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Data-enabled Predictive Repetitive Control

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***Abstract*— Many systems are subject to periodic disturbances and exhibit repetitive behaviour. Model-based repetitive control employs knowledge of such periodicity to attenuate periodic disturbances and has seen a wide range of successful industrial implementations. The aim of this paper is to develop a data-driven repetitive control method. In the developed framework, linear periodically time-varying (LPTV) behaviour is lifted to linear time-invariant (LTI) behaviour. Periodic disturbance mitigation is enabled by developing an extension of Willems' fundamental lemma for systems with exogenous disturbances. The resulting Data-enabled Predictive Repetitive Control (DeePRC) technique accounts for periodic system behaviour to perform attenuation of a periodic disturbance. Simulations demonstrate the ability of DeePRC to effectively mitigate periodic disturbances in the presence of noise.**

I. INTRODUCTION

Periodic disturbances and repetitive behaviour are encountered in many systems such as wind turbines [1] and semiconductor manufacturing [2]. The periodic nature of disturbances is exploited by repetitive control to obtain improved reference tracking performance compared to conventional feedback control. Model-based repetitive control uses the internal model principle [3] to model a periodic disturbance by means of a memory loop, thereby facilitating complete attenuation of errors that share the same periodicity as the disturbance [4].

The pursuit of fast and accurate repetitive controllers has prompted many model-based forms of repetitive control. Modelling has allowed the design of potentially noncausal filters to enhance robustness and learning [5]. In the frequency domain, Frequency Response Function (FRF) data enables uncertainty modelling for robust controller designs [6]. Repetitive control of multiple-input multiple-output (MIMO) systems has been facilitated by means of, for example, $\mathcal{H}_\infty$ techniques [7]. Unfortunately, repetitive control applications often rely on parametric models that are costly and hard to obtain due to the complexity that arises with, e.g., underdamped mechanical systems [8].

The combination of widespread availability of data and increasing system complexity motivates direct data-driven repetitive control designs. Unlike model-based control, direct data-driven techniques do not rely on an intermediate synthesis of a parametric model [9], thereby alleviating the need for the aforementioned costly modelling of systems with complex dynamics [10]. Recently, the development of the direct data-driven predictive control technique Data-enabled Predictive Control (DeePC) [11] has garnered significant interest. DeePC applies Willems' fundamental lemma [12] in a receding horizon framework. An important feature of DeePC is its innate ability to handle constraints due to its reliance on optimization to solve an optimal control problem.

Despite attracting considerable attention, typical DeePC approaches do not account for periodic system behaviour and cannot incorporate periodic exogenous disturbance generators. DeePC has recently been incorporated in Iterative Learning Control (ILC) [13], [14], in which field periodic disturbances feature prominently, but by the nature of ILC these applications are limited to cases where the system's state resets periodically. In addition, to mitigate a periodic disturbance, standard DeePC based on Willems' fundamental lemma is insufficient because it assumes system controllability. Whilst DeePC has been extended to linear parameter-varying [15] and linear periodically time-varying (LPTV) [16] systems, periodic disturbance attenuation for repetitive control applications is not considered.

Although DeePC has seen considerable development, its use to attenuate periodic disturbances and accommodate periodic dynamics is not adequately addressed. This paper's main contribution is the development of a DeePC-inspired repetitive control framework named Data-enabled Predictive Repetitive Control (DeePRC) that attenuates the influence of periodic disturbances and accommodates LPTV system behaviour. Building on [16] a technique called 'lifting' is used to transform LPTV to linear time-invariant (LTI) dynamics. A suitable relaxation of the system controllability assumed by Willems' fundamental lemma is developed to faciliate the mitigation of periodic disturbances. Furthermore, by incorporating Closed-loop Data-enabled Predictive Control (CL-DeePC) [17] in a lifted framework, DeePRC relies on a computationally efficient implementation that can adequately mitigate noise, including during closed-loop operation.

This paper is organized as follows. Section II introduces the employed LPTV model, notation and definitions. The DeePRC framework is developed in Section III. To this end, the LPTV system is first lifted to an LTI representation in Section III-A. Then, in Section III-B, the internal model principle is used to motivate augmenting the lifted state with a constant disturbance. Section III-C explains how the controllability assumption of Willems' fundamental lemma may be relaxed to accomodate such disturbances in a DeePC framework, which motivates the DeePRC formulation presented in Section III-D. Thereafter, a simulation case study is presented in Section IV, and conclusions and suggestions for future work are provided in Section V.

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II. PRELIMINARIES

A. Periodic System Model

This paper considers the signal generating plant to be a discrete-time LPTV system $\mathcal{S}$ in innovation form to capture the effects of process and measurement noise [18]

$$\mathcal{S}:\begin{cases} x_{k+1} = A_k x_k + B_k u_k + F_k d_k + K_k e_k & \text{(1a)}\\ y_k = C_k x_k + D_k u_k + G_k d_k + e_k & \text{(1b)}\end{cases}$$

where the subscript $k \in \mathbb{Z}$ is used as a discrete time index, $\{A_k, B_k, F_k, K_k, C_k, D_k, G_k\}$ are periodic system matrices, $x_k \in \mathbb{R}^n$ represents the system's states, $u_k \in \mathbb{R}^r$ are inputs, $d_k \in \mathbb{R}^m$ are periodic disturbances, $y_k \in \mathbb{R}^l$ are outputs, and $e_k \in \mathbb{R}^l$ is zero mean white innovation noise. The disturbances d_k and the system $\mathcal{S}$ are assumed to be P-periodic. For disturbances this entails that $d_k = d_{k+P}$ whilst for the system $\mathcal{S}$ the definition below applies.

Definition 1. (P-periodic LPTV system) [19]: a causal system $\mathcal{S}$ is said to be P-periodic if it commutes with the delay operator $\mathscr{D}_P$ such that $\mathscr{D}_P\mathcal{S} = \mathcal{S}\mathscr{D}_P$, where $(\mathscr{D}_P f)(k) := f(k-P)$ for a function of time f.

In effect, for the state space representation of (1), P-periodicity entails that for all of the system matrices $A_k = A_{k+P}$, $B_k = B_{k+P}$, etc.

B. Notation and Definitions

This section introduces notation and definitions that are used throughout this paper. For discrete time indices $k_1, k_2 \in \mathbb{Z}$ with $k_2 \geq k_1$ we start by defining a monodromy matrix $\Phi_{k_1}^{k_2}$ and Markov parameters $\mathcal{G}_{k_1}^{k_2}$:

$$\Phi_{k_1}^{k_2} := \begin{cases} A_{k_2-1}A_{k_2-2}\cdots A_{k_1}, & k_2 > k_1\\ I_n, & k_2 = k_1,\end{cases} \tag{2}$$

$$\mathcal{G}_{k_1}^{k_2}(\mathfrak{B}_k, \mathfrak{D}_k) := \begin{cases} \mathfrak{D}_{k_1}, & k_2 = k_1\\ C_{k_2}\Phi_{k_1+1}^{k_2}\mathfrak{B}_{k_1}, & k_2 > k_1,\end{cases} \tag{3}$$

where $I_n \in \mathbb{R}^{n\times n}$ represents an identity matrix, and $\mathfrak{D}_k$ and $\mathfrak{B}_k$ represent (periodic) matrices of the system as in (1). Specific types of Markov parameters are defined by

$$\begin{aligned} {}^{\mathrm{u}}\mathcal{G}_{k_1}^{k_2} &:= \mathcal{G}_{k_1}^{k_2}(B_k, D_k) \quad {}^{\mathrm{d}}\mathcal{G}_{k_1}^{k_2} := \mathcal{G}_{k_1}^{k_2}(F_k, G_k)\\ {}^{\mathrm{e}}\mathcal{G}_{k_1}^{k_2} &:= \mathcal{G}_{k_1}^{k_2}(K_k, I_l)\end{aligned} \tag{4}$$

where I_l is not a periodic matrix but an identity matrix. Using (2) and (3), furthermore define matrices with Markov parameters $\mathcal{T}_{k_1}^{k_2}(\mathcal{G})$, reversed extended controllability matrices $\mathscr{C}_{k_1}^{k_2}(\mathfrak{B}_k)$, and an extended observability matrix $\mathcal{O}_{k_1}^{k_2}$ as

$$\mathcal{T}_{k_1}^{k_2}(\mathcal{G}) := \begin{bmatrix} \mathcal{G}_{k_1}^{k_1} & 0 & \cdots & 0\\ \mathcal{G}_{k_1}^{k_1+1} & \mathcal{G}_{k_1+1}^{k_1+1} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \mathcal{G}_{k_1}^{k_2} & \mathcal{G}_{k_1+1}^{k_2} & \cdots & \mathcal{G}_{k_2}^{k_2}\end{bmatrix}, \tag{5}$$

$$\mathscr{C}_{k_1}^{k_2}(\mathfrak{B}_k) := \left[\Phi_{k_1+1}^{k_2+1}\mathfrak{B}_{k_1}\ \Phi_{k_1+2}^{k_2+1}\mathfrak{B}_{k_1+1}\ \cdots\ \Phi_{k_2+1}^{k_2+1}\mathfrak{B}_{k_2}\right], \tag{6}$$

$$\mathcal{O}_{k_1}^{k_2} := \left[C_{k_1}\Phi_{k_1}^{k_1};\ C_{k_1+1}\Phi_{k_1}^{k_1+1};\ \cdots;\ C_{k_2}\Phi_{k_1}^{k_2}\right]. \tag{7}$$

Block-Hankel data matrices are defined as

$$\mathcal{H}_{i,s,N}(u_k) = \begin{bmatrix} u_i & u_{i+1} & \cdots & u_{i+N-1}\\ u_{i+1} & u_{i+2} & \cdots & u_{i+N}\\ \vdots & \vdots & \iddots & \vdots\\ u_{i+s-1} & u_{i+s} & \cdots & u_{i+N+s-2}\end{bmatrix}, \tag{8}$$

where u_k can be replaced with different types of data, $i \in \mathbb{Z}$ indicates the start of the used sequence, and $N, s \in \mathbb{Z}$ respectively indicates the number of columns and block rows of the matrix. The notion of persistency of excitation is defined using block-Hankel matrices as follows.

Definiton 2. (Persistency of excitation) [12]: The signal given by the sequence $\{w_k\}_{k=i}^{i+N+s-2}$ is called persistently exciting of order s if its block-Hankel matrix $\mathcal{H}_{i,s,N}(w_k)$ is full row rank.

Furthermore, column vectors of concatenated data samples are exemplified by

$$u_{[k_1,\,k_2]} := \begin{bmatrix} u_{k_1}^\top & u_{k_1+1}^\top & \cdots & u_{k_2}^\top\end{bmatrix}^\top, \quad k_2 \geq k_1. \tag{9}$$

III. DATA-ENABLED PREDICTIVE REPETITIVE CONTROL

This section presents the development of Data-enabled Predictive Repetitive Control (DeePRC). The periodic system model is first transformed to an LTI system by means of lifting. To mitigate periodic disturbances, it is shown that the controllability assumption that underpins Willems' fundamental lemma may be relaxed, thereby facilitating data-driven use of the internal model principle.

A. Lifting the LPTV to an LTI system

Lifting is a technique that is used to transform LPTV systems into a higher-dimensional LTI representation. This section lifts the P-periodic system from (1) along the lines of [16], from which we obtain the following definition.

Definition 3. (Lifting): For a P-periodic LPTV system $\mathcal{S}$ as in (1) the corresponding LTI, lifted system representation $\mathcal{S}_{\mathsf{L}}(k_0)$ of $\mathcal{S}$ with initial time $k_0 \in \mathbb{Z}$ is

$$\mathcal{S}_{\mathsf{L}}(k_0):\begin{cases} \mathbf{x}_{j+1} = A\mathbf{x}_j + B\mathbf{u}_j + F\mathbf{d}_j + K\mathbf{e}_j & \text{(10a)}\\ \mathbf{y}_j = C\mathbf{x}_j + D\mathbf{u}_j + G\mathbf{d}_j + H\mathbf{e}_j & \text{(10b)}\end{cases}$$

with iteration index $j \in \mathbb{Z}$, and state $\mathbf{x}_j \in \mathbb{R}^n$, inputs $\mathbf{u}_j \in \mathbb{R}^{rP}$, outputs $\mathbf{y}_j \in \mathbb{R}^{lP}$, disturbance $\mathbf{d}_j \in \mathbb{R}^{mP}$, and innovation noise $\mathbf{e}_j \in \mathbb{R}^{lP}$. The following relations exist between these quantities in the P-periodic LPTV system $\mathcal{S}$ and the lifted LTI system $\mathcal{S}_{\mathsf{L}}(k_0)$

$$\mathbf{x}_j := x_{k_0+jP}, \qquad \mathbf{u}_j := u_{[k_0+jP,\ k_0+(j+1)P-1]}$$

with $\mathbf{y}_j$, $\mathbf{d}_j$, $\mathbf{e}_j$ defined akin to $\mathbf{u}_j$. For the lifted system matrices $\{A, B, F, K, C, D, G, H\}$ in (10) note the lack of a subscript to distinguish them from LPTV counterparts. These system matrices are defined using (2) to (7) as follows:

$$\begin{aligned} A &:= \Phi_{k_0}^{k_0+P} & C &:= \mathcal{O}_{k_0}^{k_0+P-1}\\ B &:= \mathscr{C}_{k_0}^{k_0+P-1}(B_k) & D &:= \mathcal{T}_{k_0}^{k_0+P-1}({}^{\mathrm{u}}\mathcal{G})\\ F &:= \mathscr{C}_{k_0}^{k_0+P-1}(F_k) & F &:= \mathcal{T}_{k_0}^{k_0+P-1}({}^{\mathrm{d}}\mathcal{G})\\ K &:= \mathscr{C}_{k_0}^{k_0+P-1}(K_k) & H &:= \mathcal{T}_{k_0}^{k_0+P-1}({}^{\mathrm{e}}\mathcal{G}).\end{aligned}$$

B. Applying the Internal Model Principle

The internal model principle is the main mechanism behind repetitive control to mitigate disturbances. The essence of the internal model principle is that the effect of a disturbance that is generated by some signal generating model may be asymptotically attenuated by means of feedback if the controller includes the dynamics of the disturbance generating model [3].

In the case of a P-periodic disturbance $d_k = d_{k+P}$, as is the case here, this implies that the lifted disturbance $\mathbf{d}_j$ is constant: $\mathbf{d}_{j+1} = \mathbf{d}_j \ \forall j \in \mathbb{Z}$. Combining this disturbance model with the lifted system dynamics of (10) obtains the augmented system description

$$\mathcal{S}_{\mathsf{L}}^{\mathrm{d}}(k_0): \begin{cases} \begin{bmatrix} \mathbf{x}_{j+1} \\ \mathbf{d}_{j+1} \end{bmatrix} = \begin{bmatrix} A & F \\ 0 & I_{mP} \end{bmatrix} \begin{bmatrix} \mathbf{x}_j \\ \mathbf{d}_j \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \mathbf{u}_j + \begin{bmatrix} K \\ 0 \end{bmatrix} \mathbf{e}_j & \text{(11a)} \\ \mathbf{y}_j = \begin{bmatrix} C & G \end{bmatrix} \begin{bmatrix} \mathbf{x}_j \\ \mathbf{d}_j \end{bmatrix} + D\mathbf{u}_j + H\mathbf{e}_j. & \text{(11b)} \end{cases}$$

In data-driven control applications, minimality of the controlled system is often assumed, requiring that the system is both controllable and observable. Whilst the augmented lifted system (11) may be observable, the modes corresponding to the dynamics of the disturbance are not controllable. The next section derives an extension of Willems' fundamental lemma for such uncontrollable systems in a (not necessarily lifted) LTI domain that is subsequently applied in Section III-D for the development of a data-driven repetitive control method that operates on lifted data from a system like (11).

C. Disturbances with Willems' Fundamental Lemma

DeePC applies Willems' fundamental lemma to make data-driven predictions in a receding horizon optimal control framework. For the purpose of the following discussion of Willems' fundamental lemma, consider a generic deterministic state-space LTI model $\mathcal{P}$

$$\mathcal{P}: \begin{cases} x_{k+1} = \mathcal{A}x_k + \mathcal{B}u_k & \text{(12a)} \\ y_k = \mathcal{C}x_k + \mathcal{D}u_k, & \text{(12b)} \end{cases}$$

with states $x_k \in \mathbb{R}^{n_{\mathrm{x}}}$, inputs $u_k \in \mathbb{R}^{n_{\mathrm{u}}}$, outputs $y_k \in \mathbb{R}^{n_{\mathrm{y}}}$ and system matrices $\{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$. For the system $\mathcal{P}$ consider the following lemma.

Lemma 1 (Willems' fundamental lemma [12, Th. 1]). *Consider the deterministic LTI system $\mathcal{P}$ from* (12) *and assume it to be controllable*[1]*. Collecting input u_k^{m} and output measurements y_k^{m} during an experiment, if the input signal $\{u_k^{\mathrm{m}}\}_{k=0}^{N+L+n_{\mathrm{x}}-2}$ is persistently exciting of order $L+n_{\mathrm{x}}$, then any L-long input-output trajectory of $\mathcal{P}$ is described by*

$$\begin{bmatrix} u_{[0,\ L-1]} \\ y_{[0,\ L-1]} \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{0,L,N+n_{\mathrm{x}}}(u_k^{\mathrm{m}}) \\ \mathcal{H}_{0,L,N+n_{\mathrm{x}}}(y_k^{\mathrm{m}}) \end{bmatrix} g, \tag{13}$$

with $g \in \mathbb{R}^{N+n_{\mathrm{x}}}$.

[1]Note that [12, Th. 1] employs a behavioural definition of controllability (see, e.g., [20]) that is implied by classical state controllability. This latter notion of state controllability is used in [21] to demonstrate the fundamental lemma for state-space representations, as well as in this work.

A proof of Lemma 1 relies on the fact that if $\mathcal{P}$ is controllable and if u_k^{m} is persistently exciting of order $L+n_{\mathrm{x}}$ then [12, Cor. 2]

$$\operatorname{rank}\left(\begin{bmatrix} \mathcal{H}_{0,L,N+n_{\mathrm{x}}}(x_k^{\mathrm{m}}) \\ \mathcal{H}_{0,L,N+n_{\mathrm{x}}}(u_k^{\mathrm{m}}) \end{bmatrix}\right) = Ln_{\mathrm{u}} + n_{\mathrm{x}}, \tag{14}$$

i.e. the matrix in (14) is full row rank. It then follows from the Rouché-Capelli theorem, that there exists a vector g as above such that

$$\begin{bmatrix} x_0 \\ u_{[0,\ L-1]} \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{0,1,N+n_{\mathrm{x}}}(x_k^{\mathrm{m}}) \\ \mathcal{H}_{0,L,N+n_{\mathrm{x}}}(u_k^{\mathrm{m}}) \end{bmatrix} g, \tag{15}$$

with x_0 as initial state corresponding to the input-output trajectory on the left-hand side of (13). As is furthermore shown in [22, Lem. 2], (13) follows directly from (15).

Unlike is assumed in Willems' fundamental lemma, the lifted augmented system $\mathcal{S}_{\mathsf{L}}^{\mathrm{d}}(k_0)$ from (11) is not controllable due to the uncontrollable disturbance modes. This raises the question of what conditions are sufficient to guarantee that (15), and therefore (13) on which DeePC relies, still hold in the presence of a periodic disturbance.

1) Imposing full row rank on the state-input data matrix: Given the developments in the preceding section, a natural answer is to seek guarantees for the equivalent of (14) using an extended state to incorporate an uncontrollable disturbance. To this end, $\mathcal{P}$ is augmented with an observable disturbance as

$$\mathcal{P}_{\mathsf{d}}: \begin{cases} \bar{x}_{k+1} = \underbrace{\begin{bmatrix} \mathcal{A} & \mathcal{B}_{\mathrm{d}} \\ 0 & \mathcal{A}_{\mathrm{d}} \end{bmatrix}}_{:=\mathcal{A}_{\mathrm{L}}} \bar{x}_k + \underbrace{\begin{bmatrix} \mathcal{B} \\ 0 \end{bmatrix}}_{:=\mathcal{B}_{\mathrm{L}}} u_k & \text{(16a)} \\ y_k = \begin{bmatrix} \mathcal{C} & \mathcal{C}_{\mathrm{d}} \end{bmatrix} \bar{x}_k + \mathcal{D}u_k, & \text{(16b)} \end{cases}$$

where $\bar{x}_k = \begin{bmatrix} x_k^\top & d_k^\top \end{bmatrix}^\top \in \mathbb{R}^{\bar{n}}$ is the augmented state containing the disturbance $d_k \in \mathbb{R}^{n_{\mathrm{d}}}$, with $\bar{n} = n_{\mathrm{x}} + n_{\mathrm{d}}$, and $\{\mathcal{B}_{\mathrm{d}}, \mathcal{A}_{\mathrm{d}}, \mathcal{C}_{\mathrm{d}}\}$ are extra system matrices with respect to $\mathcal{P}$ that indicate effects of the disturbance.

What follows is a theorem that specifies sufficient conditions to guarantee that, analagous to (14),

$$\operatorname{rank}\left(\begin{bmatrix} \mathcal{H}_{0,1,N+\bar{n}}(\bar{x}_k^{\mathrm{m}}) \\ \mathcal{H}_{0,L,N+\bar{n}}(u_k^{\mathrm{m}}) \end{bmatrix}\right) = Ln_{\mathrm{u}} + \bar{n}, \tag{17}$$

such that (15) and therefore (13) also hold for the augmented system $\mathcal{P}_{\mathsf{d}}$.

Theorem 1. *Consider the deterministic LTI system $\mathcal{P}_{\mathsf{d}}$ from* (16)*. Let the superscript* m *denote data obtained from an experiment. If a measured input signal $\{u_k^{\mathrm{m}}\}_{k=0}^{N+L+\bar{n}-2}$ is persistently exciting of order $L+\bar{n}$, and $(\mathcal{A}, \mathcal{B})$ and $(\mathcal{A}_{\mathrm{d}}, d_0^{\mathrm{m}})$ are controllable, then the resulting augmented state-input trajectories are such that* (17) *holds.*

A proof of Theorem 1 follows along the same lines as [21, Th. 1.i], and is provided in the extended report [23].

The above Theorem 1 provides sufficient conditions for the rank condition stipulated by (17) to hold. As explained in the beginning of this section, this rank condition is an important building block of typical DeePC implementations. Unfortunately, the conditions posed by Theorem 1 are rather restrictive. To see this, compare $\mathcal{P}_\mathsf{d}$ from (16) to the non-stochastic component of $\mathcal{S}_\mathsf{L}^\mathrm{d}(k_0)$ from (11). In the lifted framework the periodic disturbance is constant such that $\mathcal{A}_\mathrm{L} = I_{mP}$. With reference to Theorem 1, this implies that the lifted disturbance $\mathbf{d}_j \in \mathbb{R}^{mP}$ must be full row rank, which is highly unlikely given that this would require that $m = 1$ and that $P = 1$. Particularly this latter condition would obviate the need for lifting in the first place since the considered LPTV system is then actually LTI.

2) Relaxing the assumption of controllability: Since the controllability of $(\mathcal{A}_\mathrm{d}, d_0^\mathrm{m})$ seems like a difficult condition to satisfy for many kinds of exogenous disturbance generators, it makes sense to look into whether this condition, and consequently (17), may be relaxed. As it turns out, the conditions presented by Theorem 1 are sufficient but not necessary to ensure an equivalent of (15) upon which DeePC ultimately relies. The following theorem formalizes this insight.

Theorem 2 (**Fundamental lemma for systems with an exogenous disturbance**). *For the LTI system $\mathcal{P}_\mathsf{d}$ from* (16)*, if the pair $(\mathcal{A}, \mathcal{B})$ is controllable, the controllability matrix of $(\mathcal{A}_\mathrm{d}, d_0^\mathrm{m})$ has rank ν with $1 \leq \nu < n_\mathrm{d}$, and a measured input signal $\{u_k^\mathrm{m}\}_{k=0}^{N+L+\bar{n}-2}$ is persistently exciting of order $L + \bar{n}$, then the following three properties hold:*

(i) *the matrix from* (17) *is $n_\mathrm{d} - \nu$ rank deficient:*

$$\operatorname{rank}\left(\begin{bmatrix}\mathcal{H}_{0,1,N+\bar{n}}(\bar{x}_k^\mathrm{m}) \\ \mathcal{H}_{0,L,N+\bar{n}}(u_k^\mathrm{m})\end{bmatrix}\right) = Ln_\mathrm{u} + n_\mathrm{x} + \nu, \quad (18)$$

(ii) *$\exists g$ such that*

$$\begin{bmatrix}\mathcal{H}_{0,1,N+\bar{n}}(\bar{x}_k^\mathrm{m}) \\ \mathcal{H}_{0,L,N+\bar{n}}(u_k^\mathrm{m})\end{bmatrix} g = \begin{bmatrix}\bar{x}_0 \\ u_{[0,\, L-1]}\end{bmatrix}, \quad (19)$$

(iii) *$\exists g$ such that for measured inputs u_k^m and outputs y_k^m,*

$$\begin{bmatrix}\mathcal{H}_{0,L,N+\bar{n}}(u_k^\mathrm{m}) \\ \mathcal{H}_{0,L,N+\bar{n}}(y_k^\mathrm{m})\end{bmatrix} g = \begin{bmatrix}u_{[0,\, L-1]} \\ y_{[0,\, L-1]}\end{bmatrix}. \quad (20)$$

For proof of Theorem 2, see the extended report [23]. The above theorem demonstrates that the controllability and rank condition posed by Willems' fundamental lemma can be relaxed to facilitate the application of DeePC to systems of the form given by (16).

3) Implications for a constant disturbance: The following corollary states the implications of Theorem 2 for a constant disturbance, as is the case in the lifted domain of (11).

Corollary 1. *Let the conditions of Theorem 2 hold for a nonzero constant disturbance such that $\mathcal{A}_\mathrm{d} = I_{n_\mathrm{d}}$, then the results of Theorem 2 hold with $\nu = 1$.*

LTI systems with a constant disturbance are a type of affine system, for which an alternative proof of Corollary 1 can be found in the combination of [24, Th. 1] and [25, Th. 1]. The above corollary enables direct data-driven control of the lifted system on the basis of (20) using DeePC if the predicted output is additionally unique. This is the case when the past window length that is used to approximate the initial state is larger than the lag of the lifted system [16, Lem. 7.ii]. The next section presents the control problem formulation and noise mitigation strategy.

D. DeePRC Formulation and Noise Mitigation

Having lifted the LPTV system $\mathcal{S}$ of (1) to the LTI form $\mathcal{S}_\mathsf{L}^\mathrm{d}(k_0)$ of (11), and having shown how DeePC may accommodate exogenous disturbances in such an LTI domain in the previous section, this section develops a data-driven repetitive control method that operates on lifted data to accommodate periodic behaviour and disturbances and mitigates noise.

1) Closed-loop DeePC applied to a lifted system: Use will be made of the computationally efficient CL-DeePC framework developed in [17, Sec. 4] for several reasons. Firstly, the method uses the available data relatively efficiently to suppress noise when compared to DeePC. Secondly, the dimension of the identification task is considerably reduced w.r.t. DeePC, which is especially significant for lifted systems. Thirdly, we note the potential of this method to obtain consistent output predictions from noisy closed-loop data,[2] which is a problem for regular DeePC [27]. For clarity we provide an analytically equivalent, simpler representation of the employed CL-DeePC framework from [17, Sec. 4] as

$$\underbrace{\begin{bmatrix}\mathcal{H}_{i,p,N}(\mathbf{u}_j) \\ \mathcal{H}_{i_p,1,N}(\mathbf{u}_j) \\ \mathcal{H}_{i,p,N}(\mathbf{y}_j)\end{bmatrix}}_{:=\mathcal{Z}} \mathcal{Z}^\top G = \begin{bmatrix}\mathcal{H}_{\hat{i},p,f}(\mathbf{u}_j) \\ \mathcal{H}_{\hat{i}_p,1,f}(\mathbf{u}_j) \\ \mathcal{H}_{\hat{i},p,f}(\bar{\mathbf{y}}_j)\end{bmatrix}, \quad (21a)$$

$$\mathcal{H}_{i_p,1,N}(\mathbf{y}_j)\mathcal{Z}^\top \underbrace{\begin{bmatrix}g_1 & \cdots & g_f\end{bmatrix}}_{:=G} = \mathcal{H}_{\hat{i}_p,1,f}(\hat{\mathbf{y}}_j), \quad (21b)$$

where i and $\hat{i}$ are starting indices of the relevant data matrices, p is a window length of past data, f is the prediction window length, $i_p := i + p$, $\hat{i}_p := \hat{i} + p$, $\hat{(\cdot)}$ indicates predictions, and $\bar{(\cdot)}$ indicates that the data is composed in part of predictions. With this notation, the first future sample is found at $\hat{i}_p$. Furthermore, $G \in \mathbb{R}^{((p+1)Pr+pPl)\times f}$ is a collection of f vectors that are found in DeePC with instrumental variables [28]. This paper uses $\mathcal{Z}$ as an instrumental variable to mitigate the effects of noise whilst preserving the rank of the matrix pre-multiplying G. Subsequent columns of (21) correspond to DeePC with an instrumental variable matrix and a prediction window length of one, applied to the same past data matrix to find trajectories that are each subsequently shifted one (lifted) sample into the future.

2) Optimal Control Problem Formulation: The optimal control problem formulation that is used here is

$$\min_{\mathbf{u}_{[\hat{i}_p,\, \hat{i}_p+f-1]}} ||\hat{\mathbf{y}}_{[\hat{i}_p,\, \hat{i}_p+f-1]}||_Q^2 + ||\mathbf{u}_{[\hat{i}_p,\, \hat{i}_p+f-1]}||_R^2 \quad (22a)$$

$$\text{s.t. } (21), \quad (22b)$$

$$\mathbf{u}_j \in \mathcal{U},\ \hat{\mathbf{y}}_j \in \mathcal{Y},\ \forall j \in [\hat{i}_p,\ \hat{i}_p + f - 1], \quad (22c)$$

in which, R and Q are respectively positive(semi)-definite weighting matrices, and $\mathcal{U}$ and $\mathcal{Y}$ are allowed sets constraining the inputs and outputs.

[2]For lifted systems this necessitates using a suitably chosen instrumental variable $\mathcal{Z}$. An optimal choice is non-trivial, so for simplicity (21) uses a common choice for open-loop data that is sub-optimal otherwise. For dedicated alternatives for closed-loop data see [26].

3) Receding horizon implementation: With regards to the receding horizon implementation, consider implementing either the entire sequence of inputs corresponding to the first computed lifted input $\mathbf{u}_{\hat{i}_p}$, or the first input of this sequence $u_{k_0+\hat{i}_p P}$. A significant advantage of the latter method is the availability of more frequent feedback by which to improve performance. However, note that for data-driven control applications that seek to learn LTI behaviour (e.g. by updating RQ data factorizations at each time step as in [29]) the latter method also has an important drawback: since the lifted LTI system varies periodically with the starting point $\mathcal{S}^{\mathrm{d}}_{\mathsf{L}}(k_0) = \mathcal{S}^{\mathrm{d}}_{\mathsf{L}}(k_0 + P)$, such applications would need to learn the behaviour of P different lifted LTI systems.

IV. Simulation Results

Having presented the DeePRC framework, this section will demonstrate the superior performance it can achieve for a periodic system with a periodic disturbance.

A. Simulated LPTV System

The simulated periodic system was obtained from [30] and constitutes a linear parameter-varying system with a periodic scheduling parameter $\mu_k = \cos(\frac{2\pi}{P}k)$, with period $P = 20$. The periodic matrices from (1) are defined by

$$\left[\, A^{(1)} \mid A^{(2)} \,\right] = \left[\begin{array}{ccc|ccc} 0 & 0.9 & 0.2 & 0.6 & 0.5 & 0.5 \\ -0.9 & 0.5 & 0 & 0.5 & 0.6 & 0 \\ -0.2 & 0 & 0.2 & -0.5 & 0 & 0.6 \end{array}\right],$$

$$\left[\, B^{(1)} \mid B^{(2)} \,\right] = \left[\begin{array}{c|c} 1 & 0.4 \\ 1 & 0.2 \\ 1 & 0.12 \end{array}\right], \left[\, D^{(1)} \mid D^{(2)} \,\right] = \left[\begin{array}{c|c} 0.1 & 0.2 \\ 0.2 & 0.1 \end{array}\right],$$

$$\left[\, C^{(1)} \mid C^{(2)} \,\right] = \left[\begin{array}{ccc|ccc} 0.2 & 1 & 0.5 & 0.2 & 0.1 & 1 \\ 0.2 & 0.1 & 1 & 0.3 & 0.4 & 0.8 \end{array}\right],$$

$$\left[\, K^{(1)} \mid K^{(2)} \,\right] = \left[\begin{array}{cc|cc} 0.0130 & 0.0225 & 0 & 0 \\ 0.0089 & 0.0060 & 0 & 0 \\ 0.0002 & -0.0010 & 0 & 0 \end{array}\right],$$

with $A_k = A^{(1)} + \mu_k A^{(2)}$, and likewise for B_k, C_k, D_k, and K_k. An unknown input disturbance will be applied such that $F_k = B_k$, and $G_k = D_k$. The periodic disturbance is given by $d_k = \sin(\frac{2\pi}{P}k)$. Use is made of zero-mean white innovation noise e_k with a variance of 0.05. In the lifted domain represented by (11) this system has controllable (A, B) and is observable.

B. Controller Settings

Two different controllers are simulated. One controller makes use of the DeePRC framework with optimal control problem formulation given by (22) with $p = 1$, $f = 2$ periods, $Q = 100$, $R = 1$. This controller computes a lifted future input sequence and implements the first sample $u_k \in \mathbb{R}^r$ thereof. A second controller uses CL-DeePC and operates fully in the non-lifted domain, likewise implementing the first sample u_k that it computes. The optimal control problem solved by the second controller is comparable to that solved by DeePRC. Furthermore Q and R are the same for the second controller, as are the effective window lengths in terms of the non-lifted domain. Both controllers implement constraints of the form $|u_k| \leq 10$ and $|y_k| \leq 20$ and are initialized with 1000 periods of open-loop data where u_k is zero-mean white noise with a variance of 1.

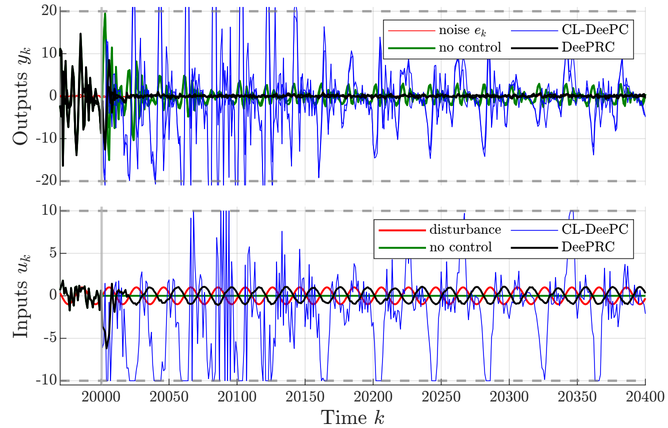

Fig. 1. Performance of DeePRC and CL-DeePC controllers using respectively data from the lifted LTI and the non-lifted periodic domain under the influence of a periodic disturbance and noise. DeePRC effectively compensates the input disturbance, whilst CL-DeePC performs worse here w.r.t. the case without control. Dashed grey lines indicate contraints.

C. Simulation Results

With the above simulation model and controller settings, the obtained results are shown in Fig. 1. The controllers are enabled at the grey vertical line when the open-loop data collection ends. The figure clearly demonstrates that the DeePRC controller, which operates in the lifted domain, outperforms the CL-DeePC controller in attenuating the effect of the disturbance to regulate the output channels to zero. This is because the DeePRC controller effectively operates on LTI data and is therefore better able to form an implicit internal disturbance model, as explained in Section III-C. Furthermore, notice that after initialization the CL-DeePC controller violates the output constraints. This is possible because the constraints are formulated only for future samples and are, if necessary to ensure feasibility, relaxed. Moreover, there is considerable mismatch for the CL-DeePC controller between the data-driven output predictor that it employs and the true system. This contributes to a deterioration of the performance compared also to the case where no control action is applied.

The obtained iteration cost, as specified by

$$\mathcal{J}(j) = ||\mathbf{y}_j||_Q^2 + ||\mathbf{u}_j||_R^2 \tag{23}$$

can be calculated for each iteration index j. The results are shown in Fig. 2 for both the setting with noise shown in Fig. 1 and in the same setting, but without noise. In the noiseless case, the DeePRC iteration cost demonstrates convergence that is, at least initially, quite fast. Moreover, the iteration cost of DeePRC is considerably lower than would be the case without control. This is not the case for the CL-DeePC controller, which only appears to achieve a somewhat better iteration cost than no control would in the absence of noise and after a considerable number of iterations.

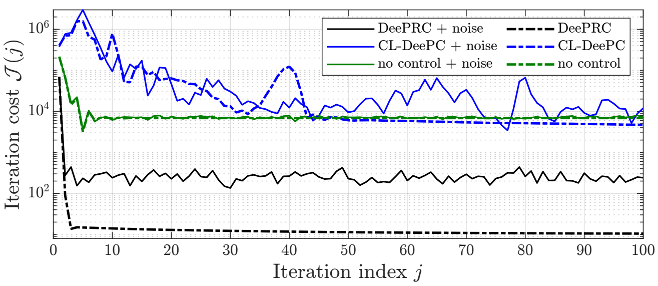

Fig. 2. Obtained iteration cost as specified by (23) of the DeePRC and CL-DeePC controllers under conditions with and without noise. The iteration cost of DeePRC is lower than that of CL-DeePC and illustrates faster convergence.

V. Conclusions and Future Work

A new control framework is presented that is able to address both dynamics and disturbances of a known period in the presence of noise. Moreover, it is shown under what conditions Willems' fundamental lemma can accommodate autonomous, uncontrollable disturbance dynamics that arise from the application of the internal model principle. In particular, it is shown how despite a loss of the generally assumed controllability, a constant disturbance may still be accommodated by DeePC formulations. Simulation results indicate superior performance of the DeePRC controller compared to a CL-DeePC controller that respectively use data from the lifted LTI and periodic system domains. Future work considers the effect of an uncertain period as well as periodic data differencing to remove the effect of the disturbance from the data.

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