Topology Optimization of Launcher Structures under Multiple Critical Load Cases

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by

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Abstract

The launch vehicle industry is undergoing a major shift as new commercial organizations are staking claim to large market shares with bold technologies. To stay competitive in this new era, launch vehicle companies must develop new technologies to reduce launch costs while serving the growing demand for space accessibility. A possible solution is to use additive manufacturing in combination with topology optimization to relieve manufacturing and assembly burdens in addition to reducing launch mass. Topology optimization is a mathematical method that optimizes the distribution of material in a design domain for a given combination of loads, boundary conditions and constraints with the objective of maximizing mechanical performance. Now an increasingly popular design tool in the aerospace and automotive industries, topology optimization is used to redesign structures to reduce mass while increasing stiffness. To effectively apply this design tool to launcher structures, the optimization process must take into account multiple load cases to represent the large number of load cases such structures are subjected to, throughout the various flight phases. In this thesis, a design methodology was established for topology optimization of launcher structures under multiple load cases by redesigning a launcher structure produced by Airbus Defence and Space Netherlands (Airbus DS NL). The thesis used two analytical models; a simple cantilever beam model to set baseline expectations of design methodologies which were then verified on the launcher structure demonstrator model. To achieve the objective, a suitable multiple load case objective function was identified for the design problem at hand by testing different objective functions on both analytical models. In the process, efficient ways to accommodate the large number of load cases in the objective were also studied through which promising methods to reduce the number of load cases were identified. The final optimized design was compared to the original launcher structure which showed that the optimized design exhibited organic design features, higher stiffness with the same mass. This thesis provided insight into multiple load case topology optimization which is expected to accelerate its adoption in the launch vehicle industry and pave a new path for more efficient space travel.

Keywords: Topology Optimization, Launch vehicle, Multi-Objective Optimization, Multiple Load Case

Preface

As I write this, I am sitting facing the three windows in my room which have been my only view of the outside world during the majority of my MSc thesis. The COVID-19 pandemic forced me out of the office and into my room – an island where it was far too easy to lose context. Mere steps away from my bed, my workstation displays the Airbus logo on a blue screen- a sight I have grown almost too familiar with, one which reminds me how my journey began. After a successful internship with Airbus, I jumped headfirst into my thesis project without much prior knowledge on the topic. I knew it would be an uphill battle but I found myself drawn to the opportunity of exploring a new field, and was excited enough to ignore my cautious self and buckle up for the ride. This thesis marks the end of a very inspiring and memorable time at TU Delft. The experiences here pulled me right out of my comfort zone, made me challenge what I knew about myself and imparted invaluable knowledge.

I would like to express my sincere gratitude towards Dieter for giving me the opportunity to work on exciting projects and meet inspiring people at Airbus. His unwavering support allowed me to find strength in my abilities and work through difficult days. I am grateful to Airbus Defence and Space, Netherlands for their warm welcome and for always making me feel at home at the offices. I will forever cherish the time I spent in the studentenkamer and the friendships that I will carry with me into the future.

I would like to thank my supervisors, Sergio and Jian, for their steady guidance throughout the project. Their kind and patient nature gave me the confidence to tackle a new subject without getting overwhelmed.

I am thankful for my friends who helped me maintain some semblance of a social life during these unique times. I am especially grateful to Srikanth and Harsha for their willingness to lend a hand whenever I needed it. This past year has been challenging for all of us but having shared it with them made it quite bearable.

Finally, and most importantly, I would like to express my heartfelt appreciation for my parents and my sister for being by my side (and on the other side of the screen) throughout my journey these past two and a half years. Their constant love and encouragement gave me the courage to be my best self – I hope I have made them proud.

Siddharth Bharteeya
Delft, February 2021

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Nomenclature

Acronyms

AM Additive Manufacturing

CB Cantilever Beam

FEM Finite Element Method

LSD Launcher Structure Demonstrator Model

MLC Multiple Load Case

OLS Original Launcher Structure

SLC Single Load Case

TO Topology Optimization

Greek symbols

β Upper bound value

ρ Density

θ Angle between two vectors

Roman symbols

$\overline{C}$ Normalized compliance

C Compliance

C^* Final optimized compliance

$C^{initial}$ Compliance at design cycle 0

C^{target} Target compliance

CL Combination load case

D Relative difference from minimum compliance

E Material properties

E_e Stiffness of element e

f Load vector

G	Global load case
K	Stiffness matrix
K_e	Global level element stiffness matrix
L	Local load case
M	Relative importance of load case
p	Penalty value
S	Performance score
u	Displacement vector
V	Volume
v	Single load case vector
w	Weight assignment
x	Design variable

1
INTRODUCTION

1.1. BACKGROUND

The space industry is quite unrecognizable from what it used to be a few years back and it can definitely be attributed to the commercialization of space related activities. The entry of companies such as SpaceX, Rocket lab, Blue Origin and many more have brought with them much innovation in the launch vehicle industry. The resulting increase in access to space based assets and associated data have enabled the expansion of existing businesses as well as seeded several new entrepreneurial ventures.

The original reigning organizations of the space industry such as NASA, Arianespace, ROSCOSMOS, etc. must invest in new technologies to face the stiff competition from the newcomers defining the new space age. In this context, additive manufacturing (AM) is gaining popularity in the launch vehicle industry to manufacture engine components and in some extreme cases - entire launch vehicles [4]. AM allows complex design, lightweight components, shorter manufacturing times and ease of manufacturing [5]. Recently AM is being increasingly used for structural components for space missions as well. To extract the maximum benefit from AM, it can be coupled with a structural optimization technique which optimizes material distribution within a component's design envelope called topology optimization.

Topology optimization is a structural optimization method used together with FEM tools to generate optimized component designs with the objective of reducing weight whilst increasing mechanical performance. The result designs often exhibit mass savings of 40%-60% as well as higher stiffness by similar proportions. Since the design process is not naturally guided by manufacturing constraints, the optimized designs are not manufacturable through traditional techniques such as machining, casting, forging, etc., as the designs exhibit fine structures, hollow channels and features which are inaccessible by standard tools [6]. The compatibility between AM and TO is then clear and so is the ensuing benefits for producing structural components for flight using such techniques.

One of the major obstacles in using TO for launch vehicle structures is the shear number of load cases such structures are designed for. During its operational lifetime, different sets of loads (load cases) are experienced by any launch vehicle structure due to the multiple flight phases and multiple operation modes of its systems. For a feasible design to be produced, each of the load cases must be taken into account during the design process, therefore, currently the applications of TO are largely limited to academic problems. This thesis project aims to put forth a methodology for topology optimization of a launcher structure which is subjected to multiple flight load cases.

This Master thesis project is conducted at Airbus Defence and Space Netherlands under the guidance of research groups, Space Systems Engineering and Aerospace Structures and Computational Mechanics at TU Delft. The project achieves its goals by redesigning a structural component from a launch vehicle using topology optimization. The author hopes that this would accelerate the adoption of topology optimization and additive manufacturing in the launch vehicle industry.

1.2. THEORY AND BASICS

1.2.1. TOPOLOGY OPTIMIZATION

Structural design optimization problems may come in the form of sizing, shape or topology optimization problems. The process of which involves finding the optimal design features in the structure with the goal of minimizing or maximizing physical responses of the structure such as stiffness, stress, deflection, etc. In the case of size optimization, the design features addressed may be geometrical attributes such as thickness, length, width, etc., whereas in shape optimization the shape of the design domain itself is addressed. Topology optimization pushes further and aims to find the optimal number, locations and size of holes - in other words, the connectivity of the structure [7]

After the landmark paper introducing the methodology for generating optimal topologies through a material distribution method by Bendsoe et al. [8], the field has expanded drastically. Comprehensive reviews on various approaches can be found in Patel et al. [9], Sigmund et al. [10] and Rozvany [11]. A topology optimization problem aims to find the best lay-out of a structure within a defined domain so as to optimize mechanical performance [9]. Specifically, the objective of this optimization can be to minimize compliance (maximize stiffness), minimize mass, minimize stress, or any other structural response. The structure is subject to certain loading conditions and will also have constraints imposed on volume, mass, stress as well as shape, size and location of design features. A common formulation of a topology optimization problem is designing for minimum compliance (or maximum stiffness). The compliance of a structure gives information about the deformation of the structure, stress, strain and can also be used to approximate the natural frequencies[12]

To implement topology optimization problems computationally, the component to be optimized is discretized using the finite element method. A minimum compliance formulation is thus given by Equation 1.1:

$$\min_{\mathbf{u},E_e} \mathbf{f}^T \mathbf{u} \tag{1.1}$$

$$s.t. \quad \mathbf{K}(E_e)\mathbf{u} = \mathbf{f}$$

$$\mathbf{K} = \sum_{e=1}^{N} \mathbf{K}_e(E_e)$$

where **K** is the stiffness matrix at the element level and it depends on the stiffness of element e, E_e . E_e can take the value of 0 or E; 0 for material not being present and E for material of stiffness E being present for that element. This formulation results in a binary valued problem, which is computationally expensive due to the direct search methods which have to be applied. Therefore it is computationally more efficient to replace the integer variable with a continuous variable [11] as given by Equation 1.2

$$E_e = \rho(x)E \tag{1.2}$$

$$where, \quad E_e = \begin{cases} 0, & \text{if } x = 0 \\ E, & \text{if } x = 1 \end{cases}$$

The design variable, x, can take values 0 (material does not exist) and 1 (material exists) with E_e mapping from 0 to E. However, the optimal solutions from the optimization problem as stated above consist of designs with most elements with densities between 0 and 1. Since the requirement of this design optimization is to obtain a final design with distinct structural features - an optimal connectivity - the intermediate density values must be penalized [7] [11]. To force the solution to a "1-0" solution, a penalty for intermediate values of x can be introduced. The power-law approach that was introduced by Bendsoe et al. [13] or how it's more commonly known now – the Solid Isotropic Material with Penalization or SIMP method - can be employed. The formulation of the SIMP method is given in Equation 1.3:

$$E_e = \rho(x)^p E \tag{1.3}$$

Density ρ is the design function, which defines the material properties to be between 0 and E. The penalty value p is usually kept as $p > 1$ such that the intermediate densities are unfavourable to the optimizer compared to the cost of the material added. Rozvany et al. [14] put forward the conditions for the use of the interpolation method to effectively imitate material properties which include a penalty value of 3 and a Posisson's ration of $\frac{1}{3}$.

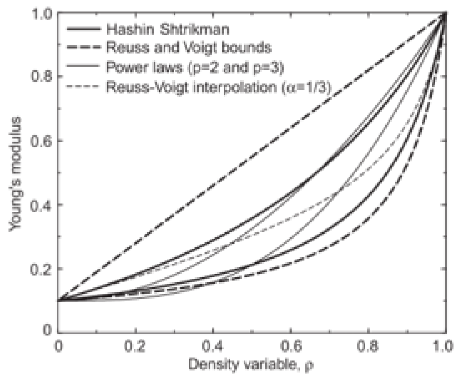


Figure 1.1: Various interpolation schemes for SIMP [1]

1.2.2. MULTIPLE LOAD CASES

The formulation of TO problems described in the last section is tailored for a single objective function, and therefore is largely limited for use in academic problems only, since in most real world applications, components are subject to multiple loading conditions. This is especially true for aerospace applications where for example, an aircraft part must be designed to withstand multiple load cases during different flight phases. The part, therefore, has to be optimized for multiple load cases. Apart from these loads, dynamic effects may also need to be considered and so the part must be designed to reduce vibration load levels in addition to the static loads. First off, let us clarify some recurring terms which will follow:

Multiple Loads: In the document, this term simply refers to the multiplicity of individual loads or types of loads. For example, we may say that the following cantilever beam in Figure 1.2 is subjected to multiple loads namely load 1, 2 and 3.

Figure 1.2: Cantilever beam with loads 1, 2 & 3 constitute one load case

Load Case: A load case refers to a configuration of loads (may be one load or multiple) that may be applied to a component simultaneously at a given instance. The above example of a cantilever beam subjected to 3 loads can be considered one load case.

Multiple Load Cases: During the operational life of a component, it may be subjected to different configurations of loads or load cases at different phases of its life. Therefore, it can be said that the component is subjected to multiple load cases. For example, the following cantilever beam in Figure 1.3 is subjected to multiple load cases and more specifically 2 load cases. Each load case refers to a different use case of the beam during its operational life.

(a) (b)

Figure 1.3: (a)Load case A with loads 1,2 & 3 and (b) Load case B with loads 4 & 5

1.3. LITERATURE REVIEW

Following the initial intention of topology optimization to be used for minimum weight designs, it is now used to optimize structures in the aerospace and automobile industries [2]. Components designed for space applications, like aviation, are subjected to multiple load cases, and therefore the topology optimization design process must account for these cases to obtain a feasible structural design. Previous work in the field of multiple load case TO has been discussed below in this section.

1.3.1. MULTIPLE LOAD CASE TO

The importance of multiple load case TO is highlighted in the example of design of a Danish government sponsored satellite[2]. The design of the satellite was driven by 4 load cases - vertical acceleration force, two for lateral vibrations and one for simulating the ground handling. Additionally, the design required to have a minimum resonant frequency. The example is a basic one as most aerospace components would be designed for many more load cases.

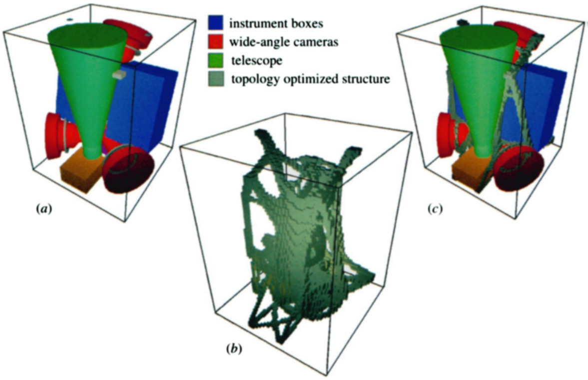


Figure 1.4: TO design of a satellite structure [2] : (a) Design domain and instruments; (b) topology optimized structure; (c) optimized structure with instruments

Many approaches exist to tackle multi-objective optimization problems[15]. Presented below are some methods found in the literature for multiple load case TO problems:

WEIGHTED-SUM

The most popular multi-objective optimization strategy is the weighted-sum method. Through it, a combined objective function is created by using a weighted sum of the individual objectives.

$$minimize \quad \sum_{i=0}^{N} w_i l_i(u_i) \tag{1.4}$$

Equation 1.4 depicts the weighted sum objective function when formulated to minimize a weighted sum of individual compliance values associated with each load case ([3],[7]). In the equation, $l_i(u_i)$ represents the compliance for load case i, w_i is the respective weight and the sum is calculated for a total of N load cases. The weights are decided *a priori* and are based on the designer's intuitive choice or systematic approaches like ranking methods, categorization methods, evolutionary algorithms, etc.[15].

COMPLIANCE VOLUME PRODUCT

Although used for its simplicity and efficiency in engineering problems, the weighted sum has been criticized for being unable to obtain all pareto solutions which depends on the strategy to select the weights[16]. A combined objective function with compliance and volume using the Normalized Exponential Weighted Criterion is put forward by li et al. [17],[18] to obtain all pareto solutions. The weights in this case were derived using fuzzy set theory. The result was successfully tested on 2D and 3D examples however it was not compared against the traditional weighted sum approach.

BOUND FORMULATION

The bound formulation introduced by Bendsoe et al. [19] circumvents the process of assigning weights by using a re-formulated min-max function. In cases where a component must be designed for a worst case load, a weighted approach may not produce adequate results.

$$minimize \quad \beta \tag{1.5}$$

$$s.t. \quad w_i l_i - \beta \le 0, i = 1, ..., M$$

Initially employed in small analytical problems, this method is now adopted in large scale problems as well. It outperformed the weighted-sum approach when applied to multi-loaded disk plate structures[20]. It was also investigated for aircraft wing box ribs [21] but was criticized for the design not being well balanced for all loads. This can be deduced, considering it is formulated in preference of the most critical and therefore the largest loads.

MINIMUM MASS WITH COMPLIANCE CONSTRAINTS

In the last example of aircraft wing box ribs[21], the weighted sum method was also criticized for being driven by non-critical load. The author then presents an alternate approach of setting constraints on the compliance values for each load case and using a minimum mass objective. The method works well in cases where the target compliance values are known beforehand which is the case when a component is being re-designed. In Equation 1.6, the mass ($\rho_n V_n$) is minimized while keeping the compliance of load case C_i below target compliance C^{target}.

$$\underset{\rho_n}{\text{Minimize}} \quad \sum_{n=1}^{N} \rho_n V_n \tag{1.6}$$

$$s.t. \quad C_j \le C_j^{target} \quad j = 1, ..., J$$

$$0 \le \rho_n \le 1 \quad n = 1, ..., N$$

CRITICAL LOAD IDENTIFICATION

Even with the various objective functions designed to deal with multi-objective TO problems, TO for aerospace components is burdened with the presence of hundreds of load cases [12] resulting in a significant computational power requirements. The reduction of these numerous load cases can benefit the design process. A.R. Faria [12] put forward a technique to obtain a subset of load cases which contains the most critical loads for the structure. The technique based on convex modelling is used to reduce the number of load cases to include an extremal load case set in terms of its compliance values.

Iwamura et al [22], with a similar motivation, presented a strategy to reduce the computational effort by reducing the number of load cases evaluated during the sensitivity analysis. Three variations were investigated where the number of load cases was restricted by : 1 worst case load case, 2 worst case load cases or all load cases with 90% or higher magnitude than the worst case load case. The latter criterion was concluded to produce the best results overall in terms on stiffness and convergence.

1.3.2. TOPOLOGY OPTIMIZATION APPLICATIONS

Table 1.1 lists the previous work in the field of topology optimization of aerospace components. Notable applications of topology optimization for space vehicles include the payload adapter support bracket [23] and 3D printed brackets for a lunar spacecraft [24] - both of which are using minimum mass formulations. Five of the papers are confirmed to be for multiple load cases and it can be seen that a minimum weighted compliance is used in most of them considering only a few load cases ([25], [26] & [27]). Only one paper, by Krog et al. [21] investigates the relative performances of various multi-objective formulations to deal with a high number of load cases; applied to aircraft wing box ribs, TO is performed for a structure subjected to an order of 10 flight load cases in addition to local loads. The methodology being for a very specific component and in a unique load environment, cannot be directly applied to other structures.

The high number of papers post 2015 can be attributed to the increasing popularity of TO in the aerospace industry which is also facilitated by development of compatible manufacturing and development processes such as additive manufacturing which makes this approach a feasible one. However, to truly adopt TO in this industry further research in TO under multiple load cases is still required.

1.3.3. SUMMARY OF LITERATURE REVIEW

The literature survey presents the theoretical concepts behind topology optimization and its use with multiple load cases. Different ways of tackling multi-objective topology optimization problems are presented. Multiple load case objective formulations are described which have been developed and previously employed in engineering applications and in most cases, aerospace engineering applications. These case studies have been reviewed in context of understanding multiple load case TO problems. Previous work in the field relating closely to this thesis project are also summarized. From the literature review it is evident that although TO has been explored for aerospace applications, not much work exists in the study of dealing with multiple load cases. Even in the instances found in the literature only one explores different objective formulations and none in the context of launcher structures. To advance the adoption of TO design processes in the launch vehicle industry, more research is required investigating topology optimization applied to launcher structures under multiple load cases.

Table 1.1: Previous work on topology optimization of aerospace structures

Year	Title	Objective	Constraint	Multiple load cases	Software	Source
2019	Topology optimization and additive manufacturing for aerospace components	Min Mass	N/A		N/A	Berrocal et al. [23]
2019	Optimization Design of Configuration and Layout for Queqiao Relay Satellite	N/A	N/A		N/A	Gao et al. [28]
2018	Topology Optimization Design of Launcher Bracket Based on Multi-body Dynamics	Min Volume	Max Displacement, Natural Frequency		N/A	Li et al. [29]
2017	Designing for Additive Manufacturing: Lightweighting Through Topology Optimization Enables Lunar Spacecraft	Min Mass	Natural Frequency, Max Stress	Yes	Hyperworks	Orme et al. [24]
2017	Structural Optimization of Space Components Adapted for 3D Printing	Min Weighted Compliance	N/A	Yes	Inspire	Munteanu et al. [26]
2017	Multi-objective topology optimization to reduce vibration of micro-satellite primary supporting structure	Min Weighted Compliance and Random Response	Volume	Yes	OptiStruct	Tan et al. [25]
2016	A Demonstration of Additive Manufacturing as an Enabling Technology for Rapid Satellite Design and Fabrication	Min Mass	Natural Frequency, AM		OptiStruct	Orme et. Al [30]
2016	Topology Optimization of Spacecraft Transfer Compartment	Min Mass	N/A		OptiStruct	Borovikov et al.[31]
2016	Aerospace Case Study on Topology Optimization for Additive Manufacturing	Min Compliance	Volume		N/A	Süß et al. [32]
2013	Topology Optimization in Support of Spaceborne Device Based on Variable Density Method	Min Compliance	Volume		Ansys	Wu et al. [33]
2011	Topology optimization of aeronautical structures with stress constraints: general methodology and applications	Min Mass	Stress		N/A	Paris et al. [34]
2011	Topology Optimization of an Additive Layer Manufactured (ALM) Aerospace Part	Min Mass	Max Displacement		OptiStruct	Tomlin et al. [35]
2008	Topology optimization of compressor bracket	Min Combined Strain Energies from Static and Dynamic	Natural Frequency	Yes	OptiStruct	Chang et al. [27]
2004	Topology Optimization of Aircraft Wing Box Ribs	Min Weighted Compliance, Min Max Compliance, Min Mass	Compliance	Yes	OptiStruct	Krog et al.[21]

1.4. RESEARCH QUESTIONS

The following research question follows which will be answered during the thesis:

"How could topology optimization be performed for a launch vehicle structure which is subjected to multiple load cases?"

The above question can be subdivided into the following sub-questions:

1. Which multiple load case (MLC) objective functions can be used for optimization under the multiple load cases?

   (a) *How can the MLC objective functions identified in the literature study be implemented for this problem?*
   The objective function of a multiple load case TO problem must be formed in a way that it leads to a feasible design. This is done by determining the choice of loads to be included into the formulation, the respective weights to be assigned (if any) as well as including load cases in the objective or as constraints.

   (b) *What are the relative performances of each of the MLC objective functions implemented?*
   Different MLC formulations would provide different results and in addition to selecting the appropriate objective function for the design problem in this thesis, a comparative study will be performed to assess the performance of each objective function relative to the rest. This would provide insight to determine whether they could be used for other design problems as well as identify their benefits or limitations.

   (c) *Which MLC objective functions implemented offer better solutions?*
   The mechanical performance and qualitative design features will be evaluated to determine which objective function provides better design results. Based on this, the final objective function can be chosen for the final TO design.

   (d) *Is the performance of the MLC objective function dependent on the structure?*
   The performance of the objective functions could be dependent on the structure to be optimized and therefore the relative performance of these objective functions could vary for different design domains or different types of structures. This sensitivity must be evaluated to comment upon whether there is universality in the application of these objective functions for different structures.

2. *How does the final TO design compare to the original structure designed for conventional manufacturing processes?*

   (a) *How does the TO design compare to a design led by design engineer's experience?*

Design of new structures are generated by engineers based on the prior experience of designing and analyzing structures. The design process is an iterative one; the initial design taking into account loads, constraints, manufacturability, etc. is produced through the intuition of an engineer which is then checked through FE analyses and testing. The topology optimization design process aims to produce the optimal structural design which might be different than one which is designed by a design engineer and therefore it could indicate a better way to design the structure or similar ones in the future.

(b) *How does the new design compare in terms of mass and mechanical performance?*
The TO design will be compared to the original structure that is to be redesigned in terms of mass and mechanical performance to comment on the feasibility of the new design as well as to study the benefits or limitations of using TO.

3. *How can the TO design methodology be implemented into the existing design process of Airbus DS NL?*
The overarching motivation for this thesis is to assist Airbus DS NL to eventually employ topology optimization to its launcher structures and therefore the TO design process must be implemented into the existing design process. Implementation will be easier if the TO software tools can be easily interfaced with or are already used in the existing design process at Airbus DS NL.

1.5. RESEARCH OBJECTIVE AND METHODOLOGY

The research objective of this thesis is:

“To develop a design methodology for topology optimization of launch vehicle structures taking into account multiple load cases”

This thesis will address the research questions by generating topology optimized designs for a simple cantilever beam model and a launcher structure demonstrator model (based on a launcher structure produced at Airbus DS NL) both of which are subjected to multiple load cases. Multiple TO design experiments are set up to study the behaviour of the design process under different loading conditions, objective functions, and constraints. In most cases, the cantilever beam model is used to set expectations of the experiment results which are then verified on the launcher structure demonstrator model. Analytical FE tools are used to create the experiment models and in-built topology optimization packages are used to generate the optimized designs. To comment on the feasibility of the optimized design and the design process, a comparison is made against the original launcher structure being produced at Airbus DS NL.

1.6. THESIS STRUCTURE

The thesis report consists of 7 chapters and two appendices. A brief description of the chapters is as follows:

Chapter 2, provides the description of the tools and models used for the analyses performed during the thesis. The chapter introduces the two analytical models that will be used for performing topology optimization and testing out the different objective functions. The description includes details about the geometry, the design loads, constraints, the FEM model and the optimization parameters that are used for the TO design studies.

Chapter 3, aims to select the design load cases that will be used to perform TO in the subsequent chapter. The chapter dives into the idea of load case combinations which significantly increase the total number of load cases in consideration. The effect of loads acting as separate load cases or as combined load cases is studied on both analytical models to understand the difference in design and mechanical performance.

Chapter 4, describes the comparative study between different objective functions by performing topology optimization for the two analytical models. Four objective functions are described which will be used to generate the optimized designs. The designs obtained are then compared to select the objective function which generates the best design based on design features and mechanical performance. To aid in the comparison, a performance metric is introduced to assign a performance score to each objective function.

Chapter 5, introduces the concept of inadvertent stiffening and load case similarity. By identifying similar load cases, the total number of load cases to be considered can be reduced while not adversely affecting the resulting design. The chapter describes the application of a common data science metric to identify similar load cases effectively. The method is verified by performing topology optimization using single load cases. The reduced set of load cases is then used to generate the reduced-set TO design.

Chapter 6, provides the final topology optimized design of the launcher structure and sensitivity studies. The final design is compared with the original launcher structure produced by Airbus DS NL based on design features and mechanical performance. The sensitivity of the TO process to changing design domain and goal mass fraction are studied and the results presented.

Chapter 7, provides the conclusions of the thesis project, evaluation of the research questions set at the beginning, and recommendations for future work.

2

Analysis tools and Models

This section contains the description of the analysis tools and models that were used to conduct all the experiments which are described in the later sections. MSC Nastran and Patran were used to create the FE models as well as perform the FE analysis. Experiments were conducted on two analysis models, namely, the cantilever beam model and the launcher structure demonstrator model. The cantilever beam model was chosen to set baseline expectations which would then be assessed when working with the launcher structure demonstrator model.

2.1. Software tool

2.1.1. MSC Nastran Sol200

MSC Nastran is a multidisciplinary structural analysis software used to perform finite element analysis. It exhibits multiple "solution sequences" to solve for different types of problems such as for buckling, heat transfer, aeroelasticity, optimization, etc. SOL200 is the solution sequence used to solve design optimization problems such as sizing optimization, shape optimization and topology optimization. In this thesis, Topology Optimization is implemented through the SIMP or the Solid Isotropic Material with Penalization method as introduced in Section 1.2.1.

Implementation

Figure 2.1 shows a representation of the complete design optimization process as implemented in MSC Nastran. Starting with the initial design, it is a combination of the FE analysis model along with the definition of design variables and designed properties. The initial design is defined by the user, specifying the starting point of the design optimization. The initial design is subjected to a structural analysis (FE analysis) after which those constraints which are likely to drive the redesign process are identified (Active Constraints) in Constraint Screening. A design sensitivity analysis is performed which calculates the sensitivity of the structural responses with respect to the design variables. Only those structural responses are considered which either contribute to the

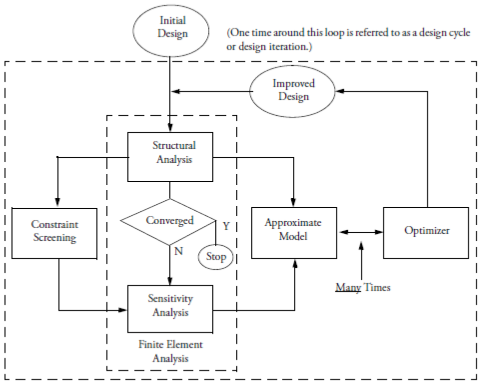


Figure 2.1: Implementation of structural optimization in MSC Nastran

objective function or the active constraints, thereby reducing the computational load. An approximate model is generated which is then used by the optimizer to generate the improved design. The improved design undergoes a new FE analysis and convergence tests determine whether the design has settled sufficiently. Design optimization is an iterative process and therefore Figure 2.1 shows a closed loop cycle which is only broken when convergence is achieved.

2.1.2. OPTIMIZER

MSC Nastran uses a variety of optimization algorithms however, in this thesis only the IPOPT algorithm [36] is used due to its ability to address design tasks with a large number of variables and therefore its topology optimization tasks. The IPOPT algorithm implements an interior point line search filter method which aims to finding a local optima for large scale nonlinear optimization problems.

2.2. CANTILEVER BEAM MODEL

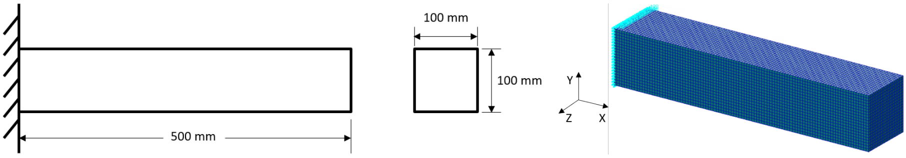
The cantilever beam (CB) model is used for straightforward load application and analysis of results. The simplicity of the model enables an intuitive assessment of the results in order to understand the relationship between problem set-up and the topology optimized results.

2.2.1. GEOMETRY

A basic cantilever beam model is used with dimensions of $500mm \times 100mm \times 100mm$ as shown in Figure 2.2a.

2.2.2. FINITE ELEMENT MODEL

The cantilever beam geometry is uniformly meshed with 40,000 Hex8 elements as shown in Figure 2.2b. Nodal displacements constraints are applied on the -X face with translations in x, y and z constrained to zero.



(a) Cantilever beam dimensions (b) Cantilever beam FE mesh

Figure 2.2: Cantilever beam model

2.2.3. LOAD CASE DEFINITION

Two set-ups are described using the cantilever beam model. Each set-up contains a set of load cases which are distinct in their nature. Similar loads are avoided in order to assess the influence of each load in the resulting topology.

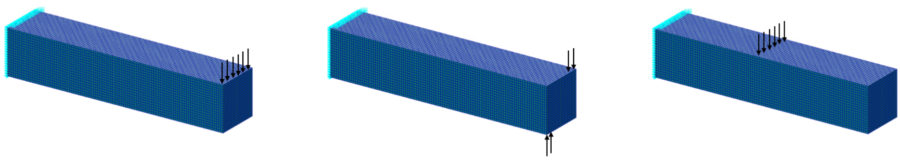
CB-1

In CB-1, three distinct load cases are defined - shear, torsion and normal.

a. In the shear load case, uniform load on the free end is applied in the -Y direction.

b. In the torsion load case, torsional load on the free end is applied to cause a torque along the X axis.

c. In the normal load case, uniform load in the middle of the beam on the +Z face is applied in the -Z direction.



(a) Shear load case (b) Torsion load case (c) Normal load case

Figure 2.3: CB-1 load case definitions

CB-2

In CB-2, two distinct load cases are defined - moment and normal.

a. In the moment load case, a moment is applied on the free end along the +Y direction

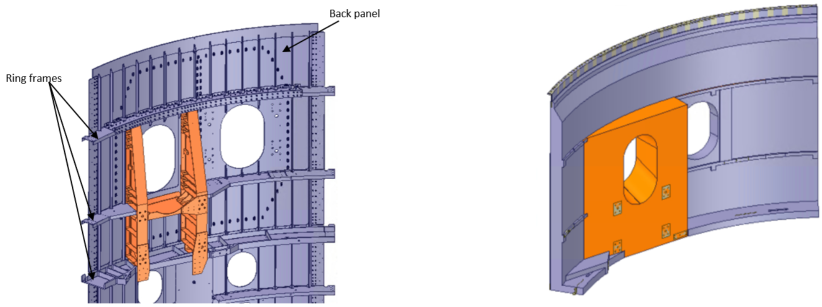
b. In the normal load case, uniform load in the middle of the beam on the +Y face is applied in the -Y direction.

(a) Moment load case (b) Normal load case

Figure 2.4: CB-2 load case definitions

2.3. LAUNCHER STRUCTURE DEMONSTRATOR MODEL

A structure from the Ariane 6 launch vehicle will be considered for the project. The structure will be used as a demonstrator for the topology optimization methodology that is to be developed as part of the thesis. The chosen structure (shown in Figure 2.5a), named Original Launcher Structure (OLS) for the project, is an assembly of multiple machined aluminium components embedded onto the fuselage of the launch vehicle. OLS is attached to the fuselage panels on the back as well as three ring frames on the top, middle and bottom. The launcher structure demonstrator(LSD) model is created from the design envelope (shown in Figure 2.5b) of the OLS which incorporates an access port for equipment and four distinct regions for local load application.



(a) Original Launcher Structure (b) Design Envelope

Figure 2.5: Original Launcher Structure and corresponding design envelope

2.3.1. GEOMETRY

A simplified geometric model is constructed which incorporates the design envelope and the fuselage section to which the final structure must attach. The fuselage section contains the curved back panel with vertical stiffeners, three horizontal panels representing the three ring frames along with perpendicular curved panels attached to the free end of the ring frames.

The full design envelope of the LSD model is considered. The OLS can be seen to be much smaller than the design envelope being considered, this is because the design envelope is provided to the designer's at the very beginning of the design phase and therefore represents the larger volume which the particular part is allowed to use. This thesis will use the larger design envelope so as to study a relatively unconstrained topology optimization task given the basic design requirements available at the beginning of the design phase.

(a) Design envelope geometry (b) Fuselage section geometry

Figure 2.6: Launcher structure demonstrator model geometry

2.3.2. DESIGN LOADS

For the Ariane 6, the total number of load cases can be in the order of 10^{179}, thus, to say that launcher structures are subjected to multiple load cases is an understatement. The design of such a structure considers local load cases and global load cases. Local load cases are loads transferred from attached equipment and sustaining these loads is often the primary function of the structure. Global load cases consist of loads which are transferred to the structure from the rest of the launcher throughout all flight phases. These loads are generated due to all the equipment on board (engines, actuators, etc.) as well as the external environment during launch. Given the various flight phases and numerous equipment on board, there is an immense number of global load cases, and ideally each of them should be taken into account during the design of a launcher structure. This thesis considers all the required local load cases and a subset of the global load cases during the topology optimization design process. It is expected that the methodology developed using this subset can be applied to the extended set of global loads in the future.

The load cases used are created by a process of maximizing or minimizing a load component in an element of the FE model. Given 6 load components, F_x, F_y, F_z, M_x, M_y and M_z, 12 load cases can be generated as shown in Table 2.1. Each load case maximizes or minimizes one of the 6 load components at a particular element or a set of elements. For example, a load case F_x MAX is created which consists of loads which result in the maximum value of F_x for the particular element or set of elements. Similarly a load case M_x MIN is created which consists of loads which result in the minimum value (usually a high negative value) of M_x. Therefore, a set of 12 extreme load cases are generated which can be used for the design process.

LOCAL LOAD CASES

12 extreme local load cases are provided following the format described in Table 2.1. The table describes that F_x MAX represent a load cases consisting 3 static force components

Table 2.1: Extreme load case generation

Load Component	**Load case maximizing load component**	**Load case minimizing load component**
F_x	F_x MAX	F_x MIN
F_y	F_y MAX	F_y MIN
F_z	F_z MAX	F_z MIN
M_x	M_x MAX	M_x MIN
M_y	M_y MAX	M_y MIN
M_z	M_z MAX	M_z MIN

F_x, F_y and F_z for each load application region which maximizes the local load in x direction (F_x) for the component.

GLOBAL LOAD CASES

Multiple flight phases exist and the structure should be designed by taking into account all the load cases associated with each flight phase. However, for this thesis a only a single flight phase is considered. This decision is made to have a manageable number of load cases in terms of computational power and result analysis. The flight phase Gamma max is chosen which is considered to be one of the most critical flight cases and takes into account loads from the main engine, boosters, actuators, etc. The global load cases are created for the bottom and left edges of the fuselage section. The loads are applied at these edges which are then expected to propagate through the entire structure. A total of 24 extreme load cases are generated with 12 extreme load cases generated for each edge as shown in Table 2.2.

Each global load case ideally consists of 6 load components - F_{radial}, $F_{tangential}$, F_{axial}, M_{radial}, $M_{tangential}$ and M_{axial}. Due to the manner in which the loads were defined and provided, high radial deflections in the fuselage section were produced which is not characteristic of the actual fuselage structure. Majority of the radial load is to be taken by the ring frames and multiple iterations in load application and boundary condition were made to reconcile the inconsistency which led to quite some delay. Since the load generation was a tedious process, a decision was made to exclude load components which cause high radial deflections in the fuselage section instead of re-generating the load cases at that stage in the project. Finally, each global load case consists of 3 load components - $F_{tangential}$, F_{axial} and M_{radial}. Therefore the load cases do not account for the full gamma max flight phase load and should just be considered as multiple representative load cases.

2.3.3. FINITE ELEMENT MODEL

The FE model is constructed using an FE pre-processor, MSC Patran. The fuselage section is meshed using shell elements with the appropriate thicknesses and vertical stiffeners are modelled using 1D beam elements spanning the length of the fuselage section. The design envelope is meshed using TET4 (Tetrahedral) solid elements which has been chosen for a better mesh given the complicated geometry and the 4 node configuration allows for smaller element sizes to stay within the computational load constraints.

Table 2.2: Extreme load cases for Gamma max flight phase

Sr. no.	Load cases for Gamma max flight phase	Sr. no.	Load cases for Gamma max flight phase
1	Bottom Edge Min F_{radial}	13	Left Edge Min F_{radial}
2	Bottom Edge Min $F_{tangential}$	14	Left Edge Min $F_{tangential}$
3	Bottom Edge Min $F_{longitudinal}$	15	Left Edge Min $F_{longitudinal}$
4	Bottom Edge Min M_{radial}	16	Left Edge Min M_{radial}
5	Bottom Edge Min $M_{tangential}$	17	Left Edge Min $M_{tangential}$
6	Bottom Edge Min $M_{longitudinal}$	18	Left Edge Min $M_{longitudinal}$
7	Bottom Edge Max F_{radial}	19	Left Edge Max F_{radial}
8	Bottom Edge Max $F_{tangential}$	20	Left Edge Max $F_{tangential}$
9	Bottom Edge Max $F_{longitudinal}$	21	Left Edge Max $F_{longitudinal}$
10	Bottom Edge Max M_{radial}	22	Left Edge Max M_{radial}
11	Bottom Edge Max $M_{tangential}$	23	Left Edge Max $M_{tangential}$
12	Bottom Edge Max $M_{longitudinal}$	24	Left Edge Max $M_{longitudinal}$

LOCAL LOADS

Local loads are loads which are acting on the LSD at the four load interface regions on the design envelope. The loads are applied on nodes in space not geometrically associated with the design envelope. The loads are then transferred to the interface regions using RBE3 rigid elements as shown in Figure 2.7. RBE3 elements are used as multi-point constraints to evenly apply load from a singular load application point (called the Dependent Node) to multiple nodes (called the Independent Nodes) in the load application region. RBE2 can also be used for this purpose however, since it would create an artificial stiffness in the load application region owing to its formulation, the RBE3 elements was preferred.

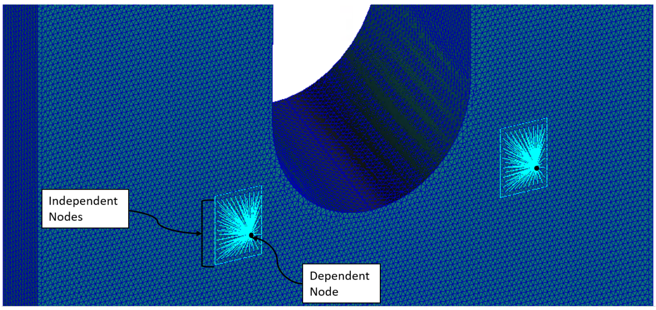


Figure 2.7: RBE3 rigid elements for load application

GLOBAL LOADS

Global loads are, similar to local loads, applied through RBE3 elements to the left and bottom edge nodes of the fuselage section.

BOUNDARY CONDITIONS

Displacement constraints are applied on the top and right edges of the fuselage section. The boundary conditions represent the stiffness of the surrounding fuselage structure and are detailed in Table 2.3

Table 2.3: Displacement constraints for fuselage section

Component	Edge	Displacement		
		Radial	Tangential	Axial
Fuselage Back Panel	Top & Right		0	0
Ring Frame - Horizontal	Left & Right	0	0	
Ring Frame - Vertical	Left & Right		0	0

CONTACT CONDITIONS

Contact between the design envelope and the fuselage section is modelled using glued contact. Permanent glued contact is applied between solid body (design envelope) and shell body (fuselage section). This contact definition allows the TO process to choose the optimal points of contact where the final part must attach itself to the fuselage section - this again follows the idea of imposing minimum constraints for the TO design process. It can be noticed that in Figure 2.8, modifications are made to the middle and bottom ring frames to accommodate the OLS. These modifications are not explicitly modelled in the LSD but rather are allowed to occur during the TO task due to the given contact conditions.

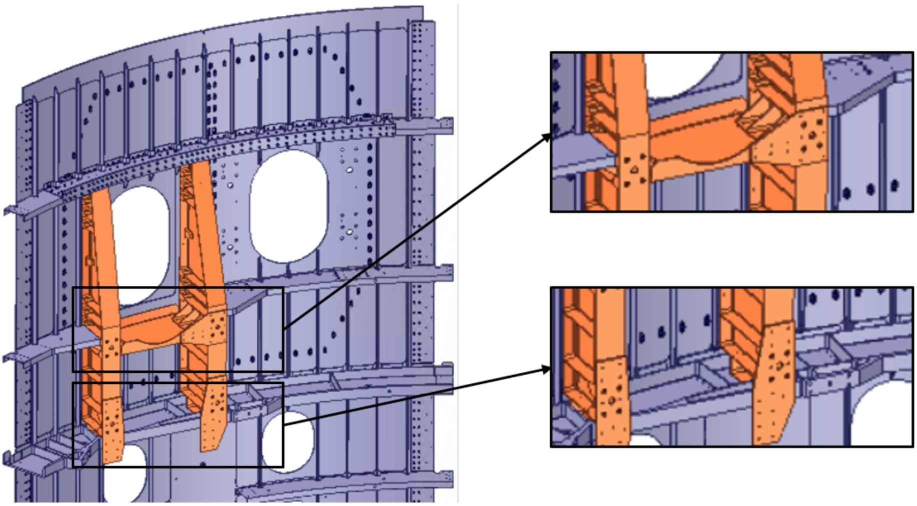


Figure 2.8: Modifications to middle and bottom ring frames to accommodate OLS

2.3.4. OPTIMIZATION PARAMETERS

MINIMUM MEMBER SIZE CONSTRAINT

A minimum member size constraint is implemented to avoid the appearance of thin structures which is possible in TO tasks. A member size constraint of 30 mm has been chosen which is the recommended value for the chosen element size (i.e. at least three times the elements size).

MASS FRACTION

Mass fraction of 0.037 has been chosen to produce an output design with the same mass as the OLS. This allows a 1 : 1 comparison of performance.

DESIGN DOMAIN

The solid mesh excluding the load application regions is set as the design domain for the problem. This ensures that material always remains in the regions highlighted in Figure 2.9 regardless of the optimization process.

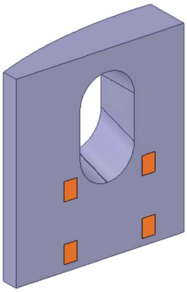


Figure 2.9: Highlighted regions show the load application regions which are exempted from the design domain

2.3.5. TOPOLOGY OPTIMIZATION OUTPUT

After optimization, Nastran provides the list of elements with their respective optimized element densities (which range from 0 to 1) which can be called the *continuous density result*. TO view the topology in Patran, a threshold value must be chosen which eliminates the elements from the FE model of the design domain which are below the set threshold value. The threshold value is chosen such that the result mesh has the same mass as the goal mass. For the LSD model, the resulting FE mesh is hard to view because of the exposed jagged edges of the elements. So, for ease of viewing, the FE mesh is smoothed using CAD software. All TO results for the LSD model shown in the document are smoothed representations of the optimized FE mesh.

2.3.6. MESH CONVERGENCE STUDY

A non-traditional approach to mesh convergence has been adopted to take into account the solid and surface meshes. First a mesh convergence study has been performed on the fuselage section mesh by varying the edge size for the surface mesh and performing static analyses. This provides a mesh size for the fuselage section which can then be used for the solid mesh convergence study. Consequently, TO tasks are set up with varying edge sizes of the solid mesh. The result of this step provides the final solid mesh size as well as the errors with regard to topology optimization.

SURFACE MESH - STATIC ANALYSIS

The mesh convergence study for the fuselage section has been performed without the design envelope in contact. The loads applied are the global loads or Gamma max flight load cases for the top and bottom edges.

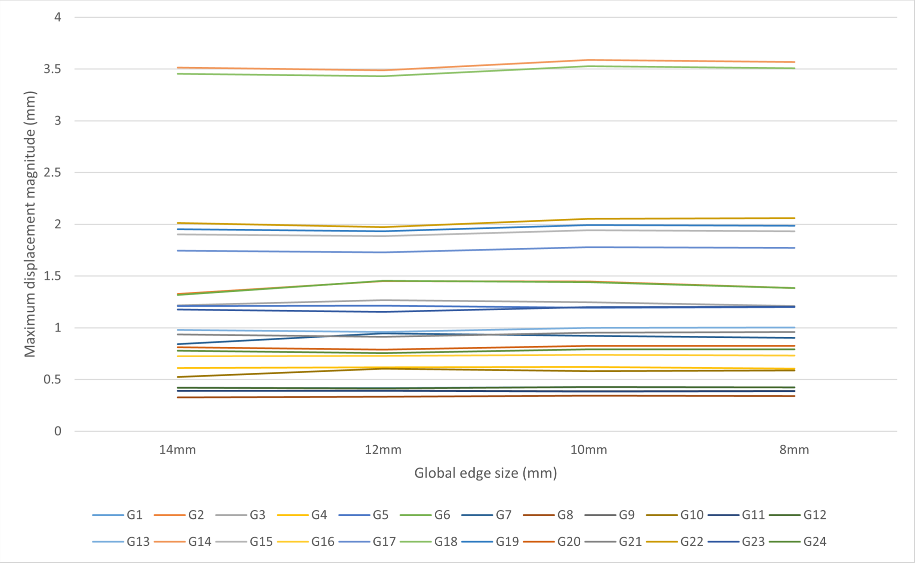


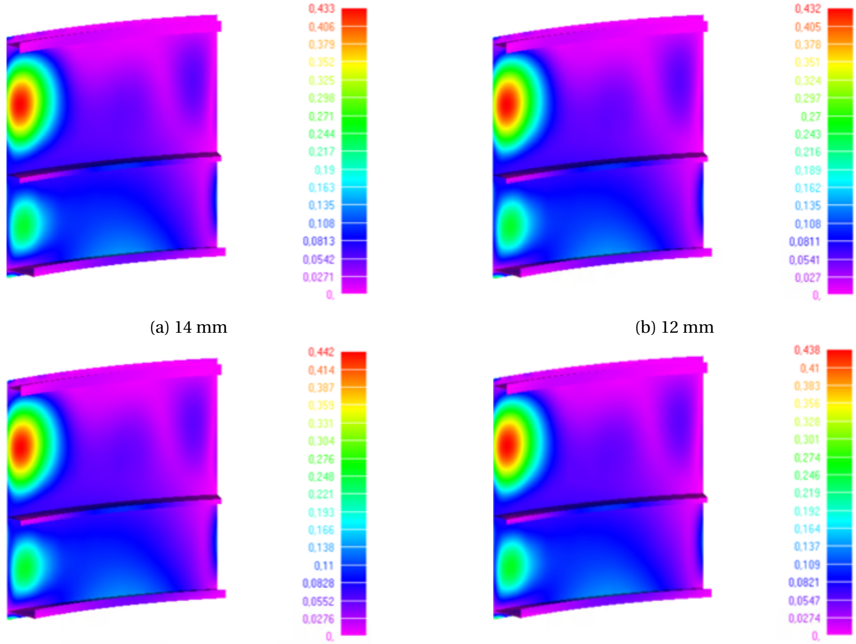
Figure 2.10: Mesh Convergence study for surface mesh on Fuselage section

Figure 2.10 shows the maximum displacement magnitude values for each load case under the gamma max flight cases. Figure 2.11 shows the contour plots for maximum displacement magnitude for load case "Bottom edge Min F_{radial}". It is concluded that a mesh with global edge length of 8mm has converged reasonably and is suitable for further study.

SOLID MESH - TOPOLOGY OPTIMIZATION

The thesis project deals with topology optimization and therefore key criteria to study are the compliance values from the TO study as well as the resulting designs. Compliance is the parameter used to study the mesh convergence considering the rest of the project will be monitoring compliance values as well. If the compliance results after TO vary significantly with changing mesh sizes, it will be hard to draw conclusions of larger patterns in the TO process. Therefore, the compliance value is studied.

Similarly, the design must not vary significantly either with changing mesh sizes which is a well studied phenomenon with TO and is called mesh dependency. Intuitively, reducing the element size would imply the design would be more detailed and



(a) 14 mm (b) 12 mm

(c) 10 mm (d) 8 mm

Figure 2.11: Contour plot with magnitude of maximum displacement (in mm) for different global edge lengths (Load Case: Bottom Edge Min F_{radial})

exhibit better definition of structure boundaries. Larger element sizes are computationally cheap, however, they are unable to provide a feasible result as the design detail is then restricted by the size of the elements. Ideally, one would desire the smallest possible element size so that a smooth design result is obtained - this is often limited by the computational resources available. Smaller elements means more number of elements which means more design variables and therefore more computational resources.

The mesh dependency of TO adds another problem to reducing element sizes- Instead of adding more detail to the final design, a finer mesh leads to a qualitatively distinct design consisting of micro-structures as it is, numerically, a more efficient use of material ([7], [37]). Therefore any mesh refinement studies should compensate for this phenomenon for a conclusive mesh convergence. To that end, a minimum member size constraint is implemented which limits the minimum size of structures to $30mm$. This forces the TO process to avoid micro-structures and produce similar designs for varying mesh sizes.

The following mesh convergence study aims to obtain the smallest element size for the highest design detail but due to a limitation on computational resources for this project, the smallest element size obtained is a global edge length of $9mm$.

A converged mesh for the rest of the study would have a converged compliance value as well as a converged final design. Mesh convergence for the solid mesh on the design envelope has been performed through topology optimization tasks. The surface mesh on the fuselage section is kept constant while the solid mesh global edge length is varied. The objective function described in Equation 2.1 is formulated to minimize the average compliance of all the 36 load cases with a constraint on the mass fraction to be 0.037 (as explained in 2.3.4)

$$minimize: f(C_1, C_2, C_3, ...C_{36}) = \frac{\sum_{n=1}^{36} C_i}{36} \tag{2.1}$$

where,
C_i represents the compliance of the structure under load case i.

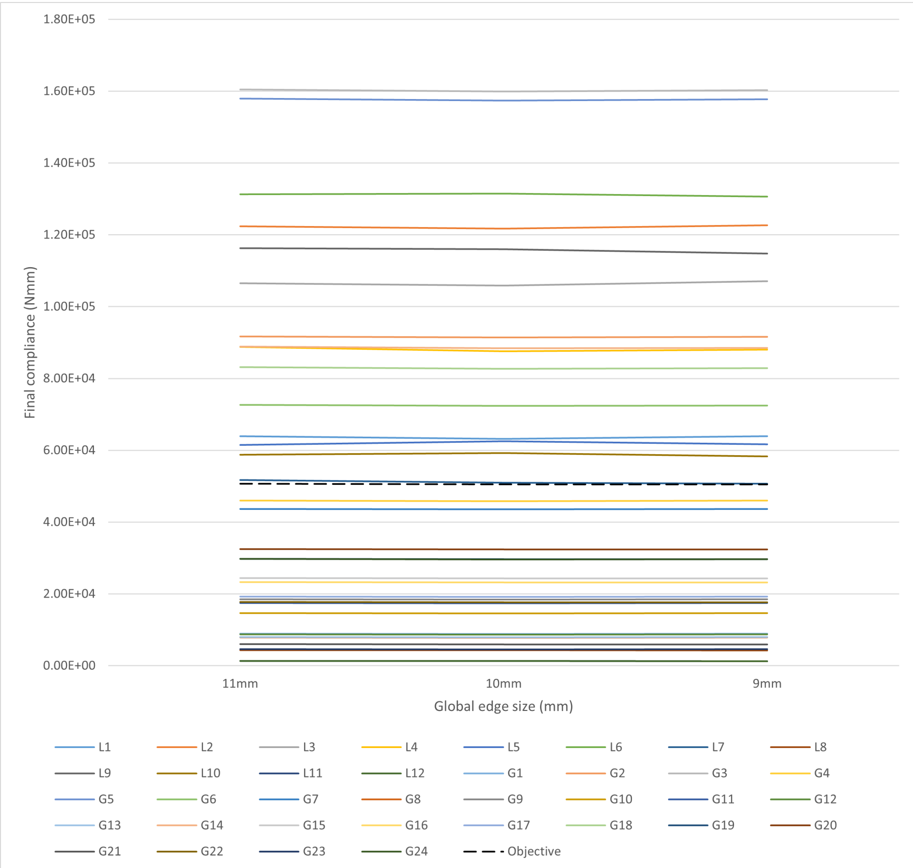
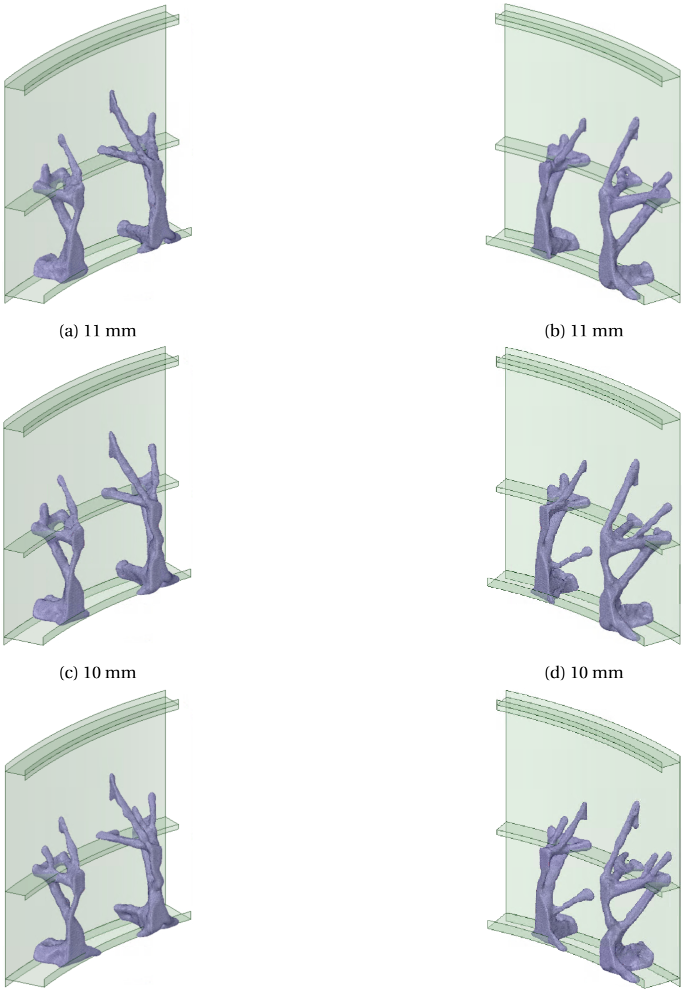


Figure 2.12: Mesh Convergence study for solid mesh on design envelope using final compliance values

In Figure 2.12, the final compliance values are provided for each of the 36 load cases (12 local load cases and 24 global load cases). It is concluded that for compliance values, a global edge length of 9mm is reasonably converged.

Figure 2.13 shows the smoothed final designs from the solid mesh convergence study. Due to the minimum member constraints, there is no appearance of micro-structures and all designs are largely similar. There are still minor differences in the design due to mesh refinement as can be seen in Figure 2.13. This implies that more detail is available in the smaller meshes without adversely affecting the micro-structure or final compliance values. The $9mm$ is considered to be reasonably converged in terms of final design as well.

The LSD model is fixed with a mesh size of $8mm$ for the fuselage section and a mesh size of $9mm$ for the design envelope.



(a) 11 mm (b) 11 mm

(c) 10 mm (d) 10 mm

(e) 9 mm (f) 9 mm

Figure 2.13: Topology optimized designs for solid mesh convergence study left and right views

3

LOAD CASE SELECTION

This section contains the description of the load case selection process. First, the concept of load case combinations is described in the context of this thesis. Subsequently, studies are performed which consider the effect of using individual load cases or combination load cases. The cantilever beam model and the LSD model are used in these studies. The section is concluded with the final set of load cases that will be considered for the next chapter.

3.1. LOAD CASE COMBINATIONS

In Section 2.3.2, 12 local load cases and 24 global load cases were listed. The local loads are applied on the component through the load application regions and the global load cases are acting on the component through the fuselage. If the local and global load cases act independently, the design will consider 36 individual design load cases. In reality however, a combination of the local and global load cases act on the component. The component is to be designed for scenarios where it experiences local load and global load simultaneously. Each design load case then consists of one local and one global load case. Therefore, under simultaneous load application, there are 12 local load cases times 24 global load cases i.e. 288 design load cases.

This significantly increases the absolute number of load cases to be designed for, as can be seen above. When considering failure criteria, it is imperative that all possible combinations be tested as different combinations produce different excitations in the component. In a compliance-based topology optimization design process however, this may not hold true. As is described by Krog et al.[21], considering a combination of local and global load cases leads to the TO process favouring local load cases. In the case of a combination load case which consists of one local and one global load case, the local load results in more displacement than the global load and is therefore a dominating value in the combination. This results in a poor stiffness for the global load cases.

3.2. SEPARATE V/S SIMULTANEOUS LOAD APPLICATION

To assess the effect of using individual load cases (separate load application) versus combination load cases (simultaneous load application), the following experiments on the CB and LSD models have been performed.

3.2.1. CANTILEVER BEAM MODEL

3

In both CB experiments, two scenarios are considered: one with all loads being applied separately and one with all loads applied simultaneously.Using the objective function of average compliance, each experiment results in two designs which are then evaluated for each load case (individual load cases and one combination load case).

LOAD CASE DEFINITION

In CB-1, four load cases are created - 3 individual load cases as described in Section 2.2.3, and 1 combination load case which consists of the 3 individual load cases acting simultaneously. Similarly for CB-2, there are 2 individual load cases, and 1 combination load case which consists of the 2 individual load cases acting simultaneously.

(a) Combination load case for CB-1 (b) Combination load case for CB-2

Figure 3.1: Combination load cases for cantilever beam model

RESULTS AND DISCUSSION

Figure 3.2 and 3.3 show the final topology optimized designs for the CB models under separate and simultaneous load applications - they are named individual design and combination design respectively. Immediately it is seen that the designs differ significantly for separate and simultaneous load application. Therefore it is certainly not an equivalent design process and different performances are expected.

In Figure 3.2b, a lot of material has been removed from the +Z face of CB-1 in the combination design which would have stiffened against the torsion load case. This can

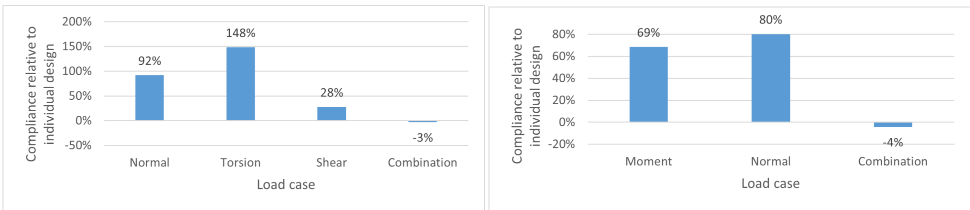
(a) Separate load application (b) Simultaneous load application

Figure 3.2: Topology optimized designs for CB-1

(a) Separate load application (b) Simultaneous load application

Figure 3.3: Topology optimized designs for CB-2

also be confirmed by the performance values in Figure 3.4a which depicts the compliance of the combination design relative to that of the individual design. The combination design has a much higher compliance for the torsion load case compared to the individual design. In fact, it is has higher compliance values for all load cases when considered individually. It is intuitive to an extent as reducing the compliance of individual load cases is not the objective of the design process. The objective is to reduce the compliance of the combination load case which it does manage to do better by 3% in CB-1. A similar trend is noticeable in CB-2 where the simultaneous load application design offers a significantly poor performance for the individual load cases and an improvement of 4% for the combination load case.



(a) CB-1 (b) CB-2

Figure 3.4: Combination design compliance performance

It can be concluded that in this case the simultaneous load application model cannot be a substitute for the separate load application model as it would lead to a much poorer performance. The converse, however is not entirely true. If the compliance performance is considered, although the designs differ significantly, both load application models result in designs which are different to each other in performance by 3% – 4% - with the combination design being the marginally better one. Therefore, when designing for the combination load case, the separate load application model can be a substitute for the simultaneous load application model - given that a compliance performance is the driving factor and a difference of 3% – 4% in compliance is not considered significant. With the CB models, there is no need to make that substitution but in the next section since there are many more combination load cases than individual ones, this possibility will become an attractive alternative.

3.2.2. LAUNCHER STRUCTURE DEMONSTRATOR MODEL

The LSD model is used to assess the effect of using the 36 individual load cases versus using 288 combination load cases for the topology optimization task. The objective is formulated as one of average compliance similar to the one described in Equation 2.1 with the constraint that the final mass be equal to the mass of the OLS. The goal of the following experiment is to finalize the design load cases for the next step where the different objective functions are compared.

LOAD CASE DEFINITION

Two scenarios are considered, one with the local and global load cases taken as individual load cases and the second considering combination load cases where each combination load case consists of one local and one global load case. Figure 3.5 visualizes the different scenarios and also how the objective function is formulated for each. When considering individual load cases, a total of 36 load cases are taken into account and the resulting design is hereafter referred to as "individual design". Using the realistic loading conditions, however, the local and global load cases are applied simultaneously on the component and a total of 288 load cases are considered. Each load case consists of one local and one global load case. The resulting design is hereafter referred to as "combination design".

Given the large number of combination load cases, they have been organized into *combination groups* which are depicted in Figure 3.6. Each combination group refers to the collection of 12 combination load cases which share a global load case. The first combination group, for example, consists of combinations of each local load case with the first global load case. Results will be presented as a function of the combination group and not 288 individual cases.

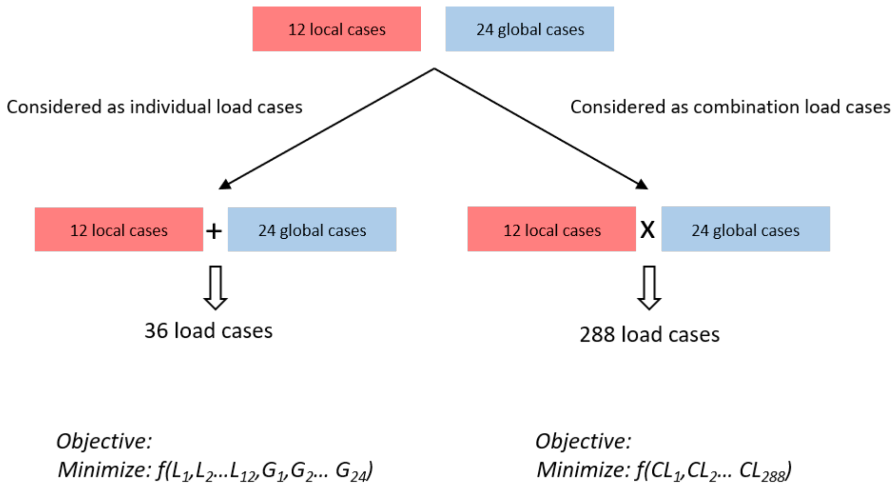


Figure 3.5: Load case definitions for LSD model

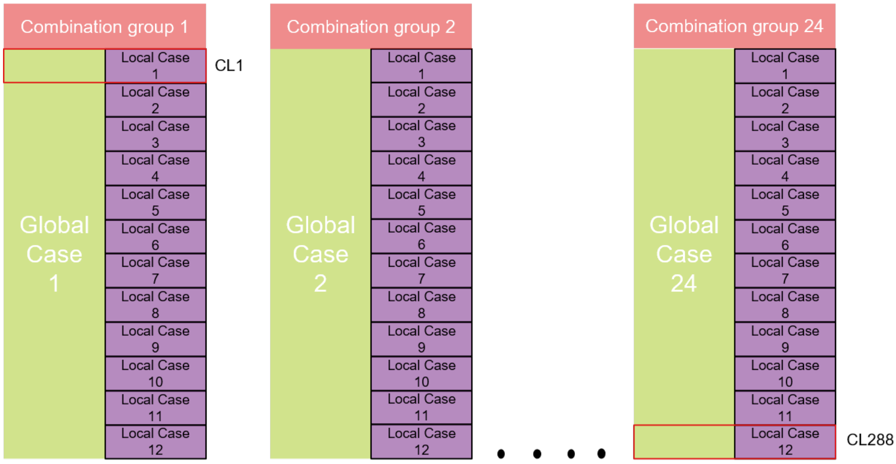
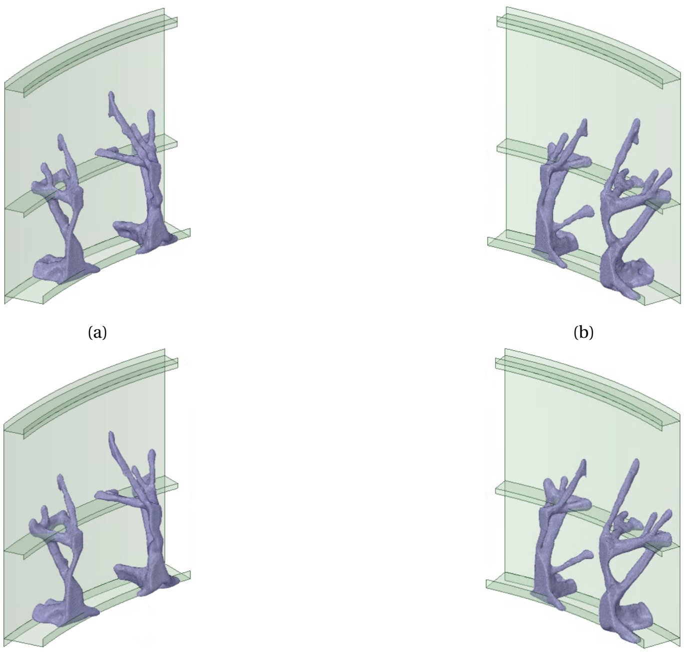


Figure 3.6: Load case definitions for LSD model



(a) (b)

(c) (d)

Figure 3.7: Topology optimized results for individual design (a,b) and combination design(c,d)

DESIGN RESULTS

The combination design takes 3× the time to run compared to the individual design and double the scratch memory. Figure 3.7 shows the topology optimized designs for individual and combination load cases. Unlike the CB models, no significant design differences can be noticed between the individual load case design and the combination load case design.

Both structures exhibit truss members connecting the load application areas to the fuselage panel as well as the ring frames. The number, positions and orientations of the truss members seem to be the same in both designs. The designs appear as if both structures have been designed for the same loads. Some minor differences can still be seen but largely the designs are almost identical.

COMPLIANCE RESULTS - INDIVIDUAL LOAD CASES

Figure 3.8 shows the reduction in compliance for each load case from its initial value (given by the grey line) and it is clear why the designs look so similar. In both designs, the local loads have been given preference in the optimization process with the compliance of global cases being barely reduced. The reduction in compliance for the local load cases are in the range of 98% – 99% whereas the global load case compliance values have been reduced by 1% – 5%. The reduction in compliance appears to follow the magnitude of the load cases - higher magnitude load cases get higher reduction in compliance. This disparity will be further discussed in the next chapters, till then it is important to note that both designs have addressed both types of load cases in a similar way and have thus arrived at a similar topological result.

The combination design is analyzed for the individual load cases and the compliance relative to the individual design is presented in Figure 3.9. In the figure, the Y axis shows the relative difference of the compliance with respect to the individual design. The positive values indicate that the combination design has a higher compliance than its counterpart and similarly a negative value indicates a lower compliance. The combination design exhibits a lower compliance for all local load cases and most global load cases. This is contrary to the results obtained with the CB model and also a counter intuitive result as these individual load cases are not the objective of the combination design during the topology optimization process.

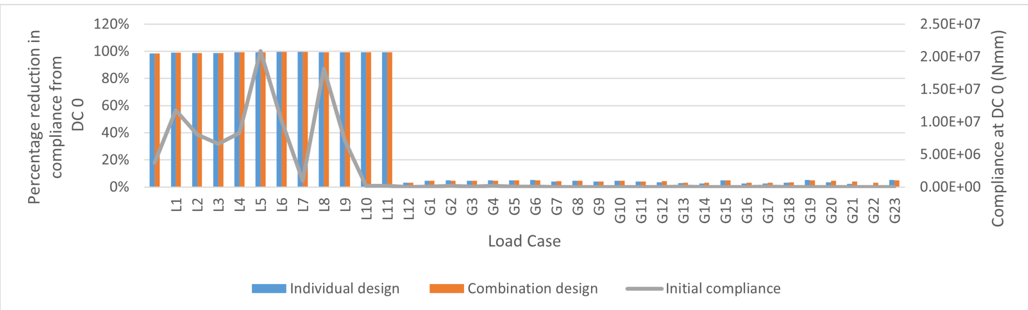
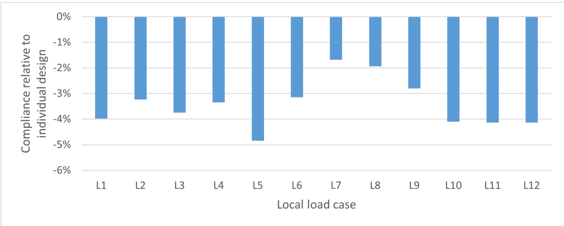


Figure 3.8: compliance reduction from initial for individual and combination design

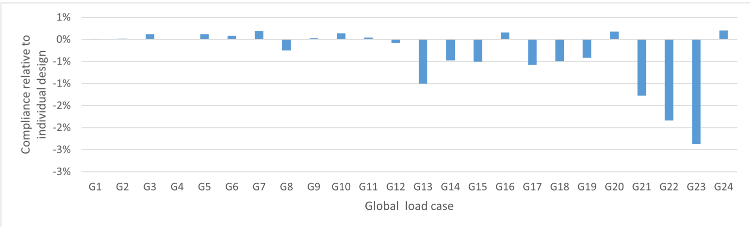
The values are somewhat low - in the range of -1% to -5% for local load cases and in the range of 0.2% to -3% for global load cases. It is clear that both the structures have addressed the load cases in a similar manner and if the values were alternating between small positive and small negative values, the deviations could have been ignored and the designs concluded to be the same. Instead, The occurrence of systematically negative relative compliance values indicates that the combination design has a marginally better design for the individual load cases and may indicate that the individual design has settled into a local optima.

RESULTS - COMBINATION LOAD CASES

To assess the combination load cases, again the relative compliance metric is used but the combination groups are used to simplify the results for the 288 load cases. Two measurements are taken from each combination group -the average relative compliance and the minimum relative compliance.



(a) Local load cases

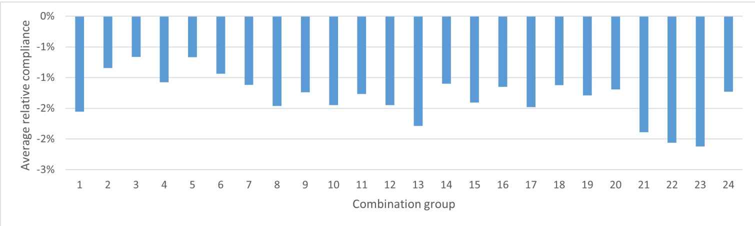


(b) Global load cases

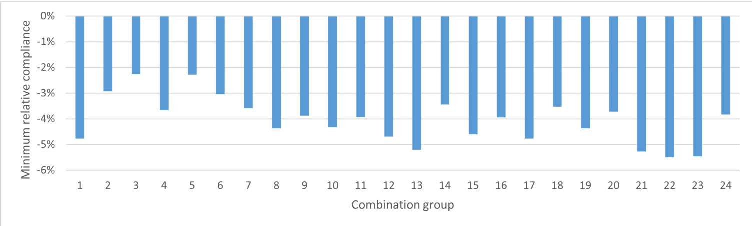
Figure 3.9: Combination design compliance relative to individual design

Figure 3.10a shows the average relative compliance for each combination group which shows that on average the combination load case design exhibits a lower compliance for the combination load cases by up to approximately 2%. Figure 3.10b shows the minimum relative compliance for each combination group. The combination design is able to obtain a stiffer design by at most 5% for some combination load cases. This is in line with the results obtained with the CB model where a marginal improvement of 3% – 4% was obtained for the combination design.

The combination design does offer a better performance but the improvement in compliance is marginal with no major design differences. For the 288 load cases and 3× the computational time, both the resulting designs have no major differences as both are mainly designed to stiffen against the local load cases. This indicates that a feasible design for the realistic loading conditions can be produced by using just the individual load cases. The results from the above comparison show that the number of load cases to be considered can be reduced if combination load cases exist.



(a)



(b)

Figure 3.10: Average (a) and minimum (b) relative compliance values for each combination group

3.2.3. SUMMARY

This section investigated the difference between separate and simultaneous load application models. Experiments were conducted with simple CB models where loads were considered as separate load cases and also as a single combination load case. It was shown that while comparing final compliance values, both models would produce a final result within a difference of 3%−4% for the combination load cases. This showed that when considering a combination load case, both separate and simultaneous load application models can be used. Along the same lines, a similar experiment was conducted for the LSD model. Two scenarios were considered - one with 36 individual load cases and one with 288 combination load cases (created from the 36 individual load cases). Although the combination design takes three times as long to run due to the large number of load cases, the resulting designs did not exhibit any significant design differences and the benefit in compliance was low. The results show that the trade-off of using the 36 individual load cases (instead of 288) is marginal taking into account the minor qualitative design differences, reduced computational effort and a much more manageable number of load cases. It is concluded that moving forward the individual load cases should be used in the design process.

4

Objective function selection

This section describes the selection process for the objective function to be used in the topology optimization process. Some objective functions were identified in the literature study which are commonly used to deal with multiple load cases. A description is provided of the different objective functions which are used for topology optimization under multiple load cases and how they will be implemented in the design process. Following which, a comparative study is performed on the cantilever beam and launcher structure demonstrator models to assess the objective functions. A performance metric is introduced which is used to assign performance scores to each objective function and a final selection of objective function is made.

4.1. MLC objective functions

In compliance-based topology optimization, the standard objective function is one of structural compliance minimization with a constraint on the volume fraction. When considering a component under a single load case, the objective is to reduce the structural compliance of the component under that load case. When considering multiple load cases, the objective function must be formed as a function of the compliance of the structure under each load case. The function must aggregate the structural compliance under each load case in an appropriate way which would lead the optimization algorithm to minimize the compliance of each load case.

4.1.1. Average (AVG)

The most straightforward objective function for multiple load cases is minimizing the average compliance of all the load cases and was introduced in Section 2.3.6 as Equation 2.1. It assigns equal weight to each load case; in this thesis it is taken as the most basic one and is used to conduct the mesh convergence studies as well as some preliminary experiments in Chapter 3.

4.1.2. WEIGHTED-SUM (WS)

The previous objective function can be considered as a special case of the weighted sum function given in equation if all the weights are equal to $1/N$. WS allows the designer to assign appropriate weights to each load case to represent their individual importance. The weights are chosen as per the requirement of the designer from the component to be optimized. In the situation where some load cases are more critical than others, larger weights can be assigned. One of the limitations of this function is the choice of weights which is to be done by the designer based on experience or some prior knowledge of the component and the load cases.

$$minimize: f(C_1, C_2, C_3, ...C_N) = \sum_{n=1}^{N} w_i C_i \tag{4.1}$$

where,
C_i represents the compliance of the structure under load case i.
w_i represent the weight assigned to load case i

For this thesis, weights are chosen based on the results from the TO result using AVG. The final compliance values of the load cases after optimization are used to calculate the respective weights for WS. The reason for selecting weights in such a way is to improve the design based on previous results. Through this procedure, load cases which exhibit higher compliance even after optimization, will be assigned larger weights.

$$w_i = \frac{C_i^*}{\sum_{n=1}^{N} C_i^*} \tag{4.2}$$

where,
w_i represent the weight assigned to load case i
C_i^* represents the final optimized compliance of the structure under load case i using AVG.

4.1.3. DYNAMIC WEIGHTED-SUM (DWS)

Dynamic weighted sum introduces a way to automatically assign weights to the different load cases based on their relative magnitude. The name is so chosen because the weights constantly change during the course of the optimization process. The objective function's formulation is able to update the weights for the load cases based on how the optimization proceeds.

$$minimize: f(C_1, C_2, C_3, ...C_N) = \frac{\sum_{n=1}^{N} C_i^2}{\sum_{n=1}^{N} C_i} \tag{4.3}$$

where,
C_i represents the compliance of the structure under load case i.

4.1.4. MINMAX (MM)

Often aerospace components must take into account the worst case loading conditions and therefore must be designed to withstand those. The objective can be formulated such that it minimizes the largest compliance of all load cases. This is a way of identifying the most critical load cases so that the component can be designed for the worst case scenario.

$$minimize: f(C_1, C_2, C_3, ...C_N) = max((C_1, C_2, C_3, ...C_N) \tag{4.4}$$

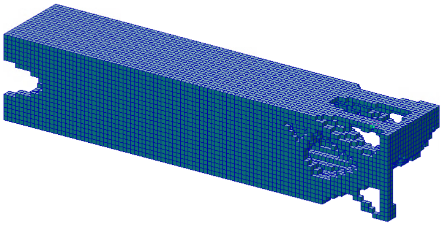
where,
C_i represents the compliance of the structure under load case i.

4.2. CANTILEVER BEAM MODEL

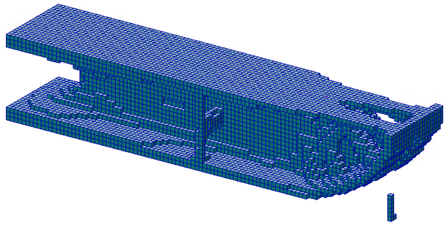
In the two CB models, the different objective functions were used to perform topology optimization. Comparison of different objective functions was performed on the basis of how much the compliance of each load case is reduced from the initial value.

4.2.1. RESULTS

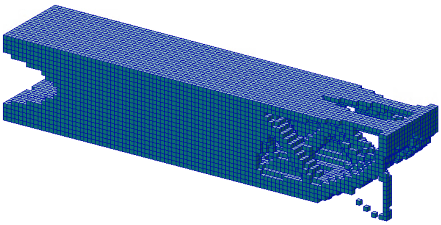
The result designs after topology optimization using the different MLC objective functions is provided in Figure 4.1 and 4.2 for CB-1 and CB-2 respectively. Additionally, single load case topology optimization designs were also generated for both CB models for comparison in Appendix A.1. As can be seen, each objective function leads to a different final design and therefore the compliance values are also expected to vary as is confirmed by figures 4.3b & 4.3c. There is a trend of a higher compliance reduction associated with larger loads and smaller loads receiving less compliance reduction. Figure 4.3a shows the initial compliance value for each load case i.e the compliance of the structure when all the elements are initialized to a density value of 0.4. These initial compliance values give a sense of the magnitude of each of the load cases. Looking at the CB-1 results in Figure 4.3b, the shear load case which has the highest magnitude receives the maximum compliance reduction under each objective function, next comes the normal load case and finally the torsion load case with the lowest compliance reduction values. Similarly, for CB-2 Figure 4.3c shows that the normal load case receives more reduction in compliance than the moment load case.



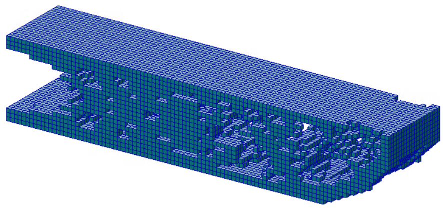
(a) AVG



(b) DWS

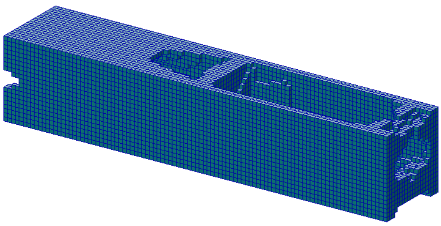


(c) WS

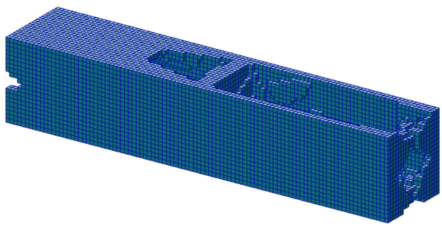


(d) MM

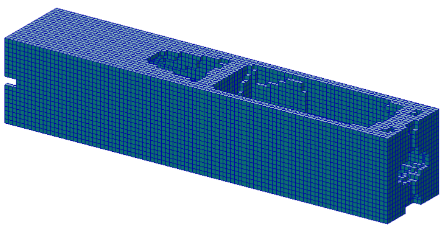
Figure 4.1: Result designs for topology optimization of CB-1 model using different objective functions



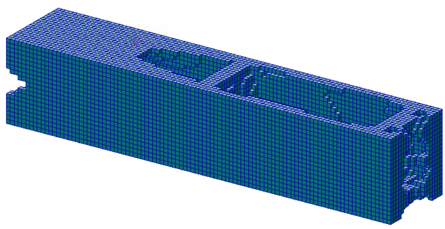
(a) AVG



(b) DWS

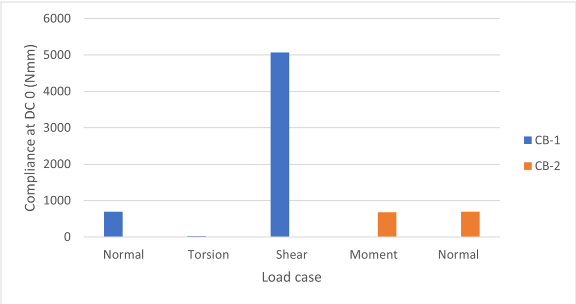


(c) WS

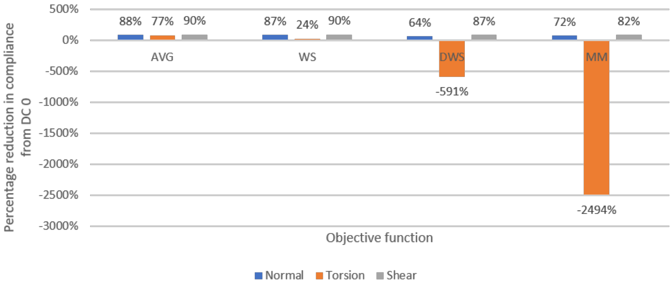


(d) MM

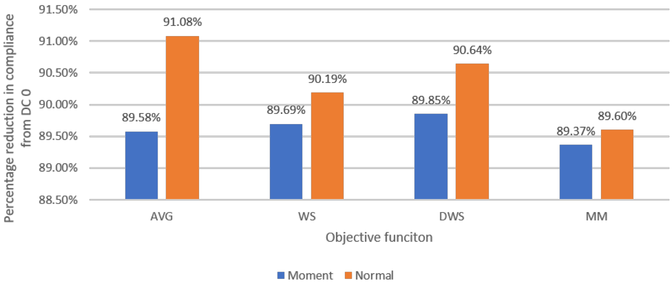
Figure 4.2: Result designs for topology optimization of CB-2 model using different objective functions



(a) CB-1

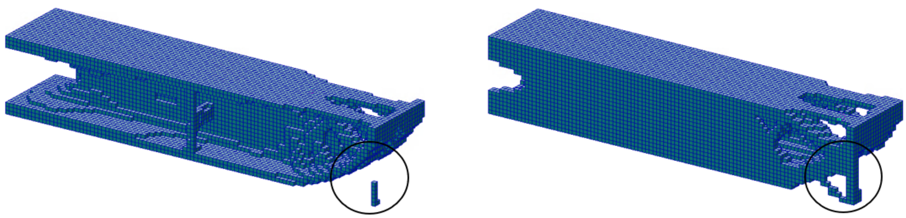


(b) CB-1



(c) CB-2

Figure 4.3: Compliance reduction from initial value at DC0 using different objective functions



(a) DWS (b) AVG

Figure 4.4: Absence of vertical members for torsion load application in (a) DWS result as compared to (b) AVG result

4

DWS and MM exhibit drastically poorer performance for the torsion load case and this can be seen in Figure 4.4 which highlights a vertical member present in the AVG result but not in DWS. The vertical member is for the torsion load application, and the neglect of this load case manifests as reducing the density for the vertical member elements. It is important to note that, since the result is obtained by retaining elements with a density value of 0.4 and above, the material for the vertical members still exist in the solution but is just of lower density. This is also the reason why floating members can be seen in the designs - the connecting elements have been removed due to the threshold value of 0.4.

A similar trend is seen in Figure 4.3c which shows the compliance results for CB-2 and Figure 4.2 which shows the result designs. Though difference between the compliance reduction values of the two load cases are much less compared to CB-1 which is also seen through the similarity of designs, the larger magnitude load case still receives a higher compliance reduction. The normal load case is the larger magnitude load case and therefore receives a better compliance reduction. Clearly, the focus a load case gets is directly related to its relative magnitude. The lack of stark differences in design and disparities in compliance values with CB-2 can be attributed to the relative load case magnitudes - The difference between the CB-2 load cases is far smaller than those of CB-1. Therefore, A higher difference in the load magnitudes leads to a higher difference in compliance reduction.

In CB-2, AVG results are better for the normal load case and DWS is better for the moment load case but the performances of all the objective functions do not vary significantly due to the magnitude of both load cases. In CB-1, it can be seen that AVG offers the stiffest design under normal and torsion load cases with the shear load case compliance being slighter higher than that of WS. DWS and MM have drastically poor performance for the load cases which have a lower relative magnitude due to the nature of the objective function.

Looking at the two poorly performing objective functions: MM aims to minimize the largest load case compliance and therefore, the compliance of the torsion load case, due to its much lower magnitude, is never the objective. The goal of the MM objective function remains the shear load case throughout the optimization for CB-1, and any change in the other load cases is purely inadvertent. The neglect of the torsion load case results in a higher compliance than the initial value. In the case of DWS, the weights calculated during the optimization for the torsion load case are always much smaller due to its relative magnitude to the other load cases. As the optimization progresses, the difference between the torsion load case and the others reduces and the weight assigned for torsion increases. Overall, there is a bias towards the higher magnitude cases and the torsion load case is still neglected leading to an increase in compliance - but the effect is not as bad as in the case of MM. It should also be noted that AVG results in a lower compliance for the shear load case overall and not the MM, even though the formulation of MM is focused on minimizing the largest load case. This is repeated in CB-2 where AVG results in a better design for the normal load case compared to MM.

Given that the relative magnitudes of the load cases directly influence their respective reduction in compliance, the compliance values can be normalized. Such a normalization should improve the result especially for load cases of different orders of magnitude as is seen in CB-1. Taking a uniform normalization value would not serve the purpose as the differences in magnitude will still remain so each load case must be normalized by a different value. The respective initial compliance values are chosen for the normalization of the objective function and the formulation is described in Equation 4.5. The weights for the normalized weighted sum in this case are chosen from the results of the average objective function.

$$minimize: f(\overline{C_1}, \overline{C_2}, \overline{C_3}, ...\overline{C_N}) \quad ; \overline{C_i} = \frac{C_i}{C_i^{initial}} \tag{4.5}$$

where,
$C_i^{initial}$ represents the compliance of the structure under load case i at design cycle 0.

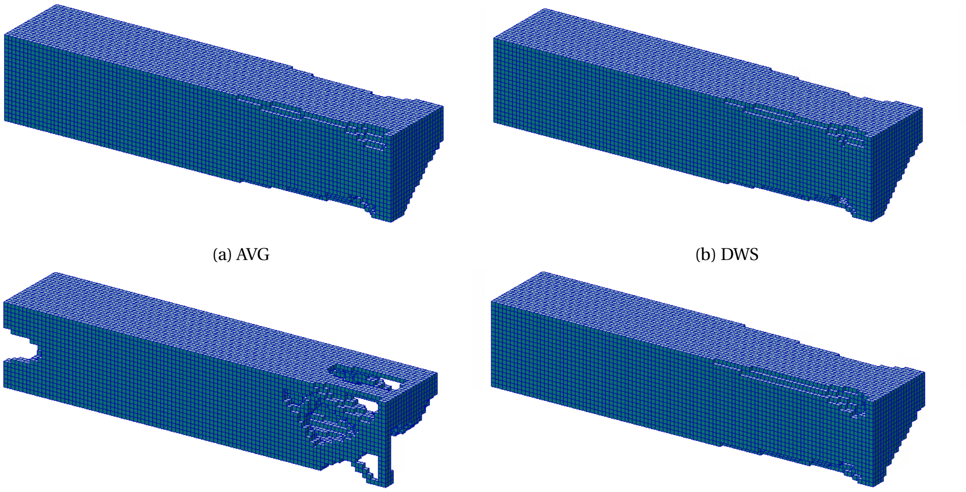
The objective functions are named N-AVG, N-WS, N-DWS and N-MM to distinguish them from the non-normalized functions.

4.2.2. RESULTS - NORMALIZED OBJECTIVE FUNCTIONS

Figures 4.5 & 4.6 show the results of the topology optimization design process when each compliance in the objective is normalized by its respective initial compliance value. The effect of normalization is instantly seen as the performance of each objective function is improved in CB-1. While N-AVG, N-DWS and N-MM have similar final values due to the normalization, the N-WS result stands apart as the final compliance values represent the relative weights assigned to each load case.

Figure 4.7 shows the relative change in compliance values compared to the results using non-normalized objective functions. For CB-1, apart from the reduction of compliance due to the normalization, it can also be noticed that the improvement in the values is a trade-off with the compliance for the shear load case, however, it is not nearly as much as the improvement achieved. An improvement in the compliance values of 15% and 58% for the normal and torsion load cases is achieved at a cost of 4% for the shear load case using N-AVG. In CB-2 however, similar results are not seen as the relative change in compliance remains between 0.7% – 1.5%.

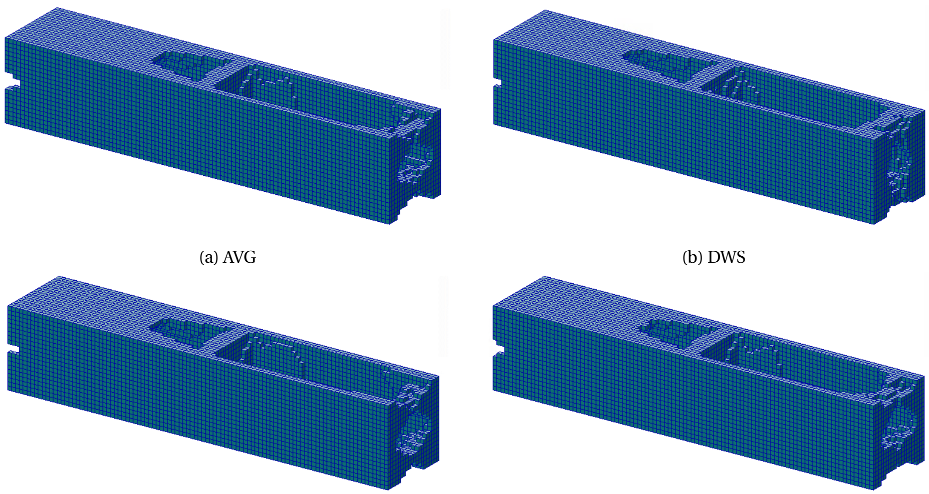
We can compare all the objective functions by comparing the compliance value for each load case with the minimum compliance achieved for that respective load case. Figure 4.8 lists out the percentage difference between the load case compliance and the minimum compliance for a load case. A value of 0% implies that objective function re-



(a) AVG (b) DWS

(c) WS (d) MM

Figure 4.5: Result designs for topology optimization of CB-1 model using normalized objective functions

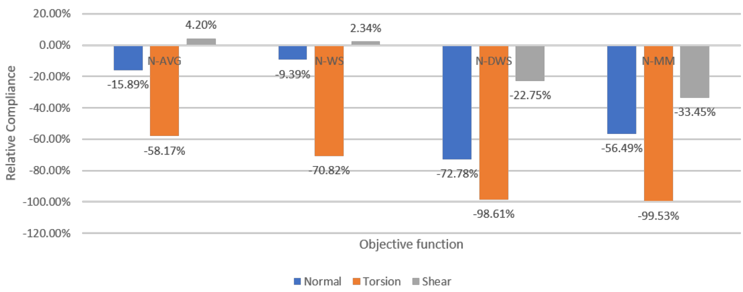


(a) AVG (b) DWS

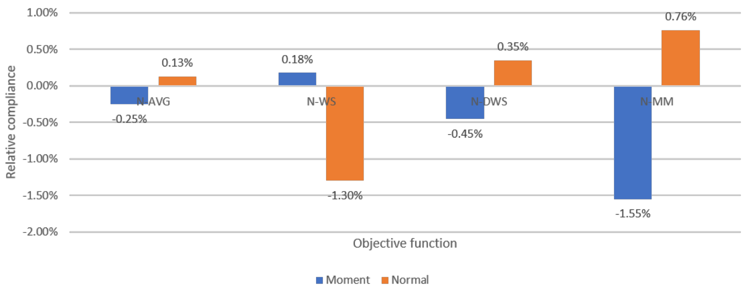
(c) WS (d) MM

Figure 4.6: Result designs for topology optimization of CB-2 model using normalized objective functions

sults in the lowest compliance compared to all other objective functions. A positive value represents a worse performance and a negative value represents a better performance. From this list we can conclude that N-AVG and N-DWS achieve better results in the case of CB-1 and AVG coming out on top for CB-2. It can be concluded that when there are load cases of varying magnitudes present, normalization as put forward in Equation 4.5 can be used to remove the heavy bias towards large magnitude load cases. In the case that the load cases are of similar magnitude already, normalization is not required.



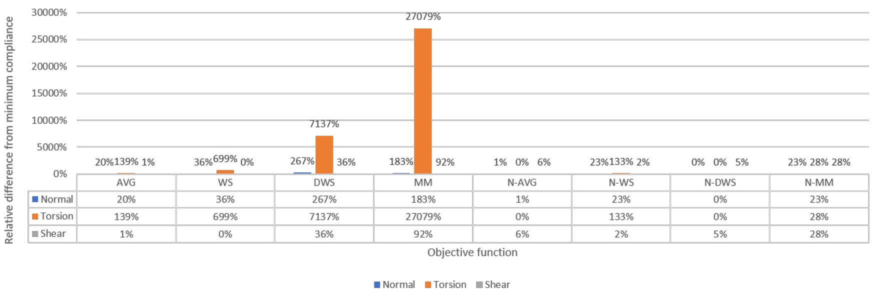
(a) CB-1



(b) CB-2

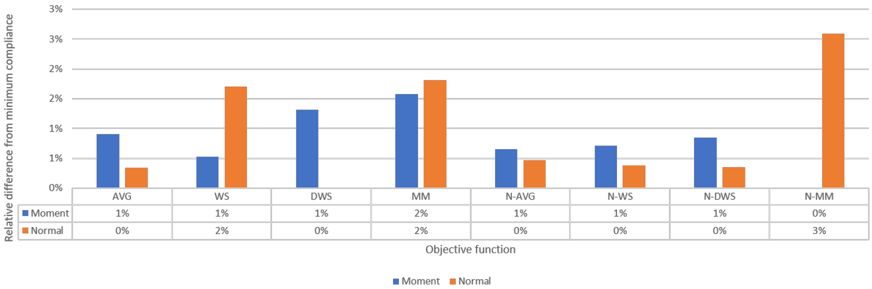
Figure 4.7: Compliance of the designs produced using normalized objective functions relative to designs produced using non-normalized objective functions

4



	AVG	WS	DWS	MM	N-AVG	N-WS	N-DWS	N-MM
Normal	20%	36%	267%	183%	1%	23%	0%	23%
Torsion	139%	699%	7137%	27079%	0%	133%	0%	28%
Shear	1%	0%	36%	92%	6%	2%	5%	28%

(a) CB-1



	AVG	WS	DWS	MM	N-AVG	N-WS	N-DWS	N-MM
Moment	1%	1%	1%	2%	1%	1%	1%	0%
Normal	0%	2%	0%	2%	0%	0%	0%	3%

(b) CB-2

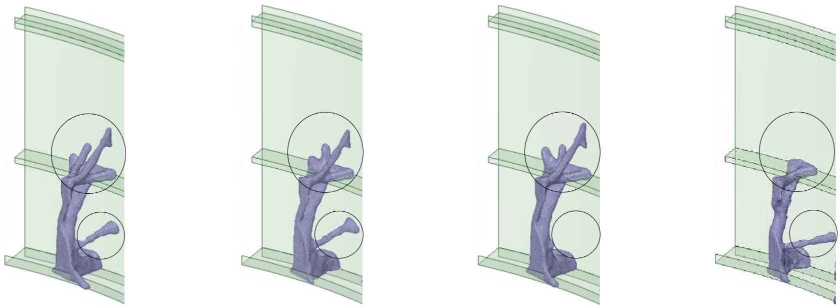
Figure 4.8: Relative difference from minimum compliance

4.3. LAUNCHER STRUCTURE DEMONSTRATOR MODEL

Topology optimization of the LSD model is run with all the objective functions discussed in Section 4.1 and the results are presented. The resulting designs as well as the final compliance results are studied to gain insight into the topology optimization process and the performances of each objective function. Finally a performance metric is put forward in an effort to compare the performances of the objective functions. Using the performance metric, a final selection of the objective function is made which will be used to obtain the final design.

4.3.1. DESIGN RESULTS

The designs consist of load application regions connected to the fuselage section through truss structures. The new structure attaches itself to the fuselage panel along the middle and lower ring frames leaving out the top ring frame unlike the OLS which attaches to all three ring frames. The overall design characteristics remain the same with some differences that are highlighted in Figure 4.9.

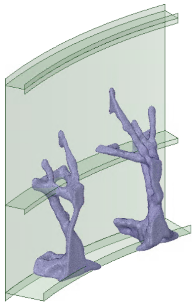


(a) AVG (b) WS (c) DWS (d) MM

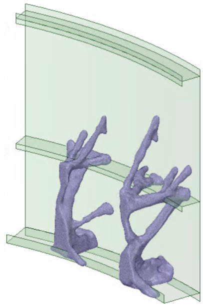
Figure 4.9: Differences in result design for TO of LSD using different objective functions

The resulting designs from the normalized objective functions are provided in Figure 4.11 which show very different designs compared to the non-normalized objective functions. The major difference compared to the previous designs is that the left part of the component has more material. More material has been allocated to the left side of the component, essentially taking it out of the right side which is seen in the much more skeletal structure on the right side. An exaggerated effect of this can be seen in Figure 4.11e & 4.11f where a lot of floating material can be seen with a large aggregation of material on the left side of the fuselage. Similarly, in figures 4.11g & 4.11h, material has aggregated in the bottom of the design volume creating a very infeasible design. The design process clearly does not work as intended.

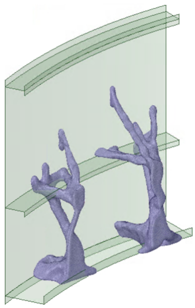
This material redistribution can be explained by the fact that the left side of the fuselage section is more compliant toward the global load cases and these designs stiffen those load cases. The normalization as was seen in Section 4.2.1 helps to eliminate the disparity in load case stiffening due to the difference in magnitude of the load cases. The normalized objectives would, therefore, similarly stiffen the smaller magnitude load



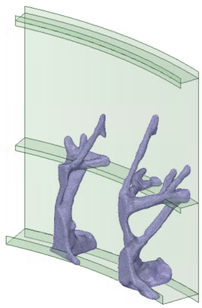
(a) AVG



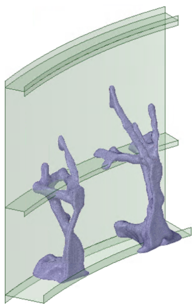
(b) AVG



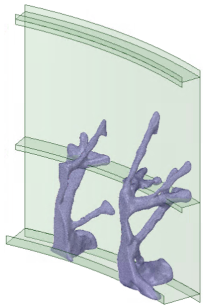
(c) WS



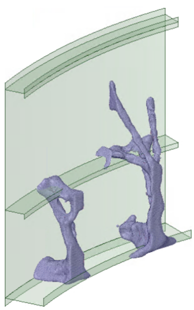
(d) WS



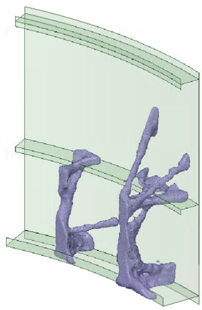
(e) DWS



(f) DWS

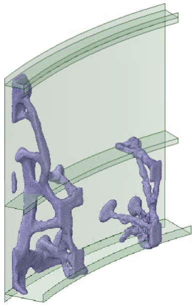


(g) MM

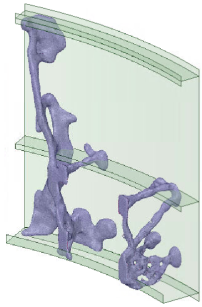


(h) MM

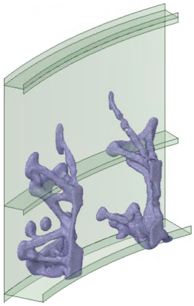
Figure 4.10: Result designs for topology optimization of LSD model using different objective functions



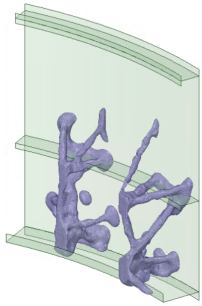
(a) N-AVG



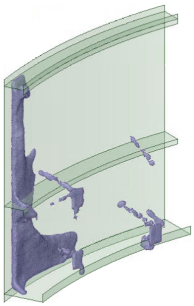
(b) N-AVG



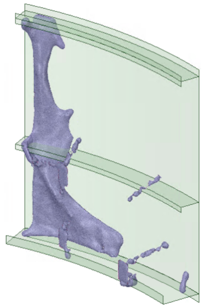
(c) N-WS



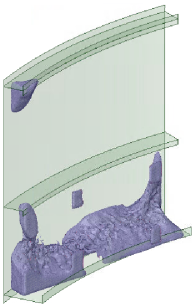
(d) N-WS



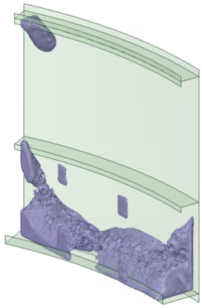
(e) N-DWS



(f) N-DWS



(g) N-MM



(h) N-MM

Figure 4.11: Result designs for topology optimization of LSD model using different objective functions using normalization

cases more which happen to be the global load cases - this will be seen in the next part as we study the resulting compliance values for these designs.

The normalization, contrary to what was seen with the CB models, does not work well with the N-MM and N-DWS designs. The design adds mass disproportionately to the edges of the design domain - presumably to address the global load cases - with a lot of intermediate density and floating elements. Removing a sense of relative magnitude from the load cases for these objective functions, does not result in a feasible design. In these two cases, a bias is seen towards the global load cases given the mass allocation at the edges of the domain.

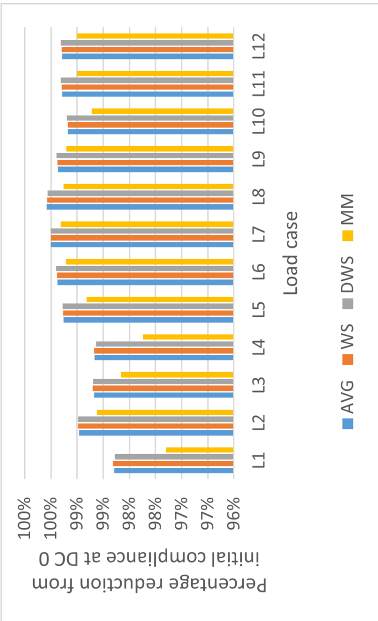
4.3.2. PERFORMANCE RESULTS

Figure 4.12 shows the compliance reduction of the local and global load cases for the different objective functions. Figure 4.12a shows that the MM design, similar to the CB model in Section 4.2.1, is inferior in almost all cases even for the largest load magnitude (L_6). The other non-normalized objective function appear to have similar performances - the differences will be discussed in the next section. For now, it is interesting to note that even with the LSD model MM does not exhibit a superior performance even for the largest load cases. It essentially does not achieve the goal of its formulation as it is outperformed by AVG, DWS and WS. Therefore if the goal is to minimize the most critical load case, it would be more beneficial to use any of the other three objective functions.

The results using the normalized objective functions are provided in Figure 4.12b. It shows a poor performance of N-DWS and N-MM which is expected, looking at the designs in Figure 4.11. The result designs from the normalized objectives show a preference of material distribution towards the left side of the component which, as previously, discussed is the side more affected by the global load cases. The compliance values confirm this assumption. All designs for the normalized objectives have more area in contact with the fuselage section. The N-DWS design exhibits the maximum compliance reduction values for the global load cases, which does come at a cost of lower values for the local load cases i.e. the focus on the global load cases adversely affects the design for the local load cases. N-MM on the other hand does not perform well in any load case and has the most number of floating and intermediate density elements. N-WS has the closest design and performance to the non-normalized objectives and that is due to the fixed weights which retain the relative magnitudes in the objective function.

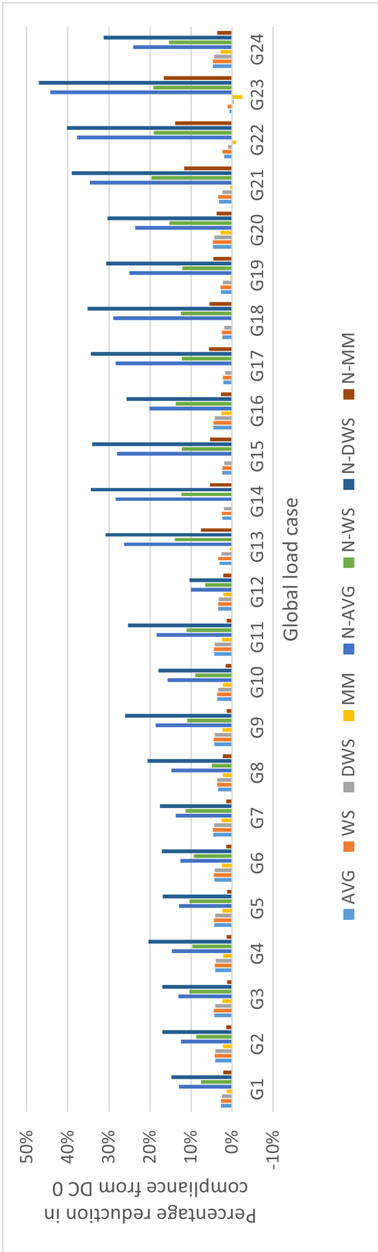
Contrary to the results with the CB model, N-DWS and N-MM do not work to produce a viable structure given the loads. The normalization in these two objective functions simply forces the optimizer to treat all load cases cases regardless of magnitude - equally. In N-DWS, the load cases with more reduction from initial would have a lower weight and therefore the weighting is equitable. In N-MM, due to the normalization all load cases start with the same "magnitude" and so the optimizer will oscillate between different load cases which can be seen in Figure 4.13 shows the oscillations in the objective value during optimization.The relatively flat gradient of the objective line also points to the problem with this formulation. Since the objective does not stick with a particular load case to minimize, it is not able to affect it significantly and ends up reducing the compliance partially for all load cases till it reaches the required mass fraction.

In Section 3, the preliminary topology optimization results showed a large difference



(a) Local load cases

(b) Local load cases - Normalized



(c) Global load cases

Figure 4.12: Compliance reduction through different objective functions

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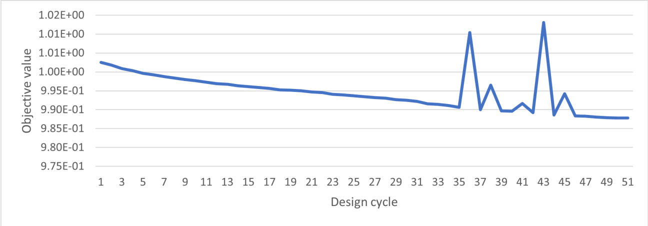


Figure 4.13: Oscillations in the optimization process using the N-MM objective formulation

in compliance reduction for the local load cases vs the global load cases which is also seen here in Figure 4.12. The compliance values of the local load cases are reduced by upto 99% whereas the global load cases are reduced by a maximum of 47% and within the global load cases, the values for the non-normalized objectives stay below 6%. Previously, the presumption was that due to the difference in load case magnitude, there is a bias towards higher magnitude load cases. This was investigated in Section 4.2 warranting the normalization method which showed improvements for the lower magnitude load cases. This is certainly true to an extent, as the figure shows the normalized objective designs result in stiffer designs for the global load cases, however, the reduction values are still low which indicates another mechanism to be present. The normalization does allow more focus toward the global load cases, but the extent of the component to stiffen against these load cases seems to be limited due to the nature of the loads and the design domain. Due to the inherent limitation, the normalized objective functions lead the design to an infeasible one as it reduces the compliance of the global load cases and as a consequence negatively affect the design for local load cases.

4.3.3. PERFORMANCE METRIC

The previous section shows the varying performances of the different objective functions for the different load cases. Studying the values in Figure 4.12, there is no clear indication of which objective function should be picked for the final design but we can disregard some based on the previous discussion: N-DWS and N-MM for their consistently poor performance and low design quality, and MM for its consistently poor performance relative to the others as well.

For the remaining objective functions, a performance metric is put forward which will help assign a performance score to each objective function. Different loads are being considered and not all of them would be considered to be equally important. Here, the importance of a load may be debated since here the component is technically to be designed to sustain the local loads while surviving the global loads. The primary purpose, in that sense can be said to be to transfer the local loads from the equipment but it must also not break under the global load environment. Objectively, a load case with high importance can be a load case which causes a high compliance in the structure. The initial compliance values of the load cases can therefore be used to indicate importance - the same value used previously as an indication of the load magnitude.

The initial compliance for a load case is the compliance of the structure (under that load case) when all its elements are set to an initial density value 0.037 in this case. The relative importance M_i of a load case i is defined by Equation 4.6. Figure 4.14 shows the relative importance values for all 36 load cases with the higher values representing higher load importance such as in local load case 6 and 9.

$$M_i = \frac{C_i^{initial}}{\sum_{i=1}^{36} C_i^{initial}} \tag{4.6}$$

We can assess the relative performance of the different objective functions for each load case by comparing the final compliance with the minimum compliance of that load case among all objective functions.
For an objective function k and load case i, the final compliance is represented by C^*_{ki}. $[C^*_i]$ represents the final compliance values of all objective functions for a load case i. The best C^*_{ki} value will be $min[C^*_i]$ (lowest compliance). Therefore, by checking their relative difference we can compare each C^*_{ki} value. The relative difference from minimum compliance for each load case is given by Equation 4.7.

$$D_{ki} = \frac{C^*_{ki} - min[C^*_i]}{min[C^*_i]} \tag{4.7}$$

Table 4.1 shows the D_{ki} values, a value of 0% represents the lowest compliance for that load case and higher the values, further away the final compliance is from the minimum compliance for that load case. N-AVG accounts for the lowest compliance values for the global load cases and DWS for most of the local load cases. The effect of the normalization can be seen in these values as the normalized objectives show the lowest values for all global load cases which comes at a cost of the local load cases.

Now we can give a performance score to each C^*_{ki} value using:

$$S_{ki} = M_i \times D_{ki} \tag{4.8}$$

We can now also give a performance score to each objective function by adding the S_{kn} values for all load cases under each objective function:

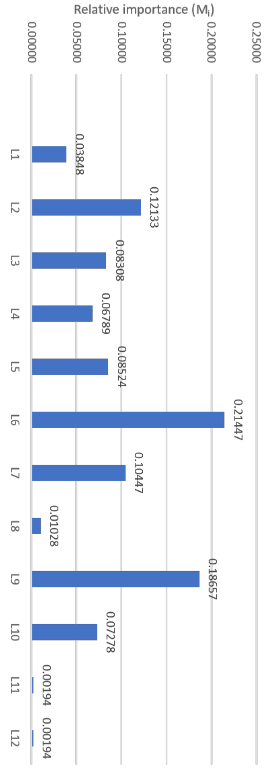
$$S_k = \left(\sum_{i=1}^{36} S_{ki} \right) \tag{4.9}$$

The lowest value of S_k indicates the best performance. If the load importance is high, a final compliance close to the minimum is preferred. Similarly, if the load importance is low, a larger difference between the final compliance and the minimum value can be tolerated. The final S_k values for each objective function are given in Table 4.2.

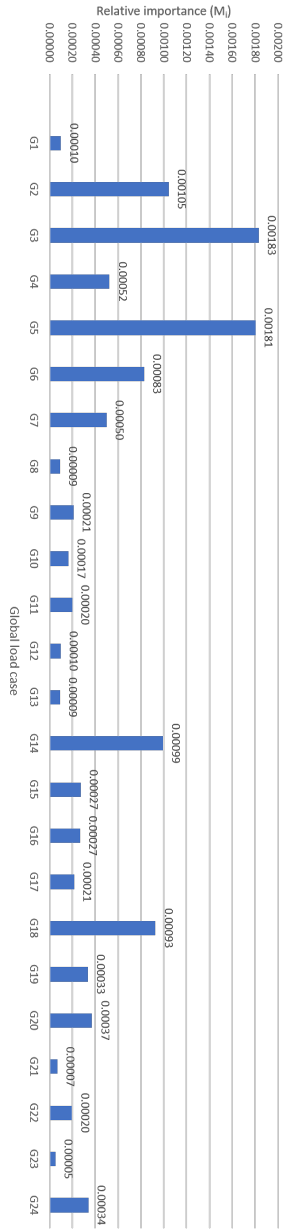
From tables 4.2 and 4.1, the following inferences can be made:

a. The advantage of using the normalized objectives for the global load cases is outweighed significantly by the adverse effect it has on the local load cases.

b. The method of choosing weights for the weighted sum method improves upon the AVG result. It could therefore be explored as a useful method to choose weights for a TO process, however, a previous TO run has to be performed to obtain the weights.



(a) Local load cases



(b) Global load cases

Figure 4.14: Load case relative importance represented by M_i values

4

Table 4.1: Relative difference of compliance values form minimum compliance of a load case (D_{ki})

Load Case	D_{ki}				
	AVG	WS	DWS	N-AVG	N-WS
L1	1%	0%	2%	145%	0%
L2	2%	0%	0%	164%	0%
L3	2%	0%	0%	140%	0%
L4	1%	0%	3%	185%	0%
L5	3%	2%	0%	302%	2%
L6	4%	2%	0%	258%	2%
L7	1%	0%	0%	310%	0%
L8	0%	1%	3%	457%	1%
L9	4%	2%	0%	235%	2%
L10	2%	1%	0%	235%	1%
L11	4%	3%	0%	263%	2%
L12	4%	3%	0%	263%	2%
G1	12%	12%	12%	0%	12%
G2	10%	9%	10%	0%	9%
G3	10%	10%	10%	0%	10%
G4	12%	12%	12%	0%	12%
G5	10%	10%	10%	0%	10%
G6	9%	9%	10%	0%	9%
G7	11%	10%	11%	0%	11%
G8	13%	13%	13%	0%	13%
G9	18%	17%	18%	0%	17%
G10	14%	14%	15%	0%	15%
G11	17%	17%	17%	0%	17%
G12	7%	7%	7%	0%	7%
G13	32%	31%	32%	0%	31%
G14	36%	36%	37%	0%	36%
G15	36%	36%	36%	0%	35%
G16	19%	19%	20%	0%	19%
G17	37%	36%	37%	0%	37%
G18	37%	37%	38%	0%	37%
G19	30%	30%	30%	0%	30%
G20	25%	25%	25%	0%	25%
G21	48%	48%	49%	0%	48%
G22	58%	57%	59%	0%	57%
G23	78%	77%	80%	0%	77%
G24	26%	26%	26%	0%	26%

Table 4.2: Final performance scores, S_k

Objective Function	AVG	WS	DWS	N-AVG	N-WS
Performance Score (S_k)	0.029	0.015	0.005	2.290	0.41

c. Using the performance metric, DWS performs the best among all the objective functions can be chosen as the objective function to be used for the final design.

The performance metric described above is certainly not the only way to evaluate the different objective functions. Certain assumptions are made based on the model definition and the application in consideration, based on which, DWS is chosen for the final designs. It performs well for both local and global load cases and is able to give a reasonable design result which can be taken forward for further evaluation. The TO result produced using 36 individual load cases, a DWS objective function and a mass target equal to that of the OLS will be referred to as the *Basic result* in the following chapters.

4.4. SUMMARY

This chapter investigated the various MLC objective functions which can be used for the TO design process. Four objective formulations were defined which were then used to produce TO results for the CB model which highlighted a bias in the optimization process towards higher magnitude load cases. This was addressed through a magnitude normalization procedure which showed improvements in the compliance values for the lower magnitude load cases without significant adverse affects on the other load cases. The objective functions were then used to produce TO results using the LSD model which showed a drastically poor performance of the normalized objective functions compared to those from the CB model. It was also noted that the nature of the design domain and the load definitions limited the extent to which the structure could be stiffened against global load cases. To compare the objective functions further, a performance metric was put forward utilising the compliance of the design as well as the magnitude of the load cases. Using the metric, DWS was chosen as the objective function to be used for the final design.

5

LOAD CASE SIMILARITY

The following section explores the idea of load case similarity which is used to reduce the total number of load cases considered. The multiple load case TO result is compared with a single load case TO result to identify inadvertent stiffening of load cases not included in the objective function. A method using the concept of cosine similarity is investigated in order to predict similar load cases which explain the inadvertent stiffening. The method is investigated by analyzing results from different SLC TO results and a correlation between load case similarity and inadvertent stiffening is confirmed. A reduced set of critical load cases is defined by identifying similar load cases and the reduced-set TO result is compared with the basic result. Finally, a method to identify similar load cases is put forward with the goal of reducing the number of load cases to be included into the objective function.

5.1. SINGLE LOAD CASE RESULT

A single load case TO design is produced with the objective of compliance minimization of the highest magnitude load case - local load case 6 (L_6). The majority of the design is similar however some truss members on the left component connecting the structure to the fuselage panel are missing. Since the SLC result only considers one load case, additional members present in the basic result which were required to transfer the load from the other load cases do not appear in the SLC result. Figure 5.1 shows the difference between the two designs for load application area A. The absent vertical member connecting load application areas A and C can be attributed to the low magnitude axial direction load applied on A under L_6 as opposed to other load cases.

Figure 5.2 shows the relative compliance of the SLC result with respect to the basic result. The compliance of the structure under the global load cases are almost the same compared to that of the basic result within a margin of +/- 1%. This indicates that including the global load cases in the MLC objective function does not make a significant difference. The previous section pointed out that there exists an extent to which the structure can stiffen against the global load cases even when the magnitudes are normalized and equal importance is given to the global loads. Figure 5.2 indicates that,

(a) (b)

Figure 5.1: (a) Basic result with highlighted vertical members on load application area A which are absent in (b) TO design with load case 6

while addressing L_6, the structure is inadvertently stiffened against the global loads and therefore they could be omitted from the objective function.

The SLC result is stiffer for L_6 than the basic result which is expected, however, it is also stiffer for L_9, L_{11} & L_{12}. This again implies that some of the loads in the objective function could be omitted and instead be accounted for by L_6. To an extent, even L_5 & L_7 are addressed but do not benefit as greatly as L_9, L_{11} & L_{12}. All other load cases in Figure 5.2 show a higher compliance. The phenomenon of inadvertent stiffening may be present in other single load case results as well, exploring this would help identify a reduced set of load cases which could account for all 12 load cases.

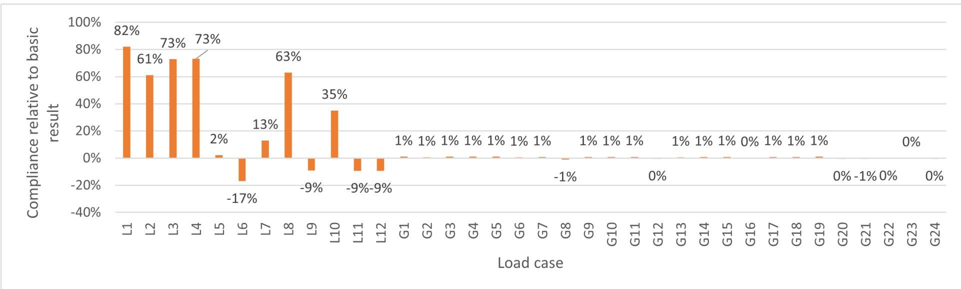


Figure 5.2: Compliance of SLC (L_6) TO result relative to basic result

5.2. PREDICTING INADVERTENT STIFFENING

5.2.1. INADVERTENT STIFFENING OF SIMILAR LOAD CASES

One way of identifying the reduced-set of load cases would be to obtain TO results for each load case separately and recognize inadvertent stiffening. This method, although accurate, is a tedious one due to the resources required for each TO run, post processing and manual analysis. Considering this brute force method for an even larger set of load cases is quite infeasible. Instead an analytical method to predict the reduced-set would be more appropriate and would limit the need to produce TO results for verification only.

Omitting the global load cases, all local load cases are applied at the same load application regions and therefore an explanation for the inadvertent stiffening could be explained by the similarity of load case definition. Similar load cases would require similar designs to emerge through TO. Figure 5.3 shows a simple cantilever beam design domain with a point load at the free end and its respective topology optimized design result [3]. Changing the magnitude of this point load would not affect the design. In such a problem, if one were looking to optimize for multiple load cases which were defined at the same application point and with the same load direction, it should be just taken as a single load case problem.

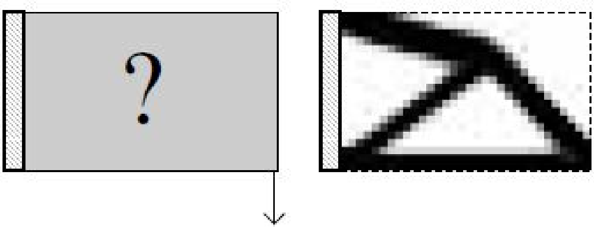


Figure 5.3: 2D topology optimization problem on cantilever beam [3]

For the launcher structure, similar load cases can thus be identified by comparing the load vectors at the application points. if the magnitude and direction of the load vectors of one load case are similar to another load case, they can be said to be similar.

5.2.2. COSINE SIMILARITY

The current problem is more complex than the previous example due to the multiple load application points and load components at each application point. To check for load case similarity a comparison of load vectors - in magnitude and direction - at the same load application regions must be made. Each load case consists of 3 load vectors per load application region giving 12 components to compare between the 12 load cases. To check for similarity and patterns manually would be a challenging task and not a scalable solution when larger number of load cases are considered and even harder if more load application points are present. Instead, the concept of cosine similarity is used.

Cosine similarity is a metric commonly used in Data Science to identify clusters in large data sets. Given a multidimensional space with multiple data points, the "similarity" between data points is ascribed to whether the vectors associated to these data points have similar orientations. For two data point vectors v_1 & v_2, the calculation of the similarity "score" is simply a calculation of the cosine of the angle between them and is given by Equation 5.1. Smaller the angle, higher the similarity score. The metric takes values from -1 (exactly opposite) to $+1$ (exactly same).

$$cos(\theta) = \frac{v_1 \cdot v_2}{|v_1||v_2|} \tag{5.1}$$

Table 5.1: Cosine similarity scores for each local load case pair

Load Case	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12
L1	1.000	-0.411	-0.623	0.859	0.182	-0.040	0.122	-0.257	0.072	-0.013	0.087	0.087
L2	-0.411	1.000	0.969	0.114	-0.516	0.817	0.823	-0.275	0.748	-0.328	-0.695	-0.695
L3	-0.623	0.969	1.000	-0.135	-0.493	0.712	0.673	-0.165	0.623	-0.279	-0.621	-0.621
L4	0.859	0.114	-0.135	1.000	-0.092	0.415	0.596	-0.437	0.498	-0.199	-0.295	-0.295
L5	0.182	-0.516	-0.493	-0.092	1.000	-0.841	-0.549	0.526	-0.842	0.971	0.950	0.950
L6	-0.040	0.817	0.712	0.415	-0.841	1.000	0.897	-0.565	0.992	-0.764	-0.964	-0.964
L7	0.122	0.823	0.673	0.596	-0.549	0.897	1.000	-0.658	0.895	-0.474	-0.758	-0.758
L8	-0.257	-0.275	-0.165	-0.437	0.526	-0.565	-0.658	1.000	-0.612	0.577	0.544	0.544
L9	0.072	0.748	0.623	0.498	-0.842	0.992	0.895	-0.612	1.000	-0.794	-0.963	-0.963
L10	-0.013	-0.328	-0.279	-0.199	0.971	-0.764	-0.474	0.577	-0.794	1.000	0.898	0.898
L11	0.087	-0.695	-0.621	-0.295	0.950	-0.964	-0.758	0.544	-0.963	0.898	1.000	1.000
L12	0.087	-0.695	-0.621	-0.295	0.950	-0.964	-0.758	0.544	-0.963	0.898	1.000	1.000

To apply this concept to the load cases, data points need to be formulated for each load case. Each data point represents a single load case and is defined by 12 variables - 3 load vectors each in 4 load application areas. Therefore each data point can be defined by the 12 dimensional vector given in Equation 5.2. Using these, a cosine similarity matrix is generated which is a table representing the cosine similarity scores for each load case which is shown in Table 5.1. A high positive score implies that the load cases are very similar in magnitude and direction at the respective load application areas. A high negative score implies that the load cases are similar in magnitude but opposite in directions.

$$v_i = (A_x, A_y, A_z, B_x, B_y, B_z, C_x, C_y, C_z, D_x, D_y, D_z) \tag{5.2}$$

where,
A, B, C, D represent the load application regions and x, y, z represent the load application directions
Similar in magnitude here does not only imply that magnitude of respective vector components are equal or in the same range. Two vectors may have a high difference in their overall magnitudes but still be considered similar if the vectors are similarly oriented due to the ratios of the components. In this way L_6 and L_{11} can be similar even though according to Figure 4.1, magnitude of L_6 is significantly higher than L_{11}.

5.2.3. COMPARISON WITH SINGLE LOAD CASE DESIGNS

To verify if the cosine similarity metric works, two additional SLC designs were produced for L_1 and L_5. The compliance results from the single load case TO designs along with a comparison with the similarity scores are presented in Figure 5.4. Figure 5.4a shows the final compliance of each load case relative to the basic result which helps identify the load cases which are getting inadvertently stiffened. Figure 5.4b helps visualize the relationship between inadvertent stiffening and the cosine similarity score. In both graphs, a value of relative compliance close to 0 implies that the load case is being addressed as well as it is addressed in a multiple load case formulation, a negative value implies it has a lower compliance compared to the basic result and a positive value implies a higher compliance compared to the basic result. The result designs are provided in Appendix

A.2 for reference. The results with respect to load case $L_1, L_5 \& L_6$ are now discussed below:

LOAD CASE 6

The SLC result for L_6 is analyzed with the similarity scores obtained. L_6 has a value of 1 which means it is exactly same to itself, and the other load cases have a value ranging between -1 and 1. Table 5.1 shows high positive scores for L_7 & L_9 along with high negative scores for $L_5, L_{11} \& L_{12}$. Comparing this with the results shown in Figure 5.4a, it indicates that load cases with high positive or negative scores get inadvertently addressed in the SLC result. Load cases $L_9, L_{11} \& L_{12}$ - ones with lower compliance than the basic result - have similarity scores of > 0.9 or < -0.9. $L_5 \& L_7$ have relatively lower scores than $L_9, L_{11} \& L_{12}$ (between $0.8-0.9$) and are seen to have a compliance closer to the basic result compared to the rest of the load cases with low similarity scores.

LOAD CASE 5

From the previous SLC result, it is expected that $L_{10}, L_{11} \& L_{12}$ would be inadvertently addressed due to a score of > 0.9 while the structure exhibits higher relative compliance values for all other load cases. The values match the expectation - only $L_{10}, L_{11} \& L_{12}$ are inadvertently addressed. Moreover, load cases with similarity scores close to $+/-0.9$ such as for $L_7, L_6 \& L_9$ are not visibly addressed.

5

LOAD CASE 1

The highest similarity score with L_1 is with L_4 but it is still below .9 and so the SLC result is not expected to exhibit a better compliance value for L_4 compared to the basic result. The values presented in Figure 5.4a meet expectation with no other load case than L_1 having a lower compliance than the basic result. L_4 has a compliance value close to the one obtained in the basic result owing to its relatively higher similarity score.

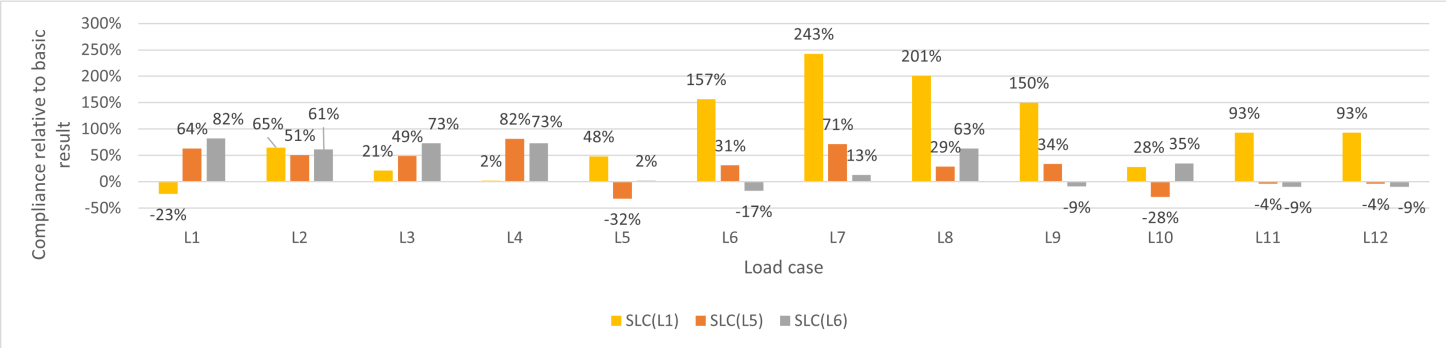
5.3. REDUCING LOAD CASES BASED ON SIMILARITY

The cosine similarity metric is able to predict similar load cases and also give an insight into the inadvertent stiffening which was discussed in Section 5.1. This is now used to reduce the total number of load cases included in the objective function for a TO design.

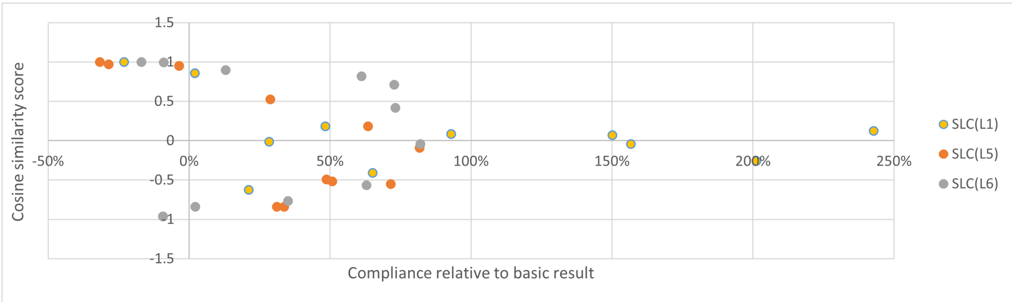
5.3.1. REDUCED-SET TO DESIGN

Based on the results from the previous section, a reduced set of load cases is identified. Figure 5.4b shows that load cases with similarity scores of > 0.9 or < -0.9 have a compliance equal to or better than when a multiple load case formulation is used. Therefore. a threshold value for cosine similarity score of $+/-0.9$ is chosen, the reduced-set is defined with L_1, L_2, L_4, L_5, L_6, L_7, L_8 and global load cases are omitted. In the reduced-set, the highest magnitude load case among similar load cases was chosen where: L_2 accounts for L_3, L_5 accounts for L_{10}, and L_6 accounts for L_9, L_{11} & L_{12}.

Figure 5.5 shows the reduced-set TO design and the basic result. Although the reduced-set result takes into account only 7 load cases as opposed to the 36 in the Basic result, the reduced-set result takes slightly more time (30min) to complete the design process.



(a)



(b)

Figure 5.4: (a) Compliance of SLC TO results relative to basic result (b) Relationship between cosine similarity score and compliance relative to basic result indicating link between higher cosine similarity score and inadvertent stiffening

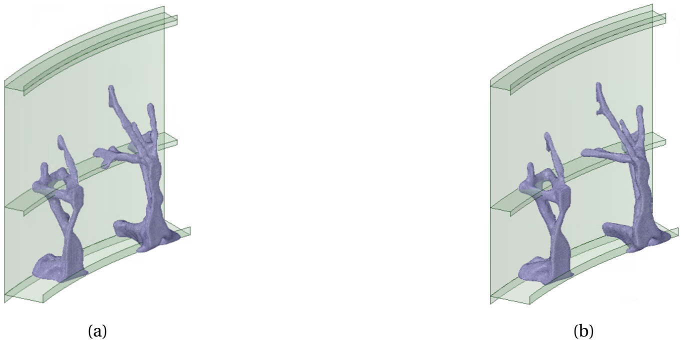


Figure 5.5: (a) Basic result and (b) Reduced-set design result exhibit minimal design differences

Perhaps the reduction of load cases from 36 to 7 was not significant enough of a reduction in computational power requirements to translate into much time difference. The reason for this is not clear currently and requires further investigation but it will not be explored during this thesis.

Qualitatively, there are no major design differences compared to the basic result which shows that the reduced-set is able to account for the omitted load cases. This is confirmed by the compliance values of this design relative to the basic result shown in Figure 5.6. The reduced-set design is similar to the basic result, and marginally stiffer in most local load cases. Though the compliance is marginally higher (max 3.6%), it is still interesting that it is the case for almost all local load cases consistently. This indicates that the basic result is a local optima just like the individual design was presumed to be in Section 3. The combination design and the reduced-set design are able to achieve a more optimal result. This may be because both formulations in a way don't have global load cases as separate load cases, which cause the design to settle into a local optima. However, more investigation into the optimizer and the optimization design space is required to effectively comment on this aspect which is not in the scope for this thesis.

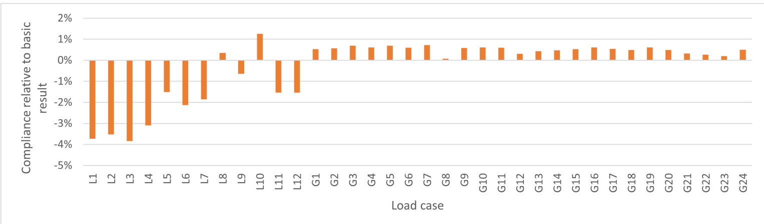


Figure 5.6: Compliance values of reduced-set design with respect to basic result

5.3.2. COSINE SIMILARITY FOR LOAD CASE REDUCTION

In a multiple load case TO problem, there can exist load cases which, due to their similarity with each other, produce a similar final structure. In this case, the number of load cases can be reduced by using a subset of all the load cases to represent the initial set.

5

In a complex enough problem - indicated by high number of load cases, multiple load application points and multiple load components - a manual assessment of load case similarity is a challenging task. Instead, the concept of cosine similarity can be applied by representing load cases as n-dimensional vectors where each dimension represent a load vector at a load application point. Similarity scores of > 0.9 and < -0.9 have been shown to effectively predict similar load cases and thus a reduced set of final load cases to be considered. The final TO design using the reduced-set has been shown to produce a similar design without negatively affecting the compliance for the omitted load cases.

This method is able to predict a reduced load case set which can be used to obtain a final structure which is similar in topology and performance as one which is obtained with the larger initial set of load cases. It can, therefore, potentially be used to reduce computational effort significantly. The use of this method is however limited to load cases which share load application points, load components and type of load. The use of cosine similarity is inspired by its popular use in data science applications where alternative metrics also exist which may be useful here as well but have not been explored.

5.4. SUMMARY

The SLC TO result for L_6 pointed to inadvertent stiffening of other load cases, which indicated the possibility of reducing the number of load cases to be included in the objective function. It was hypothesized that the inadvertent stiffening may be caused by similarity in load case definition and therefore to identify similar load cases they must be compared in magnitude and direction. Due to complexity of comparing the local load cases due to multiplicity of load vectors and load application points, the concept of cosine similarity was used to compare 12 dimensional vectors which represent each of the load cases. SLC TO results and the similarity scores obtained were analyzed and the correlation of high similarity scores with inadvertent stiffening was confirmed. A reduced set of load cases was determined by taking a reasonable threshold value for the similarity score. The reduced-set TO design offered a design similar to the basic result and showed that all load cases were accounted for by the reduced-set. Finally, based on the results, a method to reduce the number of load cases in a multiple load case TO problem is proposed which identifies similar load cases using the cosine similarity metric.

6

FINAL MODELS AND SENSITIVITY STUDIES

This section presents three TO designs developed for the LSD model, a stress evaluation and sensitivity studies. First, the basic result, the reduced-set result and a combination load case result are put forward which have the same mass as the OLS and are using the DWS objective function. The resulting designs are then evaluated in comparison to the OLS on the basis of structural compliance. Subsequently, a design domain reduction study is performed wherein the design domain is systematically reduced and it's effect on the structure is studied. Finally, a mass reduction study is performed wherein the goal mass is gradually reduced to study its effect on structural compliance and design features.

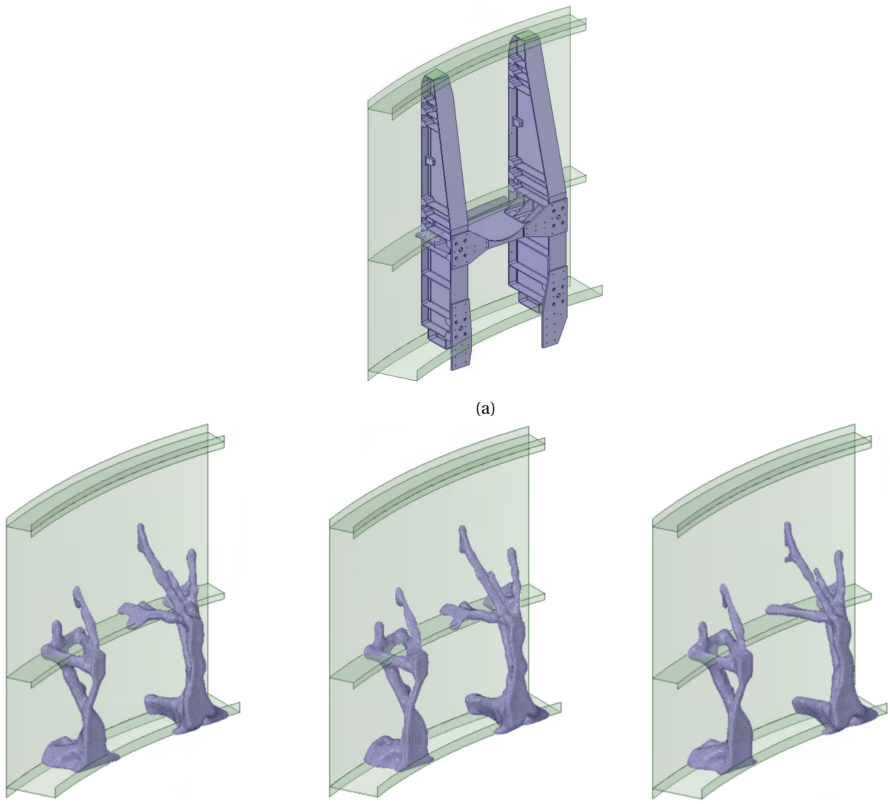
6.1. FINAL MODELS

The final models are generated using the DWS objective function and have the same mass as the OLS. The basic result includes 36 load cases (12 local and 24 global), the combination result includes 288 combination load cases and the reduced-set result includes 7 load cases in their objective functions. This section presents the design results and its structural compliance values compared to the OLS. The result topology is obtained by setting a threshold value for the mass fraction value, resulting in an irregular solid mesh. Although not ideal, the mesh quality has been deemed sufficient to conduct a preliminary FE stress analysis to get an idea of the range of values that can be expected. For ease of viewing, the mesh has been smoothed through CAD software, since feature identification is challenging with just the irregular solid mesh.

6.1.1. FINAL TOPOLOGY

The topology result is smoothed using CAD design software for ease of viewing and can be seen in Figure 6.1 along with the OLS. It is seen that the smoothing operation adds mass to the structure in an effort to fill the gaps and the result is a heavier structure. The smoothed result therefore cannot be taken as the final structure, and should just be considered as a representation of the solid mesh.

As discussed in the previous chapters, all three designs consist of the load application regions being connected to the fuselage section through truss-like protrusions. Unlike the OLS, the structures does not attach to the top ring frame and limit the contact area with the fuselage section to only a few points but spread over a wider area. Contact area with the lower ring frame is much more than the rest of the attachment points and is possibly due to it being near the global load case application region, however a similar distribution is not seen on the left edge of the fuselage section. This design could give an indication of the optimal points at which the structure should connect to the fuselage section for a high stiffness design. The result designs from all three approaches appear



(a)

(b) (c) (d)

Figure 6.1: (a) Original launcher structure; Result designs for topology optimization of LSD model using: DWS with (b) 288 combination load cases, (c) 36 individual load cases and (d) reduced set of 7 load cases

the same qualitatively with minor differences. The reduced-set approach is able to produce more or less the same design with just 7 load cases. Given the design similarity, the performance of the structure is not expected to vary too much either.

6.1.2. MECHANICAL PERFORMANCE

COMPLIANCE

The structural compliance values of the basic and combination designs are compared to those of the reduced-set result for the 36 individual load cases in figure is compared with that of the OLS for all 324 load cases (individual and combination) and the results are presented in Figure 6.2. For most local load cases, the reduced-set result exhibits a lower compliance and the maximum deviation seen is 4%. For the global load cases, the reduced-set design exhibits a higher compliance by a maximum value of 6%. Overall, it can be concluded that the reduced-set design produces basically the same, if not a better, structure as would have been produced if 36 or even 288 load cases were taken.

Figure 6.3 shows the compliance values of the reduced-set result with respect to the OLS. The optimized result has a lower compliance for most of the load cases except for L_1, L_3, G_{21}, G_{22} & G_{23}. The global load cases can be assessed through combination groups. The optimized results exhibit lower compliance values for each of the combination groups. On average the reduced-set result exhibits a compliance value 25.5% lower than the OLS with a minimum value of −62%. In 21 out of the 288 combination load cases, the reduced-set result has a higher compliance with a maximum value of 20%. These 21 load cases are combinations consisting of at least one of the 5 poorest performing individual load cases i.e. L_1, L_3, G_{21}, G_{22} & G_{23}. The compliance values of the OLS are not a requirement for the design, though it is a good indicator of the performance relative to the baseline. For a better understanding of performance a stress analysis must be done.

Overall, the optimized designs have a stiffer design than the OLS. It is also deduced that the compliance under the individual load cases can give a fair indication of the structure's performance under all the combination load cases.

The stiffness of the structure under these load cases determine the displacements in the model, which are not taken as hard requirements during the design process but by comparing the compliance values a check is performed. The topology optimized results being stiffer in almost all load cases, would have smaller displacements than the OLS. Apart from the stiffness, stress within the structure under the load cases is an important parameter to assess the mechanical performance.

STRESS

Stress analysis is performed on the result FEM for all 288 combination load cases. Figure 6.4a shows the maximum stress value which occurs under combination load case 243 (CL_{243}) with a value of 139.34 MPa. High stress points occur at the attachment points where the solid elements come into contact with plate elements whereas the rest of the structure exhibits a low stress distribution. To better visualize the majority of the structure under the load case, Figure 6.4b shows a contour plot which has a maximum level of 50 MPa. The contour plot shows that the majority of the structure has a stress value of less than 40 MPa. It is interesting to note that even though the design process did not contain constraints on stress, the final design has low stress values overall.

The results from the stress analysis indicate that not only does the structure survive the load cases, but also that the structure is to an extent, over-designed due to its large margin of safety. Mass can be further reduced and therefore a final design can be pro-

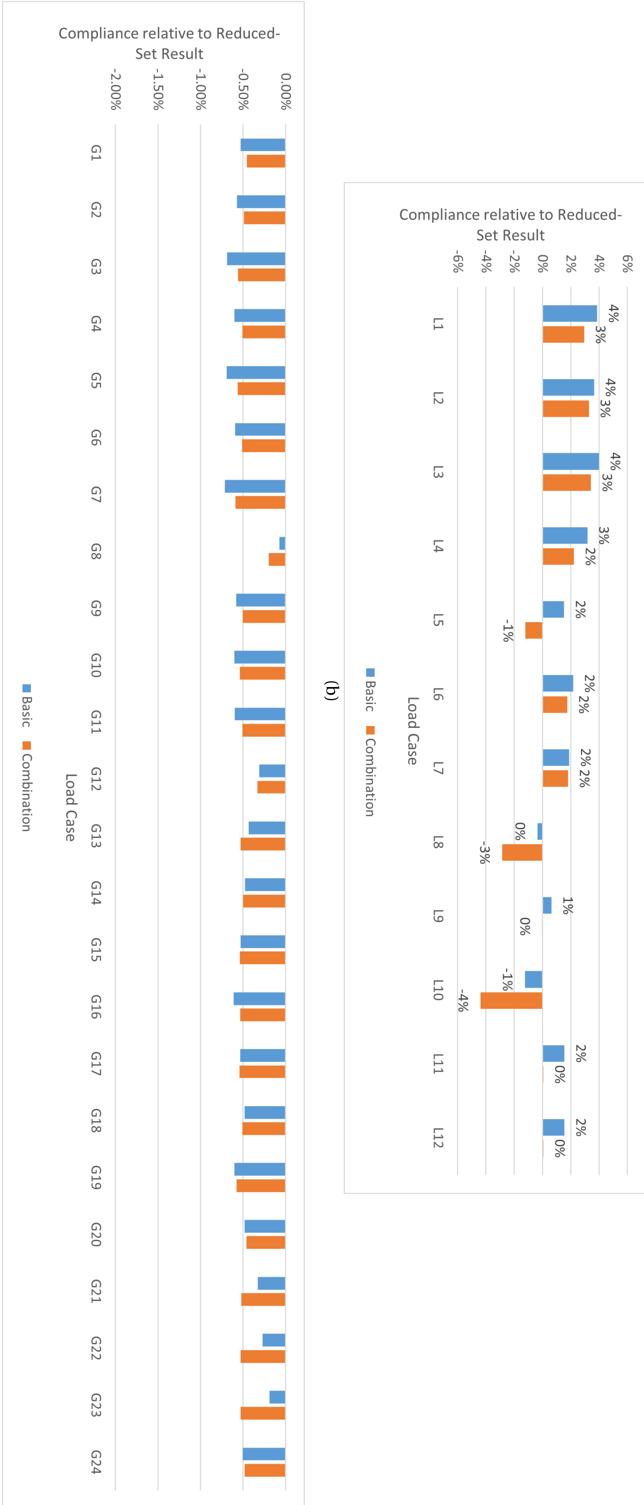


Figure 6.2: Compliance of the basic and combination results relative to the reduced-set result for (a) local load cases and (b) global load cases

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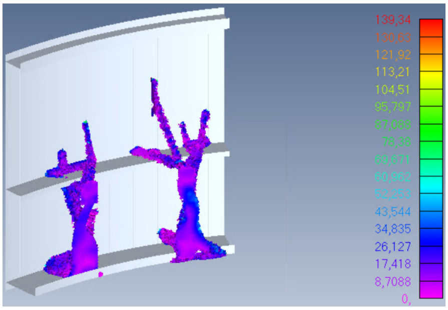
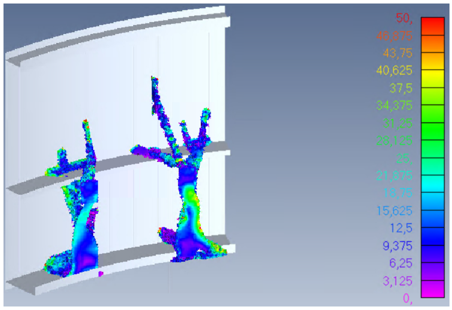
Figure 6.3: Compliance of reduced-set TO result relative to the OLS for (a) individual load cases and (b) combination load cases



(a)



(b)

Figure 6.4: Stress contour plot for reduced-set result under combination load case CL_{243} with (a) unconstrained max/min stress levels (b) maximum stress level constrained to 50 MPa

duced which has a lower mass than the OLS for the given design loads. It is important to note that the design loads do not take into account all the load vectors as was mentioned in Section 2.3.2 and loads from other launch phases but just a subset - therefore a true analysis of the structure would be fair only with all load cases considered.

These stress analyses are used to get an idea of the stress distribution but cannot be compared too extensively with the OLS since they do not represent the final designs. The topology optimized designs should be taken as an input to the next step which would be to use the optimized design as a guide to make the final design. The final design thus obtained would be a much better model to make a comparison with the OLS. The process of producing the final geometry requires additional software capabilities which do not currently exist in MSC Nastran/Patran and are therefore not explored in this thesis.

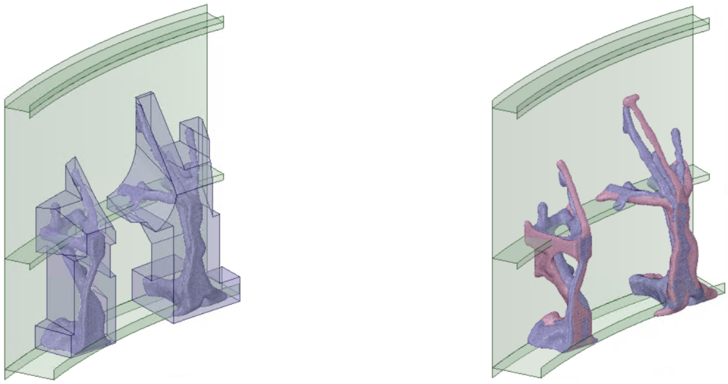
6

6.2. DESIGN DOMAIN SENSITIVITY

6.2.1. REDUCED DESIGN DOMAIN BASED ON BASIC RESULT

Through the multiple TO designs that were generated, it is seen that the final design is usually produced in a particular region of the design domain. It could be possible to reduce the design domain according to the resulting design in an effort to be able to reduce the mesh size and therefore obtain a better design definition. Using the design of the basic result, a reduced design domain is produced as shown in Figure 6.5a and is meshed with the same 9mm TET mesh. Figure 6.5b shows the design superimposed with the basic result and quite some design differences can be seen.

A major difference is that the reduced domain design allocates material at the edges of the design domain. This is an unexpected result as the basic result design is a subset (in terms of mass distribution) of the reduced design domain and intuitively the new domain should also produce the same design. A possible reason for the distinct design can be that the smooth meshed edges of the design domain offer an artificially stiffer design compared to the "rougher" members created within the design envelope. On the other hand, this could just be an indication of the non-convexity of the problem and sensitivity to design domain changes. To further investigate these questions, the design domain is increased gradually and each increase is motivated by keeping the TO process



(a) (b)

Figure 6.5: (a) Reduced design domain created around basic result design (b) RD-1 design (red) superimposed with basic result design (grey)

unrestricted by the design domain. A total of 3 iterations are done for increasing the design domain to accommodate the design. The reduced design domain models are named RD-1 through RD-4.

6.2.2. TO DESIGN RESULTS FOR REDUCED DESIGN DOMAIN MODELS

The design domains and designs obtained for all the iterations are provided in Appendix A.3. Figure 6.6 shows the differences between RD-1 (red) and RD-2 (grey) results. It is seen that the RD-2 result deviates from that of RD-1 and material distribution moves inwards for the optimal solution - which indicates that the smooth edges of the RD-1 domain were providing an artificially high stiffness.

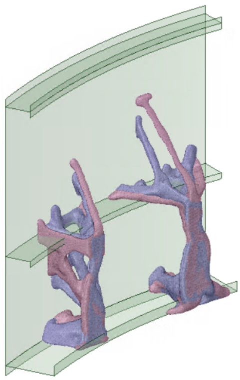


Figure 6.6: RD-1 design (red) superimposed with RD-2 design (grey)

In the RD-2 and RD-3 result, the material distribution moves inwards and much more resembles the basic result with differences highlighted in Figure 6.7. The design results show that the problem is non-convex and is hypersensitive to the design domain.

Even with slight changes to the design domain the final design can differ drastically. Between RD-2 and RD-3, only minor extensions to the design domain are made, which not only changes the topology obtained in the extended region but in other unaltered areas as well. The TO process explores the entire domain to find the optimal solution and even the slightest change could result in a different solution. Moreover, the design domain should not visibly restrict the final design as was seen with RD-1, the results of such a domain may lead to artificially better compliance values.

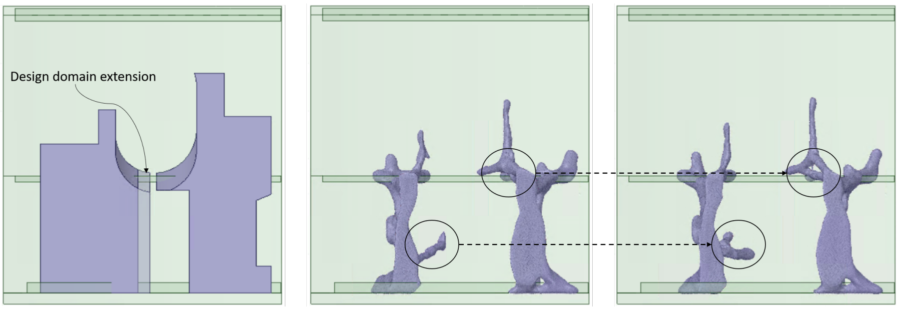


Figure 6.7: Extension to RD-2 design domain (left) leading to differences between RD-2 design (middle) and RD-3 design (right)

6.2.3. COMPLIANCE RESULTS FOR REDUCED DESIGN DOMAIN MODELS

Figure 6.8 presents the compliance of the reduced design domain TO results and the volume as a fraction of the respective properties for the basic result. The compliance is calculated as the average value across all load cases to give a macro level view. It is seen that the compliance increases as the volume increases. A different compliance value is expected for each design since they are quite different from each other qualitatively. The low compliance for the RD-1 was rationalized by artificial stiffness provided by smooth edges of the design domain, however, that effect should not be present in the results from RD-2 and RD-3 which still seem to have a lower compliance than the basic result. Since these domains are not restrictive, it is clear that the TO process is settling into different minima with different design domains. Theoretically, the designs obtained in RD-2 and RD-3 are achievable with the larger LSD design domain, but it settles for a sub-optimal solution which has a higher compliance. These results indicate the non-convex nature of the problem and its susceptibility to settle into local minima. It may be that reducing the design domain reduces the chances of settling into a sub-optimal solution, but a confirmation requires more investigation.

There surely exist multiple phenomena which dictate the sensitivity of the final topology to the design domain namely - artificial stiffness of smooth sections and non-convexity of the optimization problem. Based on these deductions, an equivalent comparison cannot be easily drawn between different design domains. Such a study should be conducted at the initial stages of the design process so as to finalize the design domain. In order to improve design definition in that case, greater computational resources are required to reduce the element size without changing the design domain.

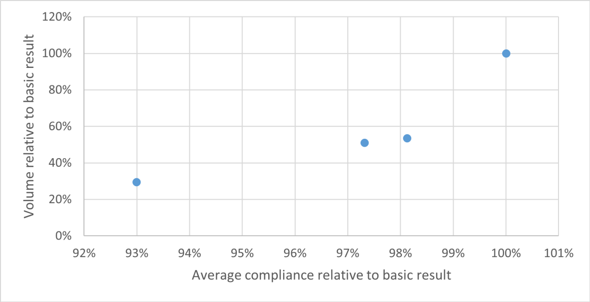
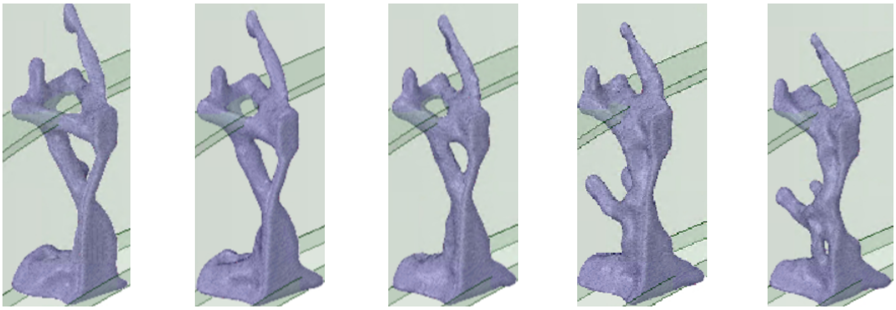


Figure 6.8: Scatter plot showing relationship between design domain and compliance. Values indicate smaller design domains offer more optimal designs

6.3. MASS FRACTION SENSITIVITY

The final models presented earlier have a mass that matches the OLS mass which is a result of setting the appropriate goal mass fraction value during the TO process. One of the major motivations for performing TO especially for launcher structures is to achieve a lower mass and thereby reducing launch costs. A lighter component design can be produced if the appropriate goal mass fraction value is set. Since designs will be produced for any mass fractions that are set, the results must be evaluated if they meet the design requirements and if they can be accepted as a feasible component design. A parametric study must therefore be performed by producing TO results for different goal mass fraction values and then performing a trade-off study to finalize the required mass fraction value.



(a) Basic Result (b) MR-10 (c) MR-20 (d) MR-30 (e) MR-40

Figure 6.9: Evolution of design with reduction in overall mass

A similar parametric study is conducted starting with the basic result model and then reducing the mass fraction to achieve mass savings of 10%, 20%, 30% and 40% and the respective designs are named MR-10, MR-20, MR-30 & MR-40 respectively. All the final design results can be found in Appendix A.4. Figure 6.9 compares the different designs obtained and shows that the major difference in design is the changing thickness of the members and in some cases removal of a truss member completely. The nature of the design does not change drastically as the interface points and orientation of the truss members produced are optimal for stiffness.

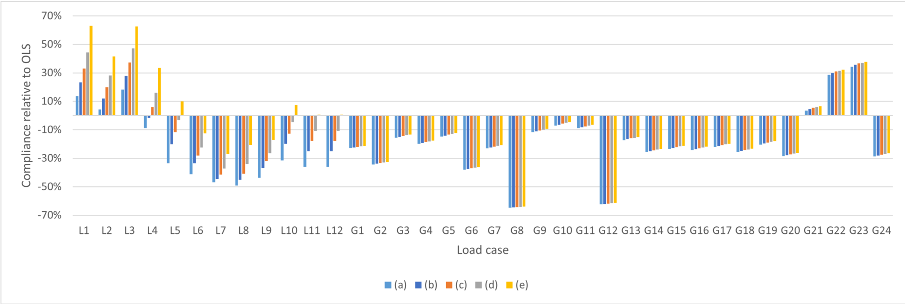


Figure 6.10: Compliance of TO design relative to OLS with (a) equal mass and (b) 10% (c) 20% (d) 30% & (e) 40% mass savings

Figure 6.10 shows the comparison of compliance values for the different reduced mass results with the OLS which shows that the mass savings comes with a corresponding reduction in stiffness. The pattern is quite intuitive as a systematic increase in compliance is seen with a reduction in mass. Under most load cases, the compliance of the structure even with a 40% reduction mass is below the value for the OLS but in L_1, L_2, L_3, L_4, L_5, L_{10}, G_{21}, G_{22} & G_{23} at least one of the reduced mass results have a higher compliance than the OLS. The OLS values are not minimum requirements for the feasibility of the optimized structure - additional analyses must be performed. The result designs, as previously mentioned in the beginning of the chapter, are representations and therefore additional post processing work must be conducted for an appropriate trade-off analysis and subsequent design selection. If less stiff design is acceptable, significant mass savings can be achieved - a trade-off must be made with respect to the mass, the compliance of the structure while meeting all mechanical requirements of stress, buckling, etc.

7

CONCLUSIONS AND RECOMMENDATIONS

7.1. CONCLUSIONS

The aim of this thesis was to understand the procedure of conducting topology optimization of launcher structures which are subjected to multiple load cases. Two analytical models were created to test out the design process: the cantilever beam (CB) model and the launcher structure demonstrator (LSD) model. The CB model was used to set baseline expectations of the design experiments which were then evaluated with those of the LSD model. Several multiple load case objective formulations were identified from the literature and a comparison of performance was made based on design features and final compliance performance. Combinations of individual load cases and similarity of load cases were studied which resulted in methods for reducing the total number of load cases to reduce the computational load during optimization. Based on the work, a well suited objective function and reduced set of critical load cases are chosen for producing a topology optimized design. Finally, the sensitivity of the TO process to design domain and goal mass fraction was studied. First some salient conclusions are discussed before discussing the findings for each research question:

a. *Load case combinations*
When performing a TO design process for a structure subjected to design load cases which are combinations of distinct individual load cases, the problem can be reformulated to consider the individual load cases only. In the CB and LSD models, it was shown that a TO design considering individual load cases was able to produce a structure which was 3% – 6% higher in structural compliance than a design produced considering a combination of the individual load cases. Using this, the number of load cases for the LSD was reduced from 288 to 36 and the associated runtime was reduced by 3x along with lower memory usage.

b. *Normalization of objective function with initial compliance*
The final design of a multiple load case TO design process is heavily influenced by the relative magnitude of the different load cases. Depending on the objective function formulation, the compliance performance of the structure is biased towards higher magnitude load cases and thus produces a stiffer design for higher magnitude load cases compared to the lower magnitude load cases. While using objective functions such as AVG, DWS and MM, large differences in load case magnitudes led to a neglect of the lower magnitude load cases and thus a poor final design. This neglect can be addressed by eliminating these relative magnitude differences by normalizing the load case compliance responses in the objective function by their respective initial compliance values (Compliance at design cycle 0):

(a) For the CB model, this method improved the performance of the final design for the lower magnitude load cases without adversely affecting other load cases.

(b) For the LSD model, the normalization method did not produce satisfactory results where an increase in performance for the global load cases drastically deteriorated the performance for the local load cases. This has been ascribed to the particular problem definition which limited the extent to which the structure could be stiffened against global load cases.

c. *Load case similarity*

- Due to similarity of load cases, inadvertent stiffening of the structure under load cases not included in the objective function was identified. By identifying similar load cases, certain load cases could be eliminated from the objective function while not adversely affecting the final design. Load cases could be compared on the basis of load application points, load directions and magnitude to identify similar load cases.
- A method using the cosine similarity metric was used to predict these similar load cases where it was shown that a similarity score of > 0.9 or < -0.9 was a good indicator of identifying load cases to be eliminated from the final objective function. Using this method, the number of load cases was reduced from 12 to 7.
- The method of using cosine similarity for identifying and eliminating similar load cases, is a scalable method for larger number of load cases but is limited to load cases which share load application points, type of load and number of load components.

d. *Design domain dependency*
The smallest possible design domain should be used to have smaller elements with the least computational load. However, it is shown that a restricted TO design may have artificially high stiffness, and therefore be a misleading design. To avoid such effects, the design domain should be carefully selected avoiding design restriction.

e. *Non convexity of TO problem*
The TO problem addressed in the thesis is a non convex problem and as such is susceptible to settling in local minima during the optimization process. The optimization process is also very sensitive to changes in the changes to the design domain, goal mass fraction, load case definitions and other model parameters.

f. *Feasible topology optimized design of LSD model*
The thesis put forward the final TO problem formulation to obtain an optimized design for the LSD which matched the baseline mass and on average had a compliance 25.5% lower. The design obtained should be used as a guide to produce a final design for further analysis.

7.1.1. ADDRESSING RESEARCH QUESTIONS

The conclusions can be summed up by discussing how each research question was addressed:

a. *Which multiple load case (MLC) objective functions can be used for optimization under the multiple load cases?*

   (a) *How can the MLC objective functions identified in the literature study be implemented for this problem?*
   For the LSD model, the dynamic weighted-sum objective function was used to generate the final topology optimized design. The load cases were taken into account in the objective function with a constraint on the goal mass fraction. A reduced set of critical load cases were selected by eliminating load case combinations and similar load cases.

   (b) *What are the relative performances of each of the MLC objective functions implemented?*
   A thorough comparison was made between the different multiple load case objective functions on the basis of final design result and the compliance performances achieved. The salient notes are as follows:
   - DWS was shown to produce the best results for the LSD model and was used to produce the final design.
   - MM, despite it's formulation geared to address the most critical load case, consistently performed poorly even for the highest magnitude load cases relative to other objective functions.
   - The method of choosing weights for the weighted-sum objective function was shown to improve the performance compared to the AVG result in the LSD model and can be a useful method to choose weights.
   - AVG has been shown to give consistently satisfactory results across different problem definitions. For a basic topology optimization design process with multiple load cases, the AVG function can be used.

   (c) *Which MLC objective functions implemented offer better solutions?*
   For the LSD model, DWS, WS and AVG produced better solutions (in that order) in terms of design features and mechanical performance. Normalized

objective functions failed to offer feasible designs as they tended to redistribute material in favour of the global load cases, which adversely affected the overall design.

(d) *Is the performance of the MLC objective function dependent on the structure?* As seen from the difference in results between the CB and LSD model, the performances of the objective functions are dependent on the structure being optimized as well as the load definition. The performance of normalized objective functions greatly improved the designs for the CB model but not the LSD model. It was seen that due to the model and load definition, the normalized objective functions produced infeasible designs. Additionally on a broader level, the TO process was highly dependent on the design domain and goal mass fraction as well, which leads to the conclusion that the exact methodology used in the thesis cannot be universally applied and must be assessed for each structure.

b. *How does the final TO design compare to the original structure designed for conventional manufacturing processes?*

(a) *How does the TO design compare to a design led by design engineer's experience?*
The final TO design of the LSD model is drastically different than the OLS. While the OLS is constructed as an assembly of planar components attached to the fuselage section and all three ring frames, the optimized design exhibits truss like structures extending from the load application regions to the fuselage section and the middle and bottom ring frames. The attachment points were chosen automatically during the optimization process and may indicate optimal attachment points for similar structures.

(b) *How does the new design compare in terms of mass and mechanical performance?*
The final topology optimized design for the LSD model has the same mass as the OLS (Baseline component) and on average a lower compliance (higher stiffness) by 25.5%. A preliminary stress analysis showed majority of the component with a low stress value of $< 40MPa$ which may indicate that mass can be further reduced.

c. *How can the TO design methodology be implemented into the existing design process at Airbus DS NL?*
The software tools MSC Nastran and Patran were chosen for the thesis as they are already part of the Airbus DS NL design process, therefore implementing a TO design process would have been more natural to the workflow. During the thesis however, limitations with the TO tools of MSC Natran & Patran were identified which make it unsuitable (without the aid of additional software tools) for implementing a TO design process at Airbus DS NL. The limitations are detailed in Section 7.2.

7.2. RECOMMENDATIONS

For future work on the topic, the following recommendations are provided:

a. A general guideline is provided in Appendix B.1 for producing topology optimized designs of launcher structures under multiple load cases.

b. In many experiments the optimization process seemed to settle into local minima such as in the case of individual v/s combination load cases and with changes in the design domain. This could be investigated in other problem definitions.

c. Due to the sensitivity of the TO process to the design domain, mass fraction, load definitions, etc., the model should ideally be finalized before any designs are produced and should be kept constant throughout the design process.

d. The reduced-set TO design result presented in the thesis should be used as a guide to produce a refined model for re-analyses. In the thesis, only structural compliance was evaluated in comparison to the baseline structure (OLS). A feasible final design can only be produced after extensive evaluation of mechanical performance.

e. Stress constraints can be added in the TO formulation to keep a check on high stresses in the structure through the design process. The addition of such constraints will invariably increase computational effort as well as add to the problem's non-covexity.

f. Comparing the TO results using the continuous density result is more robust than extracting a result topology and re-importing for analysis. Continuous density result being what is directly obtained from the optimization run which contains element densities from 0 to 1, as opposed to the model extracted by deleting lower density elements from the continuous density result.

g. In the case of a TO design restricted by the design domain, artificially higher stiffness was observed which may be due to the design incorporating smooth faces of the design domain into the final design. Such a phenomenon can lead to misleading designs even for other topology optimization design tasks dissimilar to the one present in the thesis, and must be investigated further.

h. Before starting a TO process with multiple load cases, it would be beneficial to run it with single load cases to get an idea of the optimal designs as well as to identify inadvertent stiffening of other load cases.

i. In this thesis, only the cosine similarity method was used to identify similar load cases and only for the LSD model. This method should be explored with other multiple load case structures to check dependency on the structure. Other methods/metrics can also be explored which might be able to give a better prediction of load case similarity.

j. The optimized design produced in this thesis is expected to be used as an input to create a final design. The TO result should be taken as a guide to generate a CAD design which must then be used for all mechanical analyses. The process of finalizing the design would involve multiple iterations of TO, CAD model generation and FE analyses - since the TO process will not take all possible constraints such as stress, buckling, vibration performance, etc. into account.

k. The preliminary stress analysis results indicates that the optimized result with the mass of the OLS may be over-designed due to low stress levels. In this case, a final component with a lower mass can be obtained and thus appropriate studies should be performed.

l. MSC Nastran and Patran are equipped to perform topology optimization of large 3D structures. However, there were some limitations which should be known:

   (a) During the optimization process, only those structural responses are evaluated which are either active constraints in the TO process or are contributing to the objective function. Therefore, an evaluation of the structure under additonal load cases cannot be obtained from the optimization run itself.

   (b) There is no provision for re-analysis of the continuous density result from the TO process. The final density values cannot be re-imported into the model so that the model with optimized densities can be re-analyzed.

   (c) A re-analysis of the result topology can only be done by extracting an irregular finite element model (Using MSC Patran) containing elements after setting a certain mass fraction threshold. The FE model in this case is prone to discontinuities and erroneous connections which lead to problems in re-analysis. The extracted FE model in this case has to be manually and repeatedly modified.

   (d) Patran does not have a provision to smooth the topology results if TET meshes are used.

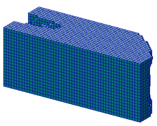
   To overcome the above limitations for Patran, additonal tools exist such as MSC Apex which have the post processing capabilities lacking in Patran.

m. Other Software tools exist such as Optistruct and Ansys which should be explored, given the limitations with MSC Nastran/Patran.

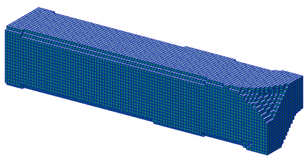
A

Appendix A

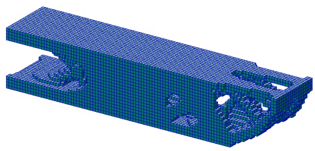
A.1. Cantilever beam single load case results



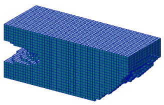
(a) CB-1 : Normal



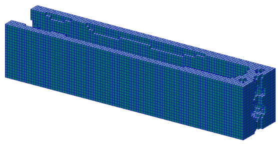
(b) CB-1 : Tosion



(c) CB-1 : Shear



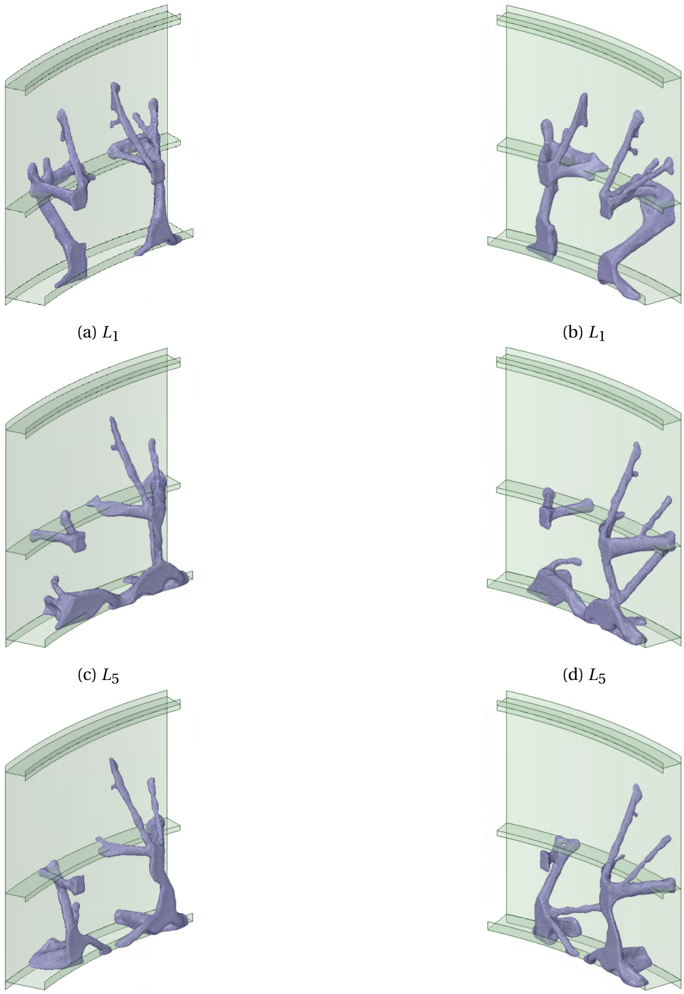
(d) CB-2 : Normal



(e) CB-1 : Moment

Figure A.1: Design results for CB-1 and CB-2 under individual load cases

A.2. LSD SINGLE LOAD CASE RESULTS



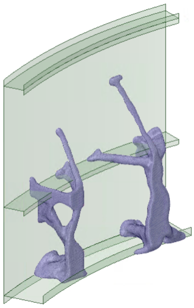
(a) L_1 (b) L_1

(c) L_5 (d) L_5

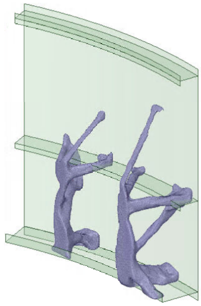
(e) L_6 (f) L_6

Figure A.2: TO results of LSD model under load cases L1, L5 & L6

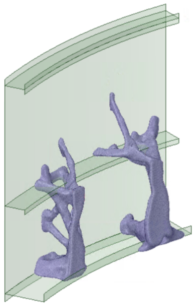
A.3. Design Domain Sensitivity



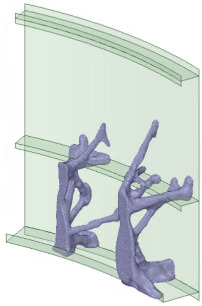
(a) RD-1



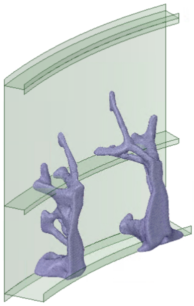
(b) RD-1



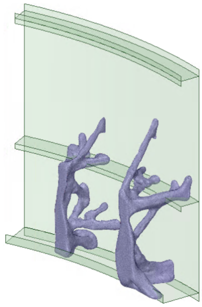
(c) RD-2



(d) RD-2



(e) RD-3

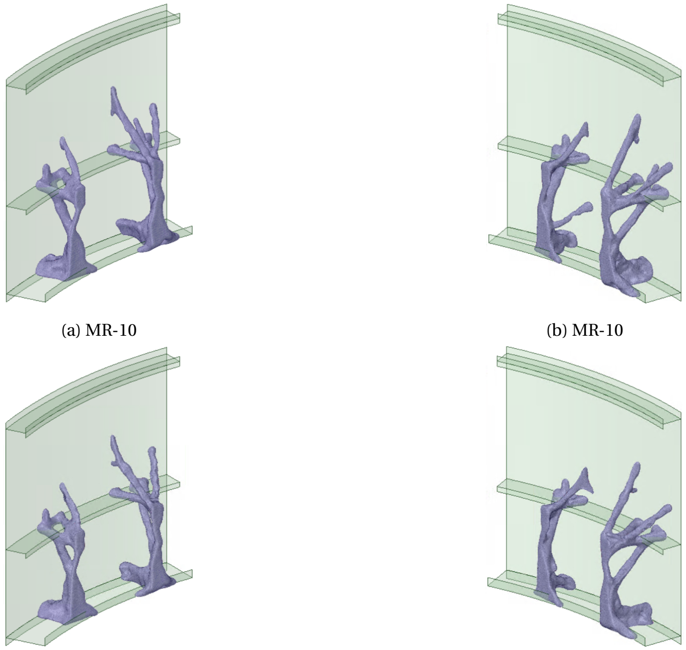


(f) RD-3

Figure A.3: Design results for MR-10 & MR-20 for design domain sensitivity study

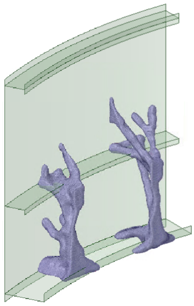
A

A.4. MASS FRACTION SENSITIVITY

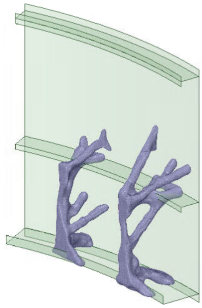


(a) MR-10
(b) MR-10
(c) MR-20
(d) MR-20

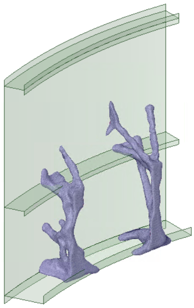
Figure A.4: Design results for mass fraction sensitivity study



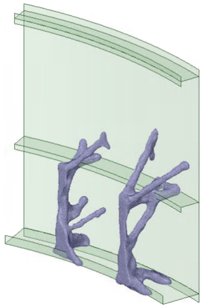
(a) MR-30



(b) MR-30



(c) MR-40



(d) MR-40

Figure A.5: Design results for MR-30 & MR-40 for mass fraction sensitivity study

B

Appendix B

B.1. General guideline for TO

From all the experiments conducted, a general guideline of producing topology optimized designs for structures under multiple load cases can be outlined.

a. *Identify appropriate design domain*
   The design domain should envelope the entire volume where the topology optimized design can be produced. It should at the same time be the smallest design volume given the above condition. A small design domain enables the use of smaller elements which increases design definition.

b. *Identify loads and boundary conditions*
   Since the optimization is highly sensitive to the load definition, the loads and boundary conditions must be fixed through the optimization study. Additionally, design load cases should be studied so as to identify combination load cases and similar load cases. As explored in Chapter 3 and Chapter 5, this could aid in reducing the total number of load cases which have to be included in the objective function

c. *Basic TO design*
   Using the objective function of average compliance, a basic topology optimized result can be produced. Through the experiments in the thesis it was seen that the AVG objective function produced adequate results with different problem definitions and thus can be used as a quick and effective method. If a better performance is required or if there is a need to test with different objective functions the next steps can be followed otherwise one may stop here.

d. *Test effect of normalization*
   Normalization may help provide better results for low magnitude load cases but it may not always work depending on the problem definition. Before diving into

testing different objective functions, a check can be performed to see if an objective function normalized by initial compliance values could produce feasible results. For this, the N-AVG objective function can be utilized. The result design should be checked for floating members and abnormal mass distributions which may indicate that the design is going in an infeasible direction. If the design shows improvement in lower magnitude load cases without significant penalty for higher magnitude load cases - normalization may work for the particular problem at hand.

e. *Compare different objective functions*
Different objective functions including the normalized formulations (After negative effects are ruled out) can be used to generate topology optimized designs. A comparative study can be conducted using the qualitative design features and compliance performance of the structure. To compare the compliance, the performance metric introduced in Chapter 4 can be used or another metric based on the designer's discretion.

f. *Final model analysis*
The design produced using the finalized set of loads and objective function can then be used for further analysis or as a guide to produce the final design.

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