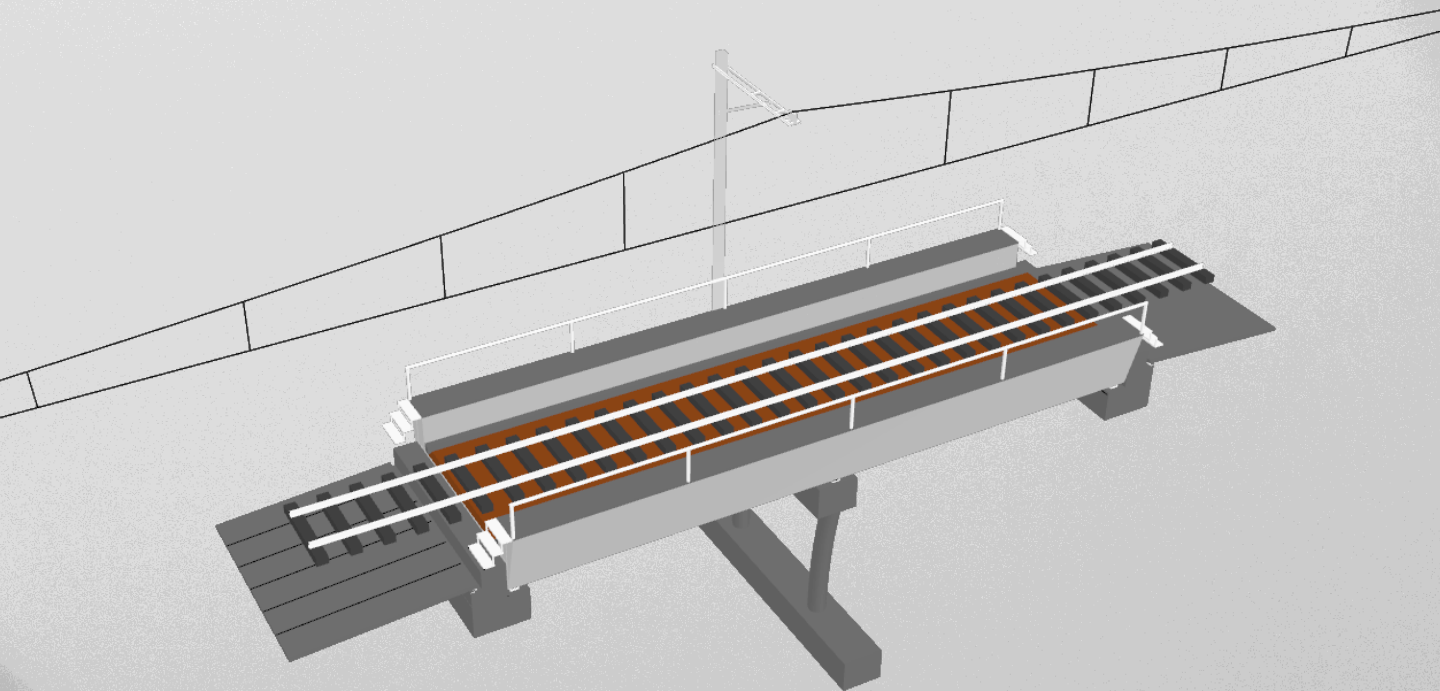




# A study into the application of a FRP deck in railway bridges

H.P. Vijn





# A Study Into the Application of a FRP Deck in Railway Bridges

by

H.P. Vijn

to obtain the degree of Master of Science  
at the Delft University of Technology,  
to be defended publicly on Thursday February 4, 2020 at 1:30 PM.

Student number: 4208234

|                   |                                |                                                |
|-------------------|--------------------------------|------------------------------------------------|
| Thesis committee: | Dr. ir. M.A.N. (Max) Hendriks, | TU Delft, chairman                             |
|                   | Dr. M. (Marko) Pavlovic,       | TU Delft                                       |
|                   | Dr. ir. J.E.P. (Joris) Smits,  | TU Delft                                       |
|                   | Ir. J.A. (Johan) Bolhuis,      | BAM Infraconsult, Hochtief AG (since 1-nov-19) |

Cover photo by Janusz Maniak, *The Lovers' Bridge, Krakow*

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Fibre reinforced polymer is a new material that is more and more used in civil engineering works. This lightweight and high-strength material comes with a lot of advantages. The application of this lightweight material in bridge structures can result in a reduction of the out-of-service time of railways. On the other hand, the low stiffness of this material offers a big challenge. The stiffness requirements of railway bridges are strict to ensure safe and comfortable passing of trains. This thesis looks at the application of a FRP deck in railway bridges.

The literature study describes (1) the material FRP, (2) state-of-the-art in (FRP) bridges and (3) a dynamic model for (railway) bridges. The main properties of FRP materials are listed including common manufacturing processes. According to a state-of-the-art study two designs are presented: (1) A sandwich deck, commonly used in FRP bridges, and (2) a through deck, commonly used in railway bridges. Furthermore, the requirements and loadings for railway bridges are listed. The dynamic model for railway bridges is based on a mass-spring rigid body system with Rayleigh damping. This model is used to verify the finite element models.

A material study is performed to determine the most suitable material properties for FRP railway bridges. Glass fibres and polyester resin are chosen based on individual material properties, market prices and common use in civil engineering structures. Foam has no structural integrity but is used as permanent form work. The lay-up of fibres is based on the literature study and differ for flanges and webs. The lay-up for flanges is:

$$[12.5\% - 90^\circ | 12.5\% - \pm 45^\circ | 62.5\% - 0^\circ | 12.5\% - \pm 45^\circ]_s$$

. The lay-up for webs is:

$$[25\% - 90^\circ | 20\% - \pm 45^\circ | 35\% - 0^\circ | 20\% - \pm 45^\circ]_s$$

The volume fraction of both laminates is 50%. These lay ups in combination with the conversion factors and safety factors result in a FRP material suitable for FRP railway bridges.

The design study describes the sandwich and through design with the materials from the material study. Two finite element model are made using Sofistik. These finite element models are verified using analytical models. A static and dynamic analysis is performed on both designs. Based on the results from the static analysis the maximum deflection and maximum stresses of the Sandwich Design do not exceed the limits, and the maximum deflection and maximum stresses of the Through Design do exceed the limits. Based on the dynamic analysis both designs do exceed the limits. Furthermore, in consultation with a FRP manufacturer, the laminate thickness must be reduced to ensure both designs are manufacturable.



Based on the results of the design study and the requirements presented in the literature study a new design is created. This design includes the following improvements: (1) reduction of the span by transferring the forces in transverse direction, (2) extra height of the design, (3) more webs, (4) extra supports, (5) reduction of the laminate thickness to 30 mm and (6) the use of pre-camber.

This final design is transformed into a finite element model. A static and dynamic analysis is performed. Based on the results of the static analysis the maximum deflection and maximum stresses do not exceed the limits. The dynamic analysis results in maximum acceleration of  $19.48 \text{ m/s}^2$ , which exceeds the limit and results in a unity check of 5.57. The final design doesn't meet the requirements based on the results of the dynamic analysis.

An important remark on the dynamic results is made in the discussion. The dynamic load model, consisting of point loads, is applied directly on the bridge deck, omitting the dynamic behaviour of the ballast bed. Taking into account the ballast bed in the dynamic analysis might result in a significant reduction of the maximum acceleration of the bridge deck due to the spreading and damping effect of the ballast bed.

In conclusion, according to the results of the static analysis of the final design it is possible to apply a FRP deck in railway bridges. Improvement of the dynamic numerical model is needed before drawing conclusions on the dynamic behaviour of the final design.

H.P. Vijn  
Gouda, January 2020

*"The first sign of wisdom is to get wisdom;  
go, give all you have to get true knowledge."*

- Proverbs 4: 7



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## Preface

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Before you lies my graduation report. This report is the result of my research into the application of Fibre Reinforced Polymers in railway bridges. This 12-month study is the final step to obtain the Master's degree in Structural Engineering at the Delft University of Technology.

The subject of this research arose from my passion for bridges in combination with the challenge of investigating a new material. This research has taught me a lot about the application of the FRP material and the design of (railway) bridges. I have to admit I considerably underestimated the extent of such a thesis.

This research was conducted at BAM Infraconsult, where I worked as a graduate trainee. I want to thank BAM Infraconsult for the services provided.

I want to thank my colleagues at BAM Infraconsult, for both a listening ear and a critical note during my research. In addition, I would like to thank Fibercore Europe for answering all my questions. Many thanks go to my graduation committee; Max Hendriks, Marko Pavlovic, Joris Smits and Johan Bolhuis. They taught me a lot during the project and, where necessary, guided me in the right direction.

Finally, I want to thank my family and friends. During my study they were the ones who supported me during setbacks, but especially the ones who made me enjoy my study. Thank you.

I wish you all a pleasant reading.

H.P. Vijn  
Gouda, January 27, 2020



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# Part I

## Introduction



# CHAPTER 1

## Introduction

The situation of Dutch bridges has often been critically reviewed ([Cobouw, 2017], [Een Vandaag, 2018], [NH Nieuws, 2018]). Due to increasing traffic loads and the approaching end of service life many bridges are prone to collapse. The maintenance report of the Dutch Ministry of infrastructure (Rijkswaterstaat), which was made public halfway 2018, do confirm this once again (see Figure 1.1). An increasing demand for maintenance and replacement is the logical result. Thereby, travellers of public transport expect as little inconvenience as possible, which is why Prorail wants limit the so-called buitendienststelling (out of service) of the railways. In this contrast, the railway manager appeals to innovation.

### Rapporten Rijkswaterstaat: risico op instorten bruggen



Volgens het onderzoek is er bij knooppunt Amstel een onacceptabele kans op instorten. © ANP



Tientallen bruggen en viaducten in Noord-Holland vertonen ernstige problemen. Door achterstallig onderhoud is er op sommige plekken een onacceptabel risico op instorten.

DOOR: HET PAROOL 11 APRIL 2018, 16:50

Figure 1.1: Article about the maintenance report of Rijkswaterstaat  
Source: Het Parool

One of the innovations in bridge construction is the use of fibre reinforced polymers. This compos-

ite, blown over from the automotive and aerospace industry, is finding its way more and more into the civil engineering industry and is regularly used in bicycle and pedestrian bridges. The high strength and durability makes FRP a suitable building material. Thereby, the lightweight nature of this composite offers new opportunities in transportation and assembly of large span bridges, resulting in a reduced buitendienststelling.

One of the first made FRP bridges is located in Kolding, Denmark (Figure 1.2). This assymetric cable-stayed bridge is positioned over a busy railway line, which restricts the installation work and time of the bridge. Use of fibre reinforced polymers has ensured that the bridge could be built within a very short time and shows that FRP is a suitable building material for bridges.



Figure 1.2: Fiberline FRP bridge in Kolding, Denmark  
Source: fiberline.com

## 1.1 Problem definition

The high strength, durability and low weight of the material offers many advantages for the contractor. On the other hand, the use of FRP has one major drawback; FRP has a low stiffness compared to other common building materials. Table 1.1 provides an overview of the strength and stiffness of common building materials.

| Material   | Strength [MPa] | Stiffness [GPa] |
|------------|----------------|-----------------|
| Concrete   | 25 - 90        | 30 - 40         |
| Timber     | 18 - 30        | 9 - 12          |
| Steel      | 360 - 770      | 210             |
| Glass FRP  | 380 - 17250    | 20 - 45         |
| Carbon FRP | 750 - 3250     | 53 - 190        |

Table 1.1: Strength and stiffness of several common building materials

Low stiffness may result in large deflections and uncomfortable vibrations of a bridge. Especially within railway engineering the criteria for deflection and vibration are strict. The effect of dynamic train loads on FRP railway bridges is an ongoing topic.

## 1.2 Research objective

Based on the problem definition in 1.1 the main research objective for this thesis is:

*A study into the application of a FRP deck in railway bridges*

## 1.3 Research questions

The research objective can be divided into several research questions.

- *What are common designs for FRP railway bridges*
- *What are the material properties of a FRP laminate suitable for railway bridges*
- *Is the static behaviour of common FRP railway bridge decks loaded by trains safe?*
- *How to model the dynamic loads of a train on a railway bridge*
- *Is the dynamic behaviour of common FRP railway bridge decks loaded by trains safe?*
- *How to improve the behaviour of common designs in railway bridges*

## 1.4 Aim and limitations

The scope of this thesis is limited by the following aims and limitations:

- The main focus of this thesis is a railway bridge deck. Other elements, like rails, ballast bed, bearings and foundation are mentioned, but won't be considered in detail.
- This thesis focusses on the application of fibre reinforced polymers in railway bridge decks. No other materials are taken into account for this bridge deck design. The lay-up, type of fibre, type of resin, type of core and type of manufacturing process are based on a literature study. Both ULS and SLS are taken into account. The long term behaviour of FRP and effect of the environment aren't treated in detail, but are taken into account using conversion factors
- This bridge is continuously over two equal spans and carrying one rail track. All requirements for load and resistance are based on Dutch environment and regulations.
- Both static and dynamic analysis are performed using Sofistik software.
- The speed limit of trains is set to 140 km/h.
- The dynamic behaviour of a train is not part of the scope.
- The maximum height of the bridge is limited to 2 meter in order to prevent unrealistic high bridges

## 1.5 Method

This report is divided into four parts. (1) Literature study, (2) Conceptual design, (3) final design and (4) reflection. The outline of this report is depicted in Figure 1.3.

### 1.5.1 Literature study

The literature study provides a review on the most important topics for this thesis. chapter 2 gives an introduction on the material, including the material properties, environmental effects and manufacturing methods. State-of-the-art in FRP railway bridges is discussed in chapter 3, as well as the requirements for bridge designs. A model for the dynamic analysis of railway bridges is described in chapter 4.

### 1.5.2 Conceptual design

Part III describes the analysis of two railway bridge designs. The first design is based on common bridge designs made out of FRP. The second design is based on common railway bridge designs. A static and dynamic analysis are performed, whereafter the results are verified and discussed. This results in a list of criteria for a final design, and the possible measures to meet the criteria.

### 1.5.3 Final design

Part IV describes the static and dynamic analysis of a final design. This final design is based on the list of measures presented in Part III.

### 1.5.4 Reflection

The conclusion and recommendations of this report are discussed in Part V.

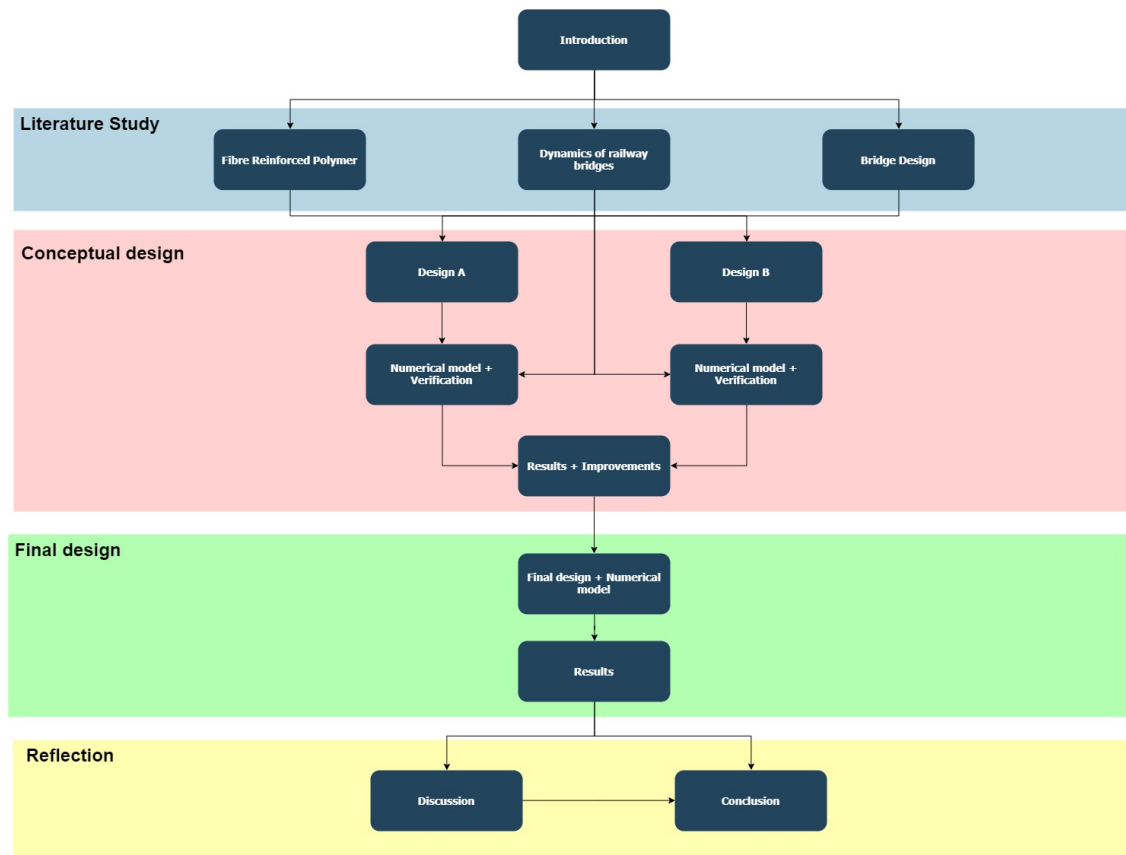


Figure 1.3: Outline of this report

# Part II

## Literature





### 2.1 Introduction to FRP

Fibre reinforced polymer, or FRP, is a composite material. It is build up of multiple materials, resulting in a synergy of the material properties. This build-up is easy adjustable by the engineers to get the best material properties for its purpose. Composites have been used since the beginning of time, when twigs and loams were combined in the construction of houses. FRP as we know it now has its origins in the aviation industry. Since then it is often used in e.g. aircrafts, sporting goods, water sports and in the transport sector. FRP composites are made of four main parts; fibres, resins, cores and additives. Fibres are the load-bearing parts of the material. Resin is the glue that keeps the fibres together and transfers the forces between the fibres. Four main tasks of resins are (1) keeping the fibres in their position, (2) transferring the forces to the fibres, (3) to prevent buckling and (4) protecting the fibres against humidity and other environmental influences. Cores are often used in sandwich panels and can have both a structural and/or non-structural purpose. The most common cores are solids, foams and honeycombs, and can be made out of metal, wood, plastic or other materials, depending on the application. Additives are used to facilitate the production process or to improve the properties of the material. An addition of pigment is possible too.

The application of FRP in civil structures is increasing in popularity, mainly due to its lightweight character. Furthermore, FRP requires low maintenance and the design is free to form. However, the characteristic high strength is accompanied by a low stiffness. Thereby, the material is orthotropic and layered and vulnerable for environmental influences.

The orthotropic properties of FRP make it difficult to work with. For civil structures the material is made homogenized using the following steps:

- Determine the material properties
- Determine the stacking of a laminate
- Calculate the ABD matrix
- Determine the homogenized laminate properties
- Apply conversion and safety factors

These homogenized laminate properties can be used in further calculations. However, the following failure modes must be taken into account:

- Matrix cracking  
Matrix cracking refers to cracks between the fibres. It does not endanger the structural

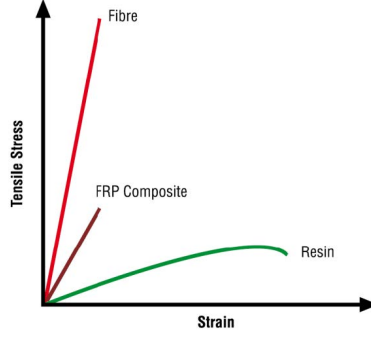


Figure 2.1: Stress-strain curve FRP  
Source: [Kolstein, 2008]

integrity, but it may initiate delamination. This mode usually is the first failure process. Matrix cracking in load direction is called splitting.

- **Delamination**  
Delamination is failure of the interface, the layers between the resin and fibres. It can lead to a reduced capacity of compressive bearing loads and stress concentrations on the interface edges. Delamination is not visible which makes it complicated to deal with.
- **Fibre failure**  
Fibre failure mostly leads to global failure. If fibres fail in tension, the energy from reloading the fibres can destroy the matrix leading to failure of the composite. Fibre kinking is a failure phenomenon for fibres under compression and leads to instability.

## 2.2 Properties

The strength and stiffness of fibres is higher than resins. Therefore, the properties of the composite must be somewhere in between [Meer, 2018], as shown in Figure 2.1. The FRP properties depend on the properties of the fibres, properties of the resin, the fibre volume fraction  $V_f$  and the geometry and orientation of the fibres. [Gurit Holding AG, 2000]. The fibre volume fraction is the ratio between the volume of fibres and the total volume of the composite (Equation 2.1). The geometry and orientation of the fibres are discussed in section 2.4.

$$V_f = \frac{V_{fibres}}{V_{total}} \quad (2.1)$$

A ply, or layer, in a laminate is build up of fibres, resins and additives. The combinations of fibres and resins creates a synergy, but are dependent on the Fibre Volume Fraction  $V_f$ . The total strength and stiffness of a ply are a combination of properties of the individual materials. The rule of mixture is used to calculate the actual stiffness of a ply, which depends on the direction of the fibres, and is shown in Equation 2.2.

$$\frac{1}{\left(\frac{V_f}{E_f} + \frac{1-V_f}{E_m}\right)} < E_{FRP} < V_f E_f + (1 - V_f) E_m \quad (2.2)$$

Where:

|             |                            |         |
|-------------|----------------------------|---------|
| $V_f =$     | Fibre volume fraction      | $[-]$   |
| $E_f =$     | Elastic modulus of fibre   | $[MPa]$ |
| $E_m =$     | Elastic modulus of matrix  | $[MPa]$ |
| $E_{FRP} =$ | Elastic modulus of FRP ply | $[MPa]$ |

The stiffness lies between an upper and lower bound, dependent on the fibre direction, see Figure 2.2. The three main directions of the local coordinate frame are (1) in fibre direction and (2) and (3) perpendicular to the fibres. The upper and lower bound approach, used to determine the stiffness, cannot be used in case of strength calculations, as it results in an overestimation of the strength. The real values can only be determined using tests, however, the JRC document [Ascione et al., 2016] provides tables with reference values.

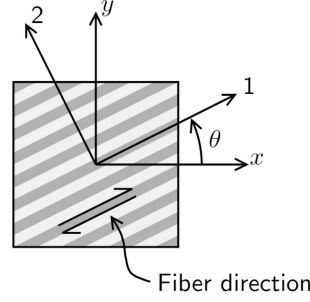


Figure 2.2: Fibre direction in local (123) and global (xyz) coordinate frame  
Source: [Gurit Holding AG, 2000]

The behavior of an orthotropic material is described using the orthotropic elasticity matrix, as shown below. As internal stresses cannot be redistributed by means of material ductility, only elastic analysis are used.

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{23})}{E_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{12}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix}$$

## 2.3 Materials

The three main material types are (1) fibres, (2) resins and (3) cores. An overview of the properties of fibres and other FRP materials is given by Table 2.1.

### 2.3.1 Fibres

The load-bearing part of the composite are fibres. Commonly used fibres are glass fibres, carbon fibres, aramid fibres, polyester fibres and natural fibres.

**Glass fibres** The first one is the most commonly used due to its high strength and low price. Glass fibres are made by melting sand, kaolin and limestone at 1600° and pulling it through drawing points resulting in continuous filaments. There are mainly five types of glass fibres; A, C, E, R and S glass and differ on material properties and resistance. Most commonly used are E-glass and S-glass.

**Carbon fibres** Carbon fibre is the most expensive reinforcement and is rarely used in civil structures. On the other hand, it has the highest strength and stiffness. The carbon fibres are grouped in different groups, depending on their modulus band; (1) intermediate modulus (IM), (2) high modulus (HM) (3) ultra high modulus (UHM) and (4) high strength (HS). To get carbon fibres, carbon-rich organic precursors which are already in form are produced by controlled

oxidation, carbonisation and graphitisation. The latter determines the type of carbon, namely a high strength (HS) fibre, a high modulus (HM) fibre or somewhere in between.

**Aramid fibres** Aramid fibres are produced by spinning a solid fibre from a liquid chemical blend, creating filaments of about 10 to 12  $\mu\text{m}$  in diameter. It is well-known under its commercial name called Kevlar, but other companies offer the same product under different names [Gurit Holding AG, 2000]. This man-made material combines high strength with low density.

**Other fibres** Besides the regular carbon, aramid and glass fibres, more fibres are available. However, the use of these fibres in civil structures is limited due to the disadvantages of the material. Some of these fibres are:

|                |                                                                                                                 |
|----------------|-----------------------------------------------------------------------------------------------------------------|
| Polyester      | Mainly used for its high impact resistance, as the modulus is low                                               |
| Polyethylene   | Has a high tensile strength, but the high price and difficulty in fabrication makes it an expensive alternative |
| Quartz         | Resistant to very high temperatures, but very expensive                                                         |
| Boron          | Very strong and resistant to very high temperatures, but extremely expensive                                    |
| Natural fibres | Cheap, but low mechanical properties and unknown long-term behavior                                             |

### 2.3.2 Resin

Polymer resins, or matrix, are the glue of a composite. It connects the fibres and transfers the forces between fibres. Furthermore, the matrix fixes fibres in the desired geometrical arrangement, prevents buckling of the fibres and protects them from environmental effects. A polymer resin consists of long chains of molecules, consisting of repeating subunits. Two types of polymers are classified: (1) thermoplastic polymers and (2) thermoset polymers. A thermoplastic polymer softens every time it is heated, while thermoset polymers are converted into a rigid structure due to the formation of crosslinks between the molecules. The mechanical properties only decrease when reaching the Glass Transition Temperature ( $T_g$ ) [Pavlovic and Csillag, 2018]. This  $T_g$  differs per resin material and effects among others the stiffness, compressive and tensile strength and water resistance. Figure 2.3 shows the determination of the Transition Glass Temperature [Pavlovic and Csillag, 2018]. The most common resins are polyester, vinyl(ester) and epoxy. The material properties are given by Table 2.1.

### 2.3.3 Cores

The structural efficiency of an FRP material is improved by adding a core in the middle creating a sandwich structure. The outer layers are made of fibres and resins and fulfil the main load bearing function of the composite. The core bonds these two outer layers. Three types of cores are used in sandwich panels: (1) Lightweight solid materials, (2) foams and (3) honeycombs [Kolstein, 2008], see Figure 2.4.

**Solid core** Balsa-wood is the most common solid core used in bridge design. It is perfectly applicable for thermal insulation, acoustic absorption and structural situations due to its high compressive strength. However, the high density of the material counteracts the low weight character of FRP materials. Also synthetic materials are used.

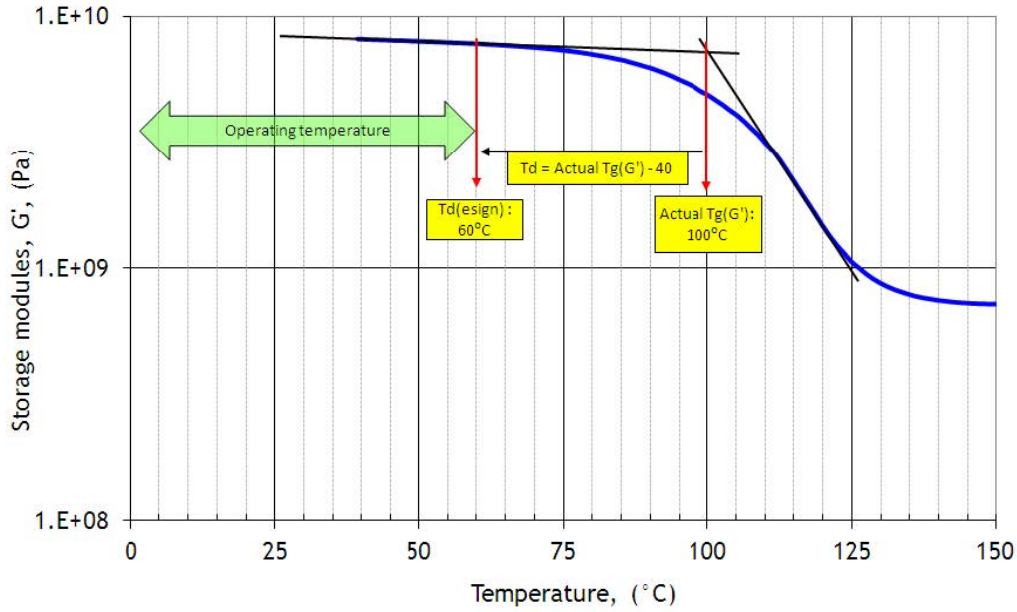


Figure 2.3: Glass Transition Temperature  
Source: Slides Lecture 2 CIE5128

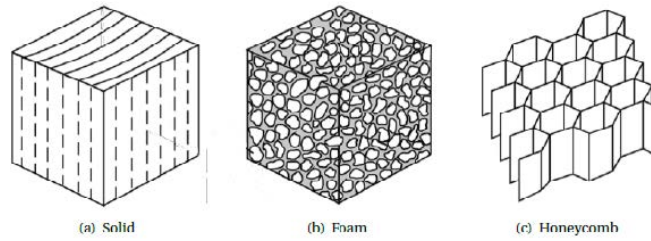


Figure 2.4: Main core types  
Source: [Pavlovic and Csillag, 2018]

**Foam core** Foam cores are used in both structural and non-structural situations. High-density foams are used in the former situation as they require high compression and shear strength. In non-structural situations the foam is used as an inner mould, using low-density foams. Foam cores have a high compressive strength, high stiffness-to-weight ratio and a low price, making them mostly used in sandwich panels [Pavlovic and Csillag, 2018]. Typical materials are polyurethane (PUR), polyvinyl-chloride (PVC) and polymethacryllmides (PMI). The properties of these materials are given in Table 2.1. The market prices are provided by Fibrecore Europe on April 30th 2019.

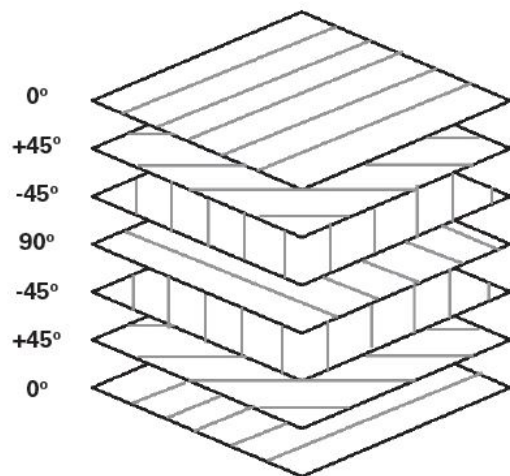
**Honeycomb** Honeycombs combine high compressive and shear strength with low weight. The hexagonal shape with thin walls is commonly used, where the walls are made out of aluminium, glass fabric, paper, aramid or even steel. The downside of honeycomb configuration is the high cost, compared to foam cores.

## 2.4 Lay-up

A FRP material is build up of different layers or plies. Each ply has its own main direction. Stacking multiple plies on top of each other creates a laminate. The geometrical arrangement of these layers determines the final properties of the FRP laminate, see Figure 2.5.

| Fibre              | Strength [MPa] | Stiffness [GPa] | Density [kg/m <sup>3</sup> ] | Price [euro/m <sup>2</sup> ] |
|--------------------|----------------|-----------------|------------------------------|------------------------------|
| E-glass            | 2750           | 73              | 2570                         | 2.52                         |
| R-glass            | 3450           | 86              | 2520                         | -                            |
| LM Aramid          | 3600           | 60              | 1450                         | -                            |
| HM Aramid          | 3100           | 120             | 1450                         | 21                           |
| UHM Aramid         | 3400           | 180             | 1470                         | -                            |
| HS Carbon          | 3600           | 238             | 1790                         | 35                           |
| IM Carbon          | 4500           | 350             | 1750                         | 37                           |
| HM Carbon          | 4700           | 410             | 1880                         | 80                           |
| UHM Carbon         | 2000           | 440+            | 2000                         | 55                           |
| <b>Resins</b>      |                |                 |                              | [euro/kg]                    |
| Polyester          | 55             | 3.55            | 1200                         | 2.00                         |
| Vinylester         | 75             | 3.35            | 1100                         | 4.00                         |
| Epoxy              | 75             | 3.10            | 1250                         | 7.00                         |
| <b>Solid cores</b> |                |                 |                              |                              |
| PUR                | 0.3 - 1.0      | 0.006 - 0.03    | 50 - 100                     | -                            |
| PVC                | 0.5 - 2.0      | 0.02 - 0.09     | 40 - 80                      | -                            |
| PMI                | 0.5 - 1.5      | 0.03 - 0.09     | 30 - 70                      | -                            |

Table 2.1: Properties of FRP materials

Figure 2.5: Layers in a FRP material  
Source: [Gurit Holding AG, 2000]

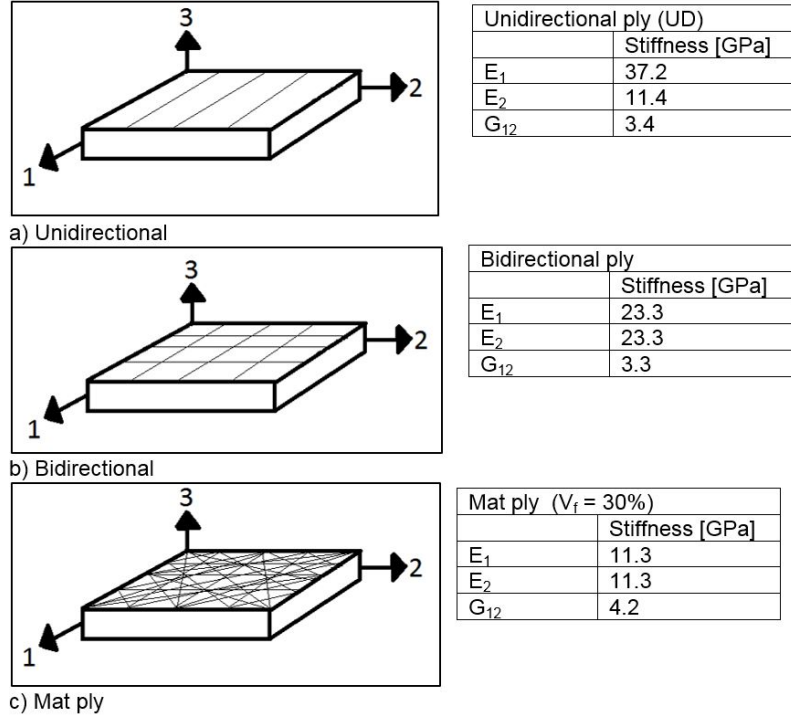


Figure 2.6: Lay-up of plies. The values refer to glass fibre with a fibre volume fraction of 50%, and 30% for the mat ply lay-up.

There are three main types of ply lay-up: (1) unidirectional (UD), (2) bidirectional and (3) a mat ply. The fiber direction in global coordinate frame is described in terms of angle with respect to the global x-axis, which is in the primary load direction. A 0-ply is in the primary load direction as shown by Figure 2.6a. A 90-ply is perpendicular to the primary load direction. Figure 2.6b shows a combination of both 0 and 90-ply, a so called cross ply. 0-ply are used to take the load in longitudinal direction. The loads in lateral direction are taken by the 90-ply. The shear loads are taken by  $\pm 45$  plies.

The ply lay-up mainly has a local influence on the behaviour of bridges. Most fibres are in main load direction to carry the loads. To prevent surface cracking  $\pm 45$  degrees plies are placed at the surface.

The behavior of a laminate is described using the classical laminate theory (CLT). The extension and curvature of a laminate are connected to its normal forces and bending moments using the ABD matrix, as shown by Equation 2.4a and 2.4b. The ABD matrix represents the properties of the laminate and is determined using Equation 2.3a, 2.3b and 2.3c, where  $z_i$  is the coordinate along the thickness and  $Q$  the plate stress stiffness matrix.

$$\mathbf{A} = \sum_{i=1}^n (z_i - z_{i-1}) \mathbf{Q}_i \quad (2.3a)$$

$$\mathbf{B} = \sum_{i=1}^n \frac{1}{2} (z_i^2 - z_{i-1}^2) \mathbf{Q}_i \quad (2.3b)$$

$$\mathbf{D} = \sum_{i=1}^n \frac{1}{3} (z_i^3 - z_{i-1}^3) \mathbf{Q}_i \quad (2.3c)$$

$$(2.3d)$$



$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix} \quad (2.4a)$$

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \quad (2.4b)$$

Delft University of Technology has developed a tool to easily calculate the ABD matrix and can be found at [delftxtools.tudelft.nl/frp](http://delftxtools.tudelft.nl/frp).

**Suitable lay-up for bridge design** CUR96 [CUR-aanbeveling 96, 2018] recommends to apply at least 12.5% of the fibres in 90°-, +45°- and -45°- direction. A parametric study performed by Zarifis confirms this. The ply orientation in the webs has a different distribution, according to Zarifis [Zarifis, 2018]:

35% in 0° direction, 25% in 90° direction, 20% in +45° direction and 20% in -45° direction.

Other criteria to take into account when designing FRP laminates are the angles between two fibre layers. They must be limited, as it can lead to delamination. Furthermore, the structure must ensure that the bending moments, shear forces, normal forces and torsional moments can be taken, without unacceptable distortions.

For flanges the 90°-direction fibres must be placed at the outer skin to ensure transverse force distribution. The outer skin of the webs are in 90-direction as well, followed by a ±45°-direction and the 0 direction fibres. Both are recommended by Zarafis.

According to these criteria the following lay-ups are suitable for bridge design, where the subscript s stand for symmetric:

Flanges:

[12.5% - 90°|12.5% - ±45°|62.5% - 0°|12.5% - ±45°]<sub>s</sub>

Webs:

[25% - 90°|20% - ±45°|35% - 0°|20% - ±45°]<sub>s</sub>

## 2.5 External influences

Several external influences do have impact on a FRP civil structure. Temperature, moisture and UV radiation are the main environmental influences. Furthermore, long term static loading cause creep and the dynamic loads can cause fatigue. Conversion factors are used to take these influences into account when designing a FRP structure.

**Influence of temperature** The fibres in FRP civil structures are mostly made of non-organic materials. These materials cannot be influenced by temperature differences. Temperature differences, on the other hand, do have impact on the matrix. An increase of the temperature leads to softening of the resin. It turns from a glossy phase into a rubbery phase. The transition temperature is given by the Glass Transition Temperature range  $T_G$ , provided by the supplier, see Figure 2.3.

The influence of temperature on the material is taken into account using a thermal conversion factor  $\eta_{ct}$ :

**Influence of moisture** Moisture influences the matrix and fibre/matrix interface as it causes delamination due to high osmotic pressure. Temperature has an enhancing effect on this process. However, the effect of moisture is reversible.

The influence of moisture is taken into account using the humidity conversion factor  $\eta_{cm}$ :

**Influence of UV-radiation** The wavelength spectrum of UV is in most cases similar to the UV absorption spectra of thermosetting resins. This causes degradation of the matrix in form of surface oxidation, brittleness and microcracking of the matrix. It has only effect on the aramid fibres. UV and moisture have a synergetic effect. UV stabilizers, coats, gels or UV resistance matrix improve the UV resistance of a FRP material.

The influence of UV-radiation isn't taken into account.

**Creep** Long term static loading cause a time dependent and permanent deformation of the material, better known as creep. This elongation of the fibres causes stresses and strains in the matrix, eventually leading to creep-rupture. Creep is influenced by material related factors, such as the type of resin, quality of the interface and the form of reinforcement, and by external and environmental related factors such as the type of loading, moisture content and temperature [Pavlovic and Deursen, 2018].

The influence of creep is taken into account using the creep conversion factor  $\eta_{cv}$  and depends on the the type and duration of the loading and vary for strength and stiffness. These values are determined using tests. Reference values, creep values after 20 years ( $\eta_{cvm,20}$ ) are given as result. Equation 2.5 is used to calculate the creep factor.

$$\eta_{cv}(t_v) = (\eta_{cvm,20})^T \quad (2.5)$$

where  $T = 0.253 + \log(t_v)$

**Fatigue** If the loading exceeds 500 load cycles, or if the cyclic loading leads to stresses of more than 40% ultimate strength, the fatigue conversion factor  $\eta_{cf}$  is used. The same values are used for SLS and ULS;

**Total conversion factor** The total conversion factor is a multiplication of all conversion factors;

$$\eta_c = \eta_{ct} * \eta_{cm} * \eta_{cv} * \eta_{cf} \quad (2.6)$$

**Material factors** The material safety factors are two-folded:

$$\gamma_M = \gamma_{M1} * \gamma_{M2} \quad (2.7)$$

Where:

$\gamma_{M1}$  is linked to uncertainties in obtaining the correct material properties

$\gamma_{M2}$  is linked to uncertainties in production processes

## 2.6 Manufacturing

The manufacturing of FRP materials consists of at least three steps: (1) placement of reinforcement (fibres), (2) getting the resin (with additional additives) between the fibres and (3) curing the resin. The way in which these steps are executed have an impact on the final properties of the material. All the steps are performed in a mold. This mould defines the final shape of the composite. In

the manufacturing of FRP three processes can be distinguished: (1) open mold process, (2) closed mold process and (3) continuous process. The former isn't covered with a film or other mold, but is exposed to the environment. It is vulnerable to emission of unwanted substances, the pressure during curing cannot be manipulated and it is labor expensive. On the other hand, no expensive molds are needed and this method has a high flexibility. Therefore, it is still widely used in FRP manufacturing. Common examples of open mold manufacturing processes are:

- Centrifugal casting
- Filament winding
- Hand lay-up
- Spray lay-up

To manipulate the internal environment of a mold, a closed mold manufacturing process can be used. High pressure or vacuum inside the mold has a positive effect on the curing of the resin. Common examples of closed mold manufacturing processes are:

- Pressure bag molding
- Compression molding
- Injection molding
- Resin Transfer molding (RTM)
- Vacuum infusion (VARTM)

One of the downsides of closed mold manufacturing is the limited web and flange thickness. Increasing the thickness can lead to a situation where the filling time is larger than the curing time.

The continuous manufacturing process enables for continual production. This process is suitable for FRP profiles with constant cross sections and material properties. Common examples of continuous processes are:

- Pultrusion
- Continuous laminating
- Continuous filament winding

### 3.1 Current situation

Railinfratrust B.V. is the owner of the Dutch railway network, which consists of more than 7000 kilometers of rail tracks and almost 3000 railway bridges. Concession of management and maintenance of the Dutch railway network is granted to ProRail B.V., a subsidiary of Railinfratrust B.V.. The main activities of ProRail B.V. are:

1. Maintenance of the main railway infrastructure
2. Preparation and execution of the expansion of the main railway infrastructure
3. Fair and non-discriminatory and transparent allocation of the capacity of the main rail infrastructure
4. Directing traffic over the main railway infrastructure

Although ProRail B.V. is responsible for the activities mentioned above, these activities are executed by contractors. The contracts signed with a contractor are performance based; the contractors have to ensure good performance of the assets, while minimizing the costs. Due to these contracts the out of service time of railway tracks are minimized. The use of lightweight bridge decks has a positive effect on the out of service time of tracks.

### 3.2 Common bridge designs

The span of a bridge has a great influence on the structural form. An overview of the different structural forms and their corresponding main span ranges is given in Figure 3.1. For small span bridges a plate girder or box section beam bridge is commonly used. These bridge types are loaded by shear forces and bending moments. Common bridge deck designs for small spans are given in the following sections.

#### 3.2.1 Concrete and steel bridge decks

Plate girders or box section beams are mainly made out of (prestressed) concrete. Common shapes are solid decks, beam bridges, box beam bridges or through girders, see Figure 3.2. In latter design the flooring is provided at the bottom of the superstructure. In this way the height of the superstructure can be used without reducing the clearance under the bridge. This bridge type is commonly used in railway bridges due to the limited required free space, compared to traffic bridges.

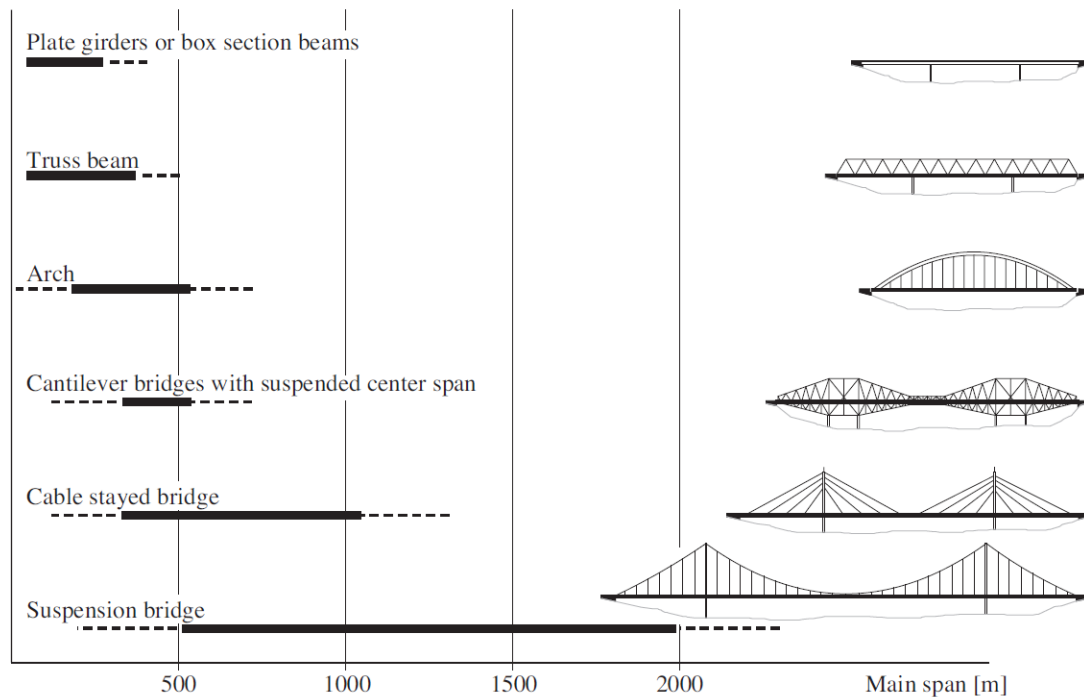


Figure 3.1: Main type of longitudinal structural forms and corresponding main span ranges  
Source: [Lebet et al., 2013]

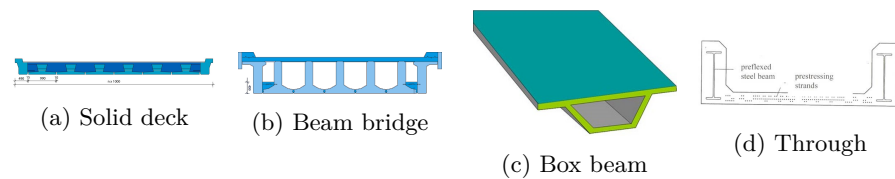


Figure 3.2: Bridge types  
Source: Lecture slides CT5127

### 3.2.2 Fibre Reinforced Polymer bridge decks

The positive effects of FRP in infrastructure has been discovered, resulting in an increase in application of the material. The high strength and low weight makes it widely applicable. The material can be used in different ways:

- As cables and tendons
- As beams and girders
- As trusses
- During repair work
- As bridge deck systems
- In hybrid applications

This thesis is limited to the application of FRP in bridge deck systems. The following paragraph gives an overview of the state-of-the-art in FRP bridge deck systems.

**FRP bridge deck system** Various types of FRP bridge deck systems are developed all over the world, which confirms the fast development of FRP within the infrastructure. An overview of some of the FRP bridge deck systems is given by Table 3.1 and /autoreffig: bridge deck system.

| Manufacturer                 | Bridge deck system | Material                    | Production process |
|------------------------------|--------------------|-----------------------------|--------------------|
| Fiberline Composites         | ASSET system       | Glass fibres and polyester  | Pultrusion         |
| Martin Marietta Composites   | DuraSpan           | Glass fibres and polyester  | Pultrusion         |
| Maunsell Structural Plastics | ACCS               | Glass fibres and polyester  | Pultrusion         |
| Hardcore Composites          | Hardcore system    | Glass fibres and vinylester | VARTM              |
| Creative Pultrusion          | Superdeck system   | Glass fibres and vinylester | Pultrusion         |
| InfraCore Company            | InfraCore Inside   | Glass fibres and vinylester | VARTM              |

Table 3.1: Overview of different FRP bridge deck systems

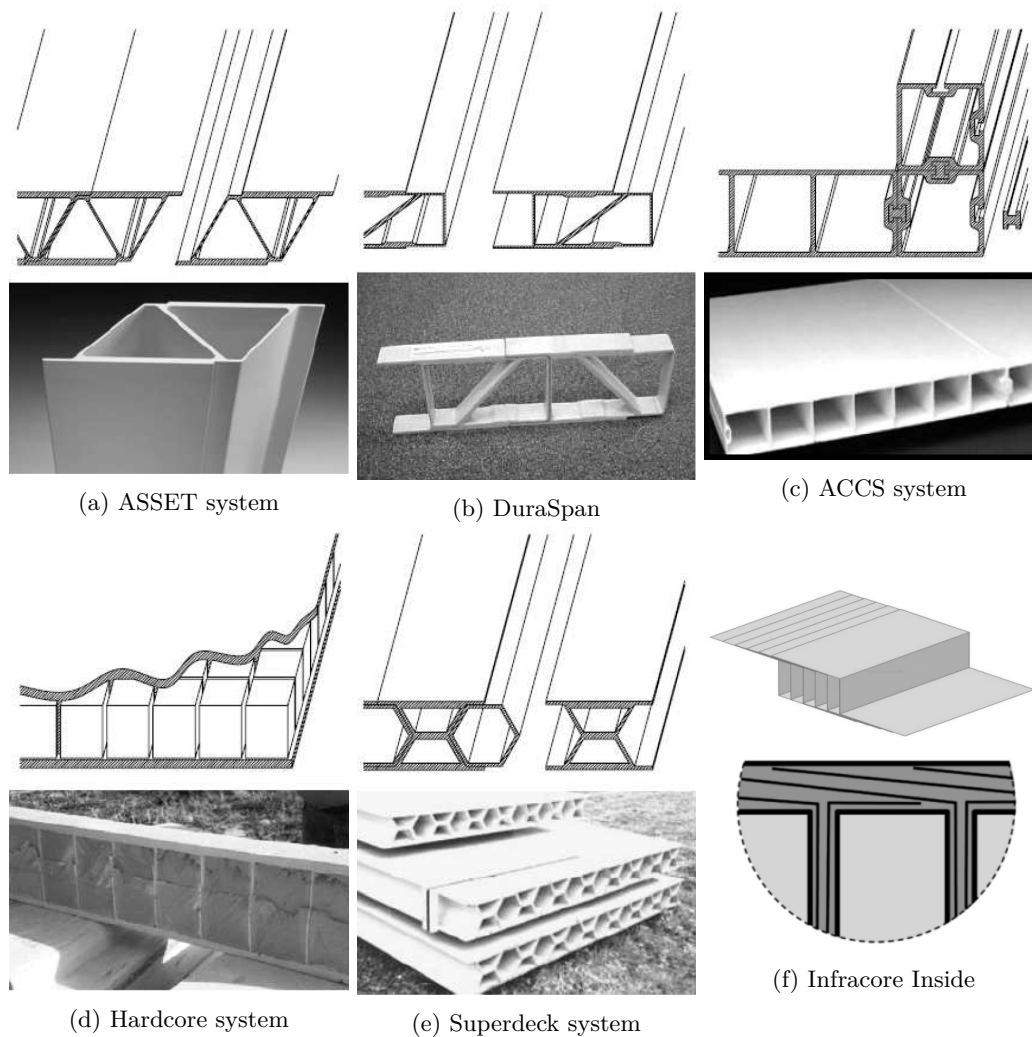


Figure 3.3: FRP bridge deck systems

Source: [Potyrała et al., 2011], fibercore-europe.com

**Sandwich systems** All bridge deck systems in Figure 3.3 are sandwich designs. A sandwich is build up of three parts (see Figure 3.4); (1) a top face sheet, (2) a core section and (3) a bottom face sheet. Both face sheets are made of FRP materials providing the strength and stiffness. The lightweight core is made of a foam, honeycomb or solid material and mostly acts as non-structural support formers.

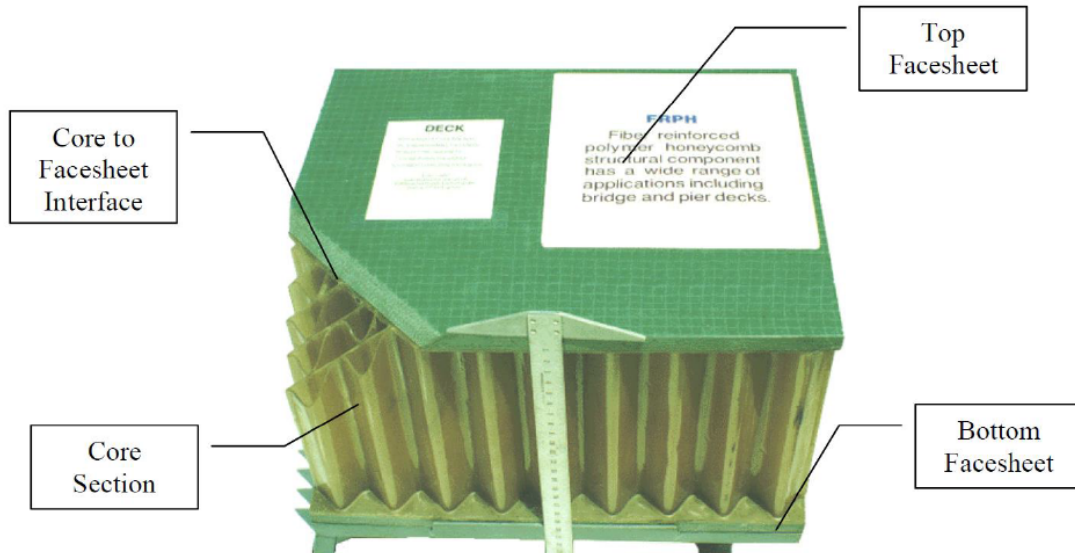


Figure 3.4: Sandwich block  
Source: [Peeters, 2015]

### 3.3 Standards and Guidelines

Civil structures are expected to be safe and do not collapse. Various standards have been drawn up to assay the structural safety of structures. These standards are made to establish uniform criteria for the construction of structures.

Different standards apply to railway bridges in the Netherlands. A list of some of the general standards used in the design of railway bridges is given below:

**NEN 1990** Basis of structural design

**NEN 1991** Actions on structures

**OVS** Ontwerpvoorschriften

The NEN standards are a translation of the European Eurocodes. The OVS standards are prescribed by ProRail and mainly relate to railway structures. These design regulations mainly build on the NEN standards.

In this chapter the various standards and guidelines are described.

#### 3.3.1 Geometric specifications

The geometric specifications are described in OVS documents. Important parts are the Profiel van Vrije Ruimte (PVR), maintenance path and ballast bed.





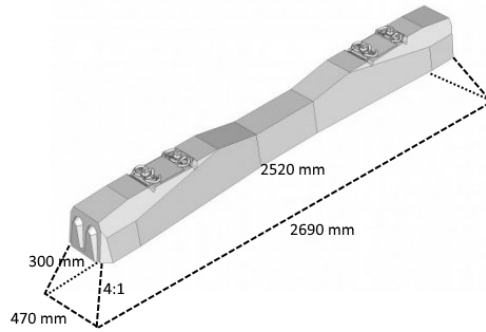


Figure 3.6: Spread under a sleeper  
Source: meteoer.nl

- SW/0
- SW/2
- Rail train model

In addition to these vertical load models horizontal start and brake forces are part of the train load models.

**LM71** LM71 describes the static effect of the vertical load caused by normal rail traffic [CEN, 2018]. This load model consists of four point loads of 250 kN and a distributed load of 80 kN/m, see Figure 3.7.

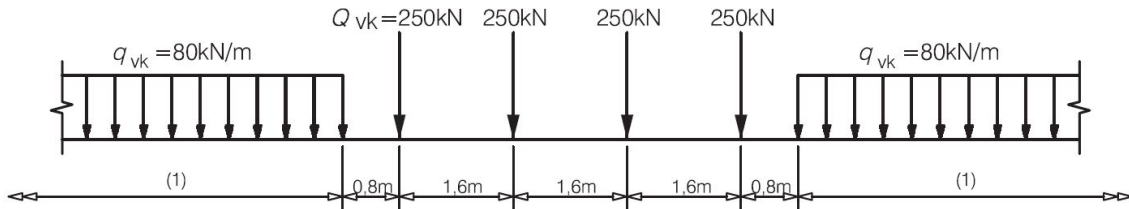


Figure 3.7: Load model 71  
Source: [CEN, 2018]

**Additional factors** The effect of lighter or heavier traffic than LM71 is taken into account using an  $\alpha$  factor. This factor is given by the National Appendix (NEN 1991-2 NB) and is  $\alpha = 1.21$ .

**SW/0 and SW/2** These load models describe the effect of a train load on a continuous bridge. SW/0 describes normal rail traffic, SW/2 is used for heavy rail traffic. Load model SW/0 and SW/2 consist of two distributed loads. The magnitude of the load for SW/0 is 133 kN/m and 150 kN/m for SW/2. Furthermore, the distributed load has a gap of 5.3 m for SW/0 and 7.0 m for SW/2 (Figure 3.8).

**Start and brake force** The start and brake forces are given by NEN 1991-2 6.5.3. The start forces are 33.0 kN/m, the brake forces are 20 kN/m. The additional  $\alpha$  factor also applies to the start force.

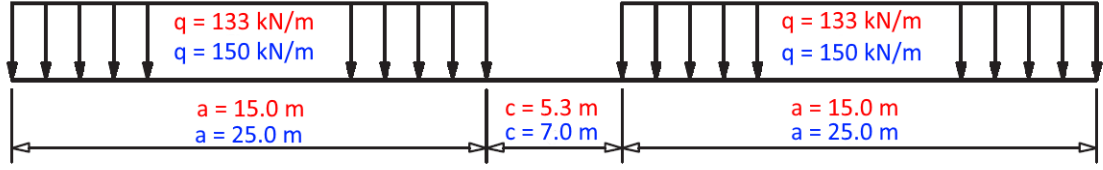


Figure 3.8: Load model SW/0 (red) and SW/2 (blue)  
Source: [CEN, 2018]

**Dynamic factor** The dynamic factor  $\Phi$  takes into account the dynamic enlargements of the results. This factor depends on the way of maintenance. A distinction is made between (1) a carefully maintained track and (2) a standard maintained track. For a standard maintained track Equation 3.1 describes how to determine the dynamic factor. This dynamic factor is multiplied with the static loads and therefore may not be used with the real train model.

$$\Phi = \frac{2.16}{\sqrt{L_\Phi} - 0.2} + 0.73 = 1.39 \quad (3.1)$$

Where  $L_\Phi = 12m$  (NEN-EN 1991-2 Table 6.2 case 5.1)

**Real train model** A dynamic analysis must be performed using a real train model. For this thesis a VIRM4 train model is used. This type of intercity train is commonly used in the Netherlands. The maximum speed is 140 km/h, the maximum axle load ( $F_t$ ) is 21.3 ton and the total length of a train unit is 28.5 meter [Vlootmanagement NS Techniek, 2018]. The wheel spacings of a train play an important role in the dynamic calculations. Figure 3.9 shows a picture of a VIRM4 train including the wheel spacings.

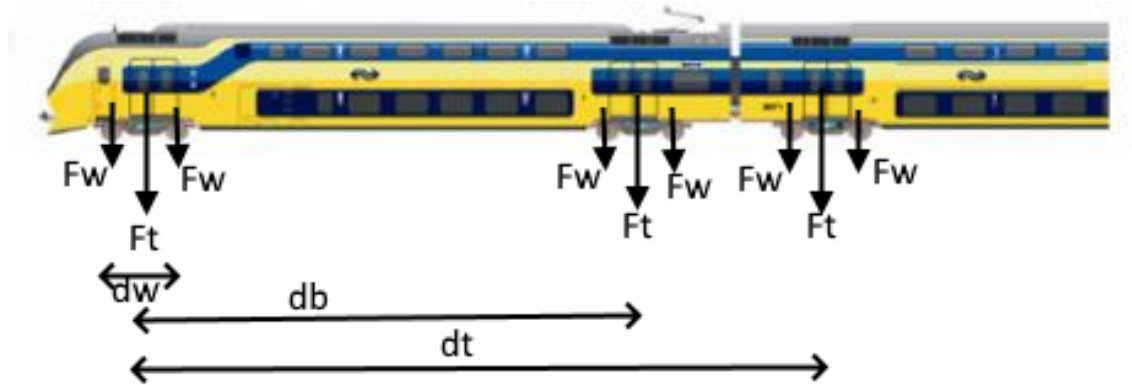


Figure 3.9: Wheel spacing of VIRM4  
Source: michawertheim.nl

### 3.3.3 Guidelines

The guidelines describe the limitations on the output. These guidelines are given in NEN 1990 and are mainly prescribed for comfort and safety reasons. The following limitations are taken into account:

- Deflection
- Acceleration

**Deflection** The maximum deflection of a bridge is limited to  $L/800$ , according to NEN 1990 NB A2.4.4.2.3.

**Maximum acceleration** The maximum vertical acceleration is limited to  $3.5m/s^2$  and given by NEN 1990 A2.4.4.2.1. This maximum acceleration must be calculated along the rails, for a bridge with ballast bed.

## CHAPTER 4

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### Dynamics

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The dynamic behaviour of a bridge can cause uncomfortable situations. Two famous examples are the Millennium Bridge in London and the Tacoma Bridge in the American state Washington. Both examples experienced large movements due to the dynamic load on the bridge. The swaying of the Millennium Bridge was caused by the pedestrians. This problem was solved using additional dampers. The extreme torsional motions of the Tacoma Bridge were primarily caused by the aeroelastic flutter and a bad bridge design. Unfortunately, the Tacoma bridge collapsed [Harish, 2019].

This chapter describes the dynamic theory to analyse the dynamic loads on a bridge and the response of the bridge [Blaauwendraad, 2010].



(a) Millennium bridge.  
Source: Wikipedia.nl



(b) Tacoma bridge.  
Source: allthatsinteresting.com

### 4.1 Equation of Motion

The dynamic behaviour of a bridge is hard to describe. A complex geometry, variable load forces and the material properties are some factors influencing this behaviour. Therefore, engineers make use of simplified models. One of these models is a mass-spring rigid body system, where the total mass of the bridge is reduced to a mass block which doesn't deform, and the stiffness of the bridge is modelled as a spring, see Figure 4.2. This system is an undamped system. Thereby, the bridge has some damping. After all, an oscillating bridge comes to rest after a certain time period. Such

a system is called a damped system, where the rigid body mass block is connected to a spring and a damper.

The dynamic behaviour is described using several formulas. These formulas are based on Newton's second law, see Equation 4.1.

$$\sum F = m * \ddot{u} \quad (4.1)$$

The undamped system is discussed in subsection 4.1.1. subsection 4.1.2 describes the damped system.

### 4.1.1 Undamped vibration

The undamped system describes the oscillating behaviour of a structure without damping. Such a system is shown in Figure 4.2.

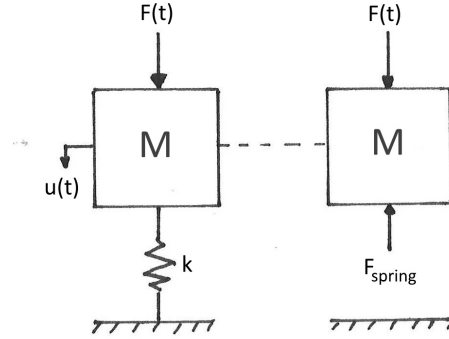


Figure 4.2: Undamped system

The equation of motion for an undamped system, based on Newton's second law and Figure 4.2, is given by Equation 4.2.

$$F(t) - F_{spring} = m * \ddot{u}(t) \quad (4.2)$$

This can be written as:

$$m\ddot{u} + ku = F(t) \quad (4.3)$$

Where:

|                |                       |         |
|----------------|-----------------------|---------|
| $m =$          | Mass of the structure | $[kg]$  |
| $k =$          | Stiffness             | $[N/m]$ |
| $u =$          | Displacement          | $[m]$   |
| $F(t) =$       | Load equation         | $[N]$   |
| $F_{spring} =$ | Spring force          | $[N]$   |

The movement of a system without any loading is called the steady state response. The movement of the system only depends on the initial conditions. The steady state response of an undamped system is given by Equation 4.4 and Equation 4.5. The values  $A$  and  $\theta$  depend on the initial conditions.

$$u(t) = A * \cos(\omega_0 t + \theta) \quad (4.4)$$

$$\ddot{u}(t) = -\omega_0^2 * A * \cos(\omega_0 t + \theta) \quad (4.5)$$

Where:

|              |                                            |           |
|--------------|--------------------------------------------|-----------|
| $A =$        | Amplitude                                  | $[m]$     |
| $\theta =$   | Phase                                      | $[rad]$   |
| $\omega_0 =$ | Natural angular frequency of the structure | $[rad/s]$ |

In most cases the force can be modelled as a sinusoidal load with a specific load frequency, see Equation 4.6.

$$F(t) = \hat{F} * \sin(\omega * t) \quad (4.6)$$

Where:

|            |                        |           |
|------------|------------------------|-----------|
| $\omega =$ | Load angular frequency | $[rad/s]$ |
|------------|------------------------|-----------|

The displacement and acceleration equations for a mass-spring system loaded by a sinusoidal load, without initial conditions ( $u(0)=\dot{u}(0)=0$ ), are given by Equation 4.7 and Equation 4.8.

$$u(t) = \frac{\hat{F}}{\omega_0^2 * m - \omega^2 * m} * (\sin(\omega * t) - \sin(\omega_0 * t)) \quad (4.7)$$

$$\ddot{u}(t) = \frac{-\omega^2 * \hat{F}}{\omega_0^2 * m - \omega^2 * m} \sin(\omega * t) + \frac{\omega_0^2 * \hat{F}}{\omega_0^2 * m - \omega^2 * m} \sin(\omega_0 * t) \quad (4.8)$$

Where:

|              |                                        |           |
|--------------|----------------------------------------|-----------|
| $\hat{F} =$  | Force                                  | $[N]$     |
| $m =$        | Mass of the structure                  | $[kg]$    |
| $\omega_0 =$ | Natural angular frequency of structure | $[rad/s]$ |
| $\omega =$   | Angular load frequency                 | $[rad/s]$ |

#### 4.1.2 Damped vibration

The damping of a structure is introduced using a dashpot in the SDOF system, see Figure 4.3. The corresponding equation is given by Equation 4.9.

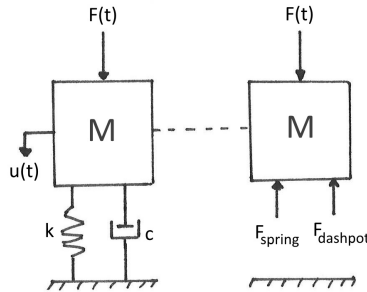


Figure 4.3: Damped system

$$F(t) - F_{dashpot} - F_{spring} = m\ddot{u} \quad (4.9)$$

This can be written as:

$$m\ddot{u} + c\dot{u} + ku = F(t) \quad (4.10)$$

Where:

$$c = \text{Damping} \quad [Ns/m]$$

The steady state behaviour of a damped SDOF system is given by Equation 4.11 and Equation 4.12. The values  $A$  and  $\theta$  depend on the initial conditions.

$$u(t) = A * e^{-\zeta\omega_0 t} \cos(\omega_e t + \theta) \quad (4.11)$$

$$\begin{aligned} \ddot{u}(t) = A\zeta^2\omega_0^2 e^{-\zeta\omega_0 t} \cos(\omega_0 t + \theta) + 2A\zeta\omega_0 e^{-\zeta\omega_0 t} \omega_e \sin(\omega_e t + \theta) \\ - A e^{-\zeta\omega_0 t} \omega_e^2 \cos(\omega_e t + \theta) \end{aligned} \quad (4.12)$$

Where:

|              |                                            |           |
|--------------|--------------------------------------------|-----------|
| $A =$        | Amplitude                                  | $[m]$     |
| $\zeta =$    | Damping parameter of the structure         | $[-]$     |
| $\omega_0 =$ | Natural angular frequency of the structure | $[rad/s]$ |
| $\theta =$   | Phase                                      | $[rad]$   |
| $\omega_e =$ | Natural angular frequency of damped system | $[rad/s]$ |

The natural angular frequency of a damped system depends on the natural angular frequency of structure and the damping parameter of a structure according to:

$$\omega_e = \omega_0 \sqrt{1 - \zeta^2}$$

The damping parameter of the structure depends on the damping, natural angular frequency of the structure and the mass of the structure according to:

$$\zeta = \frac{c}{2\sqrt{\omega_0^2 * m^2}}$$

The displacement and acceleration of a damped SDOF system loaded by a sinusoidal load (see Equation 4.6) is given by Equation 4.13 and Equation 4.14.

$$u(t) = \frac{\hat{F}}{m\omega^2 \sqrt{(1 - \frac{\omega^2}{\omega_0^2})^2 + (2\zeta \frac{\omega}{\omega_0})^2}} \cos(\omega t - \arctan(\frac{2\zeta \frac{\omega}{\omega_0}}{1 - \frac{\omega^2}{\omega_0^2}})) \quad (4.13)$$

$$\ddot{u}(t) = -\omega^2 * \frac{\hat{F}}{m\omega^2 \sqrt{(1 - \frac{\omega^2}{\omega_0^2})^2 + (2\zeta \frac{\omega}{\omega_0})^2}} \cos(\omega t - \arctan(\frac{2\zeta \frac{\omega}{\omega_0}}{1 - \frac{\omega^2}{\omega_0^2}})) \quad (4.14)$$

The effect of damping is shown in Figure 4.4

**Rayleigh damping** The damping of a structure consists of two components; (1) Mass damping and (2) stiffness damping, and is defined as:

$$C = \alpha M + \beta K \quad (4.15)$$

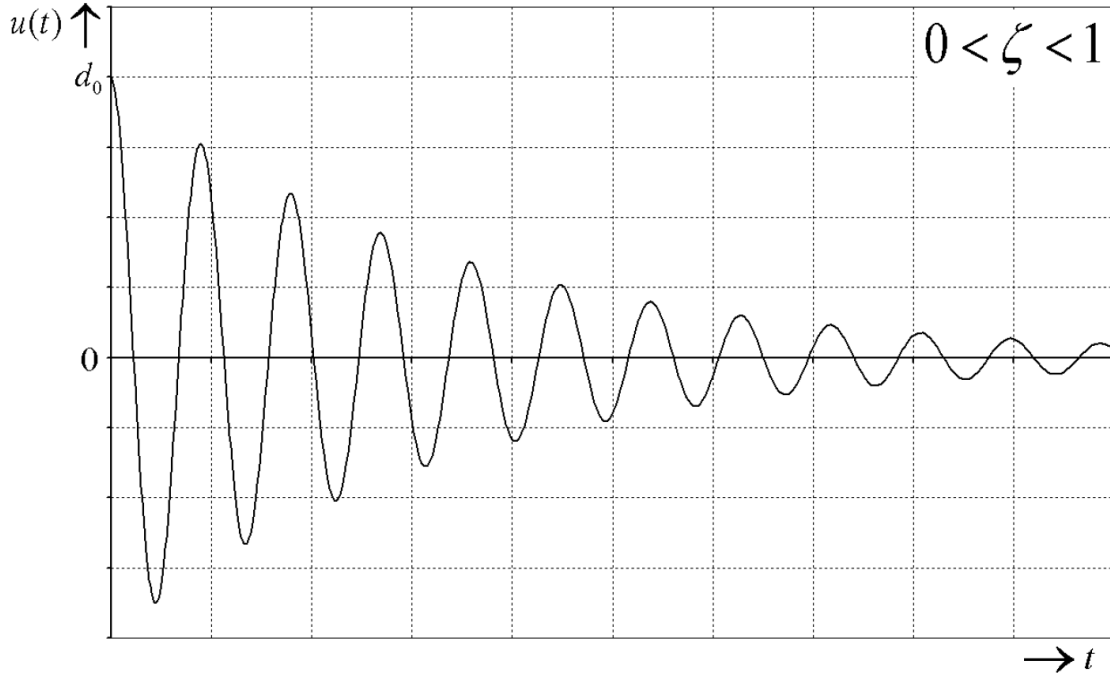


Figure 4.4: Effect of damping on an oscillating response  
Source: [Blaauwendraad, 2010]

Where:

|            |                                                     |          |
|------------|-----------------------------------------------------|----------|
| $C =$      | System damping                                      | $[Ns/m]$ |
| $\alpha =$ | Mass proportional Rayleigh damping coefficient      | $[Hz]$   |
| $M =$      | Mass of system                                      | $[kg]$   |
| $\beta =$  | Stiffness proportional Rayleigh damping coefficient | $[s]$    |
| $K =$      | Stiffness of system                                 | $[N/m]$  |

The  $\alpha$  and  $\beta$  coefficients are dependent on the natural frequency of the structure and respond in the contrary way: For a larger natural frequency, the mass damping decreases and the stiffness damping increases, as shown in Figure 4.5. The combined proportional damping (Rayleigh damping) is defined as follows (Equation 4.16):

$$\zeta = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \quad (4.16)$$

Normally speaking the values  $\alpha$  and  $\beta$  are unknown. These values can be determined by looking at a specific frequency domain. Furthermore,  $\zeta$  must be known or being estimated. In this report the first five natural frequencies of the structure determine the frequency domain.  $\alpha$  and  $\beta$  are defined as follows:

$$\alpha = \frac{2\omega_1\omega_5\zeta(\omega_5 - \omega_1)}{\omega_5^2 - \omega_1^2}$$

$$\beta = \frac{2\zeta(\omega_5 - \omega_1)}{\omega_5^2 - \omega_1^2}$$



Where:

|              |                                 |      |
|--------------|---------------------------------|------|
| $\omega_1 =$ | First natural angular frequency | [Hz] |
| $\omega_5 =$ | Fifth natural angular frequency | [Hz] |
| $\zeta =$    | Damping ratio                   | [-]  |

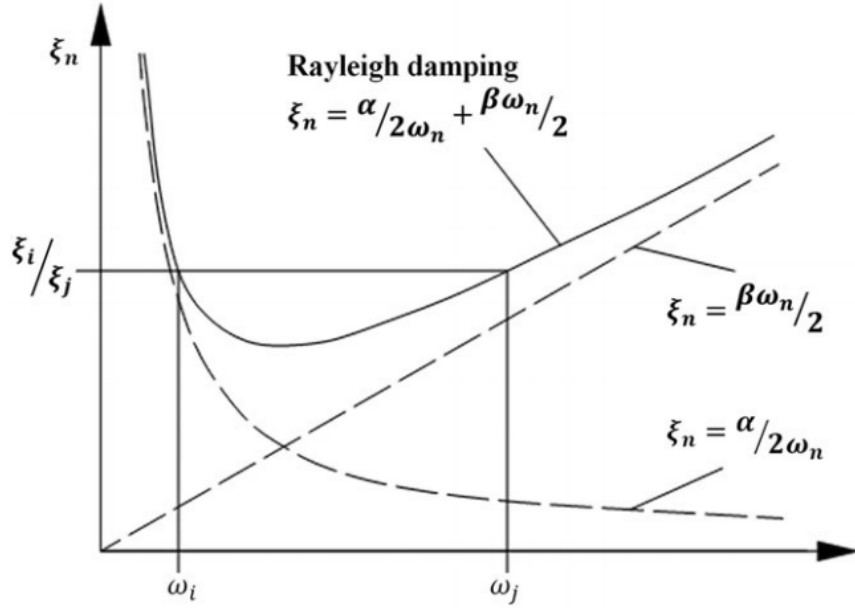


Figure 4.5: Mass, stiffness and Rayleigh damping  
Source: researchgate.net

## 4.2 Natural frequency and resonance

### 4.2.1 Natural frequency

The natural frequency of a structure describes the oscillation of a structure when the load has been released.

The natural frequency for a mass-spring system can be calculated using Equation 4.17.

$$f_i = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{EI}{mL^4}} \quad (4.17)$$

The frequency parameter  $\lambda_i$  describes the mode shape of the frequency. These mode shapes are highly correlated with the support conditions. The first three mode shapes including frequency parameters for a double span beam are given in Figure 4.6.

The natural frequency of a damped system is given by Equation 4.18.

$$f_{id} = f_i * \sqrt{1 - \zeta^2} \quad (4.18)$$

### 4.2.2 Resonance

The load has its own frequency; the load frequency  $\omega$ . If the load frequency  $\omega$  and natural frequency  $\omega_0$  coincide it results in large forces and displacements. The amplitude of the response

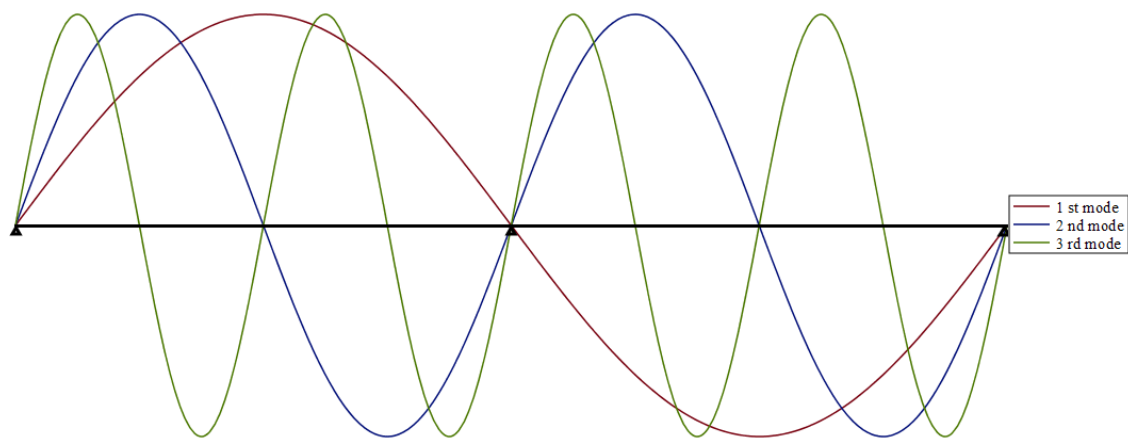


Figure 4.6: First three modes shapes for a double span beam  
 $\lambda_1 = \pi$ ,  $\lambda_2 = 1.25\pi$ ,  $\lambda_3 = 2\pi$

becomes very large, see Figure 4.7. This is called resonance and might lead to an unsafe structure. Therefore, the structures has to be designed in such a way that the natural frequency  $\omega_0$  does not coincide with the load frequency  $\omega$ .

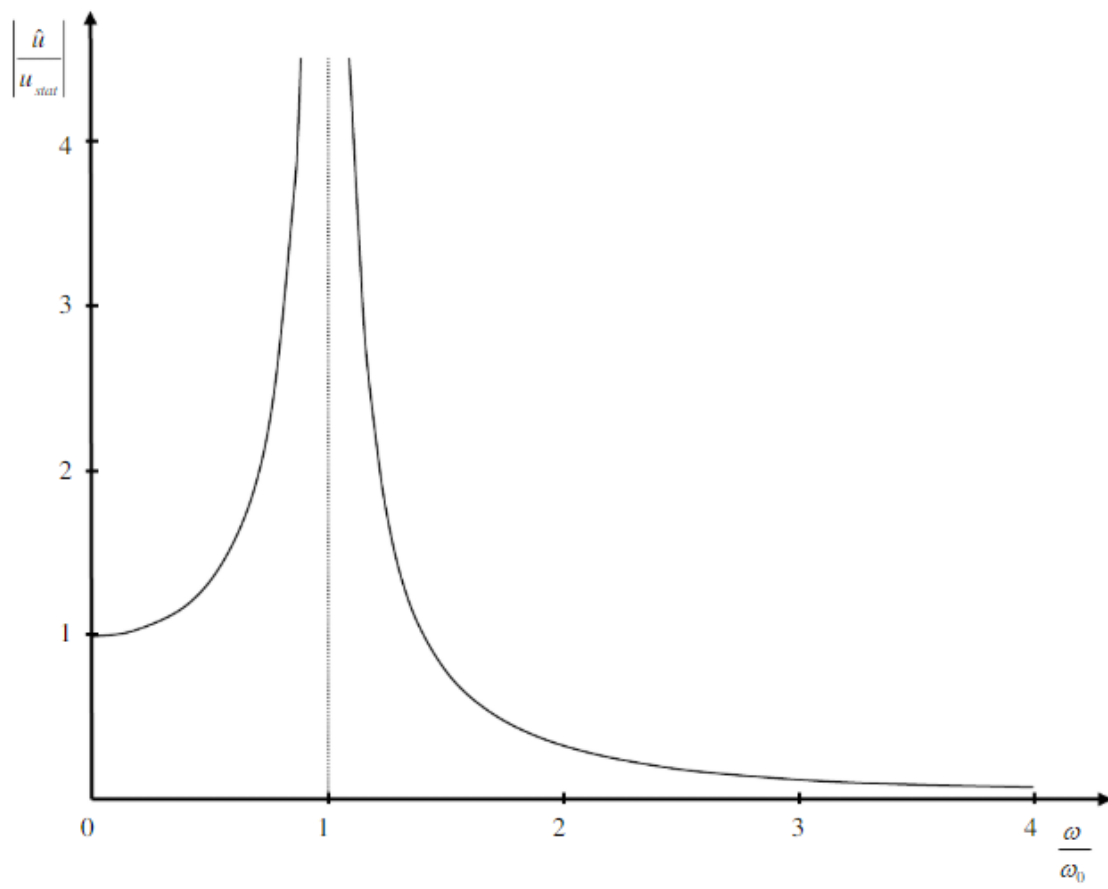


Figure 4.7: Resonance  
Source: [Blaauwendraad, 2010]

# Part III

## Conceptual designs



In chapter 3 a few common bridge types are presented. For railway bridges a through bridge design is commonly used. In FRP bridge design mainly sandwich bridge systems are used. Based on these designs two conceptual designs are made. This chapter provides an overview of both designs. These designs are made of FRP. Design A is based on common FRP bridge deck systems. Design B is based on common railway bridge designs.

Both designs are double span continuous bridge decks, with two equal spans of 10 meter. The mechanical scheme is given by Figure 5.1.

**Design A** Sandwich bridge deck design

**Design B** Through bridge deck design

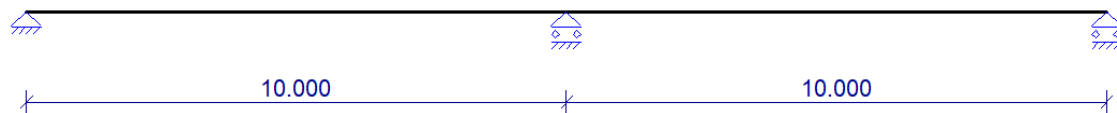


Figure 5.1: Mechanical scheme for both designs [m]

## 5.1 Design A: Sandwich bridge

Design A is based on common FRP bridge deck systems and partly inspired by internal documents from BAM Infraconsult. It consists of two plates (flanges) of 100 mm thick connected by 35 longitudinal and 21 transverse webs of 30 mm thick, see Figure 5.4. The main span direction is in longitudinal direction. The transverse webs provide stability. Foam between the webs act as lost form work. The dimensions of the bridge is presented in Table 5.1. The drawings of Design A can be found in chapter A.

| Dimension                       | sign  | Sandwich          |                 |
|---------------------------------|-------|-------------------|-----------------|
| Length                          | L     | 20000             | mm              |
| Width                           | W     | 6800              | mm              |
| Height                          | H     | 1000              | mm              |
| Center of gravity (from bottom) | y     | 500               | mm              |
| C.t.c. distance long. webs      | $a_l$ | 200               | mm              |
| C.t.c. distance trans. webs     | $a_t$ | 1000              | mm              |
| Structural Area                 | A     | 2200000           | mm <sup>2</sup> |
| Moment of inertia               | I     | $3.21 \cdot 10^4$ | mm <sup>4</sup> |

Table 5.1: Dimensions of design A

**Fibre lay up** The 0° direction of the fibres is in main span direction; the longitudinal direction. The fibres are wrapped around the foam blocks to ensure fibres in each direction. The longitudinal fibres are placed in between the foam blocks. The top and bottom layers are placed on top and underneath the blocks. Figure 5.2 given an impression of the fibre lay up. The red fibres are wrapped around the foam blocks. The green fibres are placed in between the foam blocks. Not all fibres are drawn. Furthermore, the resin and  $\pm 45^\circ$  fibres have been omitted in this figure.

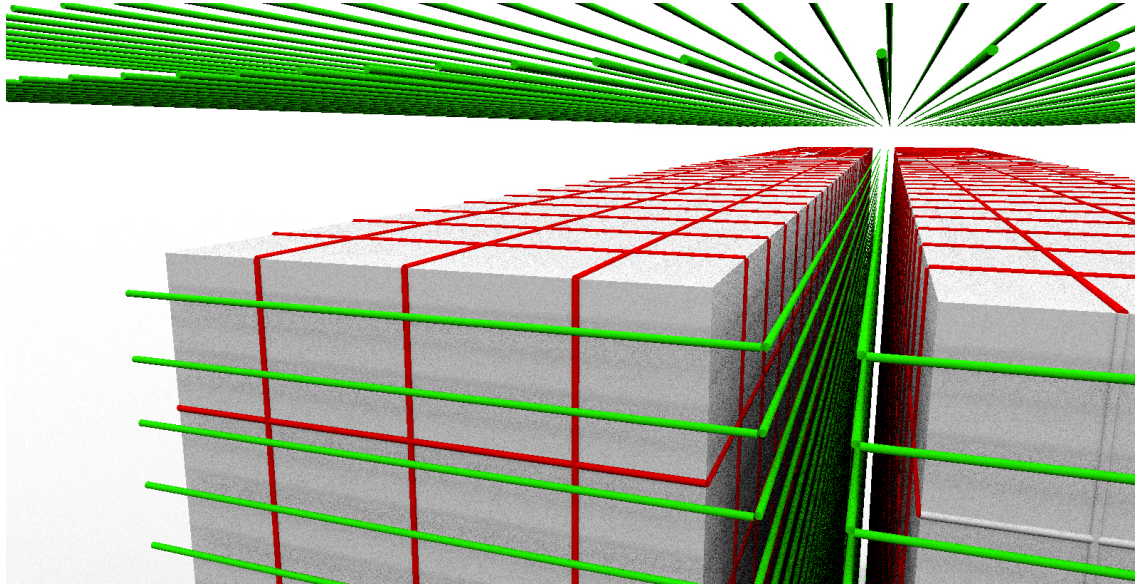


Figure 5.2: Detail of the lay up of the fibres around the foam blocks

**Maintenance path** The maintenance path is placed on a platform on both sides of the rail. This platform is made of FRP and a grating and is placed on top of the platform. The maintenance

path has no structural identity. The maintenance path is at BS level, resulting in a height of the platform of 730 mm. The width of the path is 1000 mm. With this maintenance path the structure meets the guideline for maintenance paths stated in section 3.3.

**Ballast bed** The ballast bed consists of a sub grade, NS90 sleepers and UIC54 rails. The sub grade has a height of 343 mm. The NS90 sleepers are c.t.c. 1.00 meter. The dimensions of the sleepers are LxWxH 2520x300x233 mm (see Figure 3.6). On top of the sleepers two UIC54 rails are placed.

**Cable trays** The wiring of the bridge is tucked away in cable trays. These cable trays, made of concrete, are placed against the platforms on top of the bridge, as shown in Figure 5.4a. The ballast bed is sloping to ensure the accessibility of the cable trays for maintenance.

**Supports** The bridge is supported by twelve elastomeric bearings. The position and degrees of freedom of each bearing is shown in Figure 5.3.

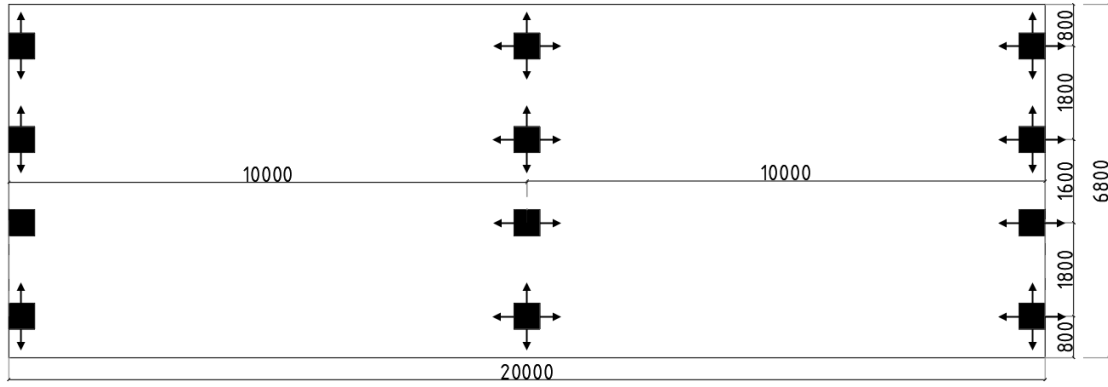
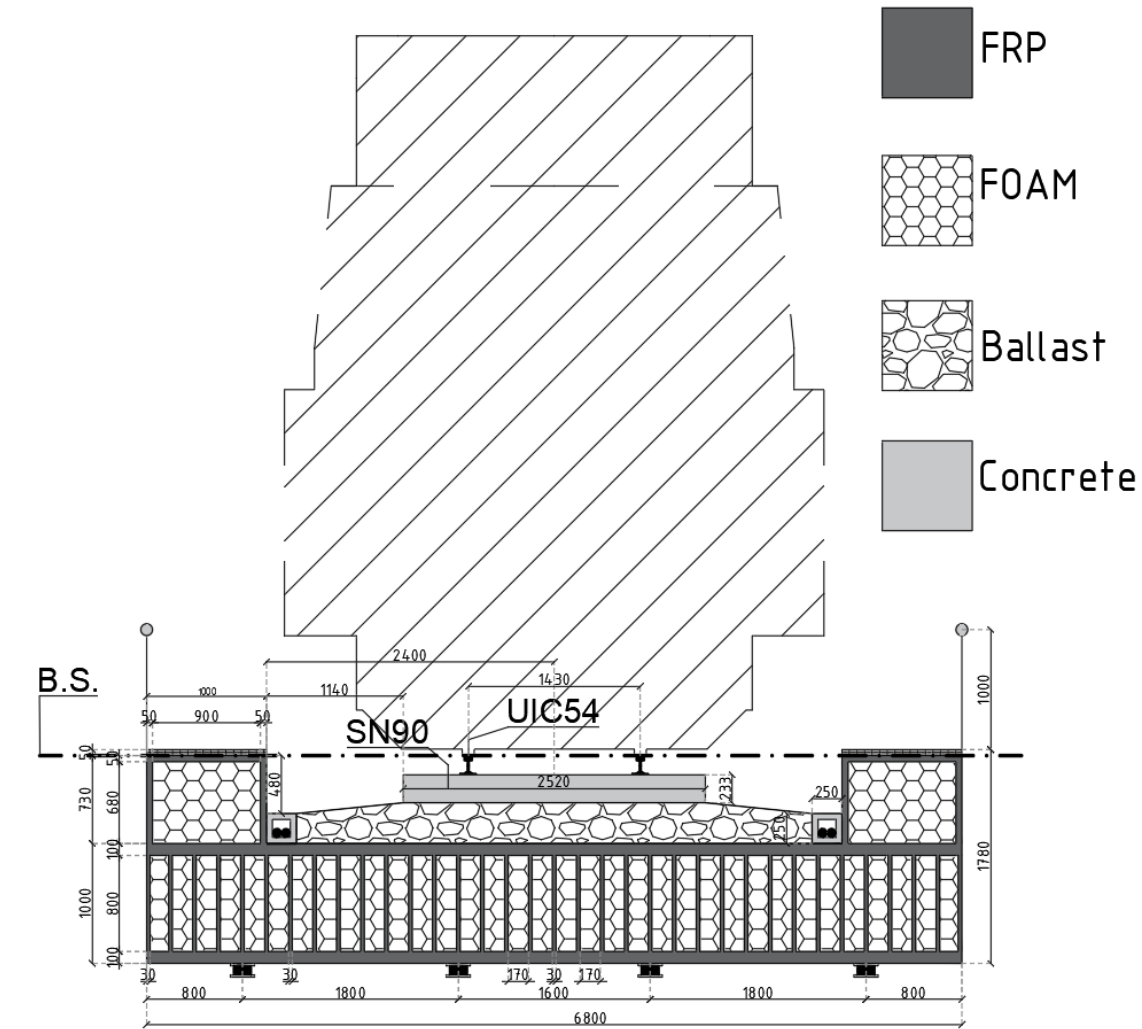


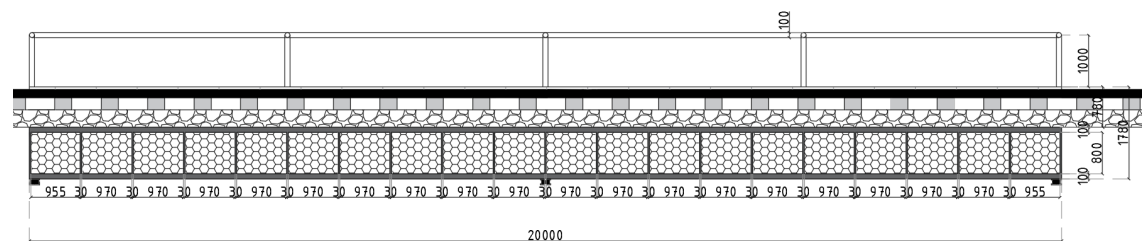
Figure 5.3: Positioning and degrees of freedom for the bearings in Design A

**PVR** This design does not intersect the PVR, as shown in Figure 5.4a, and therefore meets the guidelines given in section 3.3.





(a) Cross section of design A: sandwich bridge



(b) Side view of design A: sandwich bridge

Figure 5.4: Views of Design A

## 5.2 Design B: Through bridge

The second design is a through bridge, see Figure 5.7a. This design is based on common bridge deck designs for railway bridges and partly inspired by internal documents from BAM Infraconsult. This typical U-shaped geometry is often used in railway bridges. A through bridge exists of two parts; (1) the main load-carrying part, which are the relatively high beams on both sides of the bridge and (2) the transverse load-carrying part, which is the plate section in between the two beams. This plate section is designed to transfer the train loads to the beams sections, which transfers the forces subsequently to the bearings, and has a cross section area of 300x4500 mm. The plate section consists of a 30 mm thick top and bottom flange, connected by 101 transverse webs with a thickness of 30 mm. The beam section consists of a 100mm thick top and bottom flange, connected by 21 transverse webs with a thickness of 30mm. The cross section area of the beam parts is 1430x550 mm, resulting in a total bridge width of 5600 mm. The maintenance path is placed on top of the beam sections, resulting in a maximum height of the beam of 1430 mm. The dimensions of the through bridge are given in Table 5.2. Drawings of Design B are shown in chapter B.

| Dimension                       | sign  | Through              |                 |
|---------------------------------|-------|----------------------|-----------------|
| Length                          | L     | 20000                | mm              |
| Width                           | W     | 5600                 | mm              |
| Height beams                    | H     | 1430                 | mm              |
| Height plate                    | h     | 300                  | mm              |
| Center of gravity (from bottom) | y     | 486.50               | mm              |
| C.t.c. distance trans. webs     | $a_t$ | 200                  | mm              |
| Structural Area                 | A     | 652000               | mm <sup>2</sup> |
| Moment of inertia               | I     | $1.67 \cdot 10^{11}$ | mm <sup>4</sup> |

Table 5.2: Dimensions of design B

**Fibre lay up** The fibre lay up differs between the plate and beam section. The 0°-direction in the plate section is in transverse direction. The 0°-direction of the fibres in the beam sections is in longitudinal direction. The fibres are wrapped around the foam blocks, just like Design A. The 0°-fibres lie in between the blocks. To transfer the forces from the plate section to the beam sections (and finally to the supports) some fibres are wrapped around the structure. They connect the plate section to the beam sections. These fibres are shown in Figure 5.5, where the red lines represent the fibres. The fibres are wrapped around the foam blocks and lie around the cross section.

**Maintenance path** The shape of the bridge makes it possible to place the maintenance path both next to the rail track and on the beam section. However, there are some restrictions. When placing the path besides the track, a narrow bridge with high beam section can make it difficult to escape. Placing the path on the beam section solves this problem. On the other hand, the distance between the maintenance path and the rails must be limited to make the railway accessible during maintenance. The requirements are given in section 3.3. For Design B the maintenance path is placed on the beam section. In this way the height of the bridge can be used without widening the design. Figure 5.4a shows that the maximum distances as prescribed in section 3.3 are not exceeded. The maintenance path is made of a aluminium frame with gratings. The connection between maintenance path and FRP deck is out-of-scope.

**Ballast bed** The ballast consists of a sub grade, NS90 sleepers and UIC54 rails. The sub grade has a height of 324 mm. The NS90 sleepers are c.t.c. 1.00 meter. The dimensions of the sleepers are LxWxH 2520x300x233mm (see Figure 3.6). Two UIC54 rails are placed on top of the sleepers.

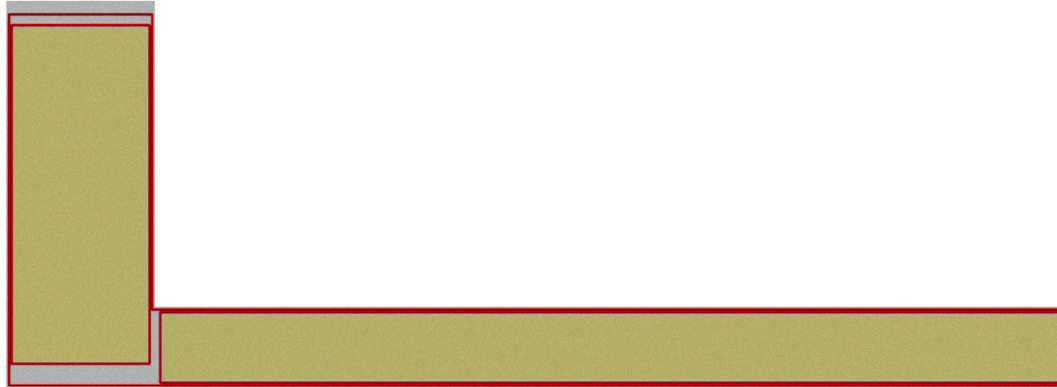


Figure 5.5: Fibre lay-up of the fibres connecting the plate and beam sections in Design B

**Cable trays** The wiring of the bridge is tucked away in cable trays. These cable trays are also made of concrete and placed against the beam sections. The ballast bed is sloping towards the trays to ensure the accessibility of the cable trays for maintenance, shown in Figure 5.7a.

**Supports** Design B is supported by six elastomeric bearings. The bearings are placed below the beam section to ensure the transverse load-carrying behaviour of the plate section. The position and degrees of freedom for each bearing is shown in Figure 5.6.

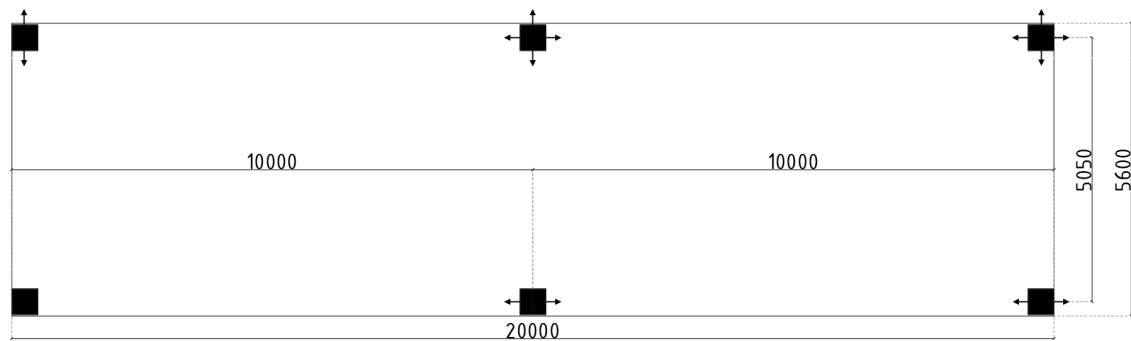
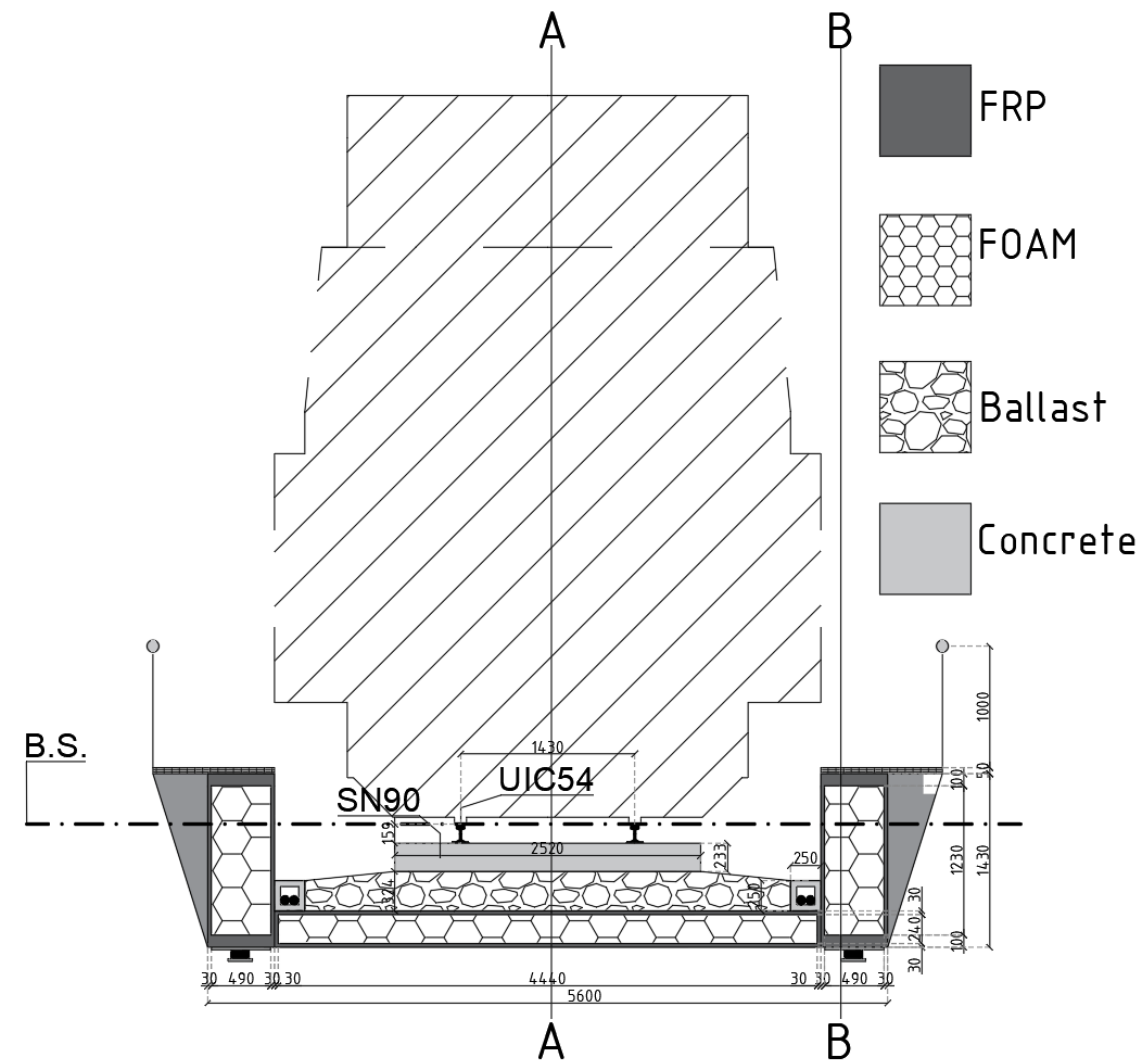
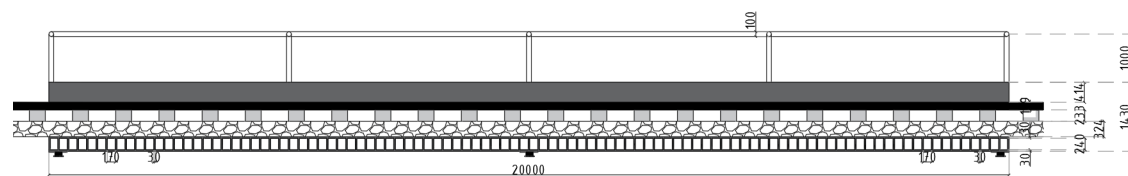


Figure 5.6: Positioning and degrees of freedom of the bearings in Design B

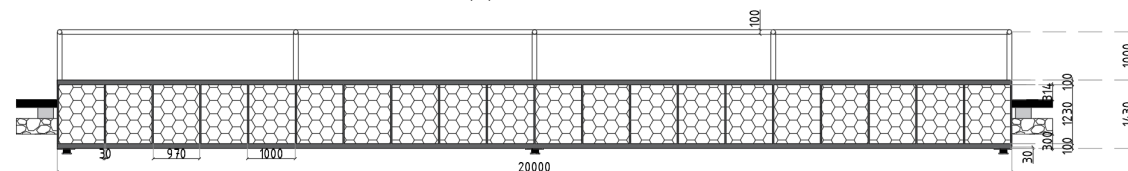
**PVR** This design does not intersect the PVR, as shown in Figure 5.7a, and therefore meets the guidelines given in section 3.3.



(a) Cross section of design B: Through bridge



(b) Side view at A-A



(c) Side view at B-B

Figure 5.7: Views of Design B



Both designs of chapter 5 are made of FRP. The material properties are discussed in this chapter. First of all, it is decided which fibre, resin, foam and manufacturing process are the most suitable for a railway bridge design. Subsequently, the material properties are determined based on the literature study. The conversion and safety factors are named successively. Finally, an overview is given of the final material properties.

### 6.1 Selection of materials

The selection of materials and manufacturing process is described in this section.

#### 6.1.1 Type of fibres

The designs in this thesis are made of E-glass fibres. These fibres are cheap and relatively strong and stiff (see Table 2.1) and are most widely used in civil engineering structures ([Kolstein, 2008]). The material properties of glass fibres are given in Table 2.1.

#### 6.1.2 Type of resin

The most common resins in bridge construction are polyester, vinyl(ester) and epoxy. The first is most commonly used in bridge construction, due to the good all-round composite properties [Potyrała et al., 2011]. Furthermore, the market price of polyester is the lowest, as given in Table 2.1. Based on this data polyester is chosen as type of resin. The material properties of polyester resin are given in Table 2.1.

#### 6.1.3 Type of foam

The foam has no structural identity but is used as permanent form work. Therefore, no extensive comparison is made. However, as permanent form work the self weight of the material must be taken into account. For simplicity, and based on Table 2.1, the self weight of the foam is set to 100 kg/m<sup>3</sup>.

#### 6.1.4 Type of manufacturing process

The designs in chapter 5 are complex 3D designs. These design are difficult to fabricate using continuous manufacturing process. This process only allows for fibres in one main direction. The

open mould process comes with high free-formability, but is labour intensive and difficult to control, resulting in low fibre volume fraction. Higher fibre volume fractions can be reached using closed mould process. Furthermore, the internal environment of a closed mould can be manipulated to gain a better final result.

Common techniques in bridge design are pultrusion, a continuous manufacturing process, and VARTM, a closed mould manufacturing process.

Based on this data VARTM is chosen as manufacturing process. The downside of this choice is the limited thickness of the webs and flanges, in order to limit the filling time of the mould. After all, the filling time must be lower than the curing time.

## 6.2 Laminate properties

The material properties of a laminate depend on the used materials and stacking of these materials; the so-called laminate lay-up. The used materials are described in previous section. The laminate properties are determined in this section.

### 6.2.1 Laminate lay-up

The lay-up for the webs and flanges are determined in section 2.4. The lay-up for flanges is:

$$[12.5\% - 90^\circ | 12.5\% - \pm 45^\circ | 62.5\% - 0^\circ | 12.5\% - \pm 45^\circ]_s$$

The lay-up for webs is:

$$[25\% - 90^\circ | 20\% - \pm 45^\circ | 35\% - 0^\circ | 20\% - \pm 45^\circ]_s$$

### 6.2.2 Characteristic properties

The characteristic properties of E-glass fibres and polyester resins with a volume fraction  $V_f = 50\%$  can be found in the JRC document, chapter 11 [Ascione et al., 2016]. The lay-up in subsection 6.2.1 consists of  $0^\circ$ - and  $90^\circ$ -plies, which are unidirectional, and  $\pm 45^\circ$ -plies, which are bi-directional. The characteristic properties of E-glass fibres and polyester resin with a volume fraction  $V_f = 50\%$  for unidirectional and bi-directional plies are given in Table 6.1.

|                | $E_1$ [GPa] | $E_2$ [GPa] | $G_{12}$ [GPa] | $\nu_{12}$ [-] |
|----------------|-------------|-------------|----------------|----------------|
| UD-ply         | 37.2        | 11.4        | 3.4            | 0.29           |
| Bi-directional | 23.3        | 23.3        | 3.3            | 0.19           |

Table 6.1: Characteristic properties of E-glass with polyester ( $V_f = 50\%$ )

### 6.2.3 ABD matrix

The material properties of the laminate can be determined using the ABD matrix. This ABD matrix is made using the FRPtool developed by Delft University of Technology.

**FRPtool** The FRPtool gives the opportunity to create different materials and lay-ups, whereafter the tool calculates the ABD matrix. The interface of the tool is shown in Figure 6.1. The tool can be found via <https://delftxtools.tudelft.nl/frp/>.

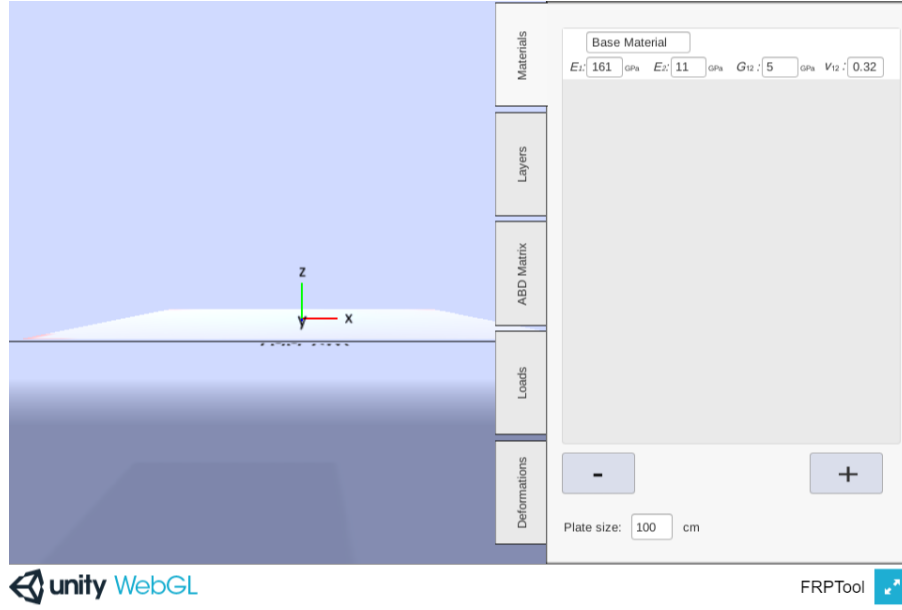


Figure 6.1: Interface of FRPtool

**Input** The required input for the tool is:

- Materials
  - $E_1$
  - $E_2$
  - $G_{12}$
  - $\nu_{12}$
- Layers
  - Thickness
  - Direction
  - Material

**Material input** The material input can be found in Table 6.1. This results in two materials; (1) UD-ply for  $0^\circ$ - and  $90^\circ$ - plies and (2) Bi-directional ply for  $\pm 45^\circ$  plies. The input is shown in Figure 6.2.

|                  |                  |                    |                   |
|------------------|------------------|--------------------|-------------------|
| UD ply           |                  |                    |                   |
| $E_1$ : 37.2 GPa | $E_2$ : 11.4 GPa | $G_{12}$ : 3.4 GPa | $\nu_{12}$ : 0.29 |
| Bi-directional   |                  |                    |                   |
| $E_1$ : 23.3 GPa | $E_2$ : 23.3 GPa | $G_{12}$ : 3.3 GPa | $\nu_{12}$ : 0.19 |

Figure 6.2: Material input for the tool

**Layer input** The layer input depends on the lay-up. This lay-up is given in subsection 6.2.1. The thickness of each layer depends on the thickness of the laminate and the percentage of the layer in a laminate. The layer input is shown in Figure 6.3. All layers are symmetric.

**Output** The output of the tool, the ABD matrix for both designs, is shown in Table 6.2.



|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------------|-----------|-----------|------|--|--|----------|--------|--|--|--|---------|-----------|--------|-----------|------|--|--|----------|----------------|--|--|--|---------|-----------|--------|-----------|-----|--|--|----------|--------|--|--|--|---------|-----------|--------|-----------|------|--|--|----------|----------------|--|--|--|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|-----------|--------|-----------|------|--|--|----------|--------|--|--|--|---------|-----------|------|-----------|------|--|--|----------|----------------|--|--|--|---------|-----------|--------|-----------|-----|--|--|----------|--------|--|--|--|---------|-----------|------|-----------|------|--|--|----------|----------------|--|--|--|
| <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;">Layer 1</td> <td style="width: 15%;">Thickness</td> <td style="width: 15%; text-align: center;">6.2 mm</td> <td style="width: 15%;">Direction</td> <td style="width: 15%; text-align: center;">90 °</td> <td style="width: 30%;"></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 2</td> <td>Thickness</td> <td style="text-align: center;">6.2 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> <tr> <td>Layer 3</td> <td>Thickness</td> <td style="text-align: center;">31. mm</td> <td>Direction</td> <td style="text-align: center;">0 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 4</td> <td>Thickness</td> <td style="text-align: center;">6.2 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> </table> <p style="text-align: center;">(a) Sandwich flanges</p> | Layer 1   | Thickness      | 6.2 mm    | Direction | 90 ° |  |  | Material | UD ply |  |  |  | Layer 2 | Thickness | 6.2 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  | Layer 3 | Thickness | 31. mm | Direction | 0 ° |  |  | Material | UD ply |  |  |  | Layer 4 | Thickness | 6.2 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  | <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;">Layer 1</td> <td style="width: 15%;">Thickness</td> <td style="width: 15%; text-align: center;">3.7 mm</td> <td style="width: 15%;">Direction</td> <td style="width: 15%; text-align: center;">90 °</td> <td style="width: 30%;"></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 2</td> <td>Thickness</td> <td style="text-align: center;">3 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> <tr> <td>Layer 3</td> <td>Thickness</td> <td style="text-align: center;">5.2 mm</td> <td>Direction</td> <td style="text-align: center;">0 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 4</td> <td>Thickness</td> <td style="text-align: center;">3 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> </table> <p style="text-align: center;">(b) Sandwich webs</p> | Layer 1 | Thickness | 3.7 mm | Direction | 90 ° |  |  | Material | UD ply |  |  |  | Layer 2 | Thickness | 3 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  | Layer 3 | Thickness | 5.2 mm | Direction | 0 ° |  |  | Material | UD ply |  |  |  | Layer 4 | Thickness | 3 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  |
| Layer 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 6.2 mm         | Direction | 90 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 6.2 mm         | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 31. mm         | Direction | 0 °       |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 6.2 mm         | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 3.7 mm         | Direction | 90 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 3 mm           | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 5.2 mm         | Direction | 0 °       |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 3 mm           | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;">Layer 1</td> <td style="width: 15%;">Thickness</td> <td style="width: 15%; text-align: center;">1.8 mm</td> <td style="width: 15%;">Direction</td> <td style="width: 15%; text-align: center;">90 °</td> <td style="width: 30%;"></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 2</td> <td>Thickness</td> <td style="text-align: center;">1.8 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> <tr> <td>Layer 3</td> <td>Thickness</td> <td style="text-align: center;">9.3 mm</td> <td>Direction</td> <td style="text-align: center;">0 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 4</td> <td>Thickness</td> <td style="text-align: center;">1.8 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> </table> <p style="text-align: center;">(c) Through flanges</p>  | Layer 1   | Thickness      | 1.8 mm    | Direction | 90 ° |  |  | Material | UD ply |  |  |  | Layer 2 | Thickness | 1.8 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  | Layer 3 | Thickness | 9.3 mm | Direction | 0 ° |  |  | Material | UD ply |  |  |  | Layer 4 | Thickness | 1.8 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  | <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%;">Layer 1</td> <td style="width: 15%;">Thickness</td> <td style="width: 15%; text-align: center;">3.7 mm</td> <td style="width: 15%;">Direction</td> <td style="width: 15%; text-align: center;">90 °</td> <td style="width: 30%;"></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 2</td> <td>Thickness</td> <td style="text-align: center;">3 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> <tr> <td>Layer 3</td> <td>Thickness</td> <td style="text-align: center;">5.2 mm</td> <td>Direction</td> <td style="text-align: center;">0 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">UD ply</td> </tr> <tr> <td>Layer 4</td> <td>Thickness</td> <td style="text-align: center;">3 mm</td> <td>Direction</td> <td style="text-align: center;">45 °</td> <td></td> </tr> <tr> <td></td> <td>Material</td> <td colspan="4" style="text-align: center;">Bi-directional</td> </tr> </table> <p style="text-align: center;">(d) Through webs</p>  | Layer 1 | Thickness | 3.7 mm | Direction | 90 ° |  |  | Material | UD ply |  |  |  | Layer 2 | Thickness | 3 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  | Layer 3 | Thickness | 5.2 mm | Direction | 0 ° |  |  | Material | UD ply |  |  |  | Layer 4 | Thickness | 3 mm | Direction | 45 ° |  |  | Material | Bi-directional |  |  |  |
| Layer 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 1.8 mm         | Direction | 90 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 1.8 mm         | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 9.3 mm         | Direction | 0 °       |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 1.8 mm         | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 3.7 mm         | Direction | 90 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 3 mm           | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 5.2 mm         | Direction | 0 °       |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | UD ply         |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
| Layer 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Thickness | 3 mm           | Direction | 45 °      |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Material  | Bi-directional |           |           |      |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |        |           |      |  |  |          |                |  |  |  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |         |           |        |           |      |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |         |           |        |           |     |  |  |          |        |  |  |  |         |           |      |           |      |  |  |          |                |  |  |  |

Figure 6.3: Layer input for the tool

### 6.2.4 Material properties

The final material properties of each laminate are determined using the ABD matrix and Equation 6.1. The values are listed in Table 6.3.

$$E_1 = \frac{1}{t} * (A_{11} - \frac{A_{12}^2}{A_{22}}) \quad (6.1a)$$

$$E_2 = \frac{1}{t} * (A_{22} - \frac{A_{12}^2}{A_{11}}) \quad (6.1b)$$

$$G_{12} = \frac{1}{t} * A_{66} \quad (6.1c)$$

$$\nu_{12} = \frac{A_{12}}{A_{22}} \quad (6.1d)$$

$$\nu_{21} = \frac{A_{12}}{A_{11}} \quad (6.1e)$$

$$\sigma_1 = E_1 * 1.2\% \quad (6.1f)$$

$$\sigma_2 = E_2 * 1.2\% \quad (6.1g)$$

$$\tau_{12} = G_{12} * 1.6\% \quad (6.1h)$$

## 6.3 Conversion and material safety factors

Conversion factors are used to describe the environmental impact on the material. A distinction is made between SLS and ULS, and between long term and short term behaviour. For short

| Sandwich design |  |                      |      |      |                  |                  |      |      |      |                  |
|-----------------|--|----------------------|------|------|------------------|------------------|------|------|------|------------------|
|                 |  | Flanges (t = 100 mm) |      |      |                  | Webs (t = 30 mm) |      |      |      |                  |
| A               |  | 2.97                 | 0.53 | 0.00 | $\ast 10^6 N/mm$ |                  | 3.50 | 0.97 | 0.00 | $\ast 10^5 N/mm$ |
|                 |  | 0.53                 | 1.65 | 0.00 |                  |                  | 0.97 | 3.11 | 0.00 |                  |
|                 |  | 0.00                 | 0.00 | 0.50 |                  |                  | 0.00 | 0.00 | 0.89 |                  |
| B               |  | 0.00                 | 0.00 | 0.00 | $N$              |                  | 0.00 | 0.00 | 0.00 | $N$              |
|                 |  | 0.00                 | 0.00 | 0.00 |                  |                  | 0.00 | 0.00 | 0.00 |                  |
|                 |  | 0.00                 | 0.00 | 0.00 |                  |                  | 0.00 | 0.00 | 0.00 |                  |
| D               |  | 2.03                 | 0.44 | 0.00 | $\ast 10^9 Nmm$  |                  | 4.91 | 1.52 | 0.00 | $\ast 10^6 Nmm$  |
|                 |  | 0.44                 | 1.83 | 0.00 |                  |                  | 1.52 | 8.04 | 0.00 |                  |
|                 |  | 0.00                 | 0.00 | 0.42 |                  |                  | 0.00 | 0.00 | 1.43 |                  |

| Through design |  |                     |      |      |                  |                  |      |      |      |                  |
|----------------|--|---------------------|------|------|------------------|------------------|------|------|------|------------------|
|                |  | Flanges (t = 30 mm) |      |      |                  | Webs (t = 30 mm) |      |      |      |                  |
| A              |  | 8.92                | 1.59 | 0.00 | $\ast 10^5 N/mm$ |                  | 7.01 | 1.94 | 0.00 | $\ast 10^5 N/mm$ |
|                |  | 1.59                | 4.95 | 0.00 |                  |                  | 1.94 | 6.21 | 0.00 |                  |
|                |  | 0.00                | 0.00 | 1.50 |                  |                  | 0.00 | 0.00 | 1.79 |                  |
| B              |  | 0.00                | 0.00 | 0.00 | $N$              |                  | 0.00 | 0.00 | 0.00 | $N$              |
|                |  | 0.00                | 0.00 | 0.00 |                  |                  | 0.00 | 0.00 | 0.00 |                  |
|                |  | 0.00                | 0.00 | 0.00 |                  |                  | 0.00 | 0.00 | 0.00 |                  |
| D              |  | 5.47                | 1.20 | 0.00 | $\ast 10^7 Nmm$  |                  | 3.93 | 1.22 | 0.00 | $\ast 10^7 Nmm$  |
|                |  | 1.20                | 4.94 | 0.00 |                  |                  | 1.22 | 6.43 | 0.00 |                  |
|                |  | 0.00                | 0.00 | 1.12 |                  |                  | 0.00 | 0.00 | 1.14 |                  |

Table 6.2: ABD matrices for sandwich and through designs

term behaviour the effect of creep can be neglected. The four conversion factors are  $\eta_{ct}$  (thermal conversion factor),  $\eta_{cm}$  (humidity conversion factor),  $\eta_{cv}$  (creep conversion factor) and  $\eta_{cf}$  (fatigue conversion factor).

### 6.3.1 Thermal conversion factor

The thermal conversion factors for SLS and ULS situations:

$$\text{For verification of strength (ULS)} \quad \eta_{ct,ULS} = 0.9$$

$$\text{For verification of deformability and stability} \quad \eta_{ct,SLS} = 1.0$$

### 6.3.2 Humidity conversion factors

The conversion factors for humidity depend on the Media Class and whether it is cured or not. In this case, the Media Class is Class II and the material is cured.

$$\text{For verification of strength (ULS)} \quad \eta_{cm,SLS} = 0.9$$

$$\text{For verification of deformability and stability (SLS)} \quad \eta_{cm,ULS} = 0.9$$

### 6.3.3 Fatigue conversion factor

The fatigue conversion factors are the same for ULS and SLS situations.

$$\text{For verification of strength (ULS)} \quad \eta_{cf,ULS} = 0.9$$

$$\text{For verification of deformability and stability (SLS)} \quad \eta_{cf,ULS} = 0.9$$

|             |     | Sandwich flange | Sandwich web | Through flange | Through web |
|-------------|-----|-----------------|--------------|----------------|-------------|
| Thickness   | mm  | 100             | 30           | 30             | 30          |
| $E_1$       | MPa | 27998           | 21346        | 28031          | 21346       |
| $E_2$       | MPa | 15554           | 18910        | 15555          | 18910       |
| $G_{12}$    | MPa | 5000            | 5967         | 5000           | 5967        |
| $\nu_{12}$  | -   | 0.32            | 0.31         | 0.32           | 0.31        |
| $\nu_{21}$  | -   | 0.18            | 0.28         | 0.18           | 0.28        |
| $\sigma_1$  | MPa | 336             | 256          | 336            | 256         |
| $\sigma_2$  | MPa | 187             | 227          | 187            | 227         |
| $\tau_{12}$ | MPa | 80              | 95           | 80             | 95          |

Table 6.3: Strength and stiffness properties of the flanges and webs for both designs

### 6.3.4 Creep conversion factor

The conversion factors for creep depend on the load duration, limit state and the direction of the fibres. An overview of all creep conversion factors is given in Table 6.4. The X-direction is in  $0^\circ$  direction, the Y-direction is in  $90^\circ$  direction and the XY-direction is in  $\pm 45^\circ$  direction.

|     |         | Direction | Dead load | Live Load |
|-----|---------|-----------|-----------|-----------|
| ULS | Flanges | X         | 0.62      | 0.65      |
|     |         | Y         | 0.51      | 0.54      |
|     |         | XY        | 0.53      | 0.56      |
|     | Webs    | X         | 0.55      | 0.58      |
|     |         | Y         | 0.53      | 0.56      |
|     |         | XY        | 0.56      | 0.59      |
| SLS | Flanges | X         | 0.59      | 0.62      |
|     |         | Y         | 0.41      | 0.45      |
|     |         | XY        | 0.45      | 0.48      |
|     | Webs    | X         | 0.48      | 0.51      |
|     |         | Y         | 0.45      | 0.48      |
|     |         | XY        | 0.49      | 0.53      |

Table 6.4: Creep conversion factors

### 6.3.5 Safety factors

The safety factors for material uncertainty ( $\gamma_{M1}$ ) and production uncertainty ( $\gamma_{M2}$ ) are given below.

Material uncertainty  $\gamma_{M1} = 1.35$

Production uncertainty  $\gamma_{M2} = 1.35$

Total safety factor  $\gamma_M = 1.82$

### 6.3.6 All factors

The conversion factors and safety factors for SLS and ULS situation are listed in Table 6.5. For simplicity, the creep conversion factor for Dead Load is used, as these values are more conservative. Characteristic values ( $X_k$ ) are converted into design values ( $X_d$ ) using Equation 6.2.

$$X_d = \eta_c \frac{X_k}{\gamma_M} \quad (6.2)$$

| Checks | Conversion factors     | Safety factors          |
|--------|------------------------|-------------------------|
| SLS    | $\eta_c = 0.81$        | $\gamma_{M,SLS} = 1.0$  |
| ULS    | $\eta_{c,f,x} = 0.50$  | $\gamma_{M,ULS} = 1.82$ |
|        | $\eta_{c,f,y} = 0.41$  |                         |
|        | $\eta_{c,f,xy} = 0.43$ |                         |
|        | $\eta_{c,w,x} = 0.45$  |                         |
|        | $\eta_{c,w,y} = 0.43$  |                         |
|        | $\eta_{c,w,xy} = 0.45$ |                         |

Table 6.5: Overview of all conversion factors and safety factors

## 6.4 Final laminate properties

The final laminate properties are calculated using Equation 6.2. The characteristic values  $X_k$  are given in Table 6.3 and the factors  $\gamma$  and  $\eta_c$  are given in Table 6.5. The final laminate properties are presented in Table 6.6 and Table 6.7.

|          |         | Thickness<br>mm | $E_1$<br>MPa | $E_2$<br>MPa | $G_{12}$<br>MPa | $\nu_{12}$<br>- | $\nu_{21}$<br>- | $\sigma_1$<br>MPa | $\sigma_2$<br>MPa | $\tau_{12}$<br>MPa |
|----------|---------|-----------------|--------------|--------------|-----------------|-----------------|-----------------|-------------------|-------------------|--------------------|
| Design A | Flanges | 100             | 22678        | 12599        | 4050            | 0.32            | 0.18            | 272               | 151               | 65                 |
|          | Webs    | 30              | 17291        | 15317        | 4833            | 0.31            | 0.28            | 207               | 184               | 77                 |
| Design B | Flanges | 30              | 22705        | 12600        | 4050            | 0.32            | 0.18            | 272               | 151               | 65                 |
|          | Webs    | 30              | 17291        | 15317        | 4833            | 0.31            | 0.28            | 207               | 184               | 77                 |

Table 6.6: SLS Laminate properties

|          |         | Thickness<br>mm | $E_1$<br>MPa | $E_2$<br>MPa | $G_{12}$<br>MPa | $\nu_{12}$<br>- | $\nu_{21}$<br>- | $\sigma_1$<br>MPa | $\sigma_2$<br>MPa | $\tau_{12}$<br>MPa |
|----------|---------|-----------------|--------------|--------------|-----------------|-----------------|-----------------|-------------------|-------------------|--------------------|
| Design A | Flanges | 100             | 7715         | 3526         | 1178            | 0.32            | 0.18            | 93                | 42                | 19                 |
|          | Webs    | 30              | 5218         | 4454         | 1485            | 0.31            | 0.28            | 63                | 53                | 24                 |
| Design B | Flanges | 30              | 7724         | 3526         | 1178            | 0.32            | 0.18            | 93                | 42                | 19                 |
|          | Webs    | 30              | 5218         | 4454         | 1485            | 0.31            | 0.28            | 63                | 53                | 24                 |

Table 6.7: ULS Laminate properties



The designs are loaded by the following load cases:

|                   |                                         |
|-------------------|-----------------------------------------|
| Load case 1       | Dead load                               |
| Load case 2       | Load Model 71 + start force             |
| Load case 3       | Dead Load + Load Model 71 + start force |
| Dynamic load case | Real train model for dynamic analysis   |

### 7.1 Load case 1

Load case 1 describes the permanent load on the structure. These permanent loads consist of the dead load of the FRP deck, the self weight of the ballast bed and the self weight of the foam. Furthermore, the maintenance path has no structural identity and is therefore implemented as a permanent line load on the bridge. The safety factor for permanent loads is  $\gamma_G = 1.35$ . The safety factor for FRP self weight is 1.2 [Mottram and Henderson, 2018]. An overview of the loads in Load Case 1 is presented in Table 7.1.

#### 7.1.1 Dead load deck

The dead load of the FRP deck is determined by the software package. Therefore, it requires the input of the material density. The density of the material is determined using Equation 7.1. The densities of fibres and resins are given in Table 2.1.

$$\rho_{FRP} = V_f * \rho_{fibre} + (1 - V_f) * \rho_{resin} = 18.5 \text{ kN/m}^3 \quad (7.1)$$

#### 7.1.2 Ballast bed

A characteristic value for the ballast bed (subgrade, sleepers and rails) is given by OVS30-6: 22 kN/m<sup>3</sup>. The thickness of the ballast bed is 0.73 m, resulting in an area load of 16.06 kN/m<sup>2</sup>.

#### 7.1.3 Self weight foam

The foam has no structural integrity but is used as permanent form work. Therefore, the foam blocks are not modelled but applied as dead loads. The density of the foam is 0.981 kN/m<sup>3</sup>. In the sandwich design all the foam blocks have the same thickness resulting in the same area load. The thickness of the foam blocks in the through design differ. At the beam elements, multiple blocks

are stacked on top of each other, resulting in a higher area load compared to the plate section. The values are given in Table 7.1.

#### 7.1.4 Maintenance path

The maintenance path on the sandwich bridge has no structural identity and is therefore applied as a permanent line load. The value is based on the self-weight of the path, which is made of the same material as the webs and flanges. Therefore, the maintenance path has a density of  $18.5 \text{ kN/m}^3$  (see Equation 7.1), resulting in four line loads of  $5.72 \text{ kN/m}$ .

| Load    |                 | Sandwich bridge        |       | Through bridge |       |
|---------|-----------------|------------------------|-------|----------------|-------|
|         |                 | SLS                    | ULS   | SLS            | ULS   |
| Deck    |                 | Calculated by Sofistik |       |                |       |
| Ballast | $\text{kN/m}^2$ | 16.06                  | 21.68 | 16.06          | 21.68 |
| Foam    | $\text{kN/m}^2$ | 0.64                   | 0.86  | 0.20           | 0.28  |
|         |                 |                        |       | 1.03           | 1.39  |
| Path    | $\text{kN/m}$   | 5.72                   | 7.72  |                |       |

Table 7.1: Loads in load case 1

## 7.2 Load case 2

Load case 2 describes the live load on the bridge. These live loads are LM71 and the brake/start force. The safety factor for live loads is  $\gamma_Q = 1.45$ . An overview of the loads is presented in Table 7.2.

### 7.2.1 Load model 71

Load model 71 is described in subsection 3.3.2. This load model consists of four point loads of  $250 \text{ kN}$  and two distributed loads of  $80 \text{ kN/m}$ , multiplied with the  $\alpha$  and dynamic factor. However, these loads are spread by the ballast bed, as shown in Figure 3.6. This results in an area load of  $50.02 \text{ kN/m}^2$  and  $332.58 \text{ kN/m}^2$ .

### 7.2.2 Start and brake force

The horizontal start force, generated by an accelerating train, is  $33 \text{ kN/m}$ . This force must be multiplied with the  $\alpha$  factor and is spread by the ballast bed, resulting in an area load of  $14.84 \text{ kN/m}^2$  in horizontal direction.

An overview of all loads in LC2, including the additional factors, is given by Table 7.2.

| Loads       |                 | SLS    | ULS    |
|-------------|-----------------|--------|--------|
| Point load  | $\text{kN/m}^2$ | 332.58 | 482.24 |
| Line load   | $\text{kN/m}^2$ | 50.02  | 72.53  |
| Start force | $\text{kN/m}^2$ | 14.84  | 21.52  |

Table 7.2: Loads of Load case 2 including additional factors

## 7.3 Load case 3

Load case 3 combines the dead loads in LC1 and the live loads in LC2. The combinations differ for SLS and ULS state.

### 7.3.1 Serviceability Limit State

The load combination equation for serviceability limit state is given by Equation 7.2.

$$\sum G_k + P + Q_{k,1} + \psi_{0,1} Q_{k,i} \quad (7.2)$$

Where:

- $G_k$  the permanent loads; Load case 1.
- $P$  the prestressing force, which is zero.
- $Q_{k,1}$  Load Model 71.
- $Q_{k,i}$  the start force.
- $\Psi_{0,1}$  the combination factor, in this case 1.0.

### 7.3.2 Ultimate limit state

The load combination equation for ultimate limit state is give by Equation 7.3.

$$\sum \gamma_G G_k + \gamma_P P_k + \gamma_{Q,1} Q_{k,1} + \gamma_{Q,i} \psi_{0,1} Q_{k,i} \quad (7.3)$$

Where:

- $\gamma_G$  safety factor for permanent loads; 1.35
- $G_k$  the permanent loads; Load case 1.
- $\gamma_P$  safety factor for prestressing loads; 1.00
- $P$  the prestressing force, which is zero.
- $\gamma_Q$  safety factor for variable loads; 1.45
- $Q_{k,1}$  Load Model 71.
- $Q_{k,i}$  the start force.
- $\Psi_{0,1}$  the combination factor, in this case 1.0.

## 7.4 Dynamic load case

The dynamic load case is based on a real train model travelling at a certain speed. As presented in subsection 3.3.2 this real train model is a VIRM4 train travelling at 140 km/h. Figure 7.2 gives a technical drawing of the VIRM4, including dimensions. The dynamic load case is shown in Figure 7.1. The axle load is spread over both wheels per axis, resulting in eight point loads of 107 kN. The distance between two wheels is 2.5 meter. The centre to centre distance between two axis is 20 meter, see Figure 7.2.

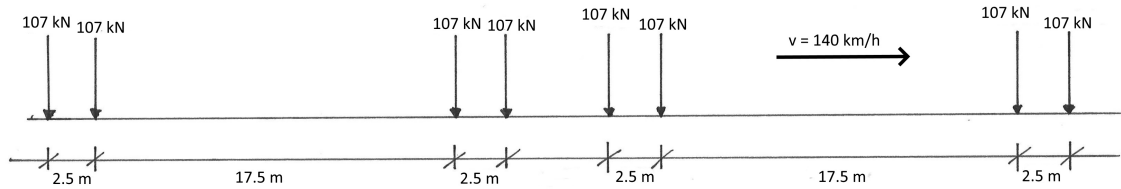


Figure 7.1: Dynamic load case



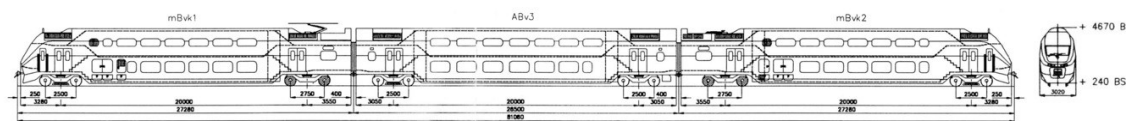


Figure 7.2: Technical drawing VIRM4  
Source: NS Reizigers B.V.

This chapter describes the numerical models used to analyse both designs. First, a small introduction is given on the used software. Secondly, the input used to create the models is discussed whereafter the load models are given. Lastly, an overview of the natural frequencies of both designs is given.

Each design presented in chapter 5 comes with two numerical models; a SLS and ULS variant. The models in ULS and SLS variant do not only differ in loadings, but also material properties are different for each variant, resulting in the following four models:

- Design A SLS
- Design A ULS
- Design B SLS
- Design B ULS

### 8.1 Introduction

All designs are modelled using the software packages of Sofistik. These software packages combine Finite Element Analysis with 3D CAD drawing software and is widely used in the field of civil and structural engineering. The CAD component, 'Sofiplus', is based on 'Autodesk Autocad'. These CAD models are exported to the FEA component of Sofistik. The finite elements have a mesh of  $\pm 100mm$ . This mesh size is determined by the software itself, with mesh refinement around critical points. Sofistik uses four noded shell elements which can displace in three directions  $u_x$ ,  $u_y$  and  $u_z$ . The through thickness stresses ( $\sigma_z$ ) are zero. The properties of the finite elements are given in Table 8.1.

### 8.2 Input

The input of the numerical models consists of the designs and the corresponding material properties.

#### 8.2.1 Material input

The material properties of all four designs is given in section 6.2. Sofistik needs the following material properties input:

|                    | Sandwich bridge             | Through bridge              |
|--------------------|-----------------------------|-----------------------------|
| Element size       | $\pm 100$ mm                | $\pm 100$ mm                |
| Element type       | 4 noded shell elements      | 4 noded shell elements      |
| Degrees of freedom | $u_x, u_y$ and $u_z$        | $u_x, u_y$ and $u_z$        |
| Stresses           | $\sigma_{xx}, \sigma_{yy}$  | $\sigma_{xx}, \sigma_{yy}$  |
|                    | $\sigma_{xy} = \sigma_{yx}$ | $\sigma_{xy} = \sigma_{yx}$ |
|                    | $\sigma_{yz} = \sigma_{zy}$ | $\sigma_{yz} = \sigma_{zy}$ |
|                    | $\sigma_{xz} = \sigma_{zx}$ | $\sigma_{xz} = \sigma_{zx}$ |
| Number of elements | 106586                      | 26242                       |
| Number of nodes    | 92547                       | 22832                       |
| EI                 | $7037437kNm^2$              | $3746325kNm^2$              |
| GA                 | $10582650kN$                | $2484270kN$                 |

Table 8.1: Properties of the finite element models

- Self weight  $[kN/m^3]$
- Elastic modulus  $[N/mm^2]$
- Poisson ratio  $[-]$
- Shear modulus  $[N/mm^2]$
- Elastic modulus 90  $[N/mm^2]$
- Poisson ratio 90  $[-]$

### 8.2.2 Design input

Both designs are modelled using 'Sofiplus'. This program is limited to 2D elements with a certain thickness, so called 4 noded shell elements. Therefore, each design is build up using area elements. The horizontal elements are named flanges, the vertical elements are named webs.

**Design A: Sandwich bridge** The 3D Sofiplus model of Design A is shown in Figure 8.1.

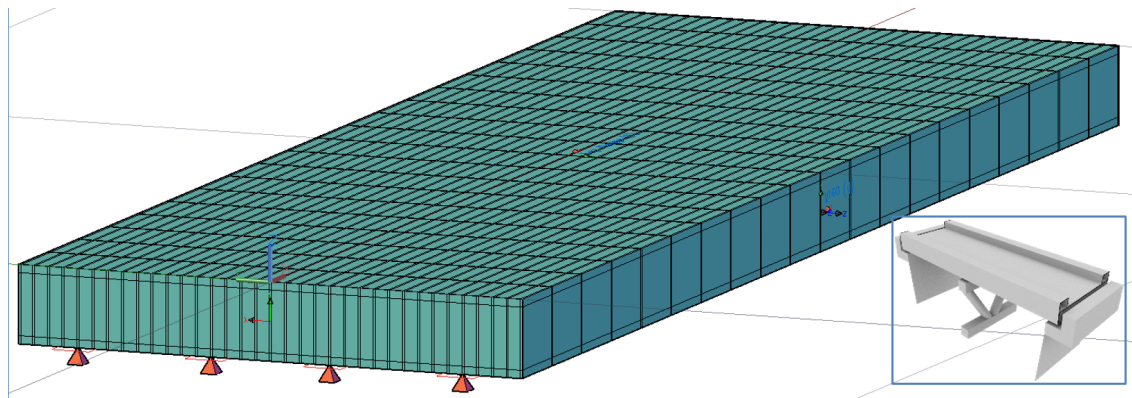
The different components of design A are shown in Figure 8.2. The material properties of the flanges and webs is given in section 6.2. The supports are modelled as point supports with the degrees of freedom shown in Figure 5.3.

This Sofiplus model is transformed into a FE model. The mesh size is determined by the software, with mesh refinement around critical points. Figure 8.3 shows the finite element model of Design A.

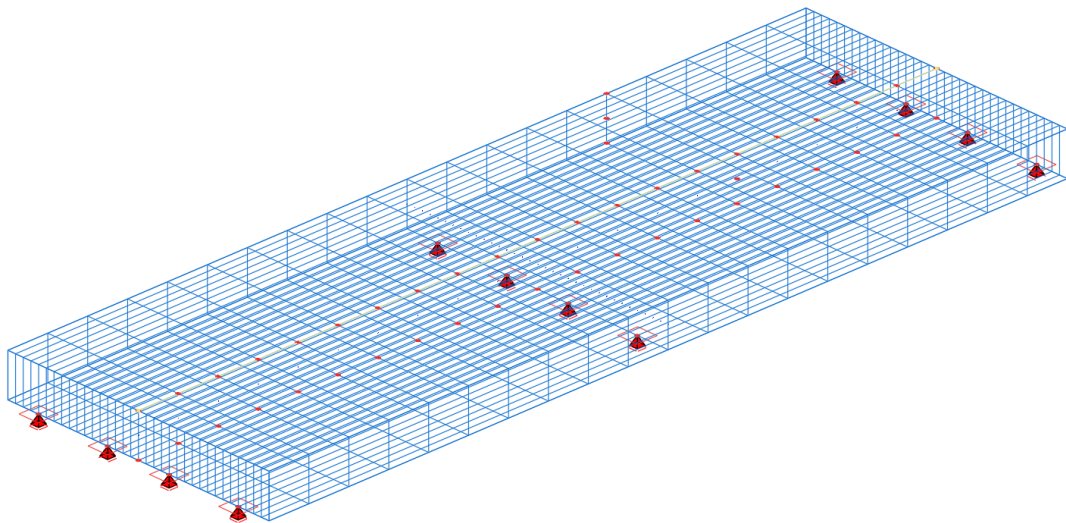
**Design B: Through bridge** The 3D Sofiplus model of Design B is shown in Figure 8.4.

The different components of design B are shown in Figure 8.5. The material properties of the flanges and webs are given in section 6.4. The supports are modelled as point loads with the degrees of freedom as shown in Figure 5.6.

The Sofiplus model is transformed into a FE model. The mesh size is determined by the software, with mesh refinement around critical points. Figure 8.6 shows the finite element model of Design B.



(a) 3D Structural view



(b) 2D Wireframe view

Figure 8.1: 3D view of Sofiplus model of Design A

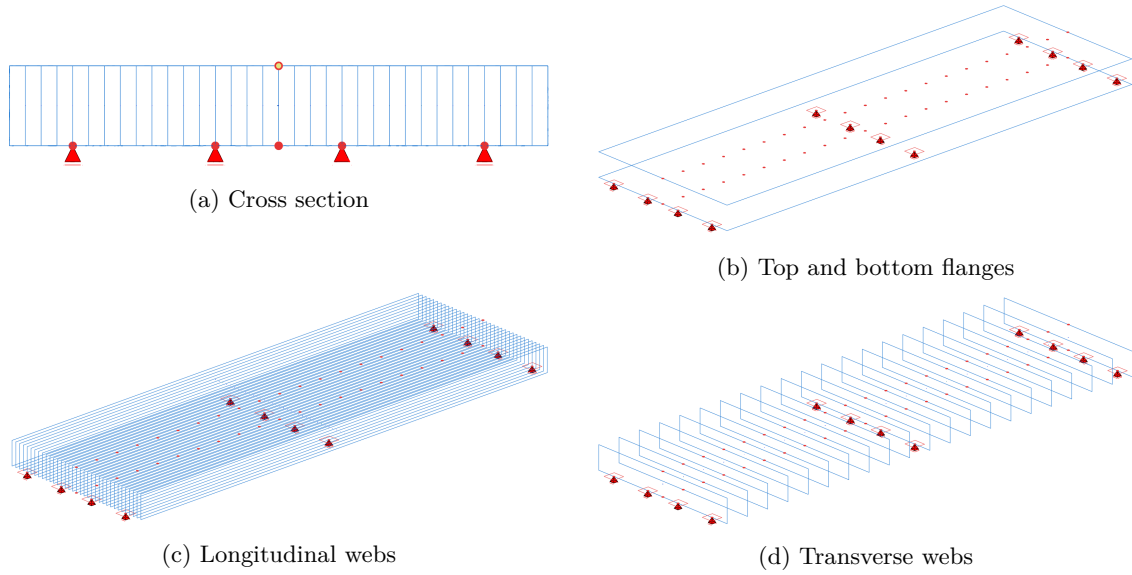


Figure 8.2: Different components of design A

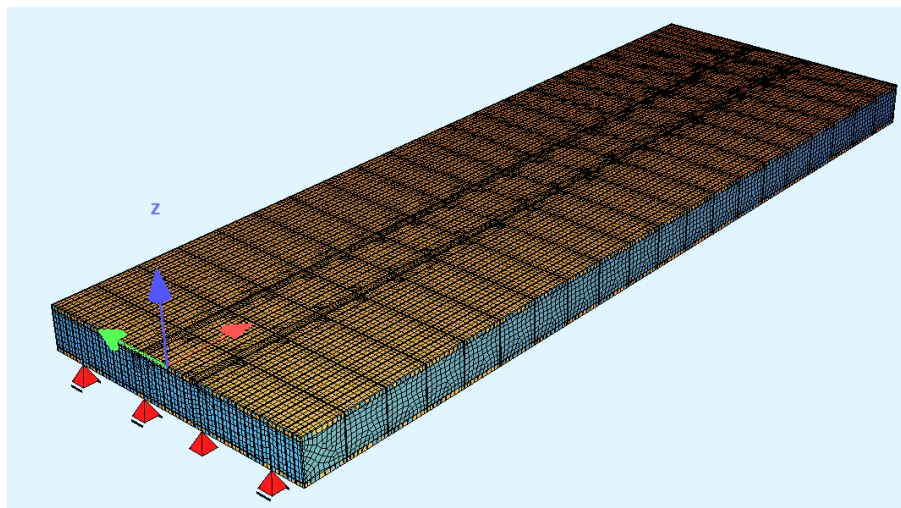
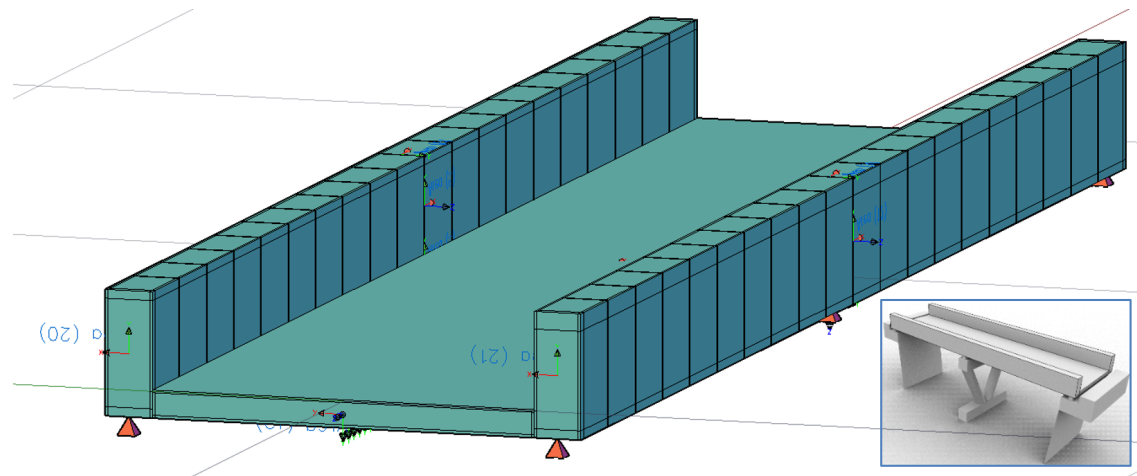
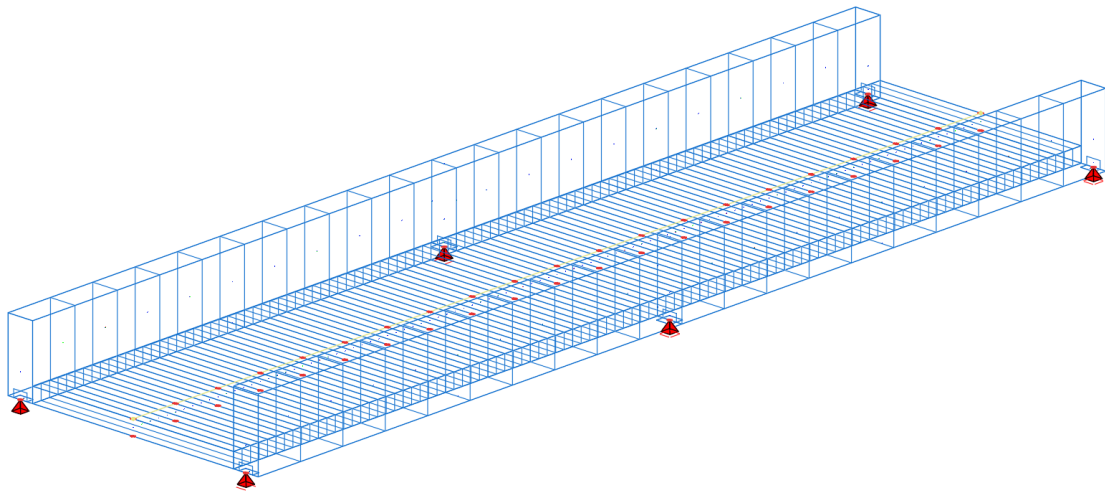


Figure 8.3: Finite element model of Design A



(a) 3D Structural view



(b) 2D Wireframe view

Figure 8.4: 3D view of Sofiplus model of Design B

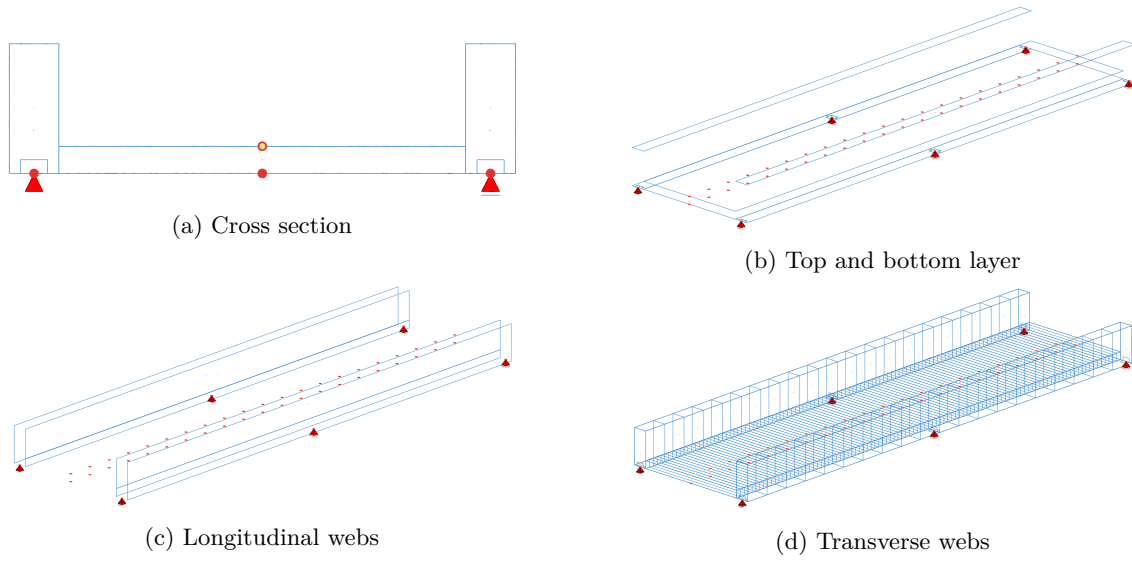


Figure 8.5: Different components of Design B

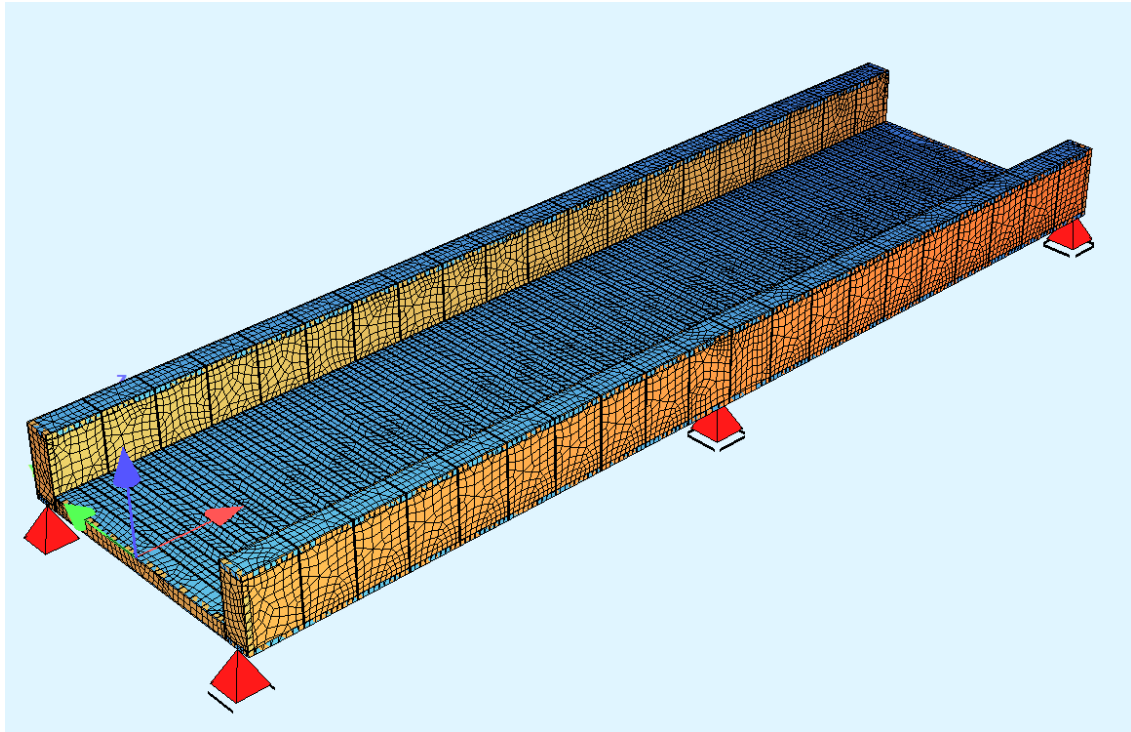


Figure 8.6: Finite element model of Design B



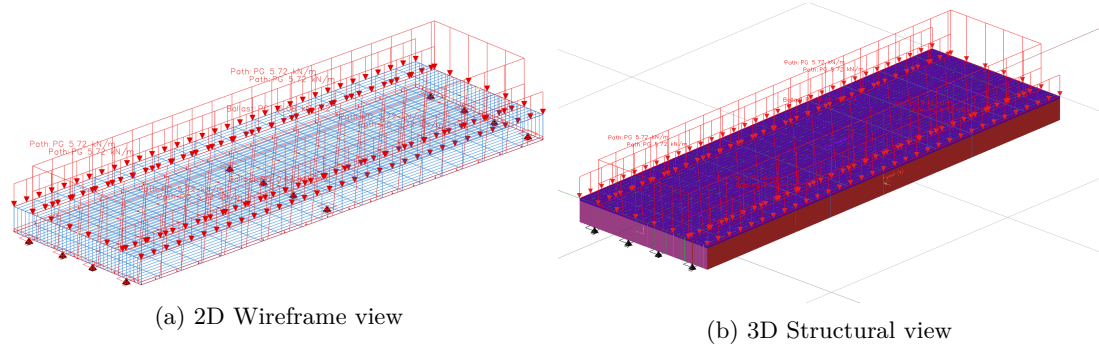


Figure 8.7: Load model 1 for design A

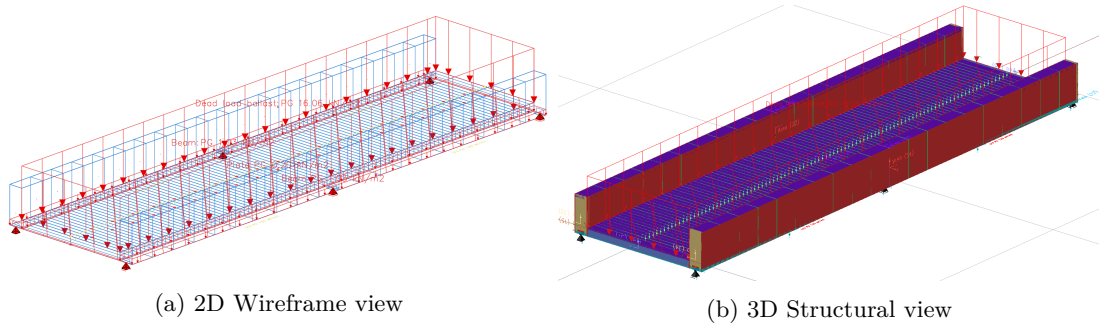


Figure 8.8: Load model 1 for Design B

### 8.3 Modelling the loads

A total of four different load cases are applied. These load cases are discussed in subsection 3.3.2. This paragraph describes how these load cases are modelled. Only the SLS load models are discussed. For the ULS models, the SLS models must be multiplied with the load safety factor.

- LM1 Load case 1 - Dead load
- LM2 Load case 2 - LM71 + start force
- LM3 Load case 3 - LC1 + LC2
- LM4 Dynamic load case

#### 8.3.1 Load model 1

Load model 1 describes the dead load on the bridge and exists of the self-weight of the deck, the ballast and the foam. The safety factor for LM1 is  $\gamma_G = 1.35$ .

**Design A: Sandwich bridge** Load model 1 for Design A is shown in Figure 8.7. The self-weight of the deck is calculated by the software. The ballast load is modelled as an area load of  $16.06 \text{ kN/m}^2$  on the top layer. The foam is loaded as an area load of  $0.64 \text{ kN/m}^2$  on the bottom layer. The maintenance path is modelled as four line loads of  $5.72 \text{ kN/m}$  on the top layer.

**Design B: Through bridge** Load model 1 for Design B is shown in Figure 8.8. The self-weight of the deck is calculated by the software. The ballast load is modelled as an area load of  $16.06 \text{ kN/m}^2$  on the top layer. The foam is loaded as an area load of  $0.20 \text{ kN/m}^2$  on the bottom layer of the beam sections and as an area load of  $1.03 \text{ kN/m}^2$  on the bottom layer of the plate section.



### 8.3.2 Load model 2

Load model 2 describes the train loads on the bridge and exists of LM71 including start force, as presented in section 7.2. The safety factor for LM2 is  $\gamma_Q = 1.45$ . Load model 2 is the same for both designs. Therefore, only Load Model 2 for Design A is presented.

Load model 2 for Design A is shown in Figure 8.9. LM71 is modelled as area loads on the top layer of the bridge. The area loads are  $332.58kN/m^2$  for the point loads and  $50.05kN/m^2$  for the distributed load. The start force is modelled as a horizontal area load of  $14.84kN/m^2$  on the top layer of the bridge.

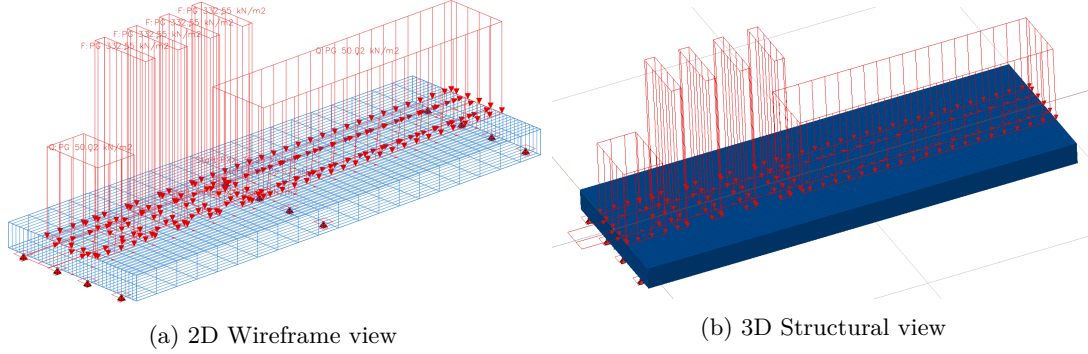


Figure 8.9: Load model 2 for Design A

### 8.3.3 Load model 3

Load model 3 combines LC1 and LC2. Sofistik provides a module to combine different loads into a new Combined Load case. For Load Model 3, LC1 and LC2 are combined into a new load model. The module is shown in Figure 8.10.

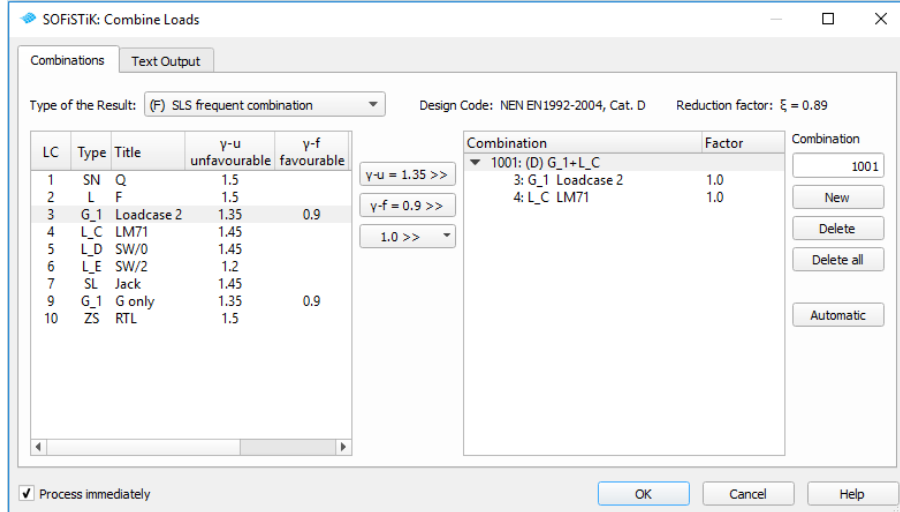


Figure 8.10: Load combination module of Sofistik

### 8.3.4 Load model 4

The dynamic analysis requires a specific dynamic load model, in this case LM4. This real train model is modelled as eight point loads travelling over the bridge with a constant velocity. This

```

+PROG SOFILOAD urs:24.2
head BOOGIE LOADS

lc 125  TYPE NONE  TITL 'REAL TRAIN MODEL'
      trai user
      trpl #F
      trpl #F A -2.5
      trpl #F A -17.5
      TRPL #F A -2.5
      TRPL #F A -4.5
      TRPL #F A -2.5
      TRPL #F A -17.5
      TRPL #F A -2.5
end

```

Figure 8.11: Numerical input for the point loads in LM4

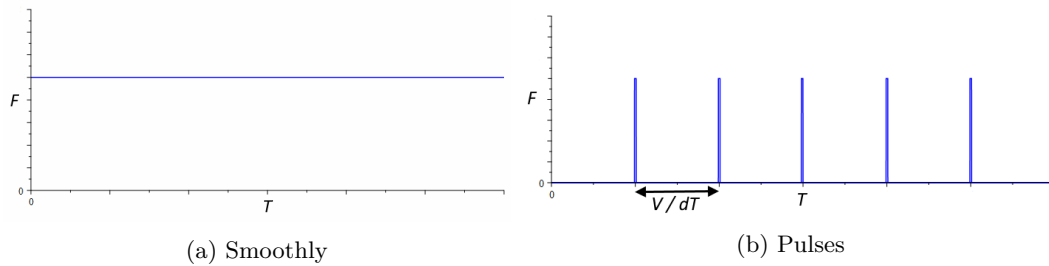


Figure 8.12: Moving load models

load is modelled using Teddy, a command center of Sofistik to enter numerical input. The real train model is presented in section 7.4.

**Modelling the movement** Sofistik is unable to model a smoothly moving load because time steps are used. Therefore, this moving load is modelled as a series of pulses acting on the deck and shifts at the velocity of the train. The distance between two pulses depends on the velocity of the train and the used time steps of the analysis. Figure 8.12 shows the difference between a smoothly moving load model and a series of pulses.

## 8.4 Natural frequencies

The first five natural frequencies of both designs are listed in Table 8.2. These values are based on a bridge with full dead load and 50% of the real train model load. For the angular frequencies, the natural frequencies must be multiplied with  $2\pi$ .

| Natural frequency | Sandwich design | Through design |
|-------------------|-----------------|----------------|
| First mode        | 12.74 Hz        | 5.86 Hz        |
| Second mode       | 13.82 Hz        | 6.23 Hz        |
| Third mode        | 14.77 Hz        | 7.90 Hz        |
| Fourth mode       | 16.96 Hz        | 8.05 Hz        |
| Fifth mode        | 17.84 Hz        | 11.45 Hz       |

Table 8.2: Natural frequencies of both bridges

## 8.5 Verification of the numerical models

Numerical models are hand-made and therefore prone to error. Therefore, the results of the numerical models must be verified on the basis of analytical results. The static verification of the numerical models is handled in chapter C. The dynamic verification is handled in chapter D.

According to the conclusions in chapter C and chapter D the numerical models provide realistic results for static and dynamic analysis. The dynamic results provided at the top nodes of both numerical models are high. Therefore, it is advised to look at the dynamic results at the bottom of both numerical models to get realistic results.

### 9.1 Dynamic load case

The dynamic load case is presented in section 7.4. The dynamic load model is presented in subsection 8.3.4.

#### 9.1.1 Damping

The damping of both designs is defined by the Rayleigh damping, described in subsection 4.1.2. The Rayleigh damping depends on the first and fifth natural angular frequency of the structure and the damping value. This damping value is 5%, according to JRC [Ascione et al., 2016]. The natural frequencies are given below.

**Natural frequencies** The first five natural frequencies of both designs are listed in Table 8.2 and shown below. These values are based on a bridge with full dead load and 50% of the real train model mass. For the angular frequencies, the natural frequencies must be multiplied with  $2\pi$ .

| Natural frequency | Sandwich design | Through design |
|-------------------|-----------------|----------------|
| First mode        | 12.74 Hz        | 5.86 Hz        |
| Second mode       | 13.82 Hz        | 6.23 Hz        |
| Third mode        | 14.77 Hz        | 7.90 Hz        |
| Fourth mode       | 16.96 Hz        | 8.05 Hz        |
| Fifth mode        | 17.84 Hz        | 11.45 Hz       |

Table 9.1: Natural frequencies of both bridges

**Rayleigh coefficients** Sofistik needs the Rayleigh coefficients ( $\alpha$  and  $\beta$  factor) to include damping in the analysis. The formulas to determine these values is presented in subsection 4.1.2. For the given natural frequencies and damping factor the  $\alpha$  and  $\beta$  factor are given in Table 9.2.

#### 9.1.2 Input numerical model

The input parameters for Design A are shown in Figure 9.1. The input parameters for Design B are almost the same, except for the 1st Eigenform and 5th Eigenform. These values are given in

| Design   | $\alpha$ | $\beta$   |
|----------|----------|-----------|
| Design A | 4.61164  | 0.0006340 |
| Design B | 2.43548  | 0.0009195 |

Table 9.2: Alpha and beta values for both designs

Table 9.1.

```

+prog TEMPLATE urs:54.2
head DEFINITION OF TRAIN LOAD PARAMETERS
let#V 140          $ Train speed [km/h]
sto#V 140/3.6      $ Train speed [m/s]
sto#F 213/2        $ Wheel load [kN]

end

+PROG TEMPLATE urs:15.1
head DEFINITION OF DAMPING PARAMETERS
let#xsi 0.05       $ Material damping
let#f1 12.47       $ 1st Eigenform
sto#w1 #f1*2*3.1415 $ 1st angular frequency
let#f2 17.84       $ 5th Eigenform
sto#w2 #f2*2*3.1415 $ 5th angular frequency
sto#a (2*#w1*#w2*(#xsi*#w2-#xsi*#w1))/(#w2^2-#w1^2) $ alpha value
sto#b (2*(#xsi*#w2-#xsi*#w1))/(#w2^2-#w1^2) $ beta value

end

```

Figure 9.1: Parameter input Teddy

The code to calculate the dynamic response of the structure loaded by LM4 is shown in Figure 9.2. LM4 (see Figure 8.11) is inserted as LC 125. Both Rayleigh coefficients are inserted as RADA and RABD. The mass of the bridge is automatically inserted, however the mass of the dead load components (ballast, foam) must be added manually. The mass of the dead load components without the self weight of the bridge is modelled in LC 9. LM4 travels over a line at the centre of the bridge, NR 101 (Figure 10.5), with velocity V. The time step is 5% of the load frequency; DT 0.005.

## 9.2 Load frequencies

The natural frequency of the bridge becomes critical when it coincides with the load frequency, leading to resonance. The load frequencies are caused by passing trains and depends on the velocity

```

PROG DYNA urs:25.1
HEAD ROLLING STOCK ANALYSIS
CTRL RLC 6000+7
CTRL STYP MAX

GRP - FULL RADA #a RABD #b ; CTRL MCON 2

STEP DT 0.005 N 800
LC 125
MASS LC 9
CONT NR 101 V 140/3.6

HIST U-Z FROM 9000 TO 9009 LCST 202
HIST A-Z FROM 9000 TO 9009 LCST 201

HIST U-Z FROM 6000 TO 6009 LCST 204
HIST A-Z FROM 6000 TO 6009 LCST 203

END

```

Figure 9.2: Code for dynamic analysis

of the train and the distance between the wheels, the so-called wheel spacing. The formula used to calculate the load frequency is given by Equation 9.1.

$$f_{train} = \frac{v}{d} \quad (9.1)$$

Where:

|             |                                  |
|-------------|----------------------------------|
| $f_{train}$ | Frequency of the train load [Hz] |
| $v$         | Velocity of the train [m/s]      |
| $d$         | Wheel spacing [m]                |

The wheel spacing is given by the dimensions of the train. The wheel spacing of a VIRM4 train has already been showed in chapter 4 and section 7.4, see Figure 7.1. The values of the wheel spacing with corresponding load frequency, period and angular load frequency can be found in Table 9.3. The velocity of the train is 140 km/h.

| Wheel spacing | Load frequency | Period | Angular frequency |
|---------------|----------------|--------|-------------------|
| 2.5 m         | 15.56 Hz       | 0.06 s | 97.74 rad/s       |
| 7.0 m         | 5.56 Hz        | 0.18 s | 34.93 rad/s       |
| 9.5 m         | 4.09 Hz        | 0.24 s | 25.70 rad/s       |
| 17.5 m        | 2.22 Hz        | 0.45 s | 13.96 rad/s       |
| 20.0 m        | 1.95 Hz        | 0.51 s | 12.25 rad/s       |
| 22.5 m        | 1.73 Hz        | 0.58 s | 10.87 rad/s       |
| 24.5 m        | 1.59 Hz        | 0.63 s | 9.99 rad/s        |
| 27.0 m        | 1.44 Hz        | 0.69 s | 9.05 rad/s        |
| 29.5 m        | 1.32 Hz        | 0.76 s | 8.29 rad/s        |
| 44.5 m        | 0.87 Hz        | 1.14 s | 5.47 rad/s        |
| 47.0 m        | 0.83 Hz        | 1.21 s | 5.22 rad/s        |
| 49.5 m        | 0.79 Hz        | 1.27 s | 4.96 rad/s        |

Table 9.3: Load frequencies of LM4



This chapter gives the results for the loadings in chapter 7 on both designs. At first, the SLS situation is analysed by checking the deflection. Thereafter the stresses in both designs are analysed in ultimate limit state. Thirdly, the results of the dynamic analysis are given. Finally, a conclusion on the results is given.

### 10.1 SLS results

The unity check in SLS situation are based on the maximum deflection of the models. The limit is given by OVS30, which prescribes a maximum deflection of  $L/800$ . The results including unity checks are given in Table 10.1.

| Bridge          | Load case | Deflection | Limit   | Unity check |
|-----------------|-----------|------------|---------|-------------|
| Sandwich bridge | LC 1      | 2.26 mm    | 12.5 mm | 0.18        |
|                 | LC 2      | 4.94 mm    | 12.5 mm | 0.40        |
|                 | LC 3      | 7.06 mm    | 12.5 mm | 0.56        |
| Through bridge  | LC 1      | 10.2 mm    | 12.5 mm | 0.82        |
|                 | LC 2      | 38.4 mm    | 12.5 mm | 3.07        |
|                 | LC 3      | 49.2 mm    | 12.5 mm | 3.94        |

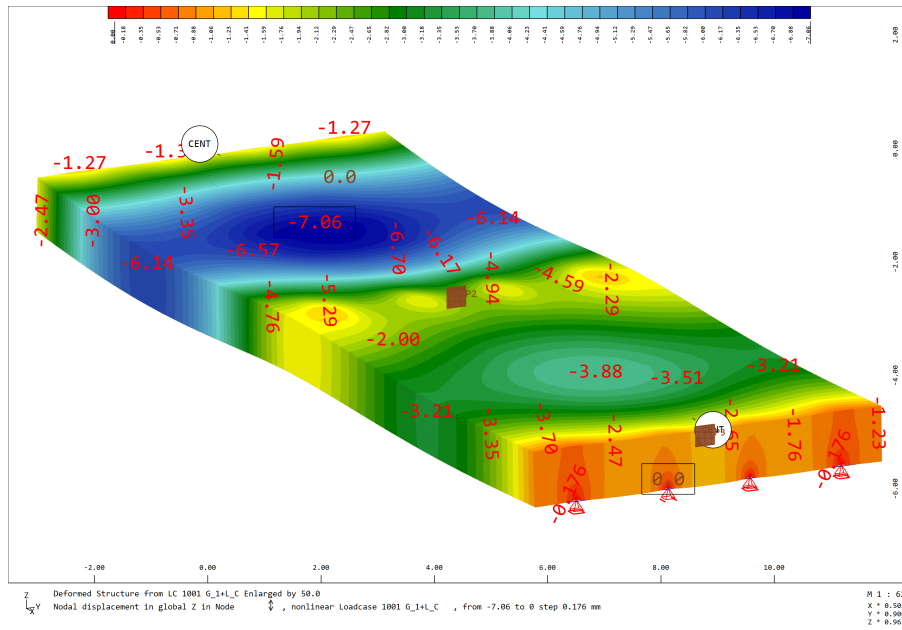
Table 10.1: SLS results conceptual designs

According to Table 10.1 the unity checks for the Design A are lower than 1, which means this design meets the requirements for SLS situation. The unity checks of Design B are lower than 1 for LC 1, but higher than 1 for LC 2 and LC 3. This means Design B does not meet the requirements for SLS situation.

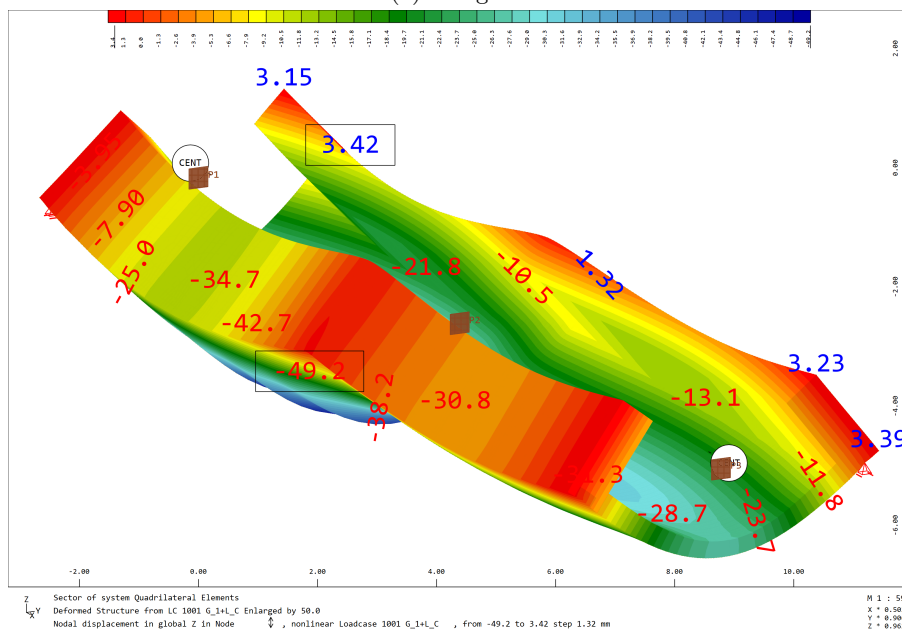
The deflection of Design A for LC 3 is given in Figure 10.1a. The deflection of Design B for LC 3 is given in Figure 10.1b. The deflection for all load cases is given in chapter F.

According to Figure 10.1a the deflection of Design A is in two directions. The design acts as two plates supported at each corner, with maximum deflection in the middle of the plate. One span is more deflected than the other one, caused by the points loads of LC 2 only acting on one span. The mid-supports are visible by the small peaks in the figure, caused by modelling the supports as point supports.





(a) Design A



(b) Design B

Figure 10.1: Deflection of both designs for LC 3

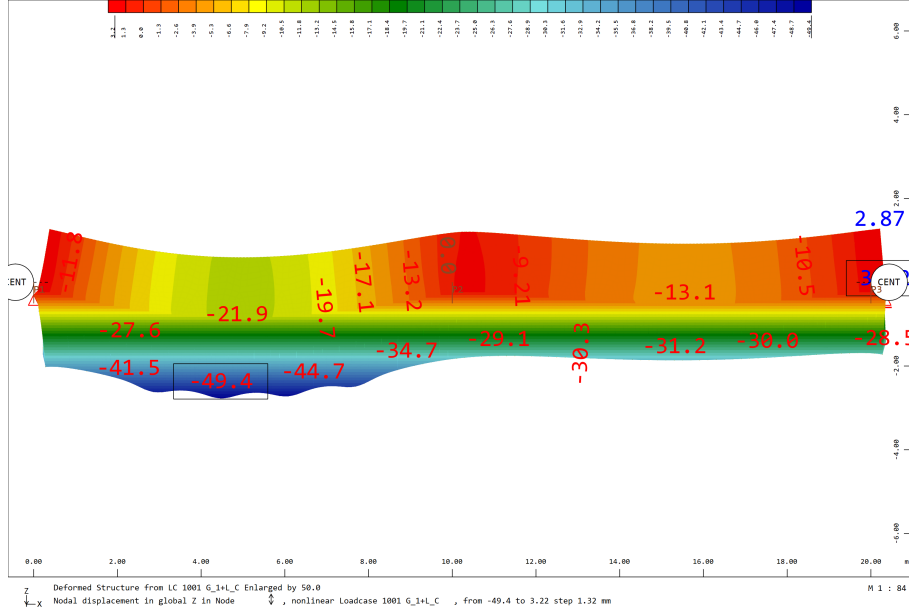


Figure 10.2: Side view of Design B for LC3

The deflection of Design B is mainly in transverse direction, according to Figure 10.1b. The high beam elements are less deflected, with a maximum of approximately 12 mm. The maximum transverse deflection is 49.2 mm. The beam elements rotate in transverse direction, caused by the deflection of the plate section and the connection of this plate section with the beam sections. Furthermore, one span is more deflected than the other span, caused by the large point loads of LC2 only acting on one span. The impact of these large point loads is even visible at the bottom of the plate section, showing large deflections underneath the point loads, see Figure 10.2. The deflection at mid-support is large regardless the supports. However, these supports are positioned underneath the beam sections causing the plate section to deflect freely.

The large transverse deflections can be caused by the limited height of the plate section. Furthermore, the limited amount of supports can have a negative effect on the reduction of the deflections. Furthermore, the absence of longitudinal prevents the longitudinal stress distribution.

## 10.2 ULS results

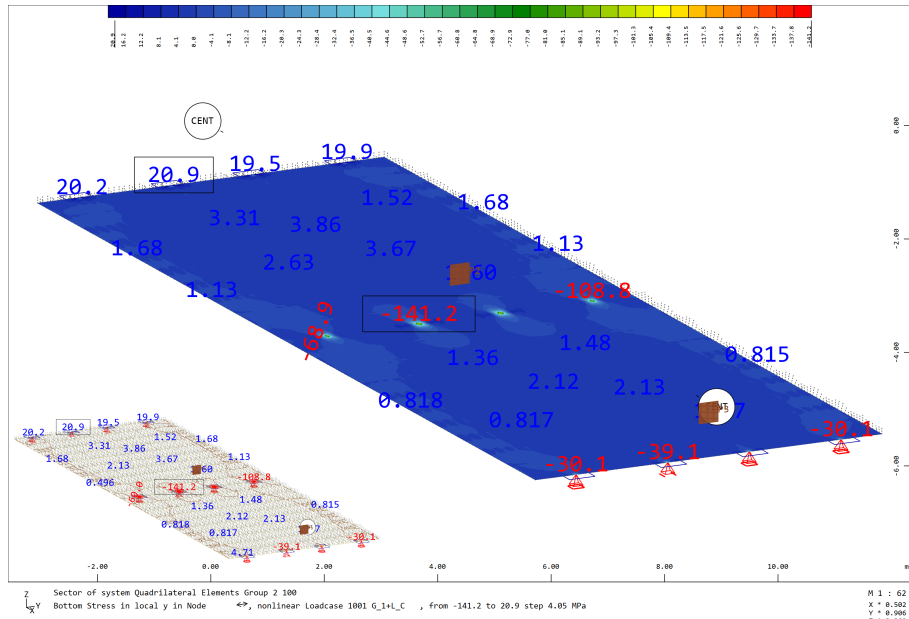
The unity checks in ULS situation are based on the maximum stresses in both models. The limits are given by the material properties; tensile/compressive strength and shear strength. The maximum tensile/compressive stresses are expected in the flanges, due to the bending moments. The maximum shear stresses are expected in the webs. The results including unity checks are given in Table 10.2.

According to Table 10.2 both designs have at least one unity check higher than 1 for any of the load cases. Especially the stresses in the through bridge are much higher than allowed, with unity checks up to 6.50 for  $\sigma_{yy}$  in the bottom flange for LC 3. The highest unity check for the sandwich bridge is 3.33 for the same situation ( $\sigma_{yy}$  in the bottom flange for LC 3). This means that both designs do not meet the requirements for ULS situations.

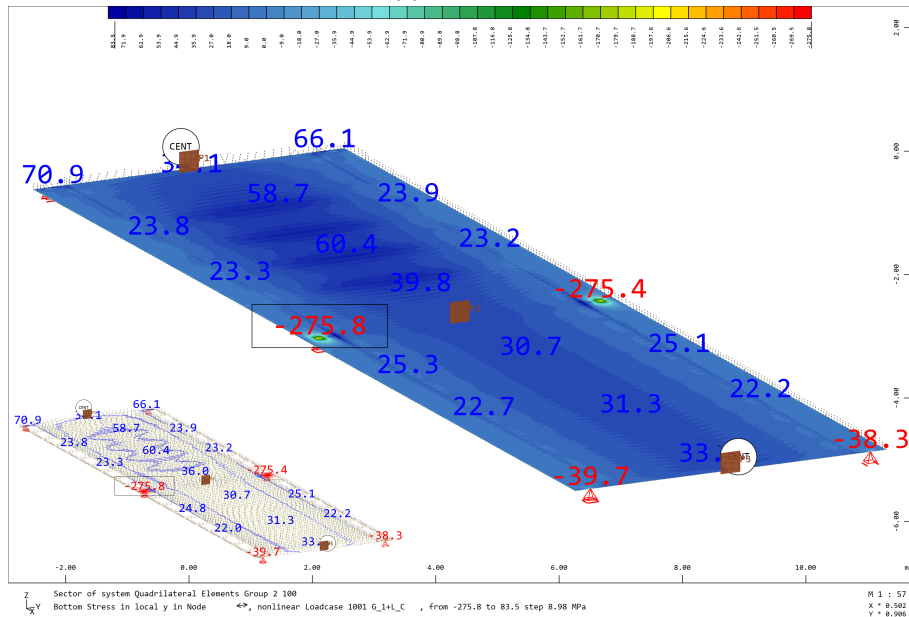
**Singularity** When taking a closer look at the results (Figure 10.3), it is visible that the high stresses occur at the supports. This principle, where high stresses occur at critical points, is called stress singularity. According to Pacoste [Pacoste et al., 2012] these stresses are unrealistic high and can be neglected. Pacoste suggests to look at elements at a physical distance of the point

| Bridge          | Load case | Stresses             | Results [MPa] | Limit [MPa] | UC   |
|-----------------|-----------|----------------------|---------------|-------------|------|
| Sandwich bridge | LC 1      | $\sigma_{xx}$ Top    | 11.8          | $\pm 92.6$  | 0.13 |
|                 |           | $\sigma_{xx}$ Bottom | -78.7         | $\pm 92.6$  | 0.85 |
|                 |           | $\sigma_{yy}$ Top    | -10.2         | $\pm 42.4$  | 0.24 |
|                 |           | $\sigma_{yy}$ Bottom | -89.8         | $\pm 42.4$  | 2.12 |
|                 |           | $\tau_{xy}$          | -26.6         | $\pm 23.9$  | 1.11 |
|                 | LC 2      | $\sigma_{xx}$ Top    | 12.2          | $\pm 92.6$  | 0.13 |
|                 |           | $\sigma_{xx}$ Bottom | 279.5         | $\pm 92.6$  | 3.02 |
|                 |           | $\sigma_{yy}$ Top    | 6.5           | $\pm 42.4$  | 0.07 |
|                 |           | $\sigma_{yy}$ Bottom | -85.2         | $\pm 42.4$  | 2.01 |
|                 |           | $\tau_{xy}$          | -25.6         | $\pm 23.9$  | 1.07 |
|                 | LC 3      | $\sigma_{xx}$ Top    | 19.9          | $\pm 92.6$  | 0.21 |
|                 |           | $\sigma_{xx}$ Bottom | 286.9         | $\pm 92.6$  | 3.10 |
|                 |           | $\sigma_{yy}$ Top    | 11.2          | $\pm 42.4$  | 0.26 |
|                 |           | $\sigma_{yy}$ Bottom | -141.2        | $\pm 42.4$  | 3.33 |
|                 |           | $\tau_{xy}$          | -41.8         | $\pm 23.9$  | 1.75 |
| Through bridge  | LC 1      | $\sigma_{xx}$ Top    | 11.2          | $\pm 92.6$  | 0.12 |
|                 |           | $\sigma_{xx}$ Bottom | -74.8         | $\pm 92.6$  | 0.81 |
|                 |           | $\sigma_{yy}$ Top    | -9.83         | $\pm 42.4$  | 0.23 |
|                 |           | $\sigma_{yy}$ Bottom | -85.3         | $\pm 42.4$  | 2.25 |
|                 |           | $\tau_{xy}$          | -25.4         | $\pm 23.9$  | 1.06 |
|                 | LC 2      | $\sigma_{xx}$ Top    | -34.8         | $\pm 92.6$  | 0.38 |
|                 |           | $\sigma_{xx}$ Bottom | 333.4         | $\pm 92.6$  | 3.60 |
|                 |           | $\sigma_{yy}$ Top    | -45.3         | $\pm 42.4$  | 1.07 |
|                 |           | $\sigma_{yy}$ Bottom | -188.9        | $\pm 42.4$  | 4.46 |
|                 |           | $\tau_{xy}$          | 53.6          | $\pm 23.9$  | 2.24 |
|                 | LC 3      | $\sigma_{xx}$ Top    | 41.9          | $\pm 92.6$  | 0.45 |
|                 |           | $\sigma_{xx}$ Bottom | 358.9         | $\pm 92.6$  | 3.88 |
|                 |           | $\sigma_{yy}$ Top    | -53.9         | $\pm 42.4$  | 1.27 |
|                 |           | $\sigma_{yy}$ Bottom | -275.8        | $\pm 42.4$  | 6.50 |
|                 |           | $\tau_{xy}$          | 77.4          | $\pm 23.9$  | 3.24 |

Table 10.2: ULS results with unity checks



(a) Design A



(b) Design B

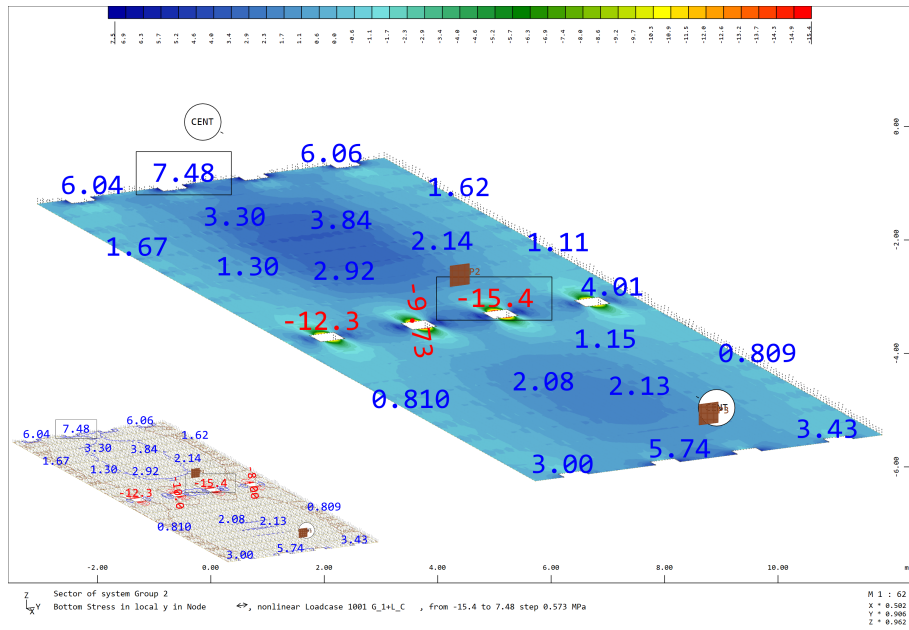
Figure 10.3:  $\sigma_{yy}$  in the bottom flange of both designs for load case 3

loads, like a column or support. In this case the dimensions of a bearing block are used resulting in a physical distance of 500 mm. Table 10.3 gives the unity checks for the stresses without the effect of stress singularity.

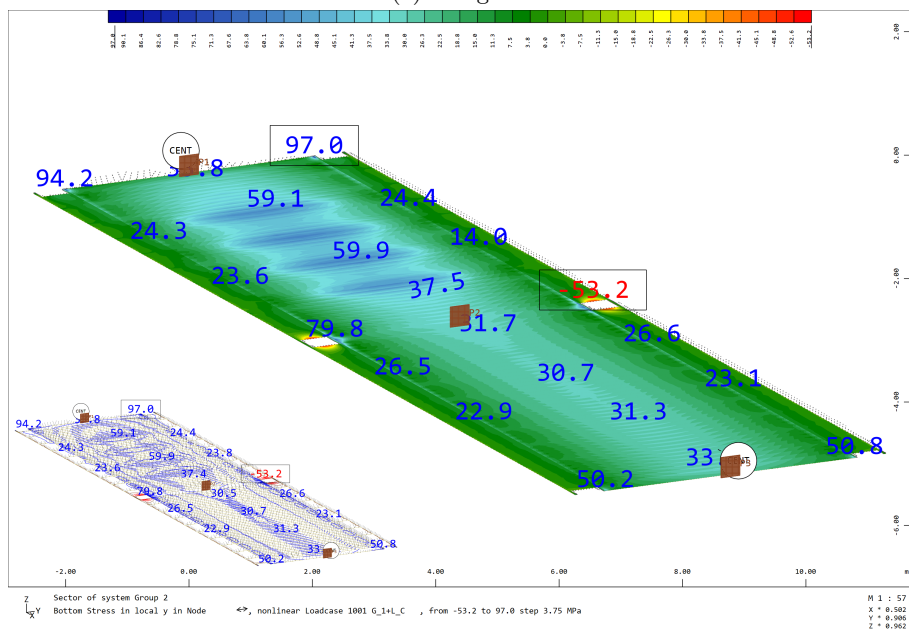
According to Table 10.3 the unity checks of Design A for all load cases are lower than 1. This means that Design A meets the requirements for the ULS situation. The unity checks of Design B for LC 1 are lower than 1, but the unity checks for LC 2 and LC 3 are higher than one. According to these unity checks it means that Design B does not meet the requirements for ULS situation. According to Figure 10.4a the maximum tensile stresses at the bottom flange of Design A occur

| Bridge          | Load case | Stresses             | Results [MPa] | Limit [MPa] | UC   |
|-----------------|-----------|----------------------|---------------|-------------|------|
| Sandwich bridge | LC 1      | $\sigma_{xx}$ Top    | 11.8          | $\pm 92.6$  | 0.13 |
|                 |           | $\sigma_{xx}$ Bottom | -25.2         | $\pm 92.6$  | 0.27 |
|                 |           | $\sigma_{yy}$ Top    | -10.2         | $\pm 42.4$  | 0.24 |
|                 |           | $\sigma_{yy}$ Bottom | -30.4         | $\pm 42.4$  | 0.72 |
|                 |           | $\tau_{xy}$          | 16.8          | $\pm 23.9$  | 0.70 |
|                 | LC 2      | $\sigma_{xx}$ Top    | 12.2          | $\pm 92.6$  | 0.13 |
|                 |           | $\sigma_{xx}$ Bottom | 43.7          | $\pm 92.6$  | 0.47 |
|                 |           | $\sigma_{yy}$ Top    | 6.5           | $\pm 42.4$  | 0.15 |
|                 |           | $\sigma_{yy}$ Bottom | -9.1          | $\pm 42.4$  | 0.21 |
|                 |           | $\tau_{xy}$          | 12.5          | $\pm 23.9$  | 0.52 |
|                 | LC 3      | $\sigma_{xx}$ Top    | 19.9          | $\pm 92.6$  | 0.21 |
|                 |           | $\sigma_{xx}$ Bottom | 49.0          | $\pm 92.6$  | 0.53 |
|                 |           | $\sigma_{yy}$ Top    | 11.2          | $\pm 42.4$  | 0.26 |
|                 |           | $\sigma_{yy}$ Bottom | -15.4         | $\pm 42.4$  | 0.36 |
|                 |           | $\tau_{xy}$          | 19.8          | $\pm 23.9$  | 0.83 |
| Through bridge  | LC 1      | $\sigma_{xx}$ Top    | 11.2          | $\pm 92.6$  | 0.12 |
|                 |           | $\sigma_{xx}$ Bottom | -23.9         | $\pm 92.6$  | 0.26 |
|                 |           | $\sigma_{yy}$ Top    | -9.83         | $\pm 42.4$  | 0.23 |
|                 |           | $\sigma_{yy}$ Bottom | -28.9         | $\pm 42.4$  | 0.68 |
|                 |           | $\tau_{xy}$          | -24.4         | $\pm 23.9$  | 0.67 |
|                 | LC 2      | $\sigma_{xx}$ Top    | -34.8         | $\pm 92.6$  | 0.38 |
|                 |           | $\sigma_{xx}$ Bottom | 106.8         | $\pm 92.6$  | 1.15 |
|                 |           | $\sigma_{yy}$ Top    | -45.3         | $\pm 42.4$  | 1.07 |
|                 |           | $\sigma_{yy}$ Bottom | 76.3          | $\pm 42.4$  | 1.80 |
|                 |           | $\tau_{xy}$          | 31.7          | $\pm 23.9$  | 1.33 |
|                 | LC 3      | $\sigma_{xx}$ Top    | 41.9          | $\pm 92.6$  | 0.45 |
|                 |           | $\sigma_{xx}$ Bottom | 115.7         | $\pm 92.6$  | 1.25 |
|                 |           | $\sigma_{yy}$ Top    | -53.9         | $\pm 42.4$  | 1.27 |
|                 |           | $\sigma_{yy}$ Bottom | 97.0          | $\pm 42.4$  | 2.29 |
|                 |           | $\tau_{xy}$          | 45.5          | $\pm 23.9$  | 1.90 |

Table 10.3: ULS results without the effect of stress singularities



(a) Design A



(b) Design B

Figure 10.4:  $\sigma_{yy}$  in the bottom flange for LC 3 without stress singularity

at mid-supports. This is in accordance with theory, where the maximum bending moment and maximum shear force for a double span beam are located at the mid-support, resulting in the highest stresses. When taking a closer look, the peak stresses are located close to the supports. The stress resulting in the highest unity check for Design B is located around the outer supports, according to Table 10.3 and Figure 10.4b. One of the reasons for these higher stresses in Design B, compared to Design A, is the limited amount of supports used in Design B. The full load is only transferred to six supports, compared to twelve supports in Design A. This results in higher support reaction and therefore higher stresses. Furthermore, these higher forces are transferred via thinner flanges, resulting in even higher stresses.

### 10.3 Dynamic results

This section shows the results of the dynamic analysis presented in chapter 9. For both designs the deflection and acceleration are given for specific nodes on the deck. These nodes are shown in Figure 10.5. Only the bottom node are analysed, according to the advise giving in section 8.5.

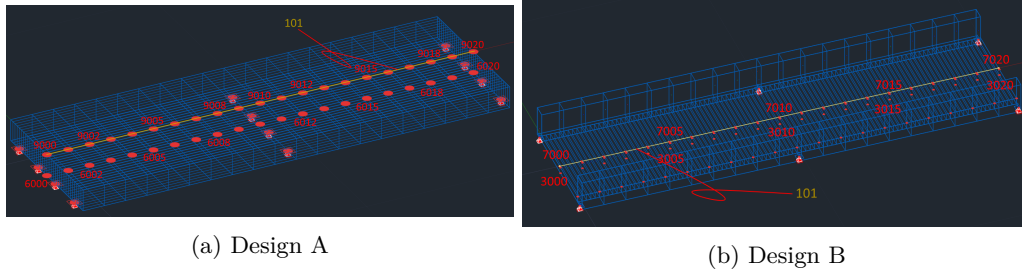


Figure 10.5: Position of output nodes

The maximums of both designs are given in Table 10.4. According to this table the deflection of both designs are within the limits given by the unity checks lower than 1. On the other hand, the accelerations of the deck of both designs are too high, given by the unity checks being more than 1. Therefore, both designs meet the requirements for deflection but do not meet the requirements for the maximum acceleration of the deck.

| Design                   | Results  |          | Limit | Unity Check |          |
|--------------------------|----------|----------|-------|-------------|----------|
|                          | Design A | Design B |       | Design A    | Design B |
| Displacement [mm]        | 0.709    | 7.76     | 12.5  | 0.06        | 0.62     |
| Acceleration [ $m/s^2$ ] | 13.12    | 237.91   | 3.5   | 3.75        | 68.0     |

Table 10.4: Maximum results of dynamic analysis

The dynamic results of both designs are discussed in the following paragraphs.



### 10.3.1 Sandwich bridge

The maximum acceleration and deflection of the bottom nodes of Design A are given in Figure 10.6.

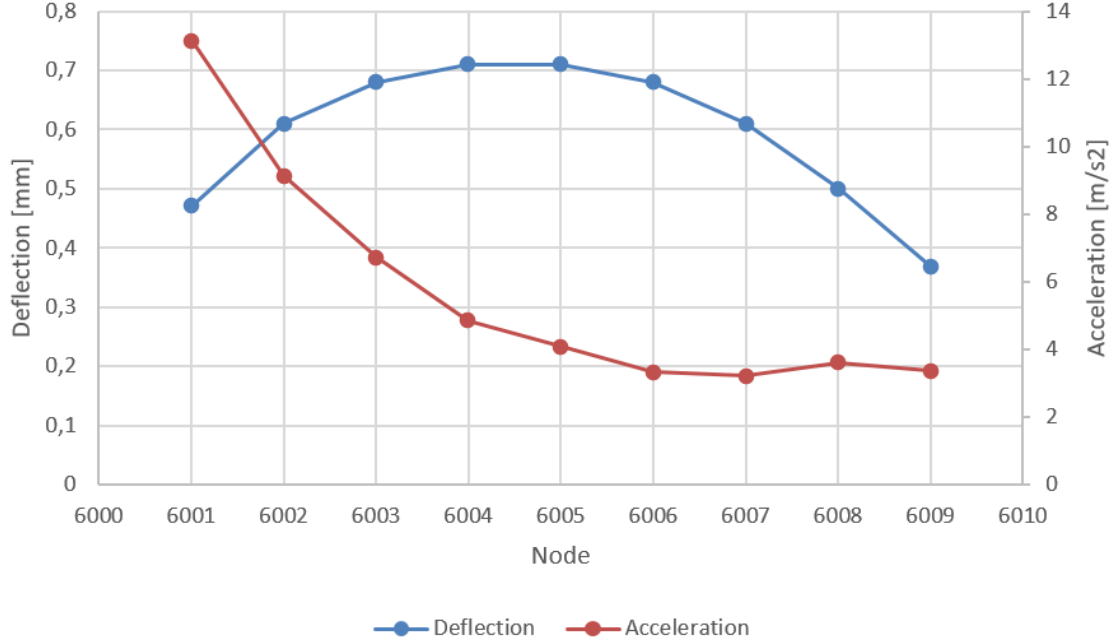


Figure 10.6: Maximum acceleration and deflection of Design A

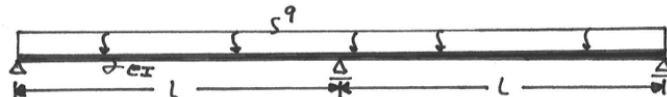
According to Figure 10.6 the maximum deflection is between node 6004 and node 6005. This is in accordance with the theory, where the maximum deflection is at  $0.4215 \cdot L$  (see Figure 10.7). The maximum acceleration is at node 6001.

**Deflection** The deflection at node 6001, 6003, 6005 and 6007 of Design A are presented in Figure 10.8.

The impact of the point loads on the deflection of the bridge is clearly visible. The time between the peaks in the results is in accordance with the time steps between the point loads of LM 4, see Figure 10.9. It takes the train 1.26 s to pass. The time between the two outer peaks at each node is about 1.2 s.

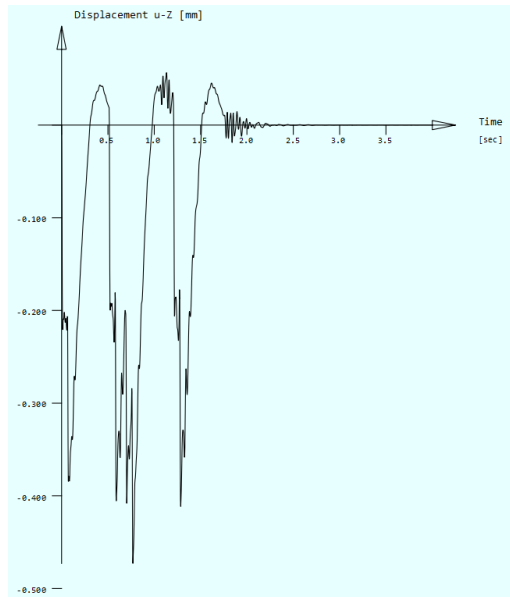
**Acceleration** The acceleration at node 6001, 6003, 6005 and 6007 of Design A are presented in Figure 10.10.

According to Figure 10.10 the maximum accelerations are at the point loads. The time between the peaks is in accordance with the time steps between the point loads of LM 4, see Figure 10.9.

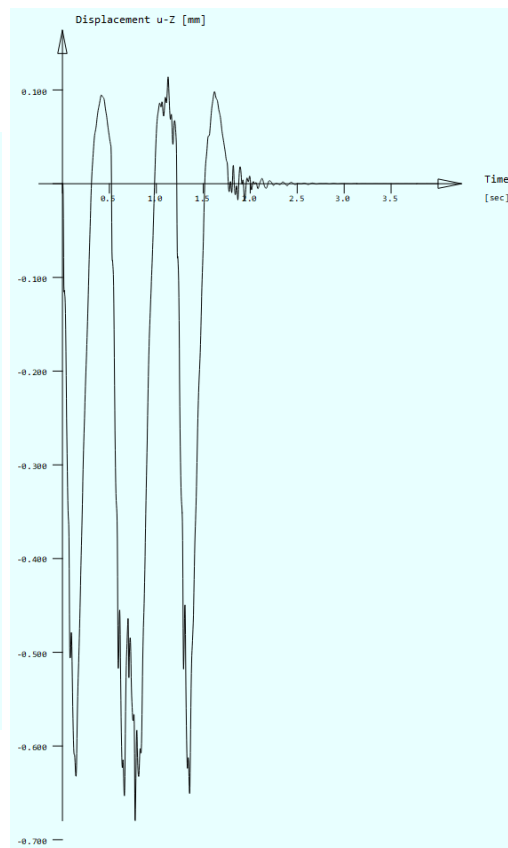


$$w_{max} = \frac{qL^4}{185EI} \text{ at } x = 0.4125 L$$

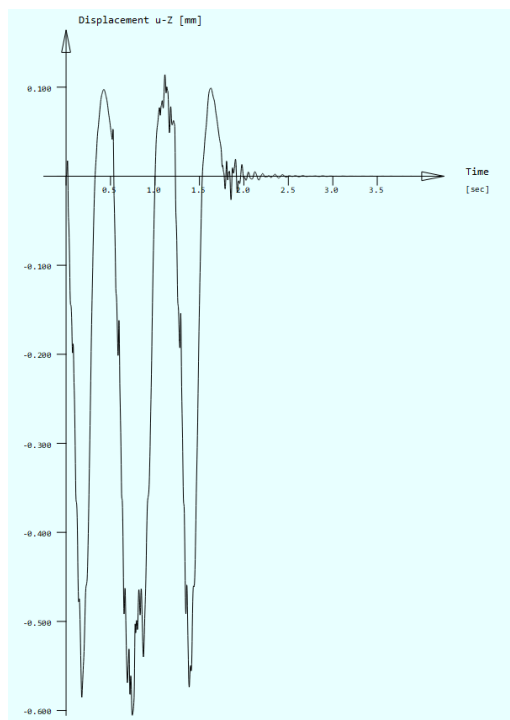
Figure 10.7: Maximum deflection for double-span beam



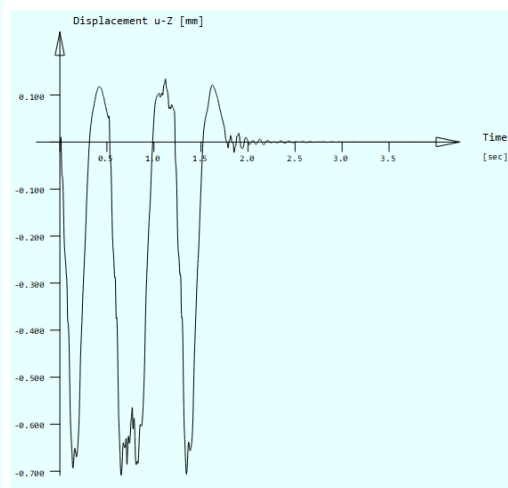
(a) Node 6001



(b) Node 6003



(c) Node 6007



(d) Node 6005

Figure 10.8: Deflection of Design A at different nodes

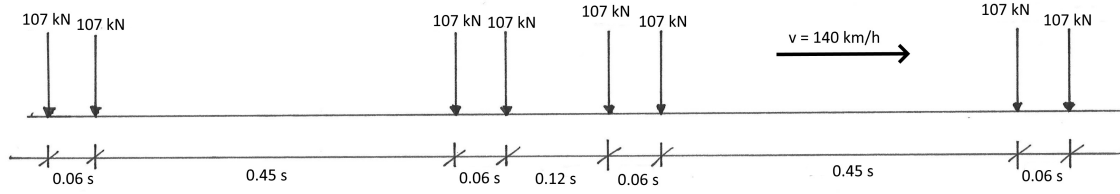
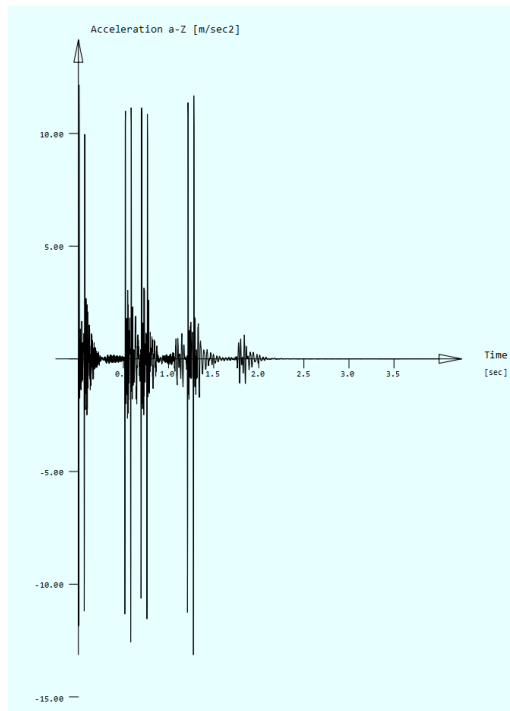
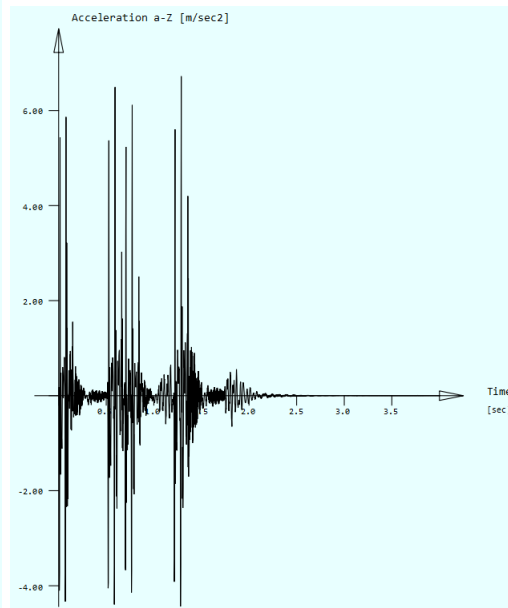


Figure 10.9: Time steps between point loads of LM 4

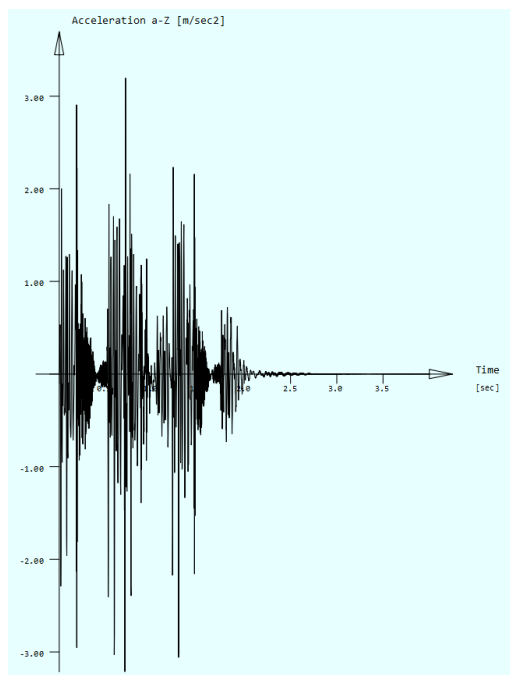
The effect of damping is visible, as the response of the structure tends to zero after every point load. The maximum acceleration of the steady state response is at about  $1 \text{ m/s}^2$ . One of the reasons for the high peaks might be the effect of singularity. The effect of singularity can be neglected by looking at elements at a physical distance. Looking directly underneath the point loads might have insufficient physical distance between the critical point and the nodes. The maximum acceleration is at node 6001 and then decreases, as shown in Figure 10.6. This can be explained by the way of modelling the moving train load, explained in subsection 8.3.4. The pulses act within a short time period of 0.005 sec, resulting in very high accelerations. The other nodes are already influenced by the load at the first time step when the load reaches them. Therefore, the maximum acceleration decreases over the length of the bridge.



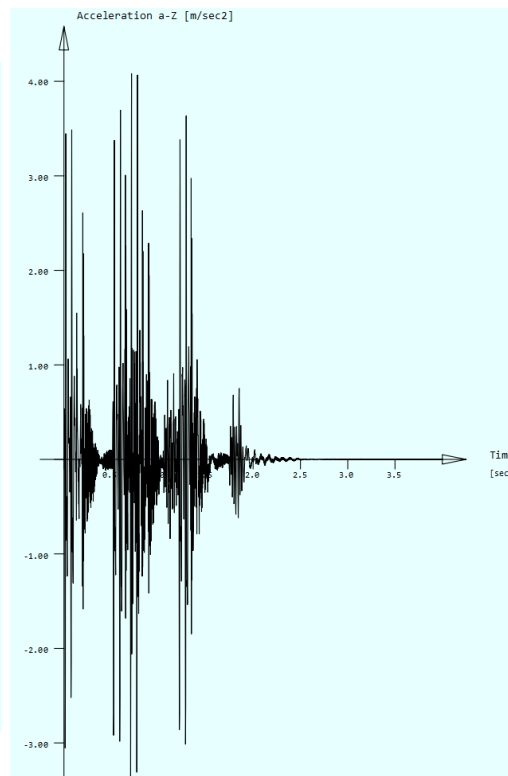
(a) Node 6001



(b) Node 6003



(c) Node 6007



(d) Node 6005

Figure 10.10: Acceleration of Design A at different nodes

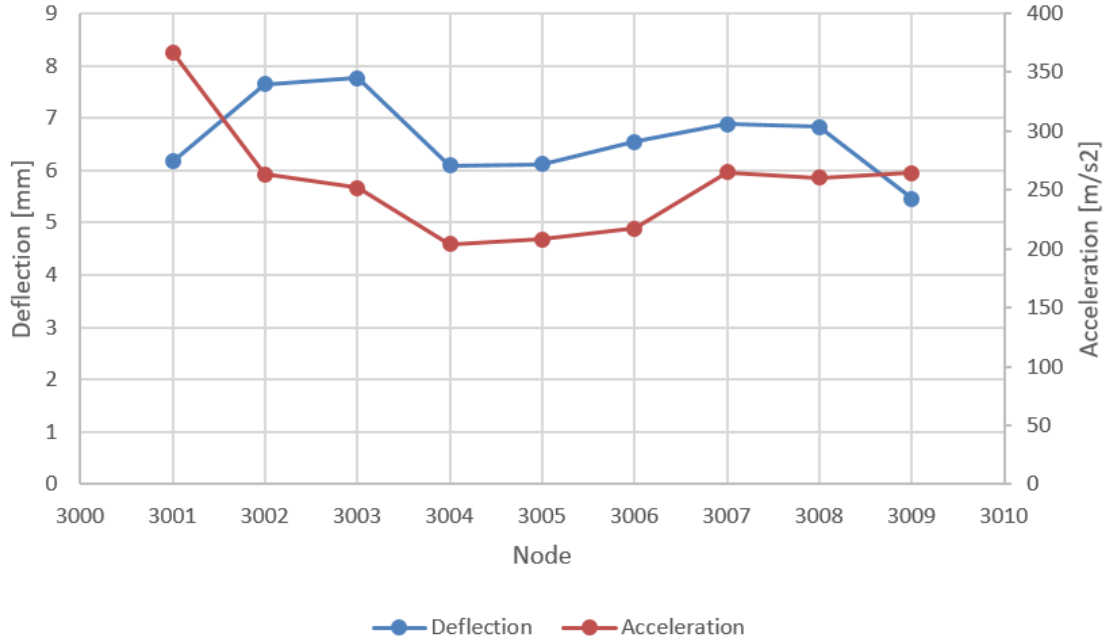


Figure 10.11: Maximum acceleration and deflection of Design B

### 10.3.2 Through bridge

The maximum acceleration and deflection at the bottom nodes of Design B are given in Figure 10.11.

According to Figure 10.11 the maximum deflection is at node 3003. This isn't in accordance with the theory, where the maximum deflection is expected between node 3004 and 3005 (see Figure 10.7). One cause might be the difference between the theory and the numerical results. The location of the maximum deflection is based on an equally distributed load, where LM 4 consists of only four point loads.

**Deflection** The deflection at node 3001, 3003, 3005 and 3007 of Design B are presented in Figure 10.12.

The impact of the eight point loads of LM 4 are clearly visible. The time between the peaks in the results is in accordance with the time steps between the point loads in LM4, see Figure 10.9. The damping of the structure is visible, as the steady state response of the structure becomes zero after a short period of time. However, this time period is larger for Design B compared to Design A. According to Figure 10.8a the damping time of Design A is approximately 0.7 seconds. The damping time of Design is approximately 1.5 seconds, according to Figure 10.12a. This is caused by the different natural frequencies of both designs. According Equation 4.11 a higher natural frequency leads to more damping. The natural frequencies of Design A are larger than the natural frequencies of Design B (Table 8.2) resulting in more damping.

**Acceleration** The acceleration at node 3001, 3003, 3005 and 3007 of Design B are presented in Figure 10.13.

According to Figure 10.13 the maximum accelerations occur at the point loads. These maximum acceleration are extremely high. One of the reasons is the effect of singularity. The effect of singularity can be neglected by looking at elements at a physical distance. Due to the limited thickness of the bridge including the position of the nodes directly underneath the point loads this might be an insufficient physical distance. Another reason for the extreme high peaks is the effect of resonance. The natural frequencies of Design B (Table 8.2) and the load frequencies

(Table 9.3) are close together leading to resonance. The effect of damping is visible; the response of the structure tends to zero after every point load.

## 10.4 Conclusion

**SLS** The deflection of Design A is within the limits and therefore meets the requirements; Design A is safe. However, the large laminate thickness might cause problems during manufacturing. The deflection of Design B exceeds the limits and therefore does not meet the requirements; Design B is unsafe. One of the reasons for the large deflection of Design B is the limited height of the plate section. Furthermore, by positioning the supports only underneath the beam sections the plate section isn't supported and can freely deflect, resulting in large displacements.

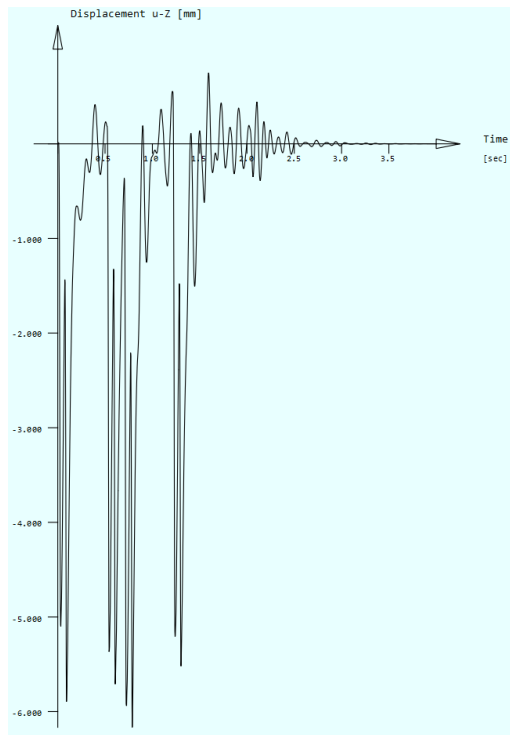
**ULS** The stresses of Design A, without the effect of singularity, are all within the limits. Therefore, this design meets the requirements; Design A is safe. The stresses of Design B, without the effect of singularity, aren't within the limits. Therefore, this design does not meet the requirements; Design B is unsafe. The limited amount of supports in Design B can cause these high stresses in combination with the reduced thickness of the flanges.

**Dynamic** Both designs meet the requirements for deflection but exceed the acceleration limit. Therefore, both designs are unsafe. The large accelerations can be caused by the effect of singularity, the way of modelling the moving load and/or the effect of resonance.

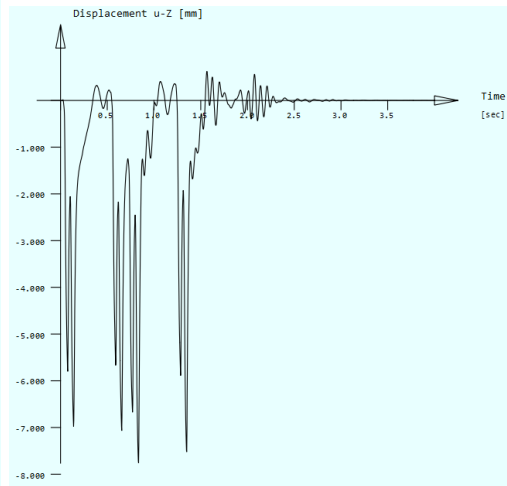
The main results of the conceptual designs are presented in Table 10.5.

| Designs         | SLS    | ULS    | Dynamic |
|-----------------|--------|--------|---------|
| Sandwich design | Safe   | Safe   | Unsafe  |
| Throughh design | Unsafe | Unsafe | Unsafe  |

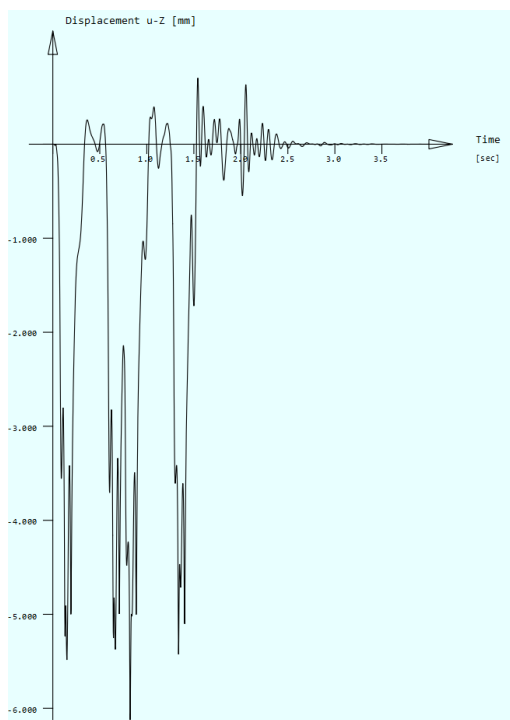
Table 10.5: Conclusive results of both conceptual designs



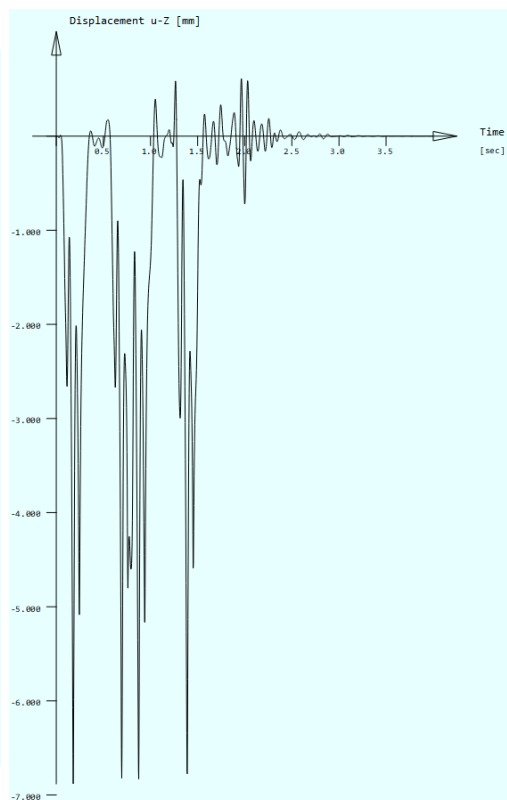
(a) Node 3001



(b) Node 3003



(c) Node 3005



(d) Node 3007

Figure 10.12: Deflection of Design B at different nodes



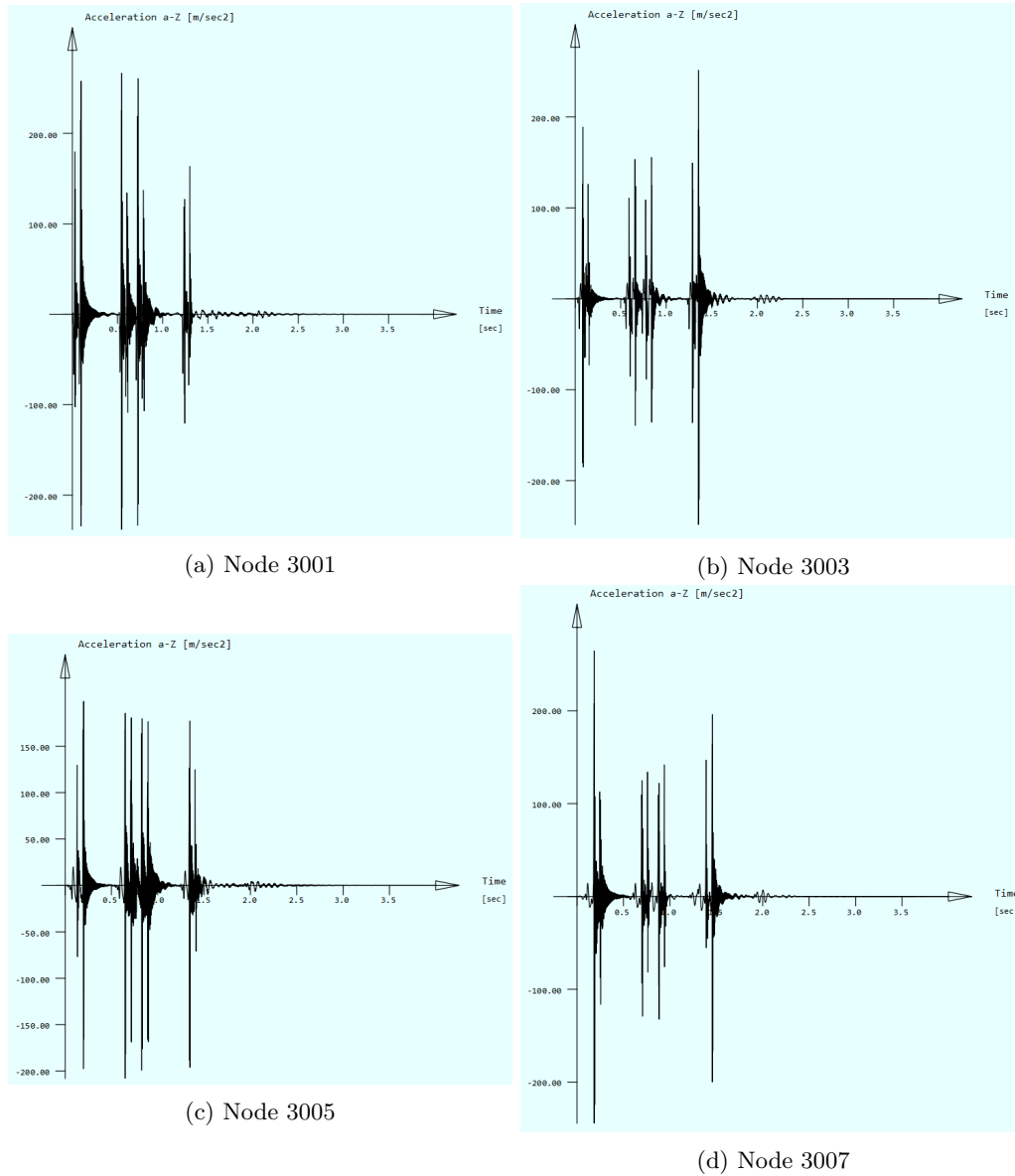


Figure 10.13: Acceleration of Design B at different nodes

# CHAPTER 11

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## Improvement of conceptual designs

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The results in previous chapter show that both designs are unsafe. A few possible measures are given to improve both designs, such as increasing the height of the plate section of design B, applying more supports and increase the thickness of the web and flanges. In this chapter a list of possible measures to improve both designs is given. Firstly, the most important criteria for a new design are given whereafter the possible measures are discussed.

### 11.1 Criteria

This list provides an overview of some of the criteria for a final design. These criteria are based on guidelines in section 3.3 and the results of both conceptual designs discussed in previous chapter.

- Maximum deflection is limited
- Maximum stresses are limited
- Maximum oscillation and resonance is limited
- Maximum acceleration is limited
- The design must be economically advantageous
- The design must be manufacturable
- Practicability must be taken into account
- Transverse rotation must be limited
- Deflection at the abutments must be limited

### 11.2 Measures

This list provides an overview of possible measures that can help to meet the criteria mentioned above. Each measure is discussed in the following subsections.

- Reduce the span
- Apply more height
- Adding more webs
- Use of carbon fibres
- Apply more supports
- Increase the mass
- Reduce the loadings
- Limit the laminate thickness

- Generative design
- Apply pre-camber
- Clamped supports at the abutments

The effect of the measures on each criteria is given by Table 11.1. A + sign means that the measure has a positive effect on the criteria. A - sign means that the measure has a negative effect on the criteria. When a measure has no significant or unknown effect on the criteria, a 0 is given. The effect of each measure on a bridge design are analysed using eqs. (11.1) to (11.5).

$$w_{max,lon} = \frac{q * L^4}{185 * EI} \quad (11.1)$$

$$w_{max,tran} = \frac{5 * q * L^4}{384 * EI} \quad (11.2)$$

$$\sigma = \frac{M}{W} = \frac{M}{\frac{I}{0.5 * h}} \quad (11.3)$$

$$f = \frac{\pi^2}{2\pi} \sqrt{\frac{EI * g}{q * L^4}} \quad (11.4)$$

$$a = \frac{4 * L * \cos(\frac{\pi^2 \sqrt{E} \sqrt{I} * t}{L^2 * \sqrt{M}}) * Q * \sin(\frac{\pi * x}{L})}{\pi * M} \quad (11.5)$$

### 11.2.1 Reduce the span

The first measure is to reduce the span. The deflection in both longitudinal and transverse direction decreases when reducing the span, according to Equation 11.1 and Equation 11.2. The same results occur for the stresses. Reducing the span results in lower maximum bending moment, which results in lower maximum stresses, according to Equation 11.3. The effect of resonance occurs when the load frequency coincides with the natural frequency of the bridge. Reducing the span leads to a lower natural frequency, according to Equation 11.4. For both designs, the natural frequencies and load frequencies coincide. However, the dynamic load model comes with a lot of load frequencies. Therefore, lowering the natural frequencies might cause resonance with other load frequencies. Therefore, this measure has unknown effect on the resonance. Based on Equation 11.5 [Xia and Zhang, 2005] reducing the span has a positive effect on the maximum acceleration.

Smaller decks are easier to manufacture and to transport due to the smaller sizes. Thereby, a smaller deck is cheaper.

### 11.2.2 Apply more height

Applying more height in the design results in a higher moment of inertia. Based on eqs. (11.1) to (11.5) an increase of the moment of inertia has a positive effect on the maximum deflection, vibrations and maximum stresses, and no effect on the maximum acceleration. Thereby, applying extra height makes transportation more complex and a larger mould is needed for manufacturing.

### 11.2.3 More webs

Adding extra webs has the same effect as applying more height; the moment of inertia increases. Therefore, the maximum deflection and maximum stresses become less and the maximum acceleration increases. The manufacturing of extra webs takes more time as more webs need to be filled.

#### 11.2.4 Use of carbon fibres

Carbon fibres have a higher strength and stiffness compared to glass fibres, see Table 2.1. According to eqs. (11.1), (11.2) and (11.5) higher stiffness has a positive effect on the maximum deflection, but a negative effect on the maximum acceleration. Higher strength automatically leads to higher maximum stresses. However, carbon fibres are more expensive compared to glass fibres. Therefore, replacing glass fibres by carbon fibres isn't economically advantageous.

#### 11.2.5 More supports

One of the biggest problems in the through bridge design was the transverse deflection. One of the reasons for this behaviour is the amount of supports. By placing only two supports at each side, close to the edge, the bridge becomes a plate structure. A plate structure deflects in 2 directions leading to larger maximum deflections. By adding extra supports the structure describes a more 2D behaviour, resulting in smaller deflections. Furthermore, the transverse deflections at the abutments are limited. However, adding extra supports becomes more expensive and leads to extra detailing.

#### 11.2.6 Increase the mass

One of the biggest problems in railway bridges is the acceleration of the deck. According to Equation 11.5, increasing the mass leads to a decrease of the maximum acceleration. However, based on eqs. (11.1) and (11.2) an increase of the mass leads to an increase of the maximum deflection.

#### 11.2.7 Limit laminate thickness

Limiting the laminate thickness is one of the solutions to ease the manufacturing of FRP bridge structures. The filling time cannot exceed the curing time, as mentioned in section 2.6. Assuming that the moment of inertia remains unchanged by adding extra webs, limiting the laminate thickness has only effect on the manufacturing. However, without adding extra webs the moment of inertia decreases, leading to higher maximum deflections and stresses and lower maximum acceleration.

#### 11.2.8 Generative design

Generative design can be used to minimise the material use. Using an automatic optimisation tool material can be added where needed, but especially omitted where not needed. In this way extra height is applied at high bending moment areas. For high shear force areas the effective area is increased. In this way, generative design leads to economically advantageous design. The downside is the complex shape of the design which is difficult to manufacture.

#### 11.2.9 Pre-camber

The maximum deflection consists of a self weight and variable load component. When applying precamber, the deflection due to self weight can be compensated. In this way the maximum deflection only consists of the variable load component. However, the manufacturing is more complex due to the cambered shaped.

#### 11.2.10 Clamped supports

The maximum deflection of a double-span simply supported beam loaded by a distributed load is given by Figure 11.1a. Figure 11.1b shows the maximum deflection of a clamped double-span beam loaded by a distributed load. Based on these figures the maximum deflection with clamped supports is 2 times less than the maximum deflection of a simply supported beam. Furthermore,

due to the clamped supports the maximum velocity of the bridge is reduced. The direct effect on the maximum acceleration is hard to tell and must be investigated. The maximum stresses are reduced when applying clamped supports, as given by Figure 11.1. The maximum shear forces and bending moments are less for the clamped supports compared to the simply supported system. A clamped support comes with a more complex design resulting in a more complex manufacturing. The clamped supports have no impact on the transverse rotation. The deflection at the abutments automatically become zero.

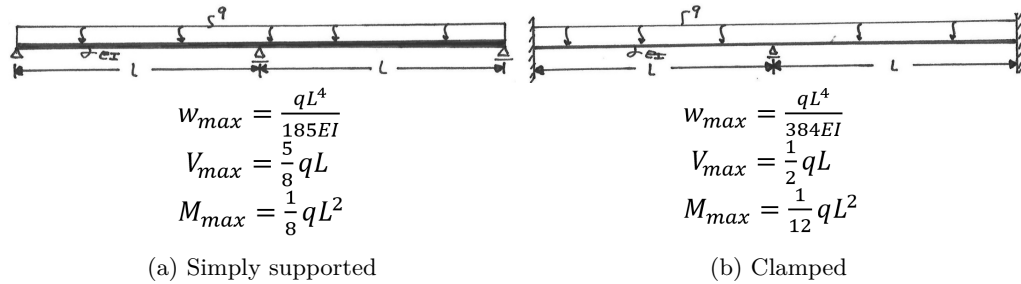


Figure 11.1: Difference between simply supported and clamped systems

|                                   | Maximum deflection | Maximum stresses | Maximum oscillation and resonance | Maximum acceleration | Economically advantageous design | Manufacturability | Practicability | Transverse rotation | Deflection at the abutments |
|-----------------------------------|--------------------|------------------|-----------------------------------|----------------------|----------------------------------|-------------------|----------------|---------------------|-----------------------------|
| Reduce the span                   | +                  | +                | +                                 | +                    | +                                | +                 | +              | +                   | +                           |
| Apply more height                 | +                  | +                | +                                 | 0                    | -                                | -                 | -              | +                   | +                           |
| Adding more webs                  | +                  | +                | +                                 | -                    | -                                | -                 | 0              | +                   | +                           |
| Use of carbon fibres              | +                  | +                | +                                 | -                    | -                                | 0                 | 0              | +                   | +                           |
| Apply more supports               | +                  | +                | +                                 | +                    | -                                | -                 | -              | +                   | +                           |
| Increase the mass                 | -                  | -                | +                                 | +                    | -                                | 0                 | -              | -                   | -                           |
| Reduce the loadings               | +                  | +                | +                                 | +                    | 0                                | 0                 | 0              | +                   | +                           |
| Limit the laminate thickness      | 0                  | -                | 0                                 | 0                    | +                                | +                 | 0              | 0                   | 0                           |
| Generative design                 | +                  | +                | +                                 | 0                    | +                                | -                 | 0              | +                   | +                           |
| Apply pre-camber                  | +                  | 0                | 0                                 | 0                    | 0                                | -                 | 0              | +                   | -                           |
| Clamped supports at the abutments | +                  | +                | +                                 | 0                    | -                                | -                 | -              | 0                   | +                           |

Table 11.1: The effect of the measures on the various criteria

# Part IV

## Final design



This chapter describes the final design. This final design is based on both conceptual designs and the measure-criteria table in Part III. At first, the used measures from chapter 11 are listed and discussed. Secondly, the final design is presented whereafter the used material properties are given. Finally, the load cases for this new design are given.

### 12.1 Used measures

Both designs in Part III have a number of drawbacks. The results in chapter 10 show that design B doesn't meet the static requirements and both designs do not meet the dynamic requirements. Furthermore, the web thickness in design A are difficult to manufacture. A combination of both designs can result in a functional design, as long as the correct measures are taken against the drawbacks of both designs. Some of these measures are listed in chapter 11. The following measures are included in the final design:

- Reduce the span
- Apply more height
- Adding more webs
- Apply more supports
- Limit the web thickness
- Apply pre-camber

#### 12.1.1 Reduce the span

According to Table 11.1 reducing the span has no negative effects. However, the total span of the bridge is set. By using a through design, like Design B, the main span of certain elements can be reduced. The forces of the train are transferred via the plate section, in transverse direction, to the beam sections, whereafter the forces are transferred to the support. In this way, the main span of the plate section is reduced to the width of the bridge.

#### 12.1.2 Apply more height

The extra height results in larger moment of inertia and cross section area. The height of the plate section is increased, extra elements are applied at the supports to increase the height and the height of the beam sections are maximized without exceeding the limits.



### **12.1.3 Adding more webs**

The extra webs are added in transverse direction. A total of 109 transverse webs is applied with a c.t.c. distance varying between 125 mm and 250 mm.

### **12.1.4 More supports**

The results in chapter 10 clearly show that more supports lead to less deflection. Therefore, a total of 15 elastomeric bearings is used.

### **12.1.5 Limit the laminate thickness**

It is decided, in consultation with a manufacturer of FRP bridges, based on his experience, to limit the laminate thickness to 30 mm. In this way the filling time of the mould won't exceed the curing time of the resin.

### **12.1.6 Pre-camber**

The deflection caused by the dead load (load case 1) will be compensated by pre-cambering the bridge.

## 12.2 Design

The combination of both conceptual designs and the used measures in section 12.1 result in this final design. Some views of the final design are shown in Figure 12.2. A 3D view of this new design is shown in Figure 12.3. All drawings can be found in Appendix C. This final design consists of two beam sections and one plate section in between, like Design B. The top and bottom flange of the plate section are connected by 19 longitudinal and 109 transverse webs with a thickness of 30 mm. The top and bottom flange of the beam sections are connected by 41 transverse webs at each beam. The foam between the webs act as lost form work. The dimensions of the bridge are given in Table 12.1. Drawings of the final design can be found in chapter E.

| Dimension               | Sign     | Final design |    |
|-------------------------|----------|--------------|----|
| Length                  | L        | 20000        | mm |
| Total width             | W        | 6000         | mm |
| Total height            | H        | 2000         | mm |
| Width plate section     | $W_p$    | 5000         | mm |
| Height plate section    | $H_p$    | 600          | mm |
| Width beam section      | $W_b$    | 500          | mm |
| Height beam section     | $H_b$    | 1950         | mm |
| C.t.c. long. webs plate | $a_{lp}$ | 250          | mm |
| C.t.c. tran. webs plate | $a_{tp}$ | 250          | mm |
| C.t.c. tran. webs beam  | $a_{tb}$ | 500          | mm |

Table 12.1: Dimensions final design

**Height** The shape of the bridge is mostly based on Design B. The span of the bridge is partly reduced by dividing the design in two beam elements and one plate element; the forces in the plate element are transferred to the beam element, whereafter the forces are transferred to the supports. In this way the span of the plate element is reduced to the width of the bridge. The height of the plate element was one major drawback of Design B. The limited height in the through design resulted in large deflections of the plate section. By doubling the height of this plate section the deflection will be reduced. The height of the beam elements is limited to 2 meter, including the maintenance path. At the supports and at mid-span the height of the plate is increased by adding extra material underneath the bridge, see Figure 12.2b.

**Webs** The main direction of the webs in the plate section is in transverse direction. In this direction extra webs are added at locations with high bending moments to increase the effective area and moment of inertia.

**Fibre lay-up** The fibre lay-up is the same as Design B (section 5.2 and Figure 5.5).

**Bearings** A total of fifteen elastomeric bearings is used in the final design, divided over three supports. The positioning and degrees of freedom of the bearings is given in Figure 12.1.

**Web thickness** The thickness of all webs is limited to 30 mm.

**Pre-camber** The bridge is slightly cambered to compensate for the dead load deflection. The cambering isn't visible in the figures, but is used to determine the unity checks for the static deflection.

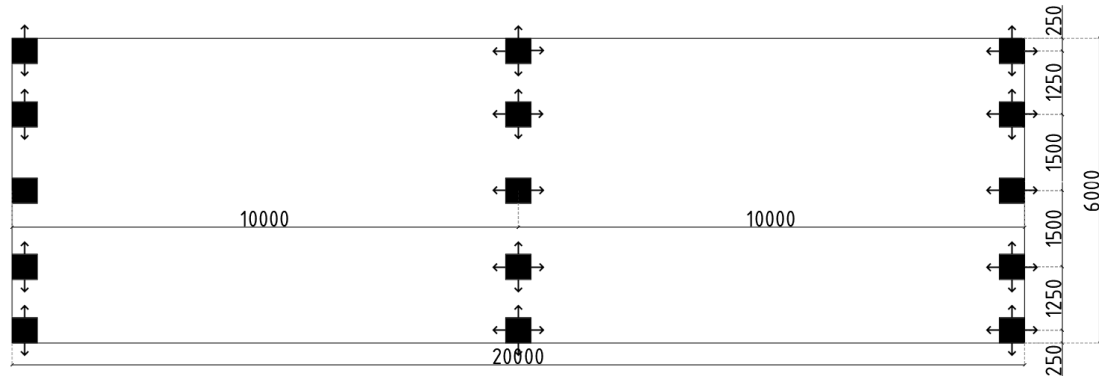


Figure 12.1: Positioning and degrees of freedom for the bearing in the final design

**PVR** A train can freely pass as the PVR isn't blocked by any material, shown in Figure 12.2a. The dimensions of the maintenance path are in accordance with the requirements given by section 3.3. The wiring of the bridge is tucked away in cable trays. These cable trays, made of concrete, are placed against the beam sections. The ballast bed is sloping to ensure the accessibility of the cable trays.

**Ballast** The ballast bed consists of sub grade, NS90 sleepers and two UIC54 rails. The sub grade has a height of 340 mm. The NS90 sleepers are c.t.c. 0.90 meter. The dimensions of the sleepers are LxWxH 2520x300x233 mm (see Figure 3.6).

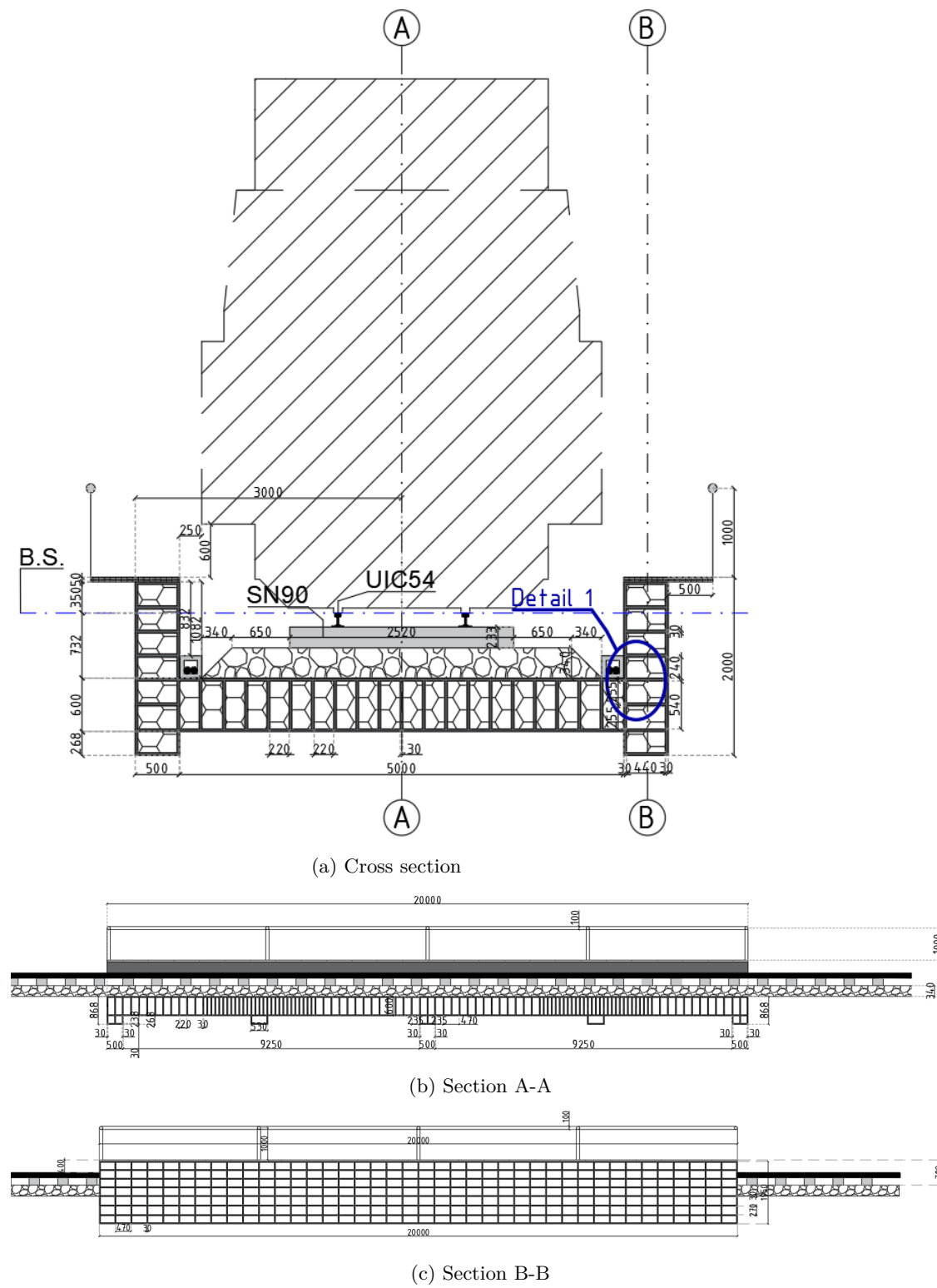


Figure 12.2: Views of final design

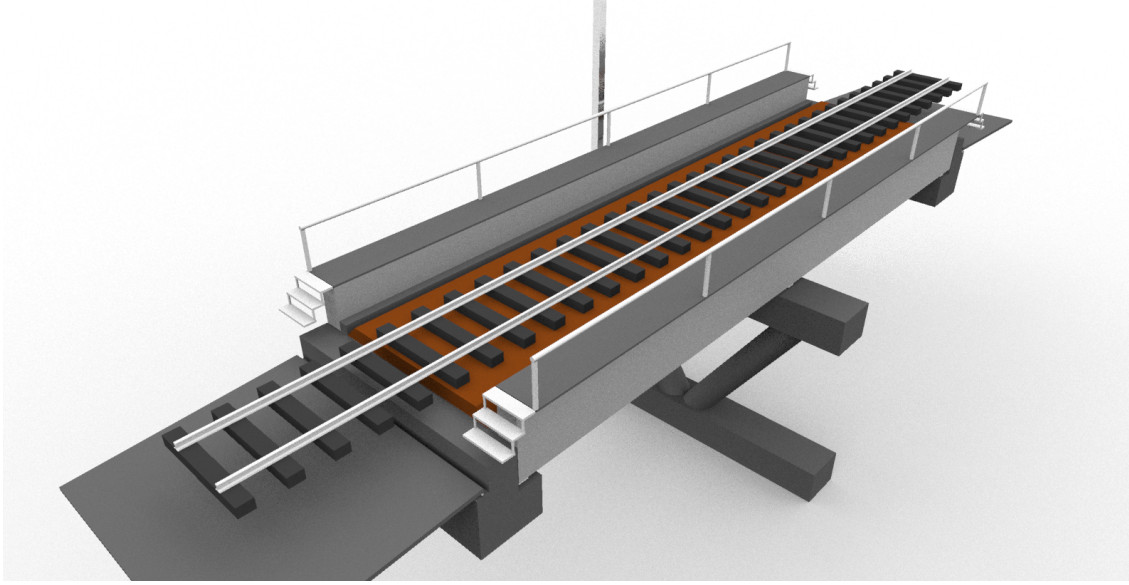


Figure 12.3: 3D view of final design

### 12.3 Material

This final design uses the material properties and conversion factors determined in chapter 6. The thickness of the webs and flanges for Design B and the final design are the same. Therefore, the material properties of Design B are used in the final design. The material properties of the final design are given in Table 12.2.

|     |         | Thickness<br>mm | $E_1$<br>MPa | $E_2$<br>MPa | $G_{12}$<br>MPa | $\nu_{12}$<br>- | $\nu_{21}$<br>- | $\sigma_1$<br>MPa | $\sigma_2$<br>MPa | $\tau_{12}$<br>MPa |
|-----|---------|-----------------|--------------|--------------|-----------------|-----------------|-----------------|-------------------|-------------------|--------------------|
| SLS | Flanges | 30              | 22705        | 12600        | 4050            | 0.32            | 0.18            | 272               | 151               | 65                 |
|     | Webs    | 30              | 17291        | 15317        | 4833            | 0.31            | 0.28            | 207               | 184               | 77                 |
| ULS | Flanges | 30              | 7724         | 3526         | 1178            | 0.32            | 0.18            | 93                | 42                | 19                 |
|     | Webs    | 30              | 5218         | 4454         | 1485            | 0.31            | 0.28            | 63                | 53                | 24                 |

Table 12.2: Material properties of final design

### 12.4 Loadings

This final design is loaded by the same load cases as Design A and Design B in Part III:

- Load case 1    Dead load
- Load case 2    Load Model 71 + start force
- Load case 3    Dead load + Load Model 71 + start force
- Dynamic load case    Real train model for dynamic analysis

The new design has no impact on the external loads. The load values in Load case 2 and 3 remain the same and can be found in chapter 7. The load values in Load case 1 depend on the design. The self-weight of the deck is calculated by Sofistik using the density of the material. The self-weight of the ballast depends on the dimensions of the ballast and the density of the ballast. Both the dimension and the density are unchanged, resulting in the same load values for the self weight of the ballast. The self weight of the foam inside the bridge is divided into three parts: (1) foam in

the plate section, (2) foam in the beam sections and (3) foam in the support sections. The self weight of the foam in the beam section is larger due to stacking of the foams resulting in a higher area load at the bottom of the beam sections. The load values are given in Table 12.3. The load values in load case 4 are given in subsection 12.4.1.

| Load         |          | SLS                    | ULS   |
|--------------|----------|------------------------|-------|
| Deck         | $kN/m^2$ | Calculated by Sofistik |       |
| Ballast      | $kN/m^2$ | 16.06                  | 21.68 |
| Foam plate   | $kN/m^2$ | 0.44                   | 0.59  |
| Foam beam    | $kN/m^2$ | 1.24                   | 1.67  |
| Foam support | $kN/m^2$ | 0.54                   | 0.73  |

Table 12.3: Load value for LC1 final design

### 12.4.1 Dynamic load case

The dynamic load case is presented in section 7.4. However, for the dynamic load case the Rayleigh coefficients are needed which depend on the natural frequency of the bridge.

**Natural frequencies final design** The natural frequencies of the final design are given in Table 12.4.

| Mode | Natural frequency [Hz] |
|------|------------------------|
| 1    | 13.86                  |
| 2    | 16.44                  |
| 3    | 17.02                  |
| 4    | 19.55                  |
| 5    | 20.50                  |

Table 12.4: Natural frequencies of final design

**Rayleigh coefficients** The Rayleigh coefficients are calculated using Equation 4.16. The coefficients for the given natural frequencies are listed in Table 12.5.

| Design       | $\alpha$ | $\beta$  |
|--------------|----------|----------|
| Final design | 5.1957   | 0.000463 |

Table 12.5: Rayleigh coefficients final design



# CHAPTER 13

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## Numerical model

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This chapter describes the numerical model used to analyse the final design. First, the input used to create the the model is discussed whereafter the load models are given.

### 13.1 Input

The input of the numerical model consists of the design and the corresponding material properties.

#### 13.1.1 Material input

The material input for the final design is given in section 12.3. Sofistik needs the following material properties input:

- Self weight [ $kN/m^3$ ]
- Elastic modulus [ $N/mm^2$ ]
- Poisson ratio [-]
- Shear modulus [ $N/mm^2$ ]
- Elastic modulus 90 [ $N/mm^2$ ]
- Poisson ratio 90 [-]

#### 13.1.2 Design input

The final design is modelled using Sofiplus. Therefore, the model is made with 2D area elements with a given thickness.

The 3D sofipplus model of the final design is shown in Figure 13.1. The different components are shown in Figure 13.2. The supports are modelled as point supports with degrees of freedom and positions as shown in Figure 12.1.

### 13.2 Loadings

A total of four different load cases are applied. These load cases are discussed in section 12.4. This paragraph describes how these load cases are modelled. Only the SLS load models are discussed. For the ULS models, the SLS models are multiplied with the corresponding load safety factor.



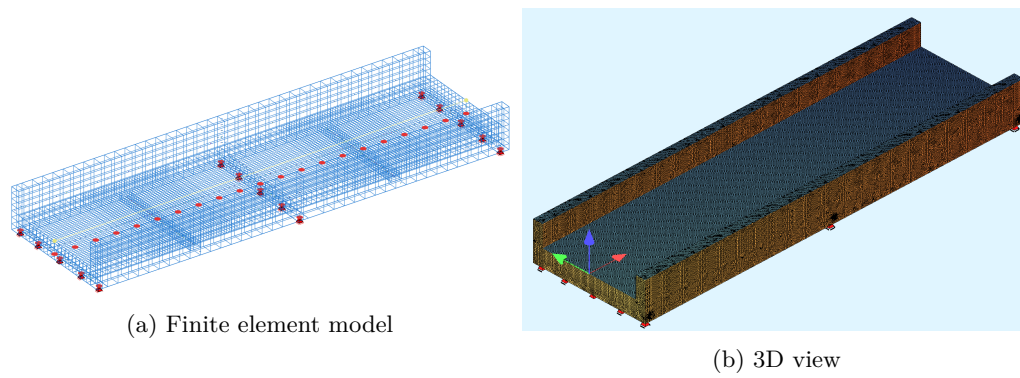


Figure 13.1: Finite element models of sandwich bridge

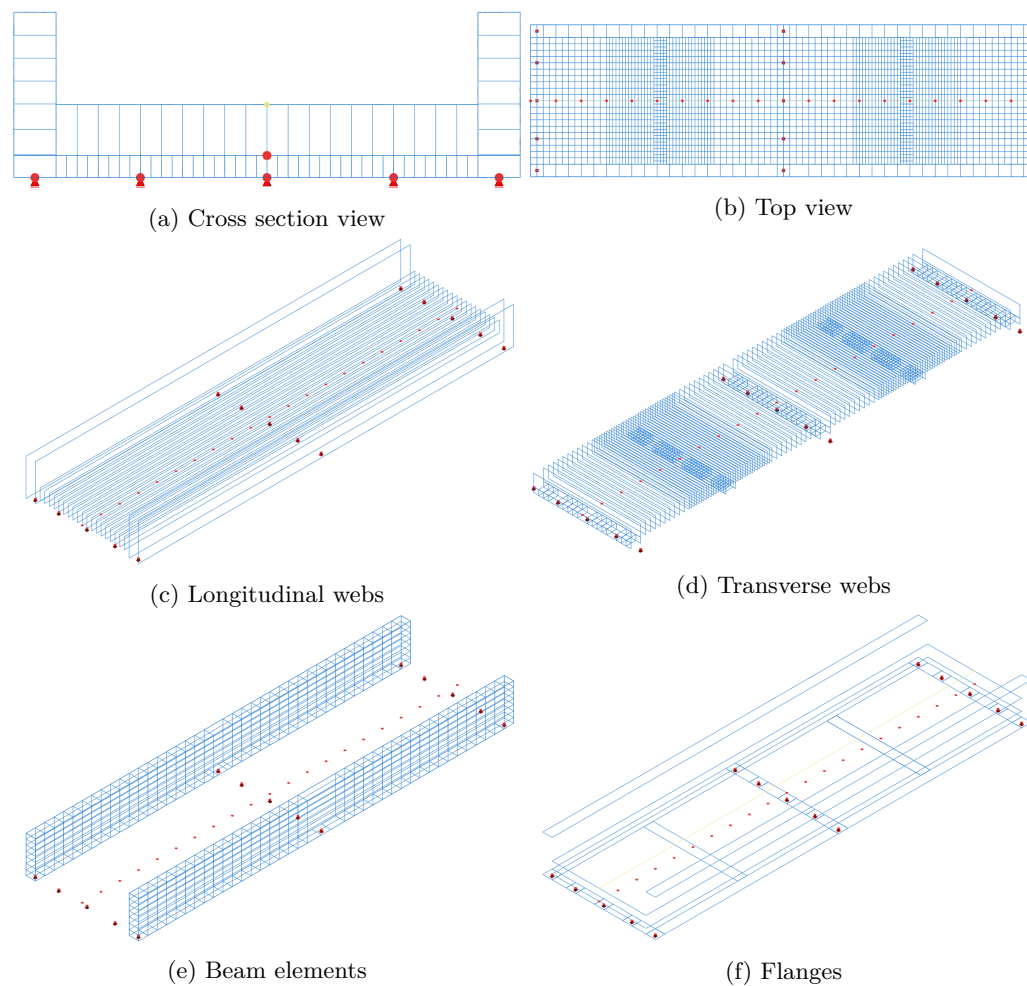


Figure 13.2: Elements in numerical model of final design

### 13.2.1 Load case 1

Load case 1 for the numerical model of the new design is shown in Figure 13.3. The self-weight of the deck is calculated by the software. The ballast load is modelled as an area load of  $16.06kN/m^2$  on the top layer of the plate section. The foam is loaded as an area load on the bottom layer of the plate section of  $0.44kN/m^2$ , on the bottom layer of the beam section of  $1.24kN/m^2$  and on the bottom layer of the support sections of  $0.54kN/m^2$ . The safety factor for load case 1 is  $\gamma_G = 1.35$ .

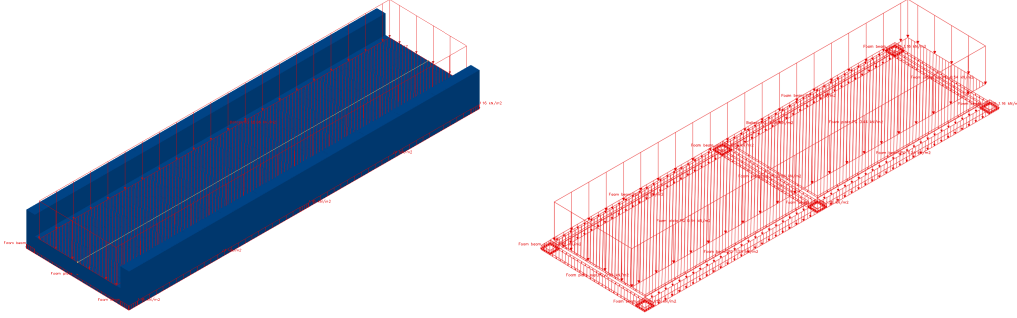


Figure 13.3: Numerical model of load case 1 on final design

### 13.2.2 Load case 2

Load case 2 for the numerical model of the new design is shown in Figure 13.4. This load case consists of LM71 and the start force. LM71 is modelled as an area load on the top layer of the plate section. The area loads are  $332.85kN/m^2$  for the point loads and  $50.05kN/m^2$  for the distributed loads. The start force is modelled as an horizontal area load of  $14.84kN/m^2$  on the top layer of the plate section. The load safety factor for LC 2 is  $\gamma_Q = 1.45$ .

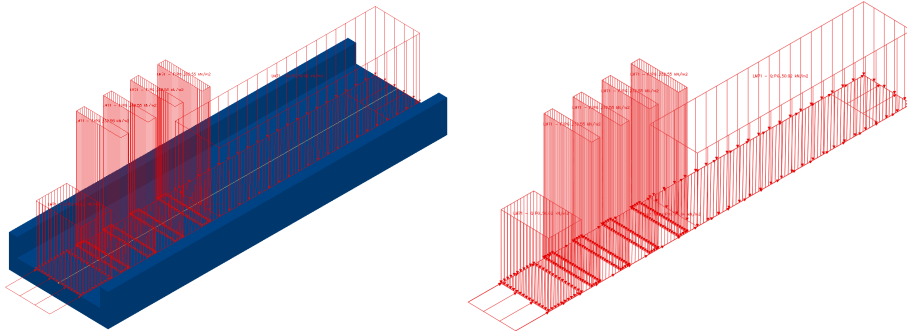


Figure 13.4: Numerical model of load case 2 on final design

### 13.2.3 Load case 3

Load case 3 combines load case 1 and 2. Sofistik provides a module to combine different loads into a new Combined Load case. For load case 3, load case 1 and 2 are combined into a new load case. The module is shown in Figure 8.10.

### 13.2.4 Load case 4

The dynamic load case is modelled using Teddy. The input is the same for the conceptual designs and shown in Figure 8.11. The natural frequencies of the final design, used to determine the Rayleigh coefficients, are given in Table 12.4.



This chapter describes the results for the load cases given in section 12.4 on the final design. At first, the maximum deflection of this design on the static load cases is analysed. Secondly, the maximum stresses caused by the static load cases are analysed, followed by the results of the dynamic analysis. Finally, a conclusion is drawn.

### 14.1 SLS results

The maximum deflection of the final design is given in Table 14.1. The deformed structures are shown in Appendix B.

| Load case   | Maximum deflection | Limit |    | Unity check |
|-------------|--------------------|-------|----|-------------|
| Load case 1 | 3.19               | 12.5  | mm | 0.26        |
| Load case 2 | 9.21               | 12.5  | mm | 0.74        |
| Load case 3 | 12.4               | 12.5  | mm | 0.99        |

Table 14.1: Maximum deflection of the final design for all load cases

According to Table 14.1 the final design meets the requirements for maximum deflection. Furthermore, by applying pre-camber the deflection caused by LC 1 can be neglected, resulting in the unity checks given in Table 14.2.

| Load case   | Maximum deflection | Limit |    | Unity check |
|-------------|--------------------|-------|----|-------------|
| Load case 2 | 9.21               | 12.5  | mm | 0.74        |
| Load case 3 | 9.21               | 12.5  | mm | 0.74        |

Table 14.2: Maximum deflection of final design with pre-camber

The deflected shape of the final design for LC 3 is given in Figure 14.1. According to this figure, the deflection is in two directions. The design acts as two plates supported at each corner, with a maximum at the centre of the plate, shown by the circular iso-lines. The impact of the point loads of LC2 is clearly visible, with a larger deflection at one of the spans. The beam sections of the final design are less rotated, compared to Design B. The deflection at the edge of the final

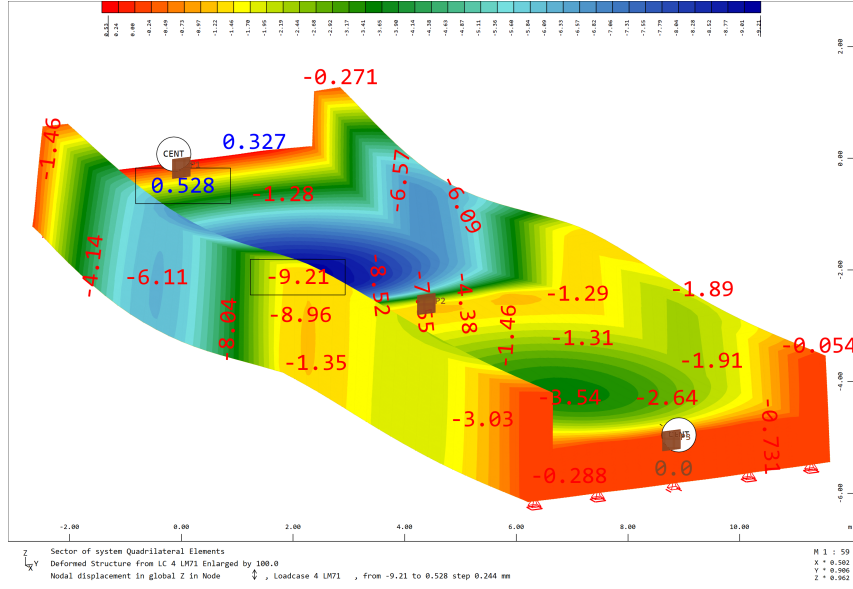


Figure 14.1: SLS results of final design for LC3

design (8.76 mm) is larger than the deflection of edge of Design A (6.14 mm). This can be caused by the limited flange thickness of the final design, resulting in lower moment of inertia.

## 14.2 ULS results

The unity checks in ULS situation are based on the maximum stresses. The largest compressive/tensile stresses are expected in the top and bottom flanges. The largest shear stresses are expected at the webs. The limits are given by the material properties presented in Table 12.2. For ULS calculations LC 3 is normative, due to the highest forces. Therefore, only the ULS results of LC3 are given. The results including unity checks are given in Table 14.3.

|               | Stresses | Results   | Limit          | Unity check |
|---------------|----------|-----------|----------------|-------------|
| $\sigma_{xx}$ | Top      | -31.2 MPa | $\pm 92.6$ MPa | 0.34        |
|               | Bottom   | -39.6 MPa | $\pm 92.6$ MPa | 0.43        |
| $\sigma_{yy}$ | Top      | -32.3 Mpa | $\pm 42.4$ MPa | 0.76        |
|               | Bottom   | -33.2 MPa | $\pm 42.4$ MPa | 0.78        |
| $\tau_{xy}$   |          | 10.6 MPa  | $\pm 23.9$ MPa | 0.44        |

Table 14.3: ULS results of final design

According to Table 14.3 all unity checks are lower than 1. The ULS output is presented and discussed in chapter G.

### 14.2.1 Longitudinal webs

Due to the transverse webs, the longitudinal fibres aren't continuous. This means the stresses between the longitudinal and transverse webs can only be transferred by the resin. Therefore, the maximum stresses in the longitudinal webs may not exceed the tensile strength of the resin. The tensile strength of polyester is based on Table 2.1 and Equation 6.2:

$$\sigma_d = \eta_c \frac{\sigma_c}{\gamma_M} = 0.45 * \frac{55}{1.82} = 14 \text{ MPa}$$

The stresses in longitudinal webs are given in Figure 14.2. The maximum tensile strength is 12.8 MPa, resulting in a unity check of 0.91. Therefore, the connection between transverse and longitudinal webs do not fail on tension. Due to the redistribution of compressive stresses between the resin and fibres, and the possible buckling of the webs under compression, further investigation into the compressive failure of the connection between the transverse and longitudinal webs is needed.

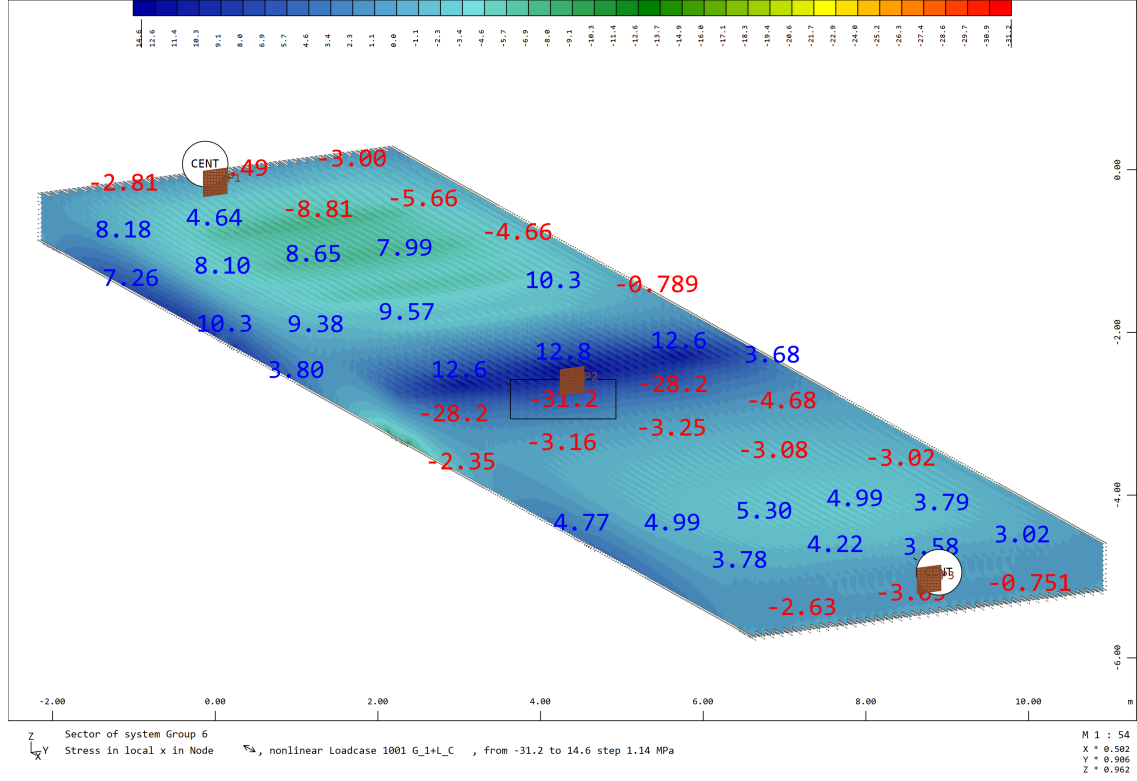


Figure 14.2: The stresses in the longitudinal webs for LC 3

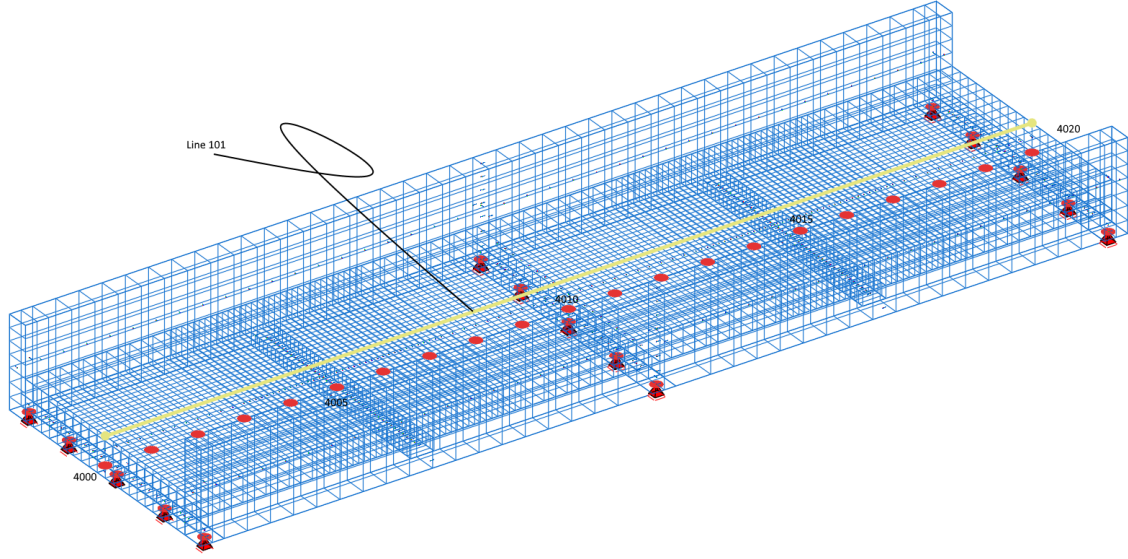


Figure 14.3: Position of the nodes in the final design

|                          | Results | Limit | Unity check |
|--------------------------|---------|-------|-------------|
| Deflection [mm]          | 1.45    | 12.5  | 0.12        |
| Acceleration [ $m/s^2$ ] | 19.48   | 3.5   | 5.57        |

Table 14.4: Maximum results of dynamic analysis for final design

### 14.3 Dynamic results

This section shows the results of the dynamic analysis presented in chapter 9 for the final design. The deflection and acceleration are calculated for specific nodes on the deck. These nodes are shown in Figure 14.3. Only the bottom nodes are analysed, based on the advice in section 8.5. The maximum deflection and acceleration including unity checks are given in Table 14.4. The limits are given by the codes.

According to Table 14.4 the maximum deflection is within the limits. The maximum acceleration results in a unity check of 5.57 and therefore exceeds the limits. The maximum deflection and acceleration at each node are given in Figure 14.4. The results for deflection and acceleration are discussed in the following paragraphs.

**Deflection** The deflection at node 4001, 4003, 4005 and 4007 are given in Figure 14.5.

According to Figure 14.4 the maximum deflection is at node 4004. This is in accordance with the theory as depicted in Figure 10.7. The impact of the point loads on the deflection of the deck is clearly visible in Figure 14.5. The time between the peaks in the results is in accordance with the time period between the point loads of LM 4, see Figure 10.9.

**Acceleration** The acceleration at node 4001, 4003, 4005 and 4007 are given in Figure 14.6.

According to Figure 14.6 the maximum acceleration occurs at the point loads, showing large peaks. One of the reasons for these high accelerations is the effect of singularity. The distance between the point load and the nodes might be insufficient. Another reason is the way of modelling the loads. The pulses of the load act within a very short time period of 0.005 sec, resulting in very high accelerations.

The highest maximum acceleration occurs at node 4008. This is in contrast to the theory given

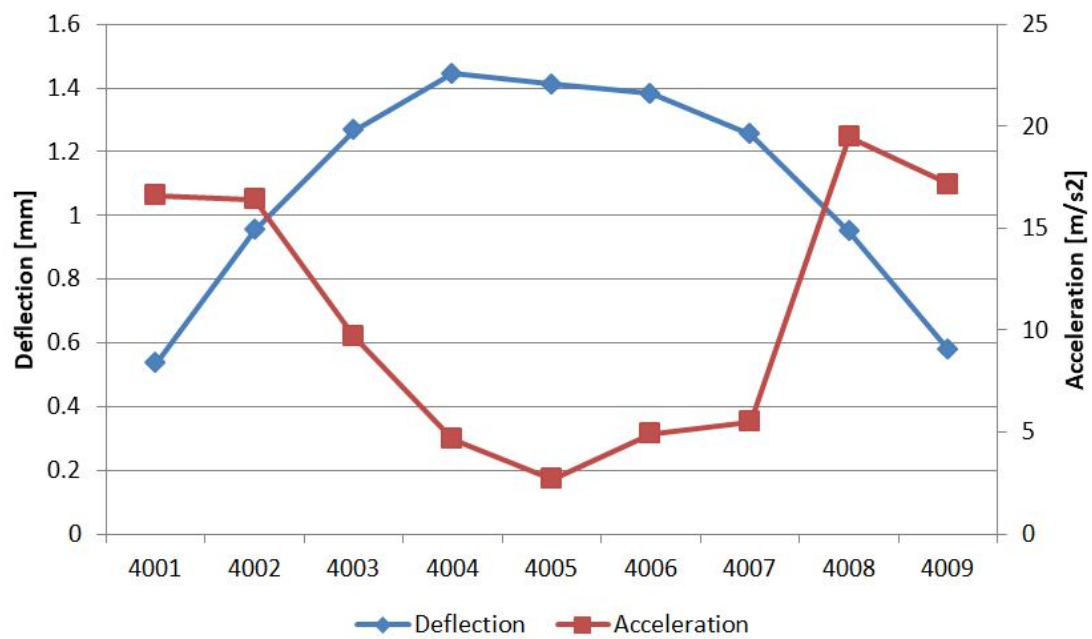
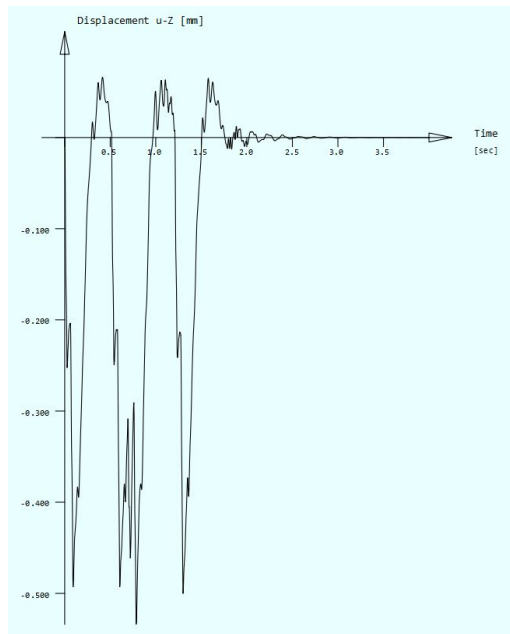


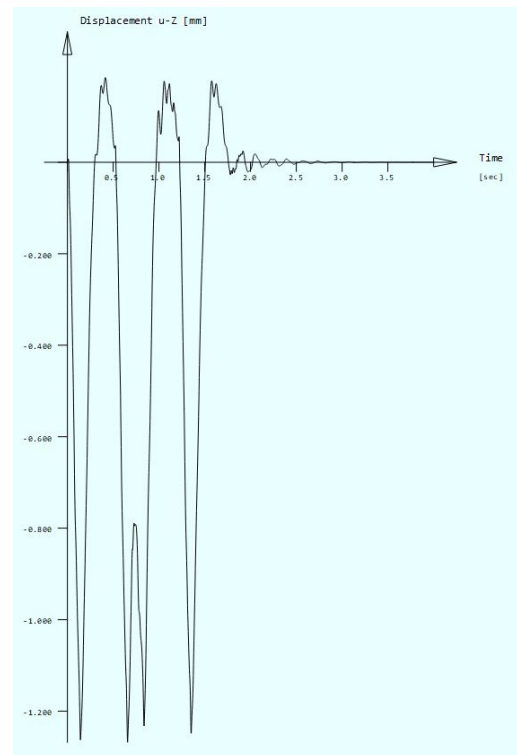
Figure 14.4: Maximum acceleration and deflection of final design

in subsection 10.3.1, where the highest maximum acceleration occurs at the first node. However, both conceptual designs do show a small peak at node X008, see Figure 10.6 and Figure 10.11. Resonance could be one of the reasons for these peaks; one of the load frequencies caused by the occurring loads might coincide with the natural frequency of the bridges.

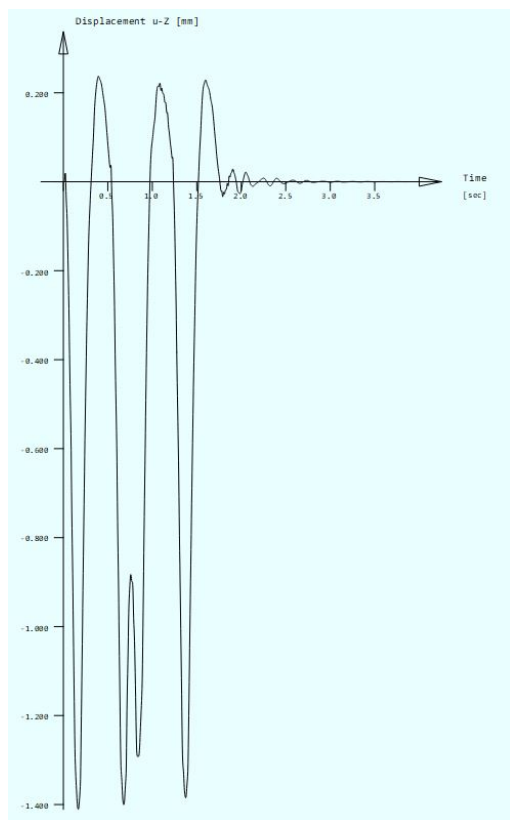




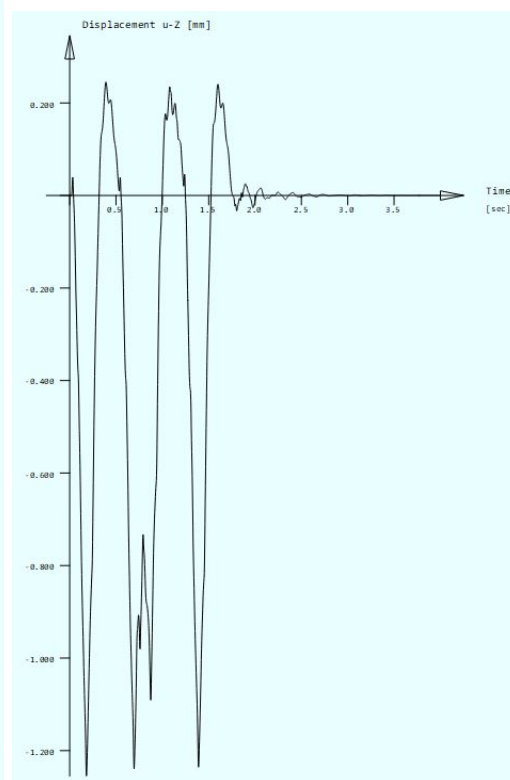
(a) Node 4001



(b) Node 4003

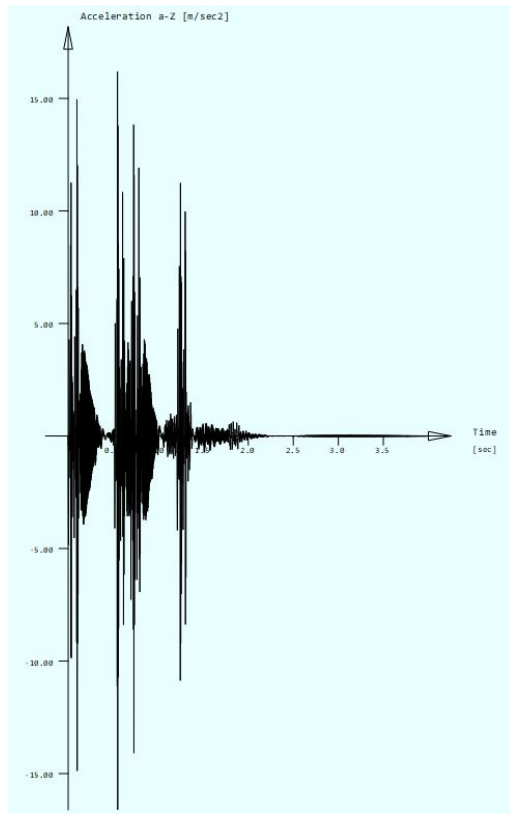


(c) Node 4005

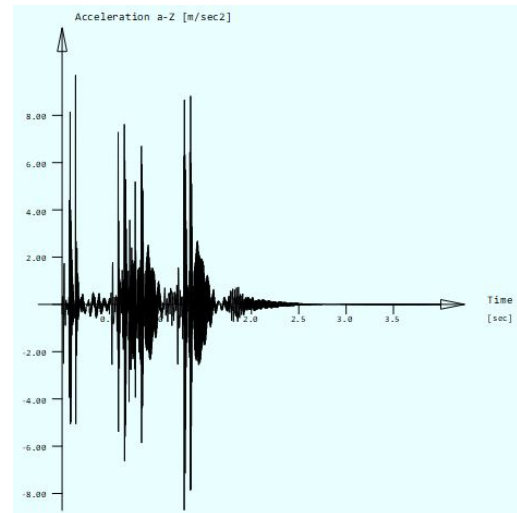


(d) Node 4007

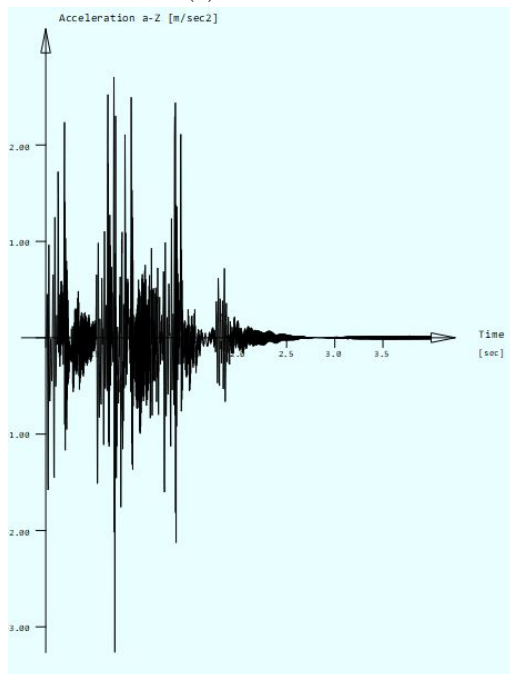
Figure 14.5: Deflection of final design at different nodes



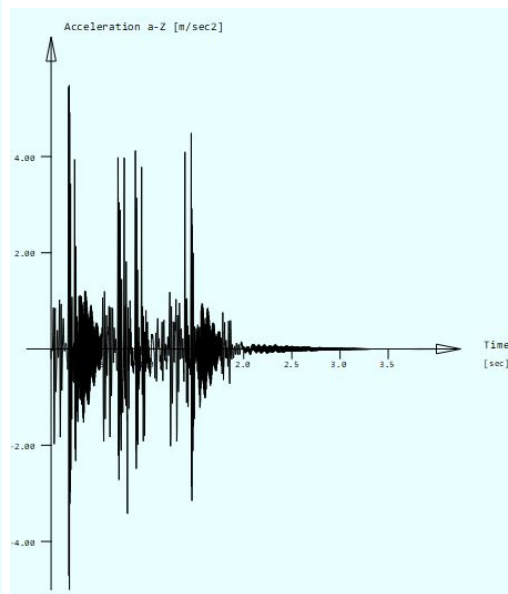
(a) Node 4001



(b) Node 4003



(c) Node 4005



(d) Node 4007

Figure 14.6: Acceleration of final design at different nodes

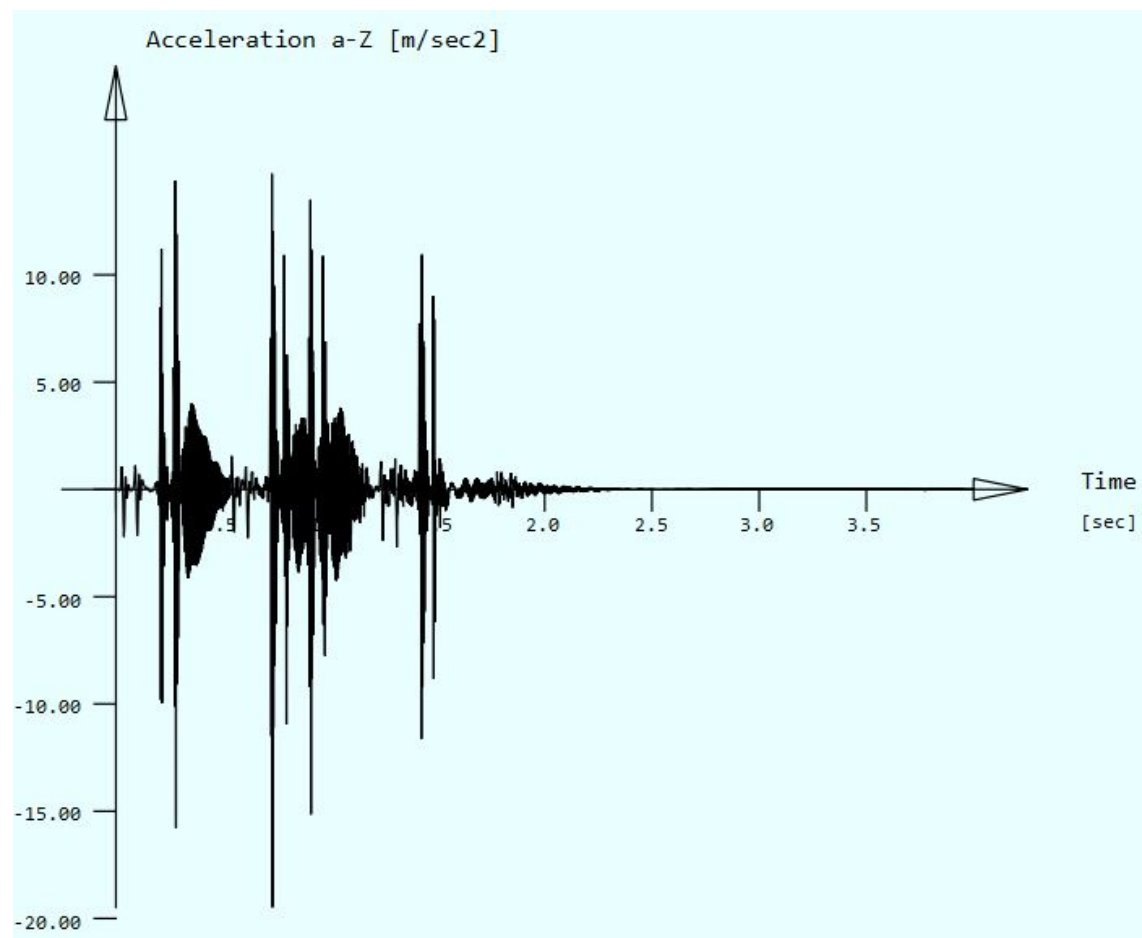
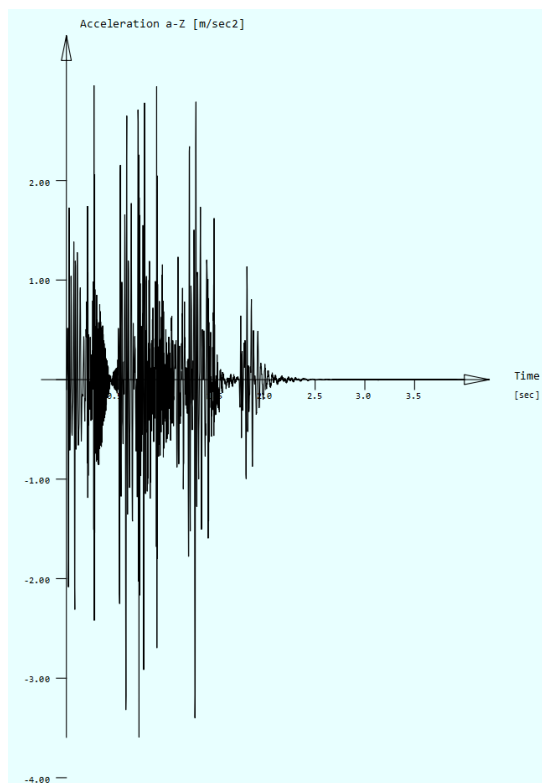
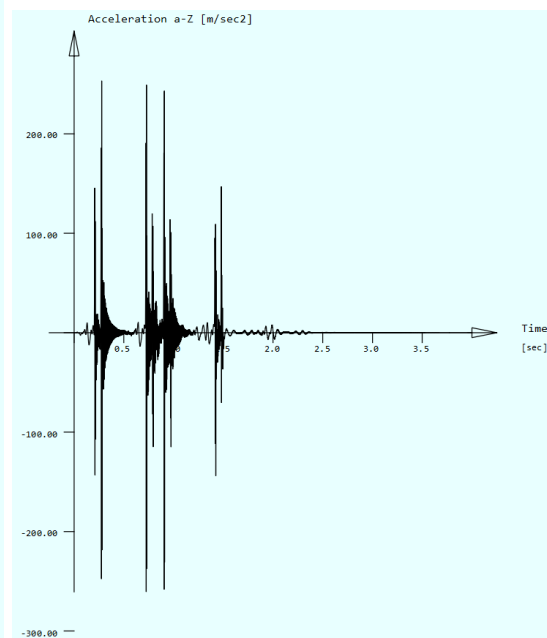


Figure 14.7: Node 4008



(a) Design A at node 6008



(b) Design B at node 3008

Figure 14.8: Acceleration of both conceptual designs at node X008

## 14.4 Conclusion

**SLS** The deflection of the final design is within the limits, according to Table 14.2, and therefore meets the requirements. Pre-camber of the final design has effect on maximum deflection. The unity check is reduced from 0.99 without pre-camber to 0.74 with pre-camber.

**ULS** All ULS unity checks are lower than 1 and therefore the final design meets the ULS requirements. A closer look at the connection between the longitudinal and transverse webs give positive results for tension. Further investigation for the compressive behaviour between the transverse and longitudinal webs is needed.

**Dynamic** The maximum acceleration of the design exceeds the limits, given by Table 14.4. The high maximum accelerations are located at the point loads. Singularity, resonance and the way of modelling the loads might explain these high accelerations.

The main results of the final design are presented in Table 14.5.

| Design       | SLS  | ULS  | Dynamic |
|--------------|------|------|---------|
| Final design | Safe | Safe | Unsafe  |

Table 14.5: Conclusive results of final design

Part V

Reflection



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### Discussion on the main results

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This chapter gives an overview of the main results, where after the results are discussed. This chapter is divided into five parts; (1) common bridge design, (2) material properties, (3) results of conceptual designs, (4) final design and (5) results of final design.

#### 15.1 Common bridge designs

**Results** In chapter 3 two common railway bridge deck designs are given: (1) sandwich deck and (2) through bridge. Based on this literature study two conceptual designs are presented: (1) a Sandwich bridge deck design and (2) a Through bridge deck design.

**Discussion** The sandwich structure of FRP materials is visible in both designs; an upper and lower flange, connected by webs. The space between the laminates is filled with foam, served as permanent form-work. The dimensions of both designs meet the requirements given by the codes. Both designs are based on literature study and internal documents from BAM Infraconsult. However, this doesn't mean that alternatives aren't possible. Due to the free formability of the material, many design alternatives are possible. The thickness of laminates and number of webs are some of the parameters to adjust. This has consequences for the stress distribution in the designs.

Both designs are purely structural, without taking into consideration the architecture of bridge design. A more architectural bridge design, often accompanied by natural forms, makes the design more complex to analyse.

The connection between the maintenance path and the bridge is not considered in detail.

#### 15.2 Material properties

**Results** Literature research has shown that glass fibre and polyester resin are the most suitable materials for use in railway bridges. This conclusion was drawn on the basis of a comparative study between different materials, taking into account the individual material properties, market price and common use in civil engineering structures.

The environmental influences and long term effects are taken into account using conversion factors. The final laminate properties can be found in Table 15.1 and Table 15.2.



|          |         | Thickness<br>mm | $E_1$<br>MPa | $E_2$<br>MPa | $G_{12}$<br>MPa | $\nu_{12}$<br>- | $\nu_{21}$<br>- | $\sigma_1$<br>MPa | $\sigma_2$<br>MPa | $\tau_{12}$<br>MPa |
|----------|---------|-----------------|--------------|--------------|-----------------|-----------------|-----------------|-------------------|-------------------|--------------------|
| Design A | Flanges | 100             | 22678        | 12599        | 4050            | 0.32            | 0.18            | 272               | 151               | 65                 |
|          | Webs    | 30              | 17291        | 15317        | 4833            | 0.31            | 0.28            | 207               | 184               | 77                 |
| Design B | Flanges | 30              | 22705        | 12600        | 4050            | 0.32            | 0.18            | 272               | 151               | 65                 |
|          | Webs    | 30              | 17291        | 15317        | 4833            | 0.31            | 0.28            | 207               | 184               | 77                 |

Table 15.1: SLS Laminate properties

|          |         | Thickness<br>mm | $E_1$<br>MPa | $E_2$<br>MPa | $G_{12}$<br>MPa | $\nu_{12}$<br>- | $\nu_{21}$<br>- | $\sigma_1$<br>MPa | $\sigma_2$<br>MPa | $\tau_{12}$<br>MPa |
|----------|---------|-----------------|--------------|--------------|-----------------|-----------------|-----------------|-------------------|-------------------|--------------------|
| Design A | Flanges | 100             | 7715         | 3526         | 1178            | 0.32            | 0.18            | 93                | 42                | 19                 |
|          | Webs    | 30              | 5218         | 4454         | 1485            | 0.31            | 0.28            | 63                | 53                | 24                 |
| Design B | Flanges | 30              | 7724         | 3526         | 1178            | 0.32            | 0.18            | 93                | 42                | 19                 |
|          | Webs    | 30              | 5218         | 4454         | 1485            | 0.31            | 0.28            | 63                | 53                | 24                 |

Table 15.2: ULS Laminate properties

**Discussion** The characteristic properties, presented in Table 6.1, are given by JRC. These characteristic are determined for a  $V_f = 50\%$ . However, the fibre volume fraction depends on the manufacturing process. This process given inaccurate fibre volume fractions Therefore, the characteristic properties are purely theoretical. Measurements must be taken to determine the real fibre volume fraction.

The choice of fibres is based on the market price. However, taking into account the total material costs of a design might result in another type of fibre, as the material properties are included by the choice of fibres.

The environmental effect on the material is described by the conversion factors, given in Table 6.5. These conversion factors result in a reduction of the material properties of at least 50%. The conversion factors are very conservative. Further investigation in the environmental effects on the material, in stead of using the conversion factors, might lead to better material properties.

### 15.3 Results conceptual designs

**Results** The unity check for deflection of Design A is lower than one and higher than one for Design B.

The stresses for both designs are high, resulting in unity checks higher than one, according to Table 10.2: both designs are unsafe. However, these high stresses are caused by the effect of singularity. Ignoring this effect leads to the results presented in Table 10.3. The unity checks for Design A are lower than one. Some unity checks for Design B are higher than one.

The dynamic unity checks of both designs are higher than one.

The main results of both conceptual designs are shown in Table 15.3.

| Designs         | SLS    | ULS    | Dynamic |
|-----------------|--------|--------|---------|
| Sandwich design | Safe   | Safe   | Unsafe  |
| Throughh design | Unsafe | Unsafe | Unsafe  |

Table 15.3: Conclusive results of both conceptual designs

**Discussion** The sandwich bridge deflects in two directions. The through bridge deflects mainly in one direction. Furthermore, the deflections of the through bridge are too large. This can be caused by the limited height of the plate section of the through bridge. Furthermore, the through has less supports what might result in larger deflections. The effect of the supports is visible by the peaks at the point loads. These peaks are caused by modelling the bearings as point supports. The peak stresses of the through design are located around the supports. This can be caused by the limited amount of supports and reduced thickness of the laminates. This results in high support reactions and therefore high stresses. Furthermore, these high stresses are transferred via thinner laminates, resulting in even higher stresses.

Both designs are unsafe due to the large accelerations. The maximum acceleration occurs at the point loads. Furthermore, the maximum acceleration of Design B is larger than the maximum acceleration of Design A. These large acceleration can be caused by the effect of singularity, the way of modelling the loads and/or the effect of resonance.

## 15.4 Final design

**Results** The final design is based on both conceptual designs and the following improvements: (1) Reduction of the span, (2) applying more height, (3) Adding more webs, (4) Applying more supports, (5) limiting the laminate thickness and (6) Applying pre-camber. The dimensions of this final design meet the requirements given by the codes.

**Discussion** Besides the six used improvements, more measures can be taken to improve the design.

The limitation of the laminate thickness is determined in consultation with a FRP manufacturer, based on his experience. However, advice based on experience gives a good impression on the expected results, but cannot be accepted as truth.

Pre-camber is taken into account by ignoring the deflection caused by LC1, instead of actually applying pre-camber in the design. When looking at the maximum deflection, there is no difference between both methods. However, the stress distribution for both methods might differ, causing different results.

## 15.5 Results final design

**Results** The main results of the final design are shown in Table 15.4. The unity checks for SLS and ULS calculations are lower than one. The dynamic unity check is higher than one. The maximum acceleration of the final design exceeds the limits. The high maximum accelerations are located at the point loads.

| Design       | SLS  | ULS  | Dynamic |
|--------------|------|------|---------|
| Final design | Safe | Safe | Unsafe  |

Table 15.4: Main results of final design

**Discussion** The deflection of the final design caused by LC3 is in 2 directions. The designs acts as a rectangular plate supported at each corner. The effect of the limited laminate thickness is visible by an increased deflection of the beam sections, compared to the sandwich design.

The maximum stresses in the final design do not exceed the limits. However, the stresses between the longitudinal and transverse webs are transferred via the resin. The tensile behaviour of the resin is within the limits. Further investigation in the compressive behaviour of the resin is needed. The dynamic analysis results in too large accelerations at the point loads, resulting in an unsafe design. However, the way of modelling the dynamic load case is unrealistic. In reality, point loads

are spread over a (small) area. Furthermore, the effect of ballast bed is ignored. This ballast bed might spread the loads over a larger area and add extra damping to the model.

In this chapter a conclusion of this research is drawn. The conclusion is based on the he research objective, stated in section 1.2. This research objective can be transformed into the following research question:

*Is it possible to apply a FRP deck in railway bridges?*

During this research a number of choices and assumptions are made that might influence the results of this research, and therefore the conclusion. At first, these assumptions and choices are discussed, resulting in a number of remarks which should be made in order for the results and conclusions to be properly interpreted.

The assumptions are divided in two parts; (1) Methodical assumptions and (2) numerical assumptions. The first focusses on the assumptions made to limit the scope. The latter focusses on the limitations of the numerical models.

## 16.1 Scope and methodical assumptions

The methodological assumptions are made to limit the scope of this research. Choices are made for the train type and choice of fibre.

### Length of span

The span of the bridge is predetermined: a continuous bridge with two spans of 10 meter. However, the span of a bridge has major influence on, among others, the deflection of the bridge, the resulting stresses and the dynamic behaviour of the bridge. Therefore, drawing a conclusion from the results only relates to a continuous bridge with two spans of 10 meter.

### Train type

The dynamic analysis is performed using a VIRM 4 train model. This train type is often used in The Netherlands for passenger transportation. Besides the VIRM4 train type other train types make use of the Dutch railways. Many cargo trains are heavier loaded and drive slower. Furthermore, the wheel spacing between cargo- and passenger trains differ. This results in different load frequencies and point load magnitude what might lead to different accelerations of the deck.

### Choice of type of fibre

The choice of type of fibre is among others based on the costs. In this research the market price of fibre materials is used, resulting in glass fibre as most suitable fibre material. Instead of using the market price, the total material cost of a design can be used. The total material cost depend on the market price of the material and the used amount of material. The used amount of material depends among others on the material properties. Taking the total material cost of a design in consideration, instead of the market price of the material, might result in another type of fibre.

## 16.2 Numerical assumptions

The application of finite element models should always be considered critically. The models are hand-made and prone to error. Analytical and numerical models are compared to verify the results of the numerical models. However, errors still might occur. The output of a numerical model is as wrong as its input.

### Comparison between numerical and analytical model

The Timoshenko beam theory is used by the comparison between the analytical model and numerical model to verify the numerical models. The Timoshenko beam theory and the numerical models differ in two ways. At first, the Timoshenko beam theory describes the behaviour of a solid beam element. The numerical models are made of several plate elements, resulting in an open beam element. Secondly, the Timoshenko beam theory is used for 2D elements. Both numerical models are 3D.

Although both models differ on crucial points, it is nevertheless assumed that a comparison between the analytical model and the numerical models is possible, especially due to a lack of better applicable alternatives.

### Application of ballast bed in dynamic analysis

For the dynamic analysis the ballast bed isn't taken into account in the numerical model. The dynamic load model, consisting of eight point loads, is directly applied on the bridge deck. Omitting the ballast bed in the dynamic analysis has consequences for the dynamic results. At first, the spreading effect of the ballast bed is ignored. This spreading effect reduces the effect of singularity, by transferring the point loads into area loads. Secondly, the damping effect of the ballast bed is left behind. Taking into account the damping effect of the ballast bed might result in a reduction of the maximum acceleration of the bridge deck.

Taking into account the ballast bed in the dynamic analysis might result in a significant reduction of the maximum acceleration of the bridge deck due to the spreading and damping effect of the ballast bed.

## 16.3 Sub-questions

The research question is divided into several sub questions, given in section 1.3. By answering these sub questions the research question can be answered. The answers of the sub questions and research question are based on the obtained results, the discussion in previous chapter and the assumptions mentioned above.

*What are common designs for FRP railway bridges?*

This question is answered in chapter 3. In this literature study two common designs are presented; (1) a Sandwich bridge deck design, commonly used in FRP bridges and (2) a Through bridge deck design, commonly used in railway bridges.

*What are the material properties of a FRP laminate suitable for railway bridges?*

The material properties of a FRP laminate suitable for railway bridges are discussed in chapter 6 and presented in Table 6.6 and Table 6.7. These material properties are suitable for railway bridges due to the used fibre lay-up, conversion factors and safety factors. Thereby, the chosen type of material and manufacturing process is based on its frequent application in FRP bridges.

*Is the static behaviour of common FRP railway bridge decks loaded by trains safe?*

The static behaviour for both common FRP railway bridge decks loaded by trains is determined using two numerical models. Both designs are loaded by corresponding load cases based on the Dutch codes and OVS. The numerical models are verified using analytical models to ensure realistic results. According to the results presented in chapter 10 the Sandwich bridge deck meets the requirements for SLS and ULS. The Through bridge deck doesn't meet the requirements for SLS and ULS. Therefore, the static behaviour of the Sandwich bridge is safe and the static behaviour of the Through bridge is unsafe.

*How to model the dynamic loads of a train on a railway bridge?*

The dynamic load model for train loads is given by the Eurocode. The dynamic loads of a train on a railway bridge must be modelled using a real train model. For this research a VIRM4 train type, a commonly used passenger train in the Netherlands, is used. This real train model consists of eight point loads with specific spacing, moving at a velocity of 140 km/h. This real train model is presented in subsection 8.3.4.

*Is the dynamic behaviour of common FRP railway bridge decks loaded by trains safe?*

The dynamic behaviour of both common FRP railway bridge decks, loaded by a real train model, is determined using the same numerical models for the static analysis. These models are verified using dynamic analytical models to ensure realistic results. According to the results presented in chapter 10 both designs exceed the acceleration limit given by OVS. Therefore, the dynamic behaviour of the Sandwich bridge design and Through bridge design loaded by train models is unsafe.

*How to improve the behaviour of common designs in railway bridges?*

A list of possible measures that can help to improve the behaviour of the Sandwich and Through bridge is given in chapter 11. The following measures are used to create a final design:

- Reduce the span
- Apply more height
- Adding more webs
- Apply more supports

- Limit the web thickness
- Apply pre-camber

The final design is based on a combination of (the results of) Design A and Design B and the used measures. The static and dynamic results of this final design are used to answer the research question.

## 16.4 Research question

Based on the results of Design A and Design B and the used measures a final design is presented in chapter 12. The results of the static and dynamic analysis of this final design are given in chapter 14.

According to these results the static behaviour of this final design (SLS and ULS) meets the requirements. Therefore, the static behaviour of the final design is safe. Furthermore, the thickness of the webs and flanges are in accordance with the advice of the FRP manufacturer.

The deflection of the final design for the dynamic analysis is within the limits. Therefore, the dynamic deflection behaviour of the final design is safe. The acceleration of the final design for the dynamic analysis exceeds the limits. Therefore, the dynamic acceleration behaviour of the final design is unsafe, based on the results presented in chapter 14. However, the numerical model for the dynamic analysis is incomplete due to omitting the ballast bed. Taken into account the ballast bed in the dynamic analysis might result in a reduction of the maximum acceleration.

### *Is it possible to apply a FRP deck in railway bridges?*

According to the results of the static analysis in combination with the limited thickness of the webs and flanges it can be concluded that it is possible to apply a FRP deck in railway bridges. According to the results of the dynamic analysis it can be said that it is not possible to apply a FRP deck in railway bridges. However, improvement of the dynamic numerical model is needed to obtain more realistic results, before drawing conclusions on the application of a FRP deck in railway bridges based on the dynamic results.

# CHAPTER 17

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## Recommendations

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This chapter gives an overview of the recommendations for further research.

- The application of a ballast bed in the dynamic analysis might have a positive effect on the maximum acceleration, as discussed in chapter 15. It is recommended to investigate the dynamic behaviour of a FRP deck with a ballast bed, by using a multi-body dynamic system.
- The longitudinal webs in the final design aren't continuous, due to the transverse webs. Therefore, the stresses between the longitudinal and transverse webs are transferred via the resin. A tensile check of on the resin has been performed in subsection 14.2.1. The compressive behaviour of resins is more complicated due to the redistribution of forces between the resin and fibres, and the possible buckling of webs under compression. Therefore, further research is recommended in the compressive behaviour of the webs.
- The dynamic analysis is performed for a VIRM4 train type. Other train types come with different wheel spacing, velocity and loading, causing different load frequencies. Further investigation in the maximum acceleration of the deck caused by different train types is recommended.
- The higher stiffness of carbon fibres, compared to glass fibres, comes with higher market prices, but offers a reduction in material use. Therefore, the total costs of a carbon-fibre bridge and glass-fibre bridge might be competitive. A comparative cost study is recommended to determine the best value design.
- The final design is based on the requirements given by the codes and some of the measures given in chapter 11. The effect of the remaining measures (Use of carbon fibres, increase the mass, reduce the loadings, use of generative design, clamped supports at the abutments) is unknown, but has a positive effect on the final behaviour of the bridge. Further investigation in the effect of the remaining measures is advised.





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## Part VI

# Appendix

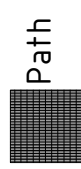
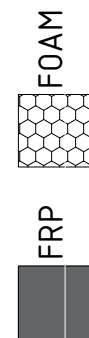
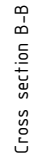


## APPENDIX A

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### Drawing Sandwich Bridge Deck Design

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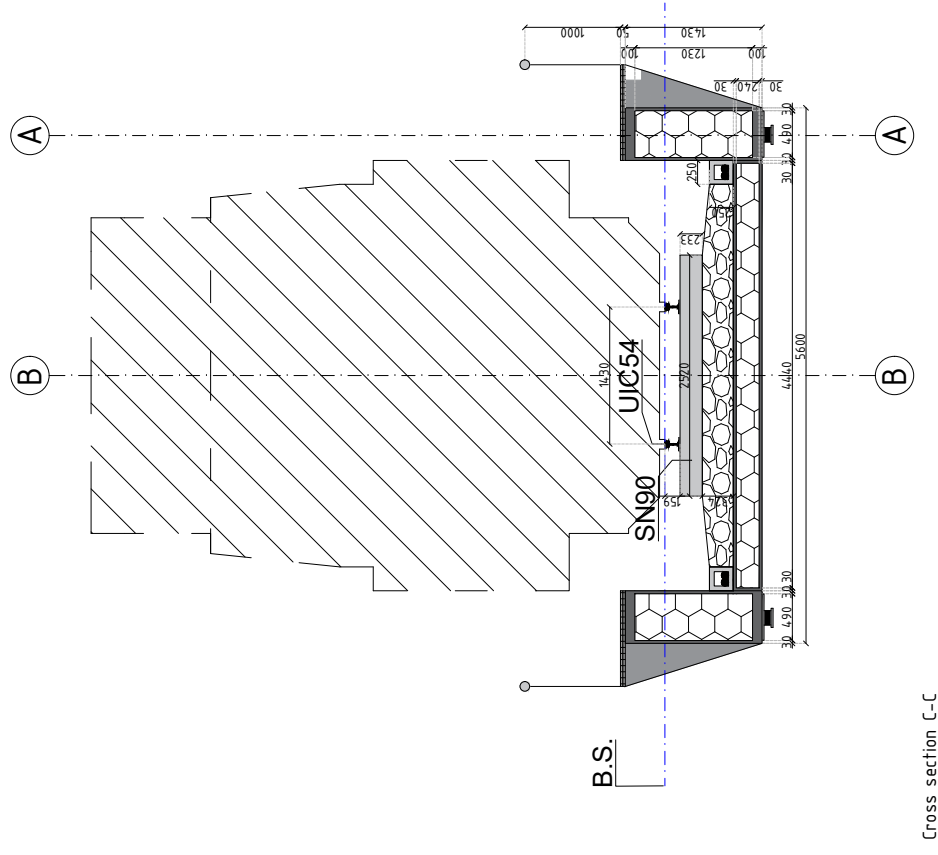
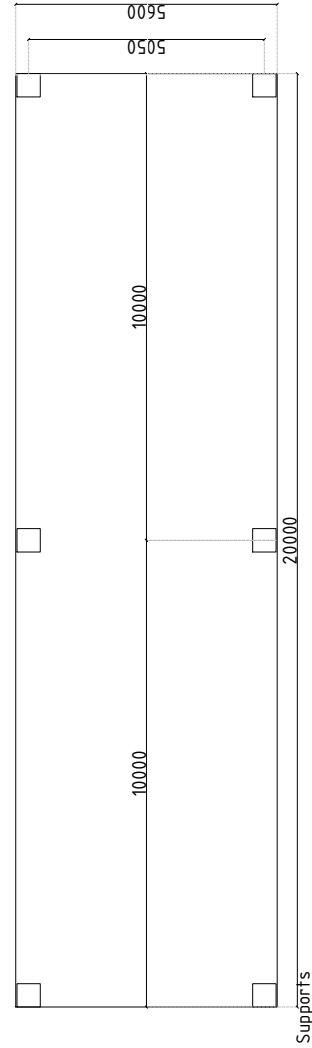
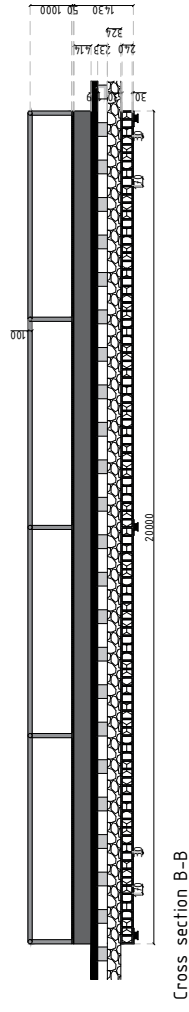
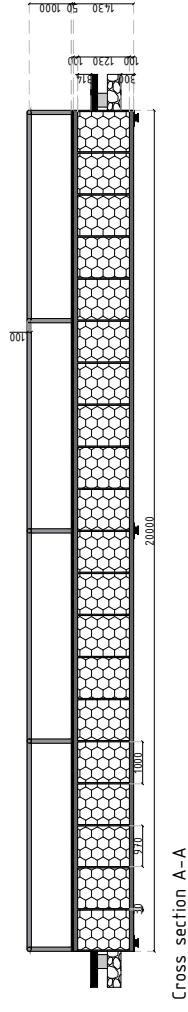
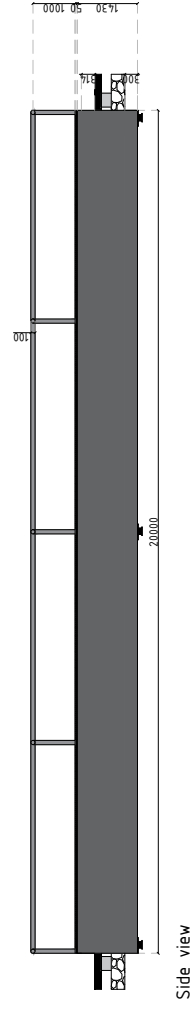
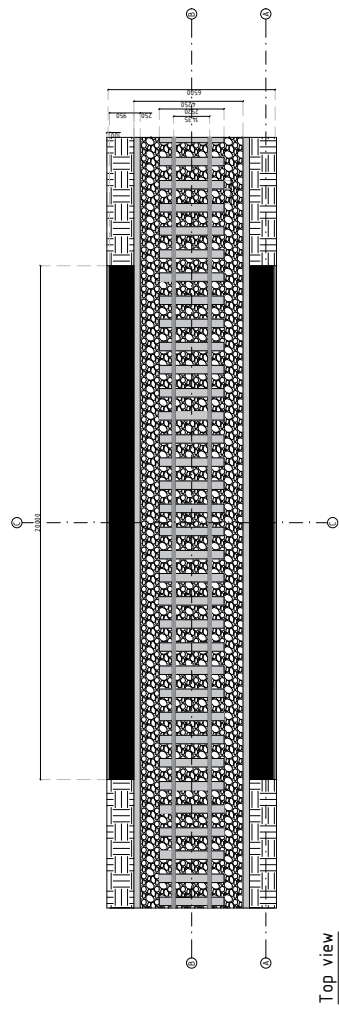
## APPENDIX B

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### Drawing Through Bridge Deck Design

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## Static verification of the numerical models

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In this appendix both models presented in chapter 8 are verified using two different load models. The verification is based on a comparison between the deflections of the numerical models and the deflection calculated with the Timoshenko beam theory. Furthermore, the total loads on both numerical models are compared with hand calculations.

First of all, the verification models are presented. Secondly, an introduction is given to the Timoshenko beam theory. Then, the load cases to verify the models are presented where after the results are given including the conclusion. Finally, the verification of the loads is discussed.

### C.1 Verification designs

The Timoshenko equation is used to determine the deflection of a beam. Both models, however, are 3D plates. In order to compare the results both models are adjusted in such a way that they describe a more 2D behaviour. This paragraph describes both adjusted models.

#### C.1.1 Verification model for the sandwich bridge

To describe a 2D behaviour the support of the sandwich bridge is adjusted. Instead of four point supports at each side a line support is applied over the full width of the deck. In this way the deflection in transverse direction is reduced. This numerical model with line supports is shown in Figure C.1.

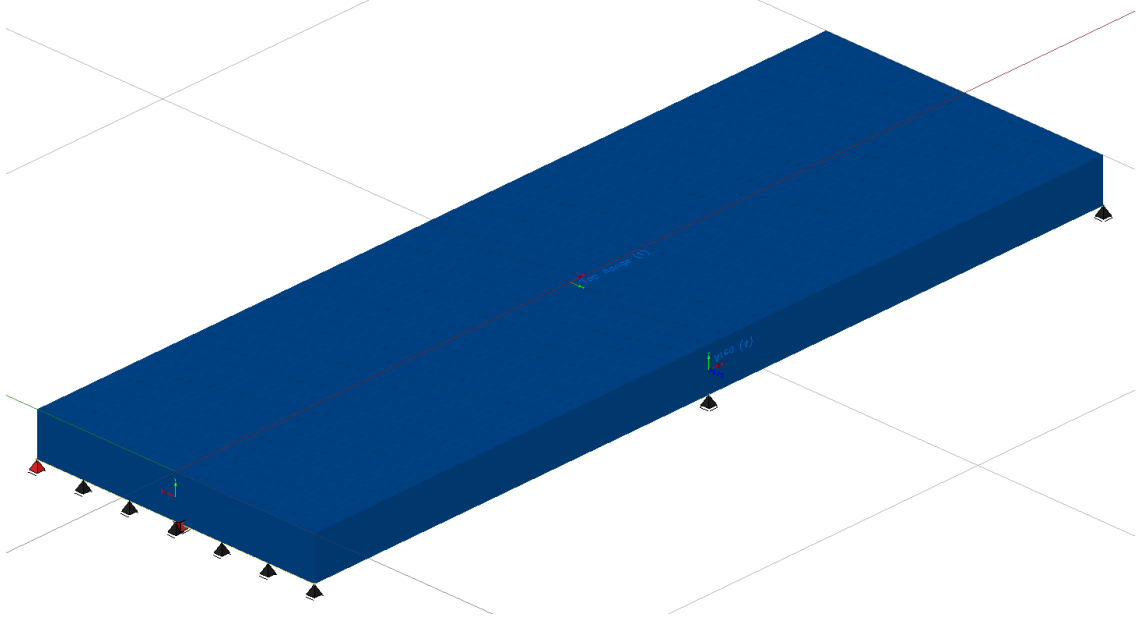


Figure C.1: Numerical model for the verification of the sandwich bridge

### C.1.2 Verification model for the through bridge

The U-shaped cross section of the through bridge makes it more complex design to calculate analytically, compared to the sandwich design. Therefore, a few changes are applied to make the model more two-dimensional. At first, the supports are changed to line supports. Secondly, the plate and beam sections are separated. In this case the beam and plate sections can act as a 2D structure. The verification model of the through design is shown in Figure C.2.

## C.2 Timoshenko equation

The Timoshenko beam theory describes the bending and shear deformation of a 2D beam, presented by Stephan Prokofyovich Timoshenko, "the father of modern engineering mechanics" [Simone, 2011]. This theory consists of two second order differential equations, see Equation C.1. To solve these equations a minimum of four boundary conditions is needed. These boundary conditions are given in section C.2.

$$\begin{aligned} EI * \phi'' - GA * (w' + \phi) &= 0 \\ GA * (w'' + \phi') &= -q \end{aligned} \tag{C.1}$$

Where:

- E = Modulus of elasticity [MPa]
- I = Second moment of inertia [mm<sup>4</sup>]
- G = Shear modulus [MPa]
- A = Effective area in shear [mm<sup>2</sup>]
- $\phi$  = Cross sectional rotation [degrees]
- w = Transverse deflection [mm]
- q = Distributed load [N/mm]

**Mechanical model and boundary conditions** The mechanical model for both designs is shown in Figure C.3. This model consists of a two span beam with a length of 2\*L. The shear stiffness GA and flexural stiffness EI for each design are given in section 5.1 and section 5.2. To use

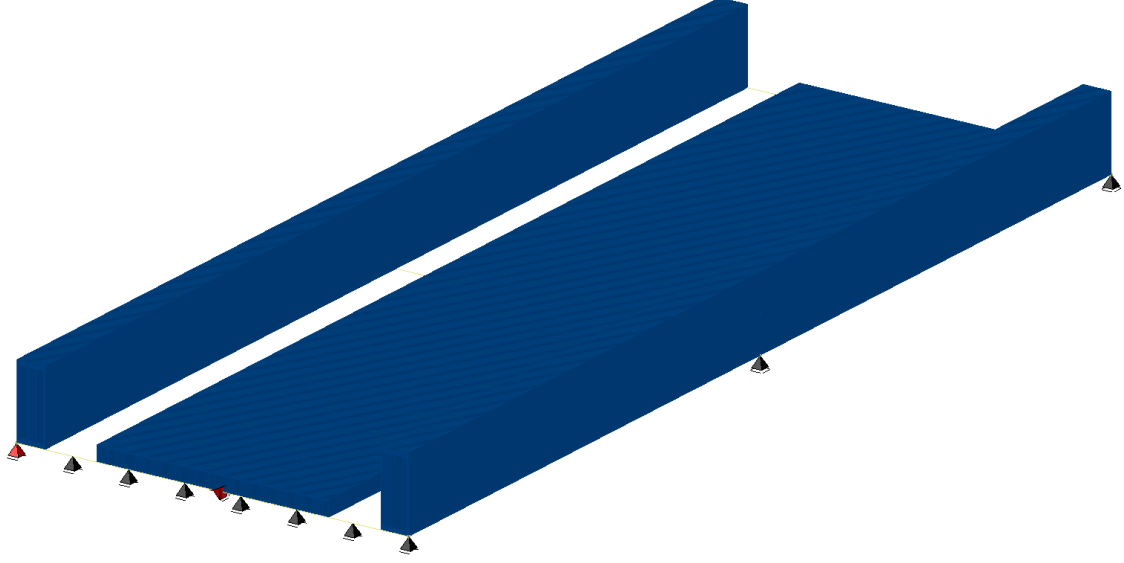


Figure C.2: Numerical model for the verification of the through bridge

the Timoshenko beam theory on this model it is divided into two sections; section 1 and section 2. To calculate the deflection of both sections at least two Timoshenko equations are needed, resulting in a total of eight boundary conditions, given below.

$$\begin{aligned} EI * \phi_1'' - GA * (w_1' + \phi_1) &= 0 \\ GA * (w_1'' + \phi_1') &= -q \end{aligned} \quad (C.2)$$

$$\begin{aligned} EI * \phi_2'' - GA * (w_2' + \phi_2) &= 0 \\ GA * (w_2'' + \phi_2') &= -q \end{aligned} \quad (C.3)$$

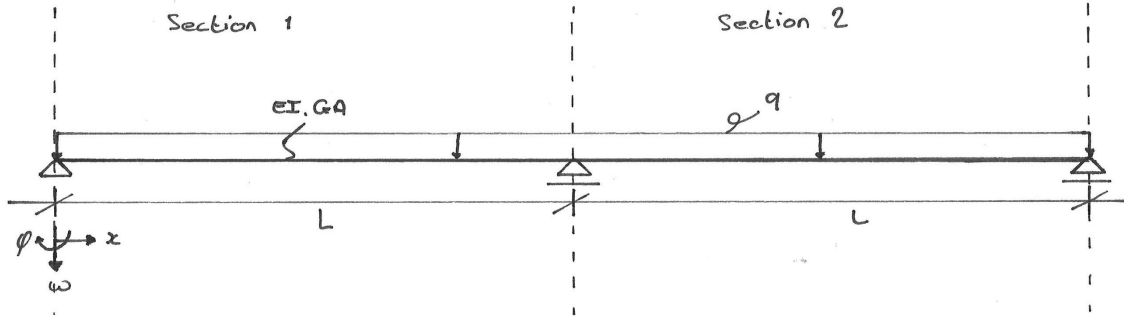


Figure C.3: Mechanical model for both designs

#### Boundary conditions

$$\begin{aligned} x = 0: \quad & w_1 = 0 \quad M_1 = 0 \\ x = L: \quad & w_1 = 0 \quad w_2 = 0 \quad M_1 = M_2 \quad \phi_1 = \phi_2 \\ x = 2L: \quad & w_2 = 0 \quad M_2 = 0 \end{aligned}$$

### C.3 Load cases for verification

For the verification of the numerical models the following load cases are used:

LC V1 A uniformly distributed load of  $1 \text{ kN/m}^2$

LC V2 A line load of  $100 \text{ kN/m}$  over the full width of the model, at the centre of one of the spans ( $x=5\text{m}$ )

For the analytical model, these 3D load cases are transformed into 2D load cases by multiplying with the width of the model. An overview of the loads is presented in Table C.1. The numerical models with both verification load cases are shown in Figure C.4 and Figure C.5.

| Load case |            | Sandwich bridge    |                    | Through bridge     |                    |
|-----------|------------|--------------------|--------------------|--------------------|--------------------|
|           |            | Numerical          | Analytical         | Numerical          | Analytical         |
| LC V1     | Figure C.4 | $1 \text{ kN/m}^2$ | $6.8 \text{ kN/m}$ | $1 \text{ kN/m}^2$ | $5.6 \text{ kN/m}$ |
| LC V2     | Figure C.5 | $100 \text{ kN/m}$ | $680 \text{ kN}$   | $100 \text{ kN/m}$ | $560 \text{ kN}$   |

Table C.1: Load cases for verification

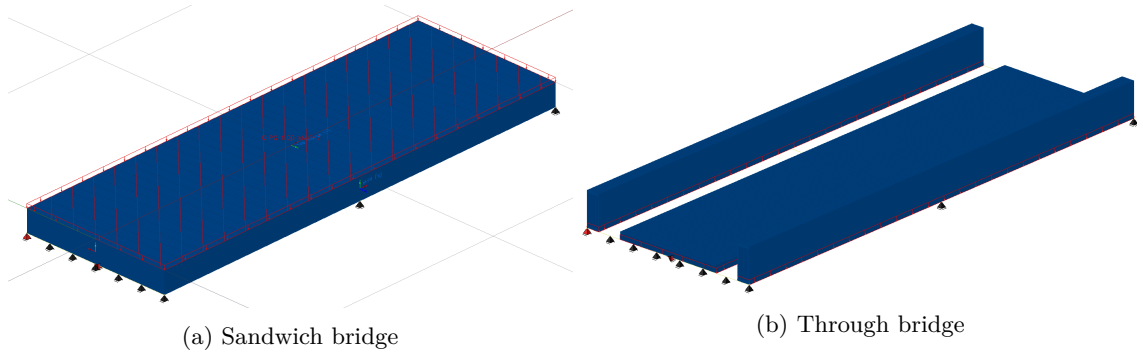


Figure C.4: LC V1

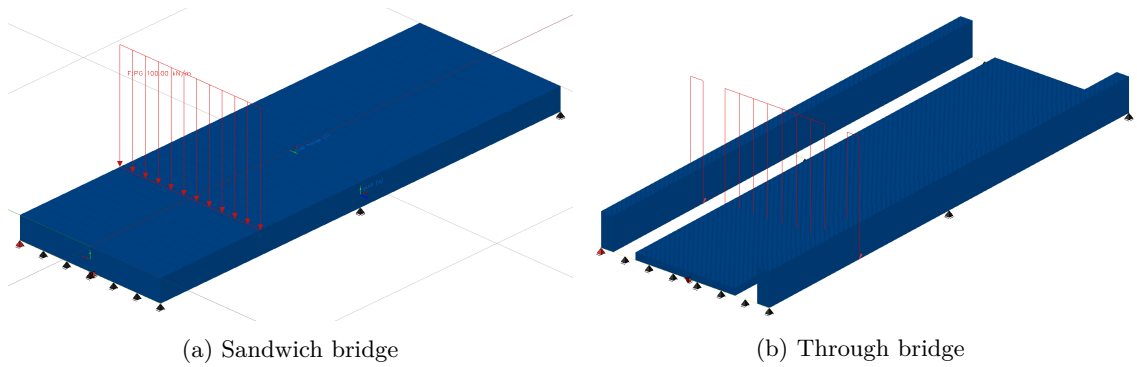


Figure C.5: LC V2

## C.4 Results

### C.4.1 Sandwich bridge

The results for the verification of the sandwich bridge are given in Table C.2.

| Load case | Sandwich bridge |            |            |
|-----------|-----------------|------------|------------|
|           | Numerical       | Analytical | Difference |
| LC V1     | 0.0673 mm       | 0.0600 mm  | 11%        |
| LC V2     | 1.64 mm         | 1.59 mm    | 3%         |

Table C.2: Maximum deflection of the numerical and analytical verification models for the sandwich bridge

According to Table C.2 the difference between the analytical calculations and numerical results for LC 2 is 3%. This difference is quite low. Therefore I assume that the deflection for LC 2 given by the numerical model is correct.

According to this same table, the difference between the analytical calculation and numerical results for LC 1 is 20%. This difference is high. However, according to Blaauwendraad [Blaauwendraad, 2010], the shear rigidity of a multi-cell element, consisting of a top and bottom flange, is an order of magnitude too large. This means that the deflection caused by the shear force is larger than calculated with the Timoshenko equations. Taking into account the reduced shear rigidity leads to a greater deflection, which means the difference between the analytical and numerical results are getting smaller.

Based on these statements I assume that the numerical model of the sandwich bridge provides realistic results.

### C.4.2 Through bridge

The results for the verification of the numerical model of the through bridge are given in tables C.3 and C.4.

| Load case | Beam section of through bridge |            |            |
|-----------|--------------------------------|------------|------------|
|           | Numerical                      | Analytical | Difference |
| LC V1     | 0.224 mm                       | 0.216 mm   | 4%         |
| LC V2     | 0.535 mm                       | 0.538 mm   | 1%         |

Table C.3: Maximum deflection of the numerical and analytical verification models for the beam section of the through bridge

| Load case | Plate section of through bridge |            |            |
|-----------|---------------------------------|------------|------------|
|           | Numerical                       | Analytical | Difference |
| LC V1     | 14.8 mm                         | 17.36 mm   | 17%        |
| LC V2     | 41.8 mm                         | 47.79 mm   | 14%        |

Table C.4: Maximum deflection of the numerical and analytical verification models for the plate section of the through bridge

According to Table C.3 the difference between the analytical calculations and numerical results for the beam sections of the through bridge are 4% for LC 1 and 1% for LC 2. These differences are close to zero. Therefore I assume that the results given by the numerical model are realistic.

The differences between the analytical calculation and numerical results for the plate section of the through bridge are 17% for LC 1 and 14% for LC 2, according to Table C.4. These differences are very high. However, according to Blaauwendraad, the shear rigidity of a multi-cell analytical element, consisting of a top and bottom flange, is an order of magnitude too large. This means that the deflection caused by the shear force is larger than calculated by the Timoshenko equations. Taking into account the reduced shear rigidity leads to a greater deflection, which means the difference between the analytical and numerical results are getting smaller.

Furthermore, both plate section and beam sections are made within the same model. When the results of the beam section are realistic, this means the model is realistic. Therefore, I assume that the results for LC 2 are realistic, as both designs are part of the same model.

### C.4.3 Conclusion

Based on the results in subsection C.4.1 I assume that both numerical models are realistic.

## C.5 Verification of the loads

The verification of the total loads applied on both numerical models is given in Table C.5.

| <b>Sandwich bridge</b> | Self weight | Dead load | LM71      | All loads | Difference |
|------------------------|-------------|-----------|-----------|-----------|------------|
| Numerical model        | 971.0 kN    | 2087.4 kN | 3511.7 kN | 6569.0 kN | 0.002%     |
| Hand calculations      | 971.0 kN    | 2087.4 kN | 3511.8 kN | 6569.1 kN |            |
| <b>Through bridge</b>  | Self weight | Dead load | LM71      | All loads | Difference |
| Numerical model        | 345.6 kN    | 1486.1 kN | 3511.7 kN | 5365.6 kN | 0.004%     |
| Hand calculations      | 344.0 kN    | 1485.8 kN | 3511.8 kN | 5341.6 kN |            |

Table C.5: Verification of the vertical loads

The differences between the numerical and analytical results for all loads, presented in Table C.5, are 0.002% and 0.004%. These differences are lower than 1% and therefore I assume that the loads are modelled correctly.

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## Dynamic verification of the numerical models

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The dynamic verification is based on an oscillating load. This dynamic load is only verified on the numerical model of Design A. The dynamic verification of Design B is omitted because (1) the dynamic load is the same for both models and (2) the numerical model of design B has already been verified in ??.

The numerical model of Design A, presented in chapter 8, and an analytical model are loaded by a sinusoidal load with given load frequency and amplitude. This sinusoidal load is described by Equation D.1:

$$F = \hat{F} * \sin(2 * \pi * f * t) \quad (D.1)$$

Where:

|             |                |        |
|-------------|----------------|--------|
| $F =$       | Load function  | $[kN]$ |
| $\hat{F} =$ | Load amplitude | $[kN]$ |
| $f =$       | Load frequency | $[Hz]$ |
| $t =$       | Time           | $[s]$  |

This load is applied at midspan, at the center of one of the spans (node 9005), see Figure D.1. The amplitude is 100 kN. Two load frequencies are used; 2 Hz and 10 Hz (Figure D.2). The models are analysed for both damped and undamped situation, resulting in the following load cases:

LC DV1 Undamped system with a load frequency of 10 Hz

LC DV2 Undamped system with a load frequency of 2 Hz

LC DV3 Damped system with a load frequency of 10 Hz

LC DV4 Damped system with a load frequency of 2 Hz

First of all, the numerical model for the dynamic verification is presented including the software input. Secondly, the analytical model is presented including the load model. Finally, the results of both models (numerical and analytical) are presented and discussed, whereafter a conclusion on the dynamic verification is given.



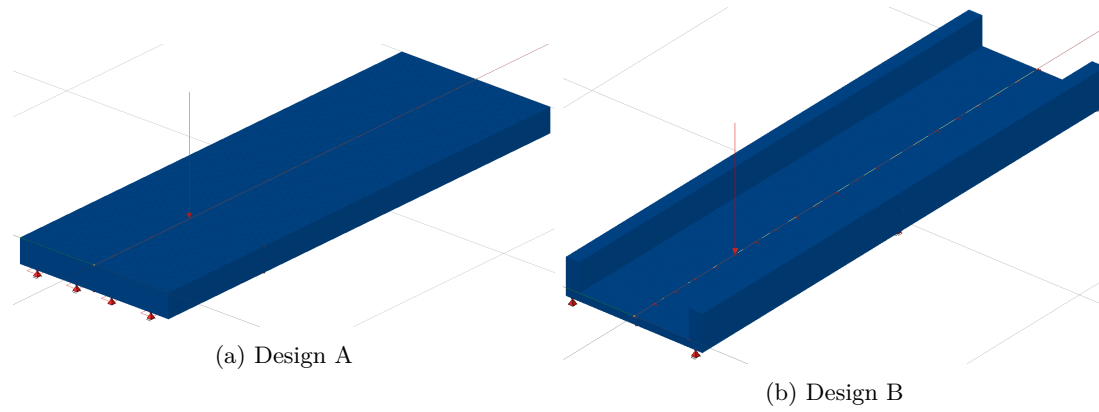


Figure D.1: Position of oscillating load on both designs

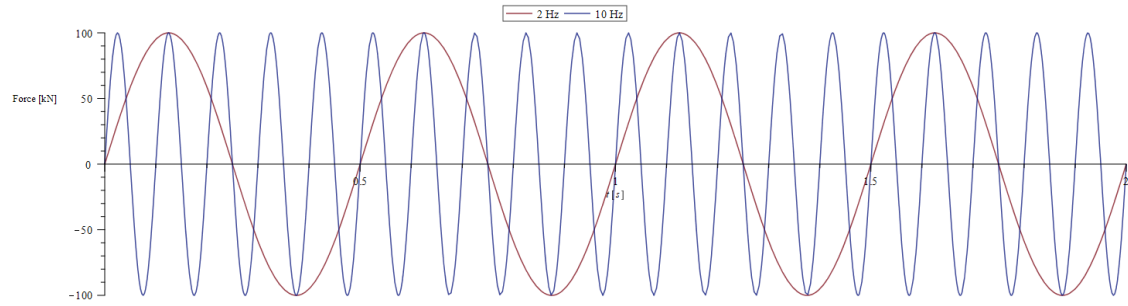


Figure D.2: Oscillating load with a load frequency of 10 Hz (blue) and 2 Hz (red)

## D.1 Numerical model

The dynamic verification is performed on the numerical model of Design A presented in chapter 8.

### D.1.1 Loading

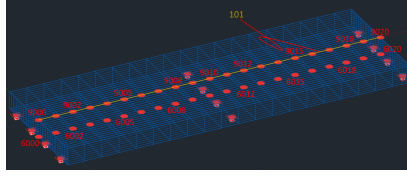
The load is modelled using Teddy and given in Figure D.3. The load in this figure is applied on node 9005 (see Figure D.4), has a load frequency of 2 Hz (period  $T = 0.5$  sec), the amplitude PG is 100.0 kN and the load is applied between T1 is 0.0 sec and TMAX is 2.0 sec. For a load frequency of 10 Hz, the period  $T = 0.1$  sec.

```
+PROG SOFILOAD urs:33.1
head OSCILATING LOAD
LC 600 TYPE NONE ; FUNC 0.50 T1 0 0.0 TMAX 2.0 ; NODE 9005 PG 100.0
end
```

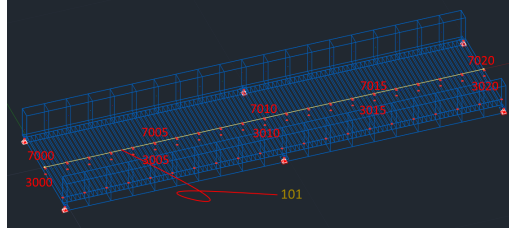
Figure D.3: Input of the oscillating load in Teddy for Design A

### D.1.2 Calculation

The displacement and acceleration of the bridge are calculated using Teddy. The input for Sofistik to calculate the displacement and acceleration is shown in Figure D.5. The time steps are 0.01 second with 200 steps. The output is given for node 9000 - 9020 and 6000 - 6020. The damping



(a) Sandwich bridge



(b) Through bridge

Figure D.4: Location of the corresponding nodes the numerical model of both designs

| Natural frequencies | Design A  |
|---------------------|-----------|
| 1st                 | 16.034 Hz |
| 2nd                 | 19.412 Hz |
| 3rd                 | 19.570 Hz |
| 4th                 | 23.520 Hz |
| 5th                 | 24.214 Hz |

Table D.1: Natural frequencies of design A only loaded by dead load

is modelled using the RABA and RABD codes, representing the alpha and beta factor of the Rayleigh damping. For the undamped system RABA and RADB are 0.

```

PROG DYNA urs:25.1
HEAD ROLLING STOCK ANALYSIS
GRP - FULL RADA #a RADB #b
CTRL MCON 2

STEP DT 0.01 N 200
LC 600

HIST U-Z FROM 9000 TO 9020 LCST 202
HIST A-Z FROM 9000 TO 9020 LCST 201

HIST U-Z FROM 6001 TO 6020 LCST 204
HIST A-Z FROM 6001 TO 6020 LCST 203

END

```

Figure D.5: Teddy input file of the calculation for the dynamic verification for Design A

**Damping** The alpha and beta factor depend on the natural frequencies of both models. The natural frequencies are calculated by Sofistik. For the verification only the dead load is taken into account. The natural frequencies of Design A are given in Table D.1.

Using the natural frequencies in Table D.1 the alpha and beta factor are calculated, see Figure D.6. The material damping  $\zeta = 5\%$ .

## D.2 Analytical model

The analytical model is based on the theory described in chapter 4. The design is modelled as a mass-spring rigid body system for an undamped system and a mass-dashpot-spring rigid body system for a damped system. These rigid body systems only describe the behaviour of one span of the bridge. Therefore, only 50% of the mass is used in the calculation (as shown in Figure D.7).

```

+PROG TEMPLATE   urs:45.2
head DEFINITION OF DAMPING PARAMETERS
let#xsi 0.05      $ Material damping
let#f1 16.034     $ 1st Eigenform
sto#w1 #f1*2*3.1415 $ 1st angular frequency
let#f2 24.21     $ 5th Eigenform
sto#w2 #f2*2*3.1415 $ 5th angular frequency
sto#a (2*#w1*#w2*(#xsi*#w2-#xsi*#w1))/(#w2^2-#w1^2) $ alpha value
sto#b (2*(#xsi*#w2-#xsi*#w1))/(#w2^2-#w1^2) $ beta value

end

```

Figure D.6: Teddy codes to calculate Rayleigh coefficients

The angular natural frequency of the bridge remains unchanged, as the natural angular frequency of a single span bridge and double span bridge with equal spans is the same.

### D.2.1 Undamped

The mass-spring rigid body system is shown in Figure D.7.

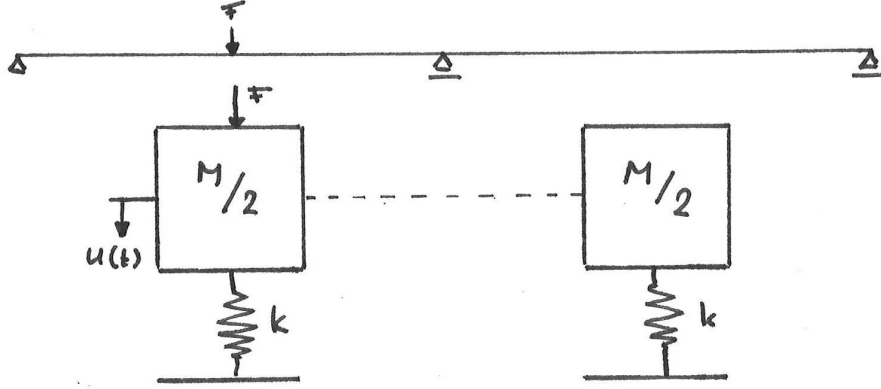


Figure D.7: Mass-spring rigid body system for both designs

The corresponding equation for this system is:

$$u = \frac{\hat{F}}{m(\omega_0^2 - \omega^2)} (\cos(\omega t) - \cos(\omega_0 t)) \quad (\text{D.2})$$

Where:

|              |                                         |           |
|--------------|-----------------------------------------|-----------|
| $u =$        | Displacement                            | $[m]$     |
| $\hat{F} =$  | Force                                   | $[N]$     |
| $m =$        | Mass                                    | $[kg]$    |
| $\omega_0 =$ | Natural angular frequency of the system | $[rad/s]$ |
| $\omega =$   | Angular frequency of the load           | $[rad/s]$ |
| $t =$        | Time                                    | $[s]$     |

The equation for the acceleration of the system is:

$$a = \frac{\hat{F}(-\omega^2 \cos(\omega t) + \omega_0^2 \cos(\omega_0 t))}{m(\omega_0^2 - \omega^2)} \quad (\text{D.3})$$

### D.2.2 Damped

The mass-dashpot-spring rigid body system is shown in Figure D.8.

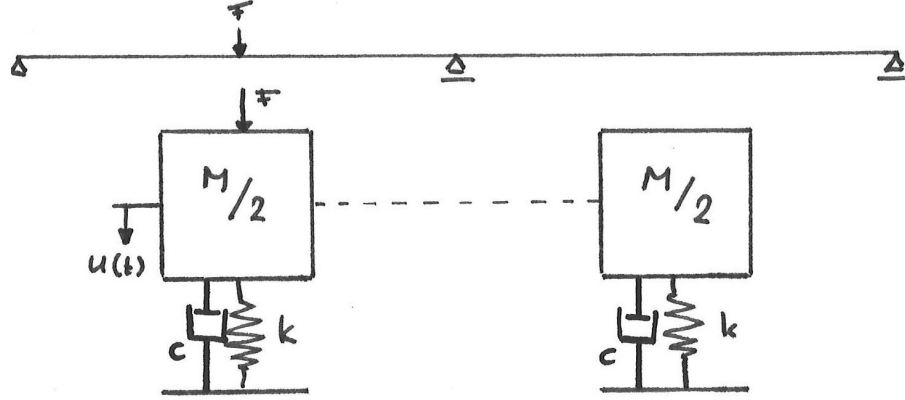


Figure D.8: Mass-dashpot-spring rigid body system for both designs

The corresponding equation for this system is:

$$u = A * e^{-\zeta * \omega_0 t} \cos(\omega_e t + \theta) + \frac{\hat{F}}{\sqrt{(m(\omega_0^2 - \omega^2))^2 + (c * \omega)^2}} * \cos(\omega t + \arctan(\frac{-c\omega}{m(\omega_0^2 - \omega^2)})) \quad (D.4)$$

The parameters A and  $\theta$  depend on the initial conditions. For the dynamic verification the initial conditions are  $u(0)=v(0)=0$ .

| Load case                |        | LC DV1 | LC DV2 | LC DV3 | LC DV4 |
|--------------------------|--------|--------|--------|--------|--------|
| Displacement [mm]        | Top    | 0.81   | 0.54   | 0.70   | 0.53   |
|                          | Bottom | 0.69   | 0.41   | 0.57   | 0.40   |
| Acceleration [ $m/s^2$ ] | Top    | 137.47 | 26.24  | 21.56  | 3.81   |
|                          | Bottom | 9.34   | 1.47   | 6.51   | 0.94   |

Table D.2: Maximums of numerical results

| Load case                | LC DV1 | LC DV2 | LC DV3 | LC DV4 |
|--------------------------|--------|--------|--------|--------|
| Displacement [mm]        | 0.67   | 0.37   | 0.50   | 0.36   |
| Acceleration [ $m/s^2$ ] | 4.7    | 2.1    | 3.5    | 2.0    |

Table D.3: Maximums of analytical results

### D.3 Results

This section shows the results of LC DV1 - LC DV4 for both numerical and analytical model. Firstly, the deflection and acceleration of the numerical model are given, whereafter the deflection and acceleration of the analytical model is presented.

**Numerical results** The maximum displacement and acceleration for node 9001-9009 (top) and node 6001-6009 (bottom) are given in Figure D.9. The blue lines represent the deflection and are linked to the left axis. The red lines represent the acceleration and are linked to the right axis.

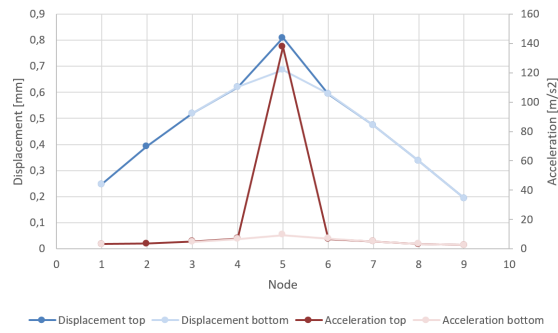
The graphs in Figure D.9 show that the deflection and acceleration of the top and bottom nodes are the same, except for node 5. At this node the results between top and bottom differ, with high peak values at the top. The load is applied on this node.

The maximums of LC DV1 and LC DV3, with a load frequency of 10 Hz, are higher than the maximums of LC DV2 and LC DV4, with a load frequency of 2 Hz. This is in accordance with the theory of resonance (given in chapter 4), as the load frequency of 10Hz is closer to the natural frequency of the bridge; 16.034 Hz.

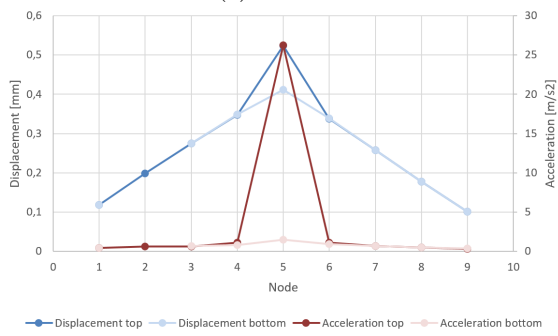
The maximum acceleration of the undamped load cases (LC DV1 and LC DV2) are higher than the maximum acceleration of the damped load cases (LC DV3 and LC DV4). This is in accordance with Equation 4.12, where the amplitude is decreased by the damping parameter of the structure. The maximum numerical acceleration and displacement of all load cases is given in Table D.2.

**Analytical results** The displacement of the analytical model for all load cases is given in Figure D.10. The acceleration of the analytical model for all load cases is given in Figure D.11.

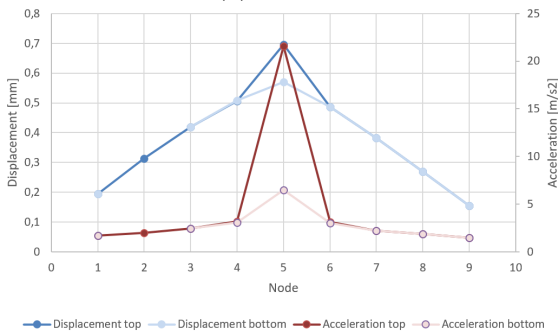
The effect of damping is clearly visible in Figure D.10 and Figure D.11. The steady state responses of LC DV3 and LC DV4, with the corresponding frequency, becomes zero. Only the forced responses with given frequencies remain. Furthermore, both load frequencies are clearly visible. Especially for both damped systems the load frequency are clearly demonstrated. The maximum acceleration and deflection of all load cases is given in Table D.3.



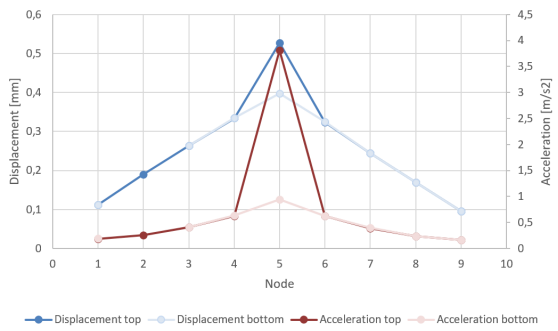
(a) LC DV1



(b) LC DV2



(c) LC DV3



(d) LC DV4

Figure D.9: Maximums of numerical model of Design A at top (node 9001-9009) and bottom (6001-6009)

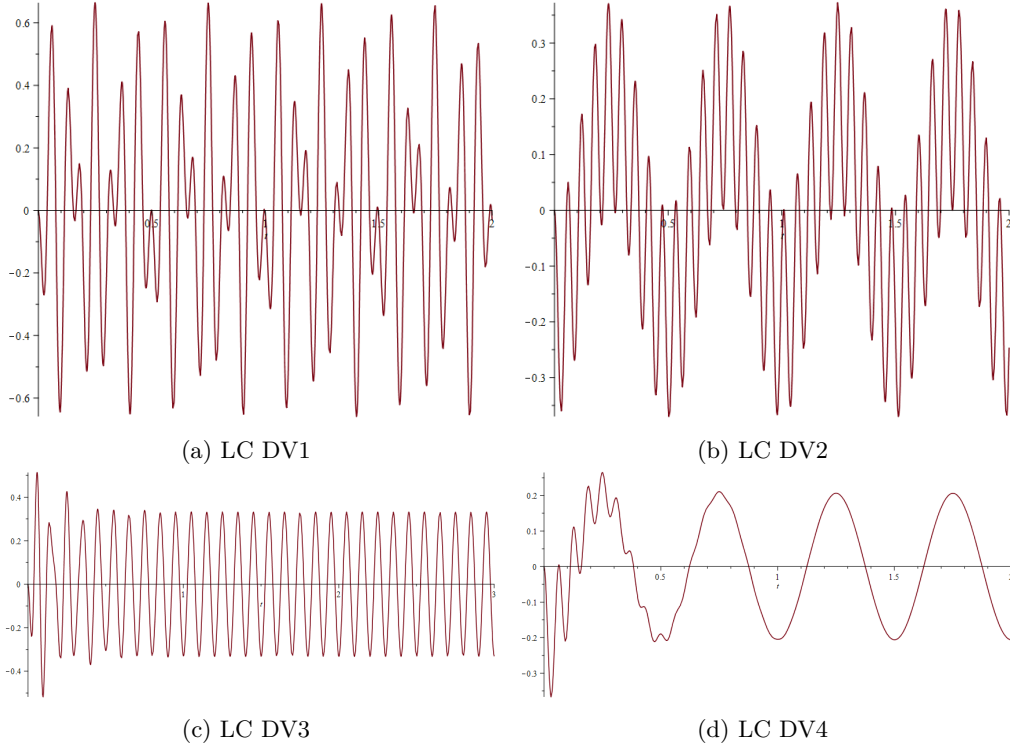


Figure D.10: Analytical results of the displacement

### D.3.1 Discussion

The maximums of the numerical and analytical models are given in Table D.2 and Table D.3. According to these tables the differences between the analytical results and the numerical results of the top nodes are very large. One of the reasons for these large differences is the effect of singularity; the results tend towards infinity at critical points, like point loads. The results in Figure D.9 affirm this.

The differences between the analytical results and the numerical results of the bottom nodes are less than the top nodes, but there are still some differences. One of the reasons is the use of only one natural frequency in the analytical model. The numerical model has multiple natural frequencies, given in section 8.4. The effect of resonance increases when the load frequency coincides with multiple natural frequencies. This effect is less when the differences between the natural frequencies and load frequency are large. The results in Table D.2 and Table D.3 affirm this. The acceleration of the numerical model are higher for the load cases with a load frequency of 10Hz; LC DV1 and LC DV3.

**Conclusion** Based on the results of section D.3 I assume that the numerical model generates realistic values. However, due to singularity the response of the top nodes is overestimated. Therefore, the bottom nodes must be used to determine the response of the structure.

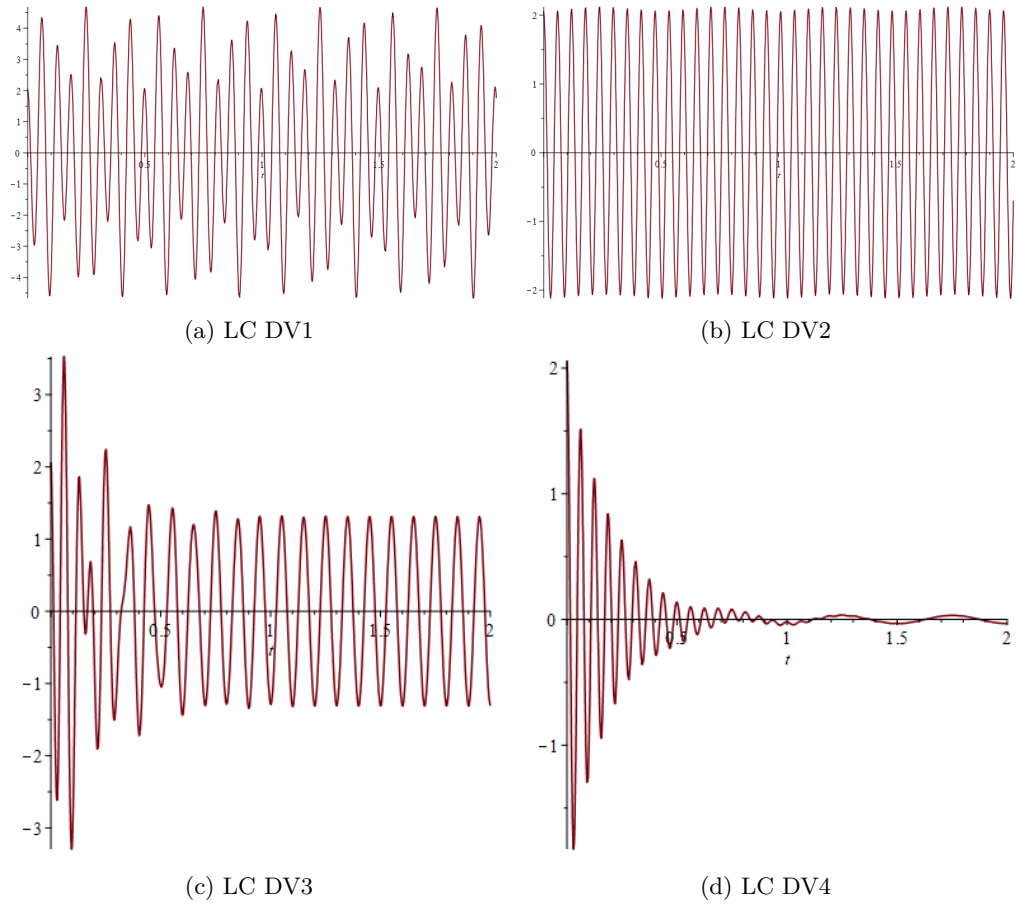


Figure D.11: Analytical results of the acceleration



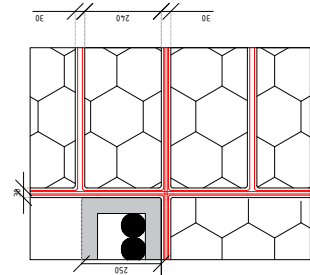
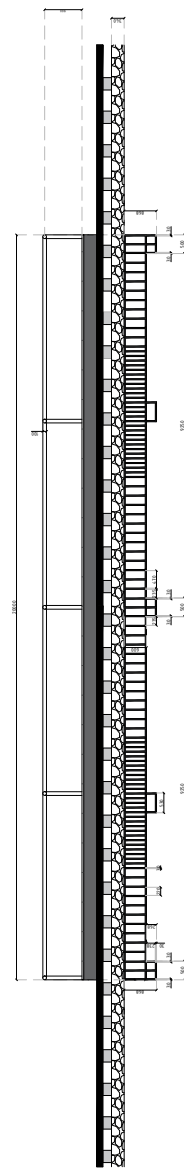
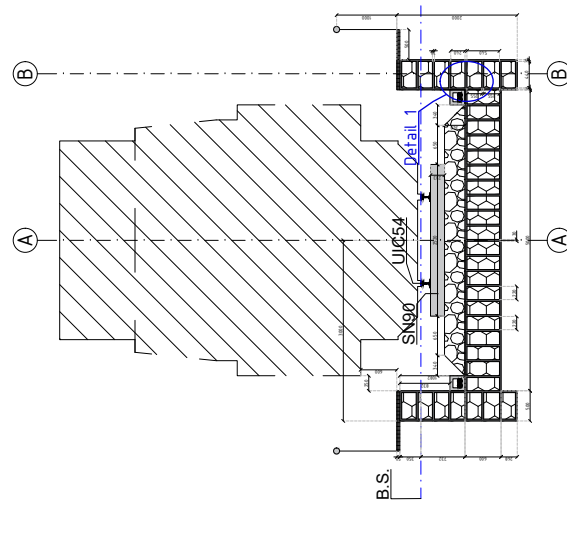
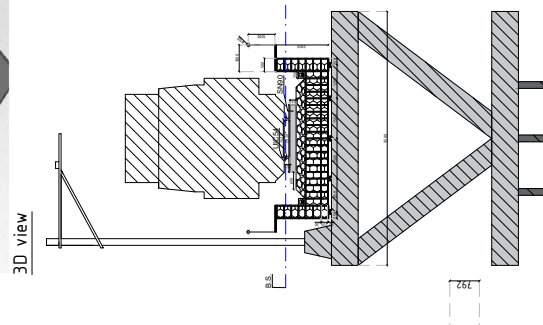
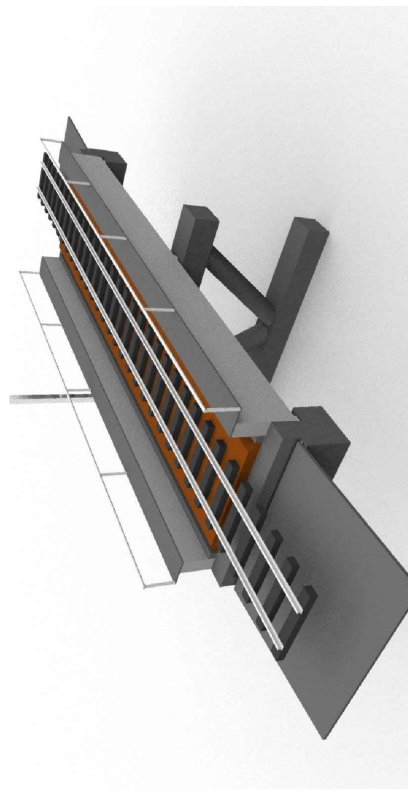
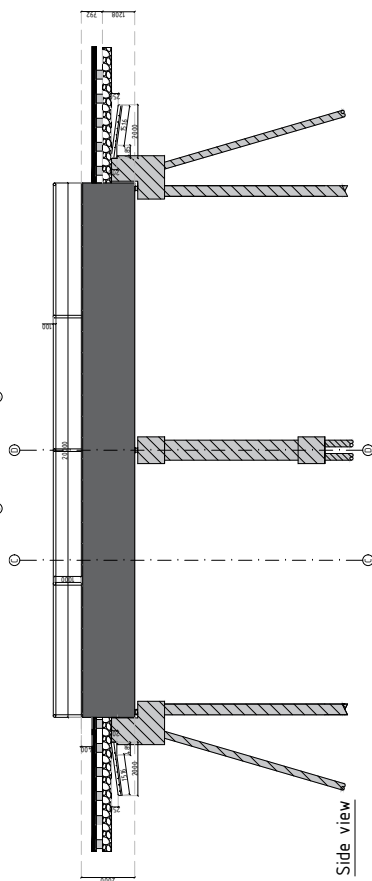
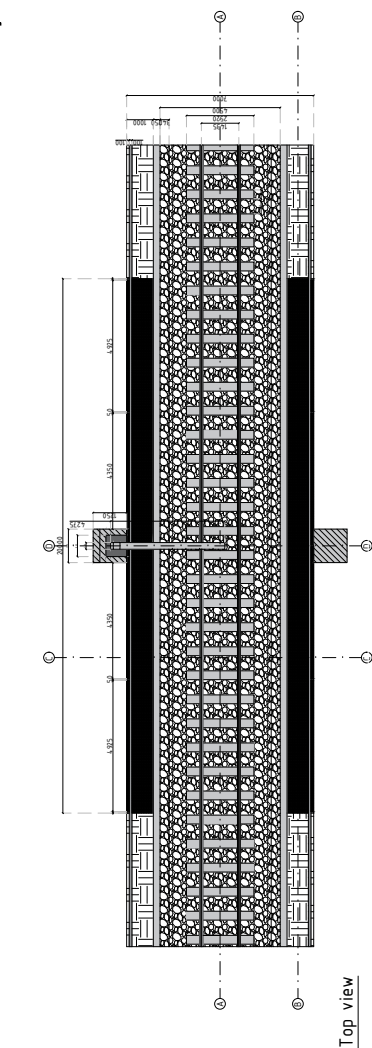


## APPENDIX E

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Drawing Final Design

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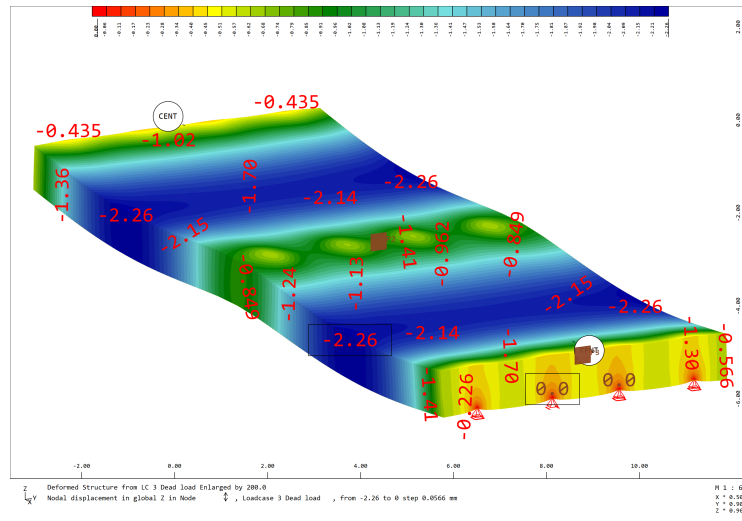
## APPENDIX F

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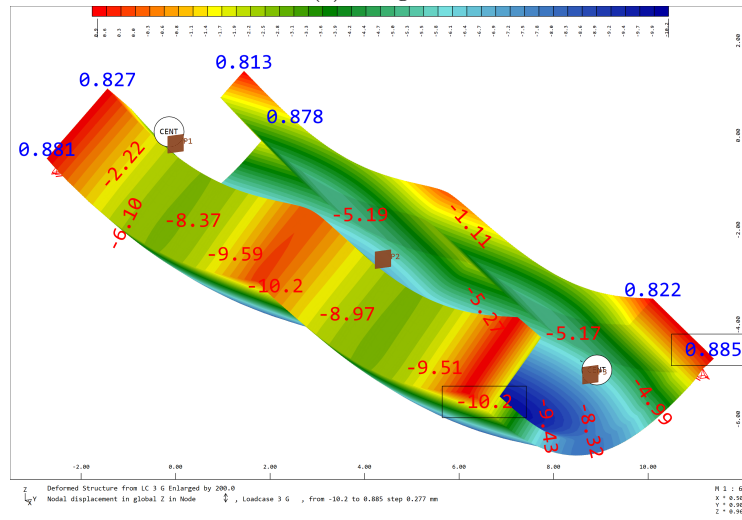
### SLS results

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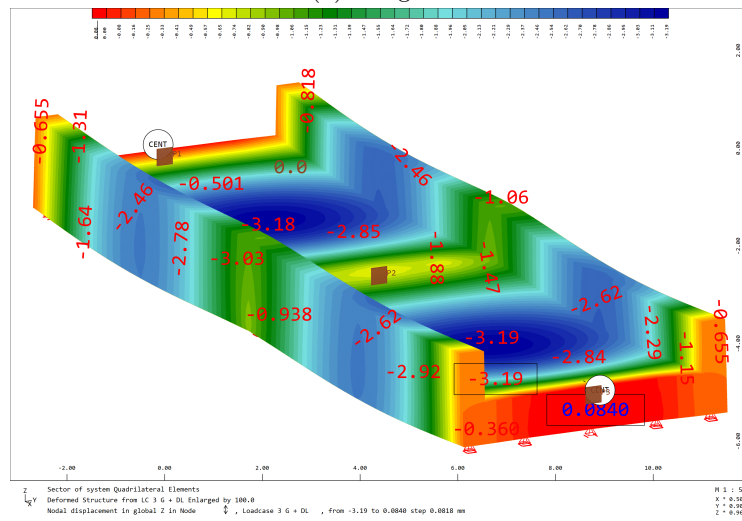
The SLS results are summarized in this chapter. The SLS results for load case 1 are presented in Figure F.1. The SLS results for load case 2 are presented in Figure F.2. The SLS results for load case 3 are presented in Figure F.3.



(a) Design A

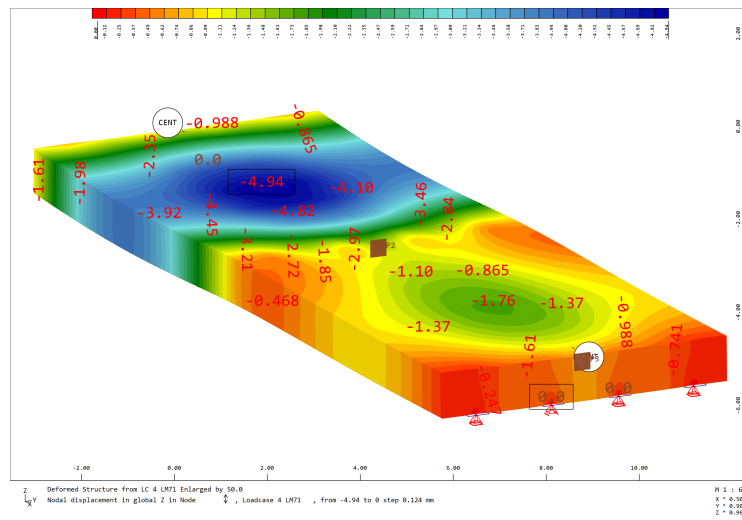


(b) Design B

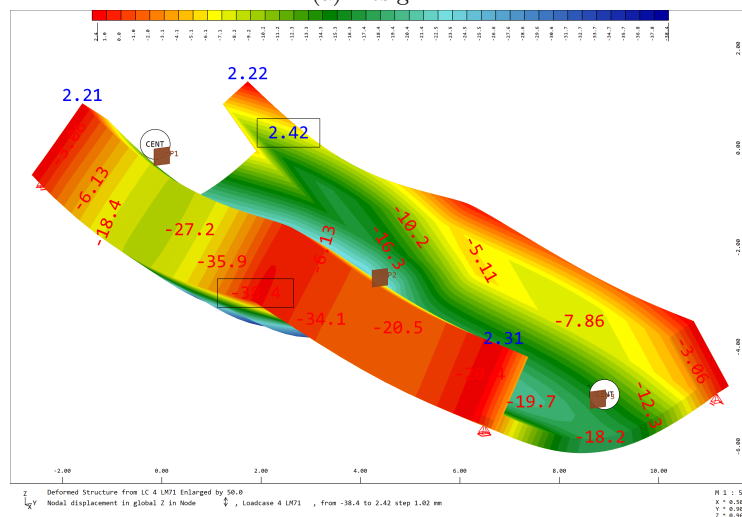


(c) Final design

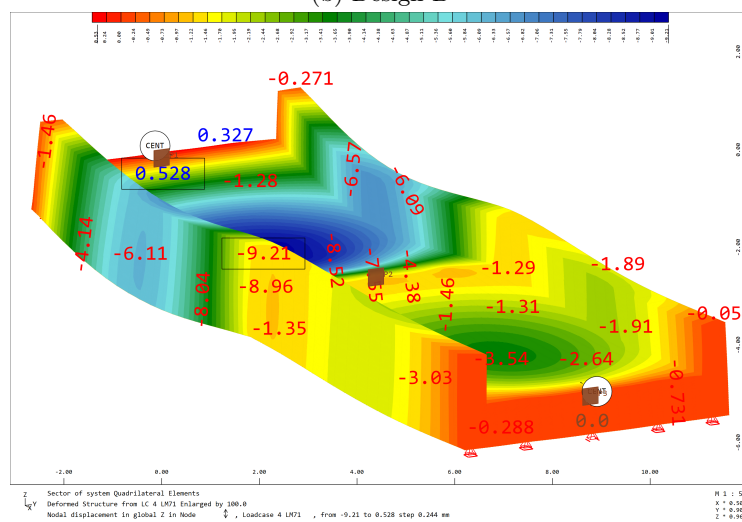
Figure F.1: SLS results for LC 1

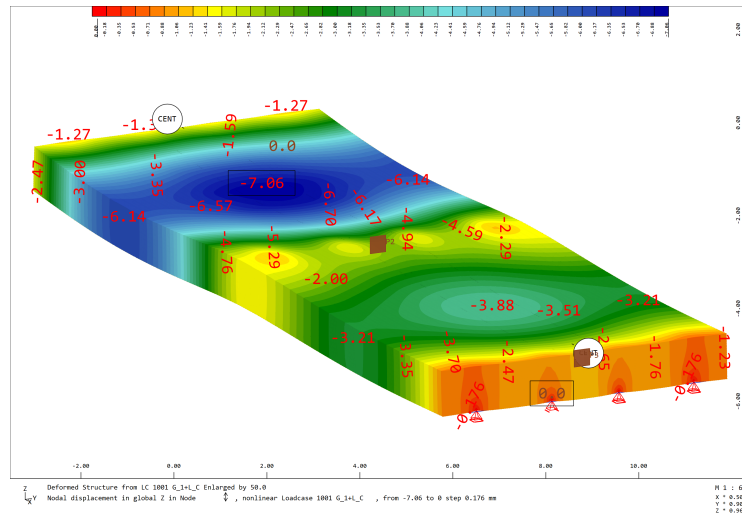


(a) Design A

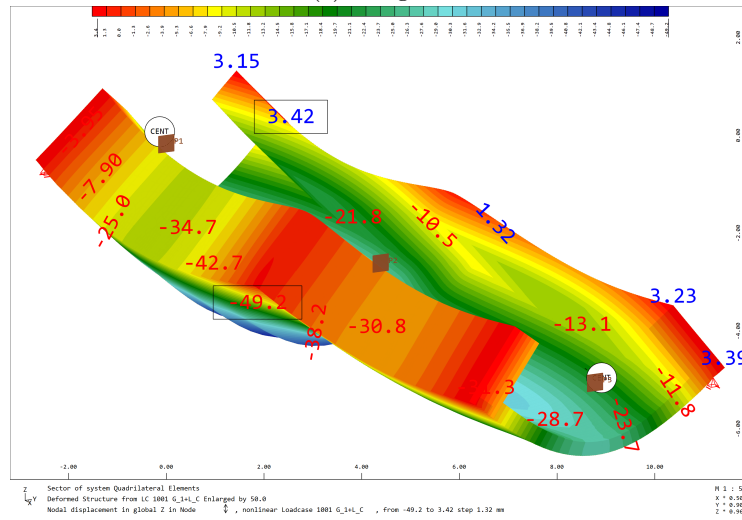


(b) Design B

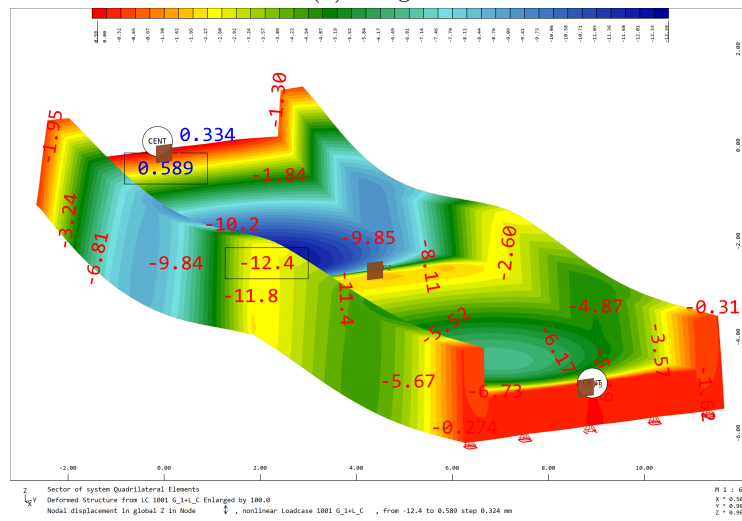




(a) Design A



(b) Design B



(c) Final design

Figure F.3: SLS results for LC 3

## Uls results of conceptual designs

## Sandwich bridge

The ULS results of the Sandwich deck bridge design are shown in figs. G.1 to G.5

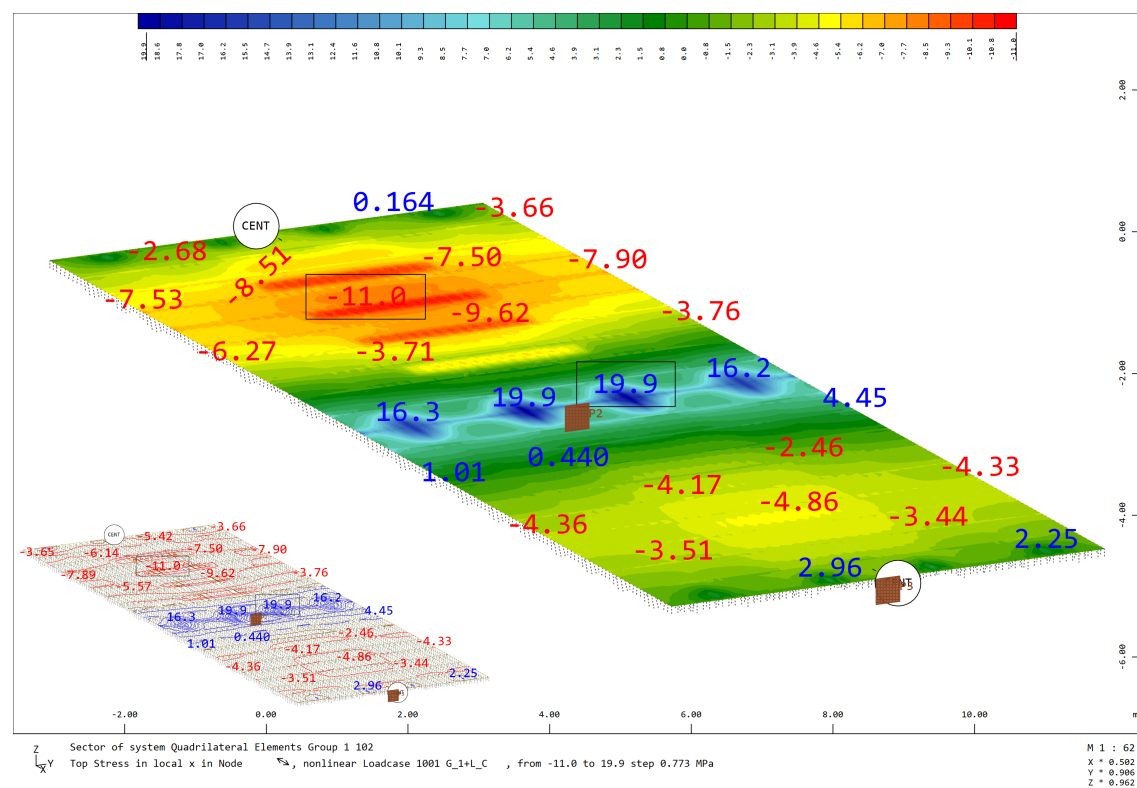
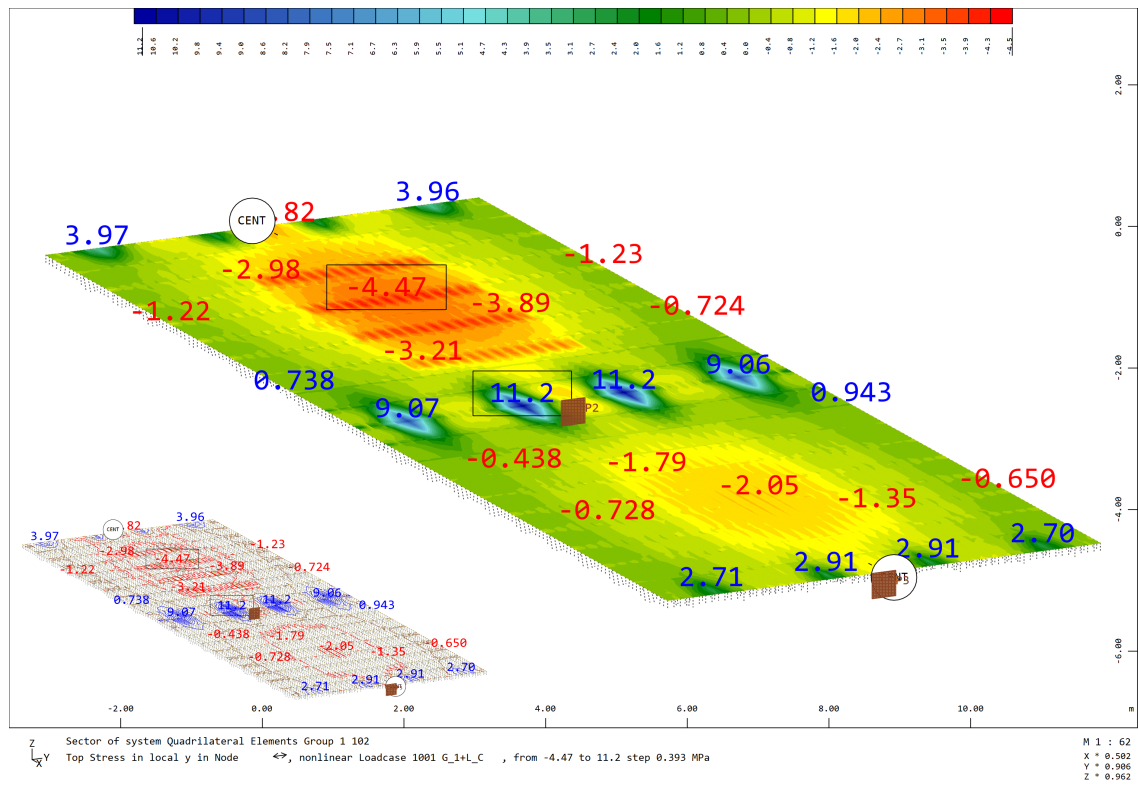


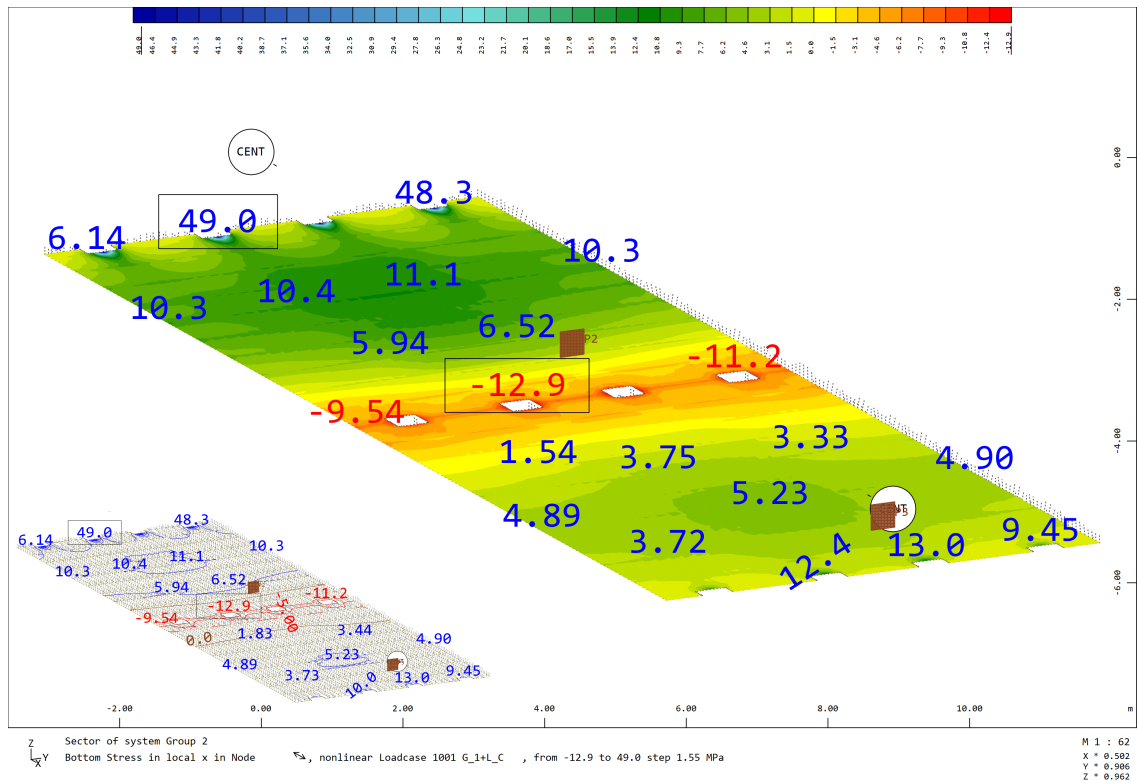
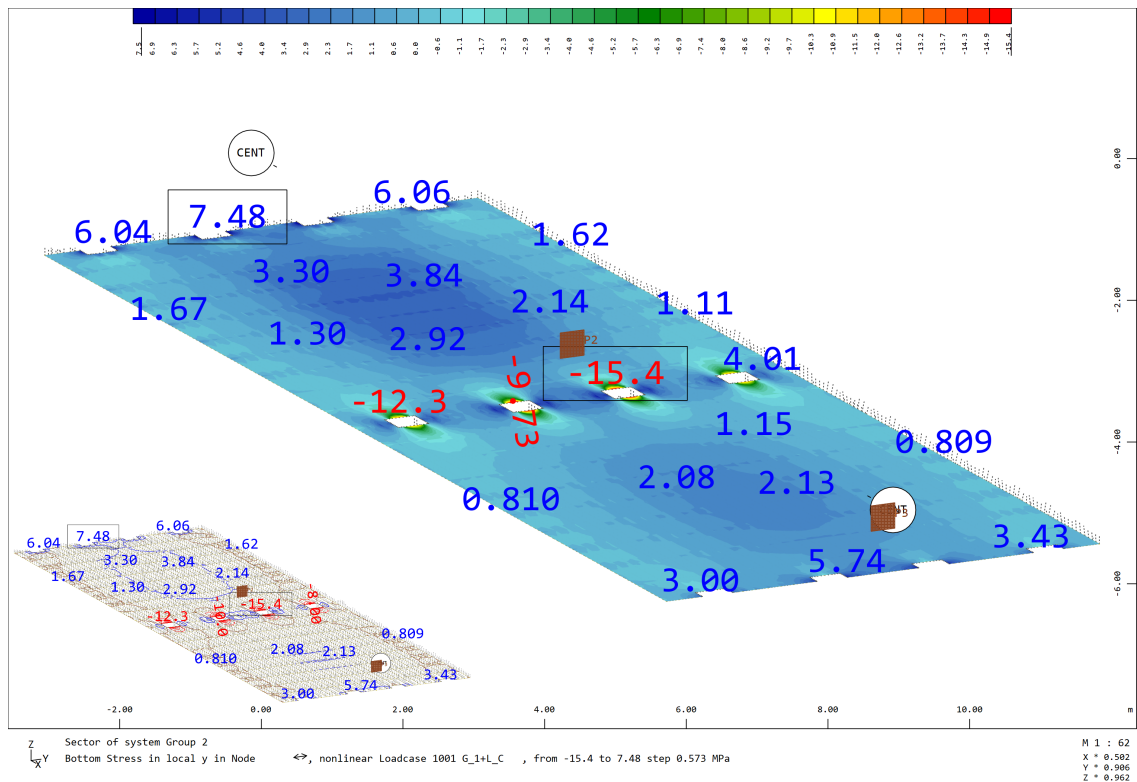
Figure G.1:  $\sigma_{xx}$  top Sandwich Bridge

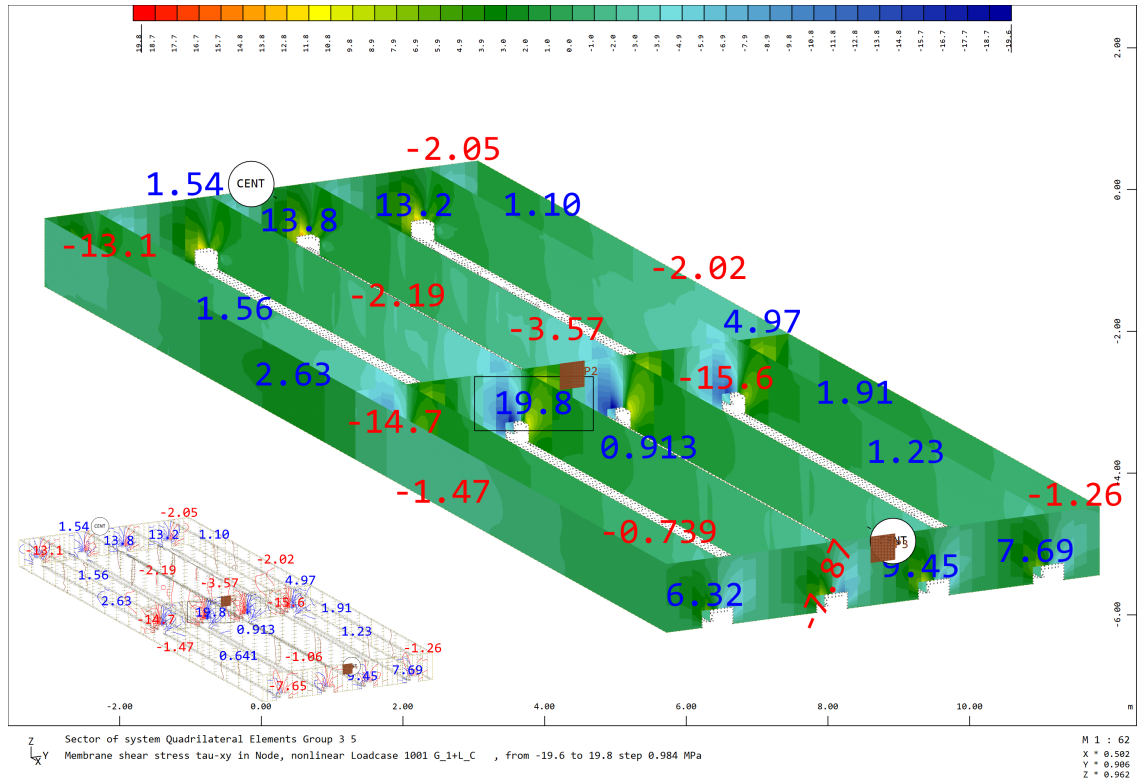
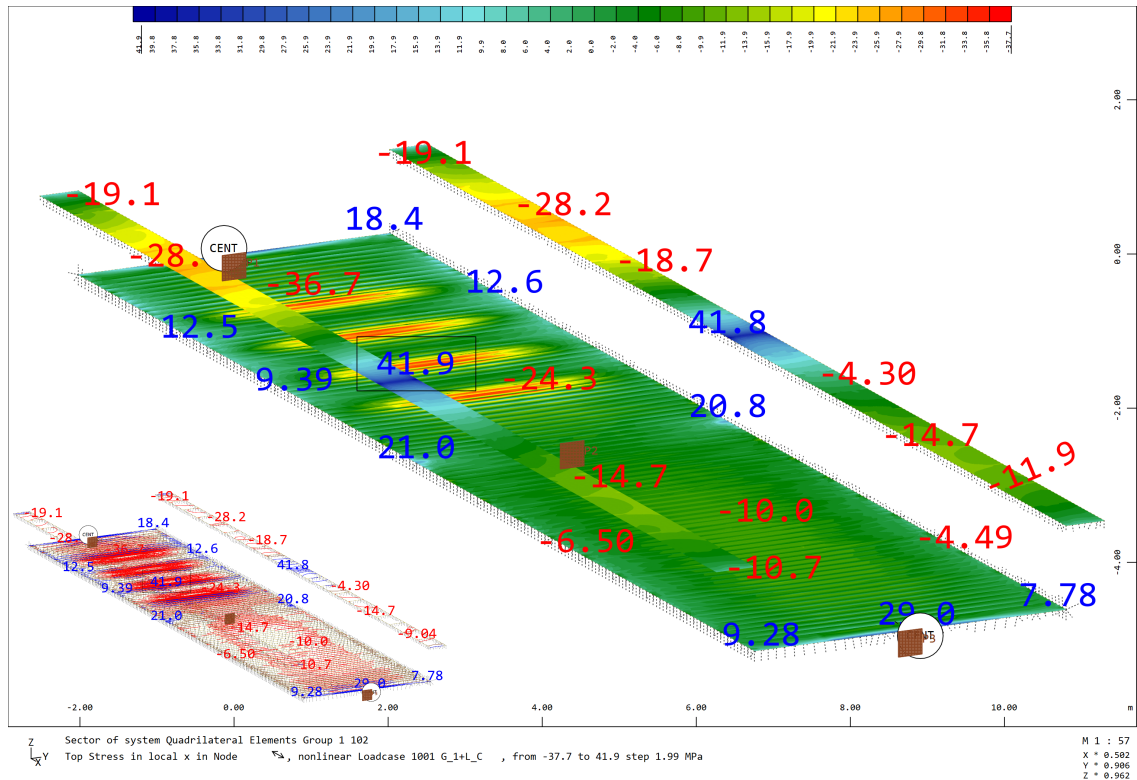


Figure G.2:  $\sigma_{yy}$  top Sandwich Bridge

## Through bridge

The ULS results of the Through deck bridge design are shown in figs. G.6 to G.10

Figure G.3:  $\sigma_{xx}$  bottom Sandwich BridgeFigure G.4:  $\sigma_{yy}$  bottom Sandwich Bridge

Figure G.5:  $\sigma_{xy}$  Sandwich BridgeFigure G.6:  $\sigma_{xx}$  top Through Bridge

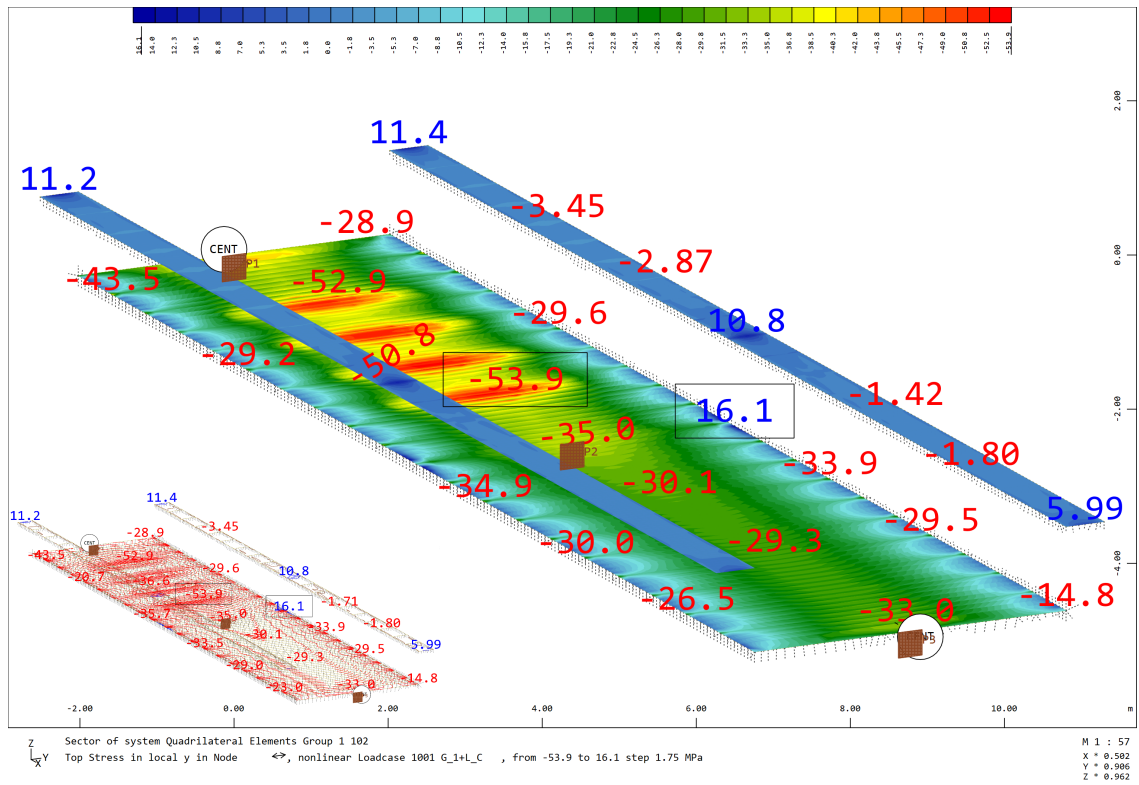


Figure G.7:  $\sigma_{yy}$  top Through Bridge

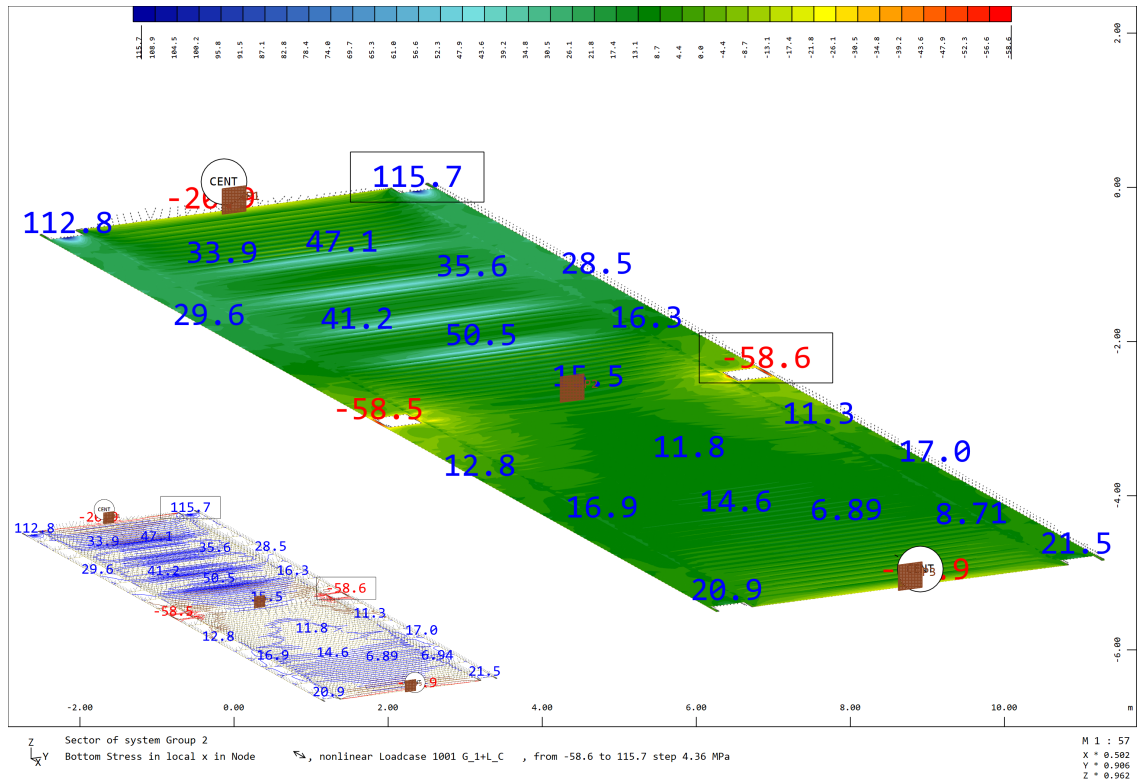
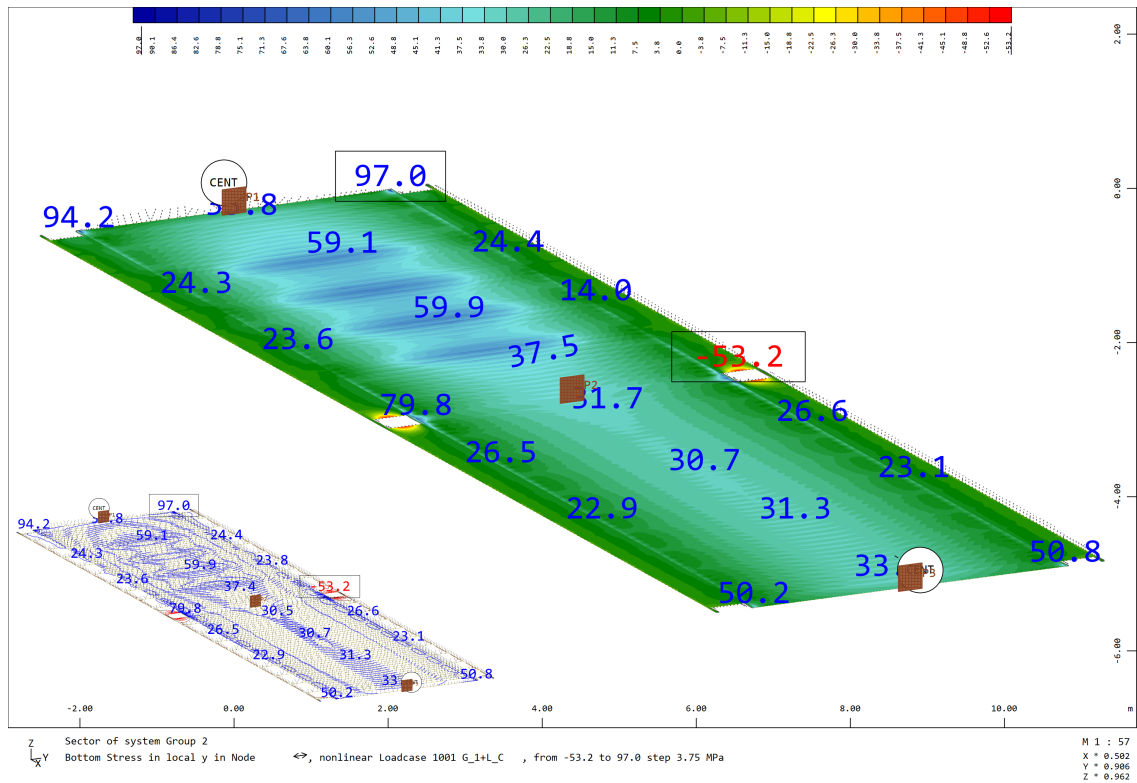
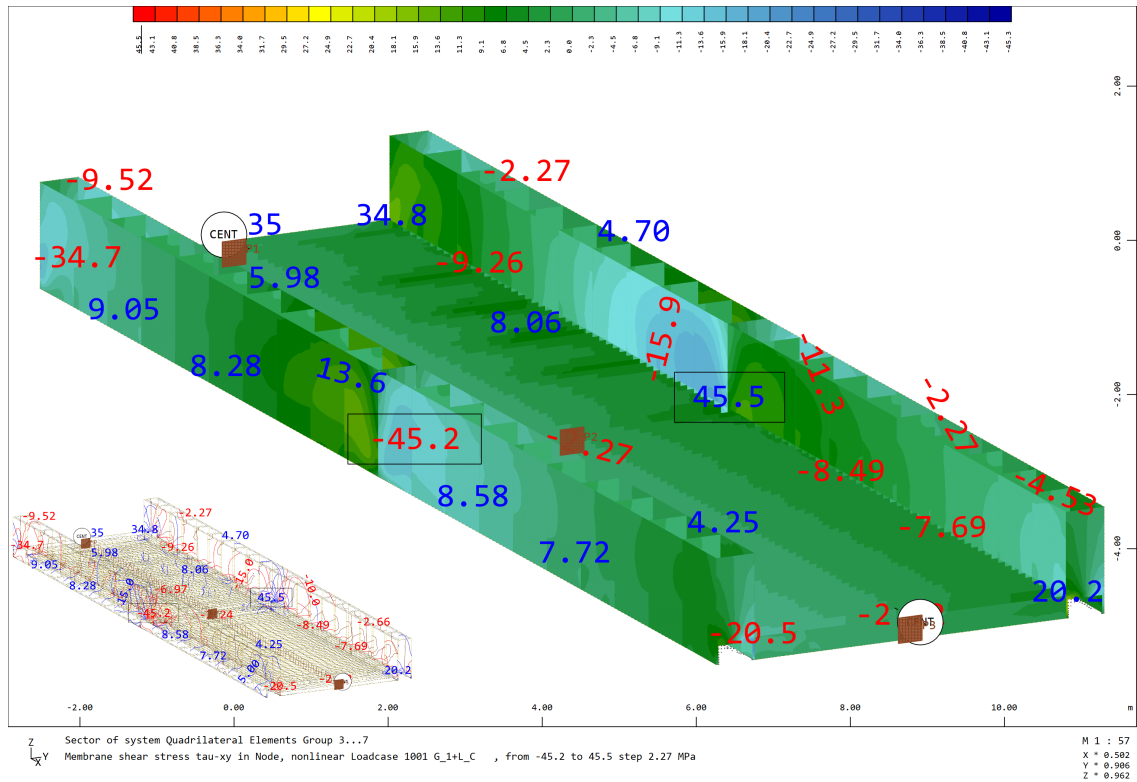


Figure G.8:  $\sigma_{xx}$  bottom Through Bridge

Figure G.9:  $\sigma_{yy}$  bottom Through BridgeFigure G.10:  $\sigma_{xy}$  Through Bridge

## APPENDIX H

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### ULS results of final design

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The ULS results of the final design are shown in figs. H.2 to H.6

According to Figure H.2 and Figure H.3 the largest compressive stresses in longitudinal direction (x-direction) occur at the top layers at mid-support and at the bottom layers at mid-span. Furthermore, large tensile stresses in longitudinal direction occur at the top layers at mid-span and at the bottom layers at mid-support. This is in accordance with theory, see Figure H.1. Large stresses are caused by large bending moments. According to Figure H.1 large bending moments are located at mid-support and mid-span, resulting in high compressive/tensile stresses at these locations. The stresses at mid-span of one of the spans is larger than the other. This is caused by the large point loads of LM71.

According to Figure H.4 and Figure H.5 high stresses in transverse direction (y-direction) occur at mid-span. The stresses at one of the spans are larger compared to the other span, caused by the large point loads of LM71. This is in accordance with theory. High stresses are caused by large bending moments. According to Figure H.1 large bending moments occur at mid-span (in transverse direction), resulting in high tensile and compressive stresses.

The largest shear stresses are located at mid-support, according to Figure H.6. The effect of the mid-supports are clearly visible by the switching of signs: compressive stresses turn into tensile stresses. This is in accordance with the theory. High shear stresses are caused by high shear forces. According to Figure H.1 high shear forces occur at mid-support. At mid-support the shear force diagram "jumps" from positive into negative.



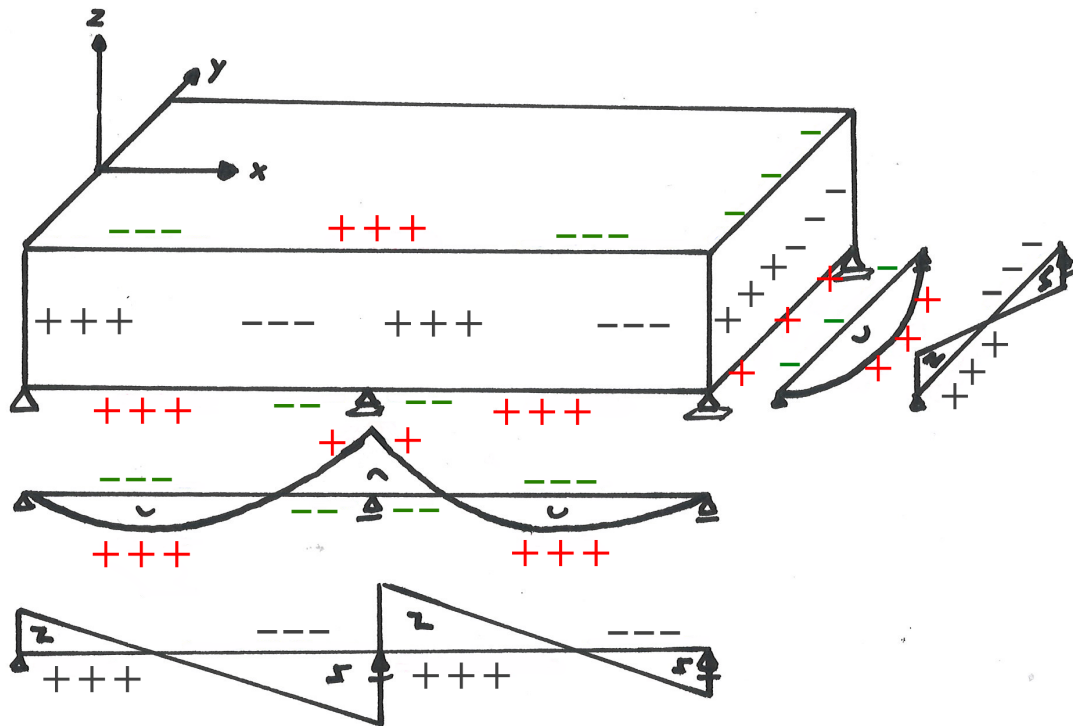


Figure H.1: Position of high bending moments and high shear forces  
Red: Compression, Green: tension, grey: shear

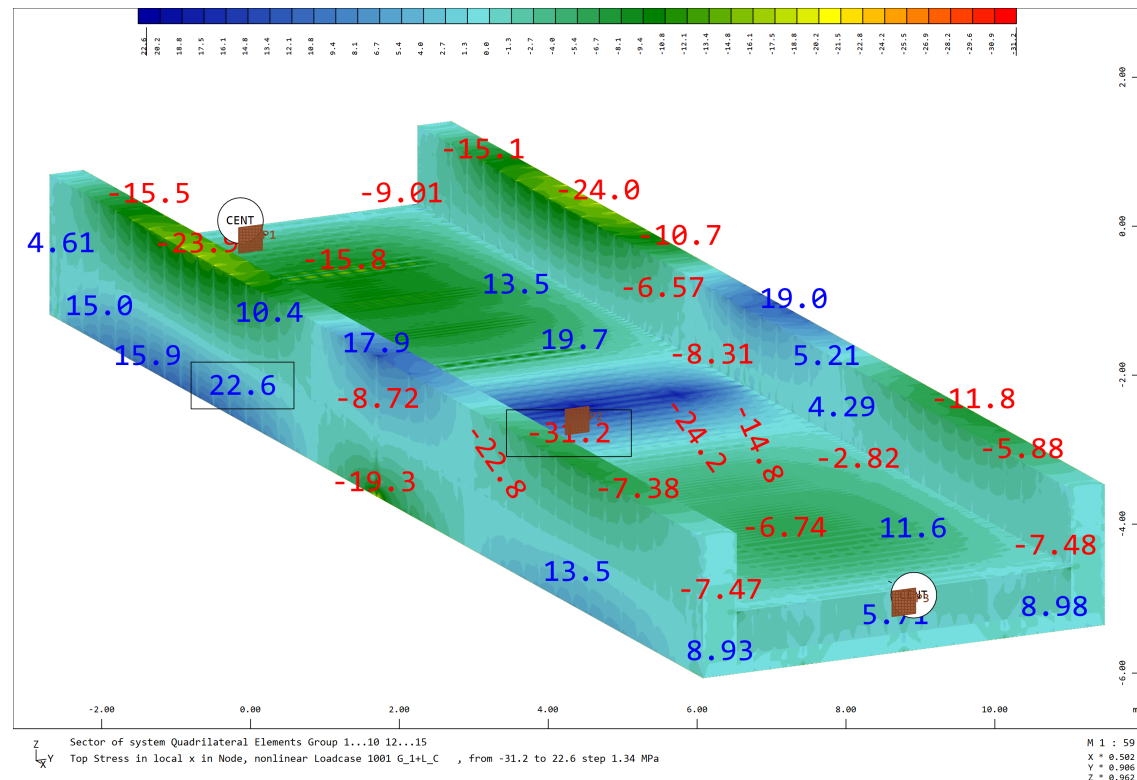
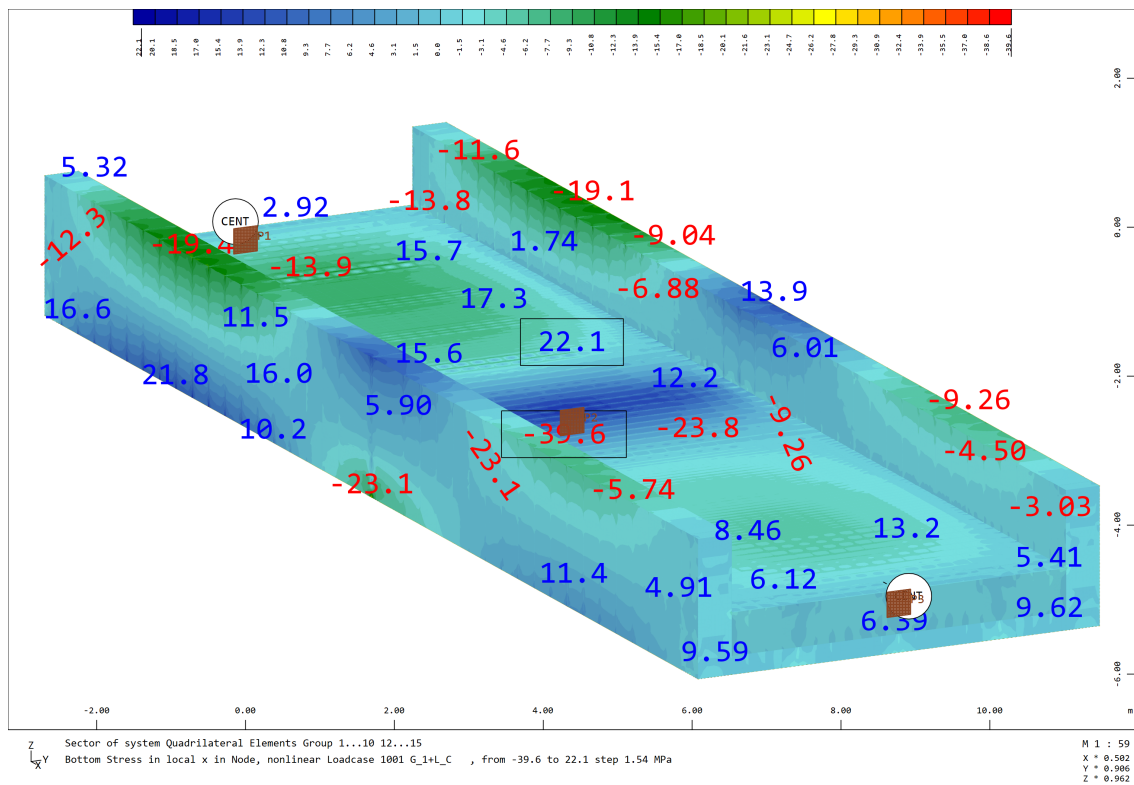
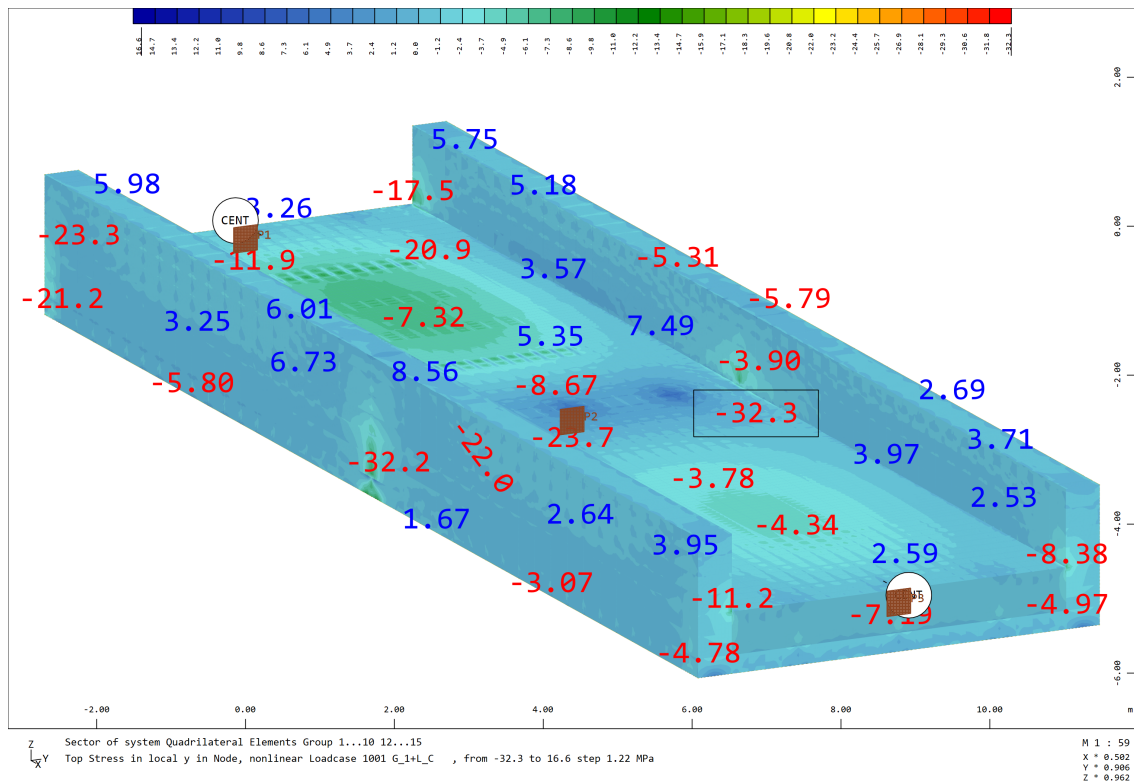
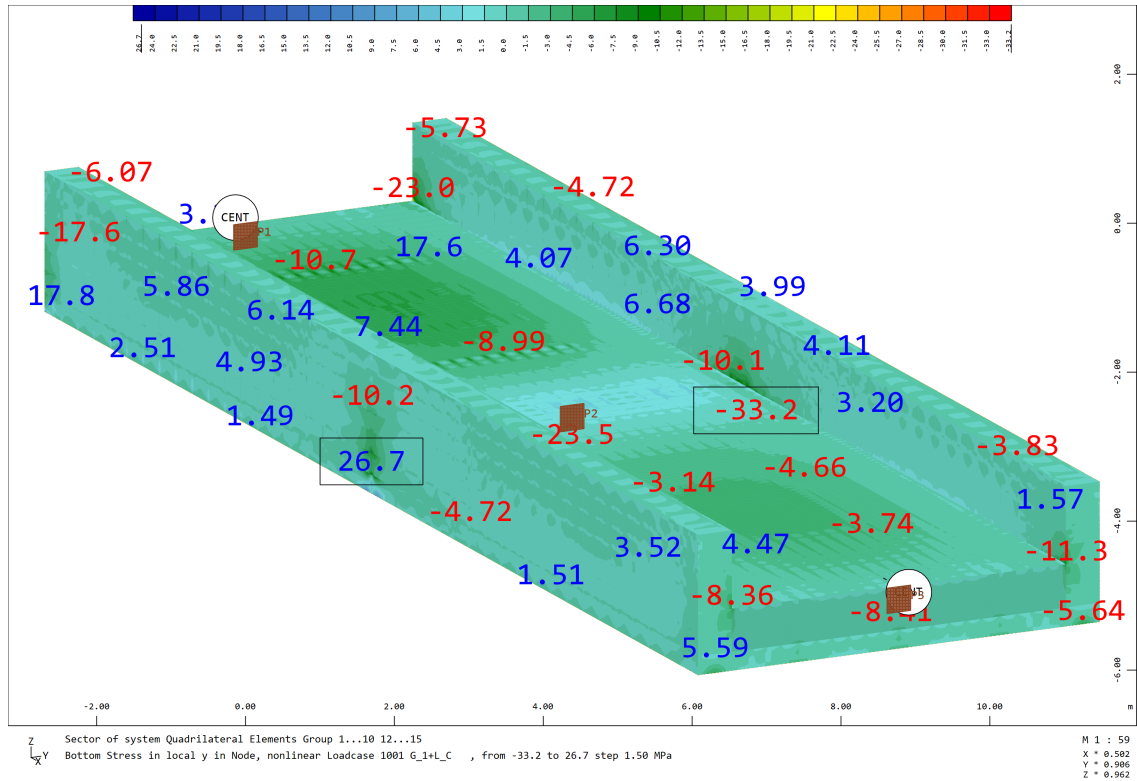
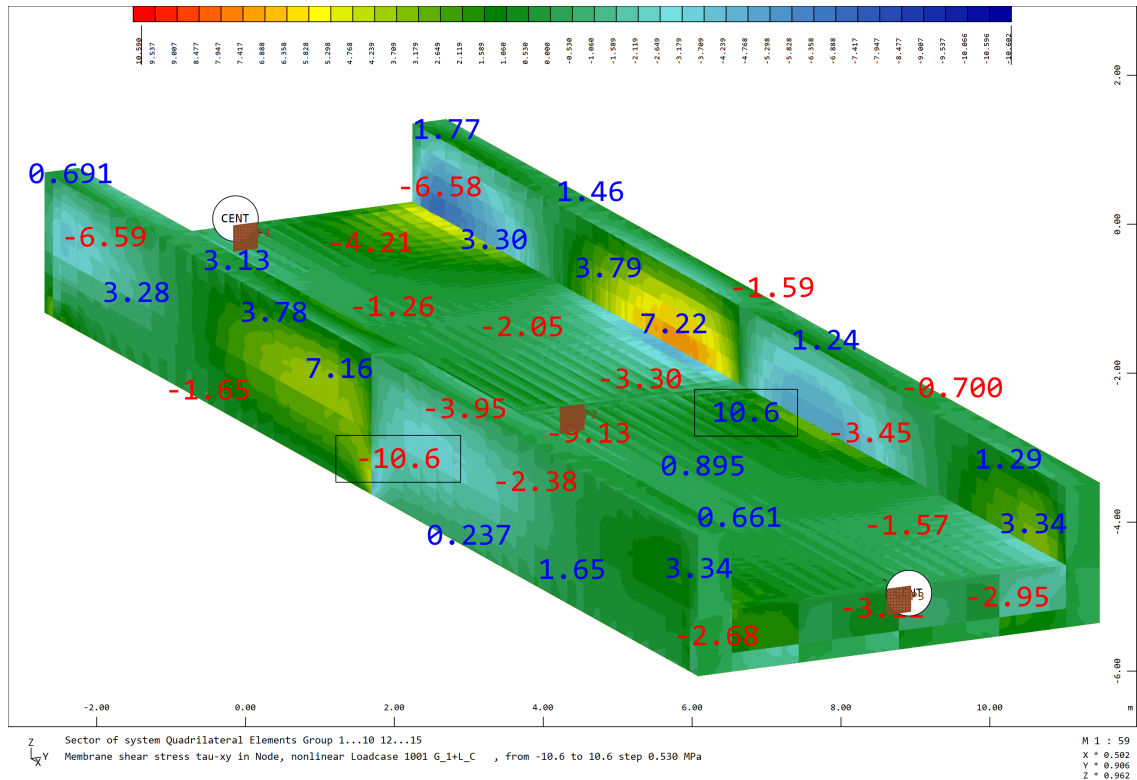


Figure H.2:  $\sigma_{xx}$  top Final design

Figure H.3:  $\sigma_{xx}$  bottom Final designFigure H.4:  $\sigma_{yy}$  top Final design



Figure H.5:  $\sigma_{yy}$  bottom Final designFigure H.6:  $\sigma_{xy}$  Final design



This work is the master thesis of H.P. Vijn as a partial fulfillment of the requirements to obtain the degree of Master of Science in Civil Engineering at the Delft University of Technology. This project was carried out in collaboration with BAM Infraconsult in the period between January 2019 and January 2020.

This thesis looks at the application of a FRP deck to be used in railway bridges. A material study is performed to obtain the best FRP material properties suitable for railway bridge decks. The static and dynamic behavior of two common FRP bridge decks is analyzed using finite element models resulting in a final design. The static and dynamic behavior of this final design is analyzed too. Based on the results it can be said that the static behavior of this final design meets the requirements. Further investigation in the dynamic model is needed in order to draw useful conclusions.