Piezoelectric Compensation of Structural Damping in Metamaterial Beams
Stability and Performance Analysis

Alimohammadi, H.; Vassiljeva, K.; HosseinNia, S. H.; Ellervee, P.; Petlenkov, E.

DOI
10.1117/12.3024120

Publication date
2024

Document Version
Final published version

Published in
Active and Passive Smart Structures and Integrated Systems XVIII

Piezoelectric Compensation of Structural Damping in Metamaterial Beams: Stability and Performance Analysis

H. Alimohammadi[a], K. Vassiljeva[a], S. H. HosseinNia[b], P. Ellervee[a], and E. Petlenkov[a]

[a]Department of Computer Systems, Tallinn University of Technology, Tallinn, Estonia
[b]Department of Precision and Microsystems Engineering, Delft University of Technology, Delft, The Netherlands

ABSTRACT

This paper examines the stability implications of integrating piezoelectric actuators into metamaterial beams, focusing on the compensation of structural damping and its effect on the system's dynamic performance. Metamaterials, characterized by their unique bandgap properties, offer potential in various engineering applications, including vibration control and energy harvesting. However, structural damping inherent in such systems can degrade these properties, prompting the use of piezoelectric actuators as a compensatory mechanism. Through a distributed parameter model and modal analysis, this study explores the temporal and spatial dynamics of the metamaterial beam and investigates how piezoelectric actuation influences the natural frequencies and mode shapes, with a particular emphasis on stability thresholds. Employing root locus analysis, the paper visualizes the transition of system stability across different levels of actuation voltage, highlighting the delicate balance between enhanced performance and stability. The findings delineate a clear operational voltage range, within which piezoelectric actuation improves bandgap properties without compromising system stability.

Keywords: Damping Compensation, Stability Analysis, Piezoelectric Actuation, Distributed parameter model of metastructures, Bandgap Engineering

1. INTRODUCTION

In this study, we delve into the stability analysis of metamaterial beams incorporating piezoelectric actuators, a subject that sits at the intersection of advanced materials science and dynamic system control. The crux of this investigation lies in understanding how the integration of active control elements affects the overall stability of these sophisticated structures.

Metamaterial beams, known for their unique mechanical properties and dynamic behaviors, present a complex challenge when augmented with piezoelectric actuators. These actuators, capable of precise manipulation of the beam's response, introduce a new dimension to the system's dynamics. Thus, a rigorous stability analysis becomes paramount to ensure that such modifications do not lead to detrimental outcomes like system instability or resonance failures.

Our approach begins with constructing a distributed parameter model for the metamaterial beam. This model, essential for capturing the continuous spatial variation of mechanical properties along the beam, is governed by a nuanced set of partial differential equations. These equations are the backbone of our analysis, allowing us to explore the temporal and spatial dynamics of the beam in detail.

We employ modal analysis to uncover the natural frequencies and mode shapes of the beam. Understanding these characteristics is crucial for predicting the emergence of bandgaps—a phenomenon where certain frequency ranges are blocked or altered by the structure of the metamaterial. Modal analysis also aids in assessing how active control via piezoelectric actuators can be used to tailor these bandgaps effectively.

To visualize and understand the implications of varying control parameters, specifically the applied voltage v_a, we utilize root locus analysis to track the movement of the poles in the complex plane as the applied voltage changes, offering a clear depiction of the stability landscape of the system under different operational scenarios.

Further author information: (Send correspondence to E. Petlenkov)
E.P.: E-mail: eduard.petlenkov@taltech.ee, Telephone: 1 234 567 890

Active and Passive Smart Structures and Integrated Systems XVIII, edited by Serife Tol, Guoliang Huang, Xiaopeng Li, Mostafa A. Nouh, Shima Shahab, Jinkyu Yang, Proc. of SPIE Vol. 12946, 129460J · © 2024 SPIE · 0277-786X · doi: 10.1117/12.3024120

A focal point of our study is the exploration of the potential risks associated with using piezoelectric actuation to counteract damping effects. We rigorously investigate whether this approach introduces new poles or zeros in the transfer function, potentially leading to instability. Through comprehensive simulations, we aim to establish stability thresholds, providing a concrete framework for evaluating the system's resilience to various levels of piezoelectric actuation.

This research is not just an academic exercise but a practical guide for engineers and designers. By identifying and understanding the boundaries of stability in such complex systems, we aim to inform the design and operational protocols of metamaterial beams, ensuring they achieve their intended dynamic performance without compromising their structural integrity.

2. LITERATURE REVIEW AND BACKGROUND

The study of bandgaps in metamaterials has garnered considerable attention from engineers and researchers due to its potential in various practical applications. This Literature Review and Background section will delve into the existing research and identify the gap that our study aims to address.

Metamaterials, known for their extraordinary mechanical properties, have been a focal point in materials science research. One of the key features of these materials is their ability to exhibit bandgaps - frequency ranges in which wave propagation is significantly reduced or entirely inhibited. The control over bandgap characteristics, such as width and depth, has important implications for noise reduction, vibration control, and energy harvesting applications.[1]

In practical scenarios, the effectiveness of these bandgaps is often compromised due to the inherent damping characteristics of the structure. Damping, a natural phenomenon in materials, leads to the dissipation of energy, thereby affecting the depth of the bandgaps and diminishing their effectiveness.[2]

To counteract this issue, one approach has been the integration of piezoelectric actuators. These actuators, leveraging the piezoelectric effect, are employed to enhance the resonator's performance, thereby compensating for the damping and potentially restoring the depth of the.[3,4]

However, the introduction of piezoelectric actuators alters the dynamic properties of the system, potentially leading to instability. The interaction between the piezoelectric elements and the mechanical structure can introduce changes in the system's natural frequencies and mode shapes, which in turn might affect its stability.[5,6]

While previous studies have explored the influence of piezoelectric actuation on metamaterial bandgaps, there is a lack of comprehensive understanding of how this actuation affects the stability of the system. This leads us to the primary research question of our study: How does the use of piezoelectric actuators to compensate for damping in metamaterial beams influence the stability of the system, and what are the implications for the practical application of these enhanced bandgaps?

Our research aims to investigate this stability, or potential instability, introduced by piezoelectric actuation in metamaterial beams. By addressing this question, we seek to bridge the gap in current understanding and provide insights that are crucial for the practical application and optimization of piezoelectrically enhanced metamaterials.

3. DISTRIBUTED PARAMETER MODEL USING MODAL ANALYSIS

In this study, a distributed parameter model is employed to analyze the behavior of a metamaterial system. This model is crucial for understanding spatial variations in physical properties and their impact on wave propagation. Modal analysis is utilized to determine the natural frequencies and mode shapes of the metastructure, which are essential for identifying and manipulating bandgaps.

The locally resonant metastructure, incorporating piezoelectric actuators, is illustrated in Fig. 1. This diagram emphasizes the deliberate placement of piezoelectric elements crucial for the actuation approach. Employing a distributed parameter model aids in formulating partial differential equations that delineate the system's dynamics, which are subsequently discretized and numerically analyzed to determine the metastructure's modal properties.

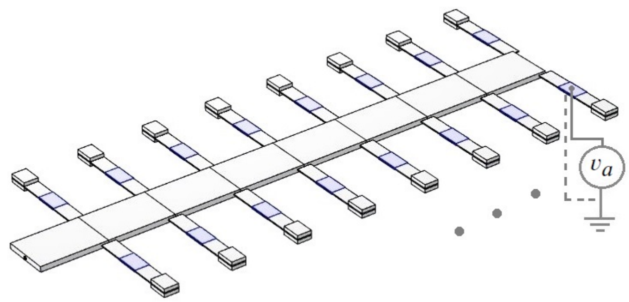

Figure 1. Schematic of a locally resonant metastructure with piezoelectric actuators. Features cantilever beams with tip masses as resonators and piezoelectric elements (with voltage v_{p_r}) bonded to alter stiffness dynamically.

The dynamics of the system are captured by a set of partial differential equations as follows:[7]

$$\mathcal{L}w(x,t) + \mathcal{C}\frac{\partial w(x,t)}{\partial t} + \mathcal{M}\frac{\partial^2 w(x,t)}{\partial t^2} - \sum_{r=1}^{N_r}\left(k_r z_r(t) + c_r\frac{\partial z_r(t)}{\partial t}\right)\delta\left(x - x_r\right) = \mathcal{F}_{b_m}(x,t) \tag{1}$$

$$m_r\frac{\partial^2 z_r(t)}{\partial t^2} + c_r\frac{\partial z_r(t)}{\partial t} + k_r z_r(t) + m_r\frac{\partial^2 w\left(x_r,t\right)}{\partial t^2} - \vartheta_{p,r}v_{p,r}(t) = \mathcal{F}_{b_r}(t) \tag{2}$$

$$C^*_{p,r}\frac{\partial v_{p,r}(t)}{\partial t} + \vartheta_{p,r}z_r(t) = 0 \tag{3}$$

Equations (1), (2), and (3) in the study address the dynamics of a metamaterial beam system with piezoelectric actuators. Equation (1) focuses on the primary beam's displacement and its interaction with resonators, while Equation (2) details the resonators' motion and the influence of the piezoelectric actuator force. Equation (3) links the actuator voltage to the resonator motion through capacitance and a coupling coefficient.

The parameters $\mathcal{L}, \mathcal{C}$, and $\mathcal{M}$ represent the system's flexural rigidity, damping, and mass distribution. The piezoelectric properties defined as electromechanical coupling (ϑ_r), capacitance ($C^*_{p,r}$), and the shunt circuit admittance (G_r) on each resonator. The functions $w(x,t)$ and $z_r(t)$ describe the main structure's and resonators' transverse vibrations, respectively. The Kronecker delta function $\delta\left(x - x_r\right)$ denotes resonator locations, with N_r being the total number.

The system's boundary conditions are determined by linear homogeneous differential operators, and damping is often modeled based on the mass and stiffness matrices. This approach simplifies analysis using common mode shapes for both damped and undamped scenarios, albeit as an approximation.

Orthogonality conditions involving these operators, as indicated by δ_{mn} in Equations (4) and (5), confirm that the mode shapes are orthogonal in terms of the system's stiffness and mass. Proportional damping links damping characteristics to these orthogonal eigenfunctions, defining the damping ratio ζ_m for each mode as a linear combination of the system's mass and stiffness.

$$\int_D \phi_m(x)\mathcal{M}\left[\phi_n(x)\right]dx = \delta_{mn} \tag{4}$$

and

$$\int_D \phi_m(x)\mathcal{L}\left[\phi_n(x)\right]dx = \delta_{mn}\omega_m^2 \tag{5}$$

In modal decomposition, the deflection of an Euler beam over the domain $D = [0, L]$ is represented as a sum of modal contributions:

$$w(x,t)=\sum_{m=1}^{N_m}\phi_m(x)z_m(t), \tag{6}$$

The deflection of an Euler beam in modal decomposition is expressed through a series of modal contributions, where $\phi_m(x)$ illustrates the spatial pattern of each mode shape, and $z_m(t)$ indicates the time-dependent development for each mode. This method effectively reduces the intricate dynamics of a flexible beam with embedded resonators to simpler, more understandable elements.

The modal expansion from Eq. (6) is substituted into Eqs. (1) and (2), using the orthogonality conditions from Eqs. (4) and (5). By reorganizing these equations and applying Laplace transforms (assuming no initial conditions), a set of linear equations in the Laplace domain is derived. This process leads to the formulation of a transfer function for the displacement of the resonator.

In this framework, the resonator masses (m_r) are proportionally defined relative to the mass distribution of the structure at resonator attachment points, scaled by a mass ratio (μ). This ratio represents the total mass of the resonators compared to the base structure's mass. The relationship is expressed as $m_r=\mu m\left(x_r\right)dx_r$, ensuring that the resonator masses are integrally linked to the structure's mass distribution.

For systems with a large number of resonators, the expression involving the sum of resonator masses can be approximated by an integral over the structure's length. This approximation aligns with the orthogonality condition represented by the Kronecker delta function δ_{mp}.

Using the defined terms and after rearranging the equations mathematically, the transfer function that describes the displacement of the resonator is constructed as follows:

$$Z_r(s)=\frac{-s^2w_b-\sum_{m=1}^{N_m}Z_m(s)s^2\phi_m\left(x_r\right)}{s^2+2\zeta_r\omega_r s+\omega_r^2+\frac{k_e^*\omega_r^2}{s}},\quad r=1,2,\ldots,N_r \tag{7}$$

where $Z_r(s)$ signifies the Laplace transform of the displacement response for the r-th resonator in a metamaterial system, where ω_b represents the base excitation. The term $\phi_m\left(x_r\right)$ indicates the mode shape at the location x_r of the resonator, N_r is the total number of resonators, ζ_r is the damping ratio signifying energy dissipation efficiency, and ω_r is the resonator's natural frequency.

The effective electromechanical coupling of the resonator, denoted as k_e^*, is defined by $k_e^*=\frac{\vartheta_{p,r}^2}{C_{p,r}^*k_r}$. When considering the voltage source as the input, this effective stiffness k_e^* can be expressed through the relationship $k_e^*=\alpha\omega_r v_0=v_a$, , where ω_r represents the natural frequency of the resonator, v_0 is the voltage applied to the piezoelectric component, and α is an empirical constant with the unit Farads per Coulomb (F/C). After some mathematical manipulation, the transfer function for the displacement of the m-th mode of the structure relative to the excitation force on the same mode is formulated, simplified, and described by Eq. (8). This equation includes terms for modal frequency, damping, and the interaction between the structure's dynamics and the resonator's properties, including the applied voltage.

$$\frac{Z_m(s)}{Q_{b_m}(s)}=\frac{1}{s^2\left(1+\frac{\mu\left(2\zeta_r\omega_r s+\omega_r^2\right)}{s^2+2\zeta_r\omega_r s+\omega_r^2\left(1+\frac{v_a}{s}\right)}\right)+2\zeta_m\omega_m s+\omega_m^2},\quad m=1,2,\ldots,N_m \tag{8}$$

Here, $Z_m(s)$ and $Q_{b_m}(s)$ represent the Laplace transforms of the displacement response and the external force applied to the m-th mode, respectively. ω_r and ω_m are the natural frequencies of the resonator and the m-th mode of the structure. The damping ratios, ζ_r for the resonator and ζ_m for the structure's mode, describe the rate of oscillation decay due to damping. v_a is the voltage applied to the piezoelectric elements, influencing the system's response. N_m indicates the number of modes in the analysis.

4. STABILITY ANALYSIS

The theoretical framework for the stability analysis of a piezoelectrically actuated metamaterial beam is rooted in a distributed parameter model and modal analysis. These models illuminate the system's natural frequencies and mode shapes, crucial for evaluating dynamic manipulation and potential bandgaps. The stability is assessed by the poles' locations of Eq. (8) in the complex plane, with stability indicated by poles residing in the left-half plane.

Root locus analysis is used to visualize how the poles shift with varying applied voltages v_a, offering a graphical perspective of stability regions against the control parameter changes. This analysis becomes particularly significant when examining the effects of piezoelectric actuation meant to counteract damping. The introduction of an additional s term in the transfer function's denominator, indicative of a new pole, poses a critical inquiry into whether such compensation might inadvertently lead to instability. Further, a specific stability criterion is set where the system is considered stable if the weighted average of the real parts of the poles is less than the threshold of $w_a(L)/w_b = 0.1$. This benchmark, both theoretical and empirical, allows for a pragmatic assessment of stability, taking into account the practical operational limits.

The methodological framework outlined in this study serves as the foundation for the subsequent numerical simulations aimed at exploring the stability characteristics of the metamaterial beam under various actuation strategies. The outcomes of this theoretical analysis will inform the design decisions and operational procedures to ensure that the system can achieve the desired dynamic behavior without succumbing to instability.

5. RESULTS AND DISCUSSION

This section presents a numerical analysis using defined geometric and material properties of a rectangular beam, detailed in Table 1. This table provides the parameters for the simulations and calculations, offering a precise depiction of the beam's properties for the study.

Table 1. Geometric and material properties of the studied rectangular aluminum beam

Parameter	Value	Parameter	Value
L_m	0.3 m	m_r	17 g
w_m	40 mm	k_r	9 kN/m
h_m	3 mm	ζ_r	0.01
ρ_m	2700 kg/m^3	N_m	8
E_m	69.5 GP	N_r	8
ζ_m	0.01		

The root locus plot in Fig. 2 demonstrates the stability of a flexible structure with varying modes. It illustrates the pole trajectories for a system considering one mode versus eight modes. As the plot shows, including more modes in the analysis captures a more complex stability behavior, indicated by the poles' paths moving into the left-half plane, which suggests increased stability. This highlights the critical importance of accounting for multiple modes in flexible structures to ensure a robust stability analysis and accurate prediction of the system's dynamic response. Neglecting higher modes could omit essential details, potentially compromising the effectiveness of stability enhancement strategies like the implementation of deep bandgaps with controlled damping.

The root locus plot depicted in Fig. 3 provides insight into the dynamic stability of a metamaterial beam with piezoelectric actuators. As shown, varying the applied voltage v_p alters the pole positions within the system, which can lead to instability. The plot reveals that the system's poles respond differently to changes in v_p, with some poles moving toward the instability region (the right-half of the s-plane) as v_p increases. This differential responsiveness underscores the nuanced control that piezoelectric actuation can exert on the system.

By finely adjusting v_p, it is possible to tune the dynamic stability of the beam, which is a critical aspect of enhancing the performance of metamaterials. The ability to control stability through piezoelectric actuation is particularly valuable in applications where material properties must be precisely managed to achieve desired dynamic behavior. Conversely, if the approach involves using piezoelectric actuators to offset the damping effect

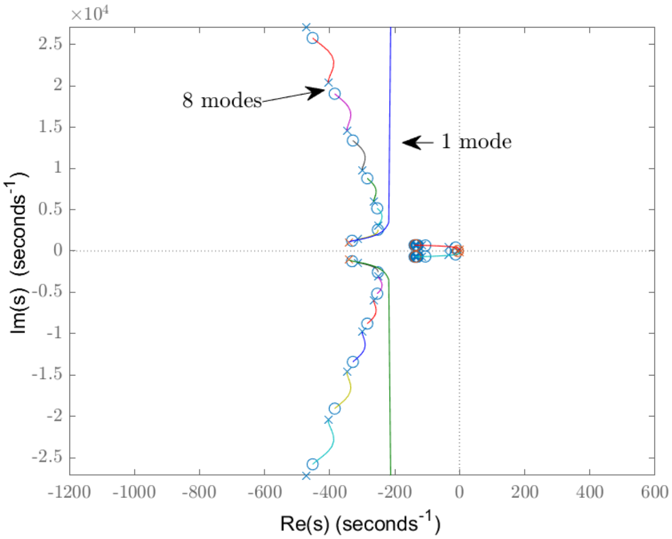

Figure 2. Root locus plot illustrating the pole trajectories for one mode versus eight modes in a piezoelectrically actuated metamaterial beam, highlighting the impact of mode inclusion on system stability. The sourced parameters from Table 1.

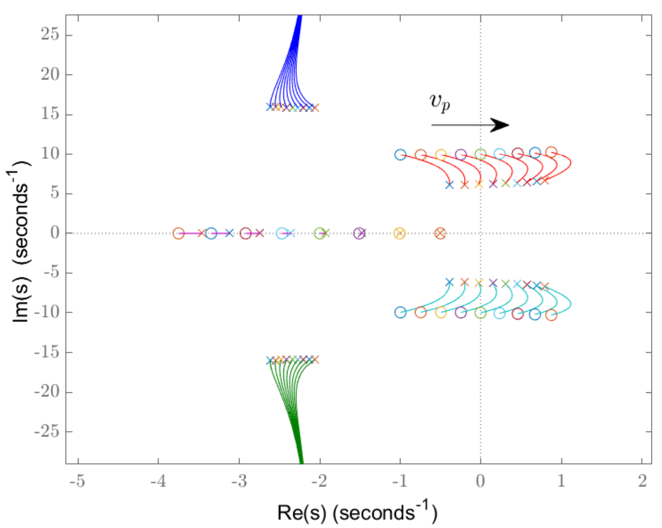

Figure 3. Root locus plot demonstrating the stabilization effect of increasing piezoelectric actuation voltage v_p on stability of metamaterial beam system. System parameters include a mass ratio $\mu = 1$, resonator damping ratio $\zeta_r = 0.1$, resonator natural frequency $\omega_r = 10$, structural damping ratio $\zeta_m = 0.1$, and the first structural natural frequency $\omega_{m_1} = 10$. The plot traces the pole movement across modes, showing enhanced stability with higher v_p, pertinent to the precise dynamic control of metamaterials.

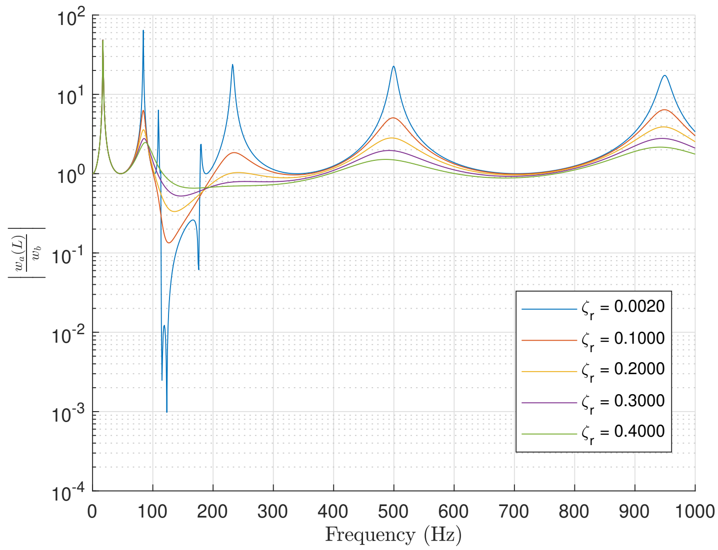

Figure 4. This figure illustrates the effect of varying resonator damping ratios ζ_r on the bandgap properties of the metastructure, considering a constant metamaterial damping ratio of $\zeta_m = 0.01$ in the frequency response analysis. The sourced parameters from Table 1.

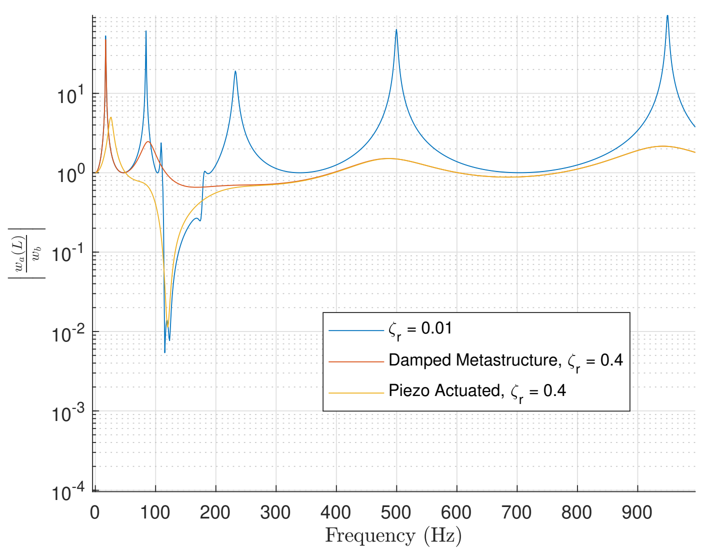

Figure 5. This figure illustrates the effect of varying resonator damping ratios ζ_r on the bandgap properties of the metastructure, considering a constant metamaterial damping ratio of $\zeta_m = 0.01$ in the frequency response analysis. The sourced parameters from Table 1.

and thus deepen the bandgap, the figure reveals that such use of piezoelectric actuators to counterbalance the damping in a metamaterial beam might result in instability. This is shown in Fig. 4, where it's clear that altering the resonator's structural damping significantly influences the behavior of the bandgap.

Fig. 5 graphically demonstrates the impact of high damping $\zeta_r = 0.4$ in a practical application, where it significantly diminishes the amplitude of vibrations across the frequency spectrum and effectively suppresses resonant peaks. When piezoelectric actuation is applied in conjunction with this high damping rate, there is a notable further reduction in peak amplitudes. Importantly, it also compensates for the reduced bandgap effect caused by increased damping and modifies the resonant frequencies, showcasing the capacity for active vibration

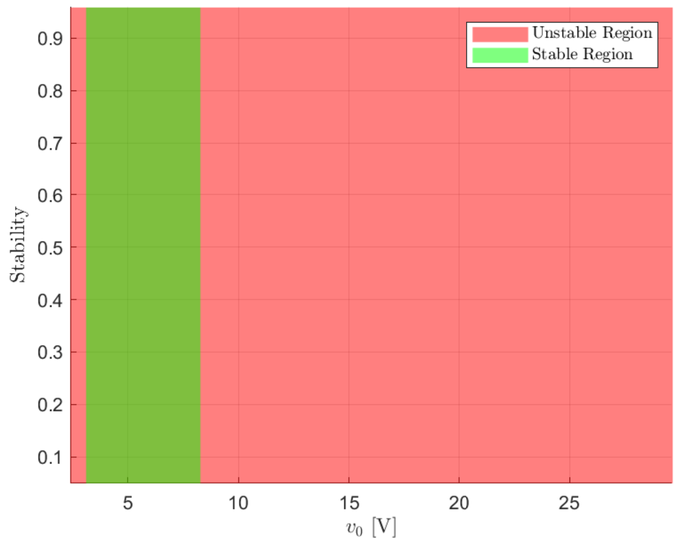

Figure 6. Stability landscape of a metamaterial beam with piezoelectric actuation, showcasing the stable (green) and unstable (red) voltage regions. Stability is quantified by the defined threshold, for balancing piezoelectric damping compensation and maintaining system stability

control. This comparative analysis confirms that piezoelectric actuation not only surpasses passive damping in vibration reduction but also implies that it can adjust the system's dynamic properties, such as stiffness or mass distribution. Consequently, the utilization of piezoelectric actuators in metamaterial beams is a promising strategy for vibration isolation and noise reduction, offering a dynamically tunable solution to control undesirable vibrations.

The chart in Fig. 6 depicts how the stability of a metamaterial beam responds to different applied voltages to piezoelectric actuators. Stability is defined by the weighted average of the pole real parts, with a threshold set at $w_a(L)/w_b = 0.1$. The green area represents stable voltage levels, while the red indicates voltages that cause instability. Maintaining applied voltage within the green region is essential to ensure stable operation while enhancing the bandgap depth using piezoelectric damping.

Fig. 7 illustrates the relationship between the cost function and the applied voltage v_0 in a piezoelectrically actuated system. It shows a rapid decline in cost as v_0 increases, which then levels off, indicating an optimal voltage range for system operation. This data is critical for determining the most efficient voltage for system performance, highlighting the effectiveness of piezoelectric actuation within a specific voltage range.

The 3D plot in Fig. 8 illustrates the effect of the applied voltage v_0 on the transmittance response of a metamaterial beam's tip displacement with length L to the base excitation displacement over a range of frequencies. The plot's valley depth signifies a pronounced attenuation, indicating the optimization of v_0 enhances the bandgap due to compensated damping effects in the presence of over-damping. The response surface reveals that v_0 not only modulates the bandgap's depth but also its frequency position, which is critical for applications that require precise control of vibrational characteristics.

Fig. 9 illustrates the variation in the maximum ratio of tip displacement $w_a(L)$ to base excitation displacement w_b as a function of applied voltage v_0. The plot shows a notable decrease in the displacement ratio when v_0 rises from 0 to about 5 volts, indicating a significant improvement in the beam's response, likely due to the influence of actuation on the system dynamics. Beyond 5 volts, the ratio levels off and increases gradually, implying limited benefits from further voltage increases. This behavior aids in determining the optimal operational voltage range where the piezoelectric actuation has the most significant impact on the beam's vibrational characteristics. The sharp peaks indicate resonant frequencies where the system's response is highly sensitive to changes in v_0. In practice, such a plot is essential for system designers to set appropriate actuation voltages that balance

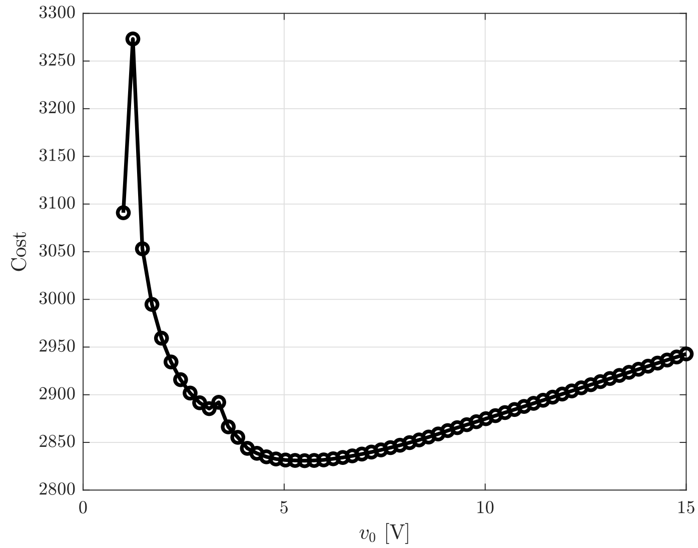

Figure 7. The cost function's variation with applied voltage v_0, displaying a sharp decrease and subsequent stabilization. This identifies an optimal voltage range for the piezoelectric actuation system, beyond which no significant cost benefit is observed.

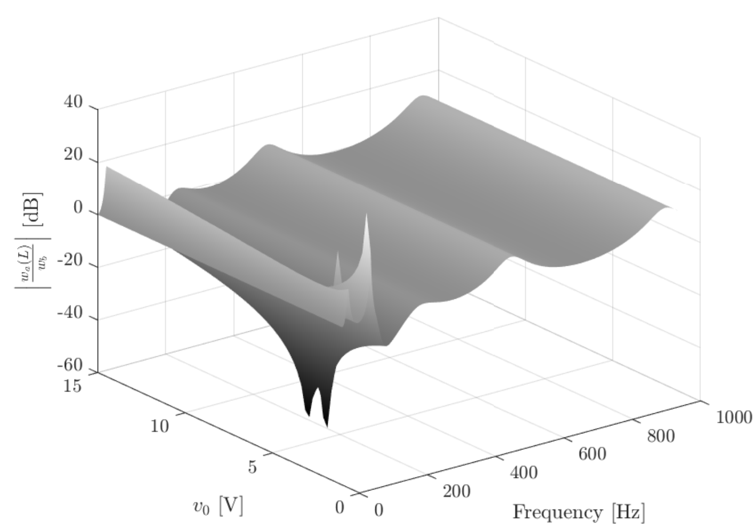

Figure 8. Transmittance response of the beam's tip displacement to base excitation across frequencies, depicting the effect of applied voltage v_0. The pronounced valley indicates an optimized bandgap within the structure, adjusted by v_0 to mitigate overdamping effects. This plot is key for identifying optimal v_0 settings to control the metamaterial's vibrational characteristics effectively.

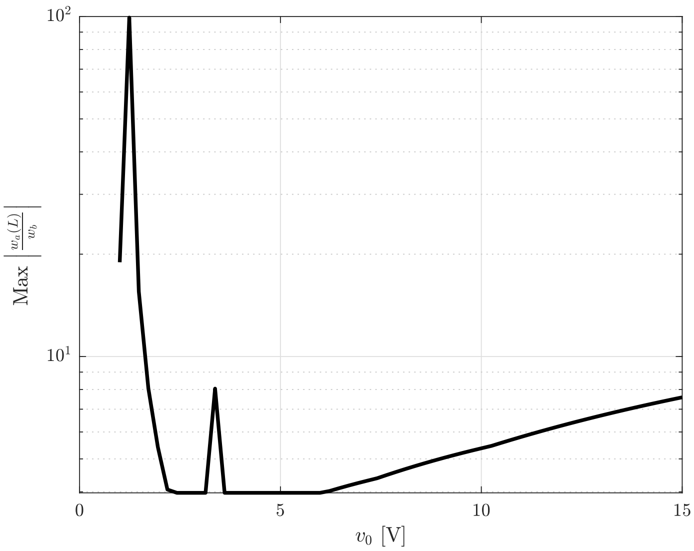

Figure 9. Variation of maximum $|w_a(L)/w_b|$ with applied voltage v_0, highlighting resonant frequencies and the impact of piezoelectric actuation on the beam's vibrational response.

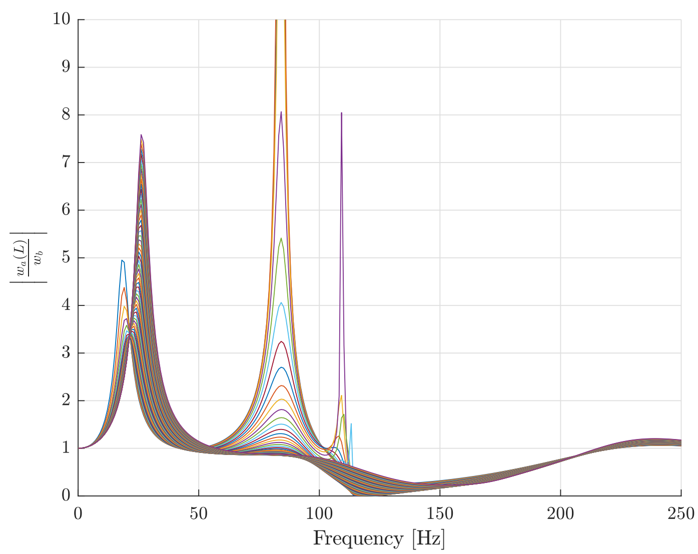

Figure 10. Transmittance variation with frequency for different applied voltages v_0 on a piezoelectrically actuated damped system, illustrating the modulation of resonant peaks and the adjustment of bandgap frequencies due to piezoelectric effects.

performance with energy efficiency and ensure the structural integrity of the beam under dynamic loading conditions.

Fig. 10 shows transmittance responses for varying applied voltages v_0, demonstrating the tunability of the system's resonant frequencies. The curves shift in response to different v_0 levels, which is indicative of the piezoelectric effect's impact on the damped system. Such control is essential for optimizing dynamic behaviors like vibration suppression and bandgap frequency adjustments. The visualization captures the nuanced influence of voltage on the system's ability to manage and refine the depth of its bandgap in response to external damping forces.

6. CONCLUSION

This research investigated the dynamic stability of a metamaterial beam system with integrated piezoelectric actuators, designed to compensate for the structural damping of resonators. Central to this investigation was the implementation of piezoelectric actuation as a means to counteract damping effects, represented by the introduction of an additional pole term in the denominator of the system's transfer function (Eq. (8)).

The study's findings indicate that while piezoelectric actuation can effectively compensate for damping and enhance the system's performance, it introduces a complexity that can lead to instability. The root locus analyses provided a clear depiction of how the system's poles migrate with varying applied voltage v_a, highlighting the delicate balance between achieving desired damping compensation and maintaining system stability.

Through numerical simulations, we delineated the stability regions, revealing that there is an optimal range of applied voltages within which the system can operate stably while benefiting from the damping compensation provided by the piezoelectric actuators. However, surpassing this range may lead to instability, as evidenced by the poles' transition across the critical boundary in the complex plane.

In conclusion, the study confirms that piezoelectric actuation, when carefully applied, is a potent tool for managing the dynamic response of metamaterial beams. The critical contribution of this work lies in the identification of stability thresholds for applied voltages, enabling the use of piezoelectric actuators to control damping without compromising the system's stability.

Acknowledgments

This paper is supported by the European Union's HORIZON Research and Innovation Programme under grant agreement No 101120657, project ENFIELD (European Lighthouse to Manifest Trustworthy and Green AI). It was also supported by the Estonian Research Council grant PRG658.

REFERENCES

[1] Cummer, S. A., Christensen, J., and Alù, A., "Controlling sound with acoustic metamaterials," *Nature Reviews Materials* **1**(3), 1–13 (2016).

[2] Smith, D. R., Pendry, J. B., and Wiltshire, M. C., "Metamaterials and negative refractive index," *science* **305**(5685), 788–792 (2004).

[3] Erturk, A. and Inman, D. J., [*Piezoelectric energy harvesting*], John Wiley & Sons (2011).

[4] Sirohi, J. and Chopra, I., "Fundamental understanding of piezoelectric strain sensors," *Journal of intelligent material systems and structures* **11**(4), 246–257 (2000).

[5] Li, F., Zhang, C., and Liu, C., "Active tuning of vibration and wave propagation in elastic beams with periodically placed piezoelectric actuator/sensor pairs," *Journal of Sound and Vibration* **393**, 14–29 (2017).

[6] Saravana Jothi, N. and Hunt, A., "Active mechanical metamaterial with embedded piezoelectric actuation," *APL Materials* **10**(9) (2022).

[7] Mead, D., "Leonard meirovitch, elements of vibration analysis , mcgraw-hill book company, new york (1986).," *Journal of Sound Vibration* **117**(3), 603–604 (1987).