Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science
Delft Institute of Applied Mathematics

Optimal ambulance locations in the Netherlands.

A thesis submitted to the
Delft Institute of Applied Mathematics
in partial fulfillment of the requirements

for the degree

MASTER OF SCIENCE
in
APPLIED MATHEMATICS

by

Daphne Looije

Delft, the Netherlands
March 2013

Copyright © 2013 by Daphne Looije. All rights reserved.



MSc THESIS APPLIED MATHEMATICS

"Optimal ambulance locations in the Netherlands."

Daphne Looije

Delft University of Technology

Daily supervisor

Ir. G.J. Kommer (RIVM)

Responsible professor

Prof. dr. ir. K.I. Aardal

Other thesis committee members

Dr. D.J.P. Lahaye

Drs. P.L. van den Berg

March 2013

Delft, the Netherlands

Summary

When an incident occurs and we need acute health care, we call the emergency number. After such a call, an ambulance should be with us within 15 minutes.

To this end, there are ambulance stations all over the Netherlands, from where ambulances can depart to the scene of an incident. In this research we determine the optimal locations of these stations, to be able to provide the best coverage with the least number of stations. In 2010 there were 203 ambulance stations in the Netherlands.

The research was carried out at the National Institute for Public Health and the Environment, the RIVM, which advises the ministry of Health, Welfare and Sports on for example ambulance care.

We consider the Netherlands as a directed graph, where the nodes represent four-digit postal code areas and the arcs represent the road network, and each arc has a value corresponding to the travel time between the tail node and the head node of the arc.

We look at different models, which are all based on two models. The first model is the Location Set Covering Model (LSCM), which determines the minimal number of location sites needed to cover all demand points. The second model is the Maximal Coverage Location Problem (MCLP), which determines the maximal coverage that can be achieved with a fixed number of location sites.

With the LSCM we determine the minimal number of ambulance stations needed to cover all areas using three different maximal travel times of 8, 12 and 15 minutes. As a result we see that if we consider the Netherlands as one region, we need 302, 165 and 100 location sites respectively. If we consider each RAV as a separate region we need 203 location sites with a maximal travel time of 12 minutes.

Using the results from the LSCM, we use the MCLP to determine the percentage of the residents we can still cover with less ambulance stations than the outcome of the LSCM. We do this for the same three different travel times. We see that we can still cover approximately 95% of the residents with 225, 95 and 60 location sites respectively.

Next we again use the LSCM and MCLP, but now distinguish between urban and rural areas, assigning a different maximal travel time to the two different areas, namely 8 and 12 minutes and 12 and 15 minutes. We see that we then need 217 and 173 location sites respectively to cover all demand points.

Next, we include the travel time from the incident to a hospital and use an MCLP-like model. We look at two sets of hospitals: all hospitals with an Emergency Department (ED) and the Trauma Centers. These sets consist of 98 and 11 hospital locations respectively. We observe that we can get 99,9% of the residents to a hospital with an ED and 83,8% to a Trauma Center within 45 minutes (and reach them within 12 minutes travel time after the call), if we use 165 location sites.

Finally we introduce the use of rapid responders, which can only provide the acute care, where an ambulance might come later if the rapid responder is already there. The results indicate that the use of these cheaper rapid responders could be advantageous.

In all the models we have looked at a greenfield scenario, where we can build an ambulance station anywhere, and assume there are no existing ambulance stations. Furthermore we consider one non-greenfield scenario for the LSCM and MCLP, where we already fix an ambulance station at each of the nine different locations of the academic hospitals.

Even though these results give several indications, some of the results are inconclusive due to the fact that we considered four-digit postal code areas, which represent city districts or small villages. To get a better insight we recommend to work with for example six-character postal code areas, which represent streets. A consequence, however, is that the computing time might increase to a level that makes it difficult to determine provably optimal solutions within reasonable time. Further research to obtain good approximations of optimal solutions is recommended.

Preface

With this master thesis I will finish my degree of Master of Science in Applied Mathematics at the faculty of Electrical Engineering, Mathematics and Computer Science at Delft University of Technology.

For this final project I wanted to apply my mathematical knowledge on a problem from real life. I found this in a project at the RIVM, the National Institute for Public Health and the Environment, in Bilthoven. All of the data used in my research was provided by the RIVM.

I want to thank my daily supervisor at the RIVM, Geert Jan Kommer, for his help and support during, and after, my time at the RIVM. His knowledge about the subject and its problems from practice has been very helpful.

Next I want to thank my supervisors at the TU Delft, Karen Aardal and Pieter van den Berg, for all the discussions we had and for all of their feedback on the many, many versions of my thesis I submitted to them.

Also I want to thank Maarten Mulder from the RIVM, for creating the maps that are used to illustrate the results and background information in this thesis.

Finally I want to thank Gytha Rijnbeek, academic counsellor at the TU Delft, for helping me to organize the last phase of my study.

Daphne Looije
Delft, March 2013.

Contents

Summary **v**

Preface **vii**

1 Introduction **1**
1.1 RIVM . . . 1
1.2 AZN . . . 2
1.3 Problem description . . . 3

2 Literature study **5**
2.1 Model introduction . . . 6
2.2 Deterministic models . . . 6
2.3 Theoretical background . . . 9
2.3.1 Branch-and-cut . . . 10
2.3.2 Bounds on the optimal objective value . . . 12

3 Data **13**
3.1 RAV regions . . . 13
3.2 Four-digit postal codes . . . 15
3.3 Travel time model . . . 15
3.4 Demand . . . 16
3.5 Maximal response time . . . 16
3.6 Current ambulance locations . . . 17
3.7 Hospital locations . . . 18
3.7.1 Reachability of the hospitals . . . 19

4 Model descriptions **21**
4.1 LSCM . . . 22
4.2 MCLP . . . 22
4.3 Adapted models: Two types of postal codes . . . 23
4.3.1 Classification of demand points . . . 24
4.4 From location site to incident to hospital . . . 25
4.5 Multiple vehicle types . . . 27

5 Results **31**
5.1 LSCM . . . 31
5.1.1 The Netherlands as one region . . . 32
5.1.2 Each RAV separately . . . 32
5.1.3 Comparison of the results for LSCM . . . 35
5.1.4 Non greenfield scenario . . . 39

5.2 Maximal Covering Location Problem (MCLP) 40
5.2.1 Non greenfield scenario 42
5.3 Adapted models: Two types of postal codes 44
5.3.1 Adapted LSCM 44
5.3.2 Adapted MCLP 46
5.4 From location site to incident to hospital 47
5.4.1 Hospital with ED 47
5.4.2 Trauma Centers 47
5.4.3 Comparison of results 47
5.5 Multiple vehicles 49
5.5.1 LSCM version 1 49
5.5.2 LSCM version 2 50
5.5.3 MCLP version 51

6 Conclusion and recommendations **53**
6.1 Conclusions 53
6.2 Recommendations for further research 55

A Notation **59**

B Models from literature **61**

Chapter 1

Introduction

In case of accidents or other life-threatening emergency situations, it is important for health care to be provided within a short time, since this can be the difference between life and death. One of the most important providers of care in such situations are ambulances and the ambulance personnel. These are the people who to go to emergency sites, provide the necessary acute care and transport the patients to a hospital for further care. In current practice, the standard is that after a call, an ambulance is at the scene in less than 15 minutes. This goal should be achieved in at least 95% of the cases for so-called A1 calls, which involve life threathening situations.
In the Netherlands the number one sector organization for ambulance care is Ambulance Care the Netherlands (AZN, in Dutch: Ambulancezorg Nederland). This organization offers various types of support to the Regional Ambulance Services (RAVs), ambulance organizations and the professionals who work there [2]. In AZN all RAVs are represented. Since 15 minutes is a short time and there are limited resources, it is important to have insight in what is the best way to utilize the available equipment. One of the research institutes that provides insight into the current performance of the ambulance care and possible improvements is the National Institute for Public Health and Environment (RIVM, in Dutch: Rijksinstituut voor Volksgezondheid en Milieu).
In this chapter we provide background information on both the RIVM and the AZN, we will describe the problem definition, and finally we give an outline of the remaining part of the thesis.

1.1 RIVM

The research described in this thesis was partly carried out at the RIVM, which is a research institute that conducts research to advice different government authorities. One of these government authorities is the Ministry of Health, Welfare and Sports (Ministerie van VWS, in Dutch: Ministerie van Volksgezondheid, Welzijn en Sport). Other commissioners are the Ministry of Infrastructure and the Environment, the Ministry of Economic Affairs, Agriculture and Innovation, the European Union and the United Nations. Approximately 1.500 people work at the RIVM. As their website states [16]:

RIVM is the premier expertise and orchestration centre in its field and its remit is to modernise, gather, generate and integrate knowledge and make it usable in the public domain. By performing these tasks RIVM contributes sustainably to promoting the health of the population and the environment by providing protection against health risks and environmental damage.

The RIVM focuses on four themes: Disease Prevention and Healthcare, Infectious Diseases, Environment & Safety and Lifestyle, Diet and Nutrition. This research was carried out in the Centre for Public Health Forecasting (or cVTV, in Dutch: Centrum voor Volksgezondheid Toekomstverkenningen). This is covered by the theme Disease Prevention and Health Care. The center collects, collates, evaluates and disseminates knowledge about public health, the factors which can influence health, and the consequences for the national health care system [16]. The cVTV conducts various types of surveys and in this way collects lots of data on various subjects. Every four years they present the Public Health Forecasting, a document that presents a lot of information and statistics about Health Care. It compares the Dutch situation with the situation in other countries. The last PHF was presented in 2010, so the next will be presented in 2014. The main purpose of the research performed at cVTV is to support policy decisions made by the ministry of VWS. This ministry is responsible for the decisions made about ambulance care in the Netherlands as well. Most of the models and results described in this thesis are also used in the advice of the RIVM to the ministry of VWS about ambulance care.

1.2 AZN

As mentioned before, AZN respresents the RAVs, which are responsible for the provision of ambulance care in the Netherlands. In the Netherlands, about 680 ambulances are available 24 hours a day, 7 days a week. Each RAV has its own control center for ambulance care (MKA, in Dutch: Meldkamer Ambulancezorg). However, since 2011 some RAVs joined there control centers. These control centers assign about one million calls per year, of which about 650.000 are emergency calls, the other 350.000 calls are so-called planned transports. Actually, we distinguish between three types of calls:

- A1 calls: in life threatening situations, where the ambulance has to be at the scene within 15 minutes.
- A2 calls: in situations that are not life threatening, but still require emergency care. The ambulance has to be at the scene within 30 minutes.
- B calls: the planned transport, for example the transportation of a patient from one hospital to another.

AZN is the representative of the ambulance care sector, it communicates with governments, organizations and other authorities (within and outside of health care). Every year AZN publishes Ambulances in Sight (in Dutch: Ambulances in Zicht), this document contains information about the current situation of the ambulance care in the Netherlands and statistics of the performance of the sector in the past year. A summary of this document from 2011 can be found in [3].

1.3 Problem description

The problems that are treated in this thesis are about the location of ambulance stations. We are interested in finding the 'optimal' configuration/distribution of locations of ambulance stations. Of course, the first question that arises now is 'what is optimal?'. We will consider different approaches to this concept and discuss the results. To this end, we introduce the concept of 'coverage': an area is considered to be covered, if it can be reached by an ambulance within a specified time. One approach to this problem, is to consider the country as a graph: we divide the country into areas, which correspond to the nodes in the graph, and the road network is represented by the arcs in the graph. This means we have a directed graph consisting of nodes and two arcs between all pairs of nodes. Every arc will have a weight corresponding to the travel time between the two corresponding postal codes. Note that the graph is directed and hence there are two arcs between every pair of nodes, because the travel times between two locations can differ in each direction. Using this representation, we will capture different settings in (integer) linear programs. The main question kept in mind will be:

What is the optimal configuration of ambulance stations in the Netherlands?

It is not possible to give a definite answer to this question, one of the reasons for this is introduced before: what is optimal depends on the objective and the constraints we impose. We will consider different objective functions, as well as different constraints, giving multiple solutions. Immediately, different questions arise, among which:

- What is the current organization of ambulance care in the Netherlands?
- What are the most important constraints that have to be considered?
- How many locations do we need to cover all areas of the Netherlands?
- What can we achieve with a smaller number of locations?

There is more that can be considered. An emergency situation is not always resolved after an ambulance arrives. The ambulance only provides the first care, after that the patient might have to be transported to a hospital. Also the time this takes is important, so another question that was considered is:

- How many patients can the ambulances get to a hospital within a specified time?

The last thing considered was the idea of having different types of emergency care. In case of an incident, it is essential that emergency care can be provided quickly, but transport is typically not immediately necessary. So we could consider using two types of vehicles. One is the so-called rapid responder, which is not suitable for transporting a patient, but only to get an emergency worker to the scene who can provide care. The other vehicle is a regular ambulance that can transport a patient to the hospital. Now the first vehicle can be less advanced than the second, so it can be cheaper. This implies we could use more of these vehicles and they might have a shorter response time. The question that arises then is:

- How can we use different types of vehicles?

These questions will be answered throughout this thesis, introducing different models and solutions.

Finally, we will give an overview of the rest of the thesis. In Chapter 2 we discuss the previous research in this area and we give a theoretical introduction into the used optimization techniques. In Chapter 3 we give an overview of the data we used in our modeling and describe how this data is obtained. In Chapter 4 we describe different models we have considered. In Chapter 5 the results of the different models introduced in Chapter 4 are given and discussed. At last, in Chapter 6 we summarize the information by answering the questions posed before and discuss further research that can be conducted in this area.

Chapter 2

Literature study

There has been quite some research on emergency service vehicle systems and the location of emergency vehicles. As Goldberg [9] points out, there are different models needed for different types of emergency services. For example fire stations are at fixed places, and fire vehicles are stationed at these places, but ambulance stations are not limited to hospitals and police cars are even patrolling around the city.
In this chapter we will focus on the models that have been developed for ambulance locations. We can distinguish between different types of models, the most important being:

- Deterministic and stochastic models: the deterministic models aim at finding the best positions for ambulance locations, where 'the best' is determined by the area or number of citizens or incidents covered. But these models do not take into account that once an ambulance is dispatched, it is no longer available for other calls. That is where stochastic models come in, they do take into account that there is a chance that an ambulance is not available.

- Static and dynamic models: the static models assume that all ambulances have a fixed station, every day, all day long, but of course the need for ambulances at a site differs during the day. Dynamic models let ambulances change their stations during the day, as to have the best coverage at different times.

- Simulation models: with the help of simulation, one can try to evaluate solutions and get insight for improvement.

In this chapter we will treat the development of different models. We will focus on the deterministic and static models only, since these are the only models that are used later on. First in Section 2.1 we will describe the theoretical background of these models, then we will give a chronological overview of existing deterministic models from the literature in Section 2.2 and finally we will give some theoretical background on the resolution of these problems in Section 2.3.

2.1 Model introduction

First we will describe some general theory that the models are based on, and introduce notation conventions. As described in Section 1.3, the models that are described here are based on graphs. We consider a graph with two sets of vertices, namely a set I with the demand points and a set J with the (potential) ambulance location sites, and arcs between all pairs of nodes from both sets. The arcs represent the road network, we assume that the shortest travel time t_{ij} from vertex j to vertex i is known. These can be given by a travel time model (rijtijdenmodel), which we will assume given at the moment. We say that a vertex $i \in I$ is *covered* by site $j \in J$ if and only if $t_{ij} \leq r$, where r is the maximal time a vehicle may need to reach a demand point. For each demand point, we define the set $J_i = \{j \in J | t_{ij} \leq r\}$ as the set of ambulance location sites that can cover that demand point.

The models that are discussed in this chapter, as well as the models that we will introduce later, are all (binary) integer linear programs, consisting of an objective function and constraints. The prefix 'linear' indicates that all the functions used in the model are linear. Furthermore, all variables in the programs are assumed to be integer, or even binary (0 or 1).

2.2 Deterministic models

In this section we will give an overview of existing deterministic models, ordered chronologically. We will give a short description of each model. There are two quite extensive review articles about this subject, namely one written in 2003 by Brotcorne et al. [4] and another one in 2011 by Li et al. [13]. An overview of the notation and the mathematical formulation of the models described in this section can be found in Appendix A and B.

The first model that is described in the literature is the **location set covering model** (LSCM), developed in 1971 by Toregas et al. [19]. This early model for EMS planning is a binary integer linear optimization problem, i.e, a linear optimization problem where the variables may only take values 0 or 1. The goal of this model is to cover all demand points with a minimum number of ambulance location sites. We introduce one variable for every $j \in J$, which indicates whether there should be an ambulance station at that location:

$$x_j = \begin{cases} 1 & \text{if a station is located at site } j, \\ 0 & \text{otherwise.} \end{cases}$$

Then the mathematical formulation in this case is straightforward:

LSCM

Minimize	$\sum_{j \in J} x_j$				(1.1)
Subject to	$\sum_{j \in J_i} x_j$	$\geq$	1	$i \in I$	(1.2)
	x_j	$\in$	$\{0,1\}$	$j \in J$	(1.3)

The constraints (1.2) make sure that each demand point $i \in I$ is covered by at least one location. The objective (1.1) is to minimize the number of locations used. And the binary constraints (1.3) make sure that we do not have a fractional part of an ambulance location somewhere.

Of course there are shortcomings to this model. The first thing to notice is that this model assumes that we have an 'infinite' amount of stations available, so it gives an indication on the number of stations that is needed to cover all demand points. In reality, maybe we have a certain number of ambulances available. Furthermore, the model only makes sure that each demand node is covered once. If the ambulance from the location that covers this node is busy with another accident, this can mean that the demand node is uncovered.

The next model that we will consider is the **maximal covering location problem** (MCLP) model, which was developed by Church and ReVelle in 1974 [5]. Here the goal is to maximize the fraction of the demand covered, given a certain number of ambulances available. For this, we introduce a new variable y_i, which indicates whether a demand point is covered:

$$y_i = \begin{cases} 1 & \text{if demand point } i \text{ is covered by an ambulance} \\ 0 & \text{otherwise} \end{cases}$$

And we need a parameter p, the number of location sites that can be built, and for each demand point i, a certain demand d_i (for example the number of incidents per hour, or the number of residents in that area). The mathematical formulation then is:

MCLP

Maximize	$\sum_{i \in I} d_i y_i$				(2.1)
Subject to	$\sum_{j \in J_i} x_j$	$\geq$	y_i	$i \in I$	(2.2)
	$\sum_{j \in J} x_j$	$=$	p		(2.3)
	x_j	$\in$	$\{0,1\}$	$j \in J$	(2.4)
	y_i	$\in$	$\{0,1\}$	$i \in I$	(2.5)

The constraints (2.2) indicate whether a demand point is covered, the constraint (2.3) is on the number of stations that is built. The objective (2.1) here is to maximize the fraction of the demand that is covered. This model still has the drawback that if accidents occur and ambulances are busy, nodes might be left uncovered.

An extension of the MCLP model is the **tandem equipment allocation model** (TEAM), developed in 1979 by Schilling et al. [18]. It models two types of vehicles: primary and special vehicles. It assumes that the coverage standard for the primary vehicles is higher than for the special vehicles, so the maximal travel time for the primary vehicles is lower than the maximal travel time for the special vehicles. The mathematical formulation of this model is:

TEAM

$$\text{Maximize} \quad \sum_{i \in I} d_i z_i \tag{3.1}$$

$$\text{Subject to} \quad \sum_{j \in J_i^P} x_j^P \geq z_i \qquad i \in I \tag{3.2}$$

$$\sum_{j \in J_i^S} x_j^S \geq z_i \qquad i \in I \tag{3.3}$$

$$\sum_{j \in J} x_j^P = p^P \tag{3.4}$$

$$\sum_{j \in J} x_j^S = p^S \tag{3.5}$$

$$x_j^S \leq x_j^P \qquad j \in J \tag{3.6}$$

$$x_j^P, x_j^S \in \{0,1\} \qquad j \in J \tag{3.7}$$

$$z_i \in \{0,1\} \qquad i \in I \tag{3.8}$$

The objective (3.1) is to maximize the covered demand, where a demand point is only covered when it is covered by both a primary as well as a special vehicle, which is indicated by constraints (3.2) and (3.3). The constraints (3.4) and (3.5) are on the number of primary and special vehicles respectively. The constraint (3.6) indicates that we can only locate a special vehicle at site j if there already is a primary vehicle located at site j. So in this model, if a demand node is only covered by a primary vehicle, this is considered just as bad as not being covered at all.

In the same article [18], a slightly adapted version of the TEAM model is proposed, the **multi-objective tandem equipment allocation model** (MOTEAM). The difference with the TEAM model is that this model does distinguish between a location not being covered at all or being covered by just a primary vehicle. For this purpose, in the objective function there is an additional parameter λ. This parameter represents the trade-off between the value of coverage by (only) a primary vehicle and by a special vehicle. It also assumes that a demand site has two demands, one for primary equipment and one for special equipment. This model still assumes that there can only be a special vehicle located at a site if there already is a primary vehicle there.

The third model proposed in this article [18] is the **facility-location equipment-emplacement technique model** (FLEET). In essence this model is the same as the TEAM model, but there are two differences. On the one hand it relaxes the constraint on the occurrence of special vehicles where there are no primary vehicles. On the other hand it has the restriction that no more than p new location sites can be built.

Another model that is known, is the **hierarchical objective set covering model** (HOSC), developed in 1981 by Daskin and Stern [7]. This model is a combination of the LSCM model and the MCLP model, it looks for a solution that covers each demand point with the minimal number of locations, and simultaneously maximizes the extent of multiple coverage of demand points.

Later, the **backup coverage problem** (BACOP) 1 and 2 were developed in 1986 [11]. The BACOP models try to cover as many demand points as possible twice. The most important difference between these models is that the first version has the constraint that every demand point has to be covered once and the objective is to maximize the demand covered twice, whereas in the second version there is no constraint on coverage, but the objective is to maximize both the demand covered once, as well as the demand covered twice. The objective function for BACOP2 is a combination of the demand covered once and the demand covered twice, where the coefficient can be specified.

The last model we will describe is the **double standard model** (DSM), published in 1997 by Gendreau et al. [8]. The DSM model looks for a solution that covers all demand points at least once, within a time r_1 and α percent within a time r_2 (where $r_2 < r_1$). Simultaneously it tries to cover as many demand points as possible twice within a time r_1.

2.3 Theoretical background

As mentioned in Section 2.1, the models described in this chapter as well as the models that are introduced later, are (binary) integer linear programs. This comes naturally, since we consider objects that cannot be split, we cannot have a fractional part of an ambulance station or a fractional part of a resident. So the models we consider are integer linear programs (ILP). For every ILP, we can consider its LP relaxation, which we obtain by deleting the integrality constraints on the variables. The information in this section about solving these types of problems comes from [10].
Intuitively, ILP problems seem to be easier than LP problems, since we have less solutions. Unfortunately this is not true. The efficiency for solving LP problems is partially based on the good characterization of potentially optimal points. We can distinguish between two types of solutions:

- Feasible solutions: solutions that satisfy all constraints
- Infeasible solutions: solutions that violate at least one constraint

An optimal solution then is a feasible solution that has the most favorable value of the objective function. Note that an optimal solution is not necessarily unique, but also that some problems have no optimal solution, either because the objective function is unbounded or because there is no feasible solution.

For an LP, we know that all feasible solutions lie within a polyhedron (the intersection of finitely many halfspaces, generated by the constraints). We even know that the best solution of the cornerpoints of the polyhedron is an optimal solution (provided that the optimal solution of the problem is bounded).
Now for an ILP, in theory we could check all solutions, since there are finitely many (if we optimize over a polytope), and then select the best. But we have to realize that even a finite number can be very large. Furthermore note that a binary ILP with n variables, potentially has 2^n solutions. So if we have 10 binary variables, we could have $2^{10} = 1024$ solutions, and if we have 100 binary variables we could have $2^{100} > 1 \cdot 10^{30}$ solutions. So in practice, it is not efficient to just check all solutions, since this would take too much time.

There are existing implementations and programs that help us to solve larger optimization problems. In this research we used two software packages: AIMMS was used for easy generation of the mathematical programs and CPLEX is the package used to solve the programs.

AIMMS is an optimization modeling technology produced by Paragon Decision Technology [15]. It offers an environment to create models and solve them. The most important reason to use AIMMS is that it provides an intuitive way to set up mathematical programs, by defining sets, parameters, variables and constraints. It also provides an easy way to set up database connections, which is handy since most of the data we use is saved in Microsoft Access databases. For the solving of (Mixed) Integer Programs, AIMMS uses CPLEX.

ILOG CPLEX (usually referred to as CPLEX) is an optimization software package. It is named after the simplex method and the C programming language [6]. It can be used to solve (large) linear programs, mixed integer programs (MIPs), quadratic programs and quadratically constrained programs. CPLEX has an implementation of several methods. It solves MIPs by solving a sequence of linear relaxations to provide bounds, using the branch-and-cut algorithm, which is described in the next sections. For solving the linear relaxations that occur in MIPs, CPLEX usually uses the dual simplex method.

2.3.1 Branch-and-cut

In this section we give a short description of the branch-and-cut algorithm that is used in CPLEX. The branch-and-cut-algorithm is based on the branch-and-bound algorithm, which we will describe first.

The idea behind the branch-and-bound algorithm is that an ILP problem has only finitely many feasible solutions. As mentioned before, finitely many can still be a lot, so it is not an option to just check all of them. But we can try to enumerate the solutions in a smart way, so we do not have to explicitly check all solutions, but can still say something about them. One approach to this idea is the branch-and-bound algorithm. An extensive description of this technique can be found in [10], here we give a short summary for the case of a binary IP. The technique basically consists of three steps:

- Branch: Divide the set of feasible solutions into subsets.
- Bound: For each subset find a bound on how good the solution is at best.
- Fathom: Decide whether a subproblem can be fathomed, thus dismissed from further consideration. There are three ways a subproblem can be fathomed:
    1. Its bound is worse than the current best integer solution.
    2. Its LP relaxation has no feasible solutions.
    3. The optimal solution for its LP relaxation has integer values for the integer variables in the original problem (so is a feasible solution for the original problem). (If this solution is better than the current best solution, it becomes the new best solution).

A summary of the algorithm is (from [10]):

Initialization: Set $Z^* = -\infty$ if you want to maximize the objective function. (If you want to minimize the objective function, set $Z^* = \infty$). Apply the bounding step, fathoming step and optimality test described below to the whole problem. If not fathomed, classify this problem as the one remaining "subproblem" for performing the first full iteration below.

Steps for each iteration:

1. *Branching:* Among the remaining (unfathomed) subproblems, select the one that was created most recently. Branch from the node for this subproblem to create two new subproblems by fixing the next variable (the branching variable) at either 0 or 1.

2. *Bounding:* For each new subproblem, obtain its bound by applying the simplex method to its LP relaxation.

3. *Fathoming:* For each new subproblem, apply the three fathoming tests summarized above, and discard those subproblems that are fathomed by any of the tests.

Optimality test: Stop when there are no remaining subproblems: the current best solution is optimal (if there is no current best solution, there is no feasible solution). Otherwise, return to perform another iteration.

Unfortunately, the branch-and-bound algorithm is not suitable for large scale problems as a stand-alone approach since in the worst case, the number of iterations grows exponentially with the number of variables. This is why we consider the branch-and-cut algorithm. Basically, the branch-and-cut algorithm is the branch-and-bound algorithm, using cutting planes to reduce the feasible region for the LP relaxation without eliminating any feasible solution for the ILP. A cutting plane is an extra constraint that is added to the LP relaxation. Suppose that the optimal solution $\bar{x}$ to the LP relaxation does not satisfy the integrality constraints of the ILP, then we would like to exclude this solution. We can do this by finding a cutting plane: we search for an inequality of the form $f(x) \leq a$, such that:

- $f(y) \leq a$ for all feasible solutions y of the ILP
- $f(\bar{x}) > a$

The set $\{x : f(x) = a\}$ is then called a cutting plane: it cuts of the optimal solution of the LP relaxation. Now if we add the constraint $f(x) \leq a$ to the LP relaxation, we can solve it again and we will find a different solution. Note that there is still no guarantee that the branch-and-cut algorithm is theoretically efficient. In the worst case scenario the number of solutions to be checked still grows exponentially with the number of variables. However, in practice it turns out that this works well for many ILPs. Furthermore, even if we have not checked all solutions yet, we already have some information on the optimal solution, this is described in the next section.

2.3.2 Bounds on the optimal objective value

Of course it would be ideal if we could solve every problem to optimality, but this is not always possible due to, for example, time limitations. Fortunately, we can give an idea of the quality of the solutions found during the resolution process.

As described earlier in this section, the solving of binary IP problems is done by solving a lot of LP relaxations, which we can do efficiently. However, we might have so many of these LP problems, that it still takes too much time. Fortunately, during this process we already obtain useful information on the optimal solution, namely an upper and lower bound on the objective value of an optimal solution, which will come closer to each other when we continue the resolution process. For now, assume that we consider a maximization problem. Then the two bounds are provided in the following way:

Lower bound. Every feasible solution for the binary IP problem provides a lower bound on the objective value: we know that we have a solution with this objective value and maybe we can find one with a higher objective value, but we cannot be sure.

Upper bound. The optimal solution for a relaxation of a problem, gives an upper bound on the objective value: if the relaxation does not have a better solution, we know for sure that the original problem, which has additional constraints, so less solutions, can not have a better solution either.

If we consider a minimization problem, the roles are reversed: a feasible solution gives an upper bound on the objective value and the optimal solution of a relaxation gives a lower bound.

Chapter 3

Data

In this chapter we describe the data that is used in the models introduced later. The sets of demands points, (potential) location sites and the travel times between those points are based on the travel time model that is described in the *Refrence framework for ambulance care 2008* [12]. This means that we consider every four-digit postal code area as a point, based on the postal codes that were designated in 2008. Furthermore we give an overview of the current organization of the ambulance care in the Netherlands. In Section 3.1 we describe the division of the Netherlands into RAVs, next in Section 3.2 we describe the four-digit postal codes areas, in Section 3.3 we describe how the travel time model was developed and in Section 3.4 we discuss the demand assigned to each demand point. In Section 3.5 we describe the background on the maximal response time, in Section 3.6 we give an overview of the current location sites, and in Section 3.7 we give an overview of the hospitals in the Netherlands.

3.1 RAV regions

For the organization of ambulance care, the Netherlands is divided into 25 regions (actually 24, since two regions are seen as one), called RAVs (in Dutch: Regionale AmbulanceVoorziening). In Figure 3.1 this division is shown. Every RAV consists of a control room (MKA) and one or more ambulance services. Every RAV is legally responsible for the ambulance care in its region, both when it comes to the functioning of the MKA as well as for the care provided by ambulance personnel [17].

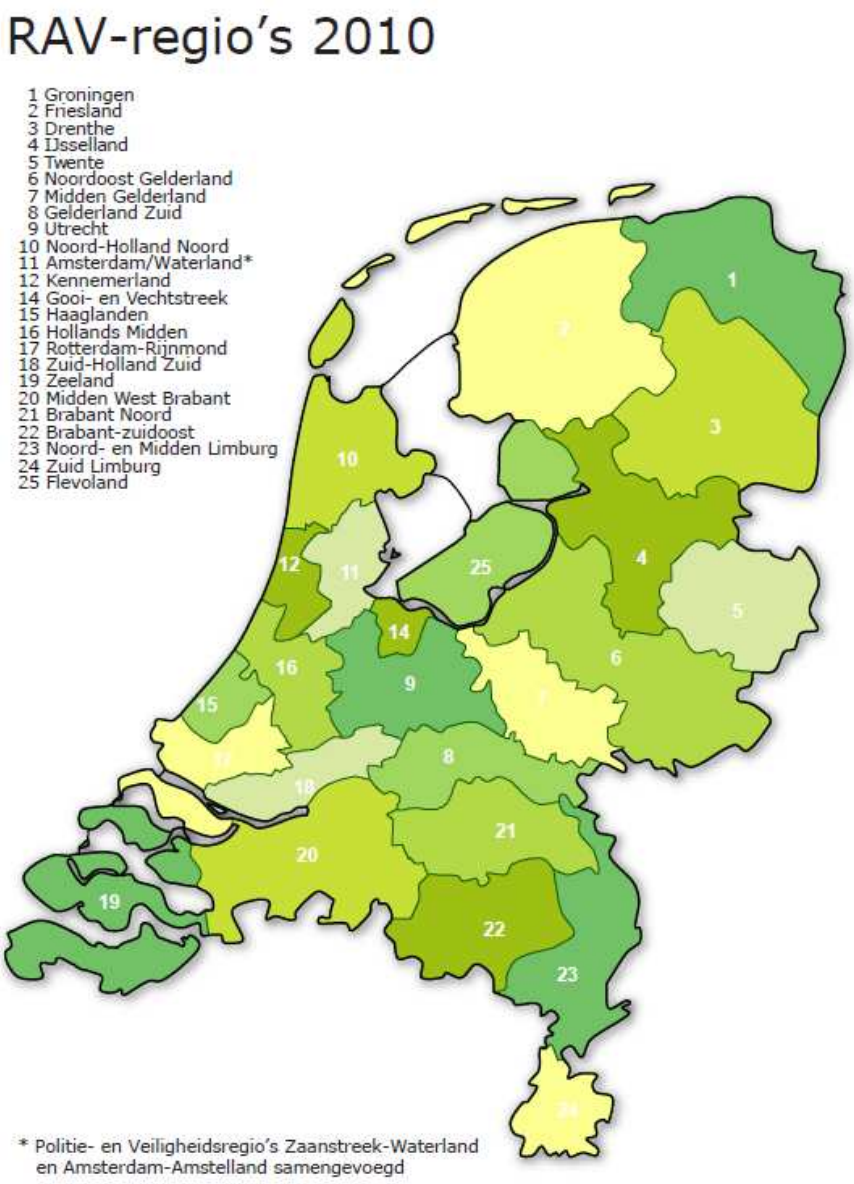


Figure 3.1: The division of the Netherlands in 24 RAVs, from [17].

3.2 Four-digit postal codes

In the Netherlands, a postal code consists of four digits and two letters. In our model, we consider regions that have the same four digits. In general this means that we consider city districts or small villages as one region.

In the travel time model [12], there are 4019 different four-digit postal codes. These postal codes are spread over the 24 RAVs, see Table 3.1 for the number of postal codes for each region [12].

RAV nr	Name	Postal codes	RAV nr	Name	Postal codes
1	Groningen	251	14	Gooi- en Vechtstreek	40
2	Friesland	474	15	Haaglanden	141
3	Drenthe	255	16	Hollands-Midden	124
4	IJssel-Vecht	170	17	Rotterdam-Rijnmond	185
5	Twente	120	18	Zuid-Holland-Zuid	98
6	Noordoost-Gelderland	202	19	Zeeland	153
7	Gelderland-Midden	134	20	Midden- en West-Brabant	217
8	Gelderland-Zuid	160	21	Brabant-Noord	146
9	Utrecht	217	22	Zuidoost-Brabant	137
10	Noord-Holland-Noord	167	23	Limburg-Noord	137
11/13	Amsterdam-Amstelland	161	24	Zuid-Limburg	141
12	Kennemerland	98	25	Flevoland	91

Table 3.1: Number of postal codes per RAV.

Note that there is no distinction between RAVs 11 and 13, they are seen as one region, since they belong to one organization.

3.3 Travel time model

The travel times used in the modeling are determined within the framework of reference for ambulance care 2008 [12]. It consists of the estimated time an ambulance needs in order to get from an ambulance location site to an emergency call. These travel times are determined using a navigation system, which distinguishes twelve different road types, and differentiates these road types to placement, whether it is in- or outside a village and the type of region it is in. The different RAVs are divided into three regions: urban, rural and intermediate (in Dutch: randstad, periferie en halfweg). The estimations for the travel times on the different road types in the different regions are based on real measurements, which were made in eight of the RAVs between August 27, 2007 and October 8, 2007. For more technical details about this model, we refer to the report [12]. It resulted in three models, one for the peak-hours, one for normal daytime and one for the nights. To have the most conservative results, we used only the model for the peak hours, since in general the travel time will be highest in the peak hours.

3.4 Demand

As the demand for each postal code area we take the number of residents in 2010 determined by Statline, part of Statistics Netherlands (in Dutch: CBS, Centraal Bureau voor de Statistiek) in each postal code area. In this file there were 16 postal codes missing, that are in our dataset. Since these postal codes are in industrial areas, we assumed that the demand at these demand points is zero for simplicity. This might not be ideal, since there can still be incidents in these areas. However, this is a choice made if we take the number of residents as demand. Another option is to take the number of incidents per hour, or per day, as the demand. In Table 3.2 we give an indication of the distribution of the number of residents per postal code area.

Number of residents	Number of postal codes
0 - 250	512
250 - 500	341
500 - 1.000	397
1.000 - 2.500	725
2.500 - 5.000	623
5.000 - 7.500	572
7.500 - 10.000	441
$>$ 10.000	408

Table 3.2: Overview of the distribution of the number of residents per postal code area.

3.5 Maximal response time

The response time of an ambulance is defined as the time between the moment that the call is taken at the call center and the arrival of the ambulance at the scene of an incident. The aim of the Dutch ambulance sector, is to reach the scene within 15 minutes under normal circumstances [1]. This time is divided into three steps: first the time of handling the call, second the time to get the ambulance ready and depart from the station and third the travel time to the scene. Using information from historical data, it is assumed that the first step takes about 2 minutes and the second step takes 1 minute. This leaves 12 minutes for the ambulance to drive to the scene. Of course, in practice it can be that the first two steps only take 2 minutes in total, which would leave 13 minutes for the ambulance to drive to the scene.

Recently, there has been a discussion about enhancing this standard, to a maximal response time of 8 minutes, since there seems to be no scientific base for the norm of 15 minutes [14]. There are situations where the norm of 15 minutes is not sufficient, for example in case of a heart attack.

3.6 Current ambulance locations

In 2010 there were 223 location sites for ambulances, of which 196 location sites are occupied 24/7. The other location sites are occupied only during day time, or only on weekdays. These location sites are spread over the 24 different regions as indicated in Table 3.3, a visual overview is given in Figure 3.2.

RAV nr	Number of 24/7 stations (total)	RAV nr	Number of 24/7 stations (total)
1	11 (13)	14	3
2	17 (19)	15	7 (8)
3	12 (13)	16	9 (10)
4	10	17	10
5	8 (9)	18	5 (6)
6	10 (13)	19	10 (11)
7	7	20	10 (13)
8	5 (11)	21	6 (7)
9	11	22	7
10	9	23	7
11/13	9	24	3 (4)
12	4 (7)	25	6

Table 3.3: Current ambulance stations per RAV.



Figure 3.2: Locations of ambulance stations in NL in 2010.

3.7 Hospital locations

In the Netherlands, we distinguish between five types of hospitals [17]:

- General and academic hospitals: These hospitals provide general care for all types of patients and they also train junior doctors and nurses.
- Specialized hospitals: These hospitals are specialized in a certain type of patient, for example rehabilitation or dialysis patients.
- Private clinics: These are clinics that provide care that is not covered by health insurance.
- Top clinical hospitals (in Dutch: Topklinische ziekenhuizen): These are hospitals that provide extra care on a higher level than the general hospitals, for example for fertility treatments or organ transplantations.
- Trauma centers: Hospitals that provide care for patients with serious injuries. These hospitals for example have a 24/7 ER, ICU and a mobile medical team (MMT, in Dutch: Mobiel Medisch Team).

3.7.1 Reachability of the hospitals

In our models we focus on two sets of hospitals, first all hospitals that have an emergency department (ED) and second the trauma centers. These sets are most relevant because when one needs care after for example a severe incident, usually transportation to an ED is enough, but in special cases where higher care is needed, one needs to be transported to a trauma center. Policy rules are that every resident should be able to reach a hospital by ambulance within 45 minutes after the emergency call. The current situation is that this is possible for 99.7% of the residents. The areas that are not within this norm are mostly at the Wadden Islands. These islands are only reachable by boat, so it is not possible to reach these places by ambulance in a short time. To reach these places quickly, we need an air ambulance. There are four air ambulances in the Netherlands. In Figure 3.3 we see the travel time to the closest hospital with an ED and the Trauma Centers by ambulance for each area.

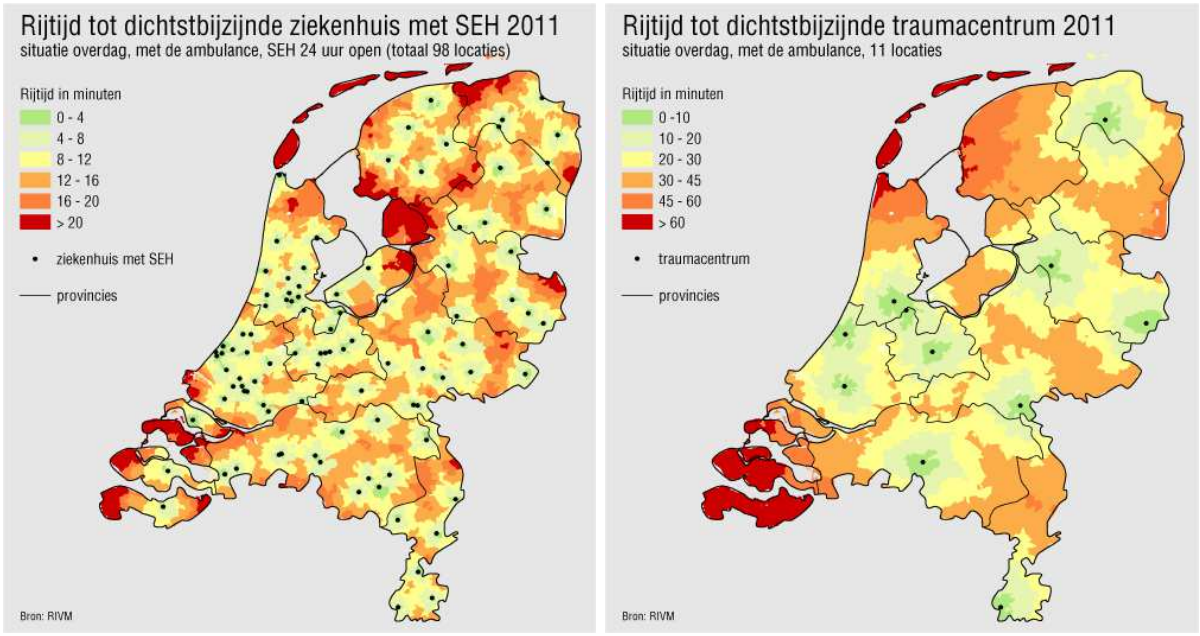


Figure 3.3: Travel time to hospital with an ED (left) and Trauma Centers (right) by ambulance.

Chapter 4

Model descriptions

The first step in this research was to work with simple, existing models. So we started by looking at the first two models described in section 2.2, the Location Set Covering Model (LSCM), described in Section 4.1, and the Maximal Covering Location Problem (MCLP), described in Section 4.2. After that, in Section 4.3, we consider an adaptation to the LSCM and MCLP, where we distinguish between urban and rural areas. In Section 4.4 we consider the connection between the location sites of ambulances and the transport to hospitals with emergency care facilities. Finally we consider multiple vehicle types in Section 4.5.

All of the models described in this chapter have some of their input in common:

- The set of demand points I.
- The set of (potential) location sites J.
- The travel times t_{ij} from $j \in J$ to $i \in I$.
- A maximal travel time r an ambulance may take to get to the scene of an incident.

Using this input we can determine for each demand point $i \in I$ which potential location sites $j \in J$ can cover it, we define:

$$J_i = \{j \in J | t_{ij} \leq r\}.$$

The sets I and J are the same, they consist of all 4019 postal codes as described in Section 3.2. The two sets are the same because we assume that we can build an ambulance station at every location point that we have. The travel times are given by the travel time model as described in Section 3.3. The maximal travel time has a different definition in different cases, this will be mentioned for each model.
For all models, we start with a greenfield scenario: we assume that there are no existing ambulance stations. Together with the assumption that J contains all 4019 postal codes, this gives a good idea of the possibilities in the ideal case, but in reality of course there are places where it is more convenient to build and places where they do not want to or cannot build.

4.1 LSCM

Recall the formulation of the LSCM from Section 2.2:

LSCM				
Minimize	$\sum_{j \in J} x_j$			
Subject to	$\sum_{j \in J_i} x_j$	$\geq$	1	$i \in I$
	x_j	$\in$	$\{0,1\}$	$j \in J$

Apart from the general input, we only have to specify the maximal travel time, which is used to define the sets $J_i, \quad i \in I$. We chose to solve the model for three different maximal travel times: 8 minutes, 12 minutes and 15 minutes. This choice is motivated by the current standard of 15 minutes after a call, of which 3 minutes are usually used for setting up the vehicle, which leaves 12 minutes driving time, and we want to see the influence of a lower or higher maximal travel time.

Furthermore we will consider two cases for this model:

- We can consider all of the Netherlands at once and solve the model, which gives us one solution for all of the country.
- We can consider all of the RAVs separately and solve the model for each RAV, then take the union of all these solutions as a solution for all of the country.

We know that the solution in the first case will be at least as good as the second solution, we even expect it to be better, especially near the boundaries of the RAVs. The results of these models are discussed in Sections 5.1.1, 5.1.2 and 5.1.3.

4.2 MCLP

Recall the formulation of the MCLP from Section 2.2.

MCLP				
Maximize	$\sum_{i \in I} d_i y_i$			
Subject to	$\sum_{j \in N_i} x_j$	$\geq$	y_i	$i \in I$
	$\sum_{j \in J} x_j$	$=$	p	
	x_j	$\in$	$\{0,1\}$	$j \in J$
	y_i	$\in$	$\{0,1\}$	$i \in I$

For this model, we have the same input as for the LSCM plus the demand for each demand point and a number, p, of location sites that can be built. The result is the maximal demand that can be covered with this number of location sites being built. The results of the LSCM give us the number of location sites needed to cover all demand, so using the MCLP we can see how much we can still cover with less location sites. As described in Section 3.4, we take the number of residents in 2010 as the demand d_i for each demand point i.

4.3 Adapted models: Two types of postal codes

In this section we describe adapted versions of the LSCM and MCLP. Here we distinguish between urban and rural areas. We define two sets of demand points: urban demand points I_u and rural demand points I_r and two maximal travel times t_u and t_r. Since it usually is busier in the urban areas and more accidents happen there, we could define the maximal travel time allowed for urban areas to be less than the maximal travel time allowed for rural areas. In this way ambulances will be occupied for less time per call. So we assume that $t_u \leq t_r$. We still have one set of potential location sites J.

Now we define an area in I_u to be covered by location $j \in J$ if $t_{ij} \leq t_u$, so we get the potential location sites $j \in J$ that cover $i \in I_u$:

$$N_i^u = \{j \in J : t_{ij} \leq t_u\}.$$

Analogously we define for $i \in I_r$:

$$N_i^r = \{j \in J : t_{ij} \leq t_r\}.$$

We can formulate two 'new' versions of the LSCM and MCLP:

Adapted LSCM

Minimize	$\sum_{j \in J} x_j$			
Subject to	$\sum_{j \in N_i^u} x_j$	$\geq$	1	$i \in I_u$
	$\sum_{j \in N_i^r} x_j$	$\geq$	1	$i \in I_r$
	x_j	$\in$	$\{0,1\}$	$j \in J$

Adapted MCLP

Maximize	$\sum_{i \in I} d_i y_i$			
Subject to	$\sum_{j \in N_i^u} x_j$	$\geq$	y_i	$i \in I_u$
	$\sum_{j \in N_i^r} x_j$	$\geq$	y_i	$i \in I_r$
	$\sum_{j \in J} x_j$	$=$	p	
	x_j	$\in$	$\{0,1\}$	$j \in J$
	y_i	$\in$	$\{0,1\}$	$i \in I_u \cup I_r$

4.3.1 Classification of demand points

For each demand site it has to be decided whether it is an urban or a rural area. We do this based on a classification by the CBS. They gave each four-digit postal code a value between 1 and 5, based on the number of addresses in the surroundings:

1. 2500 or more addresses per km^2.
2. 1500-2500 addresses per km^2.
3. 1000-1500 addresses per km^2.
4. 500-1000 addresses per km^2.
5. 500 or less addresses per km^2.

The CBS considers areas with a classification of 1 or 2 to be urban and with a classification of 4 or 5 rural. So they do not give an interpretation for classification 3. To consider the 'worst-case scenario', for which we would need the most location sites, we define the areas with classification 3 to be urban.
We noted that there are twelve postal codes that are in our model that do not have a classification from the CBS. For these postal codes, we checked the classification of neighboring and similar areas manually and added a classification for these points. Furthermore, the CBS has classified sixteen postal codes that are not in our model, but since we do not have travel times for these postal codes, we did not consider these postal codes.
An overview of the number of postal codes per class is given in Table 4.1. Here we see we have 1390 urban postal codes and 2629 rural postal codes.

Classification	Number of postal codes
1 (Urban)	366
2 (Urban)	548
3 (Urban)	476
4 (Urban)	576
5 (Urban)	2053

Table 4.1: Number of postal codes per class.

Based on the discussion mentioned in Section 3.5, we define t_u and t_r to be 8 and 12 minutes respectively, so we can see the effects if we enhance the standard in the urban areas, where most of the incidents occur. See Section 5.3 for the results.

4.4 From location site to incident to hospital

In the Netherlands, it is required by law (WTZi, Wet toelating zorginstellingen), that for every citizen, it has to be possible to reach an emergency department by ambulance within 45 minutes, including the time that it takes an ambulance to get to the scene of the incident. So again we split up the response time, as in Section 3.5. We assume that it takes two minutes to take the call and one minute to get the ambulance ready, and here we assume it takes five minutes to get the patient into the ambulance at the scene. This leaves 37 minutes driving time, which is again split up in two parts: the driving time from the location site to the incident (t_{ij}) and the the driving time from the incident to the hospital (h_i). In current practice, at this moment it is not possible for 57.000 citizens, so $0,3\%$, to get to the hospital within these 45 minutes. One of the reasons for this can be that the nearest hospital with emergency department is more than 45 minutes away.
We want to see how the inclusion of this norm influences our greenfield scenario. To prevent our solution to exclude those regions that cannot reach an emergency department in time from any ambulance care, we include an extra constraint, that within every RAV, 97% of the residents has to be within 12 minutes of an ambulance location. Note that this constraint implies that we need to have enough ambulance locations available, otherwise we will get an infeasible problem. Under this extra constraint, we want to maximize the number of residents that can reach an emergency department by ambulance within 45 minutes. For this, we define a new set, similar to J_i, which describes for each demand point $i \in I$, which potential location sites can cover this demand point such that the patient can reach the hospital within these 45 minutes:

$$K_i = \{j \in J | t_{ij} \leq 12 \text{ and } t_{ij} + h_i \leq 37\}, \tag{4.1}$$

where h_i is the travel time from i to the nearest emergency department. We introduce a new variable w_i which indicates whether a demand point i can be reached within 12 minutes by an ambulance and then transported to an emergency department within 45 minutes in total (we can see this as another definition of coverage):

$$w_i = \begin{cases} 1 & \text{can be reached within 12 minutes by an ambulance and then transported to an} \\ & \text{emergency department within 45 minutes in total,} \\ 0 & \text{otherwise.} \end{cases} \tag{4.2}$$

Now as mentioned before, not all postal code areas are within 37 minutes of a hospital with an ED. This implies that it is impossible to solve an LSCM version of this model, since there would not be a feasible solution, which is why we only formulate an MCLP version. The mathematical formulation is an adaptation of the general MCLP:

MCLP with hospitals

$$
\begin{array}{lrcll}
\text{Maximize} & \displaystyle\sum_{i\in I} d_i w_i & & & \\
\text{Subject to} & \displaystyle\sum_{j\in J_i} x_j & \geq & y_i & i \in I \\
& \displaystyle\sum_{j\in K_i} x_j & \geq & w_i & i \in I \\
& \displaystyle\sum_{j\in J} x_j & = & p & \\
& \dfrac{\sum_{i\in I_1} y_i d_i}{\sum_{i\in I_1} d_i} & \geq & 0.97 & \\
& & \vdots & & \\
& \dfrac{\sum_{i\in I_{25}} y_i d_i}{\sum_{i\in I_{25}} d_i} & \geq & 0.97 & \\
& x_j & \in & \{0,1\} & j \in J \\
& y_i & \in & \{0,1\} & i \in I \\
& w_i & \in & \{0,1\} & i \in I
\end{array}
$$

Where I_ι contains all demand points within RAV ι.

4.5 Multiple vehicle types

The last new model we introduce is a model in which we consider the possibility to reach the incident by different vehicles. Remember that in Chapter 2 we already introduced a model with two vehicle types from literature, the TEAM model. However, the theory behind this model did not correspond with the idea from the practice in the Netherlands. The TEAM model considers two vehicle types, primary and special vehicles. In this model a demand point has to be covered by both a primary as well as a special vehicle, where the special vehicle has a higher maximal travel time than the primary vehicle. Besides that, a special vehicle can only be located at a location site with a primary vehicle. We developed a model that considers two types of vehicles, a regular ambulance and a rapid responder, and two ways to cover a demand point. The ambulance is a vehicle that can provide care and can transport a patient to the hospital. The rapid responder is a vehicle that can only provide care and not transport a patient, for example a motor cycle. This implies that a rapid responder can be cheaper than an ambulance. Now if a rapid responder is cheaper, it can be advantageous to have a more dense network of these rapid responders, so they can arrive at the scene within a short time and to have a less dense network of ambulances, because they can take more time to get to the scene. However, a rapid responder cannot transport a patient to the hospital. Now we want to consider two ways to cover a demand point:

1. The standard coverage by an ambulance which is at the scene within a_1 minutes (so 12 minutes in the current practice).
2. Coverage by a rapid responder and an ambulance, where the rapid responder is at the scene within r minutes and the ambulance within a_2 minutes, where $r << a_1$ and $a_2 > a_1$.

So we introduce a second way to cover an incident, where a rapid responder arrives at the scene quickly, so an ambulance can take more time since it is mainly used for transport. Using the LSCM and MCLP models described before in this chapter, we have formulated three versions of this model:

1. An LSCM version where the objective is to minimize the costs, given a ratio between the costs of the two different locations.
2. An LSCM version where the number of rapid responder locations is fixed and the objective is to minimize the number of ambulance locations.
3. An MCLP version where the number of rapid responder locations and ambulance locations is fixed and the objective is to maximize the covered demand.

The choice of first fixing only the number of rapid responder locations comes from the fact that there are only 50 rapid responders available in the Netherlands at this moment, but the results of the LSCM version indicated a larger number of rapid responder locations.

First we introduce the new notation for these models:

Variables

y_{i1} indicates whether there is an ambulance location within a_1 minutes of demand point i.

y_{i2} indicates whether there is an ambulance location within a_2 minutes of demand point i.

r_i indicates whether there is a rapid responder location within r minutes of demand point i.

x_{j1} indicates whether there is an ambulance location at location j.

x_{j2} indicates whether there is a rapid responder location at location j.

c_i indicates whether demand point i is covered (so either by an ambulance location within a_1 minutes, or by a rapid responder location within r minutes and an ambulance location within a_2 minutes).

Sets and parameters

W_i is the set of locations that can cover demand point i within r minutes (by the rapid responder).

Y_i is the set of locations that can cover demand point i within a_1 minutes (by the ambulance, as in the normal case).

Z_i is the set of locations that can cover demand point i within a_2 minutes (by the ambulance, after the rapid responder).

p_1 is the number of ambulance locations.

p_2 is the number of rapid responder locations.

Now the formulation of the LSCM version where the objective is to minimize the costs is:

LSCM with multiple vehicle types

$$\text{Minimize} \quad \alpha \sum_{j \in J} x_{j1} + (1-\alpha) \sum_{j \in J} x_{j2} \tag{4.1}$$

$$\text{Subject to} \quad \sum_{j \in Y_i} x_{j1} \geq y_{i1} \qquad i \in I \tag{4.2}$$

$$\sum_{j \in Z_i} x_{j1} \geq y_{i2} \qquad i \in I \tag{4.3}$$

$$\sum_{j \in W_i} x_{j2} \geq r_i \qquad i \in I \tag{4.4}$$

$$\tfrac{1}{2}(y_{i2} + r_i) + y_{i1} \geq 1 \qquad i \in I \tag{4.5}$$

$$x_{j1}, x_{j2} \in \{0,1\} \qquad j \in J \tag{4.6}$$

$$y_{i1}, y_{i2}, c_i, r_i \in \{0,1\} \qquad i \in I \tag{4.7}$$

Here the objective (4.1) contains a parameter α, which determines the ratio between the costs of an ambulance location and a rapid responder location. The first three constraints (4.2), (4.3) and (4.4) indicate whether a demand point is covered, using the three different maximal travel times a_1, a_2 and r. The constraint (4.5) is the new part in this model. It makes sure that every demand point is covered. Recall that there are two ways to cover a demand point:

1. By an ambulance location within a_1 minutes, then $y_{i1} = 1$.
2. By a rapid responder location within r minutes, then $r_i = 1$, **and** an ambulance location within a_2 minutes, then $y_{i2} = 1$.

So there are two ways a demand point can be covered, we should have that either $y_{i1} = 1$ or $y_{i2} = r_i = 1$, so $y_{i2} + r_i = 2$. Combining these two requirements, we obtain the constraint $\frac{1}{2}(y_{i2} + r_i) + y_{i1} \geq 1$.

The formulation of the LSCM version where the number of rapid responders is fixed is almost the same. There is only one constraint added and the objective function changes.

LSCM with multiple vehicle types 2

$$\text{Minimize} \quad \sum_{j \in J} x_{j1} \tag{5.1}$$

$$\text{Subject to} \quad \sum_{j \in Y_i} x_{j1} \geq y_{i1} \qquad i \in I \tag{5.2}$$

$$\sum_{j \in Z_i} x_{j1} \geq y_{i2} \qquad i \in I \tag{5.3}$$

$$\sum_{j \in W_i} x_{j2} \geq r_i \qquad i \in I \tag{5.4}$$

$$\tfrac{1}{2}(y_{i2} + r_i) + y_{i1} \geq 1 \qquad i \in I \tag{5.5}$$

$$\sum_{j \in J} x_{j2} = p_2 \tag{5.6}$$

$$x_{j1}, x_{j2} \in \{0,1\} \qquad j \in J \tag{5.7}$$

$$y_{i1}, y_{i2}, c_i, r_i \in \{0,1\} \qquad i \in I \tag{5.8}$$

The constraint (5.6) is new, it defines the number of rapid responder locations that is used. Furthermore, now the objective function (5.1) is to minimize the number of ambulance locations used. Note that we still require that every demand point is covered with constraint (5.5).

Finally we formulate the MCLP version:

MCLP with multiple vehicle types

$$\text{Maximize} \quad \sum_{i \in I} c_i d_i \tag{6.1}$$

$$\text{Subject to} \quad \sum_{j \in Y_i} x_{j1} \geq y_{i1} \qquad i \in I \tag{6.2}$$

$$\sum_{j \in Z_i} x_{j1} \geq y_{i2} \qquad i \in I \tag{6.3}$$

$$\sum_{j \in W_i} x_{j2} \geq r_i \qquad i \in I \tag{6.4}$$

$$\tfrac{1}{2}(y_{i2} + r_i) + y_{i1} \geq c_i \qquad i \in I \tag{6.5}$$

$$\sum_{j \in J} x_{j1} = p_1 \tag{6.6}$$

$$\sum_{j \in J} x_{j2} = p_2 \tag{6.7}$$

$$x_{j1}, x_{j2} \in \{0,1\} \qquad j \in J \tag{6.8}$$

$$y_{i1}, y_{i2}, c_i, r_i \in \{0,1\} \qquad i \in I \tag{6.9}$$

The objective (6.1) now is to maximize the covered demand. The constraints (6.2), (6.3) and (6.4) are the same as in the previous models. In constraint (6.5) the right-hand side is replaced by c_i, which indicates whether a demand point is covered. In the other two versions of this model this was required to be greater than or equal to one, now this is no longer required. The constraints (6.6) and (6.7) define the number of ambulance locations and rapid responder locations used.

Chapter 5

Results

In this chapter we will describe the results obtained from the models described in Chapter 4. As mentioned there, we consider a greenfield scenario. This should be taken into account when interpreting the results, because this is not the most realistic situation. However, in a way these results give a reference to which we can compare the performance of other settings.

5.1 LSCM

The first model introduced was the Location Set Covering Model, which computes the minimal number of location sites needed to cover all demand points. This model was run for two different cases:

1. The minimal number of locations sites needed to cover all demand points in our set. This is the "national" case, i.e, all RAVs are combined.

2. The minimal number of location sites needed to cover all demand points for each RAV separately.

The results of these two cases are discussed in Section 5.1.1 and 5.1.2 respectively. We would expect that the first scenario requires less location sites in total than the second scenario. On the other hand, it can occur that a single RAV has less location sites when it is considered as a separate region, than when it is considered in the national case. This can happen when that RAV covers demand points in neighboring RAVs in the national scenario, whereas in the case where we consider each RAV separately this demand needs to be covered by locations in each respective region.

5.1.1 The Netherlands as one region

First the model was used to compute the minimal number of location sites needed to cover all demand points, considering the Netherlands as one region. This was done for three different maximal travel times: 8, 12 and 15 minutes. As discussed in Section 3.5, the aim is a travel time of less than 12 minutes, but there has been a discussion on enhancing this maximal travel time to 8 minutes. It is interesting to see what the influence of this enhancement would be and in this view it is also interesting to see what the influence of a less strict norm would be.
The results are summarized in Table 5.1. One can see that for the current norm of 12 minutes, there are 165 location sites needed to cover all demand points. Recall that the current number of location sites is 223, see Section 3.6. Furthermore note that the difference in number of location sites needed between 8 and 12 minutes, which is 217, is much bigger than the difference between 12 and 15 minutes, which is 65. A map of the results for the three travel times can be found in Figure 5.1

Maximal travel time	Number of location sites needed
8 minutes	382
12 minutes	165
15 minutes	100

Table 5.1: Results from LSCM, covering all demand points.

5.1.2 Each RAV separately

The current practice is that RAVs try to cover their 'own' demand points, so it is not exactly fair to compare the outcome of 165 locations of the LSCM to the current real situation. A better indication of the 'ideal scenario' given the current organizational situation with RAVs can be found by running the LSCM for every RAV separately. This was only done for a maximal travel time of 12 minutes, since it is the current norm in practice. The result of this run can be found in Table 5.2, where one can see both the number of locations needed in the scenario where we consider every RAV separately as well as the scenario where the model was run for all of the Netherlands at once. As expected, the number of location sites needed increases in this scenario, and even the number of location sites for each RAV is at least as high as the number of location sites needed in the first scenario. In case every RAV is considered as a separate region, a minimal number of 203 location sites is needed to cover all demand points. This is quite close to the current number of location sites, 223, and substantially more than 165.

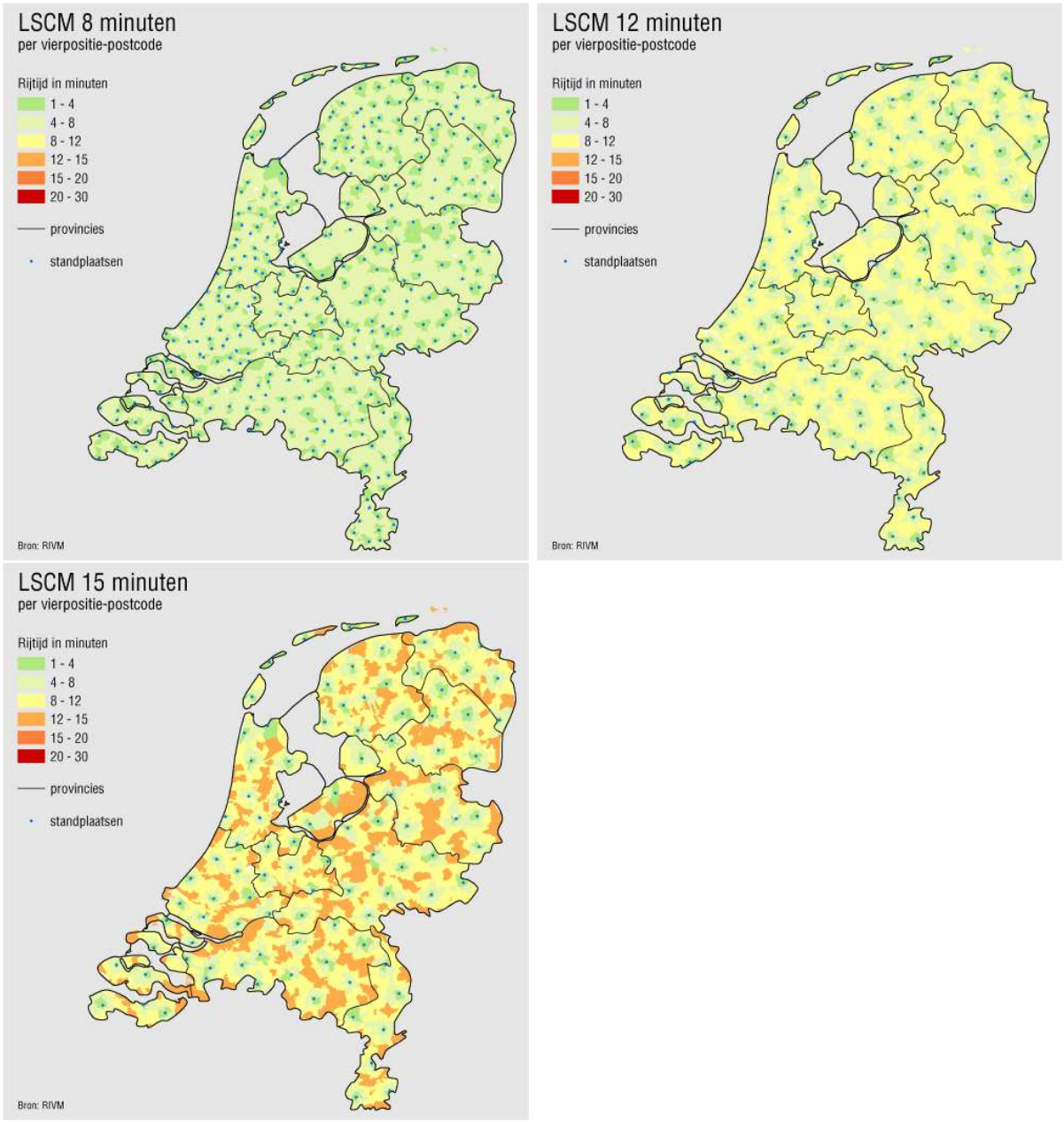


Figure 5.1: Results of the LSCM model for 8, 12 and 15 minutes maximal travel time.

RAVnr	RAV name	Number of location sites needed	Number of location sites needed in NL version
1	Groningen	12	12
2	Friesland	22	20
3	Drenthe	12	9
4	RAV IJsselland	10	8
5	Twente	6	5
6	Noord en Oost Gelderland	13	10
7	Gelderland Midden	8	7
8	Gelderland Zuid	8	6
9	Utrecht	10	6
10	Noord Holland Noord	8	6
11/13	Amsterdam-Amstelland	6	6
12	Kennemerland	4	2
14	Gooi en Vechtstreek	2	2
15	Haaglanden	4	2
16	Hollands-Midden	8	7
17	Rotterdam-Rijnmond	10	7
18	Zuid-Holland-Zuid	6	4
19	Zeeland	11	11
20	Brabant Midden-West	11	10
21	Brabant Noord	7	5
22	Brabant-Zuidoost	7	7
23	Limburg Noord	8	6
24	Zuid Limburg	4	4
25	Flevoland	6	3
	Total	203	165

Table 5.2: Results of the LSCM for all RAVs, when we consider each RAV separately and when we consider the Netherlands as one region, with maximal travel time of 12 minutes.

5.1.3 Comparison of the results for LSCM

In Table 5.2 we see the results of the LSCM for all RAVs in both the national case as well as when we consider each RAV as a separate region. We can see that the results differ per RAV, some have the same number of location sites in both cases, others have a few location sites less in the national case and some have significantly less location sites in the national case. In Table 5.3 we see the percentage of residents not covered by an ambulance location in the RAV of the resident with the solution from the national case.

RAVnr	Percentage not covered (Number of demand points)	RAVnr	Percentage not covered (Number of demand points)
1	3,03% (4)	14	3,60% (2)
2	1,37% (10)	15	15,06% (22)
3	12,36% (32)	16	2,76% (3)
4	13,45% (24)	17	14,19% (22)
5	8,26% (10)	18	14,26% (17)
6	9,59% (22)	19	0% (0)
7	3,86% (5)	20	1,37% (5)
8	3,23% (5)	21	25,02% (31)
9	15,67% (46)	22	1,18% (1)
10	7,17% (17)	23	7,45% (14)
11	2,95% (4)	24	0% (0)
12	12,21% (18)	25	14,68% (14)

Table 5.3: Percentage of residents not covered by an ambulance location in the RAV of the resident with the solution from the national case.

In Figure 5.2 we see the location sites in the national scenario and the scenario where we consider each RAV as a separate region in one figure.

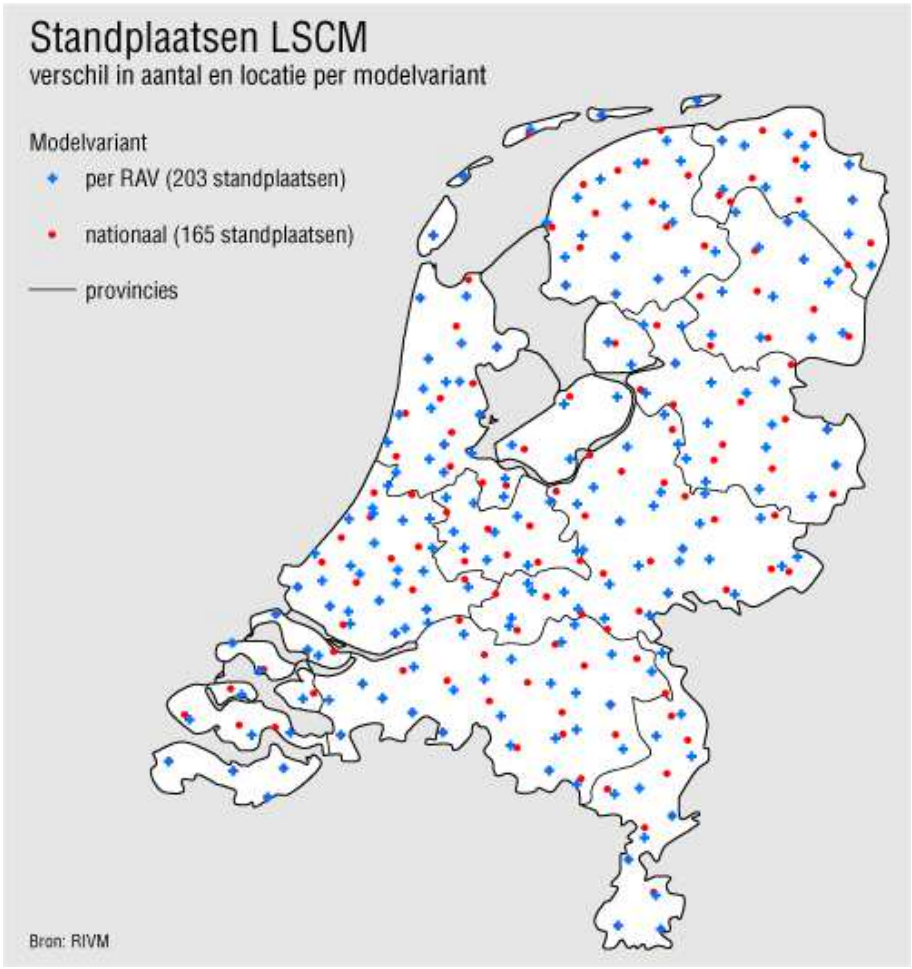


Figure 5.2: Results of LSCM 12 minutes for the two scenario's

We will consider four RAVs in more detail:

1. RAV 9, Utrecht: The difference between the number of locations sites need in the two cases is large, both relative (10 is far more than 6) and absolute (a difference of 4 location sites is the largest we see in the table).

2. RAV 15, Haaglanden: The number of location sites needed in the national case is half of the number of location sites needed when the RAV is considered as a separate region.

3. RAV 21, Brabant Noord: The percentage of residents not covered within the RAV with the national solution is the highest.

4. RAV 24, Zuid-Limburg: The number of location sites needed is the same in both cases.

Recall the location of these RAVs from Section 3.1 and Figure 3.1: RAV 9 is located in the center of the Netherlands, RAV 15 is located at the western border, RAV 21 is in the east of the Netherlands and RAV 24 is located in the very south-east.

The first RAV we will discuss is RAV 9. This is an RAV for which the difference between the number of location sites in the two scenarios is large. A visualization of the result for this RAV can be found in Figure 5.3.

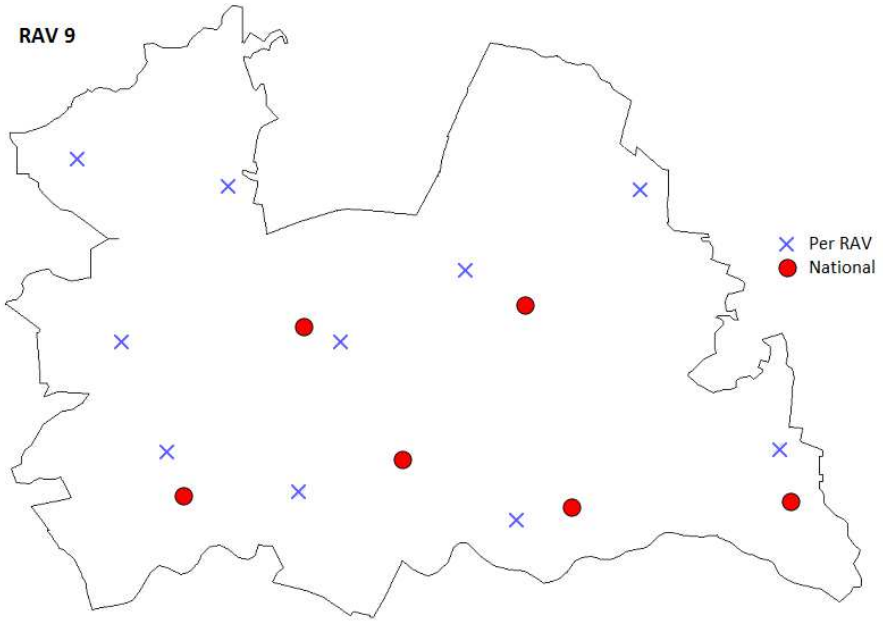


Figure 5.3: Results of the LSCM model for RAV 9, Utrecht.

We can see the most important difference occurs in the upper left corner of the RAV, where in the national case there are no location sites and in the other case there are 2. An explanation could be that this top part of the RAV is enclosed by other RAVs and that the location sites in those RAVs already cover this part of RAV 9. The remaining location sites seem to be shifted to the right in the national case, which makes that another location site can be discarded.

The next RAV we will consider is RAV 15. This is an RAV where the relative difference between the number of location sites in the two scenarios is large. A visualization of the result for this RAV can be found in Figure 5.4.

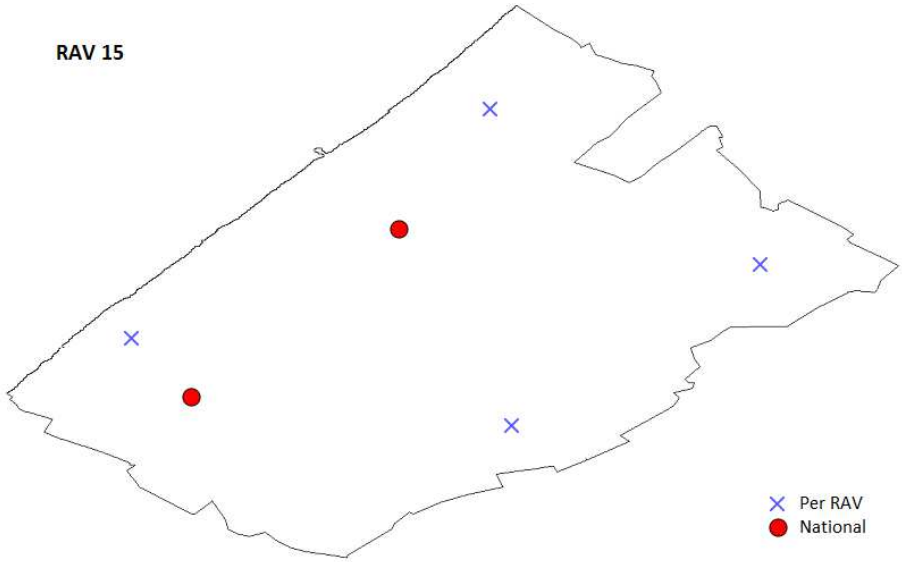


Figure 5.4: Results of the LSCM model for RAV 15, Haaglanden.

We see the result is really different in the two scenarios. The placement of the location sites in the national case can be explained by the fact that the western part of the RAV is next to the sea, so it cannot be covered by location sites in a different RAV. However, the eastern part can be covered by the neighboring RAV.

The third RAV we will consider is RAV 21. This is the RAV where the percentage of residents not covered within the RAV with the national solution is the highest. A visualization of the result for this RAV can be found in Figure 5.5.

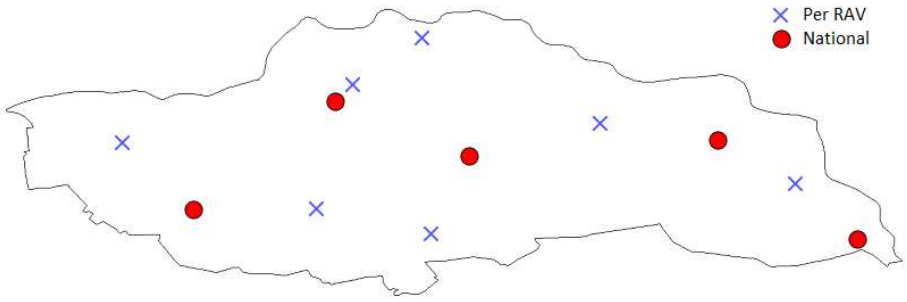


Figure 5.5: Results of the LSCM model for RAV 21, Brabant Noord.

It seems that the location sites in the national case are more to the center of the RAV, and not at the north and south of the RAV. This implies that these parts are covered by location sites in neighbouring RAVs, which seems plausible since this RAV is enclosed by other RAVs.

The last RAV we consider here is RAV 24. This is an RAV that has the same number of location sites in both cases. A visualization of the result for this RAV can be found in Figure 5.6. We can see that three out of four location sites are located at the same location and one is

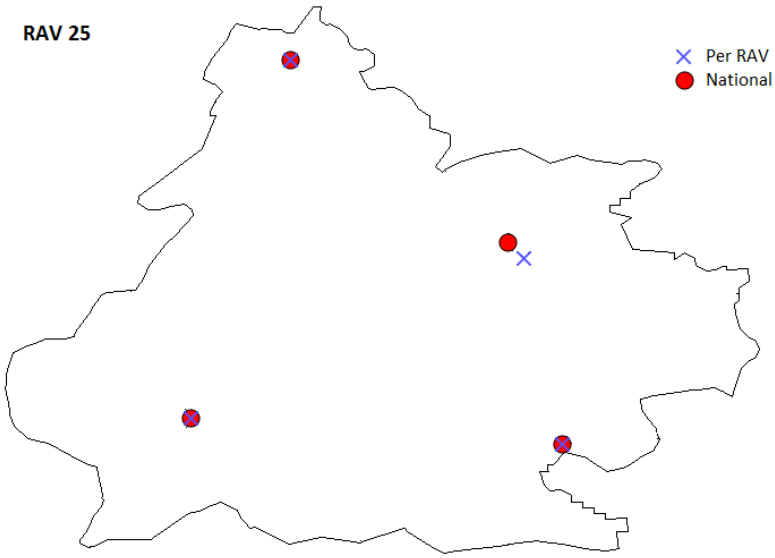


Figure 5.6: Results of the LSCM model for RAV 24, Zuid Limburg.

relocated. Actually, the location sites in the solution of the national case are also optimal if we consider the RAV as a separate region. This is because RAV 24 is an RAV at the very south of the Netherlands and has not that much connection with other RAVs, just with the one right above (RAV 23, Limburg Noord). The fact that one location site is different in the solutions is because the solutions to the model are not necessarily unique.

5.1.4 Non greenfield scenario

Here we consider one national scenario where we fix location sites at academic hospitals, which are at 9 different locations. The optimal solutions for this scenario are given in Table 5.4.

Maximal travel time	Number of location sites needed
8 minutes	388
12 minutes	168
15 minutes	107

Table 5.4: Results from LSCM, covering all demand points, with fixed locations at academic hospitals.

We see that this increases the number of location sites needed, by 6, 3 and 7 respectively. We fixed 9 locations, so the first and last increase are quite significant. Some of the academic hospitals are very close to each other, for example there are academic hospitals at two locations in Amsterdam and in Rotterdam. This may cause suboptimality in the solutions, since two hospitals can cover more or less the same area.

5.2 Maximal Covering Location Problem (MCLP)

The next model we investigated is the MCLP. Using this model we determine how much coverage can be achieved with a specified number of location sites. Obviously this is only interesting for a number of location sites less than the outcome of the LSCM model in Section 5.1. The effect of decreasing the number of location sites used in the coverage is investigated. From Section 5.1 we know how many location sites there are needed to cover 100% of the demand, but it is also interesting to see what the effect of for example 10 location sites less would be. This was investigated for the three different maximal travel times. The model was only ran for the Netherlands as one region. Given the results from the LSCM model, the following ranges were chosen for the number of ambulance locations available:

- 8 minutes: $200 - 225 - \ldots - 375$.
- 12 minutes: $90 - 100 - \ldots - 160$.
- 15 minutes: $40 - 50 - \ldots - 100$.

The choice for these ranges is made based on some test runs and a balance between the computing time and the amount of extra information it gives. It turned out that for 8 minutes travel time, 10 or 20 stations less did not have a significant influence on the covered demand, so we decided to take larger steps of 25 stations. In the case of 12 or 15 minutes travel time, a smaller step of 10 stations gave more useful information.

An overview of the results as well as a visualization can be found in Figures 5.7, 5.8 and 5.9. This gives an idea of the effect of lowering the number of location sites. One can see that in all three cases at first the influence is rather small, the coverage percentage decreases slowly, but as the number of location sites is decreasing further, the coverage percentage drops faster. This might be explained by the fact that there are areas that have very few residents (see Section 3.4, and these will be left uncovered 'first' if we decrease the number of available location sites. Later on it will be inevitable to leave demand points with a higher number of residents uncovered, which causes the faster decrease.

Unfortunately, as the covered percentage comes closer to 100%, it becomes more and more difficult to solve the problem to optimality. In these cases the lower bound on the covered percentage, as described in Section 2.3.2, is given in brackets.

Additionally, in Figure 5.11 a map of the results for the three different travel times is given, for approximately 95% coverage, which corresponds to 225, 95 and 60 location sites respectively.

Number of location sites	Covered demand
200	92.64%
225	95.14%
250	96.93%
275	98.27%
300	99.15%
325	99.68% (99.69%)
350	99.92% (99.93%)
375	99.998% (99,9996%)

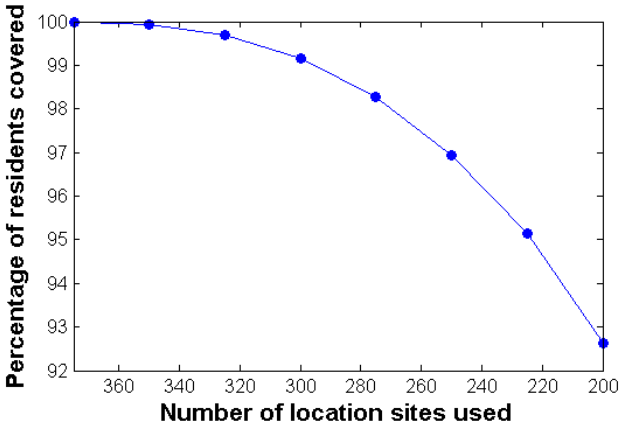


Figure 5.7: Results of MCLP for maximal travel time 8 minutes.

Number of location sites	Covered demand
90	94.34%
100	96.39%
110	97.85%
120	98.83%
130	99.49% (99.51%)
140	99.80% (99.85%)
150	99.95% (99.99%)
160	99.998% (100%)

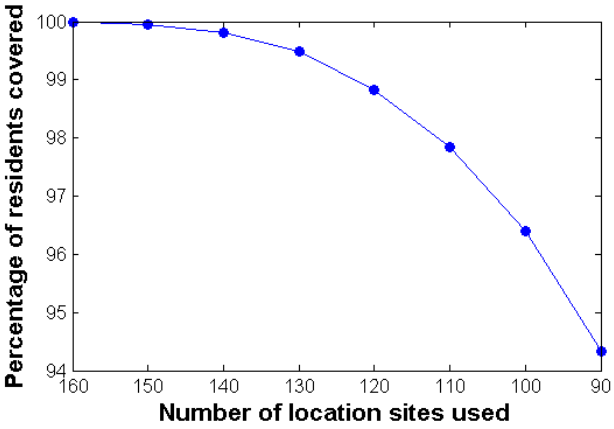


Figure 5.8: Results of MCLP for maximal travel time 12 minutes.

Number of location sites	Covered demand
40	85.66%
50	91.54%
60	95.69%
70	98.16%
80	99.44% (99.48%)
90	99.89% (99.93%)
100	100%

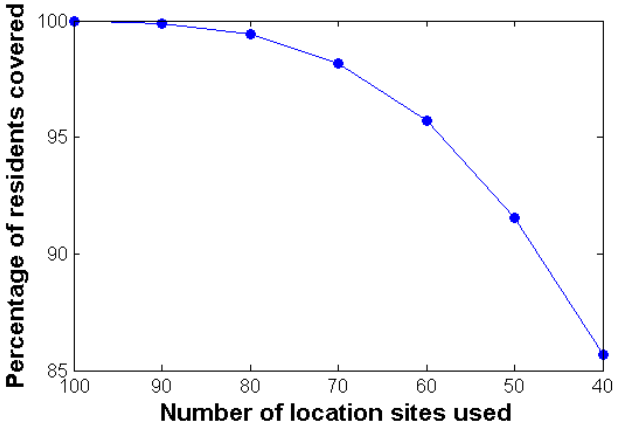


Figure 5.9: Results of MCLP for maximal travel time 15 minutes.

5.2.1 Non greenfield scenario

Again we consider one scenario where we fix location sites at academic hospitals, which are 9 locations. We only consider this scenario for 12 minutes maximal travel time. The results for this scenario are given in Table 5.10.

Number of location sites	Covered demand
90	93.24%
100	95.57%
110	97.17%
120	98.32%
130	99.16%
140	99.64% (99.68%)
150	99.89% (99.92%)
160	99.98% (99.99%)

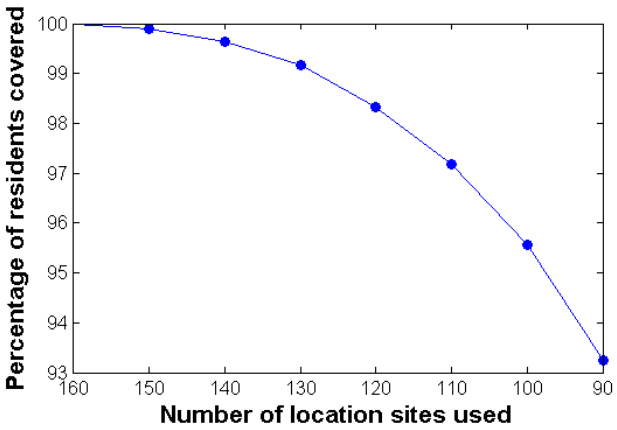


Figure 5.10: Results of MCLP for maximal travel time 12 minutes with locations fixed at academic hospitals.

As expected, we see that the percentage of the demand that is covered is slightly lower than if we do not fix these location sites. Furthermore we see the same behaviour as for the other MCLP results, the coverage percentage decreases slowly at first and faster later on.

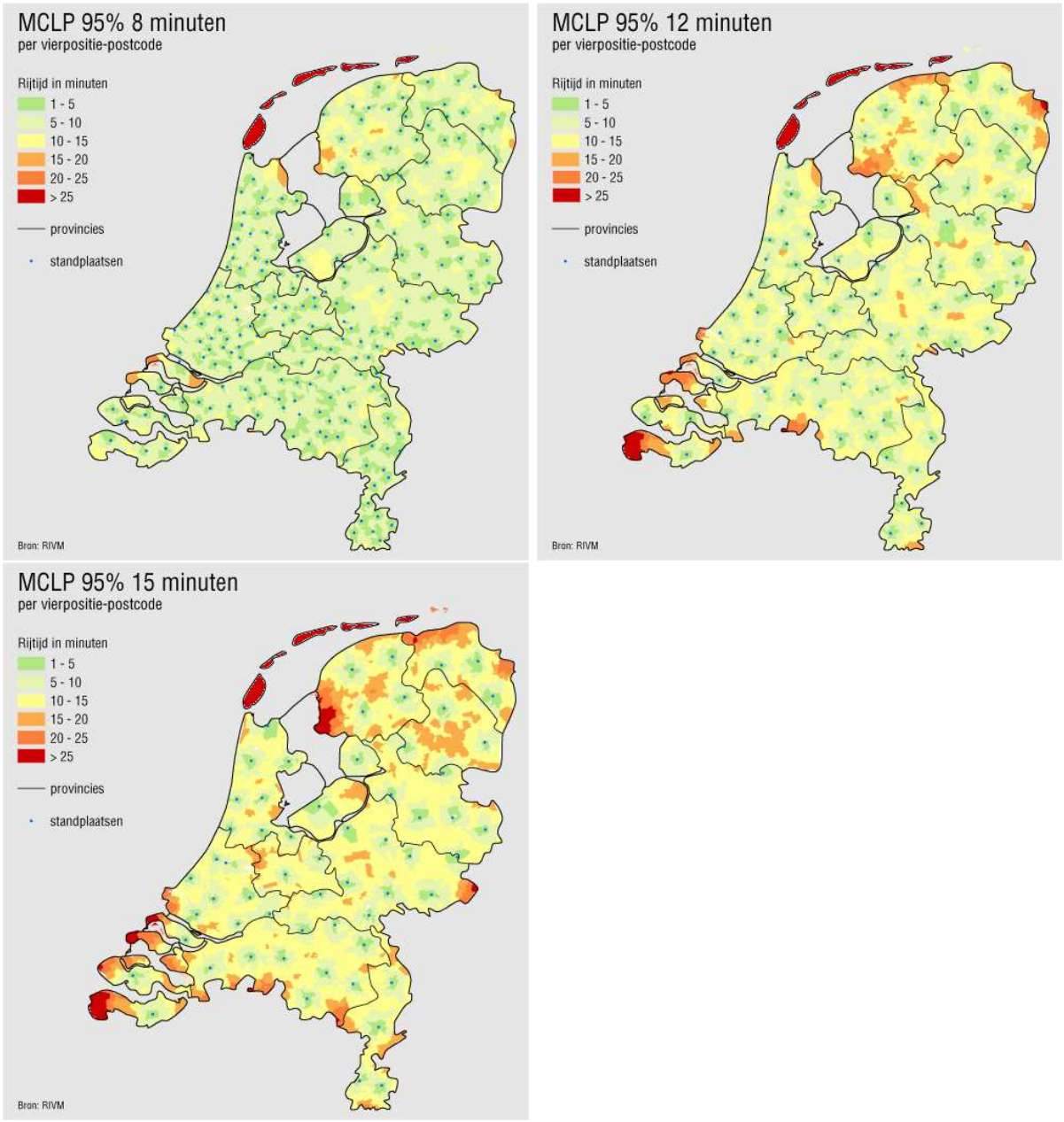


Figure 5.11: Results of the MCLP model for 8, 12 and 15 minutes maximal travel time, achieving approximately 95% coverage.

5.3 Adapted models: Two types of postal codes

As a continuation of the idea of enhancing the norm on the maximal travel time, a new case is considered, namely enhancing the norm within the urban areas, which are more densely populated, while preserving the current norm in rural areas. The idea behind this is that in the urban areas there are more incidents, due to the higher number of residents, and if the travel time is shorter, the ambulance will be available again sooner for a new call. Two scenarios are considered, one where the norm in the urban areas is enhanced to 8 minutes maximal travel time, while preserving the norm 12 minutes maximal travel time in rural areas. In the second case, again the norm in the urban areas is enhanced to 8 minutes maximal travel time, but at the same time the norm in rural areas is changed to 15 minutes maximal travel time.

As before, two versions of the model are considered. First an LSCM version in Section 5.3.1, giving the number of locations sites needed to cover all demand points, and second an MCLP version in Section 5.3.2, giving the percentage of the demand that can be covered by a given number of location sites.

5.3.1 Adapted LSCM

As described before, two scenarios are considered, with different maximal travel times. The results are summarized in Tables 5.5 and 5.6.

In the first scenario, where the maximal travel time in urban areas is 8 minutes and the maximal travel time in rural areas is 12 minutes, there are 217 location sites needed to cover all demand points. This result can be compared to the results of the LSCM model for 8 and 12 minutes maximal travel time, where 382 and 165 location sites are needed respectively. It can be seen that the result of 217 location sites is somewhere in the middle, a little closer to the result for 12 minutes maximal travel time, as expected, since there are more demand points in rural areas (with 12 minutes maximal travel time) than in urban areas (with 8 minutes maximal travel time), namely 2629 and 1390 respectively, see also Section 4.3.1.

In the second scenario, where the maximal travel time in urban areas is 8 minutes and the maximal travel time in rural areas is 15 minutes, there are 173 location sites needed to cover all demand points. This result can be compared to the results of the LSCM model for 8 and 15 minutes maximal travel time, where 382 and 100 locations sites are needed respectively. We see that the result of 173 is now not in the middle, but really closer to the result of the LSCM model for 15 minutes. Again this can be explained by the fact there are more demand points in the rural areas.

A map of the results for these two scenarios can be found in Figure 5.12

Maximal urban travel time	Maximal rural travel time	Number of location sites needed
8 minutes	12 minutes	217

Table 5.5: Results of the adapted LSCM distinguishing between urban and rural areas for 8 and 12 minutes maximal travel time.

Maximal urban travel time	Maximal rural travel time	Number of location sites needed
8 minutes	15 minutes	173

Table 5.6: Results of the adapted LSCM distinguishing between urban and rural areas for 8 and 15 minutes maximal travel time.

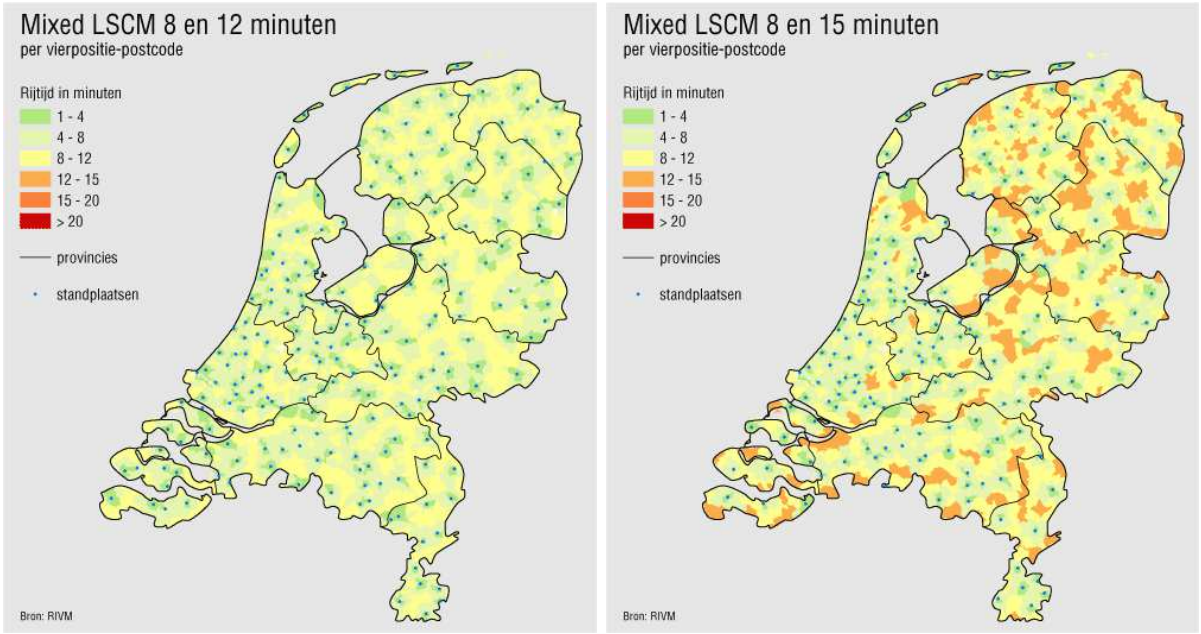


Figure 5.12: Results of the adapted LSCM model, for 8 and 12 minutes (left) and 8 and 15 minutes (right) maximal travel time.

5.3.2 Adapted MCLP

Using the results from Section 5.3.1, we again investigate the influence of decreasing the number of location sites is. This is done for two ranges:

- 8 and 12 minutes: $120 - 135 - \ldots - 210$
- 8 and 15 minutes: $95 - 110 - \ldots - 170$

In Figure 5.13 and 5.14 it can be seen that the influence of decreasing the number of location sites is similar to what was seen in Section 5.2: in the first steps the influence is rather small, but further on the coverage percentage drops faster.

Number of location sites	Covered demand
120	92.09%
135	94.84%
150	96.91%
165	98.42%
180	99.37%
195	99.84%
210	99.98%

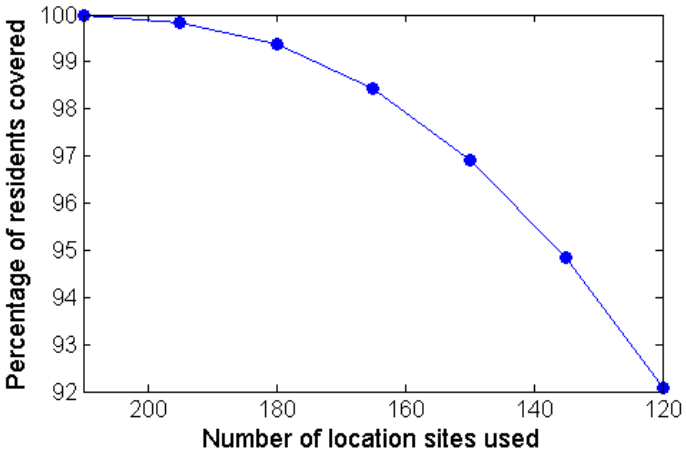


Figure 5.13: Results of the adapted MCLP distinguishing between urban and rural areas for 8 and 12 minutes maximal travel time.

Number of location sites	Covered demand
95	90.40%
110	94.33%
125	97.03%
140	98.74%
155	99.75%
170	99.996%

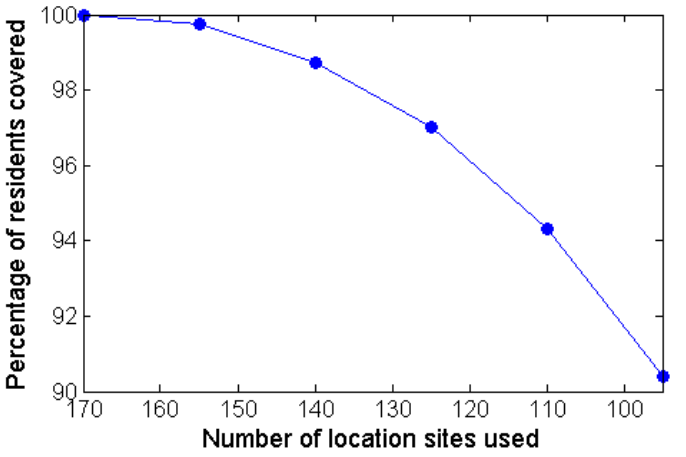


Figure 5.14: Results of the adapted MCLP distinguishing between urban and rural areas for 8 and 15 minutes maximal travel time.

5.4 From location site to incident to hospital

As described in Section 3.7.1, every citizen should be able to reach an emergency department by ambulance within 45 minutes after an emergency call. Here we consider this condition for two sets of hospitals, namely all hospitals that have an emergency department (ED) and the Trauma Centers (TC). These are the most interesting, since usually when a patient has to be transported to the hospital by ambulance, the patient has to be treated in an ED. And in some cases specialized help is necessary, in which case the patient should be treated in a Trauma Center. There are 99 hospitals with an ED and 11 Trauma Centers in the Netherlands. In this model, described in Section 4.4, we determine how many residents we can transport to such a hospital within the 45 minutes and at the same time can be reached within 12 minutes travel time by an ambulance. Since some demand points are not within 45 minutes of a hospital with an ED at all, we included as an extra requirement that in each RAV, 97% of the residents has to be within 12 minutes of an ambulance location. This is to prevent the solution from excluding all the areas which cannot be covered, i.e. which are not within 45 minutes of any hospital with an ED. Another consequence of having such areas that are not within 45 minutes of any hospital, is that we cannot determine the minimal number of location sites needed to be able to transport all residents to a hospital within 45 minutes. So the choice for the available number of locations has to be based on other information. Considering the results of the LSCM for 12 minutes travel time, see Tables 5.1 and 5.8, we decided to use 165 as the number of location sites that can be used.

5.4.1 Hospital with ED

First we consider all hospitals with an ED, which are 99 hospitals. We find that using 165 location sites, 99,85% of the residents can reach an ED by ambulance within 45 minutes (and at the same time can be reached within 12 minutes by an ambulance).

5.4.2 Trauma Centers

Next we consider only the Trauma Centers, which are 11 hospitals. We find that using 165 location sites, 83,78% of the residents can reach a Trauma Center by ambulance within 45 minutes (and at the same time can be reached within 12 minutes by an ambulance).

5.4.3 Comparison of results

We can compare the solutions of this MCLP model including the travel times to the hospitals to the solution that resulted from the original LSCM with 12 minutes maximal travel time. Note the 12 minutes maximal travel time to the demand point is a constraint for coverage here as well. Considering the solution from the LSCM, we find that with this solution 99,78% of the residents can reach an ED within 45 minutes and 77,16% of the residents can reach a Trauma Center within 45 minutes. See also Table 5.7, which gives an overview of the performance of three different solutions:

- Solution 1: from the LSCM model with 12 minutes maximal travel time.
- Solution 2: from the MCLP model described in this chapter, considering all hospitals with an ED.
- Solution 3: from the MCLP model described in this chapter, considering the Trauma Centers.

	Percentage of residents covered by ambulance	Percentage of residents covered by ED	Percentage of residents covered by TC
Solution 1	100%	99,78%	77,16%
Solution 2	99,85%	99,85%	77,40%
Solution 3	98,82%	98,69%	83,78%

Table 5.7: Results of three different solutions for reachability of hospitals.

So we see that all three solutions have a similar performance if we consider all hospitals with an ED, but the third solution gives a significantly better result if we consider only the Trauma Centers, but the consequence of this improvement is that less residents are covered by an ambulance within 12 minutes. Also we see that the second solution has the same result for coverage by an ambulance as well as for the coverage by an ED. This implies that if a demand point is covered by an ambulance, it is also covered by a hospital with an ED. An inspection of the results verifies this.

In Figure 5.15 we see the results for the two scenarios, indicating the total travel time from the closest ambulance station to the incident and then to the closest hospital. We see that if we consider all hospitals with an ED, only the Wadden Islands and a very small part of South Holland are not within 45 minutes of a hospital.

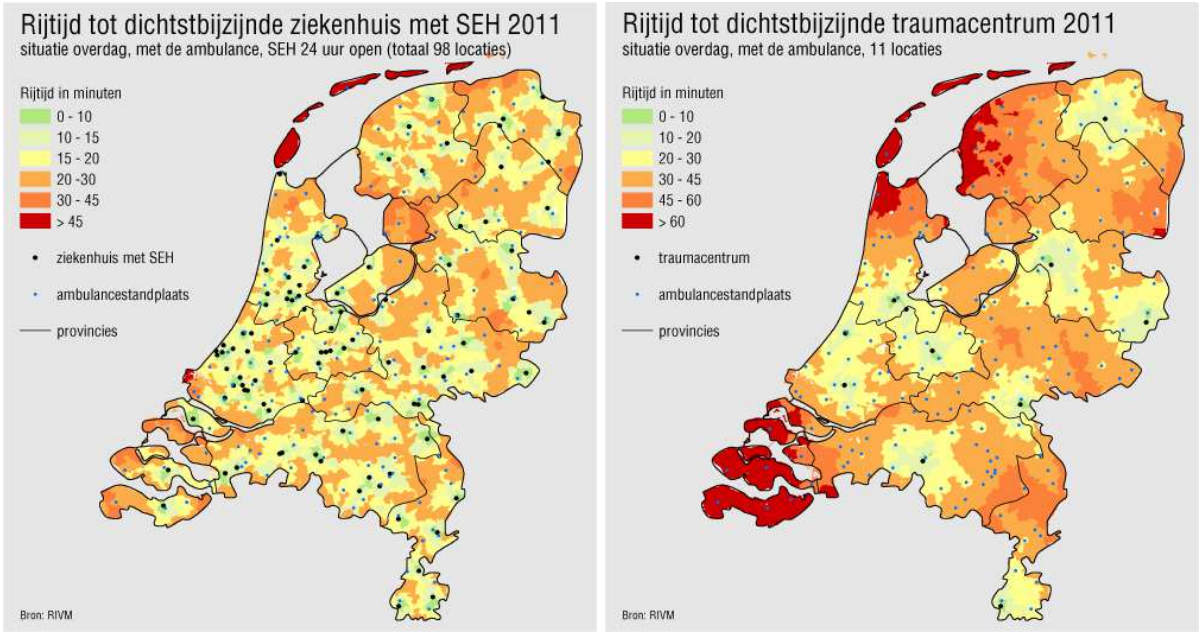


Figure 5.15: Results of the MCLP model including hospitals with an ED (left) and Trauma Centers (right)

5.5 Multiple vehicles

In this last model we consider the use of two different types of vehicles, a regular ambulance and a rapid responder. In this model, there are two ways to cover a demand point: just by an ambulance, or by a rapid responder and an ambulance.

We also consider three versions of the model, the first two are similar to the LSCM, since they can be used to determine where to locate ambulances and rapid responders to cover all demand points, by either an ambulance or both a rapid responder and an ambulance, minimizing the costs. In the first version this distribution depends on the ratio between the costs of an ambulance location and the costs of a rapid responder location. If a rapid responder location is much cheaper than an ambulance location, it is more advantageous to have a lot of rapid responders and less ambulances than when the costs are not that different.

We considered two values for a_2, the maximal travel time for the ambulance after a rapid responder arrived to the scene, namely 20 and 30 minutes. Furthermore the maximal travel time for a rapid responder is $r = 8$ minutes and the maximal travel time for coverage by only an ambulance is again $a_1 = 12$ minutes.

The second version determines the minimal number of ambulance locations needed to cover all demand with a given number of rapid responder locations.

The third version is similar to the MCLP, it determines the maximal coverage we can achieve with a specified number of ambulance locations and rapid responder locations.

Unfortunately, it turned out that these models are not easily solved to optimality within a reasonable time. Since our input values (parameter α and the number of available rapid responders and ambulances) are also just estimations, we decided to settle with near-optimal solutions.

As described in Section 2.3.2, although we do not solve these problems to optimality, we can still give an upper and a lower bound on the objective value of the optimal solution.

5.5.1 LSCM version 1

We consider two scenarios in this first LSCM version, one where the costs of an ambulance location are twice the costs of a rapid responder location and one where the costs of an ambulance location are three times the costs of a rapid responder location. These cases correspond to $\alpha = \frac{2}{3}$ and $\alpha = \frac{3}{4}$ respectively in the objective function described in Section 4.5. The results of the four runs are summarized in Table 5.8.

Parameters	Number of rapid responder locations	Number of ambulance locations	Value objective function	LP bound	Gap
$\alpha = \frac{2}{3}$ $a_2 = 20$ minutes	54	126	102	97 (96,8)	0,05%
$\alpha = \frac{3}{4}$ $a_2 = 20$ minutes	154	84	101,5	95,5 (95,4)	0,06%
$\alpha = \frac{2}{3}$ $a_2 = 30$ minutes	69	117	101	97 (96,9)	0,04%
$\alpha = \frac{3}{4}$ $a_2 = 30$ minutes	232	50	95,5	92,25 (92,2)	0,03%

Table 5.8: Results of the first LSCM version for two vehicle types.

We must note that the gap is always a 'worst case scenario'; we know that in the worst case the optimal solution is close to the current LP bound. On the other hand, it can also happen that the current best solution is actually already the optimal solution. Actually the lower bound can even be somewhat higher than the best LP bound, because of the binary constraints on the variables in the objective function. This excludes most of the fractions in the lower bound. In fact, if all objective function coefficients are integer we can round up (down) the lower (upper) LP-bound in case of minimization (maximization). In Table 5.8, we give the lower bound on the value of the objective function, the actual LP bound is given within the brackets.

We see that the two different values of α give two different results: if $\alpha = \frac{2}{3}$ there are more ambulance locations than rapid responder locations, but if $\alpha = \frac{3}{4}$ there are a lot of rapid responder locations. This effect is bigger if the travel time a_2 is higher. Furthermore, note that the optimal solution from our very first model needed 165 ambulance locations, which corresponds to objective function values of 110 and 123,75 for $\alpha = \frac{2}{3}$ and $\alpha = \frac{3}{4}$ respectively. So in all cases it is advantageous to have rapid responders and the more so when α comes closer to one (which means that the rapid responder location becomes increasingly cheaper than the ambulance location). Furthermore the maximal travel time a_2 also has a significant influence on the number of rapid responder locations, if we increase a_2 it is more advantageous to use rapid responder locations.

5.5.2 LSCM version 2

Again we consider the two scenarios with a maximal travel time of 20 or 30 minutes. Furthermore we fix the number of rapid responder locations. At this moment there are about 50 rapid responder vehicles available in the Netherlands. So we fix the number of rapid responder locations at 50, 40 and 30 and determine the number of ambulance locations needed to still cover all demand points. The results for a maximal travel time of 20 minutes can be found in Table 5.9 and for a maximal travel time of 30 minutes in Table 5.10.
Here we also could not get to optimal solutions, so we stopped the optimization when the gap between the LP bound and the best solution was smaller than 5%.

Number of rapid responder locations	Number of ambulance locations	LP bound
50	127	121 (120,9)
40	132	127 (126,3)
30	138	133 (132,3)

Table 5.9: Results of the second LSCM version for two vehicle types $a_2 = 20$ minutes.

Number of rapid responder locations	Number of ambulance locations	LP bound
50	127	121 (120,7)
40	132	126 (125,9)
30	138	132 (132,0)

Table 5.10: Results of the second LSCM version for two vehicle types, $a_2 = 30$ minutes.

Now we see interesting results. It appears that the maximal travel time for the ambulance after a rapid responder does not affect the number of ambulance locations. This might be explained by the small number of rapid responders available and the relatively small portion of the demand points they can reach. Most of the ambulances are needed to cover demand points without a rapid responder, so the value of a_2 does not have an influence on this. Another thing to note here, is that the solving of the model with $a_2 = 20$ minutes seemed to be more efficient than with $a_2 = 30$ minutes.

It is also interesting to compare these results to the results of the LSCM model, which showed that we need 165 ambulance location sites to cover all demand points. Now here we see we need 177, 172 and 168 location sites (either a rapid responder or an ambulance location sites) in total. These number are close to the 165 location sites from the LSCM.

5.5.3 MCLP version

Because of the small difference in the results between $a_2 = 20$ minutes and $a_2 = 30$ minutes, we decided to only consider $a_2 = 20$ minutes for this version. Using the results from the second LSCM version we now decreased the number of available ambulance locations to see how much we can still cover. Here we stopped the optimization once the gap was smaller than 0.1% (which is still about $0,001 \cdot 16.558.295 = 16.558$ residents), because of time limitations. The computing time varies somewhere between 4 and 12 hours for these problems.

Number of rapid responder locations	Number of ambulance locations	Covered demand	LP bound
50	120	99,94%	100%
50	100	99,52%	99,63%
50	80	97,69%	97,80%
50	60	93,83%	93,90%
40	125	99,97%	100%
40	105	99,52%	99,62%
40	85	97,74%	97,83%
40	65	93,95%	93,95%
30	130	99,97%	100%
30	110	99,53%	99,60%
30	90	97,73%	97,81%
30	70	93,91%	94,00%

Table 5.11: Results of MCLP version for two vehicle types, $a_2 = 20$ minutes.

We see that the behavior of the results is similar to that of the results we found for the MCLP model in Section 5.2. The coverage percentage decreases slowly at first, and faster later. Again the explanation might be that the first demand points that are left uncovered are demand points with few residents. However, at some point there will be demand points with more residents that are left uncovered which causes the faster decrease.

If we compare these results to the second LSCM version, we can see that for example the LP bound for the first case (50 rapid responder location and 120 ambulance locations) is too high here, since we found that we need at least 121 to cover all demand, so it cannot be that we can cover 100% with 120 locations. The same holds for two other cases (with 40 rapid responder

locations and 125 ambulance locations and 30 rapid responder locations and 130 ambulance locations), where we found that we need at least 126 and 132 ambulance locations respectively. This illustrates the note made earlier, that the gap between the LP bound and the current best solution is always a worst case gap, the real gap between the current best solution and the optimal solution can be (much) smaller.

Chapter 6

Conclusion and recommendations

In this final chapter we will answer the research questions posed in Chapter 1 and suggest ideas for further research.

6.1 Conclusions

Throughout this section we will answer the questions posed in Chapter 1 by summarizing the information presented in the thesis.

The Netherlands is divided into 24 RAVs, which all consist of a control room and one or more ambulance services. The different RAVs are represented in AZN, the representative organization for ambulance care. For more information on AZN, refer to Section 1.2. Every RAV is legally responsible for the ambulance care in its region. The division of the RAVs is given in Section 3.1. In 2010, there were 223 stations for ambulances, 196 of them are occupied 24/7 and the others are for example only occupied during day time, or only on weekdays. A visualization of the location of the current ambulance stations can be found in Figure 3.2, Section 3.6.

The most important constraint on the location of ambulance stations is that the target of the Dutch ambulance sector is to reach the scene within 15 minutes after an A1 call in 95% of the cases. Since the time after the call is answered is divided into three steps, namely handling the call, preparing the ambulance and driving to the scene, we assume this means that we have 12 minutes maximal driving time for an ambulance from its location site to the scene. Another aim of the Dutch ambulance sector is that every resident should be able to reach a hospital within 45 minutes after a call. Since this time after a call is now divided into 5 steps, we assume that the maximal driving time is 37 minutes. Of this 37 minutes, again 12 minutes should be the maximal driving time for the ambulance to the resident.

We started by considering only the first constraint, so the 12 minutes maximal driving time to the resident. The model introduced for this purpose is the LSCM (Location Set Covering Model). The result of this model is the minimal number of locations needed to cover all demand points. This was determined for three different maximal travel times, namely 8, 12 and 15 minutes. We found that the minimal number of locations needed is 382, 165 and 102 respectively. Recall that the current number of location sites is 223, which is significantly more than the 165 that is minimal for 12 minutes maximal travel time. However, it is not fair to compare these numbers one on one, because of the organization with the RAVs. To make a better comparison, we determined the minimal number of locations needed for each RAV to cover all of its own demand points. We found that in total we need at least 203 locations, which is quite close to the current 223. The small difference between those two situations might be explained by the fact that in the current situation, there is some overlap in the areas covered by different

stations, since this means the chance that an ambulance is available increases. However, the large difference between 165 and 223 location sites seems to imply that the performance of the ambulance sector could be improved by more interaction between different RAVs and a better alignment of their location sites.

It is not only interesting to see what is needed to have 100% performance, but also what is possible with less. To this end, the MCLP (Maximal Covering Location Problem) was introduced. The result of this model is the maximal coverage we can achieve with a given number of location sites. From the LSCM we know with how many locations we can perform perfectly, so we lowered this number to see the effect. We see that a little less locations can still give a fairly good result, only after decreasing the number more we see a real effect on the maximal possible coverage. For the three different maximal travel times (8, 12 and 15 minutes), we see that we can still have 95% coverage with 225, 95 and 60 locations respectively. This is significantly less than the number needed to have perfect performance.

Next we introduced a model that distinguishes between urban and rural areas, where we lower the maximal travel time within the urban area. If we take a maximal travel time of 8 minutes within the urban area, and maintain the maximal travel time of 12 minutes within the rural area, we find that we need 217 location sites. Additionally, if we replace the maximal travel time of 12 minutes by a maximal travel time of 15 minutes within the rural area, we find that we need 173 location sites. Investigating what we could achieve with a lower number of location sites, we see the same effect as with the first MCLP model, namely that slightly fewer locations still gives a high percentage of coverage.

Next we considered the second constraint introduced, reaching the hospital within 45 minutes for residents. Therefore we developed a new version of the MCLP, including the travel time to the nearest hospital in the model. Also, since there are areas that are not within 45 minutes of any hospital, there are extra constraints added to prevent these areas from being excluded. These constraints make sure that in each RAV, at least 97% of the residents can be reached within 12 minutes by an ambulance, regardless of the travel time to the hospital after that. We produced the optimal solutions for this scenario, using 165 location sites, and considering two sets of hospitals: hospitals with an ER (99) and Trauma Centers (11). We see that in the best case, with 165 location sites, we can get 99,85% of the residents to an ER within 37 minutes (and at the same time reach the resident with an ambulance within 12 minutes) and we can get 83,78% of the residents to a Trauma Center within 37 minutes (and again, at the same time reach the resident with an ambulance within 12 minutes). If we compare this result to the performance of the solution produced by the LSCM, which gives 99,78% and 77,16% respectively, we see that the result for all hospitals with an ER is not much better, but for the Trauma Centers it is.

Finally we considered the use of two different types of vehicles and two ways to cover a demand point with these vehicles. We considered a rapid responder next to the regular ambulance. A rapid responder cannot transport a patient to the hospital, it can only provide care, so it can be a cheaper vehicle than the regular ambulance. We assumed that a demand point can either be covered in the regular way, by an ambulance within 12 minutes, or by the combination of a rapid responder and an ambulance, where the rapid responder has to be there within 5 minutes and the ambulance within 20 or 30 minutes. We introduced three basic models to see the possibilities of the use of these two vehicles. The results indicate that the use of rapid responders could be a good idea, it could decrease the costs. However, since there are only 50 rapid responders available in the Netherlands and not every RAV has rapid responders, the results only give an indication.

6.2 Recommendations for further research

Finally we will give some recommendations for further research on this subject. In general, the recommendation is to try to make the models more realistic.

For example, the solutions presented here assume a greenfield scenario, so the current location sites are not included. Of course, it is not realistic to break down all existing location sites and set up new ones. A better idea would be to include the current location sites in the optimization, trying to get the best result with the least changes. For the LSCM and MCLP we considered one scenario where we fixed the location sites at academic hospitals, which showed that this already increases the number of location sites needed.

Furthermore, we used the four-digit postal codes to represent the Netherlands by a graph. However, the areas corresponding to one four-digit postal code can still be quite a large area. But if we take smaller areas, the sets of demand points and potential location sites will become very large. For example, if we consider six-digit postal codes, we have more than 400,000 areas instead of the 4019 areas we considered here. This means the number of variables and constraints in the models will also increase a lot, which might cause problems with the resolution time.

Next, the solutions presented here give only locations, but there is nothing included on capacity and (un)availability of vehicles. This implies the need of stochastic models or even simulation models, including the rate of incidents and the time an ambulance will be unavailable.

Another thing to consider is that the travel time model considers the travel times as being deterministic, but this is not the reality. The travel times are averages of real data, so there is no guarantee that the actual travel time is not a lot longer (or maybe shorter) on occasion.

In Chapter 3, we already mentioned that it might not be the best choice to take the number of residents as the demand for every demand point. Another choice could be to consider the number of incidents per hour.

Furthermore, we only made a start with the investigation of two different vehicle types. At first the costs are not very realistic, which makes the solutions not really reliable and also the maximal travel times are kind of arbitrary. More input from the actual field could make this more realistic. Next, there are different types of accidents, where some need the patient to only be treated on site and other need actual transportation to the hospital. These accidents can be treated by different vehicle types, which can be included in a model.

Finally, one should realize that more sophisticated models require more efficient solution methods. We see that most of our models could directly be solved by the standard solution methods of CPLEX. However, in the end, with the model with multiple vehicle types we could not get actual optimal solutions within a reasonable time anymore. One can imagine that including more and more information to make the models more realistic, also makes them more complicated. To this end other solution methods should be investigated. Of course, what is reasonable time depends on the context. If we solve a model multiple times to compare different scenarios, we prefer quick solutions, which can be computed within a couple of hours. On the other, if we already have done a lot of research, and want to solve a model only once to get an exact answer, it can be acceptable to wait several days for the final optimal solution.

Bibliography

[1] AZN Ambulancezorg Nederland. Ambulance in-zicht 2010, 2011.

[2] AZN. Homepage ambulancezorg nederland. `http://www.ambulancezorg.nl/engels`.

[3] AZN. Summary of ambulances in sight 2011. `http://www.ambulancezorg.nl/download/downloads/1115/summary-ambulances-in-sight-2011.pdf`.

[4] Luce Brotcorne, Gilbert Laporte, and Frédéric Semet. Ambulance location and relocation models. *European Journal of Operational Research*, 147:451–463, 2003.

[5] Richard Church and Charles ReVelle. The maximal covering location problem. *Papers in Regional Science*, 32(1):101–118, 1974.

[6] CPLEX. Homepage CPLEX. `http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/`.

[7] Mark S. Daskin and Edmund H. Stern. A Hierarchical Objective Set Covering Model for Emergency Medical Service vehicle deployment. *Transportation Science*, 15(2):137–152, May 1981.

[8] Michel Gendreau, Gilbert Laporte, and Frédéric Semet. Solving an ambulance location model by Tabu search. *Location Science*, 5(2):75–88, 1997.

[9] Jeffrey B. Goldberg. Operations Research Models for the Deployment of Emergency Services Vehicles. *EMS Mangement Journal*, 1(1):20–39, January-March 2004.

[10] F. S. Hillier and G. J. Lieberman. *Introduction to operations research*. McGraw-Hill, 2005.

[11] Kathleen Hogan and Charles ReVelle. Concepts and Applications of Backup Coverage. *Management Science*, 32(11):1434–1444, November 1986.

[12] G. J. Kommer and S. L. N. Zwakhals. Modellen referentiekader ambulancezorg 2008. RIVM, 2009.

[13] Xueping Li, Zhaoxia Zhao, Xiaoyan Zhu, and Tami Wyatt. Covering models and optimization techniques for emergency response facility location and planning: a review. *Mathematical Methods of Operations Research*, 74(3):281–310, 2011.

[14] R. Malschaert, T.H. van de Belt, and P. Giesen. Wat is de relatie tussen responstijd en gezondheidswinst. Institute for Quality of Healthcare and Acute Zorgregio Oost.

[15] Paragon Decision Technology. Homepage AIMMS. `http://www.aimms.com`.

[16] RIVM. Homepage RIVM. `http://www.rivm.nl/English`.

[17] RIVM. Nationale Atlas Volksgezondheid. `http://www.zorgatlas.nl`.

[18] David Schilling, D. Jack Elzinga, Jared Cohon, Richard Church, and Charles ReVelle. The TEAM/FLEET models for simultaneous facility and equipment siting. *Transportation Science*, 13(2):163–175, May 1979.

[19] Constantine Toregas, Ralph Swain, Charles ReVelle, and Lawrence Bergman. The location of emergency service facilities. *Operations Research*, 19(6):1363–1373, October 1971.

Appendix A

Notation

General Symbols

I		set of demand nodes
J		set of (potential) location sites
t_{ij}		travel time from node j to node i
x_j	$=$	$\begin{cases} 1 & \text{if an ambulance is located at site } j \\ 0 & \text{otherwise} \end{cases}$
y_i	$=$	$\begin{cases} 1 & \text{if demand node } i \text{ is covered} \\ 0 & \text{otherwise} \end{cases}$
r		maximal time an ambulance may take to get to the scene
J_i	$=$	$\{j \in J \mid t_{ji} \leq r\}$, the sites j that cover demand node i
d_i		population to be served at demand node i
p		the number of ambulances available

Specific to a model

TEAM

x_j^P	$=$	$\begin{cases} 1 & \text{if a primary vehicle is located at site } j \\ 0 & \text{otherwise} \end{cases}$
x_j^S	$=$	$\begin{cases} 1 & \text{if a special vehicle is located at site } j \\ 0 & \text{otherwise} \end{cases}$
z_i	$=$	$\begin{cases} 1 & \text{if demand node } i \text{ is covered by both a primary as well as a special vehicle} \\ 0 & \text{otherwise} \end{cases}$
r^P		maximal time a primary vehicle may take to get to the scene
r^S		maximal time a special vehicle may take to get to the scene
J_i^P	$=$	$\{j \in J \mid t_{ji} \leq r^P\}$, the sites j that cover demand node i with a primary vehicle
J_i^S	$=$	$\{j \in J \mid t_{ji} \leq r^S\}$, the sites j that cover demand node i with a special vehicle
p^P		number of primary vehicles available
p^S		number of special vehicles available

MOTEAM (also see TEAM)

d_i^P	$=$	population to be served by primary vehicle at demand node i
d_i^S	$=$	population to be served by special vehicle at demand node i
z_i^P	$=$	$\begin{cases} 1 & \text{if demand node } i \text{ is covered by primary equipment} \\ 0 & \text{otherwise} \end{cases}$
z_i^S	$=$	$\begin{cases} 1 & \text{if demand node } i \text{ is covered by special equipment} \\ 0 & \text{otherwise} \end{cases}$

FLEET (also see TEAM)

$w_j = \begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{otherwise} \end{cases}$

J_N the set of potential new facilities

p^w the number of facilities to be built

HOSC

W some positive weight

S_i the number of additional EMS units capable of responding to a call in zone i in a time less than or equal to r

BACOP 1,2

$u_i = \begin{cases} 1 & \text{if demand node } i \text{ is covered twice} \\ 0 & \text{otherwise} \end{cases}$

DSM

r_2 maximal time an ambulance may take to get on the scene

r_1 maximal time α percent of the cases

$y_i^k = \begin{cases} 1 & \text{if a demand node } i \text{ is covered at least } k \text{ times within radius } r_1 \\ 0 & \text{otherwise} \end{cases}$

v_j number of ambulances that is located at site j

p_j maximum number of ambulance that can be located at site j

Appendix B

Models from literature

An overview of the mathematical formulations of the models described in Chapter 2. For the used notation, please refer to Appendix A.

LSCM

Minimize	$\sum_{j \in J} x_j$			
Subject to	$\sum_{j \in J_i} x_j$	$\geq$	1	$i \in I$
	x_j	$\in$	$\{0,1\}$	$j \in J$

MCLP

Maximize	$\sum_{i \in I} d_i y_i$			
Subject to	$\sum_{j \in J_i} x_j$	$\geq$	y_i	$i \in I$
	$\sum_{j \in J} x_j$	$=$	p	
	x_j	$\in$	$\{0,1\}$	$j \in J$
	y_i	$\in$	$\{0,1\}$	$i \in I$

TEAM

$$
\begin{array}{lrcll}
\text{Maximize} & \displaystyle\sum_{i\in I} d_i z_i & & & \\
\text{Subject to} & \displaystyle\sum_{j\in J_i^P} x_j^P & \geq & z_i & i \in I \\
& \displaystyle\sum_{j\in J_i^S} x_j^S & \geq & z_i & i \in I \\
& \displaystyle\sum_{j\in J} x_j^P & = & p^P & \\
& \displaystyle\sum_{j\in J} x_j^S & = & p^S & \\
& x_j^S & \leq & x_j^P & j \in J \\
& x_j^P, x_j^S & \in & \{0,1\} & j \in J \\
& z_i & \in & \{0,1\} & i \in I
\end{array}
$$

MOTEAM

$$
\begin{array}{lrcll}
\text{Maximize} & \displaystyle\sum_{i\in I} d_i^P z_i^P + \lambda \sum_{i\in I} d_i^S z_i^S & & & \\
\text{Subject to} & \displaystyle\sum_{j\in J_i^P} x_j^P & \geq & z_i^P & i \in I \\
& \displaystyle\sum_{j\in J_i^S} x_j^S & \geq & z_i^S & i \in I \\
& \displaystyle\sum_{j\in J} x_j^P & = & p^P & \\
& \displaystyle\sum_{j\in J} x_j^S & = & p^S & \\
& x_j^S & \leq & x_j^P & j \in J \\
& x_j^P, x_j^S & \in & \{0,1\} & j \in J \\
& z_i^S & \in & \{0,1\} & i \in I \\
& z_i^P & \in & \{0,1\} & i \in I
\end{array}
$$

FLEET

$$
\begin{array}{lrcll}
\text{Maximize} & \sum_{i\in I} d_i z_i & & & \\
\text{Subject to} & \sum_{j\in J_i^P} x_j^P & \geq & z_i & i \in I \\
& \sum_{j\in J_i^S} x_j^S & \geq & z_i & i \in I \\
& \sum_{j\in J} x_j^P & = & p^P & \\
& \sum_{j\in J} x_j^S & = & p^S & \\
& \sum_{j\in J_N} w_j & = & p^w & \\
& x_j^P & \leq & z_j & j \in J_N \\
& x_j^S & \leq & z_j & j \in J_N \\
& x_j^P, x_j^S & \in & \{0,1\} & j \in J \\
& z_i & \in & \{0,1\} & i \in I \\
& w_j & \in & \{0,1\} & j \in J_N
\end{array}
$$

HOSC

$$
\begin{array}{lrcll}
\text{Minimize} & W\sum_{j\in J} x_j - \sum_{i\in I} S_i & & & \\
\text{Subject to} & \sum_{j\in J_i} x_j - S_i & \geq & 1 & i \in I \\
& x_j & \in & \{0,1\} & j \in J \\
& S_i & \geq & 0 & i \in I
\end{array}
$$

BACOP1

$$
\begin{array}{lrcll}
\text{Maximize} & \sum_{i\in I} d_i u_i & & & \\
\text{Subject to} & \sum_{j\in J_i} x_j - u_i & \geq & 1 & i \in I \\
& u_i & \in & [0,1] & i \in I \\
& x_j & \geq & 0 & i \in I
\end{array}
$$

BACOP2

$$
\begin{array}{lrcll}
\text{Maximize} & \theta\sum_{i\in I} d_i y_i + (1-\theta)\sum_{i\in I} d_i u_i & & & \\
\text{Subject to} & \sum_{j\in J_i} x_j - y_i - u_i & \geq & 0 & i \in I \\
& u_i - y_i & \leq & 0 & i \in I \\
& \sum_{j\in J} x_j & = & p & \\
& u_i & \in & [0,1] & i \in I \\
& y_i & \in & [0,1] & i \in I \\
& x_j & \geq & [0,1] & i \in I
\end{array}
$$

DSM

$$
\begin{array}{lrcll}
\text{Maximize} & \sum_{i\in I} d_i y_i^2 & & & \\
\text{Subject to} & \sum_{j\in N_i} v_j & \geq & 1 & i \in I \\
& \sum_{i\in I} d_i y_i^1 & \geq & \alpha \sum_{i\in I} d_i & \\
& \sum_{j\in N_i^2} v_j & \geq & y_i^1 + y_i^2 & i \in I \\
& y_i^2 & \leq & y_i^1 & i \in I \\
& v_j & \leq & p_j & j \in J \\
& y_i^1, y_i^2 & \in & \{0,1\} & i \in I \\
& v_j & \in & \mathbb{N} & j \in J
\end{array}
$$