

Collision risks for end-of-life satellite de-orbit trajectories

W.J. Huisman

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Cover image: Artist impression of space debris in Earth orbit
(<https://argonautnews.com/space-pollution-is-a-real-world-problem/>).

Collision risks for end-of-life satellite de-orbit trajectories

by

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Aan mijn ouders, Paul en Janneke, die oneindig veel geduld hebben.
Dank jullie wel!

Preface

Most people look at rocket launches and space based operations with awe and wonder. Space engineering has pushed the boundaries of what technology can achieve. In today's world it is hard to image a time when we did not rely on space based equipment in our everyday lives. We can exchange information at the speed of light with practically everyone on the planet by a push of a button. Modern day space technology has allowed us to greatly deepen our understanding of Earth's weather systems or the cosmological structures of distant galaxies. This leads to believe that space is a place of unlimited potential ready the taking. History has taught us however that where ever humans go, they leave their mark on the environment. With every satellite launch, space is littered with debris. And the amount of debris is increasing rapidly over the years. To me, space should be regarded just like any other fragile ecosystem here on Earth. It is a careful balance of forces, and even the slightest disruption can have major consequences. The rapidly growing number of debris objects poses a serious threat to future space operations. If we do not take action in cleaning up the debris made by humans, there is a serious possibility that space will become completely uninhabitable for us to venture out between the stars. I have chosen this topic for my thesis research project because I hope it will contribute to a sustainable future of space operations.

*W.J. Huisman
Delft, December 11, 2019*

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Abstract

Satellites that have reached their end-of-life pose a threat to the space environment. An object in orbit that no longer adds value to its user is called space debris. Collisions between objects in Low Earth Orbit (LEO) can have disastrous consequences and have the potential to create thousands of new debris objects, which in their turn can cause collisions. In order to limit the number of potential collisions it is vital to remove space debris from orbit. Removing space debris from orbit can be done via a so-called de-orbit maneuver. Satellites often have an integrated on-board propulsion system, used for stationkeeping. At the end-of-life, these systems can be used to apply a thrust force and guide the satellite into a de-orbit maneuver.

This thesis investigates the effect of a controlled low-thrust de-orbit maneuver on the propellant usage, duration and collision risk of the trajectory. This maneuver is conducted for three different objects in LEO: Zenit-2, Tsyklon-3, and Kosmos-3m. These three objects represent a wide range of different debris scenarios that form a potential danger to the space environment. Two different epochs of the debris environment are investigated: October 23, 2013, and November 7, 2014. Each trajectory is shaped by a thrust magnitude and thrust control algorithm that turns the engine of the object either on or off at different time points. The values for the thrust magnitude and thrust activation times are determined by an optimization algorithm that tries to find trajectories with optimal (minimal) values for the propellant usage, trajectory duration and collision risk. The collision risk associated with a de-orbit trajectory is determined by representing the position error of each object as an error covariance matrix. By mapping the error covariance matrix of each object pair during a close conjunction onto a common plane of reference (the B-plane), the collision probability can be described in terms of a probability density function. Integrating this function over the occupied area of the two objects in the B-plane results in a collision probability associated with that close conjunction. The total collision probability associated with a de-orbit trajectory is then the result of the accumulation of the collision probabilities of all close conjunctions.

The result of this analysis is a set of different de-orbit trajectories with varying values for the propellant usage, duration and collision probability. Trajectories with a high collision risk have collision probabilities ranging from 10^{-1} to 10^{-3} . The low-risk trajectories have collision probabilities ranging from 10^{-6} to 10^{-16} . A general trend is observed where trajectories with a short duration have relatively low collision probabilities. However, by observing the accumulation of the collision probability over time, it can be seen that the value of the total accumulated collision probability is largely determined by a low number of high-risk conjunctions. A sensitivity analysis is performed to assess the robustness of the obtained trajectories. It is found that a small variation in an initial state vector element results in a difference in the associated collision probability ranging from 10^{-3} to 10^{-6} for some cases, and 10^{-2} to 10^{-48} for other cases. This shows that the obtained results are not robust and are highly sensitive to variations in either the initial state vector or the state derivative (i.e. changes in the environment model). Additionally a high thrust is applied in order to further assess the statistical effects of multiple conjunctions on the accumulated collision probability. Again, the associated collision probability was determined by a low number of risk-risk conjunction events. These results lead to the conclusion that no robust method was found for a de-orbit trajectory that limits the collision probability. The accumulated collision probability is not smoothly determined by the number of encountered conjunctions but rather a limited number of high-risk conjunction events. Additionally, the collision probability is highly sensitive to small position offsets of either of two objects in a conjunction. Therefore, a prediction of a de-orbit trajectory that limits the collision probability is unreliable.

Nomenclature

List of notations

The nomenclature used in this research includes acronyms, symbols and subscripts/superscripts. Different notations will be used throughout the report to indicate specific mathematical properties. As an example, Q is taken as an arbitrary symbol.

Symbol	Description
Q	Scalar
$\vec{Q}$	Vector
$\hat{Q}$	Matrix
$\hat{Q}^T$	Transpose
$\hat{\mathbf{Q}}$	Unit vector
$\dot{Q}$	Time derivative
dQ/dt	Time derivative
Δ	Difference

List of acronyms

Acronym	Definition
API	Application programming interface
ASCII	American Standard Code for Information Interchange
CIO	Conventional International Origin
CPU	Central processing unit
DOPRI8(7)	Dormand-Prince integration method
ECI	Earth-centered inertial
ESA	European Space Agency
ESSS	European Space Surveillance System
GMT	Greenwich Mean Time
GPS	Global Positioning System
GRACE	Gravity Recovery and Climate Experiment
HS	Harmony Search
IHS	Improved Harmony Search
ISS	International Space Station
LEO	Low Earth Orbit
NASA	National Aeronautics and Space Administration
NORAD	North American Aerospace Defense Command
PDF	Probability density function
RAAN	Right ascension of the ascending node
RAM	Random-access memory
SGP	Simplified General Perturbations model
SH	Spherical Harmonics
SST	Space Surveillance and Tracking
tca	Time of Closest Approach
TLE	Two-Line Element
Tudat	Technical University Delft Astrodynamics Toolbox
UNOOSA	United Nations Office for Outer Space Affairs
USA	United States of America
USSR	Union of Soviet Socialist Republics
UTC	Universal Time Coordinated

List of symbols

Symbol	Description	Units
α	Azimuth	rad
γ	Flight-path angle	rad
δ	Elevation	rad
δ	Thrust angle	rad
θ	True anomaly	rad
Λ	Geographic longitude	rad
λ	Eigenvalues	-
μ	Earth's gravitational parameter	m^3/s^2
ρ	Debris number density	objects/ m^3
ρ	Atmospheric density	kg/m^3
ρ	Dimensionless radius	-
ρ	Separation distance	m
σ_U	Standard deviation in radial direction	m
σ_V	Standard deviation in along-track direction	m
σ_W	Standard deviation in cross-track direction	m
τ	Dimensionless time	-
ϕ	Geocentric latitude	rad
Ω	Right ascension of the ascending node	rad
ω	Argument of pericenter	rad
A	Collision cross-sectional area	m^2
A	Satellite cross-sectional area	m^2
A_c	Critical area	m^2
a	Semi-major axis	m
a	Dimensionless acceleration	-
C	Error covariance matrix	-
C_D	Drag coefficient	-
C_R	Coefficient of reflectivity	-
C_{sat}, C_{deb}	Error covariance matrix of satellite and debris	-
$C_{U,V,W}$	Error covariance matrix in Hill frame	-
c	Speed of light	m/s
d	Distance or separation distance	m
E	Eccentric anomaly	rad
$\mathcal{E}$	Specific energy	m^2/s^2
e	Eccentricity	-
F	Force	N
f	Acceleration	m/s^2
f_0	Initial acceleration	m/s^2
f_d	Acceleration exerted by perturbing body d	m/s^2
f_E	Acceleration exerted by Earth	m/s^2
G	Gravitational constant	$\text{m}/\text{kg s}^2$
g_0	Gravitational acceleration at sea level	m/s^2
g_{max}	Maximum gravitational acceleration	m/s^2
H	Specific angular momentum	m^2/s
$\hat{I}$	Identity matrix	-
i	Inclination	rad
I_{sp}	Specific impulse	s
$J_{n,m}$	Gravitational model parameters (degree n , order m)	-
M	Mean anomaly	rad
M_0	Initial mean anomaly	rad
M_0	Initial satellite mass	kg
M_p	Consumed propellant mass	kg
m	Mass	kg

Symbol	Description	Units
m_E	Mass of the Earth	kg
m_d	Mass of the perturbing body	kg
N_{skip}	Number of skips	-
n	Mean motion	rad/s
P_c	Collision probability	-
$P_{n,m}$	Legendre polynomial (degree n , order m)	-
p	Probability density	-
R	Radius of the Earth	m
R_{acc}	Acceleration volume radius	m
R_c	Critical radius	m
R_{cr}	Critical radius	m
R_{th}	Threshold radius	m
$R_{th,fine}$	Fine threshold radius	m
$R_{U,V,W}$	Transformation matrix between Hill and inertial frame	-
R_{X_B,Y_B}	Transformation matrix between inertial frame to B-plane	-
r	Distance or radius	m
r_0	Initial distance or radius	m
r_a, r_p	Radius of apocenter and pericenter	m
r_B	Conjunction position in B-plane	m
r_d	Distance to center of mass of a perturbing body	m
r_i	Distance to center of mass of the Earth	m
r_{min}	Minimum distance	m
t	Time	s
t_0	Initial time	s
t_{tca}	Time of closest approach	s
U	Gravitational potential	m^2/s^2
V	Velocity	m/s
V_c	Critical volume	m^3
V_j	Exhaust velocity	m/s
V_{rel}	Relative velocity between objects	m/s
v	Velocity	m/s
v_{esc}	Escape velocity	m/s
W	Energy flux of light	J/s
w	Dimensionless velocity	-
w_j	Dimensionless exhaust velocity	-
x_B, y_B	Unit vectors of probability ellipse in B-plane	-

Introduction

Ever since the launch of the first satellite, Sputnik-1, in 1957, humankind sought to explore and populate the new frontier of space. This event marked the start of the Space Race between the United States of America and the Soviet Union. The Space Race birthed many new milestones such as the first photograph of the planet Earth, the first animal in orbit, and later the first human in orbit. A well-known event during the Space Race was the first manned Moon landing in 1969. Sending satellites or living organisms to space was a major endeavour, which required a large number of resources. Over the years, the technology and the reliability of launching objects in orbits improved, which caused the commercial business to invest in space-based technology. With this increased interest in the space environment, commercial launch providers started growing to meet the demand of the new commercial space market. Nowadays it is hard to imagine a world without a vastness of space applications, in a majority provided by private companies and institutions.

1.1. Background

Inserting a satellite into orbit requires a large amount of resources. Multi-stage rockets are often used to overcome the Earth's gravitational pull and reach orbital velocities. During an ascent the multiple stages are used consecutively for different parts of the trajectory. The first stage of the launch vehicle is used to rapidly increase the altitude, whereas later stages of the launch vehicle are used to steer the payload (satellite) into an (elliptical or circular) orbit around the Earth. Most parts of the launch vehicle return back to the Earth after completing its task. Sometimes however, the last stage in a launch vehicle is used to steer the payload into its designed orbit and does not return to the Earth after completing its task. This object then starts to also orbit the Earth.

Satellites in orbit do not have an eternal lifetime. Just like instruments on Earth, they are prone to system failures. Failures can occur due to the harsh space environment they are orbiting in. Due to a highly reduced atmosphere, there is no protection from the harmful solar radiation and large temperature differences which can damage sensitive electronics on-board the satellite. After the satellite has reached the end of its lifetime, and can no longer perform its designed duty, it continues to orbit the Earth for relatively long periods of time (depending on the orbital altitude). This happens because there is no natural mechanism to slow the vehicle down and return it back to the Earth. When an object orbits the Earth without performing its designed purpose, it is referred to as *space debris*.

Space debris no longer serves a purpose in its orbit and is typically uncontrollable. This can lead to potentially dangerous situations when coming close to other objects. In order to avoid collisions with space debris, collision avoidance maneuvers are sometimes performed by active vehicles. The International Space Station (ISS) in February 2018 had performed 25 debris collision avoidance maneuvers, since 1999 [20]. Objects orbiting in Low Earth Orbit (LEO)¹, at altitudes of 400 km above the Earth's surface have an orbital velocity of approximately 7.7 km/s. At these velocities, even a collision with a small object can cause a large amount of damage to active vehicles. Even though larger debris objects can be tracked, objects with a radius less than 10 centimeters cannot be tracked, and also pose a serious threat for sensitive satellite equipment (<https://www.space-track.org/documentation/faq>). In 2017, the Copernicus Sentinel-1A satellite was hit by

¹The LEO region extends from an altitude of 180 to 2000 km above the Earth.

a millimeter-sized debris particle [16]. This particle hit one of the solar panels, which caused a sudden decrease in power of the satellite. With an increase of space activities, so does the number of space debris increase. Figure 1.1 shows a visualisation of the (trackable) number of objects orbiting the Earth (in LEO). This is a combination of active satellites and debris. The data shown in this visualisation is obtained from www.space-track.org.

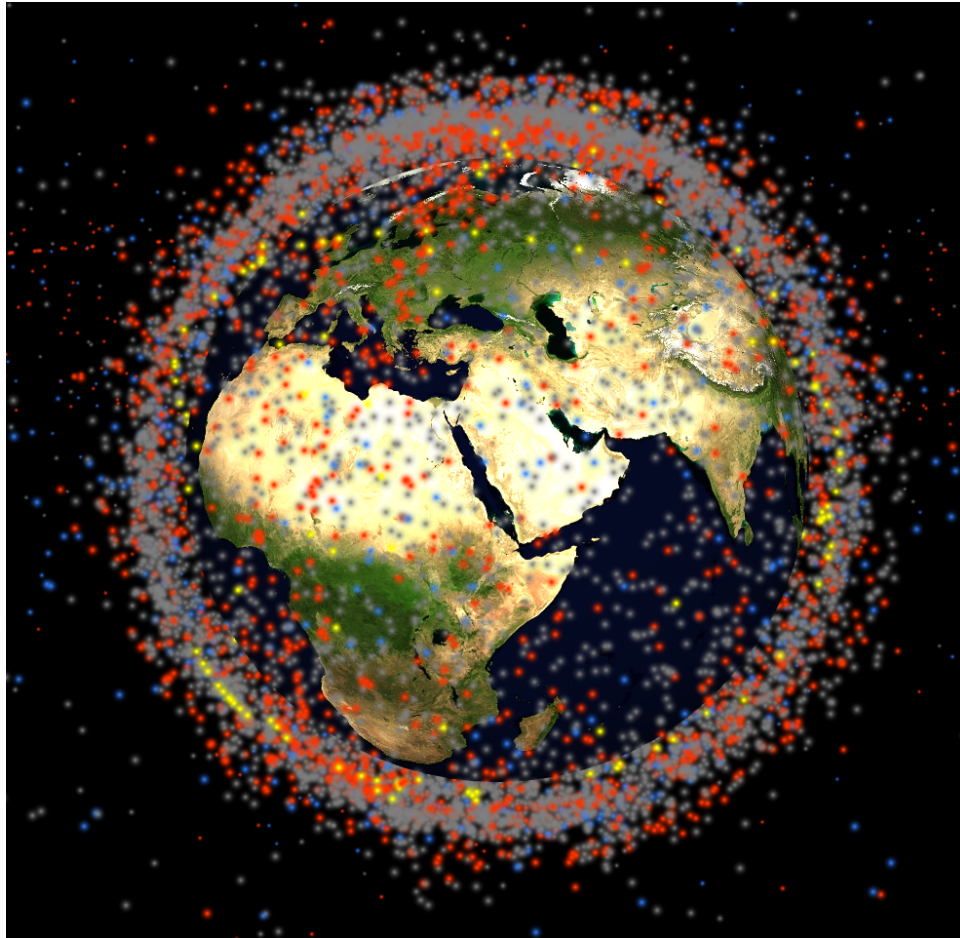


Figure 1.1: Visualisation of the number of objects in orbit. The different colors represent either a satellite (red), rocket body (blue) or debris object (grey). The data is updated daily from www.space-track.org².

With rising number of objects orbiting the Earth, collision events are expected to occur more frequently in the future. Figure 1.2 shows the number of objects in Earth orbit. It can be clearly seen that the fragmentation debris contributes most to the total number of catalogued objects in orbit. This also means that there is also a number of (fragmentation) objects in orbit that are not catalogued because they are too small to be tracked. In 1978, NASA scientist David J. Kessler proposed a scenario in which the density of orbital debris objects in LEO would become large enough that collisions will cause a cascade of debris objects, which, in turn, will collide further with other objects, rendering some orbital altitudes unfeasible for any space activity in the future. This is called the Kessler Syndrome. In their paper, Kessler and Cour-Palais [14] estimated that the flux of artificial debris will rise exponentially with time.

In the past, several fragmentation events have happened that greatly increased the number of fragments in orbit. In Figure 1.2, a clear increase in fragmentation debris can be seen in 2007 as a result of an anti-satellite missile test conducted by China, resulting in more than 2000 new trackable objects. In the same figure in 2009, another increase in fragmentation debris can be seen as a result from a collision between two satellites. Cosmos-2251, an inactive Russian military communications satellite, and Iridium 33, an operational commercial telephone communications satellite, collided, resulting in an estimated number of 2000 trackable debris objects [19].

²www.stuffin.space, retrieved: September 18, 2019

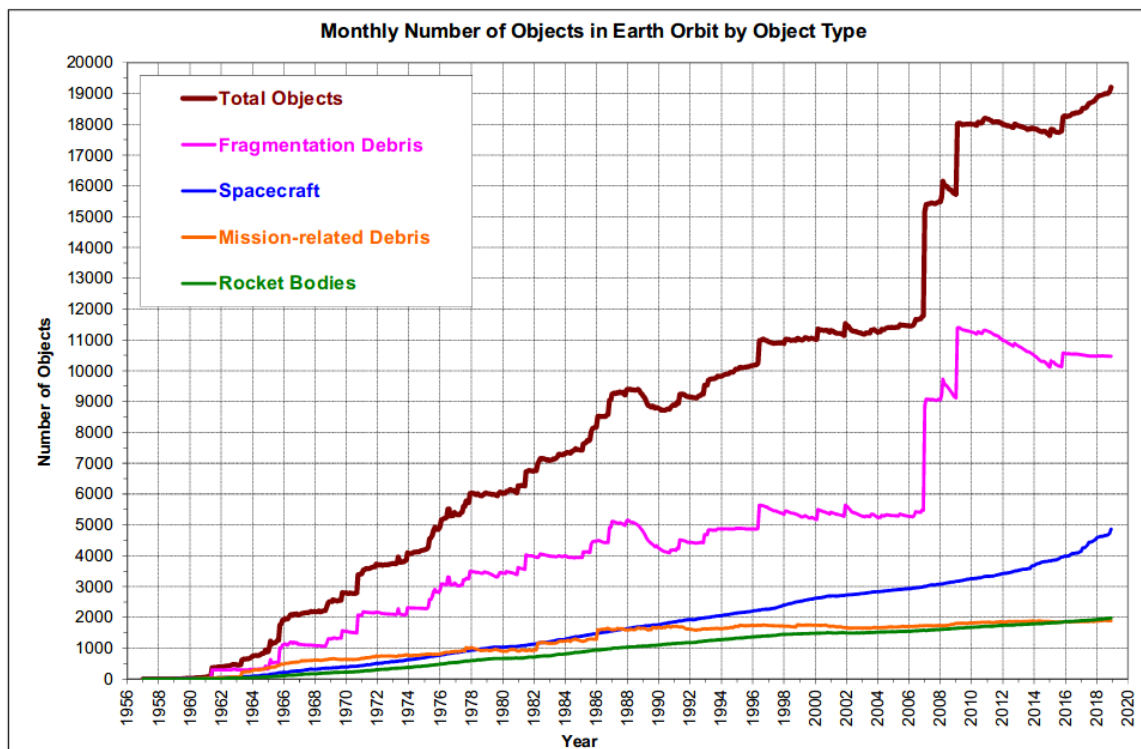


Figure 1.2: Number of objects in Earth orbit (May 2019) officially catalogued by the United States Space Surveillance Network (USSSN) [4]

With an increase of potentially dangerous space debris, the need for suitable ways to clean up the debris increases as well. Removing an object from its orbit can happen in two different ways: either its orbit decays and the object re-enters the Earth's atmosphere where it will either burn up or impact in a desolated area (e.g. the ocean); or the object is sent to a so-called graveyard orbit where the number of operational satellites is low so that the debris does not pose any threat. When an object is placed in a graveyard orbit, it is thus not removed from the space environment, just relocated. Graveyard orbits usually have a large orbital radius and are thus not feasible for objects orbiting in LEO, because it would require a large amount of energy to reach these altitudes. For objects in LEO, a more obvious solution is to let the orbit decay and re-enter the Earth's atmosphere. An orbital decay can either be passive or active. In a passive decay, perturbing forces of the object's environment (e.g. atmospheric drag) cause the object to lose altitude until it finally re-enters (see Section 2.4). In an active decay, the object is artificially guided into an orbit where it will decay and re-enter. The process of actively decaying an orbit of an object to re-enter the Earth's atmosphere is called a *de-orbit*, and for the purpose of this research it will be defined as follows:

A de-orbit is the active removal of artificial space debris from its current orbit without it permanently entering a new closed orbit about the main body or creating more debris after the de-orbit maneuver has finished.

Guidelines to encourage satellite operators to take the de-orbit strategy into account have been created by the United Nations Office for Outer Space Affairs (UNOOSA). The UNOOSA "works to promote international cooperation in the peaceful use and exploration of space, and in the utilisation of space science and technology for sustainable economic and social development." (<http://www.unoosa.org/oosa/en/aboutus/index.html>) One of these guidelines is stated by the European Code of Conduct for Space Debris Mitigation: "The operator of a space system should perform disposal manoeuvres at the end of the operational phase to limit the permanent or periodic presence of its space system in the protected regions to a maximum of 25 years." [3]. This code of conduct is signed by the European Space Agency (ESA) and several other European national space agencies.

1.2. Research framework

Many debris objects currently in orbit are uncontrollable and cannot be communicated with. This is because they are either part of a launch vehicle (rocket body) or they are old satellites that seized to work. These uncontrollable objects pose a serious threat to the current space environment and future space operations. A large amount of research is being conducted to find sustainable de-orbit methods. Mark and Kamath [23] review different methods that are currently being investigated. As an example, some of these methods are: laser-based, tether-based, or sail-based³. Laser-based methods use the radiation pressure of a laser beam to exert a force on the debris, slowing it down and thus lowering its orbital altitude. This method can only de-orbit small debris objects and is currently limited in range. Tether-based methods use a long wire attached to the debris on which the Earth's magnetic field exerts a force, slowly pulling back the debris towards the Earth. The force acting on the wire is however very small, which results in a large de-orbit time. Sail-based methods make use of solar sails which interact with the radiation pressure of the photons emitted by the Sun. This method however is also rather slow and the debris object cannot be controlled, which potentially can be very dangerous for other satellites in its environment.

Ideally before a satellite has reached its end-of-life (the end of the operational use of the satellite), it uses its own instruments to enter a de-orbit trajectory. This saves the costs of having to use external satellites to assist in the de-orbit maneuver, potentially creating even more debris because an extra satellite is required. On-board propulsion systems ensure that the responsibility lies with the satellite operator and reduces the cost of having to use extra resources to remove uncontrollable space debris. Past research has been conducted using the on-board propulsion system of objects in LEO to perform a de-orbit maneuver. In their research, Lidtke et al. [18] look at de-orbit strategies for three different rocket-bodies, orbiting in LEO. They compare different propulsion methods with each other and assess the risk involved with each trajectory. One of the propulsion methods they investigate is the low-thrust propulsion method. This method uses a low-thrust propulsion system that slowly lowers the altitude of the object. The advantage of using low-thrust propulsion systems is that it requires much less propellant mass than high-thrust propulsion systems. Therefore, de-orbit trajectories using low-thrust propellant will be investigated in this thesis.

Three different objectives are important to consider when performing a de-orbit maneuver: the amount of propellant used, the probability of collisions with other objects, and the duration of the de-orbit.

Amount of propellant When a satellite has reached its end-of-life, it no longer has any economic value for its operator. Preferable, when an object has reached its end-of-life there are no more costs tied to the satellite for the operator. As mentioned previously, the cost for putting satellites in orbits requires a large amount of resources and is therefore very expensive. These costs are often tied to the mass of the satellite. For satellite operators, it is therefore preferable to limit the mass of the satellite as much as possible in order to cut costs. A part of the satellite mass is the on-board propellant, used to perform small maneuvers, like stationkeeping. These maneuvers ensure the satellite to stay in its designed orbit. To avoid having to bring excess propellant into space, the de-orbit of a satellite must be performed in such a way to limit the propellant usage. Logically, this would mean that the satellite would have to perform a de-orbit maneuver which limits the amount of propellant used.

Collision probability The number of satellites in LEO is growing each year and thus the environment gets more crowded. With this growing number of satellites, the chances of colliding with other objects also increases. Collisions between objects with orbital speeds of approximately 7.7 km/s can have catastrophic consequences. Not only can such collisions cause large structural damage to spacecraft, it can also endanger human lives. As mentioned before, the ISS has to maneuver several times per year to avoid a potential collision with space debris. During a de-orbit, an operator has to take into account that the object traverses regions where the satellite number density can be relatively large, or where an object of extremely high value is orbiting (like the ISS). The probability of collisions is not smoothly defined by a continuous function, but is rather discretely defined. When a de-orbit occurs in as little time as possible, the satellite might traverse very densely populated orbital regions populated by high-risk objects such as the ISS quickly. Even in low-density orbital regions, one single conjunction event might account for a high collision probability. An important objective in designing a de-orbit is therefore to limit the collision probability between the object in de-orbit and other objects in the space environment.

³More methods, including on-board propulsion methods, are investigated. The number of methods explained here serves as an indication of the variety of different methods that can be used to de-orbit a piece of debris.

Amount of time in orbit In general, the longer a satellite is in orbit, the higher the probability of it colliding with another object. This is because there is more time available for potential collisions to occur. Over time, the amount of space debris also grows due to continuous space activities (launching new objects) or due to break-up events. Limiting the time in orbit therefore in general reduces the probability of potential collisions. This would suggest that the safest way to de-orbit a satellite is to reduce the time of the de-orbit as much as possible. This does not always have to be the case, however.

1.2.1. Research gap

Past research of low-thrust de-orbit strategies has focused primarily on the associated collision risk for trajectories with a *constant* thrust [10] [21] [38] [6]. There are however more factors to consider, when designing a de-orbit, as mentioned above. The objects in LEO are not distributed homogeneously across all different altitude regions, however. Some orbital regions are very attractive for certain satellite applications and the satellite density therefore varies significantly for different orbital regions. Using a constant thrust ignores the effect these varying satellite densities might have on the associated risk of collisions. The probability of a collision P_c per orbital altitude can be estimated by treating the objects in LEO as a statistical distribution of particles. The probability of a collision can then be estimated using the kinetic theory of gasses [35]:

$$P_c = 1 - \exp^{-\rho A t V_{rel}} \quad (1.1)$$

where ρ is the object density, A the collision cross-sectional area, t the time in orbit, and V_{rel} the relative velocity between a satellite and debris objects. Figure 1.3 shows the density of objects per orbital altitude and its corresponding collision risk (using the values $A = 5 \text{ m}^2$, $t = 1 \text{ year}$, $V_{rel} = 10 \text{ km/s}^4$). From this figure it can be seen that this estimation of the probability of collisions varies significantly per orbital altitude. These low-density orbital regions can be used as an advantage when designing de-orbit trajectories. A satellite during a de-orbit might choose to coast in low-density orbital regions before crossing high-density regions. In this way, the satellite can choose the optimal moment in time when the object density at lower orbital altitudes is favourable enough to continue the de-orbit. This can be done effectively by adding a controlled thrust force. This research will therefore focus on adding a *controlled low-thrust force in order to find an optimal de-orbit strategy*.

1.2.2. Research question

As explained previously, there are different objectives to consider when finding an *optimal de-orbit strategy*. The optimal strategy therefore does not have a single solution. A low thrust means that the object will stay in orbit longer, increasing the de-orbit time. A high thrust means that the de-orbit time is reduced. Both of these strategies can yield different associated collision risks. A research question is formulated to address the problem stated above:

Research question: *What is a robust method to de-orbit an object in Low Earth Orbit at its end-of-life, using on-board propulsion methods?*

In order to provide insight into the meaning of the research question, it is broken down in multiple parts. A *robust method* means that the researched method will not be sensitive to small changes in various parameters used during the analysis. It means that the obtained method will work for various cases and not only for very specific cases (such as varying satellite states or debris configurations). The *on-board propulsion methods* points to the propulsion system integrated in the object. This means that the object has a means of propulsion it can use without assistance from external sources (other than maybe communication methods) in order to guide itself into a de-orbit trajectory. The on-board propulsion methods do not use passive propulsion systems such as a solar sail or an electrodynamic tether, but rather active propulsion systems, which can be toggled on or off. *Low Earth Orbit* refers to the region of space where objects orbit the Earth below 2000 km altitude and have eccentricities below 0.25.

How optimal the de-orbit is will be assessed in terms of the propellant usage, the propagation time and the associated collision risk. This research will compare the results obtained with the results found by Lidtke et al. [18]. In their paper, Lidtke et al. investigate several cases where different rocket bodies use propulsive thrust to conduct a de-orbit. They compare both high-thrust and low-thrust cases of constant thrust. Additionally they

⁴These values are taken arbitrarily to represent a real-life scenario of two objects with orbital planes at an angle during a collision.

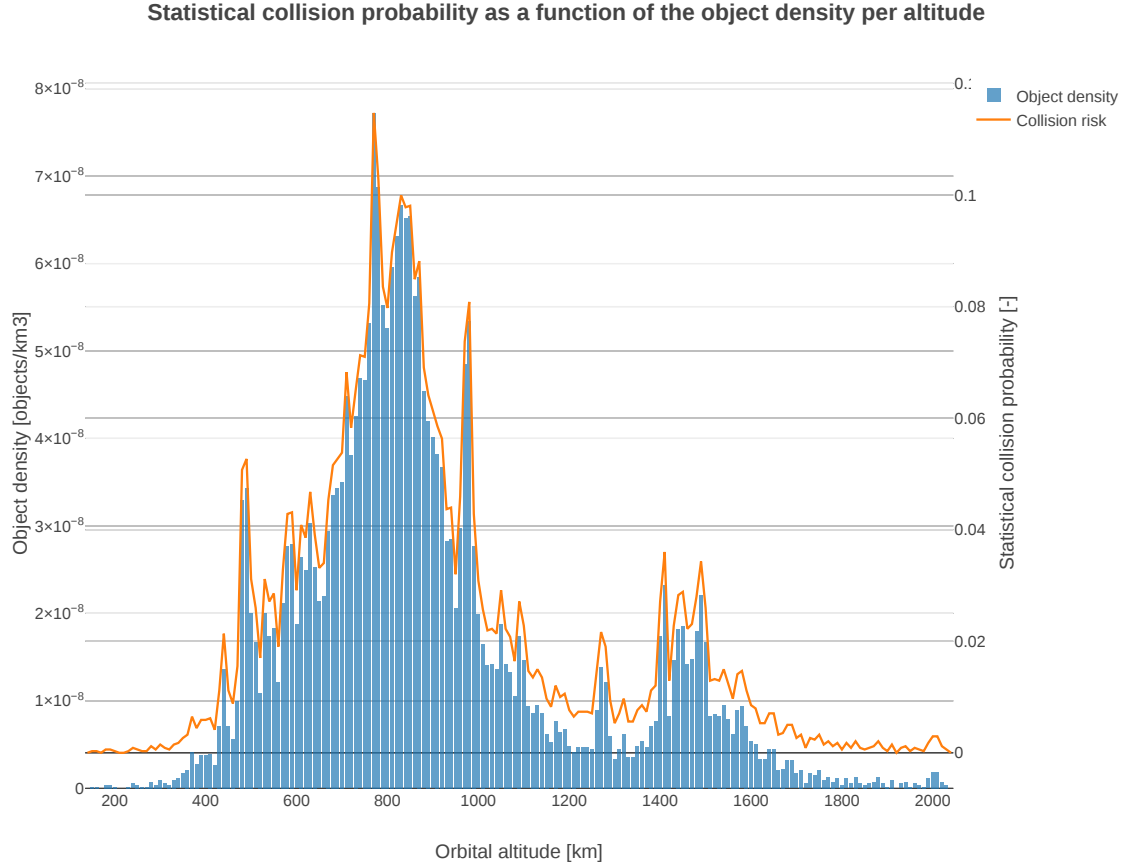


Figure 1.3: The density of objects (blue) and its associated collision probability (orange) per orbital altitude. The data was obtained from www.space-track.org

look at the collision risk associated with a high-thrust de-orbit that includes a preliminary coasting phase. The three objects they use for their analysis are Zenit-2, Tsyklon-3 and Kosmos-3m, which are rocket bodies that have the potential to cause collisions, suggested by McKnight et al. [24]. These objects are chosen to represent a wide-enough range of different orbital configurations where debris objects can cause collisions. The results of their research are shown in Table 1.1.

Target	Trajectory	Propellant mass [kg]	Duration [days]	Accumulated P_c [-]
Zenit-2	Constant low-thrust	34.8	593.14	1.6×10^{-3}
Tsyklon-3	Constant low-thrust	6.6	174.25	3.6×10^{-5}
Kosmos-3m	Constant low-thrust	7.6	121.24	4.1×10^{-4}

Table 1.1: Used propellant mass, duration and associated collision probability P_c of the de-orbit trajectories for a constant low-thrust trajectory. The duration of the orbit was determined by a lower altitude limit (in kilometers) specific for each object, selected by the researchers [18].

Although the three objects are rocket bodies, left over from a previous launch, they will be referred to as *satellites* in the remainder of this thesis. All other objects in the LEO environment will be referred to as *debris*. This is because the optimization of the trajectories of these rocket bodies can be applied to satellites that have reached their end-of-life. Making the clear distinction between satellite and debris will prevent confusion during the remainder of the thesis.

1.3. Report structure

The structure of this thesis will be as follows. First, in Chapter 2, the orbital dynamics involved in a de-orbit maneuver are introduced. Some key concepts about the shape of satellite orbits and the effect of thrust are explained. Different low-thrust propulsion systems are introduced, and the effect of orbital perturbations is presented. Next in Chapter 3, the theory behind the conjunction analysis is presented. The geometry of close conjunctions and the concept of applying filters and sieves to reduce the computational time is given. Also, the derivation of the calculation of the collision probability is presented. Chapter 4 presents the methodology of this thesis. The general outline of the workflow is presented and the transformation from the raw data to the Kepler elements used in the analysis is explained, along with a visualization of the distribution of objects in LEO. Model specifications used in this thesis are given and explained. The results of this thesis are presented in Chapter 5. Here, a selection of the results obtained in this thesis are presented, and discussed. The conclusions of this thesis are presented in Chapter 6, together with the recommendations on further research.

2

De-orbit Dynamics

Objects in the space environment are subject to a number of different forces, which determine the shape of satellite orbits. A de-orbit maneuver requires the addition of an extra force, thrust, to guide the satellite back to the Earth's atmosphere (and eventually the surface). Adding a thrust force to a satellite may sometimes have a counter-intuitive effect on its orbital shape. In order to perform a safe and a cost-effective de-orbit, the dynamics that govern the motion of satellites has to be understood. This chapter presents theory about the orbital dynamics of the satellite during its de-orbit maneuver. First, the laws of motion will be presented, along with the introduction of Kepler orbits. Next, adding low thrust to a satellite will be presented. The theory in this chapter is largely summarized from Wakker [32], unless indicated otherwise.

2.1. Gravitational motion

The dynamics of orbiting satellites is largely governed by gravitational forces. The Earth exerts a gravitational force on the satellite, pulling it towards the surface. The horizontal velocity of the satellite then ensures that the satellite stays in its orbit. Planetary motion has been described by both Isaac Newton (1643 - 1727) and Johannes Kepler (1571 - 1630). Both descriptions are discussed to provide insight into the orbits of objects under the influence of gravity.

2.1.1. Newton

Newton's second law of motion states that the sum of forces $\vec{F}$ acting on an object equals the mass m times the acceleration $\vec{a}$ of that object: $\vec{F} = m\vec{a}$. An object in LEO is subject to several forces, keeping the object in its orbit around the Earth. The most dominant force is the force of gravity exerted by the Earth. When two bodies are attracted to each other under the influence of gravity, the force acting between the two bodies can be described by Newton's law of gravity:

$$\vec{F} = -G \frac{m_1 m_2}{r^3} \vec{r} \quad (2.1)$$

where G is the gravitational constant, m_1 , m_2 are the masses of the two bodies attracting each other, and r is the relative distance between the two objects (and $\vec{r}$ is the position vector between the two bodies). The minus sign indicates that the force is attractive. In the case that one of the two bodies has a mass m much greater than the other body, the motion of the smaller body can be described by

$$\frac{d^2 \vec{r}}{dt^2} = -\frac{\mu}{r^3} \vec{r} \quad (2.2)$$

where $\frac{d^2 \vec{r}}{dt^2}$ is the acceleration of the smaller body, and $\mu = Gm$ is the gravitational parameter of the larger body. In the case of the Earth, this parameter is $\mu = Gm_{Earth} = 3.986004418 \times 10^{14} \text{ m}^3/\text{s}^2$. Newton's equations of motion are described using a conventional Cartesian (right-handed) coordinate system, where the position of an object is indicated by an X, Y, and Z coordinate. Although Newton's laws describe the (non-relativistic) gravitational motion of objects in space, they are not very insightful in understanding the shape and configuration of different orbits. A more insightful method is to use Kepler elements to describe satellite orbits.

2.1.2. Kepler elements

Kepler elements are six elements that fully describe the geometry of an orbit. Figure 2.1 shows an object P in the orbital plane, orbiting about the focal point F , tracing out an elliptical trajectory. The focal point is the gravitational body about which the object is orbiting (Earth is the focal point of objects orbiting about the Earth). Points A and A' indicate the pericenter and apocenter of the orbit (which are the points in the orbit where P is closest to and furthest from the focal point F), respectively. The first Kepler element is the semi-major axis a of the orbit, which is equal to the radius of the pericenter and the apocenter divided by two. The second Kepler element is the eccentricity e of the orbit. The eccentricity describes the ellipticity of the orbit. An eccentricity of 0 describes a circular orbit. An eccentricity between 0 and 1 describes an elliptical orbit. The angle θ between the pericenter and the position vector r of the object is the true anomaly. The true anomaly indicates the angular position of the object during the orbit and varies between 0 and 2π .

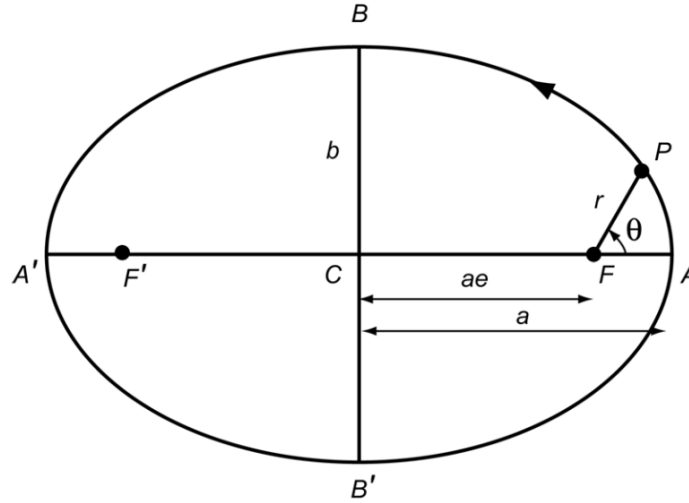


Figure 2.1: Geometry of an elliptical orbit, viewed in the orbital plane. The object P orbits around the focal point F of the elliptical orbit. Points A and A' indicate the peri- and apocenter of the orbit (peri- and apogee in the case when the focal point is the Earth) [32].

Figure 2.2 shows $1/8$ section of a sphere, displaying an object and its orbital trajectory about the Earth. The object's position at this point in time on this spherical section is shown in spherical coordinates: radius r , azimuth α , and elevation δ . The geometry of the orbit is given in Kepler elements. The x-axis of this inertial coordinate system points to the vernal equinox (Υ), the z-axis points to the Conventional International Origin (CIO pole), and the y-axis completes this right-handed coordinate system.

The inclination i of the orbit is the angle between the equatorial plane of the Earth and the orbital plane of the object, evaluated at the point in the orbit where the object crosses the equatorial plane to the northern hemisphere of the celestial sphere¹. This specific is called the ascending node (A.N.). The angle between the A.N. and the pericenter of the orbit is the argument of pericenter ω . The angle between the vernal equinox and the ascending node of the orbit is called the right ascension of the ascending node Ω .

If no other forces act on the satellite in orbit, the only time-varying parameter will be the true anomaly, which will vary between 0 and 2π in one orbital period. An orbit where the only varying element is the true anomaly is referred to as a Kepler orbit. Kepler orbits have the property that they are completely stable over time, meaning that the shape and orientation of the orbit do not change over time. Due to orbital perturbations present in the space environment, pure Kepler orbits do not exist. Some orbits however can be closely represented by Kepler orbits.

2.2. Propulsive thrust

Objects in LEO can maneuver by using their on-board propulsion system to apply a thrust acceleration to their equation of motion. Thrust in this case is generated in the rocket engine by expelling mass through a nozzle, in the form of propellant. Due to Newton's third law, this expelled mass generates a force in the opposite direction, which is the thrust force. The effect of the thrust can differ significantly for different thrust

¹The celestial sphere is an abstract sphere with an arbitrary large radius and is concentric to the Earth. All objects in the sky can be viewed as a projection on the inside of the surface of the celestial sphere.

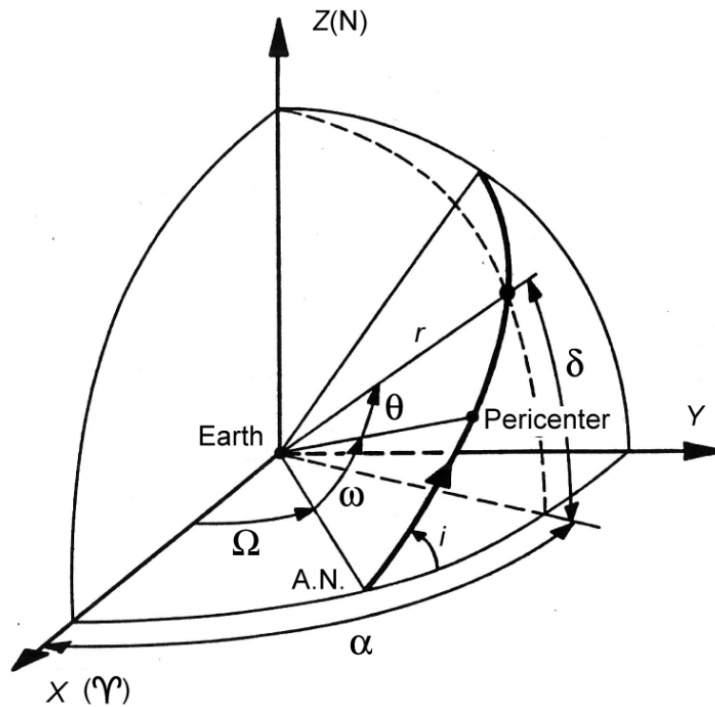


Figure 2.2: Visualization of the orbital elements ω , Ω , i , and θ . The position of the object is given in spherical coordinates $(r, \alpha, \text{and } \delta)$ [32, p. 261].

mechanisms. Propulsive thrust can be divided in two different groups: high thrust and low thrust. Traditionally, most people view a propulsion system for a rocket or a satellite as a chemical propulsion system. In such propulsion systems, a fuel and an oxidizer are burned to create hot gasses, which then accelerate through a nozzle and produce a thrust force. Launch vehicles use this type of propulsion to send a payload into orbit from the Earth's surface, needed to overcome the gravitational pull. Large fuel tanks are required to carry the large amounts of propellant needed in the chemical reaction. For now, chemical propulsion systems are the only way to send a payload into orbit, due to the high thrust force associated with this type of propulsion. Satellites in orbit also use propulsion systems. These propulsion systems are used for stationkeeping, which are propulsive maneuvers to correct for the environmental perturbations (see Section 2.4). These propulsion systems can also use chemical propulsion to produce thrust, but there are also other options available. Because a lower thrust force is needed for stationkeeping, *low-thrust* propulsion systems such as cold-gas propulsion, or electric propulsion can also be used. These propulsion systems provide a lower thrust, but sometimes a much higher specific impulse, and in addition may need less propellant than chemical propulsion systems. This reduced propellant mass means that the launch vehicle lifts a lighter payload, which reduces the cost of launch.

2.2.1. High thrust

High-thrust propulsion systems rely on the expanding gasses generated by the chemical reaction of burning propellant with an oxidizer. These expanding gasses are forced through a small nozzle, further accelerating them. These gasses reach very high velocities and are expelled to generate a thrust force. High-thrust propulsion, like the name suggests, produces a high thrust magnitude. This high thrust magnitude allows the object to perform drastic maneuvers in a relatively short time (like a transfer between two planetary bodies). High-thrust propulsion requires a large amount of propellant and is therefore (in satellite maneuvers) only used for relatively short time periods (burnt time < 20 minutes). Using high-thrust propulsion in a de-orbit maneuver may require a large amount of propellant.

2.2.2. Low thrust

Low-thrust propulsion systems also expel mass to produce a thrust force, but this mass is not expelled by a chemical reaction. These systems can use pressurized tanks which expel a cold gas to produce thrust, or accelerate ionized particles with an electromagnetic field. For the removal of large (mass ≥ 1500 kg) debris objects (e.g. satellites, rocket bodies), the most effective propulsion system, used in a de-orbit, is an electrical propulsion system. The energy delivered by electric propulsion systems is determined by an external power source (e.g. solar panels). For larger objects, these propulsion systems can be used in regular stationkeeping maneuvers and due to the large object mass, the impact of the electrical system is not as large as for smaller satellites (see Table 2.1). Electrical propulsion systems become more and more favourable with increasing propagation times [13].

The equation of motion of an unperturbed satellite about the Earth is described by Eq. (2.2). When a satellite applies a thrust force $\vec{F}$, the acceleration is added to the equation of motion

$$\frac{d^2 \vec{r}}{dt^2} = -\frac{\mu}{r^3} \vec{r} + \vec{f} \quad (2.3)$$

where $\vec{f} = \vec{F}/M$, and M denotes the instantaneous mass of the satellite. From Eq. (2.3), the specific orbital angular momentum $\vec{H}$ and specific energy $\mathcal{E}$ of the satellite can be found by applying a series of operations [32]. This results in

$$\frac{dH}{d\tau} = a\rho \cos \delta \quad (2.4)$$

$$\frac{d\mathcal{E}}{d\tau} = aw \cos(\delta - \gamma) \quad (2.5)$$

where $\tau = \sqrt{\frac{\mu}{r_0^3}} t$ (dimensionless time), $a = \frac{r_0^2}{\mu} f$ (dimensionless acceleration), $w = \sqrt{\frac{r_0}{\mu}} V$ (dimensionless velocity), and 0 indicates the state of the initial (circular) orbit.

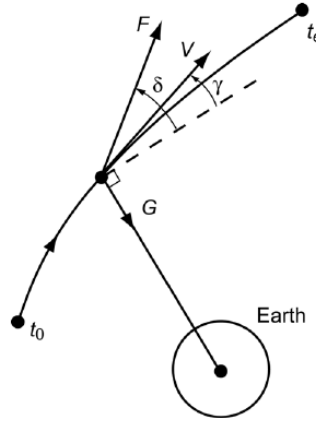


Figure 2.3: Geometry of powered flight of a satellite in orbit about the Earth. The angles δ and γ represent the thrust angle and flight-path angle, respectively [32].

From Eqs. (2.4) and (2.5) it can be seen that the instantaneous change in orbital energy is maximum if the thrust angle δ is aligned with the flight-path angle γ , i.e., the thrust is acting tangentially to the trajectory ($\delta = \gamma$). This thus means that for a certain change in orbital energy, the least amount of thrust is required if the thrust is directed along the velocity vector. This effect can be visualized by observing the dimensionless radial distance parameter ρ , as shown in Figure 2.4. It can be seen that for a radial thrust, the radial distance only decreases periodically, but does not decrease over longer periods of time. When the same thrust is applied tangentially, however, the dimensionless radial distance decreases steadily over time.

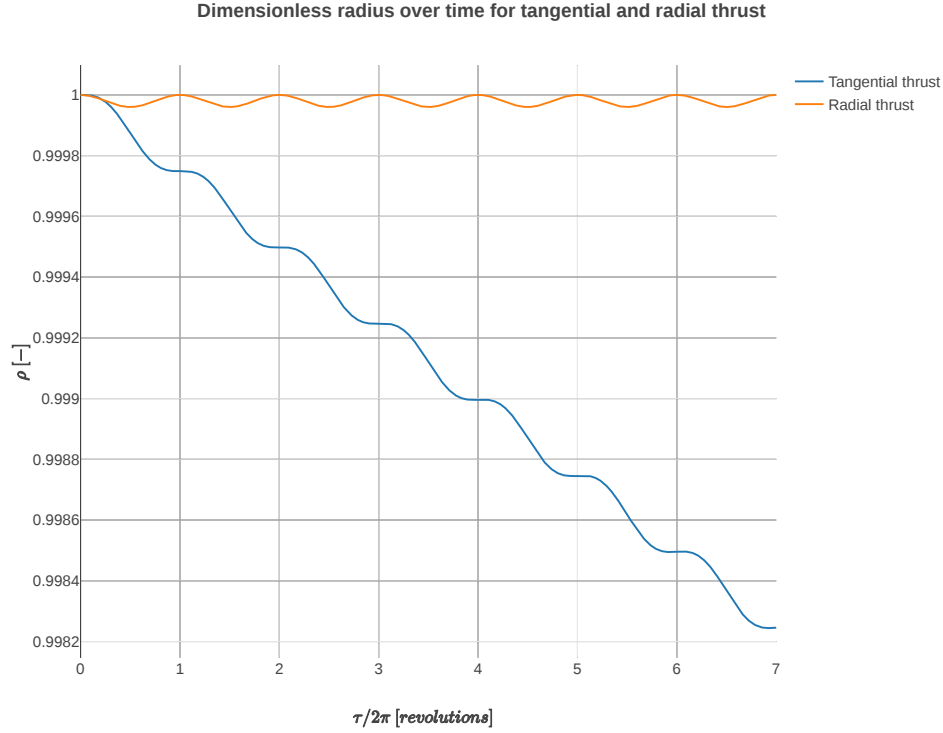


Figure 2.4: Dimensionless radial distance parameter ρ as a function of dimensionless time, when adding the same amount of thrust in both the radial and the tangential direction.

2.3. Propulsion systems

Not all propulsion systems are the same. Different propulsion systems are suitable for distinct applications. The specifications of the propulsion system determined the force of thrust and the propellant mass flow.

2.3.1. Propellant usage

Applying a thrust force requires propellant. Propulsive thrust is generated by expelling mass through a nozzle in the propulsion system. The high velocities of the expelled mass generate a thrust force. The standard rocket equation $F = \dot{m}V_j$ states that the thrust force F produced equals the mass flow $\dot{m}$ times the exhaust velocity V_j . The mass of a vehicle after a thrust M_e is equal to the initial mass M_0 minus the propellant mass M_p . The ratio of the initial mass over the final mass can be written as

$$\ln \frac{M_0}{M_e} = \int_0^{t_e} \frac{f}{V_j} dt \quad (2.6)$$

where t_e is the time of engine cut-off, and $f = F/M$ (where M is the instantaneous mass). The exhaust velocity is often described in terms of the specific impulse I_{sp} , where $I_{sp} = V_j/g_0$. The propellant usage varies for different kind of trajectories. High-thrust trajectories (i.e. trajectories defined by high-thrust maneuvers) usually have a large mass flow and thus high propellant use. Electric propulsion systems usually have a small mass flow, which makes them more attractive for satellite use, since they do not have to carry large propellant masses, which increases the cost at launch for the operator.

2.3.2. Systems

Several electric propulsion systems exist to serve different mission objectives. Some propulsion systems are suitable for orbit maintenance maneuvers where others are more suited for interplanetary trajectories. Some state-of-the-art electric propulsion systems are resistojets, electrosprays, ion engines, pulsed plasma thrusters, and Hall effect thrusters. Each of these systems provide a different range of thrust magnitude and specific impulse. The specifications for these different propulsion systems are shown in Table 2.1.

One of the more suitable low-thrust propulsion systems for de-orbit maneuvers is the ion engine system.

Propulsion system	Thrust	Specific impulse [s]	Power [W]
Resistojets	10 mN - 0.45 N	50 - 150	15 - 50
Electrosprays	74 μ N - 1.5 N	800 - 2300	1.5 - 25
Ion engines	10 μ N - 0.4 mN	300 - 6000	40 - 60
Pulsed plasma thrusters	1 μ N - 90 μ N	536 - 3000	1.5 - 100
Hall effect thrusters	5 mN - 13 mN	1139 - 1390	175 - 200

Table 2.1: System specifications for different types to low-thrust propulsion systems [37].

This system ionizes its propellant and produces thrust by accelerating those ions using helical coils. A major advantage of this system over other low-thrust propulsion systems is that it limits potential thruster lifetime threats due to the absence of electrodes [37]. In the paper by Lidtke et al. [18], the low-thrust propulsion system uses a thrust magnitude of 25 mN and 30 mN, with a specific impulse of 3400 s for both thrust levels, which corresponds to the thrust and specific impulse ranges of ion engines.

2.4. Orbital perturbations

Kepler orbits are stable over time, where only the true anomaly varies from 0 to 2π in one orbit. That is because the only force acting on the object is the central force of gravity, exerted by the Earth. Pure Kepler orbits are hypothetical, however. In the real space environment, there are many other forces acting on the object, like the gravitational force of the Moon or atmospheric drag. Since these forces are often much smaller than the dominating central gravitational force, they are called *perturbing forces*. Like the name suggests, these forces perturb the orbit of the object, and cause variations in the Kepler elements. On short timescales, these perturbing forces may not influence the orbit of the object significantly, but their effect may become important on larger timescales. In order for an accurate simulation of the motion of a satellite, these forces may have to be accounted for. Not all perturbing forces account for the same acceleration on an object. Figure 2.5 shows the acceleration of some of the main perturbing forces acting on a satellite in LEO, as a function of the orbital radius. The three different objects used in this research start out with orbital radii below 7200 km. As can be seen from Figure 2.5, at these orbital radii, all perturbing forces visualized here will affect the orbits of these three objects.

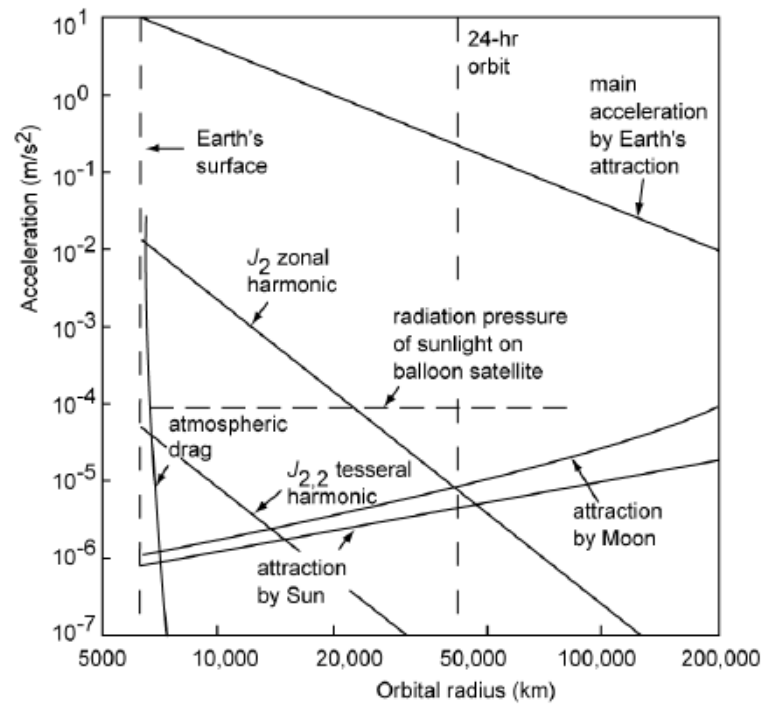


Figure 2.5: Acceleration of the main perturbing forces on a satellite as a function of the orbital radius [32].

2.4.1. Atmosphere

The atmosphere of the Earth does not have a sharp boundary but rather gets thinner at higher altitudes. At LEO, the density of the atmosphere ρ is not equal to zero. This means that there are still atmospheric particles that can collide with the spacecraft as it flies through the atmosphere. The area A of the spacecraft that is exposed to the atmospheric particles accounts for a large part of the perturbing force that the atmosphere exerts on the spacecraft. The acceleration experienced by the satellite due to the atmospheric drag is given by

$$\vec{f} = -\frac{1}{2} C_D \rho \frac{A}{m} \vec{v} |\vec{v}| \quad (2.7)$$

where C_D is the drag coefficient of the satellite, which determines the resistance of an object in a (fluid) environment, m is its mass, and v its velocity relative to the environment. The higher the drag coefficient, the larger the deceleration that acts on the satellite. The atmospheric drag is an important perturbing force at low-enough altitudes. It is sometimes even used in so-called aerobraking maneuvers, where the object enters a highly elliptic orbit around a planetary body with an atmosphere. The object experiences a drag force at pericenter, which causes the apocenter of the orbit to decrease over time, until the orbit of the object has reached its desired orbital shape. Such aerobraking maneuvers may also be used in (high-thrust) de-orbit maneuvers.

The atmospheric density is not constant over time. Seasonal changes can cause the local atmosphere to heat up and expand to higher altitudes. Also diurnal changes due to the rotation of the Earth with respect to the inflow of solar energy causes the atmosphere to expand and move. Solar activities such as the 11-year solar cycle, accompanied by sunspots, also have an effect on the density of the atmosphere. Atmospheric tides induced by the Moon and the Sun cause the atmosphere to bulge as well. An accurate atmospheric model is the NRLMSISE-00 atmospheric model. This model incorporates data from satellite accelerometers, temperature from incoherent radar scattering, and molecular oxygen number density [29] in order to provide a most realistic representation of the atmospheric density. Such accurate models however sometimes lead to a large computational effort during the simulation of the satellite orbit. Less computationally heavy models are therefore sometimes also used such as the exponential atmosphere model, where the atmospheric density decreases exponentially with increasing altitude.

2.4.2. Gravitational irregularities

At first glance, the Earth looks spherical. A close observation however reveals that this is actually not the case. The Earth's surface is littered with numerous irregularities, such as high mountain ranges and deep ocean troughs. All of these irregularities affect the gravitational field of the Earth as *gravitational anomalies*. But most of these gravitational anomalies are not directly visible to the naked eye. The largest gravitational anomaly comes from the equatorial bulge of the Earth. Due to the Earth's rotation about its polar axis, the centrifugal force is greater at the equator than it is near the poles. This centrifugal force then causes the mass of the Earth to bulge by approximately 21 kilometers more at the equator than it does near the poles [34]. This bulge has a significant effect on the Earth's gravitational field. Additionally, the mantle below the Earth's crust is constantly moving, displacing large masses. This movement underneath the Earth's crust causes local accumulation of material, which leads in turn to anomalies in the Earth's gravitational field. Figure 2.6 shows the shape of the Earth, visualized as the gravitational potential at each point on the Earth (also called a geoid) as observed by the Gravity Recovery and Climate Experiment (GRACE) mission, launched by NASA.

The Earth's gravitational field can be described by so-called spherical harmonics, which are special functions defined on the surface of a sphere. A point in space can be represented by the spherical coordinates r , ϕ , Λ . These coordinates represent the distance, geocentric latitude, and geographic longitude, respectively. The gravitational potential U at any point in space, outside the Earth, can be described using associated Legendre functions of the first kind $P_{n,m}(\sin \phi)$ and Legendre polynomials $P_n(\sin \phi)$. The degree n and order m determine the complexity of the representation of the Earth's gravitational field. The gravitational potential of the Earth is given by

$$U = -\frac{\mu}{r} \left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R}{r} \right)^n P_n(\sin \phi) + \sum_{n=2}^{\infty} \sum_{m=1}^n J_{n,m} \left(\frac{R}{r} \right)^n P_{n,m}(\sin \phi) \{\cos m(\Lambda - \Lambda_{n,m})\} \right] \quad (2.8)$$

where μ is the gravitational parameter of the Earth, J_n , $J_{n,m}$ and $\Lambda_{n,m}$ are gravitational model parameters, and R is the radius of the Earth. These spherical harmonics can be divided into three distinct groups: zonal harmonics ($n \neq 0, m = 0$), sectorial harmonics ($m = n \neq 0$), and tesseral harmonics ($m \neq n \neq 0$). Figure 2.7

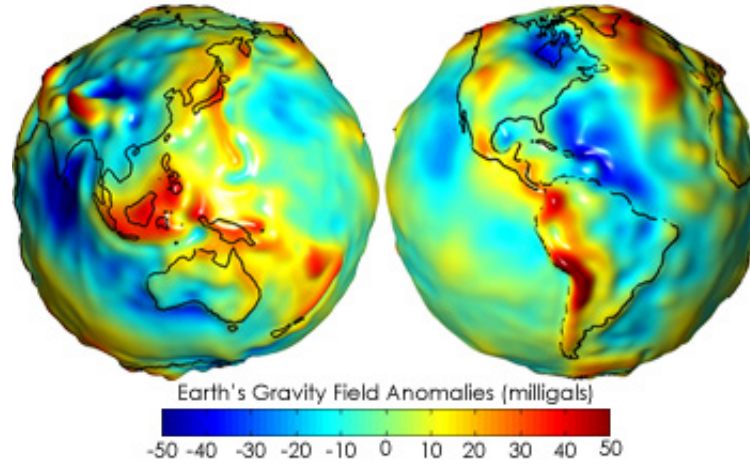


Figure 2.6: The geoid of the Earth as measured by the GRACE mission of NASA. Milligals is a measure of the strength of the gravity field. As an example: in the right figure at the bottom center, the South American Andes mountain range can be seen in red, representing the strength of the gravity field, as there is a large accumulation of mass, which results in a stronger gravitational field [33].

shows a visualization of these three kinds of harmonics. The equatorial bulge of the Earth is a zonal harmonic and is often referred to as the J_2 -effect.



Figure 2.7: From left to right: zonal harmonics, sectorial harmonics, and tesseral harmonics. White areas represent a value above the mean spherical surface and black areas represent a value below the mean spherical surface [32].

2.4.3. Third-body attraction

The Earth is not the only body in the Solar System which exerts a gravitational pull on the satellite during its de-orbit. As the Moon orbits the Earth, its gravitational attraction influences the orbit of satellites. Even the Sun, which is approximately 150 million km away, exerts a significant gravitational pull on satellites over a longer period of time. The strength of this perturbing acceleration depends on the mass of the Earth, the mass of the perturbing body and the distances to the Earth and the perturbing body. The maximum value of the ratio between the acceleration of the perturbing body over the acceleration of the Earth is given by

$$\left(\frac{f_d}{f_E}\right)_{max} = 2 \frac{m_d}{m_E} \left(\frac{r_i}{r_d}\right)^3 \quad (2.9)$$

In this equation, m_d and m_E are the masses of the perturbing body and the Earth, respectively. r_i and r_d are the distances to the center of mass of the Earth and the perturbing body, respectively. Not all bodies in the Solar System have a significant effect on the orbit of a satellite. Some bodies are just too small and/or too far away to have any noticeable effect.

2.4.4. Solar radiation pressure

As the Sun radiates, it emits photons, which travel with the speed of light. As these photons hit a satellite, they transfer some of their momentum onto the satellite, resulting in an acceleration of the satellite. This effect is even larger when the momentum of the reflected photons is taken into account. This effect is called solar radiation pressure. The acceleration that the satellite experiences is

$$\vec{f} = -C_R \frac{WA}{mc} \vec{e}_s \quad (2.10)$$

where C_R is the satellite's reflectivity, W is the energy flux of the light at the position of the satellite, A is the effective cross-sectional area of the satellite, m and c are the mass of the satellite and the speed of light, respectively, and $\vec{e}_s$ is the unit vector from the satellite pointing to the Sun.

2.5. Validation and verification

The reference trajectories computed by Lidtke et al. [18] are recreated and compared. The trajectories use a constant thrust, which varies per object. The propagation of the object is terminated when a specific altitude is reached, which varies per object. The specific impulse the the trajectory with thrust is equal for all objects: 3400 seconds. The orbital frame of reference is the J2000 frame, and is equal for all objects. The initial state, orbital parameters, physical parameters and trajectory parameters object can be seen in Table 2.2. No values were given for the drag coefficient and the cross-sectional area of the objects. The drag coefficient is set to 2.6 for all bodies, representing the drag coefficient of a cylinder (since they are rocket bodies) for a flow perpendicular to the long axis of the cylinder [26]. The cylindrical radius of Zenit-2, Tsyklon-3 and Kosmos-3m is 1.95, 1.5 and 1.2 m, respectively. The cross-sectional area is then the area of the sides of half a cylinder, which is 39.1, 9.8 and 13.0 m², for Zenit-2, Tsyklon-3 and Kosmos-3m, respectively.

	Zenit-2	Tsyklon-3	Kosmos-3m
Frame of reference	J2000	J2000	J2000
Initial epoch (UTC)	1 Jan 2020 00:00:00	2 Jun 2015 00:00:00	1 Jan 2017 00:00:00
X (km)	7183.000	6997.000	7019.860
Y (km)	0.000	0.000	0.000
Z (km)	0.000	0.000	0.000
V _x (km/s)	0.000	0.000	0.000
V _y (km/s)	-1.076	0.986	-1.051
V _z (km/s)	7.375	7.492	7.475
Inclination [deg]	98.3	82.5	98.0
Altitude [km]	805	619	667
Eccentricity	0.0011	0.0024	0.0027
Mass [kg]	8900.0	1407.0	1434.0
Radius [m]	6.377	2.082	3.447
Thrust [mN]	25	30	25
Specific impulse [s]	3400	3400	3400
Termination altitude [km]	550	450	300

Table 2.2: Specifications of the three different rocket bodies, provided by Lidtke et al. [18].

The trajectories are validated for their mission duration and propellant use. This is to ensure that the computed trajectories are the same as the reference trajectories presented by Lidtke et al. [18]. The values for the mission duration and the used propellant mass are presented in Table 2.3 (the specifics about the integration and propagation method can be seen in Section 4.8).

Target	Low-thrust	Duration [days]	Propellant mass [kg]
Zenit-2	Reference	593.14	34.8
	Computed	547.72	35.4
Tsyklon-3	Reference	174.25	6.6
	Computed	48.02	3.7
Kosmos-3m	Reference	121.24	7.6
	Computed	120.19	7.8

Table 2.3: Comparison between the computed and reference low-thrust trajectories for the three different objects. The values for the reference trajectories are obtained from Lidtke et al. [18].

As can be seen in Table 2.3, the computed values for the propellant use and duration do not match the reference values obtained by Lidtke et al. exactly. The values for Zenit-2 and Kosmos-3m match the reference data within an error margin of 10%. The computed durations for Tsyklon-3 do not match the reference values with a factor of almost 4. Some of these divergences may be explained due to unknown environment models used in the reference data. These divergences are further elaborated upon in Section 5.4. Additionally, due to computational restrictions, the analysis for the Zenit-2 object is performed using a time-step of 180 seconds instead of the time-step of 90 seconds as for the other two objects. This results in a value for the duration of the orbit of 427.64 days with a used propellant mass of 27.7 kg. Since the computed data does not match the reference data, no proper comparison can be made between the obtained results and the reference data. Therefore, the comparison will be made against the computed trajectories with a constant thrust. The reason for choosing to continue with the computed trajectories is that the three objects still represent a wide spectrum of possible de-orbit scenarios.

3

Conjunction Analysis

When two objects come close to each other, it is called a close- conjunction. The computation of the collision risk associated with close conjunctions between two objects is called a *conjunction analysis*. During the de-orbit of a satellite, the conjunction analysis is performed continuously between the satellite and a debris object, which is referred to as an object-pair. In this analysis, multiple object pairs are analyzed, and the collision risk between those pairs is determined at every time-step. Due to the heavy computational effort associated with analyzing thousands of object pairs at every time-step, multiple filters are introduced that filter out object-pairs for certain time-steps, based on their relative distance. The collision risk between object-pairs that pass through these filters is then calculated. The first part of this chapter explains the different filters used in the conjunction analysis and the motivation behind their usage. The second part of this chapter explains the calculation of the collision risk between object-pairs. The theory in this chapter is largely summarized from Klinkrad [15], unless otherwise indicated.

3.1. Filtering methods

Analyzing thousands of object-pairs (debris and active satellites in LEO) for each time-step over a period of multiple years requires a large computational effort. However, for most of the time in the trajectories of the object-pairs it is unnecessary to analyze the collision probability. When the two objects in a pair are on opposite sides of the Earth at a certain time-step, it is obvious that they will not collide with each other at this particular time-step. This principle holds the same for many other object-pair positions. In order to reduce the computational effort, *filtering methods* are introduced to filter out object-pairs for which it can be stated with certainty that they will not collide within that time-step. There exist many different filtering methods to help reduce the computational effort. It is however important that no information is lost by reducing for instance the wrong object-pair are certain time-steps. Leloux [17] reviewed several filtering techniques and assessed their accuracy and computational efficiency. He suggests using a sequence of filters with improving level of detail: the perigee-apogee filter and a number of smart-sieve filters. These filters are described in the following sections.

3.1.1. Perigee-apogee filter

The perigee-apogee filter is the first filter in the sequence of filtering methods. This filter uses the respective perigee and apogee radii of the object-pair. Plotted in two 2D: if the orbit of one of both objects lies completely within the orbit of the other object, their orbits do not overlap, and thus their objects will never cross, meaning that there is no chance of a collision. If so, the object-pair is filtered. If, on the other hand, the orbits of the objects do overlap, their orbits might cross, and thus there is a chance of a collision, meaning that the pair is not filtered. Figure 3.1 shows the geometry of these two situations. The primary orbit describes the orbit of the satellite that is analyzed. The two candidates represent two different debris orbits. The orbit of the filtered candidate never crosses the orbit of the primary orbit, and the pair is therefore filtered out for the rest of the analysis. The orbit of the accepted candidate crosses the primary orbit two times, and the pair is therefore not filtered out and proceeds to the next step of the analysis. The advantage of the perigee-apogee filter is that it only requires the perigee and apogee data of both object-pairs. The orbits of the active LEO satellites are assumed to be Keplerian, meaning that their perigee and apogee stay constant over time (see Section 4.4).

The perigee and apogee of the debris object might change over time (due to introduced thrust), but keeping track of the perigee and apogee of one object over time requires less computational effort than keeping track of all active satellites in LEO.

The perigee-apogee filter was first described by Hoots et al. [12] and states that the pair of the debris and the satellite can be discarded when the separation distance d is smaller than the difference between the larger of the two perigees r_p and the smaller of the two apogees r_a .

$$d < r_p - r_a \quad (3.1)$$

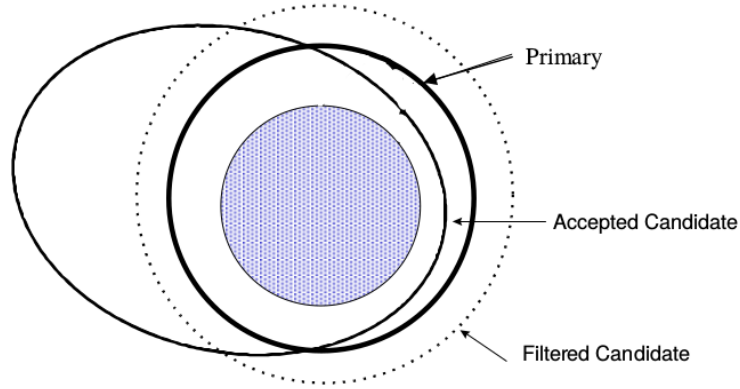


Figure 3.1: Geometry of the perigee-apogee filter. The blue circle represents the central body [36].

The perigee and apogee of the debris objects can be derived directly from the TLE data (see Appendix A). This is also the fastest way to derive the perigee and apogee values [17]. These values do not change over time, since the orbits of the debris objects are assumed to be Kepler orbits. The perigee-apogee filter is for this analysis not applied to pairs with Kepler orbits however, but between debris objects and the satellite, which is performing a de-orbit maneuver. The perigee and apogee of the satellite changes over time when a thrust is applied. This means that the orbit of the satellite might cross altitude regions where the perigee and apogee previously had not crossed with certain debris objects. The perigee-apogee filter is therefore constantly evaluated during the de-orbit maneuver, in order to include pairs that might have been filtered out in previous time-steps.

3.1.2. Smart-sieve filters

The second type of filtering methods are the *smart-sieve methods*. These methods are applied to the object pairs that pass the perigee-apogee filter. The smart-sieves filter out object-debris pairs based on their ephemerides, reducing the number of pairs that pass to the conjunction analysis, thus reducing the computational effort. These sieves are largely based on the position and velocity of both objects per pair, and check whether the objects are in close conjunction of each other per time-step. A sequence of sieves is applied in order to reduce the search space as quickly as possible. The smart sieves discussed below are largely summarized from Alarcón Rodríguez et al. [1], unless otherwise indicated.

The sieves described below make use of the geometry between two objects during the close conjunction. The critical distance between objects R_{cr} corresponds to the residual accepted risk level, and defines the radius of the critical volume surrounding each object. The value of the critical distance is set to 25 km [15]. The threshold radius R_{th} of an object is the radius of the volume required for another object to come within the critical volume between time-steps. This means that, if another object is located just outside the threshold volume of a particular object, then after one time-step, it can never reach the critical volume of the particular object. The threshold volume is therefore dependent on the relative velocity between the two objects and the size of the time-step. Figure 3.2 shows the geometry of a satellite surrounded by its critical and threshold volume. An improved description of the threshold volume is described by Leloux [17] and is defined as

$$R_{th}^2 = R_{cr}^2 + v_{esc}^2 (\Delta t)^2 \quad (3.2)$$

where v_{esc} is the escape velocity with respect to the Earth¹ and Δt is the time-step.

¹The escape velocity serves as an upper bound for closed orbits around the Earth. The real velocity of a spacecraft will always be smaller.

Step skipping When the radius r of the satellite-debris pair is very large at a particular time-step (e.g. they are located at opposite sides of the Earth), it can be safely assumed that they will not collide within that time-step or in a number of following time-steps. In order to limit the computational effort, a number of time-steps N_{skip} can be skipped. This method does not filter out satellite-debris pairs, but rather limits the computational effort of the complete analysis. This dimensionless integer is defined by [17] as

$$N_{skip} = \text{int} \left(\frac{r - R_{th}}{2v_{esc}\Delta t} \right) \quad (3.3)$$

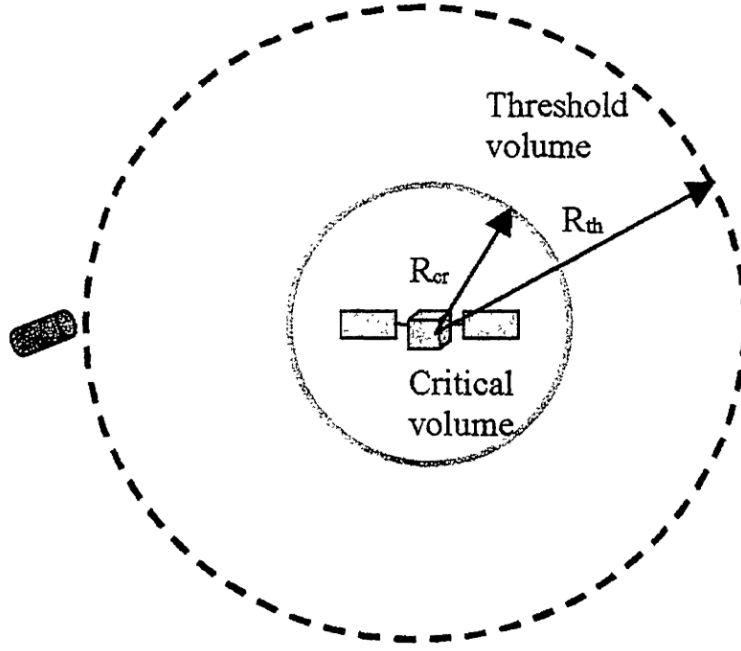


Figure 3.2: Schematic of the definition of the threshold radius and the critical radius defined by the maximum velocity of two objects. The object in the middle of the schematic is a satellite, and the cylindrical shape on the left is a piece of debris [1].

r^2 sieve The first sieve is the r^2 sieve. This sieve filters out pairs if the square of the relative distance Δr is larger than the square of the threshold radius of the satellite.

$$(\Delta r)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 > R_{th}^2 \quad (3.4)$$

Minimum distance sieve The second sieve is the minimum-distance sieve. This sieve describes an extra volume around the critical volume which is the acceleration safety volume. The principle of this sieve is that the relative acceleration between two objects can never exceed the combined maximum gravitational acceleration g_{max} of both objects

$$g_{max} = \frac{g_{max,debris} + g_{max,satellite}}{2} \quad (3.5)$$

The radius of this acceleration volume R_{acc} is described by Leloux [17] as

$$R_{acc} = R_{cr} + \frac{1}{4} g_0 (\Delta t)^2 \quad (3.6)$$

The minimum-distance sieve filters out object pairs if their relative minimum distance Δr_{min} squared is larger than the acceleration radius squared.

$$(\Delta r_{min})^2 > R_{acc}^2 \quad (3.7)$$

The minimum distance between two objects was determined by Healy [11] and is

$$(\Delta r_{min})^2 = (\Delta r)^2 - \left(\Delta \vec{r} \cdot \frac{\Delta \vec{v}}{\Delta v} \right)^2 \quad (3.8)$$

where Δv is the relative speed between the two objects. Figure 3.3 shows the geometry of the acceleration safety volume and the minimum distance radius.

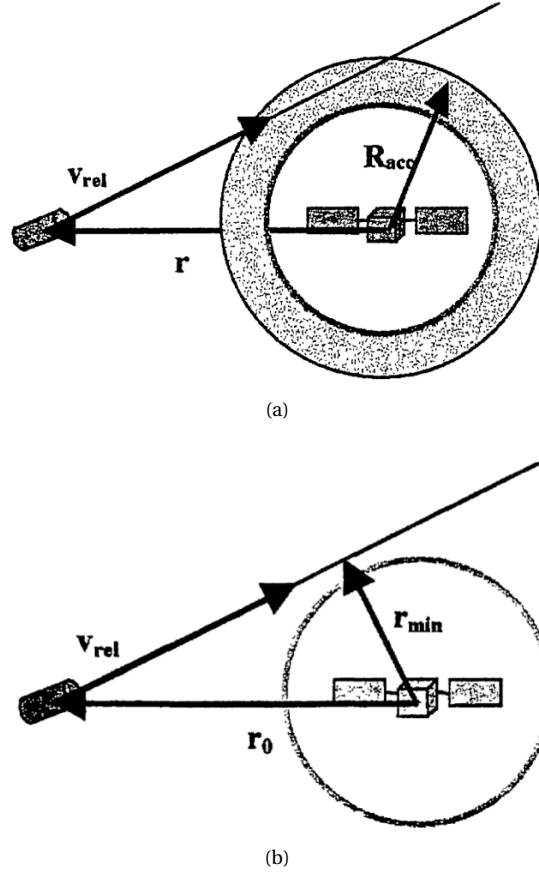


Figure 3.3: Acceleration safety volume and minimum radius: (a) shows a schematic of the geometry indicating the acceleration safety volume with radius R_{acc} ; and (b) shows a schematic of the minimum radius between two objects where the subscript 0 indicates the time when the states of both objects are determined. The object to the left is a piece of debris [1].

Fine r^2 sieve The fine r^2 sieve is similar to the r^2 sieve in that it filters object pairs based on a threshold radius. This fine threshold radius $R_{th,fine}$ uses the acceleration radius and is defined as [15]

$$R_{th,fine} = R_{acc} + \frac{1}{2} \left| \Delta \vec{v} \cdot \frac{\Delta \vec{r}}{\Delta r} \right| \Delta t \quad (3.9)$$

The fine r^2 sieve filters pairs when

$$(\Delta r)^2 > R_{th,fine}^2 \quad (3.10)$$

3.2. Collision probability

The next step in the conjunction analysis is to determine the collision probability for pairs that are not filtered out using the previously described methods. This probability is calculated using the relative position and velocity of the pair at the time of closest approach (tca). The following calculation is based on a number of assumptions which will be repeated in the analysis below [15].

- The uncertainty in the position of the object can be described as a three-dimensional Gaussian distribution, which means that the collision probability can be described by a probability density function.
- The uncertainty in the position of both objects is constant during the close conjunction, which means that the error covariance matrix is constant at close conjunction.
- The objects in close conjunction move in rectilinear paths at constant velocities, which means that the objects in close conjunction can be treated as a snapshot in time.
- The uncertainty in the position of the two objects in a pair is not correlated, which means that the covariance matrices of the two objects in a pair can be added to form a single covariance matrix.

The probability of a collision is calculated using an *error covariance matrix* of the object-pair. This matrix describes the correlation between the uncertainties in the ephemerides of the objects. The covariance matrix of both objects is determined and then combined to form a single error covariance matrix of the pair. Next, a probability density function is determined, which describes the probability distribution of a collision between the two objects. This three-dimensional problem is reduced to a two-dimensional problem, by mapping the error covariance matrix onto a B-plane (body-plane) located at the center of the satellite. Using the B-plane representation, probability can be determined by performing a double integration of the (now two-dimensional) probability density function. The full derivation of this calculation is described below and is largely summarized from Klinkrad [15], unless stated otherwise.

3.2.1. Error covariance matrix

An error covariance matrix describes the correlation between the uncertainties in the observed parameters of the ephemeris of an object (e.g. position, velocity). This matrix is separately determined for both the debris object and the user satellite. As the ephemerides of both objects propagate over time, so does the error in the ephemerides. However, propagating the error covariance matrix over longer periods of time (e.g. 25 years) will result in errors that are several orders of magnitude larger than the errors determined at the start of the analysis (the order of magnitude can even grow to the physical size of the orbit). The ephemerides of objects in LEO are known by a Space Surveillance and Tracking (SST) system to some accuracy. Periodic updates allows the error covariance matrix to remain relatively constant over time. ESA has defined a maximum position error of an object in LEO of 40, 200 and 100 m in the radial, along-track and cross-track direction, respectively, which the European Space Surveillance System (ESSS) will provide. This standard allows the assumption that the error covariance matrix remains constant over time [5]. Since the errors in the covariance matrix are assumed to be constant over time, it is also assumed that the uncertainty between the elements of the state vector are uncorrelated, resulting in the following covariance matrix of object i

$$\hat{C}_i = \begin{bmatrix} \sigma_U^2 & 0 & 0 \\ 0 & \sigma_V^2 & 0 \\ 0 & 0 & \sigma_W^2 \end{bmatrix} \quad (3.11)$$

where σ_U , σ_V and σ_W are the standard deviations in the radial, along-track and cross-track direction, respectively.

The individual error covariance matrices are then combined to form a single covariance matrix, under the assumption that the errors between the ephemerides of both objects are not correlated with each other. This combined covariance matrix includes the position error of both objects in one common frame of reference. Each of the covariance matrices are described in Hill's frame of reference [30]. A transformation is therefore required between their local frame of reference to a common inertial frame of reference, which is a mean equatorial system of 2000.0 (J2000) in this case. The J2000 frame is described by the coordinates X, Y and Z, where the X-axis points to the vernal equinox, the Z-axis points to the Conventional International Origin (CIO pole), and the Y-axis completes the right-handed reference frame. The orientation of the satellite's local frame of reference is described by U, V and W, where U points in the radial outward direction, V points in

the along-track direction, and W points to the cross-track direction, completing the right-hand coordinate system. Figure 3.4 shows the geometry of this coordinate system.

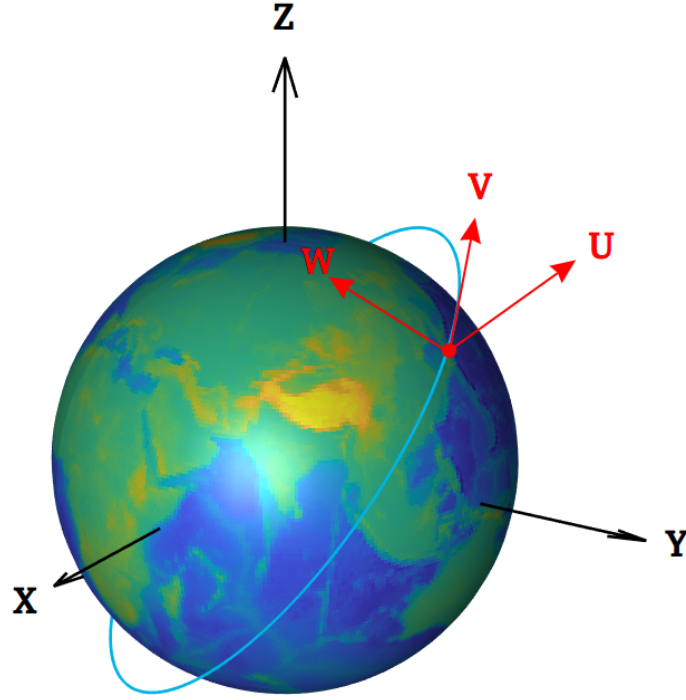


Figure 3.4: Geometry of Hill's frame of reference. The black coordinate system indicates the inertial reference frame, and the red coordinate system indicates the local Hill frame of the object in orbit.

The unit vectors of the object at an arbitrary location in its orbit can be found by

$$\vec{U} = \frac{\vec{r}}{|\vec{r}|}, \quad \vec{W} = \frac{\vec{r} \times \vec{v}}{|\vec{r} \times \vec{v}|}, \quad \vec{V} = \vec{W} \times \vec{U} \quad (3.12)$$

where $\vec{r}$ and $\vec{v}$ are the satellite's position and velocity vector in the inertial reference frame. It should be noted that the transverse direction does not always point in the direction of the velocity vector. This is only the case for circular orbits and for eccentric orbits at peri- and apocenter. These unit vectors can be used to construct an orthogonal transformation matrix $\hat{R}_{U,V,W}$ that allows a transformation of the error covariance matrix between the local and common reference frame. The error covariance matrix in the inertial reference frame of an object i is then given as

$$\hat{C}_i = \hat{R}_{U,V,W}^T \hat{C}_{U,V,W} \hat{R}_{U,V,W} \quad (3.13)$$

where $\hat{R}_{U,V,W}$ is the transformation matrix

$$\hat{R}_{U,V,W} = \begin{bmatrix} U_X & U_Y & U_Z \\ V_X & V_Y & V_Z \\ W_X & W_Y & W_Z \end{bmatrix} \quad (3.14)$$

After the transformation of both covariance matrices from their local Hill frame to a common inertial frame, they can be combined to form one single covariance matrix. It is assumed that the position errors between two objects are uncorrelated [7]. This means that the individual covariance matrices of both objects can be combined by addition. If $\hat{C}_{sat}$ is the covariance matrix of a satellite in the ECI frame and $\hat{C}_{deb}$ is the covariance matrix of a debris object in the ECI frame, then the combined error covariance matrix is

$$\hat{C} = \hat{C}_{sat} + \hat{C}_{deb} \quad (3.15)$$

3.2.2. Probability density function

When two objects are in near conjunction, their paths can be assumed to be rectilinear, ignoring the bending effect of gravity. This is an important assumption in order to simplify the problem, and it allows for the trajectories at closest approach to be seen as a snapshot in time. This assumption allows for the projection of the covariance matrix onto the B-plane [5].

Using this assumption, a probability density function (PDF) can be described. This PDF uses the relative conjunction position and velocity at the time of closest approach t_{tca} , which is described in the inertial frame by

$$(\Delta \vec{r}_{tca})_{X,Y,Z} = (\vec{r}_{sat}(t_{tca}) - \vec{r}_{deb}(t_{tca}))_{X,Y,Z} \quad (3.16)$$

$$(\Delta \vec{v}_{tca})_{X,Y,Z} = (\vec{v}_{sat}(t_{tca}) - \vec{v}_{deb}(t_{tca}))_{X,Y,Z} \quad (3.17)$$

where $\vec{r}_{sat}(t_{tca})$, $\vec{v}_{sat}(t_{tca})$, $\vec{r}_{deb}(t_{tca})$, and $\vec{v}_{deb}(t_{tca})$ are the position and velocity vectors of the satellite and debris object at t_{tca} , respectively. The time of closest approach t_{tca} is determined as the time point where the range-rate $\dot{\rho}$ between a satellite and the debris object is zero. The accuracy of this, however, is constrained by the size of the time-step (see Section 3.3). The range-rate at t_{tca} is defined as

$$\dot{\rho}_{tca} = \frac{\Delta \vec{r}_{tca} \cdot \Delta \vec{v}_{tca}}{\Delta r_{tca}} = 0 \quad (3.18)$$

For readability, the subscript indicating the inertial frame of reference will be left out in future equations. The PDF uses as inputs the relative position between a satellite and the debris object at close conjunction

$$\Delta \vec{r}(t) = \Delta \vec{r}_{tca} + \Delta \vec{v}_{tca} \Delta t_{tca} \quad (3.19)$$

where $\Delta t_{tca} = t - t_{tca}$, and is described as

$$p(\Delta \vec{r}) = \frac{1}{\sqrt{(2\pi)^3 \det(\hat{C})}} \exp \left[-\frac{1}{2} \Delta \vec{r}^T \hat{C}^{-1} \Delta \vec{r} \right] \quad (3.20)$$

A collision between two objects can be described as an intersection of their respective volumes. This volume is determined by the combined radii of the two objects, which is called the critical radius R_c , and is determined by the physical size of the object. It should be noted that this critical radius R_c is different from the critical radius R_{cr} described in Section 3.1.2. An intersection of two respective volumes can be described as one larger volume, the critical volume V_c , in which a close-conjunction occurs. The top-left image in Figure 3.5 shows a schematic view of the collision cross-section between two objects. The parameters that describe this critical volume are

$$R_c = R_{sat} + R_{deb}, \quad A_c = \pi R_c^2, \quad V_c = \frac{4}{3} \pi R_c^3 \quad (3.21)$$

The probability that a collision occurs can be determined by applying a volume integral to the PDF shown in Eq. (3.20) over the critical volume described in Eq. (3.21) and is given as

$$P_c = \frac{1}{\sqrt{(2\pi)^3 \det(\hat{C})}} \int_{(V_c)} \exp \left[-\frac{1}{2} \Delta \vec{r}^T \hat{C}^{-1} \Delta \vec{r} \right] dV \quad (3.22)$$

In order to further reduce the number of dimensions from three to two, this volume integral can be reduced to a surface integral by mapping the error ellipsoid shown in Figure 3.5 onto the B-plane (body plane), which is perpendicular to the relative velocity vector at t_{tca} and parallel to the relative position vector at t_{tca} . The contour lines shown in Figure 3.5 represent the lines of constant probability. A transformation matrix $\hat{R}_{X_B, Y_B}$ is constructed which uses the unit vectors

$$\vec{X}_B = \frac{\Delta \vec{r}_{tca}}{|\Delta \vec{r}_{tca}|} \quad \text{and} \quad \vec{Y}_B = \frac{\Delta \vec{r}_{tca} \times \Delta \vec{v}_{tca}}{|\Delta \vec{r}_{tca} \times \Delta \vec{v}_{tca}|} \quad (3.23)$$

to get

$$\hat{R}_{X_B, Y_B} = \begin{bmatrix} X_{B,X} & X_{B,Y} & X_{B,Z} \\ Y_{B,X} & Y_{B,Y} & Y_{B,Z} \end{bmatrix} \quad (3.24)$$

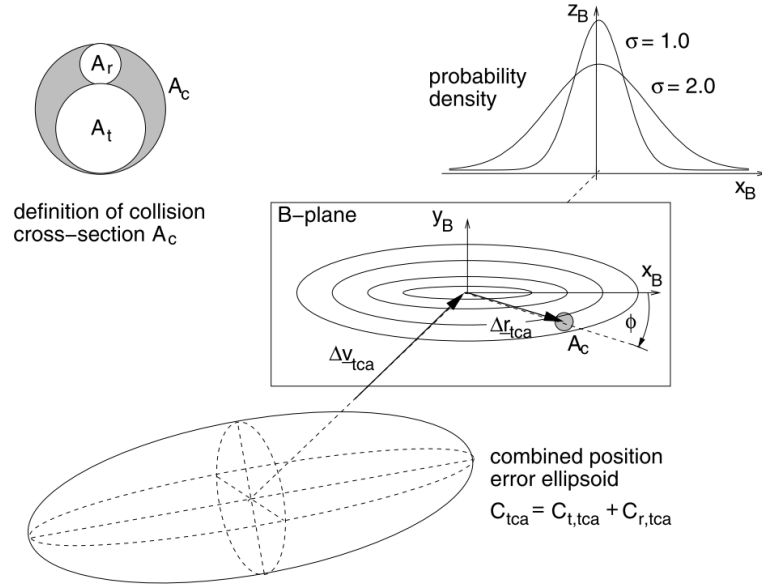


Figure 3.5: Geometry of the mapping of the three-dimensional error ellipsoid, created using the error covariance matrix onto the B-plane. The collision cross-section is defined as the critical cross-sections of a target and a risk object (user satellite and space debris). The contour lines of the two-dimensional error ellipse on the B-plane represent contours of constant probability [15].

which maps the three-dimensional covariance matrix onto the two-dimensional B-plane as

$$\hat{C}_B = \hat{C}_{X_B, Y_B} = \hat{R}_{X_B, Y_B} \hat{C} \hat{R}_{X_B, Y_B}^T \quad (3.25)$$

The orientation of the main axes of the elliptical contours are determined from the eigenvalues $\lambda_{i,B}$ and corresponding eigenvectors $\vec{e}_{i,B}$, which solve

$$(\hat{C}_B - \lambda_{i,B} \hat{I}) \vec{e}_{i,B} = \vec{0} \quad (3.26)$$

where $\hat{I}$ is the identity matrix. The semi-major axis $a_{i,B}$ and semi-minor axis $b_{i,B}$ are defined as

$$a_{i,B} = \sqrt{\max(\lambda_{1,B}, \lambda_{2,B})} \quad b_{i,B} = \sqrt{\min(\lambda_{1,B}, \lambda_{2,B})} \quad (3.27)$$

The unit vectors of the orientation of the equal-probability ellipse in the B-plane are defined as

$$\vec{x}_B = \frac{\vec{e}_{a,B}}{|\vec{e}_{a,B}|} \quad \text{and} \quad \vec{y}_B = \frac{\vec{e}_{b,B}}{|\vec{e}_{b,B}|} \quad (3.28)$$

where $\vec{e}_{a,B}$ and $\vec{e}_{b,B}$ are the directions of the semi-major and semi-minor axes of the ellipse, respectively. The volume integral shown in Eq. (3.22) can be reduced to a two-dimensional integral centered at the predicted location of closest approach $\Delta \vec{r}_{tca}$,

$$P_c = \frac{1}{2\pi \sqrt{\det(\hat{C}_B)}} \int_{-R_c}^{R_c} \int_{-\sqrt{R_c^2 - x_B^2}}^{\sqrt{R_c^2 - x_B^2}} \exp \left[-\frac{1}{2} \Delta \vec{r}_B^T \hat{C}_B^{-1} \Delta \vec{r}_B \right] dy_B dx_B \quad (3.29)$$

where $\Delta \vec{r}_B$ is a conjunction position in the B-plane, which results from the transformation

$$\Delta \vec{r}_B = \hat{R}_{X_B, Y_B} \Delta \vec{r} \quad (3.30)$$

Assuming that the probability density is constant over the circle defined by R_c , the collision probability can then be approximated by [2]

$$P_c = \frac{R_c^2}{2\sqrt{\det(\hat{C}_B)}} \exp \left[-\left(\frac{\Delta \vec{r}_B^T \hat{C}_B^{-1} \Delta \vec{r}_B}{2} \right) \right] \quad (3.31)$$

During the de-orbit of the satellite, many conjunctions occur with various different objects. Most of these conjunctions yield a relatively low collision probability (Figure 3.6). During the de-orbit the satellite accumulates the probability of collisions, which results in a final *accumulated collision probability*.

3.3. Close approach

Using large time-steps during the state propagation of the satellite and debris decreases the CPU time, but is also less accurate. According to Leloux [17], the optimal time-step, using the filters shown in Section 3.1 is $\Delta t = 90$ seconds. This time-step ensures the best CPU time, while retaining a relatively high accuracy. Although a relative high accuracy is retained, when two objects move at relative speeds of ~ 14 km/s, in 90 seconds they would have travelled a relative distance of 1260 kilometer. Finding the point of close conjunction between two objects is therefore done by calculating the transition rate, using an iterative Newton scheme.

Assume that two objects pass through all the filters and sieves, and thus at some point come into close conjunction. At time $t = t_0$, they are at an arbitrary distance moving towards each other, and at time $t = t_1$ they have passed each other and moving away from each other. In between these two time-steps of 90 seconds, the close conjunction occurred. To find this close conjunction, the relative position of the objects can be determined by the relative position $\Delta \vec{r}$ and relative velocity $\Delta \vec{v}$ at t_0 . Because of rectilinearity (see Section 3.2.2) the objects are assumed to move in straight lines at constant velocities, and the relative position between the two objects can be calculated by

$$\Delta r(t) = \Delta r(t_0) + \Delta t \Delta v(t_0) \quad (3.32)$$

Unlike the regular propagation of states, Δt in Eq. (3.32) uses a time-step of 1 second. This allows for a better accuracy of the separation distance between the two objects. Because this equation is only applied on objects that pass all the filters, the algorithm retains most of its speed. The time when two objects are in close conjunction with each other is determined by the range-rate between the objects. The range-rate is the rate at which objects move away or towards each other. At a zero-transition of the rate-rate time history (Eq. (3.33)), the objects are closest to each other.

$$\Delta \vec{v}_{tca} \cdot \frac{\Delta \vec{r}_{tca}}{\Delta r_{tca}} = \dot{\rho}_{tca} = 0 \longrightarrow t_{tca} \quad (3.33)$$

Figure 3.6 shows an example of the range-rates and separation distances between several satellite-debris pairs, using this method.

3.4. Validation and verification

Before the theory explained in this chapter can be applied to the de-orbit strategy, it needs to be validated and verified. First, the filters and sieves will be tested against data obtained from several papers using the same technique. Then, the collision risk will be calculated and tested for the same data set, as shown in the papers discussed in this section.

3.4.1. Filters

The filters can be tested against data from Alarcón Rodríguez et al. [1]. In their paper they test the filters for the entire catalog of objects against itself for a 24-hour period on May 15, 2002, using a critical distance R_{cr} of 25 kilometers. A time-step of 180 seconds was used, and the sieves were analyzed in consecutive order. The size of the catalog used by Alarcón Rodríguez et al. is not known, and therefore the results do not necessarily match. In their research, Alarcón Rodríguez et al. also apply extra filters, which were not used in this report. Table 3.1 shows the number of rejected pairs over the number of analyzed pairs (pairs that passed the filters) in percentages of the total number of pairs as computed using the theory described above and the data from Alarcón Rodríguez et al.. It can be seen that the percentages of the computed filters match the percentages of the filters by Alarcón Rodríguez et al. reasonably well (deviations within 12%) except for the fine r^2 filter. This may be caused by the small catalog size used in this validation. Additionally, Alarcón Rodríguez et al. use three extra filters before applying the fine r^2 filter, which influences the results of the filters.

Another validation of the sieves can be made by looking at the number of conjunctions detected in one month for the ISS against a snapshot of the entire LEO catalog. The results obtained by Alarcón Rodríguez et al. [1] used the catalog and position of the ISS at May 15, 2002. The results are shown in Table 3.2. It can be

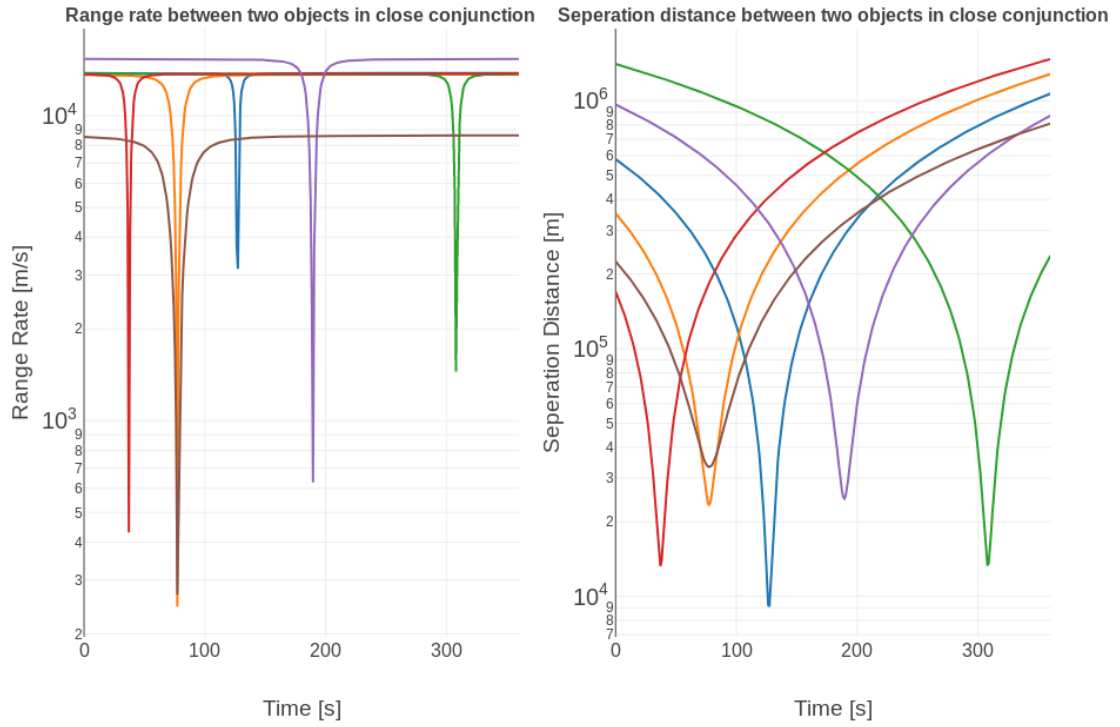


Figure 3.6: Range-rate and separation distance between several satellite-debris pairs.

Method	Rejected/analyzed [%]	Alarcón Rodríguez et al. [1] [%]
Perigee-Apogee	62.9	59.4
r^2	38.6	34.5
r_{min}	91.8	90.4
fine r^2	7.5	41.9

Table 3.1: Rejected over analyzed pairs in percentages of 311 satellites, investigated against themselves.

Catalog	Number of conjunctions	Alarcón Rodríguez et al.
2002-05-15	87	7

Table 3.2: Number of conjunctions detected for the reference and computed data using two different catalogs.

seen that the computed number of detected conjunctions varies with a order of magnitude with the number of conjunctions detected by Alarcón Rodríguez et al..

From the results shown in Tables 3.1 and 3.2, it can be seen that there are some discrepancies between values of the computed filters and sieves and the filters and sieves used by Alarcón Rodríguez et al.. The reason for these discrepancies can be explained by a number of things. First, Alarcón Rodríguez et al. use four more filters in their analysis, than used in this report. Second, the exact database from May 15, 2002 used by Alarcón Rodríguez et al. is unknown. The used data was obtained from www.space-track.org, but small differences in the data may be present. Third, the numerical tools used by Alarcón Rodríguez et al. is unknown (e.g. the used environment model or integration method), which may lead to variations in the state history of the computed and reference orbits. These differences in the analysis may all contribute to the differences observed in Tables 3.1 and 3.2.

3.4.2. Collision probability

The validation of the collision probability can be done using the data from Lidtke et al. [18]. The TLE's of the user satellites were from a snapshot at October 23, 2013, and November 7, 2014. They used three differ-

Target	Trajectory	23 October 2013			7 November 2014		
		Acc. P_c	Highest single contrib. [%]	No. conj.	Acc. P_c	Highest single contrib. [%]	No. conj.
Zenit-2	Computed	4.3×10^{-3}	76.2	14304	2.2×10^{-3}	25.4	15045
	Reference	1.9×10^{-3}	43.7	9645	8.3×10^{-4}	69.0	9484
Tsyklon-3	Computed	3.8×10^{-4}	99.0	9057	1.8×10^{-3}	56.8	11685
	Reference	2.1×10^{-4}	27.1	10102	2.5×10^{-4}	59.1	11492
Kosmos-3m	Computed	1.6×10^{-3}	52.6	7256	5.3×10^{-3}	38.2	10374
	Reference	2.5×10^{-4}	30.0	9422	7.4×10^{-4}	30.2	9805

Table 3.3: Comparison with the reference data from Lidtke et al. [18] against the computed trajectories. The percentages shown in this table are rounded to one decimal point.

ent debris objects, with specified initial states. Table 3.3 shows the accumulated collision probabilities ('Acc. P_c '), the highest single-event contribution ('Highest single contrib.') and the number of conjunctions ('No. conj.') for the computed and reference trajectories for the different objects using two different TLE snapshots. The accumulated collision probability is the collision probability of each single conjunction added to obtain a final collision probability associated with the entire trajectory. The conjunction event that produced the highest individual collision probability compared to all other conjunction events is called the highest single event contribution. The total number of encountered conjunctions (after passing the filters) for the entire trajectory is called the number of conjunctions. The objects were propagated for a (sidereal) year without any thrust. It can be seen that there is a good correspondence between the accumulated collision probabilities of the computed trajectories and the reference trajectories. Additionally, there is a good correspondence between the number of encountered conjunctions for all objects for both epochs. This is an indication that the filters and sieves perform the reasonably the same for the computed and reference trajectories. An exception may be seen in the number of encountered conjunctions of the Zenit-2 object for both epochs, where the number of encountered conjunctions is higher for the computed trajectories than for the reference trajectories. The small variations observed in Table 3.3 can be explained by differences in the numerical tools used for the reference trajectories (e.g. the used environment model), which were unknown. These differences may lead to variations in the state history of the objects over time. As will become clear in Chapter 5, the collision probability is very sensitive to small changes in the state vector, leading to variations between the reference and computed trajectories.

4

Methodology

This chapter presents the methodology used for this research. The goal of this research is to determine a robust method to de-orbit end-of-life satellites. Different trajectories of three different objects (Zenit-2, Tsyklon-3 and Kosmos-3m) will be investigated and the associated propellant mass, trajectory duration and collision probability are determined. This analysis will be performed twice, using two different snapshots of the debris population. The details of this analysis are presented in the following chapter.

4.1. General outline

The process from obtaining TLE's for objects in LEO to propagating the de-orbit trajectory and determining the collision risk requires several steps. This section will go through each of the steps taken in the process in chronological order. The general steps taken in this research are shown in Figure 4.1.

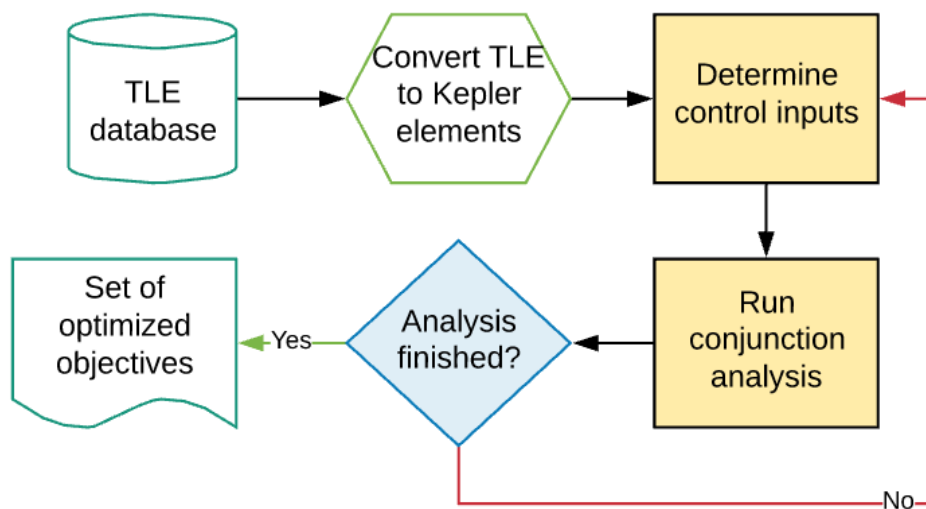


Figure 4.1: Flow-diagram of the general outline of the research.

The available information about the position and velocity of the debris objects in LEO is stored in a database, managed by NORAD, which can be accessed via an Application Programming Interface (API). The information about this database and API can be found on www.space-track.org. The API request can be modified specifically to the types of TLE's specific to the needs of the user. In this case, the TLE's of all objects in LEO that have not yet decayed (i.e. the potential collision objects) is requested. The data are stored in a text file, where the information is presented in the TLE format. For usability, the TLE format is converted to a Kepler element format. This is done via the SGP4 python package (see Section 4.7). The Kepler elements of each object are stored in an array and saved to a text file. This text file now contains the initial Kepler states of each object in LEO, and can be used as an input in the remainder of the analysis. The analysis is performed

using a control algorithm that controls the thrust magnitude and the engine activation times. The values for the inputs of the control algorithm are determined by an optimization algorithm (see Section 4.5). This optimization algorithm tries to find the best combination of inputs to the trajectory (thrust magnitude, time at which the engine is turned on or off), which results in trajectory solutions with a minimum propellant usage, propagation time, or risk. Because trajectories with a shorter propagation time often use a higher thrust magnitude (and thus more propellant), no absolute optimal trajectory is possible. A trade off between the propellant usage, duration and collision probability is made. The optimization algorithm runs for several generations, trying to find trajectories with better combinations of these objectives. The text file with the Kepler elements is loaded into the program and the conjunction analysis is started. The conjunction analysis uses the debris Kepler elements and the computed trajectory of the satellite (using the input parameters defined by the optimization algorithm) as inputs and calculates the associated collision risk. The details of the collision probability calculation can be found in Chapter 3. After the conjunction analysis has completed, the results are fed back to the optimizer, which can start a new optimization loop. If the final optimization loop has been completed, the loop ends and the final optimized trajectories are saved into a text file. This text file consists of several different solutions of trajectories (thrust magnitude and different epochs of turning the engine on/off) and corresponding values of the objectives (propellant usage, propagation time and collision probability).

This entire process from retrieving the TLE's to determining the collision risk is repeated several times during this analysis. The analysis is performed on two different snapshots of the debris environment, since the results are expected to depend on the exact debris population geometry. The first set of TLE data is the data from 23 October 2013, and the second set of TLE data is the data from 7 November 2014. These sets can then be used multiple times for different objects. The optimization is then conducted for three different objects: Zenit-2, Tsyklon-3, and Kosmos-3m, for both TLE snapshots. This means that the entire analysis is run six times in total.

4.2. Debris distribution

Objects in LEO are not homogeneously distributed over all orbital altitudes (or other orbital parameters). There are specific orbital configurations that are popular for certain satellite applications. Figure 4.2 shows the distribution of objects in LEO for different values of the orbital altitude and inclination.

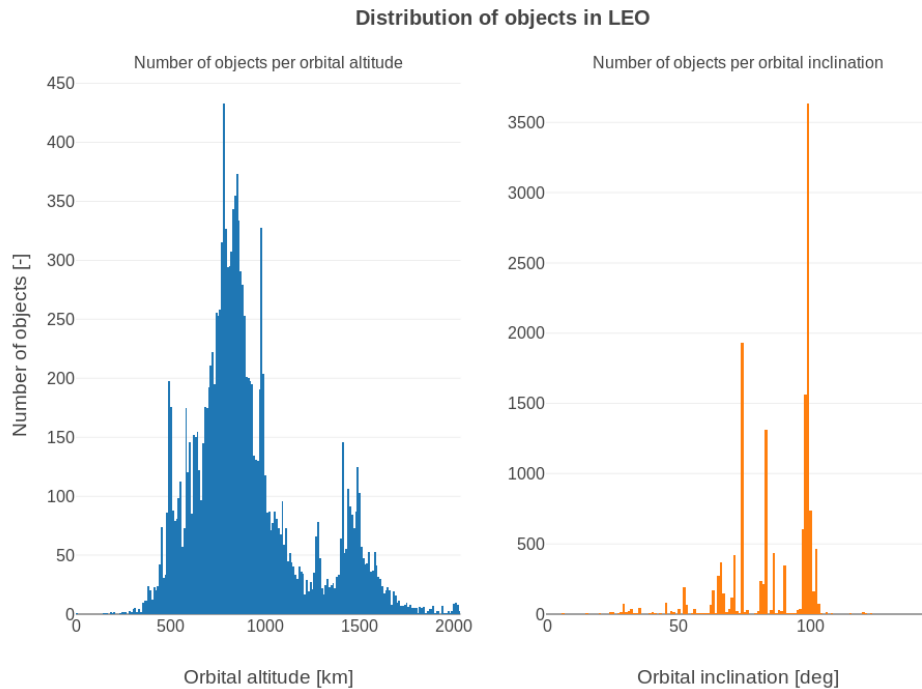


Figure 4.2: Distribution of objects in LEO for different orbital altitudes and inclinations (obtained at July 12, 2019). It can be seen that an orbital altitude at about 800 km and an orbital inclination of 100 degrees are popular orbital regions.

From this figure it can be seen that the satellite number density varies greatly for different orbital parameter configurations. In relatively empty orbital configurations, the probability for collisions is naturally lower than for orbital configurations with a high number density of objects.

4.3. TLE snapshots

During this analysis, the debris states at two different dates are used. These dates provide a *snapshot* of the TLE's of the debris objects at that particular date. The two snapshots are October 23, 2013 and November 7, 2014. Since not all TLE's are updated simultaneously to the database, the TLE's of the objects for a period of 24 hours were requested. For instance, the TLE's of the snapshot of 2013 were all the requested TLE's between 23 and 24 October 2013. Possible duplicate TLE's were filtered out. Using two different TLE snapshots ensures a better understanding of the obtained results. The two dates were chosen in accordance to the analysis performed by Lidtke et al. [18]. More information about tracking methods for objects in Earth orbit and the TLE format is presented in Appendix A.

4.4. State propagation

All objects in LEO are subject to more forces than just the central force of gravity of the Earth (see Section 2.4). This causes the orbits of the objects to perturb from a Kepler orbit over time. In order to obtain the exact positions of all the objects at every time-step, a precise orbit propagation is necessary. To propagate thousands of objects over time which include detailed environment models requires a large amount of computational effort. Due to time constraints and limited computing power used in this thesis, it is vital to limit the computational effort required during the computation. In order to reduce the computation time of the analysis, the orbits of the debris objects are assumed to be Keplerian (i.e. unperturbed). A large number of objects in LEO are active satellites which perform stationkeeping maneuvers to stay in their designed orbit, which make their orbits appear to be Keplerian. By assuming that the orbits of the debris objects are Kepler orbits, their state at an arbitrary point in time can be determined much faster than by means of numerical integration. The mean anomaly M of an object in a Kepler orbit can be determined by

$$M = M_0 + n(t - t_0) \quad (4.1)$$

where n is the mean motion and t is an arbitrary point in time. M_0 and t_0 are the mean anomaly and time at the reference epoch. M_0 and n can be determined from the initial Kepler states gained from the TLE data. The mean anomaly can then be converted into the eccentric anomaly E by

$$E_{k+1} = M + e \sin E_k \quad (4.2)$$

where e is the eccentricity of the orbit. The value for E is determined iteratively (with k the iteration step) via the Newton-Raphson method, using the value for M as the initial guess for E . With the converged value of the eccentric anomaly, the true anomaly can then be found by

$$\tan \frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \quad (4.3)$$

If $\theta \geq E$, the debris moves from pericenter to apocenter, and if $\theta \leq E$ the debris moves from apocenter to pericenter [32].

4.5. Optimization process

The optimization of the satellite trajectory consists of two parts: the control algorithm and the optimization algorithm. The control algorithm determines the various controls the satellite can use to perform a de-orbit maneuver, such as thrust magnitude, and/or engine activation time. The optimization algorithm uses these controls to find the optimal combination of inputs for this algorithm. The two algorithms thus work together to find the optimal trajectory.

4.5.1. Control algorithm

The navigation of the satellite during the de-orbit is determined by a number of varying parameters. The combination of these parameters results in the designed trajectory. The algorithm that controls what the values of these input parameters is, and what the effect is of these values is called the *control algorithm*. It is

essential to determine the right controls that the satellite can use for the maneuverability of the satellite. Too few or wrong controls will lead to the situation that the satellite has not enough freedom to steer appropriately through the debris environment. Too many controls might cause the optimization algorithm to have trouble converging to the optimal control settings. In order to determine what controls the satellite can have, three parts are important to consider.

Part 1 The propellant usage of a satellite is determined by the thrust magnitude and the specific impulse of the engine. Limiting either one of these results in a lower propellant usage. By letting the satellite control the thrust magnitude, the optimization algorithm can use this control to find the trajectory where the propellant usage is limited as much as possible.

Part 2 One of the main objectives in this thesis is to limit the collision risk caused by the de-orbit trajectory. In orbital altitudes where the debris density is high, there are more objects to collide with, thus increasing the collision probability. By letting the satellite control the time spent in certain orbital altitudes, it can avoid high collision risk regions. The time spent at certain orbital altitudes is determined by the choice whether the engine of the satellite is active or not. If the satellite has to navigate quickly through some orbital altitude region, it can choose to activate the engine, whereas when the satellite can afford to stay a certain period at a specific low-density orbital altitude, it can deactivate the engine. Choosing to stay for a certain period in some orbital altitude region can be motivated by the object density in the region below.

Part 3 As mentioned previously, the duration of the de-orbit is coupled with the collision risk associated with it. A shorter time-of-flight in general results in a lower collision risk. It is therefore important to limit the de-orbit duration.

The control algorithm consists of six different inputs, provided by the optimization algorithm. The first input is the thrust magnitude, which determines the rate at which the altitude of the satellite is decreased when the engine is turned on. The other inputs are different (consecutive) time points during the trajectory. The control algorithm treats these time points as moments to switch the engine either on or off. For instance, when two time-points lie close together, the control algorithm switches the engine from on to off (or vice-versa) in a relatively small time period. When two time points lie relatively far apart, the control leaves the engine turned on or off for a relatively large time period. The engine of the satellite always starts out at the initial epoch as turned off. The ability for the control algorithm to turn the engine on or off allows the optimization algorithm to search for the best combination of engine activation times to steer the satellite through the debris environment. Figure 4.3 shows the flow-diagram of the control algorithm. In this figure, t corresponds to the present time during the computation and $T(k)$ refers to a specific time point where the engine of the satellite is turned on or off.

4.5.2. Multi-objective optimization algorithm

The control algorithm uses the input values as control inputs in order to maneuver the satellite during the de-orbit. The values of the control inputs are determined by the optimization algorithm. The optimization algorithm tries to find the optimal combination of input values for the satellite to conduct the most optimal de-orbit. How optimal a de-orbit is, is determined by the fitness values of the problem. In this case, the fitness values of the optimization algorithm are: the amount of propellant used, the collision risk associated, and the duration of the de-orbit. The optimizer then tries to find solutions that minimize all these fitness values. An algorithm where multiple objective values are optimized is called a *multi-objective optimization algorithm*. As mentioned previously, there is no absolute optimum where all these objectives are minimum, since the objectives are coupled to each other (a high thrust corresponds to a short duration, likely). The optimizer thus finds a set of *most optimal* solutions. The multi-objective optimization algorithm used in this research is the Improved Harmony Search (IHS) algorithm. The IHS is the improved version of the Harmony Search (HS) algorithm that mimics the tuning process of musicians, where a band of musicians are tuning their instruments to find the correct harmony of tones. The IHS algorithm is one of four multi-objective optimization algorithms, provided by Pagmo (see Section 4.7). The HS algorithm has shown its effectiveness in the past dealing with multi-objective optimization problems [28]. The IHS enhances the fine-tuning characteristic and the rate of convergence with respect to the HS [22]. The algorithm optimizes the inputs for the trajectories over a number of generations and for a population size set by the user. The population size and

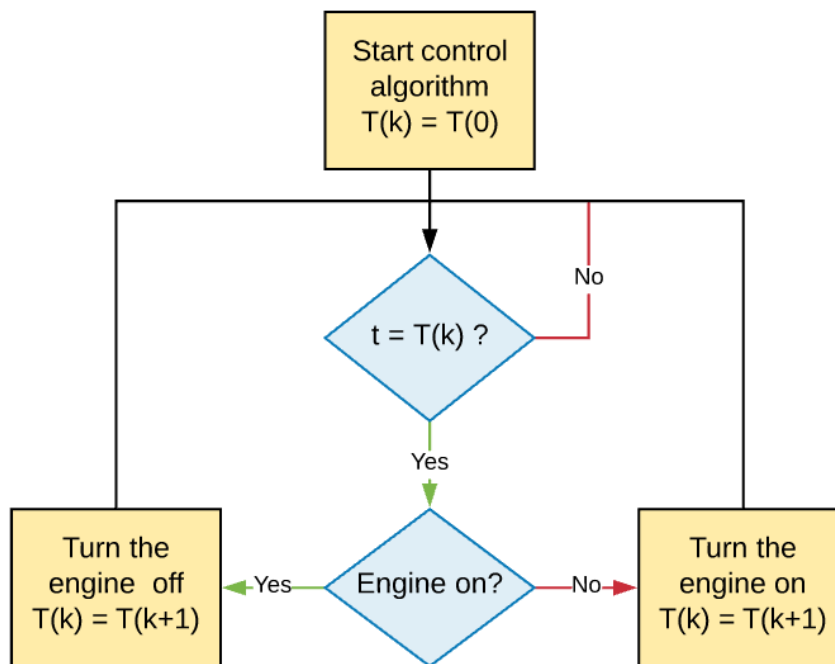


Figure 4.3: Flow-diagram of the control algorithm. The control algorithm constantly checks to see if a time point $T(k)$ provided by the optimization algorithm has been reached during the trajectory ($t = T(k)$). If so, the engine is either switched on or off, depending on its previous configuration. The time point is then updated to the next one in the sequence ($T(k) = T(k+1)$), and the control algorithm continues the loop.

number of generations is set to 128 and 100, respectively. The population size is chosen based on the limited available computational power (a larger population size means a longer computation time). The number of generations is chosen based on the convergence of the results.

The trajectory objectives will be presented as a set of points distributed over the available design space. The x-axis and y-axis represent the values of two of the trajectory objectives (propellant usage and propagation time, respectively), which are also called 'design parameters'. The third design parameter (collision probability) will be represented by a color. The points on the x-axis and y-axis will be situated along an imaginary diagonal line. This line represents the trajectories with the best possible combination of design parameters. This also means that no solutions are possible beyond this diagonal. This diagonal is often referred to as a *Pareto front*, after the Italian engineer and economist Vilfredo Pareto (1848–1923). Figure 4.4 shows an example of a set of solutions represented as a Pareto front. The solutions to the right of this front represent solutions that are not most optimal (meaning that one or more design parameters can be improved). The solutions on the front represent the most optimal combination of solutions. As can be seen from this figure, no single optimal solution can be found, rather a set of most optimal solutions is presented.

4.6. Model development

The final model used for this analysis underwent a number of iterations during the design process. A model with very accurate state elements of the debris objects was initially designed. This model would determine the state of each debris object using the same method as the state of the user satellite, with accurate state propagation methods and environment models. Additionally, a longer propagation time was suggested, where the optimization algorithm would have the option to use thrust magnitude values that resulted in trajectory durations up to 25 years. It was envisioned that these prolonged trajectories would represent the statistical effects of the collision probability more than short-term trajectories.

The model was designed as such that the state history of all the debris objects and the user satellite could be used where at each time-step the collision probability between each object and the user satellite would be determined. Accurately propagating the states of multiple thousands of debris objects resulted in a large computational effort, leading to very long computation times. However, this was not a major constraint since the debris states only had to be computed once per epoch. The major constraint was that the debris state

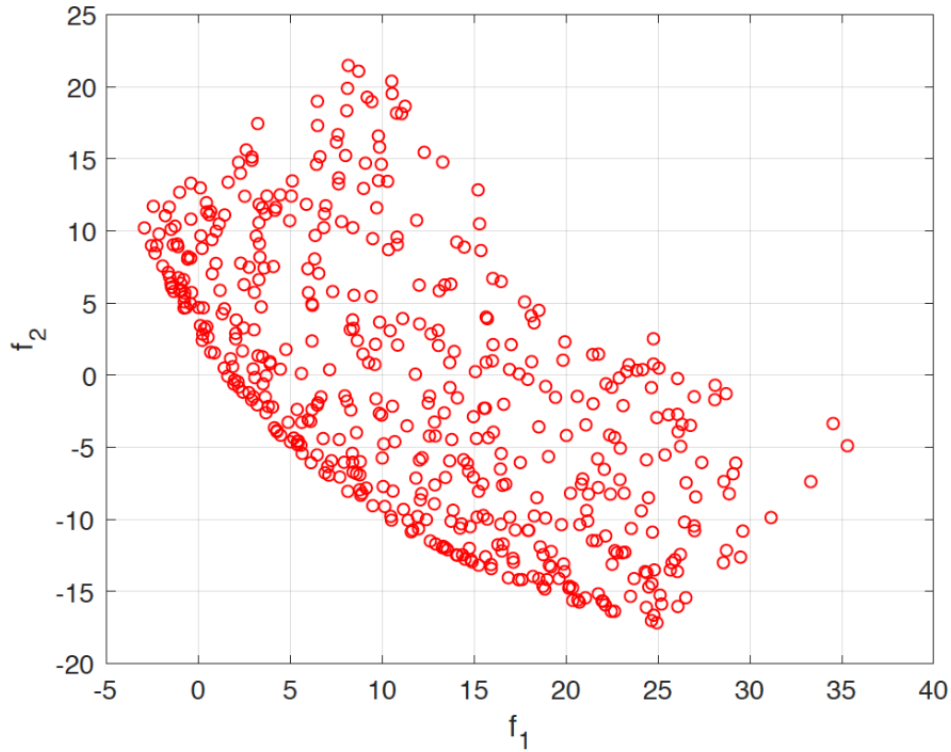


Figure 4.4: Example of a two-dimensional Pareto front for arbitrary design parameters f_1 and f_2 . The imaginary front of solutions, presented as the diagonal curve on the left is called the Pareto front. This front represents the most optimal solutions. Optimizing one objective is not possible whilst retaining the quality of the other objective [27].

history had to be saved in the random-access memory (RAM) of the computer. If six state elements (three position elements and three velocity elements) of approximately 8000 debris objects (depending on which epoch was used) for one year (365.25) days with a time-step of 90 seconds, then the amount of data stored¹ in the RAM would be $8 \times 6 \times 8000 \times 365.25 \times 86400/90 \approx 134$ Gigabyte. This greatly exceeded the specifications of the available equipment. Therefore, instead of storing the complete state history of the debris objects, a better way was to calculate the states simultaneously with the conjunction analysis. This however led to a large increase in computational time, since now the debris states had to be computed for the entire population in the optimization process for multiple generations. Because of these developments, the decision was made to assume the orbits of the debris objects as stable Kepler orbits. As explained in Section 4.4, these orbits can be computed analytically in a fraction of the time required for the numerical state propagation. These realizations during the design process and adaptations to the model required a considerable amount of time and effort.

4.7. Numerical tools

4.7.1. Python

The transformation from the raw TLE data to a set of Kepler elements is done using the SGP4 python package (<https://github.com/brandon-rhodes/python-sgp4>), which implements the most recent version of the SGP4 model.

4.7.2. Tudat

This numerical analysis will be performed using the TUDelft Astrodynamics Toolbox (Tudat), which provides a set of C++ libraries that supports astrodynamics and space research. These libraries are publicly available on <https://github.com/tudat>. Tudat includes a variety of different numerical tools such as numerical integrators and a wide choice of acceleration models. This provides a major advantage since a large number of ‘standard’ astrodynamical models, like the planetary bodies, are already included in the software. This reduces

¹Where each state element required 8 byte of information.

the development time of the software code. Though C++ is usually criticized for its complexity, it is known as a high-performance programming language with mature tool sets. In their paper, Eichhorn et al. [9] discuss different programming languages for astrodynamics problems. In their study they found that compiled languages, like C++, provide the best performance for astrodynamics applications.

4.7.3. Pagmo

Pagmo, created by ESA, is a scientific library for massively parallel optimization (<https://esa.github.io/pagmo2/>). Pagmo focuses on global optimization methods and is integrated in Tudat.

4.8. Problem setup

4.8.1. Reference frame

A convenient choice of reference frame is the Earth-centered inertial (ECI). This right-handed reference frame is located at the center of mass of the Earth and does not rotate². The ECI frame uses a right-handed coordinate system, where the xy-plane coincides with the equatorial plane of the Earth, and the z-axis points north. The reference frame of the orbit and other celestial bodies is described in the J2000 frame, where the x-axis points in the direction of Aries (Υ). The J2000 reference frame is the epoch precisely defined at January 1, 2000 at 12:00 hours GMT. The ECI reference frame is very convenient as visualizing satellite orbits is straightforward, since no apparent forces such as the Coriolis force³ has to be taken into account when conducting a propagation of a satellite state.

4.8.2. Acceleration models

The choice of which acceleration models to incorporate in the propagation of a satellite is driven by the trade-off between accuracy and computational speed. The more (accurate) acceleration models are used, the less the errors in the state derivative of the object, which leads to a higher accuracy in the computed states of the object. However, more accurate acceleration models also means that the integrator has to perform more calculations per time-step, which leads to a longer computation time. Every problem requires different levels of accuracy or computational speed. In order to choose the right combination of acceleration models, this trade-off between accuracy and speed has to be made.

Figure 4.5 shows the difference in the satellite altitude over one year of propagation time using different acceleration models. The difference is calculated with respect to an orbit where just the central gravity acceleration is used on the satellite orbit (which is a Kepler orbit). From this figure, it can be seen that even small perturbing forces such as the radiation pressure of the Sun results in a position difference of approximately 100 meters over the period of one year.

²An inertial reference frame is defined as a reference frame that is at rest or moves with a constant velocity with respect to distant stars, which is not rotating. An inertial reference frame describes space homogeneously and time independent.

³The Coriolis force is an apparent force which is induced when observing motion in a rotating reference frame.

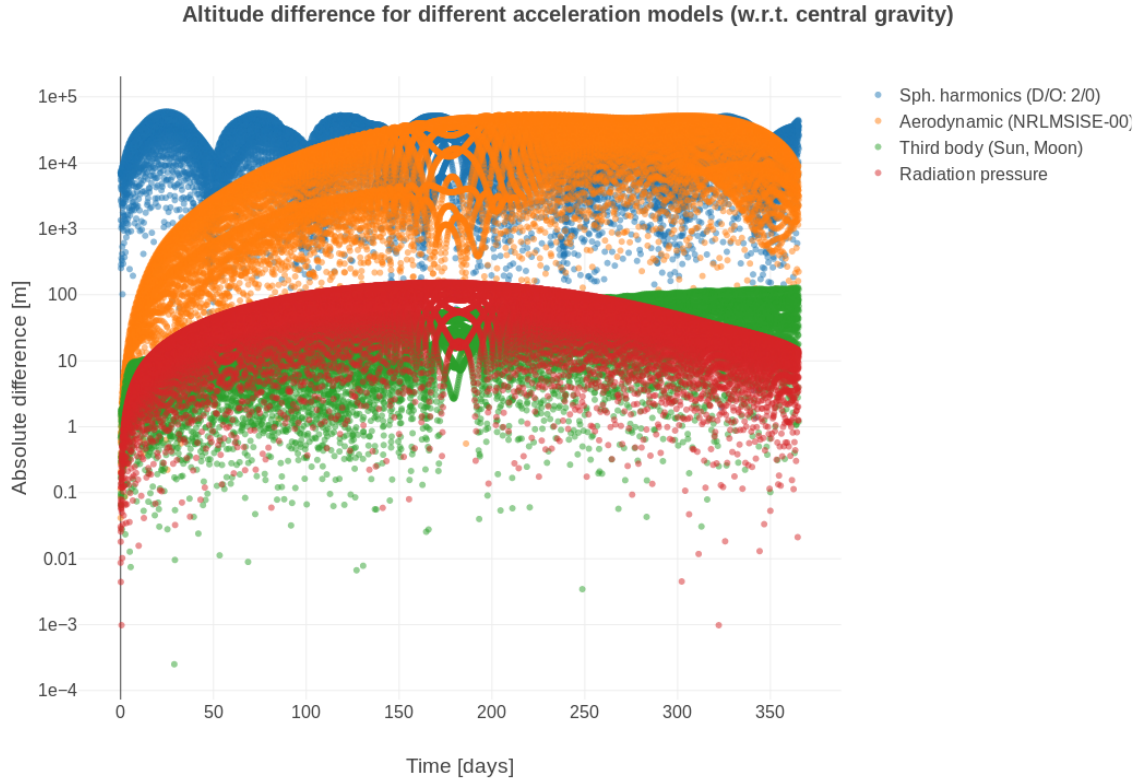


Figure 4.5: Position difference of the satellite using different acceleration models with respect to the central gravity model of the Earth. Each dot represents the position difference every 30 minutes.

As mentioned in Section 2.4, there are different models to represent the gravitational field or the atmospheric density of the Earth. Figure 4.5 shows the effects of the NRLMSISE-00 atmospheric model and the spherical harmonic gravity model with degree and order 2 and 0, respectively (spherical harmonics: 2/0). In order to assess whether these models provide an accurate-enough representation of the environment of the satellite, the position differences between these models and similar models have been investigated. Figure 4.6 shows the position difference between the 2/0 model and other combinations of degree and order of the spherical harmonic gravity field model. From this figure it can be seen that the 2/1 model does not account for a large difference in the position of the satellite (approximately 1 meter after one year). The other degree and order combinations do account for a large position difference however. Figure 4.7 shows the position difference of the exponential atmosphere model, compared to the NRLMSISE-00 model. It can be seen that the difference built up to almost 100 kilometers after one year of propagation.

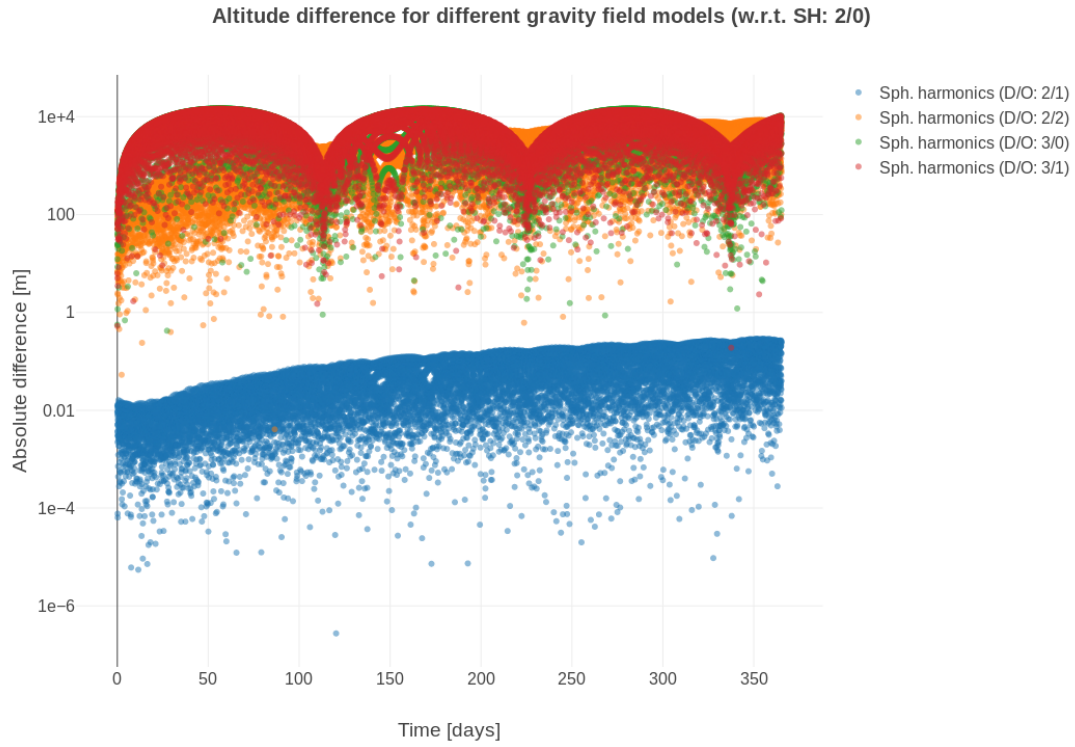


Figure 4.6: Position difference of the satellite using different spherical harmonic accelerations models, with respect to the SH: 2/0 degree and order spherical harmonic acceleration model. Each dot represents the position difference every 30 minutes.

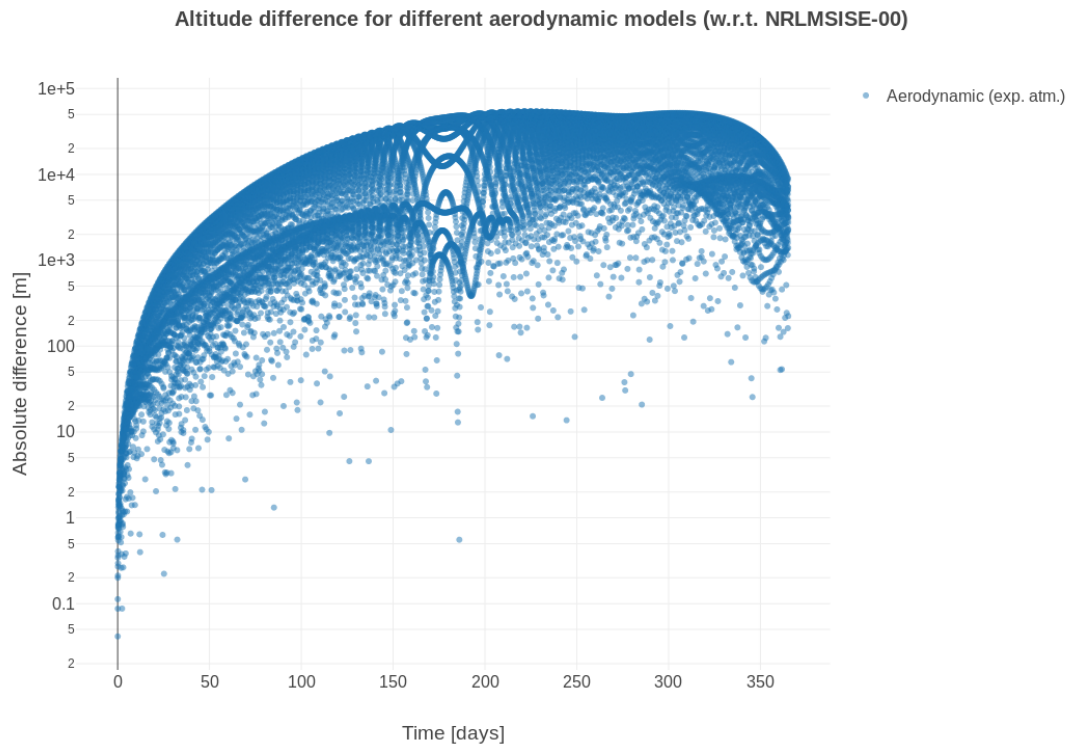


Figure 4.7: Position difference of the satellite using an exponential atmosphere model, with respect to the NRLMSISE-00 atmosphere model. Each dot represents the position of the satellite every 30 minutes.

The computation times of these environment models are presented in Table 4.1. It can be seen that all acceleration models perform similarly in computation time. The only visible outlier is the NRLMSISE-00 aerodynamic acceleration model, which takes approximately 60% longer than the exponential atmosphere model.

Model	Average computation time [s]
Central gravity	75.4
Spherical harmonics (2/0)	75.6
Spherical harmonics (2/1)	80.2
Spherical harmonics (2/2)	79.1
Spherical harmonics (3/0)	73.6
Spherical harmonics (3/1)	82.0
Aerodynamics (NRLMSISE-00)	141.4
Aerodynamics (Exp. atm.)	88.0
Third-body (Sun, Moon)	76.0
Radiation pressure	81.9

Table 4.1: Average computation times using different acceleration models.

Using the results presented in Figures 4.5, 4.6, 4.7 and in Table 4.1, the chosen acceleration models for the user satellite are: central gravity, spherical harmonics up to degree and order 3/2, NRLMSISE-00 atmospheric model, third-body central gravity of the Sun and the Moon, and the solar radiation pressure. The accuracy that the NRLMSISE-00 model provides over the exponential atmosphere model outweighs the extra computation time. An exception is made in the choice of the atmospheric model, however. The Zenit-2 object is propagated from the following epoch: 1 January 2020, 00:00:00. This epoch lies in the future at the moment of writing this report. This means that the solar activity data in the NRLMSISE-00 model is based on predictions, rather than observations, causing the validity of the model to decline. The atmospheric model for this epoch will therefore be the exponential atmosphere model. When propagating the Kosmos-3m object (with epoch 1 January 2017) for a year, the available model in Tudat also indicated that there was not data beyond a certain point in the orbit. These two objects are therefore propagated using the exponential atmosphere model.

4.8.3. Integrator

The choice in numerical integrator is determined by the error made during the integration. The error made by an integrator can be divided in two parts: a truncation error and a rounding error. A truncation error is the result of the integrator trying to estimate the dynamics of the problem. A rounding error is the result of the limited numerical precision of the computer. A truncation error can often be reduced by reducing the size of the time-step of the propagation. However, smaller time-steps means also more time-steps which increases the effect of rounding errors. Smaller and more time-steps also means that the computation time increases. In this analysis, the variation of the dynamics of the satellite do not change rapidly over time. The applied thrust has a low magnitude and the altitude of the satellite decreases slowly. The integrator used in this analysis is the Dormand-Prince integrator (DOPRI8(7)). This high-order method has a high single time-step accuracy, but less robust time-step control. Since the dynamics of the low-thrust trajectories do not change rapidly over time, this time-step control is not relevant [8]. This integrator is readily available in Tudat.

The step size of the integrator is chosen based on the trade-off between accuracy and speed. In the analysis performed by Leloux [17], the optimal step-size was chosen to be 90 seconds. This resulted in the best combination of filter accuracy and lowest computation time. The propagation of the Zenit-2 object was performed using a step size of 180 seconds. This step size was chosen to reduce the required computation time.

4.8.4. Propagator

The propagation method used in this analysis is the Encke propagation method. In this method, the true orbit of the satellite is viewed as a small deviation from a fictitious Kepler orbit, touching the true orbit at an arbitrary point in time. This fictitious orbit is called an osculating orbit. Propagating the orbit in this way means that the majority of the behaviour can be described analytically and only a small deviation Δr between the true orbit and the osculating orbit has to be computed [8], reducing the errors due to large variations in the state vector. Figure 4.8 shows this geometry, where the position of the true orbit r only deviates a small

amount from the osculating orbit. This propagation method is readily available in Tudat.

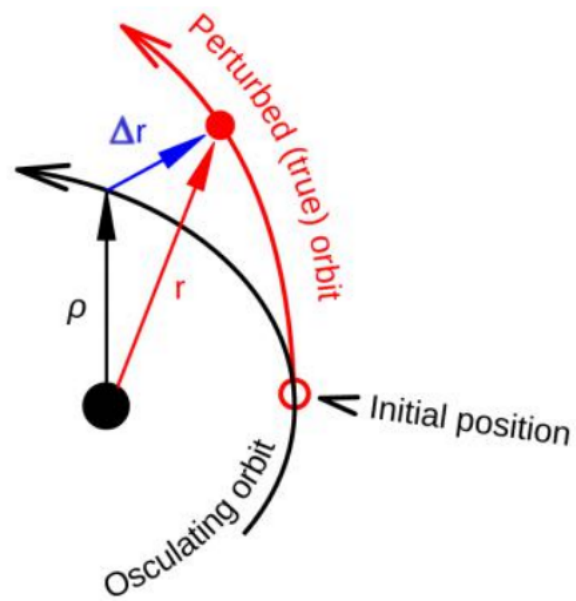


Figure 4.8: The geometry of the difference in position between the (black) osculating orbit and the (red) perturbed, true, orbit [8].

5

Results

This chapter presents the results of this research for the three different objects: Zenit-2, Tsyklon-3 and Kosmos-3m. The results are divided into two parts. The first part presents the results using the TLE snapshot of 23 October, 2013. The second part presents the results of the research using the TLE snapshot of 7 November, 2014. These parts are referred to as the ‘2013 snapshot’ and ‘2014 snapshot’, respectively. The obtained collision probabilities for the trajectory without thrust and with a constant thrust have been computed and presented in Table 5.1. Here, the reference values obtained by Lidtke et al. [18] (using the same snapshots) are also presented.

Target	Trajectory	23 October 2013			7 November 2014		
		Acc. P_c	Highest single contrib. [%]	No. conj.	Acc. P_c	Highest single contrib. [%]	No. conj.
Zenit-2 (low-thrust)	Computed	2.8×10^{-3}	99.7	2491	8.0×10^{-7}	92.6	3192
	Reference	1.6×10^{-3}	84.6	14580	1.6×10^{-3}	23.1	15481
Zenit-2 (no thrust)	Computed	4.3×10^{-3}	76.2	14304	2.2×10^{-3}	25.4	15045
	Reference	1.9×10^{-3}	43.7	9645	8.3×10^{-4}	69.0	9484
Tsyklon-3 (low-thrust)	Computed	3.1×10^{-5}	99.7	622	7.9×10^{-6}	87.3	946
	Reference	3.6×10^{-5}	99.9	1929	9.3×10^{-8}	82.0	2034
Tsyklon-3 (no thrust)	Computed	3.8×10^{-4}	99.0	9057	1.8×10^{-3}	56.8	11685
	Reference	2.1×10^{-4}	27.1	10102	2.5×10^{-4}	59.1	11492
Kosmos-3m (low-thrust)	Computed	2.5×10^{-5}	100.0	983	3.1×10^{-4}	82.4	1563
	Reference	4.1×10^{-4}	99.0	1169	6.3×10^{-6}	80.5	1101
Kosmos-3m (no thrust)	Computed	1.6×10^{-3}	52.6	7256	5.3×10^{-3}	38.2	10374
	Reference	2.5×10^{-4}	30.0	9422	7.4×10^{-4}	30.2	9805

Table 5.1: Comparison with the reference data from Lidtke et al. [18] against the computed trajectories. The accumulated collision probability is indicated by ‘Acc. P_c ’, the contribution of the conjunction event that added the most to the accumulated collision probability is indicated as ‘Highest single contrib.’ and the total number of conjunctions encountered by the trajectory is indicated as ‘No. conj.’

This table shows that the accumulated probability of the computed trajectories are typically in the same order of magnitude as the reference values. For the trajectories without thrust, this was also previously shown in Table 3.3. The accumulated probabilities and the highest single contributions for the low-thrust trajectories using the 2013 snapshot also show a good correspondence with the reference values. By observing the number of conjunctions, it can be seen that there are some discrepancies between the computed and reference trajectories, however. These same discrepancies can be observed in the values for the 2014 snapshot. Here, it can be seen that the computed values for the trajectories without thrust show good correspondence with the reference values. The computed values of the accumulated collision probability and the number of conjunctions for the low-thrust trajectories for this snapshot show some discrepancies with the reference values, however. The computed values for the propellant usage and duration of the constant low-thrust trajectories do not compare, as was shown in Table 2.3. This means that the computed trajectories are different

from the reference trajectories, resulting in different (numbers of) conjunctions, thus resulting in different accumulated collision probabilities. As an example, the number of conjunctions encountered by the computed low-thrust trajectory of the Zenit-2 object using the 2014 snapshot is approximately 5 times lower than for the reference trajectory. As was shown in Section 2.5, the duration of the computed trajectory for the Zenit-2 object is 427.64 days, whereas the duration of the reference trajectory is 593.14 days. This means that the computed trajectory spent less time in orbit, resulting in a lower number of conjunctions. The discrepancy in the values for the accumulated collision probabilities or possible discrepancies in the highest single-event contribution can be accounted for by the highly sensitive nature of the value of the collision probability. This will be further explained in the following chapter. Because there are some discrepancies between the computed and the reference low-thrust trajectories, the results presented in this chapter will be compared to the computed low-thrust trajectories. These trajectories will be referred to as the ‘nominal’ trajectories for the remainder of this chapter.

5.1. 2013 snapshot

This section shows the results of the analysis using the TLE set of 23 October, 2013. In Section 5.1.1 the objectives of the trajectories with different thrust controls are presented. Here, the results obtained for these three objectives will be shown in a scatter plot, indicating the outcome of the objectives for each trajectory. Section 5.1.2 presents a qualitative insight into some trajectories of interest shown in the previous section. Here, the altitude profile over time is shown and high-risk conjunction events are indicated, showing at which altitudes these conjunctions occurred. The accumulation of the collision probability over time is shown in Section 5.1.3. Here, the conjunctions over time and per altitude are presented, indicating at what times the conjunctions with the highest collision probability occurred and at which altitudes most conjunctions occurred. The sensitivity of the results presented in Section 5.1.1 is shown in Section 5.1.4. In this section a Gaussian distributed variation in three initial state elements of the objects is applied after which the analysis is performed. The distribution of the different associated collision probabilities for each variation is shown, indicating the robustness of the obtained results of Section 5.1.1.

5.1.1. Trajectory objectives

Multiple trajectories were computed using different input parameters, each with an associated propellant usage, propagation time¹ and accumulated collision probability. These values are referred to as the objectives of each trajectory, which are all three tried to minimize. Figures 5.1, 5.2 and 5.3 show the values of these objectives for the most favourable elements in the final generation of the optimization: the Pareto front. Each point represents the associated objective values for a distinct trajectory. The x-axis represents the propellant used during the trajectory (in kilogram) and the y-axis represents the propagation time of the trajectory (in days). The accumulated collision probability P_c per trajectory is indicated with a color. The color gradient ranges from green (which represents a low collision probability) to red (which represents a high collision probability). The values of the collision probabilities are indicated on the right-hand side of the figure by the colorscale. The values of this scale represent the order of magnitude of the collision probability (e.g. a value of -4 means that the collision probability is 10^{-4}). Note that the range of the scale in the three plots is different, which means that a relatively low collision probability for the Zenit-2 object might not be as low relative to a low collision probability for the Kosmos-3m object. The points in these plots are situated along a diagonal from the top left to the bottom right. This trend makes sense from a physical perspective: when the satellite uses a relatively large amount of propellant (right-hand side of the x-axis) the altitude decreases more quickly, which results in a shorter propagation time (bottom side of the y-axis). When a satellite uses a relatively low amount of propellant (left-hand side of the x-axis) the altitude decreases more slowly, which results in a longer propagation time (top side of the y-axis). This reciprocity between the propellant usage and the propagation time results in the points being situated along this diagonal line. Another trend can be seen in these figures regarding the collision probability. Points situated on the bottom right part of the figure have in general a lower associated collision probability than points situated on the top left part of the figure. This is an indication that shorter propagation times result in lower collision probabilities.

¹The propagation time of a trajectory is the time it takes the trajectory to reach the lower-altitude limit. This term should not be confused with the computation time, which is the time it takes for the computer simulation to run.

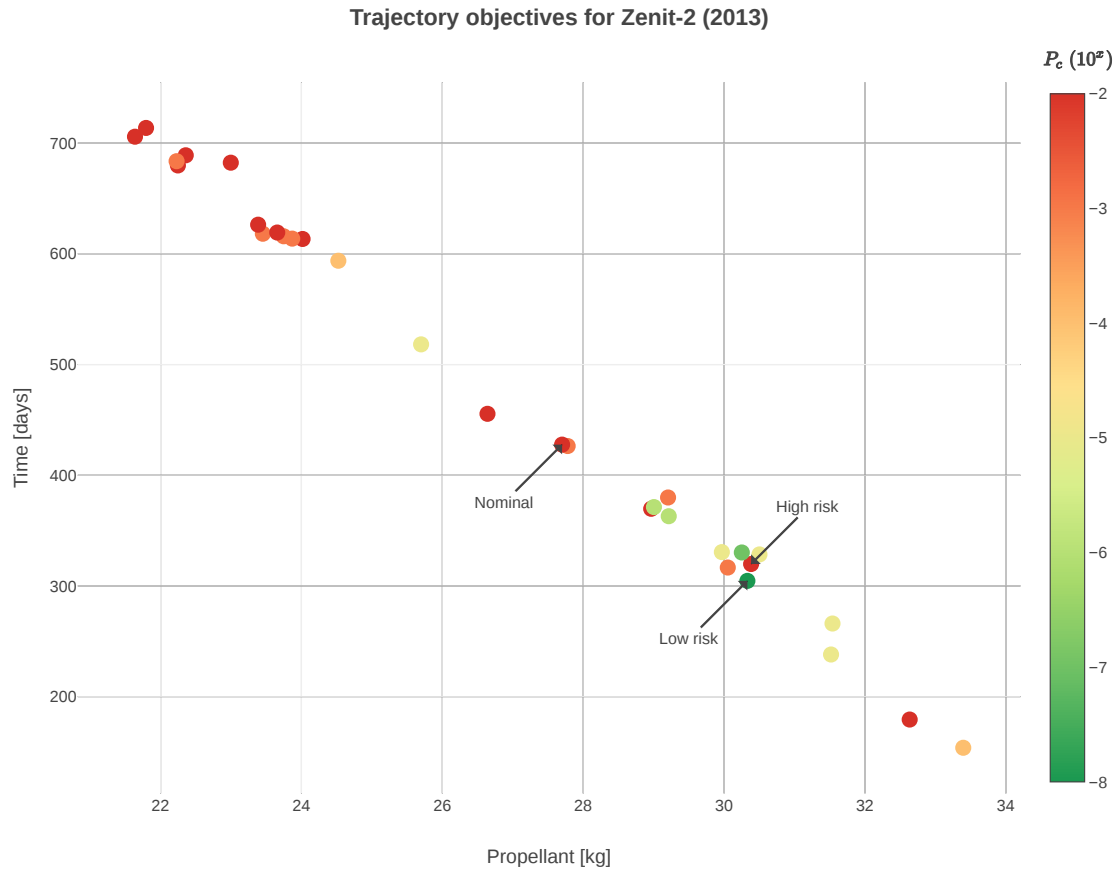


Figure 5.1: Trajectory objective for the Zenit-2 object using the TLE set of 2013. Each point represents a single trajectory. The x-axis represents the used propellant mass and the y-axis represents the propagation time of each trajectory. The collision probability P_c of each trajectory is represented as a color, where the color scale on the right indicates the order of magnitude of the collision probability. A red color represents a relatively high collision probability and a green color represents a relatively low collision probability. Trajectories of interest are indicated by arrows.

Figure 5.1 shows the values of the objectives for different trajectories of Zenit-2. This object is the heaviest object of the three and its trajectories on average have a longer propagation time than the trajectories of the other objects (of Figures 5.2 and 5.3). Although the trajectories with a long propagation time have a higher accumulated collision probability, not all trajectories with a short propagation time have a low accumulated collision probability. The trajectories with the lowest collision probability have propagation times between 300 and 400 days. Some points of interest are indicated with arrows in this figure. The nominal trajectory has a relatively high collision probability, which can be attributed to a single high-risk conjunction event (see Table 5.1). Two other points of interest are indicated in this figure, which are annotated as “high-risk” and “low-risk”. These points are of interest because they share similar values for the propellant usage and the propagation time, but have large varying collision probabilities. This leads to believe that the collision probability is not solely determined by the path of the trajectory.

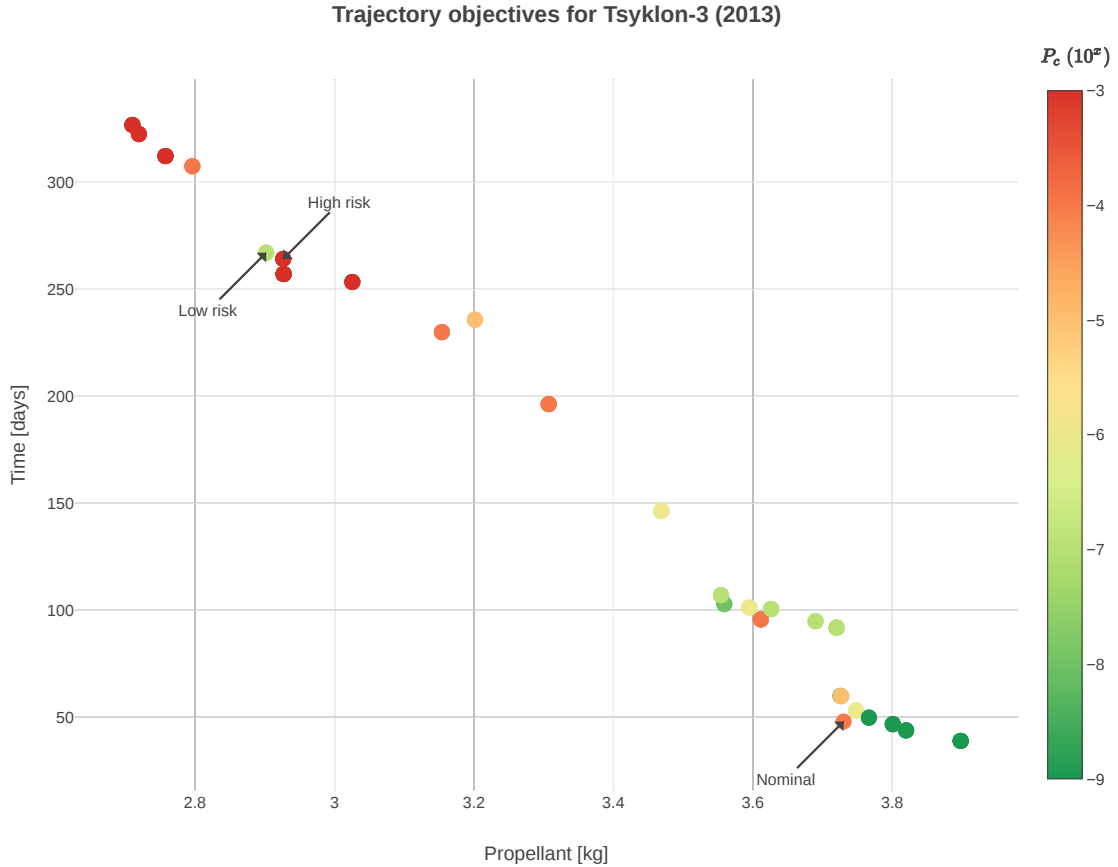


Figure 5.2: Trajectory objective for the Tsyklon-3 object using the TLE set of 2013. Each point represents a single trajectory. The x-axis represents the used propellant mass and the y-axis represents the propagation time of each trajectory. The collision probability P_c of each trajectory is represented as a color, where the color scale on the right indicates the order of magnitude of the collision probability. A red color represents a relatively high collision probability and a green color represents a relatively low collision probability. Trajectories of interest are indicated by arrows.

Figure 5.2 shows the results for Tsyklon-3. This object is much lighter than Zenit-2 and its trajectories on average thus have a shorter propagation time. In this figure it is clearly visible that trajectories with a short propagation time have a lower associated collision probability. This effect is much more clearly visible because the short trajectories of the Tsyklon-3 object have propagation times below 100 days, whereas the short propagation times of the Zenit-2 object are approximately twice as long. Although this trend is visible for the trajectories with a short propagation time, there is also a trajectory with a long propagation time with a relatively low collision probability. This trajectory is annotated as “low-risk” in this figure. This low-risk trajectory, although having a longer propagation time, requires less propellant and has a lower collision probability. That is why this point seems to be located in front of the high-risk trajectory. Two other points of interest are indicated in this figure which are the “nominal” trajectory (bottom-right part of the figure) and the “high-risk” trajectory located next to the low-risk trajectory on the top-left part of the figure. Again, just as in the case of the Zenit-2 object, the low-risk and high-risk trajectories are of interest because they share similar propellant usages and propagation time but have large varying collision probabilities. The nominal trajectory is of interest because it has a relatively large collision probability compared to trajectories with similar propellant usage and propagation times.

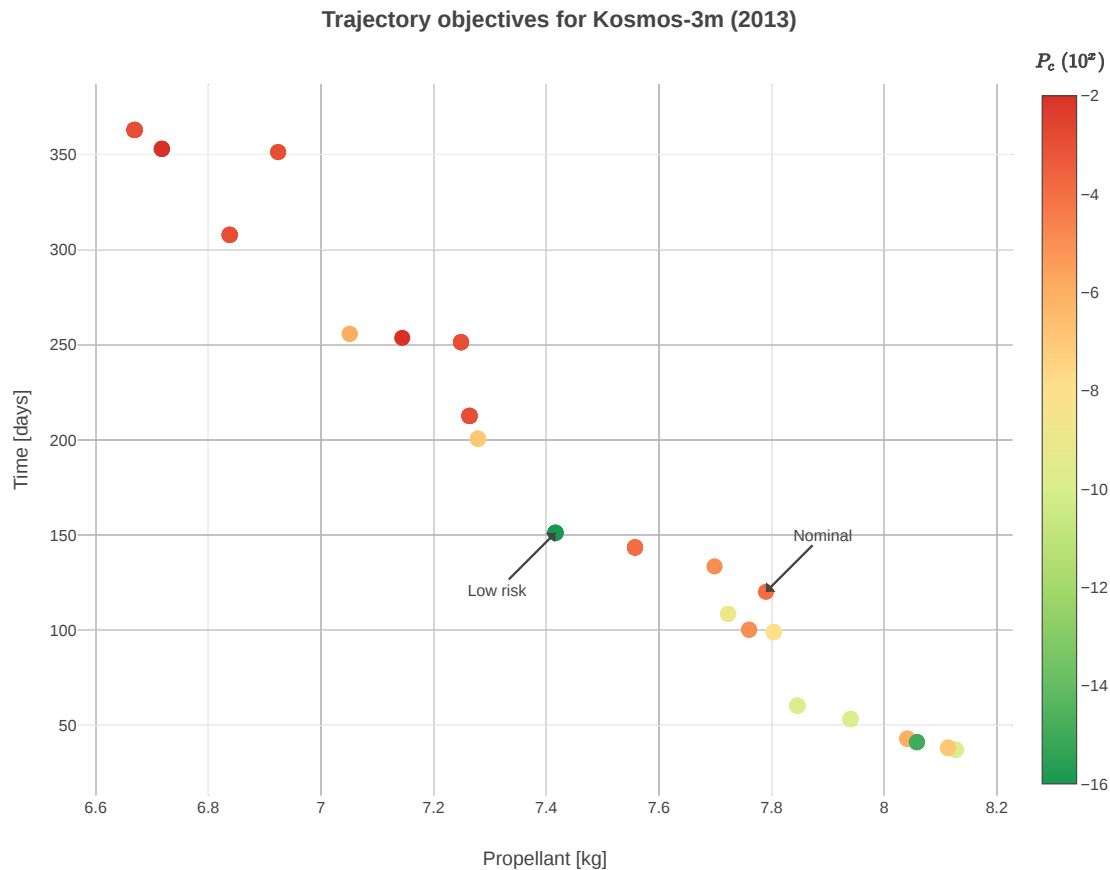


Figure 5.3: Trajectory objective for the Kosmos-3m object using the TLE set of 2013. Each point represents a single trajectory. The x-axis represents the used propellant mass and the y-axis represents the propagation time of each trajectory. The collision probability P_c of each trajectory is represented as a color, where the color scale on the right indicates the order of magnitude of the collision probability. A red color represents a relatively high collision probability and a green color represents a relatively low collision probability. Trajectories of interest are indicated by arrows.

The results for Kosmos-3m are shown in Figure 5.3. This object has a similar mass as Tsyklon-3 and its trajectories thus have similar propagation times. The trend where trajectories with a shorter propagation time have lower associated collision probabilities is also visible in this figure, although not as prominent as in the previous figure. It can be seen in Figure 5.3 that the points are not located along the diagonal as clearly as for the Zenit-2 and Tsyklon-3 objects. Even though they do not lie as close on the diagonal, each point still represents a trajectory where one of the objectives is lower than its neighbouring points, let it be the propellant usage, propagation time or collision probability. Two points of interest are indicated in this figure: the “nominal” trajectory (with a high collision probability) and a “low-risk” trajectory. The low-risk trajectory is a point of interest since it represents the trajectory with the lowest collision probability of all trajectories and does not have any immediate neighbouring trajectories. Comparing this trajectory with the nominal trajectory (which share similar propagation times) could provide insight into favourable thrust controls compared to a constant thrust. It should be noted that the range of the collision probabilities is much wider than in the previous two cases, which means that there is a large difference between the collision probabilities of the nominal and the low-risk trajectory.

In order to provide a more qualitative insight into the results presented above and in particular the points of interest, the trajectories of some of the points of interest are shown in the following section.

5.1.2. Trajectory profiles

Figures 5.4, 5.5 and 5.6 show the altitude profile over time of some of the points of interest highlighted in the previous section. In these figures, the bottom x-axis represents the time of the trajectories (in days) and the y-axis represents the altitude of the object (in kilometers). Each figure shows two different trajectories, one in blue (the trajectory with the lower collision probability of the two) and one in orange (the trajectory with the higher collision probability of the two). Additionally, the number of debris objects in LEO is represented by the green bar chart, where the top x-axis represents the number of objects. This bar chart shares the same y-axis as the trajectories of the objects. The two trajectories both start with the same state and after a period diverge from each other due to different thrust controls (the low-risk trajectory activates its engine at different epochs than the high-risk trajectory). Since the initial orbit of the objects is not perfectly circular, the trajectories are slightly eccentric. In one day, each object might orbit the Earth fourteen times and since the trajectories last for more than one hundred days, this periodic motion is projected as if the path of the trajectory is one thick line. For example, in Figure 5.4, it appears as if the path of the trajectory is presented as a thick line ranging between 800 and 825 km altitude. By zooming in, the periodic motion of the trajectory becomes visible and it can be seen that the altitude of the object actually changes between these values during one revolution. Another long-term periodic motion can be seen in the altitude profile of the trajectories with a period of about 50 days. This motion is caused by the fact that the satellite does not orbit around a point mass but rather a rotating oblate spheroid Earth, which means that there are periodic variations in the altitude of the object above the Earth's surface. In particular, variations in the argument of perigee (induced by the J-2 effect as explained in Section 2.4.2) cause the orientation of the orbit to rotate, and the flattened Earth (which radius is latitude dependent) cause the behaviour shown here. During the de-orbit, each object encounters a number of conjunctions. Most of these conjunctions barely contribute to the accumulated collision probability. The accumulated collision probability is largely determined by a low number of high-risk conjunctions. The close conjunctions that account for 100.0% of the accumulated collision probability (rounded to one decimal precision) are shown in Figures 5.7, 5.8 and 5.9 for both the low-risk and the high-risk trajectory. The time and altitude where these conjunctions occurred are indicated by arrows in Figures 5.4, 5.5 and 5.6.

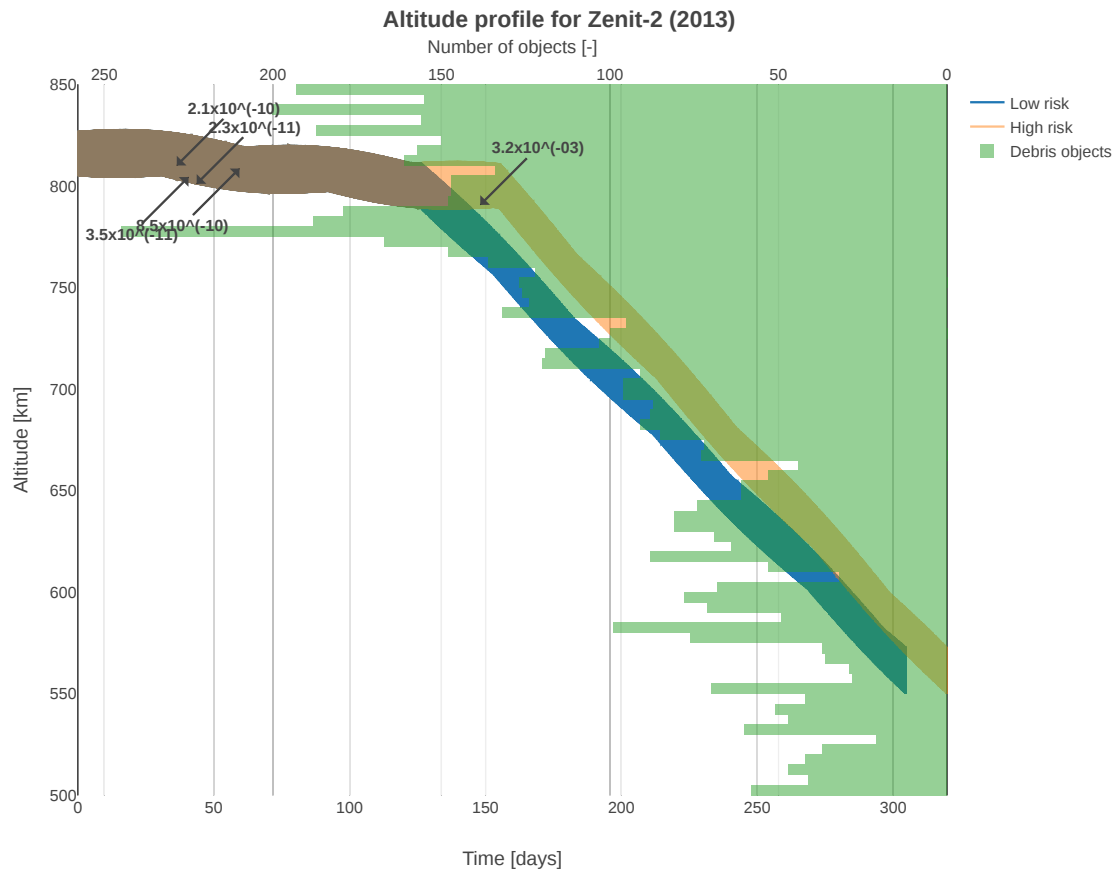


Figure 5.4: Altitude profiles of two trajectories of interest for the Zenit-2 object using the TLE set of 2013. The trajectory with a low collision probability is indicated in blue, and the trajectory with a high collision probability is indicated in orange. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.4 shows the altitude profile over time for the low-risk (in blue) and high-risk (in orange) trajectories for Zenit-2, which were indicated in Figure 5.1. As can be seen from this figure, the profile of both trajectories is quite similar. Both trajectories leave their engines off for quite some time and they both have similar thrust magnitudes, resulting in a similar decline of the altitude but also similar lifetimes and propellant consumption (as can be seen in Figure 5.1). The high-risk trajectory starts its engine after approximately 155 days, 25 days later than the low-risk trajectory. It can be seen that during this time, the high-risk trajectory encounters a close conjunction with a high collision probability (3.2×10^{-3}). The low-risk trajectory activates its engine after approximately 130 days and does not encounter this high-risk conjunction, but rather a number of low-risk conjunctions earlier on in the trajectory (2.1×10^{-10} , 3.5×10^{-11} , 2.3×10^{-11} , and 8.5×10^{-10}). Of course, the initial trajectories being identical, the high-risk trajectory also encounters these conjunctions but they do not add significantly to the accumulated collision probability. During the remainder of the time, both trajectory gradually decrease their altitude and they do not encounter significant close conjunctions anymore. The outcome in terms of collision risk is driven by a single (high-risk) conjunction event.

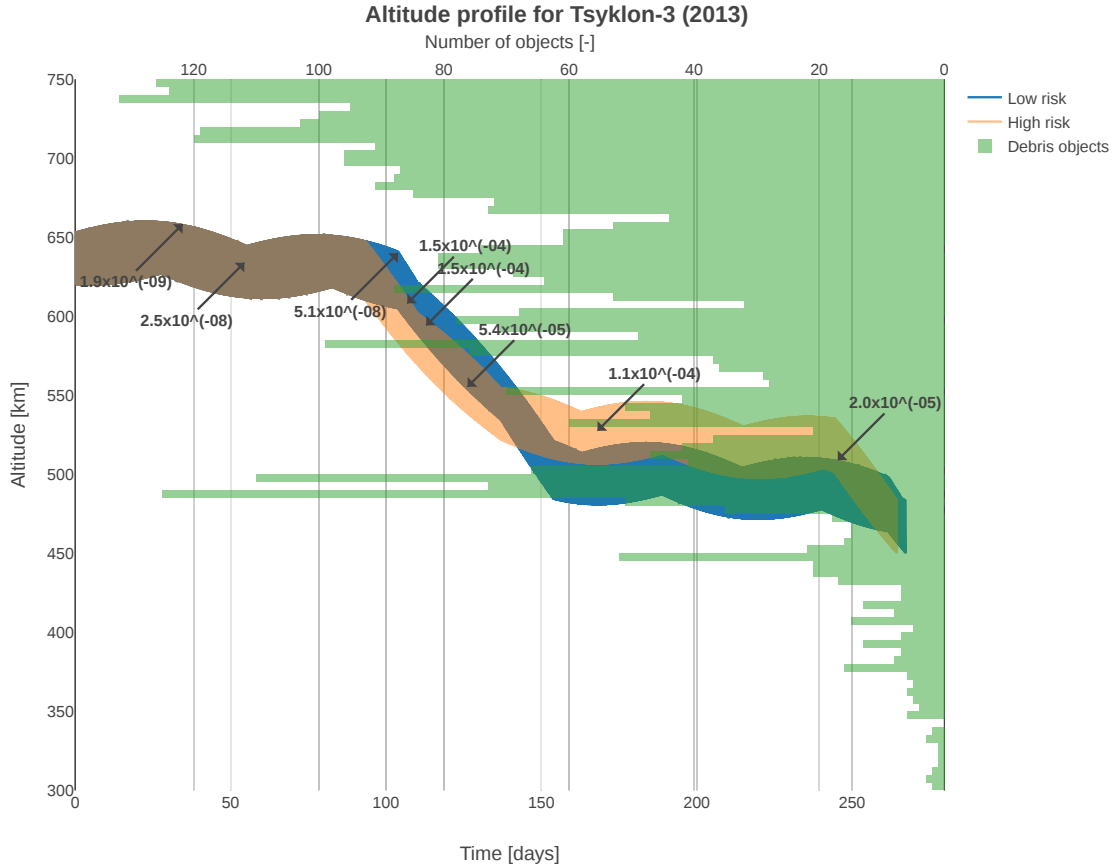


Figure 5.5: Altitude profiles of two trajectories of interest for the Tsyklon-3 object using the TLE set of 2013. The trajectory with a low collision probability is indicated in blue, and the trajectory with a high collision probability is indicated in orange. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.5 shows the altitude profile over time for the low-risk (in blue) and high-risk (in orange) trajectories for Tsyklon-3, which were indicated in Figure 5.2. These trajectories start out at a lower altitude than the trajectories of the Zenit-2 object. Just like the case for the Zenit-2 object, the trajectories follow very similar paths but have varying collision probabilities. These similarities in the trajectory are explained by the corresponding propellant usage and propagation time, which caused the two points in Figure 5.2 to lie close to each other. The altitude profile of the trajectories in this figure are different than the profiles in Figure 5.4. Both trajectories activate their engine after approximately 100 days and deactivate it about 40-50 days later. A final engine activation is initiated just before then end of the trajectory in order to reach the lower altitude limit.

It can be observed that both trajectories encounter conjunctions before the engine is activated (1.9×10^{-9} and 2.5×10^{-8}), but these only contribute significantly to the accumulated collision probability of the low-risk trajectory. The low-risk trajectory encounters another conjunction (5.1×10^{-8}) right at the point of engine activation, which contributes most to the accumulated collision probability. For the remainder of the trajectory, it does not encounter any more conjunctions that add significantly to the accumulated collision probability. The high-risk trajectory encounters a relatively large number of conjunctions that contribute to the accumulated collision probability. After the engine is activated, the trajectory encounters two high-risk conjunctions (both 1.5×10^{-4}). Another conjunction event occurs (5.4×10^{-5}) at an altitude of approximately 560 km, where the local number of debris objects is relatively high compared to surrounding altitudes. Two more conjunctions occur (1.1×10^{-4} and 2.0×10^{-5}) at altitudes where the number of debris objects is relatively low. These conjunctions account for 100.0% (rounded to one decimal precision) of the total accumulated collision probability. When the low-risk trajectory cuts off its engine, it orbits at around 500 km altitude. During this same

period, the high-risk trajectory also deactivated its engine and orbits at around 520 km altitude. It should be noted that even though at the altitude of the low-risk trajectory, the number of debris objects is much higher than at the altitude of the high-risk trajectory, it does not encounter any significant conjunctions during this period, while at the same time the high-risk trajectory does encounter a high-probability conjunction. This seems to be counter-intuitive since, from a statistical point of view, it is expected that at a higher number density of debris objects, more conjunctions would occur and also more high-risk conjunctions, but does not necessarily apply for individual cases.

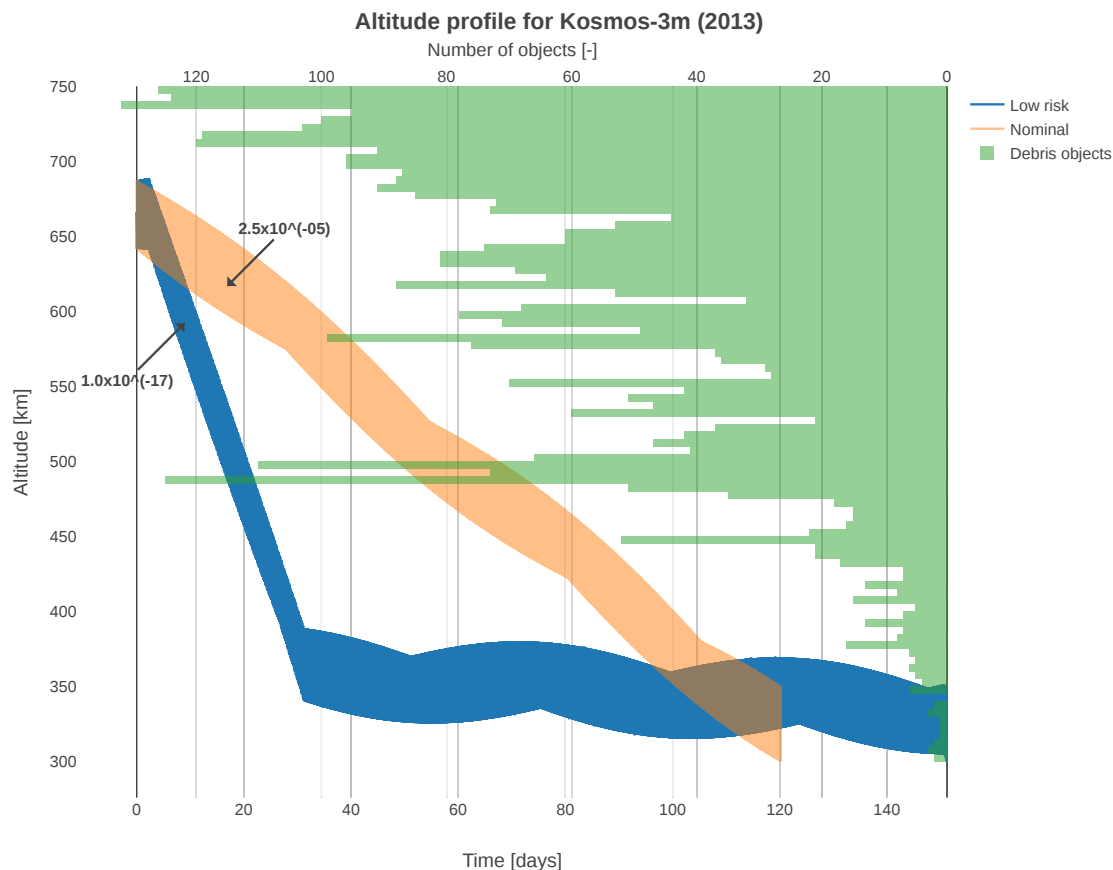


Figure 5.6: Altitude profiles of two trajectories of interest for the Kosmos-3m object using the TLE set of 2013. The trajectory with a low collision probability is indicated in blue, and the trajectory with a high collision probability is indicated in orange. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.6 shows the altitude profile over time for the low-risk (in blue) and the nominal (in orange) trajectories for Kosmos-3m, which were indicated in Figure 5.3. Immediately it can be seen that the altitude profile of these trajectories are very different from each other. The nominal trajectory immediately activates its engine and its altitude is gradually lowered during the trajectory. The low-risk trajectory activates its engine after approximately four days with a thrust magnitude that is much higher than that of the nominal trajectory. This results in a very fast reduction in the altitude of the orbit. After approximately 30 days, the engine is cut off and the object orbits the Earth at an approximate altitude of 350 km. Because its altitude is lowered very quickly, the object in the low-risk trajectory spends less time at altitudes where the number of debris objects is relatively high, thus avoiding the area with a high potential for conjunctions. The nominal trajectory spends a longer time at altitudes where the debris number density is relatively high, and encounters a conjunction with a high collision probability (2.5×10^{-5}). The high-risk trajectory does not encounter any other significant conjunctions for the rest of the propagation, however. The low-risk trajectory encounters one conjunction (1.0×10^{-17}) at the start and does not encounter any other significant conjunctions during the rest of the trajectory. It should be noted that this conjunction has a low collision probability, also relative

to the trajectories of the previous objects (Figures 5.4 and 5.5).

Even though the results presented above give an insight into the trajectories of the specific points of interest, it does not show how the altitude actually affects the number of conjunctions, or how the accumulated collision probability is dominated by a few number of high-risk conjunctions. This will be presented in the next section.

5.1.3. Conjunctions

The results presented in the previous figures show the objectives and the altitude profiles of the trajectories taken by the different objects. These figures show that there can be similar trajectories results with various outcomes for the collision probability. They do not present a clear trend on how the accumulated collision probability is determined during the de-orbit. Figures 5.7, 5.8 and 5.9 give a clear insight in the accumulation of the collision probability during the different trajectories. These figures are split into two parts: part (a) shows the cumulative collision probability over time for the low-risk (blue) and the high-risk (orange) trajectory. The x-axis represents the propagation time of each trajectory (in days) and the y-axis represents the collision probability. Part (b) shows the number of conjunctions that occurred per orbital altitude. Here, the x-axis represents the orbital altitude (in kilometers) and the y-axis represents the number of conjunctions.

In part (a), each point represents a single conjunction event. Since there are many different conjunction events, multiple points in a row might appear as a thick line. The number in the bottom right of the graphs represents the total number of conjunctions. From this it immediately becomes clear that there are many more conjunctions than the annotations in Figures 5.4, 5.5 and 5.6 did seem to appear, but that the accumulated collision probability is largely determined by a few number of high-risk conjunction events.

In part (b), the number of conjunctions per orbital altitude are represented by a bar chart. A trend can be observed where at higher altitudes, the number of conjunctions is higher than at lower altitudes. This corresponds to the number of debris objects per orbital altitude shown in the green bar chart in Figures 5.4, 5.5 and 5.6, which indicates that at altitudes with a higher number of debris objects, more conjunctions occur.

Figure 5.7 shows the accumulation of the collision probability during the low-risk (in blue) and the high-risk (in orange) trajectory of Zenit-2. As can be seen in Figure 5.4, the low-risk trajectory encounters multiple conjunctions and the high-risk trajectory encounters one conjunction that account for 100.0% (rounded to one decimal precision) of the accumulated collision probability. How the collision probability accumulates over time can be seen in Figure 5.7(a). Multiple conjunctions occur which add to the collision probability, but only a handful collisions account for the majority of the accumulated collision probability. By observing Figure 5.7(b), it can be seen that the majority of these conjunctions occur at higher altitudes for both the low-risk and the high-risk trajectory. By comparing Figure 5.7(b) with Figure 5.4, it can be seen that both trajectories remain for multiple days at altitudes just above 800 km where the number of debris objects is the largest, which results in the majority of the conjunctions also occurring at these altitudes. It is interesting to note that no conjunctions occur below 760 km altitude for the low-risk trajectory and no conjunctions below 7800 km occur for the high-risk trajectory. Both trajectories spend approximately 100 days orbiting below these altitudes where still a relatively large number of debris objects is present. Statistically however, it should be expected that below these altitudes, a number of conjunctions would still occur.

Figure 5.8 shows the accumulation of the collision probability during the low-risk (in blue) and high-risk (in orange) trajectory of Tsyklon-3. As can be seen in Figure 5.5, a relatively large number of conjunctions occur which contribute significantly to the accumulated collision probability. Figure 5.8(a) clearly shows the two conjunctions with an equal collision probability occurring close to each other after around 105 days. In Figure 5.8(b) it can be seen that there are two spikes of a large number of conjunctions occurring. The first spike occurs at altitudes just below 650 km. Here, both trajectories remain for a while before activating their engine, and the number of debris objects is relatively high. The engines are then activated and the altitudes of both trajectories decrease to below 550 km where the engine is deactivated. Here, a second spike in the number of conjunctions can be observed. Looking at Figure 5.5, it can be seen that the high-risk trajectory orbits around an altitude of about 520 km, which is where the spike of the number of conjunctions also occurs in Figure 5.8(b). The low-risk trajectory orbits around 500 km altitude after the engine is cut off and a spike of the number of collisions can be observed at these altitudes. No high-risk conjunction event occurs for the low-risk trajectory during this period, however. It can clearly be seen that the number of conjunctions is relatively low while the engine of both trajectories is activated. This shows that the number of conjunctions is related to the time spent at certain orbital altitudes.

Figure 5.9 shows the accumulation of the collision probability during the low-risk (in blue) and the nominal (in orange) trajectory of Kosmos-3m. Figure 5.9(a) clearly shows that the accumulated collision probability is completely dominated by single conjunction events accounting for 100.0% (rounded to one decimal precision) of the accumulated collision probability. The conjunction events for the low-risk and nominal cases occur at the start of the trajectories. Further conjunction events during the trajectories do not contribute significantly to the accumulated collision probability. The combination of Figures 5.6 and 5.9(a) clearly shows that the shape of the trajectory does not necessarily mean that there will be high-risk conjunctions. By observing the number of conjunctions however, it can be seen that the low-risk trajectory encounters approximately 2.5 times less conjunctions than the nominal trajectory. This given can also be observed in Figure 5.9(b). In this figure it can clearly be seen that the low-risk trajectory encounters almost no conjunctions between altitudes of 400 and 600 km. Looking at Figure 5.6, it can be seen that at these altitudes the object rapidly descends, quickly crossing altitudes where the number of debris objects is relatively high. A spike in the number of conjunctions can clearly be seen around 350 km altitude. At this altitude, the engine is cut off and the object orbits for a longer period of time, as can be seen in Figure 5.6. The number of conjunctions at this altitude is still relatively low since the number of debris objects is also low. The altitude of the nominal trajectory is lowered more gradually during the trajectory and from Figure 5.9(b) it can be seen that the number of conjunctions corresponds more to the number of debris objects at those altitudes, showing a slow decrease in the number of conjunctions as the altitude of the object is lowered.

These results indicate that the number of conjunctions an object encounters during the de-orbit corresponds to the number of debris objects present at those altitudes, but that the collision probability of the de-orbit trajectory is determined by a low number of high-risk conjunction events. This makes it rather difficult to design a robust de-orbit trajectory where a guarantee can be given that the collision probability remains low during the trajectory. In order to further substantiate this finding, a sensitivity analysis is performed, where some of the initial state elements of the objects are varied slightly in order to observe the impact on the collision probability of the associated trajectories.

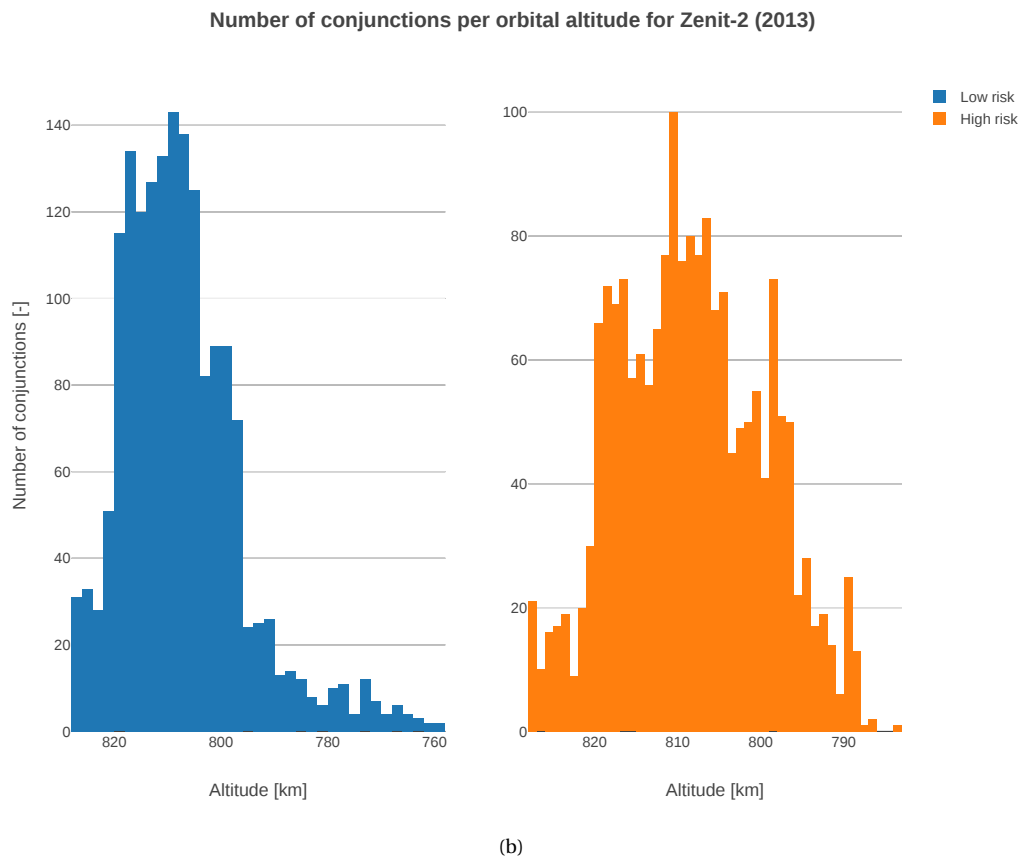
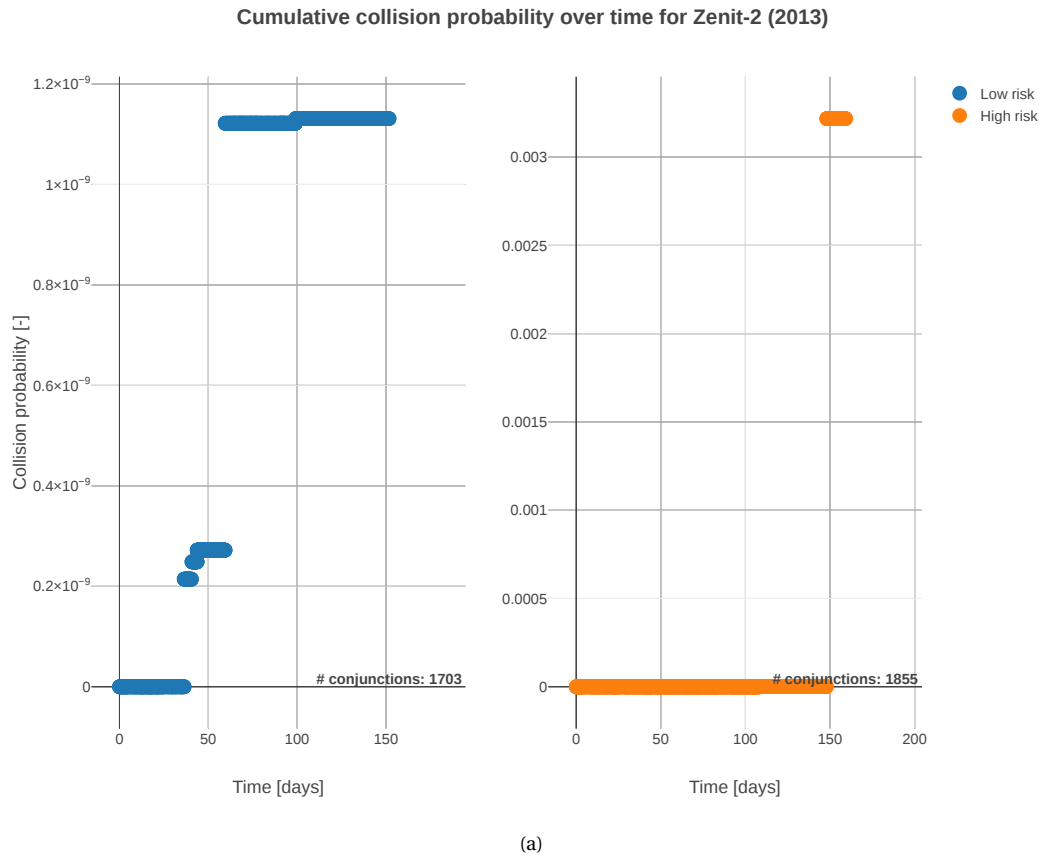


Figure 5.7: Collision probability accumulation over time and per altitude for the low-risk (blue) and high-risk (orange) trajectories of the Zenit-2 object using the TLE set of 2013: (a) shows the accumulated collision probability over time; and, (b) shows the number of conjunctions per altitude (note that the x-axes are reversed).

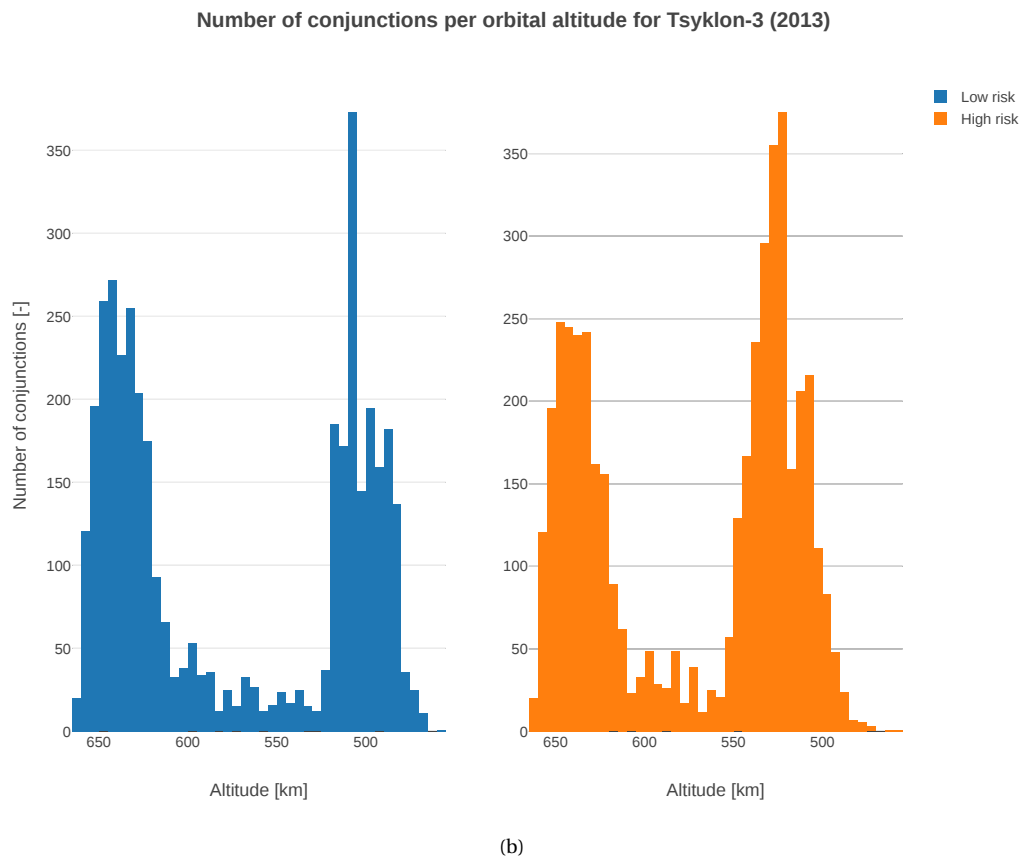
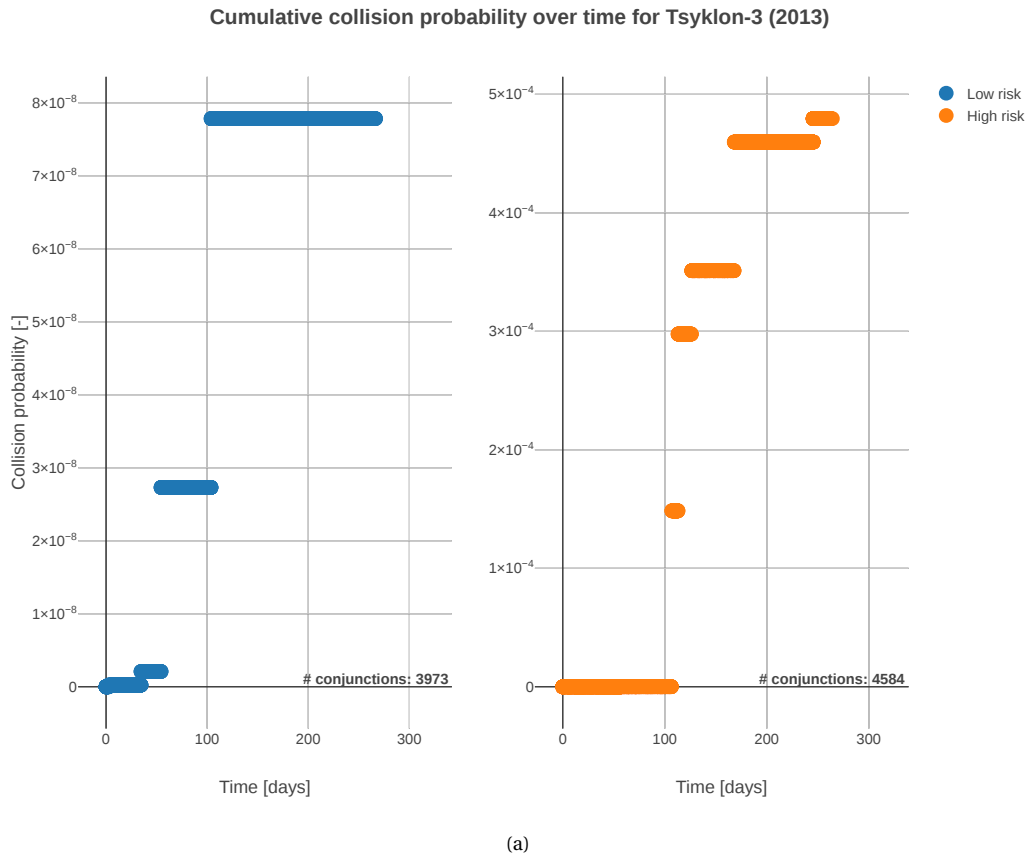


Figure 5.8: Collision probability accumulation over time and per altitude for the low-risk (blue) and high-risk (orange) trajectories of the Tsyklon-3 object using the TLE set of 2013: (a) shows the accumulated collision probability over time; and, (b) shows the number of conjunctions per altitude (note that the x-axes are reversed).

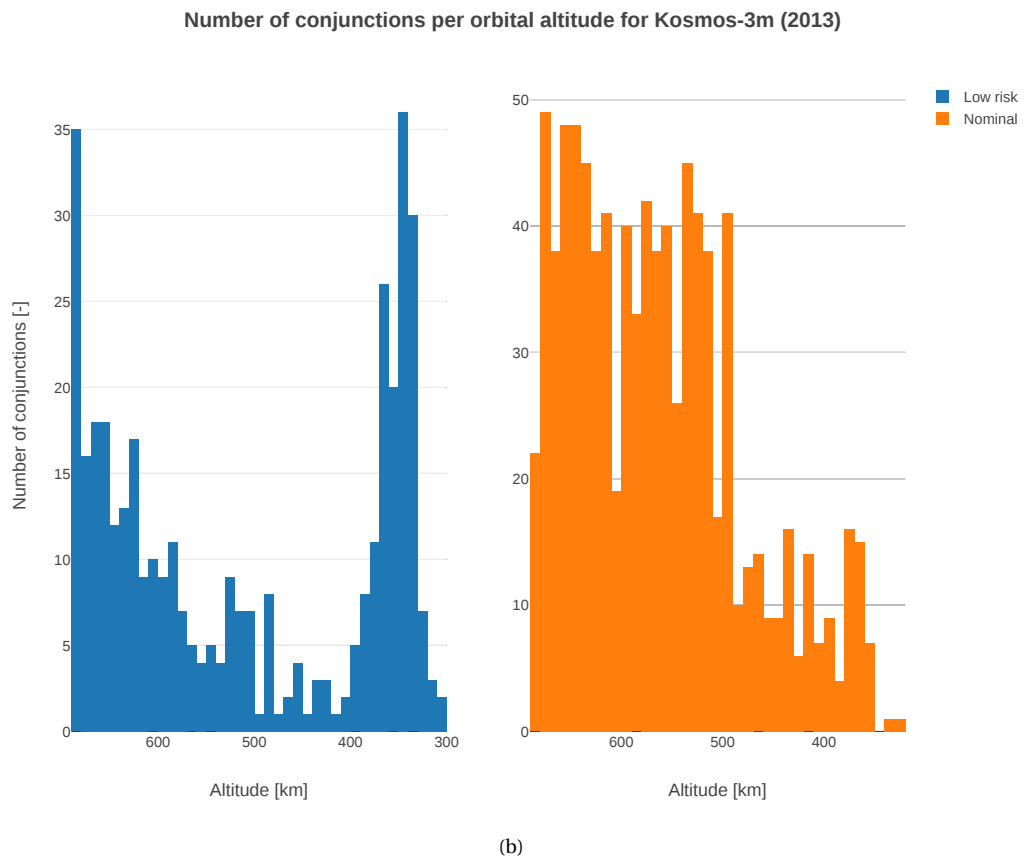
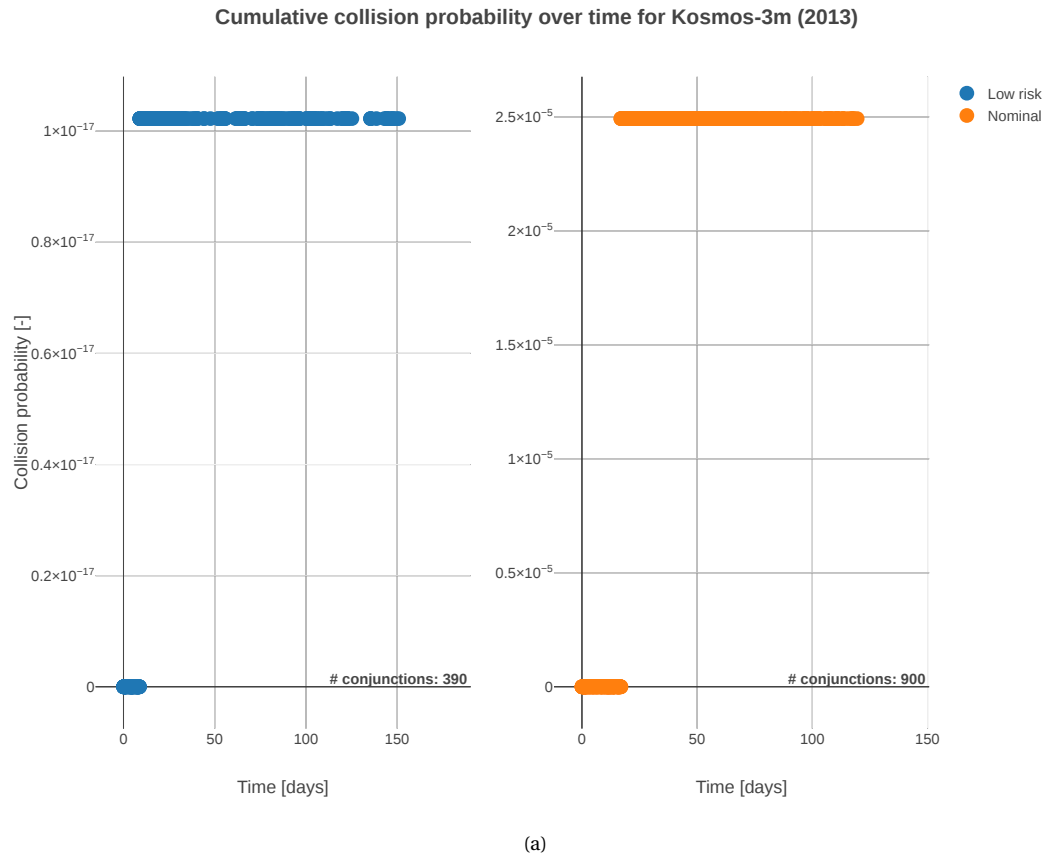


Figure 5.9: Collision probability accumulation over time and per altitude for the low-risk (blue) and nominal (orange) trajectories of the Kosmos-3m object using the TLE set of 2013: (a) shows the accumulated collision probability over time; and, (b) shows the number of conjunctions per altitude (note that the x-axes are reversed).

5.1.4. Sensitivity

Figures 5.10, 5.11 and 5.12 show the sensitivity of the accumulated collision probabilities of the trajectories presented in Figures 5.4, 5.5 and 5.6 for the three different objects. These figures are split up in two parts: part (a) shows the sensitivity of the collision probability for the high-risk trajectories; and part (b) shows the sensitivity of the collision probability for the low-risk trajectories. In this sensitivity analysis, the initial value of three different state elements of the object are varied slightly, after which the conjunction analysis is performed and the resulting collision probability of each trajectory is presented. This is performed 100 times for each different state element, resulting in 100 different outcomes of the associated collision probability. The three elements are: the semi-major axis (in blue), the right ascension of the ascending node (in orange) and the inclination (in green). For each different trajectory in this analysis, the initial value of the state element is determined by a Gaussian random number generator (with a constant seed number). This number generator uses a mean value and a standard deviation to generate random numbers that are Gaussian distributed around the mean value. The mean values of the initial elements are the original values. The standard deviation for the semi-major axis is equal to 1 km, and the standard deviation for the right ascension of the ascending node and the inclination is 1 degree. The distribution of the initial values are shown on the top part of the figure, where the x-axis represents the value of the initial element and the y-axis represents the number of hits. The values of the collision probabilities are shown in the middle part of the figure, where the x-axis represents the initial value of each parameter and the y-axis represents the value for the accumulated collision probability. The bottom part of the figure shows a box plot further indicating the spread of the collision probabilities. The points located next to the box plot represent the points shown in the part above, and correspond to the values of the y-axis, but not the x-axis. The thrust profile (thrust magnitude and turning the engine on/off) is still fixed to this reference case. From these figures it can be seen that a small variation in an initial state element can lead to large differences in the associated collision probability. Even if the variation in the initial state element is small with respect to the mean value, the collision probability can vary by many orders of magnitude.

Figure 5.10(a) shows the sensitivity of the obtained result of the high-risk trajectory of Zenit-2, shown in Figure 5.4. Here it can be seen that for an initial variation in each state element, the collision probability ranges by 6 to 9 orders of magnitude. This means that a small variation in the initial state can result in a difference of one billion times the reference collision probability. In Figure 5.10(b) this spread is even wider for an initial change in the right ascension of the ascending node. A variation in the initial inclination of the low-risk trajectory results in a smaller spread, but this spread is still six orders of magnitude, which is an indication that the obtained result is not very robust.

Figure 5.11(a) shows the sensitivity of the obtained result of the high-risk trajectory of Tsyklon-3, shown in Figure 5.5. Here the spread of the accumulated collision probability is smaller than the spread observed for Zenit-2 (Figure 5.10(a)). Observing the spread of the collision probabilities for a variation in the initial right ascension of the ascending node in Figure 5.11(b), it can be seen that the majority of the collision probabilities lie relatively close together (around 10^{-4}) but that there is a single trajectory with a relatively low collision probability. This is an indication that the trajectory is relatively robust, but since this figure shows the sensitivity of the low-risk trajectory it means that the low-risk trajectory shown in Figure 5.5 is actually an exception.

Figure 5.12(a) shows the sensitivity of the obtained result of the nominal trajectory of Kosmos-3m, shown in Figure 5.6. In this figure, it can clearly be seen that the spread of the obtained collision probabilities is very wide relative to the high-risk trajectories shown in the previous figures, indicating that the nominal trajectory is highly sensitive to variations in the initial state vector. Figure 5.12(b) shows the sensitivity of the low-risk trajectory. Here, it can be seen that some trajectories yield accumulated collision probabilities in the order of magnitude of -50 . These very low collision probabilities are the result of the object's rapid decline. Since at lower altitudes there are less debris objects to collide with (as can be seen in Figure 5.9(b)), variations in the initial state vector of the object can cause the object to avoid certain conjunction events while having less chance of creating a new high-risk conjunction event.

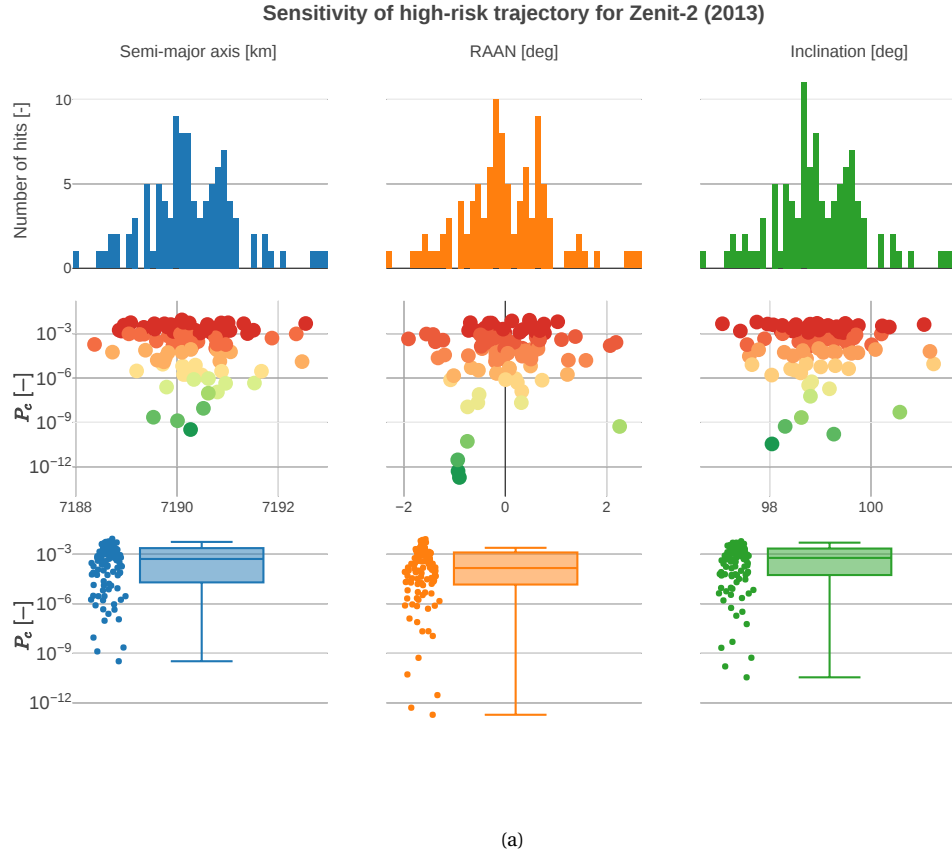


Figure 5.10: Sensitivity of the high-risk (top) and low-risk (bottom) trajectories by varying the semi-major axis (blue), inclination (orange) and right ascension of the ascending node (green) for the Zenit-2 object using the TLE set of 2013.

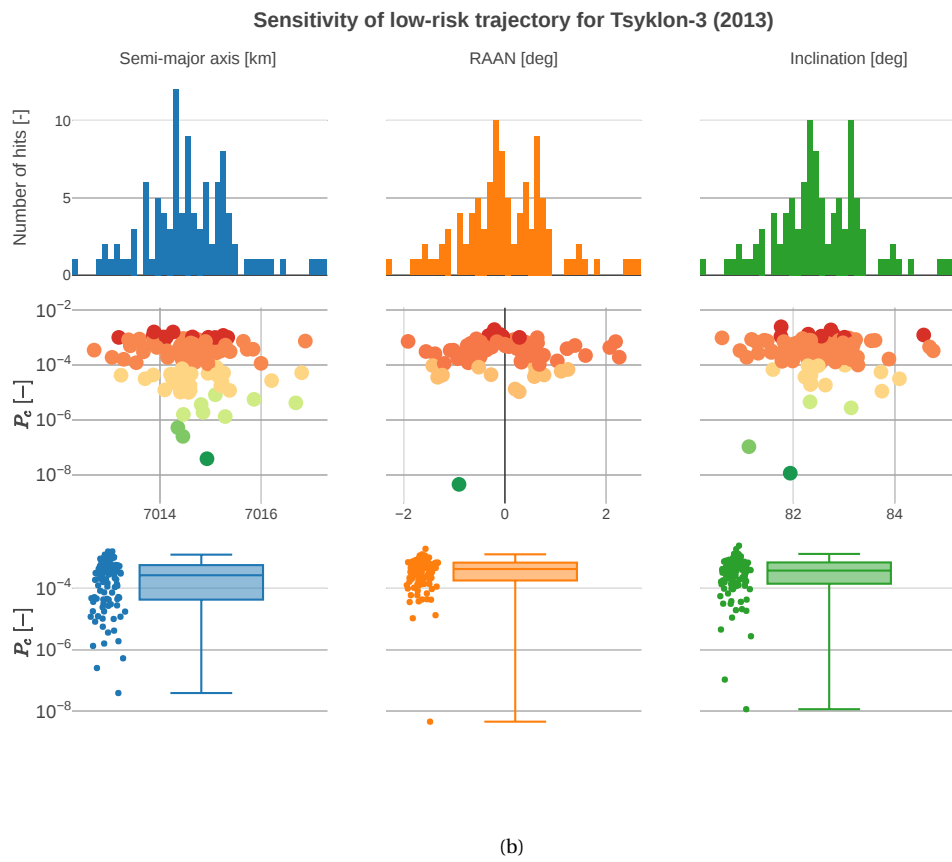
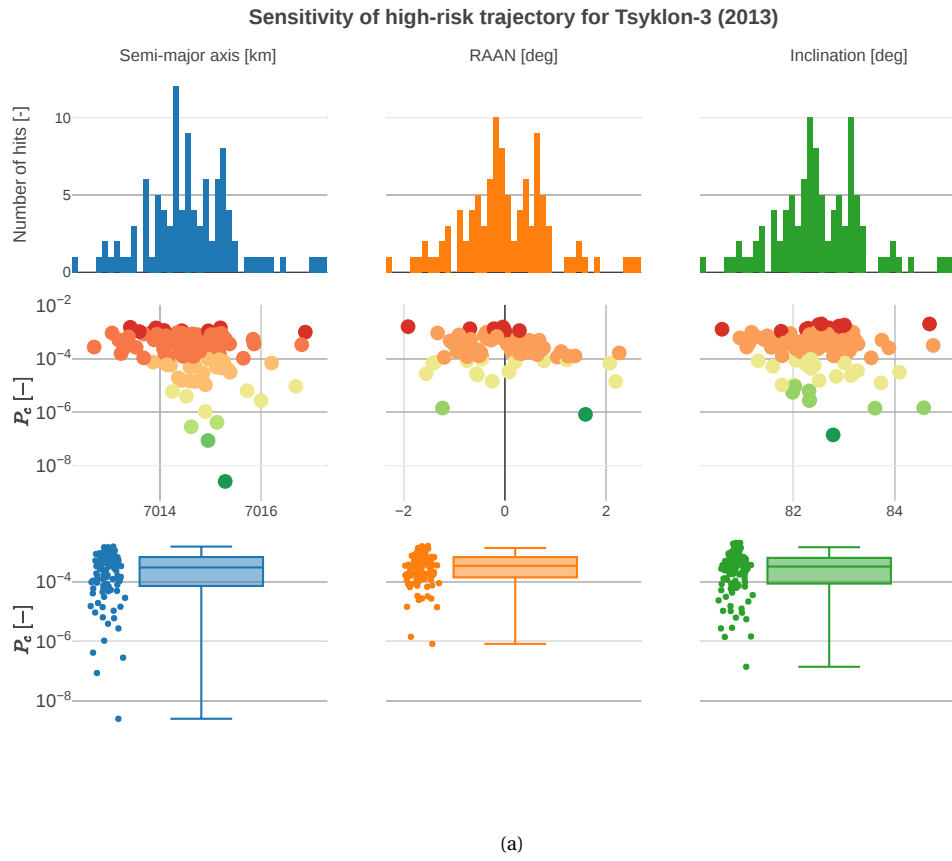


Figure 5.11: Sensitivity of the high-risk (top) and low-risk (bottom) trajectories by varying the semi-major axis (blue), inclination (orange) and right ascension of the ascending node (green) for the Tsyklon-3 object using the TLE set of 2013.

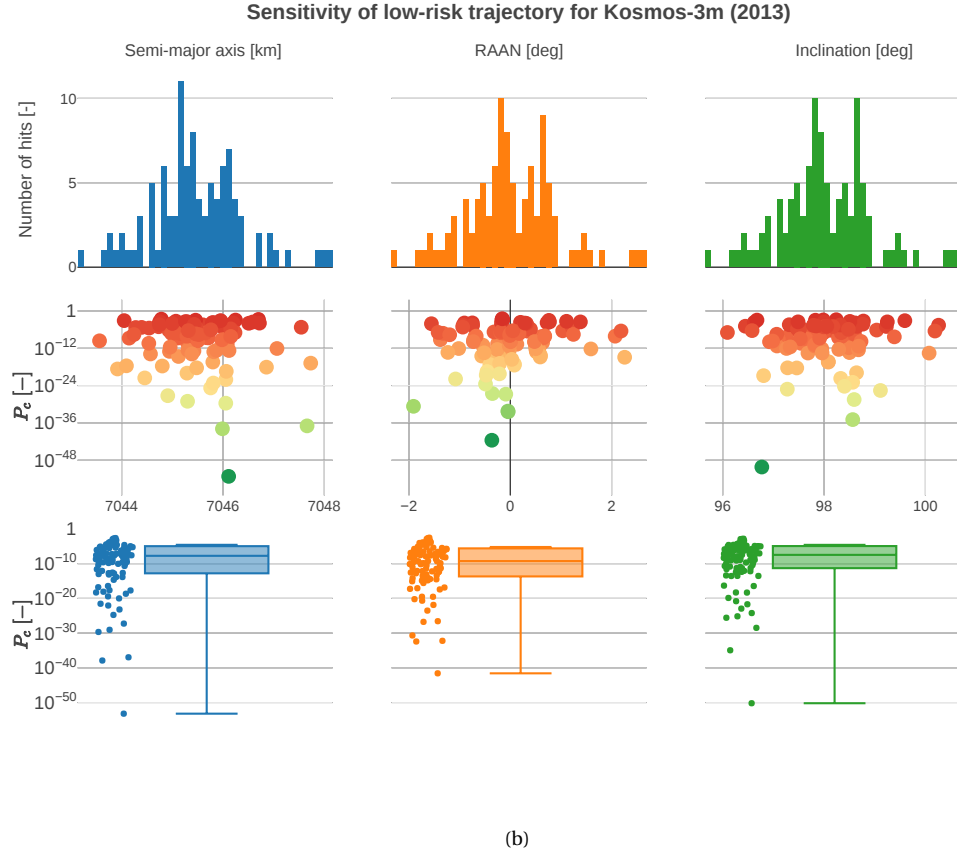
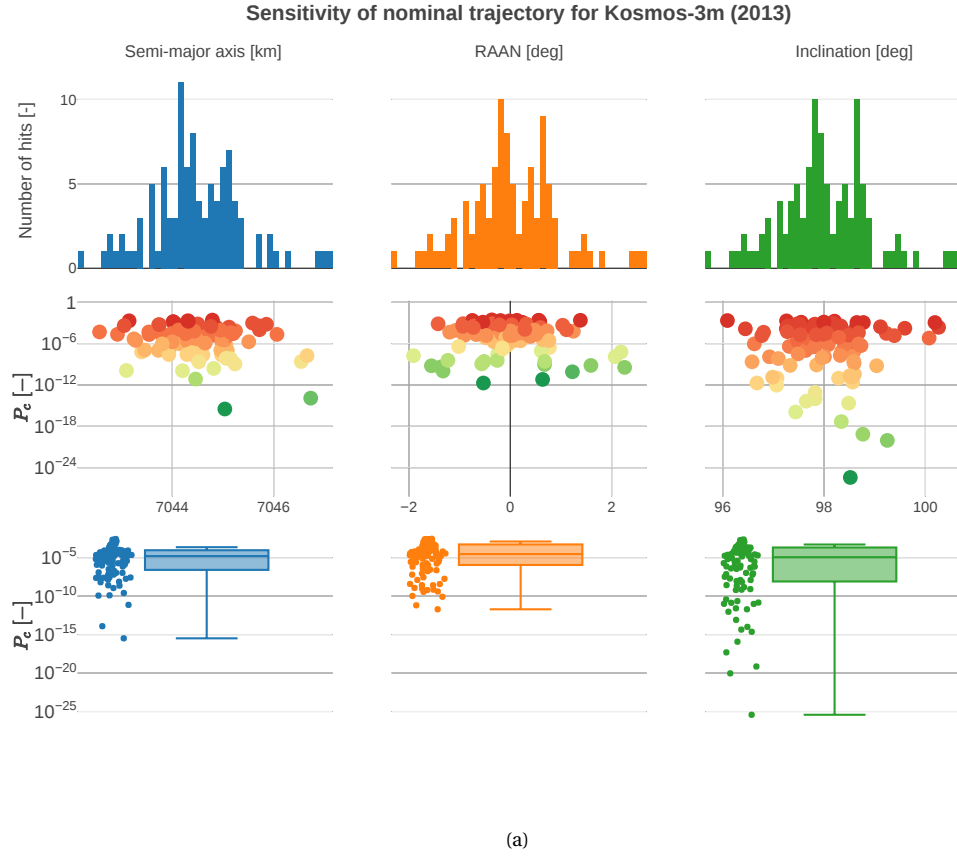


Figure 5.12: Sensitivity of the nominal (top) and low-risk (bottom) trajectories by varying the semi-major axis (blue), inclination (orange) and right ascension of the ascending node (green) for the Kosmos-3m object using the TLE set of 2013.

5.2. 2014 snapshot

This section presents the results for the TLE set of 7 November, 2014. The types of figures are the same as those presented in Section 5.1.

5.2.1. Trajectory objectives

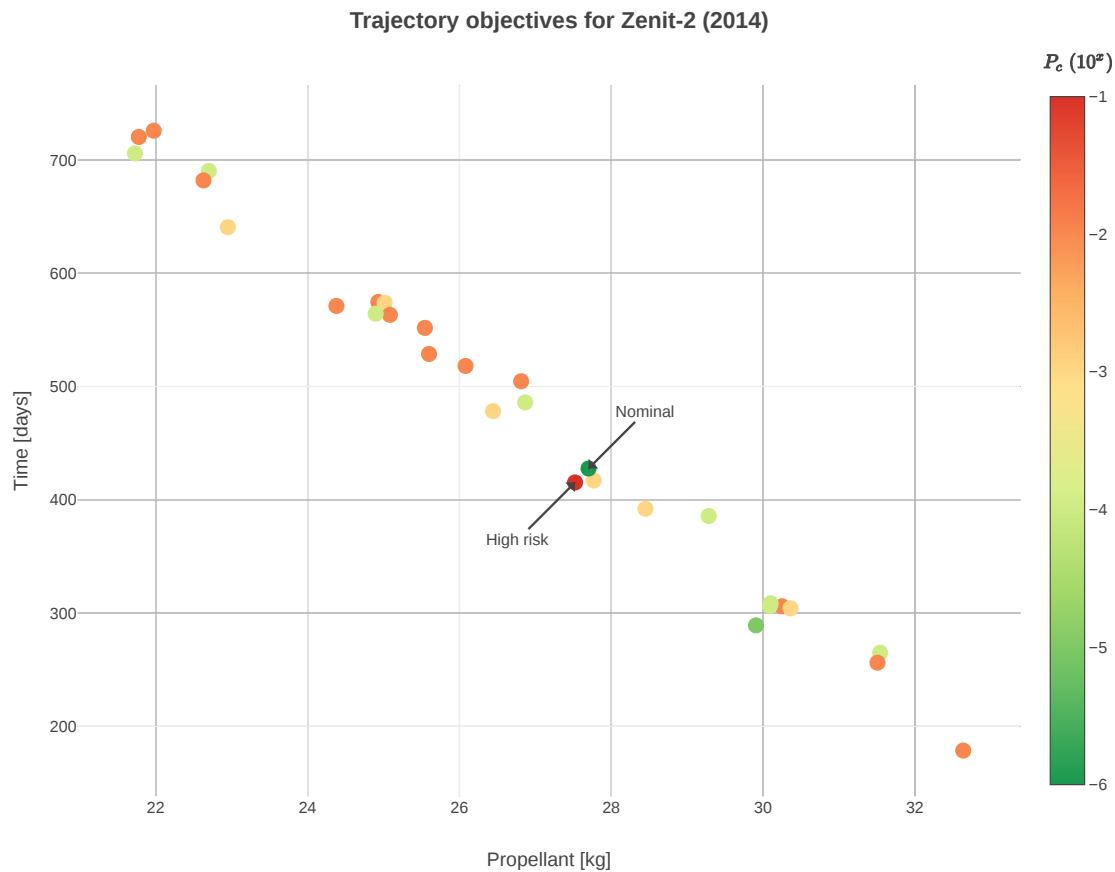


Figure 5.13: Trajectory objective for the Zenit-2 object using the TLE set of 2014. Each point represents a single trajectory. The x-axis represents the used propellant mass and the y-axis represents the propagation time of each trajectory. The collision probability P_c of each trajectory is represented as a color, where the color scale on the right indicates the order of magnitude of the collision probability. A red color represents a relatively high collision probability and a green color represents a relatively low collision probability. Trajectories of interest are indicated by arrows.

Figure 5.13 shows the values of the objectives for different trajectories of Zenit-2. The trajectories computed with the 2014 snapshot share similar values for the used propellant and propagation time as with the 2013 snapshot. The values for the accumulated collision probabilities are quite different, however. The range of the scale of the collision probabilities is smaller than the range for the 2013 snapshot, which means that most points share similar collision probabilities, resulting in points with seemingly equal collision probabilities. In reality, these different trajectories all have different collision probabilities, but many in the same order of magnitude. A striking feature of this figure is that the trajectory that results in the lowest collision probability is the nominal trajectory with constant thrust. A trajectory with a high collision probability is shown close to the point that represents the nominal trajectory. These two trajectories and the accumulation of the collision probabilities are shown in Figures 5.16 and 5.19.

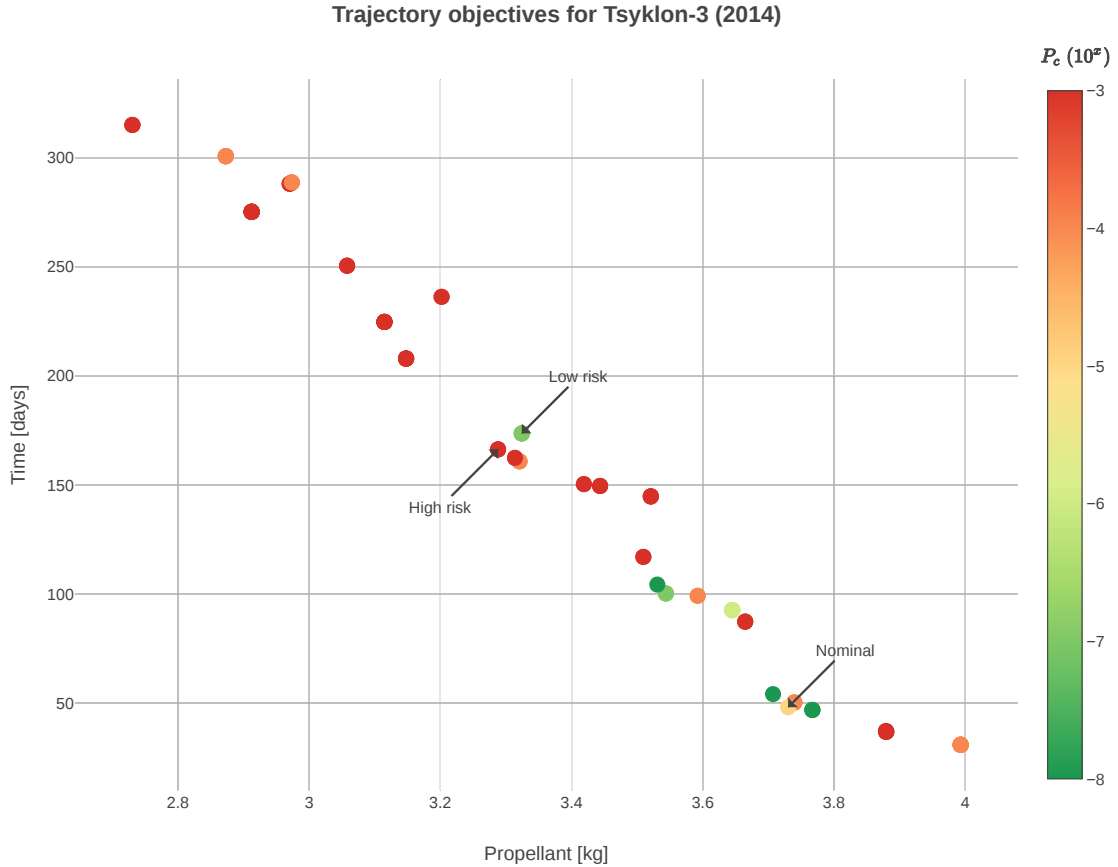


Figure 5.14: Trajectory objective for the Tsyklon-3 object using the TLE set of 2014. Each point represents a single trajectory. The x-axis represents the used propellant mass and the y-axis represents the propagation time of each trajectory. The collision probability P_c of each trajectory is represented as a color, where the color scale on the right indicates the order of magnitude of the collision probability. A red color represents a relatively high collision probability and a green color represents a relatively low collision probability. Trajectories of interest are indicated by arrows.

Figure 5.14 shows the values of the objectives for different trajectories for Tsyklon-3. It can be seen that the trajectories with short propagation times have a lower associated collision probability and that the trajectories with long propagation times have a higher associated collision probability, as usual. There are some trajectories that diverge from this trend, however. The trajectory with a propagation time of about 35 days (bottom right part of the figure) has a relatively high collision probability. Another point of interest is the low-risk trajectory annotated in the figure. This trajectory is surrounded with points that have relatively high collision probabilities, whereas this trajectory has a relatively low collision probability. The nominal trajectory in this case has an average collision probability. The low-risk and high-risk trajectories are further evaluated in Figures 5.17 and 5.20 in order to provide a qualitative insight in the differences of the trajectories and the accumulation of the collision probability.

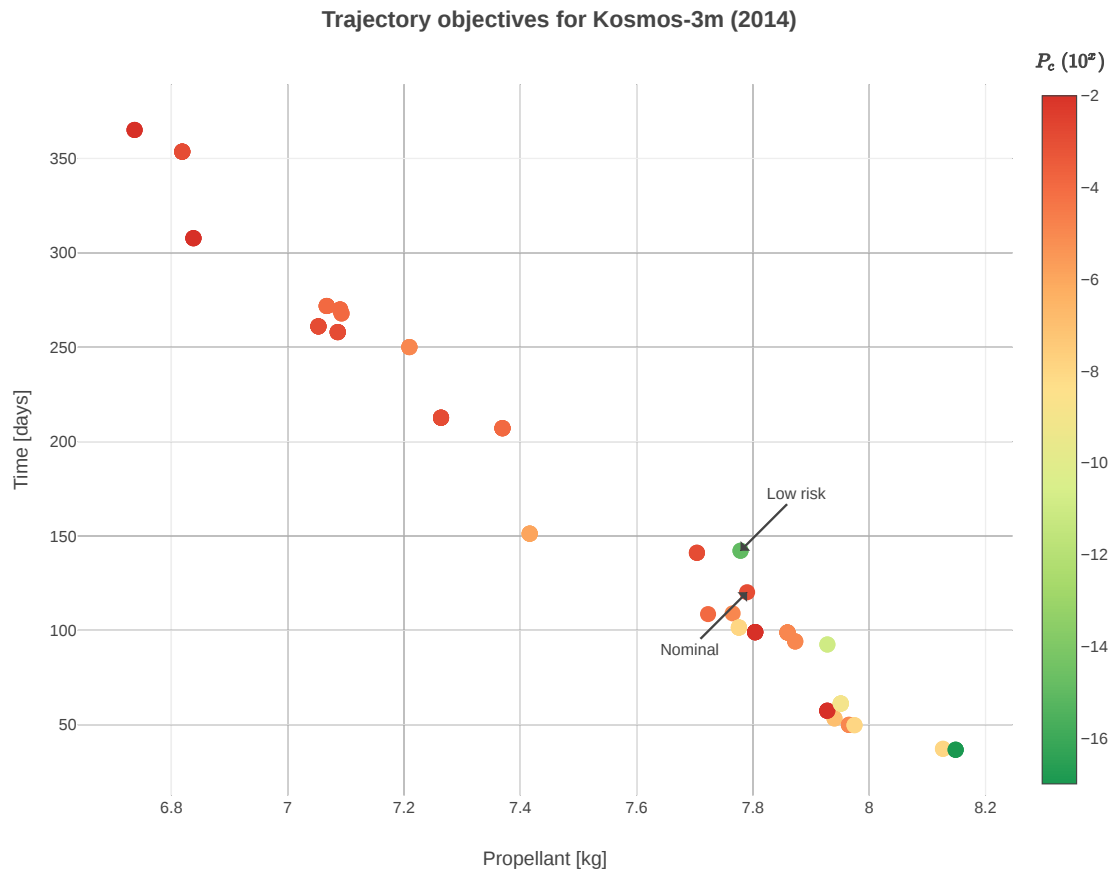


Figure 5.15: Trajectory objective for the Kosmos-3m object using the TLE set of 2014. Each point represents a single trajectory. The x-axis represents the used propellant mass and the y-axis represents the propagation time of each trajectory. The collision probability P_c of each trajectory is represented as a color, where the color scale on the right indicates the order of magnitude of the collision probability. A red color represents a relatively high collision probability and a green color represents a relatively low collision probability. Trajectories of interest are indicated by arrows.

Figure 5.15 shows the values of the objectives for different trajectories for Kosmos-3m. This figure does not clearly show the trend that shorter propagation times result in lower collision probabilities. However, the range of the scale of the collision probabilities is much wider than the range shown in Figures 5.13 and 5.14. In this figure there are some trajectories that show collision probabilities with an order of magnitude of around -8, but since they are presented by a yellow color due to the wide probability range, they do not stick out in the figure. Also, there is a larger number of high-risk trajectories visible (especially with short propagation times). There is one trajectory of interest, which is the low-risk trajectory annotated in this figure. Since this trajectory uses similar amounts of propellant as the nominal trajectory, these two trajectories are compared in Figure 5.18.

5.2.2. Trajectory profiles

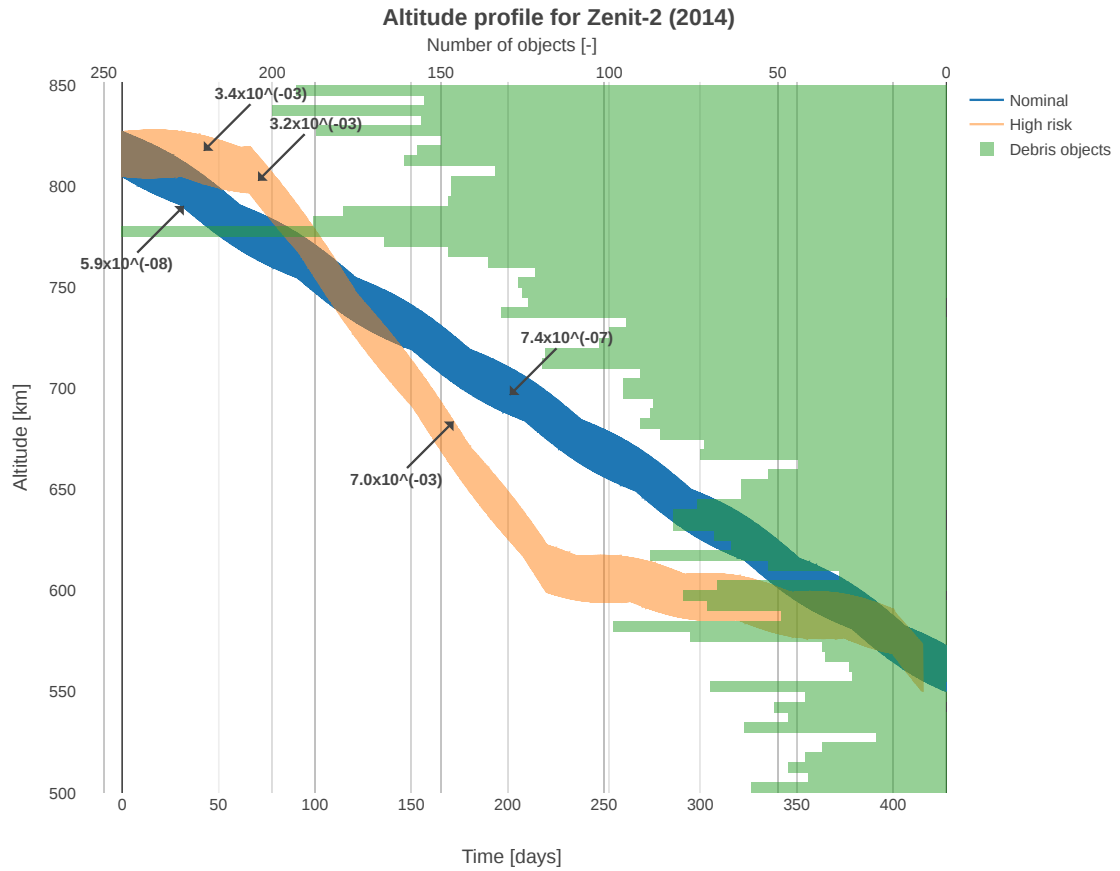


Figure 5.16: Altitude profiles of two trajectories of interest for the Zenit-2 object using the TLE set of 2014. The trajectory with a low collision probability is indicated in blue, and the trajectory with a high collision probability is indicated in orange. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.16 shows the altitude profile over time for the nominal (in blue) and high-risk (in orange) trajectory for Zenit-2, which were indicated in Figure 5.13. From this figure it can be seen that the altitude of the nominal trajectory gradually decreases over time. During this time, the trajectory encounters two conjunction events (5.9×10^{-8} and 7.4×10^{-7}) that account for 100.0% (rounded to one decimal) of the accumulated collision probability. The high-risk trajectory activates its engine after approximately 65 days after which the altitude quickly decreases. The engine is deactivated after approximately 220 days and is activated again after approximately 400 days. Just before and after the first engine activation, the trajectory encounters two conjunction events (3.4×10^{-3} and 3.2×10^{-3}) that add significantly to the accumulated collision probability. It should be noted that at the altitude where these conjunctions occur, the number of debris objects is lower than at the altitude where the first conjunction of the nominal trajectory occurs. The nominal trajectory however does not encounter a conjunction event with a relatively high collision probability. The high-risk trajectory encounters a final conjunction event (7.0×10^{-3}) during the engine activation at an altitude of approximately 680 km.

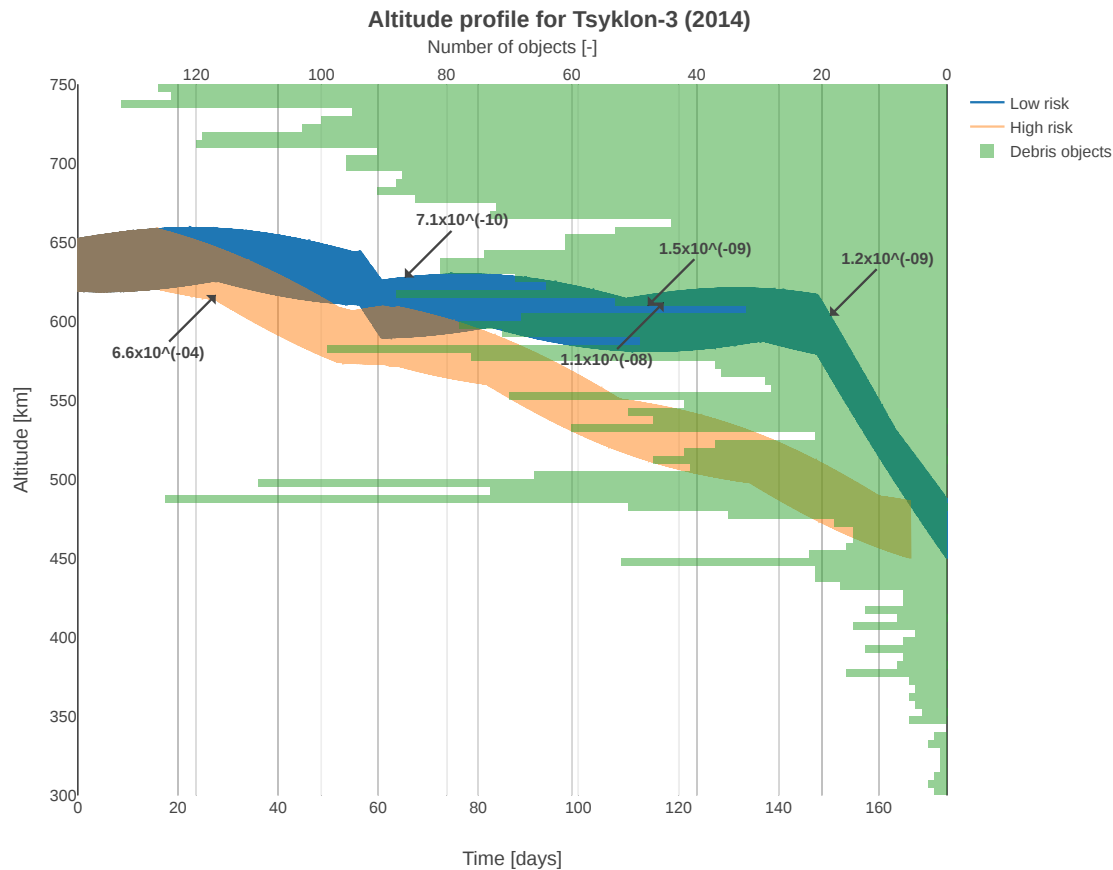


Figure 5.17: Altitude profiles of two trajectories of interest for the Tsyklon-3 object using the TLE set of 2014. The trajectory with a low collision probability is indicated in blue, and the trajectory with a high collision probability is indicated in orange. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.17 shows the altitude profile over time for the low-risk (in blue) and high-risk (in orange) trajectory for Tsyklon-3, which were indicated in Figure 5.14. In this figure it can be seen that both trajectories have a slow decrease in altitude. The low-risk trajectory uses a larger thrust magnitude than the high-risk trajectory but does only briefly activate the engine at the start of the trajectory (after approximately 55 days) and reactivates it after approximately 595 days. The high-risk trajectory after approximately 15 days, and briefly deactivates it around 55 days after the start of the trajectory. During the entire propagation, the high-risk trajectory only encounters one conjunction (6.6×10^{-4}) that accounts for 100.0% (rounded to one decimal precision) of the accumulated collision probability. The low-risk trajectory however encounters several conjunctions during its propagation but does not encounter a high-risk conjunction relative to the high-risk trajectory. The first conjunction (7.1×10^{-10}) is encountered just after the deactivation of the engine. Then, after approximately 115 days after the start of the trajectory, it encounters two conjunctions (1.1×10^{-8} and 1.5×10^{-9}) close after each other. The final conjunction (1.2×10^{-9}) is encountered just after the engine is reactivated again. It should be noted that, even though the low-risk trajectory orbits at an altitude with a relatively high number of debris objects, it does not encounter any high-risk conjunction events.

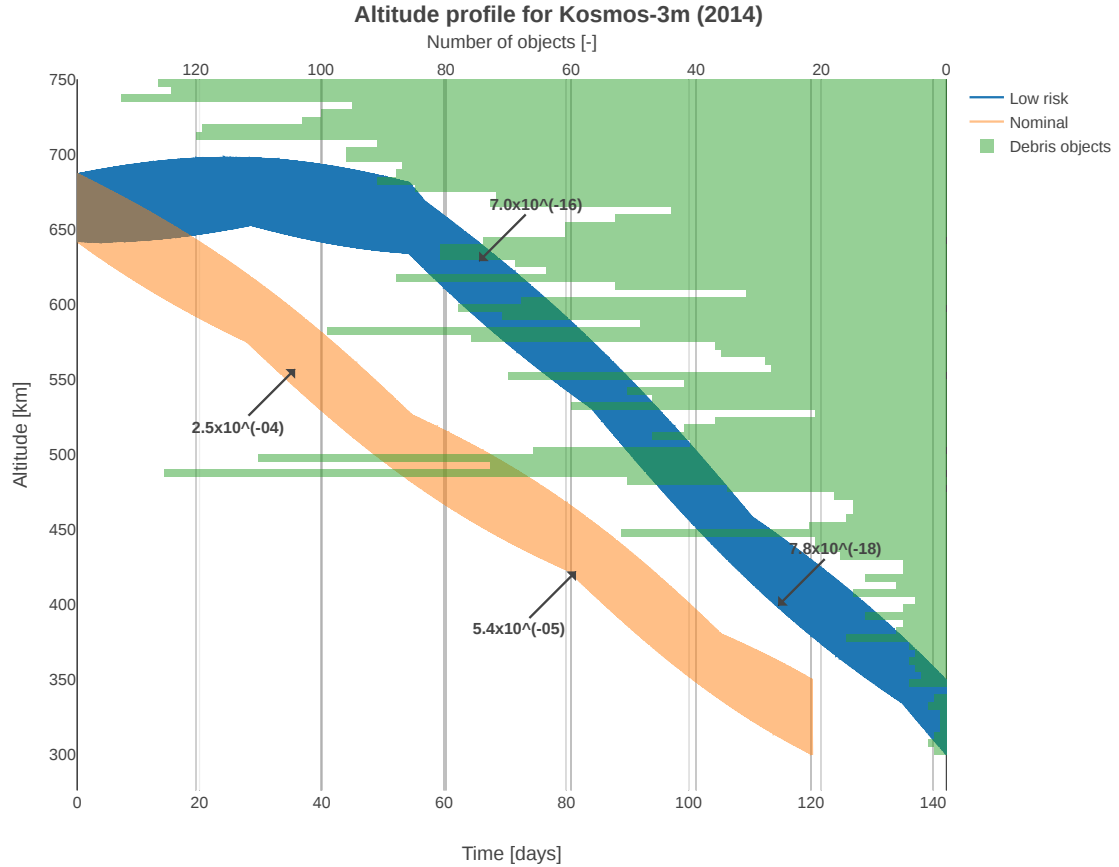


Figure 5.18: Altitude profiles of two trajectories of interest for the Kosmos-3m object using the TLE set of 2014. The trajectory with a low collision probability is indicated in blue, and the trajectory with a high collision probability is indicated in orange. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.18 shows the altitude profile over time for the low-risk (in blue) and nominal (in orange) trajectory for Kosmos-3m, as indicated in Figure 5.15. It can be seen that the altitude of the nominal trajectory slowly decreases over time. The low-risk trajectory activates its engine after approximately 55 days and is kept turned on during the remainder of the trajectory. The low-risk trajectory uses a thrust magnitude which is slightly higher than that of the nominal trajectory. The nominal trajectory encounters two conjunctions (2.5×10^{-4} and 5.4×10^{-5}) that account for 100.0% (rounded to one decimal precision) of the accumulated collision probability. These conjunctions occur at altitudes where the number of debris objects is relatively low. The low-risk trajectory also encounters two conjunctions (7.0×10^{-16} and 7.8×10^{-18}) that account for the majority of the accumulated collision probability. It should be noted that the low-risk trajectory remains for 55 days at an altitude where the number of debris objects is high relative to lower altitudes but does not encounter any significant conjunctions in this period. A more qualitative insight into the accumulation of the collision probability is presented in Figure 5.21.

5.2.3. Conjunctions

Figure 5.19 shows the accumulation of the collision probability during the nominal (in blue) and the high-risk (in orange) trajectory for Zenit-2. As can be seen in Figure 5.19(a), the nominal trajectory encounters two conjunction events that contribute significantly to the accumulated collision probability and the high-risk trajectory encounters three conjunction events. Figure 5.19(b) shows the number of conjunctions per orbital altitude. By comparing this figure to Figure 5.16, it can be seen that the nominal trajectory encounters conjunctions during the entire trajectory, where the number of conjunctions decreases with altitude, which is to be expected since the number of debris objects also decreases with altitude. It can clearly be seen in Figure 5.19(b) that the high-risk trajectory encounters a large number of conjunctions at altitudes around 810 km. By looking at Figure 5.16 it can be seen that the engine is not yet activated and the object orbits at an altitude where the number of debris objects is relatively high. The number of conjunctions decreases at lower altitudes when the engine is turned on and the altitude is decreasing rapidly. Again, Figure 5.19(b) shows that both trajectories of the Zenit-2 object do not encounter conjunctions below 680 km (for the nominal trajectory) and 630 km (for the high-risk trajectory) altitude. As for the 2013 snapshot, both trajectories orbit a considerable amount of time below these altitudes, where the number of debris objects is still relatively high. This is an interesting feature of both trajectories of the Zenit-2 object for the 2013 and 2014 snapshot.

Figure 5.20 shows the accumulation of the collision probability during the low-risk (in blue) and high-risk (in orange) trajectory for Tsyklon-3. In Figure 5.20(a) it can clearly be seen that the low-risk trajectory encounters four conjunctions that account for the majority of the accumulated collision probability, whereas the collision probability of the high-risk trajectory is determined by a single high-risk conjunction event. Looking at Figure 5.20(b) it can be seen that the number of conjunctions for the low-risk trajectory is high around 625 km altitude. By comparing this figure to Figure 5.17 it can be seen that the object orbits at this altitude for a relatively long period, thus increasing its chances for encountering multiple conjunctions. Another small peak can be seen around 525 km altitude. By looking at Figure 5.17, it can be seen that at this altitude, the number of debris objects shows a peak, which can explain the increased number of conjunctions. The number of conjunctions for the high-risk trajectory shows a correspondence to the number of debris objects per altitude, shown in Figure 5.17. Two peaks are visible: one between 600 and 650 km altitude and one around 500 km altitude. At these altitudes there are also peaks visible in the number of debris objects. This is an indication that the number of conjunctions is related to the number of debris objects per altitude.

Figure 5.21 shows the accumulation of the collision probability during the low-risk (in blue) and nominal (in orange) trajectory for Kosmos-3m. In Figure 5.21(a) it can be seen that the majority of the collision probability for the low-risk trajectory is accounted for by one single conjunction event. The accumulated collision probability for the nominal trajectory is accounted for by two high-risk conjunction events. Figure 5.21(b) shows that the largest number of conjunctions occur at higher altitudes for both trajectories. The low-risk trajectory encounters a large number of conjunctions at altitudes around 650 km. Looking at Figure 5.18, it can be seen that at these altitudes the engine of the object is not yet activated and the object remains for a longer period at this altitude, thus encountering multiple conjunctions. The nominal trajectory encounters a large number of conjunctions around 600 km altitude. This does not necessarily correspond to the distribution of the debris objects as seen in Figure 5.18. The same peak also is not observed in the same figure for the low-risk trajectory. This might suggest that there does not necessarily have to be a correspondence between the statistical number of debris objects and the number of conjunctions encountered by the object during the de-orbit.

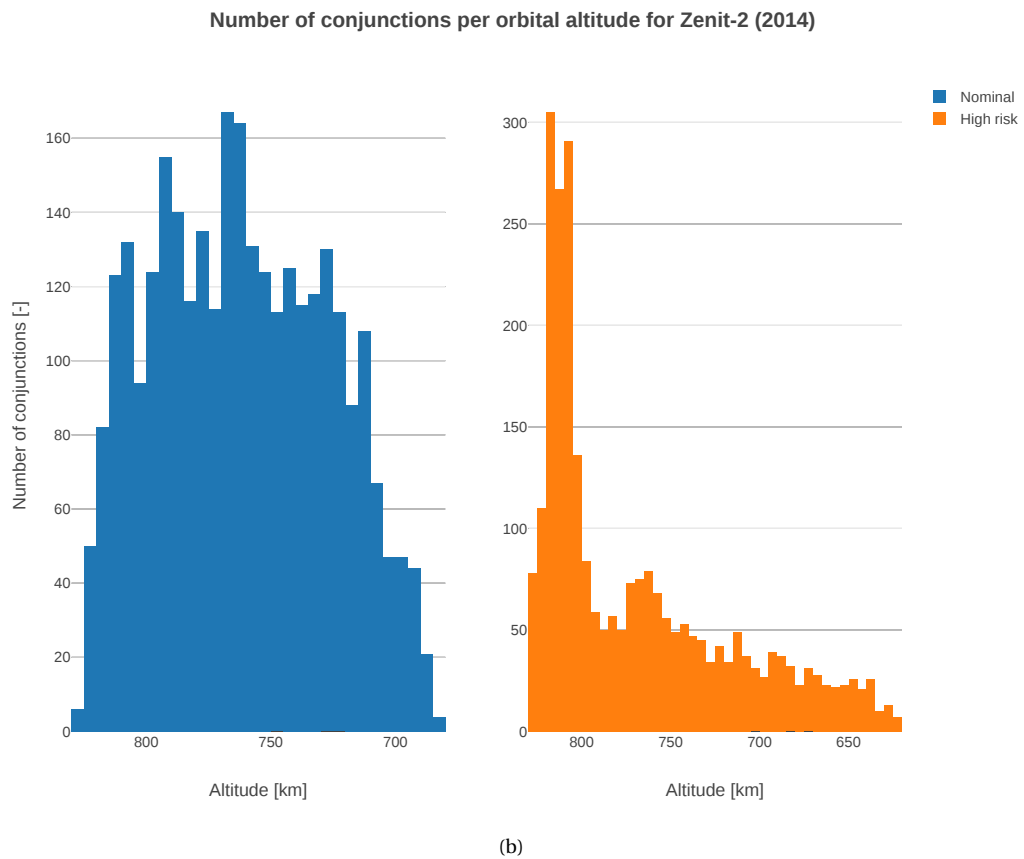
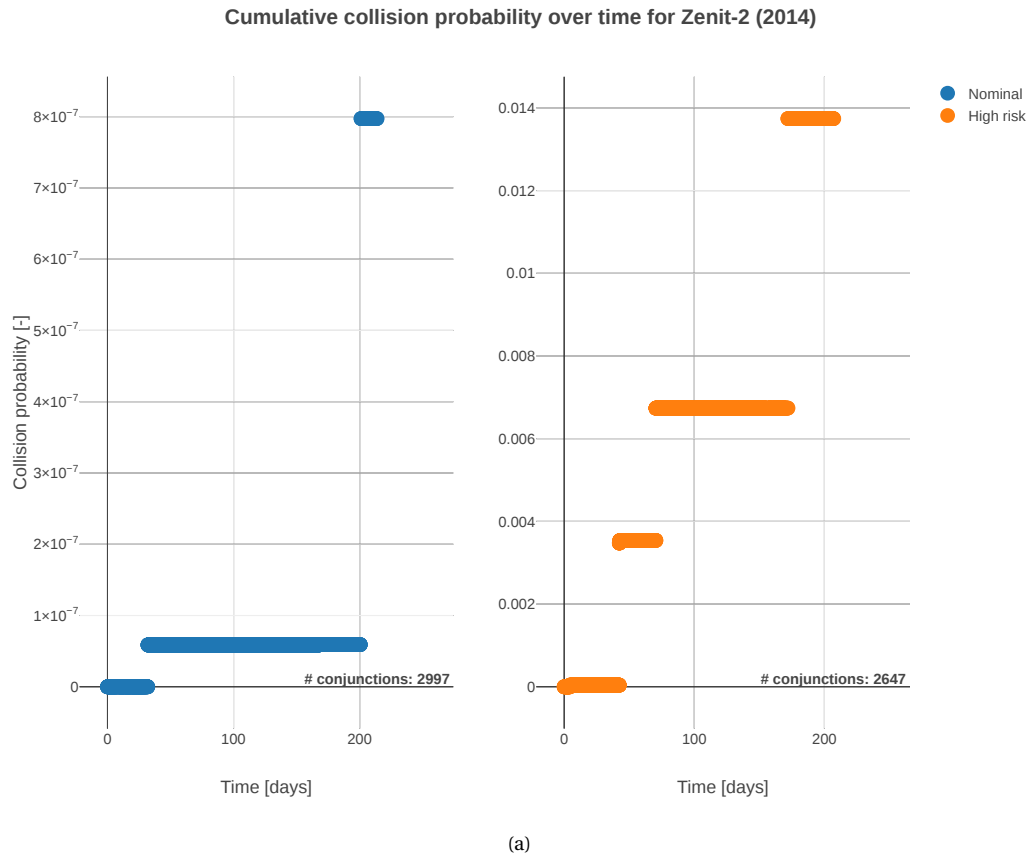


Figure 5.19: Collision probability accumulation over time and per altitude for the nominal (blue) and high-risk (orange) trajectories of the Zenit-2 object using the TLE set of 2014: (a) shows the accumulated collision probability over time; and, (b) shows the number of conjunctions per altitude (note that the x-axes are reversed).

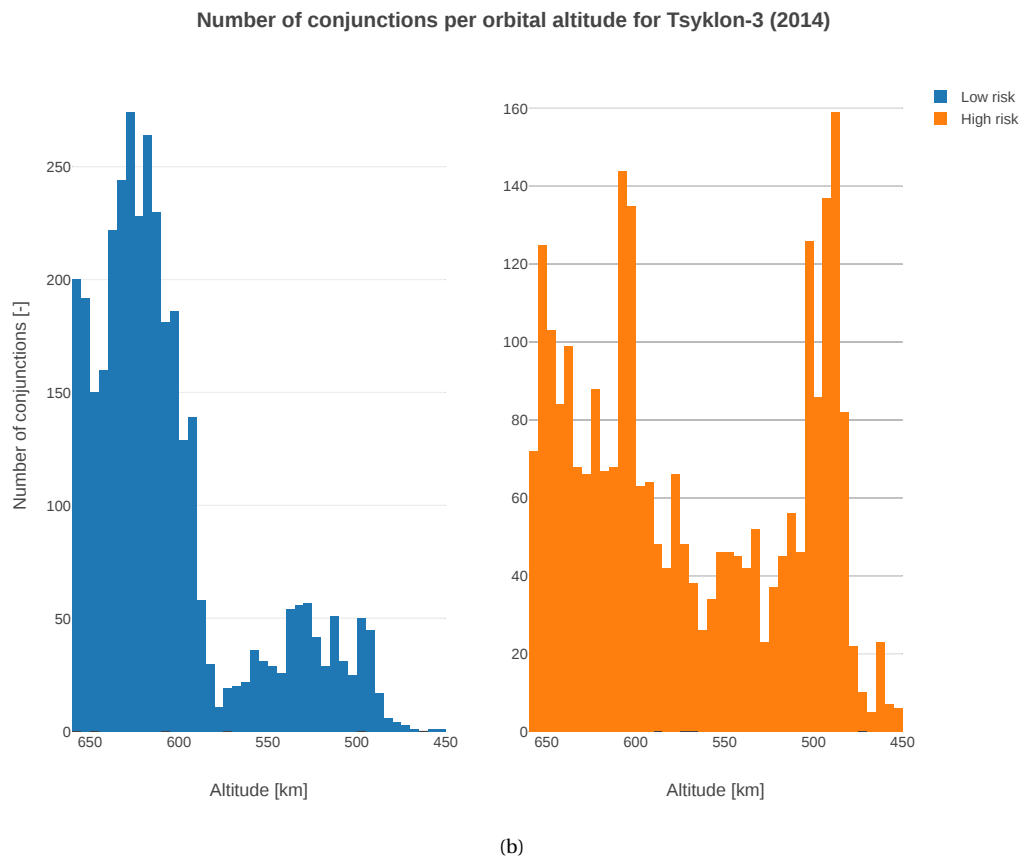
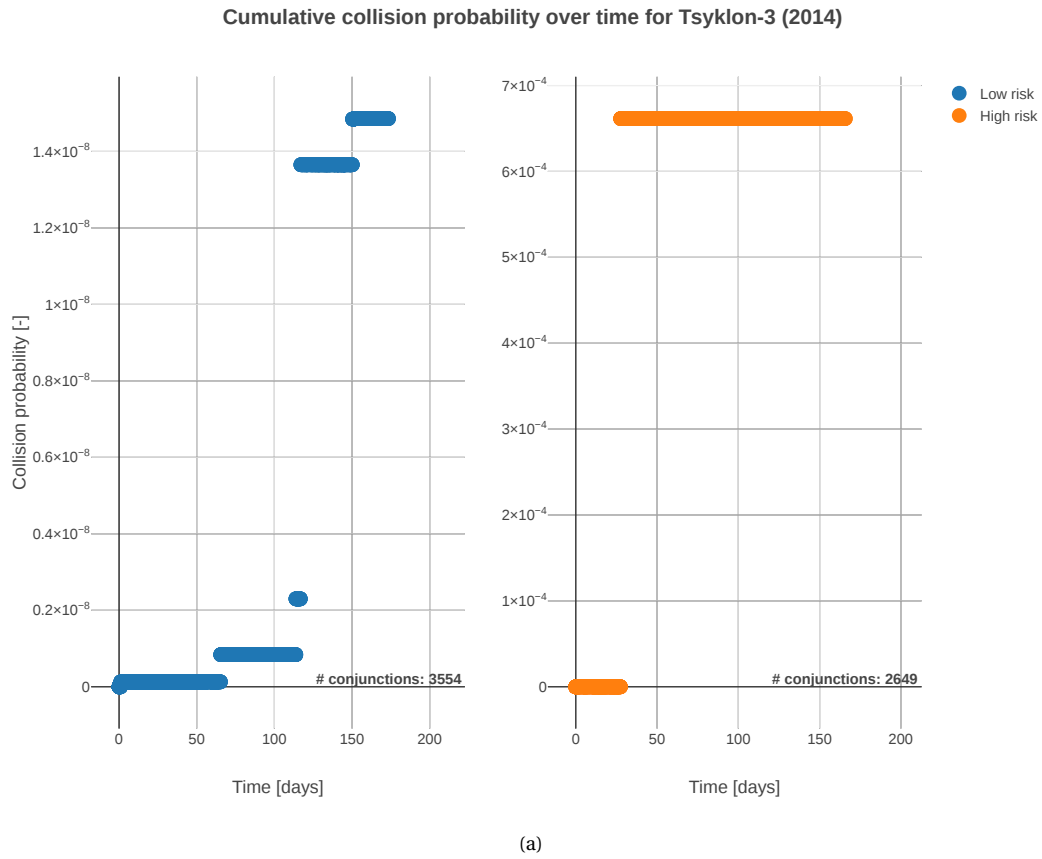


Figure 5.20: Collision probability accumulation over time and per altitude for the low-risk (blue) and high-risk (orange) trajectories of the Tsyklon-3 object using the TLE set of 2014: (a) shows the accumulated collision probability over time; and, (b) shows the number of conjunctions per altitude (note that the x-axes are reversed).

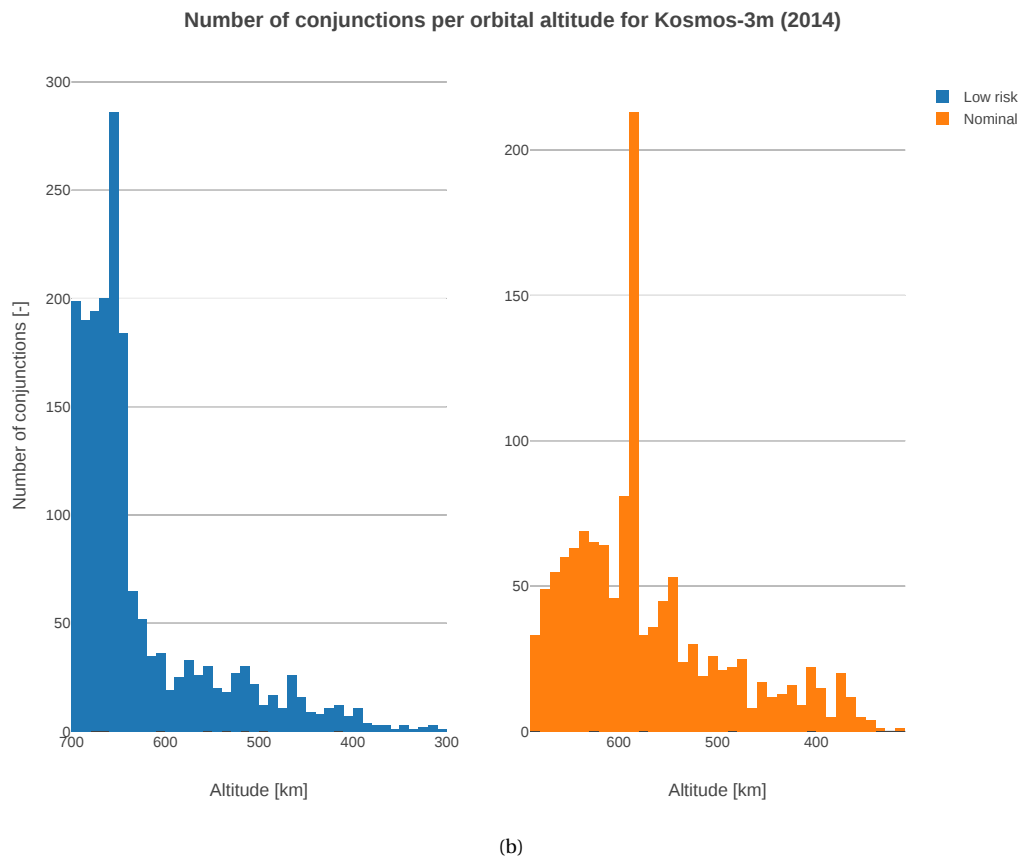
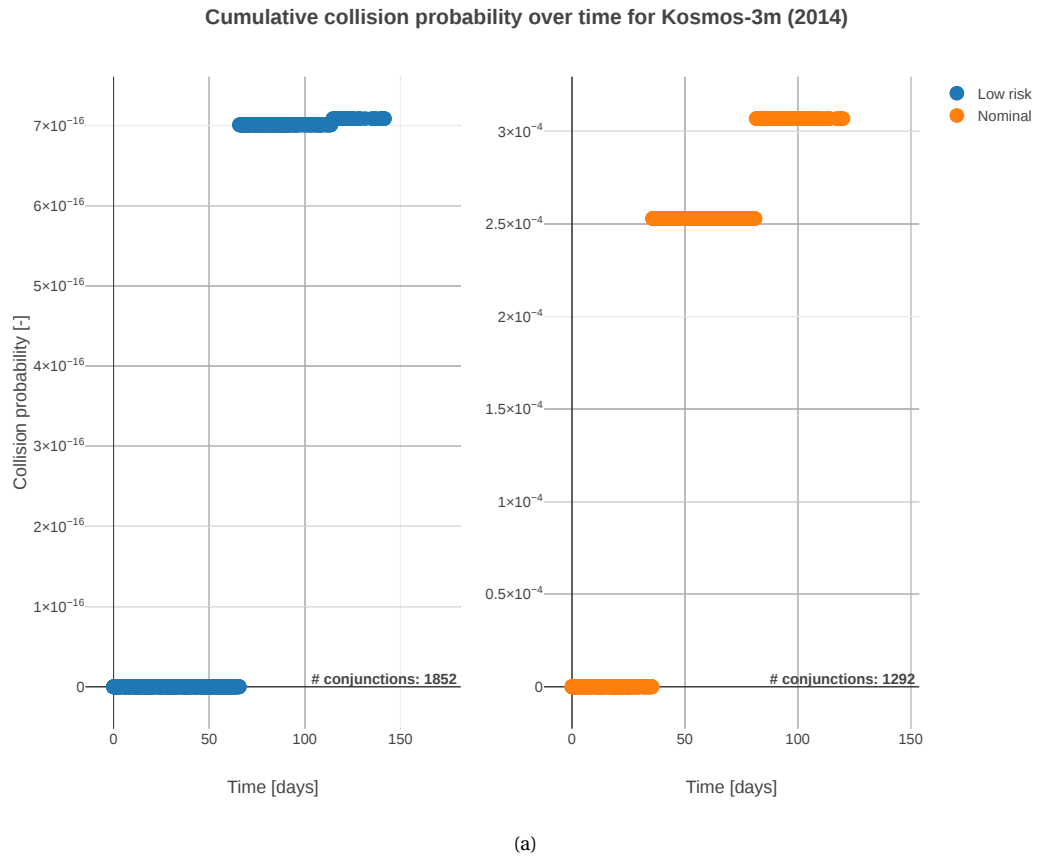


Figure 5.21: Collision probability accumulation over time and per altitude for the low-risk (blue) and nominal (orange) trajectories of the Kosmos-3m object using the TLE set of 2014: (a) shows the accumulated collision probability over time; and, (b) shows the number of conjunctions per altitude (note that the x-axes are reversed).

5.2.4. Sensitivity

Figure 5.22(a) shows the sensitivity of the obtained result of the high-risk trajectory for Zenit-2, shown in Figure 5.16. Like the 2013 case, a wide spread in the collision probabilities is visible. Even a very small variation in the initial semi-major axis results in a collision probability of approximately 10^{-14} . This is a clear indication that the obtained result for the high-risk trajectory shown in Figure 5.13 is not robust. The same observation can be made in Figure 5.22(b), where a small change in the inclination leads to a collision probability of approximately 10^{-11} .

Figure 5.23(a) shows the sensitivity of the obtained result of the high-risk trajectory for Tsyklon-3, shown in Figure 5.17. It can be seen that the spread in the collision probability is very wide for variations in all three initial state elements, indicating that the obtained results from Figure 5.14 are not very robust. The same observation holds for the sensitivity of the obtained result of the low-risk trajectory. Here, the spread in the collision probability is smaller than for the high-risk trajectory, but three orders of magnitude difference in the collision probability for a small change in an initial state element still indicates that the obtained low-risk trajectory shown in Figure 5.14 is not robust.

Figure 5.24(a) shows the sensitivity of the obtained result of the nominal trajectory for Kosmos-3m, shown in Figure 5.15. It can be seen that there is a large spread in the collision probability and that there are multiple trajectories for which the collision probability is relatively low after a small variation in an initial state element. The same observation holds for the sensitivity of the obtained result for the low-risk trajectory, shown in Figure 5.24(b). The associated collision probabilities vary by 9 orders of magnitude, which means that a small variation in an initial state element can lead to a billion times difference in the value of the collision probability.

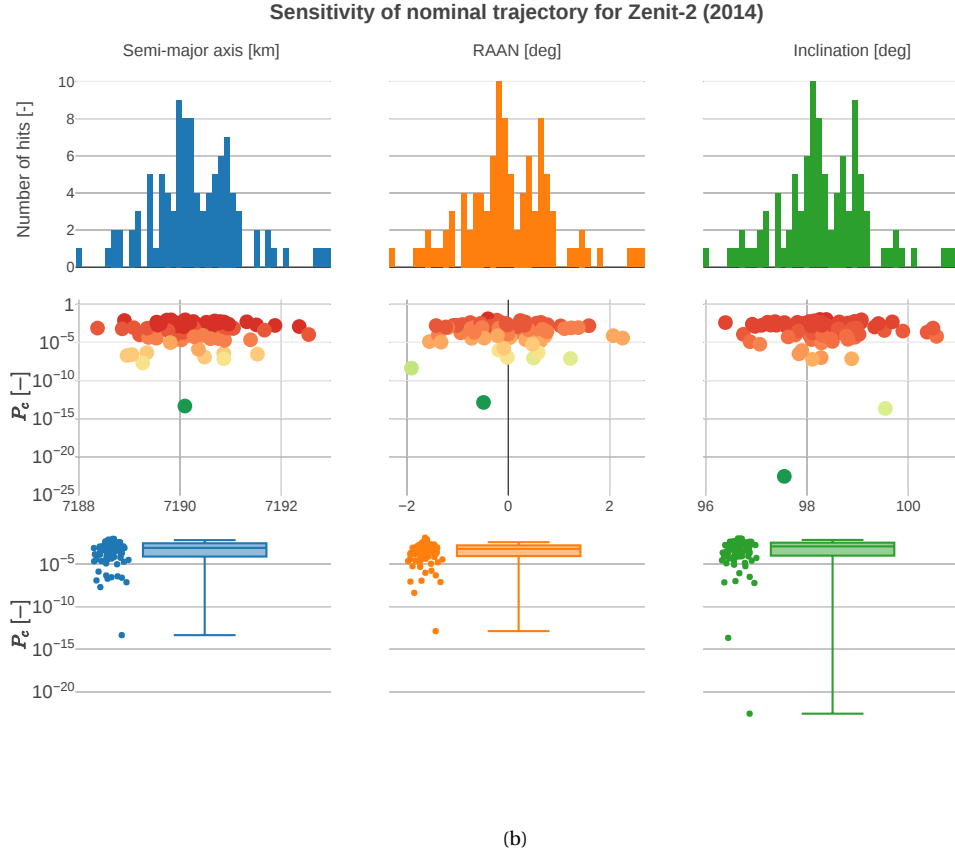
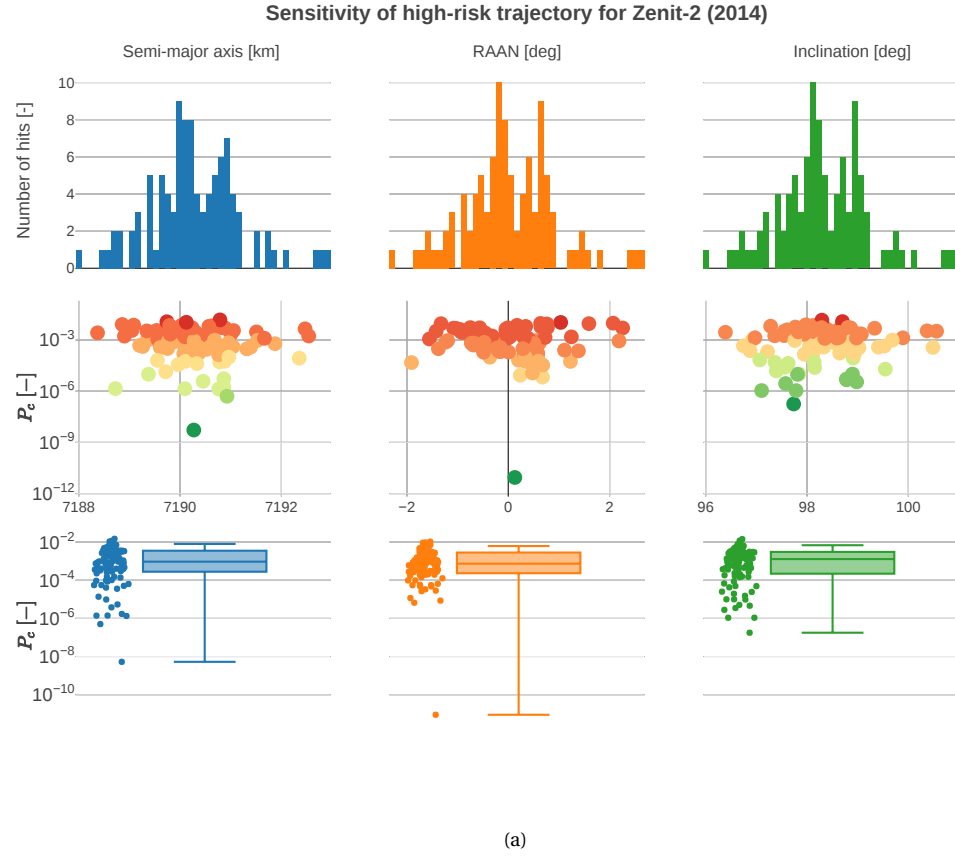
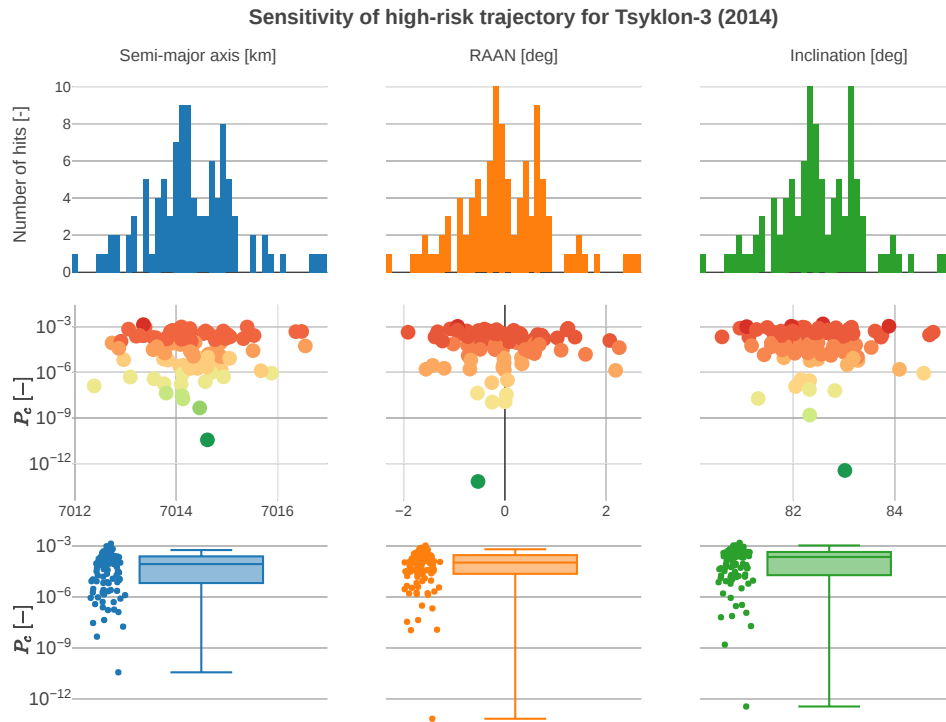
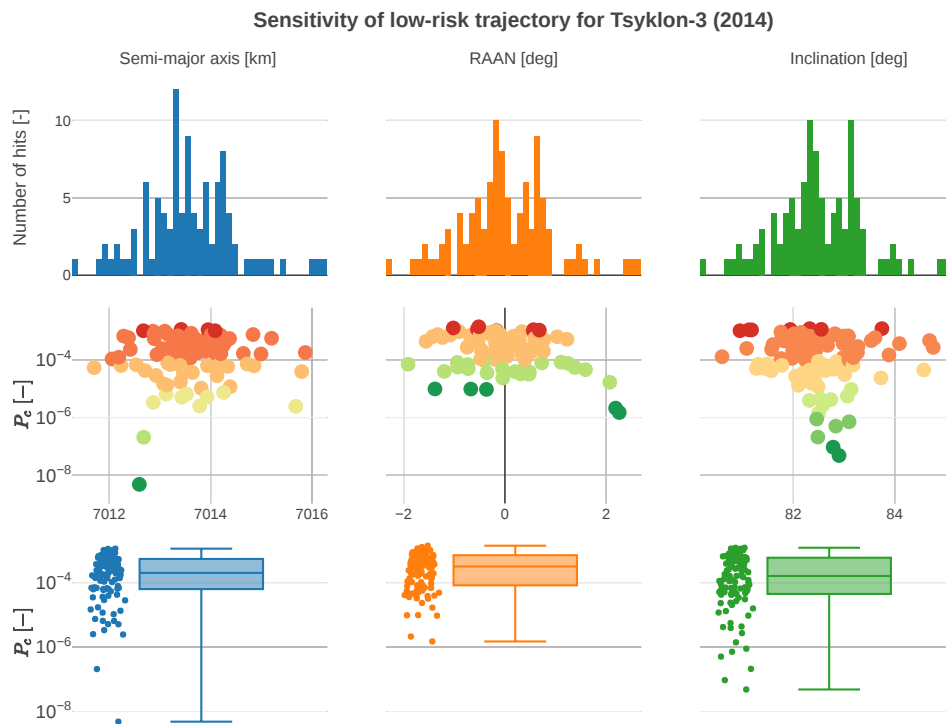


Figure 5.22: Sensitivity of the low-risk (top figure) and high-risk (bottom figure) trajectories by varying the semi-major axis (in blue), inclination (in orange) and right ascension of the ascending node (in green) for the Zenit-2 object using the TLE set of 2014: (a) shows the sensitivity of the high-risk trajectory; and, (b) shows the sensitivity of the low-risk trajectory.



(a)



(b)

Figure 5.23: Sensitivity of the low-risk (top figure) and high-risk (bottom figure) trajectories by varying the semi-major axis (in blue), inclination (in orange) and right ascension of the ascending node (in green) for the Tsyklon-3 object using the TLE set of 2014: (a) shows the sensitivity of the high-risk trajectory; and, (b) shows the sensitivity of the low-risk trajectory.

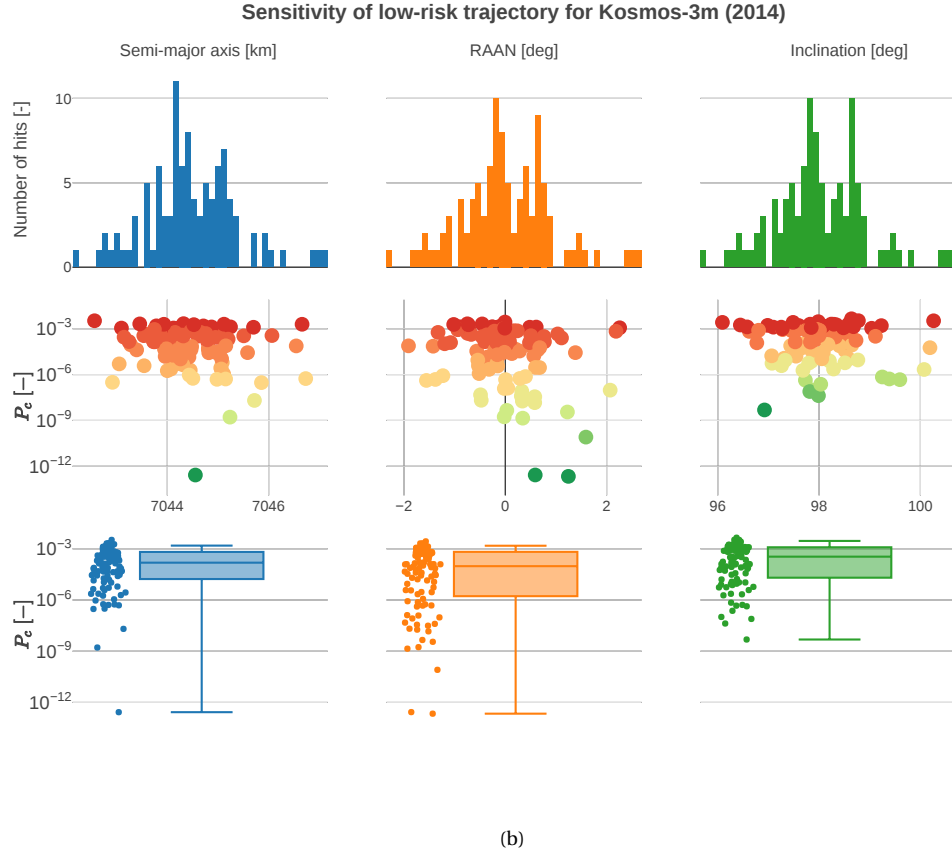
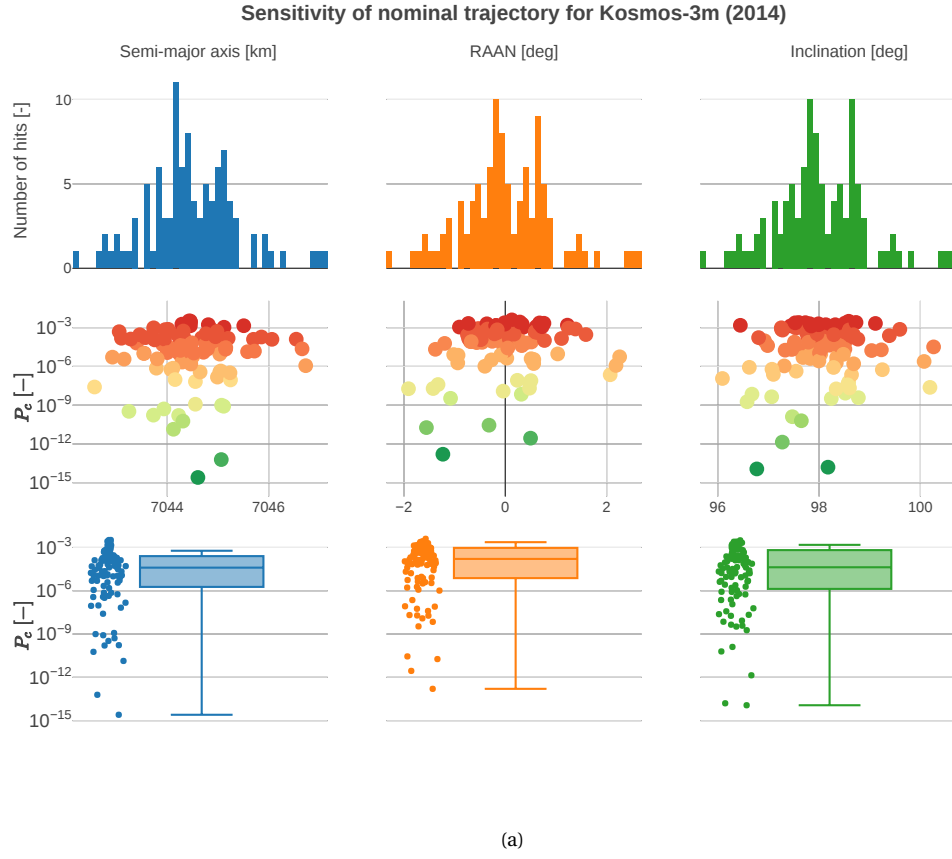


Figure 5.24: Sensitivity of the low-risk (top figure) and high-risk (bottom figure) trajectories by varying the semi-major axis (in blue), inclination (in orange) and right ascension of the ascending node (in green) for the Kosmos-3m object using the TLE set of 2014: (a) shows the sensitivity of the high-risk trajectory; and, (b) shows the sensitivity of the low-risk trajectory.

5.3. High-thrust trajectories

In the results presented in this chapter, low-thrust trajectories were applied for different de-orbit maneuvers. As explained in Chapter 2, these low-thrust trajectories use electrical propulsion systems which typically require a low amount of propellant. No high-thrust trajectories were researched that used chemical propulsion systems, since it is assumed that these trajectories require a large amount of propellant, which means higher costs for the satellite operator. However, it would be interesting to observe the accumulation of the collision probability during these (very elliptical) trajectories. A straightforward high-thrust trajectory uses a large thrust magnitude in order to direct the object to the Earth's surface. These trajectories also use a large amount of propellant and have very short propagation times (< 1 day). As could be seen in the results presented in this chapter, trajectories with short propagation times spend less time in orbit and thus have less time to encounter conjunctions with other objects. Statistically, this would result in less high-risk conjunction events, thus leading to lower collision probabilities. These trajectories would not provide a lot of insight into the application of chemical propulsion systems for de-orbit trajectories, since it is expected that they would have a large propellant usage, a short propagation time and a low collision probability. In order to assess the different objective values for low-thrust and high-thrust propulsion systems, a high-thrust is applied to the three objects resulting in very elliptical orbits. Over the length of the de-orbit, the objects cross a large altitude range during each revolution. Because the objects do not spend long periods of time at a small altitude range but the entire trajectory at a large altitude range, it is expected that the number of conjunctions encountered by the object would reflect the distribution of debris objects in LEO, leading to a more statistical interpretation of the associated collision probability. The thrust is applied at the start of the trajectory for 20 minutes for each object after which the engine is cut off for the remainder of the trajectory. This results in a very elliptical trajectory which decays slowly over time until the object has reached its respective lower altitude limit. The specific impulses are 300 seconds for all three objects. The propagation times of the high-thrust trajectories were matched to the propagation times of the (high-risk) low-thrust trajectories of the objects in order for a proper comparison between the accumulated collision probabilities. The thrust magnitudes per object were chosen accordingly to the required propagation times, and thus vary per object. Table 5.2 shows the trajectory objective values for the (high-risk) low-thrust and high-thrust trajectories for the three objects using the 2013 snapshot. The values for the low-thrust trajectories are the values for the high-risk trajectories, shown in Figures 5.1, 5.2, and 5.3. It can be seen that the values of the collision probability for the high-thrust trajectories agree with the values for the low-thrust trajectories. A much larger propellant mass is required however for the high-thrust trajectories.

Target	Trajectory	Propellant mass [kg]	Duration [days]	Accumulated P_c [-]
Zenit-2	Low thrust	30.4	319.80	3.3×10^{-3}
	High thrust	181.3	344.29	3.0×10^{-3}
Tsyklon-3	Low thrust	2.9	264.26	4.8×10^{-4}
	High thrust	20.1	269.17	1.1×10^{-4}
Kosmos-3m	Low thrust	7.8	120.19	4.1×10^{-4}
	High thrust	49.5	111.20	9.4×10^{-4}

Table 5.2: Trajectory objective values for the (high-risk) low-thrust and high-thrust trajectories, using the 2013 snapshot.

Figure 5.25, 5.27 and 5.29 show the altitude profile of the low-thrust (orange) and high-thrust (purple) trajectory. The green bar chart represents the number of debris objects and uses the top x-axis which reads from right to left. The conjunction events that account for 100.0% (rounded to one decimal precision) are annotated with arrows in each figure. The accumulation of the collision probability and the number of conjunctions per altitude are shown in Figures 5.26, 5.28 and 5.30.

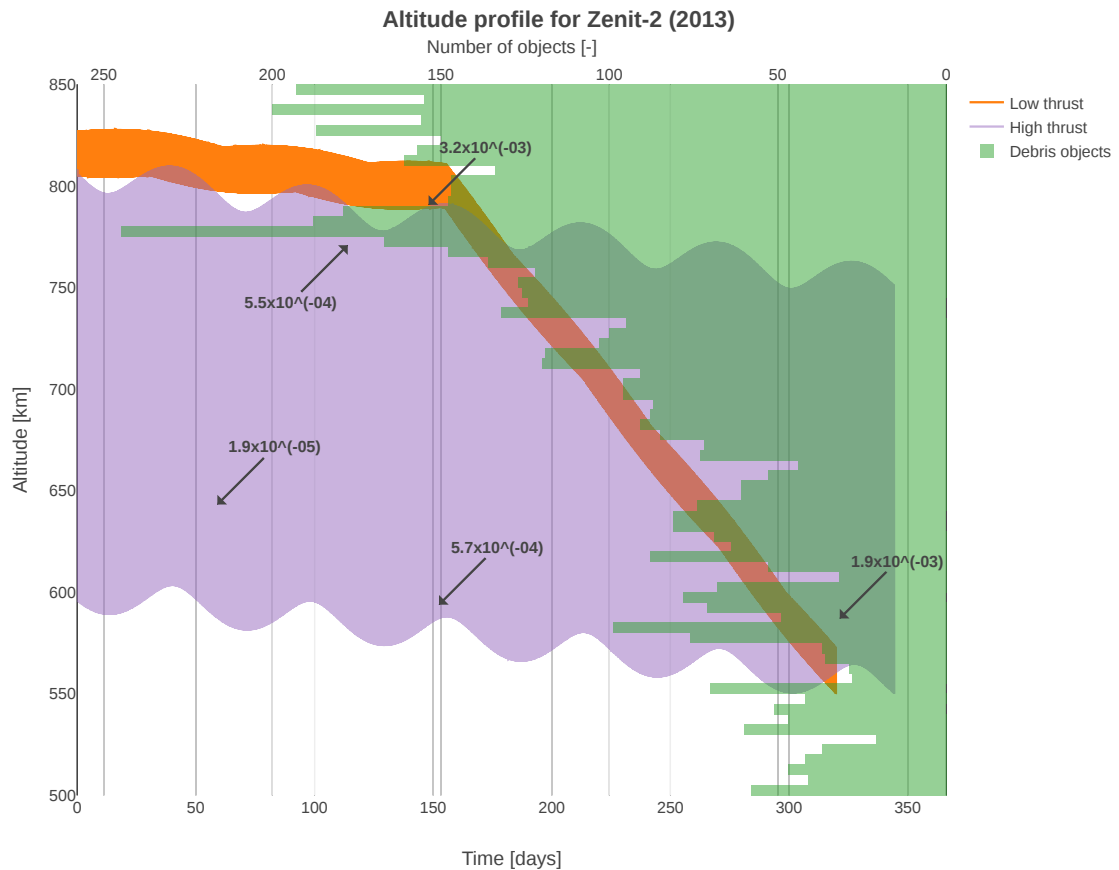


Figure 5.25: Altitude profile of the low-thrust (orange) and high-thrust (purple) trajectory for Zenit-2 using the TLE set of 2013. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.25 shows the altitude profile over time for the low-thrust and high-thrust trajectory for Zenit-2 using the 2013 snapshot. The low-thrust trajectory is the same trajectory as the high-risk trajectory presented in Figure 5.4. The thrust magnitude is 450 N, which results in the propagation time shown in Table 5.2. The high-thrust trajectory encounters four conjunctions (1.9×10^{-5} , 5.5×10^{-4} , 5.7×10^{-4} and 1.9×10^{-3}) that account for 100.0% (rounded to one decimal precision) of the accumulated collision probability. As can be seen in Figure 5.25, these conjunctions occur at different times during the trajectory and at different altitudes.

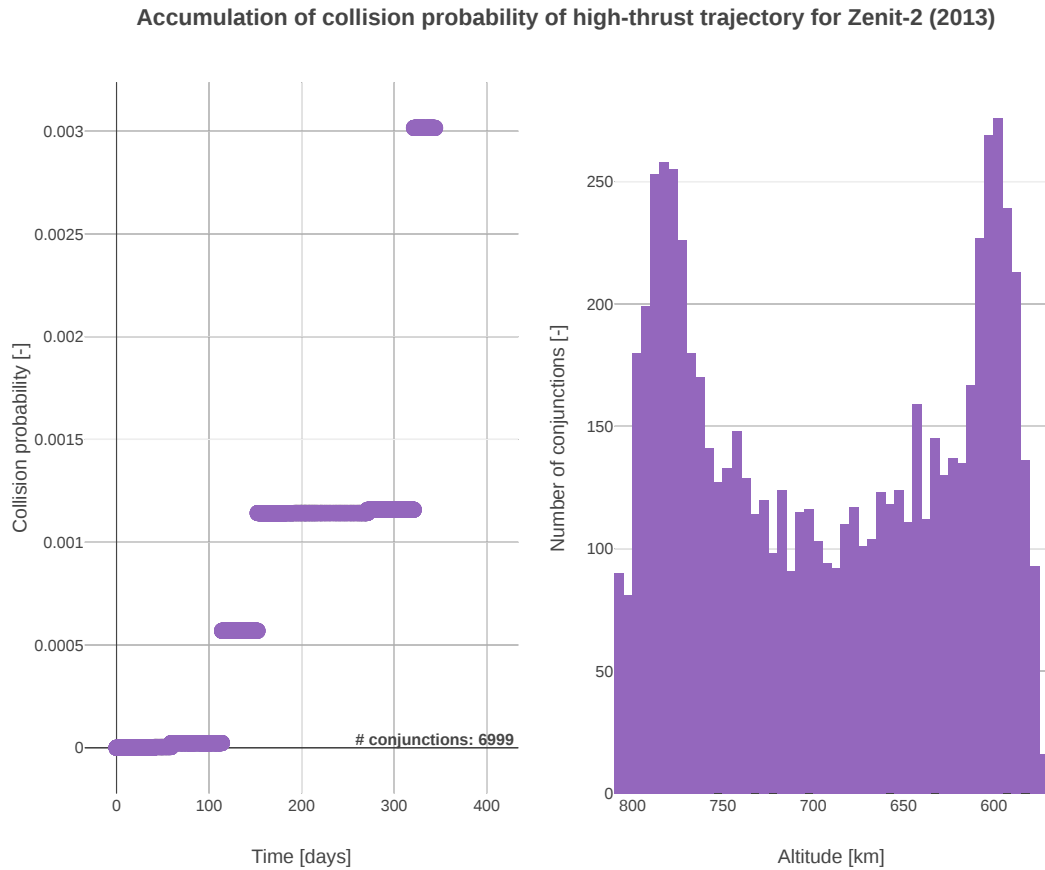


Figure 5.26: Collision probability accumulation over time and per altitude for the high-thrust trajectory of Zenit-2 using the TLE set of 2013.

Figure 5.26 shows the accumulation of the collision probability over time (left) and the number of conjunctions per altitude (right) for Zenit-2. It can be seen in this figure that four conjunctions account for the majority of the accumulated collision probability. Two peaks in the number of encountered conjunctions can be seen around 780 and 600 km altitude. The peak at 780 km altitude can be explained by the high number of debris objects at that altitude. At 600 km altitude there is no clear peak in the number of debris objects, and the trajectory also does not remain for longer periods of time at this altitude. The number of encountered conjunctions of the high-thrust trajectory is approximately four times higher than for the low-thrust trajectory (as can be seen in Figure 5.7(a)).

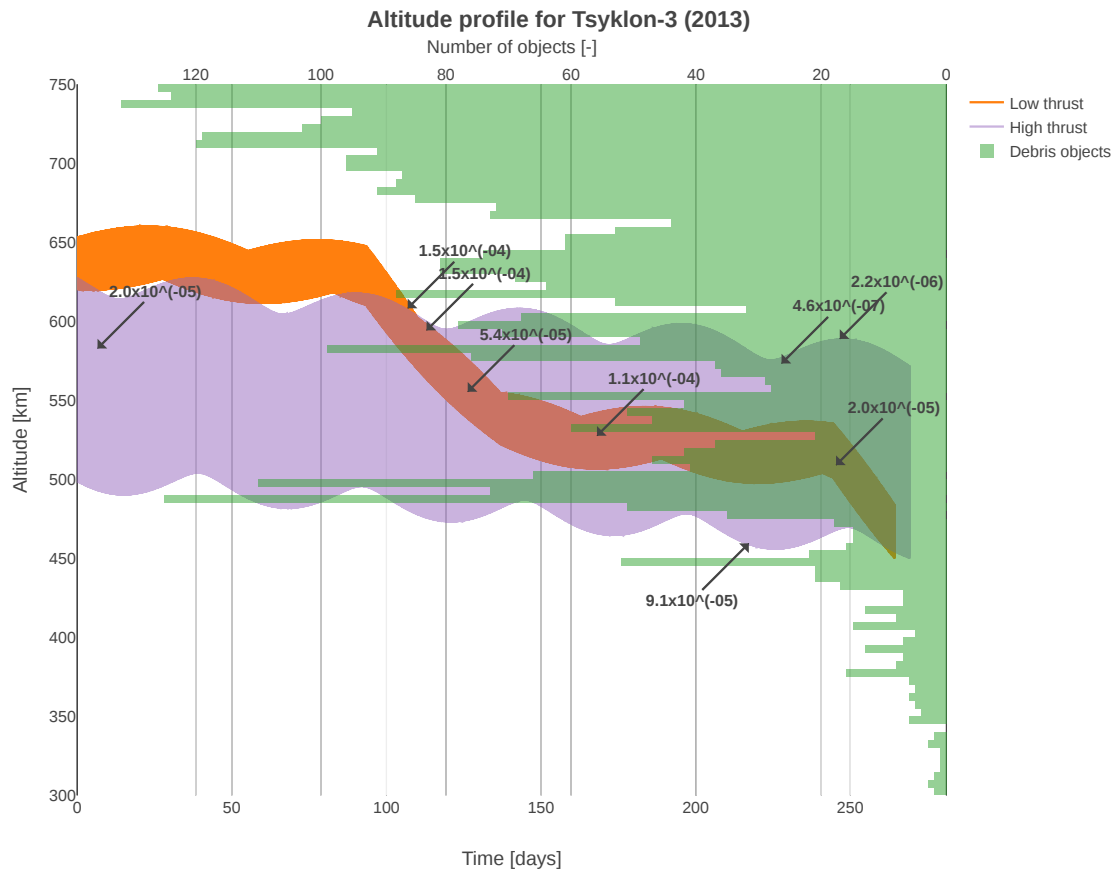


Figure 5.27: Altitude profile of the low-thrust (orange) and high-thrust (purple) trajectory for Tsyklon-3 using the TLE set of 2013. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.27 shows the altitude profile over time for the low-thrust (orange) and high-thrust (purple) trajectory for Tsyklon-3. The low-thrust trajectory is the same trajectory as the high-risk trajectory presented in Figure 5.5. The thrust magnitude of the high-thrust trajectory is 50 N, which results in the elliptical trajectory. During the trajectory, the object encounters four conjunction events (2.0×10^{-5} , 9.1×10^{-5} , 4.6×10^{-7} and 2.2×10^{-6}) that account for 100.0% (rounded to one decimal precision) of the accumulated collision probability. Just like the high-thrust trajectory of Zenit-2, the conjunctions occur at different times and altitudes in the trajectory. No significant conjunctions occur at 500 km altitude, where a large peak in the number of debris objects is located.

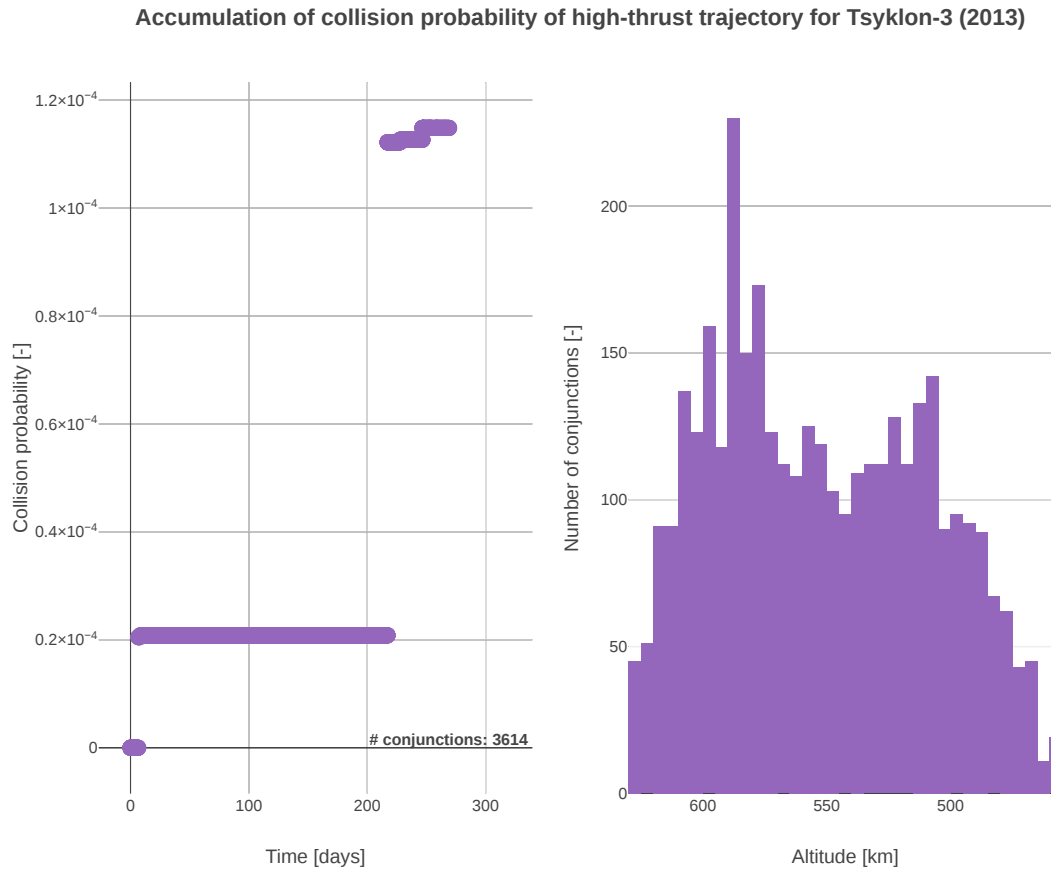


Figure 5.28: Collision probability accumulation over time and per altitude for the high-thrust trajectory of Tsyklon-3 using the TLE set of 2013.

Figure 5.28 shows the accumulation of the collision probability over time (left) and the number of conjunctions per altitude (right) for Tsyklon-3. It can be seen that the object encounters a significant conjunction very early during the trajectory and one nearing the end of the trajectory. Looking at the distribution of the number of conjunctions per altitude, no clear correspondence with the distribution of the debris objects can be seen. A peak in the number of conjunctions at around 580 km altitude is visible which can also be seen at the same altitude in the number of debris objects. However, no clear peak in the number of conjunctions at 500 km altitude is visible. The number of encountered conjunctions of the high-thrust trajectory is lower than the number of conjunctions encountered by the low-thrust trajectory (as can be seen in Figure 5.8(a)).

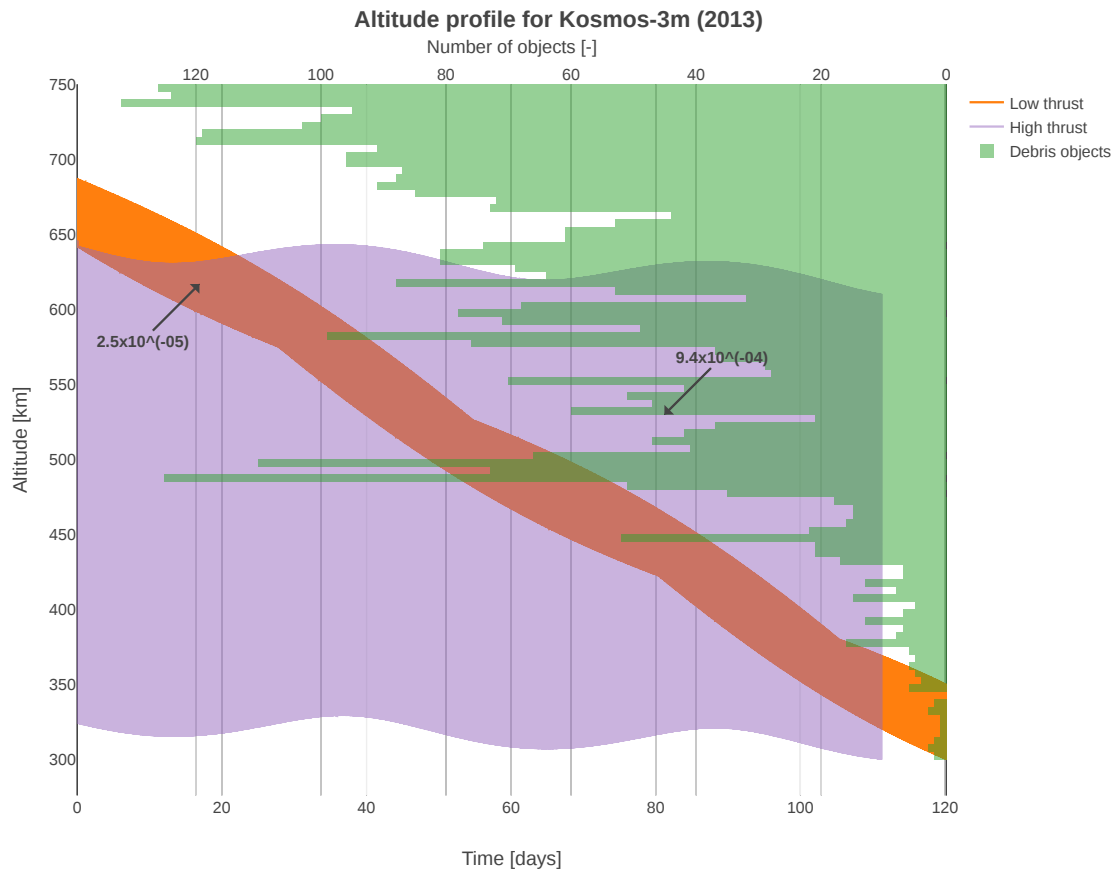


Figure 5.29: Altitude profile of the low-thrust (orange) and high-thrust (purple) trajectory for Kosmos-3m using the TLE set of 2013. The (bottom) x-axis represents the propagation time and the y-axis represents the altitude of the trajectories. The green bar chart represents the number of debris objects per orbital altitude (and uses the top x-axis reading from right to left). The conjunction events that account for 100.0% (rounded to one decimal precision) of each trajectory are indicated by arrows.

Figure 5.29 shows the altitude profile of the low-thrust (orange) and high-thrust (purple) trajectory for Kosmos-3m. The low-thrust trajectory corresponds to the nominal trajectory (using a constant thrust) as shown in Figure 5.6. The magnitude of the high-thrust trajectory is 123 N, which results in the highly elliptical trajectory. The high-thrust trajectory traverses a large altitude range during its trajectory, but only encounters one high-risk conjunction (9.4×10^{-4}) that accounts for 100.0% (rounded to one decimal precision) of the accumulated collision probability.

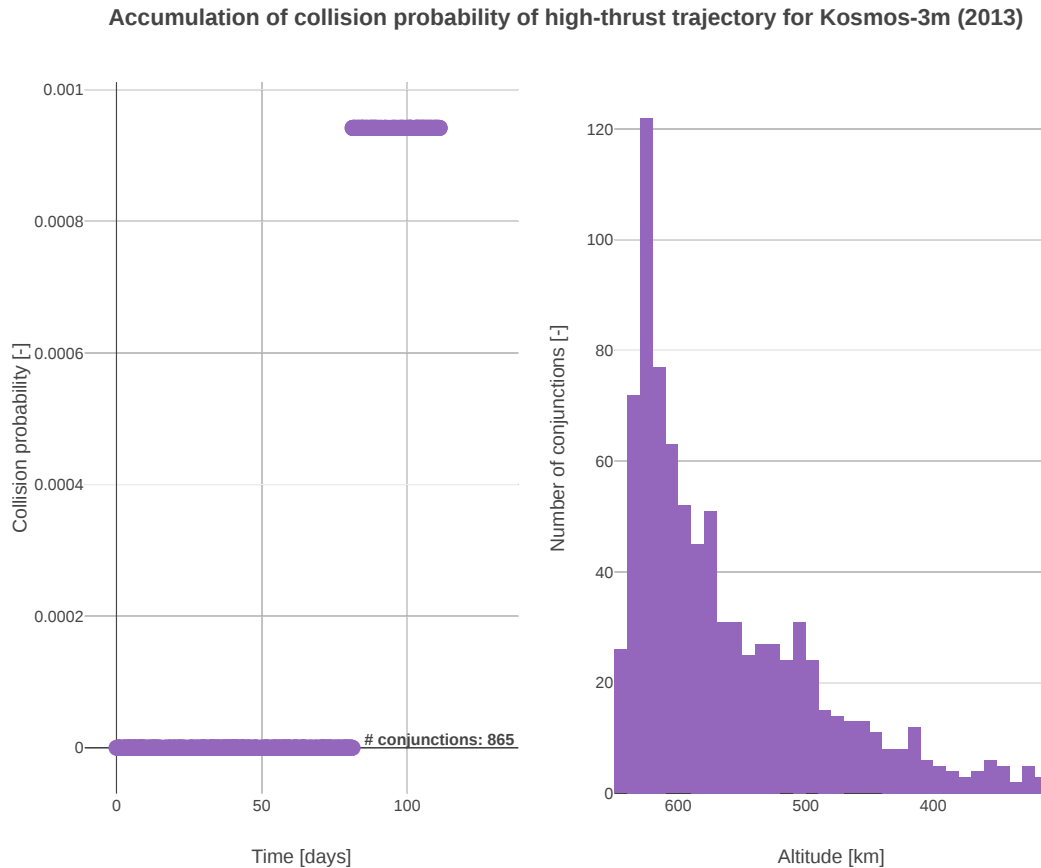


Figure 5.30: Collision probability accumulation over time and per altitude for the high-thrust trajectory of Kosmos-3m using the TLE set of 2013.

Figure 5.30 shows the accumulation of the collision probability over time (left) and the number of conjunctions per altitude (right) for Kosmos-3m. It can clearly be seen that the majority of the accumulated collision probability is determined by a single high-risk conjunction event. Looking at the number of conjunctions per altitude, an interesting pattern can be observed. The number of conjunctions seems to increase exponentially with increasing altitude, resulting in a large spike at around 600 km altitude. The expected spike at 500 km altitude, where there is a large number of debris objects, can not clearly be observed from this figure. Although the number of debris objects does decrease per orbital altitude, it does not seem to be decreasing with the same pattern as is observed in Figure 5.30. This large difference between the number of conjunctions at low and high altitudes can be caused by the elliptical nature of the orbit, however. The objects orbital motion is slower than at perigee, meaning that it spends more time at apogee. Because of this there is also more time for conjunctions to occur at apogee.

5.4. Discussion

As seen in Table 2.3, the values for the duration and the used propellant mass of the reference and computed trajectories do not match. Performing an analytical calculation using Eq. 2.6 of the associated propellant mass for a low-thrust trajectory with a certain object mass and trajectory duration results in the values presented as the ‘computed values’. The analytically computed values for the used propellant mass for low-thrust trajectories with certain durations is presented in Table 5.3. After contacting the authors of the paper by Lidtke et al., no further information could be provided about the origin of their results and why this deviation is observed.

Type	Object	Duration [days]	Propellant mass [kg]	Analytical value [kg]	Difference [kg]
Reference	Zenit-2	593.14	34.8	38.3	3.5
	Tsyklon-3	174.25	6.6	13.5	6.9
	Kosmos-3m	121.24	7.6	7.8	0.2
Computed	Zenit-2	547.72	35.4	35.4	0.0
	Tsyklon-3	48.02	3.7	3.7	0.0
	Kosmos-3m	120.19	7.8	7.8	0.0

Table 5.3: Reference and computed propellant masses for trajectories with a constant thrust for Zenit-2, Tsyklon-3 and Kosmos-3m. The analytical values of the propellant masses are determined for trajectories with said durations using Eq. 2.6 and the object specifics shown in Table 2.2.

As could be seen in Figure 1.3, the majority of the debris objects in LEO are located between 700 and 1000 km altitude. The objects that were analyzed in this thesis all had initial orbits below 750 km altitude. This means that the majority of the debris objects in LEO were located at higher altitudes than the three objects analyzed in this thesis. During the de-orbit trajectories, these objects would not cross higher altitude regions, and would therefore not come into possible close conjunctions with these objects. Objects at their end-of-life located at higher altitudes would have to traverse these very high satellite density regions during the de-orbit maneuver. This would potentially increase the associated collision probability, accompanied with possible longer propagation times (up to 25 years).

The computed trajectories presented in this chapter used a set of inputs determining the thrust magnitude and the (de)activation pattern. These inputs were determined by an optimization algorithm (see Section 4.5) that used the information of previous generations in the optimization process to generate a new set of inputs to be used in the propagation. The optimization algorithm tried to optimize the three objectives (propellant usage, propagation time and collision probability), driven by the inputs given to the control algorithm. After a number of generations the propellant usage and propagation time converge along a diagonal, which is visible in Figures 5.1, 5.2, 5.3, 5.13, 5.14, and 5.15. The collision probability converges to low values for trajectories with short propagation times. This is clearly visible in Figures 5.2, 5.3 and somewhat visible in Figure 5.14. It can also clearly be seen that trajectories with long propagation times have high collision probabilities. However, there are some exceptions to this convergence pattern. Figures 5.1 and 5.14 show two trajectories that have a shorter propagation time but higher collision probabilities than the low-risk trajectories ‘higher’ on the diagonal. Figure 5.2 shows a low-risk trajectory with a long propagation time that is surrounded by trajectories with high collision probabilities. Additionally, Figure 5.1 shows a high-risk trajectory which has a higher propellant usage, a longer propagation time and a higher collision risk than the low-risk trajectory. This means it underperforms for every objective compared to the low-risk trajectory. The reason for these exceptions is the highly sensitive nature of the collision probability, which makes it difficult for the optimization algorithm to find robust solutions with low collision probabilities. This sensitivity of the collision probability can be observed in Figure 5.31. Here, the collision probability is shown as a function of the separation distance between two objects. It can be seen that an offset of a few kilometers can lead to disparities in the collision probability of multiple orders of magnitude. Thus, changing the trajectory profile only slightly can result in many more or much less conjunctions with high collision probabilities. Another method can be used to find optimal trajectory objectives, which is a random sampling method such as the Monte Carlo method. By applying a large number of random inputs, various different trajectories can be created leading to different de-orbit trajectories. It is not believed however that this will improve the analysis. Using this method would require a large number of samples, which would not benefit the computational effort and random sampling does not use the information of previous found solutions (generations) available for optimization algorithms.

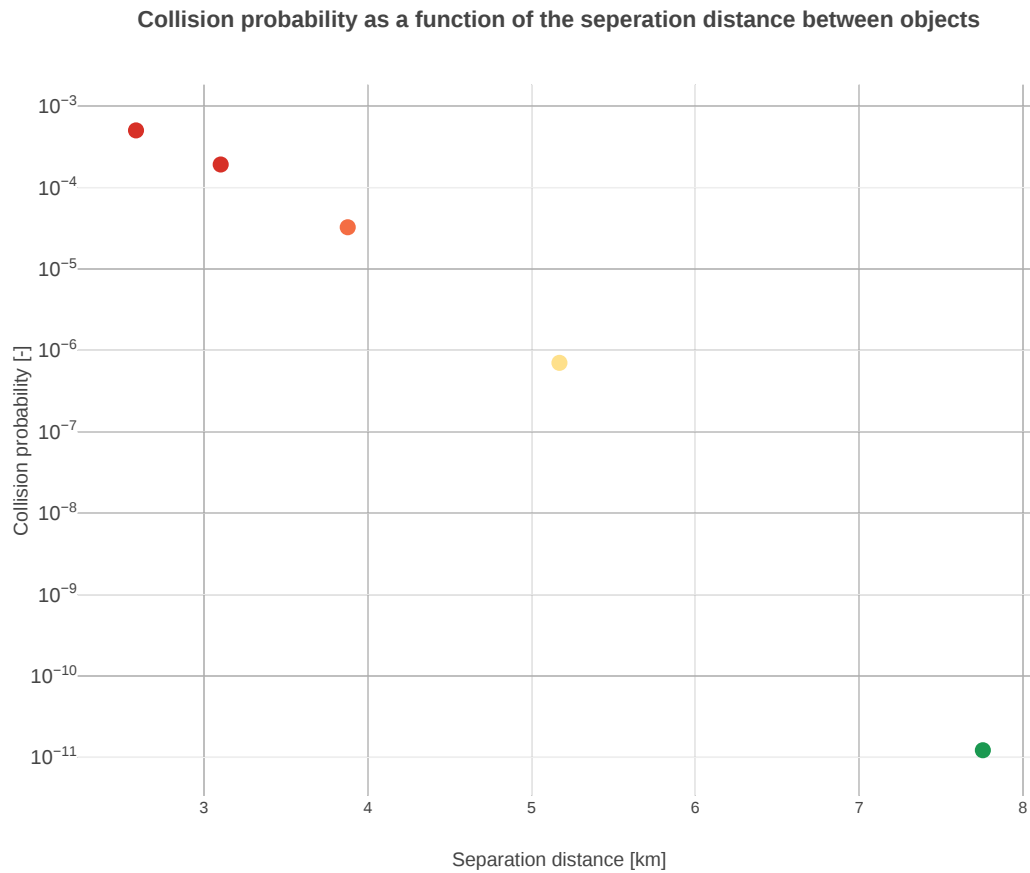


Figure 5.31: Collision risk associated with a varying separation distance for a certain object-pair. The colors serve as an additional aid to represent the collision probability.

As can be seen in Figures 5.4, 5.7(b) and 5.16, 5.19(b), both the low-risk and the high-risk trajectories do not encounter any conjunctions below certain altitudes. The objects still orbit for a considerable amount of time (approximately 100 days) below these altitudes. This is not expected because the number of debris objects below these altitudes is still relatively high. The high number of debris objects combined with the relatively long propagation time should result in an increased number of conjunctions, as can be seen in Figures 5.8(b), 5.9(b), 5.20(b), and 5.21(b).

Figures 5.32 and 5.33 show the distribution of objects in LEO for different inclinations and right ascensions of the ascending node per altitude for Zenit-2. The red rectangles indicate the region of the orbital parameters of the Zenit-2 trajectories.

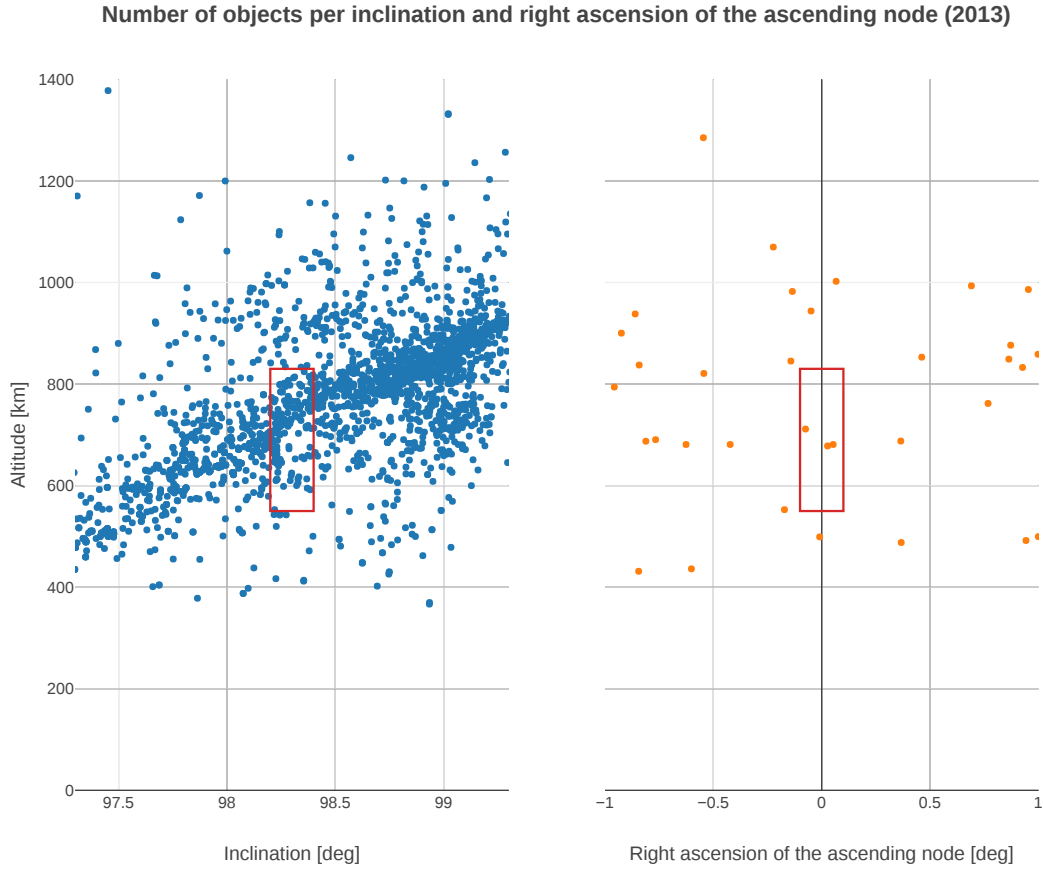


Figure 5.32: Distribution of objects in LEO for different inclinations and right ascensions of the ascending node per altitude for Zenit-2, using the 2013 snapshot. The red rectangles indicate the region of the orbital parameters of the trajectories.

From Figure 5.7(b) it can be seen that the low-risk trajectory does not encounter conjunctions below 760 km altitude and that the high-risk trajectory does not encounter conjunctions below 780 km altitude. Figure 5.32 shows that below these altitudes the number of debris objects with similar inclinations decreases. However, observing the number of debris objects per right ascension of the ascending node, it can be seen that there are almost no debris objects with similar values as Zenit-2. This means that even if Zenit-2 would share values of the inclination with debris objects, they would not orbit in similar planes, since the right ascension of the ascending node would not match. This means that the occurred conjunctions happen for the majority with objects that do not orbit in a similar plane (close to the orbital plane of Zenit-2). Therefore, it is expected that the trajectories of Zenit-2 would still encounter conjunctions at lower altitudes during its trajectory.

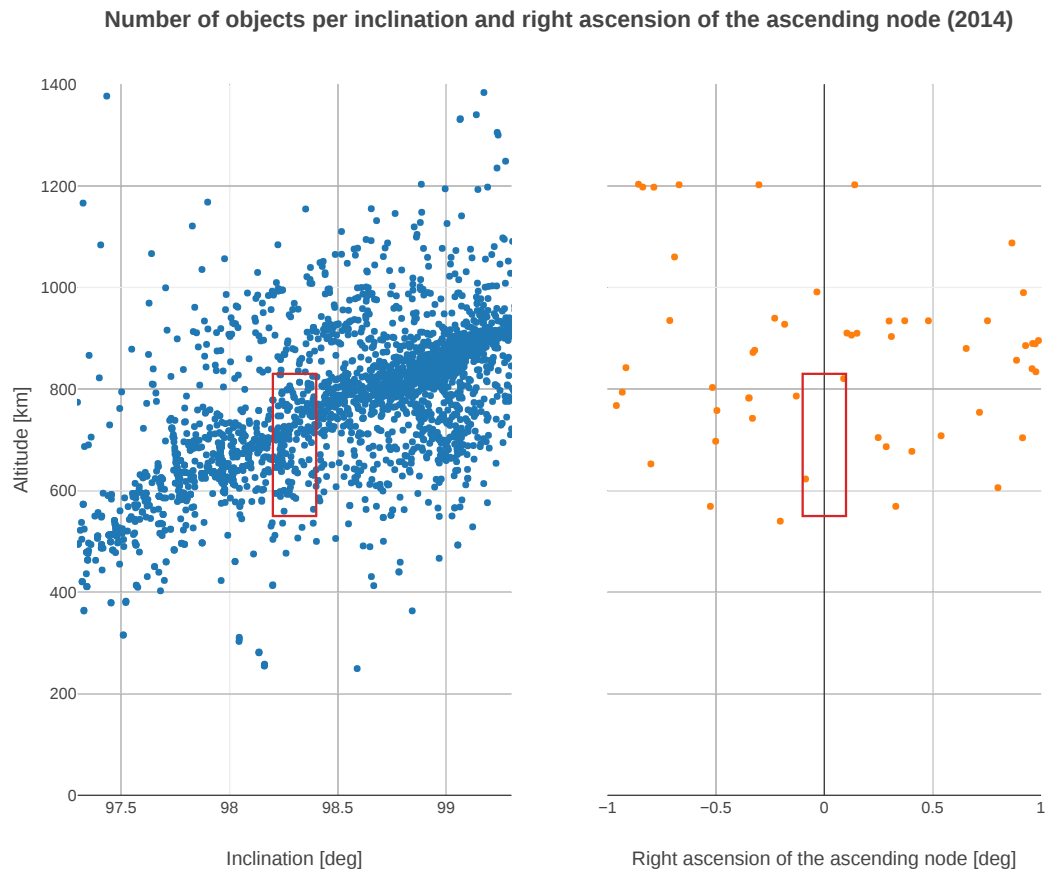


Figure 5.33: Distribution of objects in LEO for different inclinations and right ascensions of the ascending node per altitude for Zenit-2, using the 2014 snapshot. The red rectangles indicate the region of the orbital parameters of the trajectories.

Figure 5.33 shows the distribution of objects in LEO for different inclinations and right ascensions of the ascending node per altitude for Zenit-2 using the 2014 snapshot. It can be seen that the inclinations of the debris objects are relatively evenly distributed over the different altitudes (within the red rectangle). From Figure 5.19(b) it can be seen that the nominal trajectory does not encounter conjunctions below 780 km altitude and that the high-risk trajectory does not encounter conjunctions below 610 km altitude. From the distribution of the debris objects shown in Figure 5.33 it is expected that both trajectories of Zenit-2 would encounter conjunctions below these altitudes, following the same reasoning as for the case of the 2013 snapshot. The reason why the trajectories of Zenit-2 do not encounter conjunctions below specific altitudes remains unclear.

It can be seen in Figures 5.7, 5.8, 5.9, 5.19, 5.20 and 5.21 that the number of conjunctions sometimes depends on the time spent at certain altitudes and the number of debris objects per altitude and sometimes no clear pattern can be observed. Figure 5.28 does not show a peak at 500 km altitude where the number of debris objects is relatively high, but a peak does appear around 580 km altitude where the number of debris objects is not relatively high. The collision probability seems to be determined by single high-risk conjunction events instead of being determined smoothly as a function of the number of conjunctions. It is assumed that for longer propagation times the statistical effects will become more prominent, but this is not clearly visible for trajectories with these propagation time length. Because of this it is difficult to state which types of trajectories would yield low collision probabilities. Another factor that plays a role in this is the influence of a small number of high-risk conjunctions on the accumulated collision probability.

A major constraint in this research was the computational effort required by the simulation. Because of these limitations, a number of assumptions have been made: the orbits of the debris objects were assumed to be Keplerian; and, the objects were propagated for up to two years, instead of the full 25 years. The orbits

of the debris objects were assumed to be Keplerian in order to reduce the computational effort required during the simulation. This assumption greatly reduces the accuracy of the positions of the debris objects. As was observed in the sensitivity analysis, the results are highly sensitive to changes in either the initial state or the state derivative. Because the orbits of the debris objects were not propagated accurately, this might have resulted in solutions that would have been completely different from solutions where the states of the debris objects were computed with a high accuracy. It is believed however, that even with highly accurate positions of the debris objects, the solutions would still not have shown robust trajectories that limit the collision probability. This is due to the highly sensitive nature of the solutions and the inability of the optimization algorithm to converge to solutions for the collision probability. If the computational effort would have allowed for propagations of 25 years, it is believed that the statistical effects of the collision probability would have been more prominent, since the number of conjunctions during each trajectory is relatively high and shows some correspondence to the number of debris objects per altitude and the time spent at certain altitudes.

The propagation time and propellant usage of the computed trajectories shown in Table 2.3 do not agree with the reference values shown in this same table. These values correspond to trajectories with a constant low thrust with defined thrust magnitudes and specific impulses. As can be seen in this table, the computed values for Zenit-2 and Kosmos-3m vary within a margin of 10% with respect to the reference values. This discrepancy can be accounted for by the use of different environment models, leading to offsets in the trajectory duration or the use propellant mass. The computed values for Tsyklon-3 vary with almost a factor of 4. This discrepancy is too large to be accounted for by differences in the environment models. With a close observation it can be seen that the reference trajectories for Zenit-2 and Kosmos-3m have longer propagation times but require less propellant than the computed trajectories. Using the standard rocket equation, it can be seen that the values for the propagation times do not correspond to the used propellant masses for the reference trajectories. The computed values for the duration and the propellant mass do correspond to this rocket equation. The reason why the reference values do not follow the theoretical calculations is unknown.

Conclusions and Recommendations

6.1. Conclusions

End-of-life satellites pose a threat to other objects in the space environment. In order to remove these satellites from orbit a controlled thrust maneuver can be performed which lowers the altitude of the satellite until a lower altitude limit is reached. During this de-orbit maneuver, the propellant usage, trajectory duration and collision probability with other objects is observed.

Three objects were investigated that represent a wide spectrum of possible different de-orbit scenarios. These objects also pose serious threats to active spacecraft. These objects are rocket bodies used in previous satellite launches. The three objects are Zenit-2, Tsyklon-3 and Kosmos-3m. Each of these objects have different physical properties and orbit in different (densely populated) regions in LEO. Constant low-thrust trajectories were compared to controlled low-thrust trajectories by their propellant usage, propagation time and collision probability. The controls of the thrust were determined by a control algorithm that chose several epochs during the trajectory to switch the engine either on or off, along with the thrust magnitude used by the object. These controls allowed the object to traverse through the different orbital altitudes, possibly avoiding high debris density altitude regions. The controls were determined by an optimization algorithm that tried to optimize the propellant usage, propagation time and collision probability for each trajectory. This resulted in a range of possible different trajectories, each with associated values of the optimization objectives.

In Figures 5.2 and 5.3 there is a trend visible where trajectories where a short propagation time have low collision probabilities. This trend is also visible in Figure 5.14 but in lesser fashion than in the other two figures. A number of exceptions of this trend are also clearly visible however. Figure 5.14 shows that there are some trajectories with short propagation times (for instance the trajectory with a propagation time of about 40 days) that yield relatively high collision probabilities (order of magnitude of -3). In all figures showing the trajectory objectives (Figures 5.1, 5.2, 5.3, 5.13, 5.14 and 5.15) it can be seen that trajectories with long propagation times also have relatively high collision probabilities. In this case there are also some exceptions visible. Figure 5.2 shows a low-risk trajectory with a propagation time of around 260 days which has a collision probability of approximately 8×10^{-8} . Due to the range of the color scheme, it appears as if a number of trajectories for the Zenit-2 object using the TLE set of 2014 (Figure 5.13) also have low collision probabilities (colored green). However, by observing the color bar it can be seen that these trajectories have a collision probability with an order of magnitude of -4 . The trajectories of interest with a low collision probability appear on different parts of the trajectory objective figures. There is no clear indication about the combination of the propagation time and propellant usage that should lead to a trajectory with a low collision probability. This is a first indication that the collision probability associated with a de-orbit trajectory does not necessarily depend on the type of orbit. It can be seen that the nominal trajectories in Figures 5.1, 5.2, 5.3, 5.14 and 5.15 all have a high collision probability relative to the other trajectories. One exception is the nominal trajectory shown in Figure 5.13, where it has the lowest collision probability (approximately 8×10^{-7}) compared to the other trajectories. This is a second indication that the collision probability does not necessarily depend on the shape of the trajectory.

Looking at the altitude profiles of the trajectories of interest indicated in the previous figures, it can be seen that the maximum number of conjunctions that a trajectory encounters, which accounts for 100.0% (rounded to one decimal precision) of the accumulated collision probability is five. This can be seen in Figure 5.5. For many trajectories (in Figures 5.4, 5.6 and 5.17) the total accumulated collision probability is determined by a single high-risk conjunction event. This makes predicting de-orbit trajectories with a low collision probability rather difficult, which is an indication that the obtained trajectories are not very robust, since one single conjunction can drastically affect the associated collision probability. Some trajectories activate their engines after a period of time. Before the engine activation, the objects orbit at altitudes with a relatively high number of debris objects, without encountering conjunctions that add significantly to the accumulated collision probability (figures 5.4 and 5.18). The number of conjunctions that occurred during these periods are shown to increase however, which is elaborated upon below. The lack of high-risk conjunctions during these periods is an indication that the obtained trajectories are not robust. This feature shows another indication that the collision probability does not necessarily depend on the number of debris objects. Other trajectories also show this feature. Figure 5.5 shows the low-risk and high-risk trajectory altitude profile. After the engine is cut off at approximately 145 days, the high-risk trajectory orbits at altitudes where the number of debris objects ranges from 20 to 60 objects and encounters two high-risk conjunctions. The low-risk trajectory orbits at an altitude just lower than the high-risk trajectory where the number of debris objects ranges from 60 to 120 objects and does not encounter a single conjunction that adds to the accumulated (low) collision probability.

The accumulation of the collision probability over time in Figures 5.7(a), 5.8(a), 5.9(a), 5.19(a), 5.20(a) and 5.21(a) show that the collision probability of each trajectory is mostly determined by a low number of high-risk conjunction events. Most of these trajectories encounter several thousands of conjunctions but only a handful of these conjunctions contribute significantly to the collision probability. This is an indication that the collision probability for each trajectory is not smoothly determined by the total number of conjunctions but rather by a few high-risk conjunctions. By observing Figures 5.8(b), 5.9(b) and 5.20(b) it can be seen that the number of conjunctions per orbital altitude is related to the time spent by an object at those altitudes. In these figures, it can be seen that there are peaks of encountered conjunctions occurring at altitudes where the object remained for a period of time, as can be observed from Figures 5.5, 5.6 and 5.17. Additionally, a low number of conjunctions can be observed at altitudes where the engine of the trajectories were active, and the altitude of the object was decreasing. Another correlation is visible between the number of debris objects per altitude and the number of encountered conjunctions. This feature can clearly be seen in Figure 5.17, where the high-risk trajectory, although using a controlled thrust, slowly decreases its altitude over time. During this time it traverses through different altitude regions where the number of debris objects shows a peak at the initial altitude (between 600 and 650 km altitude), then decreases at lower altitudes and reaches a peak again just before the trajectory has reached the lower altitude limit (between 460 and 510 km altitude). Observing Figure 5.20(b), this pattern can also be seen in the number of conjunctions per altitude. An increased number of conjunctions is seen between 600 and 650 km altitude and another increase can be seen between 480 and 510 km altitude. This is an indication that the number of conjunctions corresponds to statistical models which are dependent on the time spent in orbit and the number density of the debris objects at those altitudes.

The sensitivity analysis presented in Figures 5.10, 5.11, 5.12, 5.22, 5.23 and 5.24 show that the obtained results for both the low-risk and high-risk trajectories are very sensitive to small variations in the initial state elements. A small initial variation might lead to an offset in the position of multiple kilometers after some time. As can be seen in Figure 5.31, a relatively small offset of a few kilometers can lead to multiple orders of magnitude difference in the collision probability. Such an offset can also lead to new conjunctions with other objects, which were not potential conjunctions in the original trajectory. Additionally, variations in the environment model can also lead to position offsets over time, which also heavily influences the associated collision probability of a trajectory. This highly sensitive nature of the obtained collision probabilities for the different trajectories is a strong indication that the obtained results are not robust.

As can be seen in Figures 5.25, 5.27 and 5.29, the orbits of the high-thrust trajectories are highly elliptical compared to the low-thrust trajectories. The accumulation collision probability for these trajectories is also determined by a handful high-risk conjunction events. During the propagation time, the trajectories traverse large altitude ranges and thus cross multiple altitude regions with a relatively high number of debris

objects. As can be seen from Figure 5.28, crossing large altitude regions results in a somewhat evenly distributed number of conjunctions per altitude. No peak at 500 km is observed however, whereas the number of debris objects itself does show a peak. Looking at Figure 5.26, it can be seen that there is an increase in the number of conjunctions at around 600 and 780 km altitude. The second peak can be explained by the peak shown in the number of debris objects at that altitude, but the first peak can not directly be linked to the number of debris objects. The number of conjunctions for the Kosmos-3m object shown in Figure 5.30 increases exponentially with increasing altitude. Also, this rapid increase in the number of conjunctions can not be linked to the number of debris objects at those respective altitudes. This unpredictable behaviour in the number of conjunctions for these highly elliptical orbits is an indication that (for these timescales) the collision probability is not subjective to the statistical effects but is determined by single high-risk conjunction events.

The research question that was formulated in Chapter 1 reads: What is a robust method to de-orbit an object in Low Earth Orbit at its end-of-life, using on-board propulsion methods? This thesis sought to answer this question using the research presented in the previous chapters. This chapter summarized and analyzed the obtained results. It was observed that the collision probability associated with a trajectory does not necessarily depend on the shape of the trajectory, since multiple trajectory shaped resulted in various different values for the collision probabilities. Also, because the collision probability is highly dependent on a handful of individual high-risk conjunction events, it is difficult to predict the actual collision risk associated with certain trajectories. Even when a high-thrust is applied, and the object traverses a large range of altitudes during its trajectory, there is no clear indication that this results in an increased collision probability since the collision probability is still determined by only a handful of high-risk conjunction events. Additionally, for these timescales the collision probability does not reflect the statistical model of the kinetic gas theory. For longer periods of time however, it is assumed that these statistical effects become more prominent, thus putting more emphasis on the type of trajectories (avoiding altitude regions with a high number of debris objects).

All of these findings result in the conclusion that *no robust method was found to safely de-orbit an object in Low Earth Orbit*. The collision probability is highly dependent on high-risk single events and the highly sensitive nature of the problem does not allow for a predictable outcome of the trajectory risk.

6.2. Recommendations

Many assumptions were made in this thesis that improved the computational speed but affected the accuracy of the results negatively. Increasing the accuracy of the results can be achieved by e.g. applying more accurate environment models, using smaller time-steps, or propagating the debris objects with a higher accuracy. In this thesis, the orbits of the debris objects were assumed to be Keplerian, greatly reducing the computational effort required to determine their position at arbitrary times. This also means that the positions of the debris objects were not represented accurately. By propagating the debris objects with a greater accuracy (e.g. using the same propagation method as the user satellite), their positions can also be determined more accurately.

As can be observed from the results presented in Chapter 5, the number of conjunctions for each trajectory sometimes showed correspondence to the number of debris objects at certain altitudes. This correspondence was not visible in all cases, however. by increasing the propagation time from one or two years to multiple years (reaching the upper limit of 25 years for the removal of space debris), it is assumed that the statistical effects of the time spent and the object density at certain altitudes will become more prominent. It can be interesting to observe these effects and the resulting optimal de-orbit trajectories that follow from this effect.

Increasing the accuracy and/or the propagation time of the results also greatly increases the required computational effort, however. The analysis was performed in C++, which is known as a relatively fast computer language, ideal for these type of computations. The individual steps during the calculation have been optimized for computational speed, but the full calculation was not optimized for multithreading capabilities. Multithreading is the ability for the central processing unit to provide multiple threads (instructions) simultaneously. Doing so, the capabilities of the computer can be used advantageously for speed. During this thesis, multithreading was not performed since this requires extensive knowledge on how to design the

simulation so that it is suitable for multithreading calculations. It is believed that redesigning the calculations so that it would be capable of applying multithreading would greatly increase the speed of the entire simulation, which would make a more accurate and longer propagation possible.

Improving the accuracy of the results presented in this thesis is not recommended, however. It can be observed that the collision probability of the results presented in this thesis is very sensitive to position offsets of either of the two objects. This means that the obtained results are not very robust. Increasing the accuracy of the analysis does not make the obtained solutions more robust. This means that even an analysis performed with high accuracy would not present a true depiction of a real-life scenario. Long-term predictions of the collision probability with debris objects can be performed using statistical measures in order to give an indication about the risk associated with de-orbit trajectories. However, these would only provide an indication of the collision probability and not an accurate estimation.

Instead, it is recommended to investigate the effects of the collision probabilities on short-term trajectories. Many highly critical objects (such as the ISS or the Hubble space telescope) orbit at LEO and should be avoided at all cost during a de-orbit maneuver. The ISS has avoidance capabilities, but it would be advantageous if the object that conducts a de-orbit maneuver will apply an avoidance maneuver. The estimation of the collision probability presented in this thesis can identify high-risk trajectories and avoidance maneuvers can be applied accordingly. Because relatively short periods of time are involved with these maneuvers and only a limited debris database size is used, the numerical errors involved will be relatively small. This creates the capability of obtaining solutions that represent a real-life scenario with high accuracy.

A

Object Tracking and TLE Database

Objects in orbit around the Earth are tracked continuously by a network of tracking devices on the Earth's surface and in orbit. These objects are part of the United States Space Surveillance Network (USSSN). This network is responsible for detecting and cataloging artificial objects in orbit around the Earth. Conventional state estimation methods for active objects are satellite laser ranging, Doppler tracking, and GPS navigation. Debris objects in LEO are often tracked by high-performance radar facilities whereas further away objects (in the geostationary ring) are often tracked by optical telescopes [25]. Figure A.1 shows the detection systems in use by the USSSN to track objects in space.

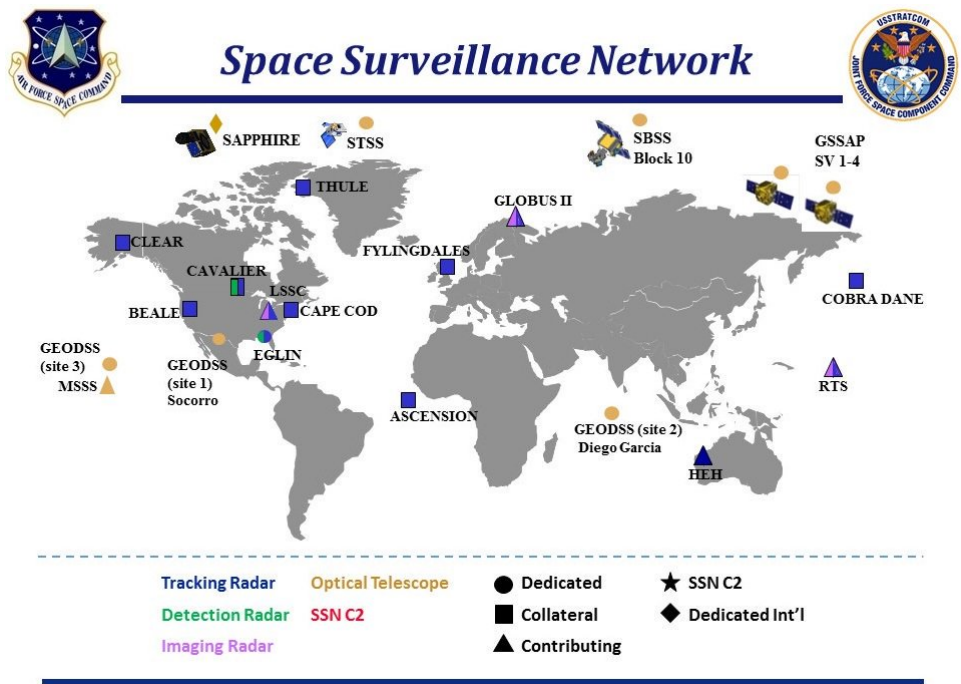


Figure A.1: Global distribution of the SSN tracking systems¹.

The observations taken by this network determine the state of each object at a reference epoch which is then published in a satellite database by the North American Aerospace Defense Command (NORAD). This database contains the whereabouts of any (publicly available) object. NORAD is a cooperation between the United States of America and Canada, tasked to provide aerospace warnings and protection for the North American continent. This need arose during the early days of the space race between the USA and the USSR, after the launch of Sputnik-1. This database contains the state history of any (non-secret) satellite, allowing

¹18th Space Control Squadron Twitter page, www.twitter.com/18SPCS.com, retrieved: July 10, 2019.

anyone to track any object. To access this data and request satellite state information, a free user account can be created at www.space-track.org. The data on this website is provided by the 18th Space Control Squadron, which is responsible for maintaining the space catalog. The data provided by www.space-track.org comes in the form of the so-called Two-Line Element (TLE) set. This data format contains internal information about the satellite and external information about its orbital parameters. In order to obtain the orbital parameters from the TLE data, the format needs to be understood.

The TLE format originated from the late 1960s when the first models were created that could predict satellite locations. This resulted in a Simplified General Perturbations (SGP) model that was widely used by the US military. Several versions of the SGP all were compatible with the TLE format. Since the source code of the SGP model was compatible with the TLE format, the format became widely used [31].

The TLE format uses two lines of 69 ASCII code characters containing both the "internal" (e.g. satellite catalog number, time stamp) and the "transmission" data (orbital parameters such as the inclination). The information in the TLE data is then used to construct a Keplerian state of each satellite. Figure A.2 shows the format of a TLE set.

```
1 NNNNNC NNNNNAAA NNNNN.NNNNNNNN +.NNNNNNNN +NNNN-N +NNNN-N N NNNNN
2 NNNNN NNN.NNNN NNN.NNNN NNNNNNNN NNN.NNNN NNN.NNNN NN.NNNNNNNNNNNNNNN
```

Figure A.2: TLE format: columns with an 'N' can have any number from 0-9 or a space; columns with 'A' can have any letter from A-Z or a space; the column with a 'C' can have only the classification symbol U (unclassified) or S (secret); columns with '+' can either have a plus sign, a minus sign or a space; and, columns with '-' can either have a plus sign or a minus sign².

As an example, the requested TLE set of the ISS is shown in Figure A.3. On the first line, columns 19-32 show the date and time at which the TLE set was updated (19191.39587963). The first two columns represent the epoch year (last two digits of 2019) and the rest of the columns represent the epoch day and the fractional portion of the day (191.39587963). For this specific TLE set, this corresponds to 10 July, 2019 at 09:30:04. The second line of the TLE set is used to convert this format into a Keplerian state of the ISS. Columns 9-16 represent the inclination in degrees (51.6431), columns 18-25 represent the right ascension of the ascending node in degrees (246.1231), columns 27-33 represent the eccentricity (0.0007092), columns 35-42 represent the argument of perigee in degrees (125.9716), and, columns 53-63 represent the mean motion in revolutions per day (15.50964659178859). Using Kepler's third law (see Section 2.1) the mean motion can be converted into a semi-major axis, completing the Kepler element set.

```
1 25544U 98067A 19191.39587963 -.00000057 00000-0 69029-5 0 9995
2 25544 51.6431 246.1231 0007092 125.9716 61.2218 15.50964659178859
```

Figure A.3: TLE set of the ISS on 10 July, 2019 at 09:30:04.

²<https://celestrak.com/columns/v04n03/>, retrieved: September 16, 2019

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