TECHNISCHE HOGESCHOOL DELFT

LUCHTVAART- EN RUIMTEVAARTTECHNIEK

DELFT UNIVERSITY OF TECHNOLOGY
DEPARTMENT OF AEROSPACE ENGINEERING

Memorandum M-368

SOME FORMULAS FOR THE CRACK OPENING STRESS LEVEL

by

J. Schijve

Delft, The Netherlands
April 1980

CONTENTS

ABSTRACT

Crack growth data for 2024-T3 sheet material were analysed with different formulas for ΔK_{eff} as a function of the stress ratio R. The data covered R values from - 1.0 to 0.54. A good correlation was obtained for $\Delta K_{eff}/\Delta K = 0.55 + 0.33\ R + 0.12\ R^2$. The relation between log da/dn and log ΔK_{eff} was non-linear for high crack rates (> 1 μm/c).

1 INTRODUCTION

Fatigue crack growth data for 2024-T3 sheet material, tested under constant-amplitude loading, were recently published in an NLR-report by Van der Linden [1]. Six different R values, varying from - 1 to + 0.54 were adopted, see table 1. The results of these tests will be used here to check some equations for the crack opening stress level (S_{op}).

2 EVALUATION FO THE CRACK GROWTH DATA

In [1] the crack growth data are presented in graphical form only, i.e. crack growth curves (crack length a vs number of cycles) and crack growth rate data (da/dn vs ΔK, with $\Delta K = C\,\Delta S\sqrt{\pi a}$ and $C = \sqrt{\sec \pi a/W}$). From the latter graphs the value of ΔK at da/dn = 10^{-5} m/c has been derived here (see table 1) for the six R values involved. For these ΔK values the so-called ΔK_{eff} will be calculated according to different formulas. A formula can be useful only if the same ΔK_{eff} values are obtained for all R values.

Originally Elber [2] proposed for 2024-T3 material:

$$U = \frac{\Delta S_{eff}}{\Delta S} = \frac{\Delta K_{eff}}{\Delta K} = 0.5 + 0.4\,R \qquad (1)$$

where $\Delta S_{eff} = S_{max} - S_{op}$, $\Delta S = S_{max} - S_{min}$ and $R = S_{min}/S_{max}$.
Elber checked this equation for R values from - 0.1 to + 0.7. In [3] the present author has shown that Eq. (1) could well account for the effect of R if R > 0. However, for negative R values Eq. (1) becomes unrealistic. Defining the ratio γ:

$$\gamma = \frac{S_{op}}{S_{max}} \qquad (2)$$

the relation to U is easily obtained as:

$$\gamma = 1 - (1 - R)\,U \qquad (3)$$

For Elber's equation this gives:

$$\gamma = 0.5 + 0.1\ R + 0.4\ R^2 \tag{4}$$

As shown by Figure 1 this function is increasing for $R \to -1$, which is unrealistic. Analytical work of Newman [4] has shown that γ should be a decreasing function for $R \to -1$. For this reason a new equation was proposed in [3] based on trends as predicted by Newman:

$$U = 0.55 + 0.35\ R + 0.1\ R^2 \tag{5}$$

which leads to $\gamma = 0.45 + 0.2\ R + 0.25\ R^2 + 0.1\ R^3$ (6)

To obtain a more flexible form these equations can be replaced by:

$$U = 0.55 + (0.45 - \alpha)\ R + \alpha\ R^2 \tag{7}$$

and $\gamma = 0.45 + (0.1 + \alpha)\ R + (0.45 - 2\ \alpha)\ R^2 + \alpha\ R^3$ (8)

For $\alpha = 0.10$ the latter equations return into Eqs. (5) and (6). The γ (R) function in Eq. (8) has been plotted in Figure 1 for α values varying from 0.10 to 0.15, which gives negligible differences for $R > 0$, but noticeable differences for $R \to -1$.

In [5] De Koning has suggested the following relations for 7075-T6:

For $R > 0$: $\gamma = 0.45 + 0.2\ R - 0.15\ R^2 + 0.9\ R^3 - 0.4\ R^4$ (9a)

For $R \leq 0$: $\gamma = 0.45 + 0.2\ R$ (9b)

These functions have also been plotted in Figure 1. For positive R values Eq. (9a) gives somewhat lower γ values than Eq. (8), while for negative R values the γ values with Eq. (9b) (linear relation) are much similar to those obtained with Eq. (8) for $\alpha = 0.14$.

Values of ΔK_{eff} have been calculated for the empirical ΔK values (for $da/dn = 10^{-5}$ m/c) employing Elber's equation (1), the present equation (8) for $\alpha = 0.10$ to 0.15 and De Koning's equations (9a) and (9b). The results are presented in table 1. If a $\gamma(R)$ equation correctly represents the R-effect, the calculated ΔK_{eff} values should be similar. The results of all equations show some variability as indicated by the variation coefficient, which is the ratio between the standard deviation of ΔK_{eff} and its mean value. The lowest scatter is obtained with Eq. (8) and $\alpha = 0.12$. If the data for R = - 1 are omitted the lowest scatter is obtained for $\alpha = 0.10$, i.e. for equation (6). In Reference [3] this equation was found to agree with crack growth data covering an R range from - 0.50 to + 0.73. For this reason Van der Linden's crack rate data have been plotted as a function of ΔK_{eff} calculated with both $\alpha = 0.10$ and $\alpha = 0.12$. The results are presented in Figures 3 and 4, while the original data from [1] plotted as a function of ΔK are reproduced here in Figure 2. It turns out again that ΔK_{eff} is capable to correlate the crack growth data of different R values. A good correlation is observed in Figure 4 ($\alpha = 0.12$). The same applies to Figure 3 ($\alpha = 0.10$) with the exception of the data for R = - 1. Apparently $\alpha = 0.12$ better fits all data.

The data in Figure 4 are in the da/dn range of 0.1 to 100 μm/c, thus covering fairly high crack growth rates. In this range a non linear behaviour between log da/dn and log ΔK_{eff} was also observed in [3] for da/dn > 1 μm/c. Similar to equations adopted in [3] the present crack growth data can be described by (da/dn in μm/c):

$$\Delta K_{eff} < 12 \text{ MPa}\sqrt{m} \rightarrow \log da/dn = -3.606 + 3.341 \log \Delta K_{eff}$$
$$\Delta K_{eff} > 12 \text{ MPa}\sqrt{m} \rightarrow \log da/dn = 0.599 - 4.451 \log \Delta K_{eff} + 3.610 (\log \Delta K_{eff})^2$$

The slope factor 3.341 was borrowed from [3].

3 CONCLUSIONS

1. For 2024-T3 sheet material a second order polynomial for U (R) (where $U = \Delta K_{eff}/\Delta K$) allowed a good correlation between crack growth data for both positive and negative R values (as low as R = - 1).
2. For high crack rates (> 1 μm/c) the log da/dn - log ΔK_{eff} relation deviated significantly from a linear function (Paris equation).

4 REFERENCES

[1]	H.H. van der Linden	NLR test results as a data base to be used in a check of crack propagation prediction models. A Garteur activity. NLR TR 79121, Nov. 1979.
[2]	W. Elber	The significance of crack closure. ASTM STP 486, 1971, pp. 230-242.
[3]	J. Schijve	The stress ratio effect on fatigue crack growth in 2024-T3 Alclad and the relation to crack closure. Delft Un. of Tech., Dept. of Aerospace Eng., Memorandum M-336, Aug. 1979.
[4]	J.C. Newman, Jr.	A finite-element analysis of fatigue crack closure. Mechanics of crack growth, ASTM STP 590, 1976, p. 281.
[5]	A.U. de Koning	A simple crack closure model for prediction of fatigue crack growth rates under variable amplitude loading. NLR MP 80006, Jan. 1980.

able 1 Empirical ΔK values [1] and calculated ΔK_{eff}

	Empirical data						Coefficient of variation $\sigma_{\Delta K_{eff}} / \overline{\Delta K_{eff}}$	
est No.	1	3	4	5	6	7		
nax (MPa)	130	130	225	130	130	130		
nin (MPa)	70	12.5	-25	-23.5	-50	-130		
	0.54	0.10	-0.11	-0.18	-0.38	-1		
at da/dn = 10^{-5} m/c (MPa$\sqrt{m}$)	26.5	33.5	37.5	43	49	60		
	Calculated $\Delta K_{eff} = U\,\Delta K$ (MPa$\sqrt{m}$)						all tests	Excluding R = - 1
q. of Elber (1)	18.97	18.09	17.10	(18.40)	(17.05)	(6.00)	5.2 %(a)	
. (8), α = 0.10	20.36	19.63	19.23	21.08	21.14	18.00	6.1 %	4.2 %
α = 0.11	20.29	19.60	19.27	21.17	21.40	19.20	4.8 %	4.6 %
α = 0.12	20.22	19.57	19.32	21.26	21.65	20.40	4.5 %	5.0 %
α = 0.13	20.16	19.54	19.36	21.35	21.91	21.60	5.4 %	5.5 %
α = 0.14	20.09	19.51	19.41	21.45	22.17	22.80	6.9 %	6.0 %
α = 0.15	20.03	19.48	19.46	21.54	22.43	24.00	8.6 %	6.5 %
q. of De Koning 9a and b)	21.78	19.75	19.32	21.35	22.23	22.50	6.1 %	6.1 %

a) based on first 3 ΔK_{eff} values only.

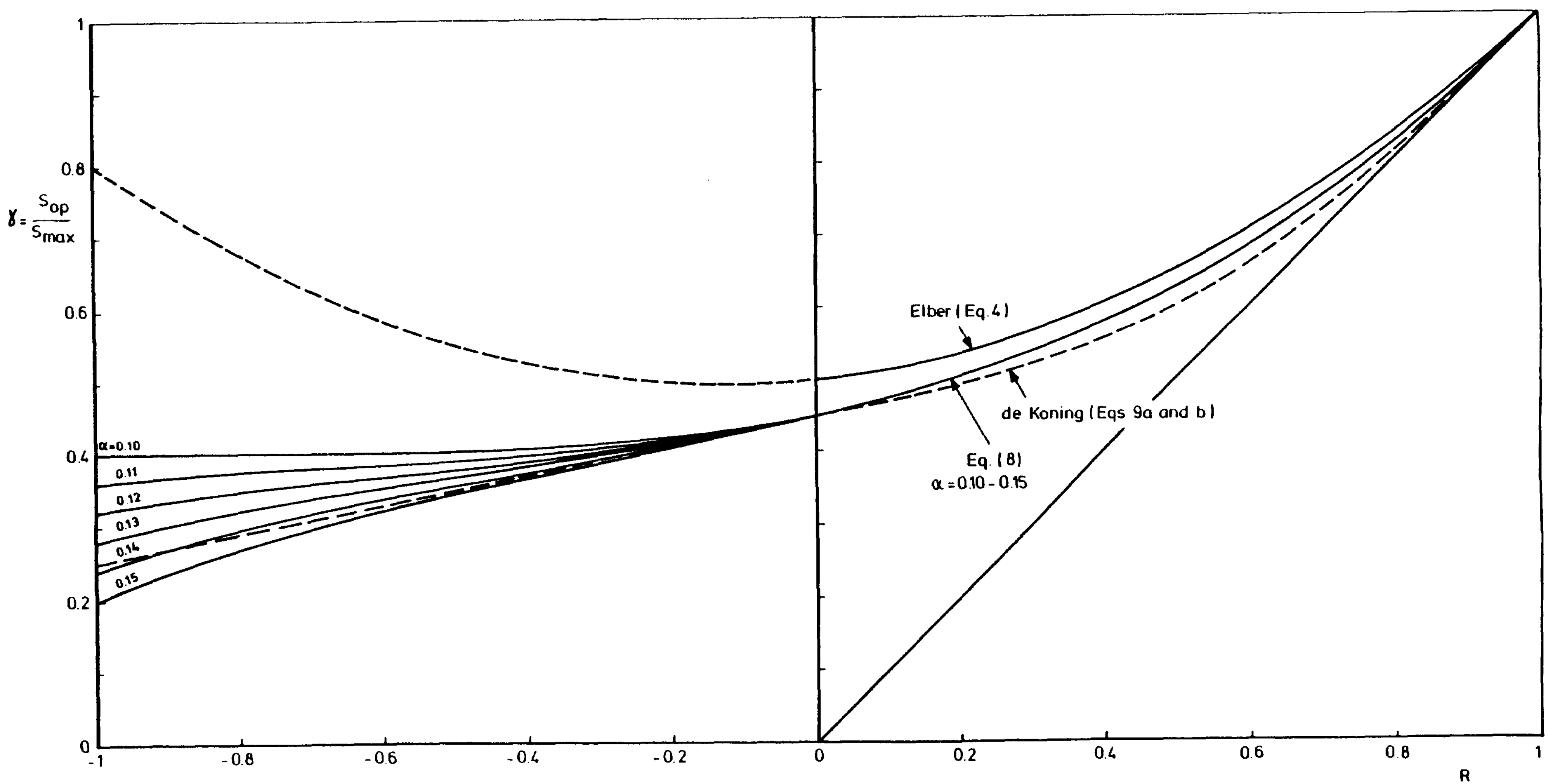

Figure 1 The crack opening stress level according to different formulas

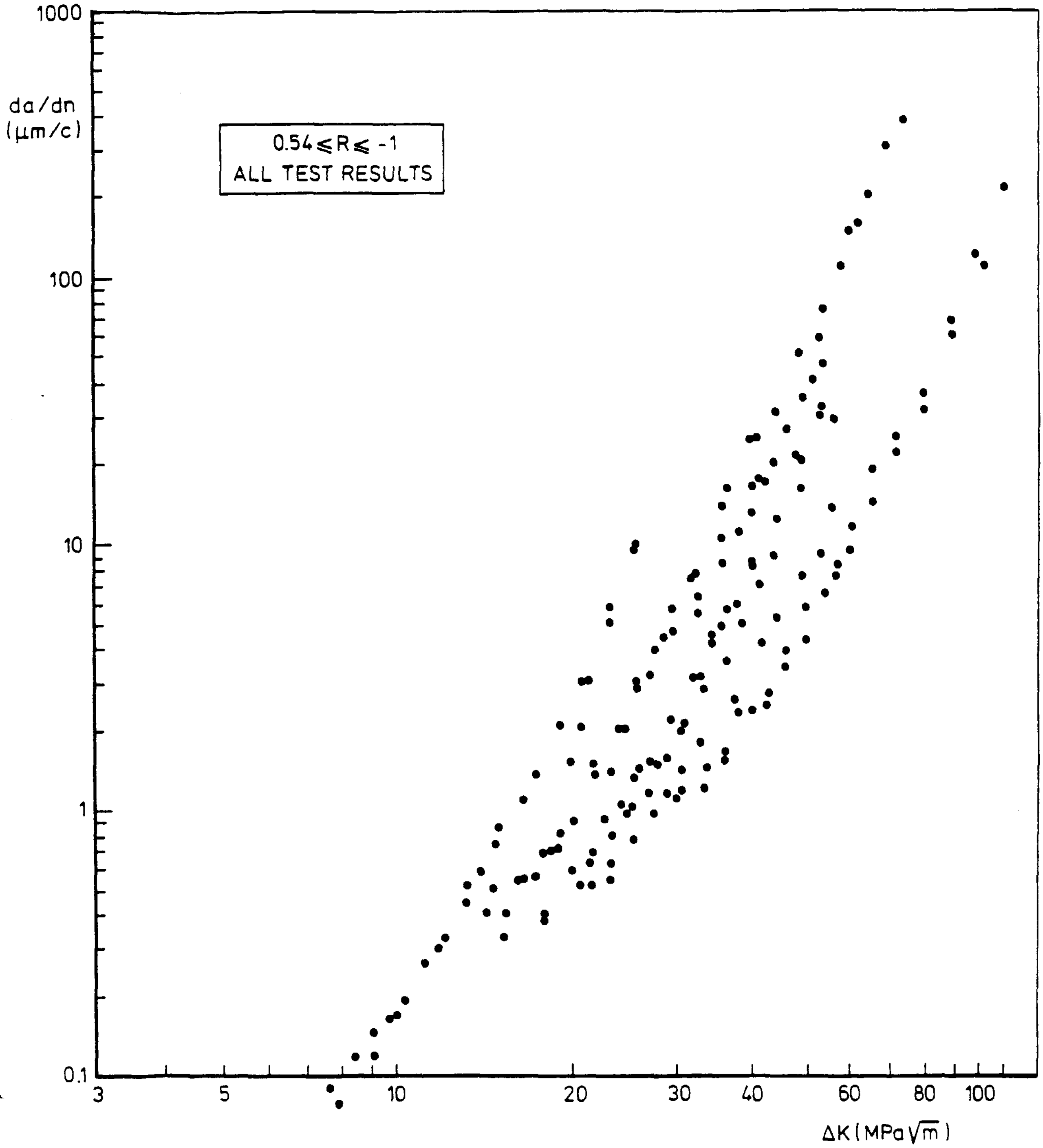

Figure 2 Crack growth data for 2024-T3 from Ref. [1]

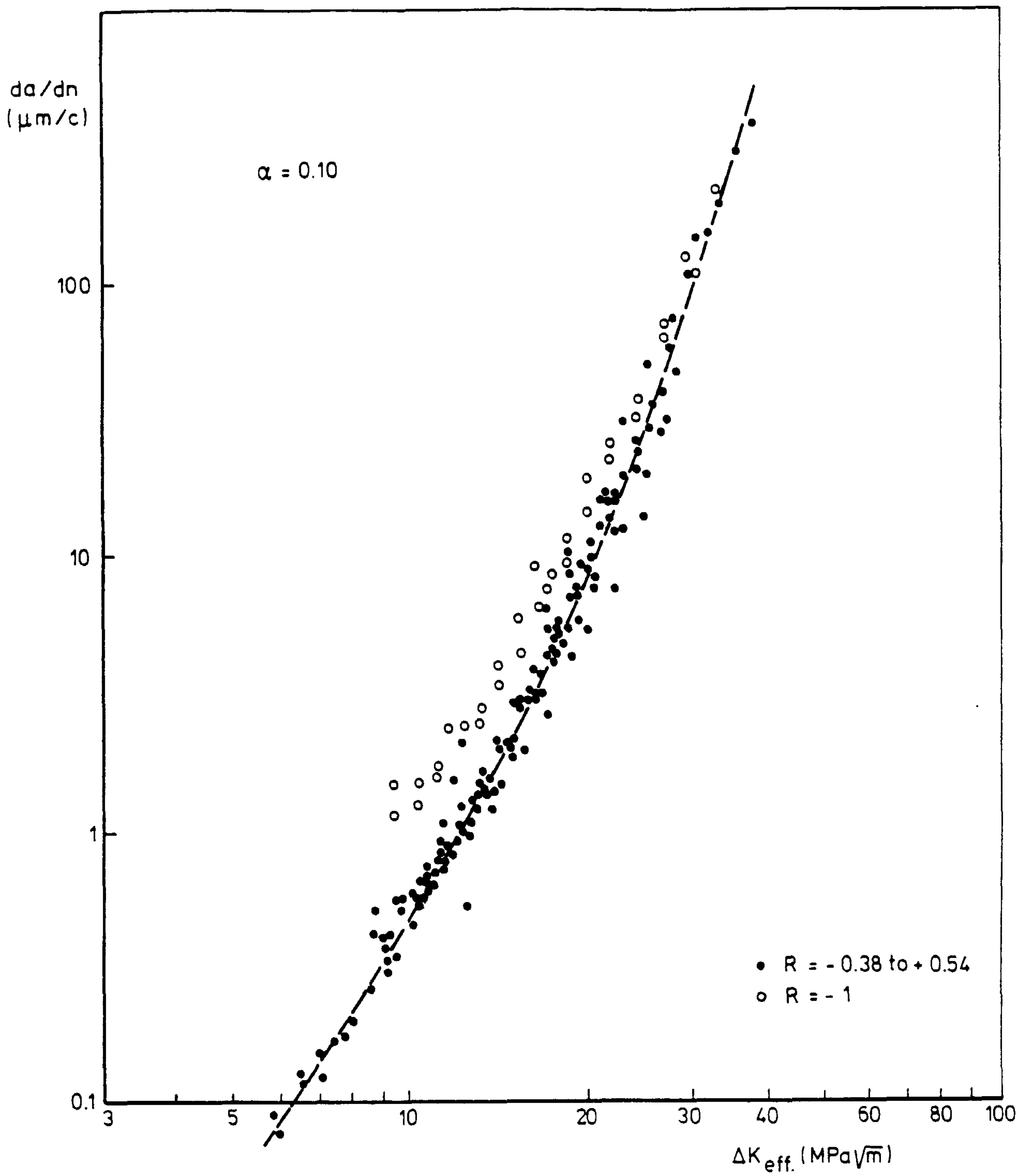

Figure 3 Crack growth data for 2024-T3 [1]

ΔK_{eff} calculated with Eq. (7) and α = 0.10

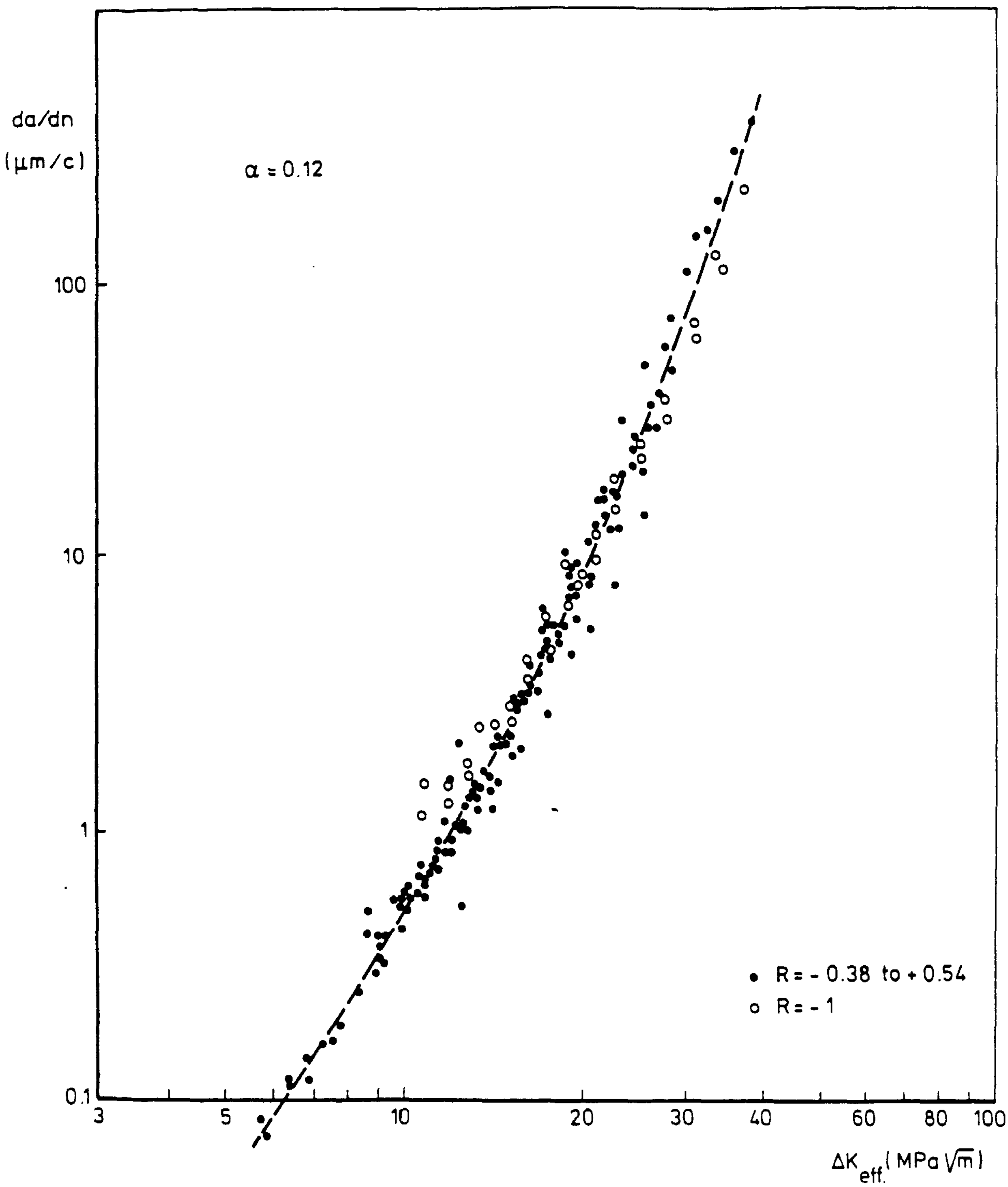

Figure 4 Crack growth data for 2024-T3 [1]

ΔK_{eff} calculated with Eq. (7) and $\alpha = 0.12$