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Quantum corrections to conductivity of disordered electrons due to inelastic scattering off magnetic impurities

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We study the quantum corrections to the conductivity of the two-dimensional disordered interacting electron system in the diffusive regime due to inelastic scattering off rare magnetic impurities. We focus on the case of very different g factors for electrons and magnetic impurities. Within the Born approximation for the inelastic scattering off magnetic impurities we find additional temperature-dependent corrections to the conductivity of the Altshuler-Aronov type. Our results demonstrate that the low-temperature transport in interacting disordered electron systems with rare magnetic impurities is more interesting than it was commonly believed on the basis of treatment of magnetic impurity spins as classical ones.

I. INTRODUCTION

As it is well known, the low-temperature properties of an electron system are significantly affected by electron scattering off rare magnetic impurities. The simplest approach is to treat a magnetic impurity classically as a random three-dimensional vector of a fixed length. Despite that such a model ignores a quantum dynamics of the spin, i.e., treats the scattering off a magnetic impurity elastically, the model is powerful enough to produce a number of interesting, nontrivial effects, e.g., suppression of the superconducting transition temperature due to elastic electron spin flip [1], suppression of temperature dependence of the weak-localization correction to conductivity [2,3], etc.

The quantum dynamics of the spin of a magnetic impurity is responsible for the Kondo effect: renormalization of the exchange coupling between an electron and impurity spins that leads to nonmonotonic temperature dependence of resistivity [4]. Physically, the quantum dynamics of the spin allows an electron to scatter inelastically off a magnetic impurity [5–7]. For example, the Zeeman splitting of the magnetic impurity levels results in energy dependence of the spin-flip scattering even within the Born approximation [8]. The presence of potential elastic scattering together with the inelastic spin-flip scattering results in modification of the Kondo effect and the behavior of the quantum corrections to conductivity [9–11]. For example, in disordered electron systems the inelastic spin-flip scattering affects the weak-localization correction and mesoscopic conductance fluctuations via the energy-dependent dephasing time induced by spin-flip scattering [8,12–15]. In addition to the influence on the weak-localization correction, the inelastic scattering off magnetic impurities results in appearance of the Altshuler-Aronov–type corrections to the conductivity [9,16,17]. These temperature-dependent corrections have been found in the third order in the exchange

interaction. It can be easily argued why this is the lowest order in which such corrections can arise. Indeed, in order to have inelastic scattering off a magnetic impurity within the Born approximation, one needs to have the Zeeman splitting. However, the Zeeman splitting induces a cutoff for the relevant diffusive modes. This forbids temperature-dependent corrections to conductivity in the second order in the exchange interaction. However, the above arguments assume that the Zeeman splitting for a magnetic impurity and for an electron are the same, which is true provided the Lánde factors are the same.

In this paper, we consider the case of very different g factors of an electron g_e and a magnetic impurity g_i: $|g_i| \gg |g_e|$. In this case, the impurity Zeeman splitting $b_i = g_i \mu_B H$ can be much larger than the electron Zeeman splitting $b_e = g_e \mu_B H$. Here, μ_B stands for the Bohr magneton and H denotes the external magnetic field. For sake of concreteness we consider a two-dimensional electron system in parallel magnetic field H. Then, as we shall demonstrate, there exists the logarithmic-in-T correction to the conductivity due inelastic scattering off magnetic impurities within the Born approximation provided the temperature satisfies inequalities:

$$|b_e| \ll T \ll |b_i|. \tag{1}$$

Also, we study how inelastic scattering off magnetic impurities interferes with the electron-electron interaction. We find that, on the one hand, the inelastic scattering off magnetic impurities modifies the Altshuler-Aronov correction, and, on the other hand, the electron-electron interaction affects the correction to the conductivity due to the inelastic scattering off magnetic impurities (which also exists in the absence of electron-electron interaction).

The paper is organized as follows. In Sec. II we remind the formalism of the Finkel'stein nonlinear sigma model. The

perturbative expansion of the nonlinear sigma model and the structure of diffusive modes are discussed in Sec. III. In Sec. IV we present our results for the temperature dependence of conductivity in two-dimensional electron system. We conclude the paper with the discussion of our findings (Sec. V).

II. FINKEL'STEIN NONLINEAR SIGMA MODEL

We consider a two-dimensional interacting electron system in the presence of short-ranged potential disorder. In addition, we assume the presence of weak spin-flip scattering due to an exchange interaction between rare magnetic impurities and electrons described by the following Hamiltonian:

$$H_{\text{mag}} = J \sum_j \psi^\dagger(\boldsymbol{r}_j) \boldsymbol{S}_j \boldsymbol{\sigma} \psi(\boldsymbol{r}_j). \tag{2}$$

Here, $\boldsymbol{\sigma}$ and $\boldsymbol{S}_j$ stand for the Pauli matrices and the spin operator of a magnetic impurity at the position $\boldsymbol{r}_j$, respectively. The electron creation and annihilation operators are denoted as $\psi^\dagger(\boldsymbol{r})$ and $\psi(\boldsymbol{r})$. We shall treat rare magnetic disorder under the following assumptions: (i) the magnetic scatterers are uniformly distributed with the Poisson distribution, i.e., the probability density to have k impurities at positions $\boldsymbol{r}_1, \dots, \boldsymbol{r}_k$ is proportional to $n_s^k/k!$ where n_s denotes the average concentration of magnetic impurities; (ii) impurity spins $\boldsymbol{S}_j$ are independent but have their own quantum dynamics.

In the absence of magnetic scattering, the effective field theory for disordered interacting electrons in the diffusive regime, $T \ll 1/\tau$, where τ denotes the elastic mean-free time, is defined in a standard way (for a review, see [18,19]). In the absence of magnetic field and magnetic impurities, the Hamiltonian of the system preserves spin-rotational and time-reversal symmetries. Then, the effective field theory is formulated in terms of a matrix field $Q \in G/K$ with $G = \text{Sp}(2N)$ and $K = \text{Sp}(N) \times \text{Sp}(N)$. The rank of G is given by $N = 4 N_r N_m$ where N_m denotes the number of Matsubara frequencies involved and N_r stands for the number of replica. For computation of physical observables, one needs to take two limits: $N_m \to \infty$ and $N_r \to 0$, at the end of calculations. We note that the limit $N_m \to \infty$ should be taken in a way consistent with the gauge invariance (see Ref. [20] for details). The factor 4 appears since one needs to take into account the spin and Nambu (particle-hole) spaces. Taking into account Zeeman splitting due to external magnetic field, the effective action can be written as follows [21–23]:

$$\mathcal{S}_\sigma = -\int d\boldsymbol{r} \, \text{Tr} \left[\frac{g}{32} (\nabla Q)^2 - 4\pi T Z_\omega \eta Q - i Z_s b_{\text{e}} t_{33} Q \right]$$
$$- \frac{\pi T}{4} \sum_{\alpha, n, r, j} \Gamma_j \int d\boldsymbol{r} \, \text{Tr} \, I_n^\alpha t_{rj} Q \, \text{Tr} \, I_{-n}^\alpha t_{rj} Q. \tag{3}$$

Here, the 16 matrices t_{rj}, $j, r = 0, 1, 2, 3$, act in a tensor product of the spin (subscript j) and Nambu (subscript r) spaces:

$$t_{rj} = \tau_r \otimes s_j, \quad r, j = 0, 1, 2, 3. \tag{4}$$

Here, matrices τ_0 and s_0 stand for the 2×2 unit matrices and

$$\tau_1/s_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2/s_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3/s_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The effective action (3) involves the following matrices:

$$\eta_{nm}^{\alpha\beta} = n \delta_{nm} \delta^{\alpha\beta} t_{00}, \quad \left(I_k^\gamma\right)_{nm}^{\alpha\beta} = \delta_{n-m,k} \delta^{\alpha\beta} \delta^{\alpha\gamma} t_{00}, \tag{5}$$

where $\alpha, \beta = 1, \dots, N_r$ stands for replica indices and indices n, m correspond to the Matsubara fermionic frequencies $\varepsilon_n = \pi T(2n + 1)$. The total (including spin) dimensionless (in units e^2/h) Drude conductivity is denoted by g. The interaction amplitudes Γ_j (for the singlet channel, $\Gamma_0 = \Gamma_s$, and for the triplet channel, $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_t$) describe electron-electron interaction in the particle-hole channel. In what follows, it will be also convenient to use $\gamma_j = \Gamma_j/Z_\omega$ and $\gamma_{s,t} = \Gamma_{s,t}/Z_\omega$. We note that in the case of Coulomb interaction γ_s is fixed to the value -1 (see Refs. [18,19] for details).

The parameter Z_ω takes into account nontrivial frequency renormalization under the renormalization group [21–23]. We note that the bare value of the parameter Z_ω is equal to $\pi \nu/4$ where ν denotes the density of states at the Fermi level. The last term in the first line of Eq. (3) describes the effect of the parallel magnetic field on electrons. This term violates explicitly time-reversal symmetry. The quantity $Z_s = Z_\omega + \Gamma_t$ describes the Fermi-liquid-type enhancement of the g factor (see reviews [18,19] for details). The bare value of Z_s is given as $\pi \nu(1 + \gamma_{t0})/4$ where γ_{t0} is the bare value of the interaction in the triplet channel which can be expressed in terms of the Fermi-liquid parameter F_0^σ as follows: $\gamma_{t0} = -F_0^\sigma/(1 + F_0^\sigma)$. Although in this work we do not study the renormalization of the parameters of the nonlinear sigma model action, g, Γ_j, Z_ω, and Z_s, in the presence of magnetic impurities, it is convenient to express the final results in terms of g, Γ_j, and Z_ω rather than their bare values.

We note that in this paper we neglect the electron-electron interaction in the Cooper channel. By construction, the matrix $Q(\boldsymbol{r})$ describes local rotations around the spatially independent matrix Λ:

$$Q = \mathcal{T}^{-1} \Lambda \mathcal{T}, \quad \Lambda_{nm}^{\alpha\beta} = \text{sgn} \, \varepsilon_n \, \delta_{nm} \delta^{\alpha\beta} t_{00}. \tag{6}$$

Here, the matrices $\mathcal{T} \in G$ obey the following symmetry relations:

$$C(\mathcal{T}^{-1})^{\text{T}} = \mathcal{T}C, \quad \mathcal{T}^{\text{T}} C = C \mathcal{T}^{-1}, \tag{7}$$

where $C = i t_{12}$. The symbol $\mathcal{T}^{\text{T}}$ denotes the matrix transpose of $\mathcal{T}$. As the consequence of Eqs. (6) and (7), the matrix Q is subjected to the local nonlinear constraint $Q^2(\boldsymbol{r}) = 1$, satisfies the condition $\text{Tr} \, Q = 0$, and obeys charge-conjugate relation

$$Q = Q^\dagger = C^T Q^T C. \tag{8}$$

In the presence of magnetic impurities, the full effective action $\mathcal{S}$ is the sum of the Finkel'stein nonlinear sigma model $\mathcal{S}_\sigma$ and the additional part $\mathcal{S}_{\text{mag}}$, i.e., $\mathcal{S} = \mathcal{S}_\sigma + \mathcal{S}_{\text{mag}}$. For rare magnetic impurities, the latter can be written as a sum over contributions of individual magnetic impurities [24]:

$$\mathcal{S}_{\text{mag}} = \frac{1}{2} \sum_j \text{Tr} \ln(1 + i\pi \nu J \, Q(\boldsymbol{r}_j) \tau_3 \boldsymbol{\sigma} \hat{\boldsymbol{S}}_j). \tag{9}$$

Here, we introduce the following notations:

$$\hat{\boldsymbol{S}}_j = \sum_n \boldsymbol{S}_j(i\omega_n) I_n, \quad \boldsymbol{S}_j(i\omega_n) = \int_0^\beta d\tau \, \boldsymbol{S}_j(\tau) e^{i\omega_n \tau}, \tag{10}$$

where $\beta = 1/T$, $\omega_n = 2\pi T n$, and the matrix I_n is defined as follows:

$$(I_k)_{nm}^{\alpha\beta} = \delta_{n-m,k}\delta^{\alpha\beta}t_{00}. \qquad (11)$$

We note that the form (9) of the action $\mathcal{S}_{\text{mag}}$ is equivalent to the self-consistent T-matrix approximation for magnetic scattering, i.e., it is derived by taking into account all orders in scattering off a single magnetic impurity but by neglecting contributions with intersecting impurity lines.

We perform the averaging over the uniformly distributed magnetic impurities, the number of which is determined by the Poisson distribution with the help of the following relation [25]:

$$\left\langle \exp \sum_j f(\boldsymbol{r}_j) \right\rangle = \exp \left\{ n_s \int d\boldsymbol{r} [e^{f(\boldsymbol{r})} - 1] \right\}. \qquad (12)$$

Then, we find that the contribution to the effective action due to magnetic impurities becomes

$$\mathcal{S}_{\text{mag}} \rightarrow n_s \int d\boldsymbol{r} (\langle e^{\frac{1}{2}\operatorname{Tr}\ln(1+i\pi\nu J\, Q(\boldsymbol{r})\tau_3\boldsymbol{\sigma}\hat{\boldsymbol{S}})} \rangle_S - 1). \qquad (13)$$

Here, $\langle \ldots \rangle_S$ stands for the averaging over dynamics of a single magnetic impurity.

In this paper we restrict our consideration by the Born approximation for the scattering off a single magnetic impurity. Therefore, we can expand $\operatorname{Tr}\ln$ in Eq. (13) up to the second order in J. Then, we find

$$\mathcal{S}_{\text{mag}} = \frac{n_s \pi \nu J}{2} \int d\boldsymbol{r} \left\langle i \operatorname{Tr} Q\tau_3\boldsymbol{\sigma} \langle\hat{\boldsymbol{S}}\rangle_S + \frac{\pi\nu J}{2} \operatorname{Tr}(Q\tau_3\boldsymbol{\sigma}\hat{\boldsymbol{S}})^2 \right.$$
$$\left. - \frac{\pi\nu J}{4}(\operatorname{Tr} Q\tau_3\boldsymbol{\sigma}\hat{\boldsymbol{S}})^2 \right\rangle_S. \qquad (14)$$

In order to proceed further, we need to perform averaging over dynamics of the spin of a magnetic impurity in $\mathcal{S}_{\text{mag}}$. In what follows, we neglect a back-action of electrons on the spin of a magnetic impurity. This allows us to write the impurity Hamiltonian as follows: $H_i = b_i S_z$. We note that there are several ways of how electrons affect the impurity spin. The first one is the Kondo effect which implies renormalization of the exchange interaction due to interaction with electrons. The Kondo renormalization is small at temperatures much higher than the Kondo temperature $T \gg T_K$. Second, the interaction between the magnetic impurity and electrons results in generation of the local anisotropy which will be of the second order in the exchange interaction [26,27]. In principle, the local anisotropy can become important for the spin dynamics but we neglect it in comparison with the Zeeman splitting b_i. This is possible in the case of small enough exchange interaction $\nu J \ll \sqrt{b_i/E_F}$, where E_F denotes the Fermi energy. Finally, the electrons can mediate the indirect exchange interaction between the spins of magnetic impurities situated nearby. In order to neglect the indirect exchange interaction we assume that the magnetic impurities are rare enough such that the distance between them exceeds the length scale associated with the temperature in disordered system, i.e., $n_s L_T^2 \ll 1$ where $L_T = \sqrt{D/T}$ [28].

Then, we need the corresponding Matsubara spin-spin correlation functions

$$\chi_\pm(\tau_1,\tau_2) = \frac{1}{S(S+1)} \begin{cases} \langle S_\pm(\tau_1)S_\mp(\tau_2)\rangle_S, & \tau_1 > \tau_2 \\ \langle S_\mp(\tau_2)S_\pm(\tau_1)\rangle_S, & \tau_2 > \tau_1 \end{cases} \qquad (15)$$

where $S_\pm = S_x \pm iS_y$, and

$$\chi_{zz}(\tau_1,\tau_2) = \frac{1}{S(S+1)} \begin{cases} \langle S_z(\tau_1)S_z(\tau_2)\rangle_S, & \tau_1 > \tau_2 \\ \langle S_z(\tau_2)S_z(\tau_1)\rangle_S, & \tau_2 > \tau_1. \end{cases} \qquad (16)$$

Using the equations of motion for a free spin in a magnetic field we find the following results:

$$\chi_\pm(i\omega_n) = -e^{i\omega_n 0^+}\frac{2M_1}{i\omega_n \pm b_i}, \quad \chi_{zz}(i\omega_n) = \delta_{n,0}\beta M_2. \qquad (17)$$

Here, we introduced

$$M_n = \frac{1}{S(S+1)} \sum_{m=-S}^{m=S} m^n e^{-\beta b_i m} \Big/ \sum_{m=-S}^{m=S} e^{-\beta b_i m}. \qquad (18)$$

We note the following useful relations: $e^{-\beta b_i}\langle S_- S_+\rangle_S = \langle S_+ S_-\rangle_S$ and $M_2 = 1 + M_1 \coth(b_i/2T)$. Using the results (17), we obtain

$$\mathcal{S}_{\text{mag}} = \int d\boldsymbol{r} \left\{ \frac{i}{2}n_s\pi\nu J \langle S_z\rangle_S \operatorname{Tr} t_{33}Q + \frac{Z_\omega T}{4\tau_{s0}}\sum_n \chi_+(i\omega_n) \right.$$
$$\times \left[\operatorname{Tr} t_- I_n Q t_+ I_{-n}Q - \frac{1}{2}\operatorname{Tr} t_- I_n Q \operatorname{Tr} t_+ I_{-n}Q \right]$$
$$\left. + \frac{Z_\omega}{2\tau_{s0}}M_2 \left[\operatorname{Tr} t_{33}Q t_{33}Q - \frac{1}{2}\operatorname{Tr} t_{33}Q \operatorname{Tr} t_{33}Q \right] \right\}, \qquad (19)$$

where $t_\pm = t_{31} \pm it_{32}$ and

$$\frac{1}{\tau_{s0}} = \frac{n_s(\pi\nu J)^2 S(S+1)}{2Z_\omega} \qquad (20)$$

denotes the classical spin-flip rate at zero magnetic field. We mention that the parameter Z_ω does not appear originally in the action (19) for magnetic impurities. However, it is convenient to define the spin-flip rate at zero magnetic field in accordance with Eq. (20) in order to preserve a standard form of the diffusion propagators.

The first term in the right-hand side of Eq. (19) corresponds to additional Zeeman splitting of electrons due to magnetization of magnetic impurities. The second term in the right-hand side of Eq. (19) describes the contribution due to inelastic spin-flip scattering off magnetic impurity. We emphasize that contrary to the term due to electron-electron interaction [see the second line in Eq. (3)], the inelastic term due to scattering off magnetic impurities mixes different replica channels.

III. PERTURBATIVE EXPANSION

For the perturbative treatment (in $1/g$) of the action $\mathcal{S}_\sigma + \mathcal{S}_{\text{mag}}$ we need to resolve the constraint $Q^2 = 1$. In order to do it, we use the square-root parametrization:

$$Q = W + \Lambda\sqrt{1 - W^2}, \quad W = \begin{pmatrix} 0 & w \\ \bar{w} & 0 \end{pmatrix}. \qquad (21)$$

In what follows, we shall adopt the following notations: $W_{n_1 n_2} = w_{n_1 n_2}$ and $W_{n_4 n_3} = \bar{w}_{n_4 n_3}$ where $n_{1,3} \geq 0$ and $n_{2,4} < 0$.

The two blocks of the matrix W are related by the following symmetry relation as

$$\bar{w} = -C w^T C. \tag{22}$$

We note that here the matrix transposition acts on the Matsubara space indices. Expansion of $\mathcal{S}_\sigma + \mathcal{S}_{\text{mag}}$ to the second order in W yields the following Gaussian action:

$$
\begin{aligned}
\mathcal{S}_\sigma^{(2)} + \mathcal{S}_{\text{mag}}^{(2)} = -4 \int \frac{d\boldsymbol{p}}{(2\pi)^d} \sum_{rr';jj';\alpha_l,n_l} & [w_{rj}(\boldsymbol{p})]_{n_1 n_2}^{\alpha_1 \alpha_2} [\bar{w}_{r'j'}(-\boldsymbol{p})]_{n_4 n_3}^{\alpha_4 \alpha_3} \delta_{n_{12},n_{34}} \Big\{ \delta_{n_1 n_3} \delta_{n_2 n_4} \delta^{\alpha_1 \alpha_3} \delta^{\alpha_2 \alpha_4} \Big[\delta_{jj'} \delta_{rr'} Z_\omega \Big(Dp^2 \\
& + \Omega_{12}^\varepsilon + \frac{1}{\tau_{rj}^{\text{sf}}} + \frac{1}{\tau_\perp^{\text{sf}}} (h(i\varepsilon_{n_1}) + h(-i\varepsilon_{n_2})) \Big) - Z_s \tilde{b}_{\text{e}}(\delta_{r0}\delta_{r'3} + \delta_{r3}\delta_{r'0})\mu_{jj'}^{(d)} - Z_s \tilde{b}_{\text{e}}(\delta_{r1}\delta_{r'2} - \delta_{r2}\delta_{r'1})\mu_{jj'}^{(c)} \Big] \\
& - 2\pi T \Gamma^{\text{sf}} \delta^{\alpha_1 \alpha_3} \delta^{\alpha_2 \alpha_4} (1 - \delta_{n_1 n_3})\delta_{rr'}\lambda_r \Big[\delta_{jj'}(\delta_{j0} - \delta_{j3}) \operatorname{Re} \widehat{\chi}\left(i\Omega_{13}^\varepsilon\right) + (\delta_{j0}\delta_{j'3} - \delta_{j3}\delta_{j'0})i \operatorname{Im} \widehat{\chi}\left(i\Omega_{13}^\varepsilon\right) \Big] \\
& + 2\pi T \delta_{jj'}\delta_{rr'}\delta^{\alpha_1 \alpha_2}\delta^{\alpha_3 \alpha_4} \Big[\Gamma_j(\delta_{r0} + \delta_{r3})\delta^{\alpha_2 \alpha_3} + \Gamma^{\text{sf}}\delta_{r3}(\delta_{j1} + \delta_{j2}) \operatorname{Re} \widehat{\chi}\left(i\Omega_{12}^\varepsilon\right) \Big] \Big\}. \tag{23}
\end{aligned}
$$

Here, we introduced the following notations: $w_{rj} = \text{sp}[wt_{rj}]/4$, where sp denotes the trace over spin and particle-hole indices, $\lambda_r = \{1, -1, -1, 1\}$, $\Omega_{12}^\varepsilon = \varepsilon_{n_1} - \varepsilon_{n_2}$, $\Omega_{13}^\varepsilon = \varepsilon_{n_1} - \varepsilon_{n_3}$, and $\widehat{\chi}(i\omega) = \chi_+(i\omega)/\chi_+(i0)$. The diffusion coefficient is given as $D = g/(16 Z_\omega)$. The parameter $\Gamma^{\text{sf}} = n_s(\pi \nu J)^2 S(S+1)\chi(i0)/(4\pi)$ characterizes the strength of interaction due to the inelastic spin-flip scattering. The effective Zeeman splitting for electrons is given as $\tilde{b}_{\text{e}} = b_{\text{e}} + \pi n_s \nu J \langle S_z \rangle_S/(2Z_s)$. The matrices $\mu_{jj'}^{(d)}$ and $\mu_{jj'}^{(c)}$ are defined as follows:

$$
\mu_{jj'}^{(d)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{jj'}, \quad
\mu_{jj'}^{(c)} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}_{jj'}. \tag{24}
$$

The second line in Eq. (23) involves the elastic spin-flip time τ_{rj}^{sf}. It can be expressed in terms of the static spin susceptibilities as follows:

$$\frac{1}{\tau_{rj}^{\text{sf}}} = \frac{1}{\tau_\parallel^{\text{sf}}} \zeta_{rj}^\parallel + \frac{1}{\tau_\perp^{\text{sf}}} \zeta_{rj}^\perp, \tag{25}$$

where $1/\tau_\parallel^{\text{sf}} = 2M_2/\tau_{s0}$, $1/\tau_\perp^{\text{sf}} = T\chi_+(i0)/\tau_{s0}$, and

$$
\zeta_{rj}^\parallel = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}_{rj}, \quad
\zeta_{rj}^\perp = \begin{pmatrix} 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 \end{pmatrix}_{rj}. \tag{26}
$$

For sake of convenience, we note that

$$
\chi_+(i0) = -\frac{2M_1}{b_{\text{i}}} = \begin{cases} 2/(3T), & |b_{\text{i}}| \ll T \\ 2/[|b_{\text{i}}|(S+1)], & |b_{\text{i}}| \gg T. \end{cases} \tag{27}
$$

In the limit of zero Zeeman splitting $b_i \to 0$, the elastic spin-flip rate becomes equal $1/\tau_{rj}^{\text{sf},(0)} = 2\zeta_{rj}/(3\tau_{s0})$, where the matrix ζ_{rj} is defined as follows:

$$
\zeta_{rj} = \begin{pmatrix} 0 & 2 & 2 & 2 \\ 3 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \end{pmatrix}_{rj}. \tag{28}
$$

Taking into account that the bare value of the parameter Z_ω is equal $\pi \nu/4$, we obtain the well-known values for the elastic spin-flip rates in different diffusive modes (see, e.g., Ref. [3]).

The function $h(i\varepsilon_n)$ in the second line of Eq. (23) describes the effect of the inelastic scattering off magnetic impurities on the part of the propagator of the diffusive modes which is diagonal in the Matsubara space. This function is defined as $(\varepsilon_n > 0)$

$$
\begin{aligned}
h(i\varepsilon_n) = \sum_{\varepsilon_n > \omega_k > 0} \operatorname{Re} \widehat{\chi}(i\omega_k) = \frac{b_{\text{i}}}{2\pi T} \operatorname{Im} \Big[& \psi\left(1 + \frac{ib_{\text{i}}}{2\pi T}\right) \\
& - \psi\left(1 + n + \frac{ib_{\text{i}}}{2\pi T}\right) \Big]. \tag{29}
\end{aligned}
$$

Here, $\psi(z)$ denotes the Euler digamma function. The function $h(i\varepsilon_n)$ appears as the self-energy correction to the diffusive modes in the diagrammatic approach [9]. In particular, the function h contains the additional contribution due to inelastic spin flip on magnetic impurities to decay rate of "cooperons" which has been studied recently in Ref. [15] in detail. In order to discuss this effect, it is convenient to make analytic continuation $i\varepsilon_{n_1} \to \varepsilon_+ = \varepsilon + \Omega/2$ and $i\varepsilon_{n_2} \to \varepsilon_- = \varepsilon - \Omega/2$. The retarded function $h^R(\varepsilon)$ corresponding to the Matsubara function $h(i\varepsilon_n)$ is given as

$$
\begin{aligned}
h^R(\varepsilon) = \frac{b_{\text{i}}}{2\pi T} \Big[& \frac{1}{2} \sum_{\sigma = \pm} i\sigma \psi\left(\frac{1}{2} - \frac{i\varepsilon}{2\pi T} + \frac{ib_{\text{i}}\sigma}{2\pi T}\right) \\
& + \operatorname{Im} \psi\left(1 + \frac{ib_{\text{i}}}{2\pi T}\right) \Big]. \tag{30}
\end{aligned}
$$

The real part of $h^R(\varepsilon)$ determines the additional contribution to the decay rate of the diffusive modes:

$$\frac{1}{\tau^{\text{sf}}_{\text{inel}}(\varepsilon)} = \frac{2}{\tau^{\text{sf}}_\perp} \operatorname{Re} h^R(\varepsilon) = -\frac{1}{\tau^{\text{sf}}_\perp}\left[1 - \frac{b_{\text{i}}}{4T}\left(2\coth\frac{b_{\text{i}}}{2\pi T}\right.\right.$$
$$\left.\left. - \tanh\frac{b_{\text{i}}+\varepsilon}{2\pi T} - \tanh\frac{b_{\text{i}}-\varepsilon}{2\pi T}\right)\right]. \quad (31)$$

Here, we took into account that $\operatorname{Re} h^R(\varepsilon)$ is even function of ε. Interestingly, the function $h^R(\varepsilon)$ produces also the imaginary correction which is linear in Ω at $\Omega \to 0$:

$$\frac{1}{\tau^{\text{sf}}_\perp}[h^R(\varepsilon_+) + h^R(\varepsilon_-)] = \frac{1}{\tau^{\text{sf}}_{\text{inel}}(\varepsilon)} - i(z(\varepsilon)-1)\Omega + \cdots. \quad (32)$$

Here, the frequency renormalization factor is given as follows:

$$z(\varepsilon) = 1 + \gamma^{\text{sf}}\frac{b_{\text{i}}}{4\pi T}\sum_{\sigma=\pm}\operatorname{Im}\psi'\left(\frac{1}{2} + \frac{i(b_{\text{i}}+\sigma\varepsilon)}{2\pi T}\right), \quad (33)$$

where $\gamma^{\text{sf}} = \Gamma^{\text{sf}}/Z_\omega$. In the case $|\varepsilon|, T \ll |b_{\text{i}}|$ the renormalization factor becomes $z(\varepsilon) = 1 + \gamma^{\text{sf}}$ where the parameter γ^{sf} is given as $\gamma^{\text{sf}} = 1/[\pi(S+1)\tau_{s0}|b_{\text{i}}|] \ll 1$ [see Eq. (27)]. We note that in the case $|\varepsilon|, T \ll |b_{\text{i}}|$ the expansion (32) holds for $|\Omega| \ll |b_{\text{i}}|$.

Since in what follows we are interested in the regime $|b_{\text{e}}| \ll T \ll |b_{\text{i}}|$, we neglect terms with the spin-flip rates and Zeeman splitting in the second line of Eq. (23). Then for frequencies which are much smaller than $|b_{\text{i}}|$, we find the following result for the propagators of different diffusive modes:

$$\langle[w_{rj}(\boldsymbol{p})]^{\alpha_1\alpha_2}_{n_1n_2}[\bar{w}_{rj}(-\boldsymbol{p})]^{\alpha_4\alpha_3}_{n_4n_3}\rangle = \frac{2}{g}\delta_{n_{12},n_{34}}\left\{\delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}\left[\delta_{n_1n_3}\mathcal{D}_p\left(i\Omega^\varepsilon_{12}\right) - \frac{2\pi T\gamma_j}{D}\delta^{\alpha_1\alpha_2}(\delta_{r0}+\delta_{r3})\mathcal{D}^{(rj)}_p\left(i\Omega^\varepsilon_{12}\right)\widetilde{\mathcal{D}}^{(rj)}_p\left(i\Omega^\varepsilon_{12}\right)\right.\right.$$
$$\left.\left. + \frac{2\pi T\gamma^{\text{sf}}}{D}\delta^{\alpha_1\alpha_3}\delta^{\alpha_2\alpha_4}\lambda_r(\delta_{j0}-\delta_{j3})\mathcal{D}_p\left(i\Omega^\varepsilon_{12}\right)\mathcal{D}^{(rj)}_p\left(i\Omega^\varepsilon_{12}\right)\right] - \frac{2\pi T\gamma^{\text{sf}}}{D}\delta^{\alpha_1\alpha_2}\delta^{\alpha_3\alpha_4}\delta_{r3}(\delta_{j1}+\delta_{j2})\left[\widetilde{\mathcal{D}}^{(rj)}_p\left(i\Omega^\varepsilon_{12}\right)\right]^2\right\}. \quad (34)$$

Here, the following propagator

$$[\mathcal{D}_p(i\omega_n)]^{-1} = p^2 + (1+\gamma^{\text{sf}})\frac{|\omega_n|}{D} \quad (35)$$

describes "diffuson" (for $r = 0,3$) and "cooperon" (for $r = 1,2$) modes in the absence of electron-electron interaction. The factor $1 + \gamma^{\text{sf}}$ appears as a result of taking into account the self-energy contributions due to scattering off magnetic impurities (see Fig. 1). The propagator

$$\left[\mathcal{D}^{(rj)}_p(i\omega_n)\right]^{-1} = p^2 + [1+\gamma^{\text{sf}} - \gamma^{\text{sf}}\lambda_r(\delta_{j0}-\delta_{j3})]\frac{|\omega_n|}{D} \quad (36)$$

(a)

(b)
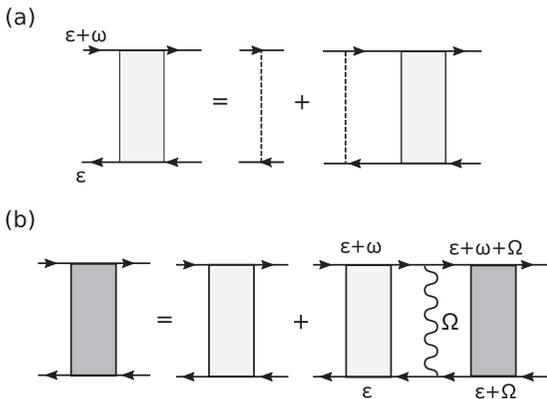
FIG. 1. The diagrammatic representation of equations for the diffusive propagators $\mathcal{D}_p(i\omega)$ (a) and $\mathcal{D}^{(rj)}_p(i\omega)$ (b). The solid line stands for the electron Green's function averaged over potential disorder and with self-energy correction due to scattering off magnetic impurities. The dashed line denotes the scattering off the potential disorder. The wavy line stands for the inelastic scattering off magnetic impurities. The Matsubara energies ε, ω, and Ω are assumed to satisfy the following conditions: $\varepsilon < 0$, $\varepsilon + \omega > 0$, $\varepsilon + \Omega < 0$, and $\varepsilon + \omega + \Omega > 0$.

accounts for the vertex insertions of the scattering off a magnetic impurity into the "diffuson" and "cooperon" ladder (see Fig. 1). The electron-electron interaction appears in the propagator of "diffuson" modes ($r = 0,3$) dressed by electron-electron scattering (see Ref. [18] for details):

$$\left[\widetilde{\mathcal{D}}^{(rj)}_p\left(i\Omega^\varepsilon_{12}\right)\right]^{-1} = \left[\mathcal{D}^{(rj)}_p\left(i\Omega^\varepsilon_{12}\right)\right]^{-1} + \frac{\gamma_j\Omega^\varepsilon_{12}}{D}. \quad (37)$$

For frequencies larger than $|b_{\text{i}}|$ the propagators are given by Eq. (34) with γ^{sf} set to zero.

We note that the form (34) of the propagators for the diffusive modes should guarantee that the form of the polarization operator is independent of the scattering on magnetic impurities. Indeed, the form of the polarization operator is restricted by Ward identity and should be consistent with the continuity equation for electron density. Therefore, one expects that the self-energy and vertex corrections in polarization bubble due to scattering off a magnetic impurity cancel each other. In order to check it, we write the density-density response (reducible polarization operator with respect to the Coulomb interaction) as follows (see Ref. [18]):

$$\Pi(\boldsymbol{q},i\omega_n) = -\frac{4}{\pi}(Z_\omega + \Gamma_s)\Big[1 - \pi T(Z_\omega + \Gamma_s)$$
$$\times \left\langle\operatorname{Tr} I^\alpha_n Q(\boldsymbol{q})\operatorname{Tr} I^\alpha_{-n}Q(-\boldsymbol{q})\right\rangle\Big]. \quad (38)$$

Evaluation of the average with the help of Eq. (34) in the lowest order with respect to $1/g$ (this approximation corresponds to the random phase approximation) results in the following form of the polarization operator:

$$\Pi^{\text{RPA}}(\boldsymbol{q},i\omega_n) = -\frac{4}{\pi}\frac{Z_\omega(1+\gamma_s)Dq^2}{Dq^2 + (1+\gamma_s)|\omega_n|}. \quad (39)$$

As expected, the parameter γ^{sf} drops from the expression for Π^{RPA} due to cancellation of self-energy and vertex contributions.

IV. EVALUATION OF THE CONDUCTIVITY

A. Kubo formula

Within the formalism of the nonlinear sigma model the static conductivity can be computed by means of the following Kubo formula:

$$
g' = -\frac{g}{16n}\big\langle \mathrm{Tr}\,[J_n^\alpha, Q(\boldsymbol{r})][J_{-n}^\alpha, Q(\boldsymbol{r})]\big\rangle + \frac{g^2}{64dn}\int d\boldsymbol{r}'
$$
$$
\times \langle \mathrm{Tr}\, J_n^\alpha Q(\boldsymbol{r})\boldsymbol{\nabla} Q(\boldsymbol{r})\, \mathrm{Tr}\, J_{-n}^\alpha Q(\boldsymbol{r}')\boldsymbol{\nabla} Q(\boldsymbol{r}')\rangle, \quad (40)
$$

where d stands for dimensionality, the limit $n \to 0$ is assumed, and

$$
J_n^\alpha = \frac{t_{30} - t_{00}}{2} I_n^\alpha + \frac{t_{30} + t_{00}}{2} I_{-n}^\alpha. \quad (41)
$$

The average $\langle \dots \rangle$ in Eq. (40) is defined with respect to the total action $\mathcal{S}_\sigma + \mathcal{S}_{\mathrm{mag}}$. Evaluating the averages in Eq. (40) with the help of Eq. (34), we find that the conductance in the one-loop approximation can be written as

$$
g' = g + \delta g^{\mathrm{wl}} + \delta g^{\mathrm{AA}} + \delta g_1^{\mathrm{sf}} + \delta g_2^{\mathrm{sf}}. \quad (42)
$$

Here, δg^{wl} represents the interference correction. It has the standard form [3,29,30]

$$
\delta g^{\mathrm{wl}} = \sum_{r=1,2}\sum_j (2\delta_{j0} - 1)\int \frac{d\boldsymbol{p}}{(2\pi)^d}\mathcal{D}_p^{(rj)}(0). \quad (43)
$$

Since the weak-localization correction involves cooperon modes at zero frequency, the spin-flip scattering affects δg^{wl} only via decay rate of cooperon modes (see Ref. [15] for detailed discussion).

The next term, δg^{AA}, in the right-hand side of Eq. (42) is the Altshuler-Aronov correction due to electron-electron interaction [21,22,31]:

$$
\delta g^{\mathrm{AA}} = \frac{128\pi T}{ngd}\sum_{r=0,3}\sum_j \Gamma_j \int \frac{d\boldsymbol{p}}{(2\pi)^d}\, p^2 \sum_{m>0}\min\{m,n\}
$$
$$
\times \mathcal{D}_p^{(rj)}(i\omega_m)\widetilde{\mathcal{D}}_p^{(rj)}(i\omega_m)\mathcal{D}_p(i\omega_{m+n}). \quad (44)
$$

Here, the limit $n \to 0$ is assumed. We emphasize that for $|\omega_m| \ll |b_{\mathrm{i}}|$ the spin-flip scattering *does* enter the expression for δg^{AA} via the frequency renormalization factors in the diffusion propagator. We mention that the Altshuler-Aronov correction involves two types of propagators of diffusive modes $\mathcal{D}_p(i\omega_m)$ and $\mathcal{D}_p^{(rj)}(i\omega_m)$ (see Fig. 2).

Performing analytic continuation to the real frequencies, $i\omega_n \to \omega + i0$, and taking the limit $\omega \to 0$ we obtain the following result:

$$
\delta g^{\mathrm{AA}} = \frac{64}{gd}\,\mathrm{Im}\sum_j \Gamma_j \int \frac{d\boldsymbol{p}}{(2\pi)^d}\, p^2 \int d\Omega\, \partial_\Omega\Big(\Omega\coth\frac{\Omega}{2T}\Big)
$$
$$
\times \mathcal{D}_p^R(\Omega)\mathcal{D}_p^{(0j),R}(\Omega)\widetilde{\mathcal{D}}_p^{(0j),R}(\Omega). \quad (45)
$$

Here, we took into account that diffusion propagators with $r = 0$ and 3 coincide. The propagators $\mathcal{D}_p^R(\Omega)$, $\mathcal{D}_p^{(rj),R}(\Omega)$, and $\widetilde{\mathcal{D}}_p^{(rj),R}(\Omega)$ denote for the retarded propagators corresponding to $\mathcal{D}_p(i\Omega)$, $\mathcal{D}_p^{(rj)}(i\Omega)$, and $\widetilde{\mathcal{D}}_p^{(rj)}(i\Omega)$, respectively.
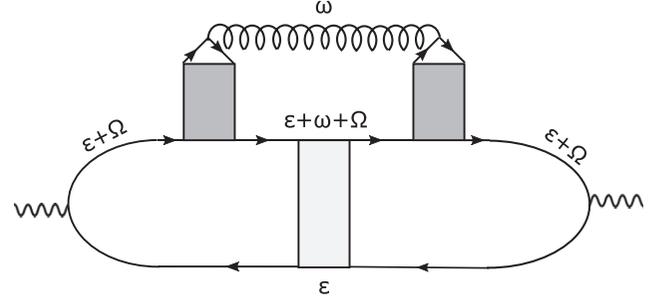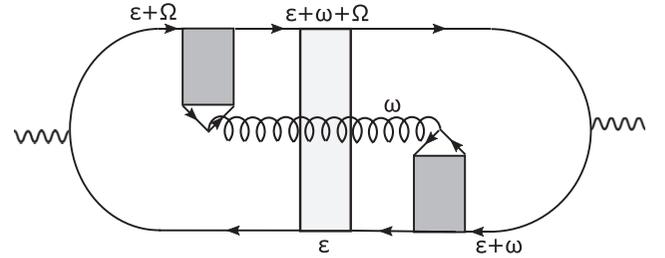
FIG. 2. The sketch of diagrams contributing to the Altshuler-Aronov corrections. The springlike line stands for the dynamically screened electron-electron interaction $[\mathcal{D}_p^{(rj)}(i\omega_m)]^{-1}\widetilde{\mathcal{D}}_p^{(rj)}(i\omega_m)$. The other elements have the same meaning as in the previous figure.

Next, there is the following correction due to inelastic spin-flip scattering:

$$
\delta g_1^{\mathrm{sf}} = -\frac{64\pi T\Gamma^{\mathrm{sf}}}{ng}\sum_{j=0,3}(-1)^j \int \frac{d\boldsymbol{p}}{(2\pi)^d}\sum_{m>0} m\,\mathcal{D}_p(i\omega_{m+n})
$$
$$
\times \sum_{r=0,3}\mathcal{D}_p^{(rj)}(i\omega_{m+n})[1 - p^2[\mathcal{D}_p(i\omega_m) + \mathcal{D}_p(i\omega_{m+2n})]].
$$
$$
(46)
$$

We remind a reader that the limit $n \to 0$ is assumed. On first glance, it seems that this limit is not finite such that correction δg_1^{sf} violates the gauge invariance. However, taking into account that the diffusion propagators $\mathcal{D}^{(rj)}$ are the same for $r = 0$ and 3, we can rewrite this correction δg_1^{sf} as the sum of two corrections $\delta g_1^{\mathrm{sf}} = \delta g_{1,\omega}^{\mathrm{sf}} + \delta g_{1,f}^{\mathrm{sf}}$, where $\delta g_{1,\omega}^{\mathrm{sf}}$ seems to have no finite limit at $n \to 0$ and $\delta g_{1,f}^{\mathrm{sf}}$ has a smooth $n \to 0$ limit. In particular, we find

$$
\delta g_{1,\omega}^{\mathrm{sf}} = \frac{2}{dn}\sum_{j=0,3}\sum_{m>0}\int \frac{d\boldsymbol{p}}{(2\pi)^d}\frac{\partial}{\partial\boldsymbol{p}}\frac{\partial}{\partial\boldsymbol{p}}\ln\frac{\mathcal{D}_p(i\omega_m)}{\mathcal{D}_p^{(0j)}(i\omega_m)}. \quad (47)
$$

Since $\delta g_{1,\omega}^{\mathrm{sf}}$ has the form of the second derivative with respect to the momentum, this correction is determined by the ultraviolet of the low-energy effective theory. Therefore, we cannot accurately compute it within the nonlinear sigma model approach. However, this correction is second order in γ^{sf} so taking it into account is accuracy excess.

After analytic continuation to the real frequencies, $i\omega_n \to \omega + i0$, and taking the limit $\omega \to 0$ the finite correction $\delta g_{1,f}^{\mathrm{sf}}$ can be written as

$$
\delta g_{1,f}^{\mathrm{sf}} = \frac{1}{d}\,\mathrm{Re}\sum_{j=0,3}\int \frac{d\boldsymbol{p}}{(2\pi)^d}\, p^2\int d\Omega\, \partial_\Omega\Big(\coth\frac{\Omega}{2T}\Big)
$$
$$
\times \big[\mathcal{D}_p^R(\Omega) - \mathcal{D}_p^{(0j),R}(\Omega)\big]^2. \quad (48)
$$

The last correction in Eq. (42) is also due to the inelastic spin-flip scattering represented by the last term in Eq. (34). It has the following form:

$$\delta g_2^{\text{sf}} = \frac{128\pi T \Gamma^{\text{sf}}}{ngd} \sum_{j=1,2} \int \frac{d\boldsymbol{p}}{(2\pi)^d} \, p^2 \sum_{m>0} \min\{m,n\}$$
$$\times \left[\widetilde{\mathcal{D}}_p^{(3j)}(i\omega_m)\right]^2 \mathcal{D}_p(i\omega_{m+n}). \quad (49)$$

Here, again, the limit $n \to 0$ is assumed. Diagrammatically, this correction has the structure similar to diagrams shown in Fig. 2 in which the electron-electron interaction line should be substituted by the dynamical spin susceptibility. Performing analytic continuation to the real frequencies, $i\omega_n \to \omega + i0$, and taking the limit $\omega \to 0$, we obtain the following result:

$$\delta g_2^{\text{sf}} = \frac{64\Gamma^{\text{sf}}}{gd} \, \text{Im} \int \frac{d\boldsymbol{p}}{(2\pi)^d} \, p^2 \int d\Omega \, \partial_\Omega \left(\Omega \coth \frac{\Omega}{2T}\right)$$
$$\times \left[\widetilde{\mathcal{D}}_p^{(31),R}(\Omega)\right]^2 \mathcal{D}_p^R(\Omega). \quad (50)$$

Here, we took into account the equivalence of diffusion propagators with $j = 1$ and 2. It is worthwhile to mention that the correction δg^{sf} involves triplet diffusive modes with the total spin projection equal ± 1. We note that the correction (50) is similar to the quantum correction due to electron-paramagnon scattering [32].

B. Logarithmic corrections to conductance due to inelastic spin-flip scattering

As we mentioned above, in this paper we focus on the case $T \ll |b_i|$. Also, we are interested in corrections of the second order in J and in two-dimensional case. Then, expanding the correction (45) to the first order in γ^{sf}, we find

$$\delta g^{\text{AA}} = -\frac{1}{\pi} \sum_{j=0}^{3} \left[1 - \frac{1+\gamma_j}{\gamma_j} \ln(1+\gamma_j)\right] \ln \frac{1}{2\pi T \tau}$$
$$- \frac{\gamma^{\text{sf}}}{\pi} \left[\frac{1}{2} + \frac{1}{\gamma_s} - \frac{1+\gamma_s}{\gamma_s^2} \ln(1+\gamma_s)\right] \ln \frac{|b_i|}{2\pi T}$$
$$- \frac{2\gamma^{\text{sf}}}{\pi} \left[1 - \frac{1}{\gamma_t} \ln(1+\gamma_t)\right] \ln \frac{|b_i|}{2\pi T}$$
$$- \frac{\gamma^{\text{sf}}}{\pi} \left[\frac{3}{2} - \frac{1}{\gamma_t} + \frac{1-\gamma_t}{\gamma_t^2} \ln(1+\gamma_t)\right] \ln \frac{|b_i|}{2\pi T}. \quad (51)$$

Here, the first line represents the standard Altshuler-Aronov correction to the conductivity. Since the corresponding contribution exists for frequencies larger than $|b_i|$, the ultraviolet cutoff for this correction is inverse transport mean-free time $1/\tau$. The second line describes the correction due to the effect of the inelastic scattering off magnetic impurities on the singlet particle-hole channel. It is worthwhile to mention that the electron Zeeman splitting b_e does not affect this contribution such that the logarithmic temperature dependence survives at $T < |b_e|$.

The third line corresponds to the correction from triplet particle-hole channel with the total spin projection equal ± 1. The fourth line describes the correction from the triplet

particle-hole channel with the zero total spin projection. We note that the corrections proportional to γ^{sf} involve $\ln(|b_i|/2\pi T)$ and vanish in the absence of electron-electron interaction. We mention that in the standard Altshuler-Aronov correction [the first line of Eq. (51)], the singlet channel favors localization (since $\gamma_s \leq 0$) whereas the triplet channel favors antilocalization (since $\gamma_t \geq 0$) at low temperature. The corrections proportional to γ^{sf} work in the opposite direction, i.e., the presence of inelastic scattering off magnetic impurities decreases the effect of localization (antilocalization) in the singlet (triplet) channels, respectively. In the case of Coulomb interaction, one needs to set $\gamma_s = -1$ in Eq. (51).

The correction (48) does not produce logarithmic terms since the integral over frequencies is restricted by $|\Omega| \lesssim T$. The other correction due to inelastic scattering, Eq. (50), reads as

$$\delta g_2^{\text{sf}} = \frac{\gamma^{\text{sf}}}{\pi \gamma_t} \left[1 - \frac{1}{\gamma_t} \ln(1+\gamma_t)\right] \ln \frac{|b_i|}{2\pi T}. \quad (52)$$

We note that this correction is positive, i.e., works in favor of antilocalization at low temperatures. In the absence of electron-electron interaction, δg_2^{sf} is the only correction to the conductivity due to the inelastic scattering off magnetic impurities. It acquires the following form:

$$\delta g_2^{\text{sf}} \to \frac{\gamma^{\text{sf}}}{2\pi} \ln \frac{|b_i|}{2\pi T} = \frac{1}{2\pi^2(S+1)\tau_{s0}|b_i|} \ln \frac{|b_i|}{2\pi T}. \quad (53)$$

This quantum correction works in opposite direction with respect to the weak-localization correction.

V. DISCUSSIONS AND CONCLUSIONS

The temperature-dependent corrections to the conductivity discussed above were derived within the Born approximation for scattering off magnetic impurities. We remind that standard Kondo correction to the conductance in the clean system appears beyond Born approximation: in the third order in the exchange interaction. In the case $T \ll |b_i|$, this correction is temperature independent since the infrared cutoff for the Kondo logarithm is given by $|b_i|$ rather than T. In the disordered case, the inelastic corrections to the conductance studied previously [9,16,17] have been also of the third order in the exchange interaction. Since the corrections (51) and (52) are of the second order in the exchange interaction they are more important for small enough νJ. We note that the corrections of the third order in J for the case $|b_e| \ll T \ll |b_i|$ have not been computed yet. Therefore, we cannot compare the second and third order corrections quantitatively.

In the absence of electron-electron interaction, the structure of the correction (50) is similar to correction to the conductivity of disordered electron system in diffusive regime due to electron-electron interaction mediated by inelastic scattering off paramagnons [32]. The difference between paramagnons and magnetic impurities is in the form of the induced electron-electron interaction. In the latter case, it is short ranged and is independent of the transferred frequency for small frequencies. Away from the ferromagnetic quantum phase transition point, the induced electron-electron interaction due to paramagnons becomes also momentum and frequency independent, and,

consequently, results in the logarithmic-in-T correction to the conductivity in two dimensions [32].

In this paper, we consider the case of the electron system in the absence of spin-orbit splitting. If the spin-orbit splitting is present, then it will cut off the diffusion poles of triplet diffusons. Therefore, this results in suppression of the temperature dependence of the correction (52) and the contributions which involve γ_t in Eq. (51). The only temperature-dependent contribution due to inelastic scattering off magnetic impurities which remains in the case of spin-orbit coupling is the term in the second line of Eq. (51) which describes modification of the Altshuler-Aronov correction in the singlet channel.

Experimentally, the influence of magnetic impurities on the weak-localization correction via the dephasing time induced by the spin-flip scattering has been intensively studied in two-dimensional electron systems for many decades starting from seminal papers [33,34]. We are not aware of any systematic experimental studies of the effect of magnetic impurities on the Altshuler-Aronov correction to the conductivity in two-dimensional electron systems. In general, clear separation of the interference and interaction corrections is a difficult experimental problem (see for example, recent papers [35–37]). The effects described in this paper obviously complicate this formidable task.

The large g factor of magnetic impurity is natural to expect for impurities embedded into the electron system close to the Stoner instability. Since the Stoner instability corresponds to $F_0^\sigma = -1$, i.e., $\gamma_t = \infty$, the closeness to the Stoner instability implies large value of the interaction parameter in the triplet channel γ_t. In this case, the g factor of magnetic impurity can be estimated as $g_i \sim 2\nu J \gamma_t$ [38]. For example, the atom of Fe in Pd host acquires g factor of the order of 12 [39]. In such case, one would have approximately a decade between b_i and b_e. The other option could be to have usual g factor of magnetic impurity $g_i \approx 2$, but anomalously small g factor of electrons $g_e \approx 0$. For example, such situation can be achieved in 2D electron system based on GaAs/AlGaAs quantum wells with the width of the order of 60–70 Å [40].

We note that potential scattering affects also the spin susceptibility of a magnetic impurity resulting in additional (with respect to usual Kondo renormalization) temperature-dependent corrections [9,41,42]. Therefore, it would be interesting to consider the corrections to the spin susceptibility of a magnetic impurity in the case of different g factors and in the presence of electron-electron interaction.

To summarize, we studied the quantum corrections to the conductivity of the two-dimensional disordered interacting electron system in the diffusive regime due to inelastic scattering off magnetic impurities. Contrary to previous works, (i) we considered the case of different g factors for electrons and magnetic impurities $|g_e| \ll |g_i|$; (ii) we focused on the intermediate temperature range $|g_e|\mu_B H \ll T \ll |g_i|\mu_B H$; (iii) we took into account electron-electron interaction in the particle-hole channel. We found that within the Born approximation the inelastic scattering off magnetic impurities results in additional temperature-dependent correction to the conductivity [cf. Eq. (52)]. Also, the inelastic scattering modifies the Altshuler-Aronov corrections to the conductivity [cf. Eq. (51)]. Our predictions present a challenge for experimental studies of low-temperature transport in electron disordered systems with rare magnetic impurities.

[1] A. A. Abrikosov and L. P. Gor'kov, Contribution to the theory of superconducting alloys with paramagnetic impurities, Zh. Eksp. Teor. Fiz. **39**, 1781 (1960) [Sov. Phys.–JETP **12**, 1243 (1961)].

[2] P. Lee, Scaling studies of localization, J. Non-Cryst. Solids **35-36**, 21 (1980).

[3] K. B. Efetov, A. I. Larkin, and D. E. Khmelnitskii, Interaction of diffusion modes in the theory of localization, Zh. Eksp. Teor. Fiz. **79**, 1120 (1980) [Sov. Phys.–JETP **52**, 568 (1980)].

[4] J. Kondo, Resistance minimum in dilute magnetic alloys, Prog. Theor. Phys. **32**, 37 (1964).

[5] G. Zaránd, L. Borda, J. von Delft, and N. Andrei, Theory of Inelastic Scattering from Magnetic Impurities, Phys. Rev. Lett. **93**, 107204 (2004).

[6] M. Garst, P. Wölfle, L. Borda, J. von Delft, and L. Glazman, Energy-resolved inelastic electron scattering off a magnetic impurity, Phys. Rev. B **72**, 205125 (2005).

[7] L. Borda, L. Fritz, N. Andrei, and G. Zaránd, Theory of inelastic scattering from quantum impurities, Phys. Rev. B **75**, 235112 (2007).

[8] M. G. Vavilov and L. I. Glazman, Conductance of mesoscopic systems with magnetic impurities, Phys. Rev. B **67**, 115310 (2003).

[9] F. J. Ohkawa, H. Fukuyama, and K. Yosida, Kondo effect in disordered two-dimensional systems, J. Phys. Soc. Jpn. **52**, 1701 (1983).

[10] K. Vladár and G. T. Zimányi, J. Phys. C.: Solid State Phys. **18**, 3739 (1985).

[11] S. Kettemann and E. R. Mucciolo, Disorder-quenched Kondo effect in mesoscopic electronic systems, Phys. Rev. B **75**, 184407 (2007).

[12] S. Kettemann and E. R. Mucciolo, Free magnetic moments in disordered systems, JETP Lett. **83**, 240 (2006).

[13] T. Micklitz, A. Altland, T. A. Costi, and A. Rosch, Universal Dephasing Rate Due to Diluted Kondo Impurities, Phys. Rev. Lett. **96**, 226601 (2006).

[14] T. Micklitz, T. A. Costi, and A. Rosch, Magnetic field dependence of dephasing rate due to diluted kondo impurities, Phys. Rev. B **75**, 054406 (2007).

[15] O. Kashuba, L. I. Glazman, and V. I. Fal'ko, Influence of spin dynamics of defects on weak localization in paramagnetic two-dimensional metals, Phys. Rev. B **93**, 045206 (2016).

[16] F. J. Ohkawa and H. Fukuyama, Kondo effect and magnetoresistance in weakly localized regime, J. Phys. Soc. Jpn. **53**, 2640 (1984).

[17] S. Suga, H. Kasai, and A. Okiji, Effect of a magnetic impurity on the conductivity in the weakly localized regime, J. Phys. Soc. Jpn. **55**, 2515 (1986).

[18] A. M. Finkelstein, in *Electron Liquid in Disordered Conductors*, edited by I. M. Khalatnikov, Soviet Scientific Reviews, Vol. 14 (Harwood Academic, Reading, UK, 1990).

[19] D. Belitz and T. R. Kirkpatrick, The Anderson-Mott transition, Rev. Mod. Phys. **66**, 261 (1994).

[20] A. M. M. Pruisken, M. A. Baranov, and B. Škorić, (Mis-) handling gauge invariance in the theory of the quantum hall effect. I. Unifying action and the $\nu = 1/2$ state, Phys. Rev. B **60**, 16807 (1999).

[21] A. M. Finkel'stein, On the frequency and temperature dependence of the conductivity near a metal-insulator transition, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 436 (1983) [JETP Lett. **37**, 517 (1983)].

[22] A. M. Finkel'stein, Influence of coulomb interaction on the properties of disordered metals, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys.–JETP **57**, 97 (1983)].

[23] A. M. Finkel'stein, Metal-insulator transition in a disordered system, Zh. Eksp. Teor. Fiz. **86**, 367 (1984) [Sov. Phys.–JETP **59**, 212 (1984)].

[24] F. M. Marchetti and B. D. Simons, Tail states in disordered superconductors with magnetic impurities: the unitarity limit, J. Phys. A: Math. Gen. **35**, 4201 (2002).

[25] R. Friedberg and J. M. Luttinger, Density of electronic energy levels in disordered systems, Phys. Rev. B **12**, 4460 (1975).

[26] L. De Leo A. Schiller, Phase diagram of the anisotropic multichannel Kondo Hamiltonian revisited, Phys. Rev. B **77**, 075114 (2008).

[27] R. M. Konik, H. Saleur, and A. W. W. Ludwig, Interplay of the scaling limit and the renormalization group: Implications for symmetry restoration, Phys. Rev. B **66**, 075105 (2002).

[28] B. Z. Spivak and A. Yu. Zyuzin, Friedel oscillations and Ruderman-Kittel interaction in disordered conductors, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 185 (1986) [JETP Lett. **43**, 234 (1986)].

[29] F. Wegner, The mobility edge problem: Continuous symmetry and a conjecture, Z. Phys.B: Condens. Matter **35**, 207 (1979).

[30] L. P. Gor'kov, A. I. Larkin, and D. E. Khmelnitskii, Particle conductivity in a two-dimensional random potential, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].

[31] B. L. Al'tshuler and A. G. Aronov, Contribution to the theory of disordered metals in strongly doped semiconductors, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys.–JETP **50**, 968 (1979)].

[32] I. Paul, Interaction correction of conductivity near a ferromagnetic quantum critical point, Phys. Rev. B **77**, 224418 (2008).

[33] R. P. Peters, G. Bergmann, and R. M. Mueller, Kondo Maximum of Magnetic Scattering, Phys. Rev. Lett. **58**, 1964 (1987).

[34] C. Van Haesendonck, J. Vranken, and Y. Bruynseraede, Resonant Kondo Scattering of Weakly Localized Electrons, Phys. Rev. Lett. **58**, 1968 (1987)

[35] G. M. Minkov, A. V. Germanenko, O. E. Rut, A. A. Sherstobitov, and B. N. Zvonkov, Disorder and temperature renormalization of interaction contribution to the conductivity in two-dimensional $In_xGa_{1-x}As$ electron systems, Phys. Rev. B **79**, 235335 (2009).

[36] G. M. Minkov, A. V. Germanenko, O. E. Rut, A. A. Sherstobitov, A. K. Bakarov, and D. V. Dmitriev, Interaction correction to conductivity of $Al_xGa_{1-x}As/GaAs$ double quantum well heterostructures near the balance, Phys. Rev. B **84**, 075337 (2011).

[37] G. M. Minkov, A. V. Germanenko, O. E. Rut, and A. A. Sherstobitov, Interaction correction to the conductivity of two-dimensional electron gas in $In_xGa_{1-x}As/InP$ quantum well structure with strong spin-orbit coupling, Phys. Rev. B **85**, 125303 (2012).

[38] A. I. Larkin and V. I. Melnikov, Magnetic impurities in almost magnetic metal, Zh. Eksp. Teor. Fiz. **61**, 1231 (1971) [Sov. Phys.–JETP **34**, 656 (1972)].

[39] J. Crangle and W. R. Scott, Dilute ferromagnetic alloys, J. Appl. Phys. **36**, 921 (1965).

[40] R. Winkler, *Spin-orbit Coupling Effects in Two-dimensional Electron and Hole Systems* (Springer, Berlin, 2003).

[41] A. G. Aronov and A. Yu. Zyuzin, Thermodynamics of electrons in disordered conductors with kondo impurities, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 369 (1984) [JETP Lett. **39**, 444 (1984)].

[42] S. Suga, H. Kasai, and A. Okiji, Susceptibility of a magnetic impurity in weakly localized regime, J. Phys. Soc. Jpn. **56**, 4522 (1987).