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DAMPING MODELS FOR FLEXIBLE COMMUNICATIONS SATELLITES

BY SUBSTRUCTURAL DAMPING SYNTHESIS

by

P. C. Hughes

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SUMMARY

Most modern spacecraft are structurally flexible and, moreover, these spacecraft can naturally and profitably be analysed as a collection of attached substructures (solar array panels, antennas, thermal radiators, etc.). This report shows how to combine various models for substructural energy dissipation so that an overall damping model for the spacecraft results. (Four such substructural damping models are discussed, two of which are shown to produce the same results.) Such a synthesis procedure proves valuable when substructural damping data is known, either from ground tests or detailed analysis.

However, even if substructural damping data is not known but merely guessed at (as is often the case) this report shows that it is better to do one's guessing at the substructural modal level that at the overall spacecraft modal level; the explanation for this, in a nutshell, is that, in the former case, 'reality' (in the form of the relative sizes, connections, elasticities and inertias of the various substructures) is invoked in the synthesis procedure: better to pass the substructural guesses through some sort of 'reality filter' (the synthesis procedure) than to simply make guesses about the overall spacecraft damping properties. Furthermore, as a numerical example for a spacecraft of topical complexity shows, the two alternatives can produce quite different results.

ACKNOWLEDGEMENTS

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1. INTRODUCTION

The object of this report is two-fold:

(a) to show how to combine damping data from the individual sub-structures of a spacecraft to form a damping model for the spacecraft as a whole;

(b) to explain why, if exact damping data is not available, it is better to estimate the modal damping factors for the individual substructures than to try to estimate the damping factors associated with the overall (unconstrained) spacecraft modes.

Two 'structures' will be considered throughout this report: the rather complex spacecraft shown in Fig. 1.1, and a very simple--but nevertheless informative--mechanical system, to be introduced in Section 2.

Figure 1.1 shows a 'mobile communications' satellite which possesses significant flexibility in its solar array, in its antenna dish reflector, and in the tower that supports the reflector. Structurally it has the topology shown in Fig. 1.2a, in which E_1 represents the tower, E_2 the reflector, and E_3 the solar array.

In this report, the emphasis will be on how to handle an *internal* substructure, such as E_1. The case of a single flexible appendage (such as E_3) attached to a rigid body has been often treated on previous occasions. In simple terms, the question that arises in dealing with an internal flexible body such as E_1 is: What should be done with E_2? In this report two answers to this question are given, either of which provides a rigorous method for synthesizing overall damping characteristics from the damping data for E_1.

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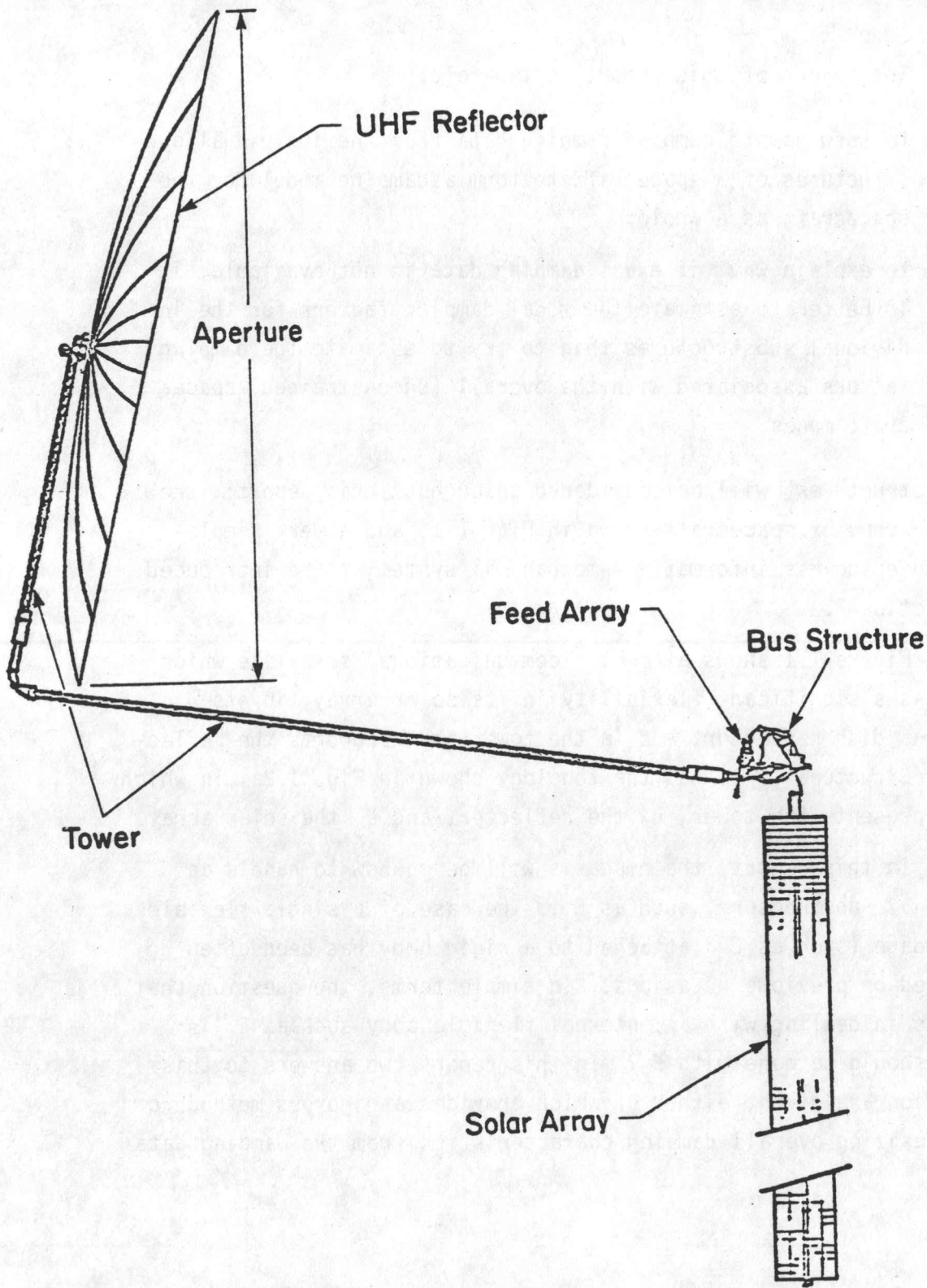UHF Reflector

Aperture

Feed Array

Bus Structure

Tower

Solar Array

Figure 1.1: "ZSAT"--A Flexible Mobile Communications Satellite

(a) A Four-Body System
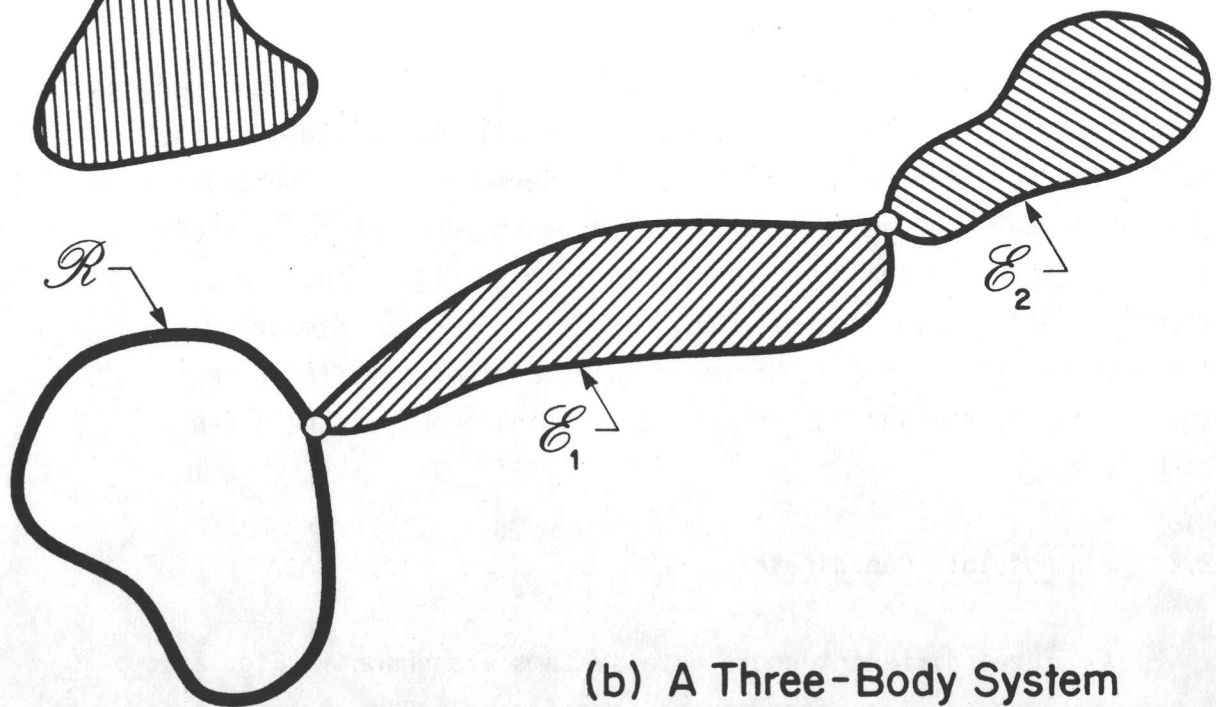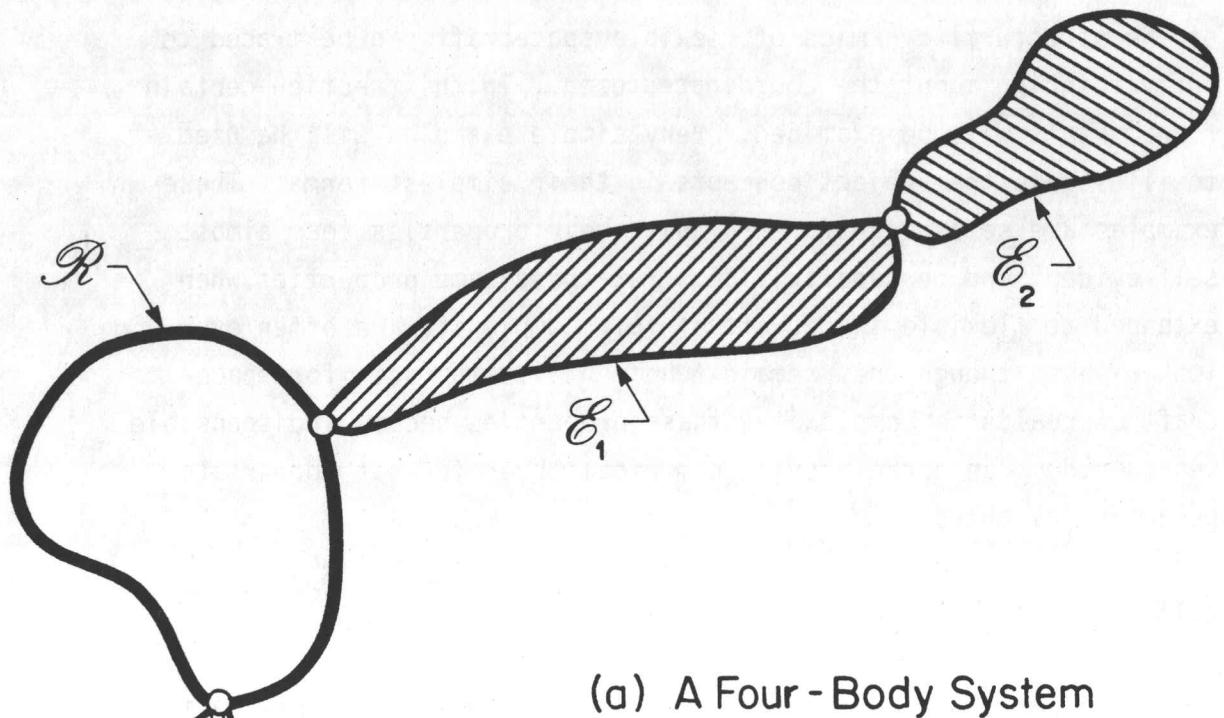
(b) A Three-Body System

Figure 1.2: Two Possible General Topologies for Flexible Spacecraft

2.   COORDINATE FUNDAMENTALS

Much of the confusion that sometimes accompanies discussion of the structural dynamics of flexible spacecraft can be traced to fuzzy thinking about the coordinates used. In this section certain fundamentals will be examined. Very simple examples will be used to illustrate the subject concepts in their simplest terms. These examples are so straightforward that their properties seem almost self-evident and perhaps trivial. Yet these same properties when extended to flexible spacecraft of full complexity are often over-looked--even though they remain fundamental. In fact, for space-craft of realistic complexity, these properties become indispensible because they can often provide numerical order in what appears to be numerical chaos.

2.1   A Three-Mass Analogy

Consider the simple three-mass system shown in Fig. 2.1. The analogy with Fig. 1.2b is fully intended: m_r is a point mass (shown with finite size for visibility) and is intended to repre-sent a rigid spacecraft 'bus'; the other two point masses, m_1 and m_2, together represent a flexible appendage (such as the offset dish antenna assembly shown in Fig. 1.1).

Elasticity within the 'appendage' is provided by the springs (of stiffness k_1 and k_2), and damping is similarly provided by the two dampers within the 'appendage'. Furthermore, the 'appendage' can be further divided into two 'flexible substructures', as shown in Fig. 2.1c. The analogy to be drawn is with Fig. 1.2b. Thus, with the ZSAT of Fig. 1.1 in mind, the {mass, spring, damper} combination $\{m_1, k_1, d_1\}$ is intended to be analogous to the antenna tower, and the combination $\{m_2, k_2, d_2\}$ is analogous to the antenna dish.

2.2   Absolute Coordinates

Three different coordinate systems are shown in Fig. 2.1. A set of three *absolute* coordinates is defined in Fig. 2.1a. It is
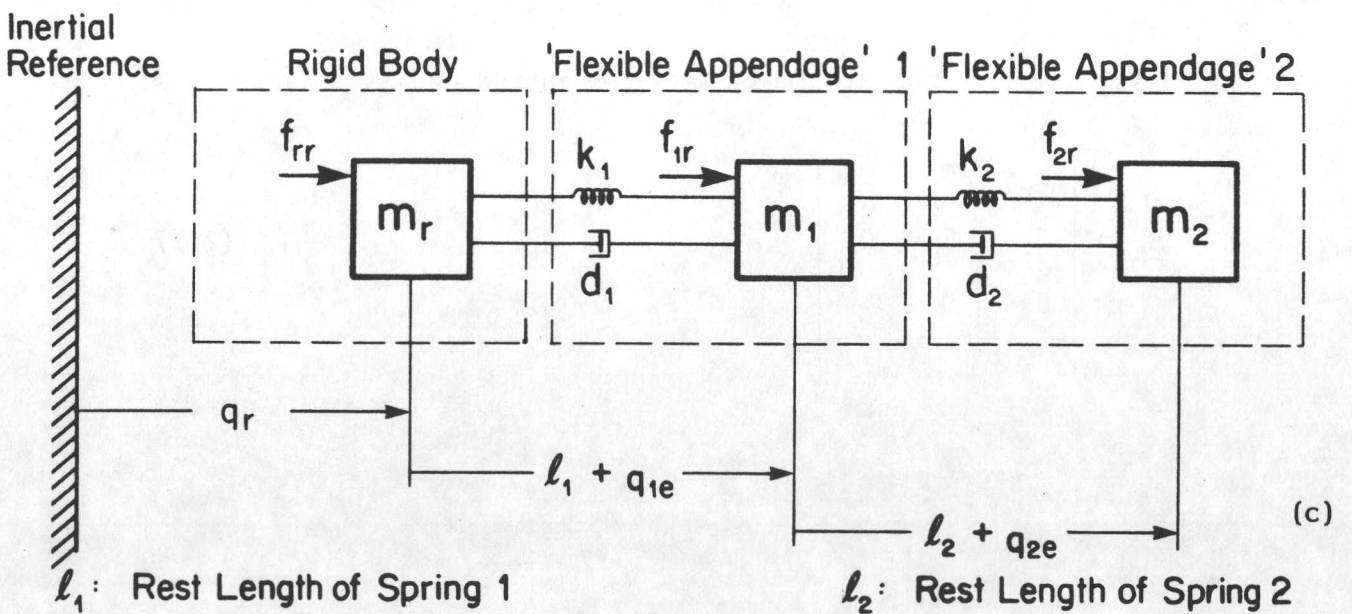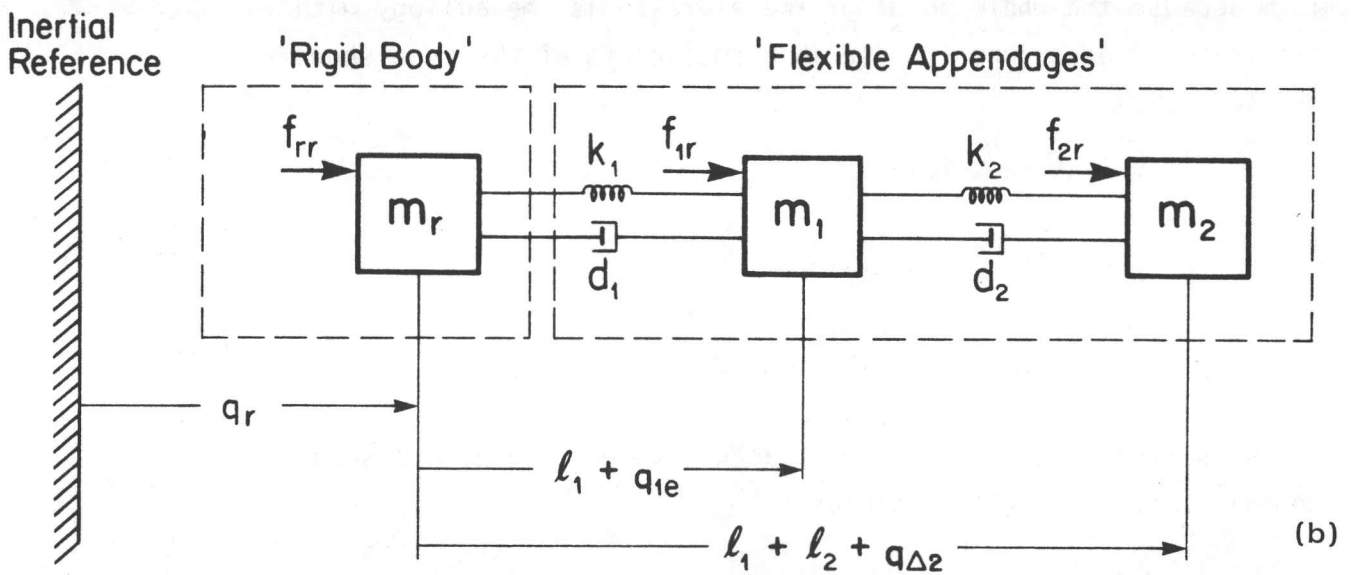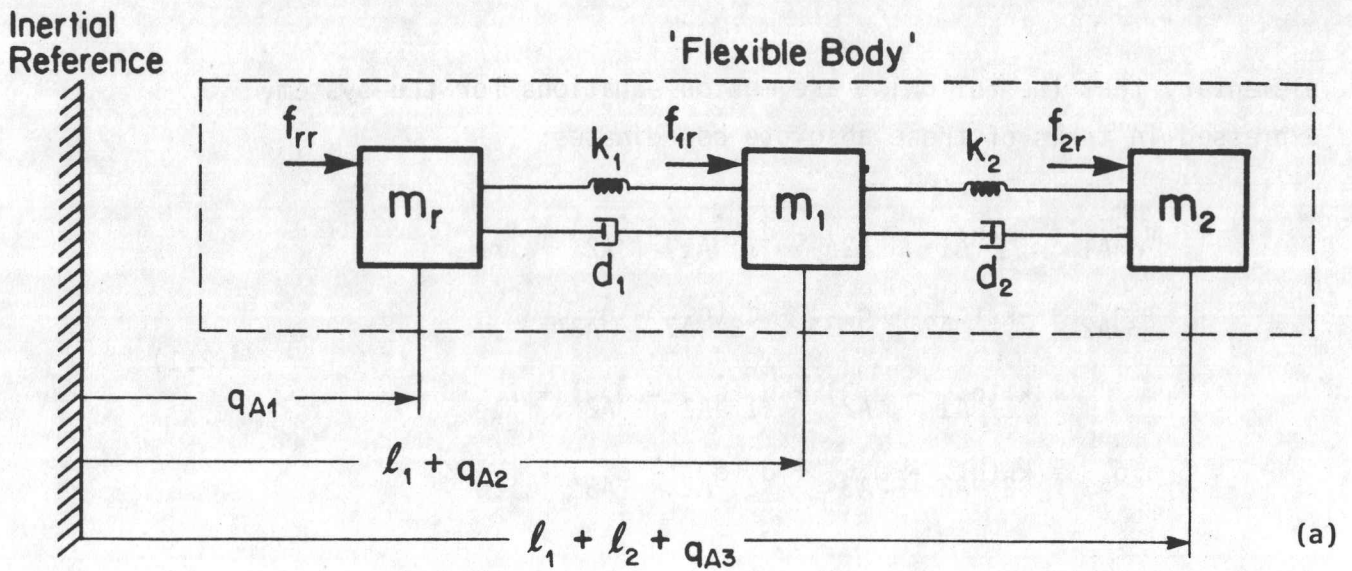
Figure 2.1: Simple Three-Mass Analogy

ℓ_1: Rest Length of Spring 1 ℓ_2: Rest Length of Spring 2

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elementary that the following are motion equations for the system, expressed in terms of these absolute coordinates:

$$m_r \ddot{q}_{A1} = -k_1(q_{A1} - q_{A2}) - d_1(\dot{q}_{A1} - \dot{q}_{A2}) + f_{rr}$$

$$m_1 \ddot{q}_{A2} = -k_2(q_{A2} - q_{A3}) - d_2(\dot{q}_{A2} - \dot{q}_{A3})$$

$$+ k_1(q_{A1} - q_{A2}) + d_1(\dot{q}_{A1} - \dot{q}_{A2}) + f_{1e}$$

$$m_2 \ddot{q}_{A3} = k_2(q_{A2} - q_{A3}) + d_2(q_{A2} - q_{A3}) + f_{2e}$$

(2.1)

We are especially interested in expressing these equations in matrix form because the whole point of the exercise is the analogy with the (matrix) equations for flexible spacecraft of the type shown in Fig. 1.2.

To that end, let

$$\underline{q}_A \triangleq col\{q_{A1}, q_{A2}, q_{A2}\}$$

(2.2)

$$\underline{f}_A \triangleq col\{f_{rr}, f_{1r}, f_{2r}\}$$

(2.3)

The subscript 'A' is a reminder that we are dealing with absolute coordinates. Then (2.1) becomes

$$\underline{M}_A \ddot{\underline{q}}_A + \underline{D}_A \dot{\underline{q}}_A + \underline{K}_A \underline{q}_A = \underline{f}_A$$

(2.4)

with the mass, damping, and stiffness matrices introduced as follows:

$$\underline{M}_A = \begin{bmatrix} m_r & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix}$$

(2.5)

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$$\underline{D}_A \triangleq \begin{bmatrix} d_1 & -d_1 & 0 \\ -d_1 & d_1 + d_2 & -d_2 \\ 0 & -d_2 & d_2 \end{bmatrix} \tag{2.6}$$

$$\underline{K}_A \triangleq \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \tag{2.7}$$

The simple diagonal form of $\underline{M}_A$ makes it immediately clear that

$$\underline{M}_A > 0 \tag{2.8}$$

a notation which means that $\underline{M}_A$ is positive definite. (In a similar fashion, $\underline{M} \geqq 0$ means "$\underline{M}$ is positive semidefinite.")

Note that $\underline{D}_A$ is *semi*definite:

$$\underline{D}_A \geqq 0 \tag{2.9}$$

This follows from the fact that the eigenvalues of $\underline{D}_A$ are

$$\lambda\{\underline{D}_A\} = 0, \quad d_1 + d_2 \pm (d_1^2 - d_1 d_2 + d_2^2)^{\frac{1}{2}} \tag{2.10}$$

And, since

$$d_1^2 - d_1 d_2 + d_2^2 \equiv (d_1 - d_2)^2 + d_1 d_2 \tag{2.11}$$

we see that two of the three eigenvalues in (2.11) are positve (un-less $d_1 = d_2 = 0$, in which case there is, of course, no damping at all).

Exactly the same remarks apply to the stiffness matrix $\underline{K}_A$:

$$\underline{K}_A \geqq 0 \qquad\qquad (2.12)$$

In fact, the zero eigenvalues of $\underline{D}_A$ and $\underline{K}_A$ are associated with the same eigenvector:

$$\underline{D}_A [1 \quad 1 \quad 1]^T = \underline{0}$$

$$\underline{K}_A [1 \quad 1 \quad 1]^T = \underline{0} \qquad\qquad (2.13)$$

Physically, this eigenvector corresponds to a 'rigid-body' mode, in which all three masses are displaced equally to the right.

2.3 Global Relative Coordinates

We turn now, in our simple three-mass system, to consider the system of coordinates shown in Fig. 2.1b. Here, the absolute displacement of m_r is given by q_r. (The subscript 'r' denotes the Reference Rigid Body, represented in our simple system by the mass m_r. Obviously, q_r in the present system of coordinates, and q_{A1} in the last set of coordinates, are identical:

$$q_r \equiv q_{A1} \qquad\qquad (2.14)$$

Thus we see that even though our present system of coordinates is said to be a system of *relative* coordinates, the Reference Rigid Body itself, though used as a reference for all other coordinates, has its own displacement characterized by *absolute* coordinates.

All other displacements in the system, however, are specified as *relative* coordinates; they represent the displacements of all other parts of the system, *relative to the Reference Rigid Body* (m_r in this case), *due to elasticity*. For the simple three-mass system of Fig. 2.1b, these coordinates are denoted q_{1e} and $q_{\Delta 2}$, and are

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defined as shown. If the entire system were rigid ($k_1 \rightarrow \infty$, $k_2 \rightarrow \infty$). All positions would be uniquely determined by the simple coordinate q_r.

We denote the current set of coordinates by

$$\underline{q}_B \triangleq \mathrm{col}\{q_r, q_{1e}, q_{\Delta2}\} \tag{2.15}$$

and note that

$$\underline{q}_A = \underline{\Gamma}_{AB}\underline{q}_B \tag{2.16}$$

where

$$\underline{\Gamma}_{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \tag{2.17}$$

Then, on insertion of (2.16) in (2.47) and after premultiplication by $\underline{\Gamma}_{AB}^T$, we have a set of motion equations in the new coordinates:

$$\underline{M}_B\ddot{\underline{q}}_B + \underline{D}_B\dot{\underline{q}}_B + \underline{K}_B\underline{q}_B = \underline{\delta}_B \tag{2.18}$$

where the definitions

$$
\begin{aligned}
\underline{M}_B &\triangleq \underline{\Gamma}_{AB}^T \underline{M}_A \underline{\Gamma}_{AB} \\
\underline{D}_B &\triangleq \underline{\Gamma}_{AB}^T \underline{D}_A \underline{\Gamma}_{AB} \\
\underline{K}_B &\triangleq \underline{\Gamma}_{AB}^T \underline{K}_A \underline{\Gamma}_{AB} \\
\underline{\delta}_B &\triangleq \underline{\Gamma}_{AB}^T \underline{\delta}_A
\end{aligned}
\tag{2.19}
$$

have been introduced.

The elements of the new system matrices are easily calculated from the definitions (2.19):

$$\underline{M}_B \triangleq \begin{bmatrix} m & | & m_1 & m_2 \\ \hline m_1 & | & m_1 & 0 \\ m_2 & | & 0 & m_2 \end{bmatrix} \qquad (2.20)$$

$$\underline{\mathcal{D}}_B \triangleq \begin{bmatrix} 0 & | & 0 & 0 \\ \hline 0 & | & d_1 + d_2 & -d_2 \\ 0 & | & -d_2 & d_2 \end{bmatrix} \qquad (2.21)$$

$$\underline{K}_B \triangleq \begin{bmatrix} 0 & | & 0 & 0 \\ \hline 0 & | & k_1 + k_2 & -k_2 \\ 0 & | & -k_2 & k_2 \end{bmatrix} \qquad (2.22)$$

$$\underline{\mathit{6}}_B \triangleq \begin{bmatrix} f \\ \hline f_{1r} \\ f_{2r} \end{bmatrix} \qquad (2.23)$$

where

$$m \triangleq m_r + m_1 + m_2 \qquad (2.24)$$

$$f \triangleq f_{rr} + f_{1r} + f_{2r} \qquad (2.25)$$

Obviously m is the total system mass and f is the total external force on the system.

The partitioning indicated in (2.20) - (2.23) corresponds to 'rigid' and 'elastic' coordinates; it will be useful in later

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comparisons with more general cases. Also, because $\underline{\Gamma}_{AB}$ is nonsingular, the sign definiteness properties of $\underline{M}_A$, $\underline{D}_A$, and $\underline{K}_A$ carry over also to $\underline{M}_B$, $\underline{D}_B$ and $\underline{K}_B$:

$$\underline{M}_B > 0$$

$$\underline{D}_B \geqq 0 \tag{2.26}$$

$$\underline{K}_B \geqq 0$$

The rigid body made is $\underline{q}_B = \text{col}\{1,0,0\}$.

It is also useful to compare the degrees of complexity of $\{\underline{M}_B, \underline{D}_B, \underline{K}_B\}$ as compared with $\{\underline{M}_A, \underline{D}_A, \underline{K}_A\}$. Basically, $\underline{M}_B$ is *more* complicated than $\underline{M}_A$, but $\{\underline{D}_B, \underline{K}_B\}$ are *less* complicated than their counterparts $\{\underline{D}_A, \underline{K}_A\}$. It could be argued that there is a slight overall simplification in matrix elements in that $\{\underline{M}_A, \underline{D}_A, \underline{K}_A\}$ contain, between them, 10 zero elements, while $\{\underline{M}_B, \underline{D}_B, \underline{K}_B\}$ contain 12 zero elements. However, this is really grasping at straws at this early stage of the discussion. It is best to wait until the Section 3, when each mass-spring-damper is replaced by a general lightly-damped elastic body, to form more definitive conclusions.

2.4   Local Relative Coordinates

We return again to Fig. 2.1, and consider now the third and final set of coordinates. The coordinate for the Reference Rigid Body is still q_r, which is, in fact an *absolute* coordinate, as observed earlier. The 'flexible appendage', however, is now thought of as a set of (two) substructures, as shown in Fig. 2.1c. Flexible Appendage 2 is an appendage to Flexible Appendage 1, which is, in turn, an appendage to the Reference Rigid Body. Therefore, although the coordinate associated with m_1, namely q_{1e}, is still referred to m_r, the coordinate associated with m_2, denoted q_{2e}, is with reference to m_1, not m_r. For this reason, this set of coordinates will be called *local* relative coordinates, not *global* relative coordinates, as in Section 2.3 and Fig. 2.1b. Thus, whereas global

elastic coordinates are all referred to a common Reference Rigid Body, local relative coordinates are referred to a local reference point in each local flexible body.

The relationship between the global and local relative coordinates is this:

$$q_{1e} \equiv q_{1e}$$
$$q_{2e} \equiv q_{\Delta 2} - q_{1e}$$

(2.27)

Furthermore, we denote by $\underline{q}_C$ our set of local relative coordinates:

$$\underline{q}_C \triangleq \text{col}\{q_r, q_{1e}, q_{2e}\}$$

(2.28)

And we note that

$$\underline{q}_B = \underline{\Gamma}_{BC}\underline{q}_C$$

(2.29)

where

$$\underline{\Gamma}_{BC} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(2.30)

Then, on insertion of (2.29) in (2.18), and after premultiplication by $\underline{\Gamma}_{BC}^T$, the motion equations in the new coordinates are:

$$\underline{M}_C \ddot{\underline{q}}_C + \underline{D}_C \dot{\underline{q}}_C + \underline{K}_C \underline{q}_C = \underline{\delta}_C$$

(2.31)

where the new system matrices are

$$\underline{M}_C \triangleq \underline{\Gamma}_{BC}^T \underline{M}_B \underline{\Gamma}_{BC}$$

$$\underline{D}_C \triangleq \underline{\Gamma}_{BC}^T \underline{D}_B \underline{\Gamma}_{BC} \quad ; \quad \underline{\delta}_C = \underline{\Gamma}_{BC}^T \underline{\delta}_B$$

$$\underline{K}_C \triangleq \underline{\Gamma}_{BC}^T \underline{K}_B \underline{\Gamma}_{BC}$$

(2.32)

In other words,

$$\underline{M}_C = \begin{bmatrix} m & m_{r1} & m_{r2} \\ m_{r1} & m_{11} & m_{12} \\ m_{r2} & m_{12} & m_{22} \end{bmatrix} \tag{2.33}$$

$$\underline{D}_C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & d_1 & 0 \\ 0 & 0 & d_2 \end{bmatrix} \tag{2.34}$$

$$\underline{K}_C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_1 & 0 \\ 0 & 0 & k_2 \end{bmatrix} \tag{2.35}$$

$$\underline{b}_C = \begin{bmatrix} f \\ f_{1e} \\ f_{2e} \end{bmatrix} \tag{2.36}$$

where

$$m_{11} \triangleq m_1 + m_2 \qquad m_{r1} \triangleq m_1 + m_2 \tag{2.37}$$
$$m_{12} = m_{22} \triangleq m_2 \qquad m_{r2} \triangleq m_2$$

$$f_{1e} \triangleq f_1 + f_2 \tag{2.38}$$
$$f_{2e} \triangleq f_2$$

One may also go directly from the absolute coordinates q_A to the local relative coordinates q_C via the single transformation

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$$\underline{q}_A = \underline{\Gamma}_{AC}\underline{q}_C$$

where

$$\underline{\Gamma}_{AC} = \underline{\Gamma}_{AB}\underline{\Gamma}_{BC}$$

That is,

$$\underline{\Gamma}_{AC} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

from (2.17) and (2.30).

It is again of interest to compare the complexity of the system matrices $\{\underline{M}_B,\underline{D}_B,\underline{K}_B\}$ to that of $\{\underline{M}_C,\underline{D}_C,\underline{K}_C\}$. While the former have 12 zero elements, the latter have 14 zero elements. The progression of coordinates $\underline{q}_A \rightarrow \underline{q}_B \rightarrow \underline{q}_C$ tends to simplify the damping and stiffness matrices at the expense of adding complexity to the mass matrix. As we shall now see in the next section, this trend is valid also when the 'appendages' of Fig. 2.1 are generalized from one-degree-of-freedom appendages to general elastic bodies.

3. RIGID REFERENCE BODY WITH A TWO-SUBSTRUCTURE APPENDAGE

Consider again the mechanical system shown in Fig. 1.2b, reproduced in Fig. 3.1 for convenience. The analogy with the three-mass system of Fig. 2.1 should be quite plain: R, E_1, and E_2 are, respectively, generalizations of m_r, $\{m_1, k_1, d_1\}$ and $\{m_2, k_2, d_2\}$.

3.1 Kinetic Energy

The velocity distribution in the system is

$$\underline{v} = \begin{bmatrix} \dot{\underline{r}} - \underline{r}^x\dot{\underline{\theta}}, & \underline{r} \, \varepsilon \, R; & \text{expressed in } F_r \\[2mm] \dot{\underline{r}}_1 - \underline{r}_1^x\dot{\underline{\theta}}_1 + \underline{\Psi}_{1e}(\underline{r}_1)\dot{\underline{q}}_{1e}, & \underline{r}_1 \, \varepsilon \, E_1; & \text{expressed in } F_1 \\[2mm] \dot{\underline{r}}_2 - \underline{r}_2^x\dot{\underline{\theta}}_2 + \underline{\Psi}_{2e}(\underline{r}_2)\dot{\underline{q}}_{2e}, & \underline{r}_2 \, \varepsilon \, E_2; & \text{expressed in } F_2 \end{bmatrix} \qquad (3.1)$$

Here, $\underline{r}(t)$ is the absolute displacement of O, expressed in F_r; $\underline{r}_1(t)$ is the absolute displacement of O_1, expressed in F_1; and $\underline{r}_2(t)$ is the absolute displacement of O_2, expressed in F_2. In a similar fashion, $\underline{\theta}$ is the absolute rotation of R at O, expressed in F_r; $\underline{\theta}_1$ is the absolute rotation E_1 at O_1, expressed in F_1; and $\underline{\theta}_2$ is the absolute rotation of E_2 at O_2, expressed in F_2. The shape functions $\underline{\Psi}_{1e}$ and $\underline{\Psi}_{2e}$, and their associated coordinates $\underline{q}_{1e}(t)$ and $\underline{q}_{2e}(t)$, represent the elastic displacements within E_1 and E_2, respectively. $\underline{\Psi}_{1e}$ is expressed in F_1, and $\underline{\Psi}_{2e}$ is expressed in F_2.

The total kinetic energy of the system is

$$T = T_r + T_1 + T_2 \qquad (3.2)$$

where

$$T_r = \tfrac{1}{2}\int_R \underline{v}^T\underline{v} \, dm \qquad (3.3)$$
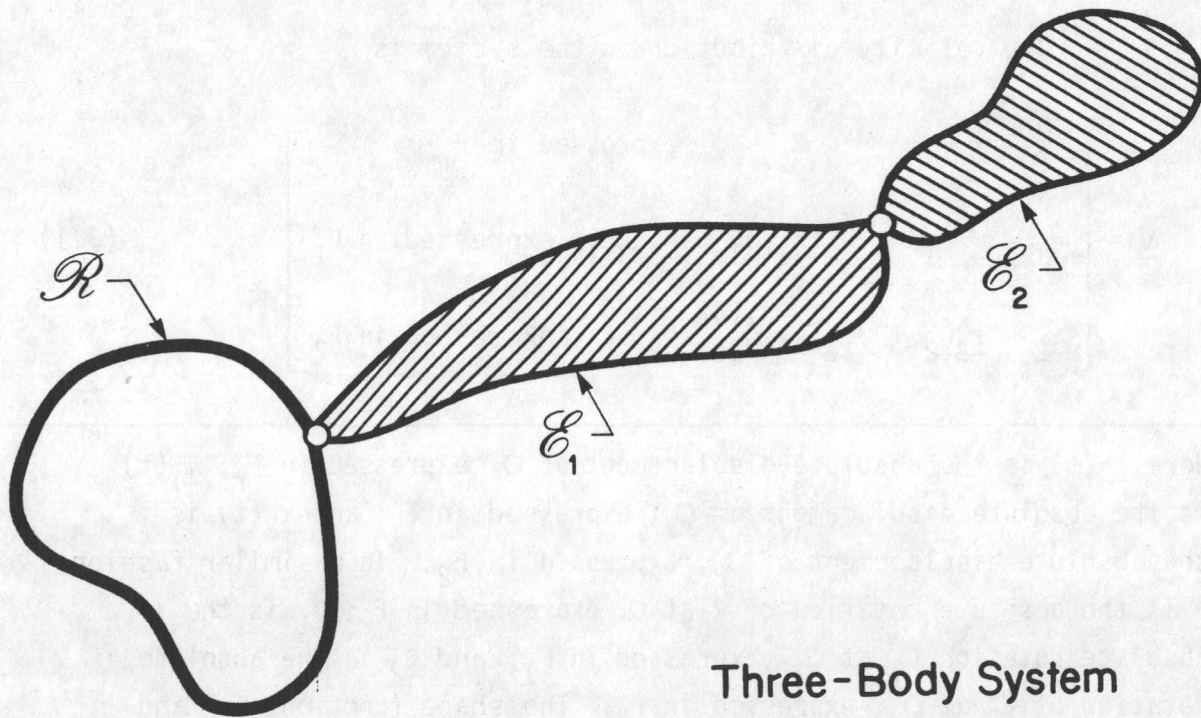
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Three-Body System

Figure 3.1: The General Three-Body Model for a Flexible Spacecraft

$$T_1 = \tfrac{1}{2}\int_{E_1} \underline{v}^T \underline{v} \ dm \tag{3.4}$$

$$T_2 = \tfrac{1}{2}\int_{E_2} \underline{v}^T \underline{v} \ dm \tag{3.5}$$

From (5.1),

$$T_r = \tfrac{1}{2}\underline{\dot{q}}_r^T \underline{M}_r \underline{\dot{q}}_r \tag{3.6}$$

where

$$\underline{q}_r \triangleq \mathrm{col}\{\underline{r}, \underline{\theta}\} \tag{3,7}$$

$$\underline{M}_r \triangleq \begin{bmatrix} m_r \underline{1} & -\underline{c}_r^x \\ \underline{c}_r^x & \underline{J}_r \end{bmatrix} \tag{3.8}$$

$$m_r \triangleq \int_R dm$$

$$\underline{c}_r \triangleq \int_R \underline{r} \ dm$$

$$\underline{J}_r \triangleq - \int_R \underline{r}^x \underline{r}^x \ dm \tag{3.11}$$

In a similar fashion, from (3.1),

$$T_1 = \tfrac{1}{2}\underline{\dot{q}}_1^T \underline{M}_1 \underline{\dot{q}}_1 \tag{3.12}$$

where

$$\underline{q}_1 \triangleq \mathrm{col}\{\underline{q}_{1r}, \underline{q}_{1e}\} \tag{3.13}$$

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$$\underline{M}_1 \triangleq \begin{bmatrix} \underline{M}_{1rr} & \underline{M}_{1re} \\ \underline{M}_{1re}^T & \underline{M}_{1ee} \end{bmatrix} \tag{3.14}$$

$$\underline{M}_{1ee} \triangleq \int_{E_1} \underline{\Psi}_{1e}^T \underline{\Psi}_{1e} \, dm \tag{3.15}$$

$$\underline{M}_{1re} \triangleq \begin{bmatrix} \underline{P}_1 \\ \underline{H}_1 \end{bmatrix} \tag{3.16}$$

$$\underline{P}_1 \triangleq \int_{E_1} \underline{\Psi}_{1e} \, dm \tag{3.17}$$

$$\underline{H}_1 \triangleq \int_{E_1} r_1^x \underline{\Psi}_{1e} \, dm \tag{3.18}$$

$$\underline{q}_{1r} \triangleq \text{col}\{\underline{r}_1, \underline{\theta}_1\} \tag{3.19}$$

$$\underline{M}_{1rr} \triangleq \begin{bmatrix} m_1 \underline{1} & -\underline{c}_1^x \\ \underline{c}_1^x & \underline{J}_1 \end{bmatrix} \tag{3.20}$$

$$m_1 \triangleq \int_{E_1} dm \tag{3.21}$$

$$\underline{c}_1 \triangleq \int_{E_1} r_1 \, dm \tag{3.22}$$

$$\underline{J}_1 \triangleq -\int_{E_1} r_1^x r_1^x \, dm \tag{3.23}$$

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In the same fashion,

$$T_2 = \tfrac{1}{2}\underline{\dot{q}}_2^T \underline{M}_2 \underline{\dot{q}}_2 \qquad (3.24)$$

with a set of definitions identical to (3.13-23), but with
$(\ )_1 \rightarrow (\ )_2$, and all quantities expressed in F_{2e} instead of F_{1e}.

3.2 The System Mass Matrix

In looking over the kinetic expressions just derived,
we see that the list of coordinates reads like this:

$$\{\underline{r},\underline{\theta},\underline{r}_1,\underline{\theta}_1,\underline{q}_{1e},\underline{r}_2,\underline{\theta}_2,\underline{q}_{2e}\} \qquad (3.25)$$

There is redundancy here, however. Assuming that E_1 is fixed
rigidly in R, the displacements $\{\underline{r}_1,\underline{\theta}_1\}$ are determined once
$\{\underline{r},\underline{\theta}\}$ are known. In fact,

$$\underline{r}_1 = \underline{C}_{1r}(\underline{r} - \underline{r}_{o1}^X \underline{\theta})$$

$$\underline{\theta}_1 = \underline{C}_{1r}\underline{\theta} \qquad (3.26)$$

where $\underline{r}_{o1}$ is the vector from O to O_1, expressed in F_r, and $\underline{C}_{1r}$
is the 'rotation matrix' from F_r to F_1. The constraints (3.26)
can be compactly summarized thus:

$$\underline{q}_{1r} = \underline{\Gamma}_{1r}\underline{q}_r \qquad (3.27)$$

with

$$\underline{\Gamma}_{1r} \triangleq \begin{bmatrix} \underline{C}_{1r} & -\underline{C}_{1r}\underline{r}_{o1}^X \\ \underline{O} & \underline{C}_{1r} \end{bmatrix} \qquad (3.28)$$

Looking further at the list (3.25) we see that $\underline{r}_2$ and $\underline{\theta}_2$ are known once the displacement of O_2 in E_1 is known. Thus,

$$\underline{r}_2 = \underline{C}_{21}(\underline{r}_1 - r_{12}^{X}\underline{\theta}_1 + \underline{\Psi}_{12}\underline{q}_{1e})$$

$$\underline{\theta}_2 = \underline{C}_{21}(\underline{\theta}_1 + \underline{\Theta}_{12}\underline{q}_{1e})$$

(3.29)

where $\underline{\Psi}_{12}$ and $\underline{\Theta}_{12}$ are related to the elastic displacements, translational and rotational, in E_1 at O_2:

$$\underline{\Psi}_{12} \triangleq \underline{\Psi}_{1e}(\underline{r}_{12})$$

(3.30)

$$\underline{\Theta}_{12} \triangleq \tfrac{1}{2}\underline{\nabla}^{X}\underline{\Psi}_{1e}(\underline{r}_{12})$$

(3.31)

Also, as shown in Fig. 3.1, $\underline{r}_{12}$ is the vector from O_1 to O_2, expressed in F_1, and $\underline{C}_{21}$ is the rotation matrix from F_1 to F_2. We write (3.29) as

$$\underline{q}_{2r} = \underline{\Gamma}_{21}\underline{q}_{1r} + \underline{\Xi}_{21}\underline{q}_{1e}$$

(3.32)

with

$$\underline{\Gamma}_{21} \triangleq \begin{bmatrix} \underline{C}_{21} & -\underline{C}_{21}r_{12}^{X} \\ \underline{0} & \underline{C}_{21} \end{bmatrix}$$

(3.33)

$$\underline{\Xi}_{21} \triangleq \begin{bmatrix} \underline{C}_{21}\underline{\Psi}_{12} \\ \underline{C}_{21}\underline{\Theta}_{12} \end{bmatrix}$$

(3.34)

Finally, combining (3.27) and (3.32), we have

$$\underline{q}_{2r} = \underline{\Gamma}_{2r}\underline{q}_r + \underline{\Xi}_{21}\underline{q}_{1e}$$

(3.35)

where

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$$\underline{\Gamma}_{2r} \triangleq \underline{\Gamma}_{21}\underline{\Gamma}_{1r} \tag{3.36}$$

Having found these constraint equations we are now in a position to find the final form for the kinetic energy and the system mass matrix.

Setting (3.27) and (3.35) in the kinetic energy expressions of Section 3.1, we have

$$T = \tfrac{1}{2}\underline{\dot{q}}^T \underline{M}\underline{\dot{q}} \tag{3.37}$$

where

$$\underline{q} \triangleq \text{col}\{\underline{q}_r, \underline{q}_{1e}, \underline{q}_{2e}\} \tag{3.38}$$

$$\underline{M} \triangleq \begin{bmatrix} \underline{M}_{rr} & \underline{M}_{r1} & \underline{M}_{r2} \\ \underline{M}_{r1}^T & \underline{M}_{11} & \underline{M}_{12} \\ \underline{M}_{r2}^T & \underline{M}_{12}^T & \underline{M}_{22} \end{bmatrix} \tag{3.39}$$

and the mass partitions are given by

$$\underline{M}_{rr} \triangleq \underline{M}_r + \underline{\Gamma}_{1r}^T \underline{M}_{1rr}\underline{\Gamma}_{1r} + \underline{\Gamma}_{2r}^T \underline{M}_{2rr}\underline{\Gamma}_{2r}$$

$$\underline{M}_{r1} \triangleq \underline{\Gamma}_{1r}^T \underline{M}_{1re} + \underline{\Gamma}_{2r}^T \underline{M}_{2re}\underline{\Xi}_{21}$$

$$\underline{M}_{r2} \triangleq \underline{\Gamma}_{2r}^T \underline{M}_{2re} \tag{3.40}$$

$$\underline{M}_{11} \triangleq \underline{M}_{1ee} + \underline{\Xi}_{21}^T \underline{M}_{2rr}\underline{\Xi}_{21}$$

$$\underline{M}_{12} \triangleq \underline{\Xi}_{21}^T \underline{M}_{2re}$$

$$\underline{M}_{22} \triangleq \underline{M}_{2ee}$$

An analogy can be drawn between the structure of the mass matrix in (3.39) and the earlier mass matrix in (2.33) for the much simpler system of Fig. 2.1: the upper-left partition reflects the rigid-only mass properties; the right-most column (and therefore the bottom row also) depends on the most outboard substructure; and so on. This analogy will be more fruitful, however, when we examine in a similar way the stiffness and damping matrices, to which we now turn.

3.3   Potential Energy and Stiffness Matrix

The potential energy is much easier to work with using the present coordinates because it depends only on the elastic coordinates, $\underline{q}_{1e}$ and $\underline{q}_{2e}$. In fact

$$V = V_r + V_1 + V_2 \tag{3.41}$$

where

$$V_r = 0 \tag{3.42}$$

$$V_1 = \tfrac{1}{2}\underline{q}_{1e}^T \underline{K}_1 \underline{q}_{1e} \tag{3.43}$$

$$V_2 = \tfrac{1}{2}\underline{q}_{2e}^T \underline{K}_2 \underline{q}_{2e} \tag{3.44}$$

and $\underline{K}_1$ and $\underline{K}_2$ can be calculated (finite element method) once $\underline{\Psi}_{1e}$ and $\underline{\Psi}_{2e}$ are chosen.

Therefore, the stiffness matrix for the system is extracted as follows:

$$V = \tfrac{1}{2}\underline{q}^T \underline{K} \underline{q} \tag{3.45}$$

where

$$\underline{K} \triangleq \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{K}_1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{K}_2 \end{bmatrix} \tag{3.46}$$

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The analogy between the stiffness matrix in (3.46) for the quite general system of Fig. 3.1, and the stiffness matrix in (2.35) for the very simple system of Fig. 2.1 is now reasonably evident. (Indeed, the sole purpose of Section 2 was to lay the foundation for this analogy.) In both cases, the stiffness matrix is block-diagonal (the 'blocks' for the simple system being, of course, simply individual elements); in both cases, the upper-left block--the one associated with the rigid coordinates--is zero, the remaining blocks being positive definite stiffness matrices, each associated with a particular substructure in the chain. This is the form one should always expect when 'local relative coordinates' are used, and it is this simple form that makes local relative coordinates an attractive set to use.

3.4   Damping Matrix

Likewise also damping can be associated with only the coordinates $\underline{q}_{1e}$ and $\underline{q}_{2e}$:

$$\underline{D} \triangleq \begin{bmatrix} \underline{O} & \underline{O} & \underline{O} \\ \underline{O} & \underline{D}_1 & \underline{O} \\ \underline{O} & \underline{O} & \underline{D}_2 \end{bmatrix} \qquad (3.47)$$

We shall discuss extensively later the role of this damping matrix.

3.5   Generalized Forces

To complete a specification of all the dynamical elements necessary to write motion equations, the generalized forces are needed. These follow from the external force distribution. We shall take this to be a body force, but extension to surface forces, point forces, or even torque distributions, is not difficult. We have

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$$f_{ext} = \begin{bmatrix} \underline{f}_{ext}(\underline{r},t) & , & \underline{r} \ \epsilon \ R \ ; \ \text{expressed in } F_r \\ \underline{f}_{ext}(\underline{r}_1,t) & , & \underline{r}_1 \ \epsilon \ E_1; \ \text{expressed in } F_1 \\ \underline{f}_{ext}(\underline{r}_2,t) & , & \underline{r}_2 \ \epsilon \ E_2; \ \text{expressed in } F_2 \end{bmatrix} \qquad (3.48)$$

Then the virtual work done by this force distribution is [consult (3.1)]

$$\delta W = \delta W_r + \delta W_1 + \delta W_2 \qquad (3.49)$$

where

$$\delta W_r = \int_R \underline{f}_{ext}^T (\delta \underline{r} - \underline{r}^x \delta \underline{\theta}) \ dV \qquad (3.50)$$

$$\delta W_1 = \int_{E_1} \underline{f}_{ext}^T (\delta \underline{r}_1 - \underline{r}_1^x \delta \underline{\theta}_1 + \underline{\Psi}_{1e} \delta \underline{q}_{1e}) \ dV \qquad (3.51)$$

$$\delta W_2 = \int_{E_2} \underline{f}_{ext}^T (\delta \underline{r}_2 - \underline{r}_2^x \delta \underline{\theta}_2 + \underline{\Psi}_{2e} \delta \underline{q}_{2e}) \ dV \qquad (3.52)$$

Thus

$$\delta W_r = \underline{f}_r^T \delta \underline{r} + \underline{g}_r^T \delta \underline{\theta} \qquad (3.53)$$

$$\delta W_1 = \underline{f}_1^T \delta \underline{r}_1 + \underline{g}_1^T \delta \underline{\theta}_1 + \underline{\delta}_{1ee}^T \delta \underline{q}_{1e} \qquad (3.54)$$

$$\delta W_2 = \underline{f}_2^T \delta \underline{r}_2 + \underline{g}_2^T \delta \underline{\theta}_2 + \underline{\delta}_{2ee}^T \delta \underline{q}_{2e} \qquad (3.55)$$

where

$$\underline{f}_r \triangleq \int_R \underline{f}_{ext} \ dV \qquad (3.56)$$

$$\underline{g}_r \triangleq \int_R \underline{r}^x \underline{f}_{ext} \ dV \qquad (3.57)$$

$$\underline{f}_1 \triangleq \int_{E_1} \underline{f}_{ext} \ dV \qquad (3.58)$$

$$\underline{g}_1 \triangleq \int_{E_1} \underline{r}_1^x \underline{f}_{ext} \ dV \qquad (3.59)$$

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$$\underline{f}_{1ee} \triangleq \int_{E_1} \underline{\Psi}_{1e}^T \underline{f}_{ext} \, dV \tag{3.60}$$

$$\underline{f}_2 \triangleq \int_{E_2} \underline{f}_{ext} \, dV \tag{3.61}$$

$$\underline{g}_2 \triangleq \int_{E_2} \underline{r}_2^X \underline{f}_{ext} \, dV \tag{3.62}$$

$$\underline{f}_{2ee} \triangleq \int_{E_2} \underline{\Psi}_{2e}^T \underline{f}_{ext} \, dV \tag{3.63}$$

The expressions (3.53-55) can be further contracted thus:

$$\delta W_r = \underline{f}_{rr}^T \delta \underline{q}_r \tag{3.64}$$

$$\delta W_1 = \underline{f}_{1r}^T \delta \underline{q}_{1r} + \underline{f}_{1ee}^T \delta \underline{q}_{1e} \tag{3.65}$$

$$\delta W_2 = \underline{f}_{2r}^T \delta \underline{q}_{2r} + \underline{f}_{2ee}^T \delta \underline{q}_{2e} \tag{3.66}$$

where

$$\underline{f}_{rr} \triangleq \text{col}\{\underline{f}_r, \underline{g}_r\} \tag{3.67}$$

$$\underline{f}_{1r} \triangleq \text{col}\{\underline{f}_1, \underline{g}_1\} \tag{3.68}$$

$$\underline{f}_{2r} \triangleq \text{col}\{\underline{f}_2, \underline{g}_2\} \tag{3.69}$$

After using (3.27) and (3.35) to re-state $\delta \underline{q}_{1r}$ and $\delta \underline{q}_{2r}$ in terms more basic coordinates, we find the final expression for the total virtual work

$$\delta W = \underline{f}_r^T \delta \underline{q}_r + \underline{f}_{1e}^T \delta \underline{q}_{1e} + \underline{f}_{2e}^T \delta \underline{q}_{2e}$$

where

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$$\underline{\delta}_r \triangleq \underline{\delta}_{rr} + \underline{\Gamma}_{1r}^T \underline{\delta}_{1r} + \underline{\Gamma}_{2r}^T \underline{\delta}_{2r} \tag{3.70}$$

$$\underline{\delta}_{1e} \triangleq \underline{\delta}_{1ee} + \underline{\Xi}_{21}^T \underline{\delta}_2 \tag{3.71}$$

$$\underline{\delta}_{2e} \triangleq \underline{\delta}_{2ee} \tag{3.72}$$

That is,

$$\delta W = \underline{\delta}^T \delta \underline{q} \tag{3.73}$$

where

$$\underline{\delta} \triangleq \text{col}\{\underline{\delta}_r, \underline{\delta}_{1e}, \underline{\delta}_{2e}\} \tag{3.74}$$

Again, the analogy with (2.36) can be drawn.

3.6   Motion Equations

The motion equations for the three-body system shown in Fig. 3.1 are therefore

$$\underline{M}\ddot{\underline{q}} + \underline{D}\dot{\underline{q}} + \underline{K}\underline{q} = \underline{\delta} \tag{3.75}$$

which, as can be seen from (3.39), (3.46), (3.47), and (3.74), may be expanded to give the set shown in Table 3.1.

3.7   Reduction to Simple System of Section 2

The three-body system of this section is a substantial generalization of the simple three-mass system discussed in Section 2, and to make this point very clear, the present system will now be reduced to the former system as the simplest special case.

First, there is translation in only one direction, and no rotation at all:

Table 3.1

Motion Equations for the Three-Body System Shown in Fig. 3.1

$$
\begin{bmatrix} \underline{M}_{rr} & \underline{M}_{r1} & \underline{M}_{r2} \\ \underline{M}_{r1}^T & \underline{M}_{11} & \underline{M}_{12} \\ \underline{M}_{r2}^T & \underline{M}_{12}^T & \underline{M}_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_{1e} \\ \ddot{q}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{O} & \underline{O} & \underline{O} \\ \underline{O} & \underline{D}_1 & \underline{O} \\ \underline{O} & \underline{O} & \underline{D}_2 \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{q}_{1e} \\ \dot{q}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{O} & \underline{O} & \underline{O} \\ \underline{O} & \underline{K}_1 & \underline{O} \\ \underline{O} & \underline{O} & \underline{K}_2 \end{bmatrix} \begin{bmatrix} q_r \\ q_{1e} \\ q_{2e} \end{bmatrix} = \begin{bmatrix} \underline{b}_r \\ \underline{b}_{1e} \\ \underline{b}_{2e} \end{bmatrix}
$$

cf: Fig. 2.1c

$$\underline{r} \to \underline{r} \quad ; \quad \underline{\theta} \to \underline{0}$$

$$\underline{r}_1 \to \underline{r}_1 \quad ; \quad \underline{\theta}_{1e} \to \underline{0}$$

$$\underline{r}_2 \to \underline{r}_2 \quad ; \quad \underline{\theta}_{2e} \to \underline{0}$$

Furthermore, there is only one elastic degree of freedom in each of E_1 and E_2:

$$\underline{\Psi}_{1e} q_{1e} \to 1 \cdot q_{1e}$$

$$\underline{\Psi}_{2e} q_{2e} \to 1 \cdot q_{2e}$$

Therefore, the mass matrices associated with each body reduce to:

$$\underline{M}_r \to m_r$$

$$\underline{M}_{1rr} \to m_1 \quad ; \quad \underline{M}_{1re} \to m_1 \quad ; \quad \underline{M}_{1ee} \to m_1$$

$$\underline{M}_{2rr} \to m_2 \quad ; \quad \underline{M}_{2re} \to m_2 \quad ; \quad \underline{M}_{2ee} \to m_2$$

Now the relationship between the three 'rigid' coordinates is simple: they are all identical. Therefore

$$\underline{\Gamma}_{1r} \to 1; \quad \underline{\Gamma}_{21} \to 1; \quad \underline{\Gamma}_{2r} \to 1$$

Also,

$$\underline{\Xi}_{21} \to 1$$

Hence

$$\underline{M}_{rr} \to m_r + m_1 + m_2$$

$$\underline{M}_{r1} \to m_1 + m_2$$

$$\underline{M}_{r2} \to m_2$$

$$\underline{M}_{11} \to m_1 + m_2 \quad ; \quad \underline{M}_{12} \to m_2 \quad ; \quad \underline{M}_{22} \to m_2$$

which is in accord with (2.33) for the three-mass system.

In a similar manner, the damping and stiffness matrices reduce in an obvious fashion:

$$\underline{D}_1 \rightarrow d_1 \; ; \qquad \underline{D}_2 \rightarrow d_2$$

$$\underline{K}_1 \rightarrow k_1 \; ; \qquad \underline{K}_2 \rightarrow k_2$$

The coordinates $\underline{q}_{1e}$ and $\underline{q}_{2e}$ are, in the terminology of Section 2, *local relative coordinates*. They are due entirely to *elastic* deformations in the body with which they are associated.

The general system of Fig. 3.1 can often be discussed using its three-mass analogy shown in Fig. 2.1c. This will prove true as the discussion proceeds to consider the question of how to build system damping matrices from substructure damping matrices.

4.   MODAL ALTERNATIVES

We shall assume that substructure damping information
is available in the form of modal damping factors. These damping
factors might be assigned based on experience or, better still,
based on measurements. The object of the discussion is to constuct
a damping matrix for the overall system based on known damping
factors for substructural modes. It is also stated here that
the notion of "modal damping factors," i.e., ignoring damping
cross-coupling between modes, is a good assumption if either
(a) the actual modal damping matrix is diagonally dominant, or
(b) the structure is lightly damped. Clearly, modal damping
uncoupling is an especially good assumption if the actual modal
damping matrix is both small and diagonally dominant.

There are several classes of modes that can be discussed.
There are, of course, the overall modes of the spacecraft, but
we shall not discuss these directly here. Instead, it is the
substructural modes that are the focus of attention. There are
several sets of such modes for the three-body satellite shown
in Fig. 3.1 and analysed in Section 3. These sets, shown in Fig.
4.1, are as follows:

(a) constrained modes for E_2, denoted M_2

(b) constrained modes for $E_1 + R_2$, denoted $M_{1\Delta}$

(c) constrained modes for E_1, denoted M_1

These modes are discussed in Sections 4.1, 4.2 and 4.3, respectively.

It is important to note the distinction between the
subscripts $(\ )_{1\Delta}$ and $(\ )_1$. In the former, the modes are those
of $E_1 + R_2$, i.e., modes in which E_1 is cantilevered at O_1, and
in which a body inertially identical to E_2, but rigid, not flexible,
is attached to E_1 at O_2. The attached rigid body, R_2, does not
have to be *geometrically* identical to E_2; so long as it has the
same $\{m_2, \underline{c}_2, \underline{J}_2\}$ as does E_2, it will serve as R_2. To recapitulate,
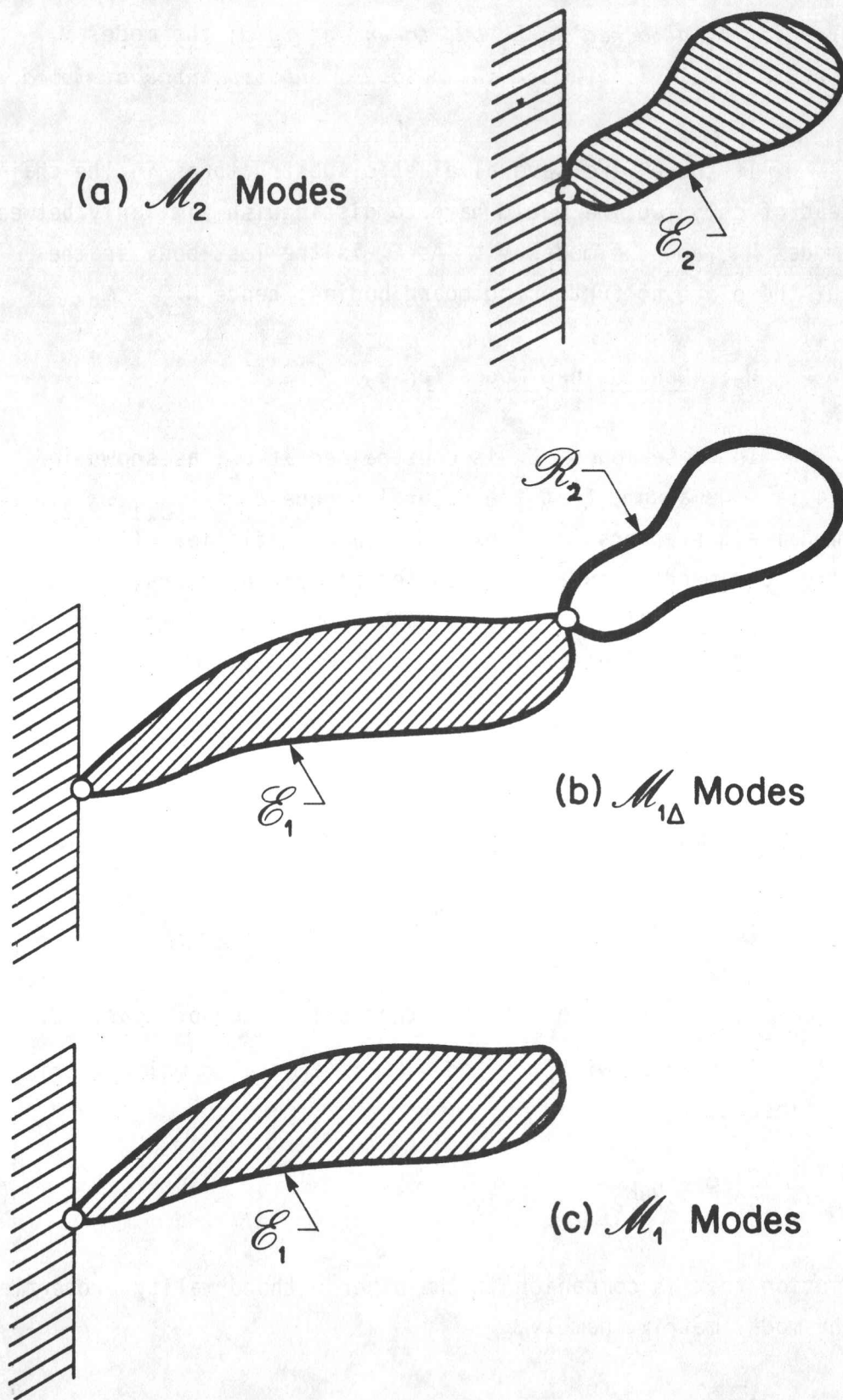the modes $M_{1\Delta}$ *correspond to an elastic E_1 with all other substurctures*

(a) $\mathcal{M}_2$ Modes

$\mathcal{E}_2$

$\mathcal{R}_2$

(b) $\mathcal{M}_{1\Delta}$ Modes

$\mathcal{E}_1$

$\mathcal{E}_1$

(c) $\mathcal{M}_1$ Modes

Figure 4.1: The Three Types of Mode Used

outboard of E_1 attached to E_1 but taken as rigid; the modes M_1
*correspond to E_1 alone, with all outboard substructures stripped
off.*

If there were several elastic substructures in the chain
instead of only two, we would have to distinguish similarly between
the modes $M_{2\Delta}$ and the modes M_2. As E_2 is the last body in the
chain, there are no further outboard bodies; hence $M_{2\Delta} \equiv M_2$.

4.1 M_2:   Constrained Modes for E_2

In these modes, E_2 is constrained at O_2, as shown in
Fig. 4.1a. We assume that the natural frequencies $\{\omega_{2,1}, \omega_{2,2}, \ldots\}$
and modal eigenvectors $\{\underline{e}_{2,1}, \underline{e}_{2,2}, \ldots\}$ are available. (These
are the 'undamped' natural frequencies and eigenvectors, since
the structural damping is assumed to be very light.) We form the
diagonal matrix of frequencies

$$\underline{\Omega}_2 \triangleq \mathrm{diag}\{\omega_{2,1}, \omega_{2,2}, \ldots\} \tag{4.1}$$

and the modal matrix

$$\underline{E}_2 \triangleq [\underline{e}_{2,1} \quad \underline{e}_{2,2} \quad \ldots] \tag{4.2}$$

from the above "available data," either calculated or measured.

Associated with the modes M_2 is a modal damping matrix,
$\hat{\underline{D}}_2$, defined to be

$$\hat{\underline{D}}_2 \triangleq \underline{E}_2^T \underline{D}_2 \underline{E}_2 \tag{4.3}$$

a relation that is companion to the other orthonormality properties
of the modal matrix, namely,

$$\underline{E}_2^T \underline{M}_{22} \underline{E}_2 = \underline{1} \tag{4.4}$$

and

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$$\underline{E}_2^T \underline{K}_2 \underline{E}_2 = \underline{\Omega}_2^2 \qquad\qquad (4.5)$$

Note that although $\underline{1}$ and $\underline{\Omega}_2^2$ are diagonal, there is no physical reason to expect $\underline{\hat{D}}_2$ to be diagonal.

In summary then, we assume that the following have been either calculated or measured: $\underline{\Omega}_2$, $\underline{E}_2$, and $\underline{\hat{D}}_2$. Often $\underline{\hat{D}}_2$ must be assigned based more-or-less on experience, and is usually made diagonal. In such cases we may set

$$\underline{\hat{D}}_2 \doteq 2\underline{Z}_2\underline{\Omega}_2 \qquad\qquad (4.6)$$

as is customary, with $\underline{Z}_2$ being the diagonal matrix of damping factors for the M_2 modes:

$$\underline{Z}_2 \triangleq \text{diag}\{\zeta_{2,1},\zeta_{2,2},\dots\} \qquad\qquad (4.7)$$

Having arrived at an agreed $\underline{\hat{D}}_2$, the damping matrix $\underline{D}_2$ needed in the motion equations is found from (4.3):

$$\underline{D}_2 = \underline{E}_2^{-T}\underline{\hat{D}}_2\underline{E}_2^{-1} \qquad\qquad (4.8)'$$

An alternate version of this equation can be inferred from (4.4):

$$\underline{D}_2 = \underline{M}_{22}\underline{E}_2\underline{\hat{D}}_2\underline{E}_2^T\underline{M}_{22} \qquad\qquad (4.8)''$$

Which version of (4.8) ultimately proves most useful probably depends on numerical algorithmic considerations beyond the scope of this report.

Reflecting on the system motion equations [shown in Table 3.1] we see that the first of the two needed damping matrices, $\underline{D}_2$, has now been specified. We now move on to the second and more interesting of the two, $\underline{D}_1$.

4.2 $M_{1\Delta}$: <u>Constrained Modes for $E_1 + R_2$</u>

Looking at the motion equations in Table 3.1, we see
that an attractive set of eigenvectors is a set that simultaneously
diagonalizes M_{11} and K_1. In other words, this set would do for
the partitions M_{11} and K_1 what the set M_2 did for the partitions
M_{22} and K_2. There is however, a big difference between these
two situations, *and this difference is the kernel idea in this
report*. While M_{22} and K_2 involve only the elastic body E_2, M_{11}
and K_1 *do not involve only the elastic body* E_1. Specifically,
M_{11} involves E_1 and E_2 (although K_1 involves only E_1). Similarly,
D_1 involves only E_1.

Thus we see that the modes associated with the matrices
$\{M_{11}, D_1, K_1\}$ have their stiffness characteristics determined by E_1
alone, but their *inertial* characteristics determined by both E_1
and E_2. Specifically, from (3.40),

$$M_{11} = M_{1ee} + \Xi_{21}^T M_{2rr} \Xi_{21} \qquad (4.9)$$

The first term is evidently the inertia matrix associated with E_1
alone. The second term, however, indicates a coupling term with
E_2. Furthermore, this coupling takes E_2 to be rigid, not flexible
(note the subscript 'rr' on M_{2rr}). Thus, as claimed, the $M_{1\Delta}$
modes correspond to O_1 constrained and E_2 rigid ($E_2 \rightarrow R_2$), with
E_1 remaining flexible.

Once this crucial fact is understood, we have a clear
route to the calculation of D_1 (not the *only* route, however--
see Section 4.3). Either from laboratory testing or by calculation,
the frequencies $\{\omega_{1\Delta,1}, \omega_{1\Delta,2}, \ldots\}$ and the eigenvectors
$\{e_{1\Delta,1}, e_{1\Delta,2}, \ldots\}$ are available, we assume. The needed matrices

$$\Omega_{1\Delta} \triangleq \text{diag}\{\omega_{1\Delta,1}, \omega_{1\Delta,2}, \ldots\} \qquad (4.10)$$

and

$$\underline{E}_{1\Delta} \overset{\triangle}{=} [\underline{e}_{1\Delta,1} \quad \underline{e}_{1\Delta,2} \quad \cdots] \tag{4.11}$$

are formed, with the properties

$$\underline{E}_{1\Delta}^T \underline{M}_{11} \underline{E}_{1\Delta} = \underline{1} \tag{4.12}$$

$$\underline{E}_{1\Delta}^T \underline{K}_1 \underline{E}_{1\Delta} = \underline{\Omega}_{1\Delta}^2 \tag{4.13}$$

Then

$$\underline{D}_1 = \underline{E}_{1\Delta}^{-T} \underline{\hat{D}}_{1\Delta} \underline{E}_{1\Delta}^{-1} \equiv \underline{M}_{11} \underline{E}_{1\Delta} \underline{\hat{D}}_{1\Delta} \underline{E}_{1\Delta}^T \underline{M}_{11} \tag{4.14}$$

which gives the needed $\underline{D}_1$ in terms of $\underline{\hat{D}}_{1\Delta}$. The latter may be either measured, calculated, or invented. If the last, it may as well be invented diagonal:

$$\underline{\hat{D}}_{1\Delta} = 2\underline{Z}_{1\Delta} \underline{\Omega}_{1\Delta} \tag{4.15}$$

where

$$\underline{Z}_{1\Delta} \overset{\triangle}{=} \mathrm{diag}\{\zeta_{1\Delta,1}, \zeta_{1\Delta,2}, \cdots\} \tag{4.16}$$

In any case, the damping matrix $\underline{D}_1$ is now established.

4.3 M_1:   Constrained Modes for E_1

The distinction between the modes $M_{1\Delta}$ and the modes M_1 is now completed: while, for modes $M_{1\Delta}$ all outboard substructures-- rigidized--remained connected to E_1 at O_2, for modes M_1 all outboard substructures are removed. These modes will no longer diagonalized $\underline{M}_{11}$ in Table 3.1, but the loss of this attractive mathematical property is more than offset by a more attractive practical property: in most cases the properties of E_1 *alone* are known, not the properties

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of E_1 with additional mysterious rigid bodies attached thereto.
After all, if E_1 and E_2 are both known, what is the basis for
separating E_2 from E_1? Better in such cases to consider $E_1 + E_2$
as a single flexible structure appended to R, and to use the methods
of Section 4.1. It is for this same reason that the modes of
$E_1 + E_2$ are not among the alternatives discussed in this report--
if we take $E_1 + E_2$ as a single elastic body, we avoid the main
issue: how to construct a damping matrix for an *internal* elastic
body.

Let us take it as granted that the frequencies and eigen-
vectors of the M_1 modes are available:

$$\underline{\Omega}_1 \triangleq \text{diag}\{\omega_{1,1}, \omega_{1,2}, \dots\} \tag{4.17}$$

$$\underline{E}_1 \triangleq [\underline{e}_{1,1} \quad \underline{e}_{1,2} \quad \dots] \tag{4.18}$$

[Incidentally note that we do not call $e_{1,i}$ the "mode shapes."
This would tend to lead to confusion since the mode *shapes* are
found from $\underline{\Psi}_{1e}(\underline{r}_1)\underline{e}_{1,i}$.] These modal parameters have the following
properties:

$$\underline{E}_1^T \underline{M}_{1ee} \underline{E}_1 = \underline{1} \tag{4.19}$$

$$\underline{E}_1^T \underline{K}_1 \underline{E}_1 = \underline{\Omega}_1^2 \tag{4.20}$$

In a similar fashion, the damping matrix $\underline{D}_1$ is trans-
formed thus

$$\hat{\underline{D}}_1 = \underline{E}_1^T \underline{D}_1 \underline{E}_1 \tag{4.21}$$

which means that, given $\hat{\underline{D}}_1$, we can calculate $\underline{D}_1$ from

$$\underline{D}_1 = \underline{E}_1^{-T} \hat{\underline{D}}_1 \underline{E}_1^{-1} \equiv \underline{M}_{1ee} \underline{E}_1 \hat{\underline{D}}_1 \underline{E}_1^T \underline{M}_{1ee} \tag{4.22}$$

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An oft-used but ill-based procedure is to set

$$\hat{\underline{D}}_1 = 2\underline{Z}_1\underline{\Omega}_1 \tag{4.23}$$

and guess at the $\underline{Z}_1$:

$$\underline{Z}_1 \triangleq \text{diag}\{\zeta_{1,1}\zeta_{1,2},\ldots\} \tag{4.24}$$

At all events we assume here that $\hat{\underline{D}}_1$ is 'known', from (4.24) or otherwise, whence $\underline{D}_1$ can be calculated from (4.22).

4.4   Motion Equations

The three types of modes defined in this section can be inserted in the motion equations of Table 3.1 in five possible ways. These result in the motion equations shown in Table 4.1.

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(a) Use M_2 for E_2:

$$\begin{bmatrix} \underline{M}_{rr} & \underline{M}_{r1} & \underline{M}_{r2}\underline{E}_2 \\ \underline{M}_{r1}^T & \underline{M}_{11} & \underline{M}_{12}\underline{E}_2 \\ \underline{E}_2^T\underline{M}_{r2}^T & \underline{E}_2^T\underline{M}_{12}^T & \underline{1} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_{1e} \\ \ddot{n}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{D}_1 & \underline{0} \\ \underline{0} & \underline{0} & \hat{\underline{D}}_2 \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{q}_{1e} \\ \dot{n}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{K}_1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{\Omega}_2^2 \end{bmatrix} \begin{bmatrix} q_r \\ q_{e1} \\ n_{2e} \end{bmatrix} = \begin{bmatrix} \underline{\phi}_r \\ \underline{\phi}_{1e} \\ \underline{Y}_{2e} \end{bmatrix}$$

where $\underline{Y}_{2e} = \underline{E}_2^T\underline{\phi}_{2e}$ and one often sets $\hat{\underline{D}}_2 \triangleq 2\underline{Z}_2\underline{\Omega}_2$

(b) Use $M_{1\Delta}$ for E_1:

$$\begin{bmatrix} \underline{M}_{rr} & \underline{M}_{r1}\underline{E}_{1\Delta} & \underline{M}_{r2} \\ \underline{E}_{1\Delta}^T\underline{M}_{r1}^T & \underline{1} & \underline{E}_{1\Delta}^T\underline{M}_{12} \\ \underline{M}_{r2}^T & \underline{M}_{12}^T\underline{E}_{1\Delta} & \underline{M}_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{n}_{1\Delta} \\ \ddot{q}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \hat{\underline{D}}_{1\Delta} & \underline{0} \\ \underline{0} & \underline{0} & \underline{D}_2 \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{n}_{1\Delta} \\ \dot{q}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{\Omega}_{1\Delta}^2 & \underline{0} \\ \underline{0} & \underline{0} & \underline{K}_2 \end{bmatrix} \begin{bmatrix} q_r \\ n_{1\Delta} \\ q_{2e} \end{bmatrix} = \begin{bmatrix} \underline{\phi}_r \\ \underline{Y}_{1\Delta} \\ \underline{\phi}_{2e} \end{bmatrix}$$

where $\underline{Y}_{1\Delta} = \underline{E}_{1\Delta}^T\underline{\phi}_{1e}$ and one often sets $\hat{\underline{D}}_{1\Delta} \triangleq 2\underline{Z}_{1\Delta}\underline{\Omega}_{1\Delta}$

(c) Use M_1 for E_1:

$$\begin{bmatrix} \underline{M}_{rr} & \underline{M}_{r1}\underline{E}_1 & \underline{M}_{r2} \\ \underline{E}_1^T\underline{M}_{r1}^T & \underline{E}_1^T\underline{M}_{11}\underline{E}_1 & \underline{E}_1^T\underline{M}_{12} \\ \underline{M}_{r2}^T & \underline{M}_{12}^T\underline{E}_1 & \underline{M}_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{n}_{1e} \\ \ddot{q}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \hat{\underline{D}}_1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{D}_2 \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{n}_{1e} \\ \dot{q}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{\Omega}_1^2 & \underline{0} \\ \underline{0} & \underline{0} & \underline{K}_2 \end{bmatrix} \begin{bmatrix} q_r \\ n_{1e} \\ q_{2e} \end{bmatrix} = \begin{bmatrix} \underline{\phi}_r \\ \underline{Y}_{1e} \\ \underline{\phi}_{2e} \end{bmatrix}$$

where $\underline{Y}_{1e} = \underline{E}_1^T\underline{\phi}_{1e}$ and one often sets $\hat{\underline{D}}_1 \triangleq 2\underline{Z}_1\underline{\Omega}_1$

(d) Use $M_{1\Delta}$ for E_1 and M_2 for E_2:

$$\begin{bmatrix} \underline{M}_{rr} & \underline{M}_{r1}\underline{E}_{1\Delta} & \underline{M}_{r2}\underline{E}_2 \\ \underline{E}_{1\Delta}^T\underline{M}_{r1}^T & \underline{1} & \underline{E}_{1\Delta}^T\underline{M}_{12}\underline{E}_2 \\ \underline{E}_2^T\underline{M}_{r2}^T & \underline{E}_2^T\underline{M}_{12}^T\underline{E}_{1\Delta} & \underline{1} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{n}_{1\Delta} \\ \ddot{n}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \hat{\underline{D}}_{1\Delta} & \underline{0} \\ \underline{0} & \underline{0} & \hat{\underline{D}}_2 \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{n}_{1\Delta} \\ \dot{n}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{\Omega}_{1\Delta}^2 & \underline{0} \\ \underline{0} & \underline{0} & \underline{\Omega}_2^2 \end{bmatrix} \begin{bmatrix} q_r \\ n_{1\Delta} \\ n_{2e} \end{bmatrix} = \begin{bmatrix} \underline{\phi}_r \\ \underline{Y}_{1\Delta} \\ \underline{Y}_{2e} \end{bmatrix}$$

(e) Use M_1 for E_1 and M_2 for E_2:

$$\begin{bmatrix} \underline{M}_{rr} & \underline{M}_{r1}\underline{E}_r & \underline{M}_{r2}\underline{E}_2 \\ \underline{E}_1^T\underline{M}_{r1}^T & \underline{E}_1^T\underline{M}_{11}\underline{E}_1 & \underline{E}_1^T\underline{M}_{12}\underline{E}_2 \\ \underline{E}_2^T\underline{M}_{r2}^T & \underline{E}_2^T\underline{M}_{12}^T\underline{E}_1 & \underline{1} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{n}_{1e} \\ \ddot{n}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \hat{\underline{D}}_1 & \underline{0} \\ \underline{0} & \underline{0} & \hat{\underline{D}}_2 \end{bmatrix} \begin{bmatrix} \dot{q}_r \\ \dot{n}_{1e} \\ \dot{n}_{2e} \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & \underline{\Omega}_1^2 & \underline{0} \\ \underline{0} & \underline{0} & \underline{\Omega}_2^2 \end{bmatrix} \begin{bmatrix} q_r \\ n_{1e} \\ n_{2e} \end{bmatrix} = \begin{bmatrix} \underline{\phi}_r \\ \underline{Y}_{1e} \\ \underline{Y}_{2e} \end{bmatrix}$$

Table 4.1: Five Possible Ways To Use The Modes of Section 4 in Conjunction with the Three Types of Mode Defined in the Section. (Note: For the definitions of M_1, M_2, and $M_{1\Delta}$, see the text.)

5.   TWO DAMPING MODELS: VISCOUS AND HYSTERETIC

To recapitulate the modeling of damping thus far, we have made the following two assumptions:

(a)   damping forces are linear combinations of the generalized velocities (i.e., of the elements of $\dot{q}$). This assumption is often called linear 'viscous' damping, having in mind the one-degree-of-freedom case where the (scalar) damping force is given by $-d\dot{q}$. We shall however, save the word 'viscous' for a slightly different purpose (see Section 5.1 below). According to this assumption, the damping force $\underline{\delta}_d$ is given by

$$\underline{\delta}_d = -\underline{D}\dot{\underline{q}} \tag{5.1}$$

(b)   the modal form of $\underline{D}$, denoted $\hat{\underline{D}}$ is either diagonal, or may as well be diagonal, due to either to diagonal dominance, or light damping, or both.

Neither of these two assumptions is untarnished for reasons given earlier but they are innocuous relative to the procedure this report points the way to avoiding: all unconstrained (overall spacecraft) modal damping factors set to 0.005. Thus, in this report, we set

$$\hat{\underline{D}} = 2\underline{Z}\underline{\Omega} \tag{5.2}$$

on a structure-by-substructure basis, with $\underline{Z}$ taken to be diagonal for each substructure. The only remaining question is How are the diagonal elements of the $\underline{Z}$ matrices chosen? In other words, How are the constrained substructural modal damping factors chosen?

5.1   Viscous Damping

The best methods for choosing modal damping factors rely

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on test results or analysis. Test results may not always be avail-
able, however, especially at the design stage, and damping analysis
for realistic structures is only recently being brought out of the
Stone Age. This leaves guesswork (hopefully based on experience)
as the only alternative. These are two types of guesses suggested
here: the 'viscous' model and the 'hysteretic' model.

In the 'viscous' model, one sets

$$\hat{\underline{D}} = \gamma_V \underline{\Omega}^2 \tag{5.3}$$

where γ_V is a constant. From (5.2), this is equivalent to setting

$$\underline{Z} = \tfrac{1}{2}\gamma_V\underline{\Omega}, \tag{5.4}$$

that is, to setting

$$\zeta_i = \tfrac{1}{2}\gamma_V\omega_i \tag{5.5}$$

on a mode-by-mode basis.

One could agree that no real progress has been made by
using (5.4) or (5.5) since, until γ_V has been specified, the ζ_i are
still unknown. However, it can be countered that there is now only
one unknown instead of many, and that a single constant, γ_V, is suffi-
cent to represent the *level* of damping. The character of viscous
damping is shown in Fig. 5.1.

5.2   Hysteretic Damping

A second possibility, generally referred to as 'hysteretic'
damping, is to use

$$\hat{\underline{D}} = \gamma_H\underline{\Omega} \tag{5.6}$$

where γ_H is a constant. Thus (5.6) replaces (5.3) and we have, on
a   mode-by-mode basis,
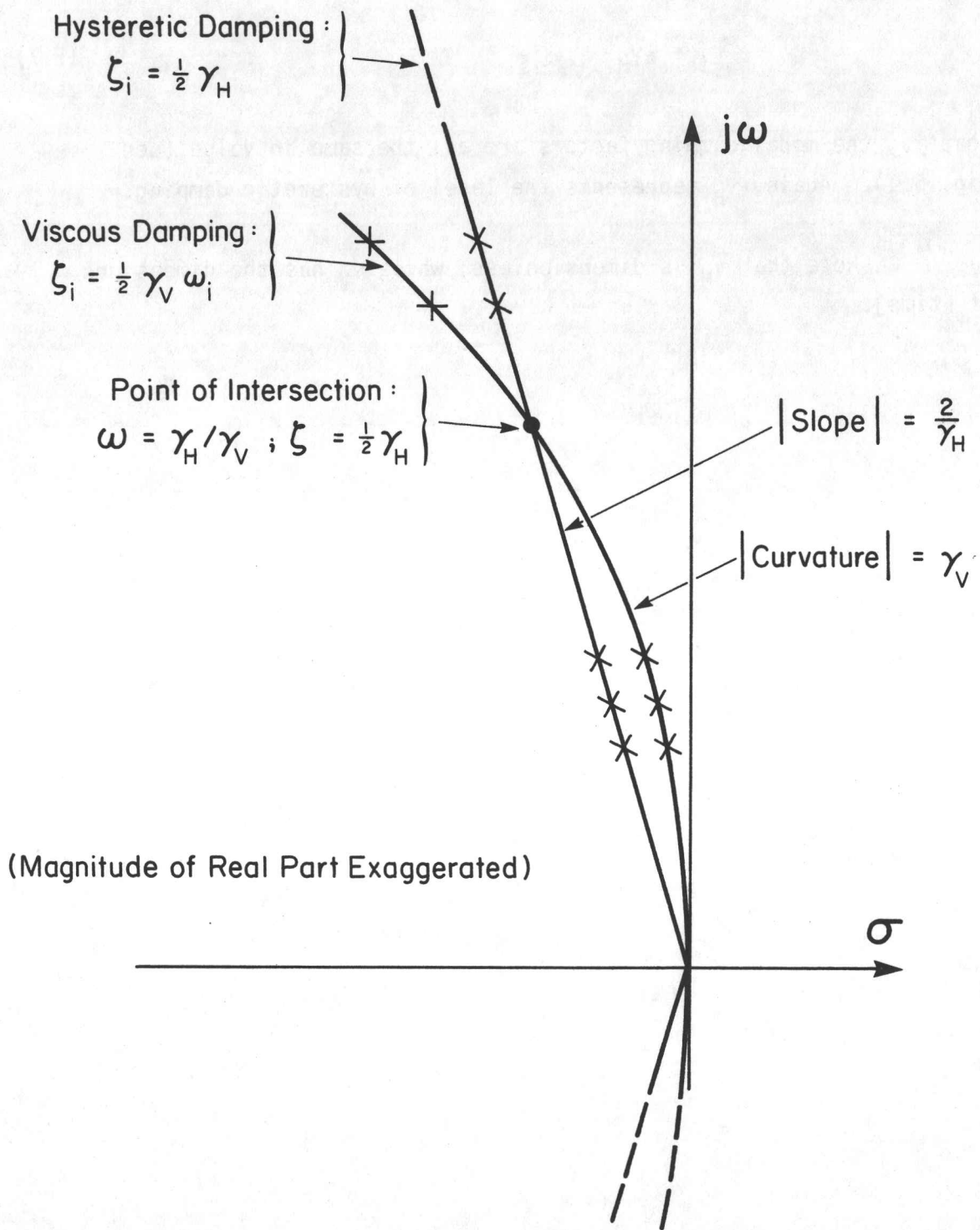
Figure 5.1: A Comparison of Viscous and Hysteretic Modal Damping

$$\zeta_i = \tfrac{1}{2}\gamma_H \tag{5.7}$$

That is, the modal damping factors are all the same in value (see Fig. 5.1). Again, γ_H represents the level of hysteretic damping.

Note that γ_H is dimensionless, while γ_V has the dimensions of [time].

6.   A NUMERICAL EXAMPLE: "ZSAT"

To use a concrete example of practical interest, consider again the 'ZSAT' satellite shown in Fig. 1.1, repeated again on the next page for convenience. The damping in the solar array will be taken as hysteretic, as will the damping in the antenna dish. The numerical comparisons will be made for hysteretic and viscous damping in the antenna tower. The tower is chosen as the object of study because it is the largest and most crucial structural component as regards the characteristics of those overall, unconstrained, spacecraft modes that are most likely to be important in attitude control and configuration integrity.

6.1   Viscous Damping in the Antenna Tower

As explained in Section 4, there are two sets of constrained (substructural) modes that can be used in respect of an internal substructure like the antenna tower: modes for the constrained tower with the reflector absent (denoted M_t), and modes for the constrained tower with the reflector rigid (denoted $M_{t\Delta}$). Damping factors can be assigned for either set of modes.

Let us begin by considering the (unaugmented) modes M_t, and let us assign a viscous damping constant γ_{Vt} for these tower modes. Then, as in (5.3),

$$\hat{\underline{D}}_t = \gamma_{Vt}\underline{\Omega}_t^2 \tag{6.1}$$

[Note that the subscript 't' for 'tower' is used in this example instead of the more ambiguous '1' for 'internal flexible body'.] The corresponding damping matrix in physical coordinates is

$$\underline{D}_t = \underline{E}_t^{-T}\hat{\underline{D}}_t\underline{E}_t^{-1} \tag{6.2}$$

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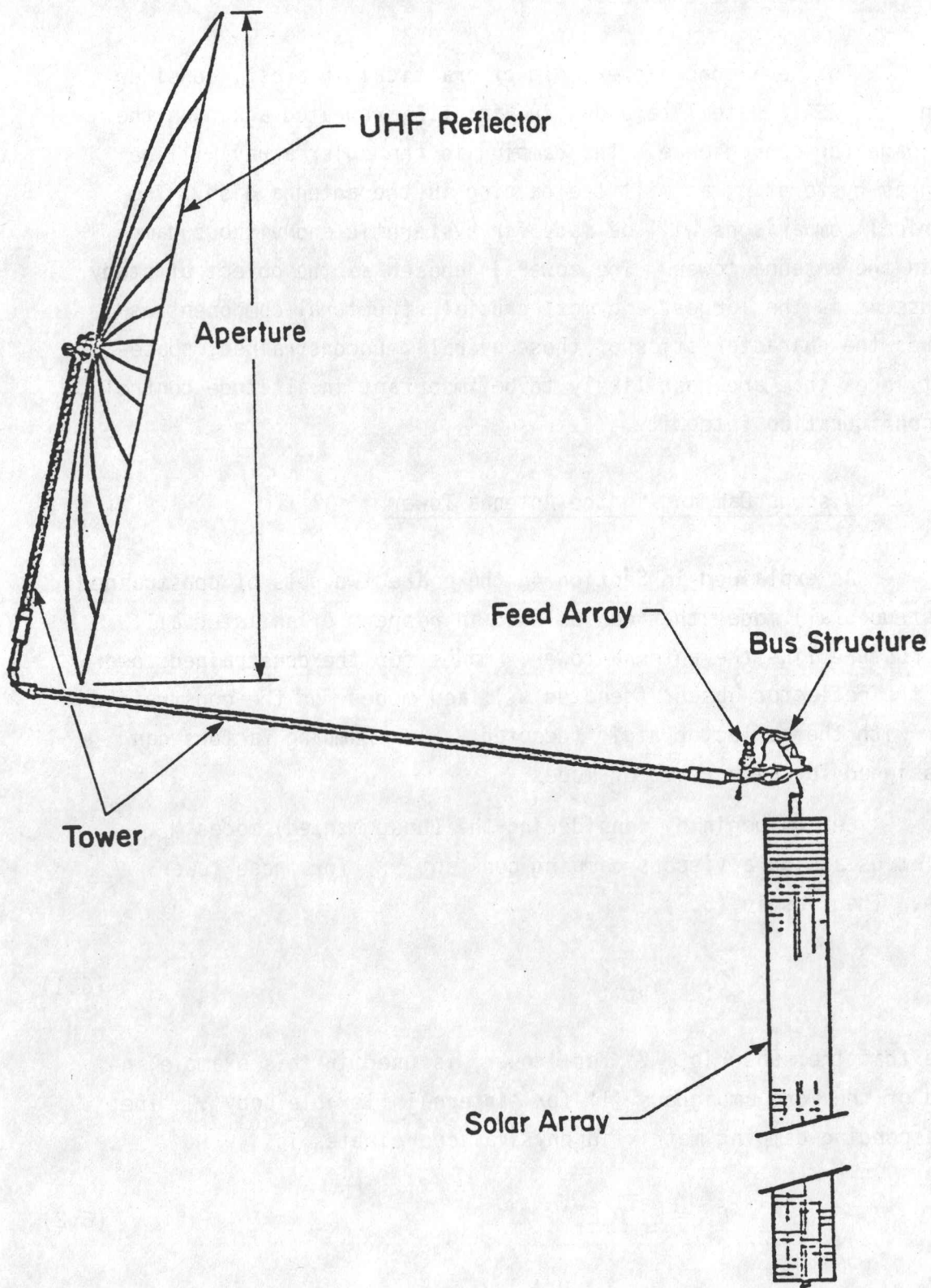
Figure 6.1: "ZSAT"--A Flexible Mobile Communications Satellite

using the first of the expressions given in (4.22). With the viscous
damping model (6.1), $\underline{D}_t$ becomes

$$\underline{D}_t = \gamma_{Vt}\underline{E}_t^{-T}\underline{\Omega}_t^2\underline{E}_t^{-1} \tag{6.3}$$

However, from (4.20), an immediate interpretation of (6.3) is this:

$$\underline{D}_t = \gamma_{Vt}\underline{K}_t \tag{6.4}$$

That is, *if one assumes 'viscous' damping (as defined in this report)
for the unaugmented interior substructure modes, the 'damping' matrix
associated with the interior substructural coordinates is simply
proportional to the stiffness matrix associated with these coordinates.*

The second 'viscous alternative' for a tower damping model
is to use the augmented modes, $M_{t\Delta}$, and to assign a viscous damping
constant, $\gamma_{Vt\Delta}$, for these (constrained-elastic-tower + rigid-reflector)
modes. Then, as in (5.3),

$$\hat{\underline{D}}_{t\Delta} = \gamma_{Vt\Delta}\underline{\Omega}_{t\Delta}^2 \tag{6.5}$$

The corresponding damping matrix in physical coordinates is

$$\underline{D}_t = \underline{E}_{t\Delta}^{-T}\hat{\underline{D}}_{t\Delta}\underline{E}_t^{-1} \tag{6.6}$$

using the first of the expressions given in (4.14). With the viscous
damping model (6.5), $\underline{D}_t$ becomes

$$\underline{D}_t = \gamma_{Vt\Delta}\underline{E}_{t\Delta}^{-1}\underline{\Omega}_{t\Delta}^2\underline{E}_{t\Delta}^{-1} \tag{6.7}$$

However, from (4.13), an immediate interpretation of (6.7) is this:

$$\underline{D}_t = \gamma_{Vt\Delta}\underline{K}_t \tag{6.8}$$

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That is, *if one assumes 'viscous' damping* (as defined in this report) *for the augmented interior substructural modes, the damping matrix associated with the interior substructural coordinates is simply proportional to the stiffness matrix associated with these coordinates.*

Moreover, from (6.4) and (6.8) we learn that *if the viscous damping constant for the unaugmented modes,* γ_{Vt}, *is chosen equal to the viscous damping constant for the augmented modes,* $\gamma_{Vt\Delta}$, *the same damping matrix,* $\underline{D}_t$, *results.*

6.2   Hysteretic Damping in the Tower

Again, there are two sets of constrained (substructural) modes that can be used in respect of an internal substructure like the antenna tower: modes for the constrained tower with the reflector absent (denoted M_t), and modes for the constrained tower with the reflector rigid (denoted $M_{t\Delta}$). Damping factors can be assigned for either set of modes.

Let us begin by considering the (unaugmented) modes M_t, and let us assign a viscous damping constant γ_{Ht} for these tower modes. Then, as in (5.31),

$$\hat{\underline{D}}_t = \gamma_{Ht}\underline{\Omega}_t \tag{6.9}$$

the corresponding damping matrix in physical coordinates is

$$\underline{D}_t = \underline{E}_t^{-T}\hat{\underline{D}}_t\underline{E}_t^{-1} \tag{6.10}$$

using the first of the expressions given in (4.22). With the hysteretic damping model (6.9), $\underline{D}_t$ becomes

$$\underline{D}_t = \gamma_{Ht}\underline{E}_t^{-T}\underline{\Omega}_t\underline{E}_t^{-1} \tag{6.11}$$

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Unlike its viscous counterpart, (6.3), there is not a proportionality between $\underline{D}_t$ and $\underline{K}_t$ and it may prove numerically convenient to use the second of the two expressions given in (4.22),

$$\underline{D}_t = \gamma_{Ht} \underline{M}_{tee} \underline{E}_t \underline{\Omega}_t \underline{E}_t^T \underline{M}_{tee},$$ (6.12)

instead.

The second 'hysteretic alternative' for a tower damping model is to use the augmented modes, $M_{t\Delta}$, and to assign a hysteretic damping constant, $\gamma_{Ht\Delta}$, for these (constrained-elastic-tower + rigid-reflector) modes. Then, as in (5.3),

$$\hat{\underline{D}}_{t\Delta} = \gamma_{Ht\Delta} \underline{\Omega}_{t\Delta}$$ (6.13)

The corresponding damping matrix in physical coordinates is

$$\underline{D}_t = \underline{E}_{t\Delta}^{-T} \hat{\underline{D}}_{t\Delta} \underline{E}_{t\Delta}^{-1}$$ (6.14)

using the first of the expressions given in (4.14). With the hysteretic damping model (6.13), $\underline{D}_t$ becomes

$$\underline{D}_t = \gamma_{Ht\Delta} \underline{E}_{t\Delta}^{-T} \underline{\Omega}_{t\Delta} \underline{E}_{t\Delta}^{-1}$$ (6.15)

Unlike its viscous counterpart, (6.7), there is not a proportionality between $\underline{D}_t$ and $\underline{K}_t$ and it may prove numerically convenient to use the second of the two expressions given in (4.14),

$$\underline{D}_t = \gamma_{Ht\Delta} \underline{M}_{tt} \underline{E}_{t\Delta} \underline{\Omega}_{t\Delta} \underline{E}_{t\Delta}^T \underline{M}_{tt}$$ (6.16)

Moreover, there is no simple relationship between the $\underline{D}_t$ calculated for 'unaugmented', hysteretically damped modes [as given by (6.11) or (6.12)] and the $\underline{D}_t$ calculated for 'augmented', hysteretically damped modes [as given by (6.15) or (6.16)].

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6.3   Numerical Results

As mentioned in the introduction to this section, the damping in the solar array will be assumed hysteretic, with hysteretic damping constant γ_{Ha}, and the damping in the antenna dish reflector will be assumed hysteretic also, with hysteretic damping constant γ_{Hr}. We shall take

$$\gamma_{Ha} = 0.01 \qquad \text{(solar array)} \qquad\qquad (6.17)$$

$$\gamma_{Hr} = 0.01 \qquad \text{(antenna reflector)} \qquad\qquad (6.18)$$

Four models for the tower damping will be studied:

(I)   Unaugmented tower modes (M_t) viscously damped, with

$$\gamma_{Vt} = 0.01 \qquad \text{(tower)} \qquad\qquad (6.19)$$

(II)   Augmented tower modes ($M_{t\Delta}$) viscously damped, with

$$\gamma_{Vt\Delta} = 0.01 \qquad \text{(tower)} \qquad\qquad (6.20)$$

(III)   Unaugmented tower modes (M_t) hysteretically damped, with

$$\gamma_{Ht} = 0.01 \qquad \text{(tower)} \qquad\qquad (6.22)$$

(IV)   Augmented tower modes ($M_{t\Delta}$) hysteretically damped, with

$$\gamma_{Ht\Delta} = 0.01 \qquad \text{(tower)} \qquad\qquad (6.22)$$

In view of the observation at the end of Section 6.1, we will have, for both Damping Models I and II, the same damping matrix in 'physical coordinates,' $\underline{D}_t$. Thus (cf. Table 3.1) the damping characteristics of the tower, and thus of the spacecraft as a whole, are indistinguishable in these two cases.

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The motion equations for the ZSAT example of Fig. 6.1 will not be developed here in detail. The interested reader has recourse to Reference 1. Suffice it to say that the motion equations for ZSAT are as shown in Table 3.1, with two alterations:

(a)   the subscript $1 \rightarrow t$;
(b)   the subscript $2 \rightarrow r$;
(c)   the equations are augmented appropriately to encompass the solar array (see Table 3.1 and Figure 1.2).

The results are given in Fig. 6.2. Damping factors for the overall spacecraft modes are shown. (The 'rigid' modes are, of course, undamped and are not included.) The logarithmic scale for these damping factors should not distract from the fact that there are substantial differences between the damping factors of different modes (for a particular substructural damping model), and that there are substantial differences (for many of the modes) between the results for different substructural damping models.

Not surprisingly, damping factors of $\frac{1}{2}\gamma_H$ (= 0.005) are quite common among many of the spacecraft modes. However, there is a substantial variation with respect to this value. It should also be kept in mind that if a spacecraft damping factor is 0.02 and it is assumed to be 0.002, this is a $\sim$1000% error. Similarly, if the spacecraft damping factor is 0.002 and it is assumed to be 0.02, this also is a $\sim$1000% error. Such an error in a modal frequency would never be tolerated, and such an error in a modal damping factor should not be tolerated either.
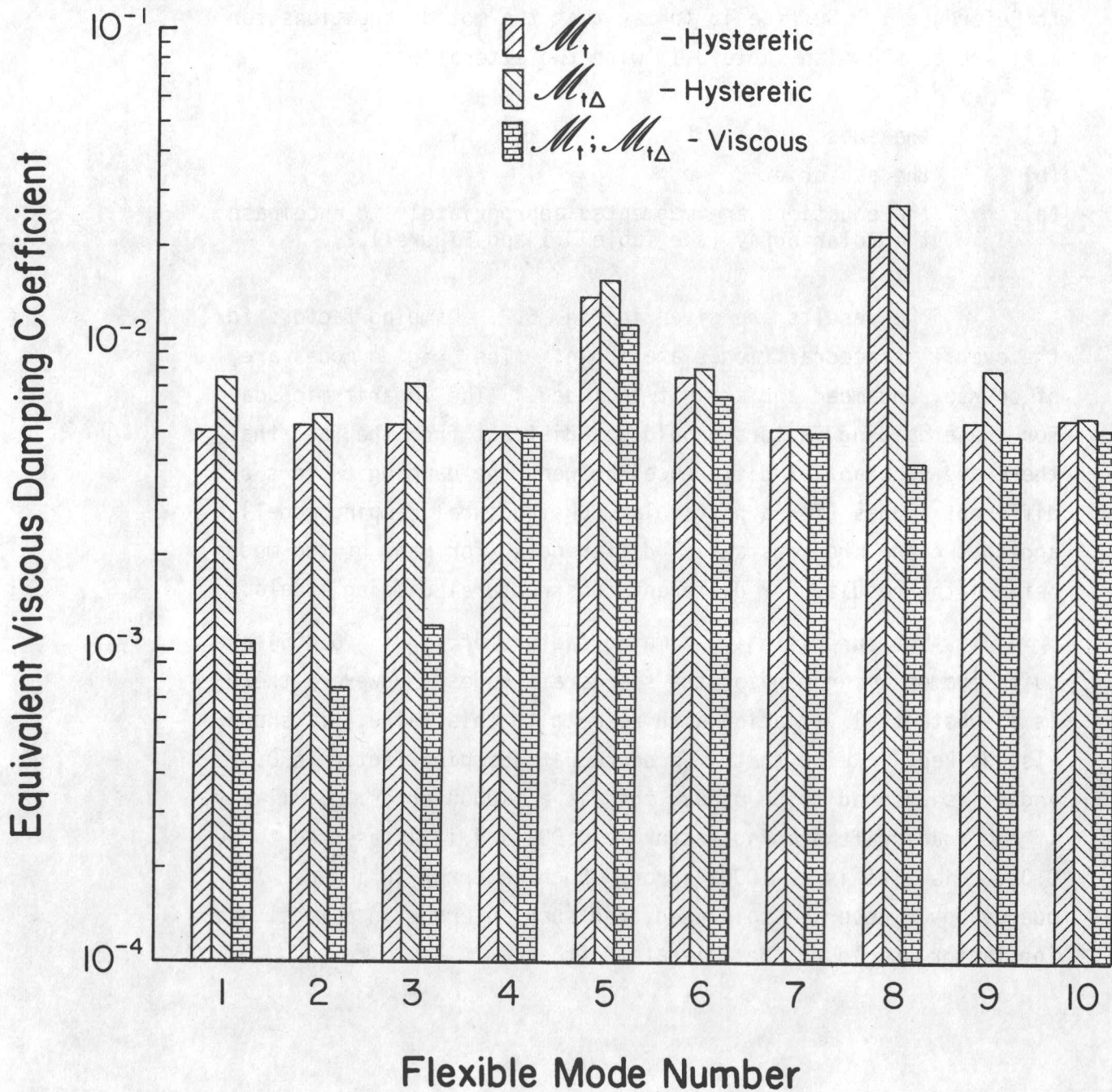
Figure 6.2: A Comparison Between the Overall Unconstrained (Spacecraft) Modal Damping Factors Obtained Using Four Different Approaches to the Substructural Damping Model for the Antenna Tower. [Note: In all cases the substructural damping models for the array and the antenna reflector are hysteretic.]

(a) Modes 1 Through 10
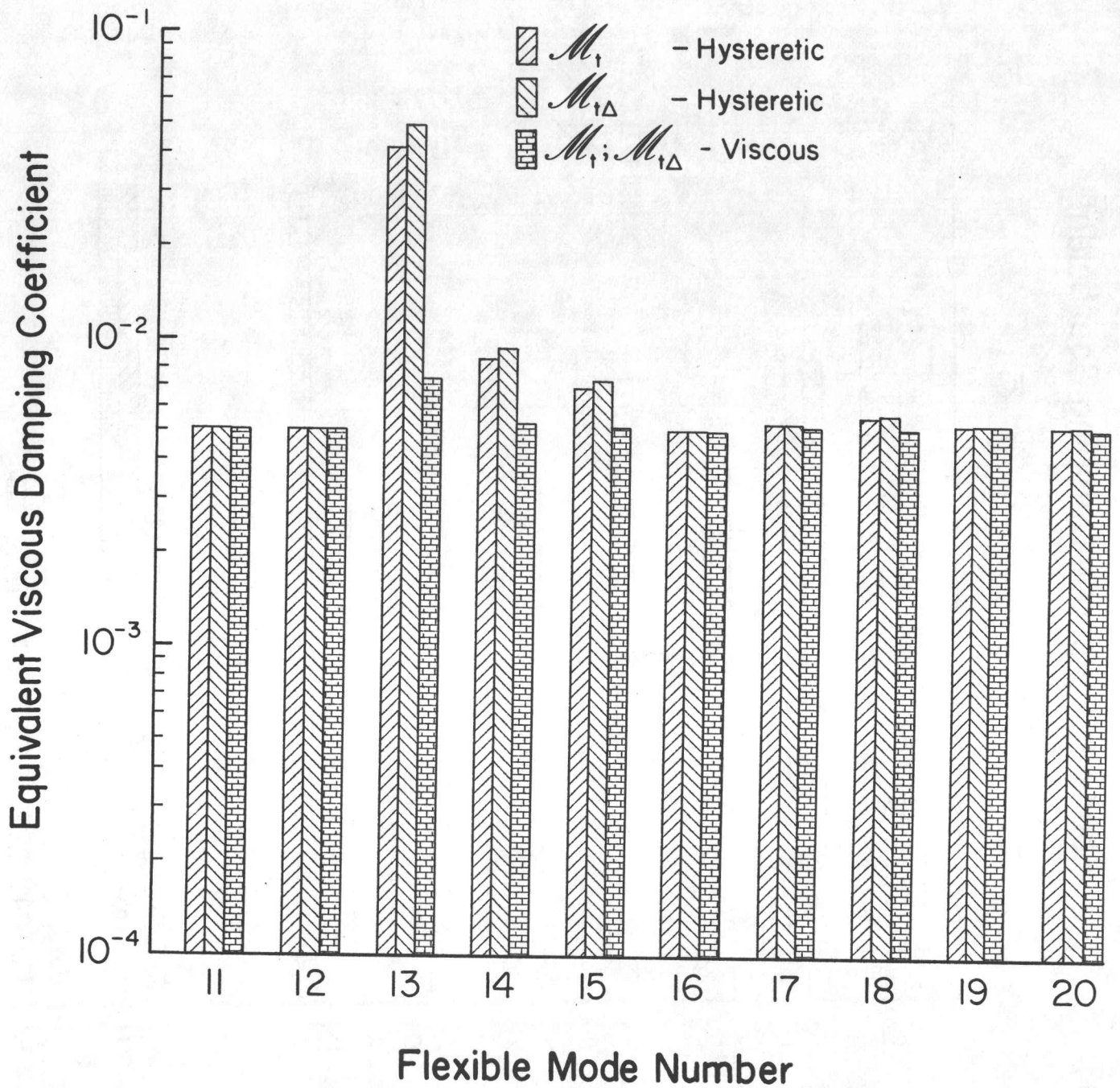
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Figure 6.2: A Comparison Between the Overall Unconstrained (Spacecraft) Modal Damping Factors Obtained Using Four Different Approaches to the Substructural Damping Model for the Antenna Tower. [Note: In all cases the substructural damping models for the array and the antenna reflector are hysteretic.] CONT'D.

(b) Modes 11 Through 20
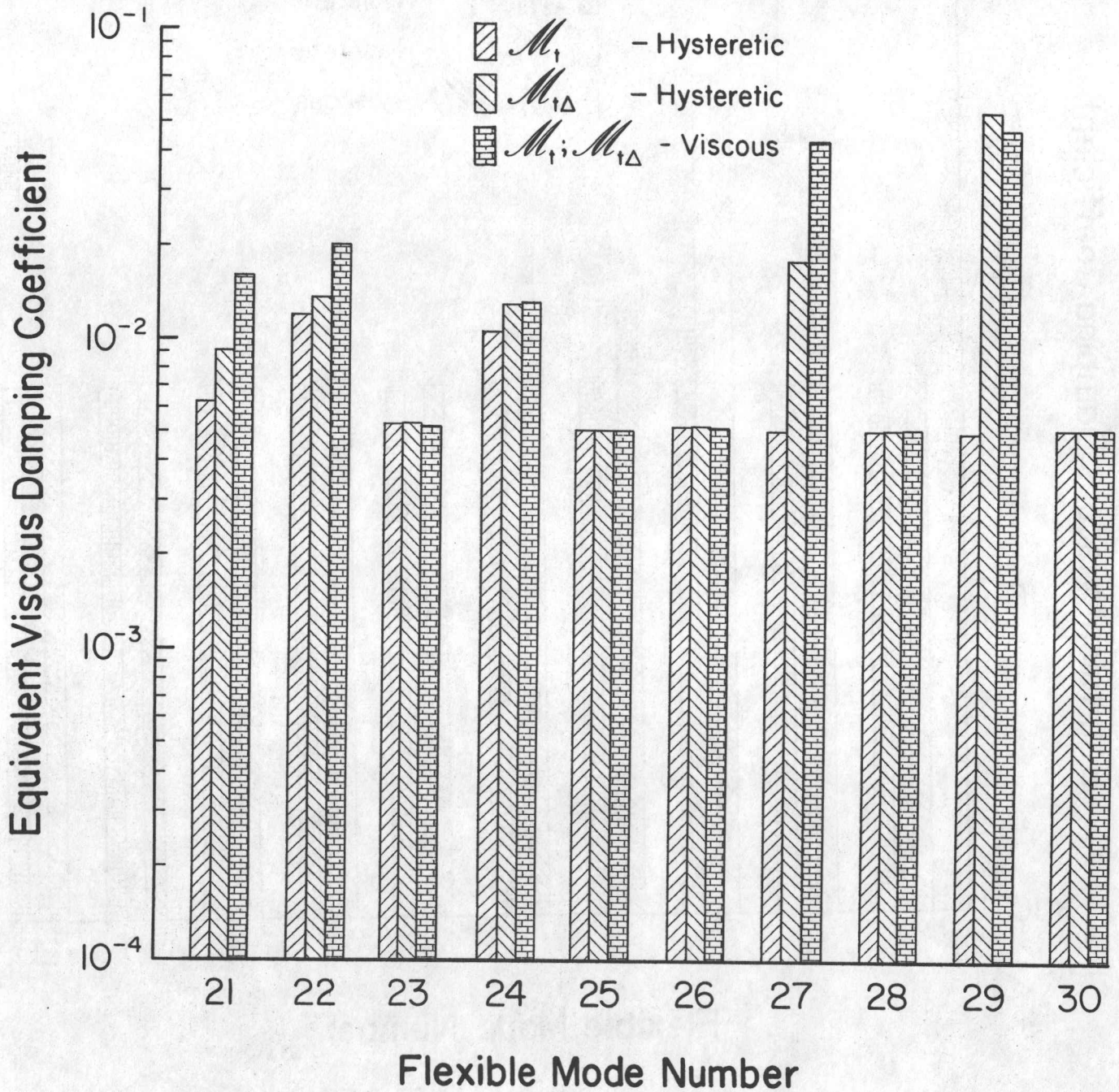
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Figure 6.2: A Comparison Between the Overall Unconstrained (Spacecraft) Modal Damping Factors Obtained Using Four Different Approaches to the Substructural Damping Model for the Antenna Tower. [Note: In all cases the substructural damping models for the array and the antenna reflector are hysteretic.] CONT'D.

(c) Modes 21 Through 30
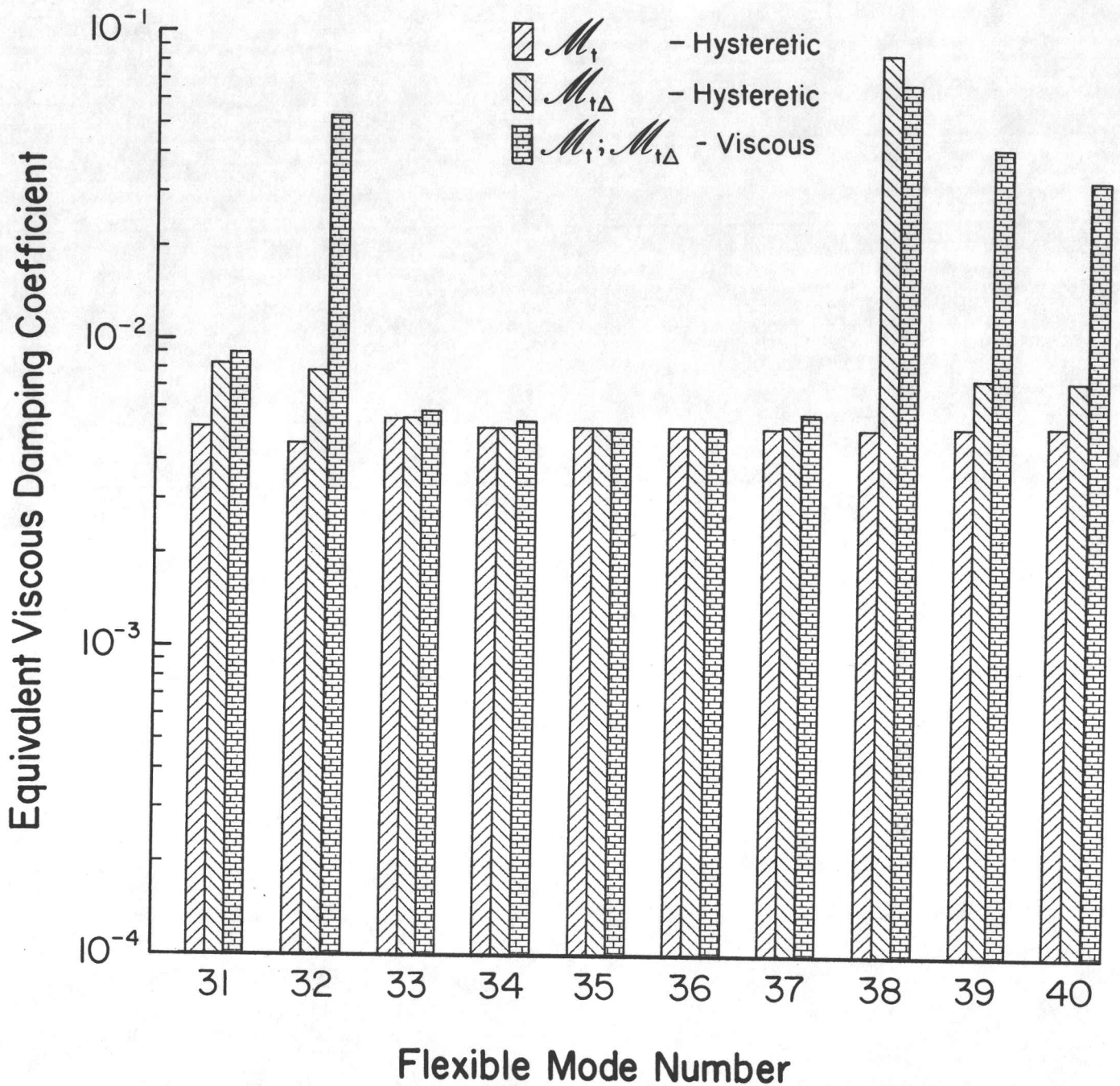
Figure 6.2: A Comparison Between the Overall Unconstrained (Spacecraft) Modal Damping Factors Obtained Using Four Different Approaches to the Substructural Damping Model for the Antenna Tower. [Note: In all cases the substructural damping models for the array and the antenna reflector are hysteretic.] CONT'D.

(d) Modes 31 Through 40

7. CONCLUDING REMARKS

In this report it has been shown how to combine the modal damping factors associated with substructural modes, to form an overall damping model for a spacecraft. Ideally the substructural modal damping factors would be measured, or calculated in terms of known structural properties.

However, even if they are merely 'estimated based on experience' this is still preferrable to estimating the damping factors of the overall, unconstrained modes of the spacecraft. This can be seen in the example in the last section, in which unconstrained modal damping factors can vary from mode to mode by more than an order of magnitude (Fig. 6.2). In this case it would clearly be a further major approximation to assume these unconstrained damping factors all to be equal.

Perhaps this lesson can be stated in loose terms the following way: it is better to make guesses at the substructural level than at the overall spacecraft level. The reason is that, in the former procedure, many important properties of the system (relative size, mass, elasticity of the substructures, etc.) are involved in the calculation whereas, in the latter procedure, only damping guesswork is used.

Even though one of these 'guesswork procedures' is better than the other, a still better procedure is to arrive at the substructural damping factors by other than guesswork, after which the synthesis procedures outlined in this report can be used to obtain a reliable damping model for the overall spacecraft.
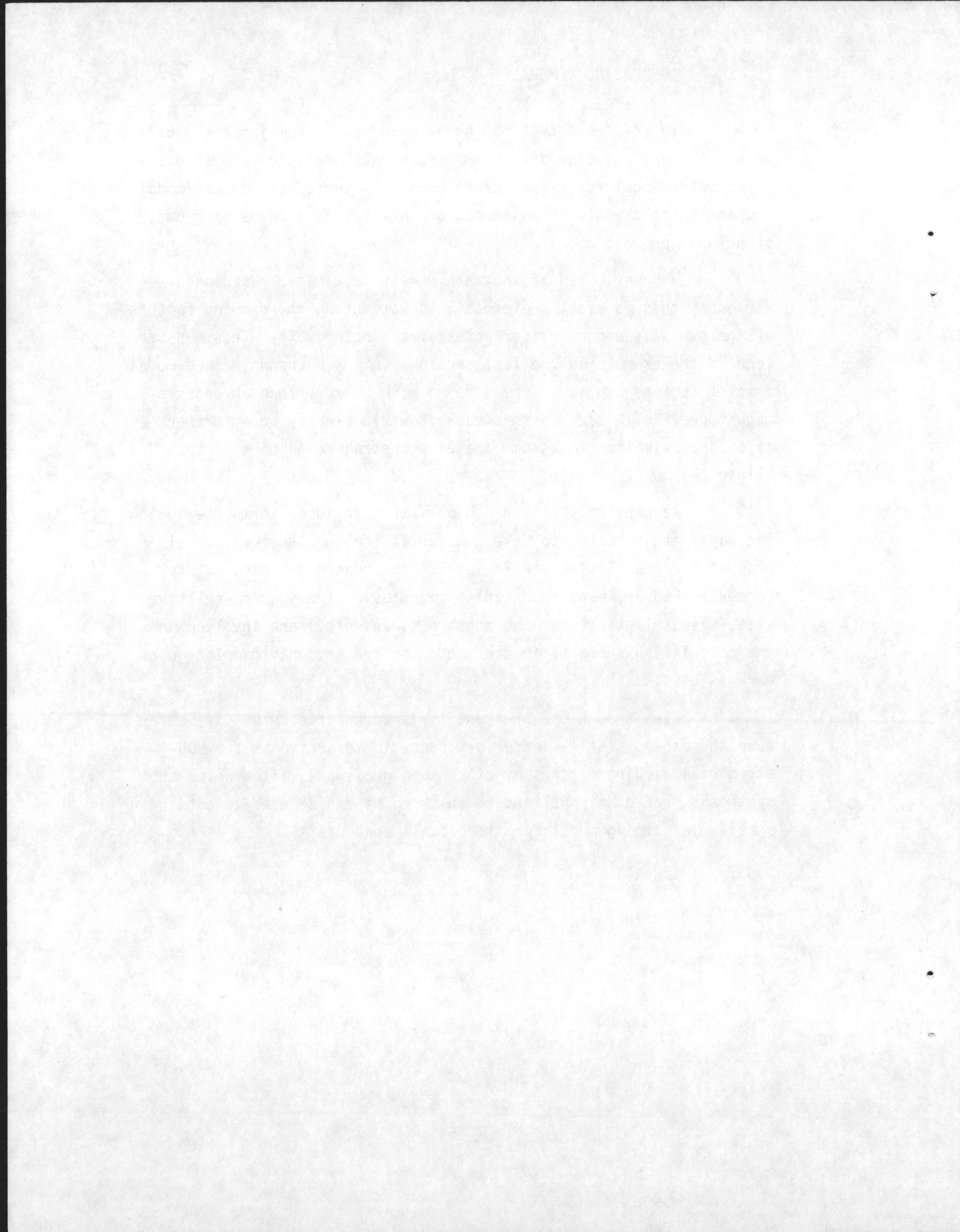
8.   REFERENCE

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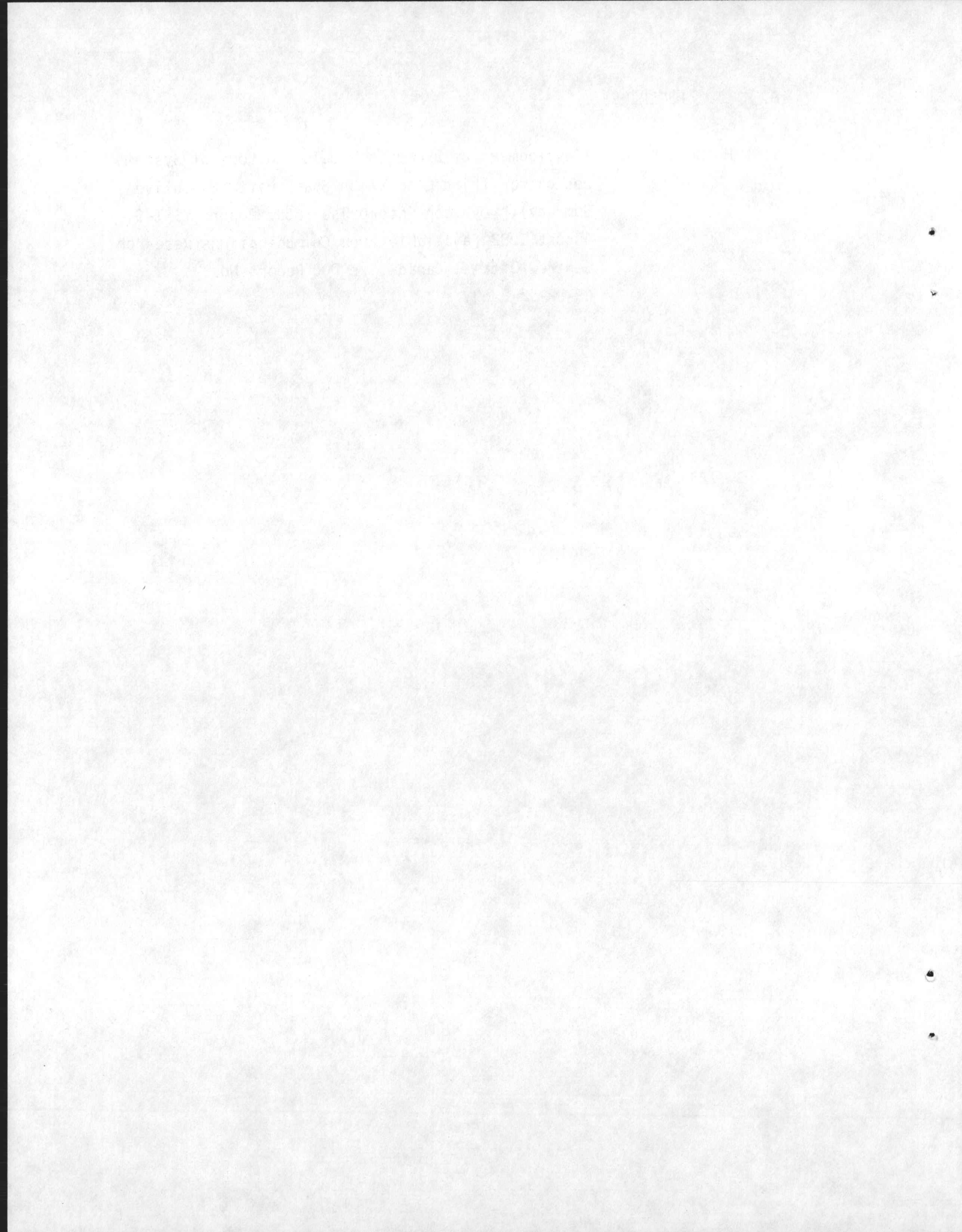
UTIAS Report No. 287

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DAMPING MODELS FOR FLEXIBLE COMMUNICATIONS SATELLITES
BY SUBSTRUCTURAL DAMPING SYNTYHESIS

Hughes, P. C.

1. Flexible spacecraft   2. Structural damping   3. Satellite dynamics

I. UTIAS Report No. 287   II. Hughes, P. C.

Most modern spacecraft are structurally flexible and, moreover, these spacecraft can naturally and profitably be analysed as a collection of attached substructures (solar array panels, antennas, thermal radiators, etc.). This report shows how to combine various models for substructural energy dissipation so that an overall damping model for the spacecraft results. (Four such substructural damping models are discussed, two of which are shown to produce the same results.) Such a synthesis procedure proves valuable when substructural damping data is known, either from ground tests or detailed analysis.

However, even if substructural damping data is not known but merely guessed at (as is often the case) this report shows that it is better to do one's guessing at the substructural modal level than at the overall spacecraft modal level; the explanation for this, un a nutshell, is that, in the former case, 'reality' (in the form of the relative sizes, connections, elasticities and inertias of the various substructures) is invoked in the synthesis procedure: better to pass the substructural guesses through some sort of 'reality filter' (the synthesis procedure) than to simply make guesses about the overall spacecraft damping properties. Furthermore, as a numerical example for a spacecraft of topical complexity shows, the two alternatives can produce quite different results.

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