a governmental organisation. Based on the current condition, the expected maximum useful life is 35 years after a thorough and expensive renovation, including reinforcement. If replaced, the bridge will be replaced by a concrete bridge with an expected useful life of 100 years. The asset owner is interested in the optimal replacement time of the existing bridge for several reasons.

First, the immediate decision for a renovation or replacement is justified. Second, the current bridge is part of a larger asset portfolio. The analysis directly contributes to the long-term capital investment planning and the required budget forecasts. Moreover, the analysis is directly applicable to other bridges in the asset portfolio. Only changes of input values are required.

Data

The cost estimates are provided by the asset owner (Table 4.3) and proportionally adjusted for confidentiality. This proportional adjustment does not affect the interpretation of results and the presented cost values could be valid for another situation.

The estimates for differential inflation rates are obtained from analysing specific producer price indices (PPI) and the consumer price index (CPI) from 1995 to 2017. The publicly available CPI data is found in the data disclosure statement at the end of this paper. The raw PPI data used for this case study is obtained from a specialised knowledge centre (CROW, 2018).

The average yearly general inflation rate over this period, 1.87%, is derived from the CPI. The slight difference in general inflation rate, in comparison with the illustrative analysis in the section on differential inflation, is caused by a more extensive data set from the same source, that allows for a longer analysis period. Due to the long-life cycles of infrastructure assets, preference goes to the longest period available.

Table 4.3 Data for defender - challenger analysis under differential inflation and ageing

	Start year (year)	Interval n (year)	Ageing g (%/year)	Differential inflation d (%/year)	Costs in price level year 0[a] (€)
Defender					
Renovation	0	-	-	-	€ 3,000,000
O&M	1	1	0.50%	0.85%	€ 26,250
Overhauls steel	10	20	-	0.96%	€ 1,500,000
Overhauls concrete	20	20	-	0.80%	€ 425,000
Overhauls asphalt	10	10	-	1.44%	€ 62,500
Challenger					
Investment	0	100	-	0.12%	€ 8,300,000
O&M	1	1	0.20%	0.85%	€ 7,500
Overhauls steel	20	20	-	0.96%	€ 178,750
Overhauls concrete	20	20	-	0.80%	€ 175,000
overhauls asphalt	10	10	-	1.44%	€ 62,500

[a]*Costs begin at first year of a cycle and repeat with interval n.*

The total inflation for different cost components in the case study is in consultation with the asset owner, derived from the respective PPI's and proportionally combined in baskets where appropriate (e.g., a combination of labour and materials). The values deviate from the values in the section on differential inflation because other cost groups are analysed, and less aggregated data is used. Equation 4.1 is used to calculate the differential inflation, as depicted in Table 4.3. The estimates for cost development as a consequence of ageing are made in consultation with cost and maintenance engineers.

Calculation

First, the economic life of the challenger as if installed today is calculated as explained in the literature on engineering economics (Hastings, 2015; Newnan et al., 2016; Park, 2011; Sullivan et al., 2012), which results in $N^* = N = 100$ years with an $EAC_C^* \; (t = 0)$ of € 448,910. Currently, the economic life is bounded by the challenger's functional life N.

Under the assumption the challenger's future life cycle costs are a perpetuity with interval N, the defender-challenger analysis is reduced to a spreadsheet calculation using the derived formulae. The combined present values for the 35 keep-replace scenarios are depicted in Figure 4.6 and in Supplemental material,

Appendix B, Table B1 in Van den Boomen, Leontaris, et al. (2019). The results of underlying calculations (including equations used) for the defender and challenger are presented in Supplemental material, Appendix B, Tables B2 and B3.

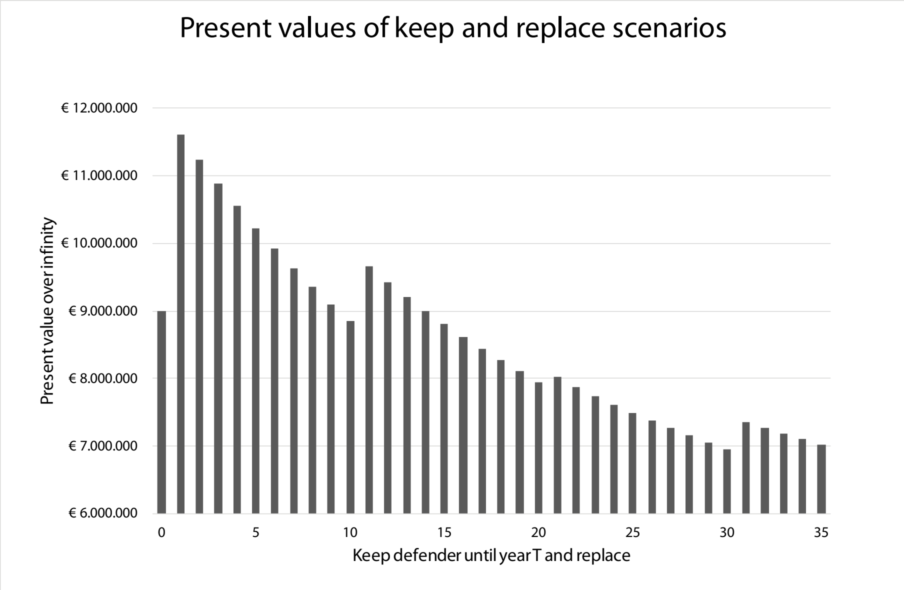

Figure 4.6 Present values for keep-replace scenarios subject to differential inflation and ageing

Analysis of results

The spikes in Figure 4.6 are caused by the defender's major overhauls. The lowest present value occurs for scenario 30; keeping the defender for 30 years and replacing the defender at the end of the 30th year, immediately before the defender's planned major overhaul. However, more conclusions can be derived. For example: the defender's renovation is only sensible if the defender can be kept in service for at least another 10 years, until then, there is no financial gain.

If an asset owner is not certain about a functional life exceeding 10 years, and the preferable 15 years in this case study, the best option may be to replace the defender immediately. Similarly, replacing the defender in year 30 is the cheapest option because its useful life is bounded by 35 years. However, if there is reason to believe the remaining life of the defender exceeds 35 years, the calculations should be extended to incorporate more maintain-replace scenarios. A graph such as Figure 4.6 supports a decision maker constructing motivated short- and long-term capital investment planning.

As discussed in the literature review, a classical capitalised equivalent analysis ignores differential inflation. This analysis is quickly simulated by letting the values for differential inflation be approximately zero in the inflation adjusted model. For the case study, this results in an optimal replacement in year 35 instead of year 30. Differential inflation makes the concrete replacement option more attractive and the steel defender less attractive, which is motivated by the relatively high differential inflation rate for steel and the defender's expensive major overhauls for steel.

Practical implications

From a practical point of view, a replacement decision in 30 years or 35 years may not seem of immediate interest. This however is also an answer to the question of this authority. Although a concrete bridge is cheaper in maintenance, replacing it for that reason would economically not be a wise decision in this case study. From a life cycle costs point of view, it is better to invest in a renovation and to incur the higher maintenance costs of the current steel bridge for at least another 10 and preferably 30 years.

Another practical use of this replacement optimisation method is that this bridge is not a stand-alone case. This authority owns over thousand bridges. Most were built around 1960 – 1970 and since then subject to increasing traffic intensity. Inspection and structural safety assessments determine the useful remaining life. The economic optimisation method efficiently determines the economic remaining life of an existing asset which is less than or bounded by its useful life. Moreover, this method accounts for differential inflation, which grows in importance because of the low discount rates used by public sector organisations. This method directly supports the long-term capital investment planning of infrastructure in a fast and efficient manner. In this context the current research shows that ignoring differential inflation will lead to a less accurate estimate of the long-term capital investment planning.

In general, low discount rates and long lives stress the importance of a careful assessment of the presence and impact of differential inflation. Historic PPI data is publicly available at for example a bureau of labor statistics. The developed formulae allow a rapid inclusion of differential inflation in a common defender-challenger replacement challenge. This research provides a pragmatic alternative to classical approaches that cannot handle inflation and case-specific DP-modelling. The inflation adjusted capitalised equivalent approach will by definition provide more accurate results than the classical capitalised equivalent approach. The assumption of a challenger's constant life N underlies both approaches. The developed inflation adjusted capitalised equivalent approach also easily accounts for ageing, which can be modelled with an underlying stochastic process. Mathematically, the application of the method is comparable to classical approaches.

4.6 Comparison with DP-solution

The DP-solution to this defender-challenger problem relaxes the assumption of a constant challenger's N since it optimises the entire challenger's replacement chain, including future economic lives. The approach of Wagner (1975) is used for comparison with the approach developed in the current research. Wagner's approach is explained in Supplemental material, Appendix A in Van den Boomen, Leontaris, et al. (2019).

A disadvantage of a DP-solution is that an infinite calculation horizon is approximated by a bounded calculation horizon, sufficiently long to capture the future cash flows that contribute to the total present value (Regnier et al., 2004; Wagner, 1975). The difficulty of applying the DP-solution to an approximated infinite calculation horizon is the size of the solution space, which is reflected in a cost matrix that contains all possible keep – maintain scenarios for the challenger's replacement chain.

Approximating infinity with a calculation horizon of 300 years requires a cost matrix with approximately 45,000 calculations of cumulative present values. Second, the DP-approach requires solving this solution matrix with a DP-algorithm to determine the least-cost route. This recursive calculation is not easily applied in practice with a spreadsheet. For an efficient implementation, a recursive DP solution requires programming.

Comparison of the DP calculation described by Wagner (1975) is performed over a calculation horizon of 300 years to determine the challenger's optimal replacement chain for the case study. The results of the comparison with the differential inflation adjusted capitalised equivalent approach are presented in Table 4.4. The results of the comparison for the case study are the following:

- Both methods deliver the same optimal defender's replacement time.
- The optimal economic lives of the challenger within the calculation period of 300 years do not differ from the fixed assumption used in the adjusted capitalised equivalent approach. The third 70-year cycle in the DP-optimisation is simply a consequence of the bounded time horizon.
- The total present values of the replacement chains for both methods are nearly equal with a difference of €9. In fact, the DP-approach underestimates the total present value due to the bounded time horizon (approximation error caused by truncation of cash flows).

Table 4.4 Comparison of the differential inflation adjusted capitalised equivalent approach with a DP-solution for the challenger's replacement chain starting at T = 30 years

	Inflation adjusted capitalised equivalent approach	**DP-approach**
Primary feature	Challenger's replacement chain with fixed life N	Challenger's replacement chain optimised for economic lives N*(t)
Calculation horizon	Infinite	Approximation of infinity by 300 years
Result: economic lives of successive challengers	Cyclic: 100 years	Optimised economic lives by applying a shortest path algorithm, for case study calculated at 100 – 100 – 70 years.
Result: present value	$P_C[0,\infty]_{T=30}$ = € 2,203,435	$P_C[0{,}300]_{T=30}$ = € 2,203,424

The case study shows that relaxing the assumption of a constant N and the application of a DP-solution does not lead to differences compared to the differential inflation adjusted capitalised equivalent approach. This is explained by the long functional life of the challenger and the high ratio between investment and O&M costs. Since this is a common characteristic in most infrastructure assets, the inflation adjusted capitalised equivalent approach will provide accurate results without requiring a DP model. However, DP-solutions are unavoidable when challengers have shorter economic lives or when multiple challengers are involved (for example, combinations of maintain, renovate or replace).

4.7 Conclusions

Public infrastructure assets are ageing and need to be replaced at the optimal time. The current study found that investments and operational expenditures of infrastructure assets are subject to differential inflation (price increases and decreases). Public sector organisations use low discount rates which magnifies the impact of differential inflation on replacement decisions.

The presence of differential inflation undermines the application of mainstream classical replacement optimisation techniques because of their underlying assumption of a repeatability of future life cycle cash flows of a replacement option. Under these circumstances, advanced linear or dynamic programming (LP/DP) techniques are required but these approaches ask for case-specific modelling, are time consuming and complex in their application in practice.

The literature does not offer a quick solution for a generic case in infrastructure replacement optimisation: an existing asset to be replaced at the optimal time by a new asset where both assets are subject to their own ageing and differential inflation rates. The current study develops a set of mathematical equations equally accurate as a full DP calculation. The alternative method builds on the classical but lesser known capitalised equivalent approach which allows for fluctuating operational expenditures caused by major overhauls.

The benefit of this approach is twofold. First, it provides, by definition, accurate results compared to classical approaches which cannot handle differential inflation due to their underlying assumption. Second, the alternative method reduces a complex DP approach to a relatively easy spreadsheet solution.

The limitation of the alternative approach is the assumption that the useful life of the replacement option equals its economic life, whether it is installed now or anytime in the future. However, this assumption in general holds for infrastructure assets which are characterised by high investment costs, long technical lives and relatively low operation and maintenance expenditures.

As a practical recommendation, the current study proposes to assess the presence and impact of differential inflation based on an analysis of construction sector producer price indices and consumer price indices. In the presence of differential inflation, the current study recommends professionals to use the developed method to determine the optimal replacement time of existing infrastructure assets challenged by a replacement option, instead of using classical methods or DP methods.

As further research is proposed to investigate the wider application of the proposed method to other asset types with shorter technical lives and different life cycle cash flow patterns than infrastructure assets, in comparison to advanced DP solutions.

Notation

A_0	first year costs of an annuity at price level year 0
d	differential inflation
EAC	equivalent annual cost
EAC^*	minimum equivalent annual cost
EAC_C^*	minimum equivalent annual cost of a challenger
EAC_D^*	minimum equivalent annual cost of a defender
F	future value or real cash flow (opposed to nominal cash flow)
f	general inflation
f_{tot}	total inflation
g	growth factor for ageing
I_0	the cost of an investment at price level year 0
K	a factor defined by $(1 + d)$ / $(1 + r)$
M_0	real costs (opposed to nominal costs) of a major overhaul at price level year 0
N	the full life cycle
N^*	the economic life
n	time interval for major overhauls
P	present value (in general)
P^*	minimum present value (in general)
P_C	present value of a challenger (in general)
$P_C^{A,d\ only}[0,\infty]_{T+1}$	present value at time 0 of a perpetuity of a challenger's annuity, starting at time $T + 1$ and subject to differential inflation (d) only
$P_C^{A,g\ only}[T, T + N]_{T+1}$	relative present value at time T of a challenger's annuity over life cycle N, starting at time $T + 1$ and subject to ageing (g) only;
$P_C^{A,g\ only}[0,\infty]_{T+1}$	present value at time 0 of a perpetuity of a challenger's annuity, starting at time $T + 1$ and subject to ageing (g) only
$P_C^{A,d\ \&\ g}\ [0,\infty]_{T+1}$	present value at time 0 of a perpetuity of a challenger's annuity, starting at time $T + 1$ and subject to both differential inflation (d) and ageing (g);
$P_C^{A,d\ \&\ g}\ [0, N]_{t=1}$	present value at time 0 of a challenger's annuity starting in year 1 over time period $[0, N]$ and subject to both differential inflation (d) and ageing (g);
$P_C^I[0,\infty]_T$	present value at t = 0 of a perpetuity of a challenger's investment costs starting at time T
$P_C^M[0,\infty]_{T+n}$	the present value at time 0 of a perpetuity of a challenger's major overhauls starting at time $T + n$

$P_C^M[T, T+N]_{T+n}$	relative present value at time T of the challenger's major overhauls in time period $[T, T+N]$ starting at time $T+n$
P_D	present value of a defender (in general)
$P_D^A[0, T]_1$	present value of a defender's annuity starting in the first year, over time period $[0, T]$
$P_D^M[0, T]_t$	present value of a defender's major overhauls, staring in year t, over in time period $[0, T]$
t	time
T	time of replacement of a defender by a challenger relative to year 0
$[0, T]$	time period from time zero to time T
$[0, \infty]$	time period from time zero to infinity
$[t, T]$	time period from time t to time T
r	real interest rate
r_{net}	real net discount rate after compensating for differential inflation
r_{nom}	nominal interest rate
x	a generic growth factor
z	a generic integer to represent a finite number of major overhauls in a defender's life

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5

Simple decision tree and real options approach

Replace an old asset with a like-for-like replacement
Non-repetitive cash flows
Uncertain future

As in Chapter 4 the replacement optimisation model in the current chapter deals with a situation where an existing infrastructure asset is challenged by a new infrastructure asset. However, extra features are added.

Instead of inflation (expected price development), price uncertainty is taken into account (a cone of uncertainty around the expected price development). In addition, probabilities are introduced for a political decision and structural failure. The current model also allows for more managerial flexibility. A decision maker will base future decisions on the development of uncertainties and has three options to choose from: maintain, replace with an option 1 or replace with an option 2.

The case study is obtained from the Municipality of Amsterdam and concerns the replacement of a bridge. Inclusion of the uncertainties requires a modelling approach where decision tree analysis and real options analysis are used. This publication basically is a blueprint for how different types of uncertainty and multiple single options in parallel can be included in replacement optimisation challenges.

Untangling Decision Tree and Real Options Analyses: a public infrastructure case study dealing with political decisions, structural integrity and price uncertainty

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Published online in Construction Management and Economics at 4 October 2018
https://doi.org/10.1080/01446193.2018.1486510

Managerial flexibility in infrastructure investment and replacement decisions adds value. Real options analysis (ROA) captures this value under uncertain market prices. The concept of ROA is that future unfavourable payoffs can be deferred as soon as more information about market prices becomes available. The popularity of ROA is seen in a growing number of case studies on real assets. Despite its increasing popularity, ROA has not gained a foothold in public infrastructure decision making. One of the difficulties in the application of ROA is the required estimation of market variables. To avoid this, a simplified but not correct version of ROA is easily applied, referred to as a Decision Tree Approach (DTA) to ROA. Another difficulty is that infrastructure assets are subject to other types of uncertainties, defined here as asset uncertainties. This study investigates the value of managerial flexibility in a public infrastructure replacement decision. The uncertainty drivers are the strength of a bridge, political decisions regarding traffic flow and the price development of construction costs. Three valuation approaches are compared: DTA, ROA and the DTA approach to ROA. Although it is complex, ROA certainly adds value in public infrastructure decision making when market price uncertainty is prevalent. However, in the absence of reasonable estimates of market variables, the DTA approach to ROA is the best alternative. In the absence of market price uncertainties, ROA should be avoided and DTA used instead.

5.1 Introduction

The real options analysis (ROA) of the last decade is advocated as a promising technique in valuing the flexibility of managerial decisions in infrastructure life cycle decisions. The theory of real options originates from financial options valuation. It values financial options that give a holder the right to defer unfavourable payoffs. Groundwork for financial option pricing was laid by Black and Scholes (1973) and Merton (1973). Their efforts brought about the famous Black-Scholes-Merton formula, which provides a closed-form solution to value European call options.

A call option provides the right but not the obligation to buy a share of common stock at a predetermined price before or at a predetermined date. An owner will only exercise the option if the option payoff is positive. The market wants to be compensated for bearing this risk, which is reflected in a risk-adjusted discount rate, or equivalently, a risk-adjusted stream of cash flows discounted at a risk-free discount rate. In response to the closed form formula for a special case of option pricing, Cox, Ross, and Rubinstein (1979) developed a discrete binominal approach, that uses basic mathematics and allows for more flexibility in exercising calls and valuing options. The approach of Cox et al. (1979) is widely adopted as a standard for option pricing.

It is easy to draw a parallel between financial options and real options analysis (ROA). Unfavourable future payoffs on tangible real assets or projects can be deferred by managerial decisions. Real options are for example expansion, replacement, switching or abandonment of real assets. The theory behind real options is well-documented by authors like Amram and Kulatilaka (1999); Brealey, Myers, and Allen (2017); Copeland and Antikarov (2003); Guthrie (2009); Peters (2016) and Mun (2006) and applied in a growing number of case studies in the literature. For example, a practical spreadsheet approach to value real options of investment strategies for a garage parking case study is presented by de Neufville, Scholtes, and Wang (2006). In this study, the uncertainty driving investment decisions is the future demand.

Cheah and Liu (2006) applied ROA to value the concessions of a government in a private sector Design, Build, Finance, Maintain and Operate (DBFMO) project. The authors demonstrated their approach using a case study on a causeway. The main uncertainty driver in this case study was traffic volume and growth. A toll road example is provided by Ford, Lander, and Voyer (2002) to encourage wider use of ROA in construction projects. Chow and Regan (2011) integrated ROA in a network design optimisation challenge with uncertain demand. Richardson, Kefford, and Hodkiewicz (2013) used ROA to determine optimal replacement cycles of heavy mobile equipment under volatile operational expenditures and long lead times of orders. Kim, Ha, and Kim (2017) applied ROA in order to value the potential damage reduction under optimal adaptation strategies and volatile future climate scenarios. Electricity demand and public acceptance is driving the uncertainty in a recent case study for nuclear power plant, investigated by Cardin, Zhang, and Nuttall (2017).

The commonality in most case studies on ROA is a sole source of an external uncertainty such as weather conditions, operational expenses or demand, which may influence future costs and benefits.

A second observation in the case studies is a lack of consistency in the use of discount rates. Some authors use the weighted average cost of capital of an organisation (WACC) with or without a risk premium. Other authors use a discount rate without explanation. Some authors use the concept of risk-neutral probabilities in combination with a risk-free discount rate on bonds, obtained from the ROA-theory. These discrepancies demonstrate the major difficulties in applying ROA theory to real assets.

A third observation is the confusion regarding approaches to value flexibility: ROA and a DTA approach. In this context, Neely and De Neufville (2001) introduce the term 'hybrid real options valuation'. The authors clearly separate non-diversifiable risks from diversifiable risks. There are no mitigating measures to avoid non-diversifiable risk. In contrast, diversifiable risk can be mitigated. ROA-theory applies to non-diversifiable market price uncertainties, which need a different type of valuation approach than diversifiable asset uncertainties. Some ROA authors like Copeland and Antikarov (2003) even state that DTA is wrong. However, DTA is not wrong; the valuation method of options in a DTA approach should be aligned to the type of uncertainty involved. Schwartz and Trigeorgis (2001) sharply describe ROA as "a special and economically-corrected version of DTA which recognises market opportunities to trade and borrow". To avoid further confusion, De Neufville and Scholtes (2011) and Cardin et al. (2013) introduce the phrase "flexibility in engineering design" to designate that ROA is not the only approach to value real options. De Neufville and Scholtes (2011) state that for applying ROA, two conditions must be met: a replicating market portfolio with shares and bonds should exist and its compositions should be tradable. In other words, the market should contain a tradable portfolio of shares and bonds that exactly mimics the cash flows of the real asset or engineering project being considered. De Neufville and Scholtes (2011) conclude that these conditions seldom apply to real-life engineering assets or projects.

A fourth observation is that ROA case studies on *public* infrastructure assets, especially maintenance and replacement decisions, are hard to find. Woodward, Kapelan, and Gouldby (2014) incorporate real options to establish adaptive flood management strategies under an uncertain sea level rise. The study emphasises that flexibility has value, which should be incorporated into the design and life cycle strategies of infrastructure assets. Herder, de Joode, Ligtvoet, Schenk, and Taneja (2011) identify several barriers as to why ROA does not seem to gain a foothold in public infrastructure investment decisions. One reason mentioned by the authors is the increased difficulty for public sector organisations to find underlying comparable market information necessary for the correct valuation of real options. The most prominent barrier identified by the authors, is the political, institutional and organisational context in which public investment decisions are made. Decisions for large public infrastructure investments are seldom driven by economic reasons only. Infrastructure investments are often driven by societal and political interests. Investment decision may be influenced by anticipation to time consuming legal environmental impact assessments. Interesting is also the authors' observation that ROA may have problems of reputation, as a consequence of the financial crises in 2008/2009. Finally, the authors suggest that public organisations may face problems of lock-in. Endemic routines in combination with the absence of a ROA-toolkit could be a barrier for the application of ROA.

To summarise the literature: there is confusion and academic debate on the correct valuation of managerial flexibility in engineering practice. In addition, there is a lag in the application of valuing flexibility in public infrastructure decision making. The purpose of this study is to investigate the valuation of flexibility in public infrastructure replacement decisions and to disentangle the debate on how to correctly value flexibility. In Section 5.2 a model is developed for the valuation of options in a case study for a common infrastructure replacement problem. A clear distinction is made between asset uncertainties and market price uncertainties. This case study reveals some of the prominent difficulties in valuing flexibility and especially in the application of ROA in public sector investment and replacement decisions. Three approaches are compared: valuing flexibility of options in the absence of price uncertainty (DTA, Section 5.3), valuing flexibility of options subject to price uncertainty (ROA, Section 5.4), and the application of the popular but not fully correct DTA approach to ROA (Section 5.5). Methods are compared (Section 5.6) and discussed (Section 5.7). Conclusions for their application are presented in Section 5.8.

5.2 Model development for a bridge replacement

The case study is an existing old bridge in the city centre of the capital city of the Netherlands. The bridge was built before 1900. Around 1925 the bridge was expanded and reinforced. Around 1980 major renovation work took place. The case study is of interest because it is exemplary for many similar ageing bridges in a urban environment. Second, the case study contains multiple uncertainties of a different nature which need different treatment in a DTA and ROA analysis. Third, the identified dominant uncertainties are difficult to quantify but influence the preservation of capital and the capital investment planning. The purpose of this case study is to develop an infrastructure replacement optimisation model that is capable of inclusion of different types of uncertainty and contains the flexibility to respond to these uncertainties. The approach or method development is generic; however, the case study and its underlying assumptions are specific.

Recently, mandatory structural safety investigations and calculations were carried out for this bridge according to national standards for structural safety assessments of existing structures (NEN 8700:2011 nl). This national standard builds on and is an extension to the well-known European reference design codes for new structures (NEN-EN 1990:2002 en). The assessment comprises structural integrity calculations based on strength-load combinations for an extended design life of 15 years and assesses for compliance to a safety limit state at disapproval level. In-depth field investigations were carried out to assess the load and bearing capacity of the structure and soil. Motorised traffic prognoses for the corridor of this bridge are stable (10,752 motorised vehicles per day in 2020) and show a slight decline towards 2030 with 10,393 motorised vehicles per day (Gemeente Amsterdam, 2018). The main concern is the strength of the piles. In Amsterdam (and other cities), bridges built around 1900 are founded on timber piles which are subject to bacteriological deterioration, a process well described by René K. W. M. Klaassen (2008) and René K. W. M. Klaassen and van Overeem (2012). This bacteriological decay of submerged timber piles is not only a Dutch problem (Nilsson & Björdal, 2008; Zelada-Tumialan, Konicki, Westover, & Vatovec, 2014). As part of the structural safety assessment, under-water samples were taken from accessible wooden piles and analysed, resulting in predictions for bacteriological decay and residual strength. Also, site exploration was carried out to evaluate the load-bearing capacity of the soil.

The results of the field investigations were used to carry out the structural safety assessment calculations for a reference period of 15 years. The theoretical calculations demonstrate current compliance to the requirements of the standard (limit state for disapproval level). However, the main uncertainty is bacteriological decay of the piles and the development of the load-bearing capacity of the piles. Therefore, deformation of the bridge will be measured yearly to guarantee safe usage. In the case that a threshold for deformation is exceeded, the bridge will be closed for traffic and immediate replacement will be initiated.

We define a generic probability function $b(t)$ to model the time-variant probability of a premature unplanned replacement (corrective replacement) when a certain threshold is exceeded. For the case study, $b(t)$ is the probability that the measured deformation exceeds the permitted threshold. As a practical assumption for $b(t)$, first a current failure probability is estimated based on actual failures. Three ageing bridges out of an initial population of 160 similar bridges under investigation, were taken out of service in the past two years because thresholds were exceeded, which is an approximated 2%. As a conservative future estimate a yearly additional percentage of 0.5% is added for the remaining reference period of 15 years. This approach can be seen as a managerial strategy of an asset manager to incorporate a certain level of additional risk costs as a consequence of a probable premature replacement. Naturally, such estimates and decisions are taken by a team of (field) experts.

Bridge failure probability modelling under specific circumstances is clearly more complex and a research field on its own. Sánchez-Silva and Klutke (2016) provide a comprehensive overview of fundamental and state of the art probabilistic degradation models for infrastructure ranging from regression analysis to modelling of degradation caused by shocks. In addition, a time-variant capacity-load approach could be considered (Leira, 2013). However, these mathematical models need data to validate their statistical properties in order to establish a time-variant probability for the remaining lifetime of a structure. Data to perform such modelling is currently absent for the case study and will only become available in the future.

That certainly does not impede the method development in the current research which aims to develop strategies and budget forecasts under uncertain conditions. Dealing with uncertainty is discussed in Section 5.7.

One uncertainty is exceedance of the deformation threshold. A second uncertainty is about urban planning. At present, cars are allowed in the city centre but banning cars from the city centre is currently a hot political issue in the city council. Its success depends on the composition of the city council and elections take place every four years. Other cities may face other types of political decisions. Political decisions are uncertain and difficult to predict.

The probability of this decision in one of these years is designated with $p(t)$. For the case study its value is based on the expected probability that one of the parties will grow. One could argue that these types of events are highly uncertain. However, these events are part of real-life uncertainties that a decision maker faces and addressed in discussions about infrastructure replacement planning. A decision to ban cars from the city centre would offer the opportunity to build a smaller bridge, which would significantly reduce the future life cycle costs (investment and maintenance costs). Operational expenses of the current and new bridges are estimated at 10% of their new investment costs based on past data. Periodic major overhauls (intermediate large maintenance works such as asphalt replacement and conservation work) are included in this figure. The last uncertainty is the development of construction costs. Exogenous market forces may influence future construction costs.

The extreme scenarios are to replace the bridge immediately with a large bridge or wait for another 15 years and build a large or small bridge depending on political decisions to ban cars from the city centre. There are two clear incentives for waiting: the benefits of postponing large investments and waiting for more information that allows for the building of a smaller bridge. There is one incentive for not-waiting: the potential risk costs, more specifically unexpected corrective replacements. The question of interest is: what is the best strategy for this municipality and what is the value of waiting for information?

Table 5.1 Data with descriptions, symbols and values (monetary amounts in € million).

Description	**Symbol**	**Value**
Investment in preventive replacement large bridge	I_L^P	5
Investment in corrective replacement large bridge	I_L^C	$1.5 \cdot I_L^P = 7.5$
Investment in preventive replacement small bridge	I_S^P	$0.6 \cdot I_L^P = 3$
Investment in corrective replacement small bridge	I_S^C	$1.5 \cdot 0.6 \cdot I_L^P = 4.5$
Yearly operational expenses of large bridges	E_L	$0.1 \cdot I_L^P = 0.5$
Yearly operational expenses of small bridges	E_S	$0.1 \cdot I_S^P = 0.3$
Present value of a perpetuity of investments and operational expenses of a preventive and corrective replacement large bridge	L^P and L^C	See Table 5.2
Present value of a perpetuity of investments and operational expenses of a preventive and corrective replacement small bridge	S^P and S^C	See Table 5.2
Probability each year for a corrective replacement	$b(t)$	$2\% + 0.5\% \cdot \text{t} \; \forall \; \text{t}$
Probability for decision to ban cars from the city centre	$p(t)$	$30\% \; if \; t = 4, 8, 12,$ $otherwise \; 0$
Maximum allowable lifetime extension of old bridge	T	$15 \; years$
Time / period to evaluate in years	t	$0 \leq t \leq T$
Technical life cycle of a new bridge	n	$100 \; years$
Risk-adjusted discount rate of the municipality	r_a	3.5%

A model is developed in two stages for this case study. The first stage is described as the DTA approach, which omits market price uncertainty. Hereafter the model is extended with market price uncertainty for which the ROA-approach is incorporated.

Finally, the incorrect application of the ROA-theory (the DTA version of ROA) is applied to evaluate the deviations.

The input data for the case study are presented in Table 5.1. Planned and unplanned replacements are designated with preventive and corrective replacements respectively.

5.3 Valuing flexibility in the absence of price uncertainty

Looking at this question from a decision tree perspective first requires identifying the two events that influence decisions. The first event is that cars are allowed in the city centre. This situation is designated as state large (L). The second event is that cars are not allowed in the city centre. This event is designated as state small (S). The current situation is state L. A decision maker cannot influence the events, but can wait and base future decisions on the outcome of political decisions made in the years 4, 8 and 12. In these years a transition from state L to S is possible with a probability of $p(t)$. A transition from state L to S does not automatically imply that a decision maker will build a smaller bridge immediately. A decision maker will maximise the value of potential decisions and chooses the best option from the range of options available. A political decision to ban cars from the city centre, is considered to be irreversible for the coming decades.

The decision nodes (not yet the decisions) and possible transitions from state L to state S are shown in Figure 5.1. For example: in year 4, there is a probability of $p(t)$ for entering state S and a probability of $1 - p(t)$ for remaining in state L. Once in state S there is no possibility of transferring to state L. Figure 5.1 is a recombined version of an extended tree. Recombining enhances the efficiency of following recursive calculations and makes the interpretation of results easier. The process of recombining decision trees is well described by Guthrie (2009).

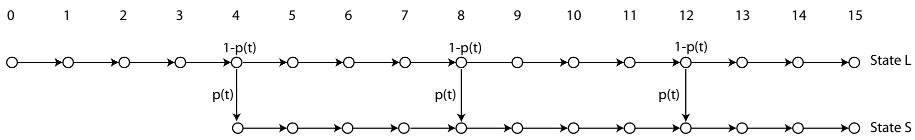

Figure 5.1 Decision nodes for the case study with recombined branches

Having identified state S and L and the possible decision nodes, the decision tree is built. Figure 5.2 shows the options in a decision node of state L and S. Figure 5.3 shows the complete decision tree. The options in any decision node in state S and L are defined as:

$$A = \{wait, replace\} \quad (5.1)$$

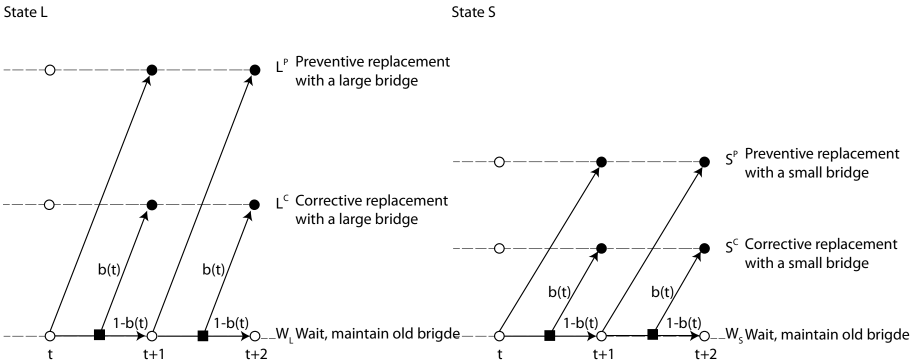

Figure 5.2 Decisions available in a large and small state

In decision nodes of a large state L, a decision maker is faced with two options each year, except for the last year. These are illustrated in Figure 5.2. A decision node is represented by a non-filled dot. The first option is to replace the old bridge with a new one. This is a preventive replacement and requires a large rebuild including all future life cycle costs L^P. The second option is to wait. Waiting comprises the probability of an unplanned (corrective) large investment $b(t)L^C$ and the benefits of postponing the investment with a probability of $1 - b(t)$, represented by W.

The chance node is designated with a square in Figure 5.2. In the last year, the only option left is to replace the old bridge with a large one: L^P. After a replacement (L^P or L^C) in any of the nodes the decisions are terminated by a perpetuity of future life cycle costs (investments and exploitation expenditures). Termination nodes are represented by a black filled dot in Figure 5.2 and Figure 5.3. In every node of the decision tree, there is a possibility that the decision tree ends, as a consequence of a planned replacement or unplanned replacement.

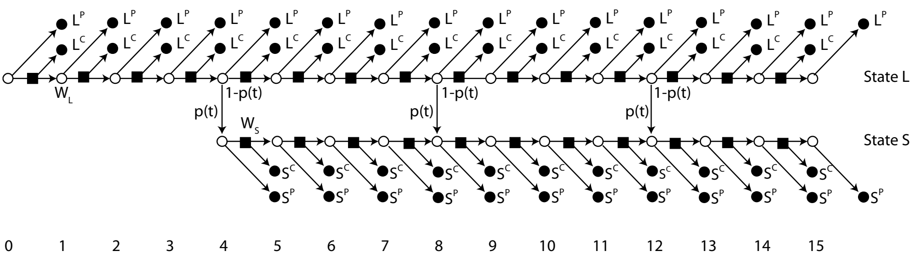

Figure 5.3 Full decision tree for the case study

In decision nodes of a small state S, cars are not allowed in the city centre. This creates an opportunity for a less expensive replacement with a small bridge. Again, a decision maker has two options as depicted in Figure 5.2: preventively replace the old bridge with a small one, including all future life cycle costs S^P, or wait. Waiting encompasses a probability $b(t)$ for a corrective replacement with a small bridge S^C, and the benefits of postponing the investment W with a probability of $1 - b(t)$. If the old bridge remains in place, the only option left at the expiration date of postponement is a preventive replacement with a small bridge S^P.

The decision tree for the case study combines Figure 5.1 and Figure 5.2 and is presented in Figure 5.3. The discounted value of perpetual life cycle costs of replacements L^P, L^C, S^P and S^C are determined after definition of the model and presented in Table 5.2.

The decision tree is solved by using backward recursion for discounting of costs. Knowing the boundary constraints at the year in which the possibility of postponement of a replacement expires (year T), allows for working back year by year until the present value at $t = 0$ is found. A clear and simplified example of this principle of backward recursion in a decision tree that contains options and probabilities is provided by Brealey et al. (2017). The boundary constraints at T for the two states are:

$$V_L(T) = L^P , \tag{5.2}$$

$$V_S(T) = S^P , \tag{5.3}$$

where $V_L(T)$ and $V_S(T)$ are the discounted values in year T of the cash flows from year 15 to infinity for a large and small state respectively. The values are calculated in Table 5.2. If the existing bridge still functions at the end of year 15 the only decision left is to replace the bridge with costs L^P or S^P, depending on the state of the bridge.

The present value of a waiting option in a small state $Q_S(t, wait)$ in a decision node at time t is given by the recursive relationship:

$$Q_S(t, wait) = b(t)S^C + (1 - b(t)) \cdot \left(E_L + \frac{V_S(t+1)}{1 + r_a} \right) \tag{5.4}$$

The waiting option comprises three cost components: (1) the more expensive corrective replacement, including all future life cycle costs, with a probability $b(t)$

resulting in costs: $b(t)S^C$, (2) the yearly regular maintenance costs of the existing bridge with a probability $(1-b(t))$ resulting in costs $\big(1-b(t)\big)E_L$, (3) and in the case of waiting, the discounted value of all future cash flows with a probability of $(1-b(t))$ resulting in costs: $((1-b(t))\cdot V_S(t+1)/(1+r_a)$. The risk costs $b(t)S^C$ and operational expenses $\big(1-b(t)\big)E_L$ are considered to be incurred at the time of the decision to wait, as a principle of prudence. It is also justified to incur these costs in the middle or end of the year, and discount them appropriately. The present value of a preventive replacement in a small state in a decision node at time *t* is:

$$Q_S(t, replace) = S^P \tag{5.5}$$

The objective in each decision node in a small state becomes:

$$V_S(t) = \min_{a \in A} Q_S(t,a) \qquad \forall\, 4 \le t \le 15 \tag{5.6}$$

$V_S(t)$ represents the discounted life cycle costs in a small state, in year t under optimal decisions. This equation minimises the discounted costs of a preventive investment in a small bridge (S^P), including all future life cycle costs and the discounted costs of waiting. As an example, the cash flows of recursive Equation 5.6, to be evaluated at $t = T-1$, are graphically depicted in Figure 5.4.

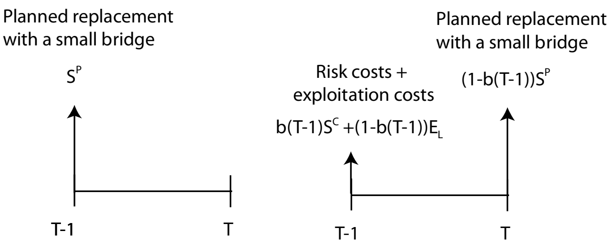

Figure 5.4 Cash flows of Equation 5.6 to be evaluated at year T-1: replace or wait

Similarly, the recursive relationships in the case of a large state are:

$$Q_L(t, wait) = b(t)L^C + (1-b(t))\cdot\left(E_L + \frac{V_L(t+1)}{1+r_a}\right), \tag{5.7}$$

$$Q_L(t, replace) = L^P \tag{5.8}$$

The objective function for a large state must incorporate the possibility of transition from a large state to a small state and becomes:

$$V_L(t) = (1 - p(t)) \min_{a \in A} Q_L(t,a) + p(t) \min_{a \in A} Q_S(t,a) \qquad \forall\, 0 \le t \le 15$$

(5.9)

For example, at the end of year 12 the decision maker will know the outcome of the political decision to ban cars from the city centre in that year. From today's point of view the decision maker will evaluate the cash flows in a small and large state with probabilities of respectively $p(t)$ and $1 - p(t)$. For solving the recursive relationships, the boundary conditions L^P, L^C, S^P and S^C first need to be determined.

Boundary conditions without price increases

The value in a termination node is estimated by the discounted value of all future replacements and exploitation expenditures. Therefore, the boundary conditions L^P, L^C, S^P and $S^C$are calculated based on a combination of traditional discounted cash flow formula for the perpetuity of replacements and the perpetuity of operational expenses. The generalised present value of a perpetuity of identical investment costs is calculated as:

$$\begin{aligned} V_I &= I \cdot \left(1 + \left(\frac{1}{1+r_a} \right)^n + \left(\frac{1}{1+r_a} \right)^{2n} + \left(\frac{1}{1+r_a} \right)^{3n} + \ldots \right) \\ &= I \cdot \frac{1}{1 - \left(\frac{1}{1+r_a} \right)^n} \end{aligned} , \qquad (5.10)$$

where n is the interval of the replacement cycles and r_a is the risk-adjusted discount rate required by the organisation.

The generalised present value of the perpetuity of identical yearly operational expenses follows from the well-known capitalised equivalent worth relationship (Park, 2011):

$$V_E = E \cdot \frac{1}{r_a}, \qquad (5.11)$$

in which E are the yearly operational expenses. When yearly operational expenses are not constant as a consequence of major overhauls or ageing, the life cycle costs should first be translated into (constant) equivalent annual costs (EAC) over the life cycle of the asset by means of the discounted cash flow annuity factor.

Combining V_I and V_E delivers the total present value for a perpetuity of preventive replacements at any time because no price increases are involved yet (price uncertainty will be included in Section 5.4). A small correction is required for calculating the present value of perpetual replacements, which start with a more expensive corrective replacement. In this case, under the assumption that subsequent future replacements will be planned, the difference between a preventive and corrective investment needs to be added to Equation 5.10. The calculations for the present values of the perpetuities L^P, L^C, S^P and S^C are presented in Table 5.2.

Table 5.2 Calculation of boundary conditions: discounted perpetual future life cycle costs without price uncertainty

	Symbol for perpetuity of replacements	Life Cycle n [years]	Investment [x million €]	Yearly operational expenses [x million €]	Present value perpetuity of replacements $V_I + V_E$ [x million €]
Perpetual preventive replacement of a large bridge	L^P	100	$I_L^P = 5$	$E_L = 10\% \; x \; I_L^P = 0.5$	5.17 + 14.29 = 19.45
Perpetual corrective replacement of a large bridge	L^C	100	$I_L^C = 1.5 \cdot I_L^P = 7.5$	$E_L = 10\% \; x \; I_L^P = 0.5$	7.75 + 14.29 = 21.95
Perpetual preventive replacement of a small bridge	S^P	100	$I_S^P = 0.6 \cdot I_L^P = 3$	$E_S = 10\% \; x \; I_S^P = 0.3$	3.10 + 8.57 = 11.67
Perpetual corrective replacement of a small bridge	S^C	100	$I_S^C = 1.5 \cdot 0.6 \cdot I_L^P = 4.5$	$E_S = 10\% \; x \; I_S^P = 0.3$	4.65 + 8.57 = 13.17

Results of DTA

With the values for the boundary conditions L^P, L^C, S^P and S^C, the recursive relationships (6) and (9) are solved. The results for the case study are presented in Table 5.3. The optimal strategy is depicted in the bottom part of Table 5.3 and in Figure 5.5. The strategy follows directly from the minimum options chosen in the recursive relationships at decision nodes. The strategy in the bottom part of Table 5.3 is read from left to right.

Table 5.3 Results of the present value calculations [x million €] for the case study without price uncertainty

Year	0	1	2	3	4	5	6	7
Option value								
$V_L(t)$	15.9	15.9	15.7	15.6	15.3	16.8	16.6	16.4
$V_S(t)$					11.7	11.7	11.7	11.7
Strategy								
Large	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C	W/L^C
Small					S^P	S^P	S^P	S^P

Table 5.3 continued

Year	8	9	10	11	12	13	14	15
Option value								
$V_L(t)$	16.1	17.8	17.6	17.4	17.1	19.5	19.5	19.5
$V_S(t)$	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7
Strategy								
Large	W/L^C	W/L^C	W/L^C	W/L^C	L^P	L^P	L^P	L^P
Small	S^P	S^P	S^P	S^P	S^P	S^P	S^P	S^P

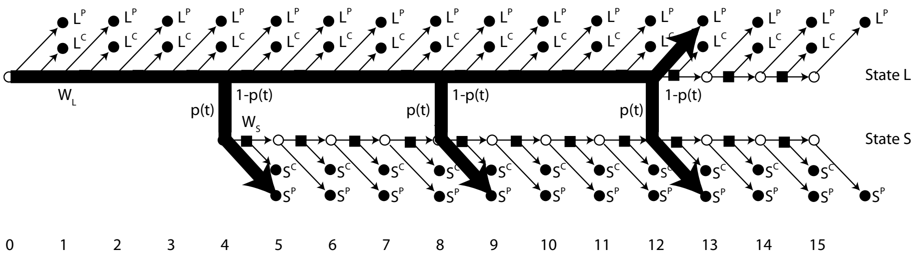

Figure 5.5 Optimal paths for case study subject to structural integrity and political decisions

In the first 4 years, cars will be allowed in the city centre. The strategy here is to wait and to accept the potential risk of a corrective large replacement (W/L^C). At the end of year 4 the politics may decide to ban cars from the city centre. In that case the strategy is to replace the old bridge immediately with a preventive small replacement (S^P) at which the options end. The explanation here is that the risk costs of a corrective small replacement, the operational expenses of the old bridge and the benefits of postponement from year 4 to 5 including future options, exceed the costs of an immediate small replacement. In contrast, if the political parties at the end of year 4 decide not to ban cars from the city centre, the best strategy is to wait.

Here the benefits of delaying the preventive replacement of a large bridge outweigh the costs of such a replacement.

The best strategies for all possible scenarios are derived from the bottom part of Table 5.3. Assume that cars are banned from the city centre at year 8 or year 12 respectively, then the best strategy for the case study is a small preventive replacement at the end of year 8 or 12 and to accept the probability of an earlier corrective large replacement. If cars are not banned from the city centre in year 8, then the best strategy is to wait and accept the probability of an earlier corrective large replacement. If cars are not banned from the city centre in year 12, then the best strategy is a preventive replacement for a large bridge. This is because the increasing risk costs do not allow a further postponement of the replacement.

It is obvious that the outcome of the calculations and the strategy depends on the ratio between the cost components and the risk function. The presented model allows for easy adaptation of input variables.

Although, backward recursion is hardly ever applied in present value calculations, the advantage of this approach is twofold. First, only a few calculations are required to calculate the expected present value of all possible scenarios. Second, the backward recursion provides the decision maker with a strategy.

5.4 Valuing flexibility in the presence of price uncertainty

Technical and political uncertainties are considered in the previous section. The current section incorporates market price uncertainty into the model. Market price uncertainty is treated differently than asset uncertainty because it is a non-diversifiable risk (Cox et al., 1979; Neely & De Neufville, 2001). For the correct valuation of options under market price uncertainty, a risk-adjustment of the discount rate or cash flows is required. There are two approaches to obtain this information from the market. The first is known as the replicating portfolio approach and the second, the risk-neutral probability approach and both deliver the same results.

The replicating portfolio approach directly obtains a risk-adjusted discount rate r_m from the market and uses actual probabilities for up and down moves of market prices. The equivalent risk-neutral probability approach obtains a risk-free discount rate r_f from the market and corrects the up and down moves of market prices with so-called risk-neutral probabilities.

Risk-neutral probabilities have no physical meaning; it is a theoretical concept that allows for discounting with a risk-free discount rate instead of a risk-adjusted discount rate. The advantage of using risk-neutral probabilities is that the risk-free discount rate can directly be observed in the market when options are to be priced over multiple periods. In contrast, the market risk-adjusted discount rate r_m will change in each period when asymmetric option payoffs are introduced. Options are asymmetric when their present values are not a common multiple of a traded security that mimics the option payoffs. Therefore, the risk-neutral probability approach often has computationally advantages over the replicating portfolio approach (Copeland & Antikarov, 2003). The risk-neutral probabilities for the case study are defined after the formulation of the mathematical model.

For correct valuation, two important assumptions underlie the ROA-theory: the payoffs of a project are spanned by traded securities in the market (called a twin security or spanning asset) and arbitrage profits do not exist. The latter means that the market is efficient and financial assets are always correctly priced. There are no possibilities for investors to achieve quick wins by exploiting price differences between similar financial assets (Cox et al., 1979; De Neufville & Scholtes, 2011; Guthrie, 2009). The validity of these assumptions for many public infrastructure projects is questioned by authors including De Neufville and Scholtes (2011) and Herder et al. (2011). Other authors like Copeland and Antikarov (2003) and Mun (2006) argue that the market will always contain replicating traded securities, even if they are not easy to find.

For the case study, construction prices per unit (X) are identified as the market state variable or spanning asset. All cost components in the case study are a common multiple of the initial construction costs (see Table 5.1 and Table 5.2). This leads to the special situation of symmetrical option payoffs for which a constant risk adjusted discount rate r_m applies. The case study, however, will use and demonstrate the more generic risk-neutral probability approach. Results have been verified with the replicating portfolio approach.

The Central Bureau for Statistics (CBS) in the Netherlands publishes historical quarterly data on construction costs for bridges and tunnels (CBS, 2017). The data are calculated based on a compiled bundle that contains labour, materials and equipment. The case study assumes that this bundle suffices as the spanning asset and that somewhere in the market a tradable replicating portfolio with this spanning asset and risk-free bonds can be found. A common convention used to estimate market prices development is the assumption of a geometric Brownian Motion (GBM). A GBM assumes that the natural logarithm of the price X follows a random walk with an annualised drift μ and volatility σ:

$$X_{t+1} = X_t \cdot EXP(\mu \Delta t_m + \sigma \sqrt{\Delta t_m} \cdot \varepsilon), \tag{5.12}$$

Δt_m is the proportional time step used in the calculation for the future price development and is 1 for yearly time steps (Δt_m would be 0.25 if quarterly time steps were used). $\varepsilon \sim N(0,1)$ is a shock, which is normally distributed with a mean of zero and a standard deviation of 1.

The drift μ and volatility σ are obtained from the mean and standard deviation of historical log price differences (equal to the log of the returns). Analysing the quarterly data of construction prices in the Netherlands from 2000 until 2017 leads to an annualised drift of 0.015 and volatility of 0.027. Based on these data, some possible scenarios for the price development of construction costs per unit are depicted in Figure 5.6.

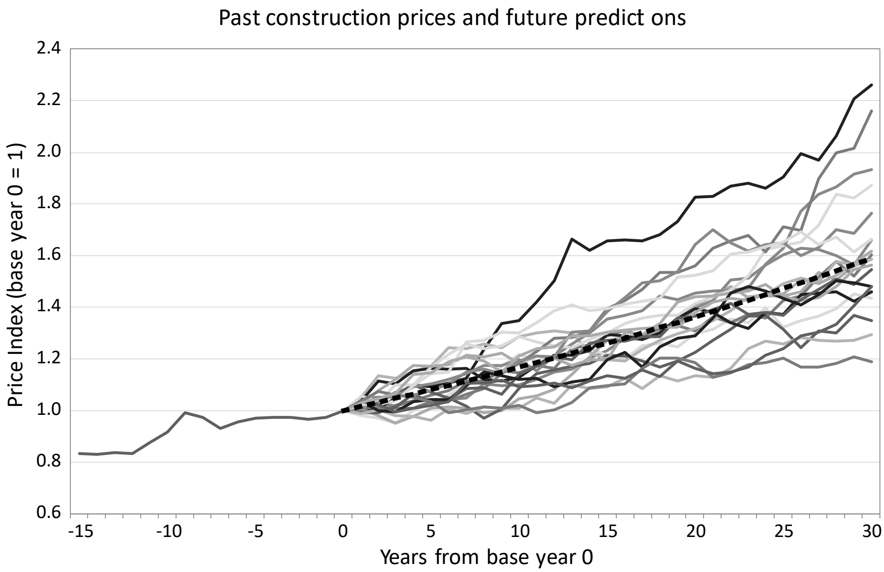

Figure 5.6 Possible scenarios for the price development of construction costs assuming a GBM with an annualised drift of 0.015 and volatility 0.027. The dotted line expresses the drift and expected value

A GBM is a continuous stochastic process, which can be converted into a discreet simulation process in the form of a recombining binominal lattice with up moves U and down moves D (Figure 5.7). The size of an up move U is calculated as:

$$U = EXP(\sigma\sqrt{\Delta t_m}) \tag{5.13}$$

For a recombining lattice, the size of a down move D must satisfy:

$$D = \frac{1}{U} \tag{5.14}$$

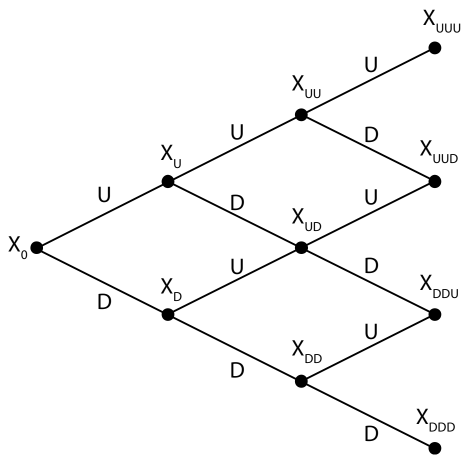

Figure 5.7 Binominal lattice with up moves U and down moves D.

Using Equations 5.13 and 5.14, the discrete simulation of the market state variable $X(i,t)$ of 1 unit of construction costs is presented in appendix A, Table A1 in Van den Boomen, Spaan, Schoenmaker, and Wolfert (2018). The index i represents the number of down moves D and the time t. For example, and with reference to Figure 5.7: $X_{UU} = X(0,2)$ and $X_{DDU} = X(2,3)$.

Equation 5.15 provides a direct relationship to calculate $X(i,t)$. This relation will be used after development of the model to derive equations for the new boundary constraints.

$$X(i,t) = X(0,0) \cdot EXP\left((t-2i)\sigma\sqrt{\Delta t_m}\right) \tag{5.15}$$

The next step is the inclusion of market price uncertainty in the model of the case study. Market price uncertainty affects all cost components which now becomes a function of the number of down moves i and time t. The adjusted boundary constraints at expiration year T are:

$$V_S(i,T) = S^P(i,T) \tag{5.16}$$

$$V_L(i,T) = L^P(i,T) \tag{5.17}$$

The discounted value of a waiting option at node (i,t) in a small state is calculated as under the new circumstances:

$$Q_S(i,t,wait) =$$
$$b(t)S^C(i,t) + (1-b(t)) \cdot \left(E_L(i,t) + \frac{\eta_U V_S(i,t+1) + (1-\eta_U)V_S(i+1,t+1)}{1+r_f} \right)$$

(5.18)

In this equation, η_U and r_f represent a risk-neutral probability for an up movement and a risk-free discount rate respectively, which will be further explored after defining the model. The symbol i represents the number of down moves. The discounted value of a preventive replacement at node (i,t) in a small state becomes:

$$Q_S(i,t,replace) = S^P(i,t) \tag{5.19}$$

The objective in each decision node in a small state now reads:

$$V_S(i,t) = \min_{a \in A} Q_S(i,t,a) \qquad \forall\, 4 \le t \le 15 \tag{5.20}$$

Similar, the discounted value of a waiting option at node (i,t) in a large state becomes:

$$Q_L(i,t,wait) =$$
$$b(t)L^C(i,t) + (1-b(t)) \cdot \left(E_L(i,t) + \frac{\eta_U V_L(i,t+1) + (1-\eta_U)V_L(i+1,t+1)}{1+r_f} \right)$$

(5.21)

The discounted value of a preventive replacement at node (i,t) in a large state becomes:

$$Q_L(i,t,replace) = L^P(i,t), \tag{5.22}$$

The objective function of a decision node in a large state needs to incorporate the probability of transferring from a large state to a small state. Recall that $p(t)$ is only > 0 for nodes 4, 8 and 12. The generic objective function for decision nodes in state L becomes:

$$V_L(i,t) = p(t)\min_{a \in A} Q_S(i,t,a) + (1-p(t))\min_{a \in A} Q_L(i,t,a) \;\; \forall\, 0 \le t \le 15 \quad (5.23)$$

The inclusion of market uncertainty quickly complicates the model. First, it significantly increases the number of calculations as a consequence of considered up and down moves. Second, the estimated fluctuations of one unit of investment costs still needs to be converted to the present values of the future life cycle costs (perpetuities) of preventive and corrective replacements at all nodes (i,t). The boundary conditions calculated in Table 5.2, need to be adjusted to incorporate price increases. Third, motivated estimations are required for the upward risk-neutral probability η_U and the risk-free discount rate r_f.

We begin with the estimations for the risk-neutral probabilities and market risk-free discount rate. The risk-neutral probability approach requires an estimation of a risk-free interest rate. The risk-free interest rate is a secure bond that serves as a standard for pricing other risky assets. Since 2009, the financial crisis has caused risk-free interest rates to decline rapidly. The current short-term risk-free interest rate is close to zero in the Euro-zone. This poses a problem for the application of ROA and other economic instruments to value derivatives (ECB, 2014, 2017; Frankema, 2016; Hull & White, 2013). This is an ongoing debate between econometrists and beyond the scope of this study, which intends to apply ROA theory in engineering practice. In accordance with the current policy of the ECB, the long-term risk-free interest rate for the case study is estimated from the Euro area yield curve that contains the long-term structure of the interest rates of AAA-rated Euro area central government bonds. Based on the instantaneous forward Euro area yield curve, an average risk-free interest rate of 0.8% is estimated for the case study.

The next step is to estimate the risk-neutral probabilities. In the absence of dividends payment or a market risk premium and systematic market risks for holding the spanning assets, the risk-neutral probability of an up move is derived by Cox et al. (1979) as:

$$\eta_U = \frac{(1+r_f)-D}{U-D}, \quad (5.24)$$

In the case study, market risk premiums and systematic market risks are present and cannot be ignored. The Dutch National Task Force for Discount Rates advises that for these types of investments (for costs and benefits) use a 3% market risk premium and a β-coefficient of 1 (Werkgroep discontovoet (National Taskforce for the Societal Discount Rate), 2015). A β-coefficient accounts for the systematic market risk. A β of 1 implies that the net benefits (benefits minus costs) of the project move along with the economy. Although benefits are not included in the case description, benefits are present. The case study just assumes that the different alternatives have equal societal benefits whether it will be a large or small bridge. It is hard to differentiate between social benefits in the case study. If cars are banned from the city centre, there will probably be benefits to tourism, the local economy (more restaurants, cafés) and improvements in air quality. If cars are not banned from the city centre, the benefits are found in better accessibility and probably shorter travel times for motorised traffic.

Guthrie (2009) provides an approach to address the market's attitude by incorporating a market risk premium and β-coefficient in the dividend free risk-neutral probability Equation 5.24:

$$\eta_U = \frac{K - D}{U - D}, \tag{5.25}$$

with:

$$K = \left(\frac{\phi_U X_U + \phi_D X_D}{X} \right) - \left(market\ \ risk\ \ premium \right) \cdot \beta \tag{5.26}$$

The first term between brackets on the right side of the equation is the expected return on the state variable using the actual probabilities of up and down moves: φ_U and φ_D.

To determine K, first the actual probability of an up move is calculated based on the observed data. Cox et al. (1979) recommend estimating φ_U as:

$$\phi_U = \frac{1}{2} + \frac{1}{2} \frac{\mu \sqrt{\Delta t_m}}{\sigma} \tag{5.27}$$

The mean μ and volatility σ were already obtained from analysing the historical data of the construction prices. Knowing that $\varphi_D = 1 - \varphi_U$, allows for calculating the risk-adjusted growth factor K and risk-neutral probabilities η_U, and $\eta_D = 1 - \eta_U$.

To complete the analysis, the one period risk-adjusted discount rate r_m for the state variable can be derived from equivalence relationship between the risk-neutral probability approach and the replicating portfolio approach. The equivalence between a market risk-adjusted discount rate r_m, the actual probabilities of up and down moves and the risk-free interest rate and risk-neutral probabilities in ROA valuation is in its simplest form, for a one-time period, represented by (Copeland & Antikarov, 2003; Cox et al., 1979; Guthrie, 2009):

$$V_0 = \frac{\eta_U \cdot V_U + \eta_D \cdot V_D}{1 + r_f} = \frac{\phi_U \cdot V_U + \phi_D \cdot V_D}{1 + r_m}, \tag{5.28}$$

where V_0 = a generic option value, V_U = option payoff after one period in an up-state and V_D = option payoff after one period in a down-state. The risk-adjusted discount rate r_m for one period follows from Equation 5.28:

$$r_m = \left(\frac{\phi_U V_U + \phi_D V_D}{V_0} \right) - 1 \tag{5.29}$$

To find r_m for the first timestep of the case study (and the following because of the special situation of symmetrical option payoffs), V_U is substituted with $X(0,1)$, V_D is substituted with $X(1,1)$ and V_0 is defined by Equation 5.28. The intermediate calculations for the market variables are presented in Table 5.4. At this point it is interesting to notice that not compensating for the market risk premium and systematic market risk would result in a risk-neutral probability for an up move of η_U = 0.643 (Equation 5.24) and a risk-adjusted discount rate r_m of 1.6% (Equation 5.29). The latter is far below even the risk-adjusted discount rate of the municipality of 3.5% and would not provide a realistic result. This observation demonstrates the difficulty in correctly estimating market variables in a infrastructure case study.

Table 5.4 Intermediate calculations of market variables

Symbol	Value	Description	Source
$X(0,0)$	1	State variable: 1 unit of construction costs	-
σ	0.027	Annualised volatility of observed historical price data of construction costs	Data analysis
μ	0.015	Annualised mean of observed historical price data of construction costs	Data analysis
U	1.027	One up move of the state variable	Eq. 5.13
D	0.974	One down move of the state variable	$1 - U$
φ_U	0.789	Actual probability of an up move	Eq. 5.27
φ_D	0.211	Actual probability of a down move	$1 - \varphi_U$
mrp	3%	Market risk premium	Data
β	-	Coefficient for systematic market risk	Data
K	0.986	Risk-adjusted growth factor	Eq. 5.26
η_U	0.228	Risk-neutral probability of an up move	Eq. 5.25
η_D	0.772	Risk-neutral probability of a down move	$1 - \eta_U$
r_f	0.8%	Risk-free interest rate	Data
r_m	0.039	Risk-adjusted discount rate	Eq. 5.29

Boundary conditions under price increases

As a result of the price uncertainty of the development of construction costs, the boundary conditions for the case study change. The case study needs values for a perpetual stream of life cycle costs indexed by (i,t) for the four types of replacements. Four additional tables like Table A1 in Van den Boomen et al. (2018) need to be constructed for $L^P(i,t), L^C(i,t), S^P(i,t)$ and $S^C(i,t)$.

Under the assumption that the yearly operational expenses $E(i,t)$ after replacement remain a fraction of the initial construction costs and are subject to the same uncertainty as the state variable $X(i,t)$, the expected discounted value at (i,t) of a continuation of operational expenses with growth rate g and a risk-adjusted discount rate r_m, is in generalised form given by:

$$E[V_E(i,t)] = E(i,t) \cdot \frac{1+g}{r_m - g} \tag{5.30}$$

This formula is derived from a standard discounted cash flow gradient annuity factor in which we allow n to approach infinity (Park, 2011; Sullivan, Wicks, & Koeling, 2012). The actual expected annual growth rate g is 0.0159 and follows from:

$$(1+g) = \frac{\phi_U X_U + \phi_D X_D}{X} \tag{5.31}$$

As a consequence of the assumptions mentioned above, its equivalent relationship in the risk-neutral world reads as (Guthrie, 2009):

$$E[V_E(i,t)] = E(i,t) \cdot \frac{K}{R_f - K}, \tag{5.32}$$

where $R_f = 1 + r_f$. In generalised form, the discounted value of a perpetuity of repeating risky replacement costs $I(i,t)$ with interval n at (i,t) is derived as:

$$\begin{aligned} E[V_I(i,t)] &= I(i,t) \cdot \left(1 + \left(\frac{1+g}{1+r_m} \right)^n + \left(\frac{1+g}{1+r_m} \right)^{2n} + \left(\frac{1+g}{1+r_m} \right)^{3n} + \ldots \right) \\ &= I(i,t) \frac{1}{1 - \left(\frac{1+g}{1+r_m} \right)^n} \end{aligned} \tag{5.33}$$

Due to the above-mentioned assumptions (all cost elements are proportional to the state variable $X(i,t)$), its risk-neutral equivalent expression is (Guthrie, 2009):

$$E[V_I(i,t)] = I(i,t) \frac{1}{1 - \left(\frac{K}{R_f} \right)^n} \tag{5.34}$$

Again, a small correction for the one-time occurrence of a more expensive corrected replacement needs to be made. To be accurate, the difference in investment costs between a preventive and corrective replacement needs to be added to Equation 5.33 or 5.34 to calculate the perpetuity of a corrective replacement, followed by future preventive replacements.

The discounted expected values capture all probable future cash flows for each i and t. This has been verified by an alternative calculation in which the actual or risk-neutral probabilities throughout the binominal lattice are used to calculate the expected values of the cash flow at year t and discounting these values to the present with r_m or r_f depending on the probabilities used (an equivalent but more time-consuming calculation).

Combining the relationships (15), (32), (34), correcting them for proportional fractions of cost components (see Table 5.1) to the state variable $X(i,n)$ as expressed in Table 5.5 and rewriting the relationships, results in direct equations for the case-specific boundary conditions under price uncertainty.

Table 5.5 Input data for case-specific boundary conditions subject to price uncertainty. The proportional values k are derived from Table 5.1.

$k_1 = 5$
$k_2 = 1.5$
$k_3 = 0.6$
$k_4 = 0.1$

$$I_L^P(i,t) = k_1 \cdot X(i,t)$$

$$I_L^C(i,t) = k_2 \cdot I_L^P(i,t) = k_1 \cdot k_2 \cdot X(i,t)$$

$$I_S^P(i,t) = k_3 \cdot I_L^P(i,t) = k_1 \cdot k_3 \cdot X(i,t)$$

$$I_S^C(i,t) = k_2 \cdot I_S^P(i,t) = k_1 \cdot k_2 \cdot k_3 \cdot X(i,t)$$

$$E_L(i,t) = k_4 \cdot I_L^P = k_1 \cdot k_4 \cdot X(i,t)$$

$$E_S(i,t) = k_4 \cdot I_S^P = k_1 \cdot k_3 \cdot k_4 \cdot X(i,t)$$

$$L^P(i,t) = k_1 \cdot X(0,0) EXP[(t-2i)\sigma] \cdot \left[\frac{1}{1-\left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K} \right] \quad (5.35)$$

$$L^C(i,t) = k_1 \cdot X(0,0)EXP[(t-2i)\sigma] \cdot \left[(k_2 - 1) + \frac{1}{1-\left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K} \right]$$

(5.36)

$$S^P(i,t) = k_1 \cdot k_3 \cdot X(0,0)EXP[(t-2i)\sigma] \cdot \left[\frac{1}{1-\left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K} \right] \quad (5.37)$$

$$S^C(i,t) = k_1 \cdot k_3 \cdot X(0,0)EXP[(t-2i)\sigma] \cdot \left[(k_2 - 1) + \frac{1}{1-\left(\frac{K}{R_f}\right)^n} + k_4 \cdot \frac{K}{R_f - K} \right]$$

(5.38)

Results of ROA

At this point the case study has all the information it needs to solve the risk-neutral recursive relationships (20) and (23). The results of the final recursive calculations are shown in appendix A, Tables A2, A3, A4 and A5 in Van den Boomen et al. (2018). The optimal strategy under price uncertainty is shown in Tables A4 and A5 and equals the strategy without price uncertainty (Figure 5.5). A decision maker should wait for political decisions on banning cars from the city centre and incur the risk costs under the current assumptions until year 12. Only when entering a small state (a decision to ban cars from the city centre) the current bridge should immediately be replaced by a small bridge.

The volatility of construction prices does not influence the strategy in the case study. This is deducted from Table A4. All the strategies in one column are identical. There is no incentive to act purely on the volatility derived from analysis of historical construction prices

5.5 The DTA approach to ROA

The difficulty in applying ROA in engineering practise lies in the establishment of reasonable assumptions for market behaviour as demonstrated in Section 5.4. A spanning asset or twin security needs to be found and analysed. Assumptions that predict future prices are required for the process (Table 5.4). Systematic market risks and risk premiums need to be estimated. A prediction of the future risk-free interest rate should be obtained from the financial market. Unfortunately, risk-free interest rates fluctuate and are only constant for an agreed term. Presently, the short-term risk-free interest is close to zero in the Netherlands. Solutions to value real options under zero or negative risk-free interest rates are not readily available and require in-depth economic expertise.

It is understandable that in engineering practice, ROA is adapted to become what is often called a DTA approach to ROA. Price development is modelled according to standard ROA practices, for example a GBM expressed in a recombining binominal lattice. However, the difference is that instead of calculating with risk-neutral probabilities (η_U, η_D) and discounting the adapted cash flows with a risk-free discount rate r_f, actual probabilities (φ_U, φ_D) are used and discounted with the minimum accepted rate of return of the organisation (r_a = 3.5% in the case study). This also affects the perpetuities of the boundary constraints which are now calculated by using the annual growth rate g and the organisation's discount rate r_a. Performing these calculations for the case study results in an option value $V_{L,DTA\ approach\ to\ ROA}\ (0{,}0)$ of 26.61 instead of 22.98. The optimal strategy does not alter in the current case study. This DTA-approach to ROA is incorrect in its definition of ROA because it allows for the possibility of arbitrage on the financial market.

5.6 Comparison

The purpose of this study is methodology development and to demonstrate how and when to apply different approaches: DTA, ROA and the DTA approach to ROA. Table 5.6 summarises the three approaches used for the case study. The approaches that value options without (DTA, Section 5.3) and with market price uncertainty (ROA, Section 5.4) show a difference in option value. This difference is a consequence of two different approaches and their underlying assumptions. The basic rule for applying ROA instead of DTA is whether or not market prices are involved.

Comparing the ROA-approach in Section 5.4 with the DTA approach to ROA in Section 5.5 also shows a difference in option values but here the explanation is clear: the DTA approach to ROA is an incorrect application of the ROA-theory. However, all these applications still result in the same strategy for the case study. This is a consequence of construction prices with low drift and volatility, and a market discount rate close to the discount rate of the organisation.

Table 5.6 Comparison of different valuation methods for the case study. Option values expressed in discounted costs [x million €]

Case study bridge replacement	**DTA-approach**	**ROA approach**	**DTA approach to ROA (wrong application of ROA theory)**
Uncertainties	- Strength of bridge - Political decisions	- Strength of bridge - Political decisions - Prices (drift & volatility)	- Strength of bridge - Political decisions - Prices (drift & volatility)
Assumptions for prices	- Constant - No drift - No volatility	- GBM - Drift and volatility obtained from historical data	- GBM - Drift and volatility obtained from historical data
Assumptions for discount rate	- Static: minimum acceptable rate of return provided by organisation	- Dynamic: influenced by market forces (trading) - Use of equivalent risk neutral probability approach and risk-free discount rate	- Static: minimum acceptable rate of return provided by organisation
Option value $V_L(0,0)$	15.9	22.9	26.6
Replacement Strategy	Identical	Identical	Identical

5.7 Discussion

The current research focusses at model development for an infrastructure replacement decision under different types of uncertainty, including the managerial flexibility to respond to these uncertainties. Although a specific case study is used to demonstrate the model development, the approach is generic and in principle applicable to a wide range of design, build and operate decisions. The approach identifies the uncertainties and managerial options, which are combined in a decision tree. The decision tree is solved with backward recursion. Each decision node is evaluated for the best option out of a range of options for all prevailing states of uncertainty at that particular time.

The underlying mathematics for solving a decision tree (the backward recursion) are generic but the modelling of a decision tree and the inclusion of different types of uncertainty in a ROA/DTA analysis is not. Probably the major barrier in the practical application of the method is the identification and quantification of uncertainties. Perminova, Gustafsson, and Wikstrom (2008) conducted a comprehensive research on defining uncertainty in projects. The authors observe the absence of a common understanding on the definition of uncertainty in and between different disciplines such as project management and economics. Uncertainty is not self-explanatory. Uncertainty is often used to designate the probability of events, but also as the probable outcome of these events to which others refer to as risk. A third definition of uncertainty is the unknown unknown: events that cannot be anticipated on because they are totally unknown. The current research follows a commonly applied convention in the discipline of project management for the definition of uncertainty and defines uncertainty as a (time-variant) probability of an event. Perminova et al. (2008) conclude that reflective learning and information sharing are methods to manage and reduce uncertainty and stress the importance of future research to develop tools that assist managers in decision making under uncertainty. The current study developed one of these tools by integrating different types of uncertainty in a DTA/ROA analysis. Neely and De Neufville (2001) referred conceptually to such an approach as 'hybrid' real options.

The current study equally emphasises the importance of separating market price uncertainty from other types of uncertainty as they require different treatment in discounting approaches. However, the case study also demonstrates the difficulty in estimating expected values of the uncertainty variables. Expected values can be obtained by wide range of approaches such as expert judgement, data-analysis, testing, using reference data of similar assets or projects and mathematical prediction modelling. Hereafter, uncertainty bounds for the expected value of variables need to be defined. Again, various approaches are available to model uncertainty bounds such as using random walks (geometric or arithmetic Brownian Motions), shock models, working with (time-variant) probability distributions or with non-probabilistic uncertainty bounds as does the info-gap decision theory and the sensitivity analysis approach. When uncertainty variables influence each other, more sophisticated techniques like Markov chains, Bayesian networks, and artificial learning come into view.

Uncertainty modelling is complex (Ilg, Scope, Muench, & Guenther, 2017; Perminova et al., 2008; Scope, Ilg, Muench, & Guenther, 2016). And even uncertainty models are subject to uncertainty. Would that be a reason for practice to refrain from the application of DTA and ROA? To answer this question, we first argue that uncertainty is inherent to every analysis conducted, including conventional LCC analysis. An extensive research on uncertainty in LCC modelling was conducted by Scope et al. (2016). The authors identify numerous approaches for dealing with uncertainty and classify these approaches in deterministic, probabilistic, possibilistic and practical methods for dealing with uncertainty. The above mentioned techniques identified by the current study are easily classified within these categories. Scope et al. (2016) also observe the absence of a holistic model in dealing with uncertainty in LCC analyses and conclude that choosing the right approach does not follow generic decision rules. Although uncertainty approaches can be grouped, their selection and application remain case specific. Therefore, the authors stress the importance of developing case studies and learning by example.

The current research is a case-specific application of uncertainty modelling. A DTA/ROA approach incorporates all possible scenario's in a condensed decision tree and the backward recursion provides for choosing the best option in any decision node. By navigating through the tables with results, the best strategy in each decision node and uncertainty state is provided (Appendix A in Van den Boomen et al. (2018)).

That still leaves the issue of selecting and quantifying uncertainties which may refrain practitioners from the application of DTA/ROA. Modelling price uncertainty is not an insuperable obstacle, because the ROA theory offers well-defined approaches and, historic price indices of construction costs and materials are often available. The estimation of boundary constraints, especially the perpetuities of replacements and life cycle costs *under price uncertainty*, are not yet available in the engineering economy discipline and only partly available in the ROA discipline. This approach has been developed in the current research.

A difficult part in the application of ROA is estimating long-term market variables, required for the calculation of the risk neutral probabilities. A pragmatic solution is to omit this process and discount with fixed discount rates. De Neufville and Scholtes (2011) provide arguments that support this pragmatic solution. Second, the discrepancy between short-term market behaviour and long-term infrastructure life cycles, also calls into question the long-term validity of these risk neutral probabilities.

The second category of uncertainty is the infrastructure asset or project related uncertainty. The current research demonstrates how these types of uncertainty can be incorporated in a DTA/ROA analysis. A pragmatic approach based on failure data and expert judgement is used to provide reasonable estimates. Although reliability modelling of infrastructure is complex, often reasonable and pragmatic estimates for the current type of calculations suffice.

Taking the case study as an example, the strategy for the first four years is to wait and see what happens in year 4. At present deformation monitoring is initiated. In four years' time, results of measurements will become available and the model can be adjusted with better predictions for the probability of exceeding a deformation threshold in the future. Deformation monitoring is also initiated on other bridges in this city, which will provide the data required for establishing uncertainty bounds. This process of managing and reducing uncertainty is an example of reflective learning as referred to by Perminova et al. (2008) and an example of a practical method for dealing with uncertainty as referred to by Scope et al. (2016).

5.8 Conclusions

This study investigates the application of DTA and ROA in a common public infrastructure challenge, that of replacing a bridge in an urban environment. The concept of DTA and ROA is an incentive to wait for more information that allows decision makers to optimise future decisions. This managerial flexibility has value, which should be incorporated into traditional investment or replacement analyses. Both DTA and ROA can capture the value of flexibility.

The theory of ROA originates from valuing financial options and is strongly tied to the behaviour of financial markets. Therefore, applying ROA requires a careful estimation of market variables such as the choice of a spanning asset whose price can be observed in the market, market risk premiums, systematic market risks and risk-free interest rates. The estimation of market variables is subject to an inherent uncertainty regarding long-term market behaviour.

In the last decade an academic debate on real options has revealed some interesting perspectives. A growing number of case studies demonstrate the application of ROA on real assets and advocate a wider application. Other literature warns against using ROA formula in the absence of price uncertainty. Two mistakes are easily made: ROA is applied to value flexibility in the absence of market price uncertainty and a DTA approach to ROA is applied to value flexibility subject to market price uncertainty. It is correct to apply ROA to value flexibility subject to market price uncertainty and apply DTA to value flexibility in the absence of market price uncertainty.

At the same time, ROA has not gained foothold in public infrastructure investment decisions. The dominant reasons are its complexity, its difficulty in estimating market variables and the political context of public decision making. Investment or replacement decisions in public infrastructure are seldom driven by economic reasons alone. The current research, however, demonstrates with a case study that ROA can be applied to public sector investment decisions when market prices are observable. Second, even after high-level political investment decisions are made, there is no reason to ignore the value of flexibility and to address the question of timing.

The complexity of ROA is easily reduced by an incorrect application of ROA (referred to as the DTA version of ROA) that partially omits the process of estimating market variables. Here experts on ROA claim that this will lead to an incorrect valuation of flexibility under uncertain market prices. However, in the case study used in this research, the differences in these monetary values resulting from the application of different methods do not result in different optimal strategies. Although the monetary values of flexibility differ, the optimal replacement strategy does not alter because the discount rate of the organisation is close to the discount rate obtained from the market. Second, the case study demonstrates that having capital-intensive options (replace or wait and accept risk costs) quickly dominates the impact of the volatility of market prices.

This leads us to the primary conclusion of this research. In the absence of market price uncertainty, ROA should be avoided and DTA used instead. In the presence of market price uncertainty ROA is the first choice to value the flexibility of engineering options. However, when market variables like market prices, systematic market risks, risk premiums and risk-free interest rates, cannot reasonably be estimated, the DTA approach to ROA is the best approximation for ROA. If the discount rate of the organisation is close to the discount rate that would be obtained from the market, and capital-intensive options are involved, then it is very unlikely that the DTA approach to ROA will result in a different strategy. These conditions often apply to public infrastructure assets.

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6

Network optimisation with dynamic programming

Optimise a sequence of distinct strategies
Non-repetitive cash flows
Certain future

This chapter presents a novel LCC optimisation model for a current infrastructure asset which is challenged by a sequence of possible intervention strategies. The model optimises the entire chain of strategies, including the option to maintain the current assets and includes ageing and inflation. The case study is a pumping station. In the Netherlands, thousands of such pumping stations, each with their own cost profiles, are operational. The intervention strategies are to maintain, renovate and replace.

Optimal intervention intervals are found by shortest path network optimisation using a nested DP algorithm or a two-step optimisation approach. This nesting allows for a truncation with properly estimated cash flows over a horizon which is long enough to approximate infinity. The research demonstrates that the application of classical replacement techniques would lead to errors.

The limitation of the current model is that it does not include price uncertainty. This feature will be addressed in the subsequent chapter. The advantage of the current model is that results also support long-term decision making and planning.

A dynamic programming approach for economic optimisation of lifetime-extending maintenance, renovation, and replacement of public infrastructure assets under differential inflation.

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Published in Structure and Infrastructure Engineering at 22 December 2018
https://doi.org/10.1080/15732479.2018.1504803

In the next decades, many public infrastructure assets will reach the end of their life that they were originally designed for. Replacement costs are high, and therefore increasing effort is put into lifetime-extending maintenance, including major overhauls and renovations. A key question is whether the investments in lifetime-extending maintenance justify the postponement of a full replacement. This question becomes more complicated when future life cycle cash flows are non-repeatable. Differential inflation and technological change, including multiple intervention strategies to maintain a desired functionality, cause such non-repeatability. In this case, classical replacement analysis techniques will not suffice in answering this question. Literature demonstrates that case-specific modelling with dynamic or linear programming techniques is required to find economic optimisation. However, such literature primarily addresses replacement interval optimisation of new investments within relative short time horizons, whereas the current research develops a nested dynamic programming (DP) approach for typical ageing infrastructure assets over long service life periods. The model can deal with multiple and various successive intervention strategies and addresses differential inflation and age-related cost increases. Finally, it is shown in an infrastructure case study that this DP approach leads to a better decision in comparison to the application of classical replacement techniques.

6.1 Introduction

Many public infrastructure assets are ageing, such as bridges, dikes, locks, pumping stations, treatment plants, and transport mains. In general, the first public assets were built around early 1900. A peak occurred in the years 1950–1970 and today increasingly more assets reach the end of the life that they were originally designed for. The technical lives of public infrastructure range from 30 to over 120 years. However, the required functionality often extends beyond the technical lives and frequently approximates infinity. A function is for example transportation, high water protection and is not restricted to the technical life of assets. The costs of replacements are high and increasing effort is therefore dedicated to lifetime-extending maintenance, including major overhauls and renovations.

The classical theories in engineering economics provide techniques for solving replacement problems, but their applicability is limited due to underlying assumptions. The most important assumption is continuous repeatability of the life cycle cash flows of a challenger (a renovation or replacement option). However, life cycle costs of many infrastructure assets are subject to differential inflation (distinct price development of cost components compared to the general inflation) and multiple successive intervention strategies with different life cycle costs often apply.

Both the characteristics reject the assumption of continuously repeating life cycle cash flows of a replacement option. Asset owners generally have several successive interventions strategies available for ageing infrastructure, for example, maintain with an initial upgrade, renovate and/or fully replace. Moreover, costs (and benefits) are subject to inflationary effects as historic consumer price and producer price indices demonstrate. For example: over the past two decades, average total inflation rates for concrete, steel, asphalt, electricity and labour, range between 1.1% to 3.8% per year in the Netherlands, with an average general inflation rate of 1.9% per year. Differential inflation is fully defined in Section 6.3 but here by approximation described as the difference between the general inflation (applicable to all goods and services) and the total inflation for specific cost components. By approximation, the differential inflation rates range between -0.8% and 1.9% per year for the same cost components. Considering low public-sector discount rates, varying between 2% and 5% in real terms (opposed to nominal), that differential inflation, if present, can significantly influence costs (and benefits), the net-discounting and potential decisions. Hence, the current research develops a realistic model that includes differential inflation and multiple successive intervention strategies for a common infrastructure replacement challenge.

To position the scope of the current research in a wider context before narrowing down, a distinction is made between component replacement and capital equipment replacement, as proposed by Campbell, Jardine, and McGlynn (2011) and Jardine and Tsang (2013). Component replacement is strongly supported by probabilistic reliability modelling and often part of a larger maintenance optimisation strategy over the life cycle of an asset (Gertsbakh, 2000). D. M. Frangopol, Kallen, and Noortwijk (2004) further classify these probabilistic optimisation models in random-variable models and stochastic process models, among which probabilistic Markov decision processes. Markov-decision processes incorporate optimised decision making by maximising multi-objective functions such as minimising life cycle costs while considering other constraints. (Adey, Burkhalter, & Lethanh, 2018; Bocchini & Frangopol, 2011; D. M. F. A. Frangopol, Estes, & Stewart, 2004; Golabi, Kulkarni, & Way, 1982).

The difficulty with probabilistic life cycle optimisation modelling is the estimation of the required statistical properties (underlying probability distributions). In practice, historical data to perform such modelling is often unavailable. Second, even if historical data is available, it may become obsolete when modern technology or new materials are introduced. Another observation is that literature on probabilistic life cycle modelling is in general less focussed on the economic aspects of life cycle costing. These aspects are better dealt with in the second class of literature to which Campbell et al. (2011) and Jardine and Tsang (2013) refer to as capital equipment replacement modelling.

Literature on capital equipment replacement modelling puts more focus on the economic aspects such as selecting a proper discount rate, incorporation of inflationary effects, using a proper calculation horizon and identifying the right cash flows in real or nominal terms. Capital equipment replacement models are often a blueprint for a larger group of similar assets. The results of these models are used for mid and long-term capital equipment replacement planning and these models are not a first choice for detailed maintenance optimisation modelling of single assets. This may explain why probabilistic failure modelling is less prevalent in capital equipment replacement models. However, in capital equipment replacement models, failures are often estimated by an increasing cost function.

In the class of capitalised equipment replacement models, the classical engineering economy approaches (de Neufville, Scholtes, & Wang, 2006; Newnan, Lavelle, & Eschenbach, 2016; Sullivan, Wicks, & Koeling, 2012) and the dynamic or linear programming optimisation approaches, including (the same) Markov-decision processes but with more emphasis on economic aspects, are found. This DP and LP literature is reviewed in Section 6.2.

The current research builds on capital equipment replacement modelling and is geared at the inclusion and impact of differential inflation and multiple successive intervention strategies (equivalent for technology change). Condition deterioration is modelled by accounting for ageing with annually increasing costs.

The outline of this paper is as follows: Section 6.2 presents the results of a literature review on capital equipment replacement decisions under differential inflation and technological change. This provides a direction for a solution using dynamic programming (DP) or linear programming (LP) techniques. Section 6.3 develops a novel DP approach for a class of problems that cannot be solved with classical replacement techniques. This approach is demonstrated for a pumping station with three alternative options: maintain with major overhauls, renovate, or fully replace (Section 6.4). All the options are subject to differential inflation. The result of the case study is compared with the incorrect application of a classical technique in Section 6.5. The paper ends with a discussion and conclusions in Sections 6.6 and 6.7 respectively.

6.2 Literature review

In addition to its treatment in classical textbooks, replacement optimisation under inflation or technological change has been investigated by several authors. Bellman (1955) laid the foundation for using DP techniques for solving this class of replacement problems, with the development of a functional equation for a single asset replacement optimisation under technological change. Wagner (1975) introduced DP techniques to solve this functional equation, designated as regeneration models. All replacement options between a source node (start decision) and a destination node (result of the final decision) are considered and visualised as a network. DP techniques are used to find the least cost route (shortest path) in such a network. The same solution is obtained using LP techniques, such as the one explained by Hillier and Lieberman (2010), as shortest path problems that are a special class of so-called transhipment models.

One of the first studies that explicitly deals with differential inflation in replacement decisions originates from Karsak and Tolga (1998). Karsak and Tolga (1998) stressed the importance of proper treatment of general and differential inflation. The authors used a DP approach to identify the optimum maintain–replace strategy for a finite 8-year time horizon under various scenarios for inflation. The short time horizon and the use of continuously increasing or decreasing cost functions limit the applicability of this model for public infrastructure assets.

Oakford, Lohmann, and Salazar (1984) conducted a similar study. DP was again used to find the optimal replacement chain of multiple challengers under total inflation for a time horizon of 25 years. The authors addressed the difference between general inflation and differential inflation, and the subsequent necessity for expressing cash flows in real and nominal terms. The term ‘real present value’ is confusing as there is no such thing as a ‘real present value’. There is merely a ‘present value’ that can be calculated by discounting real cash flows with a real discount rate or nominal cash flows with a nominal discount rate. Using either of the methods, one can arrive at the same present value.

The authors further demonstrated that the calculation horizon influences the optimised replacement chain. Oakford et al. (1984) emphasised that the current calculation power of computers enables accurate optimisation calculations, leaving no excuse for using classical replacement approaches for replacement decisions under inflation and technological change. Although this line of reasoning is plausible, it must be noted that Oakford et al. (1984) used convenient cost functions for calculating the future cash flows, and limited the computational effort by restricting the time horizon to 25 years and considering maximum asset service lives of only 10 years. The presence of salvage values also enabled appropriate and convenient truncation of cash flows.

Hartman (2004) developed a DP optimisation approach for a parallel or redundant asset replacement problem under changing demand (causing non-repeatability of future cash flows) for a finite time horizon of 50 years. Although the problem differs from the current case study which concerns a single asset replacement with successive multiple intervention strategies, Hartman (2004) demonstrates the need for a DP model formulation under conditions of non-repeatable future life cycle cash flows.

Another case-specific DP approach originates from Hartman and Murphy (2006). In their study, the single asset replacement of equipment is investigated under a finite time horizon and stationary costs (repeatability of future cash flows). Under an infinite horizon, the solution to an optimised replacement chain under stationary costs is continuously replacing the asset at its economic life. However, under a finite time horizon, this classical approach will not lead to an optimised solution as there will be a trade-off between increasing operational and maintenance (O&M) expenditures and decreasing salvage values of multiple asset replacements within a fixed time horizon. Hartman and Murphy (2006) observed that the techniques required for dealing with these types of optimisation problems, such as DP, are not learned by all engineers or financial managers. Considering this reason, classical replacement theories that assume an infinite identical repeatability of the challengers' life cycle cash flows are used in practice for this different class of problems. Hartman and Murphy (2006) again demonstrated that this will lead to errors.

The closest study to the current one is an optimisation model developed by Regnier, Sharp, and Tovey (2004), which concerns an unbounded single asset replacement problem under total inflation (combined general and differential inflation) and technological change. Starting with a new investment, an optimised replacement chain for an infinite time horizon was developed using DP techniques. Different inflationary rates were allowed for (re)investments and O&M expenditures. The authors demonstrated that under total inflation, the economic life of an asset is not a constant and this feature influences the optimised replacement chain and, in many cases, the first replacement.

The authors proved that using classical techniques will lead to suboptimal decisions in replacement problems under inflation and technological change. Regnier et al. (2004) made assumptions about total inflation and price increases for operation and maintenance expenditures and assumed a constant growth (or decline) of future cash flows to model cash flows of future technology development. This facilitates compact mathematical formulas that support an easier present value calculation of cash flows. These assumptions, however, restrict the application of their model for the case study of this research because the current research considers successive intervention strategies from which the cash flows are not proportionally connected.

Mardin and Arai (2012) used the cost model of Regnier et al. (2004) to validate an adaptation of the classical defender/challenger (existing asset versus replacement option) comparison as an alternative to the more complex DP approach presented by Regnier et al. (2004). The adaptation used an improved approximation approach first introduced by Christer and Goodbody (1980), and further used by Christer and Scarf (1994) and Scarf and Hashem (1997). This approximation approach minimises the sum of the equivalent annual cost (EAC) of the defender and challenger seen as two consecutive assets at each period in time.

Although Mardin and Arai (2012) obtained good approximation results for the case-specific studies of Regnier et al. (2004), their method does not necessarily provide optimal solutions under other circumstances as this approach ignores the impact of future challengers with different cash flow patterns. Yatsenko and Hritonenko (2011) also compared the improved approximation method with the classical economic life comparison technique and the optimal DP or LP approach (Hartman & Murphy, 2006; Regnier et al., 2004). For comparison, the authors again used technology change scenarios from Regnier et al. (2004). Considering these scenarios, Yatsenko and Hritonenko (2011) concluded that the classical economic life comparison replacement technique provided good approximation results for small technological improvement rates only (<1%). For higher technological improvement rates, improved approximation approach was sufficiently accurate for the scenarios considered. However, the optimal results were obtained by using LP or DP techniques.

LP techniques for replacement optimisation under differential inflation and technological change receive less attention in the literature than DP techniques. LP is less efficient in its computations for shortest path problems. However, the availability of solvers and their computational power make LP a good alternative. Büyüktahtakın and Hartman (2016) used LP to solve a parallel replacement optimisation problem for a finite time horizon of 100 years. Brekelmans, den Hertog, Roos, and Eijgenraam (2012); Zwaneveld and Verweij (2014), and Dupuits, Schweckendiek, and Kok (2017) provide recent examples of an LP approach to find an optimised intervention strategy for a coastal flood defense system. A time horizon of 300 years was considered as an approximation of infinity. This time horizon is of interest for the current case study, as explained in Section 3.2. The study ignored differential inflation.

The review of the literature demonstrates that replacement decisions under inflation and/or technological change require consistent calculation and discounting of future cash flows with attention to inflationary effects, in combination with a case-specific DP or LP approach to find the least cost route over a bounded or unbounded time horizon. Using classical replacement theories will lead to errors. Improved approximation methods can be used in specific circumstances, but their applicability must be assessed for each case study in comparison with DP or LP approaches.

Calculating present values of future cash flows for all possible replacement scenarios is a daunting task. Therefore, several authors generalise cost functions, which restricts the applicability of the models for the commonly observed case study of this research. The literature review showed that many authors restricted the computational effort by introducing short asset service lives and calculation horizons. Only a few authors handled inflation and none of the authors made a clear distinction between general inflation, differential inflation, and age-related cost increases. The case-specific DP models in the literature start with a new investment and do not address the common case of optimising intervention strategies for ageing existing assets.

In the literature, defender and multiple challengers' optimisation problem under differential inflation is commonly absent. The objective of the current study is to develop an approach to find the optimised intervention intervals for ageing infrastructure assets considering multiple future intervention strategies. An explicit distinction is made between general inflation, differential inflation, and age-related cost increases. A nested DP approach is developed to find an optimised maintenance, renovation, and replacement chain.

6.3 Development of model and case study

In this section, a nested DP model is developed for the optimised maintenance, renovation, and replacement chain under differential inflation and age-related cost increases. The model is explained by means of a case study: an existing and ageing polder pumping station with options for lifetime-extending maintenance, renovation, and replacement. This approach is applicable to other types of infrastructure assets and not restricted to pumping stations.

In the Netherlands, pumping stations are owned by municipalities (sewerage transport), water boards (sewerage transport, water systems management), and drinking water utilities (drinking water transport). Older pumping stations are characterised by non-automated pumping units which require labour-intensive maintenance. Revision or a full replacement allows for partial or full automation and reduces O&M expenditures. Depending on the type of pumps, energy reduction can also be achieved. O&M and energy expenditures are subject to differential inflation. Ageing also affects O&M expenditures.

Description of the case study and initial EAC* comparison

The current defender is an old non-automated pumping station that needs an immediate major overhaul. The maximum remaining technical life of the old pumping station is estimated at 15 years, provided that three major overhauls are undertaken, each at 5-year intervals. The first option is to retain the old pumping station, while the second option (first challenger) is a full renovation. This extends the technical life of the current pumping station by 30 years.

Subsequent to the initial investment for renovation, two major overhauls are required over 10 and 20 years, respectively. The regular O&M expenditures decrease after renovation, whereas the annual electricity expenditures remain the same. The third option (second challenger) is a full replacement by a modern and fully automated pumping station. This reduces the annual expenditures for both O&M and electricity. Periodic major overhauls are then required every 15 years. The maximum technical life of the new pumping station is estimated at 60 years. As a boundary constraint, the last intervention strategy in the model, is considered to be a perpetuity (the strategy, not the cash flows). This perpetuity will be optimised in a separate DP-model that will be nested in the overall DP-optimisation model. Therefore, the final intervention strategy is modelled as an optimised perpetuity of full replacements, All data are presented in Table 6.1. These three intervention strategies are designated as maintain, renovate and continuously replace in the remainder of the document. Cost data, the real interest rate, and estimates for ageing factors are obtained from a water board and are representative for many ageing polder pumping stations in the Netherlands.

Table 6.1 Data for the case study

Annual interest Real = 4.00 % Nominal = 5.94 %	Costs initial (€)	Interval (years)	Annual Inflation General (%) f_g	 Differen -tial (%) f_d	 Total (%) f_t	Ageing (%/yr) g
Maintain N_{max} = 15 years						
Overhaul t = 0	150,000		1.87	1.03	2.92	
Overhaul t = 5	150,000		1.87	1.03	2.92	
Overhaul t = 10	150,000		1.87	1.03	2.92	
O&M	70,000	1	1.87	0.85	2.74	1.5
Electricity	15,000	1	1.87	0.20	2.07	0.0
Renovate N_{max} = 30 years						
Initial investment	1,125,000		1.87	1.03	2.92	
Overhaul t = 10	150,000		1.87	1.03	2.92	
Overhaul t = 20	150,000		1.87	1.03	2.92	
O&M	35,000	1	1.87	0.85	2.74	1.0
Electricity	15,000	1	1.87	0.20	2.07	0.0
Replace N_{max} = 60 years						
Initial investment	2,400,000	60	1.87	1.03	2.92	
Overhauls	150,000	15	1.87	1.03	2.92	
O&M	25,000	1	1.87	0.85	2.74	0.5
Electricity	30,000	1	1.87	0.20	2.07	0.0

Salvage value is end-of-life cash to be received when selling an asset at a certain age (Brealey, Myers, & Allen, 2017). End-of-life demolition and scrap values are considered in this case study and incorporated in following investment costs of a successive intervention strategy. These costs are treated as fixed costs as their time-variant proportion is considered negligible for the case study. Time-variant salvage values from trading are not considered in this case study. Public infrastructure assets as investigated in the current research are mostly not tradable, and therefore generally do not have these types of salvage values. When cash flows cannot be appropriately truncated (salvaged) at the end of a calculation horizon, the convention in the domain of engineering economics is to estimate all expected future life cycle costs that contribute to the present value of a scenario (Blank & Tarquin, 2012; Park, 2011; Sullivan et al., 2012). Although technical lives of public infrastructure assets are finite, the required functionality, such as protection against high-water, is likely to be infinite. Several replacements, which can be identical or not, approximate such infinity. Although not included in the case study, equations for the calculation of time-variant salvage values from trading and time-variant demolition costs are provided in Section 3.3.

The case study uses an average real interest rate of 4%, which is common for public infrastructure assets in the Netherlands. 4% reflects the average weighted cost of capital of the water board. The long-term general inflation rate is obtained from the consumer price index (CPI) over the years 1995 until 2017 and estimated at 1.87%, based on the analysis of its past development. The relation between the nominal discount or interest rate r_{nom}, real discount rate r_{real}, and general inflation rate f_g is given by (Brealey et al., 2017; Park, 2011; Sullivan et al., 2012)

$$r_{nom} = r_{real} + f_g + r_{real} \cdot f_g = (1 + r_{real})(1 + f_g) - 1, \tag{6.1}$$

with a general inflation rate of 1.87%, real interest rate of 4%, and nominal discount rate of 5.94%.

Differential inflation is specific for each cost component. Differential inflation is additional incremental (or decremented) inflation next to general inflation (Sullivan et al., 2012). General inflation is measured by the CPI. Industrial goods and services are often subject to higher price increases and measured by the producer price index (PPI). The relation between the general inflation rate f_g, differential inflation rate f_d, and the so-called total price escalation rate f_{tot} (total inflation, PPI) is given by (Brealey et al., 2017; Sullivan et al., 2012)

$$f_{tot} = f_g + f_d + f_g \cdot f_d = (1 + f_g) \cdot (1 + f_d) - 1. \tag{6.2}$$

Equation 6.1 and Equation 6.2 mathematically define and incorporate an important engineering economics implication for discounting of cash flows. Total inflation expresses cash flows in nominal currency. Differential inflation expresses cash flows in real currency. Nominal cash flows (inflated with total inflation) need to be discounted with a nominal discount rate. Real cash flows (inflated with differential inflation) need to be discounted with a real discount rate (Park, 2011; Sullivan et al., 2012). Equation 6.1 and Equation 6.2 define that both discounting approaches are mathematically equal. The current research expresses cash flows in real values and discounts with a real discount rate.

Under differential inflation, certain cash flow components grow (or decline) faster than others and also continue to grow (or decline) after new investments. The estimates for differential inflation in Table 6.1 are subtracted from PPI data over the years 1995–2017.

Model description

This study aims to develop an optimised chain of intervention strategies under differential inflation. This section develops a nested DP optimisation model. To explain the model, first a downscaled version of the maintain, renovate and replace optimisation problem is used in the figures and tables. Hereafter, the model formulation is applied to the full-scaled case study. In the downscaled example, the maximum technical lives for the maintain, renovate and replace options are restricted to 3, 5, and 4 years, respectively in comparison to 15, 30, and 90 years, respectively in the case study. The technical lives for the downscaled example do not have a physical meaning and are only meant for explaining the structure of the model. The total time horizon in the downscaled example is restricted to 10 years instead of an approximated infinite time horizon in the case study. Replacements (but neither cash flows nor economic lives) are considered to be repeated until the end of the time horizon as motivated in Section 3.1. The renovation option is the potential first or second intervention strategy before the replacement chain. The model allows for inclusion of more in-between intervention strategies by following the same approach.

The network corresponding to this example is visualised in Figure 6.1. The node S_0 represents the source node from which the current decisions start, and Z_T represents the termination node where all replacement decisions end. The nodes X_i and Y_j represent the years in which the maintain and renovate options end, respectively. For example: X_3 is read as the year in which a maintain strategy ends (here year 3) and Y_5 is defined as the year in which a renovation strategy ends (here year 5). We start with an existing asset in place, the maintain strategy. The path $S_0 - X_3 - Y_5$ therefore represents the scenario: maintain from year 0 until year 3, renovate at year 3 and keep until year 5 (two years of a renovation strategy). From year 5 onwards, the (renovated) assets will be continuously replaced over its time-variant optimised. Note that in Figure 6.1, there are no arcs from X_i to Y_j for $j < i$ and $j > i + 5$, where 5 is the maximum number of years the asset can be renovated in this example.

A decision variable x_i is introduced to indicate whether the asset is maintained from year 0 till year i. This corresponds to the arc from node S_0 to node X_i. The parameter c_i^x represents the present value of the corresponding cost and N_x represents the maximum service life of the maintain option. Similarly, y_{ij} is introduced to indicate whether the asset is renovated from year i till year j. This corresponds to the arc from node X_i to node Y_j in the network.

Again, c_{ij}^{y} represents the present value of the corresponding cost and N_y represents the maximum service life of the renovate option. Finally, decision variable z_j indicates whether continuous replacements start in year j. This corresponds to ending the renovation of the asset in year j. The present value of the costs of continuous replacements from year j till year T (termination node Z_T) is denoted by c_j^z. The value for c_j^z is obtained by solving a separate regeneration model, which is explained at the end of this section. An overview of the present values of the costs of the arcs in Figure 6.1 is shown in Table 6.2. We emphasise that the cost variables in Table 6.2 represent the present values of life cycle costs from instalment until the time where a successive intervention strategy starts.

Table 6.2 Cost matrix for present values of costs of the arcs in the decision network in Figure 6.1

	j	0	1	2	3	4	5	6	7	8	
i	Nodes	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	S_0
0	X_0	c_{00}^y	c_{01}^y	c_{02}^y	c_{03}^y	c_{04}^y	c_{05}^y				c_0^x
1	X_1		c_{11}^y	c_{12}^y	c_{13}^y	c_{14}^y	c_{15}^y	c_{16}^y			c_1^x
2	X_2			c_{22}^y	c_{23}^y	c_{24}^y	c_{25}^y	c_{26}^y	c_{27}^y		c_2^x
3	X_3				c_{33}^y	c_{34}^y	c_{35}^y	c_{36}^y	c_{37}^y	c_{38}^y	c_3^x
	Z_T	c_0^z	c_1^z	c_2^z	c_3^z	c_4^z	c_5^z	c_6^z	c_7^z	c_8^z	

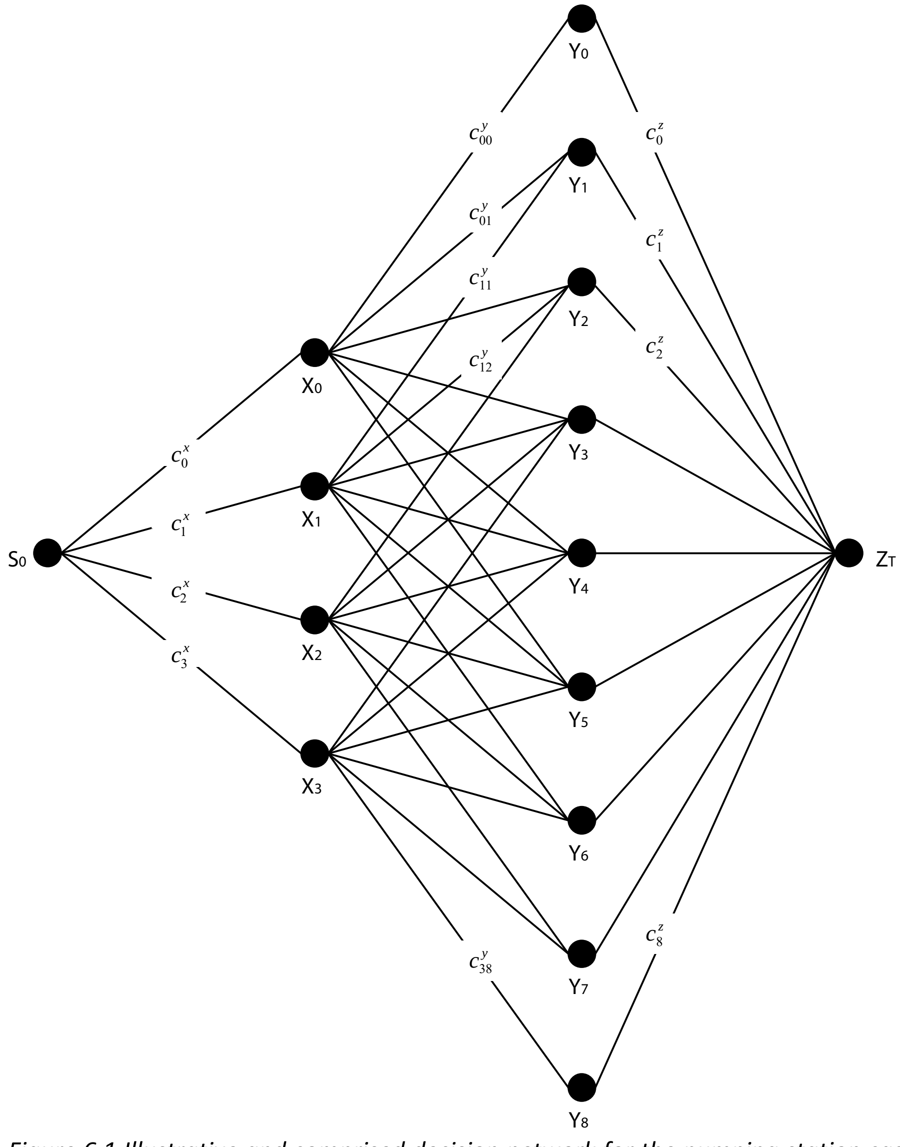

Figure 6.1 Illustrative and comprised decision network for the pumping station case study with maximum service lives for the maintain, renovate, and replacement options of respectively 3, 5 and 4 years and termination at year 10

The optimal maintain, renovate, and replace decision is given by the shortest path in this network. The shortest path in a network can be efficiently found by means of Dijkstra's algorithm (Dijkstra, 1959). However, owing to the special structure of the network, where each path between S_0 and Z_T has a fixed length, a more efficient backward recursion can be used. This is shown in the next paragraph as Equation 6.4.

Even though the problem is not solved using LP, the LP formulation helps to understand the structure of the problem. The objective of the model is to find the least cost route from S_0 to Z_T and is given by

$$\min\left[\sum_{i=0}^{N_x} c_i^x \cdot x_i + \sum_{i=0}^{N_x}\sum_{j=i}^{i+N_y} c_{ij}^y \cdot y_{ij} + \sum_{j=0}^{N_x+N_y} c_j^z \cdot z_j\right], \tag{6.3}$$

subject to the constraints

$$\sum_{i=0}^{N_x} x_i = 1,$$

$$\sum_{j=i}^{i+N_y} y_{ij} = x_i \qquad \forall i,$$

$$\sum_{i=j-N_y}^{j} y_{ij} = z_j \qquad \forall j,$$

$$x_i, y_{ij}, z_j \in \{0,1\} \qquad \forall i, j.$$

Regeneration model

The next step is to find the cost of continuous replacements from year j till year T, which is denoted by c_j^z. These continuous replacements follow from their own optimisation model, which is schematised in Figure 6.2 for a restricted maximum service life of a replacement option of 4 years (N_z = 4 years) and a restricted time horizon of $T = 10$ years. To solve the optimisation of the continuous replacements, the regeneration model explained by Wagner (1975) is expanded and solved for each start year j of the possible series of replacements.

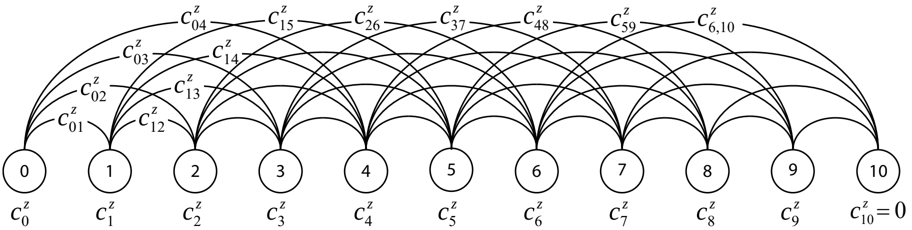

Figure 6.2 Schematic representation of the regeneration network for the continuous replacements

The decision variable z_{ij} is defined to indicate whether the asset is replaced in year i and discarded in year j. The present values of the corresponding costs are denoted by c^z_{ij}. This is again a shortest path problem that can be solved by Dijkstra's algorithm. Considering the case study, we choose to use a more efficient DP algorithm that uses the following backward recursion:

$$c^z_j = \min_{k=j+1,\dots j+N_z} (c^z_{ji} + c^z_i). \tag{6.4}$$

Here, c^z_{ij} is given as an input, c^z_i comes from the previous iterations of the algorithm, and N_z represents the maximum technical life. The recursion is initiated with $c^z_T = 0$. Again, we also provide the LP formulation of the problem. The objective function of an optimised replacement chain between year S and year T is:

$$\min \sum_{i=S}^{T-1} \sum_{j=i+1}^{T} c^z_{ij} \cdot z_{ij} \ , \tag{6.5}$$

subject to the constraints

$$\sum_{i=S}^{T-1} z_{iS} - \sum_{k=S+1}^{T} z_{Sk} = -1 \, ,$$

$$\sum_{i=S}^{T-1} z_{ij} - \sum_{k=S+1}^{T} z_{jk} = 0 \qquad \forall j \neq S, j \neq T \, ,$$

$$\sum_{i=S}^{T-1} z_{iT} - \sum_{k=S+1}^{T} z_{Tk} = 1 \, ,$$

$$z_{ij} \in \{0,1\} \qquad \forall i, j \, .$$

For the regeneration model, DP is preferable to LP as one run of the DP algorithm directly calculates the least costs from each start node S to termination node Z. When solving the problem for c_0^z, the values c_j^z for all j are found as part of the recursion. Therefore, the solutions to all the regeneration models that follow the end of a renovation option are automatically found. In contrast, an LP approach to find the minimal cost c_j^z would require solving 45 LP problems in the case study, one for each j. The costs of the arcs in Figure 6.2 are presented in a matrix structure in Table 6.3.

Table 6.3 Cost matrix for the regeneration model (continuous replacements, comprised example)

	j	0	1	2	3	4	5	6	7	8	9	10
i	Nodes	Z_0	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
0	Z_0		c_{01}^z	c_{02}^z	c_{03}^z	c_{04}^z						
1	Z_1			c_{12}^z	c_{13}^z	c_{14}^z	c_{15}^z					
2	Z_2				c_{23}^z	c_{24}^z	c_{25}^z	c_{26}^z				
3	Z_3					c_{34}^z	c_{35}^z	c_{36}^z	c_{37}^z			
4	Z_4						c_{45}^z	c_{46}^z	c_{47}^z	c_{48}^z		
5	Z_5							c_{56}^z	c_{57}^z	c_{58}^z	c_{58}^z	
6	Z_6								c_{67}^z	c_{68}^z	c_{69}^z	$c_{6,10}^z$
7	Z_7									c_{78}^z	c_{79}^z	$c_{7,10}^z$
8	Z_8										c_{89}^z	$c_{8,10}^z$
9	Z_9											$c_{9,10}^z$
10	Z_{10}											-

In theory, the decisions to be made on continuous replacements are infinite. In the case study, the solution space is reduced by choosing a finite boundary for T that approximates infinity, such that cash flows beyond T do not significantly contribute to the total present value of a maintain, renovate, and replacement chain. As long as the discount rate exceeds the total escalation rate of cash flows, the total present value is a concave and asymptotic function.

Several studies investigated how to assess a minimum approximation of infinity, such as Bean, Lohmann, and Smith (1994); Regnier et al. (2004); and Wagner (1975). These studies demonstrate that a minimum approximation of infinity is reached for a horizon length (terminal state) that is independent of the first decision to be made. To find such a minimum horizon length, successive approximations can be used. A DP algorithm is terminated as soon as an additional time period does not influence the first decision.

As a practical and safe estimate for public infrastructure assets with high investment costs and relatively low O&M expenditures, a boundary of 300 years is chosen as an approximation of infinity. The motivation is that the real costs of a full replacement in year 300 contribute to only a factor $1/(1+r_{real})^n = 1/(1+0.05)^{300} = 4.4 \cdot 10^{-7}$ to the total present value of all costs between year 0 and year 300. 300 years is in line with another case study dealing with capital-intensive infrastructure with long service lives (coastal flood protection) presented by Brekelmans et al. (2012); Zwaneveld and Verweij (2014); and Dupuits et al. (2017).

Present value calculations under inflation and age-related cost increases

This section outlines the calculation of the present values c_i^x, c_{ij}^y, and c_{ij}^z (c_j^z follows from the application of the regeneration model). The variable c_i^x represents the present value of maintaining the pumping station from year 0 to year i, c_{ij}^y represents the present value of a renovate option that starts in year i and ends in year j, and c_{ij}^z represents the present value of a replace option that starts in year i and ends in year j.

The cost calculations under total inflation, differential inflation, and age-related cost increases are rarely addressed in the literature. These factors are a real issue in practice. The relations between total inflation f_{tot}, differential inflation f_d, general inflation f_g, real interest rate r_{real}, and nominal interest rate i_{nom} are depicted in Equations 6.1 and 6.2. The general inflation rate is equal for all cost categories: investments, major overhauls, O&M expenditures, and electricity costs. The differential inflation differs across categories, and therefore the total inflation rate is also specific for a cost category.

Inflation should be treated consistently in present value analyses. Cash flows that are inflated with the total inflation rate f_{tot} are discounted with the nominal or effective discount rate r_{nom}. Cashflows that are inflated with a differential inflation rate f_d are discounted with the real interest rate r_{real} (Brealey et al., 2017; Park, 2011; Sullivan et al., 2012):

$$\frac{(1+f_{tot})}{(1+r_{nom})}=\frac{(1+f_g)(1+f_d)}{(1+r_{nom})}=\frac{(1+f_d)}{(1+r_{real})}\,. \tag{6.6}$$

Discounting of real and nominal cash flows with the appropriate discount rate leads to the same present values, as noted in Equations 6.1 and 6.2. By definition, all present values c_i^x, c_{ij}^y, and c_{ij}^z have the same baseline $t=0$. The following equations are used to calculate the present values. The equations are expressed in nominal terms, and their symbols and indices are presented in Table 6.4.

Under differential inflation, the present value ($t=0$) of an initial investment I for an asset bought in year t and disposed of at age n is modelled as

$$P_{I[t,t+n]}=\frac{I_{(0)}(1+f_g)^t(1+f_{d,I})^t}{(1+r_{nom})^t}\,. \tag{6.7}$$

The initial investment expressed in the current price level for an asset bought in year t is inflated with general inflation and differential inflation to year t, and discounted with the nominal discount rate from year t to the present.

Public infrastructure assets generally do not have salvage values from trading as motivated in Section 3.1. However, if these salvage values are relevant, the present value of a salvage value of an asset bought in year t and disposed of at age n can be modelled as

$$P_{S[t,t+n]}=\frac{-I_{(0)}(1+f_g)^t(1+f_{d,I})^t\cdot(1-b)^n(1+f_g)^n(1+f_{d,S})^n}{(1+r_{nom})^{t+n}}\,. \tag{6.8}$$

The factor b represents an annual reduction in the initial investment, expressed as a percentage. The investment is first inflated to the year of purchase, then reduced to calculate the salvage value at age n. A minus sign is added before the investment costs as a salvage value is income. The salvage value may have different inflation rates than the initial investment, and therefore the salvage value is inflated from year t to age n with its own inflation rate. Finally, the inflated salvage value at time $t+n$ is discounted to the present using the nominal discount rate.

Table 6.4 Symbols and indices used in present value equations

t	Time of purchase or instalment ($t = 0$ for c_i^x and $t = i$ for c_{ij}^y , c_{ij}^z)
I S D H M E	Initial investment cost Salvage value Demolition costs Costs of a major overhaul First year's operation and maintenance costs First year's electricity costs
$\#_I$ $\#_S$ $\#_D$ $\#_H$ $\#_M$ $\#_E$	Index for investment Index for salvage values Index for demolition costs Index for major overhauls Index for operation and maintenance costs Index for electricity costs
$\#_{(0)}$	Index for current price level, real (non-inflated) costs, constant currency
f_{tot} f_g f_d	Total inflation rate [% per year] General inflation rate [% per year] Differential inflation rate [% per year]
r_{nom} r_{real}	Nominal interest rate [% per year]; includes general inflation Real interest rate [% per year]; excludes general inflation
g	Age related price increase [% per year]
b dm	Reduction of the initial investment for calculating the salvage value [% /year] A percentage of the initial investment to calculate the end of life demolition costs
n $n_1, n_2, ..n_q$	The age of an asset when disposed of ($n = i$ for c_i^x and $n = j$ for c_{ij}^y , c_{ij}^z) The age at which the 1st, 2nd, ... qth major overhaul takes place with $n_q < n$
P	The present value (t=0) of cash flows incurred from t to $t + n$

In the case of end-of-lifetime-variant demolition costs D, the present value of these costs can be modelled in a similar manner, as shown in Equation 6.9. The time-variant demolition costs are modelled as a percentage dm of the initial investment at age n and a separate inflation rate for the demolition costs is used.

$$P_{D[t,t+n]} = \frac{I_{(0)}(1+f_g)^t(1+f_{d,I})^t \cdot dm \cdot (1+f_g)^n(1+f_{d,D})^n}{(1+r_{nom})^{t+n}} \tag{6.9}$$

However, for the infrastructure case study, demolition costs are not considered to be age-related (no significant age-related scrap value) and included as fixed costs in the successive investment costs.

Major overhauls ($H_{(0),1}, H_{(0),2}, \dots, H_{(0),q}$) are planned periodically at age $n_1, n_2, \dots, n_q$. $H_{(0),1}$ is the cost of the first major overhaul in the current price level, $H_{(0),2}$ is the cost of the second major overhaul in the current price level, and $H_{(0),q}$ is the cost of the last major overhaul in the current price level. The parameter n_q represents the age of the asset at the last major overhaul, a number of years before the end of its service life (n). The present value of major overhauls of an asset bought at time t and disposed of at age n is modelled as

$$P_{H[t,t+n]} = \frac{H_{(0),1}(1+f_g)^{t+n_1}(1+f_{d,H_1})^{t+n_1}}{(1+r_{nom})^{t+n_1}} + \frac{H_{(0),2}(1+f_g)^{t+n_2}(1+f_{d,H_2})^{t+n_2}}{(1+r_{nom})^{t+n_2}} +$$

$$\dots + \frac{H_{(0),q}(1+f_g)^{t+n_q}(1+f_{d,H_q})^{t+n_q}}{(1+r_{nom})^{t+n_q}}.$$

(6.10)

The yearly O&M expenditures (M) can be subject to both inflation and age-related price increases (g). O&M expenditures can increase with age owing to increasing failures and maintenance needs. To model the present value of O&M expenditures, the geometric gradient factor is adapted by substituting r_{nom} for r_{real}, conforming to Equation 6.1.

The generic geometric gradient with cash flows in real terms is (Newnan et al., 2016; Park, 2011; Sullivan et al., 2012)

$$P = A_{1,real}\frac{1-(1+p)^n(1+r_{real})^n}{r_{real}-p}. \quad (6.11)$$

The parameter A_1 is the first year's real costs, n is the age of the asset, and p is an annual percentage price increase. First, p is split into a part reflecting differential inflation and a part reflecting age-related price increases as follows:

$$(1+p)=(1+f_d)(1+g)\Rightarrow$$
$$p=(1+f_d)(1+g)-1.$$

Substituting $(1+p)$ and p in Equation 6.11 results in

$$P=A_{1,real}\frac{1-(1+f_d)^n(1+g)^n(1+r_{real})^{-n}}{r_{real}-(1+f_d)(1+g)+1}. \tag{6.12}$$

Expressing the right-hand side in nominal terms requires the substitution of

$$(1+r_{real})=\frac{(1+r_{nom})}{(1+f_g)},$$

$$r_{real}=\frac{(r_{nom}-f_g)}{(1+f_g)},$$

$$A_{1,real=}\frac{A_{1,nom}}{(1+f_g)^1}.$$

Performing these substitutions results in a geometric gradient with cash flows expressed in nominal terms as follows:

$$P=A_{1,nom}\frac{1-(1+f_d)^n(1+g)^n(1+f_g)^n(1+r_{nom})^{-n}}{r_{nom}-(1+f_d)(1+g)(1+f_g)+1}. \tag{6.13}$$

With this expression, the present value of O&M expenditures for an asset bought at time t and kept in service for n years is modelled as

$$P_{M[t,t+n]} = \frac{M_{(0)}(1+f_g)^{t+1}(1+f_{d,M})^{t+1}(1+g_M)^1}{(1+r_{nom})^t} \cdot \frac{1-(1+f_g)^n(1+f_{d,M})^n(1+g_M)^n(1+r_{nom})^{-n}}{r_{nom}-(1+f_g)(1+f_{d,M})(1+g_M)+1} . \tag{6.14}$$

The parameter $M_{(0)}$ represents the first-year O&M costs in the current price level (base year 0), which are inflated from year 0 to the first year after purchase $(t+1)$ of an asset. These inflated first year O&M costs are then multiplied with the classical geometric gradient factor, adapted for nominal cash flows. This results in the nominal future value of a series of n years of increasing O&M costs at the time of purchase t. To find the present value (base year 0), these costs are discounted over t using the nominal discount rate.

A similar process is followed to calculate the present value of the annual electricity costs E

$$P_{E[t,t+n]} = \frac{E_{(0)}(1+f_g)^{t+1}(1+f_{d,E})^{t+1}(1+g_E)^1}{(1+r_{nom})^t} \cdot \frac{1-(1+f_g)^n(1+f_{d,E})^n(1+g_E)^n(1+r_{nom})^{-n}}{r_{nom}-(1+f_g)(1+f_{d,E})(1+g_E)+1} . \tag{6.15}$$

The age-related price increase for electricity costs g_E could arise, for example, owing to greater electricity consumption as assets age.

Excluding salvage values from trading and time-variant demolition costs (see case study description for the underlying motivation), the total present value of an asset $(t=0)$ installed in year t and kept until year $t+n$ is calculated using Equations 6.7, 6.10, 6.14, and 6.15 as

$$c_{[t,t+n]} = P_{I[t,t+n]} + P_{H[t,t+n]} + P_{M[t,t+n]} + P_{E[t,t+n]} . \tag{6.16}$$

Equation 6.16 is used to calculate the present values c_i^x, c_{ij}^y, and c_{ij}^z. Regarding the formulation of the shortest path problems, the following is considered:

$$t = 0 \text{ for } c_i^x,$$

$$t = i \text{ for } c_{ij}^{y}, c_{ij}^{z},$$

$$n = i \text{ for } c_{i}^{x},$$

$$n = j \text{ for } c_{ij}^{y}, c_{ij}^{z}.$$

6.4 Results for the case study

The present values c_i^x, c_{ij}^y, and c_{ij}^z for the case study are obtained using Equation 6.16. The DP recursion in Equation 6.4 is first used to calculate the c_j^z values of the regeneration model (Figure 6.2), which in the current case study represent the present values of a chain of optimised continuous replacements starting at $t = 0, 1, 2, \ldots, 45$ and ending at $T_Z = 300$ years.

Hereafter, the combined maintain, renovate, and continuously replace network is solved (Figure 6.1) using the same DP recursion. The results of the regeneration model for continuous replacements that start at $t = 0$ are presented in Table 6.5. The c_j^z values for j = 0 to 45 (thus, the total present values of each optimal replacement chain starting at time j) are shown in Figure 6.3.

The current case study uses DP to calculate the future optimal service lives of the continuous replacements starting at $t = 0$ and ending at t = 45. The postponement of this optimised replacement chain will decrease its present value as depicted in Figure 6.3. These present values (c_j^Z) in Figure 6.3 represent the optimised cost values for the final paths $Y_j - Z_t$ in Figure 6.1. Table 6.5 shows that the economic lives of future challengers remain unchanged from the current viewpoint. This is a characteristic of the current case study. Different asset types with different cost profiles will result in other economic lives which are not likely to be equal. In addition, starting the replacement chain in the future instead of $t = 0$ influences the economic lives. Although economic service lives of the continuous replacements are similar in the current case study, it still needs a DP solution for continuous replacements as differential inflation rejects the repeatability assumption of the future cash flows. A comparison with a classical approach (without DP solutions) follows in Section 6.5.

Table 6.5 Results of optimal continuous replacements when starting at t = 0

Regeneration model	Replace 1st	Replace 2nd	Replace 3rd	Replace 4th	Replace 5th
Optimal life	60	60	60	60	60
Time interval	[0,60]	[60,120]	[120,180]	[180, 240]	[240, 300]
Present values (baseline t = 0)	$c^z_{0,60}$= € 3,237,632	$c^z_{60,120}$= € 547,643	$c^z_{120,180}$= € 93,695	$c^z_{180,240}$= € 16,152	$c^z_{240,300}$= € 2,798
Total present value	c^z_0= € 3,897,920				

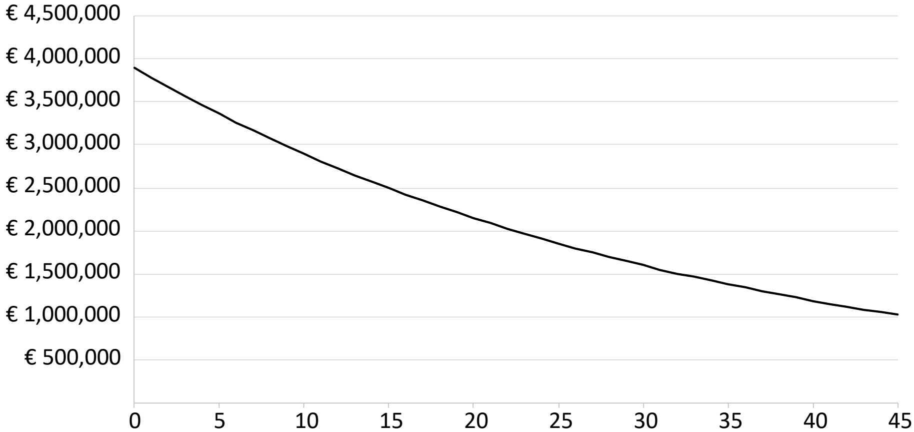

Figure 6.3 Present values c^z_j of optimal replacement chains starting at $t = j$ and ending at $t = 300$

The combined optimisation model in Figure 6.1 incorporates the current defender with the renovation option and continuous replacements. In essence, solving the mathematical model does not differ from the regeneration model. The only difference is another network structure, a different corresponding cost matrix, and different cost values. The final calculation results are presented in Table 6.6. The optimal strategy is to replace the pumping station immediately. The optimised path is X_0, Y_0, and Z_{300} (recall that the index represents the year in which the activity ends). Several additional infrastructure case studies with other realistic input data were considered in this DP model leading to different paths for optimal strategies, such as path X_{15}, Y_{25}, and Z_{300}. The path X_0, Y_0, and Z_{300} of the current case study may not appear very exciting. However, this path is an equally optimised path among many possible paths.

Table 6.6 Results of optimised maintain, renovate, and replacement chain.

Maintain, renovate, replace	Maintain	Renovate	Replace and regenerate
Node	X_0	Y_0	Z_{300}
Service life	0 years	0 years	300 years
Time	[0,0]	[0,0]	[0,300]
Present values at t = 0	c_0^x = € 0	$c_{0,0}^y$ = € 0	c_0^z = € 3,897,920
Total present value	$P_{[0,300]}$ = € 3,897,920		

6.5 Comparison with classical approach to replacement analysis

The classical approach to replacement analysis compares EAC values of the maintain, renovate, and replace options at their economic lives. The classical theory is well described in textbooks by authors such as Blank and Tarquin (2012); Hastings (2015); Newnan et al. (2016); Park (2011); and Sullivan et al. (2012).

As explained in Section 6.2, the classical economic life comparison cannot be used when differential inflation is involved. A decision maker not familiar with DP techniques could therefore choose to ignore differential inflation or to include differential inflation in the classical calculation techniques. Both situations are incorrect. In the first case, the calculations are correct but real costs caused by differential inflation are ignored. In the second case, there is an attempt to include real costs caused by differential inflation, but the calculations will be incorrect owing to the repeatability assumption of the challengers' cash flows that will not hold in the comparison of minimum EAC values. Both the cases are investigated and compared to the optimal DP solution.

Excluding differential inflation in the case study (all differential inflation is set to zero), results in the minimum EAC* values as depicted in the top part of Table 6.7. Based on these values, a decision maker would maintain the defender for 5 years. The major overhaul necessary for maintaining the defender at the end of year 5 prompts a renovation. The renovated pumping station is retained for 30 years before replacing it. The classical approach in this example searches for the least total present value which is obtained by the lowest sequence of EAC^* values as the expensive major overhauls will enforce an intervention at the calculated economic lives. Without major overhauls, optimised intervention times in a classical defender-challenger replacement analysis may occur a couple of years beyond the economic service lives (Park, 2011). However, this is not the case in the current case study.

Table 6.7 Classical EAC comparison calculated at t = 0.*

Baseline t = 0	EAC*	n* (years)
Excluding differential inflation		
Maintain now and keep for 5 years	€ 121,822	5
Renovate now and keep for 30 years	€ 129,703	30
Replace now and keep for 60 years	€ 138,430	60
Including differential inflation		
Maintain now and keep for 5 years	€ 123,765	5
Renovate now and keep for 10 years	€ 136,496	30
Replace now and keep for 60 years	€ 143,109	60

The total present value of this scenario is presented in Table 6.8 and follows from straightforward discounting of a 5 years' annuity of €121,822 starting in year 1, a 30 years' annuity of €129,703 starting in year 6, and an infinite annuity of €138,430 starting in year 36. Please note that annuities start 1 year after an investment, thus in year $t + 1$ and are first discounted to year t. Hereafter, this local present value is discounted to the present to $t = 0$.

The second case includes differential inflation in the classical economic life comparison. This leads to the EAC* values depicted in the bottom part of Table 6.7. The EAC* values are marginally higher owing to extra costs induced by differential inflation. Based on these EAC* values, a decision maker would also maintain the pumping station for 5 years, renovate and retain it for 30 before replacing it. This is not correct as the EAC* values should not be treated as constants, as a consequence of differential inflation. The total present value is presented in Table 6.8 and calculated similar to the previous case, though only the cost values differ.

Comparing the classical approach with the DP model shows a difference in optimal strategies and in their total present values. The classical approach underestimates the total cost of the case study in a range of k€500 to k€635 on an investment volume of k€2,500 in comparison to the optimal DP solution. Underestimating the real costs leads to a suboptimal strategy. The DP solution favours an early replacement, while the classical approach advises to postpone the replacement because the relatively high differential inflation on O&M expenditures make the maintain and renovate options less attractive.

Table 6.8 Comparison of DP solution and classical replacement techniques

Intervention strategies	Maintain	Renovate	Replace and regenerate	Total present value (€)
Classical without differential inflation	5 years t = [0,5]	30 years t = [5,35]	Infinite t = [35, ∞]	3,262,784
Classical with differential inflation (wrong application)	5 years t = [0,5]	30 years t = [5,35]	Infinite t = [35, ∞]	3,397,622
Optimal DP solution	0 years	0 years	Infinite t = [0, approx. ∞]	3,897,920

Under differential inflation and multiple successive intervention strategies, it is not possible to derive generic rules that estimate the deviations from the classical approach as too many variables are involved. Therefore, each case study needs to be judged on its case-specific circumstances. The number of successive intervention strategies is of importance. The cost profiles of intervention strategies may differ significantly. Differential inflation is positive in the case study considered, but it can also be negative depending on the type of costs considered. As ageing plays a role, the timing of major overhauls can be a decisive factor. Discount rates are important too. Low discount rates, as seen in public sector organisations, amplify the impact of differential inflation. The current study demonstrates that differential inflation matters and requires careful assessment.

The contribution of the current research to existing literature is twofold. First, for infrastructure assets, it confirms the case-specific conclusion of other authors on the limitations of classical replacement techniques. Second, the current research developed a novel nested DP model capable of dealing with multiple successive intervention strategies under differential inflation. Instead of three intervention strategies in the case study (maintain, renovate, and continuously replace), more intervention strategies can be included, following the same approach.

6.6 Discussion and limitations

Although DP and LP techniques provide accurate results in comparison to classical replacement techniques, there are limitations to this approach. In all likelihood, the most important one is that practitioners are not familiar with DP and LP techniques. Replacement analyses using these techniques are not found in conventional textbooks on engineering economics. Applying this approach in practice may therefore be challenging.

A second limitation to the nested DP model is its deterministic nature. This does not undermine the value of the described optimisation method. Probabilistic models are used to incorporate uncertainty in the timing and size of costs. These probabilistic models underlie the cost values in the cost matrices. Adding a probabilistic model would improve the accuracy of the cost matrices and results of the approach, but would not alter the nested optimisation method. The challenge of introducing uncertainty in the current optimisation approach is considered for further research.

A third limitation to the case study is that only two challengers are considered, a renovation option and a continuously replace option. The future may hold more than two challengers. Adding additional challengers to the described optimisation approach follows the same methodology, but will require more cost calculations of the paths in the network. This study proposes to be practical. The case study shows that for the cash flow patterns of common public infrastructure assets, decisions generally occur before one of the major overhauls or the end of the technical life of an asset. This is owing to the fact that major overhauls are expensive, and the costs of overhauls are generally much greater than the regular O&M expenditures. Cost calculation efforts can be significantly reduced by limiting the decision nodes to the intervals of the major overhauls and technical lives. This will also enhance the applicability of the model in practice.

A fourth limitation is that the future is uncertain. Nevertheless, the prime interests of a maintenance engineer are the short- and mid-term decisions, which are influenced by the long-term estimates of future costs. A reasonable estimate of the future costs is adequate in this context. The described optimisation approach already provides a more accurate estimate than the classical methods, which assume continuous repeatability of the first challenger’s life cycle cash flows.

Finally, emphasises is provided to the complexity of replacement decisions in general and least costs are just one of the replacement criteria involved.

6.7 Conclusion

Several authors have investigated the application of classical replacement techniques under inflation and technological change, and concluded in their case studies that using classical replacement techniques will lead to errors. DP and LP techniques are required to identify optimal replacement strategies when the assumption of continuous repeatability of life cycle cash flows of future intervention strategies does not hold. Case-specific modelling is applied to find the least cost route in a network of probable future scenarios.

In this study, a novel nested DP model is developed for a replacement problem that is common for many public-sector infrastructure organisations. This replacement problem is demonstrated in a case study that consists of an existing asset and multiple successive intervention strategies under differential inflation. The multiple intervention strategies include a renovate option followed by a continuously replace option as a final estimate for future cash flows. Although the last intervention strategy considers continuous replacements, the life cycle cash flows of these replacements are non-repeatable owing to differential inflation. The optimisation model can be extended with more successive intervention strategies which allows for simulating flexible technology change. Total inflation, differential inflation, and age-related cost increases are explicitly addressed as these are realistic in practice and should not be ignored. The optimisation model is applied to a case-study which demonstrates that the inclusion of differential inflation influences the optimised total intervention strategy.

The entire optimisation model is described as a nested DP approach. First, the continuously replace optimisation is solved, providing the present values of replacement chains starting at different future times. Second, the three alternatives (maintain, renovate, and continuously replace) are combined and optimised for the lowest total present value. This yields an optimal intervention chain for maintaining, renovating, and replacing the asset.

For infrastructure assets, optimal intervention decisions are very likely to occur just before a major overhaul or the end of the technical life of an asset. This feature can be used to reduce the size of the solution matrix and cost calculations. The optimisation approach provides a realistic solution for a common infrastructure asset replacement problem of an existing asset and multiple successive intervention strategies under differential inflation.

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7

Compound real options

Optimise a sequence of distinct strategies
Non-repetitive cash flows
Uncertain future

The model presented in this chapter equally optimises a sequence of intervention strategies as in the previous chapter but adds price uncertainty and includes all features considered in the current research.

The novelty of this work is an integrative approach for economic optimisation incorporating asset degradation (failure rate), structural failure (failure probability), multiple price uncertainties, multiple sequential alternatives and managerial flexibility. These features are often seen in combination in practice not combined in one optimisation approach in the literature.

A compound real options analysis is modelled with a Markov Decision Process. Such an analysis quickly suffers from an exponentially increasing number of uncertainty states which requires large computational calculation power. To prevent state explosion Portfolio Theory is applied which allows for merging multiple price uncertainties into single portfolios. Another interesting feature of this model is that transition probabilities are obtained from historical market price data.

It is demonstrated that price uncertainty may influence short- and midterm optimal decision making. The added value of the current model lies in short-term optimal decision making under uncertainty while taking long term optimal decisions into account.

Infrastructure maintenance and replacement optimisation under multiple uncertainties and managerial flexibility

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Published online in Construction Management and Economics at 15 October 2019
https://doi.org/10.1080/01446193.2019.1674450

Infrastructure maintenance and replacement decisions are subject to uncertainties such as regular asset degradation, structural failure and price uncertainty. In the engineering domain, Markov decision processes (MDPs) typically focus on uncertainties regarding asset degradation and structural failure. While the literature in the engineering domain stresses the importance of addressing price uncertainties, it does not substantiate the observations of such uncertainties through optimisation modelling. By contrast, real options analyses (ROAs) that originate from the financial domain address price uncertainties but generally disregard asset degradation and structural failure. Accordingly, this piece of current research brings both domains closer together and proposes an optimisation approach that incorporates the flexibility to choose between multiple successive intervention strategies, regular asset degradation, structural failure and multiple price uncertainties. A practical result of the current research is a realistic approach to optimisation modelling in which state space reduction is achieved by combining prices into portfolios. The current research obtains transition probabilities from existing price data. This approach is demonstrated using a case study of a water authority in the Netherlands and confirms the premise that price fluctuations may influence short-term maintenance and replacement decisions.

7.1 Introduction

Many infrastructure assets are ageing and reach the end of their useful life. Infrastructure asset owners are confronted with large scale replacements in the coming decennia (Hall, Tran, Hickford, & Nicholls, 2016; Park, Kim, & Kim, 2012). Huge capital expenditures are involved, and the planning and financing of these investments is an issue of great concern (Haffner & Gennady, 2011; Power, Burris, Vadali, & Vedenov, 2016). The American Water Works Association states in a press release: “Renewal and replacement of infrastructure and financing for capital improvements top the list of water industry concerns for the fourth year running” (AWWA, 2019). The US Federal Highway Agency warns for unprecedented challenges: “Ageing roads and bridges that carry greater traffic volumes and heavier loads than ever need extensive rehabilitation” (FHWA, 2019). Similar concerns are shared among infrastructure asset owners worldwide (Klatter, Vrouwenvelder, & van Noortwijk, 2009; Orcesi, 2016; Stewart, 2001).

Ageing infrastructure imposes the need for maintenance and replacement optimisation (Marwa Elcheikh & Burrow Michael, 2017; D. Frangopol, 2011). Such optimisation modelling is challenging for several reasons. First, acknowledging that uncertainties exist, the identification of the main uncertainty drivers is difficult given that such drivers can be related to the infrastructure integrity, the environment surrounding the infrastructure, and the costs associated with preserving the functions of the infrastructure (Ilg, Scope, Muench, & Guenther, 2017; Lange, 2018; Sinha, Labi, & Agbelie, 2017).

A second challenge is the identification of alternatives. Costs, uncertainty, changing societal demands and long technical life cycles of infrastructure assets favour lifetime extension by maintenance without compromising safety (Lange, 2018). Generally, several activities with different cost and risk profiles are available to meet the required performance demands. Activities include, for example, regular maintenance, overhauls, major overhauls, and renewals as well as the numerous sequential combinations of these activities.

A third challenge is that uncertainty drives the need for managerial flexibility. A decision maker will monitor uncertainty drivers and base future decision on the development of those drivers. This managerial flexibility has value that should be incorporated into replacement optimisation (Cardin, Neufville, & Geltner, 2015; De Neufville & Scholtes, 2011; Lander & Pinches, 1998).

The literature, which is reviewed in the following section, offers maintenance and replacement optimisation approaches but none of them covers the aforementioned challenges in an integrative manner while in practice these challenges are often found in combination. More specifically, optimisation approaches found in the literature have difficulties with quantifying uncertainty which is reflected in the substantiation of underlying transition probabilities and, price uncertainty is generally omitted.

Therefore, the aim of the current research is the development of an integrative approach towards infrastructure replacement optimisation, which includes regular asset degradation, structural integrity, price uncertainty, multiple sequential options and managerial flexibility. Such a model supports decision making in practice and is interesting from a scientific perspective as it demonstrates the importance of addressing price uncertainty.

The outline of this paper is as follows. The following section presents a literature review on the inclusion of uncertainty regarding asset integrity and prices in maintenance and replacement optimisation modelling, which identifies the research gap. Hereafter the structure of the model is presented, including the motivation of the states, actions, transition probabilities and rewards for the current research objective. The subsequent section presents in-depth motivation on the approach to modelling price uncertainty in relation to asset deterioration and structural failure. Hereafter the model is demonstrated on a case study, which is followed by a discussion and conclusions.

7.2 Literature review

The following literature review is structured along the lines of two key observations relevant for an integrative approach to maintenance and replacement optimisation under uncertainty. The first observation is that uncertainty regarding asset integrity is often modelled with transition probabilities that are difficult to substantiate in practice when condition data are unavailable. A second observation is that price uncertainty is rarely addressed in maintenance and replacement optimisation despite its importance as stressed by researchers.

Uncertainty regarding asset integrity

Optimising multiple sequential intervention strategies under uncertainty leads to probabilistic approaches to maintenance and replacement optimisation. A mainstream approach for optimising sequential decision making is the Markov Decision Process (MDP) (Baik, Jeong, & Abraham, 2006; D. M. Frangopol, Kallen, & Noortwijk, 2004; Kolobov & Kolobov, 2012; Lin, Yuan, & Tovilla, 2019). An MDP is a stochastic serial decision model that incorporates uncertainty and the managerial flexibility to optimise future decisions or actions (D. M. Frangopol et al., 2004; Kolobov & Kolobov, 2012; Puterman, 1994). MDPs are widely used in the literature to address asset integrity uncertainties in bridge and pavement systems with respect to the maintenance and replacement optimisation. For example, MDPs for optimising bridge deck and road maintenance are presented by Costello, Snaith, Kerali, Tachtsi, and Ortiz-García (2005), Robelin and Madanat (2007), Faddoul, Raphael, and Chateauneuf (2011), and Oliveira, Santos, Denysiuk, Moreira, and Matos (2017). Similar, MDP approaches have been developed for wastewater systems (Baik et al., 2006; Lin et al., 2019; Wirahadikusumah, Abraham, & Castello, 1999).

In these MDP approaches, transition probability matrices define the probabilities for transferring from one condition state to other condition states.

Moreover, the impact of maintenance activities on condition improvement is estimated.

Standardised visual inspections for bridge and pavement systems and CCTV assessments for sewer pipes provide such condition data, generally on a 1 to 5 scale. Transition probabilities are derived from this data. As an example, a dedicated solution for wastewater systems is proposed by Baik et al. (2006), who propose an ordered probit model which estimates the transition probabilities as increments in condition based on a discretisation of a continuous deterioration function. Condition data are required to validate such approach.

Another approach to obtain transition probabilities from inspection data is proposed by Marwa Elcheikh and Burrow Michael (2017) who suggest a PERT-distribution and Monte Carlo simulation to estimate the likelihood that an asset is in one of the condition states. Although the authors have a different modelling objective their approach for estimating the likelihood of an asset being in a certain condition state can be used to establish transition probabilities.

Nevertheless, when data are unavailable or unsuitable, the difficulties associated with estimating transition probabilities and the impacts of maintenance actions on the condition, limit the application of these MDP approaches. Adey, Hackl, and Lethanh (2017) identify several difficulties in data driven approaches to find such probabilities. Challenges include the possible lack of data, inconsistencies with the data, and possible biases in the data. In response, solutions to deal with inadequate data are proposed such as mechanistic empirical approaches (Adey et al., 2017), genetic algorithms (Almeida, Teixeira, & Delgado, 2015; Compare, Martini, & Zio, 2015) or specific adaptation of the modelling to available data (Adey, Burkhalter, & Lethanh, 2018). Fuzzy sets theory, a rule based expert knowledge system, is also known to be supportive to incorporate uncertainty (M. Elcheikh, Al Sheikh. D., & Burrow, 2013; Masteri, Venkatesh, & Freitas, 2018). Mohanta, Sadhu, Chakrabarti, Conference, and Exposition Atlanta (2005), for example, propose a fuzzy logic approach to establish transition probabilities in a Markov model for maintenance scheduling. Here, the knowledge of experts is used to define membership functions to address uncertainty.

Not all infrastructure assets have visible and measurable condition states like gravity sewers and pavements. For example, pump systems, machinery of movable bridges or locks, swing bridges in wastewater treatment plants and aeration facilities fail occasionally, are repaired and put into service again. In these situations deterioration is generally expressed as a rate of occurrence of failures (Harold Ascher, 2007).

Price uncertainty

Another observation is that most probabilistic MDP models for maintenance and replacement optimisation do not address price uncertainty. Faghih Sayed Amir and Kashani (2018) and Swei, Gregory, and Kirchain (2017) observe that although price fluctuations have an impact on future maintenance, rehabilitation and construction, such knowledge is generally ignored by researchers in the engineering domain. Similar conclusions are drawn by Mirzadeh, Butt, Toller, and Birgisson (2014) and Yu and Ive (2011), who emphasise the importance of proper assessments of price developments with respect to the Swedish road infrastructure and the UK construction industry, respectively. Rehan et al. (2016) and Younis, Rehan, Unger, Yu, and Knight (2016) equally emphasise the importance of a proper assessment of price indices to forecast capital expenditures. These authors stress that unit prices and producer price indices are specific for sectors and geographical locations.

Additionally, Ilbeigi, Castro-Lacouture, and Joukar (2017) observe that even previous research devoted to forecasting future prices and cost indices does not quantify price uncertainty.

Despite the observation that price uncertainty is generally ignored in MDP models for maintenance and replacement optimisation, price uncertainty is addressed in the domain of real options analysis (ROA). ROA stems from financial options. An option gives a holder the right but not the obligation to exercise a financial transaction at a future date. This right has value, the so-called option value (Black & Scholes, 1973; Brealey, Myers, & Allen, 2017; Cox, Ross, & Rubinstein, 1979; Merton, 1973).

In a ROA, price uncertainty is often modelled as a Geometric Brownian Motion (GBM) (Farida Agustini, Affianti, Putri, rd International Conference on Mathematics: Pure, & Computation, 2018; Guthrie, 2009; Mun, 2006; Younis et al., 2016). A GBM is a stochastic simulation process in which a price follows a random walk. Such random walk can be simulated with a Monte Carlo simulation or represented as a binominal lattice (Cox et al., 1979).

ROA applications dedicated to maintenance and replacement optimisation of ageing infrastructure are scarce but good ROA applications are found in related fields. Several authors have developed ROA applications for optimising investment decisions in hydropower and flood defence while addressing price uncertainty influenced by climate change scenarios (Kim, Park, Bang, & Kim, 2017; Woodward, Kapelan, & Gouldby, 2014). Another ROA application focussed on investment decisions is presented by Martani, Cattarinussi, and Adey (2018) who optimise urban real estate investments under uncertain future rental scenarios. Several ROA applications address risk allocation and regulation in public private partnerships. In this field, Liu, Gao, and Cheah Charles Yuen (2017) use ROA to establish a price mechanism for termination prices of public private partnership projects. Other ROA models focus on an optimal (win-win) risk allocation of revenues and price cap regulation in public private partnerships (Carbonara & Pellegrino, 2018; Pellegrino, Ranieri, Costantino, & Mummolo, 2011). Power et al. (2016) consider buyout, revenue sharing and minimum revenue guarantee as options in transportation public-private partnerships. ROA is also known to be a good instrument for valuing concessionaires in road toll projects (Buyukyoran & Gundes, 2018; Feng, Zhang, & Gao, 2015; Ford, Lander, & Voyer, 2002).

A ROA application for the valuation of operation and maintenance contracts for gravity sewer systems is presented by Park et al. (2012). Interesting about this research for the current objective is that an asset deterioration function is coupled with maintenance costs. A dedicated infrastructure replacement optimisation ROA, given one variable for price uncertainty, is presented by M. Van den Boomen, Spaan, Schoenmaker, and Wolfert (2018).

Similar to MDP, the application of ROA has its challenges and its critiques. In contrast to MDPs, ROAs often do not address asset deterioration, nor multiple price uncertainties. A discrete approach to ROA will become complex when multiple uncertainties are involved. Like MDP, ROA quickly suffers from state explosion. For this reason, ROA applications found in the literature dealing with multiple uncertainties generally use a Monte Carlo simulation to combine several uncertainties into one and assume a constant volatility. However, such approach provides limited insight in how distinct uncertainties contribute to the result.

Summarising the literature, MDPs are emphasised in the engineering domain when considering asset integrity uncertainty, whereas price uncertainty is generally ignored. By contrast, ROAs emphasise price uncertainty, while generally ignore asset integrity. Moreover, both approaches quickly fall prey to state explosion. The current research aims to develop an integrative approach that incorporates a sequence of intervention strategies, multiple price uncertainties, structural integrity and regular asset degradation.

7.3 Model formulation

The current research uses an MDP to model the optimisation problem because an MDP optimises sequential decisions under uncertainty (D. M. Frangopol et al., 2004; Hillier & Lieberman, 2010; Puterman, 1994). The current approach is a finite MDP but truncates the time variant final states with a discounted value representing all expected future life cycle costs belonging to the final decision. Value iteration is used to find the optimal sequence of managerial decisions in each state. Groundwork explaining MDPs and algorithms to find optimal policies are provided by Puterman (1994) and Kolobov and Kolobov (2012). The value iteration algorithm is fed with transition probability matrices which contain the transition probabilities for transferring from each state to another for each action.

The following subsections first describe the modelling approach and motivate the choices made for the states, actions, transition probabilities and rewards. This approach is visualised in Figure 7.1. The subsequent section motivates the in-depth approach to the modelling of price uncertainty, ageing and structural integrity. This modelling is the input for establishing transition probabilities and rewards.

States

The state space is defined by five discrete and finite variables, namely, asset type, time, age, price level of the operational expenditures (OPEX) and price level of the capital expenditures (CAPEX). The state space contains the Cartesian product of these variables.

The asset types are an existing asset, an overhauled asset, a major overhauled asset and a renewed asset. Time is a second variable, and the current modelling is non-stationary. Each asset has a certain age at a certain time and is bounded by a maximum life. The fourth and fifth variables are the levels for OPEX and CAPEX. These are discrete variables derived from forecasts based on analyses of long-term historic price indices as will be demonstrated in the case study section.

As example: a state designated as $[overhauled\ asset, t, n, d_{opex}, d_{capex}]$ depicts an overhauled asset, n years after the overhaul (n = relative age) in the t^{th} year from now (t = absolute time). The overhauled asset consequently has been maintained for n years. The last two variables define the price levels in the t^{th} year which will be explained in the section on price uncertainty modelling. The OPEX and CAPEX price levels follow a binominal lattice and their positions are given by respectively d_{opex} and d_{capex} number of down moves in year t.

Actions

The four allowable actions depending on the state are to maintain, overhaul, major overhaul or renew the infrastructure, as depicted Figure 7.1 and Figure 7.2.

To maintain denotes providing regular maintenance and minimal repair. Such strategy is often referred to as ‘minimal repair’ or ‘as good as old’ (Harold Ascher, 2007; Rigdon & Basu, 2000).

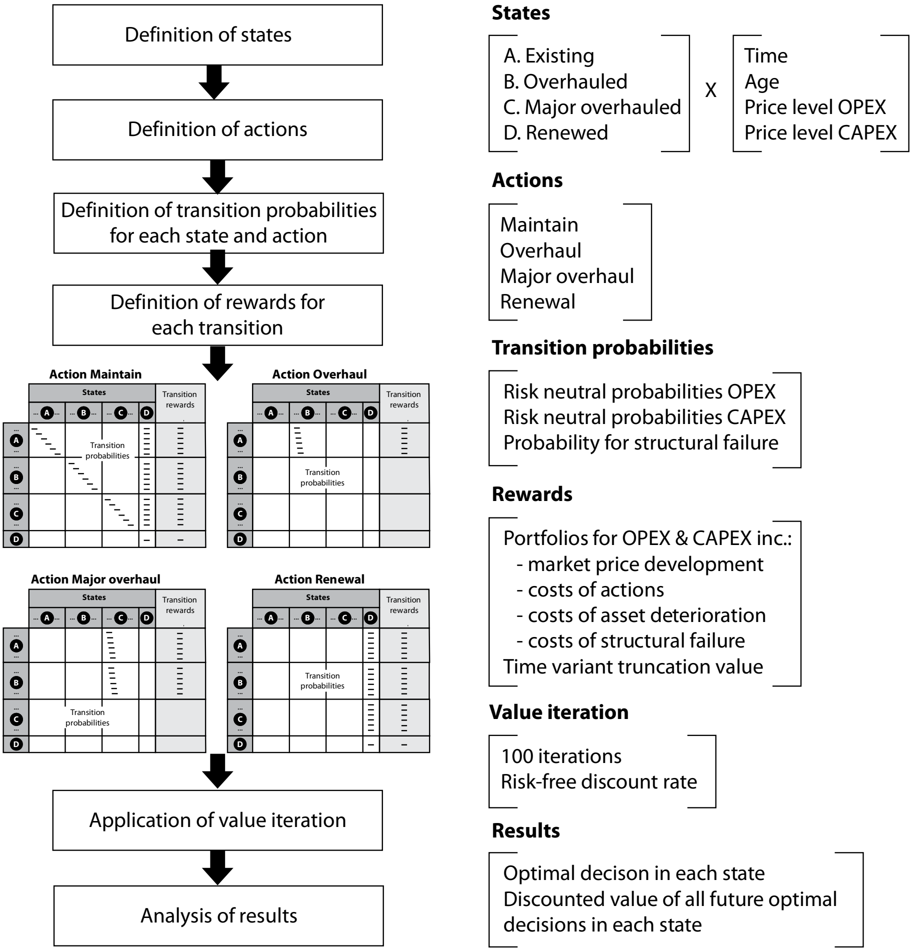

Figure 7.1 MDP approach for the current model

To overhaul refers to substantially larger maintenance efforts such as revisions, rehabilitations or partial replacements. An overhaul prolongs the remaining service life and may positively impact the failure rate. This strategy is also referred to as ‘imperfect repair’ as it brings the asset to a better condition but not ‘as good as new’ (Rigdon & Basu, 2000). A major overhaul refers to a complete renovation, refurbishment or reinforcement of the infrastructure. All of these actions are substantial and will significantly prolong the remaining service life of the infrastructure and lower the failure rate (Harold Ascher, 2007; Pascual, Ortega, & France, 2006; Shafiee, Finkelstein, & Chukova, 2011). Renewal designates a full replacement. Such strategy is denoted as 'perfect repair' or 'as good as new' (Harold Ascher, 2007; Rigdon & Basu, 2000).

Regular maintenance is valid for an existing asset, an overhauled asset and a major overhauled asset. However, inherent in this action is the probability for a structural failure, that is exceedance of a limit state. In such an event, the consequence is a penalised renewal because an unplanned renewal is generally more expensive than a planned renewal.

There is a sequence in possible intervention strategies. An existing asset can be maintained, overhauled, major overhauled and renewed, whatever is optimal under conditions like age and price levels for OPEX and CAPEX. An overhauled asset can be maintained, major overhauled or renewed. A major overhauled asset can be maintained or renewed. Finally, a renewal brings the asset to a truncation state that estimates all time variant expected future life cycle costs (infinite) after the action is taken.

A simplified representation of the MDP model is provided in Figure 7.2, whereby the four states that represent the asset type (the grey circles), contain the states for time, age, OPEX and CAPEX price levels.

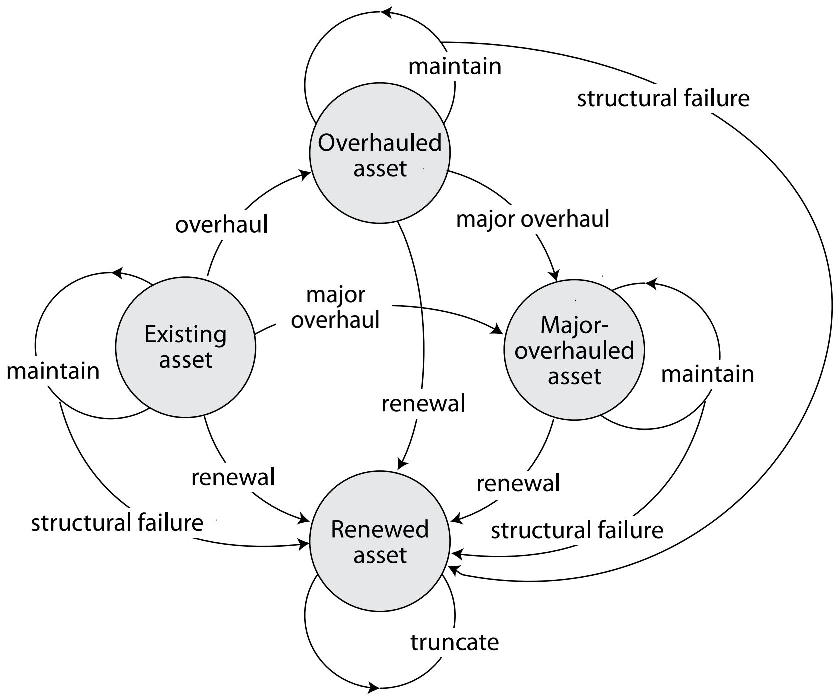

Figure 7.2 Simplified representation of the actions and states of the MDP model

Transition probabilities

Transition probabilities designate the probability of transferring from one state to another when a certain action is taken. The transition probabilities involved are the probabilities for arrival at a certain OPEX and CAPEX price level after an action is taken and the probability of a structural failure or reaching a limit state.

In contrast to MDP applications found in the literature such as Wirahadikusumah et al. (1999), Faddoul et al. (2011) and Oliveira et al. (2017), who use condition data to estimate transition probabilities to move from one condition state to another, the current study proposes to express condition degradation in maintenance repair costs substantiated with a failure rate. The current study is focused on assets which occasionally fail and are allowed to fail, and for which condition data are scarce or unavailable. The subsequent uncertainty quantification of maintenance costs can be derived from existing price data.

Price uncertainty is modelled with underlying discrete binominal lattices which represent all possible price paths as explained in detail in the following section. An illustrative visualisation of the action maintain with probabilities for one time step is presented in Figure 7.3. A maintenance action results in the following probable outcomes. One decision epoch further ensures that the asset will be one time epoch older if no structural failure occurs ($1 - P_{fs}$). In such a case, the aged asset will be subject to four probable price level states, specifically, the price level of the operational expenditures will go up ($\eta_{OPEX,up}$) or down ($\eta_{OPEX,down}$) and the price levels of the capital expenditures will go up ($\eta_{CAPEX,up}$) or down ($\eta_{CAPEX,down}$). The fifth probable outcome is that the asset will reach its limit state (threshold for a structural failure with probability P_{sf}). In that case the model truncates with a time variant corrective renewal including all estimated future life cycle costs.

In each state, the appropriate OPEX prices are traced, based on the decision to maintain, and assessed against the possible CAPEX prices of the alternative CAPEX decisions, including all future life cycle costs. The actions, overhaul, major overhaul and renewal (CAPEX actions), will result in a transfer to another asset type with relative age 0 at the moment the action is taken. Hereafter maintenance starts again, for the other asset type.

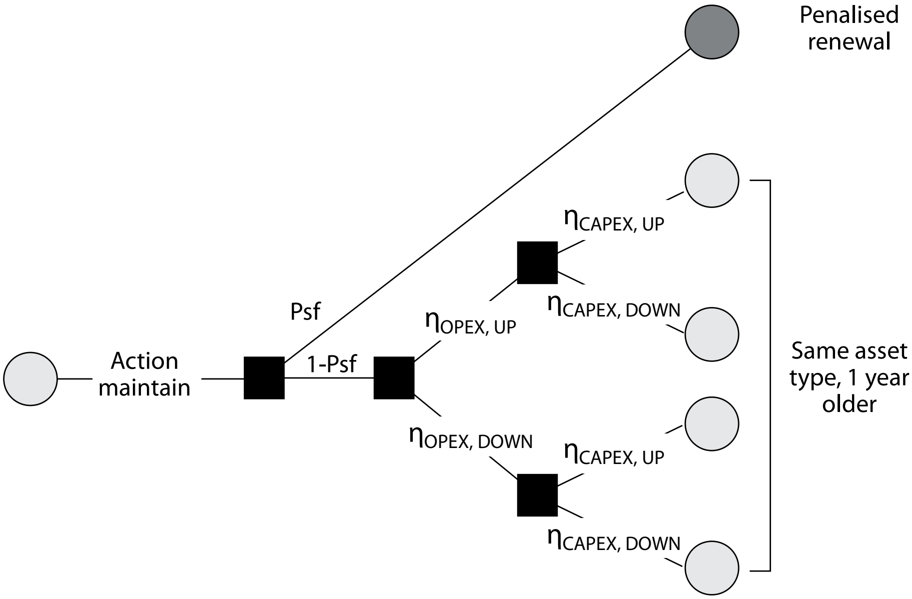

Figure 7.3 Schematised visualisation of the action maintain and probabilities for transferring to successive states

The current model assumes all CAPEX prices follow the same price path directions. For example, when prices for overhauls are high at a certain time, the expectation is that renewal prices are proportionally high as both investment actions are related to the development of market construction prices.

In the current modelling, this assumption does not hold for operational expenditures when transferring from one asset type to another as a consequence of a CAPEX decision because a new asset type can have a totally different OPEX cost profile from the previous one (demonstrated in the case study section). Accordingly, the OPEX lattices of each asset type are not expected to be connected only by up or down moves. Rather, all possible nodes one decision epoch further are considered when transferring from one OPEX lattice to another.

Rewards

The immediate rewards of each action transferring from one state to another directly follow from the underlying OPEX or CAPEX cash flow lattices.

However, special attention is required regarding the rewards when transferring to the truncation state. This is a probable outcome of the maintenance action in the case of a structural failure or a direct outcome of the renewal action. The truncation value is a time variant estimate of a perpetuity of future life cycle costs of a renewal as proposed by Guthrie (2009) and M. Van den Boomen et al. (2018). In the case of a structural failure, this truncation value is additionally penalised with a factor 1.5 for the first investment because an unplanned renewal is more expensive than a planned renewal.

7.4 Modelling price uncertainty, ageing and structural failure

The previous section formulates the overall MDP approach to optimise a sequence of maintaining, overhauling and replacing decisions, while taking multiple price uncertainties, ageing and structural failure into account. The current section motivates the approach to assess the price uncertainties, ageing and structural failure.

Price uncertainty

A common convention is to model price uncertainty using geometric Brownian motion (GBM) (Farida Agustini et al., 2018; Guthrie, 2009; Mun, 2006; Younis et al., 2016). A GBM is a continuous stochastic simulation process where prices follow a lognormal distribution with drift μ and a normally distributed shock ϵ with a volatility σ. The constants drift μ and volatility σ are estimated based on historic prices by using the mean and standard deviation of the log returns of the prices (Francis & Kim, 2013; Guthrie, 2009; Mun, 2006).

A GBM can be expressed as a binominal lattice as in Figure 7.4, which is the discrete form of the continuous stochastic simulation process. The magnitude of its up move U and down move D are defined by volatility σ and conforms to Equations 7.1 and 7.2 (Cox et al., 1979):

$$U = \exp(\sigma) \tag{7.1}$$

$$D = 1/U = \exp(-\sigma) \tag{7.2}$$

In Figure 7.4, $X(d,t)$ represents the cash flow at decision epoch t, having experienced d number of down moves.

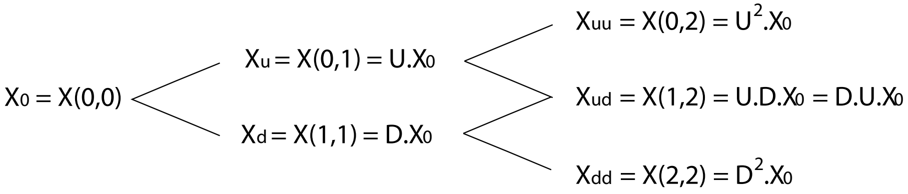

Figure 7.4 Cash flow development in a linear binominal lattice

Cox et al. (1979) propose to use drift μ and volatility σ to derive the actual probability θ_u for an up move and θ_d for a down move, respectively, according to Equations 7.3 and 7.4:

$$\theta_u = \frac{1}{2} + \frac{1}{2}\frac{\mu}{\sigma} \tag{7.3}$$

$$\theta_d = 1 - \theta_u \tag{7.4}$$

As the current research will introduce asymmetric option payoffs, estimating the actual probabilities, as in Equations 7.3 and 7.4, is not sufficient (Copeland & Antikarov, 2003; Guthrie, 2009). Rather, the interest lies in the certainty equivalent probabilities, designated as risk neutral probabilities for up and down moves, i.e., η_u and η_d, respectively. To obtain the risk neutral probabilities from the market data, the certainty equivalent of the capital asset pricing model is followed in Equations 7.5 and 7.6 (Guthrie, 2009).

$$\eta_u = \frac{K - D}{U - D}, \tag{7.5}$$

$$\eta_d = 1 - \eta_u, \tag{7.6}$$

where η_u is the risk neutral probability of an up move; η_d is the risk neutral probability of a down move; and K is the risk adjusted growth factor of the expected cash flows. The actual probabilities θ_u and θ_d are correlated to the risk neutral probabilities η_u and η_d, respectively, by the risk adjusted growth factor K, which, for one decision epoch to the next, is defined as (Guthrie, 2009):

$$K = \frac{\theta_u \cdot X_u + \theta_d \cdot X_d}{X_0} - market\ risk\ \ premium \cdot \beta_{CAPM} \qquad (7.7)$$

The first term on the right side of Equation 7.7 represents the expected growth factor of an underlying market price variable where X_0 is the price of this price variable at the beginning of the time step, $X_u = X_0 U$ and $X_d = X_0 D$. The second term represents the well-known market risk premium and the β_{CAPM}, and subtracts a risk premium from the expected growth of the underlying market price variable.

Anticipating the modelling of the current research, two direct equations are required to arrive at the predicted cash flows and their probabilities for each node (d, t), where d is the number of down moves and t is the time. The cash flow $X(d, t)$ follows from:

$$X(d,t) = X(0,0)EXP\big((t-2d)\sigma\big), \qquad (7.8)$$

and the overall probability for arriving at any node (d, t) from node (0,0) is:

$$P(d,t) = \binom{t}{d} \cdot \eta_u^{t-d} \cdot \eta_d^{d}, \qquad (7.9)$$

where $\binom{t}{d} = \frac{t!}{d!(t-d)!}$ represents the standard binominal coefficient. Figure 7.5 graphically represents the relation between the stepwise risk neutral probabilities and the overall probabilities for arriving a node (d, t) for two time steps.

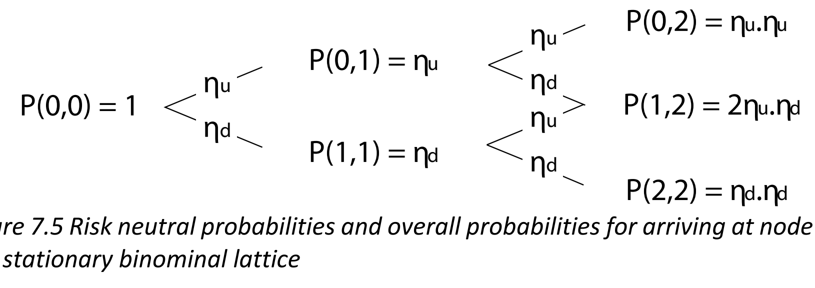

Figure 7.5 Risk neutral probabilities and overall probabilities for arriving at node (d,t) in a stationary binominal lattice

Modelling price uncertainty for each price in a ROA separately with its own binominal lattice, as depicted in Figures 7.4 and 7.5, will quickly lead to state explosion as k prices result in $(t+1)^k$ future possibilities for price levels. This illustrates the major challenge for modelling price uncertainty in a ROA and leads the current research to another domain in finance, namely, the portfolio theory. The portfolio theory was introduced by Markowitz (1952) and targets at diversification of risk by combining financial securities into a single portfolio. The portfolio theory of Markowitz (1952) allows for combining these financial securities in a single portfolio with a new portfolio drift μ_p and portfolio volatility σ_p. The portfolio mean μ_p is defined by Equation 7.10:

$$\mu_p = \sum_{i=1}^{k} w_i \cdot \mu_i \ , \tag{7.10}$$

where k is the number of constitutions in portfolio p; w_i is the proportional weight of constitution i in the portfolio; and μ_i is the mean of constitution i. The portfolio volatility σ_p is defined by Equation 7.11:

$$\sigma_p = \sqrt{\sum_{i=1}^{k} \sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_{ij}} \ , \tag{7.11}$$

where $\sigma_{ij} = COV(ij)$; $\sigma_{ii} = VAR(i) = \sigma_i^2$; and $\sigma_{jj} = VAR(j) = \sigma_j^2$. Equation 7.11 expresses portfolio volatility as a function of the proportion of constitutions in a portfolio, the separate volatilities of the constitutions and the correlations of the constitutions.

The GBM and binominal lattice are founded on log returns of prices and arithmetic mathematical operations over time, while the portfolio theory builds on compounded simple returns (the non-log version of returns), mathematical geometric operations over time and arithmetic operations over assets (Francis & Kim, 2013). For a strict interpretation, a proper mathematical conversion from log return to simple return values is required when combining both theories. These mathematical transformations follow the relationship: $log\ return\ value = \ln{(1 + simple\ return\ value)}$.

The portfolio theory allows for merging different market prices and their underlying binominal lattices in a single portfolio binominal lattice. This significantly reduces the required state space in the current ROA modelling.

Asset degradation

A challenge in the current modelling is the ageing of repairable infrastructure. In the absence of sufficient condition and failure data, the current research proposes an approach which builds on failure rates and maintenance repair costs. A similar line of reasoning is proposed by Park et al. (2012) for a situation where condition data are limited.

The power law process is a popular model applied in practice for ageing repairable assets (Harold Ascher, 2007; Harrold Ascher & Feingold, 1984; Rigdon & Basu, 2000). As such, it provides a relationship for the rate of occurrence of failures, here designated as $\lambda(n)$ in Equation 7.12:

$$\lambda(n) = \frac{\beta}{\theta}\left(\frac{n}{\theta}\right)^{\beta-1}, \tag{7.12}$$

where n is the asset age and β, θ are shape and scale parameters, respectively, that are derived from failure data or expert judgement.

While ageing leads to more repairs and affects operational expenditures, it does not affect the development of market prices of the constitutions defined by μ_i and σ_i. It will, however, affect how many units of a certain price constitution are used. Therefore, ageing directly affects the weights w_i in a portfolio. This imposes a difficulty as it introduces time variance into the portfolio binominal lattice. The time variant portfolio lattice characteristics as defined by Equations 7.1 through 7.7, are derived from $\mu_p(t)$ and $\sigma_p(t)$ for each time step, but they cannot be used to describe the move from one decision epoch to another in the combined portfolio binominal lattice. This is because these characteristics describe the next move under the assumption that each characteristic remains unchanged, which is not the case. Rather, the expected price at a successive decision epoch depends on the constitutions' price developments in the market and the proportion in which they are used, not on the current levels and proportions of operation and maintenance expenditures.

For this reason, the approach followed herein uses the estimated future market prices (Equation 7.8) and their overall arrival probabilities (Equation 7.9) as constraints and derives the corresponding combined portfolio risk-neutral probabilities $\eta_{cp,u}(d,t)$ and $\eta_{cp,d}(d,t) = 1 - \eta_{cp,u}(d,t)$ as detailed in Equation 7.13. Although this approach has similarities with the implied binominal tree approach as introduced by Rubinstein (1994), it differs in that Rubinstein (1994) only had the future exercise prices as constraints, while the current research has all future prices as constraints.. The application and validity of Equation 7.13 is demonstrated in the supplemental material belonging to this article.

$$\eta_{cp,u}(d,t)=\begin{cases}\dfrac{P_p(0,t+1)}{P_p(0,t)} & if\ d=0\\[2ex] \dfrac{\begin{array}{l}P_p(0,t+1)+P_p(1,t+1)+...+P_p(d,t+1)-\\(P_p(0,t)+P_p(1,t)+...+P_p(d-1,t))\end{array}}{P_p(d,t)}, & otherwise\end{cases} \quad (7.13)$$

Structural integrity

Structural integrity denotes the probability of a total collapse or end of life failure. The European reference design code for new structures (NEN-EN 1990:2002 en) prescribes a reliability index β_R as a design parameter for structures, which is also referred to as limit state. The reliability index is derived from load-resistance interference (Leira, 2013; Sánchez-Silva & Klutke, 2016). The probability of structural failure is defined by $p_{sf} = \Phi(-\beta_R)$, where Φ denotes a cumulative normal standard distribution.

In the Netherlands, mandatory renewed structural safety assessments are required as soon as infrastructure assets reach their design life or in the case of reconstructions (NEN 8700:2011 nl). Reliability indices for disapproval levels range from 1.8 to 3.3 corresponding with failure probabilities of 0.036 to 4.83E-04, respectively.

7.5 Case study

This case study builds on a case study presented in Martine van den Boomen, van den Berg, and Wolfert (2018). The current modelling acknowledges multiple price uncertainties, asset degradation following a power law process and structural failure following interference theory. The inclusion of multiple uncertainties requires a different and more complex model setup and follows the approach described in the previous sections.

The case study is an old pumping station with diesel engines owned by a water authority. There are still a significant number of such diesel pumping stations in the Netherlands. These diesel engines are replaced by electrical engines upon a major overhaul. Although diesel is a relatively cheap source of energy, diesel engines run on fossil energy, are labour intensive in their maintenance and require specialised maintenance engineers. The question of interest is to optimise this replacement from a cost perspective to support a wider managerial decision framework. The approach followed is applicable to similar case studies that deal with multiple successive options, several price uncertainties, asset degradation modelled with a failure rate and a probability for structural failure.

Cost data

The cost data obtained in Table 7.1 belong to a pumping station with a capacity of 4,200 litres per minute and a head of two meters. The cost data are based on registered historic costs and future estimates by maintenance engineers.

Table 7.1 Cost data in Euros

Asset type	CAPEX	OPEX		
	Construction	Labour	Gasoline	Electricity
Existing asset		75,000	125,000	
Overhauled	350,000	75,000	125,000	
Major overhaul	4,000,000	50,000		240,000
Renewed asset	15,000,000	10,000		225,000

Financial market parameters

The risk-free interest rate, market risk premium and β_{CAPM} are obtained from a study of the Dutch Authority for Consumer and Market (ACM, 2017) and provided in Table 7.2.

Table 7.2 Financial market parameters

Risk free interest rate	0.83% per year
Market risk premium	4.98% per year
CAPM β	0.71

Price uncertainty

The drift and volatility of the prices are obtained from historic prices between 1996 and 2018 and presented in Table 7.3. A clear explanation on how to obtain drifts and volatilities from market prices assuming a GBM is provided by Francis and Kim (2013), Mun (2006) and Guthrie (2009). The Dutch Central Bureau for Statistics (CBS) provides aggregated historic price indices for civil engineering works among which is included construction for water projects (CBS Stateline, 2018). The historic price indices for maintenance (labour), gasoline and electricity are obtained from CROW (2018). Conversion from log return values to simple return values is required for the following application of the portfolio theory (Francis & Kim, 2013). Table 7.3

illustrates that conversion is only significant for higher drifts and volatilities (approximately > 10%).

Table 7.3 Annualised drift and volatility derived from historical price indices 1996-2018

Parameters		**CAPEX**	**OPEX**			**Ref.**
		Construction	**Labour**	**Gasoline**	**Electricity**	
Drift[1]	μ	0.03	0.03	0.06	0.01	Data
Volatility[1]	σ	0.03	0.02	0.27	0.13	Data
Drift[2]	μ	0.03	0.03	0.06	0.01	Conversion
Volatility[2]	σ	0.03	0.02	0.32	0.14	Conversion

[1] based on log return values

[2] based on simple return values

Figure 7.6 displays the historic price developments of the cost components showing energy costs to be more volatile than construction costs and labour.

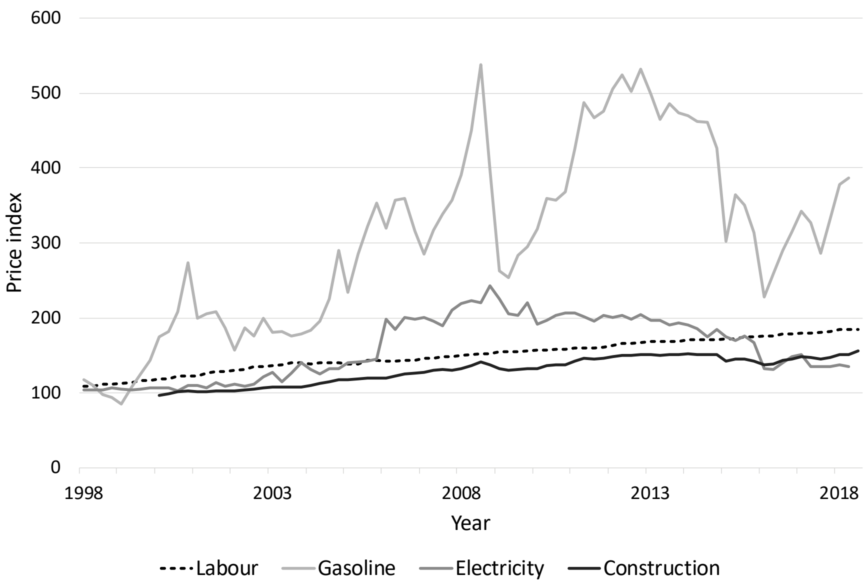

Figure 7.6 Historic development of price indices for construction, labour, gasoline and electricity

Anticipating the combining of maintenance costs in two portfolios, the covariance of labour and gasoline and the covariance of labour and electricity is derived from the log returns of the prices and presented in Table 7.4. A positive covariance indicates that the log returns of the prices move in the same direction, whereas a negative covariance indicates an opposite movement. Again, for precision, a transformation to simple return-based values is performed.

Table 7.4. Covariances of operational prices

Covariance	Log return	Simple return
COV(labour, gasoline)	8.33E-05	8.33E-05
COV(labour, electricity)	-3.88E-05	-3.98E-05

Failure rate and structural failure

In the absence of sufficient long-term failure data, the current research, in addition to trend analysis, used a pragmatic expert judgment approach for estimating the scale and shape parameters for β and θ of the power law process as depicted in Equations 7.14 and 7.15.

$$\beta = \frac{\ln(k_j / k_i)}{\ln(t_j / t_i)} + 1 \tag{7.14}$$

$$\theta = \left(\frac{\beta}{k_j}\right)^{\frac{1}{\beta}} \left(t_j\right)^{\frac{\beta-1}{\beta}} \tag{7.15}$$

where k_j is the expected number of failures in year j; k_i is the expected number of failures in year i; t_j is year j; and t_i is year i.

Expert judgement from the water authority was also used to estimate the maximum remaining service lives (N) of different asset types. Such estimates are based on a maximum allowable number of failures per year with respect to the impact on society. For example, a pumping station that fails too often, may result in negative publicity and reputation damage for a water authority. All data are presented in Table 7.5 and Figure 7.7 graphically depicts the estimated failure rates if the asset types are retained for their maximum remaining service lives.

The last column in Table 7.5 provides the rounded estimated probabilities for a structural failure (non-repairable) based on the legally demanded reliability indices for existing structures in the Netherlands.

Table 7.5.Parameter estimation for β and ϑ based on expected future failures per year

	Max N (years)	k_0 (#)	$k_{2/3}$ (#)	k_1 (#)	β (-)	θ (-)	p_{sf} (-)
Existing asset	5	3	4	5	2.7	3.1	0.005
Overhauled asset	10	2	3	5	3.7	5.7	0.005
Major overhaul	15	1	3	5	2.7	4.8	0.0005
Renewed asset	80	*Truncation with discounted perpetuity of life cycle costs*					

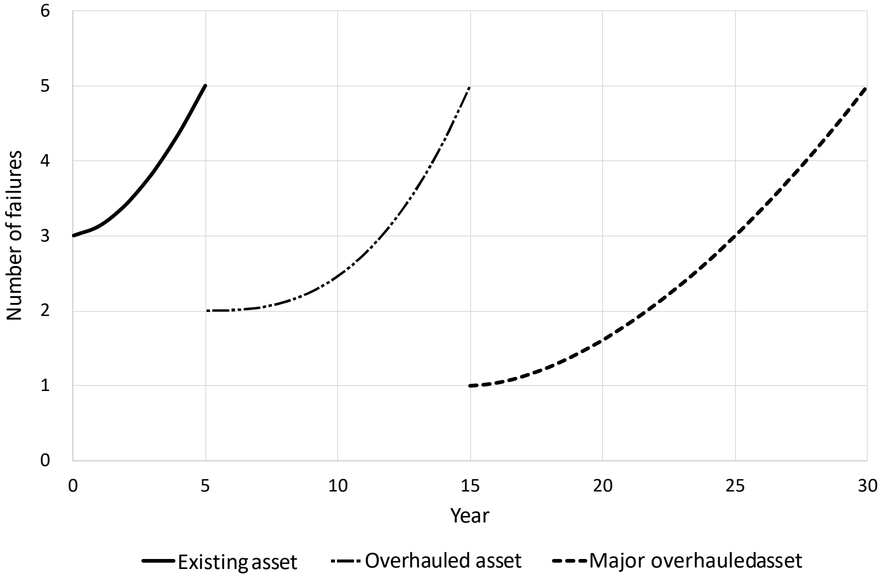

Figure 7.7 Estimates of priority 1 failure rates for the aged assets when successively maintained for their maximum remaining service lives

Combined portfolio lattices

The data in the previous sections are used to establish the combined OPEX and CAPEX binominal lattices for the price developments of each asset type. The CAPAX lattices are straightforward as they are built on aggregated construction price indices. The parameters that describe the CAPEX binominal lattices are presented in Table 7.6.

Table 7.6. Parameters describing the binominal lattice of construction prices.

Parameters	**Symbol**	**Construction**	**Reference**
Drift (log return)	μ	0.03	Data
Volatility (log return)	σ	0.03	Data
Up move	U	1.03	Eq. 7.1
Down move	D	0.97	Eq. 7.2
Actual probability up	φ_u	0.88	Eq. 7.3
Risk adjusted growth factor	K	0.99	Eq. 7.7
Risk neutral probability up	η_u	0.35	Eq. 7.5

Future CAPEX prices follow Equation 7.8 with the volatility for construction prices as depicted in Table 7.6. As illustration, Table 7.7 depicts the first five years of the probable development of the future prices of an overhaul.

Table 7.7 Price development of an overhaul with characteristics from Table 7.6 in Euros (reference Eq. 7.8).

$d \mid t$	0	1	2	3	4	5
0	350,000	361,913	374,232	386,970	400,142	413,762
1		338,479	350,000	361,913	374,232	386,970
2			327,337	338,479	350,000	361,913
3				316,562	327,337	338,479
4					306,141	316,562
5						296,064

The OPEX lattices are not straightforward because distinct operational prices are combined in a single binominal lattice and because ageing (the failure rate) causes the proportions of price constitutions to change with time. As explained in the previous sections, we propose an implied binominal tree approach to construct a time variant combined binominal lattice. While the difficulty shifts to the construction of combined binominal lattices, hereafter it allows for using the convenient characteristic of recombining branches in the MDP model, which significantly reduces state space.

Two forces drive the development of maintenance costs, namely, ageing, which is age dependent, and market price development, which includes inflationary forces and is time dependent.

First, ageing is considered. For ageing, price increases are related to the failure rate, which is a function of the age of the asset after an action is taken. This age is designated as n with a maximum remaining life of N. Price increases caused by ageing are modelled to conform to the power law process:

$$X(n) = X_0 + C_f \left(\frac{\beta}{\theta}\right)\left(\frac{n}{\theta}\right)^{\beta-1} \tag{7.16}$$

where $X(n)$ are yearly maintenance costs at age n at the current price level, X_0 are the maintenance costs just after the instalment of an asset at the current price level, C_f are the costs of a priority 1 failure, β and θ are the shape and scale parameters describing the power law process and n is the age.

The second operational cost component is energy, specifically, diesel for the existing and overhauled asset and electricity for the major overhauled asset. As no immediate changes are foreseen in energy consumption for each asset type, the expected price of diesel at *current price* level remains constant. The impact of price increases and uncertainty caused by inflationary market forces are accounted for when establishing the binominal lattices.

As an example, the proportion of maintenance expenditures and energy expenditures at the current price level are calculated for the existing asset and combined to forecast future prices (Table 7.8 and Table 7.9). A detailed calculation example for the values depicted in Table 7.8 and Table 7.9 is provided as supplemental material belonging to this article.

Table 7.8 Parameters describing the distinct binominal lattices that combine operational prices for labour and gasoline at current price levels.

Parameters	**Symbol**							**Ref.**
Age	n	0	1	2	3	4	5	
Cost labour	$C_1(n)$	75,000	76,277	79,176	83,352	88,657	95,000	Eq. 7.16
Cost gasoline	$C_2(n)$	125,000	125,000	125,000	125,000	125,000	125,000	Tab. 7.1
Cost total	$C_T(n)$	200,000	201,277	204,176	208,352	213,657	220,000	
Weight labour	w_1	0.38	0.38	0.39	0.40	0.41	0.43	
Weight gasoline	w_2	0.63	0.62	0.61	0.60	0.59	0.57	
Drift portfolio[1]	μ_p	0.05	0.05	0.05	0.05	0.05	0.05	Eq. 7.10
Vol. portfolio[1]	σ_p	0.20	0.20	0.19	0.19	0.19	0.18	Eq. 7.11
Drift portfolio[2]	μ_p	0.05	0.05	0.05	0.05	0.05	0.04	Conv.
Vol. portfolio[2]	σ_p	0.18	0.18	0.18	0.17	0.17	0.17	Conv.
Up move	U_p	1.20	1.20	1.19	1.19	1.19	1.18	Eq. 7.1
Down move	D_p	0.83	0.84	0.84	0.84	0.84	0.85	Eq. 7.2
Actual prob. up	$\varphi_{u,p}$	0.63	0.63	0.63	0.63	0.63	0.63	Eq. 7.3
Risk growth fac.	K_p	1.03	1.03	1.03	1.03	1.02	1.02	Eq. 7.7
Risk neutral prob. up	$\eta_{u,p}$	0.53	0.53	0.53	0.53	0.53	0.53	Eq. 7. 5

[1] based on simple return values

[2] based on log return values

The bottom part of Table 7.8 provides the characteristics of six separate binominal lattices that are now returned to one binominal lattice using the implied binominal lattice approach explained in the modelling section and shown in Table 7.9. Equation 7.8 provides the expected prices, Equation 7.9 provides the expected overall probabilities for arrival at each node and Equation 7.13 is used to derive the non-stationary risk neutral probabilities $\eta_{u,cp}(d,t)$ of the combined lattice as a function of time and down moves.

Table 7.9 Lattice characteristics for combined labour and gasoline prices (combined portfolio)

Price development with characteristics from Table 7.8 (reference Eq. 7.8).						
$d \mid t$	0	1	2	3	4	5
0	200,000	240,862	291,028	351,128	421,769	503,340
1		168,197	204,176	247,944	300,190	361,481
2			143,243	175,082	213,657	259,603
3				123,632	152,068	186,438
4					108,233	133,894
5						96,158
Overall probabilities for arriving at the cash flows (reference Eq. 7.9).						
$d \mid t$	0	1	2	3	4	5
0	1.00	0.53	0.28	0.15	0.08	0.04
1		0.47	0.50	0.40	0.28	0.18
2			0.22	0.35	0.37	0.33
3				0.10	0.22	0.29
4					0.05	0.13
5						0.02
Risk neutral probabilities for an up move $\eta_{u,cp}(d,t)$ (reference Eq. 7.13)						
$d \mid t$	0	1	2	3	4	5
0	0.5312	0.5304	0.5290	0.5271	0.5249	n.a
1		0.5304	0.5290	0.5271	0.5249	n.a
2			0.5290	0.5271	0.5249	n.a
3				0.5271	0.5249	n.a
4					0.5249	n.a
5						n.a

Comparing the recombined risk neutral probabilities in Table 7.9 with the non-recombined risk neutral probabilities in Table 7.8 reveals no significant differences for the current case study in the first 5 years. However, over time or with higher volatilities and increased failure rates, these differences become more explicit.

Transition probabilities and rewards

The non-stationary OPEX transition probabilities are obtained from the lattices built in the first step of the modelling and are dependent on the state, which is a function of asset type, time, age, down move position for OPEX prices and down move position for CAPEX prices (i.e., Table 7.9).

The CAPEX transition probabilities up and down are straight forward because these lattices are stationary (i.e., Table 7.6). Rewards at position (d, t) are similarly obtained from the preconstructed cash flow lattices (OPEX, i.e., Table 7.9) or directly calculated from CAPEX using Equation 7.8.

Optimisation

The model is programmed in Matlab, and the optimal policies are found by using value iteration (Puterman, 1994). The value iteration algorithm uses the transition probability matrices of the four actions (maintain, overhaul, major overhaul and renewal), their corresponding rewards and the risk-free discount rate.

The maximum service lives of the existing, overhauled, major overhaul and renewed assets are resp. 5, 10, 15 and 80 years for the case study as depicted in Table 7.5.

The value iteration algorithm calculates the optimal decision in each state. The case study has approximately 80,500 states resulting from the combinations of the state variables: asset type, time, age, price level OPEX and price level CAPEX. One-hundred iterations are performed with an error rate of 4.4E-7 and a calculation time of less than 10 seconds.

Results

The result of the calculations is a table that depicts the best decision for each state along with the discounted costs of all optimal future actions from that state forward. This includes the calculation of optimal service lives corresponding with future best decisions under future circumstances for all price levels, failure rates and probabilities for structural failures.

Table 7.10 presents the states of the first five years of the current asset, thus providing management information for the immediate decision. The optimal decision for the existing asset is found in the last column of Table 7.10. The best current decision is to maintain the existing asset until at least year 4. In year 4, the best decision depends on the development of prices. When the operational expenditures of the existing asset are in lattice position (d, t) = (0,4) while the capital expenditures are in lattice position (2,4), (3,4) or (4,4), the best decision is to overhaul the existing asset. Although prices cannot be compared individually (as all optimal future decisions are incorporated as well), Table 7.9 and Table 7.7 provide an intuitive indication that an overhaul would be optimal in this situation. Table 7.9 reveals that the operational expenditures have risen to € 421,769 at (d, t) = (0,4), whereas the costs of a major overhaul (Table 7.7) at nodes (d, t) = (2,4), (3,4) and (4,4) have decreased to € 350,000, € 327,337 and € 306,141.

Table 7.10 Results for the existing asset

Existing asset	**Time**	**Age**	**Down move OPEX**	**Down move CAPEX**	**Optimal PV of life cycle cost**	**Optimal decision**
States	0	0	0	0	-26,935,771	Maintain
	1	1	(0,1)	(0,1)		Maintain
	2	2	(0,1,2)	(0,1,2)		Maintain
	3	3	(0,1,2,3)	(0,1,2,3)		Maintain
	4	4	0	0	-30,059,775	Maintain
	4	4	0	1	-28,781,539	Maintain
	4	4	0	2	-27,570,686	**Overhaul**
	4	4	0	3	-26,433,680	**Overhaul**
	4	4	0	4	-25,368,884	**Overhaul**
	4	4	(1,2,3,4)	(0,1,2,3,4)		Maintain
	5	5	(0,…,5)	(0,…,5)		**Overhaul**

In year 5, at the maximum remaining life of the existing asset, all states indicate an overhaul to be the optimal decision.

If a decision maker wants to look further, a state can be traced into the future. For example, state $[exisiting\ asset, t = 4, n = 4, d_{OPEX} = 0, d_{CAPEX} = 2]$ indicates an overhaul. This would transfer the existing asset one decision epoch further ($t\ =\ 5$) to a 1-year-old ($n = 1$) overhauled asset. The possible down moves for the new OPEX lattice position are 0,1, 2, 3,4 or 5. The possible down moves for the new CAPEX lattice are up (2+0) or down (2+1). This results in 12 probable subsequent states, which are depicted in Table 7.11. In each of these states, the optimal decision would be to maintain the overhauled asset for another year. Similarly, all states of interest can be traced.

Table 7.11 Results of a 1-year old overhauled asset in year 5

Overhauled asset	Time	Age	Down move OPEX	Down move CAPEX	Optimal PV of life cycle cost	Optimal decision
States	5	1	0	2	-30,022,802	Maintain
	5	1	1	2	-29,169,492	Maintain
	5	1	2	2	-28,339,758	Maintain
	5	1	3	2	-27,693,206	Maintain
	5	1	4	2	-27,233,080	Maintain
	5	1	5	2	-26,915,606	Maintain
	5	1	0	3	-28,789,422	Maintain
	5	1	1	3	-27,981,822	Maintain
	5	1	2	3	-27,172,566	Maintain
	5	1	3	3	-26,531,112	Maintain
	5	1	4	3	-26,071,809	Maintain
	5	1	5	3	-25,754,408	Maintain

Navigating through the table of results (i.e. Table 7.10), based on realistic data, demonstrates that optimal decisions are dependent on price developments.

7.6 Discussion

The inclusion of uncertainty in maintenance and replacement optimisation is an area under investigation. D. Frangopol (2011), Ilg et al. (2017) and Scope, Ilg, Muench, and Guenther (2016) classify core challenges as identifying the right sources of uncertainty and selecting appropriate methods to incorporate uncertainties. These authors also conclude that uncertainty modelling remains case-specific depending on type of data and quality of data.

A source of uncertainty which is still overlooked in maintenance and replacement optimisation is price uncertainty (Ilbeigi et al., 2017; Park et al., 2012; Rehan et al., 2016; Younis et al., 2016). An important direction for further research is price forecasting, quantifying price uncertainty and the integration of these in maintenance and replacement optimisation modelling.

Adding price uncertainty quickly complicates existing MDP maintenance and replacement optimisation models because such models fall prey to state explosion. As a solution, Monte Carlo simulation could be applied to merge multiple uncertainties into one aggregated uncertainty (Pellegrino et al., 2011). However, the current research opted for an analytical solution which allows for tracing distinct price uncertainties in separate portfolios.

The current research adds to the body of knowledge on several points. First, it includes multiple price uncertainties in an MDP approach. State explosion is prevented by merging several market prices into portfolios for operational and capital expenditures by using portfolio theory (Francis & Kim, 2013; Markowitz, 1952).

Second, price uncertainty is applied to actions like regular maintenance, overhaul, major overhaul and renewal whereas the action regular maintenance incorporates repairs and a probability for reaching a limit state resulting in a subsequent unplanned renewal. Condition improvement is achieved after an overhaul, major overhaul or renewal.

The proposed approach follows a line of reasoning presented by Park et al. (2012) who integrate a condition deterioration rate in a ROA application to value operation and maintenance expenditures for sewer systems. Hereafter, these authors apply a sensitivity analysis to account for uncertainty in the condition deterioration rate. Instead of condition rate the current study uses failure rate. Both studies deal with a situation were limited data on asset deterioration are available. Further, the current study optimises a chain of actions.

Although the current approach includes multiple probabilistic price uncertainties, it does not incorporate probabilistic uncertainty on condition deterioration. When sufficient condition data are available, such uncertainty is often modelled with transition probability matrices, which express the probability of transferring from one condition state to another (Adey et al., 2017; Baik et al., 2006; Wirahadikusumah et al., 1999). Depending on the availability of data and type of data, other approaches to uncertainty modelling of condition deterioration exist such as Monte Carlo simulation (Marwa Elcheikh & Burrow Michael, 2017; Lin et al., 2019), fuzzy logic techniques (Masteri et al., 2018) and sensitivity analysis (Park et al., 2012; Scope et al., 2016).

The current study deals with infrastructure for which condition data are not available and failure data are scarce. Therefore, the current research proposes a deterministic approach to address asset deterioration related uncertainty (Ilg et al., 2017; Scope et al., 2016) combined with the probabilistic approach to model price uncertainties. An interesting direction for future research is the inclusion of a probabilistic uncertainty modelling of the failure rate. Such direction could follow a Non-Homogeneous Poisson distribution (Rigdon & Basu, 2000). The consequence, however, is the addition of states to the already impressive state space. In such case it would be interesting to investigate the applicability of more advanced algorithms to find optimal policies in large MDP models (Kolobov & Kolobov, 2012; Puterman, 1994).

Alternatively, partial observable MDP (POMDP) is a direction to proceed when limited data on condition states or failure rates are available. In a POMDP a decision maker cannot fully observe the states nor the impact of decisions. A decision maker will use his believes and update his believes when information becomes available (Faddoul et al., 2011; Walraven & Spaan, 2018).

Another important direction for future research is how to obtain useful long-term managerial decision information from results for all future uncertainty states. First year's optimal decisions under uncertainty are easily obtained and based on long-term optimal decisions. However, extracting these long-term optimal decisions from the results is hard because the inclusion of uncertainty provides an exponentially increasing number of possible paths with their own probabilities. Instead of a decision tree type of visualisation the current research proposes to investigate methods which provide a more compact representation of the optimisation problem such as influence diagrams, and their results (Lander & Pinches, 1998; Maier, Polak, & Gann, 2018). One more interesting direction is presented by McGregor et al. (2017) who developed a visual interface for a specific MDP model.

7.7 Conclusions

Huge capital expenditures are involved in the renewal of ageing infrastructure. Emphasis is put on lifetime extension of infrastructure and optimising a sequence of possible intervention strategies before renewal. Such optimisation is challenging as infrastructure is subject to asset deterioration, structural failure and price uncertainties.

Current maintenance and replacement optimisation models generally ignore price uncertainty despite researchers stressing its importance.

The current study developed an approach to include multiple price uncertainties in maintenance and replacement optimisation for repairable infrastructure assets which occasionally fail and are allowed to fail. This model includes the flexibility to choose between multiple successive intervention strategies subject to asset degradation and structural failure.

State explosion is prevented by combining multiple price uncertainties into portfolios and exploiting the convenient characteristics of recombining binominal lattices. Whereas structural failure is modelled by interference theory, asset degradation follows a time variant failure rate obtained expert judgment or trend testing, and transition probabilities are derived from available price data.

The model is demonstrated on an existing infrastructure case study with intervention strategies to maintain, overhaul, major overhaul and renew. The results indicate that price uncertainty indeed influences the first year's optimal decision making.

Hence, the current research validates the observations of previous researchers regarding the importance of addressing price uncertainty with respect to maintenance and replacement decisions. The forecast and inclusion of price uncertainty in current maintenance and optimisation modelling deserve more attention in further research and professional practice.

The main managerial implication of the current research is that price uncertainty may influence optimal decision making on maintenance and renewal. The approach developed benefits infrastructure asset owners and service contractors in accurate short- and midterm decision making under uncertainty, while taking long-term optimal decision making into account. Accurate short- and mid-term decision making and planning is very important for capital intensive infrastructure with long design lives after renewal.

However, the current model is not without limitations. Although the results contain all future optimal decisions, this long-term planning information is difficult to extract from the results because of the many uncertainty states. As future research the current research proposes to develop methods for a more compact visualisation of results. Other directions for further research are method to reduce state explosion and inclusion of a probabilistic approach to asset deterioration modelling.

Data availability statement

The data of construction prices that support the findings of this study are openly available in CBS Statline at:
https://opendata.cbs.nl/statline/#/CBS/en/dataset/81139eng/table?ts=1547989906393
The data of maintenance and energy prices that support the findings of this study are available at: https://www.crow.nl/publicaties/indexen-risicoregelingen-gww.

Acknowledgements

We owe much gratitude to Wouter van Vuuren, Ron van de Wiel, Wim Milder, Dert van Ree and their colleagues from Water Authority Rivierenland for their valuable input: a case study with cost data, failure data and intervention strategies, and for their verification of the usefulness of the results.

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8

Discussion

The previous chapters present six dedicated replacement optimisation models for infrastructure assets applicable under various circumstances. These models are meant as blueprints for common infrastructure replacement challenges in practice and can be adapted to specific circumstances. As the models range from simple to complex three important question are: “Was it worth the effort?” and if so, “What model or method should be selected under specific circumstances?”, and “What are the limitations of the presented models?”.

8.1 Was it worth the effort?

The first question justifies a positive answer based on comparison of the advanced modelling approaches with the inherently wrong application of classical approaches to infrastructure replacement optimisations in the previous chapters. The current research demonstrates with case studies that the application of classical methods leads to non-trivial errors in replacement timing and consequently to unnecessary costs or risks. These errors follow from first ignoring the proper infrastructure related features, such as inflation, and second, not selecting the proper optimisation method when dealing with these features.

Although replacement decisions are case specific, percentages of deviations in optimal costs appear as significant. As an example, the case study in Chapter 5 demonstrates that ignoring inflation and price uncertainty but using a proper evaluation method, leads to an underestimation of 44% in total discounted costs whereas inclusion of price uncertainty but choosing a wrong (but not very wrong) evaluation method leads to 16% overestimation of total discounted costs.

Moreover, Treiture et al. (2018) demonstrated that a wrong application of classical methods (premature truncation of cash flows) leads to deviations of total discounted costs ranging from 2.3% to 5.5% for three Dutch case studies when inflation is ignored. This deviation increases to a maximum of 14% when an inflation rate of 1% is included. This study did not investigate the difference between the application of classical methods and advanced optimisation methods, hence not necessarily the best scenarios were compared.

Furthermore, in Chapter 6, both inflation and method selection were investigated in another case study. This case study shows that the inherently wrong application of classical methods leads to a sub-optimised replacement strategy whether or not inflation is accounted for. Additionally, the inherently wrong classical approach, with and without inflation, underestimates the total discounted costs of the case study with 13% to 16% respectively.

As concluded in previous chapters, generic values for the contribution of different infrastructure related features and method selection to the errors found cannot be given as the case studies are specific and incorporate many variables. However, the current research shows that especially inflation, price uncertainty and method selection are significant contributors to replacement optimisation of infrastructure assets.

The current research does not support one sophisticated model applicable to all circumstances for several reasons. First, uncertainty modelling is case-specific as its core challenges are selecting the right uncertainty drivers and methods to incorporate uncertainty (Ilg, Scope, Muench, & Guenther, 2017; Scope, Ilg, Muench, & Guenther, 2016). These authors encourage researchers to develop case studies and to learn by example as the current research did.

A second reason is the practical applicability of various replacement optimisation approaches. Approaches become more complex when multiple sequential intervention strategies and uncertainties are involved. These inclusions quickly lead to numerous uncertainty states, require iterative programming and computational calculation power. In addition, these inclusions also complicate the interpretation of long-term results because many states need to be evaluated. From a professional point of view, the current research advises simple modelling approaches for simple cases and complex modelling approaches for complex cases.

Although the current research does not provide quick or easy answers to infrastructure replacement optimisation, the following section aims to provide more generic guidelines for selecting an appropriate approach for infrastructure replacement optimisation.

Table 8.1 Guidelines for selection of infrastructure replacement optimisation models

Cash flows of interventions are:	Repetitive	Non-repetitive		
The future is:	Certain	Certain	Uncertain	
			One/two uncertainties	Multiple uncertainties
Like-for-like replacement (greenfield)	Chapter 2 Classical minimum equivalent annual cost at economic life Chapter 3 Discounted age replacement Chapter 3 Discounted interval replacement	Chapter 4 Inflation adjusted capitalised equivalent Chapter 6 DP regeneration model	Chapter 5 Simple real options or decision tree analysis	Chapter 7 Compound real options or decision tree analysis (Markov decision process)
An old asset to be replaced by a like-for-like replacement (brownfield)	Chapter 2 Classical capitalised equivalent	Chapter 4 Inflation adjusted capitalised equivalent Chapter 6 Simple DP network optimisation (extended regeneration model)	Chapter 5 Simple real options or decision tree analysis	Chapter 7 Compound real options or decision tree analysis (Markov decision process)
An old asset subject to a sequence of distinct intervention strategies (brownfield)	N.A.	Chapter 6 Compound DP network optimisation	Chapter 7 Compound real options or decision tree analysis (Markov decision process)	Chapter 7 Compound real options or decision tree analysis (Markov decision process)

8.2 What model should be selected under specific circumstances?

Based on the dedicated replacement optimisation models for infrastructure, three generic guidelines are proposed for selecting a proper infrastructure replacement optimisation method for classes of infrastructure replacement challenges as depicted in Table 8.1. The guidelines are captured in three core questions:

1. What is the sequence of intervention strategies?
2. Are the cash flows of the intervention strategies repetitive?
3. Is the future certain or uncertain and to what extent?

The following paragraphs elaborate on the implications of these three questions for method selection.

What is the sequence of intervention strategies?

First one has to understand the nature of the replacement optimisation challenge.

a) Is it a like-for-like replacement (greenfield)?
b) Is it an old asset to be replaced with a like-for-like replacement (brownfield)?
c) Is it an old asset to be replaced with a sequence of possible intervention strategies for example: maintain, renovate, replace (brownfield)?

These three classifications are schematised in Figure 8.1 and the red arrows indicate the first optimal intervention times a decision maker is interested in.

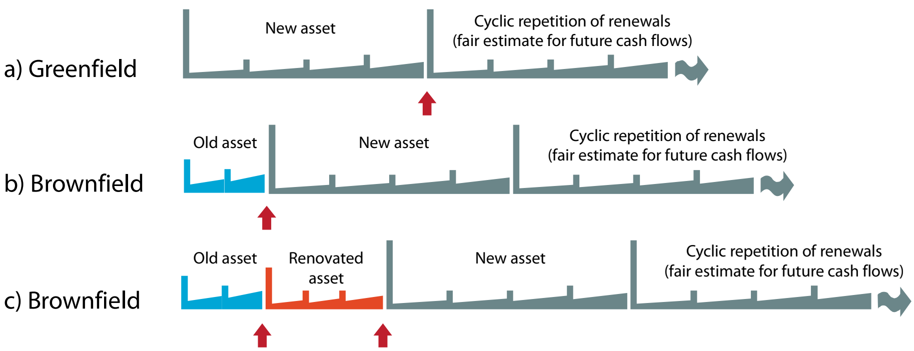

Figure 8.1 Schematised sequence of intervention strategies

The first implication of this classification is the availability of classical engineering economy approaches for classifications (a) and (b) when the future life cycle cash flows of the intervention strategy are repetitive. The current research extended these classical approaches to age and interval replacement models which are special cases of the same type of problem. Whereas the classical minimum equivalent annual cost comparison approach only incorporates a planned renewal, the discounted age and interval replacement models additionally incorporate an unplanned renewal (Campbell, Jardine, & McGlynn, 2011; Van den Boomen, Schoenmaker, & Wolfert, 2018). Classical engineering economy approaches are easily applied with spreadsheet software and do not require advanced programming.

A second implication of this classification is that the current research supports approximation methods for classifications (a) and (b) when the life cycle cash flows of the intervention strategy are non-repetitive under both a certain and uncertain future. This approximation encompasses not optimising the future chain of cyclic renewals of the new asset but instead using the cyclic design life of the new asset to estimate the time variant discounted cash flows over infinity.

This approximation is used under a certain future in the inflation adjusted capitalised equivalent approach in Chapter 4 and under an uncertain future in the simple decision tree and real options approaches in Chapter 5. It is also used as a final truncation value in the most complex Markov Decision Process in Chapter 7.

The current research demonstrates that because of typical infrastructure related features such as long design lives, high investment costs and relatively low operational expenditures, in combination with discounting, allows for such approximation (Van den Boomen, Leontaris, & Wolfert, 2019). In other words, it is futile to put many efforts in optimising a far future, if the time variant discounted value of this optimised future practically equals the time variant discounted value of a non-optimised future. As only a fair approximation of a discounted value of the future is required to determine the timing of the first replacement decision, approximations are preferred when appropriate.

Are the future life cycle cash flows repetitive?

The second step is to understand how future life cycle cash flows develop. A new asset to be replaced by itself (a) can have a repetitive future life cycle cash flow development when price increases (decreases) are insignificant, and technology change is absent. This repetitive cash flow development can also be valid for the replacement option of the second class: (b) an old asset to be replaced with a new one.

When the life cycle cash flows of the replacement option are repetitive and inclusion of uncertainty is not the prime interest, classical replacement optimisation methods like the equivalent annual cost comparison and the capitalised equivalent worth method as reviewed in Chapter 2 are very suitable for infrastructure assets. The discounted age and interval replacement models in Chapter 3 and the bridge replacement case in Treiture et al. (2018) are examples of such approaches. Although not familiar to most cost engineers, these methods provide quick and accurate answers without the need for programming skills.

However, in the absence of repetitive life cycle cash flows of a replacement option these classical techniques cannot be used. When for example price developments are significant, the first (a) and second (b) class of replacement optimisation challenges will have non-repetitive cash flows. For these situations, and in the absence of uncertainty, an inflation adjusted capitalised equivalent approach is developed as an approximation to a DP network optimisation approach (Chapter 4). The inflation adjusted capitalised equivalent approach is best applied with a spreadsheet programme. Although not optimal, the simple DP network optimisation approach (Chapter 6) can also be applied in a spreadsheet programme but some basic programming is advised here (Van den Boomen, van den Berg, & Wolfert, 2019).

Non-repetitive cash flows of the replacement option will always occur for the third class of replacement optimisation challenge: (c) an old asset challenged with a sequence of intervention strategies. Technology change may induce such non-repetitive sequence of strategies. Such strategy can also be a chain of multiple alternatives a decision maker can choose from. In the absence of uncertainty, the current research advises for these situations to use DP network optimisation approaches as developed in Chapter 6. The application of these approaches requires some basic dynamic programming skills.

The additional inclusion of uncertainty will by definition induce non-repetitive life cycle cash flow as will be elaborated on in the following paragraph.

Is the future uncertain and to what extent?

The third question of interest is targeted at the inclusion of uncertainty. By definition, life cycle cash flows of the replacement option are non-repetitive when uncertainty is involved. Uncertainty, however, adds another dimension to replacement optimisation, which is a probabilistic aspect.

The inclusion of uncertainty in replacement optimisation will result in decision tree or real options analyses whereas the difference between these approaches is that real options is an economically corrected version of a decision tree analysis as elaborated on in Chapter 5. One or two uncertainties can be solved with simple decision tree or real options analyses. Multiple uncertainties will need complex or compound decision tree or real options analyses. These compound analyses are well supported with Markov Decision Processes.

The modelling for the simple case, that is classes (a) and (b) or a new/old asset to be replaced by a new one, given one price uncertainty is demonstrated in Chapter 5. Without price uncertainty the current research advises a straightforward decision tree approach. As soon as price uncertainty is involved special economic valuation techniques from the domain of real options analysis are required. Chapter 5 demonstrates how to include this additional price uncertainty in the decision tree approach. These simple DT/ROA approaches are well conducted with a spreadsheet program.

The inclusion of multiple uncertainties, however, needs an extended decision tree or real options approach which is captured under a Markov decision process approach in Chapter 7. Chapter 7 focusses on the inclusion of multiple price uncertainties while taking infrastructure ageing, a probability for structural failure and a sequence of possible intervention strategies into account.

The inclusion of multiple uncertainties complicates the modelling and quickly leads to state explosion. The current research advises to combat state explosion by merging distinct price uncertainties into combined portfolios, one for capital expenditures (investments) and one for operational expenditures (operation and maintenance expenditures). The application of these complex models requires advanced programming skills. For these applications dedicated software should be developed and practitioners trained in using such software.

8.3 What are the limitations of the presented models?

The current research offers dedicated infrastructure replacement optimisation models ranging from simple to complex. The models are demonstrated on case studies showing the importance of a proper identification of infrastructure related features as mentioned in Chapter 1 and a corresponding method selection as elaborated on in the current chapter.

The case studies are simplifications of complex real-life mechanisms and incorporate assumptions about i.e. useful asset lives, failure rates, investments and expenditures, political decisions, inflation rates and discount rates. These assumptions are currently based on expert judgement and historic data analysis when such data is available. Unavoidably, life cycle costing requires forecasting of future events and the inclusion of uncertainty in optimisation modelling does not automatically exempt a decision maker from paying attention to the underlying assumptions. For example, results of life cycle costing calculations are known to be sensitive for changing discount rates. As such, sensitivity analyses to the assumptions made in the models presented in the current research is recommended when applying these models in practice.

Moreover, the assumptions made in the case studies do not necessarily reflect the complex reality which decision makers face. For example, the case studies assume stationary usage while future demand can change. This is especially the case for pumping stations which are subject to the consequences of climate change. More intensive rainfall, increasing water levels in rivers, accelerated settlement of soil are factors contributing to increased future capacity requirements of pumping stations and corresponding life cycle costs. Another assumption in the case studies are equal benefits when comparing current infrastructure assets with future options. Although often a reasonable assumption for infrastructure replacements, this is not necessarily valid in all circumstances.

In answer to the limitations mentioned above it is remarked that the models in the current research are meant as blueprints for typical replacement optimisation challenges, but each situation remains unique and needs to be judged on its specific expected cost or income developments. This may result in different cost forecasting. Benefits are easily included in the models i.e. as negative costs. This forecasting, however, does not alter the underlying method or model. The challenge lies in the identification of the forces that drive future cost developments and estimating their magnitude.

There are some additional considerations for the selection of an appropriate replacement optimisation approach which depends on the classification of the replacement optimisation challenge, the development of future cash flows and the identified uncertainty drivers as depicted in Table 8.1.

The first one is whether a decision maker is interested in tracing uncertainty drivers and acting upon these accordingly. Approaches which incorporate uncertainty complicate the interpretation of results as many uncertainty states need to be evaluated. These approaches are well equipped for short and mid-term investment planning. However, if a decision maker's interest lies in long-term results, for example to establish a long-term asset planning, such purpose is better served with approaches that do not include uncertainty but use its expected values. With these approaches a decision maker can opt for a sensitivity analysis to gain insight in worst- and best-case scenarios without being distracted by an abundance of information on potential uncertainty states.

A second consideration is that the type of infrastructure also indicates the choice for a replacement optimisation approach. Fixed bridges for example have high investment costs, relatively low operational expenditures and long design lives.

These characteristics imply that uncertainty related to operational expenditures are insignificant compared to the uncertainty related to the first investment costs. Moreover, long design lives relax the need for optimising the far future as the discounting process puts less weight on future cash flows. Unless special circumstances apply, replacement optimisation of fixed bridges is well supported with classical engineering approaches presented in Chapter 2 and demonstrated in Treiture et al. (2018), the inflation adjusted capitalised equivalent approach in Chapter 4 (Van den Boomen, Leontaris, et al., 2019) and the simple DT/ROA approaches in Chapter 5 (Van den Boomen, Spaan, Schoenmaker, & Wolfert, 2018).

In contrast, pumping stations have relatively high and volatile operational expenditures because of their energy consumption, in comparison to their investment costs. The design lives of the pumps are shorter than the design lives of fixed bridges. which puts more emphasis on correctly estimating future cash flows of reinvestments. Replacement optimisation approaches serving these characteristics are DP network optimisation without uncertainty in Chapter 6 (Van den Boomen, van den Berg, et al., 2019) and Markov Decision Processes with uncertainty in Chapter 7 (Van den Boomen, Spaan, Shang, & Wolfert, 2019).

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9

Conclusions, limitations and future directions

What life cycle cost modelling approaches should be applied for public infrastructure replacement optimisation taking its relevant features into account?

9.1 Results

Ageing infrastructure assets and shortage of replacement financing induce the need for optimising these replacements. Such optimisation requires identification of relevant infrastructure related features and replacement optimisation methods appropriate for these features. Chapter 1 presents the dominant infrastructure related features:

- Public sector organisations use low discount rates
- Prices are subject to inflation
- Inflation is subject to uncertainty
- Infrastructure assets have long service lives
- Infrastructure asset are repairable and can fail beyond repair
- Multiple intervention strategies are available for ageing assets
- Managerial flexibility has value

Several methods are available for replacement optimisation which are summarised in Chapter 2. The current research identifies four fundamental classical approaches and three advanced approaches. The classical approaches cannot be used when cash flows of replacement options are non-repetitive. Factors such as inflation, technology change, sequential optimisation of several interventions and the inclusion of flexibility to respond to uncertainty induce the need for advanced replacement optimisation methods. The advanced methods used in the current research are network optimisation, real options analysis and Markov Decision Process.

Based on various combinations of infrastructure related features, Chapters 3 to 7 present dedicated infrastructure replacement models for common infrastructure replacement challenges. These models are meant as blueprints and can be adjusted to specific circumstances. Chapter 8 provides a framework for selecting the appropriate model based on the infrastructure related features which are encapsulated by a classification of the type of replacement challenge, the development of the cash flows of the replacement option and the inclusion of uncertainties.

9.2 Conclusions

The conclusions are captured by five key observations emerging from this research:

- Infrastructure related features determine the LCC method
- Inflation and price uncertainty should not be neglected
- Inclusion of uncertainty complicates the long-term planning
- Transferability of LCC methods needs trained professionals
- LCC models support integral decision making

The following paragraphs elaborate on these conclusions.

Infrastructure related features determine the LCC method

The current research demonstrates that prevailing infrastructure related features are essential for the choice of an optimisation approach. This may sound obvious, but the current research observes that knowledge on selecting a sound optimisation approach is scarce and scattered among different domains like engineering economy, operations research and real options analysis.

Inflation and price uncertainty should not be neglected

The current research demonstrates that price uncertainty in infrastructure replacement optimisation is influential for short- and mid-term decision making. Although inflation and price uncertainty are influential, these are generally ignored in current infrastructure replacement optimisation. The current research emphasises the need for increased attention for price developments, price uncertainty and their inclusion in LCC modelling, especially because public infrastructure owners use low discount rates.

Inclusion of uncertainty complicates the long-term planning

The inclusion of uncertainty and the flexibility to respond complicate the interpretation of the long-term results as it generates many possible future states which needs to be evaluated by a decision maker. For that reason, optimisation models which include uncertainty like the compound real options replacement optimisation model in Chapter 7, are very appropriate for short- and midterm decision making but not well equipped to visualise a long-term infrastructure replacement planning. Long term decision making is better served with the models that do not include uncertainty but use its expected values like the network optimisation model in Chapter 6. Although more arbitrary with respect to a decision maker’s preferences, a sensitivity analysis or a Monte Carlo analysis can also provide an elemental uncertainty quantification.

Transferability of LCC methods needs trained professionals

The current research demonstrates that the application of classical approaches to find optimal replacement times often leads to wrong results. However, the current research also observes that knowledge about the more advanced optimisation methods is not common in practice and not taught in the context of life cycle costing in higher education. If public organisations want to benefit from these replacement optimisation models, it is necessary to invest in training of students and professionals to obtain new skill sets in life cycle costing and multi objective optimisation. A generic infrastructure replacement optimisation model for all types of challenges does not exist. The required case-specific modelling makes it hard and perhaps impossible to build such software. As a realistic expectation the current research foresees modular software that assists a cost engineer in constructing and shaping case-specific models.

LCC models support integral decision making

Having trained professionals, infrastructure replacement decisions are still based on multiple criteria among which economical optimisation. Therefore, the results of the current research are supportive to a wider decision-making framework. Each infrastructure asset is part of an infrastructure system and such infrastructure system depends on other infrastructure systems. Moreover, climate change, energy transition and circularity will alter current infrastructure’s technical and functional lives (Dawson et al., 2018). The current models can be shaped to account for future uncertainties and system dynamics but at present these factors are difficult to quantify.

9.3 Thesis contribution

The contribution of this thesis to the current state of knowledge is that it integrates methods from different scientific domains in novel applications for replacement optimisation of infrastructure assets.

First it identifies and structures available methods for replacement optimisation. Such integrative overview cannot be found elsewhere as these methods are scattered among different domains like Engineering Economics, Real Options Analysis and Operations Research, including subdomains within Operations Research. The current research evaluates the applicability of these methods for common infrastructure replacement challenges and concludes that infrastructure related features determine what method should be selected.

Dedicated models are designed for common infrastructure replacement challenges ranging from simple to complex and demonstrated on case studies which function as blueprints. These models integrate the relevant infrastructure related features in various combinations.

Current maintenance and replacement models in the engineering domain address asset integrity uncertainty but omit price uncertainty. Methods for investment timing decisions (ROA) in the financial domain address price uncertainty but omit asset related uncertainties and are shallow on infrastructure assets. The current research brings both fields closer together. In finding solutions while modelling, more methods from adjacent domains were integrated such as the Portfolio Theory which allows for merging diverse price uncertainties into a single portfolio. Moreover, engineering economy methods, reliability and renewal theory were merged into novel applications.

Questions about the validity of many life cycle costing applications in practice and scholarly research are seldom asked. The current research brings life cycle costing analysis to a higher maturity level and integrates uncertainty and flexibility. The methods and models developed are relevant for infrastructure asset owners, asset managers and service providers.

9.4 Limitations and future directions

The following limitations and future directions are tied to the aforementioned conclusions.

Limitations and future directions for model development

The current research did not investigate all combinations of the identified infrastructure related features. For example, price developments are not included in the age- and interval replacement models in Chapter 3. As subsequent research it is interesting to investigate the impact of price developments on the discounted age and interval replacement models. In addition, the current research did not give special attention to modular design in replacement options. A decision maker may want to base future modular investment decisions on the development of demand. Although the modelling techniques used in Chapters 6 and 7 are transferable to such approach, it may be more recognisable to develop a dedicated model for these situations.

Limitations and future directions for inclusion of inflation and price uncertainty

The current research provides optimisation models capable of dealing with inflation and price uncertainty. However, average price and price uncertainty forecasts are based on analyses of historic price indices and assumed to follow a random walk described as a Geometric Brownian Motion.

There are inherent limitations. First, trends of historic prices do not necessarily provide a proper forecast for future prices. Factors such as climate change, energy transition and price elasticity will influence future prices (Marzano et al., 2018). In addition, available price indices are often presented as aggregated baskets which are not necessarily representative for a specific project or case study. A third limitation is the forecasting method. Although a Geometric Brownian Motion is a commonly accepted method, many other methods exist (Weron, 2014).

As the current research demonstrates that price developments are influential in infrastructure replacement optimisation, it is recommended to further investigate historic trends and forecasts of relevant prices. This may lead to expansion of the current price registrations by CROW (2018) and CBS Stateline (2018) and development of different price and price uncertainty forecast methods.

Limitations and future directions for interpretation of results

A limitation of the current models which include price uncertainty is the interpretability of long-term results. As a direction for further research, the current research proposes to investigate how to subtract long-term decision-making

information from the optimisation models that include uncertainty as in the presence of sufficient computational power, it could eliminate the necessity of the less complex optimisation models.
Directions for further research are the clustering of uncertainty in scenarios (Erfani, Pachos, & Harou, 2018) and the development of interfaces to subtract key information from the results (McGregor et al., 2017). Another direction for further research is to recombine information in decision trees to reduce the number of nodes and the application of more efficient calculation algorithms.

In parallel research efforts should be targeted at reducing uncertainties for example by monitoring and improved data registration (Hall, Tran, Hickford, & Nicholls, 2016).

Limitations and future directions for transferability of LCC methods

Replacement optimisation modelling needs skilled professionals capable of selecting the right underlying methods and shaping the blueprints to case-specific situations. Similar observations are found in related domains such as water resource planning. Whereas science offers a broad array of methods and models for decision making under uncertainty, these probabilistic approaches are challenging for managerial decision makers (Hall et al., 2016). The current research advocates to invest in educating students and professionals in advanced life cycle cost calculations. Moreover, the current research proposes to investigate modular software development to assist professionals in replacement optimisation modelling. Such software should allow for building networks or decision trees, assist in uncertainty state definition and creation, support in building combined binominal price lattices, assist in allocation of probabilities and costs and contain solution algorithms.

Limitations and future directions for integrated decision making

Subsequent research is proposed, which integrates the quantitative infrastructure replacement modelling with qualitative decision criteria and acknowledges systems' thinking and the interconnectivity between infrastructure systems (Hertogh and Bakker 2016). In addition, the current research proposes to investigate the impact of climate change, energy transition and circularity on infrastructure assets life cycle management and prices, and their inclusion in the economic optimisation models provided by the current research. This also requires development of resilience-based performance metrics for the life cycle management of infrastructure assets under uncertainty (Roach, Kapelan, & Ledbetter, 2018).

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Curriculum Vitae

Martine van den Boomen was born in Eindhoven in 1970 and graduated cum laude in the department of Water Management at the Delft University of Technology in 1996. Her graduation research focused on sediment transportation modelling.

After her study Martine lived and worked in Yemen for 2 years as an associate expert in sanitary engineering. She assisted in the design of rural water supply and sanitation and with the execution of health education programs.

Unforgettable years in Yemen were followed by KWR Watercycle Research Institute where she started as a researcher in drinking water distribution and developed herself into the responsible researcher on physical asset management, a field just emerging at that time. After obtaining her master title in business administration she became chair of the group water distribution.

In 2005 Martine started her own consultancy, Colibri Advies, in infrastructure asset management. She extended her area of focus from water distribution to public infrastructure assets and has worked for water boards, drinking water utilities, municipalities, Rijkswaterstaat and some private companies since. In 2014 she wrote a practical guideline on replacement decisions for public infrastructure assets. However, the final chapter dealing with replacement decisions under inflation did not feel right. Her engineering economy knowledge was not sufficient, and this initiated her PdD research which she started in 2016.

Embarking on a PhD opened doors to knowledge which led to a growing interest in research, a seed already planted at KWR. Martine started to enjoy education as well. She supported with course development in infrastructure asset management, supervised master thesis students, published her own research, and published together with some of her students.

After her PhD Martine will continue her educational and research activities at the Delft University of Technology on a part-time basis. In addition, she is appointed as a professor (lector) in Asset Management at the University of Applied Sciences in Rotterdam. Still grounded in her own consultancy, she will be in an excellent position to bridge science and practice in her field.

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