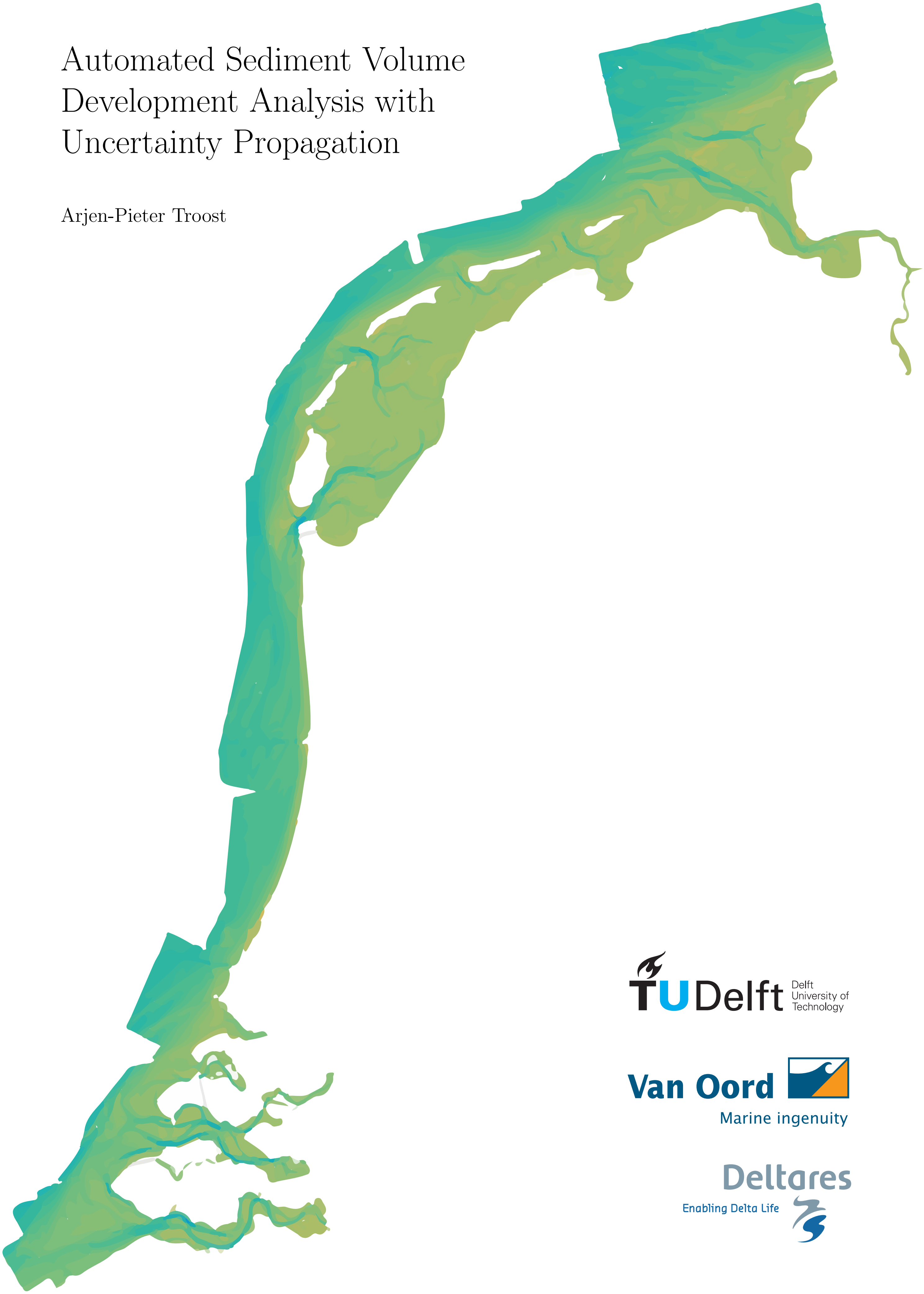
Automated Sediment Volume Development Analysis with Uncertainty Propagation

Arjen-Pieter Troost



Delft University of Technology

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Marine ingenuity

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Automated Sediment Volume Development Analysis with Uncertainty Propagation

by

Arjen-Pieter Troost

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An electronic version of this thesis is available at `http://repository.tudelft.nl/`.

Preface and Acknowledgements

This report represents my master thesis for the MSc Civil Engineering. During this project I was able to develop an algorithm that can be used for Hydraulic Engineering, the track that I followed for the MSc Civil Engineering at Delft University of Technology. The background and origination for the idea of the thesis, as well as the process of developing the algorithm are described in this paper. The use of this algorithm for the Dutch Coastal zone is visualised in various ways. It can be used as an additional way for risk assessment for contracting in the dredging industry.

This result was made possible due to the help of several persons, who I would like to thank for their help and support.
I would like to thank Mark van Koningsveld, for providing me the opportunity to start this project at Van Oord. His enthusiasm for the subject inspired me and resulted in making this research into a success. His previous research was the base for my project. Also I thank Gerben de Boer, who always challenged me and gave valuable insights in the complicated matter. I would like to express my gratitude to Mark and Gerben for giving me an interesting and up-to-date problem and giving me the confidence that I could handle it, resulting in an useful algorithm and this report.
Kees den Heijer always provided valuable comments during this research and gave critical feedback after reading this report, for which I would like to thank him. Professor Zheng Bing Wang, chairman of my graduation committee, I would like to thank for his his critical point of view during committee meetings and valuable input.

Finally, I would like to thank my friends and family that always supported me through this journey, from obtaining my VWO degree, BSc in Civil Engineering to my MSc degree from Delft University of Technology.

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Arjen-Pieter Troost

Abstract

Coastal regions change continuously due to natural processes. Human activities in coastal regions require a steady environment. Usually the dredging industry is contacted for coastal maintenance. Knowledge about the time development of sediment volume changes in specific regions is necessary for risk assessment in contracting.

Nearly a century of available bathymetric data of the Dutch coastal zone can be used. Two different approaches of calculations of sediment volume changes are currently available from literature. Both are mainly manual calculations and they do not generate confidence bands. We set out to create two automated approaches, to analyse sediment volume changes with confidence bands. Development consisted of four steps:

First, the two manual approaches were re-implemented as automated approaches. Unavailable data were calculated with a systematic and transparent algorithm filling the gaps in space and time.

Second, the automated approaches were validated by 16 different unit tests with synthetic data. The difference between the calculations of the synthetic data, manual compared to automated-approaches, were acceptably small, demonstrating the validity of the automated approaches.

Third, uncertainty was added to the automated approaches. Every step in the calculations to fill data gaps was identified and an uncertainty element added. This element merely points out the amount of uncertainty of the data. It is not a probabilistic uncertainty.

Fourth, the automated approaches with confidence bands were used for various Dutch coastal regions as case studies to show differences with the two manual approaches from literature.

These new automated approaches of sediment volume changes are validated, every step is transparent and reproducible and it generates confidence bands of the calculated data. Therefore it is a valuable additional input for risk assessment in contracting for coastal maintenance.

Samenvatting

Kustgebieden veranderen voortdurend als gevolg van van natuurlijke processen. Menselijke activiteiten in de kustgebieden vereisen een stabiele omgeving. Meestal wordt de baggerindustrie benaderd voor het kustonderhoud. Kennis over sediment volume veranderingen in specifieke gebieden is noodzakelijk voor de risicobeoordeling van aanbestedingen.

Bijna een eeuw aan bathimetrische gegevens voor de Nederlandse kust zijn beschikbaar. Twee verschillende methodes uit de literatuur worden momenteel gebruikt om sediment volume veranderingen te berekenen. Beide zijn voornamelijk handmatige berekeningen zonder onzekerheidsbanden. Wij hadden als doel gesteld om twee geautomatiseerde methodes te ontwikkelen, om sediment volume veranderingen met onzekerheidsbanden te berekenen. De ontwikkeling bestond uit vier stappen:

Ten eerste, de twee bestaande handmatige methodes zijn gebruikt om twee geautomatiseerde methodes te ontwikkelen. Een algoritme werd ontworpen om ontbrekende data op een systematische transparante manier aan te vullen in ruimte en tijd.

Ten tweede, de geautomatiseerde methodes werden gevalideerd door 16 verschillende unit tests met synthetische data. Het verschil tussen de berekeningen van de synthetische data, handmatig vergeleken met de geautomatiseerde methodes, waren aanvaardbaar klein, wat de validiteit van de geautomatiseerde methodes bevestigd.

Ten derde, onzekerheidsbanden zijn toegevoegd aan de geautomatiseerde methodes. Elke stap van de berekeningen om data aan te vullen werd geïdentificeerd waarna een onzekerheidselement werd toegevoegd. Dit element geeft alleen aan dat er een mate van onzekerheid van de gegevens aanwezig is. Het is geen probabilistische onzekerheid.

Ten vierde, de geautomatiseerde methodes met onzekerheidsbanden zijn gebruikt voor verschillende Nederlandse kustgebieden om het verschil zichtbaar te maken ten opzichte van de twee handmatige methodes uit de literatuur.

Deze nieuwe geautomatiseerde methodes van sediment volume veranderingen zijn gevalideerd, elke stap is transparant en reproduceerbaar en het genereert onzekerheidsbanden van de berekende data. Daarom is het een waardevolle extra input voor de risicobeoordeling in de aanbesteding voor kustonderhoud.

Contents

Nomenclature

2D Two-dimensional

3D Three-dimensional

4D Four-dimensional

Bathymetry Underwater topography; the study of underwater depths

BCL Basiskustlijn (Base Coast Line)

CSD Cross Shore Distance

DBFM(O) Design, Build, Finance, Maintain (and Operate)

dGPS Differential Global Positioning System

DONAR Data Opslagsysteem voor de Natte Rijkswaterstaat (Data Storage System for Wet Rijkswaterstaat)

ETL+P Extract-Transform-Load+Provide

GB Gigabyte

GHz Gigahertz

GPS Global Positioning System

GUI Graphical User Interface

JarKus Jaarlijkse Kustmeting (Yearly Coastal Survey)

KB Kaartblad (Fixed Map Grid)

Kc Kilocycle

KHz Kilohertz

LiDAR Light Detection and Ranging

Littoral zone Near shore area influenced by tide, also known as intertidal zone

LRK Long Range Kinematic

MCL Momentane Coastline

Morphology The study of migrating sediment

MLW Mean Low Water

M.Sc. Master of Science

NaN Not a Number

NAP Nieuw Amsterdams Peil (Amsterdam Ordnance Datum)

netCDF Network Common Data Form

Pixel Longitudinal and latitudinal datapoint in dataset

ROI Region of Interest

SI Standard Unit, international metric system used for measurements

TDS THREDDS Data Server

TCL Testing Coastline

VROM Ministerie van Volkshuisvesting, Ruimtelijke Ordening en Milieubeheer (Ministry of Infrastructure and the Environment)

V&W Ministerie van Verkeer en Waterstaat (Ministry of Infrastructure and the Environment)

Chapter 1

Introduction

The global increase of population is a worldwide problem that causes a more intense use of the coast. Coastal areas are a dynamic environment having constant change, while planning of human activities require the environment to be static and predictable. Erosion and accretion can therefore affect these human activities.

Coastal zones are defined as the part of land that is affected by being near the oceans and the part of the ocean that is affected by being near the land. A slow geological process developed these zones over millions of years where the coastal material can range from hard to soft material. The latter consists of sediment material ranging from mud, sand, gavel, cobbles to carbonate sands. These can be found in depositional regions like deltas, beaches and mud flats.

The coastal zone is not a static environment. Different intensities and characteristics of influences like wind, waves and tide can occur. Waves on the coast can vary in time and space (Holthuijsen, 2007).
Geomorphological timescales are large and are therefore complicated to forecast. In addition, sediment transport is a multifactorial process. Morphological changes that are of interest for coastal engineering have temporal scales of years to decades and spatial scales of 1 to 100 kilometre. This is also a timescale where human interventions can take place and have effect (Bosboom and Stive, 2013). Therefore the prediction of temporal and spatial processes in coastal engineering is based on limited knowledge of coastal processes (Marine Board, 1994).

When the total sediment volume in a coastal system is not in equilibrium and failure of this system has large economic implications, sediment management by maintenance becomes a necessity.

Sediment management is the process of studying the changes in coastal equilibrium and maintaining the coastal zone to prevent undesired consequences. Sediment management is one of the most important aspects of the dredging industry and is used as guidance for these maintenance works. Maintenance is necessary to prevent the unwanted accretion or erosion. This means that the coastal system has to be manually adjusted by in- or decreasing the sediment volume.

To manage sediment as adequately as possible, knowledge is needed about the behaviour of coastal zones. To increase the ability of the prediction of coastal morphology, temporal and spatial measurements are of utmost importance.

At present sediment management for the Dutch coastal is done with the available data. These data have been acquired since 1926 and have been edited, adapted and stored manually. These steps taken are often unclear because the procedure and measuring methods changed over the years (Rijkswaterstaat/Waterdienst, 2010). Therefore the reliability of these data is difficult to determine.

In the increasing competitive dredging industry it is crucial to invest in innovative solutions. Aside from the economics there is an increasing focus on maintaining a healthy environment for the future.

The government institution Rijkswaterstaat of The Netherlands has collected large surface bathymetric coastal data since 1926 (Elias et al., 2007). This research uses a case study of the Dutch coastal zone with almost a century of data available.

1.1 Aims of this research

A current trend is that dredging contractors start accept contracts where the maintenance responsibility for a certain coastal zone shifts from client to contractor for a long period. Therefore knowledge is required about the system and local behaviour of a particular region. This means that sediment nourishment quantity, location and interval information becomes valuable knowledge for planning, cost estimation and risk quantification of the maintenance period.

One of the challenges is to transcribe the available data in a generalised way, so these data can be used for exploitation for any given region and time period where spatial and temporal data are available.

Currently there is limited information available about the behaviour of coastal zones that are subjected to sediment management, the reliability of this information is often not clear (WL | Delft Hydraulics (2008)). Having insight in the uncertainty throughout the process is important. Therefore the attention should be focused on the source of the data, considering that the understanding can lead to improved propagation of uncertainty through the procedure.

The objective of this research is to investigate if it is feasible to create an automated data correction and extraction method (an automated-uncertainty approach) for sediment management, based on a systematic approach using the available bathymetric data. This automated approach should also be able to describe uncertainties of the calculations.

To determine the feasibility of the automated approach, a research question was formulated with three subquestions.

Research question Is there a systematic approach to extract data for the purpose of sediment management?

Subquestion How can the (un)reliability of the bathymetric data be defined, generalised and automated systematically?

Subquestion Is it possible to develop a fully data-based automated-uncertainty approach that uses only a small amount of manual intervention?

Subquestion If it is possible, how to control and even assure the quality of this automated approach?

1.2 Approach

The feasibility of the automated approach is assessed by the development of an algorithm. The (un)reliability of the data is included by isolating and accumulating assumed uncertainties. This automated approach was developed without having manual interventions, except for settings that have to be predefined. Quality of the automated approach is supported by means of software tests.

1.3 Structure of Report

In Chapter 2, the current practice of sediment management for the case study is provided. The development methodology of the generalised approach is elaborated in Chapter 3, while Chapter 4 explains the methodology and results for validating the tool. Results and application of the tool are provided in Chapter 5. In Chapter 6 a discussion is provided, together with the conclusions and recommendations of this research.

Chapter 2

Current Practice of Sediment Management: Case Study

2.1 Introduction

In this chapter the current practice of sediment management and the measured data that are available based on a case study of the Dutch Coastal Zone will be discussed. This information can be used to compare the practical application of the automated-uncertainty approach.

Figure 2.1: Kustfundament (Coastal foundation) of The Netherlands. Edited from Rijkswaterstaat (2007)

2.2 Sediment Behaviour

The Netherlands is a country in North Western Europe, bordering Belgium and Germany. Located in the Rhine-Meuse delta that is named after the two rivers flowing into it from the neighbouring countries. The delta is very well studied and has been formed by the progradation of the coast due to the output of a sediment rich Meuse and Rhine in the North Sea (Berendsen, 1998).

Human influences on the delta region have been significant since the Middle Ages and will continue. Currently it has a population density of 409 individuals per $[km^2]$. The area of the country totals 41,500 $[km^2]$ of which a large percentage is located below sea level (CBS (2012); CBS (2016)). Up to 60% of the country is prone to flooding and a system of dikes, dunes and continuous pumping prevent this (HH Rijnland, 2009).

The Netherlands has a large coastline with the North Sea that is protecting the hinterland from inundation. Ministerie van VROM (2006) states, that the kustfundament (coastal foundation) is defined starting from the z-value of -20$[m]$ with regarding to the vertical datum of NAP towards the coast on one side.

The other side is defined as the dikes, dunes including hydraulic structures present within this area. The kustfundament follows the Dutch coastal zone, including the Western Scheldt estuary in the south and the Wadden Sea in the north of The Netherlands. A visual representation is shown in Figure 2.1.

When a coastal beach profile is exposed to a regular wave motion it is expected that an equilibrium profile is reached in cross-shore direction. In case mean water level is disturbed by a storm surge level, the beach profile can fluctuate. An arbitrary cross-shore direction beach profile subjected to mean and storm surge levels with migrating sediment shown in Figure 2.2. The depth of closure is defined as the most seaward point of interest (Bosboom and Stive, 2013).

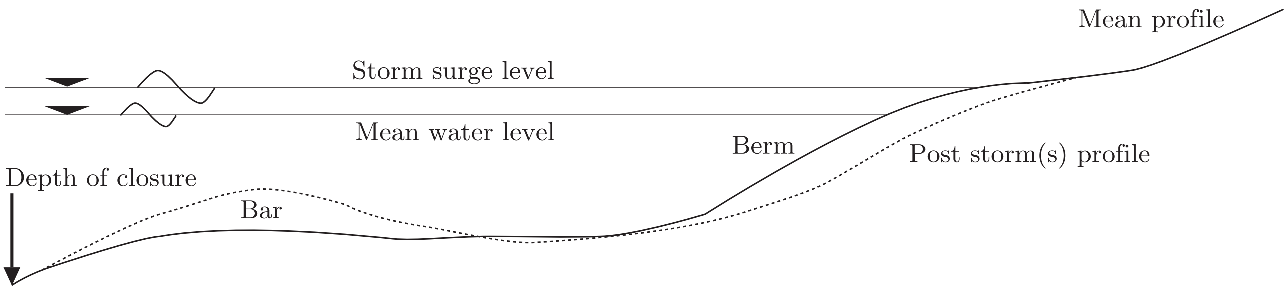


Figure 2.2: Mean and post storm(s) beach profiles. Edited from Kamphuis (2010)

The grain size of the sediment material found in the coastal zone ranges in diameter. Those grains have a different threshold of motion depending on the grain characteristics (Schiereck and Verhagen, 2012). Wave motion and flow can cause sediment to roll, slide or to go in saltation. These categories form the so called bedload transport. It is also possible for sediment to go into suspension, making it possible for grains to migrate. See Figure 2.3 for an example sediment bedload and suspended transport.

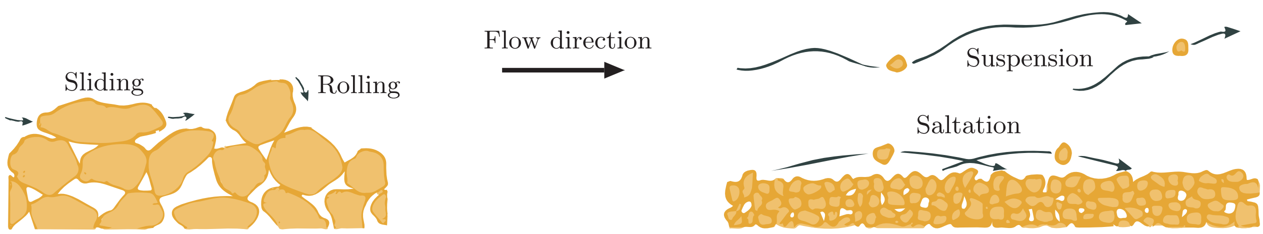


Figure 2.3: Bedload and suspended transport of sediment grains. Edited from Open University (2012)

Bedload or suspended sediment caused by the wave motion can migrate parallel to the coastline in a longshore current. This longshore current can be caused by the wave direction. Local erosion can then occur. This can only be solved when the volume of lost sediment on one side of the control volume is counterbalanced by sediment flowing into the control volume. On the left in Figure 2.4 an overview of a beach subjected to waves creating a longshore current is visualised. On the right a control volume is shown with input and output of migrating sediment on the right. The input of migrating sediment in the control volume is larger than the output, causing the migrating sediment to settle in the control volume. This leads to an enlarged sediment volume in the system.

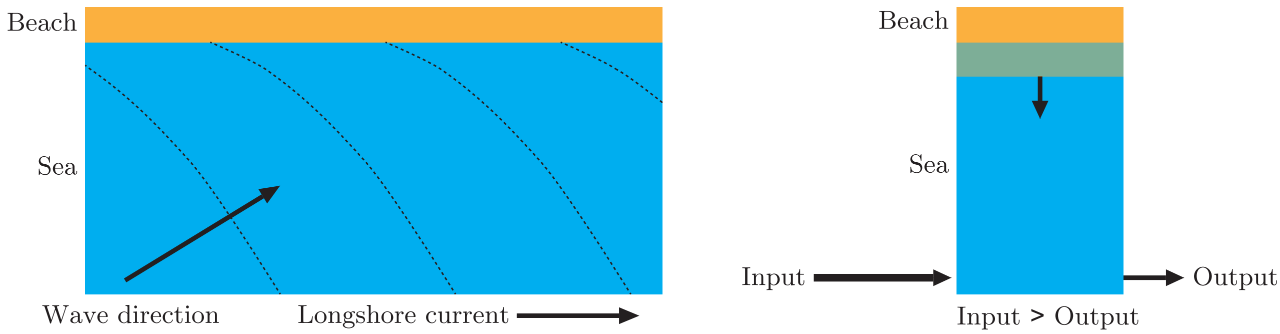


Figure 2.4: Left: Longshore current. Right: Control volume with accretion of beach

When the output exceeds the input, the total sediment volume in the system will decrease. This will lead to erosion and could have large implication on the coastal zone. Morphodynamic changes to the bathymetry can then occur when these external influences exceed the threshold of motion of the coastal sediment grain size. This change in morphology could cause changes in the external hydrodynamic influences and amplifying the process of morphological change.

The study of the change of the physical shape of the coast by natural or nourishment influences is coastal morphology.

2.3 Current Strategy

In order to create a coastal region that is protected against inundation, it is important to maintain the area between the land and the sea. This is quite a complicated task because it is a consideration between preservation of coastal developments and keeping low hinterland risk with a high safety level. Between the coastal dune, acting as a flood defence, and the sea is a location where several buildings are located. This is an area where no official safety standards are applicable.

The legal limit on a coastal region such as this is called the Base Coast Line (BCL; basiskustlijn). It is defined as the 1990 Dutch coastline and developed to maintain the coastline (Kaspers and Waanders (2002); Dillingh et al. (2010)). See Figure 2.5 for an example of a defined Base Coast Line.

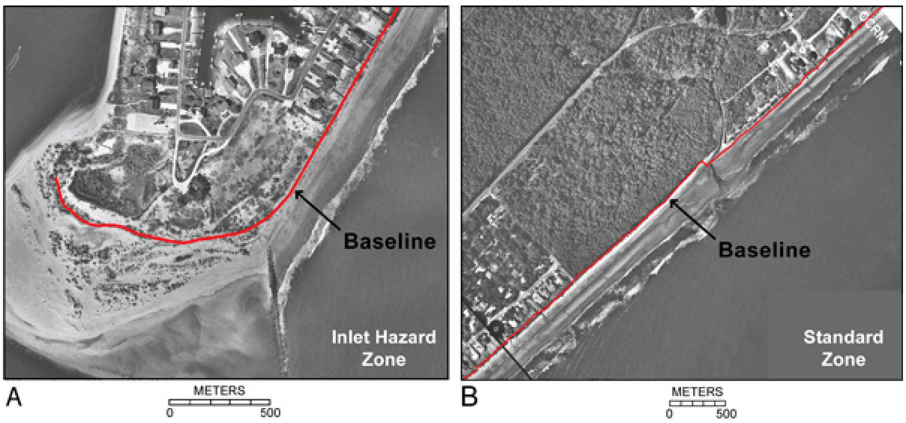


Figure 2.5: Two Base Coast Lines in South Carolina (Schwab et al., 2009)

Rijkswaterstaat is the government agency in The Netherlands responsible for maintaining infrastructure, including the coastal area. They have an annual sand budget of 12 million $[m^3]$ to be nourished along the Dutch coastline, according to a multiple year predefined program (Rijkswaterstaat (2009); Rijkswaterstaat (2015)). See Figure 2.6 for an example of a beach nourishment.

Figure 2.7 shows a schematic cross-shore representation of an arbitrary coastal zone that is subjected to erosion and accretion. After a certain time period a volumetric increase of sediment is visible that is created by a beach nourishment, which is a maintenance operation (Verhagen, 1992). This implies that sediment is relocated by a vessel from an extraction site - in seaward direction of the depth of closure from Figure 2.6 - towards the location that has to be maintained.

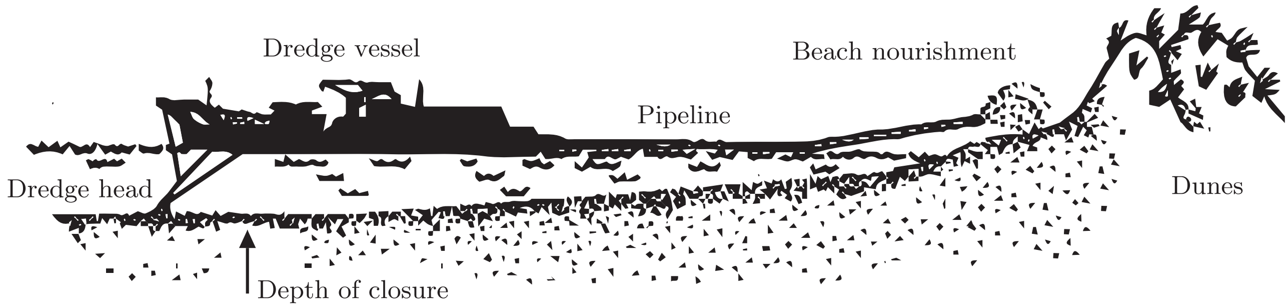


Figure 2.6: A coastal zone subjected to maintenance (not to scale). Edited from Ecomare (2015)

This is a procedure frequently used on the Dutch coastal system, to comply with the BCL. For the Dutch coastal system this level is defined as the 01 January 1990 coastline (de Ruig, 1998). It is described as the Dynamic Preservation Policy in Ministry of V&W (1990) and guarantees sustainable preservation of values, safety and functions in the coastal dune terrain.

Currently measurements are done on a regular basis of the Cross Shore Distance annually. When this dataset is visualised a trend line is created through the measurement locations. These are extrapolated so that an expected coastline position is created. This trend line is named the Testing Coastline (TCL) and if this trend line passes the minimum Base Coast Line that is defined as the 01 January 1990 coastline, intervention has to take place by means of a beach nourishment before the projected time that the trend line passes the BCL. This is called the Momentane Coastline (MCL) concept (TAW (1995); van Koningsveld et al. (2004)).

There are other reasons for beach nourishments as well, such as compensation for losses because of structural erosion or accretion. Next to that the safety of the hinterland can be maintained against natural hazards such as flooding, as well as protection for the littoral zone (Bosboom and Stive, 2013).

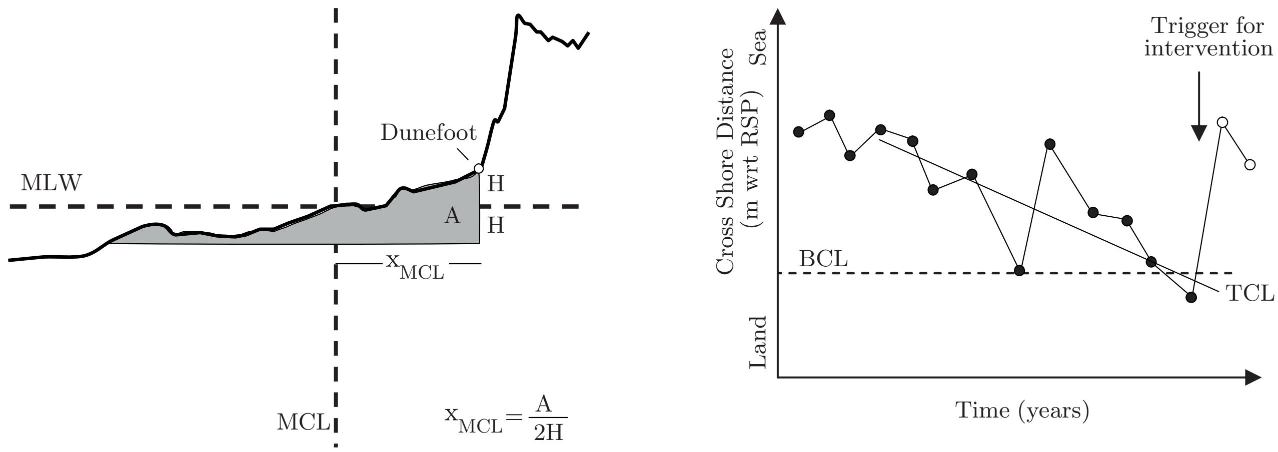


Figure 2.7: Left: Example for Momentane Coastline (MCL) concept. Right: Example for TCL compared to BCL with intervention trigger. H is the vertical difference between the dune foot and the Mean Low Water level (MLW). The value x_{MCL} is the distance between the dune foot and the Momentane Coastline (MCL). Edited from van Koningsveld et al. (2004) and van Koningsveld and Lescinski (2006).

2.4 Measurements

2.4.1 Methodology

Rijkswaterstaat performs bathymetric measurements on the Dutch coastal system to follow the erosion and accretion of the beach and nearshore seabed. This underwater topography contains information about the four-dimensional (4D) space: the three geometrical coordinates and fourth of time.

An area where research is conducted on the bathymetry in a certain period is called a campaign. This can be conducted in a large or either small area along the coast.

Research from Elias et al. (2012) states that since 1925/1936 sufficient detail is present in the bathymetric data of Rijkswaterstaat to conduct coastal volume calculations. Therefore analysis can be carried out on these data. Datasets like these are large and do not give any practical information. Processing or visualisation is necessary before they can put in to use.

Currently several datasets are available from Rijkswaterstaat: JarKus, LiDAR and Vaklodingen. These can be used for sedimentation-erosion volume calculations.

JarKus

JarKus are annual bathymetric measurements on predefined cross-shore transects. The output is a two-dimensional profile that can be used efficiently to detect erosion or sedimentation phenomena on a local level, because of the stable periodic measurements.

See Figure 2.8 for visualised JarKus data plotted in Google Earth. For more information see OET (2016) or Ministerie van BZK (2016).

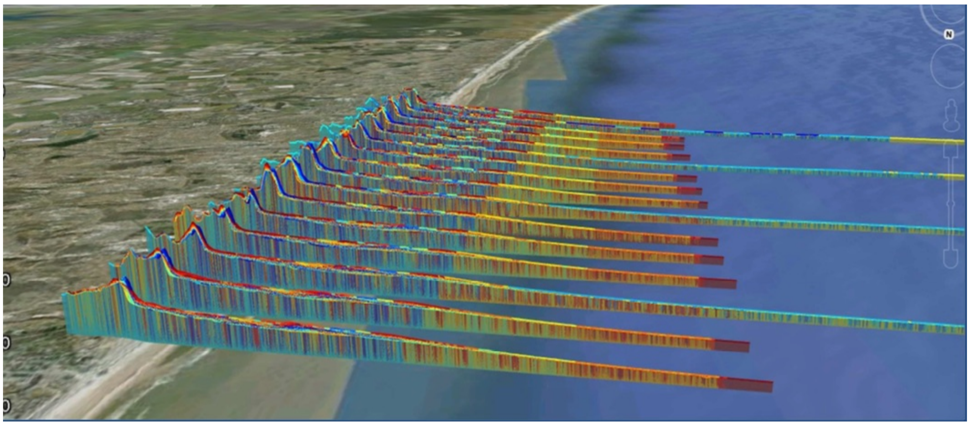


Figure 2.8: JARKUS data plotted in Google Earth (Pot, 2011)

LiDAR

The information of the coastal areas such as the beach and dunes is measured by means of LiDAR, which stands for Light Detection and Ranging. This is a remote sensing method as to be seen in Figure 2.9 that uses a pulsed laser to measure distances between the measuring device and its surroundings. Usually a device like this is placed on an aircraft and together with exact (d)GPS data a LiDAR device can generate high quality surface information about the Earth (NOAA, 2015) (Rijkswaterstaat/Waterdienst, 2010). LiDAR can only be used for areas that are not covered by water, since this technique is not fully able to penetrate water. Nevertheless, LiDAR can be used for estuaries during low water levels when a significant amount of water is drained from the estuary.

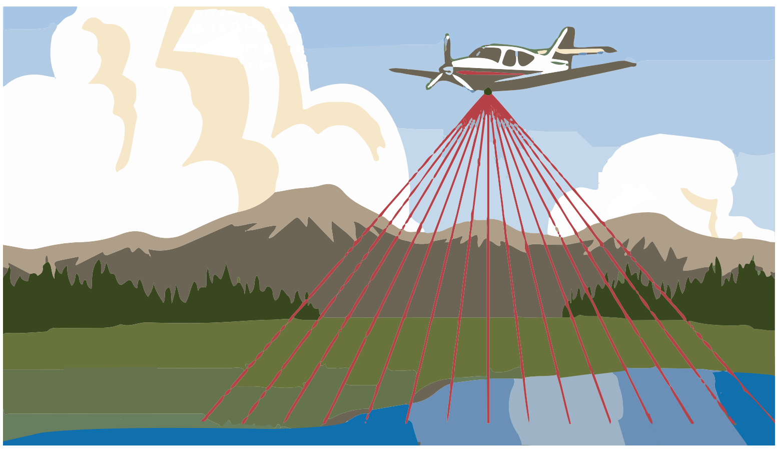


Figure 2.9: LiDAR example. Edited from Elec-intro (2016)

Vaklodingen

Vaklodingen measurements contain three dimensional data. These consist of the geographical longitude, latitude and z-value. Each vakloding covers an area of $10 \cdot 12.5[km]$, with a resolution of $20 \cdot 20[m]$. These are developed for a single time period, so many different vaklodingen create a four dimensional dataset.

The vaklodingen intervals depend on the amount of measurements, ranging between every year to once in six years. This depends on the dynamics of the specific region (van der Zijp et al., 2001).

The benefit of vaklodingen over JarKus is the higher accuracy that can be achieved with the 3D vaklodingen. See Figure 2.10 for two vaklodingen with geographical longitude, latitude and z-value. The left and right contain data from two different years, respectively 1968 and 2007.

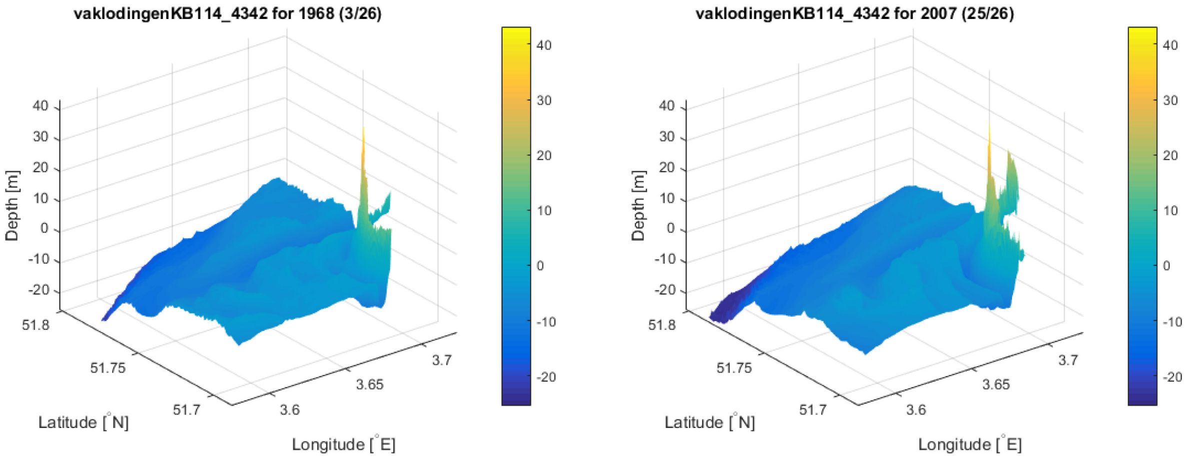


Figure 2.10: Two high-coverage datasets of vaklodingen KB114_4342 visualised

2.4.2 Reliability

In Perluka et al. (2006) and Wiegmann et al. (2002) detailed information about measurement errors that can be found are elaborated. It also provides data about the methodology of measurements.

Errors can be caused by the vessel position, the echo-sounder, the reference depth, heave/pitch/sway or settlement of the vessel, the sound speed in water or the squat. In Figure 2.11 these errors are shown. Errors created by morphological features as gully slopes are shown in Figure 2.12.

Perluka et al. (2006) and Wiegmann et al. (2002) define the reliability combination of the measurements to be between $0.11[m]$ and $0.40[m]$. The following sections will elaborate on how these values have been formulated.

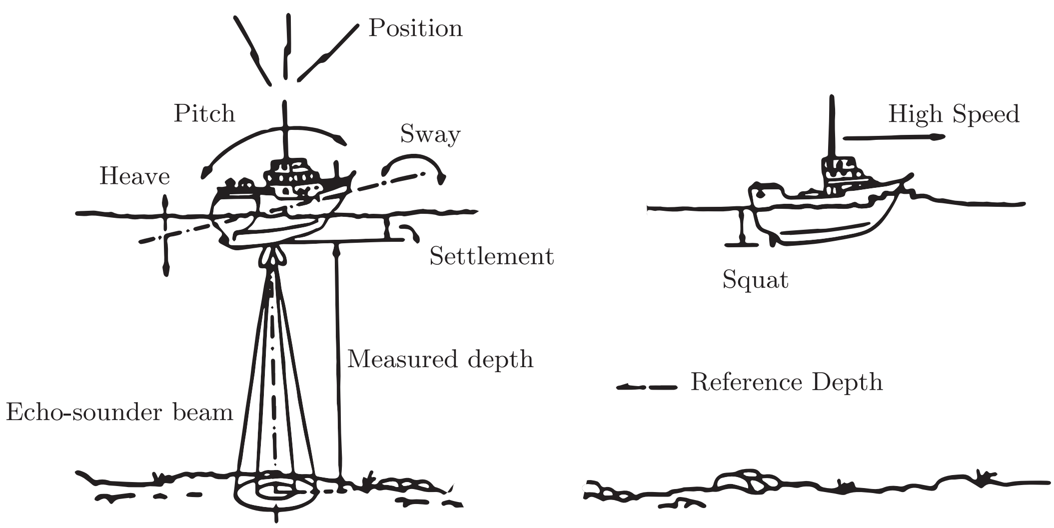


Figure 2.11: Possible errors during measurements. Edited from Marijs and Parée (2004)

Position

Wiegmann et al. (2002) found out that over time the reliability of positioning quality of the measurements increased by a factor 100. Therefore the positioning of the earliest bathymetric campaigns are much more unreliable in a positioning perspective, compared to recent survey campaigns.
Techniques improved as follows:

- Before 1979: Vessel travelling on a distance-line.
- 1979-1990: Radio-based positioning system.
- 1990-2000: (d)GPS based positioning system.
- After 2001: Satellite positioning system based on RTK (Real Time Kinematics).

Reference Depth

Calculation of the reference depth based on the local water depth used to be the standard procedure. Nowadays the reference depth is based on satellite technology (LRK) in combination with a ground support station (DGPS) (Marijs and Parée (2004); Wiegmann et al. (2002)).
Techniques improved as follows:

- Before 2000: Water depth as reference depth.
- After 2001: Reference depth based based on LRK-DGPS.

Heave, pitch and sway

Movement of the vessel can be compensated when the geometry of the vessel is taken into account. The position of the sensor compared to the vessel important to compensate this (Marijs and Parée, 2004).
Improvements were as follows:

- Before 1990: Simple calculations were done for compensations.
- At present: 3D tachymetric measuring of vessel and exact calculation of vessel to echosounder.

Echo-sounder

Accuracy of the echo-sounder needs to be as high as possible. Over time the reliability of the measurements increased by a factor 2 (Wiegmann et al., 2002).

Morphological gully slopes can induce too shallow measurements by the width of the echo-sounder. The modern measurement angles of single beam sensors are $\alpha = 2.5°$. This means that for example on a depth of $10[m]$ measurements are conducted with a width of $0.35[m]$. The measurement that should be used has to be in the middle of the beam (true depth), but this is not the case.
The registered depth is the one nearest to the sensor on the high side of the slope (shortest depth), creating an error (Marijs and Parée, 2004). See Figure 2.12 for a visualisation of this error.
The measurements were conducted as follows:

- Before 1955: Measurements by hand.
- 1955-1993: $210[Kc]$, Echo-sounding angle of measurement range between 8° and 12°. The measurement error $= 0.15[m]$ too shallow.
- After 1993: $710[Kc]$, Echo-sounding angle of measurement equals 2.5°. The measurement error $= 0.04[m]$ too shallow.

Calibration has to be conducted before using the echo-sounder. The sound speed in water is an important part of this calibration. This calibration has to be completed by measuring the speed of sound in water over a fixed measuring distance (Marijs and Parée (2004); Wiegmann et al. (2002)). The error developed by this is assumed to not vary over time because of this procedure.

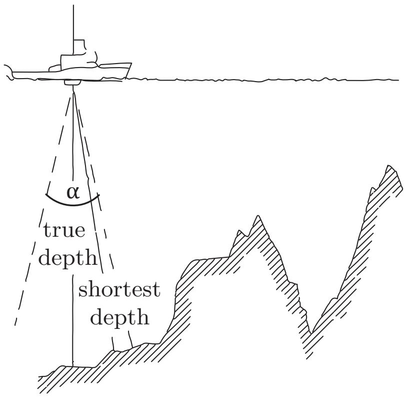


Figure 2.12: Error of echo-sounder near gully slopes. Edited from Marijs and Parée (2004)

Settlement and Squat

A vessel transporting a large mass can create settlement of the vessel and creates measurements that are too shallow. Next to that lowering of the sensor on the vessel influenced by sailing speed and water depth. The measurements register incorrect low values by result of this error. The most Rijkswaterstaat vessels have squat values between 0.05[m] for large pontoons and 0.40[m] for streamlined slim vessels. Corrections are applied in the Zeeland region of the Dutch Coastal Zone since 2001/2002 (Marijs and Parée, 2004). This correction is the continuous measurement of the height of the echo-sounder to the reference depth by using LRK-DGPS. The measurement error equals 0.25[m] for the gully and slope. For shallow areas a value of 0.15[m] needs to be applied.

2.4.3 Data

The fixed map system, in Dutch called kaartblad or KB, is a system where the campaign data are projected on. Campaign data are divided into small rectangular pieces and delivered into this fixed map system, that consists of many different blocks that have a preset and permanent name. The data in a fixed map tile has outer measurements of $10 \cdot 12.5[km]$. Within each fixed map tile an array of $500 \cdot 625$ data points can be found, which makes every measuring block $20 \cdot 20[m]$ in size. This will be further explained in Chapter 3.
See Figure 2.13 for a fixed map system of the Wadden Sea in the Dutch Coastal Zone, with Regions of Interest of the Texel (TX), Eierlandse Zeegat (ELD), Vlie (VLIE), Ameland (AME), Friesche Zeegat (FRZ) and Eems-Dollard (ED) basins and coasts (respectively inside and outside Wadden estuary).
The vaklodingen data are stored in this system, while the JarKus data has been interpolated to fit this system as well. This means that both the Vaklodingen and the JarKus information are stored in the same fixed map tile system (Elias et al., 2007). Collecting these datasets will be from public websites such as the Deltares TDS (THREDDS Data Server). Rijkswaterstaat has a digital storage facility where this information is stored in named DONAR (Data Opslagsysteem voor de NAtte Rijkswaterstaat).

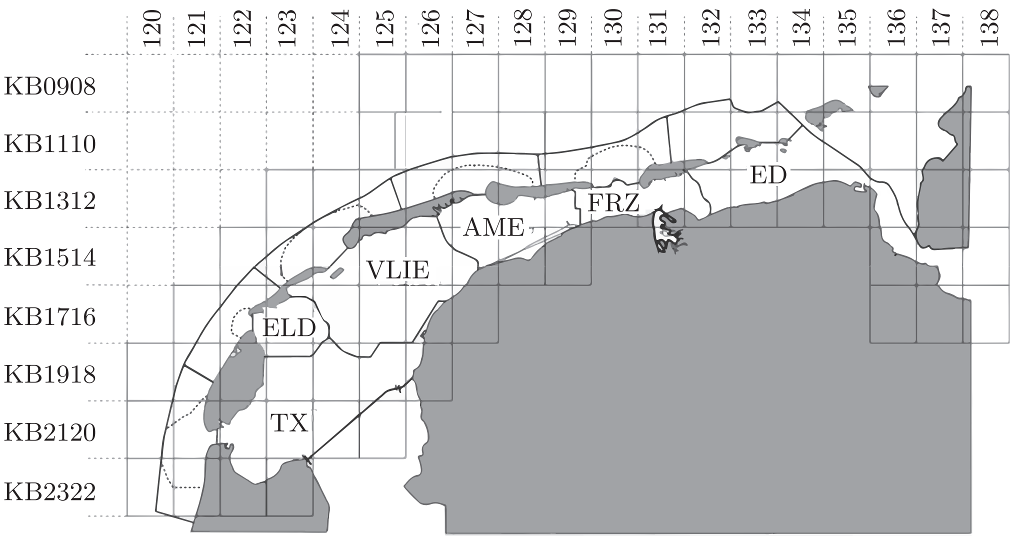


Figure 2.13: Fixed map tile for the Wadden Sea. Edited from Elias and Wang (2013)

The disadvantage of this system is that a process has been executed on the raw data that is often not completely known and only the output is saved in the fixed map tile datasets. A large number of manual steps were done and the procedure and measuring methods changed over the years (Rijkswaterstaat/Waterdienst, 2010). The approach is currently executed in the following way:

Raw data [often unavailable]	+	Procedure(t) [often unknown]	=	Data product(t) [permanently stored, unknown reliability]

It should not be a problem if the raw data retrieved from the campaigns were stored next to the data product that is actually saved in the fixed map tiles, but unfortunately that is not always the case.
The present sediment management method is based on the data product stated above, extracted from the raw data with a certain manual procedure that is not generalised. It could be of good quality but could also have manual untraceable changes, negatively benefitting the current sediment management procedure.

2.5 Previous Research

There has been research on expected sedimentation-erosion changes for the Dutch Coastal Zone. There are two different approaches to be distinguished that have been elaborated in previous research, namely the Spatial- (S) and Temporal- (T) approach. These methods are visualised in Figure 2.14.

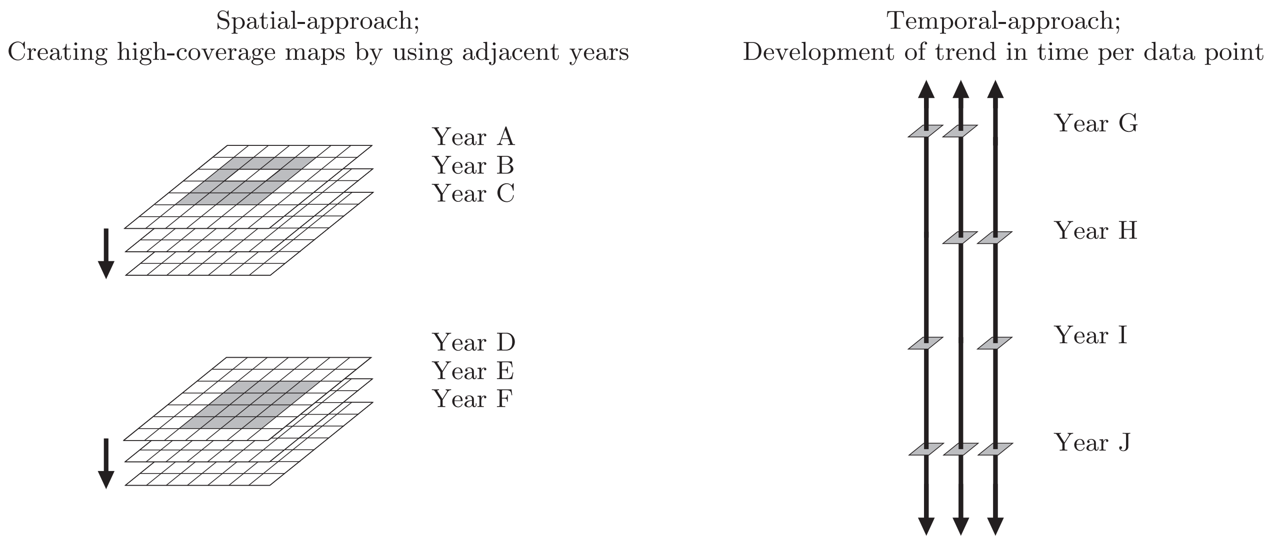


Figure 2.14: Operation procedure of S- and T-approach

2.5.1 Spatial-approach (van Koningsveld et al., 2008)

A sediment budget analysis of the Dutch Coastal System with a focus on the Wadden Sea was described by van Koningsveld et al. (2008). The research method steps were developed with the S-approach. The steps taken are applied to the example in Figure 2.14 and defined as follows:

1) All fixed map tiles within the Region of Interest were identified for a predetermined 'Year A'. One grid is created to project the identified map grids on. To create high-coverage maps, backward map filling is used to fill blank spots with a maximum search window of two years to 'Year B' and 'Year C'.
2) Repeat of step 1 but this time for 'Year D', with search window including 'Year E' and 'Year F'.
3) Data from 'Year A' are subtracted from map filled 'Year D' to generate a map of changes. No-data values returned in the difference map at grid points where both or individual data values are unavailable for map filled 'Year A' or 'Year D'.
4) Determining the overall volume change for a given period from the map of changes.
When this process is repeated for multiple years the volume development over time for a Region of Interest was determined and plotted in a graph.

Uncertainty has been added to this research by means of a discrete data coverage threshold. Three classes of data coverage were identified: Good >= 90%, Fair >= 75% & < 90% and Poor < 75%. These were plotted as different pixels in the volume development over time.
The highest data coverage year is defined as the reference map for all volume computations.
Validation has been done by comparing data to the research of Walburg (2005).

Advantages

This method is generally applicable. That means that it can be projected on any given region when data are available and a Region of Interest is defined within this region.
Discrete uncertainty is taken into account by using differences in map coverage.
No data intervention has been done manually, which makes the result reproducible and thus transparent.

Disadvantages

The article of van Koningsveld et al. (2008) refers to a debate in the accuracy of the bathymetric data. It is not possible to see uncertainties for this research for the S-approach.
The S-approach fills the data gaps in space. E.g. morphodynamic changes by tidal channels can be recognised with this approach but this has not been elaborated.
Spatial filling after map filling is not done, therefore difference quantities also depend on the coverage area and can influence the volume difference when applied to a full Region of Interest. E.g. when a low coverage map is subtracted from a high coverage map, a low map of changes is created that results in the use of a small surface and thus a small volume difference.
It might be possible that this analysis used an alternative grid resolution. This means that larger data point surfaces than $20 \cdot 20[m]$ have been used, resulting in higher calculation uncertainties. Very low coverage maps are still projected as a pixel in the volume development over time. This pixel is visually plotted in a different way, but all data coverage below 75% have the same pixel. This can develop a deviating result because low coverage means lower use of surface and thus a lower volumetric difference.
This method is partially automated, not entirely between the data/Region of Interest input and the expected sedimentation-erosion changes.
No uncertainty in confidence bands is present for (un)available data.
The model is validated with another model, not to synthetic datasets that can be analysed analytically.
This procedure created a high memory pressure for computations, because all intermediate data were stored in memory. Therefore not all computers are suitable for using this procedure.

2.5.2 Temporal-approach (Elias et al., 2012)

The research of Elias et al. (2012) focused on the morphodynamic development and sediment budget of the Dutch Wadden Sea over the last century. The research method was as follows:

1) For each 'vaklodingen' block a sequence of raw data maps were compiled for the available years.
2) Each individual map in the sequence was visually inspected and missing single data points were corrected by triangular spatial interpolation and data outliers were removed.
3) Incomplete datasets were filled in using linear interpolation between nearest datasets available in time (larger gaps) or using internal diffusion from the nearest spatial points (smaller gaps).
4) Sedimentation-erosion trends were obtained by subtracting subsequent measurements.
5) The sequences of sedimentation-erosion maps were inspected and maps with unrealistic trends in basin or ebb-tidal delta changes were manually deleted. In total approximately 10% of all data were reanalysed and corrected or deleted and therefore not traceable.

A model was developed based on the T-approach. This was based on the following steps: *Following construction, inspection and correction of the individual datasets, yearly maps for the period 1935-2005 were generated by linear interpolation between the available datasets. Since the yearly data are interpolated on the same grid, a straightforward subtraction of the datasets with a set starting year (1990) provides sedimentation-erosion values.*

Advantages

The research of Elias et al. (2012) was based on excellent knowledge implementation of the Dutch Wadden Sea. Therefore the manual deleting of uncertain data points on the datasets prior to calculation with the T-approach will probably be of good quality.
Clear volume change results for Wadden Sea are available between 1935-2005, which makes comparative research feasible.

Disadvantages

- The T-approach fills the data gaps in time. For example morphological changes by tidal channels are difficult to recognise with this approach.
- No validation has been done for the calculations. Removing data manually is not a problem when it is exactly clear where the data were deleted. When this is not reproducible, the transparency of the research becomes questionable.
- When data are removed, the total available dataset reduces. Therefore the amount of unavailable data increases. This unavailable dataset was filled and therefore created a calculation error.
- The uncertainty of manual data removal has not been included in the results. This applies for the uncertainty of temporal interpolation as well.
- The method is not automated and generally applicable.

2.5.3 OpenEarth Approach

OpenEarth is a free and open source initiative to deal with Data, Models and Tools in earth science & engineering projects, currently mainly marine & coastal. OpenEarth aims for a more continuous approach to data & knowledge management (van Koningsveld et al., 2010). The four basic criteria for OpenEarth are as follows:

- Open standards and open source
- Complete transparency
- Centralised access
- Clear ownership and responsibility

Within OpenEarth the ETL+P (Extract, Transform, Load and Provide) protocol for data collection is visualised in Figure 2.15. This will be defined as the OpenEarth Approach.

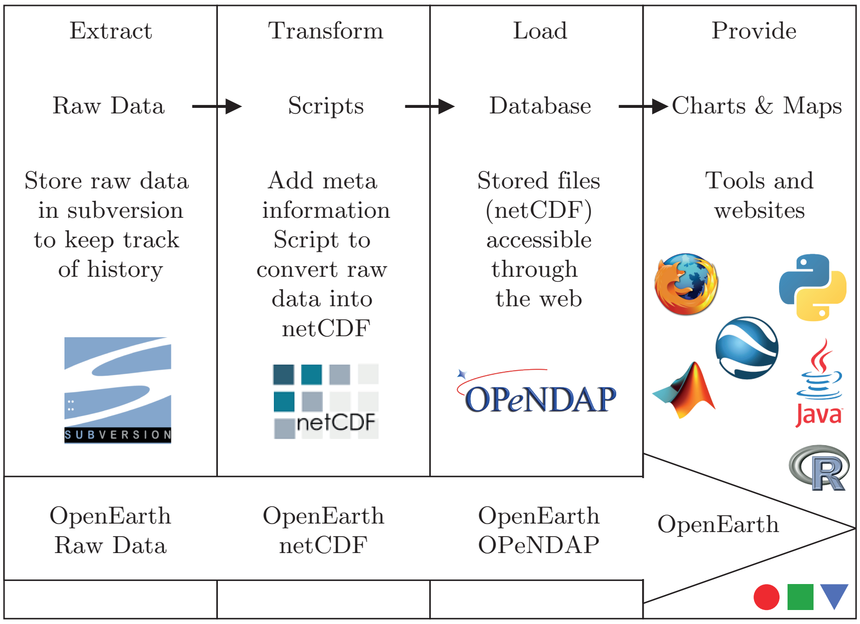


Figure 2.15: OpenEarth Approach. Edited from (van Koningsveld et al., 2010)

2.6 Summary

This section gives an elaboration on the current practice of sediment management.

Bathymetric data analysis has currently been done with two approaches. The volume analysis for the Spatial-(S) approach is based on creating high-coverage geographical maps for each year that z-value data are available. Subtraction is done based on the year with the highest coverage.

On the other hand the volume analysis for the Temporal- (T) approach develops a z-value trend for each data point through all years by interpolation. Subsequently an expected reliable reference year is used to subtract.

Since the data that are used cover almost a century, old data are bound to have a higher uncertainty because of the unavailability of high quality tools. During this century better techniques were developed and research created new insights in how to increase measurement accuracy.
The data are saved in fixed map tiles that can be used for data analysis. These consist of geographical longitude, geographical latitude, z-value and time information.

Entire datasets can potentially have absolute errors because of the unknown procedures to modify raw measurement data to data product.

Using the data products for the case of the Dutch Coastal System is inevitable. When using the data, the absence of knowledge about the quality of the data product has to be taken into account.

Knowledge from this case study will be used as a basis to develop the new automated-uncertainty approach.

Chapter 3

Model Development

3.1 Introduction

The previous chapter elaborated on the research that has been done on expected sedimentation-erosion changes for the Dutch Coastal Zone. Input for this research were 4D fixed map tiles with geographical, z-value and time information. The second input was a Region of Interest with geographical information for the specific region of interest.

The methodology of measurements of the case study over time proved to have uncertainties. After applying a certain procedure the data were stored. These were not the raw data directly from the survey, but the output after unknown manual changes had been conducted. Therefore unknown uncertainty is present in the data products, that are available as input for this research.

Previous research that was conducted based on the Spatial (S)- and Temporal (T)-approaches used respectively discrete and no uncertainty propagation. Their advantages and disadvantages have been explained in Chapter 2.

To provide a substantiated answer on the objectives of this research, an automated-uncertainty approach will be created. Within the automated-uncertainty approach the strategies of the S- and T-approaches will be used but redesigned from scratch. This chapter will provide elaboration on these automated-uncertainty S- and T-approaches.

3.2 Challenges

For the automated-uncertainty approaches the following questions need to be addressed.

Can the advantages and disadvantages of the S- and T-approaches be taken into account, to develop the best possible automated-uncertainty approach?

The automated-uncertainty approach should be made to be generally applicable; it needs to be able to be projected on any given region when data are available and a geographical Region of Interest is defined within this region. Is this possible?

To increase the persuasiveness of this automated-uncertainty approach, validation should be applied on the expected sedimentation-erosion changes. Is this feasible?

Is it possible to have no manual intervention between the in- and output of the automated-uncertainty approach?

All calculation results need to be reproducible. Can the automated-uncertainty approach be transparent?

Uncertainty propagation; previous research refers to a debate in the accuracy of the bathymetric data. Is it possible to automatically detect uncertainties in the available data? In addition, is it possible to project uncertainty on the unavailable data?

Engineers have to create advice for strategic decision makers for sediment management. Therefore these individuals should be able to use the automated-uncertainty approach, to keep the line to decision makers as short as possible. These engineers have to be able to give the correct input, run the automated-uncertainty approach and to create output. No large amount of computer knowledge should be needed to be able to understand this automated-uncertainty approach. Is this feasible?

A regular commercially available computer that is provided by organisations to their employees therefore need to be able to run the automated-uncertainty approach. To use the large datasets in the automated-uncertainty approach, computer memory management should also be taken care of to ensure the calculating ability. This should not have any implication on the process outcome. Can this be done?

An automated analysis has to be performed in an acceptable time period for practical application. Therefore the time periods for running the automated-uncertainty approach should be based on non-utilisation period of the computer during work hours. Is it possible to have the order of magnitude for calculation as follows?

- Lunch break: 30 minutes
- Night: 12 hours
- Weekend: 60 hours

Also, will it be possible to create the automated-uncertainty approach process as follows?

Input An environment where a geographical Region of Interest can be defined as input. Next to that 4D bathymetric data should be available for that particular region. Settings can be manually predefined for the procedure.

Procedure Import of relevant 4D data with a generalised method to analyse the sediment volume difference over time in that specific Region of Interest. Uncertainty detection so that reliability information about the (un)available data can be formed and presented as confidence bands.

Output Expected sedimentation-erosion changes together with uncertainty information as an output for the entire system over time.

3.3 Procedure

The process of the automated-uncertainty approach is based on the OpenEarth Approach shown in Figure 2.15. In contrast to the OpenEarth Approach, the process used for this automated-uncertainty approach will not use Raw Data as its source, but Data Products that will be extracted and transformed into Data Products 2. See Figure 3.1 for a visualisation of this process:

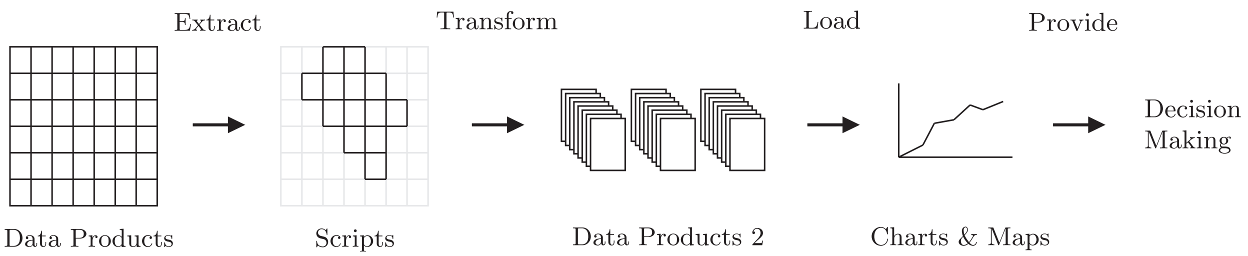


Figure 3.1: Automated-uncertainty approach process based on the OpenEarth Approach in Figure 2.15

The data used for the automated-uncertainty approach will mainly focus on the gridded 4D vaklodingen data stored in fixed map tiles. It encloses four dimensional data, consisting of the two geographical coordinates longitude and latitude, time and z-value data. These fixed map tiles will be defined as the Data Products.

Subsequently the Data Extraction will take place. During this procedure solely the necessary data will be filtered by programming Scripts. This will only be done by using a Region of Interest with geographical coordinate (longitude and latitude) data and this procedure is further elaborated in Section 3.4.

Hereafter the Data Transformation will change the data such a way that calculations can be performed for its specific goal. This procedure will use the four dimensional data within the Data Products. The output of this operation consists of Data Product 2 and will be stored. See Section 3.5 for more information.

The Data Loading can then take place. During this step the Data Product 2 will be used to find quantitative expected sedimentation-erosion changes by using the geographical coordinate (longitude and latitude), z-value and time data. The output of this procedure will be a figure, visualising the expected sedimentation-erosion changes over time including uncertainties that propagated through the calculation. See Section 3.6.

Finally the figure can be interpreted and potentially unveil a pattern. When this is done for multiple geographical coordinate Regions of Interest, systematic behaviour can potentially be discovered. This result can then be used for Decision Making.

3.4 Data Extraction

Data Extraction is a 2D methodology, where only the longitudinal (x-values) and latitudinal (y-values) data are of importance. The goal of Data Extraction (Figure 3.2) is to distinguish fixed map tiles within the geographical Region of Interest, from fixed map tiles outside the geographical Region of Interest. These fixed map tiles are defined as Data Products.

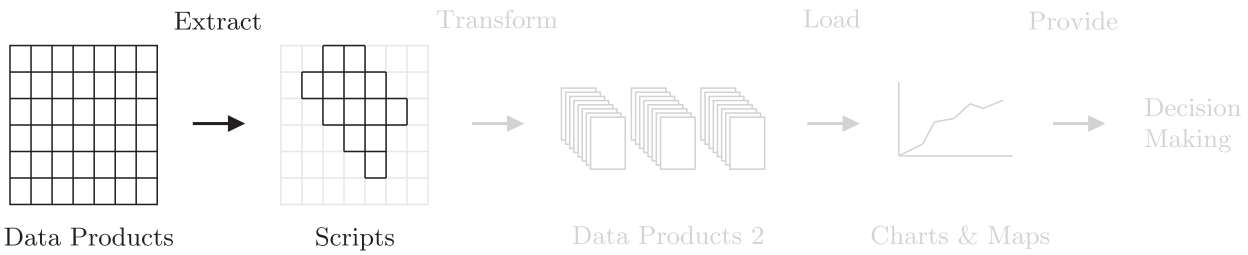


Figure 3.2: Data Extraction script process for an arbitrary task

3.4.1 Data Points

Each Data Product is stored in a netCDF Data Product. This single Data Product with dimensions based on geographical longitude and latitude information is shown in the centre of Figure 3.3. Vertically 625 Data Points are located within this netCDF container, horizontally 500 Data Points. These Data Points are saved in a grid-wise manner, each separated $20[m]$ from each other.

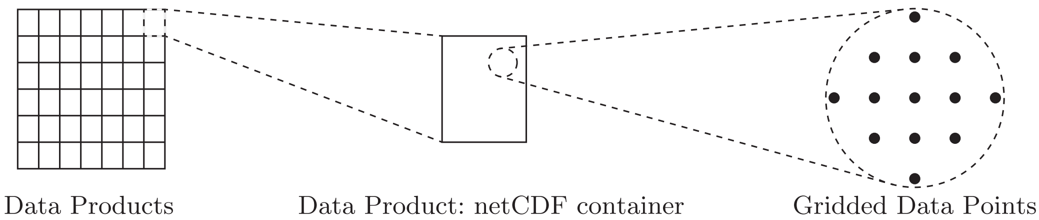


Figure 3.3: Geographical Data Points within Data Products

A Data Product as visualised in Figure 3.3 can contain a large number of Data Points. In this case these Data Points refer to certain geographical locations with z-value data on a certain time. They refer to a location that stands for a certain surface with a dx- and dy-value. This surface is visualised on the left in Figure 3.4. That surface has a z-value from a reference datum on a certain time.

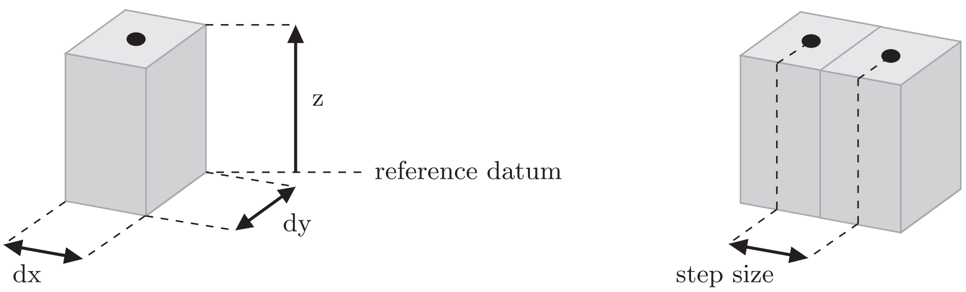


Figure 3.4: Individual Data Point with dx, dy, z-value and step size

When two Data Points are next to each other, the step size is what separates them as shown on the right in Figure 3.4. This dx and dy are defined as $20[m]$ for the Data Products. The individual step size value equals the dx and dy, so for Data Points the following applies:

$$dx = dy = \text{step size} = 20[m]. \tag{3.1}$$

The Data Points are arranged in a matrix shape as to be seen in Figure 3.3. An example is shown in Figure 3.5 for a $3 \cdot 2$ matrix; a situation, schematisation of this situation and a three dimensional schematisation of this situation. On this last image the dz (z-value) is visualised with a arbitrary reference level. This example case applies to a beach region, where the increasing z-value can be seen as the *ameland*.

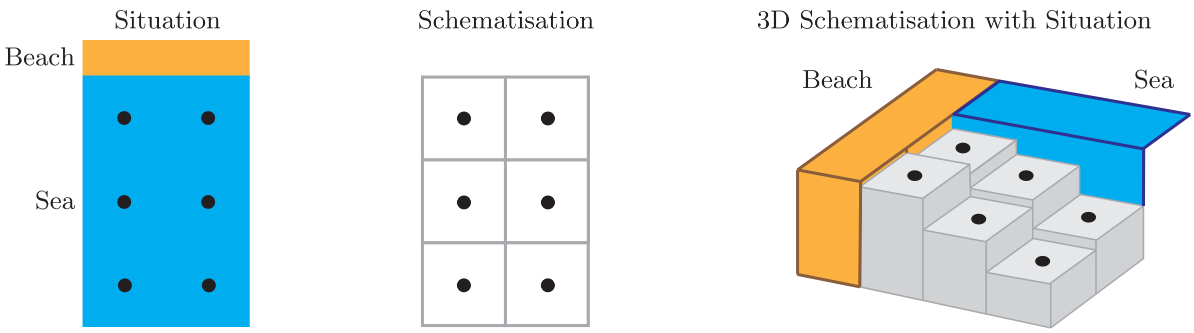


Figure 3.5: Situation, Schematisation and 3D Schematisation of 3x2 matrix with Data Points

With only six Data Points, an automated-uncertainty approach would not be necessary, because then it could have been calculated analytically. The Data Products can consist of a matrix with a large number of Data Points in both x- and y- direction. A single Data Product consists of $625 \cdot 500$ possible Data Points for each available time. Together with multiple Data Products, a manual calculation becomes a complicated and unclear task.

3.4.2 Region of Interest

Polygon

Having a large number of Data Points in Data Products does not solve the task for a specific location. That location is defined as the Region of Interest (ROI) and all data within that ROI will be used for the calculation. In practice a polygon is used to define this ROI.
This polygon saves multiple geographical coordinates in a pre-arranged way and combines these. Together they form a closed surface that can be used as an input parameter.

The Data Extraction procedure distinguishes the Data Products in polygon from Data Products outside the polygon. The definition is as follows: *If one or more Data Points are within a Data Product, the Data Product will be extracted.*
In Figure 3.6 an arbitrary polygon is given with Data Points in two separate netCDF Data Products. For this polygon both Data Product A and B will be extracted, because both meet the requirement.

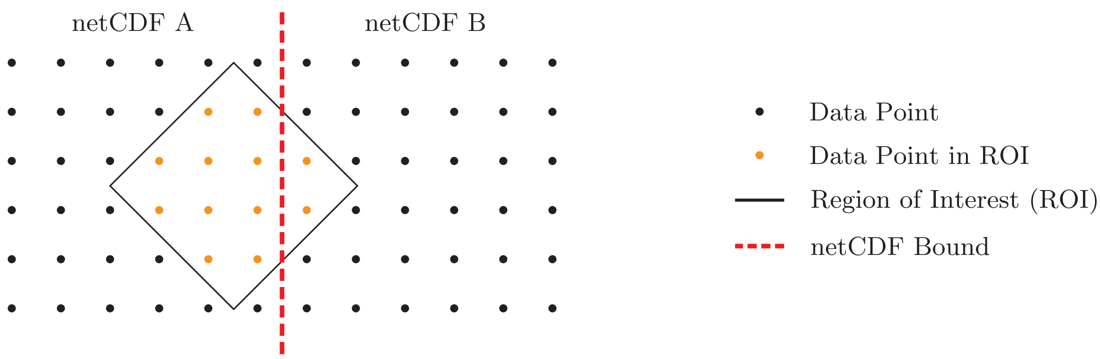


Figure 3.6: Arbitrary polygon with Data Points and two netCDF Data Products

Surface

The Data Extraction procedure is a part of the calculation of the automated-uncertainty approach. It is not developed to introduce additional uncertainties through this calculation. Therefore the predefined input polygon surface and the actual used surface within the polygon for the automated-uncertainty approach need to be equal.

If the polygon is chosen in such a way that for every single location within this polygon Data Points are available, the surface coverage of the Data Points has to be equal to the exact polygon surface. In practice this is not always realistic, since shorelines/islands/survey campaigns influence the coverage of a Data Product.
In Section 3.4 it was stated that extraction of a Data Product is defined by having one or more Data Points within the polygon. This implies that the surface of the Data Point can be used entirely, but this is not always the case when bordering a polygon. Figure 3.7 shows a simple arbitrary polygon.

Figure 3.7: Arbitrary polygon within a 6x6 dataset

The obvious methodology of defining the area of the polygon within the 6x6 Data Product is to count the amount of Data Points within the polygon and multiplying these with the $20 \cdot 20[m]$ surface of a single Data Point, as visualised in Figure 3.8 on the left. The disadvantage of this method is that with complicated or small polygons, a possible over- or underestimation of the used surface can not be excluded.

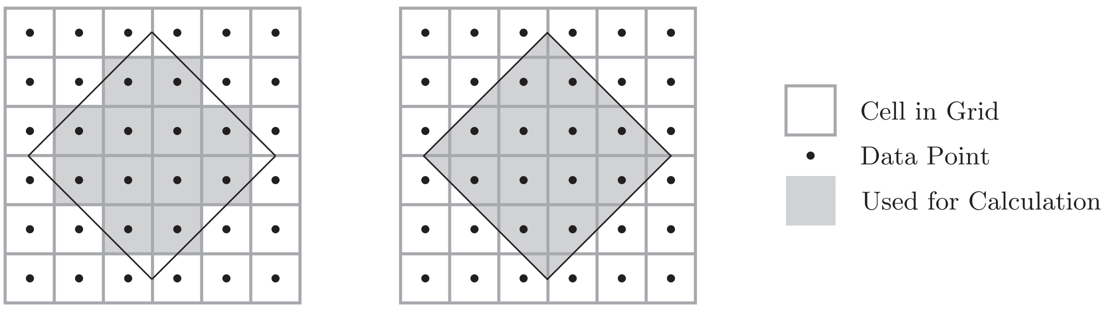


Figure 3.8: Different surface calculation approaches used with an arbitrary polygon

To elaborate on the calculation: The surface is defined as $dx \cdot dy = 20 \cdot 20[m] = 400[m^2]$, the dimensions used from Equation 3.1. The arbitrary polygon used in Figure 3.7 has 12 Data Points in the polygon and 24 outside the polygon. A short calculation puts the total surface used with this methodology at $4800[m^2]$.

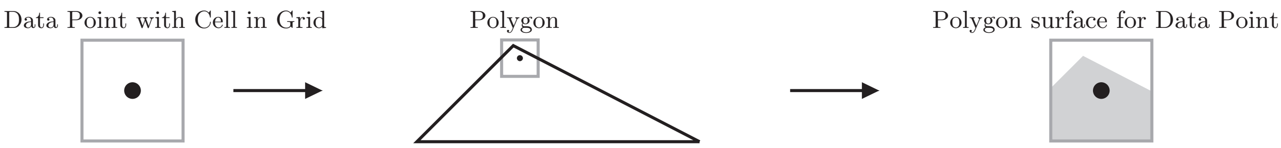


Figure 3.9: Determining surface of Data Point for an arbitrary polygon

A more precise approach is shown in Figure 3.9. This methodology defines the surface for each individual Data Point in a surface-matrix. It isolates the individual Data Point and compares it with the polygon, to extract the total surface used. This approach can be applied to the example polygon from Figure 3.7 and the surface in polygon with this methodology is shown in Figure 3.8 on the right. A quantification of this method for each

Data Point of the surface-matrix is stated in Figure 3.10. When the values of this matrix are summed, a total of 5000[m^2] can then be used for the calculation.

0	0	50	50	0	0
0	50	350	350	50	0
50	350	400	400	350	50
50	350	400	400	350	50
0	50	350	350	50	0
0	0	50	50	0	0

Cell in Grid
Region of Interest

Figure 3.10: Surface-matrix of Data Points for an arbitrary polygon, values in [m^2]

The difference between the analytical and geometrically correct example polygon is 4%. Especially for small polygons this procedure will be beneficial. Another advantage is that each individual Data Point can have its own surface to be calculated with. Therefore the uncertainty of this procedure is not only reduced in two dimensions, but in three dimensions.

3.5 Data Transformation

The Data Transformation procedure in Figure 3.11 will be elaborated for each Data Product. The procedure can be repeated for all extracted Data Products.
The output will consist of different types of Data Products 2, namely for the S- and T-approach, meaning that the data for the T-approach has to be transformed and stored in a different way than S-approach data. In this section the Data Transformation for both approaches towards Data Product 2 will be explained.

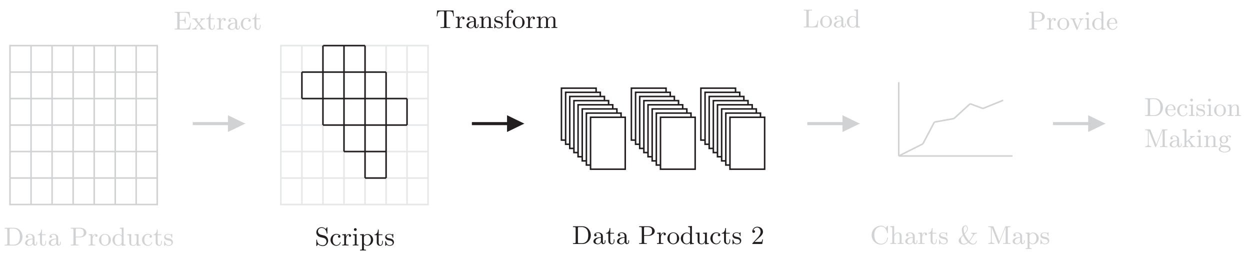


Figure 3.11: Data Transformation script process for an arbitrary task

3.5.1 Inquiry

Methodology

There are different methods to get input data from Data Products ready to do make a volumetric sedimentation-erosion changes calculation. As discussed in Section 2.5 two deterministic approaches have been used in previous research, visualised in Figure 2.14. Both the S- and T-approach will be used for the automated-uncertainty approach.
Analysing all Data Products within the polygon results in finding the oldest and most recent time within the data, as well as all times that *have* any data available in the polygon. This is defined as the Times in Polygon.

Spatial-approach The strategy of this method is to create high coverage geographical maps. These maps will be developed for all Times in Polygon that are available in the Data Product. The output of this approach is a 2D matrix sized after the amount of longitudinal and latitudinal Data Points, $625 \cdot 500$. It is attempted to fill all Data Points in this matrix with z-value data for a specific time. This map development will be repeated for all Times in Polygon available in the Data Product.
Not all maps can develop high coverage for all Times in Polygon. Therefore a tactic of using data from other maps in the Data Product is allowed within a time-bounded bandwidth. This is defined as Support Data.
The Support Data will be able to inquire z-value data from the Data Product with three different methods namely Backward, Forward and Nearest. See Figure 3.12 for an illustration of this process.

The high-coverage maps are composed by filling an array within a certain time-bounded bandwidth Δt (e.g. 2[y]). This filling process is done by entering all data within the polygon for a particular time into an array, where after the next point in time is searched from the Support Data for additional data to create a higher coverage. When Δt is reached or no higher coverage can be achieved, a new data saving point will be created and the process of Inquiry will be redone for that point. An example is visualised in Figure 3.13.

- Backward; filling from a time starting point (F.E. 2014) in backward direction (F.E. 2013... 2012...) until a maximum of Δt.
- Forward; filling from a time starting point (F.E. 2014) in forward direction (F.E. 2015... 2016...) until a maximum of Δt.
- Nearest; filling array at a starting point (F.E. 2014) in forward (2015...) and backward (2013...) direction until a maximum of $\frac{1}{2}$Δt.

Temporal-approach This methodology develops yearly vectors for all extracted Data Products. This is done by analysing all Times in Polygon and thereby finding the oldest and most recent point in time within the data. Subsequently the z-value data from a Data Product will be processed for each Data Point (Figure 3.12) over all Times in Polygon. The result is then a vector with z-value data over time for each Data Point.

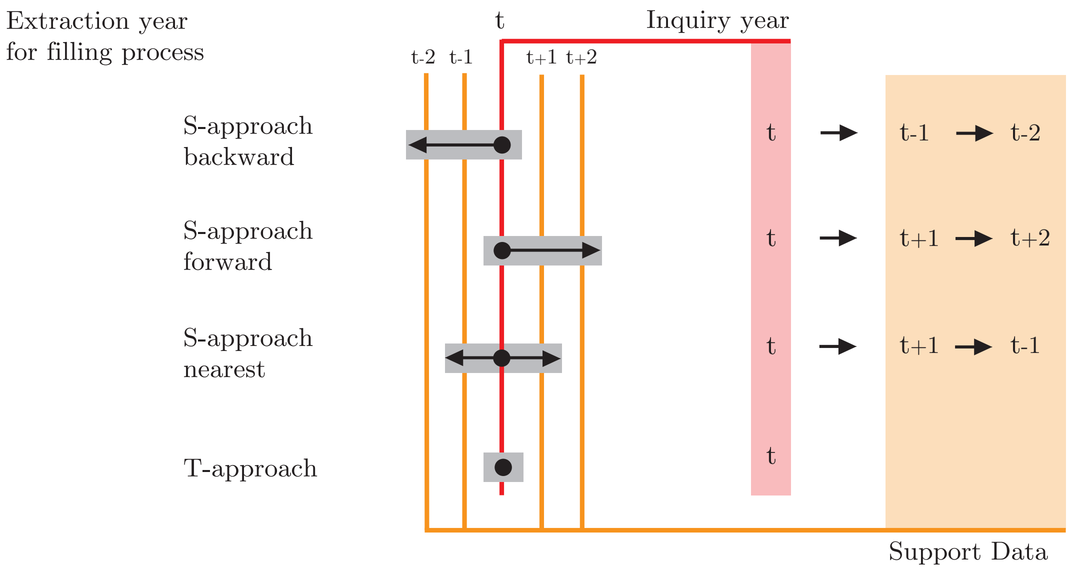


Figure 3.12: Definition of extraction year for filling process with Support Data Range Δt = 2[y]

Example

To clarify the inquiry methodology described in Section 3.5.1 an example inquiry will be performed. The example will consist of a certain period, of which data are available for a Times in Polygon of three years. This timeline is shown in Figure 3.13. Based on creating maps that are defined in an entire year this method creates yearly tiles and begins at a certain starting point (F.E. 2014... 2010...). It is prioritised by data starting from the beginning of the year (01 Jan).

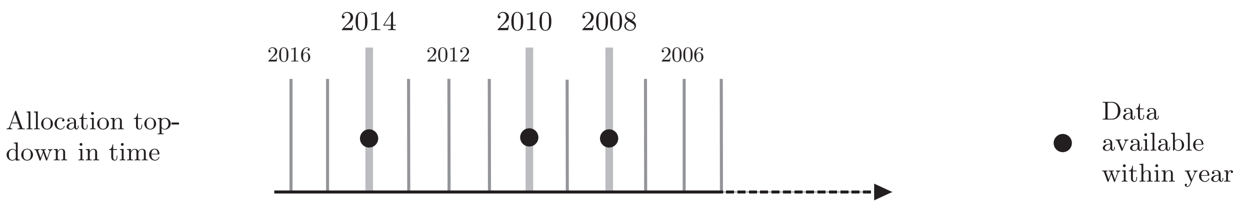


Figure 3.13: Data allocation example with data available in three Times in Polygon (2014, 2010 and 2008)

Inquiring will take place for all years that data are available, so for the example in Figure 3.13 that would be for 2014, 2010 and 2008. The first time step is 2014 and the inquiry for this particular year is visualised in

Figure 3.14, where the Support Data Range is $\Delta t = 2[y]$. In practice this means that for each and every Data Point within the polygon, this procedure will be performed.

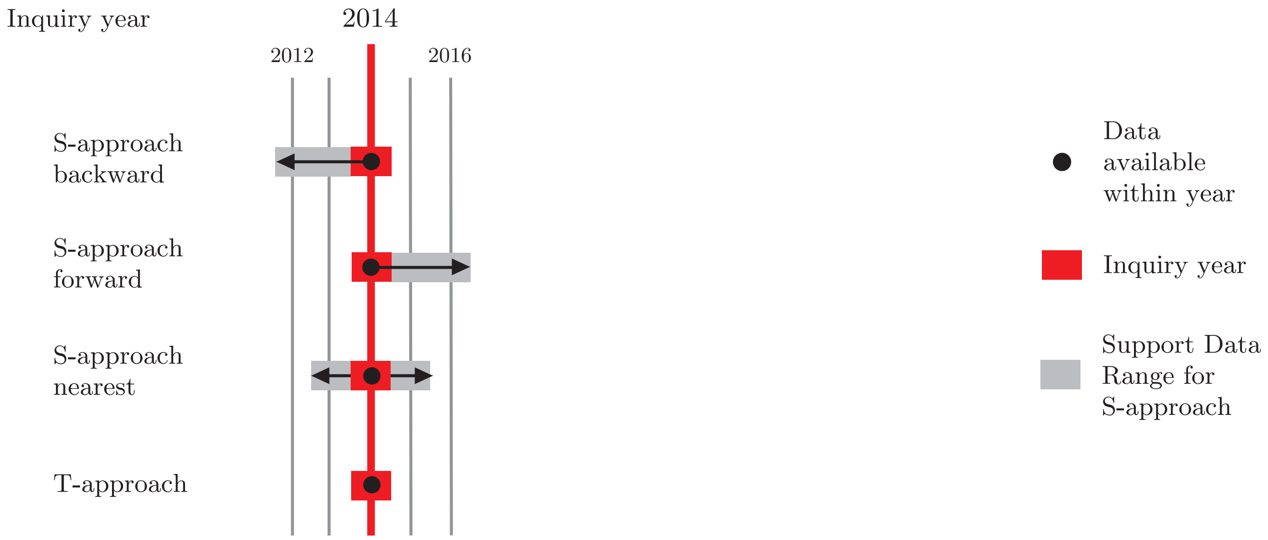


Figure 3.14: Data inquiry example for first time step (2014), Support Data Range $\Delta t = 2[y]$

The example in Figure 3.14 can examine data in 2014, because no other data in the $\Delta t = 2[y]$ Support Data Range are available. That is different for the next Times in Polygon, 2010 and 2008. In Figure 3.15 both these years are shown for the same process that has been explained in Section 3.5.1. The difference now is that for both 2010 and 2008 Support Data are available. This means that for 2010 each Data Point in the dataset is searched for data and when available, stored as 2010. When no more data is found for the year 2010, the same procedure is done for the 'Backward' S-approach for 2009. Not a single Data Point has data for 2009 so the routine goes to the next year 2008. There are data available for this year and when a Data Point is found that *does not* have any data - that means that no data was available for that Data Point for 2010 - available, the 2008 data are stored as 2010 data to create high coverage maps.
This procedure will be repeated for the other methods, whereafter the routine will repeat as well for 2008, meaning that for the 'forward' S-approach data could be found in 2010 that are *stored* as 2008 data.

A misconception could be that the forward method 'looks' forward, but 'saves' that data backwards to be used as the inquiry year.

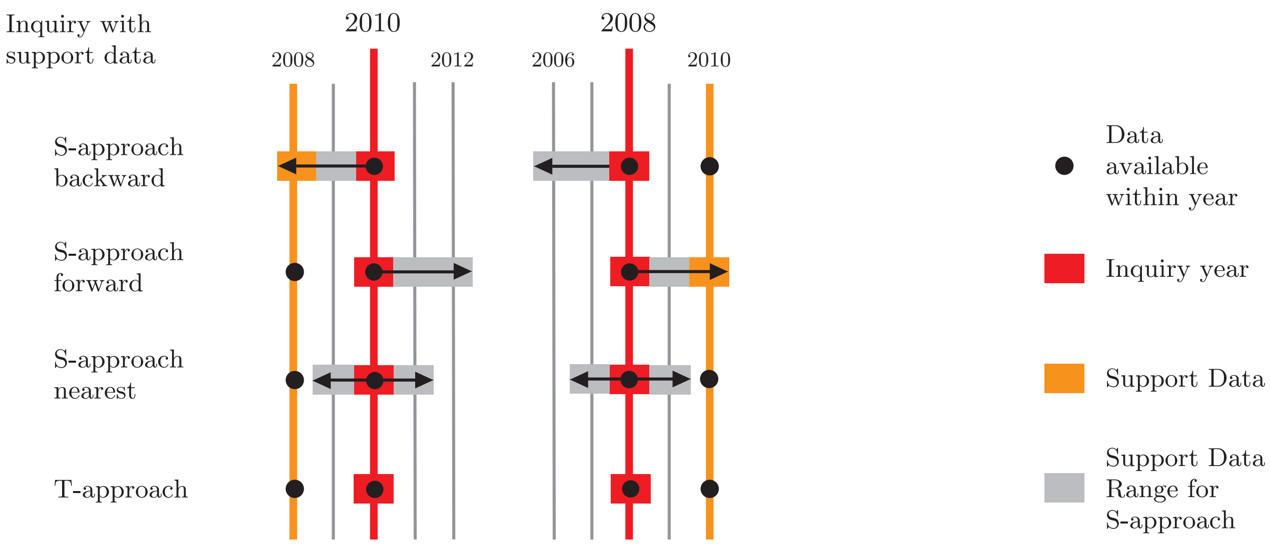


Figure 3.15: Data inquiry example for two time steps (2010 and 2008), Support Data Range $\Delta t = 2[y]$

When the procedures in Figure 3.14 and 3.15 have been completed, the data will be stored as shown in Figure 3.16. This means that the data will be saved as 2014, 2010 or 2008 but the *original* year will still be traceable.

This is important for the uncertainty propagation, that will be elaborated later in this Chapter.
To clarify the saved data even more, Table 3.1 provides possible data for each approach, for each year.

Table 3.1: Possible sources of data per method and inquiry year, data inquiry example with Δt = 2[y]

	Inquiry year 2014	**Inquiry year 2010**	**Inquiry year 2008**	**Inquiry year ...**
S-approach backward	t = 2014	t = 2010 t-2 = 2008	t = 2008	...
S-approach forward	t = 2014	t = 2010	t = 2008 t+2 = 2010	...
S-approach nearest	t = 2014	t = 2010	t = 2008	...
T-approach	t = 2014	t = 2010	t = 2008	...

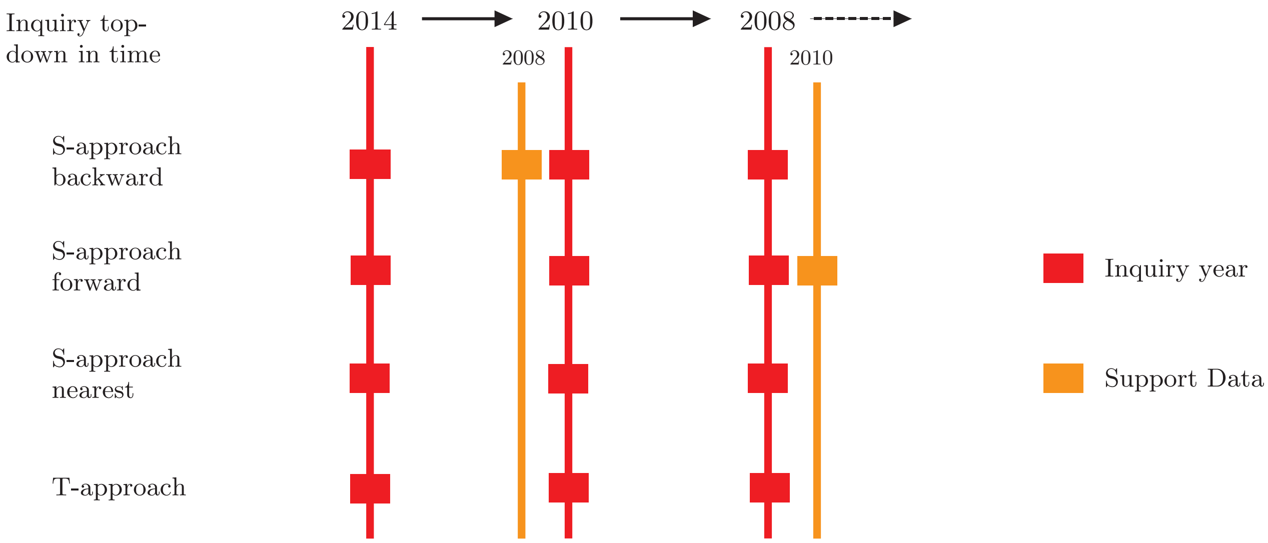


Figure 3.16: Data inquiry top-down process example for all time steps, Support Data Range Δt = 2[y]

3.5.2 Volume Difference

Spatial-approach

The S-approach uses the inquired maps with z-value data ([m^1]) that have high coverage. These maps can be subtracted from a reference map to develop a depth-difference matrix ([m^1]). This matrix can be multiplied with the surface-matrix ([m^2]) described in Section 3.4.2, in order to ensure that the correct surface is used for each Data Point. This will lead to a volumetric time difference matrix ([m^3]), that can be summed to get the volume difference for each yearly tile at a specific time.
The philosophy of this research is to set a discrete total minimum coverage of data coverage for the polygon for each Data Product *and* for the polygon.

Data for the S-approach will be imported from the applicable netCDF files with respect to geography and time. The geography will be defined by the boundary conditions and information stored in the netCDF files will be stored according to the Inquiry procedure described in Section 3.5.1.
To visualise the actual map-filling procedure, an example of the forward S-approach is visualised in Figure 3.17. A square polygon that is much larger then the available data is shown with three available Data Points. Not each Data Point has the same availability in time.

In this example the starting year is defined in 2006, whilst the maximum time difference is Δt = 2[y]. So starting from 2006, a yearly tile containing the predefined polygon can be filled by data starting in 2006. As soon as all available Data Points within the polygon are filled with 2006 data, the 2007 tile will be evaluated. This tile is empty so 2008 will be inquired. If there are still empty Data Points in the 2006 tile and these are available in 2008, the tile will use this as Support Data. This applies up to the pre-allocated maximum Support Data Range of Δt = 2[y].

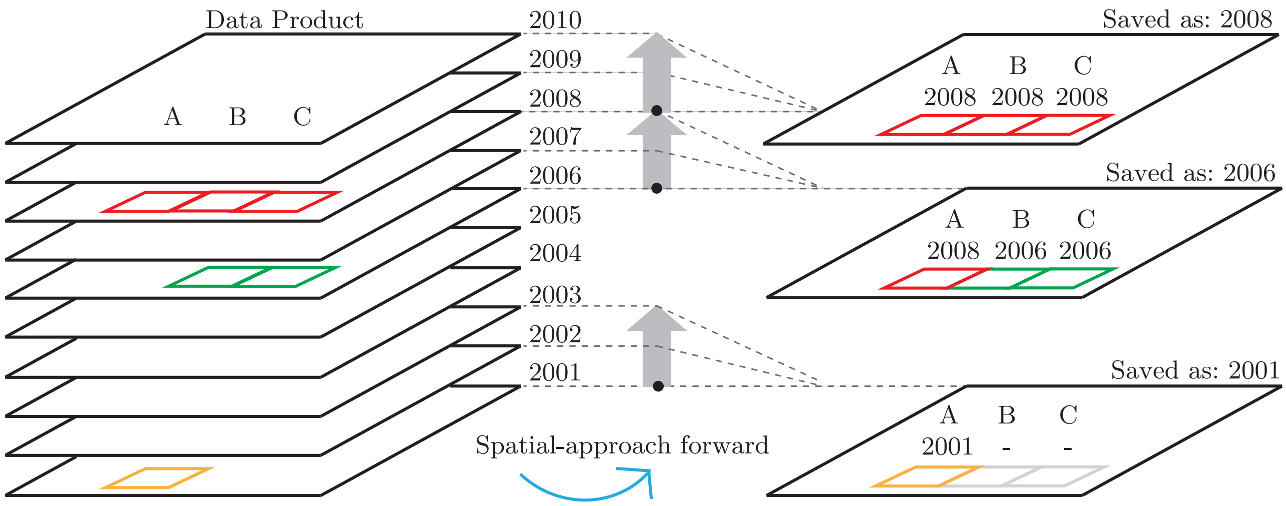


Figure 3.17: S-approach forward visualised, example for $\Delta t = 2[y]$

In the example Figure 3.17 on the right it is shown that for the saved map of 2006 only Data Points B and C received data from 2006, while Data Point A received data from 2008. For the tile saved as 2001, Data Point A received data in 2001, but no additional Support Data were available within the $\Delta t = 2[y]$ limits. Therefore the coverage of the 2001 tile is limited to $\frac{1}{3}$.

As soon as starting year + Δt is reached, the next coverage map has to be created for a new year saved in the Times in Polygon.
Subsequently the tiles that meet the minimum coverage requirement can be filled to develop 100% coverage maps, thereafter subtracted from the reference-yearly tile. This reference-yearly tile is defined as the highest-coverage yearly tile. Thereafter it is multiplied with the surface-matrix available for that Data Product. An example is provided in Figure 3.18, where four adjacent Data Points are shown for a single time with varying z-values but equal surface. The volume difference for each Data Point will therefore differ. All volume differences can be summed up for all into one value for each map in each Data Product.

The procedure will then be as follows:

- For each Data Product spatial tiles like visualised on the right in Figure 3.17 will be developed.
- Based on all Data Points that occur at least once over time within the polygon, a coverage mask will be created.
- The spatial tiles that meet the condition of minimum data coverage to the mask will be separated.
- These data will be spatially filled to fill the coverage mask using Matlab Inpaint (Garcia (2010); Wang et al. (2012)), to compensate for the disadvantage of the surface allocation described for the deterministic S-approach.
- Highest coverage map *prior* to spatially filling will be used as reference map.
- Subtraction of spatially filled maps from highest coverage map that has been spatially filled.

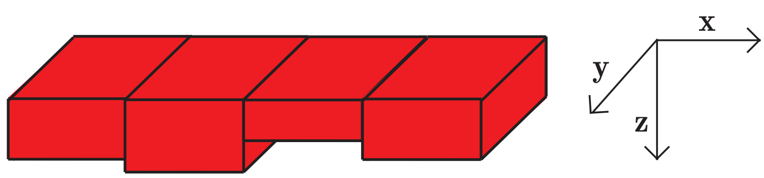


Figure 3.18: Volume difference for four random Data Points

This procedure can be elaborated for all S-approaches; for ‘backward’, ‘forward’ and ‘nearest’. After this is done the total summed volume difference for the Data Product are saved. The yearly tile 2010 of the Region of Interest Wadden Basin for the ‘nearest’ S-approach is shown in Figure 3.19. This Region of Interest is visualised in Figure 2.13.

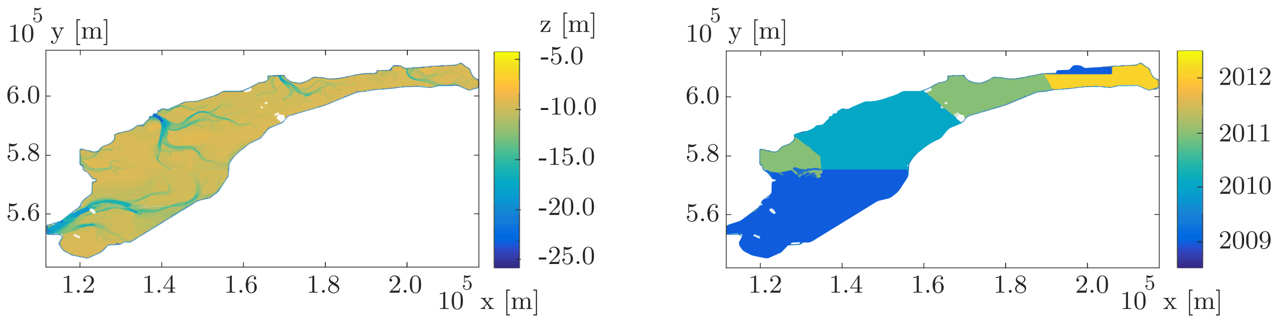


Figure 3.19: S-approach 'nearest' 2010-tile for Wadden Basin, 98.6% coverage, Support Data Range $\Delta t = 4[y]$

Temporal-approach

The T-approach is a procedure based on the principle that for every geographical location within the requested ROI (Region of Interest) a volume difference trend can be developed by summarising the volume differences from all Times in Polygon of each Data Point. Data for the T-approach will be imported from the applicable netCDF files to the Inquiry procedure from Section 3.5.1. A coverage mask will be developed for this procedure as well, based on all Data Points that occur at least twice over time within the polygon.

To elaborate on the methodology of the approach, an example is visualised on the left side of Figure 3.20, where a square polygon is shown with three different Data Points drawn in red. Not each Data Point has the same availability in time but in total there are seven yearly tiles created to fill the available data.
The Times in Polygon vector is defined as $[2010, 2008, 2006, 2004, 2002, 2000, 1998]$, since all these times have Data Points available.

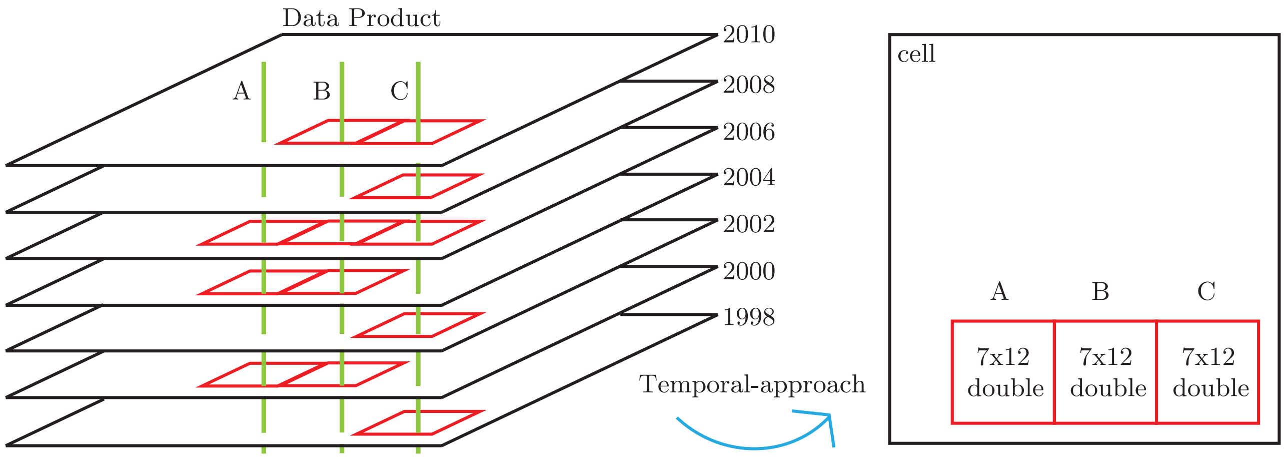


Figure 3.20: Three Data Points with data as an arbitrary example for the T-approach

This method will create a 'cell array' in Matlab. The benefit of a cell array is that for every cell a subset can be developed. The size is based on the geographical boundaries of the Data Product. The cell will have a large number of cells with each of them containing a Matlab variable of the type 'double'. This 'double' will have an equal amount of rows as there are yearly tiles, so in the example case this will be seven. The amount of columns are defined by the types of variables for each Data Point in time, which will be twelve for this Matlab script. The right side of Figure 3.20 provides information about the cell array. The 'double' defined in Figure 3.20 as 'B' has been visualised in Table 3.2. Only the first six columns have been added.

The T-approach is based on determining the trend for a single Data Point over all years available in the vector Times in Polygon. So it is defined as the time difference between the oldest and the most recent Data Point determined in the polygon. That means that there could be individual Data Points that do not have data in that bandwidth.
An example is Data Point B from Figure 3.20, where no data are available for the year 1998. Data Point C does and therefore oldest and most recent Times in Polygon are defined as 1998 and 2010. To satisfy the philosophy of the T-approach, all values for the Times in Polygon have to be filled. Therefore extrapolation is required; when high-order polynomial extrapolation is used, the incorrect assumption is made that a Data Point has a foreseeable trend in time. Therefore the simplest methodology is by using linear extrapolation.

Table 3.2: First six columns of data belonging to cell '7x12 double' B in Figure 3.20

Dates (Based on 01-Jan)	**Serial date number**	**Available z-data** $[m]$	**Linear inter-/ extrapolated z-data** $[m]$	**Relative to oldest date** $[m]$	**Relative volume diff.** $[m^3]$	...
2010	734139	-21.0	-21.0	-1.2	-120	...
2008	733408		-20.8	-1.0	-100	...
2006	732678	-20.6	-20.6	-0.8	-80	...
2004	731947	-20.4	-20.4	-0.6	-60	...
2002	731217		-20.2	-0.4	-40	...
2000	730486	-20.0	-20.0	-0.2	-20	...
1998	729756		-19.8	0	0	...

The same procedure has to be carried out for intermediate years without data points. Within this same example no data are available for 2002 and 2008, so linear interpolation would be required to create a support point. In column 4 of Table 3.2 this inter- and extrapolation filling procedure is visualised. Figures 3.21 and 3.22 give a visual representation of the data used in Table 3.2, including the inter- and extrapolated Data Points.

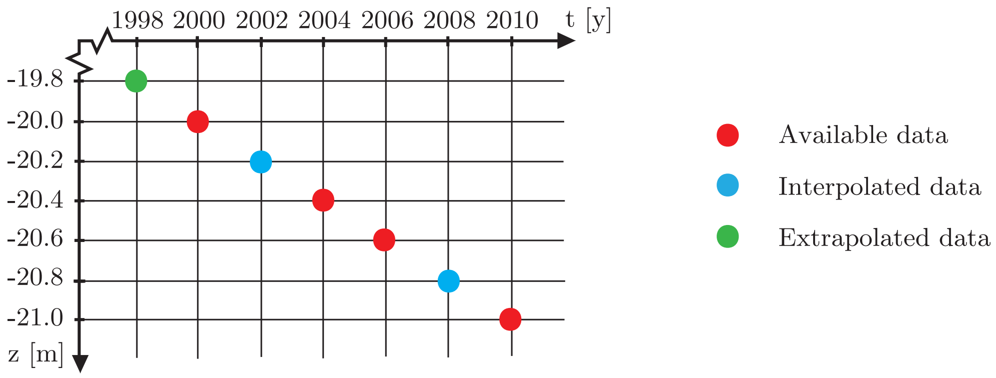


Figure 3.21: T-approach linear interpolation/extrapolation of z-data for Data Point B of example in Figure 3.20

When this is performed for all Data Points in the entire polygon, a volume difference calculation can be performed for each individual Data Point within the polygon.
This calculation consists of multiplying the the inter- and extrapolated z-value data ($[m^1]$) for each year for every Data Point with the surface-matrix ($[m^2]$) described in Section 3.4.2. When all the volumes are summed for the Times in Polygon, a total volume difference ($[m^3]$) for the entire polygon can be developed.

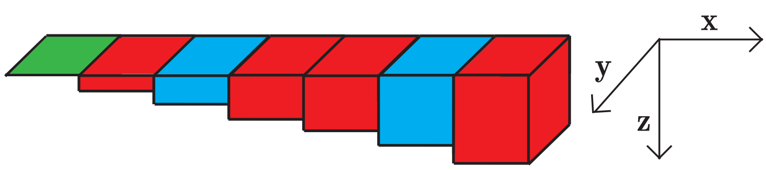


Figure 3.22: T-approach volume difference for Data Point B from example in Figure 3.20

3.6 Data Loading

The Data Loading procedure in Figure 3.23 will be elaborated for the output of the Data Transformation.

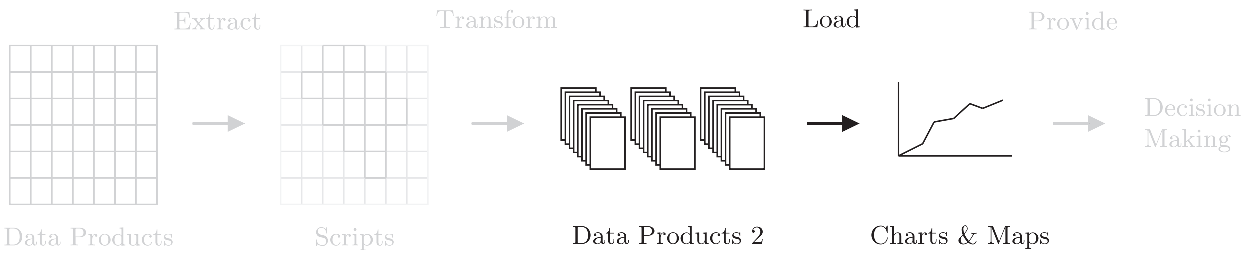


Figure 3.23: Data Loading script process for an arbitrary task

The input for the Data Loading consists of Data Product 2 that has been developed in the previous Section for both the S- and T-approach. Next to that the input consists of calibration settings.
Within the Data Loading, non-necessary data will be removed. The output will consist of separate Charts & Maps for the S- and T-approach. See Figure 3.24 for an output for the example used in Figure 3.20, when the year 1998 is used as reference.

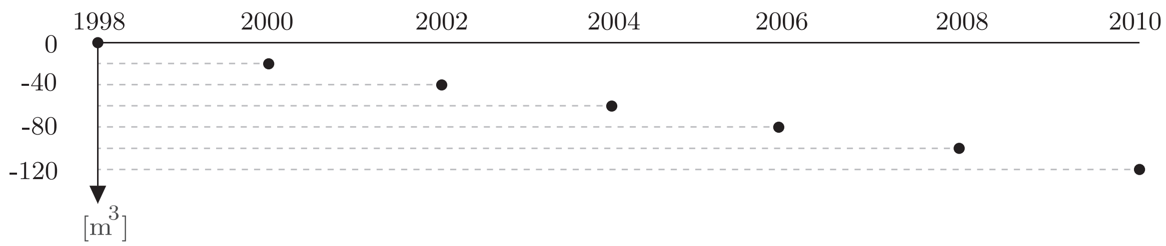


Figure 3.24: Volume difference for T-approach from example in Figure 3.17

3.7 Uncertainty Propagation

Uncertainty propagation for both S- and T-approach has been developed. Both methodologies used alternative strategies for spatial and temporal filling and therefore develop uncertainties. These uncertainty elements will be stored so propagation can be quantified.

To register the uncertainties that are detected within the volume difference calculation, the following has been elaborated. During the Data Extraction, Data Transformation and Data Loading the processes that could lead to uncertainties in the procedure have been stored. When these are combined eventually confidence bands can be created and added to the volume difference calculation. This can be defined as data-based uncertainty.

Three types of uncertainties have been defined. Uncertainty by data, by relation and by procedure. The uncertainty by data quantifies deviations by measurements and time, while uncertainty by procedure includes the deviation by map filling and coverage (spatial filling) for the S-approach. The uncertainty by relation is used for the S-approach to define an uncertain yearly tile in all tiles, as well as the deviation by interpolation and extrapolation for the T-approach. In Figure 3.25 all these identified deviations are defined.

All deviations are developed and saved in the SI unit $[m^3]$.

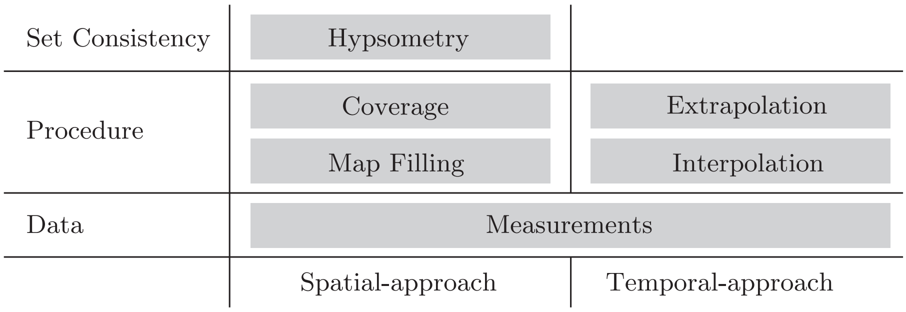


Figure 3.25: Summary of all identified deviations

Deviation by Data: Measurement

In Section 2.4 an elaboration has been made on the reliability of bathymetric data within the Dutch Coastal Zone. It is explained in Section 2.4.2 that Wiegmann et al. (2002) and Perluka et al. (2006) quantified the reliability of the data to be between 0.11$[m]$ and 0.40$[m]$. Next to that the possible sources are made known. It therefore is assumed that all Data Points have an uncertainty.

To use this uncertainty in the S- and T-approach, this deviation has to be taken into account. Therefore the following procedure will be applied for every Data Product and for all Inquiry methods discussed in Section 3.5.1:

- Defining the total surface ($[m^2]$) for every single year by using the surface-matrix from Section 3.4.2.
- Using a predefined calibration value for the vertical reliability ($[m^1]$)
- Summing and multiplication of the two previous steps ($[m^3]$) to meet the SI unit check.

The Section 2.4 also defines a difference in accuracy between recent and old bathymetric data. Therefore an uncertainty will be developed for old data, starting from a predefined year. This means that uncertainty will be added to data before this year, developing in a linear way.

The process will be applied for every Data Product and for all Inquiry methods discussed in Section 3.5.1:

- Using a predefined year that will be defined as the year where the linear uncertainty development starts.
- Using a predefined calibration value for the oldest dataset that can be found ($[m^1]$)
- Defining the total surface ($[m^2]$) for every single year by using the surface-matrix from Section 3.4.2.
- Summing and multiplication of the two previous steps ($[m^3]$) to meet the SI unit check.

In the example in Figure 3.26, the deviation by measurements are visualised. For time, the predefined calibration year is defined to be 2004. Hereafter the deviation increases linearly for data older then this year. The example is also based on Figures 3.17 and 3.20.

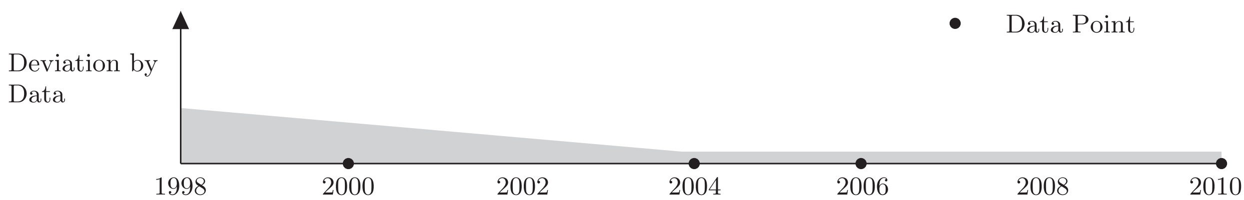


Figure 3.26: Deviation by Data Point B from example in Figures 3.17 and 3.20

3.7.1 Spatial-approach

The previous deviation by Data is applicable to the S- and T-approach. This section will only elaborate on deviations that solely apply for the S-approach.

Spatial-approach, Deviation by Procedure: Map Filling

In Section 3.5.2 the S-approach is explained. One of the steps of this process is map-filling with Support Data. This filling technique is accurate, but there is an error in the temporal direction. To make this error noticeable the error is projected in *spatial* direction.

During Inquiry, described in Section 3.5.1, Support Data has been used for filling. Together with this filling process a separate DT-matrix was created for all Data Products, Times in Polygon and S-approaches Backward, Forward and Nearest. The procedure of following the Map Filling uncertainty is as follows:

- Within the DT-matrix the discrepancy in years is logged between the actual year of inquiry ($[t^1]$) and the Support Data, for each individual Data Point.

- A predefined calibration value is used to meet the SI unit check ($[m^1\ t^{-1}]$).

- Thereafter both values above are multiplied with the surface-matrix ($[m^2]$) that is applicable to meet volumetric units ($[m^3]$).

When this process is executed for the example in Figure 3.17, the year 2006 receives uncertainty by Map Filling. This is because the 2006 tile uses a single Data Point from 2008. This deviation is visualised in Figure 3.27.

Figure 3.27: Map Filling deviation for S-approach from example in Figure 3.17

Spatial-approach, Deviation by Procedure: Coverage

Previous research of the deterministic S-approach used by van Koningsveld et al. (2008) elaborated in Section 2.5.1, provided uncertainty information by using different appearances of Data Point in the plotted volumetric difference calculation.
The methodology used in this research was by discretely dividing the coverage maps into different classes. Three classes of data coverage were identified: Good >= 90%, Fair >= 75% & < 90% and Poor < 75%. The highest data coverage year was then defined as the reference polygon for all volume difference calculations.
It is also explained in the disadvantages that this could have implications on the calculated volume difference. The reason for this is that lower coverage of z-value data implies that the use of surface is lower as well. Therefore the volume difference calculation could be under- or overestimated. To counteract the problem of under- or overestimating volumes a spatial filling technique is used in this research.

Filling gaps in data develops uncertainty, because *unavailable* data is made *available* for calculation. Therefore this uncertainty will be taken into account during the procedure.
The uncertainty developed during this filling process is assumed to grow when the coverage decreases. This will be done in a continuous way. That means that the higher the coverage, the lower the uncertainty becomes. On the other hand the lower the coverage, the higher the uncertainty becomes. The range of this uncertainty is defined between the minimum defined coverage and full coverage.

In the example of the S-approach in Figure 3.17, the tile saved as 2001 could not be filled entirely. Assuming the minimum defined coverage to be lower than $\frac{1}{3}$ (low minimum coverages like these will not be used in practice), this 2001 tile is eligible for spatial filling. During the procedure the total surface ($[m^2]$) that has been filled will be saved in a separate 'Deviation by Coverage' position. When used for uncertainty propagation, the values will be multiplied by a manual calibration value ($[m^1]$) in order to meet the SI unit check ($[m^3]$).
Figure 3.28 shows a visual representation of how the uncertainty for the year 2001 will be defined for the S-approach in this example.

Figure 3.28: Coverage deviation for S-approach from example in Figure 3.17

Spatial-approach, Deviation by Relation: Hypsometry

In Section 2.5.1 it was discussed that the accuracy of the *available* bathymetric data could be debatable. In Section 2.4 an elaboration was done on the bathymetric measurements and their potential uncertainties.

A channel within a tidal delta can potentially migrate over a certain area, caused by external influence such as tides. When exact measurements are conducted on a specific Data Point, it is possible to recognise fluctuations over time because a tidal channel migrated on that specific location. Therefore it is difficult for the T-approach to recognise a phenomenon like this.
Morphological changes like discussed above can be detected, because the total volume within a Data Product remained equal, the tidal channel just shifted location. Assuming this phenomenon does not migrate *outside* the Data Product, it can be detected by hypsometric curves.
A hypsometric curve is a cumulative length frequency curve. It provides information about the frequency of a specific length occurring. The length will be the z-value information retrieved from the Data Product.
The procedure is for each Data Product and applicable for S-approaches Backward, Forward and Nearest:

- Using input settings to define lower and highest z-value boundary for hypsometric data, e.g. ranging between $-20[m]$ and $20[m]$.
- Development of a hypsometric step range between lower and highest z-value boundary for a predefined bin width, for example $0.10[m]$.
- Extraction of separate tiles with their own specific time that have been filled spatially for the 'Forward', 'Backward' and 'Nearest' approaches.
- Analysing all tiles and saving the used surface from the surface-matrix (Section 3.4.2) in the z-value bound hypsometric step range.
- Using all tiles to create a mean value of the hypsometric step range.
- Subtracting all hypsometric step surface tiles ($[m^2]$) from the mean hypsometric step surface tile ($[m^2]$) and multiplying it by the z-value boundary ($[m^1]$)it belonged to, to create a hypsometric difference ($[m^3]$) for each tile that is stored.

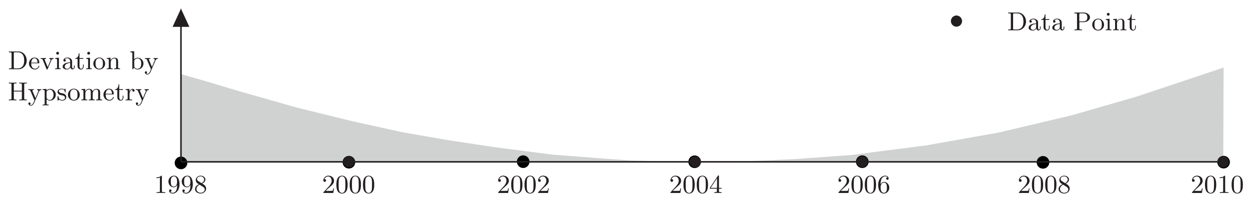


Figure 3.29: Hypsometric deviation for Unit Test 3 (Appendix A-3)

When this procedure is used, the output can be used to make a statement about the accuracy of the data within a certain time, so actual measurement errors can be detected over time.

An example is shown in Figure 3.29, where a synthetic dataset is used as input. The number of the year (e.g. 1998) is divided by -100 and the entire dataset for that year is defined as that number in a geographical way ($-19.98[m]$), which will be further explained during validation. When the procedure described above is followed for a dataset like this, the hypsometric curve will look like the one showed in the figure.

In Figure 3.30 an example of multiple hypsometric curves have been plotted. These hypsometric curves are derived from the Data Product 'vaklodingenKB114_4342.nc' (same as visualised in Figure 2.10) and only the

'Backward' approach is visualised, because theoretically a different cumulative area is possible for the 'Forward' and 'Nearest' approaches. The entire range of z-values has been utilised and divided into bins width $dz = 0.10[m]$.
Next to that the mean of all plotted hypsometric curves is plotted. A variation is noticeable, that will be quantified with the procedure above for uncertainty propagation.

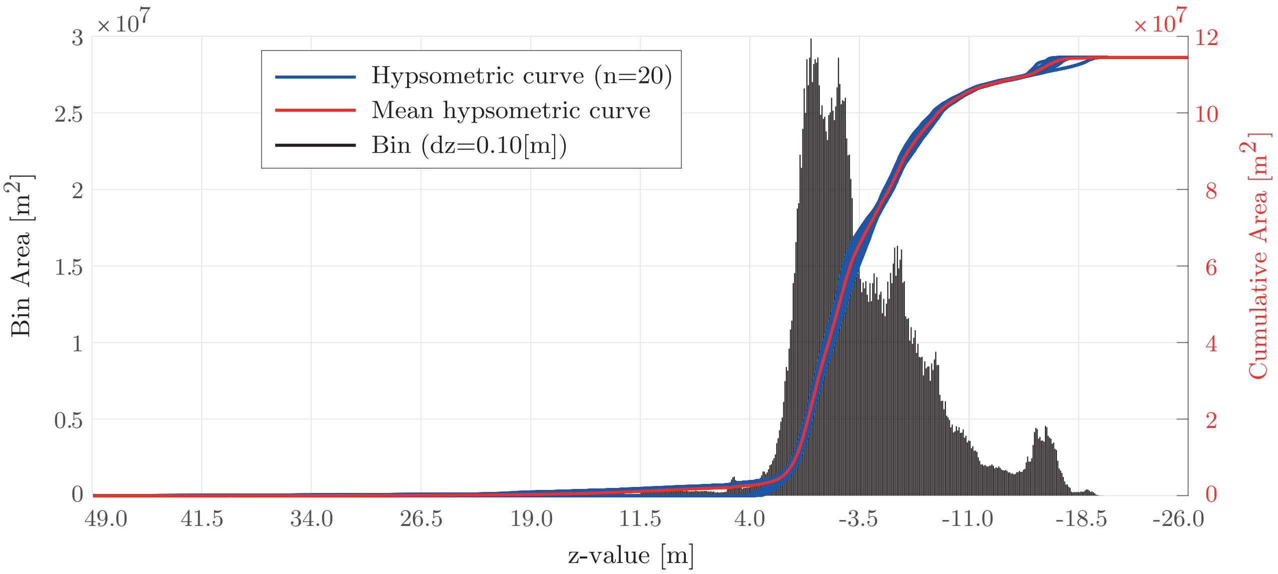


Figure 3.30: Hypsometric curves for Spatial-approach 'Backward' for Data Product 'vaklodingenKB114_4342.nc'

3.7.2 Temporal-approach

This section will elaborate on deviations that only apply to the T-approach.

Temporal-approach, Deviation by Procedure: Interpolation

During this process Data Points will be created by using interpolation. It is used as a linear process as well to create uncertainties when support Data Points are created. This does not create the large expected uncertainties as extrapolation, but because this procedure would be applicable to a large temporal space of Data Points, its eventual implication could be of large influence.

The procedure of following the interpolation uncertainty is as follows:

- Between Data Points, interpolated Data Points receive a linear counting uncertainty value ([-]). This counts up from both sides, developing the highest value between two known Data Points.
- A predefined calibration value is used to meet the SI unit check ($[m^1]$).
- Thereafter both values above are multiplied with the surface-matrix ($[m^2]$) that is applicable to meet volumetric units ($[m^3]$).

See Figure 3.31 for the created deviation by interpolation of the years 2002 and 2008 for Data Point B from the example in Figure 3.20.

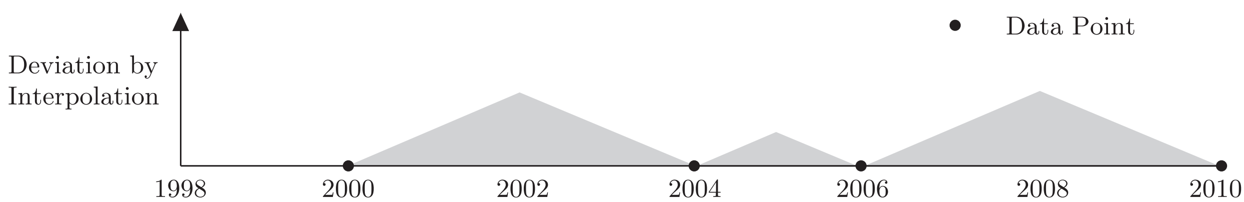


Figure 3.31: Interpolation deviation for Data Point B from example in Figure 3.20

Temporal-approach, Deviation by Procedure: Extrapolation

This is a process where large uncertainties can occur. The standard methodology is linear interpolation, but if extrapolation occurs over a large amount of years a large error could occur. Therefore the uncertainty increases with a large rate. See Figure 3.32 for a visual example of the deviation created by the extrapolation ($[m^3]$) of the year 1998 for Data Point B from the example in Figure 3.20. It causes a certain error that is unknown, but is be registered.

The procedure of following the extrapolation uncertainty is as follows:

- Starting the latest available Data Point, extrapolated Data Points will receive a linear counting uncertainty value. Every extrapolated year receives an additional value ([-]).
- A relatively high predefined calibration value is used to meet the SI unit check ($[m^1]$).
- Thereafter both values above are multiplied with the surface-matrix ($[m^2]$) that is applicable to meet volumetric units ($[m^3]$).

Because of the large uncertainties that can develop, the predefined calibration value will be significant. The reason for this is that the linearly extrapolated values are based on the latest available Data Point, and the preceding one. A visualisation of this process is visualised in Figure 3.32, based on the example from Figure 3.20.

Figure 3.32: Extrapolation deviation for Data Point B from example in Figure 3.20

3.7.3 Combining Deviations

It is assumed that the individual deviations that are defined above are uncorrelated. Therefore summing the deviations can be performed by using Equation 3.2, when n loops through the deviations applicable to the S- or T-approach. This process has to be performed for all individual time steps available in the Data Product 2 described in Section 3.5 (Dekking et al., 2005).

$$combined\ deviation = \sqrt{\sum_n deviation_n^2} \tag{3.2}$$

3.8 Summary

This chapter provided an elaboration on the methodology used to develop the automated-uncertainty approach.

The Challenges defined in Section 3.2 were addressed, to make the automated-uncertainty approach feasible for utilisation in the dredging industry for sediment management.
To structure the automated-uncertainty approach development the OpenEarth Approach was used to create an operating procedure.

The first step of the automated-uncertainty approach is the Data Extraction. This procedure will define the surface of the polygon used as input for the automated-uncertainty approach, based on the Data Products that are available in this polygon. Also the Data Products used for the calculation are extracted.
The second step of the automated-uncertainty approach is the Data Transformation. During this procedure the Inquiry of the Data Products is elaborated for the Spatial- (S) and Temporal- (T) approach. Hereafter the Volume Difference methodology is explained for these same approaches.
The third step of the automated-uncertainty approach is the Data Loading. In this step the uncertainties that propagate through the S- and T-approach are analysed and prepared for the development of volumetric confidence bands. Three types of uncertainties have been added: data, procedure and relation. The Volume Difference data developed in the second step and the uncertainty analysis data are visualised in graphs for both the S- and T-approach.

Chapter 4

Validation

4.1 Introduction

Chapter 3 provided a methodology on how the automated-uncertainty approach is developed.
Developing an automated-uncertainty approach that processes large amounts of data could potentially cause unclear results. The reason for this is that an analytical calculation of a Case Study dataset is a long and complicated task, so manual verification of the results can cause errors that cannot be used for validation. To validate and thus check whether the procedure used by the automated-uncertainty approach is correct a methodology has been developed to make an analytical calculation possible, to compare the results of the automated-uncertainty approach with the analytical solutions and thereby validating the procedure. This Chapter will provide a validation of the automated-uncertainty approach.

4.2 Synthetic Datasets

The automated-uncertainty approach uses the following procedure to develop output:

- Input of Data Products and polygon of Region of Interest.
- Calculation.
- Output of saved Data Product 2, tables and charts.

To verify the calculation, input will be developed that can be calculated manually to produce the output. This input will consist of *synthetic* data, which means that the entire dataset used as an input is created manually.

Therefore synthetic Data Products have been developed. These consist of reconfiguring existing Data Products from the Case Study. The 4D information stored in these Data Products will therefore partially be edited. Geographical data in longitudinal and latitudinal direction remains the same.

On the other hand the time and z-value data will be altered. A logical methodology is used for this: All stored time values will be entered in Equation 4.1 to retrieve the z-value value for the entire dataset within the Data Product for each year. Before this can be done multiple years have to be defined, which has been done by using values that would not create complications towards the z-values.

$$z(yr) = \frac{-yr}{100} \quad \text{for } x(yr) = 1,2,...,500; \; y(yr) = 1,2,...,625 \tag{4.1}$$

To make the above understandable the following example has been elaborated. Table 4.2 provides the defined times in the second column, while the third column gives the z-value calculated by using Equation 4.1. Together with the geographical coordinate range defined in Table A-2, the Figure 4.1 could be developed with the procedure described above.
The new Data Product described above is then stored in a netCDF file, named A1 like to be seen in Table A-2.

Table 4.1: Synthetic dataset values

	NetCDF File	**Range** $[m]$		**Difference** $[m]$	**Analytical Surface** $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000
Region of Interest y		437,500	450,000	12,500	
NetCDF x	A1	40,000	50,000	10,000	125,000,000
NetCDF y		437,500	450,000	12,500	
Net. Surface					125,000,000

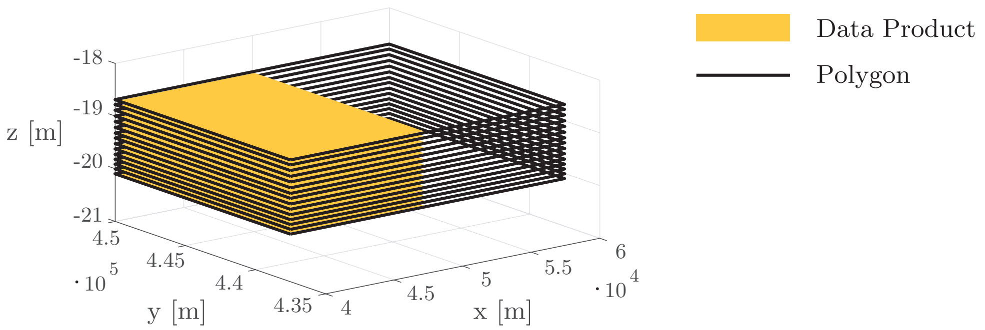


Figure 4.1: Visualised synthetic dataset with regular times

Table 4.2: Synthetic dataset by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**	
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^6\ m^3]$
1	2010	-20.1	-0.2	-25
2	2000	-20.0	-0.1	-12.5
3	1990	-19.9	0	0
4	1980	-19.8	0.1	12.5
5	1970	-19.7	0.2	25
6	1960	-19.6	0.3	37.5
7	1950	-19.5	0.4	50
8	1940	-19.4	0.5	62.5
9	1930	-19.3	0.6	75
10	1920	-19.2	0.7	87.5
11	1910	-19.1	0.8	100
12	1900	-19.0	0.9	112.5
13	1890	-18.9	1.0	125
14	1880	-18.8	1.1	137.5
15	1870	-18.7	1.2	150

Surface information $[m^2]$ and z-value data $[m^1]$ are available to calculate the total volume difference. Reference data has to be defined to calculate the total volumetric difference. This reference year is defined as 1990. In Table 4.2 in column 4 the z-value relative to 1990 is visualised, together with the *analytical* volume calculation relative by multiplying the surface and relative z-value data $[m^3]$.

This was one example to develop a synthetic dataset. Variations can be developed in e.g. time, to develop different synthetic datasets. The second column in Table 4.3 give information about a different dataset of times, with corresponding z-value data based on Equation 4.2 in the third column. The analytical volume difference calculation relative to 1990 is shown in the fifth column. For a visualisation see Figure 4.2.

$$z(n) = \left(\frac{n}{2}\right)^2 \quad \text{for } x(n) = 1,2,...,500;\ y(n) = 1,2,...,625 \tag{4.2}$$

Table 4.3: Synthetic dataset by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**	
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [10^6 m^3]
1	1992	-19.92	-0.02	-2.5
2	1991	-19.91	-0.01	-1.25
3	1990	-19.9	0	0
4	1962	-19.62	0.28	35
5	1961	-19.61	0.29	36.25
6	1960	-19.6	0.3	37.5
7	1932	-19.32	0.58	72.5
8	1931	-19.31	0.59	73.25
9	1930	-19.3	0.6	75
10	1902	-19.02	0.88	110
11	1901	-19.01	0.89	111.25
12	1900	-19.0	0.9	112.5
13	1872	-18.72	1.18	147.5
14	1871	-18.71	1.19	148.75
15	1870	-18.7	1.2	150

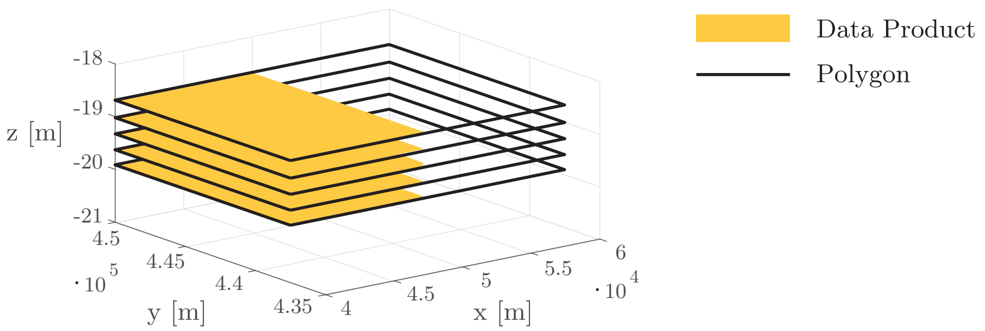


Figure 4.2: Visualised synthetic dataset with varying times

4.3 Unit Tests

To verify the accuracy of the automated-uncertainty approach methodology described in Chapter 3, different synthetic datasets as described in Section 4.2 are developed and tested by means of an Unit Test.
Unit Testing is the procedure to test software in order to verify its behaviour. These are based on the *potential* bottlenecks of the software.

In Table 4.4 all separately developed Unit Tests are shown with their corresponding characteristics. Different Regions of Interests, trends, time steps, coverages, geographical coordinates and Data Products are used for this processing. In Table 4.4 the type of synthetic data that used for each specific Unit Test is provided.

The previous Section 4.2 gave an analytical solution for a specific synthetic dataset and it is expected that all Unit Tests give the same output as the analytical solution. When these give an acceptable error, the automated-uncertainty approach can be validated for the 16 developed Unit Tests.

Table 4.4: Unit Tests

Unit Test	**Region of Interest**	**Trend**	**Time Step** [y]	**Gap**	**Amount of netCDF Files**
1	Two netCDF Files	Linear	10	No	1
2	Two netCDF Files	Linear	10	Yes	1
3	Two netCDF Files	Linear	Variable	No	1
4	Two netCDF Files	Linear	Variable	Yes	1
5	Two netCDF Files	Linear	10	No	2
6	Two netCDF Files	Linear	10	Yes	2
7	Two netCDF Files	Linear	Variable	No	2
8	Two netCDF Files	Linear	Variable	Yes	2
9	Two netCDF Files	Non-Linear	10	No	1
10	Two netCDF Files	Non-Linear	10	Yes	1
11	Two netCDF Files	Non-Linear	10	No	2
12	Two netCDF Files	Non-Linear	10	Yes	2
13	Diamond Small	Linear	10	No	1
14	Diamond Small	Linear	10	No	2
15	Triangle Large	Linear	10	No	1
16	Triangle Large	Linear	10	No	2

Table 4.5: Type of synthetic data used for each Unit Test

Unit Test	**Region of Interest**	**Trend**		**Time Step**		**Gap**		**Lon/x/ Lat/y**		**netCDF File**	
1	A	A	-	A	-	-	-	A	-	A1	-
2	A	A	-	A	-	A	-	A	-	A2	-
3	A	A	-	B	-	-	-	A	-	A3	-
4	A	A	-	B	-	A	-	A	-	A4	-
5	A	A	A	A	A	-	-	A	B	A1	B1
6	A	A	A	A	A	A	-	A	B	A2	B2
7	A	A	A	B	B	-	-	A	B	A3	B3
8	A	A	A	B	B	A	-	A	B	A4	B4
9	A	B	-	A	-	-	-	A	-	A9	-
10	A	B	-	A	-	A	-	A	-	A10	-
11	A	B	B	A	A	-	-	A	B	A9	B9
12	A	B	B	A	A	A	-	A	B	A10	B10
13	B	A	-	A	-	-	-	A	B	A1	-
14	B	A	A	A	A	-	-	A	B	A1	B1
15	C	A	-	A	-	-	-	A	B	A1	-
16	C	A	A	A	A	-	-	A	B	A1	B1

Results of Unit Tests 1-16 are detailed in Appendix 2. The Unit Tests are assembled in the following predefined structure:

- The first table consists of general information applicable to the Unit Test. What the number is of the Unit Test, what kind of Region of Interest is used, the trend, time step, coverage and amount of netCDF files that are used for the method. An example of this is shown in Table A-1.

- This figure provides a visualisation of the Region of Interest that is used, together with a representation of the trend, time step, coverage and amount of netCDF files that are used. See Figure A-4 for an example.

- The second table gives information about the range of the Region of Interest and its total surface. A difference is defined between the *analytical* and *computed* surface, to validate the surface calculation as defined in 3.4.2. See Table A-2 for an example.

- The third table gives information about the S-approach backward. In this table the *analytical* volume calculation relative to the reference year is stated against the *computed* volume calculation by the use of this method. The error is defined as the difference between the analytical and computed volume. An example is shown in Table A-3.

- The fourth table gives information about the S-approach forward. It is established in the same manner as the S-approach backward table. An example is shown in Table A-4.
- The fifth table gives information about the S-approach nearest. It is established in the same manner as the S-approach backward and forward. An example is shown in Table A-5.
- This figure provides a graphical representation of the analytical and all computed volume difference calculations. See Figure A-5 for an example.

For Unit Test 1, the 'backward' S-approach is graphically visualised in Figures 4.3, 4.4 and 4.5.
The values computed by the model and the analytical calculation of the z-values relative to 1990 are shown in Figure 4.3, while Figure 4.4 gives a representation of the volumetric differences relative to 1990. In Figure 4.5 the differences between the analytical and computed values are presented.

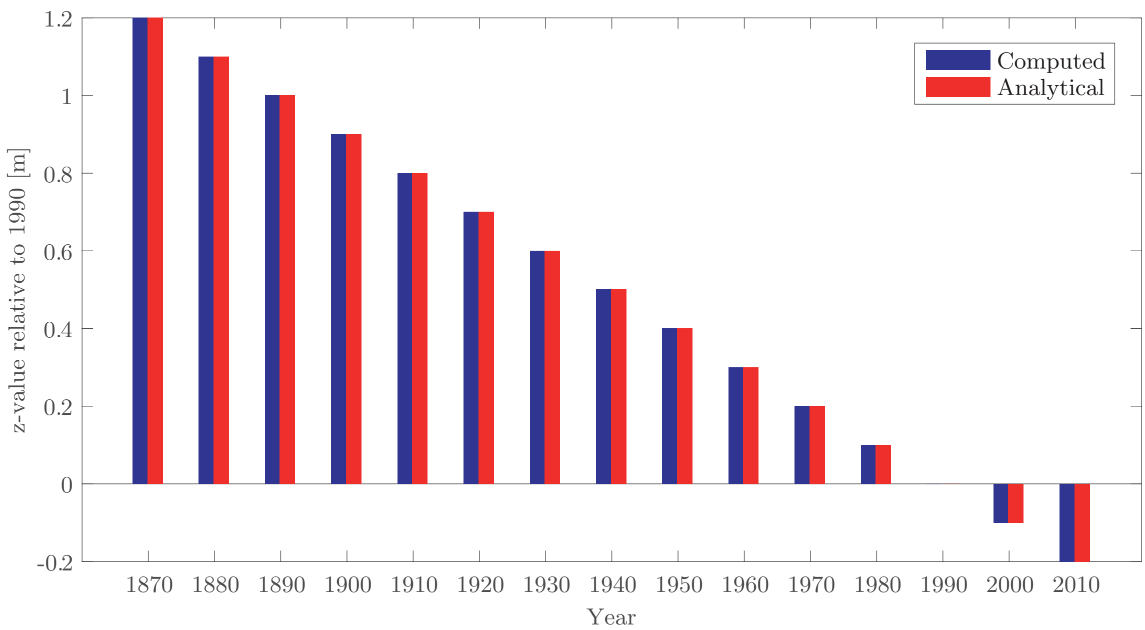


Figure 4.3: Computed and analytical z-values, S-approach 'backward' for Unit Test 1

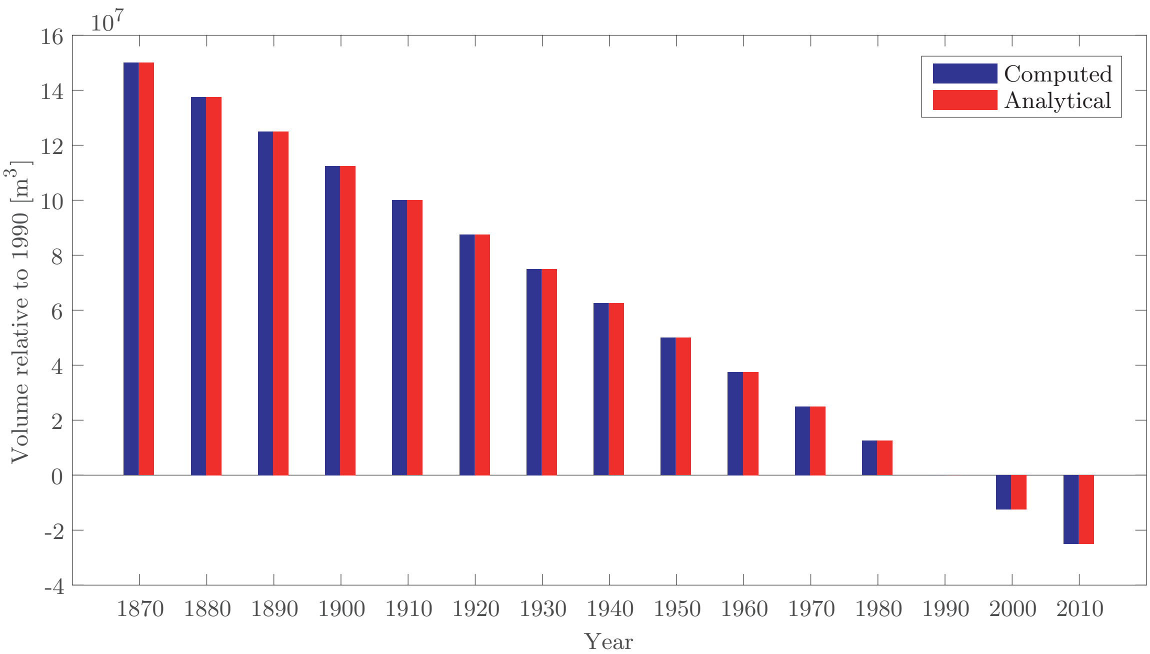


Figure 4.4: Computed and analytical volumetric differences, S-approach 'backward' for Unit Test 1

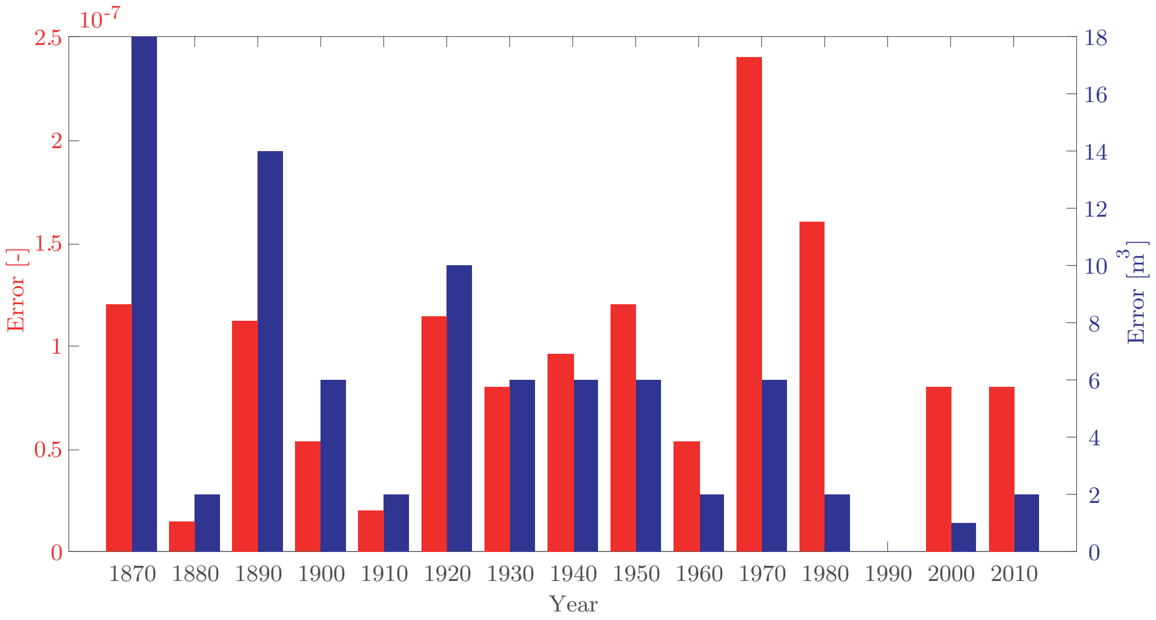


Figure 4.5: Computed and analytical errors, S-approach 'backward' for Unit Test 1

The 16 separate Unit Tests described in Section 4.3 were applied to the S- and T-approach of the automated-uncertainty approach.
A summary of the results is shown in Table 4.6. It is assumed that an error of 10^{-5} is acceptable. Therefore, all Unit Tests pass successfully.

Table 4.6: Summarised results of Unit Tests 1-16

	Smallest error [-]	Largest error [-]
S-approach	10^{-8}	10^{-5}
T-approach	0	10^{-4}

4.4 Summary

This Chapter provided a methodology to validate the automated-uncertainty approach.

Testing the automated-uncertainty approach is executed by means of Unit Tests. Unit Testing is the procedure to test software in order to verify its behaviour from start tot finish. These are based on the *potential* bottlenecks of the software.

Sixteen different Unit Tests are developed for this purpose. Synthetic Data Products that can be analytically calculated are developed. Different Regions of Interest, time trends, coverages, geographical information and Data Products are used during this process.

All Unit Tests have been elaborated in Appendix 6.3.3. The results prove that the maximum error for the Spatial- (S) approach is approximately [10^{-4}] and for the Temporal- (T) approach approximately [10^{-5}]. It is assumed that an error of 10^{-5} is acceptable, so all Unit Tests passed successfully.

Chapter 5

Application

5.1 Introduction

Chapter 3 gave detailed information about the methodology used to develop the automated-uncertainty Spatial (S)- and Temporal (T)-approaches. Subsequently in Chapter 4, the validation of the automated-uncertainty approach was described. This Chapter will provide results of the Case Study simulations.

5.2 Case Study: Dutch Coastal Zone

The deterministic S-approach of van Koningsveld et al. (2008) and deterministic T-approaches of Elias et al. (2012) and Elias and van der Spek (2014) are projected in the results of the automated-uncertainty T-approach. The results are combined with the automated-uncertainty T-approach on the reference value in the year 1990. The deterministic S- and T-approaches are projected in the results of the automated-uncertainty S-approach as well. The results are combined with the automated-uncertainty S-approach at the value of the oldest year of the deterministic S-approach. This applies for the deterministic T-approach as well.

The Figures with blue dots and grey uncertainties are defined as the automated-uncertainty T-approaches. The Figures with green, red, blue dots and cyan-purple uncertainties are defined as the automated-uncertainty S-approach. This applies for all Figures in this section.

In Appendix 1 the process of the Matlab script used for the Case Study is elaborated. It consists of predefined input settings to run the script. When the intermediate results are stored after the Data Transformation, predefined settings for the Data Loading can be used to generate the graphs for the automated-uncertainty S- and T-approaches.

The following Regions of Interest are simulated and visualised for the automated-uncertainty S- and T-approaches including comparison to the deterministic S- and T-approaches, except for the entire Dutch Coastal Zone:

Wadden Basin
Wadden Coast
Wadden: Eierlandse Zeegat Basin
Wadden: Eierlandse Zeegat Coast
Wadden: Ameland Basin
Wadden: Ameland Coast
Wadden: Friesche Zeegat Basin
Wadden: Friesche Zeegat Coast
Wadden: Texel/Marsdiep Basin
Wadden: Texel/Marsdiep Coast
Wadden: Vlie Basin
Wadden: Vlie Coast
Voordelta
Voordelta: Haringvliet
Voordelta: Grevelingen
Voordelta: Oosterschelde
Voordelta: Westerschelde
Dutch Coastal Zone

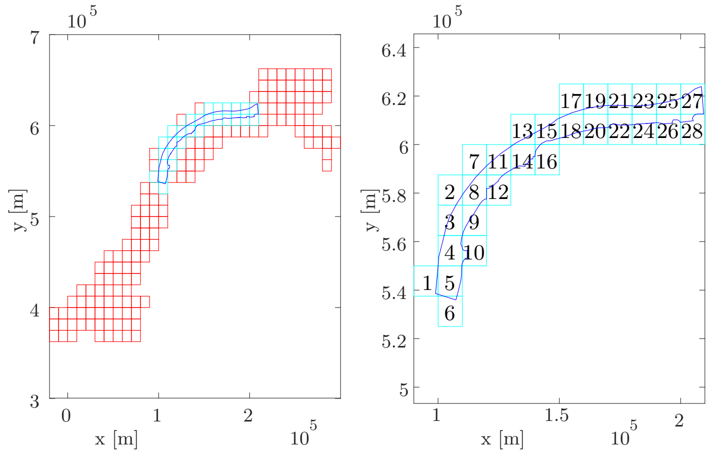


Figure 5.1: Wadden Coast Region of Interest (blue) in Dutch Coastal Zone and (un/)used Data Products (red/cyan)

5.2.1 Wadden Basin/Coast

Figure 5.2 gives a visualisation of the results of the T-approach of the automated-uncertainty approach for both the Wadden Basin and Coast Regions of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Wadden Basin Region of Interest is visualised in Figure 2.13, combining the basin and coast Regions of Interest TX, ELD, VLIE, AME, FRZ and ED. The Wadden Coast is defined as the Regions of Interest that are not located within the basin, visualised in Figure 5.1.

Calculation time for the Wadden Basin for the S-approach was 20.03[h] and for the T-approach 40.23[h]. For the Wadden Coast the calculation time for the S-approach was 12.33[h] and for the T-approach 25.25[h].

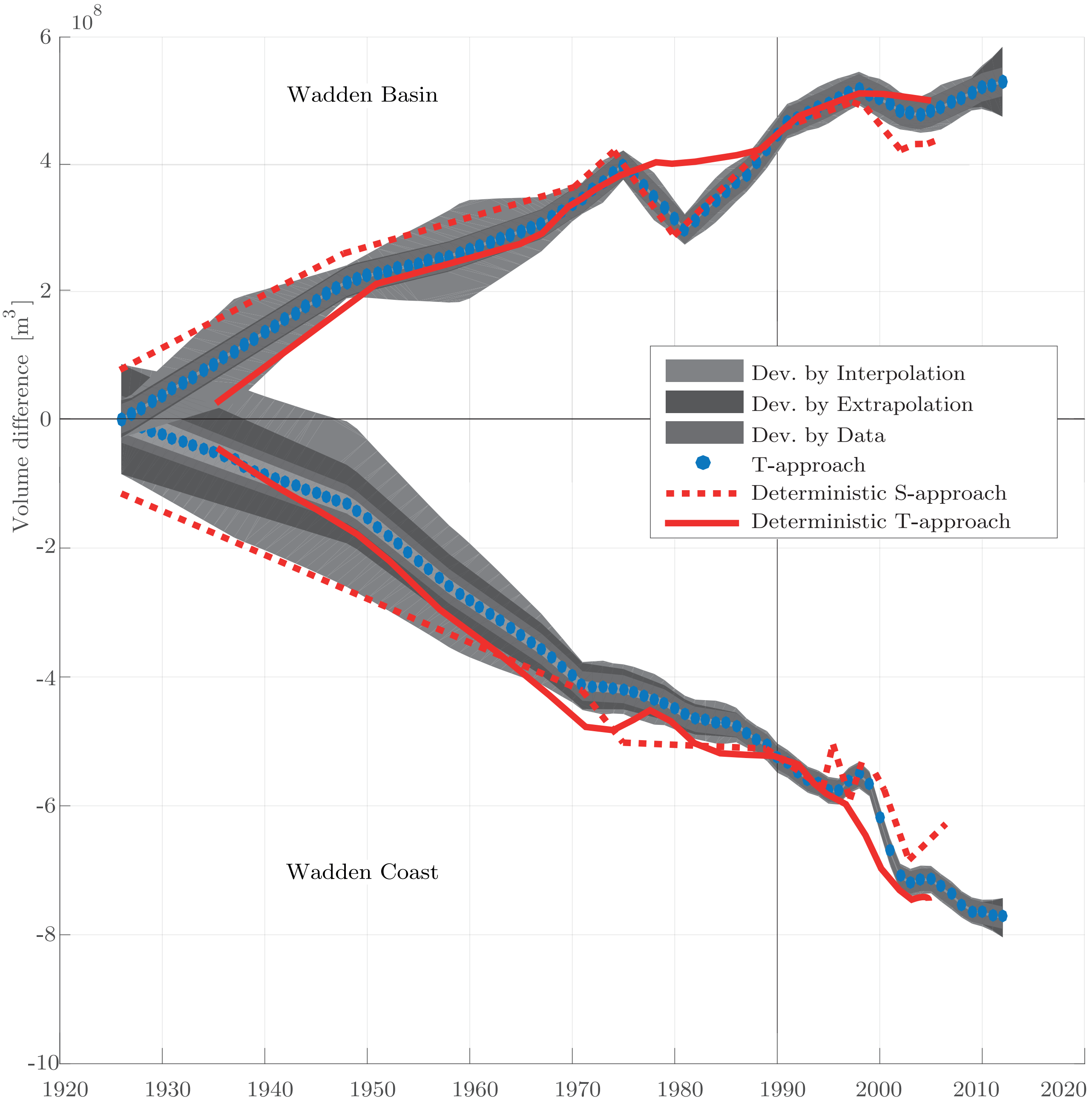


Figure 5.2: Wadden Basin/Coast for automated T-approach and deterministic S- and T-approaches

Figure 5.3 gives a visualisation of the results of the S-approach of the automated-uncertainty approach for both the Wadden Basin and Coast Regions of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Wadden Basin Region of Interest is visualised in Figure 2.13, combining the basin and coast Regions of Interest TX, ELD, VLIE, AME, FRZ and ED. The Wadden Coast is defined as the Regions of Interest that are not located within the basin, visualised in Figure 5.1.

Calculation time for the Wadden Basin for the S-approach was 20.03[h] and for the T-approach 40.23[h]. For the Wadden Coast the calculation time for the S-approach was 12.33[h] and for the T-approach 25.25[h].

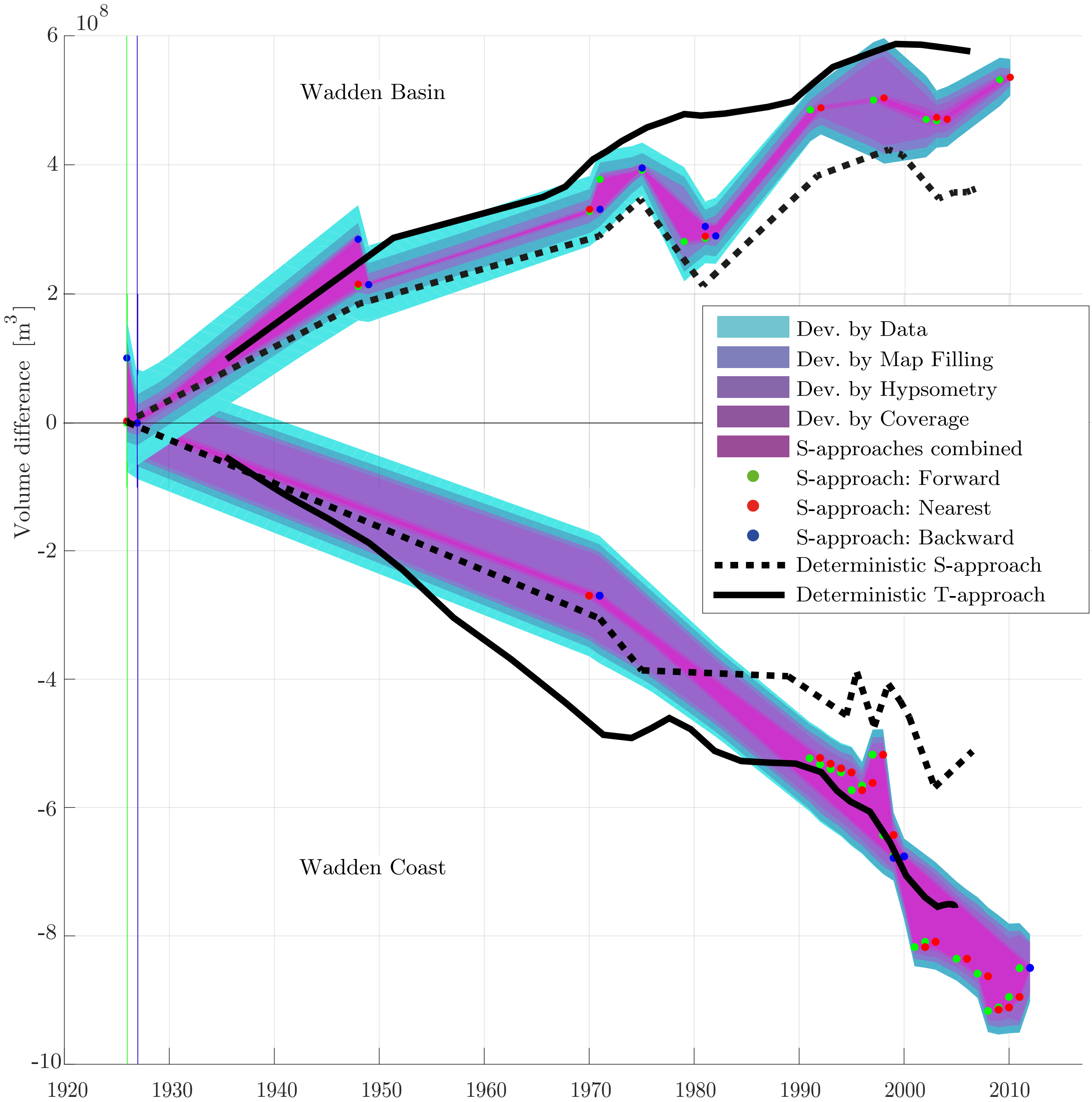


Figure 5.3: Wadden Basin/Coast for automated S-approach and deterministic S- and T-approaches

5.2.2 Wadden: Eierlandse Gat Basin

Figure 5.4 gives a visualisation of the results of the S- and T-approaches (respectively bottom and top) of the automated-uncertainty approach for the Eierlandse Gat (ELD) Basin Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Eierlandse Gat (ELD) Basin Region of Interest is visualised in Figure 2.13. Calculation time for the Eierlandse Gat Basin for the S-approach was 1.52[h] and for the T-approach 2.63[h].

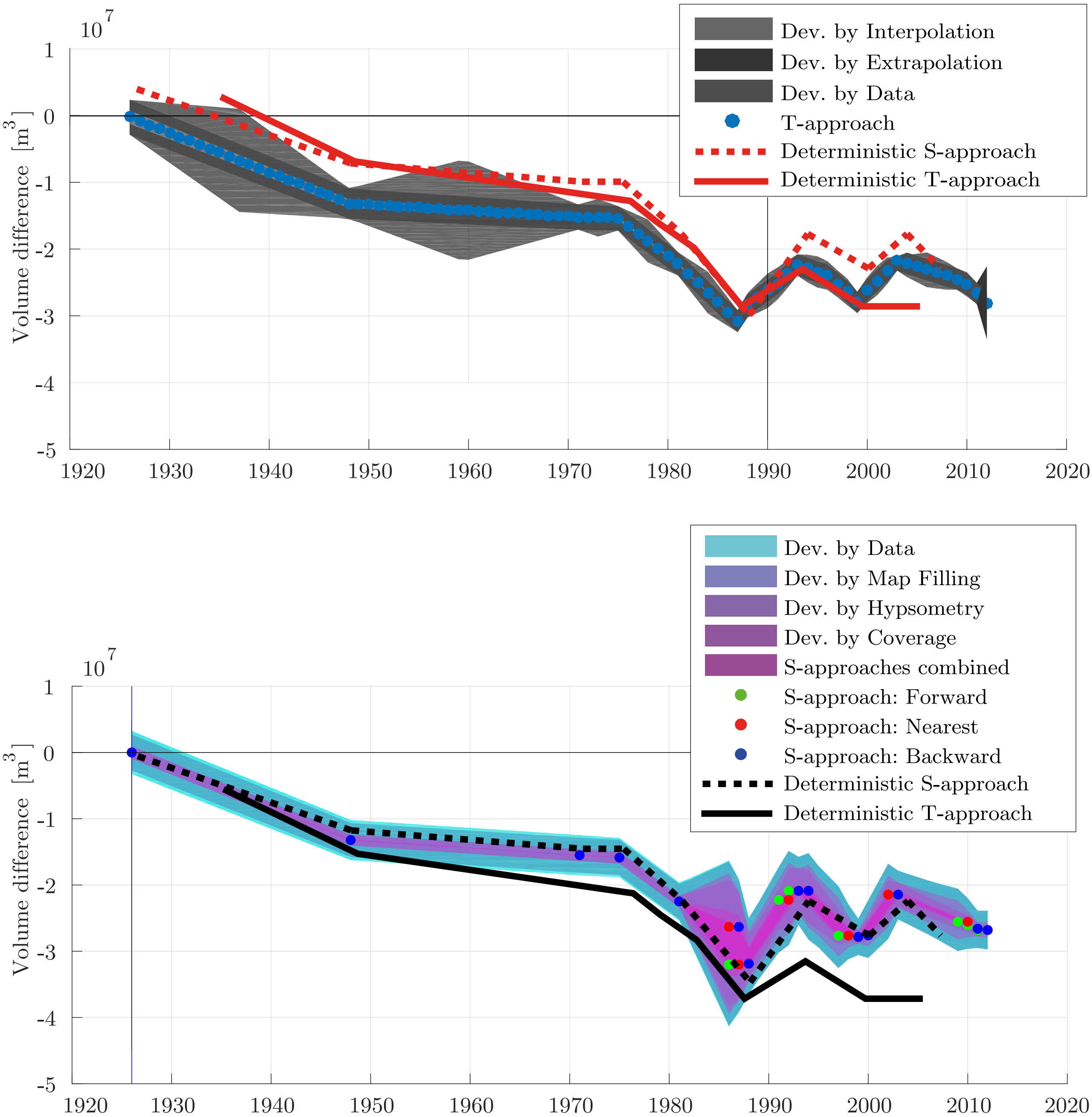
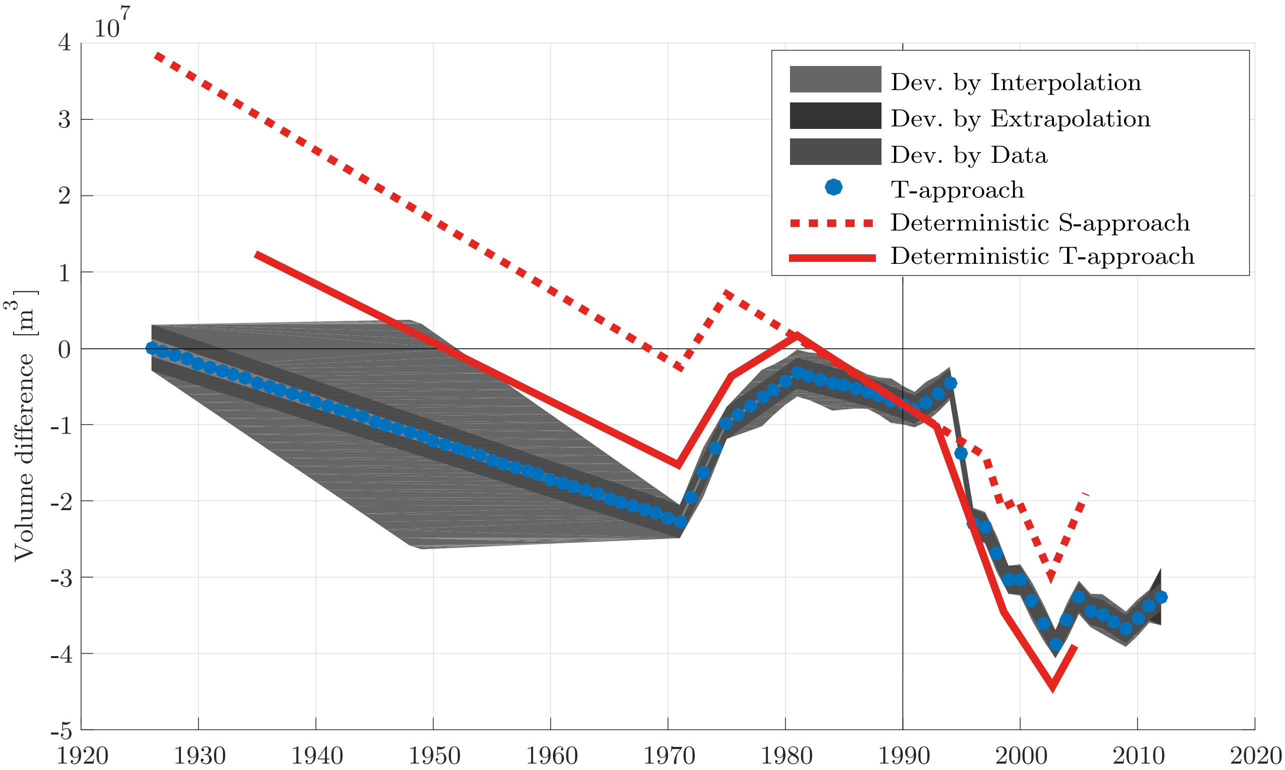


Figure 5.4: Eierlandse Gat Basin for automated and deterministic S- and T-approaches

5.2.3 Wadden: Eierlandse Gat Coast

Figure 5.5 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Eierlandse Gat (ELD) Coast Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Eierlandse Gat (ELD) Coast Region of Interest is visualised in Figure 2.13. Calculation time for the Eierlandse Gat Coast for the S-approach was 2.09[h] and for the T-approach 4.55[h].



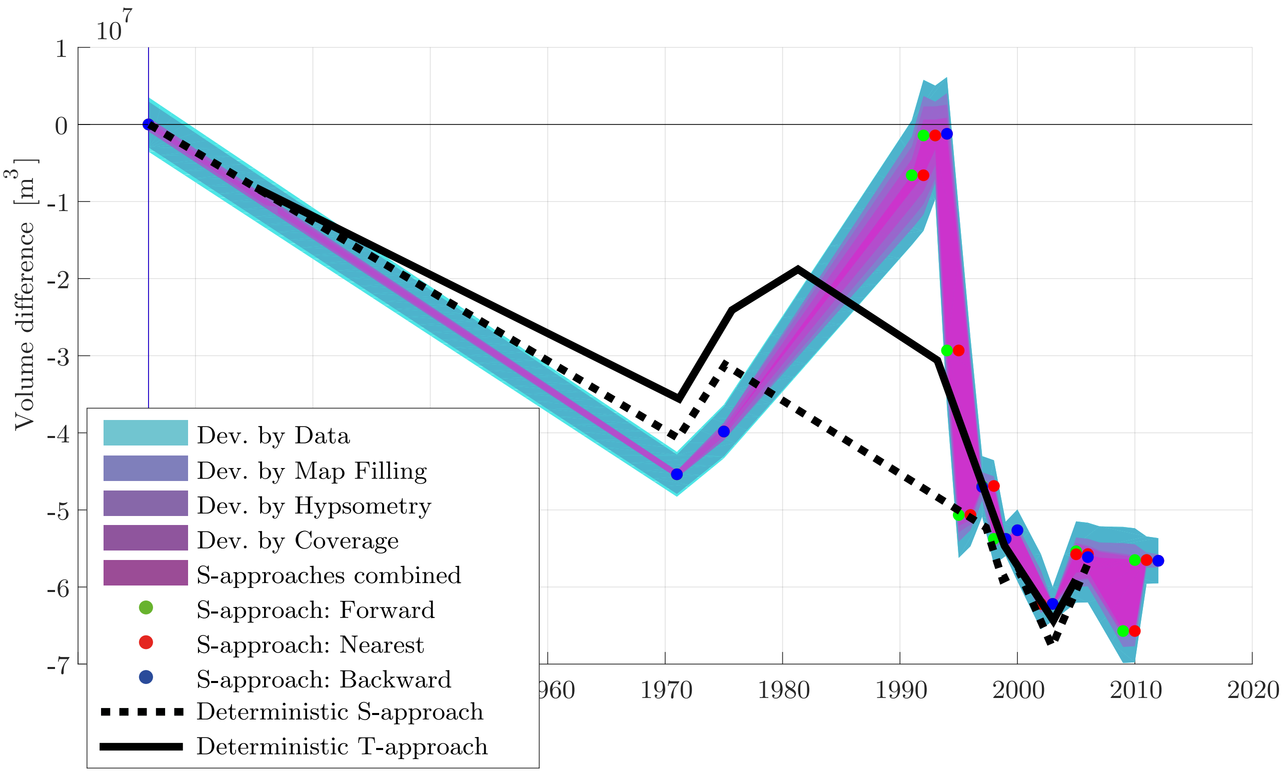


Figure 5.5: Eierlandse Gat Coast for automated and deterministic S- and T-approaches

5.2.4 Wadden: Ameland Basin

Figure 5.6 gives a visualisation of the results of the S- and T-approaches (respectively bottom and top) of the automated-uncertainty approach for the Ameland (AME) Basin Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Ameland (AME) Basin Region of Interest is visualised in Figure 2.13. Calculation time for the Ameland Basin for the S-approach was 3.26[h] and for the T-approach 5.27[h].

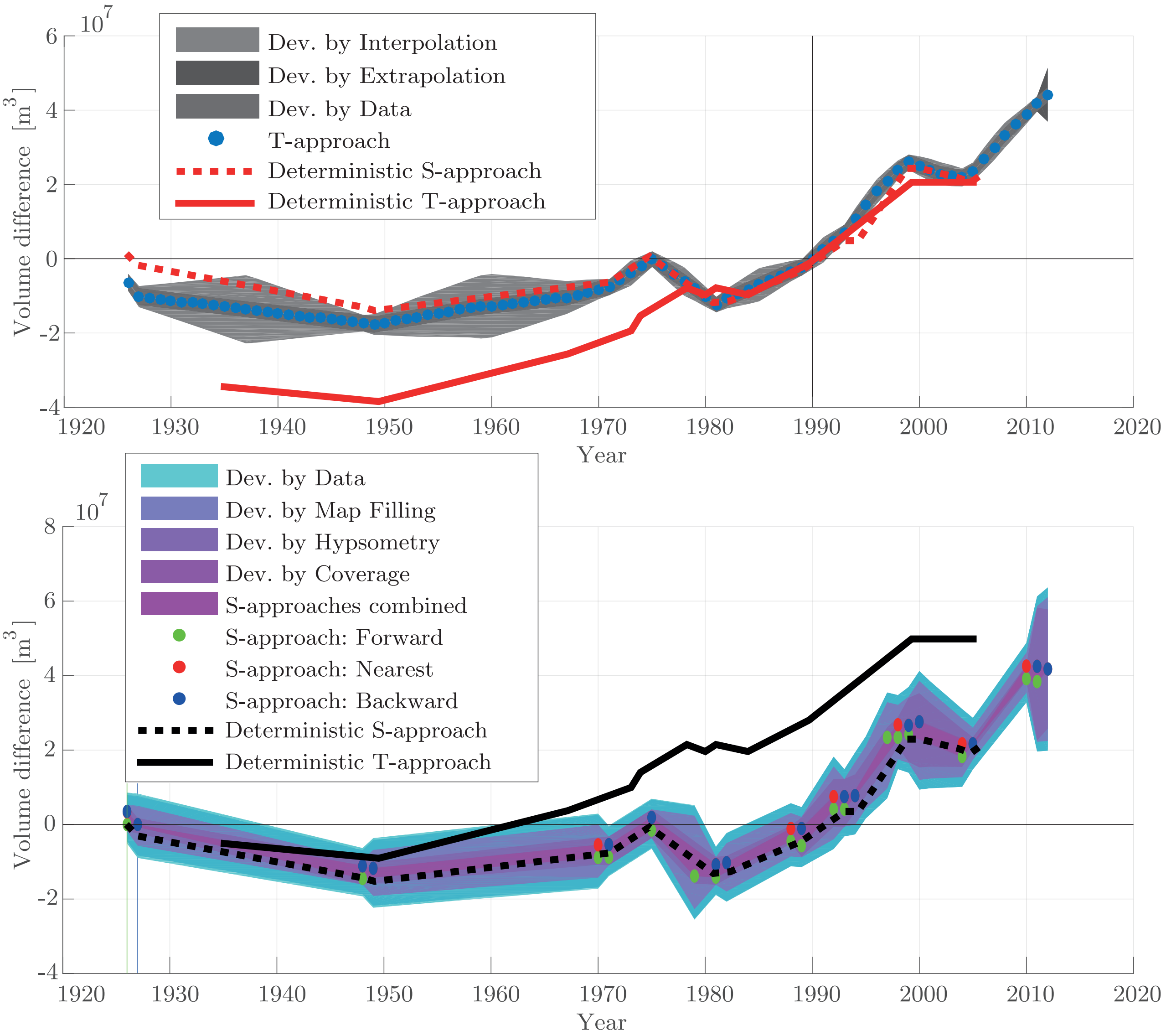


Figure 5.6: Ameland Basin for automated and deterministic S- and T-approaches

5.2.5 Wadden: Ameland Coast

Figure 5.7 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Ameland (AME) Coast Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Ameland (AME) Coast Region of Interest is visualised in Figure 2.13. Calculation time for the Ameland Coast for the S-approach was 2.75[h] and for the T-approach 6.42[h].

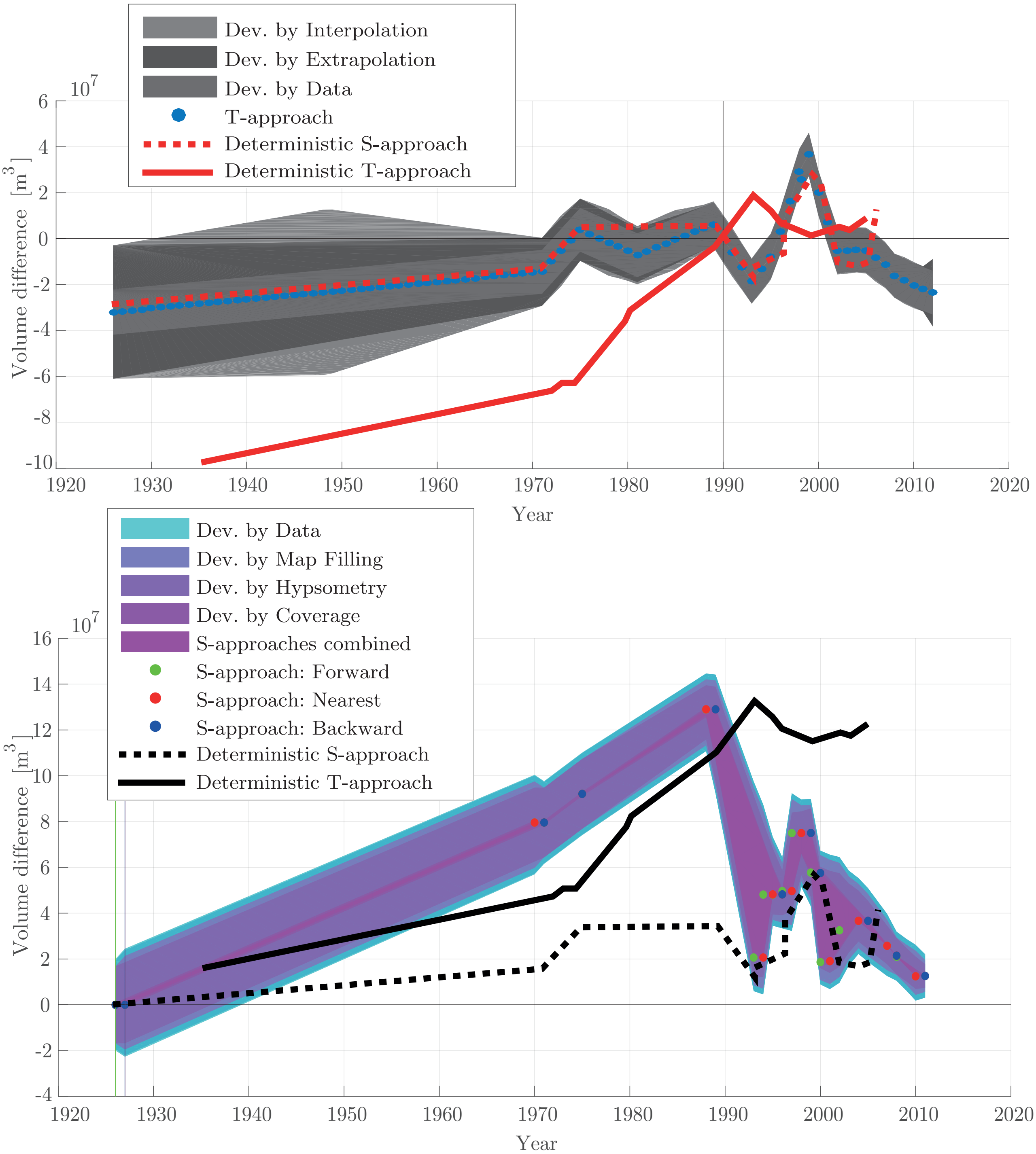


Figure 5.7: Ameland Coast for automated and deterministic S- and T-approaches

5.2.6 Wadden: Friesche Zeegat Basin

Figure 5.8 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Friesche Zeegat (FRZ) Basin Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Friesche Zeegat (FRZ) Basin Region of Interest is visualised in Figure 2.13. Calculation time for the Friesche Zeegat Basin for the S-approach was 1.97[h] and for the T-approach 5.59[h].

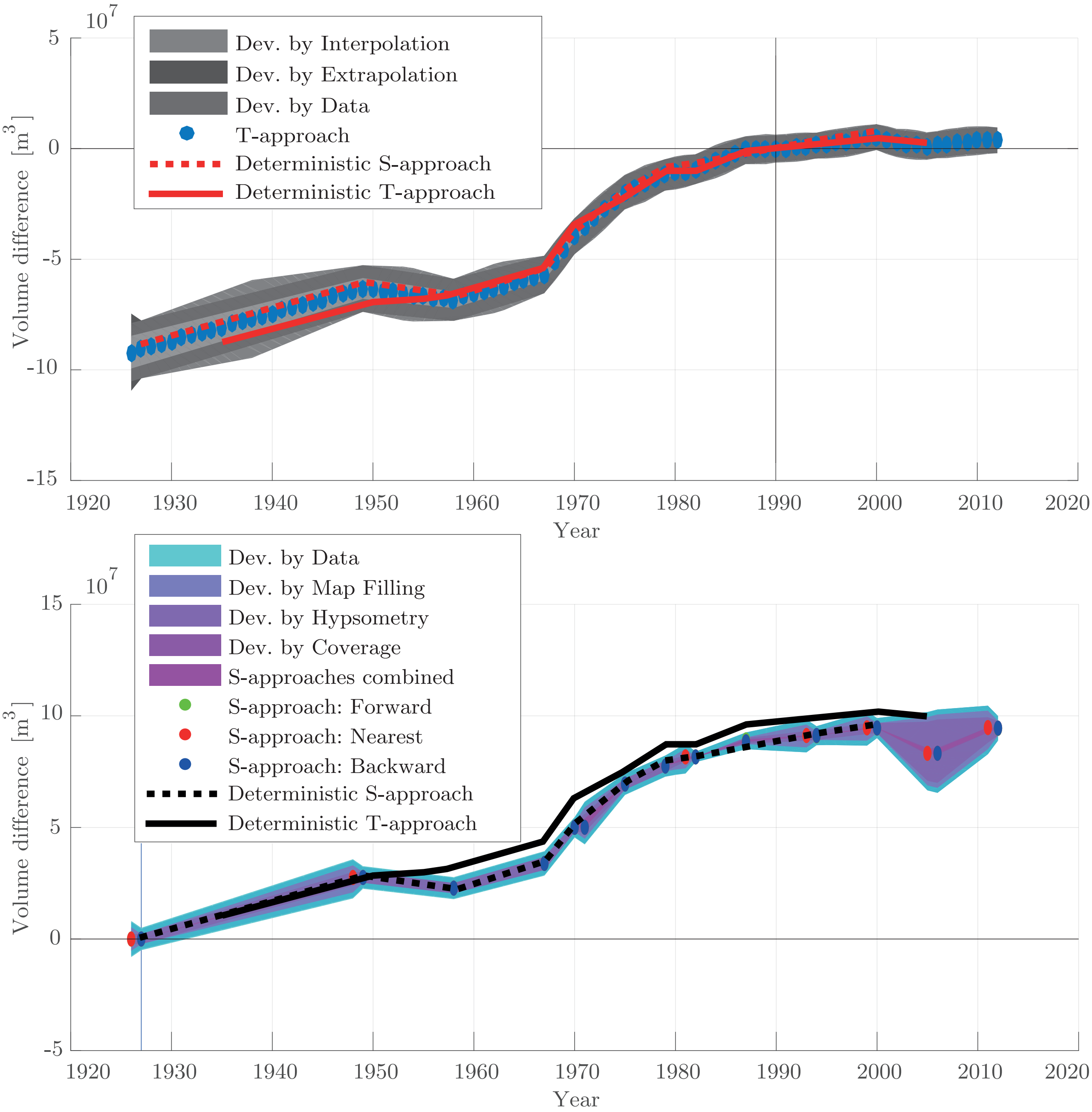


Figure 5.8: Friesche Zeegat Basin for automated and deterministic S- and T-approaches

5.2.7 Wadden: Friesche Zeegat Coast

Figure 5.9 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Friesche Zeegat (FRZ) Coast Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Friesche Zeegat (FRZ) Coast Region of Interest is visualised in Figure 2.13. Calculation time for the Friesche Zeegat Coast for the S-approach was 3.00[h] and for the T-approach 6.24[h].

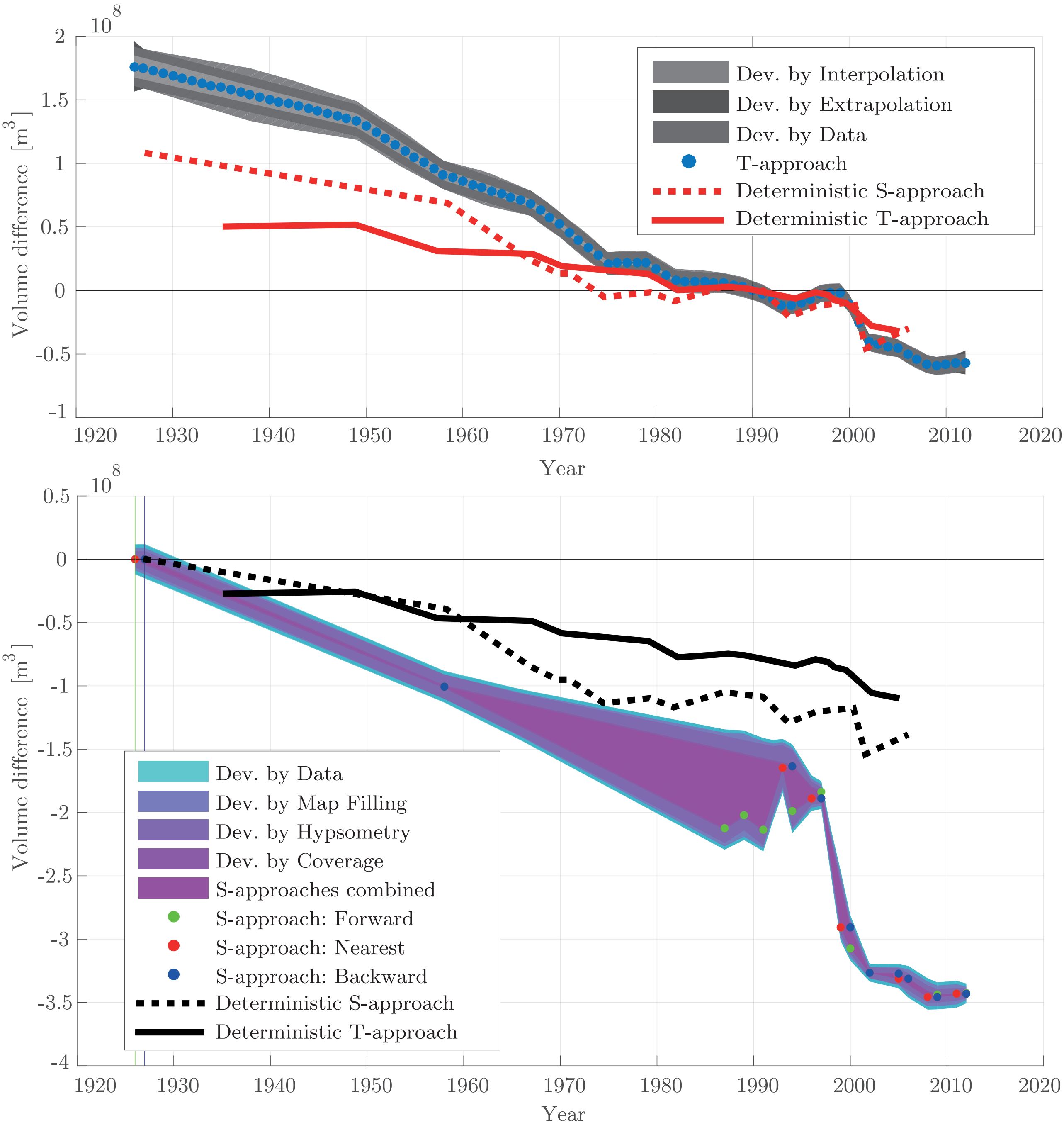


Figure 5.9: Friesche Zeegat Coast for automated and deterministic S- and T-approaches

5.2.8 Wadden: Texel/Marsdiep Basin

Figure 5.10 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Texel/Marsdiep (MD) Basin Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Texel/Marsdiep (MD) Basin Region of Interest is visualised in Figure 2.13. Calculation time for the Texel/Marsdiep Basin for the S-approach was 7.40[h] and for the T-approach 14.36[h].

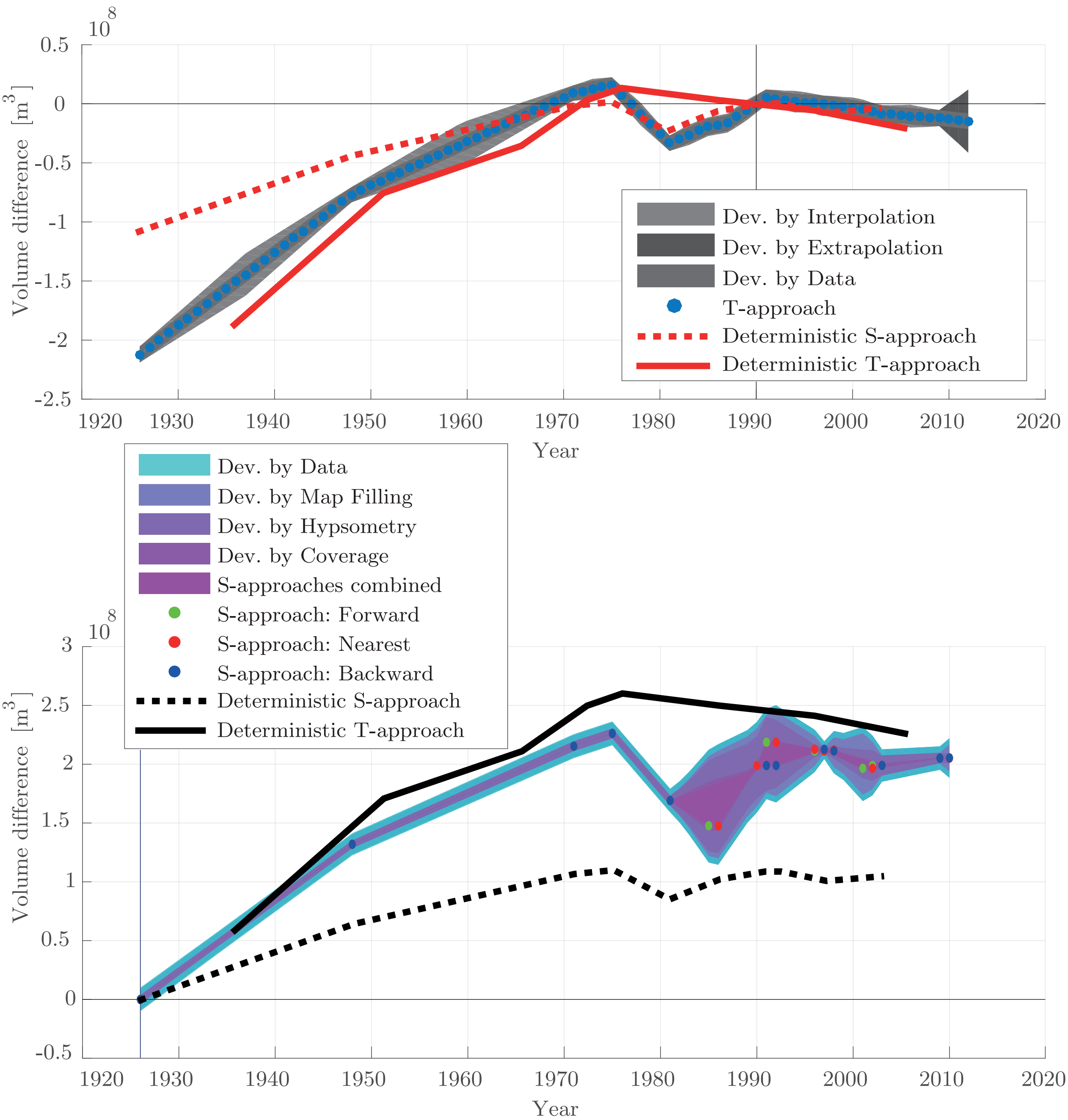


Figure 5.10: Texel/Marsdiep Basin for automated and deterministic S- and T-approaches

5.2.9 Wadden: Texel/Marsdiep Coast

Figure 5.10 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Texel/Marsdiep (MD) Coast Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Texel/Marsdiep (MD) Coast Region of Interest is visualised in Figure 2.13. Calculation time for the Texel/Marsdiep Coast for the S-approach was 3.09[h] and for the T-approach 5.57[h].

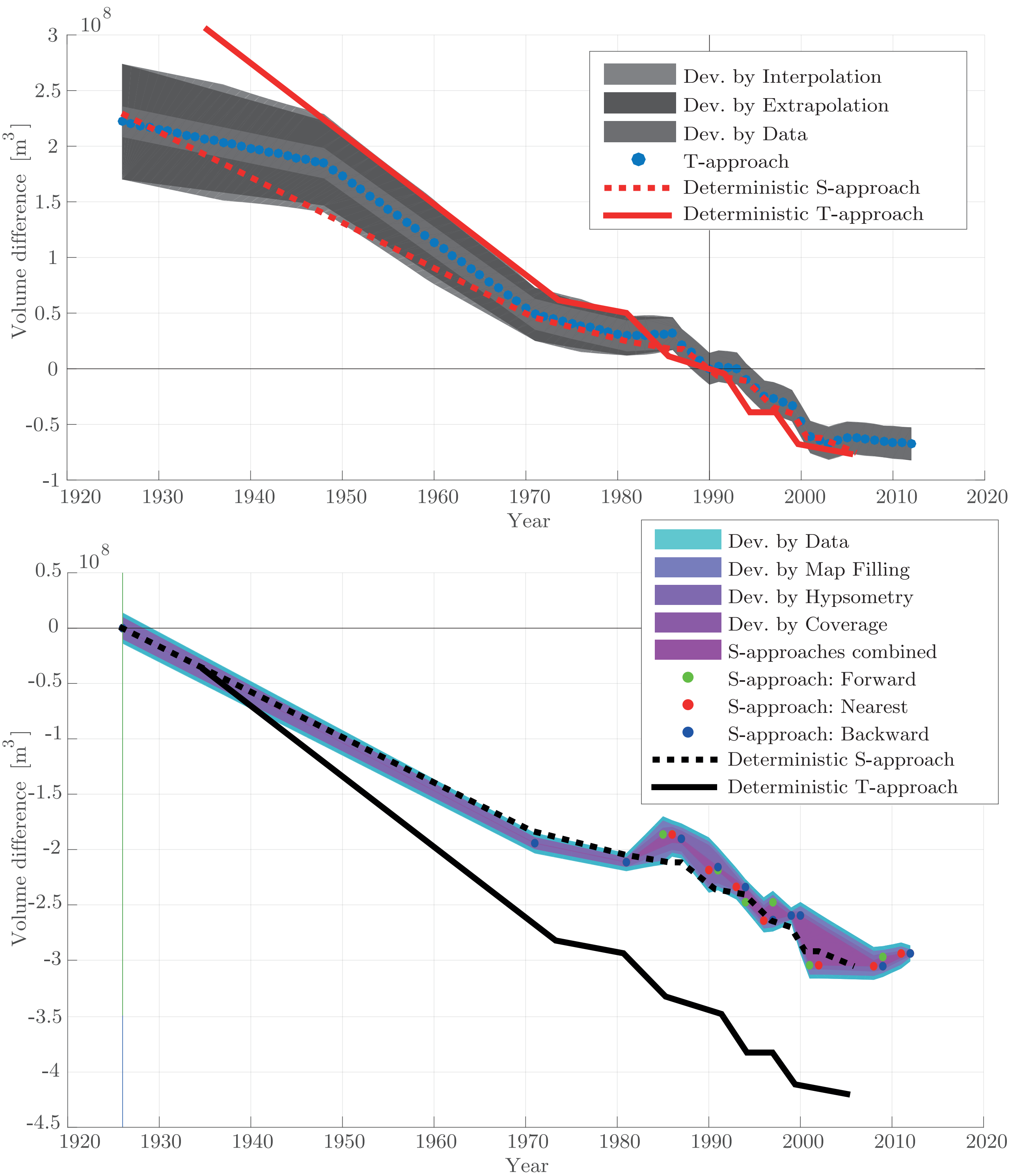


Figure 5.11: Texel/Marsdiep Coast for automated and deterministic S- and T-approaches

5.2.10 Wadden: Vlie Basin

Figure 5.12 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Vlie (VLIE) Basin Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Vlie (VLIE) Basin Region of Interest is visualised in Figure 2.13. Calculation time for the Vlie Basin for the S-approach was 6.67[h] and for the T-approach 13.16[h].

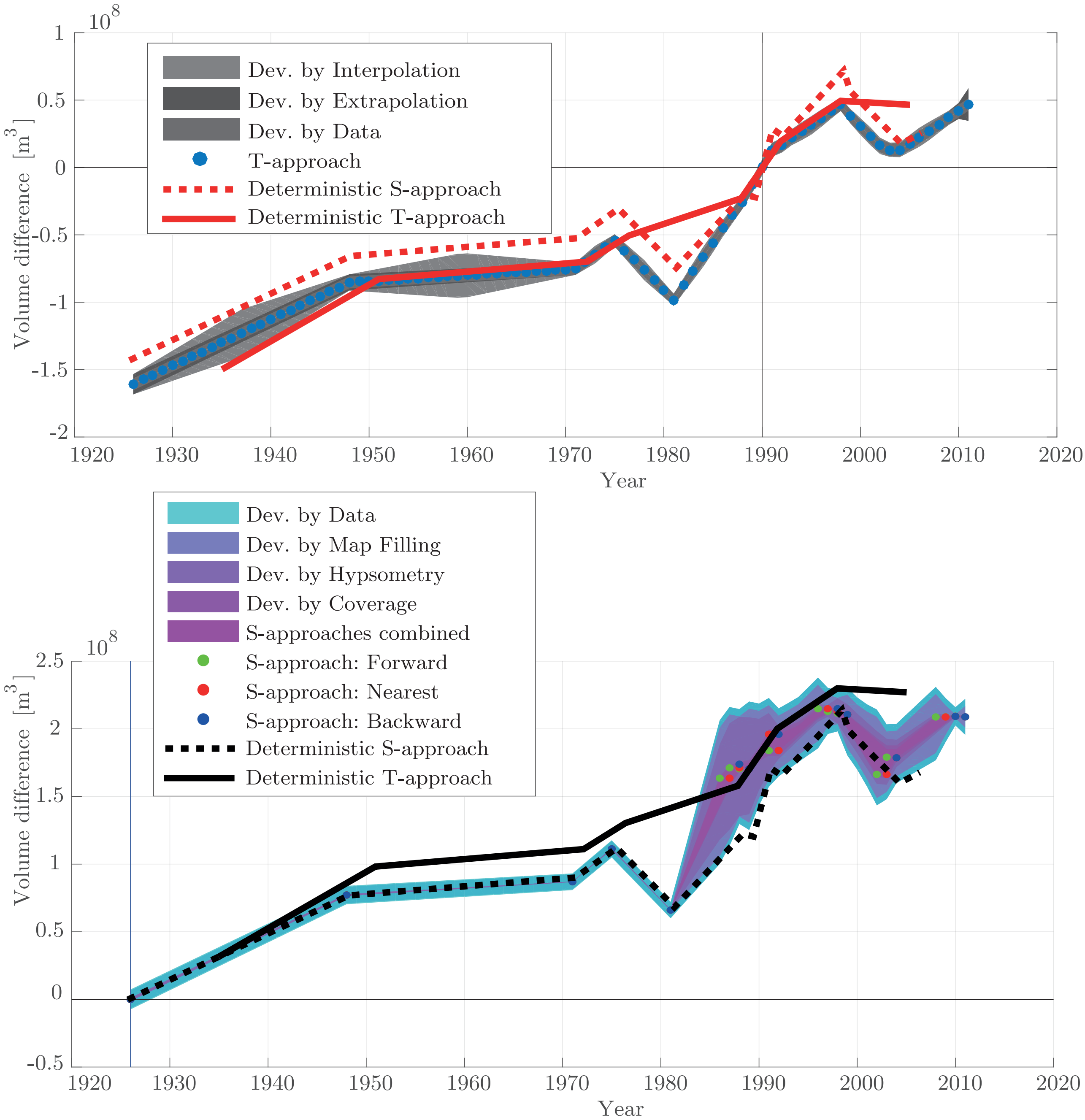


Figure 5.12: Vlie Basin for automated and deterministic S- and T-approaches

5.2.11 Wadden: Vlie Coast

Figure 5.13 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Vlie (VLIE) Coast Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012) for this same Region of Interest are included.

The Vlie (VLIE) Coast Region of Interest is visualised in Figure 2.13. Calculation time for the Vlie Coast for the S-approach was 2.89[h] and for the T-approach 5.20[h].

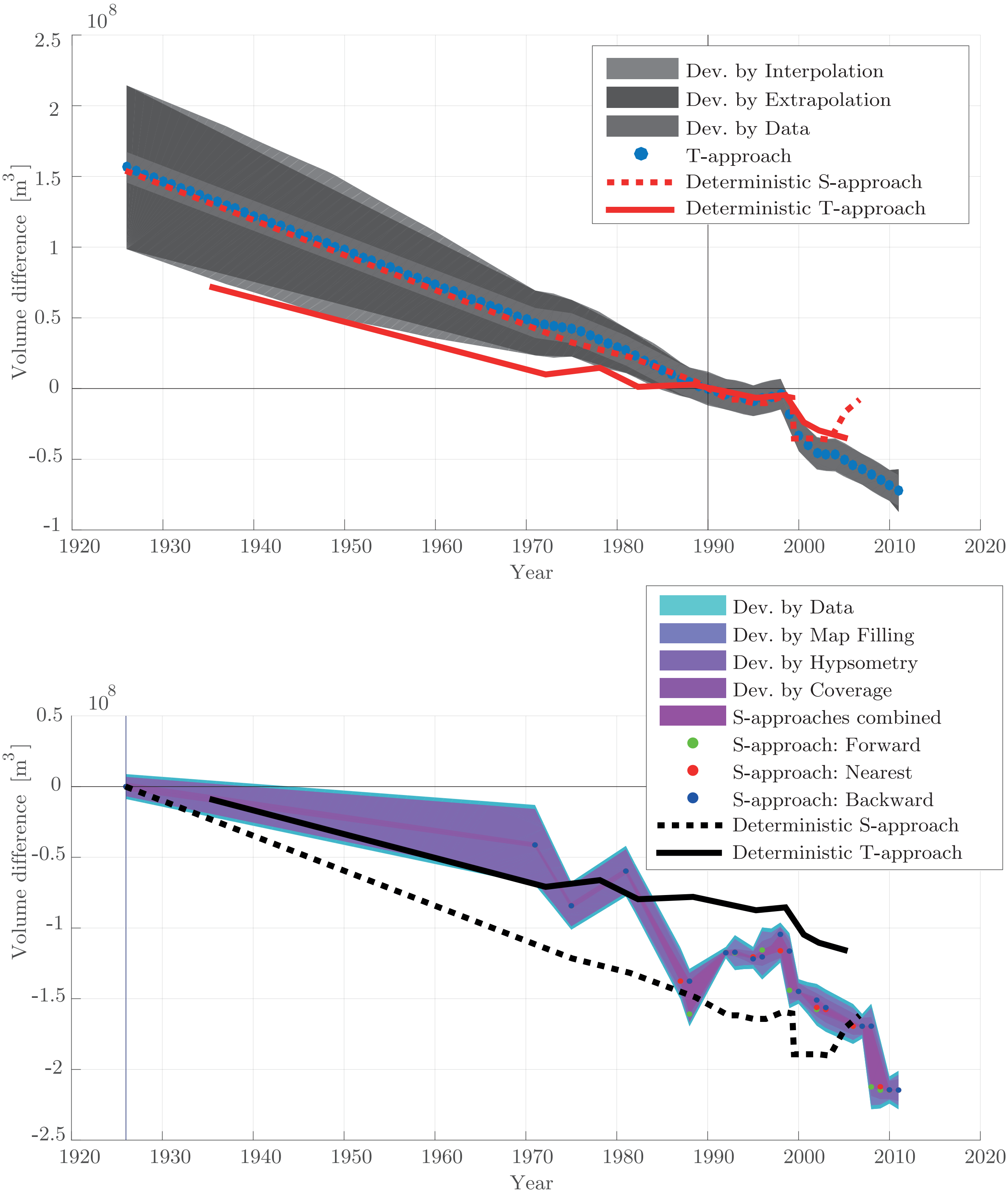


Figure 5.13: Vlie Coast for automated and deterministic S- and T-approaches

5.2.12 Voordelta

Figure 5.16 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Voordelta Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias and van der Spek (2014) for this same Region of Interest are included.

The Voordelta Region of Interest is visualised in Figure 5.14 and 5.15. Calculation time for the Voordelta for the S-approach was 13.31[h] and for the T-approach 13.99[h].

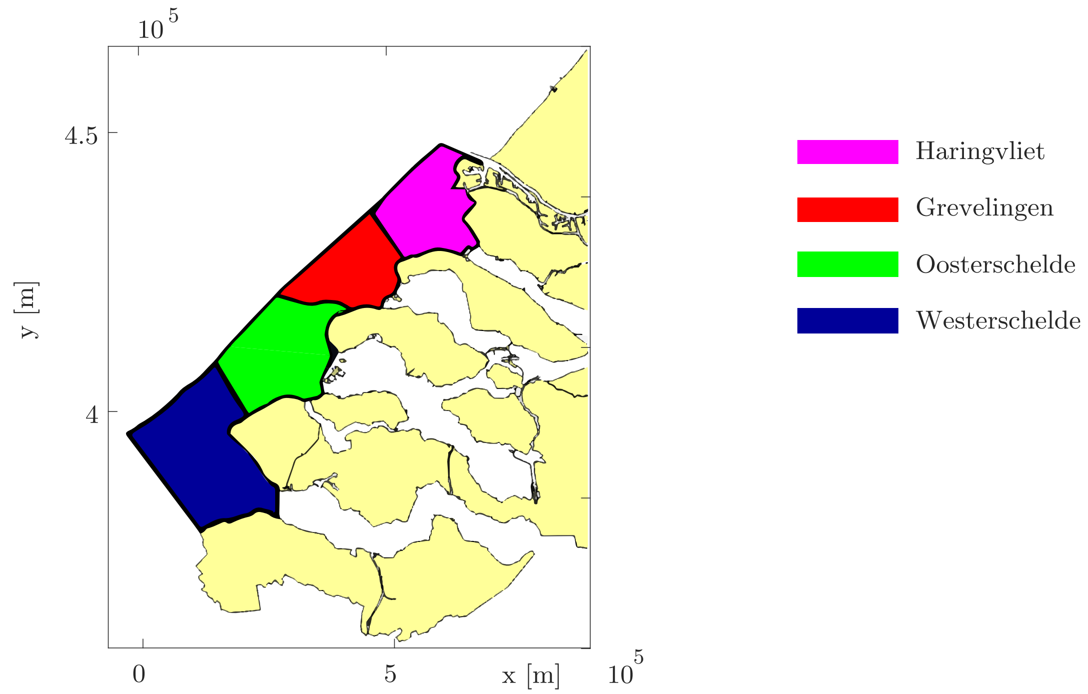


Figure 5.14: Voordelta with its subareas in Dutch Coastal Zone. Edited from van Koningsveld et al. (2008)

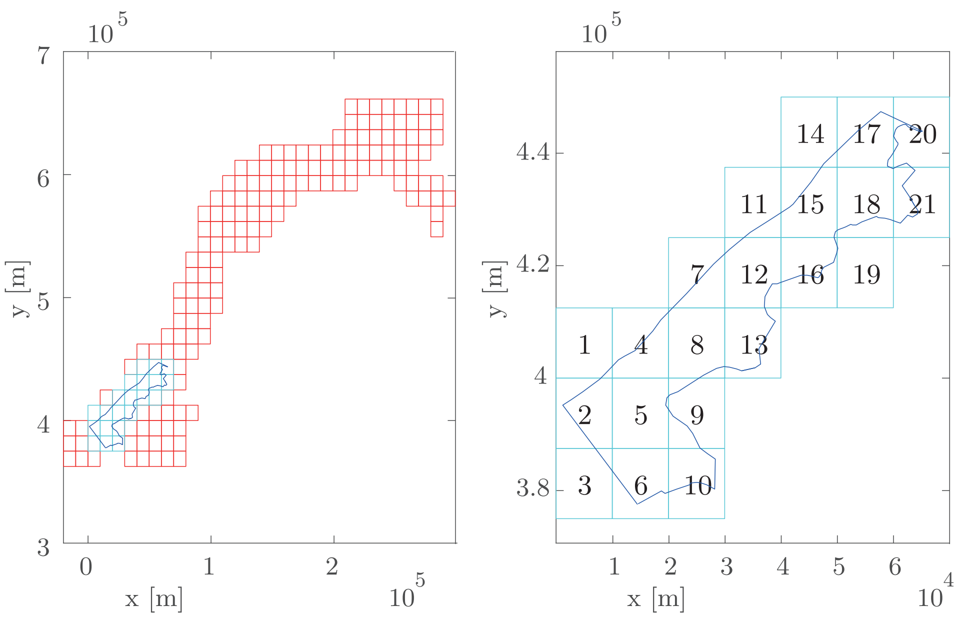


Figure 5.15: Voordelta Region of Interest (blue) in Dutch Coastal Zone and (un/)used Data Products (red/cyan)

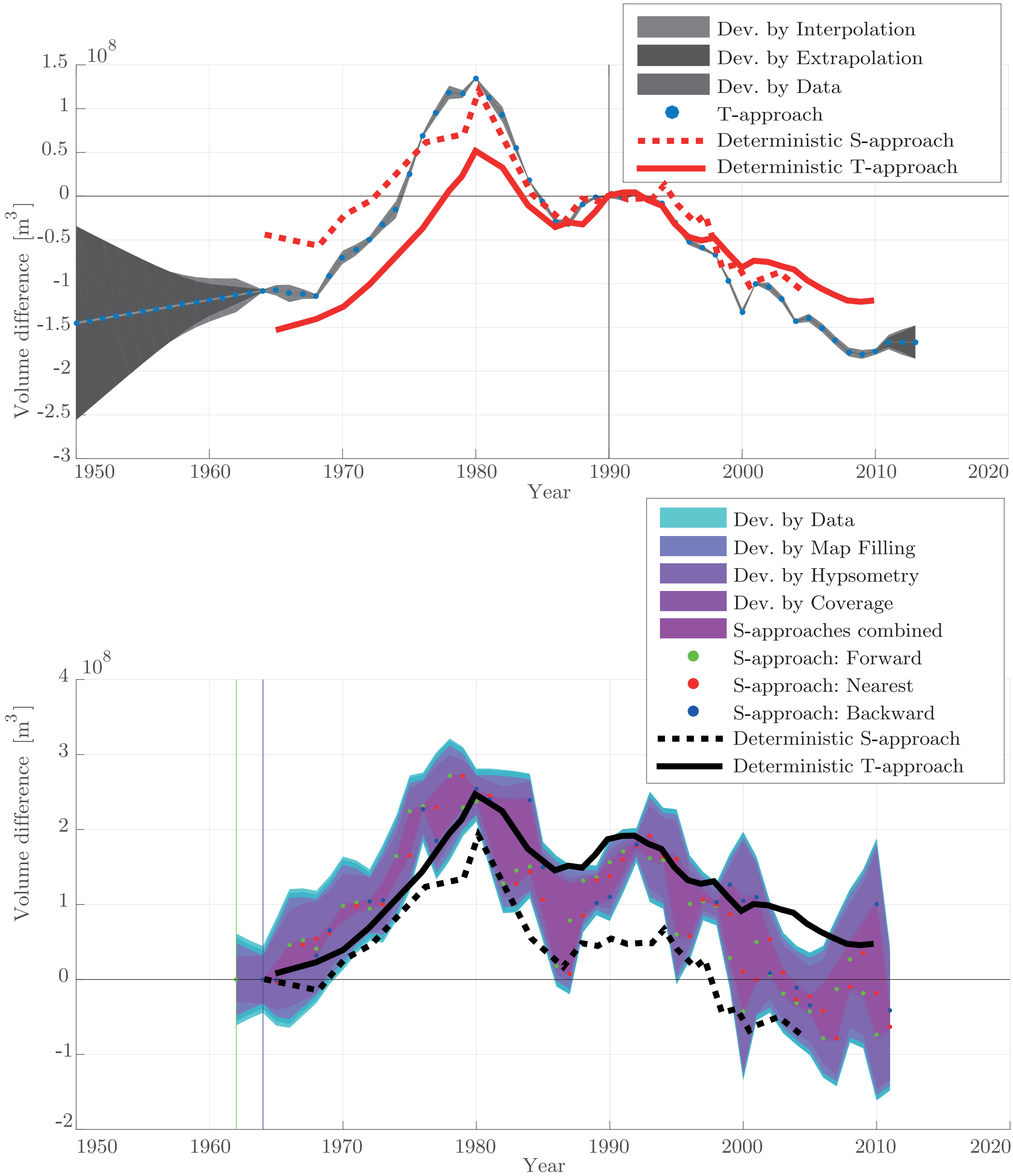


Figure 5.16: Voordelta for automated and deterministic S- and T-approaches

5.2.13 Voordelta: Haringvliet

Figure 5.17 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Haringvliet Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias and van der Spek (2014) for this same Region of Interest are included.

The Haringvliet Region of Interest is visualised in Figure 5.14. Calculation time for the Haringvliet for the S- and T-approach together was $5.41[h]$.

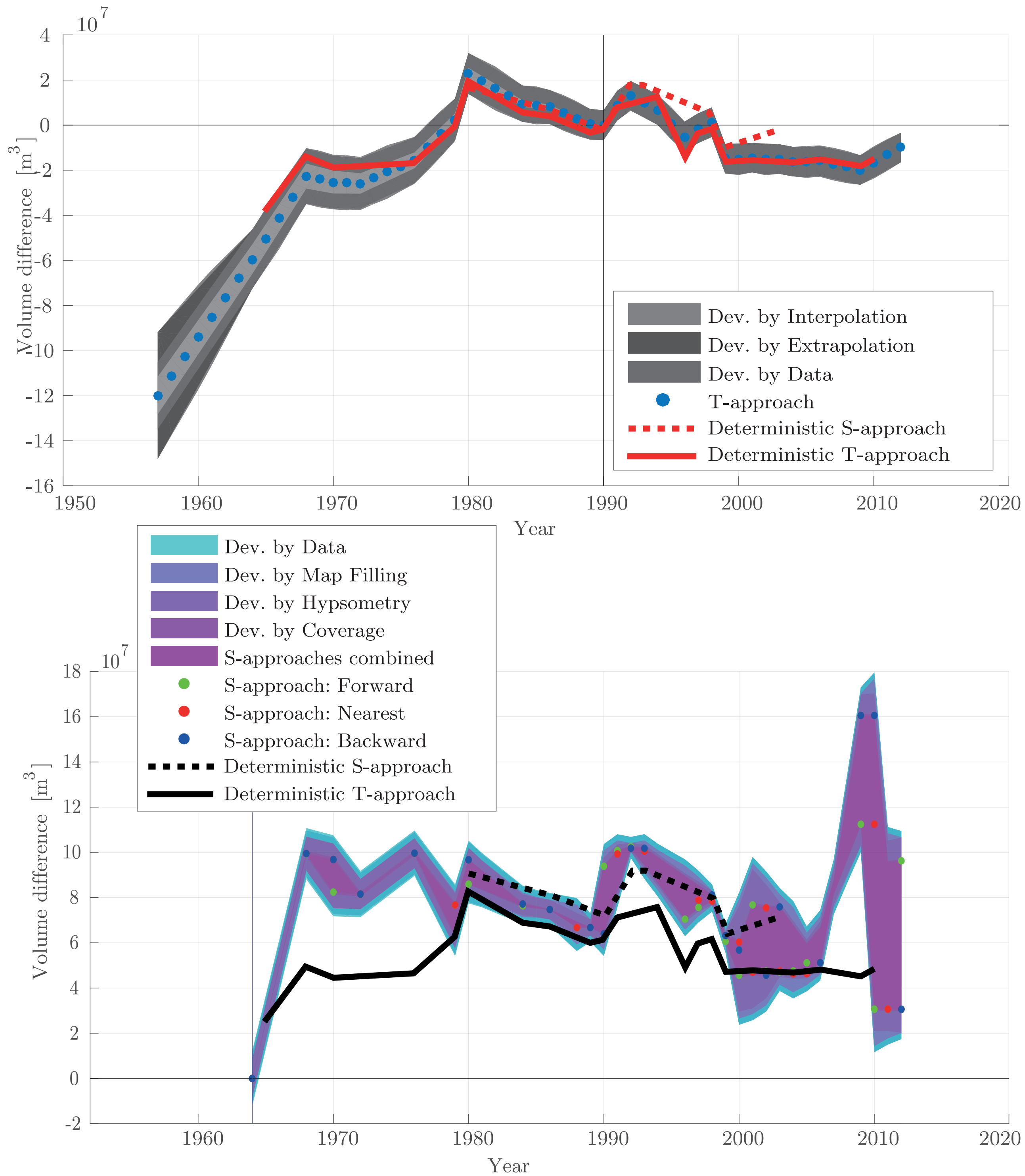


Figure 5.17: Haringvliet for automated and deterministic S- and T-approaches

5.2.14 Voordelta: Grevelingen

Figure 5.18 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Grevelingen Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias and van der Spek (2014) for this same Region of Interest are included.

The Grevelingen Region of Interest is visualised in Figure 5.14. Calculation time for the Grevelingen for the S- and T-approach together was 5.68[h].

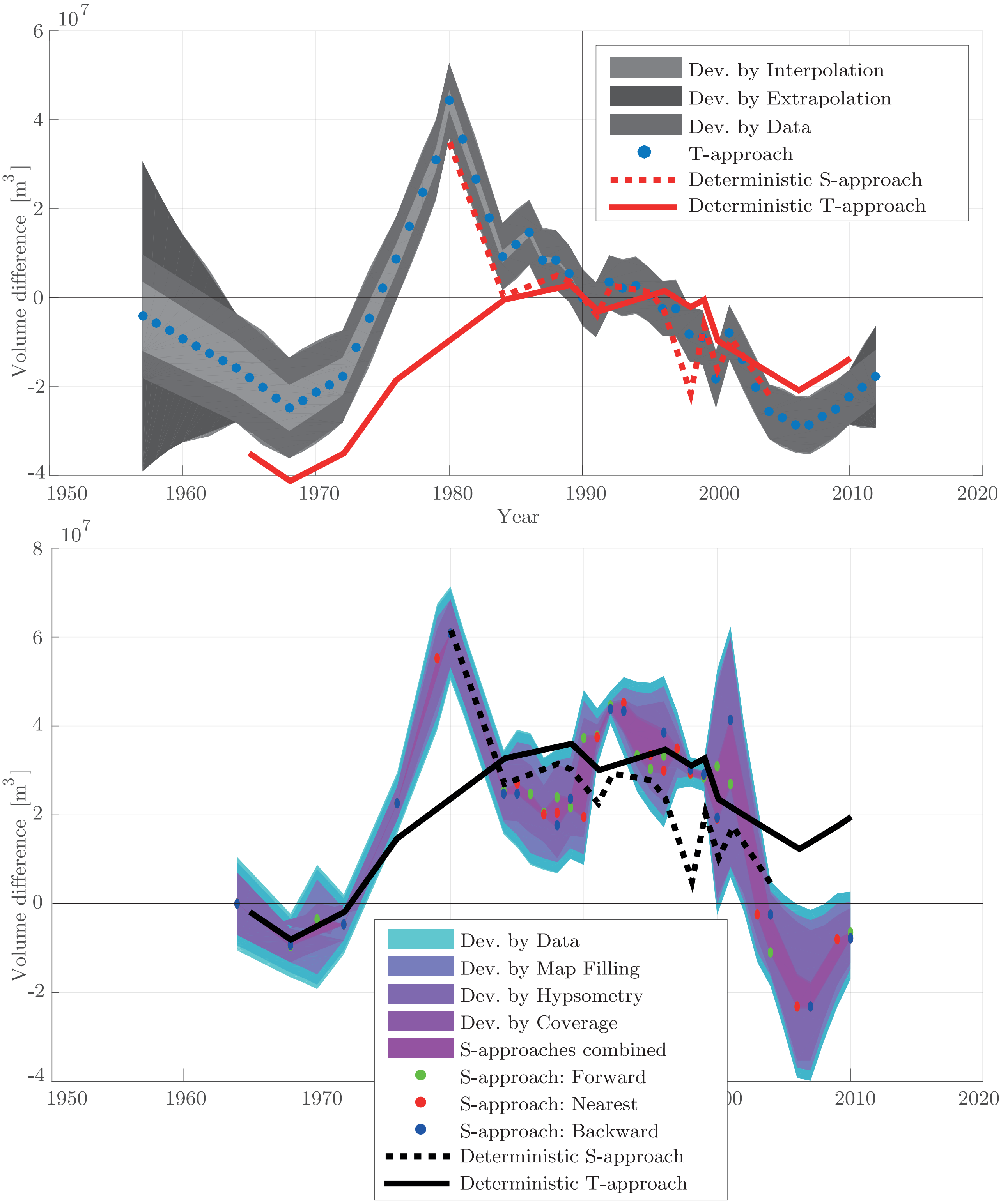


Figure 5.18: Grevelingen for automated and deterministic S- and T-approaches

5.2.15 Voordelta: Oosterschelde

Figure 5.19 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Oosterschelde Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias and van der Spek (2014) for this same Region of Interest are included.

The Oosterschelde Region of Interest is visualised in Figure 5.14. Calculation time for the Oosterschelde for the S- and T-approach together was 7.14[h].

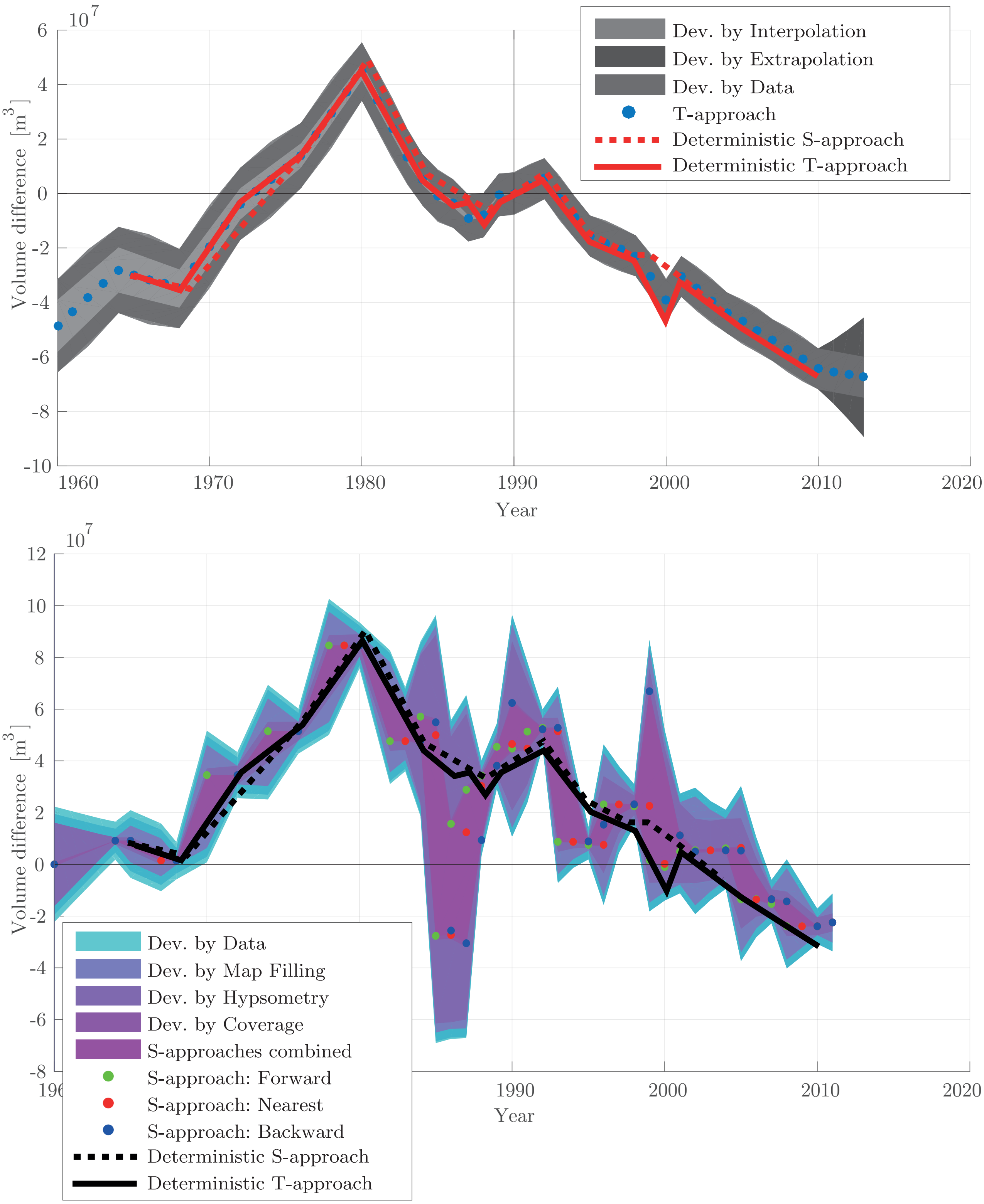


Figure 5.19: Oosterschelde for automated and deterministic S- and T-approaches

5.2.16 Voordelta: Westerschelde

Figure 5.20 gives a visualisation of the results of the S- and T-approach (respectively bottom and top) of the automated-uncertainty approach for the Westerschelde Region of Interest. The graphs of the deterministic S- and T-approaches by respectively van Koningsveld et al. (2008) and Elias and van der Spek (2014) for this same Region of Interest are included.

The Westerschelde Region of Interest is visualised in Figure 5.14. Calculation time for the Westerschelde for the S- and T-approach together was 10.63[h].

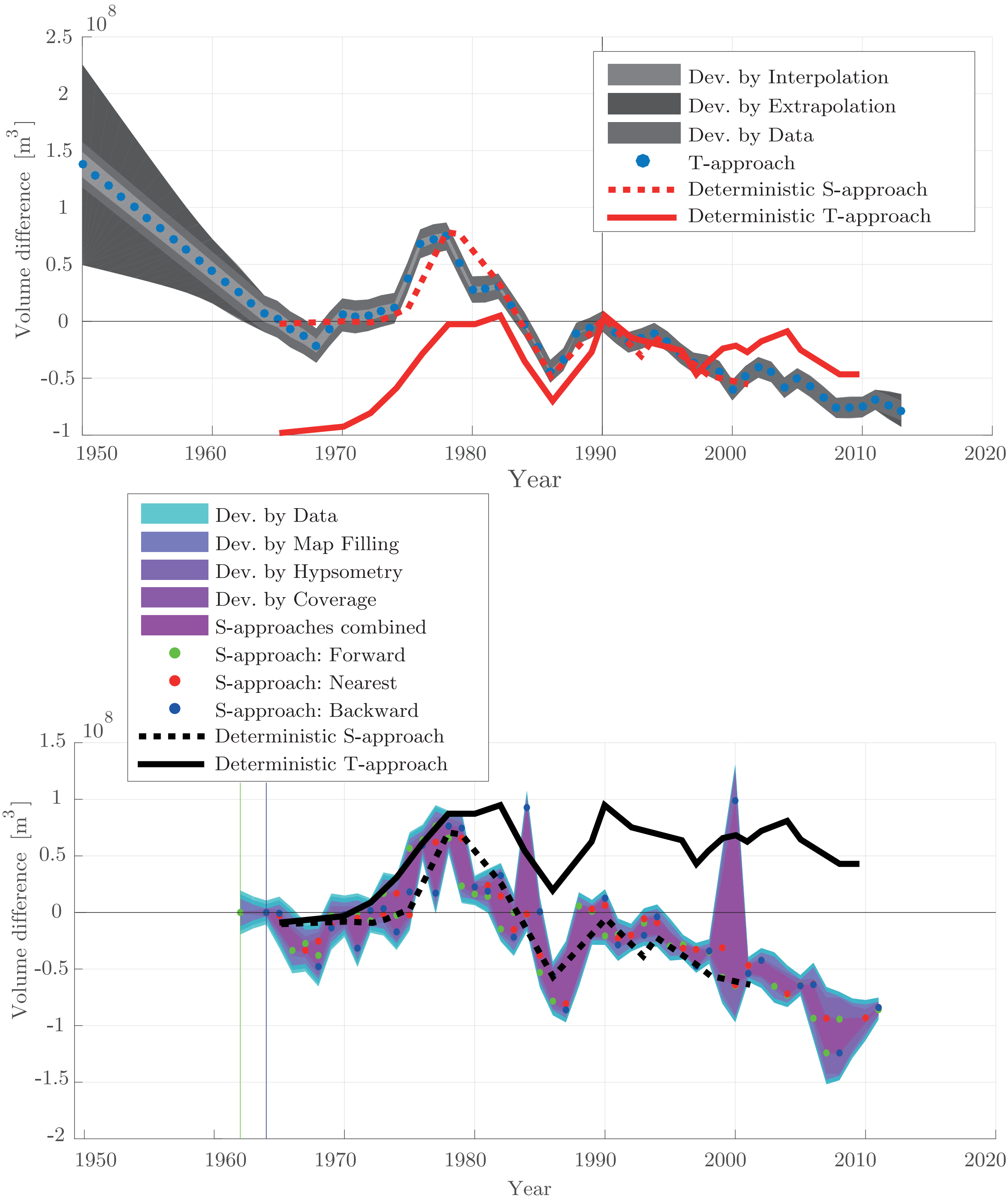


Figure 5.20: Westerschelde for automated and deterministic S- and T-approaches

5.2.17 Dutch Coastal Zone

Figure 5.22 gives a visualisation of the results of the S and T-approach of the automated-uncertainty approach for the Dutch Coastal Zone Region of Interest, visualised in Figure 5.21. Extreme Outliers are noticed for the T-approach, that are not visualised in this Figure. Calculation time for the Dutch Coastal Zone for the S- and T-approach was respectively 134.73[h] and 281.84[h]. The rectangular Region of Interest is chosen around all outer boundaries of the Data Products. Therefore for the automated S-approach a different minimal Region of Interest coverage has been used, namely OPT.spatialpolygonsurface = 0.2[-]. Because of this, the uncertainty created by this low coverage has been changed as well to OPT.spatialsurface = 0.01[m].

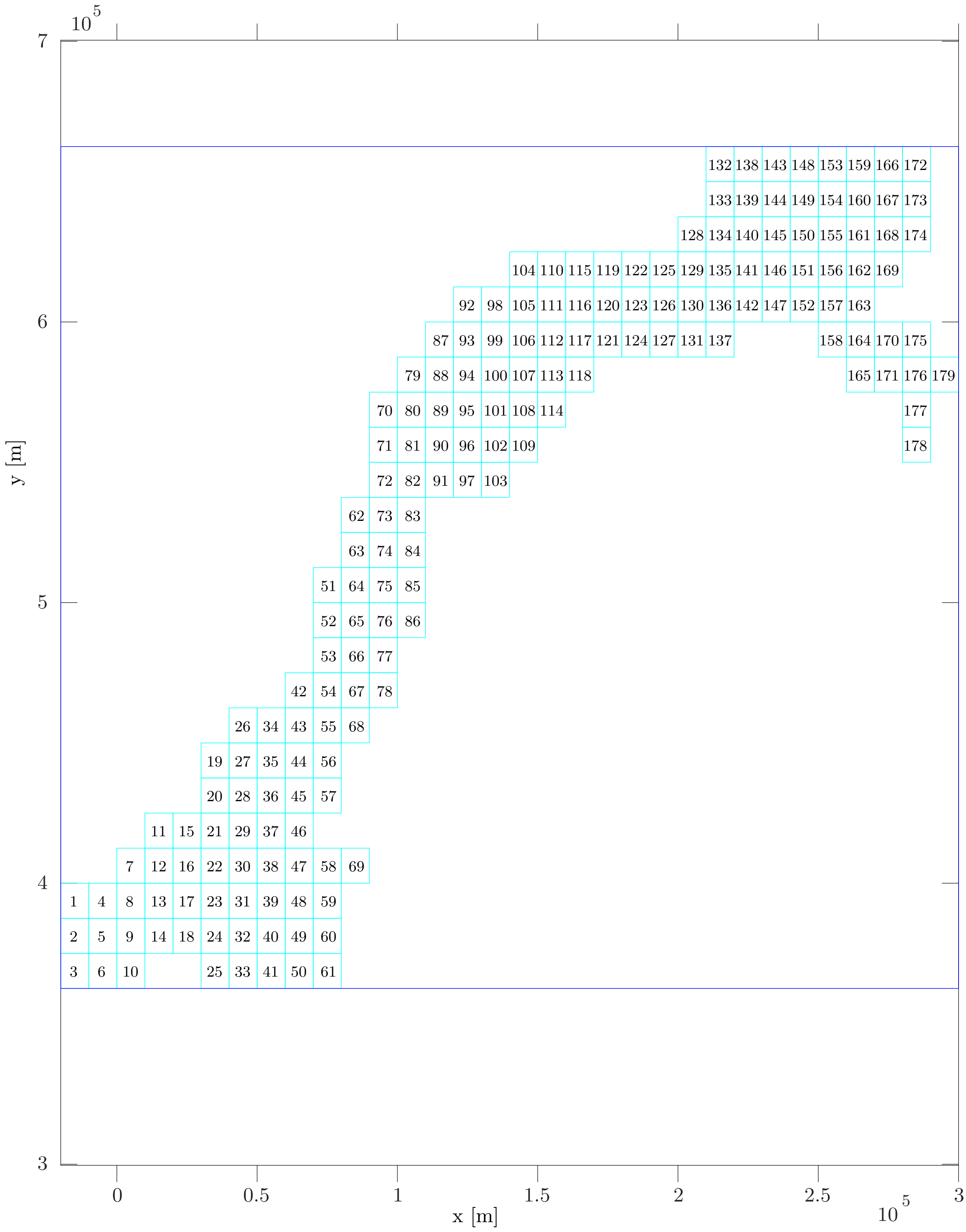
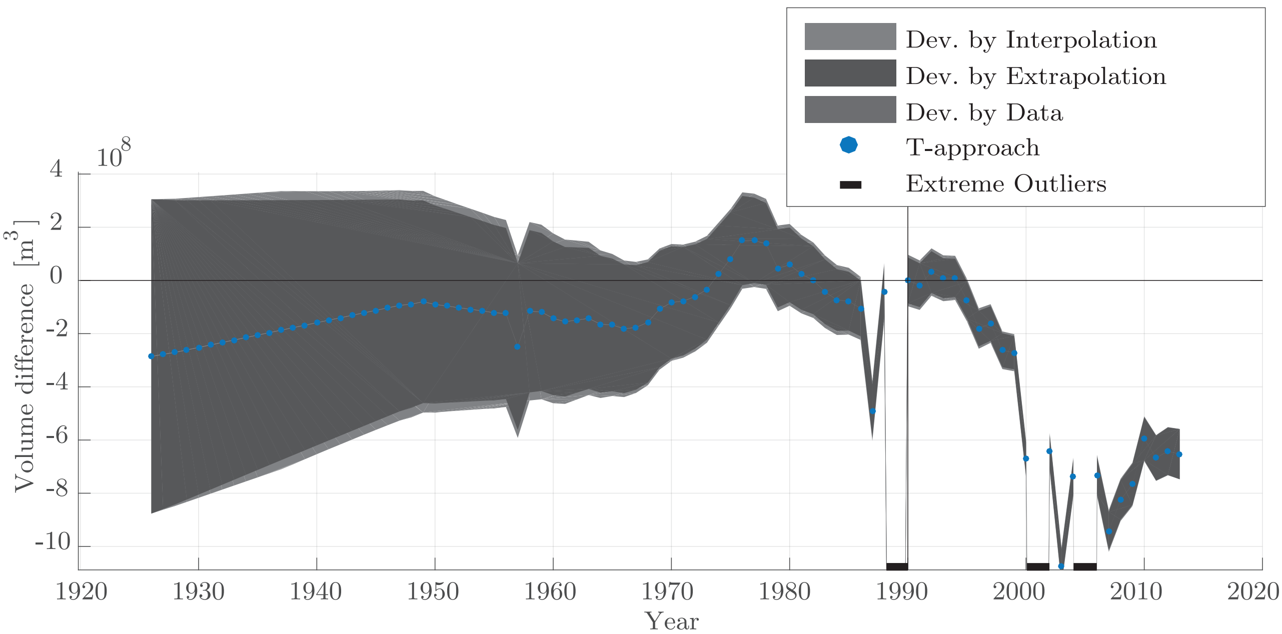


Figure 5.21: Dutch Coastal Zone Region of Interest (blue) and Data Products (cyan)



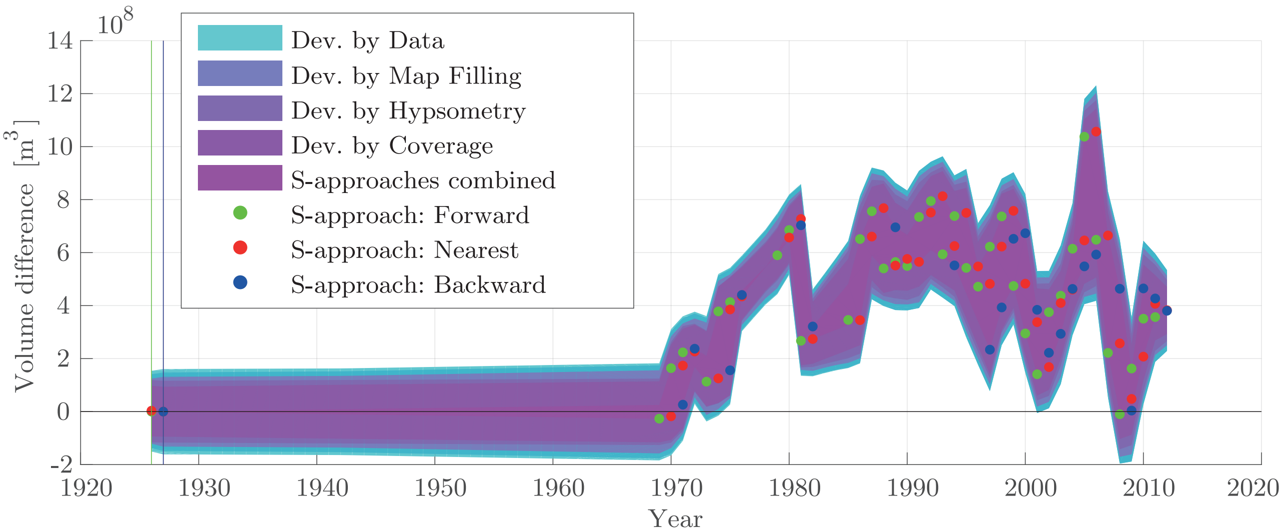


Figure 5.22: Dutch Coastal Zone for automated S- and T-approach

5.3 Summary

The Case Study simulations have been performed on the Wadden Basin and Coast, Eierlandse Gat Basin and Coast, Ameland Basin and Coast, Friesche Zeegat Basin and Coast, Texel/Marsdiep Basin and Coast, Vlie Basin and Coast, Voordelta, Haringvliet, Grevelingen, Oosterschelde, Westerschelde and the entire Dutch Coastal Zone.

Visual interpretation shows clear differences between the automated-uncertainty and deterministic T-approach. Also clear differences are shown between the automated-uncertainty and deterministic S-approach.
Clear differences are also noted between the automated-uncertainty S- and T- approach.

Chapter 6

Discussion, Conclusion and Recommendations

6.1 Discussion

Within this research, an automated approach has been developed for the purpose of sediment management. It consists of two sub-approaches, the deterministic Spatial (S)- and Temporal (T)-approaches by respectively van Koningsveld et al. (2008) and Elias et al. (2012).

With this new automated-uncertainty S- and T-approach, new data of sediment change are readily available for sediment management.

The automated-uncertainty-approach shows clear differences within the deterministic S- and T-approaches when compared to literature. Also differences are visible between the automated S-approach compared to the automated T-approach.

Both automated approaches have been validated, therefore the difference in output cannot be explained by the approach itself. Validation has been carried out by means of 16 Unit Tests that are selected to verify the behaviour of the automated-uncertainty approach. It could be possible that the simplification to calculate the synthetic data analytically influenced the results of the Unit Tests to provide the current reliability.

A possible explanation for the differences between the automated and deterministic T-approach can be that alternative calculation strategies were used on the available data. This deterministic T-approach corrected missing single data points and removed data outliers. Data-filling was done by interpolation for larger gaps and spatial internal diffusion for smaller gaps. Up to 10% of the data were removed manually in this approach.

The automated-uncertainty-approach uses no manual intervention. Every Data Point is filled in a temporal-way by linear inter- and extrapolation, while saving all filled Data Points and their uncertainty.

The differences between the automated and deterministic S-approach can be explained by different strategies used for data-filling, possibly an alternative data grid resolution and choosing the threshold of coverage maps. The approach used for the deterministic S-approach is by data-filling of actual data within a temporal bandwidth. No calculation in spatial filling is done. The actual reference was defined as the highest coverage map.

In the automated-uncertainty-approach data-filling of actual data within a temporal bandwidth was used as well. After defining a discrete minimum map coverage percentage the spatial data-filling technique Matlab Inpaint (Garcia (2010); Wang et al. (2012)) was used to develop full coverage maps. These were then subtracted from the reference map that was defined as the highest coverage map as well.

The automated S- and T-approach show a difference in outcome for the case study results. This is something that can be explained by the output of the case study results. For the case study, data are not available for every point in space and time. Data-filling in the gaps is done in different manners with different assumptions for both the S- and T-approach.

It is contradictive that both the S- and T-approach have very similar results for the Unit Tests. But this can be interpreted by the fact that the Unit Test data was simplified and unavailable data were added in such a way that it was possible to calculate analytically.

Furthermore, the automated S- and T-approach difference might be explained by the fact that the data have large gaps in respectively space and time. Because filling for the S-approach is different from T-approach filling, considerable differences can occur when large gaps have to be filled in spatial or temporal direction. These filling techniques could be of large influence for the output graph with high filling percentages.

The automated S- and T-approach provide a difference in confidence bands as well. Spatial filling results in a large 'jump' in the output graph for the different spatial approaches (forward, backward and nearest), while temporal filling gives a fine representation in temporal view because this is the filling direction.

All unavailable data have been filled with a value as realistic as possible. Based on the expected deviation of the true value, uncertainty is added in a continuous way that is used in the confidence bands. The differences in confidence bands can therefore be explained by the different uncertainty elements that have been added because of different filling techniques. The difference in confidence band between the approaches is therefore a direct result of these filling techniques.

This uncertainty approach is a way to point out, the reliability of the data. Available data can thus be interpreted more cautious if the uncertainty confidence bands are increased. It is not a probabilistic mathematical calculated uncertainty.

6.2 Conclusion

This is the first automated approach that can be applied generally for systematic coastal sediment management and allows for uncertainty calculations with confidence bands, to my knowledge.

In conclusion this is a valuable new approach that can be used for strategic decision making for sediment management.

The (un)reliability of the bathymetric data is defined, generalised and automated systematically. The output of this automated-uncertainty approach provides information in the volumetric differences over time for a specified region of interest, including confidence bands. This means that the automated-uncertainty approach provides a quantitative insight of possible uncertainties and where these uncertainties are in space and time. More realistic assumptions can be made using quantified uncertainties.

It is possible to develop a fully data-based automated-uncertainty approach that uses only a small amount of manual intervention. This automated-uncertainty approach is generally applicable to any given coastal region with available sediment data. The input parameters consist of a Region of Interest containing geographical data and spatial and temporal data of the applicable region. These data are generally available from binned data products. Only the settings have to be defined manually prior to running the script.
It is fully automated, which means that no manual intervention is needed between input and output.

It is possible to control and even assure the quality of this automated-uncertainty approach. This automated-uncertainty approach consists of two different methodologies, based on current methods for expected sedimentation-erosion changes. These are validated with 16 separate Unit Tests.
The process and subprocesses of the automated-uncertainty approach are documented and reported to provide traceable insight in the calculations.
A regular commercially available computer can be used to run the automated-uncertainty approach.

6.3 Recommendations

Three types of recommendations have been developed regarding procedure, material and policy. Recommendations for the procedure contain possible improvements for the developed automated-approach, that can possibly increase the reliability of the approach.
The material recommendations about the used hard- and software to develop and use the approach, together with the type of data and scripting that has been used for the Data Products.
Also recommendations for policy are added with possible improvements to increase the quality of sediment management.

6.3.1 Procedure

Uncertainties The deviations for the uncertainty propagation have currently not been quantified. Only an expected quantification can be provided. To increase the reliability of the automated-uncertainty approach, calibration for the assumptions used for the developing confidence bands is recommended. This can be done by performing large amounts of case study simulations of very well studied regions of interest. Calibration can also be performed on literature that is already available.
It is also recommended to create Unit Tests for the uncertainties for each method, so calibration can be performed in this way as well. This can also be done by creating synthetic datasets and running these through the automated-uncertainty approach. Hereafter comparison to the analytically calculated values can be done.

More insight into the uncertainties created by both methodologies could also be researched. Both approaches have weaknesses and by testing these possibly a quantified calibration can be done. The T-approach has a large extrapolation in time as weakness, for the S-approach this same weakness applies for spatial filling. This could be done by performing sensitivity analyses.

Developing a mathematically more formal approach could be done by combining both the S- and T-approach into a spatio-temporal 4D approach. Mathematically calculated numerical errors could then be extracted from the procedure and result in a quantified probabilistic uncertainty in the 4D space.

In addition, currently the automated S-approach uses hypsometric uncertainty for the spatial methodology. Hypsometric deviations that could find anomalies that do not represent morphological changes over time, are included. When e.g. a tidal channel migrates across multiple Data Products, an increase in hypsometric deviation is detected. Further research has to prove whether comparison to neighbouring files gives a more reliable uncertainty.

The T-approach used in the automated-uncertainty approach assumed linear inter- and extrapolation for data-filling. Further research could prove whether different interpolation techniques could benefit the reliability of the results.

In the S-approach that has been used in the automated-uncertainty approach, hypsometric deviations could be detected, combined with low coverage values. This can potentially be caused by spatial-filling and further research should prove the impact by means of a sensitivity analysis. Potentially this can be done by using Unit Tests with synthetic datasets, provided that an analytical calculation can be performed.

It is assumed that the deviations discussed in Section 3 are uncorrelated. Further research could be done on the validity of this assumption.

Uncertainty can also be added for the bounds of the survey campaigns described in Section 2.4. Specific areas were surveyed and when this data can be retrieved, uncertainty could be added.

Data Useage The entire process between survey measurements and volume difference calculation should be automated. Having an insight in the uncertainty through the process of modifying the raw data is important. It could be that a large amount of slightly inaccurate data is available at a high precision. When in that case the deviation of the accuracy can be determined, that value can be used for further calculations. Therefore the attention should be focused on the source of the data, considering it would be interesting to have an understanding of the importance of uncertainty through the procedure.

Nourishment quantities can be included in the automated-uncertainty approach. Among others, the research of Hoogervorst (2005) can be used.

For sediment management it would be ideal to have up-to-date visualised systematic and location specific knowledge available, both in history as in future projections. Therefore it would be interesting to create a generalised automated procedure to the raw data retrieved from the measurements, so that the changes in the raw data are traceable and thus transparent.

The benefit of this method is that the actual measured raw data will never be overwritten by Data Product. When the procedure is changed by any reason, the raw data can be reprocessed and stored as new modified data. NASA uses this method as well, e.g. their Ocean Colour data (NASA, 2015). This method can only work objectively if the raw data are accessible, but at the moment only the Data Products are available.
The following approach could possibly be developed:

Raw data [permanently stored]	+	Procedure(t) [automated generalised dynamic script]	=	Data Product(t) [reprocessed, demonstrable reliability]

Transformation A disadvantage of the inquiry method described in Section 3.5.1 is that the data within a single year is inquired top-down, starting with the 'youngest' dataset. That could mean that potential data stored on the exact same location as earlier defined data can be left unused. When data are stored on 01 January, it can be recognised as a 'dummy date'. Dates in August can potentially be more accurate in time. Therefore it is recommended to develop an additional uncertainty, based on the data storage date. Also it is recommended to 'start' the procedure at 01 August and inquire in top and down direction over time, to use this data over 01 January data.

While using the 'nearest' S-approach, either t-1 or t+1 has priority over the other. It is recommended to research whether high map coverage could be used over the t-1 or t+1 approach.
A disadvantage of the surface defining procedure described in Section 3.4.2 is shown in Figure 6.1. This Figure shows a Region of Interest being situated within two datasets, netCDF A and netCDF B.
When the Region of Interest has data *in* a certain dataset but *does not* have any Data Points in that Region of Interest, the entire dataset will *not* be used for Data Transformation. Therefore netCDF B in Figure 6.1 will not be used. The reason for this is time related; a more reliable method could be found but because of other priorities this has not been solved. Therefore it is recommended to define a Region of Interest carefully or to find a solution for this procedural disadvantage.

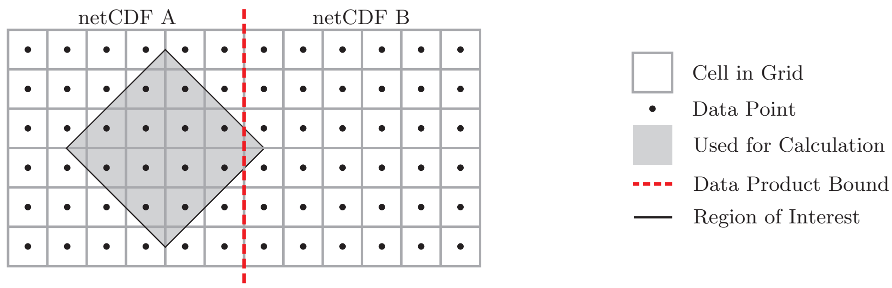


Figure 6.1: Disadvantage of the surface defining procedure (Section 3.4.2)

6.3.2 Material

Software The automated-uncertainty approach is developed in the software application MathWorks Matlab R2015b. Therefore it is only possible to use it locally on a computer. It is recommended to also used Matlab R2015b for running the automated-uncertainty approach. See Figure 6.2 for the packages that were installed, this does not mean that all packages are needed for running the automated uncertainty-approach.

The automated-uncertainty approach currently outputs visualised information, based on calculated data. It could be reprogrammed in a dynamic way for both the S- as the T-approach. When this is done the largest uncertainties within the methodologies could be changed in a dynamic way. For the S-approach the spatial filling technique could be changed dynamically, for the T-approach the reference year could be changed.

It could be reprogrammed in a GUI (Graphical User Interface) web environment in a software environment such as Python, to make it even more accessible for non-specialistic decision makers.

```
MATLAB Version: 8.6.0.267246 (R2015b)
MATLAB License Number: 329138
Operating System: Microsoft Windows 7 Enterprise  Version 6.1 (Build 7601: Service Pack 1)
Java Version: Java 1.7.0_60-b19 with Oracle Corporation Java HotSpot(TM) 64-Bit Server VM mixed mode
----------------------------------------------------------------------------------------------------
MATLAB                                                  Version 8.6         (R2015b)
Simulink                                                Version 8.6         (R2015b)
Curve Fitting Toolbox                                   Version 3.5.2       (R2015b)
Delft3D-MATLAB interface toolbox                        Version <VERSION>
Image Processing Toolbox                                Version 9.3         (R2015b)
M2HTML Toolbox - A Documentation Generator for ...      Version 1.5
M_Map - mapping toolbox (Author: rich@eos.ubc.ca)       Version 1.4f        1 Dec
Mapping Toolbox                                         Version 4.2         (R2015b)
Modelit DONAR Toolbox                                   Version 1.0.0       (R2006B)
Parallel Computing Toolbox                              Version 6.7         (R2015b)
Symbolic Math Toolbox                                   Version 6.3         (R2015b)
```

Figure 6.2: Used Matlab version with its packages

Hardware The automated-uncertainty approach needs to process large amounts of data. It requires strong computer processing power. This is of influence on the automated-uncertainty approach, since the data needs to be divided in small processing blocks to be able to cope with the available computer memory.

Currently the order of magnitude of memory use for the T-approach is 500KB/0.5MB p/Data Point p/y (for example 2,100,000 MB for 500x625x14, which is a rather large synthetic dataset of 500x625 Data Points and a 100% coverage for the z-values for 14 different years). The memory used of the S-approach is approximately 25% of the memory used T-approach.

Development of the automated approach has been carried out on a 64-bit Intel® Core™ i5-5300U CPU running on 2.3 GHz, having the availability of 8.00 GB (7.89 GB usable) of memory (RAM). It is recommended to use a computer with at least this amount of memory to be able to run the automated-uncertainty approach. When the individual datasets increase in size over time because of new saved measurements, it is recommended to use more memory.

To increase calculations, multiple Matlab runs can be executed in a parallel way. This is not possible for the automated T-approach, because the current Data Products (fixed map tiles) create a large utilisation of memory. When a Data Product can be divided into two separate tiles, the memory usage can be reduced to run parallel Matlab scripts. See Figure 6.3 for a visualisation of this process.

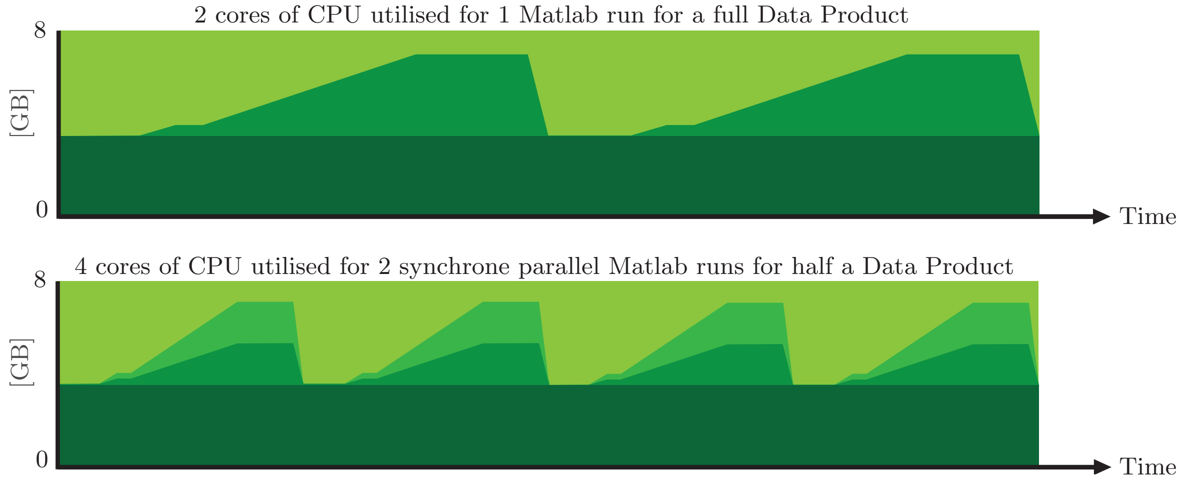


Figure 6.3: Memory utilisation with Data Products

When installed on a network computer with high processing power, the calculating time can reduce as well to lower the threshold of using the automated-uncertainty approach.

Data Products The used file format for the datasets is the netCDF file format. UCAR (2016) defines it is "a set of software libraries and self-describing, machine-independent data formats that support the creation, access, and sharing of array-oriented scientific data". This type of file format can store nD data in a single file and can be accessed by Matlab. In addition a netCDF dataset that can be accessed efficiently to extract small amounts of information from a large dataset. The automated-uncertainty approach can only use netCDF datasets stored as '.nc' files in a single folder saved on a specific location.

The data used for the automated-uncertainty approach are gridded in longitudinal and latitudinal directions, with information of time and z-value for each Data Point, meaning that it uses 4D information. It is assumed that variations can occur in coverage for the longitudinal, latitudinal, time and z-value. When this is stored differently - such as two dimensional - the automated-uncertainty approach is not able to process the information. It is recommended to develop a procedure that can use nD information as well.

Using netCDF data created in Python resulted in a Matlab error. The possible reason for this is the different methodology Python uses to store its NaN data. It is recommended to make both compatible.

6.3.3 Policy

Significance of Volume Differences With this study I have shown that it is possible to add uncertainty bands. It is possible to adopt this approach into the daily practice of engineering. The sector of coastal sediment management could shift to a probabilistic quantitative approach instead of a deterministic approach with qualitative uncertainties. Current research only focuses on the quantities of coastal sediment volume differences. Future research should focus on the significance of the volumes as well.

Measurement Uncertainty Future survey data should contain information about measurement-uncertainty. This can be of influence for the generalised uncertainty that is currently used in the automated-uncertainty approach for individual Data Points. This could be done by using uncertainty information beside the measurement data from future survey campaigns.

Research Quality Validating software should become a standard practice in research at universities and research institutes such as Delft University of Technology and Deltares. The standard product quality of the delivered results will then increase.

Risk in Contracts A commonly used engineering contract used by Rijkswaterstaat is the DBFM(O) contract. This Design, Build, Finance, Maintain (and Operate) contract creates flexibility for the contractor, but the risks are also its responsibility. An example where this resulted in an economical disaster for the contractor is the case A15 MaVa (NRC, 2015). This study demonstrated that uncertainties for sedimentation-erosion calculations are present for the Dutch Coastal System, that will also be present for sedimentation-erosion projections. This automated-uncertainty approach adds an extra dimension that can be used as input for long-term maintenance contracts. The additional information can be used to assess the possible financial risks.

References

Berendsen, H. (1998). Birds-Eye View of the Rhine-Meuse Delta (The Netherlands). *Journal of Coastal Research*, 14(3):740–752.

Bosboom, J. and Stive, M. (2013). *Coastal Dynamics 1*. Delft Academic Press (VSSD). Version 0.4, Lecture notes for CIE4305, ISBN: 978-90-6562-286-0.

CBS (2012). Bodemgebruik; uitgebreide gebruiksvorm, per gemeente. Centraal Bureau voor de Statistiek, http://statline.cbs.nl/StatWeb/publication/?VW=T&DM=SLN&PA=70262NED&D1=0-1,5,11,18,24,27,31,41&D2=0,5-16&D3=a&HD=130529-1417&HDR=G2&STB=G1,T, Accessed 1st of April 2016 (in Dutch).

CBS (2016). Cbs: Nederland telt 17 miljoen inwoners. Centraal Bureau voor de Statistiek, http://www.cbs.nl/nl-NL/menu/themas/bevolking/publicaties/artikelen/archief/2016/nederland-telt-17-miljoen-inwoners.htm, Accessed 1st of April 2016 (in Dutch).

de Ruig, J. (1998). Coastline management in The Netherlands: human use versus natural dynamics. *Journal of Coastal Conservation*, 4:127–134.

Dekking, F., Kraaikamp, C., Lopuhaä, H., and Meester, L. (2005). *A Modern Introduction to Probability and Statistics*. Springer. ISBN: 978-1-85233-896-1.

Dillingh, D., Baart, F., and de Ronde, J. (2010). Definitie zeespiegelstijgingvoor bepalingsuppletiebehoefte . Technical Report 1201993-002. Rekenmodel t.b.v. handhaven kustfundament, Deltares, (in Dutch).

Ecomare (2015). Waarvandaan en waarheen. http://www.ecomare.nl/fileadmin/ecomare/encyclopedie-nieuw/content-vleet.php?id=4299&language=0, Accessed 30th of March 2016.

Elec-intro (2016). http://www.elec-intro.com/cms/plus/view.php?aid=14290, Accessed 31st of March 2016.

Elias, E., van der Spek, A., Wang, Z., and de Ronde, J. (2012). Morphodynamic development and sediment budget of the Dutch Wadden Sea over the last century. *Netherlands Journal of Geosciences*, 91(3):293–310.

Elias, E. and van der Spek, A. F. (2014). Grootschalige morfologische veranderingen in de Voordelta. Technical report. 1207724-001, Werkdocument (in Dutch).

Elias, E., van Koningsveld, M., Tonnon, P., and Wang, Z. (2007). Sediment budget analysis and testing hypotheses for the Dutch coastal system. Technical Report Z4100.00, WL | Delft Hydraulics. Prepared RIKZ (Rijksinstituut voor Kust en Zee) for Rijkswaterstaat (in Dutch).

Elias, E. and Wang, Z. (2013). Abiotische gegevens voor monitoring effect bodemdaling. Technical report. Evaluatierapport hoogtemetingen 2006-2012, Deltares, (in Dutch).

Garcia, D. (2010). Robust smoothing of gridded data in one and higher dimensions with missing values. *Computational Statistics and Data Analysis*, 54(4):1167–1178.

HH Rijnland (2009). *Flood control in The Netherlands*. ISBN: 978-90-72381-10-1, Water Control Board of Rijnland.

Holthuijsen, L. (2007). *Waves in Oceanic and Coastal Waters*. Cambridge University Press. ISBN: 978-05-1126-717-8.

Hoogervorst, R. (2005). Inventarisatie bagger-, stort- en zandwingegevens binnen de -20 m NAP. Technical Report RIKZ/KW/2005.103W, WL | Delft Hydraulics. Prepared RIKZ (Rijksinstituut voor Kust en Zee) for Rijkswaterstaat (in Dutch).

Kamphuis, J. (2010). *Introduction to Coastal Engineering and Management*. World Scientific, 2nd edition. ISBN: 978-981-283-484-3.

Kaspers, J. and Waanders, M. (2002). Towards an Integrated Coastal Zone Policy, Policy agenda for the coast. Ministry of Infrastructure and the Environment.

Marijs, K. and Parée, E. (2004). Nauwkeurigheid vaklodingen Westerschelde en -monding: "De Praktijk". Technical Report ZLMD-04.N.004, Meetinformatiedienst Zeeland, Vlissingen. (in Dutch).

Marine Board (1994). *Restoring and Protecting Marine Habitat*. National Academy Press. ISBN: 0-309-04843-5, Commission on Engineering and Technical Systems, Division on Engineering and Physical Sciences, National Research Council, Washington (USA).

Ministerie van BZK (2016). Kustlijnkaarten: Ligging JARKUS-raaien langs de Nederlandse Kust. Accessed 2nd of May 2016, https://data.overheid.nl/data/dataset/kustlijnkaarten-ligging-jarkus-raaien-langs-de-nederlandse-kust (in Dutch).

Ministerie van VROM (2006). *Nota Ruimte, Ruimte voor Ontwikkeling*. Ministry of Infrastructure and the Environment (in Dutch), Katern over nieuw ruimtelijk kustbeleid in 2006.

Ministry of V&W (1990). *Coastal defence after 1990, a policy choice for coastal protection*. 1st coastal policy document edition. Ministry of Infrastructure and the Environment.

NASA (2015). Ocean Color and SST Product Versions and Processing History. http://eoimages.gsfc.nasa.gov/images/imagerecords/69000/69721/NorthSea.A2003352.1220.250m.jpg, Accessed 26th of September 2015.

NOAA (2015). What is lidar. National Oceanic and Atmospheric Administration, http://oceanservice.noaa.gov/facts/lidar.html, Accessed 28th of September 2015.

NRC (2015). Verbreding A15 loopt uit op drama voor bouwers. http://www.nrc.nl/nieuws/2015/04/24/verbreding-a15-waarom-lijden-de-bouwers-niet-de-overheid, Accessed 2nd of May 2016 (in Dutch).

OET (2016). Dataset documentation JarKus. Open Earth Tools, Accessed 2nd of May 2016, https://publicwiki.deltares.nl/display/OET/Dataset+documentation+JarKus.

Open University (2012). *Waves, Tides & Shallow-Water Processes*. Elsevier Science & Technology, 2nd edition. ISBN: 978-0-08-036372-1.

Perluka, R., Wiegmann, E., Jordans, R., and Swart, L. (2006). Opnametechnieken waddenzee. Technical Report AGI-2006-GPMP-004, Rijkswaterstaat Adviesdienst Geo Informatie en ICT, Derde Werelddreef 1, Delft. Ministry of Infrastructure and the Environment (in Dutch).

Pot, R. (2011). Coastal Morphologic Data: JarKus (Jaarlijkse Kustmeting) Systematics and accuracy, Yearly Coastal Survey in the Netherlands. Technical report.

Rijkswaterstaat (2007). Integraal Beheerplan Noordzee 2015. 30 195, Kamerstuk, Tweede Kamer der Staten-Generaal, https://zoek.officielebekendmakingen.nl/dossier/30195/kst-30195-22?resultIndex=41&sorttype=1&sortorder=4, Accessed 3rd of May 2016 (in Dutch).

Rijkswaterstaat (2009). Beheer- en Ontwikkelplan voor de Rijkswateren 2010-2015. Technical report. Ministry of Infrastructure and the Environment (in Dutch).

Rijkswaterstaat (2015). Beheer- en Ontwikkelplan voor de Rijkswateren 2016-2021. Technical report. Ministry of Infrastructure and the Environment (in Dutch).

Rijkswaterstaat/Waterdienst (2010). Memo Vaklodingen. Technical report. Monitoring Gaswinning Waddenzee (in Dutch), rapportages door NAM (Nederlandse Aardolie Maatschappij).

Schiereck, G. and Verhagen, H. J. (2012). *Introduction to Bed, bank and shore protection*. Delft Academic Press (VSSD). ISBN: 978-90-6562-306-5, Engineering the interface of soil and water.

Schwab, W. C., Gayes, P. T., Morton, R. A., Driscoll, N. W., Baldwin, W. E., Barnhardt, W. A., Denny, J. F., Harris, M. S., Katuna, M. P., Putney, T. R., Voulgaris, G., Warner, J. C., and Wright, E. E. (2009). Coastal Change Along the Shore of Northeastern South Carolina: The South Carolina Coastal Erosion Study. Technical Report 2008-1206, Unites States Geological Survey, Woods Hole, MA. http://pubs.usgs.gov/of/2008/1206/.

TAW (1995). Basisrapport zandige kust. Technical report. Technische Adviescommissie voor de Waterkeringen, Delft: Rijkswaterstaat. (in Dutch).

UCAR (2016). Unidata, NetCDF. University Corporation for Atmospheric Research, http://www.unidata.ucar.edu/software/netcdf/, Accessed 25th of March 2016.

van der Zijp, N., Marsman, E., Ambachtsheer, R., Dunsbergen, D., Heinen, P., van der Hoeven, W., Kalf, R., Oskam, G., Visser, W., Vreeke, J., and Walburg, A. (2001). Bodemdieptegegevens van het nederlandse kustsysteem. Technical report. MobiData.

van Koningsveld, M., de Boer, G., Baart, F., Damsma, T., den Heijer, C., van Geer, P., and de Sonneville, B. (2010). OpenEarth: Inter-Company Management Workflow for Dredging Projects. Proceedings WODCON XIX Conference : Dredging Makes the World a Better Place, 9-14 September 2010, Beijing, China.

van Koningsveld, M., de Sonneville, B., Wang, Z., and van Rijn, L. (2008). Sediment budget analysis of the Dutch coastal system, a focus on the Wadden Sea. *Proceedings 31rd International Conference on Coastal Engineering (ICCE Cardiff)*.

van Koningsveld, M. and Lescinski, J. (2006). Decadal scale performance of coastal maintenance in The Netherlands. *Shore & Beach*, 7(1):2985–2997. WL | Delft Hydraulics, Marine and Coastal Management.

van Koningsveld, M., Stive, M., and Mulder, J. (2004). Balancing research efforts and management needs. a challenge to coastal engineering. pages 2985–2997. Proceedings of the 29th International Conference of Coastal Engineering, Lisbon Portugal.

Verhagen, H. (1992). Method for artificial beach nourishment. *Proceedings 23rd International Conference on Coastal Engineering (ICCE Venice)*, pages 2474– 2485.

Walburg, L. (2005). Zandvolumes in het nederlandse kustsysteem. Technical Report RIKZ/KW/2005.133w. Rijkswaterstaat RIKZ (Rijksinstituut voor Kust en Zee) werkdocument (in Dutch).

Wang, G., Garcia, D., Liu, Y., de Jeu, R., and Dolman, J. (2012). A three-dimensional gap filling method for large geophysical datasets, application to global satellite soil moisture observations. *Environmental Modelling & Software*, 30:139–142.

Wiegmann, N., Perluka, R., and Boogaard, K. (2002). Onderzoek naar efficiency verbetering kustlodingen. Technical Report AGI/110105/GAM01, Meetkundige Dienst, Delft. (in Dutch).

WL | Delft Hydraulics (2008). Morphodynamics of the Delta coast (south-west Netherlands). Technical Report Z4407. Synthese kustlijnzorg 2007, executive summary of Cleveringa (2008) Alkyon Report A1881 (in Dutch).

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Appendix 1: S- and T-approach Script

Listing A-1: S- and T-approach Main Script *main.m*

```
%% Define input settings
OPT.disp                    = 1;      % Display progress in command window
OPT.plot                    = 1;      % Plot all figures
OPT.log                     = 1;      % Store command window to Log folder
OPT.savedata                = 1;      % Store data to Data folder
OPT.memlargedata            = 0;      % Keep all large data in memory
OPT.address                 = 'C:\opendap\vaklodingen\nc\'; % Data Product local folder
OPT.methodology             = 3;      % 1=Spatial, 2=Temporal, 3=Spatial+Temporal
OPT.refyr                   = 1990;   % Temp: Reference year for volume diff. calc.
OPT.step                    = 20;     % Spat/Temp: Step/grid size in x/y [m]
OPT.devz                    = 0.11;   % Spat/Temp: Dev. in z for all data points [m]
OPT.devzt                   = 0.14;   % Spat/Temp: Dev. in oldest z, lin. to devrefyr [m]
OPT.devrefyr                = 1990;   % Spat/Temp: Dev. in z increases to oldest year;
OPT.mincov                  = 0.7;    % Spat: Min coverage p/year p/file [-]
OPT.mindepth                = 15;     % Spat: Minimum depth for hypsometry in [m]
OPT.maxdepth                = -20;    % Spat: Maximum depth for hypsometry in [m]
OPT.hyps                    = 0.1;    % Spat: Hypsometry bin bandwidth [m]
OPT.dtmax                   = 2;      % Spat: Max time offset for spatial map [y]
OPT.dtmaxdev                = 0.05;   % Spat: deviation for each year offset [m]
OPT.xpoly                   = [-20000,-20000,300000,300000,-20000];
OPT.ypoly                   = [362500,662500,662500,362500,362500]; % Dutch Coastal Zone

% If polygon imported in Main script, override OPT.xpoly and OPT.ypoly
if exist('polygon','var')
    OPT.xpoly = polygon(:,1);
    OPT.ypoly = polygon(:,2);
end

%% Data Extraction
function(dataextraction)

%% Data Transformation
function(spatialapproach)
function(temporalapproach)
% Store Data Product 2

%% Data Loading
OPT.deviationz              = 1;      % Calibration factor for all z values [m]
OPT.deviationzt             = 1;      % Calibration factor z linear to refyr [m]

% Spatial-approach
OPT.plotspatial             = 1927;   % Year of reference
OPT.spatialpolygonsurface   = 0.75;   % Threshold mask coverage to join polygon [-]
OPT.spatialsurface          = 0.01;   % Calibration factor for coverage [m]
OPT.spatialhyps             = 0.03;   % Calibration factor for hypsometry [-]
OPT.spatialfillingdt        = 0.3;    % Calibration factor for map filling [m t^-1]

% Temporal-approach
OPT.interp                  = 1;      % Interpolation coefficient for vol diff [m]
OPT.extrap                  = 8;      % Extrapolation coefficient for vol diff [m]

function(dataloading)
```

In Listing A-1 the Main script of the automated-uncertainty S- and T-approach is visualised. Among other sub-functions, the scripts of the functions 'dataextraction', 'spatialapproach', 'temporalapproach' and 'dataloading' have not been included in this report.

Appendix 2: Unit Tests

A-1 Unit Test 1

Table A-1: Unit Test 1

Unit Test	**Region of Interest**	**Trend**	**Time Step** $[y]$	**Gap**	**Amount of NetCDF Files**
1	Two NetCDF Files	Linear	10	No	1

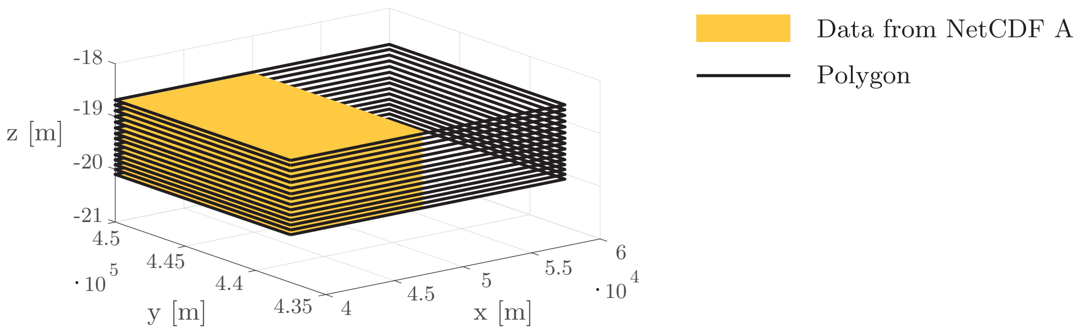

Figure A-4: Unit Test 1

Table A-2: Unit Test 1: Synthetic dataset values

	NetCDF File	**Range** $[m]$		**Difference** $[m]$	**Analytical Surface** $[m^2]$	**Computed Surface** $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A1	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					125,000,000	125,000,000

Table A-3: Unit Test 1: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-25	-25.00000	2.0	$8.00 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-12.5	-12.50000	1.0	$8.00 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50000	2.0	$1.60 \cdot 10^{-7}$
5	1970	-19.7	0.2	25	25.00000	6.0	$2.40 \cdot 10^{-7}$
6	1960	-19.6	0.3	37.5	37.50000	2.0	$5.33 \cdot 10^{-8}$
7	1950	-19.5	0.4	50	50.00000	6.0	$1.20 \cdot 10^{-7}$
8	1940	-19.4	0.5	62.5	62.50000	6	$9.60 \cdot 10^{-8}$
9	1930	-19.3	0.6	75	75.00000	6.0	$8.00 \cdot 10^{-8}$
10	1920	-19.2	0.7	87.5	87.50000	10.0	$1.14 \cdot 10^{-7}$
11	1910	-19.1	0.8	100	100.00000	2.0	$2.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	112.5	112.50000	6.0	$5.33 \cdot 10^{-8}$
13	1890	-18.9	1.0	125	125.00000	14	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	137.5	137.50000	2.0	$1.45 \cdot 10^{-8}$
15	1870	-18.7	1.2	150	149.99998	18.0	$1.20 \cdot 10^{-7}$

Table A-4: Unit Test 1: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-25	-25.00000	2.0	$8.00 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-12.5	-12.50000	1.0	$8.00 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50000	2.0	$1.60 \cdot 10^{-7}$
5	1970	-19.7	0.2	25	25.00000	6.0	$2.40 \cdot 10^{-7}$
6	1960	-19.6	0.3	37.5	37.50000	2.0	$5.33 \cdot 10^{-8}$
7	1950	-19.5	0.4	50	50.00000	6.0	$1.20 \cdot 10^{-7}$
8	1940	-19.4	0.5	62.5	62.50000	6	$9.60 \cdot 10^{-8}$
9	1930	-19.3	0.6	75	75.00000	6.0	$8.00 \cdot 10^{-8}$
10	1920	-19.2	0.7	87.5	87.50000	10.0	$1.14 \cdot 10^{-7}$
11	1910	-19.1	0.8	100	100.00000	2.0	$2.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	112.5	112.50000	6.0	$5.33 \cdot 10^{-8}$
13	1890	-18.9	1.0	125	125.00000	14	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	137.5	137.50000	2.0	$1.45 \cdot 10^{-8}$
15	1870	-18.7	1.2	150	149.99998	18.0	$1.20 \cdot 10^{-7}$

Table A-5: Unit Test 1: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^6\ m^3]$	**Computed Volume** $[10^6\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	-20.1	-0.2	-25	-25.00000	2.0	$8.00 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-12.5	-12.50000	1.0	$8.00 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50000	2.0	$1.60 \cdot 10^{-7}$
5	1970	-19.7	0.2	25	25.00000	6.0	$2.40 \cdot 10^{-7}$
6	1960	-19.6	0.3	37.5	37.50000	2.0	$5.33 \cdot 10^{-8}$
7	1950	-19.5	0.4	50	50.00000	6.0	$1.20 \cdot 10^{-7}$
8	1940	-19.4	0.5	62.5	62.50000	6	$9.60 \cdot 10^{-8}$
9	1930	-19.3	0.6	75	75.00000	6.0	$8.00 \cdot 10^{-8}$
10	1920	-19.2	0.7	87.5	87.50000	10.0	$1.14 \cdot 10^{-7}$
11	1910	-19.1	0.8	100	100.00000	2.0	$2.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	112.5	112.50000	6.0	$5.33 \cdot 10^{-8}$
13	1890	-18.9	1.0	125	125.00000	14	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	137.5	137.50000	2.0	$1.45 \cdot 10^{-8}$
15	1870	-18.7	1.2	150	149.99998	18.0	$1.20 \cdot 10^{-7}$

Table A-6: Unit Test 1: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^6\ m^3]$	**Computed Volume** $[10^6\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	-20.1	-0.2	-25	-25.00010	95.4	$3.81 \cdot 10^{-4}$
2	2000	-20.0	-0.1	-12.5	-12.50005	47.7	$3.81 \cdot 10^{-4}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50005	47.7	$3.81 \cdot 10^{-4}$
5	1970	-19.7	0.2	25	24.99986	95.4	$5.72 \cdot 10^{-4}$
6	1960	-19.6	0.3	37.5	37.49991	143.1	$2.54 \cdot 10^{-4}$
7	1950	-19.5	0.4	50	49.99995	47.7	$9.54 \cdot 10^{-5}$
8	1940	-19.4	0.5	62.5	62.50000	0	0
9	1930	-19.3	0.6	75	75.00005	47.7	$6.36 \cdot 10^{-5}$
10	1920	-19.2	0.7	87.5	87.49986	143.1	$1.63 \cdot 10^{-4}$
11	1910	-19.1	0.8	100	99.9991	95.4	$9.54 \cdot 10^{-5}$
12	1900	-19.0	0.9	112.5	112.49995	47.7	$4.24 \cdot 10^{-5}$
13	1890	-18.9	1.0	125	125.00000	0	0
14	1880	-18.8	1.1	137.5	137.50005	47.7	$3.47 \cdot 10^{-5}$
15	1870	-18.7	1.2	150	149.99985	143.1	$9.54 \cdot 10^{-5}$

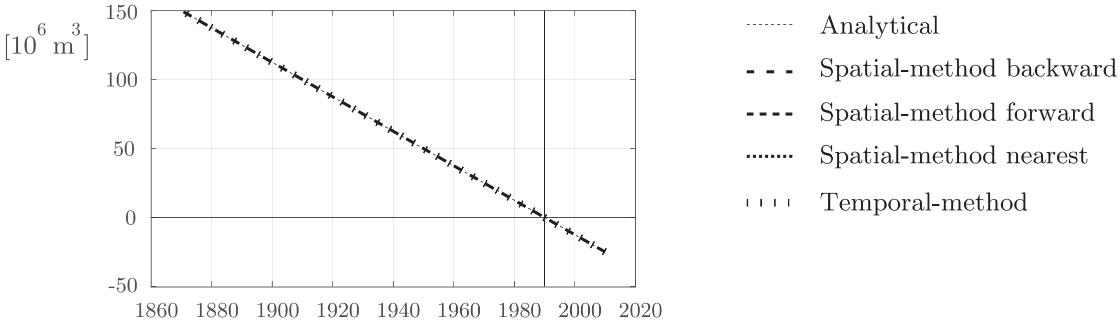

Figure A-5: Unit Test 1: Synthetic volume difference calculation relative to 1990

A-2 Unit Test 2

Table A-7: Unit Test 2

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
2	Two NetCDF Files	Linear	10	Yes	1

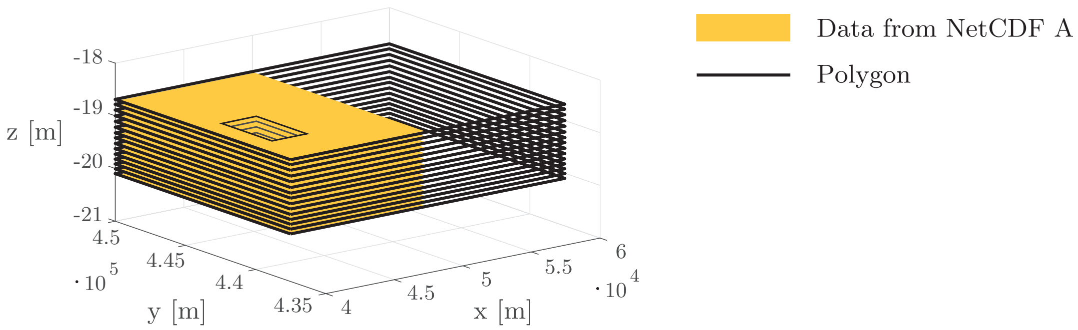

Figure A-6: Unit Test 2

Table A-8: Unit Test 2: Synthetic dataset values

	NetCDF File	Range [m]		Difference [m]	Analytical Surface [m^2]	Computed Surface [m^2]
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A2	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Gap x	A2	42,000	44,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Net. Surface					120,000,000	120,000,000

Table A-9: Unit Test 2: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [10^6 m^3]	**Computed Volume** [10^6 m^3]	**Error** [m^3]	[$-$]
1	2010	-20.1	-0.2	24	-24.00000	2.0	$8.33 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-12	-12.00000	1.0	$8.33 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12	12.00000	2.0	$1.67 \cdot 10^{-7}$
5	1970	-19.7	0.2	24	24.00000	6.0	$2.50 \cdot 10^{-7}$
6	1960	-19.6	0.3	36	36.00000	2.0	$5.56 \cdot 10^{-8}$
7	1950	-19.5	0.4	48	48.00000	6.0	$1.25 \cdot 10^{-7}$
8	1940	-19.4	0.5	60	60.00000	6.0	$1.00 \cdot 10^{-7}$
9	1930	-19.3	0.6	72	72.00000	6.0	$8.33 \cdot 10^{-8}$
10	1920	-19.2	0.7	84	84.00000	10.0	$1.19 \cdot 10^{-7}$
11	1910	-19.1	0.8	96	96.00000	2.0	$2.08 \cdot 10^{-8}$
12	1900	-19.0	0.9	108	108.00000	6.0	$5.56 \cdot 10^{-8}$
13	1890	-18.9	1.0	120	120.00000	14.0	$1.17 \cdot 10^{-7}$
14	1880	-18.8	1.1	132	132.00000	42.0	$3.18 \cdot 10^{-7}$
15	1870	-18.7	1.2	144	143.99998	18.0	$1.25 \cdot 10^{-7}$

Table A-10: Unit Test 2: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	24	-24.00000	2.0	$8.33 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-12	-12.00000	1.0	$8.33 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12	12.00000	2.0	$1.67 \cdot 10^{-7}$
5	1970	-19.7	0.2	24	24.00000	6.0	$2.50 \cdot 10^{-7}$
6	1960	-19.6	0.3	36	36.00000	2.0	$5.56 \cdot 10^{-8}$
7	1950	-19.5	0.4	48	48.00000	6.0	$1.25 \cdot 10^{-7}$
8	1940	-19.4	0.5	60	60.00000	6.0	$1.00 \cdot 10^{-7}$
9	1930	-19.3	0.6	72	72.00000	6.0	$8.33 \cdot 10^{-8}$
10	1920	-19.2	0.7	84	84.00000	10.0	$1.19 \cdot 10^{-7}$
11	1910	-19.1	0.8	96	96.00000	2.0	$2.08 \cdot 10^{-8}$
12	1900	-19.0	0.9	108	108.00000	6.0	$5.56 \cdot 10^{-8}$
13	1890	-18.9	1.0	120	120.00000	14.0	$1.17 \cdot 10^{-7}$
14	1880	-18.8	1.1	132	132.00000	42.0	$3.18 \cdot 10^{-7}$
15	1870	-18.7	1.2	144	143.99998	18.0	$1.25 \cdot 10^{-7}$

Table A-11: Unit Test 2: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	24	-24.00000	2.0	$8.33 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-12	-12.00000	1.0	$8.33 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12	12.00000	2.0	$1.67 \cdot 10^{-7}$
5	1970	-19.7	0.2	24	24.00000	6.0	$2.50 \cdot 10^{-7}$
6	1960	-19.6	0.3	36	36.00000	2.0	$5.56 \cdot 10^{-8}$
7	1950	-19.5	0.4	48	48.00000	6.0	$1.25 \cdot 10^{-7}$
8	1940	-19.4	0.5	60	60.00000	6.0	$1.00 \cdot 10^{-7}$
9	1930	-19.3	0.6	72	72.00000	6.0	$8.33 \cdot 10^{-8}$
10	1920	-19.2	0.7	84	84.00000	10.0	$1.19 \cdot 10^{-7}$
11	1910	-19.1	0.8	96	96.00000	2.0	$2.08 \cdot 10^{-8}$
12	1900	-19.0	0.9	108	108.00000	6.0	$5.56 \cdot 10^{-8}$
13	1890	-18.9	1.0	120	120.00000	14.0	$1.17 \cdot 10^{-7}$
14	1880	-18.8	1.1	132	132.00000	42.0	$3.18 \cdot 10^{-7}$
15	1870	-18.7	1.2	144	143.99998	18.0	$1.25 \cdot 10^{-7}$

Table A-12: Unit Test 2: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^6\ m^3]$	**Computed Volume** $[10^6\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	-20.1	-0.2	24	-24.00009	91.6	$3.81 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-12	-12.00005	45.8	$3.81 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12	12.00005	45.8	$3.81 \cdot 10^{-6}$
5	1970	-19.7	0.2	24	23.99986	137.3	$5.72 \cdot 10^{-6}$
6	1960	-19.6	0.3	36	35.99991	91.6	$2.54 \cdot 10^{-6}$
7	1950	-19.5	0.4	48	47.99995	45.8	$9.54 \cdot 10^{-7}$
8	1940	-19.4	0.5	60	60.00000	0	0
9	1930	-19.3	0.6	72	72.00005	45.8	$6.36 \cdot 10^{-7}$
10	1920	-19.2	0.7	84	83.99986	137.3	$1.63 \cdot 10^{-6}$
11	1910	-19.1	0.8	96	95.99991	91.6	$9.54 \cdot 10^{-7}$
12	1900	-19.0	0.9	108	107.99995	45.8	$4.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	120	120.00000	0	0
14	1880	-18.8	1.1	132	132.00005	45.8	$3.47 \cdot 10^{-7}$
15	1870	-18.7	1.2	144	143.99986	137.3	$9.54 \cdot 10^{-7}$

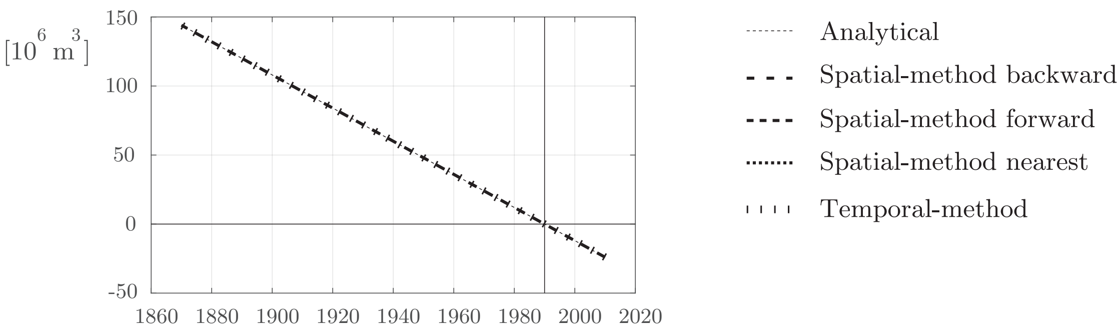

Figure A-7: Unit Test 2: Synthetic volume difference calculation relative to 1990

A-3 Unit Test 3

Table A-13: Unit Test 3

Unit Test	**Region of Interest**	**Trend**	**Time Step** $[y]$	**Gap**	**Amount of NetCDF Files**
3	Two NetCDF Files	Linear	Variable	No	1

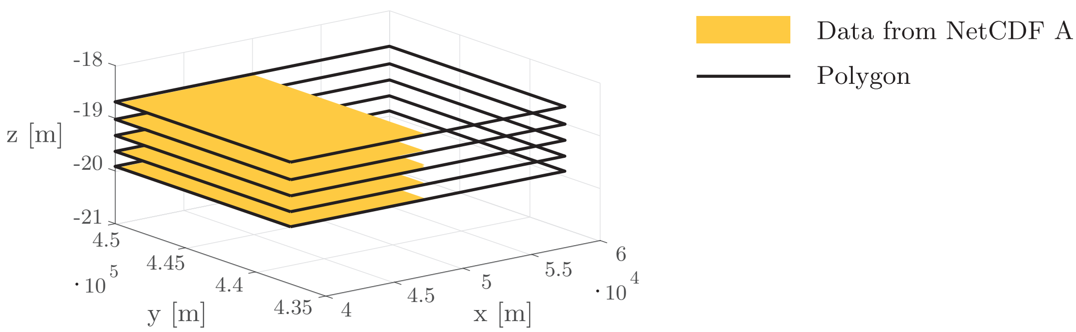

Figure A-8: Unit Test 3

Table A-14: Unit Test 3: Synthetic dataset values

	NetCDF File	**Range** $[m]$		**Difference** $[m]$	**Analytical Surface** $[m^2]$	**Computed Surface** $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A3	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					125,000,000	125,000,000

Table A-15: Unit Test 3: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-2.5	2.50005	52.25	$2.09 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.25	1.25003	26.13	$2.09 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	
4	1962	-19.62	0.28	35	34.99994	56.25	$1.61 \cdot 10^{-6}$
5	1961	-19.61	0.29	36.25	36.24994	56.25	$1.55 \cdot 10^{-6}$
6	1960	-19.60	0.30	37.5	37.49994	56.25	$1.50 \cdot 10^{-6}$
7	1932	-19.32	0.58	72.5	72.25000	44.25	$6.10 \cdot 10^{-7}$
8	1931	-19.31	0.59	73.75	73.75019	187.75	$2.55 \cdot 10^{-6}$
9	1930	-19.30	0.60	75	75.00000	44.25	$5.90 \cdot 10^{-7}$
10	1902	-19.02	0.88	110	110.99994	60.25	$5.48 \cdot 10^{-7}$
11	1901	-19.01	0.89	111.25	111.24912	876.25	$7.88 \cdot 10^{-6}$
12	1900	-19.00	0.90	112.5	112.50000	44.25	$3.93 \cdot 10^{-7}$
13	1872	-18.72	1.18	147.5	147.49996	36.25	$2.46 \cdot 10^{-7}$
14	1871	-18.71	1.19	148.75	148.75062	619.75	$4.17 \cdot 10^{-6}$
15	1870	-18.70	1.20	150	150.00000	532.25	$3.55 \cdot 10^{6}$

Table A-16: Unit Test 3: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-2.5	2.50005	52.25	$2.09 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.25	1.25003	26.13	$2.09 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	
4	1962	-19.62	0.28	35	34.99994	56.25	$1.61 \cdot 10^{-6}$
5	1961	-19.61	0.29	36.25	36.24994	56.25	$1.55 \cdot 10^{-6}$
6	1960	-19.60	0.30	37.5	37.49994	56.25	$1.50 \cdot 10^{-6}$
7	1932	-19.32	0.58	72.5	72.25000	44.25	$6.10 \cdot 10^{-7}$
8	1931	-19.31	0.59	73.75	73.75019	187.75	$2.55 \cdot 10^{-6}$
9	1930	-19.30	0.60	75	75.00000	44.25	$5.90 \cdot 10^{-7}$
10	1902	-19.02	0.88	110	110.99994	60.25	$5.48 \cdot 10^{-7}$
11	1901	-19.01	0.89	111.25	111.24912	876.25	$7.88 \cdot 10^{-6}$
12	1900	-19.00	0.90	112.5	112.50000	44.25	$3.93 \cdot 10^{-7}$
13	1872	-18.72	1.18	147.5	147.49996	36.25	$2.46 \cdot 10^{-7}$
14	1871	-18.71	1.19	148.75	148.75062	619.75	$4.17 \cdot 10^{-6}$
15	1870	-18.70	1.20	150	150.00000	532.25	$3.55 \cdot 10^{6}$

Table A-17: Unit Test 3: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [$10^6\ m^3$]	**Computed Volume** [$10^6\ m^3$]	**Error** [m^3]	[$-$]
1	1992	-19.92	-0.02	-2.5	2.50005	52.25	$2.09 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.25	1.25003	26.13	$2.09 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	
4	1962	-19.62	0.28	35	34.99994	56.25	$1.61 \cdot 10^{-6}$
5	1961	-19.61	0.29	36.25	36.24994	56.25	$1.55 \cdot 10^{-6}$
6	1960	-19.60	0.30	37.5	37.49994	56.25	$1.50 \cdot 10^{-6}$
7	1932	-19.32	0.58	72.5	72.25000	44.25	$6.10 \cdot 10^{-7}$
8	1931	-19.31	0.59	73.75	73.75019	187.75	$2.55 \cdot 10^{-6}$
9	1930	-19.30	0.60	75	75.00000	44.25	$5.90 \cdot 10^{-7}$
10	1902	-19.02	0.88	110	110.99994	60.25	$5.48 \cdot 10^{-7}$
11	1901	-19.01	0.89	111.25	111.24912	876.25	$7.88 \cdot 10^{-6}$
12	1900	-19.00	0.90	112.5	112.50000	44.25	$3.93 \cdot 10^{-7}$
13	1872	-18.72	1.18	147.5	147.49996	36.25	$2.46 \cdot 10^{-7}$
14	1871	-18.71	1.19	148.75	148.75062	619.75	$4.17 \cdot 10^{-6}$
15	1870	-18.70	1.20	150	150.00000	532.25	$3.55 \cdot 10^{6}$

Table A-18: Unit Test 3: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [$10^6\ m^3$]	**Computed Volume** [$10^6\ m^3$]	**Error** [m^3]	[$-$]
1	1992	-19.92	-0.02	-2.5	-2.50006	57.2	$2.29 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.25	-1.25003	28.6	$2.29 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	35	34.99985	152.6	$4.36 \cdot 10^{-6}$
5	1961	-19.61	0.29	36.25	36.24988	124.0	$3.42 \cdot 10^{-6}$
6	1960	-19.60	0.30	37.5	37.49990	95.4	$2.54 \cdot 10^{-6}$
7	1932	-19.32	0.58	72.5	72.49999	9.5	$1.32 \cdot 10^{-7}$
8	1931	-19.31	0.59	73.75	73.75002	19.1	$2.59 \cdot 10^{-7}$
9	1930	-19.30	0.60	75	75.00005	47.7	$6.36 \cdot 10^{-7}$
10	1902	-19.02	0.88	110	109.99990	104.9	$9.54 \cdot 10^{-7}$
11	1901	-19.01	0.89	111.25	111.24992	76.3	$6.86 \cdot 10^{-7}$
12	1900	-19.00	0.90	112.5	112.49995	47.7	$4.24 \cdot 10^{-7}$
13	1872	-18.72	1.18	147.5	147.50004	38.1	$2.59 \cdot 10^{-7}$
14	1871	-18.71	1.19	148.75	148.75007	66.8	$4.49 \cdot 10^{-7}$
15	1870	-18.70	1.20	150	149.99986	143.1	$9.54 \cdot 10^{-7}$

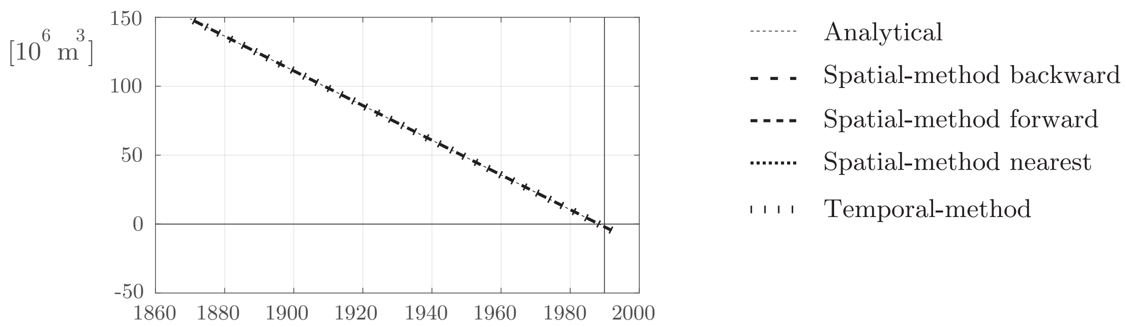

Figure A-9: Unit Test 3: Synthetic volume difference calculation relative to 1990

A-4 Unit Test 4

Table A-19: Unit Test 4

Unit Test	Region of Interest	Trend	Time Step $[y]$	Gap	Amount of NetCDF Files
4	Two NetCDF Files	Linear	Variable	Yes	1

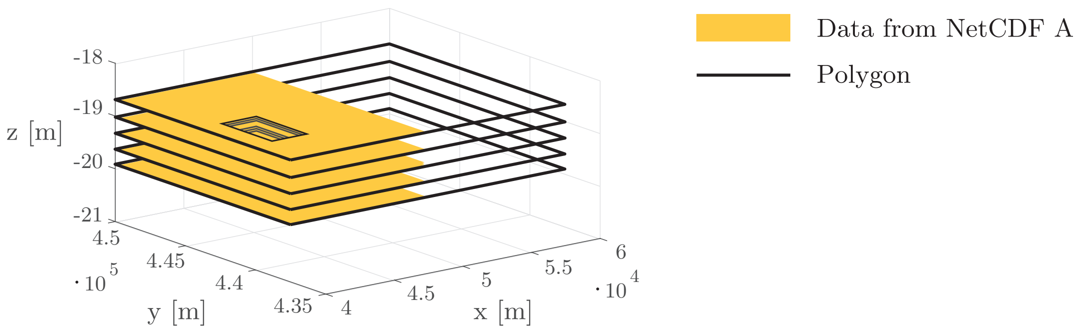

Figure A-10: Unit Test 4

Table A-20: Unit Test 4: Synthetic dataset values

	NetCDF File	Range $[m]$		Difference $[m]$	Analytical Surface $[m^2]$	Computed Surface $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A4	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Gap x	A4	42,000	44,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Net. Surface					120,000,000	120,000,000

Table A-21: Unit Test 4: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^6\ m^3]$	**Computed Volume** $[10^6\ m^3]$	**Error** $[m^3]$	$[-]$
1	1992	-19.92	-0.02	-2.4	2.40005	54.75	$2.28 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.2	1.20003	27.38	$2.28 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	33.6	33.69994	58.75	$1.74 \cdot 10^{-6}$
5	1961	-19.61	0.29	34.8	34.79994	58.75	$1.69 \cdot 10^{-6}$
6	1960	-19.60	0.30	36.0	36.79994	58.75	$1.63 \cdot 10^{-6}$
7	1932	-19.32	0.58	69.6	69.60000	46.75	$6.71 \cdot 10^{-7}$
8	1931	-19.31	0.59	70.8	70.80011	105.25	$1.49 \cdot 10^{-6}$
9	1930	-19.30	0.60	72.0	71.99995	46.75	$6.49 \cdot 10^{-7}$
10	1902	-19.02	0.88	105.6	105.99937	62.75	$5.94 \cdot 10^{-7}$
11	1901	-19.01	0.89	106.8	106.79920	798.75	$7.48 \cdot 10^{-6}$
12	1900	-19.00	0.90	108.0	108.99995	46.75	$4.33 \cdot 10^{-7}$
13	1872	-18.72	1.18	141.6	141.59996	38.75	$2.74 \cdot 10^{-7}$
14	1871	-18.71	1.19	142.8	142.80046	457.25	$3.20 \cdot 10^{-6}$
15	1870	-18.70	1.20	144.0	143.99963	374.75	$2.60 \cdot 10^{-6}$

Table A-22: Unit Test 4: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-2.4	2.40005	54.75	$2.28 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.2	1.20003	27.38	$2.28 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	33.6	33.69994	58.75	$1.74 \cdot 10^{-6}$
5	1961	-19.61	0.29	34.8	34.79994	58.75	$1.69 \cdot 10^{-6}$
6	1960	-19.60	0.30	36.0	36.79994	58.75	$1.63 \cdot 10^{-6}$
7	1932	-19.32	0.58	69.6	69.60000	46.75	$6.71 \cdot 10^{-7}$
8	1931	-19.31	0.59	70.8	70.80011	105.25	$1.49 \cdot 10^{-6}$
9	1930	-19.30	0.60	72.0	71.99995	46.75	$6.49 \cdot 10^{-7}$
10	1902	-19.02	0.88	105.6	105.99937	62.75	$5.94 \cdot 10^{-7}$
11	1901	-19.01	0.89	106.8	106.79920	798.75	$7.48 \cdot 10^{-6}$
12	1900	-19.00	0.90	108.0	108.99995	46.75	$4.33 \cdot 10^{-7}$
13	1872	-18.72	1.18	141.6	141.59996	38.75	$2.74 \cdot 10^{-7}$
14	1871	-18.71	1.19	142.8	142.80046	457.25	$3.20 \cdot 10^{-6}$
15	1870	-18.70	1.20	144.0	143.99963	374.75	$2.60 \cdot 10^{-6}$

Table A-23: Unit Test 4: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-2.4	2.40005	54.75	$2.28 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.2	1.20003	27.38	$2.28 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	33.6	33.69994	58.75	$1.74 \cdot 10^{-6}$
5	1961	-19.61	0.29	34.8	34.79994	58.75	$1.69 \cdot 10^{-6}$
6	1960	-19.60	0.30	36.0	36.79994	58.75	$1.63 \cdot 10^{-6}$
7	1932	-19.32	0.58	69.6	69.60000	46.75	$6.71 \cdot 10^{-7}$
8	1931	-19.31	0.59	70.8	70.80011	105.25	$1.49 \cdot 10^{-6}$
9	1930	-19.30	0.60	72.0	71.99995	46.75	$6.49 \cdot 10^{-7}$
10	1902	-19.02	0.88	105.6	105.99937	62.75	$5.94 \cdot 10^{-7}$
11	1901	-19.01	0.89	106.8	106.79920	798.75	$7.48 \cdot 10^{-6}$
12	1900	-19.00	0.90	108.0	108.99995	46.75	$4.33 \cdot 10^{-7}$
13	1872	-18.72	1.18	141.6	141.59996	38.75	$2.74 \cdot 10^{-7}$
14	1871	-18.71	1.19	142.8	142.80046	457.25	$3.20 \cdot 10^{-6}$
15	1870	-18.70	1.20	144.0	143.99963	374.75	$2.60 \cdot 10^{-6}$

Table A-24: Unit Test 4: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [$10^6\ m^3$]	**Computed Volume** [$10^6\ m^3$]	**Error** [m^3]	[$-$]
1	1992	-19.92	-0.02	-2.4	2.40006	54.9	$2.29 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-1.2	-1.20003	27.5	$2.29 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	33.6	33.59985	146.5	$4.36 \cdot 10^{-6}$
5	1961	-19.61	0.29	34.8	34.79988	119.0	$3.42 \cdot 10^{-6}$
6	1960	-19.60	0.30	36	35.99991	91.6	$2.54 \cdot 10^{-6}$
7	1932	-19.32	0.58	69.6	69.59999	9.2	$1.32 \cdot 10^{-7}$
8	1931	-19.31	0.59	70.8	70.80002	18.3	$2.59 \cdot 10^{-7}$
9	1930	-19.30	0.60	72	72.00005	45.8	$6.36 \cdot 10^{-7}$
10	1902	-19.02	0.88	105.6	105.59990	100.7	$9.54 \cdot 10^{-7}$
11	1901	-19.01	0.89	106.8	106.79993	73.2	$6.86 \cdot 10^{-7}$
12	1900	-19.00	0.90	108	107.99995	45.8	$4.24 \cdot 10^{-7}$
13	1872	-18.72	1.18	141.6	141.60004	36.6	$2.59 \cdot 10^{-7}$
14	1871	-18.71	1.19	142.8	142.80006	64.1	$4.49 \cdot 10^{-7}$
15	1870	-18.70	1.20	144	143.99986	137.3	$9.54 \cdot 10^{-7}$

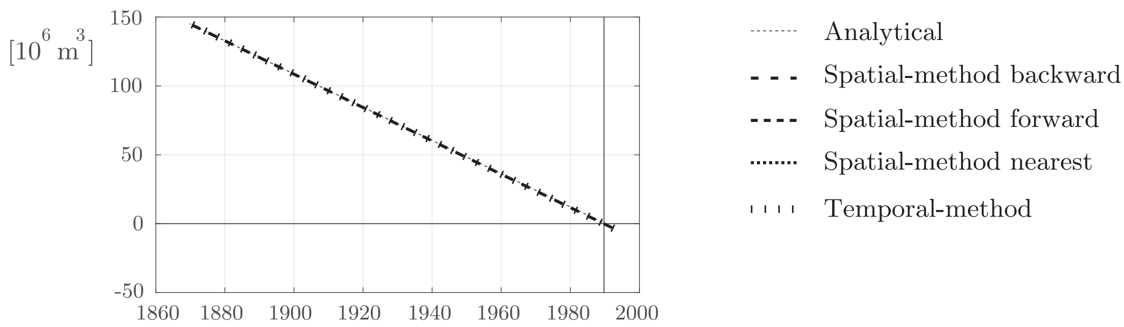

Figure A-11: Unit Test 4: Synthetic volume difference calculation relative to 1990

A-5 Unit Test 5

Table A-25: Unit Test 5

Unit Test	**Region of Interest**	**Trend**	**Time Step** [y]	**Gap**	**Amount of NetCDF Files**
5	Two NetCDF Files	Linear	10	No	2

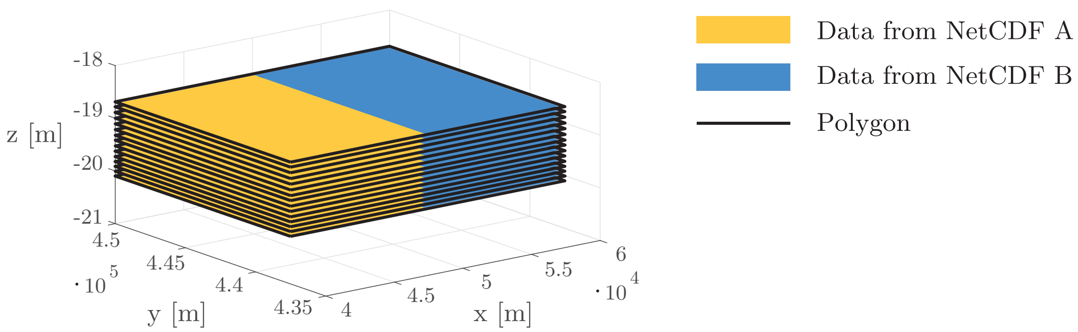

Figure A-12: Unit Test 5

Table A-26: Unit Test 5: Synthetic dataset values

	NetCDF File	Range $[m]$		Difference $[m]$	Analytical Surface $[m^2]$	Computed Surface $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A2	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B2	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					250,000,000	250,000,000

Table A-27: Unit Test 5: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-50	-50.00000	4.0	$8.00 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-25	-25.00000	2.0	$4.00 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	25	25.00000	4.0	$1.60 \cdot 10^{-7}$
5	1970	-19.7	0.2	50	50.00000	12.0	$2.40 \cdot 10^{-7}$
6	1960	-19.6	0.3	75	75.00000	4.0	$5.33 \cdot 10^{-8}$
7	1950	-19.5	0.4	100	100.00001	12.0	$1.20 \cdot 10^{-7}$
8	1940	-19.4	0.5	125	125.00001	12.0	$9.60 \cdot 10^{-8}$
9	1930	-19.3	0.6	150	150.00001	12.0	$8.00 \cdot 10^{-8}$
10	1920	-19.2	0.7	175	175.00000	20.0	$1.14 \cdot 10^{-7}$
11	1910	-19.1	0.8	200	200.00000	4.0	$2.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	225	225.00019	188.0	$8.36 \cdot 10^{-7}$
13	1890	-18.9	1.0	250	250.00003	28.0	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	275	275.00140	1404.0	$5.11 \cdot 10^{-6}$
15	1870	-18.7	1.2	300	299.99996	36.0	$1.20 \cdot 10^{-7}$

Table A-28: Unit Test 5: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-50	-50.00000	4.0	$8.00 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-25	-25.00000	2.0	$4.00 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	25	25.00000	4.0	$1.60 \cdot 10^{-7}$
5	1970	-19.7	0.2	50	50.00000	12.0	$2.40 \cdot 10^{-7}$
6	1960	-19.6	0.3	75	75.00000	4.0	$5.33 \cdot 10^{-8}$
7	1950	-19.5	0.4	100	100.00001	12.0	$1.20 \cdot 10^{-7}$
8	1940	-19.4	0.5	125	125.00001	12.0	$9.60 \cdot 10^{-8}$
9	1930	-19.3	0.6	150	150.00001	12.0	$8.00 \cdot 10^{-8}$
10	1920	-19.2	0.7	175	175.00000	20.0	$1.14 \cdot 10^{-7}$
11	1910	-19.1	0.8	200	200.00000	4.0	$2.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	225	225.00019	188.0	$8.36 \cdot 10^{-7}$
13	1890	-18.9	1.0	250	250.00003	28.0	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	275	275.00140	1404.0	$5.11 \cdot 10^{-6}$
15	1870	-18.7	1.2	300	299.99996	36.0	$1.20 \cdot 10^{-7}$

Table A-29: Unit Test 5: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-50	-50.00000	4.0	$8.00 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-25	-25.00000	2.0	$4.00 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	25	25.00000	4.0	$1.60 \cdot 10^{-7}$
5	1970	-19.7	0.2	50	50.00000	12.0	$2.40 \cdot 10^{-7}$
6	1960	-19.6	0.3	75	75.00000	4.0	$5.33 \cdot 10^{-8}$
7	1950	-19.5	0.4	100	100.00001	12.0	$1.20 \cdot 10^{-7}$
8	1940	-19.4	0.5	125	125.00001	12.0	$9.60 \cdot 10^{-8}$
9	1930	-19.3	0.6	150	150.00001	12.0	$8.00 \cdot 10^{-8}$
10	1920	-19.2	0.7	175	175.00000	20.0	$1.14 \cdot 10^{-7}$
11	1910	-19.1	0.8	200	200.00000	4.0	$2.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	225	225.00019	188.0	$8.36 \cdot 10^{-7}$
13	1890	-18.9	1.0	250	250.00003	28.0	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	275	275.00140	1404.0	$5.11 \cdot 10^{-6}$
15	1870	-18.7	1.2	300	299.99996	36.0	$1.20 \cdot 10^{-7}$

Table A-30: Unit Test 5: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-50	-50.00019	95.4	$3.81 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-25	-25.00010	47.7	$3.81 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	25	25.00010	47.7	$3.81 \cdot 10^{-6}$
5	1970	-19.7	0.2	50	49.99971	95.4	$5.72 \cdot 10^{-6}$
6	1960	-19.6	0.3	75	74.99981	143.1	$2.54 \cdot 10^{-6}$
7	1950	-19.5	0.4	100	99.99990	47.7	$9.54 \cdot 10^{-7}$
8	1940	-19.4	0.5	125	125.00000	0	0
9	1930	-19.3	0.6	150	150.00010	47.7	$6.36 \cdot 10^{-7}$
10	1920	-19.2	0.7	175	174.99971	143.1	$1.63 \cdot 10^{-6}$
11	1910	-19.1	0.8	200	199.99981	95.4	$9.54 \cdot 10^{-7}$
12	1900	-19.0	0.9	225	224.99990	47.7	$4.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	250	250.00000	0	0
14	1880	-18.8	1.1	275	275.00010	47.7	$3.47 \cdot 10^{-7}$
15	1870	-18.7	1.2	300	299.99971	143.1	$9.54 \cdot 10^{-7}$

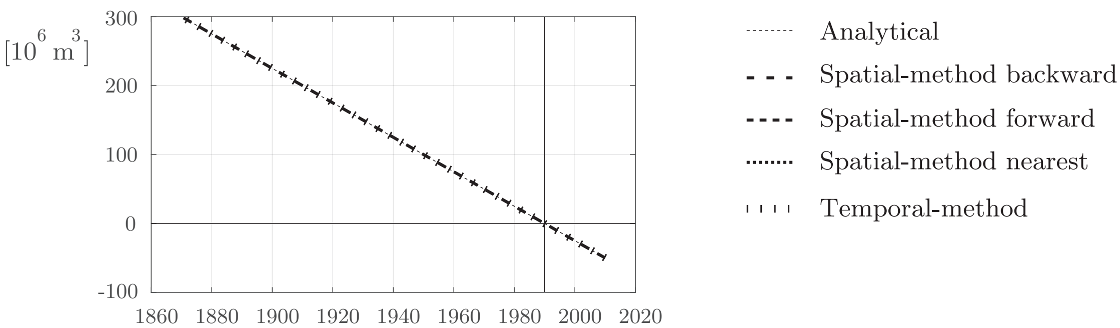

Figure A-13: Unit Test 5: Synthetic volume difference calculation relative to 1990

A-6 Unit Test 6

Table A-31: Unit Test 6

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
6	Two NetCDF Files	Linear	10	Yes	2

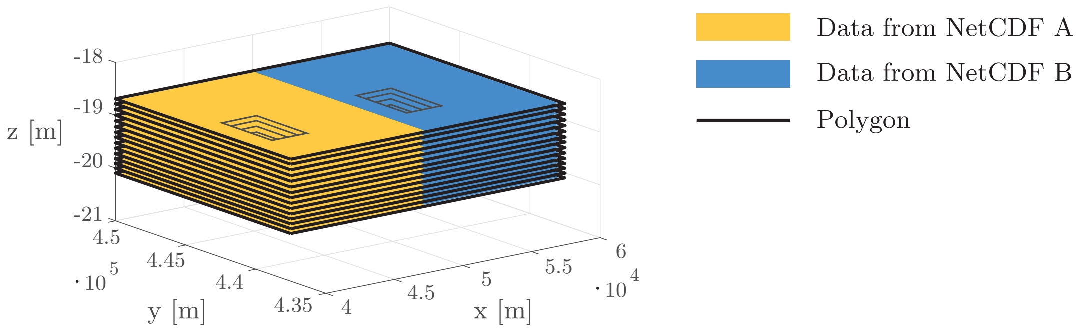

Figure A-14: Unit Test 6

Table A-32: Unit Test 6: Synthetic dataset values

	NetCDF File	Range [m]		Difference [m]	Analytical Surface [m^2]	Computed Surface [m^2]
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A2	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B2	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Gap x	A2	42,000	44,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Gap x	B2	52,000	64,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Net. Surface					240,000,000	240,000,000

Table A-33: Unit Test 6: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-48	-48.00000	4.0	$8.33 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-24	-24.00000	2.0	$8.33 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	24	24.00000	4.0	$1.67 \cdot 10^{-7}$
5	1970	-19.7	0.2	48	48.00000	12.0	$2.50 \cdot 10^{-7}$
6	1960	-19.6	0.3	72	72.00000	4.0	$5.56 \cdot 10^{-8}$
7	1950	-19.5	0.4	96	96.00000	12.0	$1.25 \cdot 10^{-7}$
8	1940	-19.4	0.5	120	120.00000	12.0	$1.00 \cdot 10^{-7}$
9	1930	-19.3	0.6	144	144.00002	12.0	$8.33 \cdot 10^{-8}$
10	1920	-19.2	0.7	168	167.99998	20.0	$1.19 \cdot 10^{-7}$
11	1910	-19.1	0.8	192	191.99998	4.0	$2.08 \cdot 10^{-8}$
12	1900	-19.0	0.9	216	215.99998	12.0	$5.56 \cdot 10^{-8}$
13	1890	-18.9	1.0	240	240.00000	28.0	$1.17 \cdot 10^{-7}$
14	1880	-18.8	1.1	264	264.00002	1084.0	$4.11 \cdot 10^{-6}$
15	1870	-18.7	1.2	288	288.00002	36.0	$1.25 \cdot 10^{-7}$

Table A-34: Unit Test 6: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-48	-48.00000	4.0	$8.33 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-24	-24.00000	2.0	$8.33 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	24	24.00000	4.0	$1.67 \cdot 10^{-7}$
5	1970	-19.7	0.2	48	48.00000	12.0	$2.50 \cdot 10^{-7}$
6	1960	-19.6	0.3	72	72.00000	4.0	$5.56 \cdot 10^{-8}$
7	1950	-19.5	0.4	96	96.00000	12.0	$1.25 \cdot 10^{-7}$
8	1940	-19.4	0.5	120	120.00000	12.0	$1.00 \cdot 10^{-7}$
9	1930	-19.3	0.6	144	144.00002	12.0	$8.33 \cdot 10^{-8}$
10	1920	-19.2	0.7	168	167.99998	20.0	$1.19 \cdot 10^{-7}$
11	1910	-19.1	0.8	192	191.99998	4.0	$2.08 \cdot 10^{-8}$
12	1900	-19.0	0.9	216	215.99998	12.0	$5.56 \cdot 10^{-8}$
13	1890	-18.9	1.0	240	240.00000	28.0	$1.17 \cdot 10^{-7}$
14	1880	-18.8	1.1	264	264.00002	1084.0	$4.11 \cdot 10^{-6}$
15	1870	-18.7	1.2	288	288.00002	36.0	$1.25 \cdot 10^{-7}$

Table A-35: Unit Test 6: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-48	-48.00000	4.0	$8.33 \cdot 10^{-8}$
2	2000	-20.0	-0.1	-24	-24.00000	2.0	$8.33 \cdot 10^{-8}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	24	24.00000	4.0	$1.67 \cdot 10^{-7}$
5	1970	-19.7	0.2	48	48.00000	12.0	$2.50 \cdot 10^{-7}$
6	1960	-19.6	0.3	72	72.00000	4.0	$5.56 \cdot 10^{-8}$
7	1950	-19.5	0.4	96	96.00000	12.0	$1.25 \cdot 10^{-7}$
8	1940	-19.4	0.5	120	120.00000	12.0	$1.00 \cdot 10^{-7}$
9	1930	-19.3	0.6	144	144.00002	12.0	$8.33 \cdot 10^{-8}$
10	1920	-19.2	0.7	168	167.99998	20.0	$1.19 \cdot 10^{-7}$
11	1910	-19.1	0.8	192	191.99998	4.0	$2.08 \cdot 10^{-8}$
12	1900	-19.0	0.9	216	215.99998	12.0	$5.56 \cdot 10^{-8}$
13	1890	-18.9	1.0	240	240.00000	28.0	$1.17 \cdot 10^{-7}$
14	1880	-18.8	1.1	264	264.00002	1084.0	$4.11 \cdot 10^{-6}$
15	1870	-18.7	1.2	288	288.00002	36.0	$1.25 \cdot 10^{-7}$

Table A-36: Unit Test 6: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-48	-48.00018	183.1	$3.81 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-24	-24.00009	91.6	$3.81 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	24	24.00009	91.6	$3.81 \cdot 10^{-6}$
5	1970	-19.7	0.2	48	47.99973	274.7	$5.72 \cdot 10^{-6}$
6	1960	-19.6	0.3	72	71.99982	183.1	$2.54 \cdot 10^{-6}$
7	1950	-19.5	0.4	96	95.99991	91.6	$9.54 \cdot 10^{-7}$
8	1940	-19.4	0.5	120	120.00000	0	0
9	1930	-19.3	0.6	144	144.00009	91.6	$6.36 \cdot 10^{-7}$
10	1920	-19.2	0.7	168	167.99973	274.7	$1.63 \cdot 10^{-6}$
11	1910	-19.1	0.8	192	191.99982	183.1	$9.54 \cdot 10^{-7}$
12	1900	-19.0	0.9	216	215.99991	91.6	$4.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	240	240.00000	0	0
14	1880	-18.8	1.1	264	264.00009	91.6	$3.47 \cdot 10^{-7}$
15	1870	-18.7	1.2	288	287.99973	273.7	$9.54 \cdot 10^{-7}$

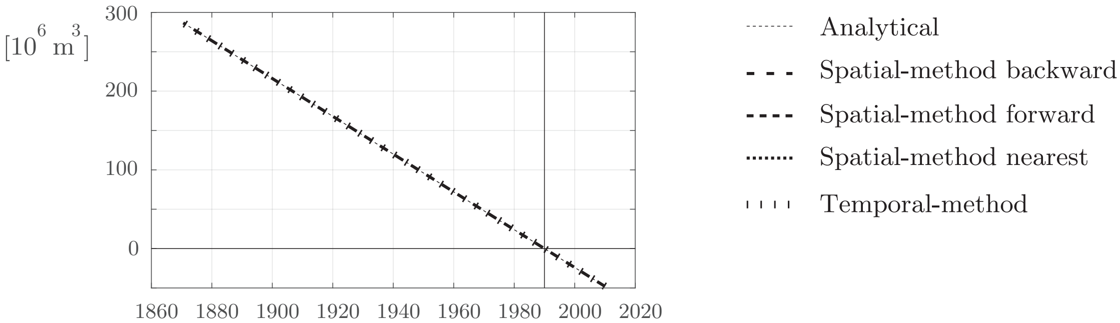

Figure A-15: Unit Test 6: Synthetic volume difference calculation relative to 1990

A-7 Unit Test 7

Table A-37: Unit Test 7

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
7	Two NetCDF Files	Linear	Variable	No	2

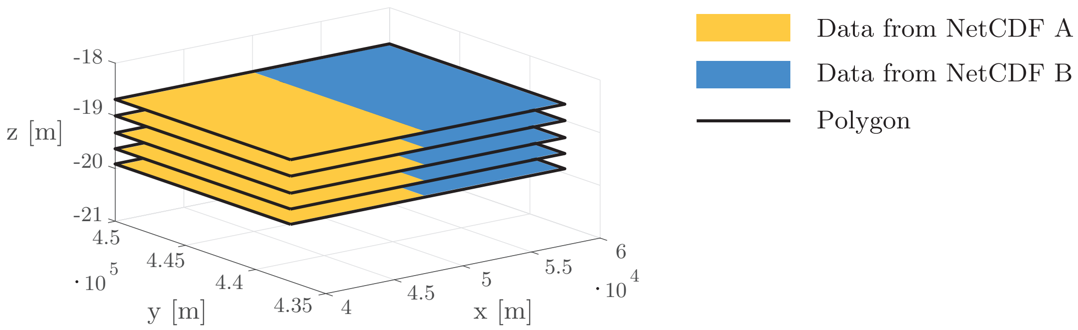

Figure A-16: Unit Test 7

Table A-38: Unit Test 7: Synthetic dataset values

	NetCDF File	Range [m]		Difference [m]	Analytical Surface [m^2]	Computed Surface [m^2]
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A3	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B3	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					250,000,000	250,000,000

Table A-39: Unit Test 7: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [10^6 m^3]	**Computed Volume** [10^6 m^3]	**Error** [m^3]	[$-$]
1	1992	-19.92	-0.02	-5.0	-5.00010	104.50	$2.09 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.50	-2.50005	52.25	$2.09 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	70	70.00000	112.50	$1.61 \cdot 10^{-6}$
5	1961	-19.61	0.29	72.5	72.50000	112.50	$1.55 \cdot 10^{-6}$
6	1960	-19.60	0.30	75	75.00000	112.50	$1.50 \cdot 10^{-6}$
7	1932	-19.32	0.58	145	145.00000	88.50	$6.10 \cdot 10^{-7}$
8	1931	-19.31	0.59	147.5	147.50038	375.50	$2.55 \cdot 10^{-6}$
9	1930	-19.30	0.60	150	149.99991	88.50	$5.90 \cdot 10^{-7}$
10	1902	-19.02	0.88	220	220.00000	120.50	$5.48 \cdot 10^{-7}$
11	1931	-19.01	0.89	222.5	222.49825	175.25	$7.88 \cdot 10^{-6}$
12	1930	-19.00	0.90	225	224.99991	88.50	$3.93 \cdot 10^{-7}$
13	1872	-18.72	1.18	295	294.99993	72.50	$2.46 \cdot 10^{-7}$
14	1871	-18.71	1.19	297.5	297.50124	123.95	$4.17 \cdot 10^{-6}$
15	1870	-18.70	1.20	300	300.99894	106.45	$3.55 \cdot 10^{-6}$

Table A-40: Unit Test 7: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-5.0	-5.00010	104.50	$2.09 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.50	-2.50005	52.25	$2.09 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	70	70.00000	112.50	$1.61 \cdot 10^{-6}$
5	1961	-19.61	0.29	72.5	72.50000	112.50	$1.55 \cdot 10^{-6}$
6	1960	-19.60	0.30	75	75.00000	112.50	$1.50 \cdot 10^{-6}$
7	1932	-19.32	0.58	145	145.00000	88.50	$6.10 \cdot 10^{-7}$
8	1931	-19.31	0.59	147.5	147.50038	375.50	$2.55 \cdot 10^{-6}$
9	1930	-19.30	0.60	150	149.99991	88.50	$5.90 \cdot 10^{-7}$
10	1902	-19.02	0.88	220	220.00000	120.50	$5.48 \cdot 10^{-7}$
11	1931	-19.01	0.89	222.5	222.49825	175.25	$7.88 \cdot 10^{-6}$
12	1930	-19.00	0.90	225	224.99991	88.50	$3.93 \cdot 10^{-7}$
13	1872	-18.72	1.18	295	294.99993	72.50	$2.46 \cdot 10^{-7}$
14	1871	-18.71	1.19	297.5	297.50124	123.95	$4.17 \cdot 10^{-6}$
15	1870	-18.70	1.20	300	300.99894	106.45	$3.55 \cdot 10^{-6}$

Table A-41: Unit Test 7: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-5.0	-5.00010	104.50	$2.09 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.50	-2.50005	52.25	$2.09 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	70	70.00000	112.50	$1.61 \cdot 10^{-6}$
5	1961	-19.61	0.29	72.5	72.50000	112.50	$1.55 \cdot 10^{-6}$
6	1960	-19.60	0.30	75	75.00000	112.50	$1.50 \cdot 10^{-6}$
7	1932	-19.32	0.58	145	145.00000	88.50	$6.10 \cdot 10^{-7}$
8	1931	-19.31	0.59	147.5	147.50038	375.50	$2.55 \cdot 10^{-6}$
9	1930	-19.30	0.60	150	149.99991	88.50	$5.90 \cdot 10^{-7}$
10	1902	-19.02	0.88	220	220.00000	120.50	$5.48 \cdot 10^{-7}$
11	1931	-19.01	0.89	222.5	222.49825	175.25	$7.88 \cdot 10^{-6}$
12	1930	-19.00	0.90	225	224.99991	88.50	$3.93 \cdot 10^{-7}$
13	1872	-18.72	1.18	295	294.99993	72.50	$2.46 \cdot 10^{-7}$
14	1871	-18.71	1.19	297.5	297.50124	123.95	$4.17 \cdot 10^{-6}$
15	1870	-18.70	1.20	300	300.99894	106.45	$3.55 \cdot 10^{-6}$

Table A-42: Unit Test 7: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [$10^6\ m^3$]	**Computed Volume** [$10^6\ m^3$]	**Error** [m^3]	[$-$]
1	1992	-19.92	-0.02	-5.0	-5.00011	114.4	$2.29 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.5	-2.50006	57.2	$2.29 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	70	69.99969	305.2	$4.36 \cdot 10^{-6}$
5	1961	-19.61	0.29	72.5	72.49975	248.0	$3.42 \cdot 10^{-6}$
6	1960	-19.60	0.30	75	74.99981	190.7	$2.54 \cdot 10^{-6}$
7	1932	-19.32	0.58	145	144.99998	19.1	$1.32 \cdot 10^{-7}$
8	1931	-19.31	0.59	147.5	147.500038	38.1	$2.59 \cdot 10^{-7}$
9	1930	-19.30	0.60	150	150.00001	95.4	$6.36 \cdot 10^{-7}$
10	1902	-19.02	0.88	220	219.99979	209.8	$9.54 \cdot 10^{-7}$
11	1901	-19.01	0.89	222.5	222.49985	152.6	$6.86 \cdot 10^{-7}$
12	1900	-19.00	0.90	225	224.99990	95.4	$4.24 \cdot 10^{-7}$
13	1872	-18.72	1.18	295	295.00008	76.3	$2.59 \cdot 10^{-7}$
14	1871	-18.71	1.19	297.5	297.50013	133.5	$4.49 \cdot 10^{-7}$
15	1870	-18.70	1.20	300	299.99971	286.1	$9.54 \cdot 10^{-7}$

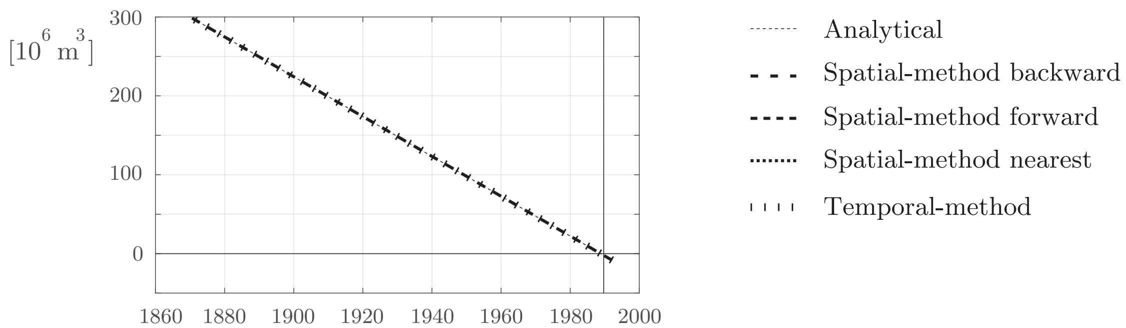

Figure A-17: Unit Test 7: Synthetic volume difference calculation relative to 1990

A-8 Unit Test 8

Table A-43: Unit Test 8

Unit Test	**Region of Interest**	**Trend**	**Time Step** [y]	**Gap**	**Amount of NetCDF Files**
8	Two NetCDF Files	Linear	Variable	Yes	2

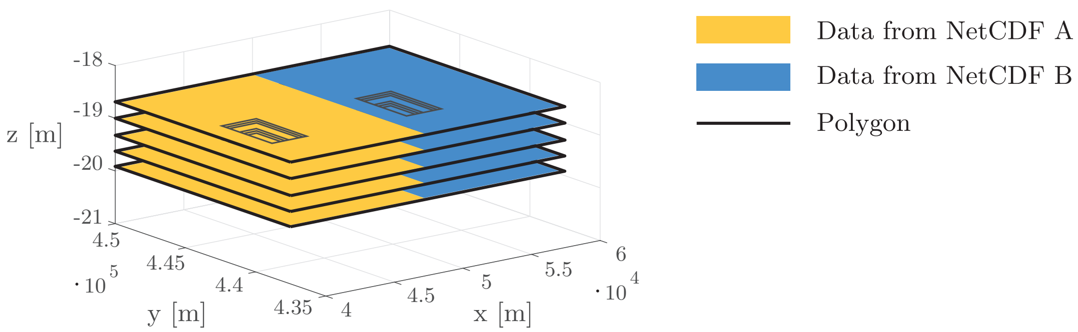

Figure A-18: Unit Test 8

Table A-44: Unit Test 8: Synthetic dataset values

	NetCDF File	**Range** $[m]$		**Difference** $[m]$	**Analytical Surface** $[m^2]$	**Computed Surface** $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A4	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B4	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Gap x	A4	42,000	44,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Gap x	B4	52,000	64,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Net. Surface					240,000,000	240,000,000

Table A-45: Unit Test 8: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-4.8	4.80011	109.50	$2.28 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.4	2.40005	54.75	$2.28 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	67.2	67.19988	117.50	$1.75 \cdot 10^{-6}$
5	1961	-19.61	0.29	69.6	69.59988	117.50	$1.69 \cdot 10^{-6}$
6	1960	-19.60	0.30	72.0	71.99988	117.50	$1.63 \cdot 10^{-6}$
7	1932	-19.32	0.58	139.2	139.19991	93.50	$6.72 \cdot 10^{-7}$
8	1931	-19.31	0.59	141.6	141.60021	210.50	$1.49 \cdot 10^{-6}$
9	1930	-19.30	0.60	144.0	143.99991	93.50	$6.49 \cdot 10^{-7}$
10	1902	-19.02	0.88	211.2	211.99875	125.50	$5.94 \cdot 10^{-7}$
11	1901	-19.01	0.89	213.6	213.59840	159.75	$7.48 \cdot 10^{-6}$
12	1900	-19.00	0.90	216.0	215.99991	93.50	$4.33 \cdot 10^{-7}$
13	1872	-18.72	1.18	283.2	283.19992	77.50	$2.74 \cdot 10^{-7}$
14	1871	-18.71	1.19	285.6	285.60091	914.50	$3.20 \cdot 10^{-6}$
15	1870	-18.70	1.20	288.0	287.99925	749.50	$2.60 \cdot 10^{-6}$

Table A-46: Unit Test 8: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-4.8	4.80011	109.50	$2.28 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.4	2.40005	54.75	$2.28 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	67.2	67.19988	117.50	$1.75 \cdot 10^{-6}$
5	1961	-19.61	0.29	69.6	69.59988	117.50	$1.69 \cdot 10^{-6}$
6	1960	-19.60	0.30	72.0	71.99988	117.50	$1.63 \cdot 10^{-6}$
7	1932	-19.32	0.58	139.2	139.19991	93.50	$6.72 \cdot 10^{-7}$
8	1931	-19.31	0.59	141.6	141.60021	210.50	$1.49 \cdot 10^{-6}$
9	1930	-19.30	0.60	144.0	143.99991	93.50	$6.49 \cdot 10^{-7}$
10	1902	-19.02	0.88	211.2	211.99875	125.50	$5.94 \cdot 10^{-7}$
11	1901	-19.01	0.89	213.6	213.59840	159.75	$7.48 \cdot 10^{-6}$
12	1900	-19.00	0.90	216.0	215.99991	93.50	$4.33 \cdot 10^{-7}$
13	1872	-18.72	1.18	283.2	283.19992	77.50	$2.74 \cdot 10^{-7}$
14	1871	-18.71	1.19	285.6	285.60091	914.50	$3.20 \cdot 10^{-6}$
15	1870	-18.70	1.20	288.0	287.99925	749.50	$2.60 \cdot 10^{-6}$

Table A-47: Unit Test 8: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	1992	-19.92	-0.02	-4.8	4.80011	109.50	$2.28 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.4	2.40005	54.75	$2.28 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	67.2	67.19988	117.50	$1.75 \cdot 10^{-6}$
5	1961	-19.61	0.29	69.6	69.59988	117.50	$1.69 \cdot 10^{-6}$
6	1960	-19.60	0.30	72.0	71.99988	117.50	$1.63 \cdot 10^{-6}$
7	1932	-19.32	0.58	139.2	139.19991	93.50	$6.72 \cdot 10^{-7}$
8	1931	-19.31	0.59	141.6	141.60021	210.50	$1.49 \cdot 10^{-6}$
9	1930	-19.30	0.60	144.0	143.99991	93.50	$6.49 \cdot 10^{-7}$
10	1902	-19.02	0.88	211.2	211.99875	125.50	$5.94 \cdot 10^{-7}$
11	1901	-19.01	0.89	213.6	213.59840	159.75	$7.48 \cdot 10^{-6}$
12	1900	-19.00	0.90	216.0	215.99991	93.50	$4.33 \cdot 10^{-7}$
13	1872	-18.72	1.18	283.2	283.19992	77.50	$2.74 \cdot 10^{-7}$
14	1871	-18.71	1.19	285.6	285.60091	914.50	$3.20 \cdot 10^{-6}$
15	1870	-18.70	1.20	288.0	287.99925	749.50	$2.60 \cdot 10^{-6}$

Table A-48: Unit Test 8: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [$10^6\ m^3$]	**Computed Volume** [$10^6\ m^3$]	**Error** [m^3]	[$-$]
1	1992	-19.92	-0.02	-4.8	-4.80011	114.4	$2.29 \cdot 10^{-5}$
2	1991	-19.91	-0.01	-2.4	-2.40005	57.2	$2.29 \cdot 10^{-5}$
3	1990	-19.90	0	0	0.00000	0	0
4	1962	-19.62	0.28	67.2	67.19971	305.2	$4.36 \cdot 10^{-6}$
5	1961	-19.61	0.29	69.6	69.59976	248.0	$3.42 \cdot 10^{-6}$
6	1960	-19.60	0.30	72	71.99982	190.7	$2.54 \cdot 10^{-6}$
7	1932	-19.32	0.58	139.2	139.19998	19.1	$1.32 \cdot 10^{-7}$
8	1931	-19.31	0.59	141.6	141.60004	38.1	$2.59 \cdot 10^{-7}$
9	1930	-19.30	0.60	144	144.00009	95.4	$6.36 \cdot 10^{-7}$
10	1902	-19.02	0.88	211.2	211.19980	209.8	$9.54 \cdot 10^{-7}$
11	1901	-19.01	0.89	213.6	213.59985	152.6	$6.86 \cdot 10^{-7}$
12	1900	-19.00	0.90	216	215.99991	95.4	$4.24 \cdot 10^{-7}$
13	1872	-18.72	1.18	283.2	283.20007	76.3	$2.59 \cdot 10^{-7}$
14	1871	-18.71	1.19	285.6	285.60013	133.5	$4.49 \cdot 10^{-7}$
15	1870	-18.70	1.20	288	287.99973	286.1	$9.54 \cdot 10^{-7}$

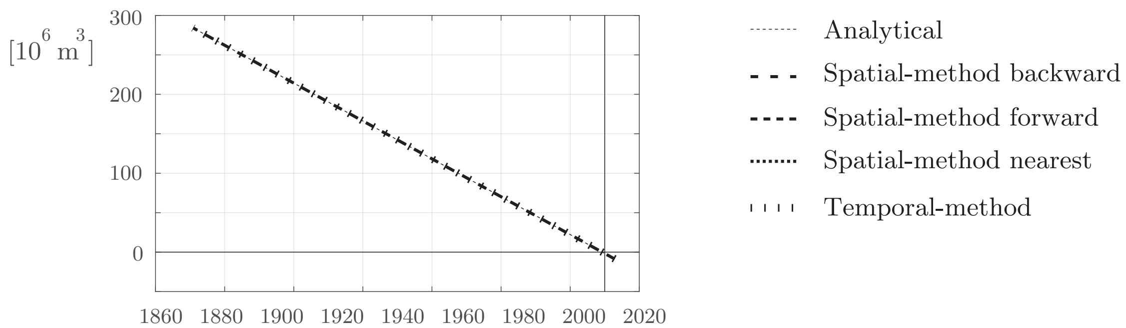

Figure A-19: Unit Test 8: Synthetic volume difference calculation relative to 1990

A-9 Unit Test 9

Table A-49: Unit Test 9

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
9	Two NetCDF Files	Non-Linear	10	No	1

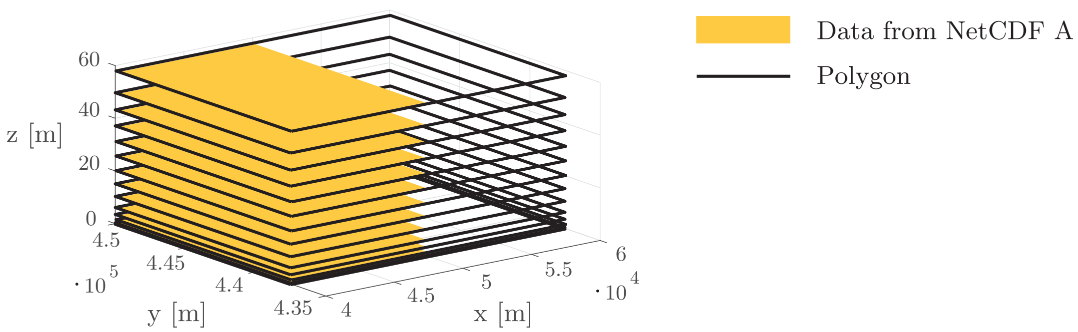

Figure A-20: Unit Test 9

Table A-50: Unit Test 9: Synthetic dataset values

	NetCDF File	**Range** $[m]$		**Difference** $[m]$	**Analytical Surface** $[m^2]$	**Computed Surface** $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A9	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					125,000,000	125,000,000

Table A-51: Unit Test 9: S-approach backward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^9\ m^3]$	**Computed Volume** $[10^9\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	56.25	14	1.75	1.75001	1,008	$5.76 \cdot 10^{-7}$
2	2000	49	6.75	0.84375	0.84376	3,712	$4.40 \cdot 10^{-6}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.78125	-0.78124	64	$8.19 \cdot 10^{-8}$
5	1970	30.25	-12	-1.5	-1.50000	2,688	$1.79 \cdot 10^{-6}$
6	1960	25	-17.25	-2.15625	-2.15626	544	$2.52 \cdot 10^{-7}$
7	1950	20.25	-22	-2.75	-2.74996	4,672	$1.70 \cdot 10^{-6}$
8	1940	16	-26.25	-3.28125	-3.28124	4,032	$1.23 \cdot 10^{-6}$
9	1930	12.25	-30	-3.75	-3.75000	3,312	$8.83 \cdot 10^{-7}$
10	1920	9	-33.25	-4.15625	-4.15624	3,840	$9.24 \cdot 10^{-7}$
11	1910	6.25	-36	-4.5	-4.49997	4,528	$1.01 \cdot 10^{-6}$
12	1900	4	-38.25	-4.78125	-4.78125	2,480	$5.19 \cdot 10^{-7}$
13	1890	2.25	-38.25	-5.0	-4.99996	448	$8.96 \cdot 10^{-8}$
14	1880	1	-41.25	-5.15625	-5.15628	1,344	$2.61 \cdot 10^{-7}$
15	1870	0.25	-42	-5.25	-5.25003	3,024	$5.76 \cdot 10^{-7}$

Table A-52: Unit Test 9: S-approach forward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^9\ m^3]$	**Computed Volume** $[10^9\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	56.25	14	1.75	1.75001	1,008	$5.76 \cdot 10^{-7}$
2	2000	49	6.75	0.84375	0.84376	3,712	$4.40 \cdot 10^{-6}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.78125	-0.78124	64	$8.19 \cdot 10^{-8}$
5	1970	30.25	-12	-1.5	-1.50000	2,688	$1.79 \cdot 10^{-6}$
6	1960	25	-17.25	-2.15625	-2.15626	544	$2.52 \cdot 10^{-7}$
7	1950	20.25	-22	-2.75	-2.74996	4,672	$1.70 \cdot 10^{-6}$
8	1940	16	-26.25	-3.28125	-3.28124	4,032	$1.23 \cdot 10^{-6}$
9	1930	12.25	-30	-3.75	-3.75000	3,312	$8.83 \cdot 10^{-7}$
10	1920	9	-33.25	-4.15625	-4.15624	3,840	$9.24 \cdot 10^{-7}$
11	1910	6.25	-36	-4.5	-4.49997	4,528	$1.01 \cdot 10^{-6}$
12	1900	4	-38.25	-4.78125	-4.78125	2,480	$5.19 \cdot 10^{-7}$
13	1890	2.25	-38.25	-5.0	-4.99996	448	$8.96 \cdot 10^{-8}$
14	1880	1	-41.25	-5.15625	-5.15628	1,344	$2.61 \cdot 10^{-7}$
15	1870	0.25	-42	-5.25	-5.25003	3,024	$5.76 \cdot 10^{-7}$

Table A-53: Unit Test 9: S-approach nearest synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[m^3]$	$[-]$
1	2010	56.25	14	1.75	1.75001	1,008	$5.76 \cdot 10^{-7}$
2	2000	49	6.75	0.84375	0.84376	3,712	$4.40 \cdot 10^{-6}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.78125	-0.78124	64	$8.19 \cdot 10^{-8}$
5	1970	30.25	-12	-1.5	-1.50000	2,688	$1.79 \cdot 10^{-6}$
6	1960	25	-17.25	-2.15625	-2.15626	544	$2.52 \cdot 10^{-7}$
7	1950	20.25	-22	-2.75	-2.74996	4,672	$1.70 \cdot 10^{-6}$
8	1940	16	-26.25	-3.28125	-3.28124	4,032	$1.23 \cdot 10^{-6}$
9	1930	12.25	-30	-3.75	-3.75000	3,312	$8.83 \cdot 10^{-7}$
10	1920	9	-33.25	-4.15625	-4.15624	3,840	$9.24 \cdot 10^{-7}$
11	1910	6.25	-36	-4.5	-4.49997	4,528	$1.01 \cdot 10^{-6}$
12	1900	4	-38.25	-4.78125	-4.78125	2,480	$5.19 \cdot 10^{-7}$
13	1890	2.25	-38.25	-5.0	-4.99996	448	$8.96 \cdot 10^{-8}$
14	1880	1	-41.25	-5.15625	-5.15628	1,344	$2.61 \cdot 10^{-7}$
15	1870	0.25	-42	-5.25	-5.25003	3,024	$5.76 \cdot 10^{-7}$

Table A-54: Unit Test 9: T-approach synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[m^3]$	$[-]$
1	2010	56.25	14	1.75	1.75000	0	0
2	2000	49	6.75	0.84375	0.84375	0	0
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.78125	-0.78125	0	0
5	1970	30.25	-12	-1.5	-1.50000	0	0
6	1960	25	-17.25	-2.15625	-2.15625	0	0
7	1950	20.25	-22	-2.75	-2.75000	0	0
8	1940	16	-26.25	-3.28125	-3.28125	0	0
9	1930	12.25	-30	-3.75	-3.75000	0	0
10	1920	9	-33.25	-4.15625	-4.15625	0	0
11	1910	6.25	-36	-4.5	-4.50000	0	0
12	1900	4	-38.25	-4.78125	-4.78125	0	0
13	1890	2.25	-38.25	-5.0	-5.00000	0	0
14	1880	1	-41.25	-5.15625	-5.15625	0	0
15	1870	0.25	-42	-5.25	-5.25000	0	0

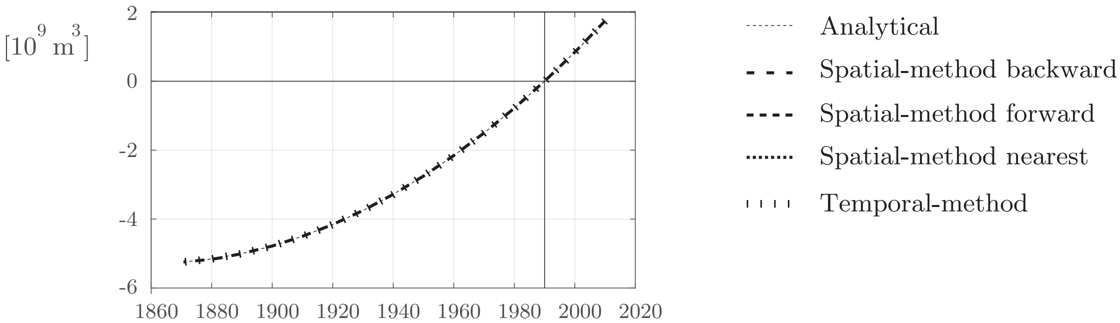

Figure A-21: Unit Test 9: Synthetic volume difference calculation relative to 1990

A-10 Unit Test 10

Table A-55: Unit Test 10

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
10	Two NetCDF Files	Non-Linear	10	Yes	1

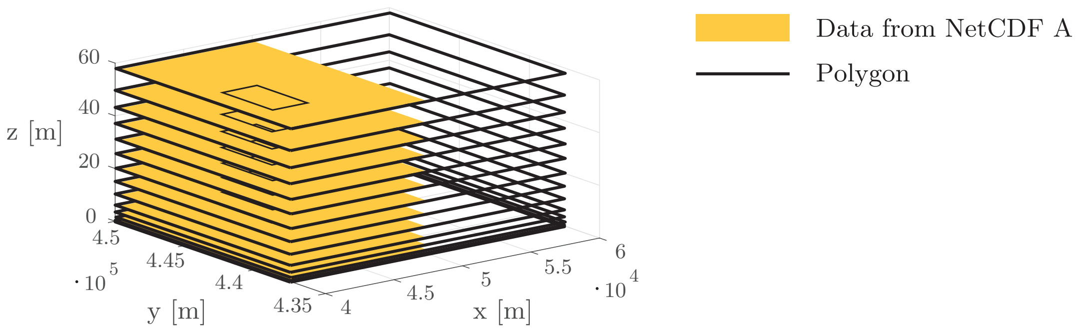

Figure A-22: Unit Test 10

Table A-56: Unit Test 10: Synthetic dataset values

	NetCDF File	Range [m]		Difference [m]	Analytical Surface [m^2]	Computed Surface [m^2]
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A10	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Gap x	A10	42,000	44,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Net. Surface					120,000,000	120,000,000

Table A-57: Unit Test 10: S-approach backward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [10^9 m^3]	**Computed Volume** [10^9 m^3]	**Error** [m^3]	[$-$]
1	2010	56.25	14	1.68	1.68001	7,808	$4.65 \cdot 10^{-6}$
2	2000	49	6.75	0.81	0.81001	1,267.20	$1.56 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.75	-0.75000	3,328.00	$4.44 \cdot 10^{-6}$
5	1970	30.25	-12	-1.44	-1.44000	1,152.00	$8.00 \cdot 10^{-7}$
6	1960	25	-17.25	-2.07	-2.07001	9,984.00	$4.82 \cdot 10^{-6}$
7	1950	20.25	-22	-2.64	-2.63997	25,728.00	$9.75 \cdot 10^{-6}$
8	1940	16	-26.25	-3.15	-3.15000	10,752.00	$3.41 \cdot 10^{-6}$
9	1930	12.25	-30	-3.6	-3.60000	11,392.00	$3.16 \cdot 10^{-6}$
10	1920	9	-33.25	-3.99	-3.98998	17,920.00	$4.49 \cdot 10^{-6}$
11	1910	6.25	-36	-4.32	-4.31997	31,872.00	$7.38 \cdot 10^{-6}$
12	1900	4	-38.25	-4.59	-4.59000	2,560.00	$5.57 \cdot 10^{-7}$
13	1890	2.25	-40	-4.80	-4.80000	14,464.00	$3.01 \cdot 10^{-6}$
14	1880	1	-40	-4.95	-4.95003	27,136.00	$5.48 \cdot 10^{-6}$
15	1870	1	-42	-5.04	-5.04002	27,136.00	$4.65 \cdot 10^{-6}$

Table A-58: Unit Test 10: S-approach forward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[m^3]$	$[-]$
1	2010	56.25	14	1.68	1.68001	7,808	$4.65 \cdot 10^{-6}$
2	2000	49	6.75	0.81	0.81001	1,267.20	$1.56 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.75	-0.75000	3,328.00	$4.44 \cdot 10^{-6}$
5	1970	30.25	-12	-1.44	-1.44000	1,152.00	$8.00 \cdot 10^{-7}$
6	1960	25	-17.25	-2.07	-2.07001	9,984.00	$4.82 \cdot 10^{-6}$
7	1950	20.25	-22	-2.64	-2.63997	25,728.00	$9.75 \cdot 10^{-6}$
8	1940	16	-26.25	-3.15	-3.15000	10,752.00	$3.41 \cdot 10^{-6}$
9	1930	12.25	-30	-3.6	-3.60000	11,392.00	$3.16 \cdot 10^{-6}$
10	1920	9	-33.25	-3.99	-3.98998	17,920.00	$4.49 \cdot 10^{-6}$
11	1910	6.25	-36	-4.32	-4.31997	31,872.00	$7.38 \cdot 10^{-6}$
12	1900	4	-38.25	-4.59	-4.59000	2,560.00	$5.57 \cdot 10^{-7}$
13	1890	2.25	-40	-4.80	-4.80000	14,464.00	$3.01 \cdot 10^{-6}$
14	1880	1	-40	-4.95	-4.95003	27,136.00	$5.48 \cdot 10^{-6}$
15	1870	1	-42	-5.04	-5.04002	27,136.00	$4.65 \cdot 10^{-6}$

Table A-59: Unit Test 10: S-approach nearest synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[m^3]$	$[-]$
1	2010	56.25	14	1.68	1.68001	7,808	$4.65 \cdot 10^{-6}$
2	2000	49	6.75	0.81	0.81001	1,267.20	$1.56 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.75	-0.75000	3,328.00	$4.44 \cdot 10^{-6}$
5	1970	30.25	-12	-1.44	-1.44000	1,152.00	$8.00 \cdot 10^{-7}$
6	1960	25	-17.25	-2.07	-2.07001	9,984.00	$4.82 \cdot 10^{-6}$
7	1950	20.25	-22	-2.64	-2.63997	25,728.00	$9.75 \cdot 10^{-6}$
8	1940	16	-26.25	-3.15	-3.15000	10,752.00	$3.41 \cdot 10^{-6}$
9	1930	12.25	-30	-3.6	-3.60000	11,392.00	$3.16 \cdot 10^{-6}$
10	1920	9	-33.25	-3.99	-3.98998	17,920.00	$4.49 \cdot 10^{-6}$
11	1910	6.25	-36	-4.32	-4.31997	31,872.00	$7.38 \cdot 10^{-6}$
12	1900	4	-38.25	-4.59	-4.59000	2,560.00	$5.57 \cdot 10^{-7}$
13	1890	2.25	-40	-4.80	-4.80000	14,464.00	$3.01 \cdot 10^{-6}$
14	1880	1	-40	-4.95	-4.95003	27,136.00	$5.48 \cdot 10^{-6}$
15	1870	1	-42	-5.04	-5.04002	27,136.00	$4.65 \cdot 10^{-6}$

Table A-60: Unit Test 10: T-approach synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [10^9 m^3]	**Computed Volume** [10^9 m^3]	**Error** [m^3]	[$-$]
1	2010	56.25	14	1.68	1.68000	0	0
2	2000	49	6.75	0.81	0.81000	0	0
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-0.75	-0.75000	0	0
5	1970	30.25	-12	-1.44	-1.44000	0	0
6	1960	25	-17.25	-2.07	-2.07000	0	0
7	1950	20.25	-22	-2.64	-2.64000	0	0
8	1940	16	-26.25	-3.15	-3.15000	0	0
9	1930	12.25	-30	-3.6	-3.60000	0	0
10	1920	9	-33.25	-3.99	-3.99000	0	0
11	1910	6.25	-36	-4.32	-4.32000	0	0
12	1900	4	-38.25	-4.59	-4.59000	0	0
13	1890	2.25	-40	-4.80	-4.80000	0	0
14	1880	1	-40	-4.95	-4.95000	0	0
15	1870	0.25	-42	-5.04	-5.04000	0	0

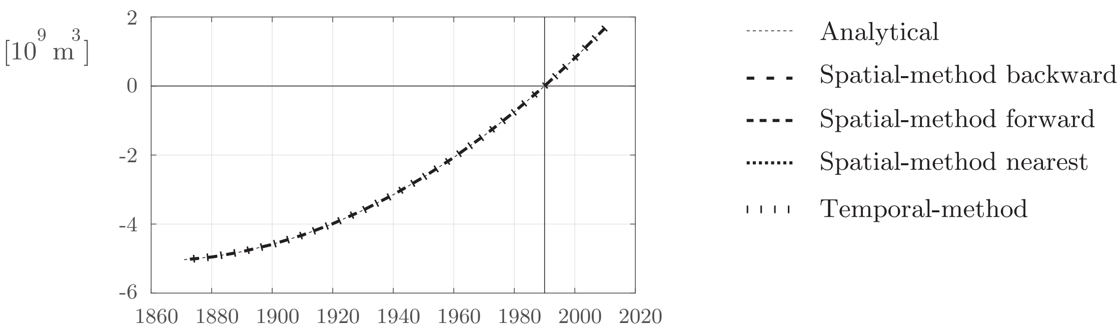

Figure A-23: Unit Test 10: Synthetic volume difference calculation relative to 1990

A-11 Unit Test 11

Table A-61: Unit Test 11

Unit Test	**Region of Interest**	**Trend**	**Time Step** [y]	**Gap**	**Amount of NetCDF Files**
11	Two NetCDF Files	Non-Linear	10	No	2

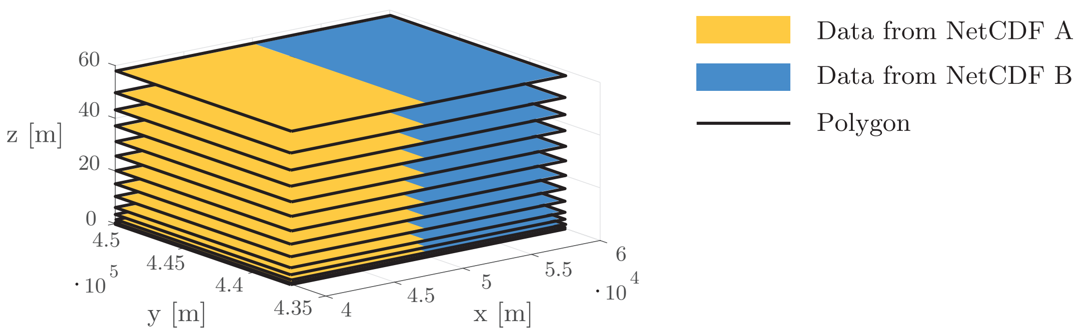

Figure A-24: Unit Test 11

Table A-62: Unit Test 11: Synthetic dataset values

	NetCDF File	**Range** $[m]$		**Difference** $[m]$	**Analytical Surface** $[m^2]$	**Computed Surface** $[m^2]$
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A9	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B9	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					250,000,000	250,000,000

Table A-63: Unit Test 11: S-approach backward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[10^3\ m^3]$	$[-]$
1	2010	56.25	14	3.5	3.50002	22,016.00	$5.29 \cdot 10^{-6}$
2	2000	49	6.75	1.6875	1.68753	27,424.00	$1.63 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5625	-1.56248	20,128.00	$1.29 \cdot 10^{-5}$
5	1970	30.25	-12	-3	-2.99995	5,376.00	$1.79 \cdot 10^{-6}$
6	1960	25	-17.25	-4.3125	-4.31252	18,912.00	$4.39 \cdot 10^{-6}$
7	1950	20.25	-22	-5.5	-5.49993	29,344.00	$5.34 \cdot 10^{-6}$
8	1940	16	-26.25	-6.5625	-6.56248	11,936.00	$1.82 \cdot 10^{-6}$
9	1930	12.25	-30	-7.5	-7.49997	26,624.00	$3.55 \cdot 10^{-6}$
10	1920	9	-33.25	-8.3125	-8.31247	27,680.00	$3.33 \cdot 10^{-6}$
11	1910	6.25	-36	-9	-8.99995	49,056.00	$5.45 \cdot 10^{-7}$
12	1900	4	-38.25	-9.5625	-9.56250	4,960.00	$5.19 \cdot 10^{-6}$
13	1890	2.25	-40	-10	-9.99992	20,896.00	$2.09 \cdot 10^{-6}$
14	1880	1	-41.25	-10.3125	-10.31256	42,688.00	$4.14 \cdot 10^{-6}$
15	1870	1	-42	-10.5	-10.50007	33,952.00	$3.23 \cdot 10^{-6}$

Table A-64: Unit Test 11: S-approach forward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[10^3\ m^3]$	$[-]$
1	2010	56.25	14	3.5	3.50002	22,016.00	$5.29 \cdot 10^{-6}$
2	2000	49	6.75	1.6875	1.68753	27,424.00	$1.63 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5625	-1.56248	20,128.00	$1.29 \cdot 10^{-5}$
5	1970	30.25	-12	-3	-2.99995	5,376.00	$1.79 \cdot 10^{-6}$
6	1960	25	-17.25	-4.3125	-4.31252	18,912.00	$4.39 \cdot 10^{-6}$
7	1950	20.25	-22	-5.5	-5.49993	29,344.00	$5.34 \cdot 10^{-6}$
8	1940	16	-26.25	-6.5625	-6.56248	11,936.00	$1.82 \cdot 10^{-6}$
9	1930	12.25	-30	-7.5	-7.49997	26,624.00	$3.55 \cdot 10^{-6}$
10	1920	9	-33.25	-8.3125	-8.31247	27,680.00	$3.33 \cdot 10^{-6}$
11	1910	6.25	-36	-9	-8.99995	49,056.00	$5.45 \cdot 10^{-7}$
12	1900	4	-38.25	-9.5625	-9.56250	4,960.00	$5.19 \cdot 10^{-6}$
13	1890	2.25	-40	-10	-9.99992	20,896.00	$2.09 \cdot 10^{-6}$
14	1880	1	-41.25	-10.3125	-10.31256	42,688.00	$4.14 \cdot 10^{-6}$
15	1870	1	-42	-10.5	-10.50007	33,952.00	$3.23 \cdot 10^{-6}$

Table A-65: Unit Test 11: S-approach nearest synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[10^3\ m^3]$	$[-]$
1	2010	56.25	14	3.5	3.50002	22,016.00	$5.29 \cdot 10^{-6}$
2	2000	49	6.75	1.6875	1.68753	27,424.00	$1.63 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5625	-1.56248	20,128.00	$1.29 \cdot 10^{-5}$
5	1970	30.25	-12	-3	-2.99995	5,376.00	$1.79 \cdot 10^{-6}$
6	1960	25	-17.25	-4.3125	-4.31252	18,912.00	$4.39 \cdot 10^{-6}$
7	1950	20.25	-22	-5.5	-5.49993	29,344.00	$5.34 \cdot 10^{-6}$
8	1940	16	-26.25	-6.5625	-6.56248	11,936.00	$1.82 \cdot 10^{-6}$
9	1930	12.25	-30	-7.5	-7.49997	26,624.00	$3.55 \cdot 10^{-6}$
10	1920	9	-33.25	-8.3125	-8.31247	27,680.00	$3.33 \cdot 10^{-6}$
11	1910	6.25	-36	-9	-8.99995	49,056.00	$5.45 \cdot 10^{-7}$
12	1900	4	-38.25	-9.5625	-9.56250	4,960.00	$5.19 \cdot 10^{-6}$
13	1890	2.25	-40	-10	-9.99992	20,896.00	$2.09 \cdot 10^{-6}$
14	1880	1	-41.25	-10.3125	-10.31256	42,688.00	$4.14 \cdot 10^{-6}$
15	1870	1	-42	-10.5	-10.50007	33,952.00	$3.23 \cdot 10^{-6}$

Table A-66: Unit Test 11: T-approach synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^9\ m^3]$	$[10^9\ m^3]$	$[10^3\ m^3]$	$[-]$
1	2010	56.25	14	3.5	3.50002	22,016.00	$5.29 \cdot 10^{-6}$
2	2000	49	6.75	1.6875	1.68753	27,424.00	$1.63 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5625	-1.56248	20,128.00	$1.29 \cdot 10^{-5}$
5	1970	30.25	-12	-3	-2.99995	5,376.00	$1.79 \cdot 10^{-6}$
6	1960	25	-17.25	-4.3125	-4.31252	18,912.00	$4.39 \cdot 10^{-6}$
7	1950	20.25	-22	-5.5	-5.49993	29,344.00	$5.34 \cdot 10^{-6}$
8	1940	16	-26.25	-6.5625	-6.56248	11,936.00	$1.82 \cdot 10^{-6}$
9	1930	12.25	-30	-7.5	-7.49997	26,624.00	$3.55 \cdot 10^{-6}$
10	1920	9	-33.25	-8.3125	-8.31247	27,680.00	$3.33 \cdot 10^{-6}$
11	1910	6.25	-36	-9	-8.99995	49,056.00	$5.45 \cdot 10^{-7}$
12	1900	4	-38.25	-9.5625	-9.56250	4,960.00	$5.19 \cdot 10^{-6}$
13	1890	2.25	-40	-10	-9.99992	20,896.00	$2.09 \cdot 10^{-6}$
14	1880	1	-41.25	-10.3125	-10.31256	42,688.00	$4.14 \cdot 10^{-6}$
15	1870	1	-42	-10.5	-10.50007	33,952.00	$3.23 \cdot 10^{-6}$

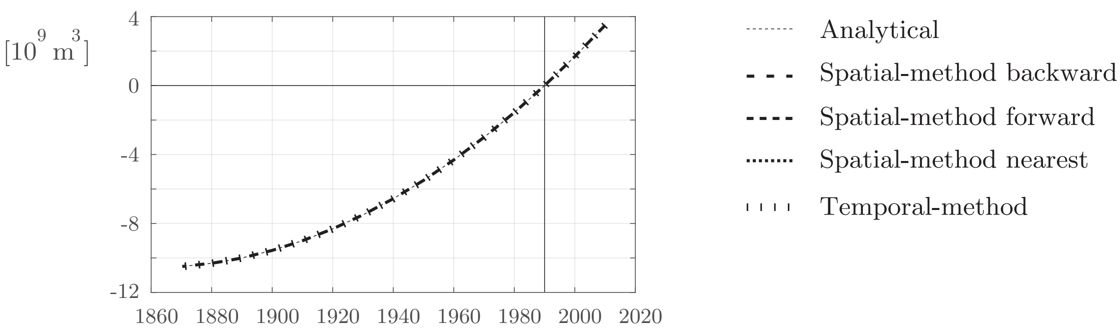

Figure A-25: Unit Test 11: Synthetic volume difference calculation relative to 1990

A-12 Unit Test 12

Table A-67: Unit Test 12

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
12	Two NetCDF Files	Non-Linear	10	Yes	2

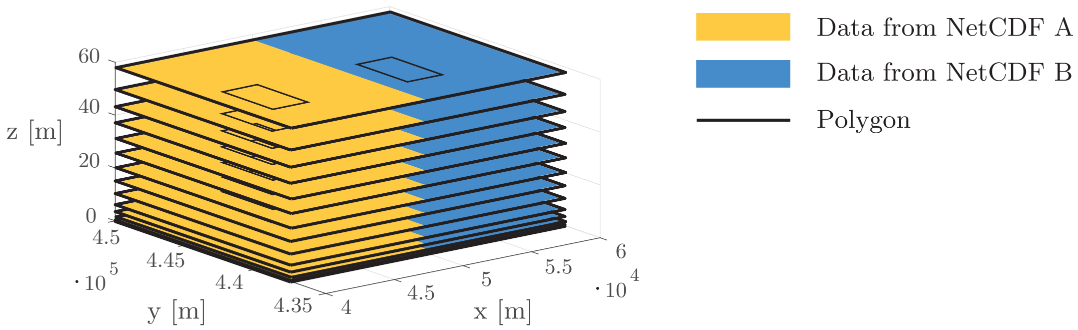

Figure A-26: Unit Test 12

Table A-68: Unit Test 12: Synthetic dataset values

	NetCDF File	Range [m]		Difference [m]	Analytical Surface [m^2]	Computed Surface [m^2]
Region of Interest x		40,000	60,000	20,000	250,000,000	250,000,000
Region of Interest y		437,500	450,000	12,500		
NetCDF x	A10	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B10	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Gap x	A10	42,000	44,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Gap x	B10	52,000	64,000	2,000	5,000,000	
Gap y		440,000	442,500	2,500		
Net. Surface					240,000,000	240,000,000

Table A-69: Unit Test 12: S-approach backward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^9\ m^3]$	**Computed Volume** $[10^9\ m^3]$	**Error** $[10^3\ m^3]$	$[-]$
1	2010	56.25	14	3.36	3.36002	15,616.00	$4.65 \cdot 10^{-6}$
2	2000	49	6.75	1.62	1.62003	25,344.00	$1.56 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5	-1.49993	6,656.00	$4.44 \cdot 10^{-6}$
5	1970	30.25	-12	-2.88	-2.88000	2,304.00	$8.00 \cdot 10^{-7}$
6	1960	25	-17.25	-4.14	-4.14002	19,968.00	$4.82 \cdot 10^{-6}$
7	1950	20.25	-22	-5.28	-5.27995	48,544.00	$9.19 \cdot 10^{-6}$
8	1940	16	-26.25	-6.3	-6.29998	21,504.00	$3.41 \cdot 10^{-6}$
9	1930	12.25	-30	-7.2	-7.19998	22,784.00	$3.16 \cdot 10^{-6}$
10	1920	9	-33.25	-7.98	-7.97996	35,840.00	$4.49 \cdot 10^{-6}$
11	1910	6.25	-36	-8.64	-8.63994	36,256.00	$4.20 \cdot 10^{-6}$
12	1900	4	-38.25	-9.1800	-9.18001	5,120.00	$5.58 \cdot 10^{-7}$
13	1890	2.25	-40	-9.6	-9.59997	28,928.00	$3.01 \cdot 10^{-6}$
14	1880	1	-41.25	-9.9	-9.90005	45.728,00	$4.62 \cdot 10^{-6}$
15	1870	0.25	-42	-10.08	-10.08005	46,848.00	$4.65 \cdot 10^{-6}$

Table A-70: Unit Test 12: S-approach forward synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^9\ m^3]$	**Computed Volume** $[10^9\ m^3]$	**Error** $[10^3\ m^3]$	$[-]$
1	2010	56.25	14	3.36	3.36002	15,616.00	$4.65 \cdot 10^{-6}$
2	2000	49	6.75	1.62	1.62003	25,344.00	$1.56 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5	-1.49993	6,656.00	$4.44 \cdot 10^{-6}$
5	1970	30.25	-12	-2.88	-2.88000	2,304.00	$8.00 \cdot 10^{-7}$
6	1960	25	-17.25	-4.14	-4.14002	19,968.00	$4.82 \cdot 10^{-6}$
7	1950	20.25	-22	-5.28	-5.27995	48,544.00	$9.19 \cdot 10^{-6}$
8	1940	16	-26.25	-6.3	-6.29998	21,504.00	$3.41 \cdot 10^{-6}$
9	1930	12.25	-30	-7.2	-7.19998	22,784.00	$3.16 \cdot 10^{-6}$
10	1920	9	-33.25	-7.98	-7.97996	35,840.00	$4.49 \cdot 10^{-6}$
11	1910	6.25	-36	-8.64	-8.63994	36,256.00	$4.20 \cdot 10^{-6}$
12	1900	4	-38.25	-9.1800	-9.18001	5,120.00	$5.58 \cdot 10^{-7}$
13	1890	2.25	-40	-9.6	-9.59997	28,928.00	$3.01 \cdot 10^{-6}$
14	1880	1	-41.25	-9.9	-9.90005	45.728,00	$4.62 \cdot 10^{-6}$
15	1870	0.25	-42	-10.08	-10.08005	46,848.00	$4.65 \cdot 10^{-6}$

Table A-71: Unit Test 12: S-approach nearest synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^9\ m^3]$	**Computed Volume** $[10^9\ m^3]$	**Error** $[10^3\ m^3]$	$[-]$
1	2010	56.25	14	3.36	3.36002	15,616.00	$4.65 \cdot 10^{-6}$
2	2000	49	6.75	1.62	1.62003	25,344.00	$1.56 \cdot 10^{-5}$
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5	-1.49993	6,656.00	$4.44 \cdot 10^{-6}$
5	1970	30.25	-12	-2.88	-2.88000	2,304.00	$8.00 \cdot 10^{-7}$
6	1960	25	-17.25	-4.14	-4.14002	19,968.00	$4.82 \cdot 10^{-6}$
7	1950	20.25	-22	-5.28	-5.27995	48,544.00	$9.19 \cdot 10^{-6}$
8	1940	16	-26.25	-6.3	-6.29998	21,504.00	$3.41 \cdot 10^{-6}$
9	1930	12.25	-30	-7.2	-7.19998	22,784.00	$3.16 \cdot 10^{-6}$
10	1920	9	-33.25	-7.98	-7.97996	35,840.00	$4.49 \cdot 10^{-6}$
11	1910	6.25	-36	-8.64	-8.63994	36,256.00	$4.20 \cdot 10^{-6}$
12	1900	4	-38.25	-9.1800	-9.18001	5,120.00	$5.58 \cdot 10^{-7}$
13	1890	2.25	-40	-9.6	-9.59997	28,928.00	$3.01 \cdot 10^{-6}$
14	1880	1	-41.25	-9.9	-9.90005	45.728,00	$4.62 \cdot 10^{-6}$
15	1870	0.25	-42	-10.08	-10.08005	46,848.00	$4.65 \cdot 10^{-6}$

Table A-72: Unit Test 12: T-approach synthetic volume difference calculation by using Eq. 4.2

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^9\ m^3]$	**Computed Volume** $[10^9\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	56.25	14	3.36	3.36000	0	0
2	2000	49	6.75	1.62	1.62000	0	0
3	1990	42.25	0	0	0.00000	0	0
4	1980	36	-6.25	-1.5	-1.50000	0	0
5	1970	30.25	-12	-2.88	-2.88000	0	0
6	1960	25	-17.25	-4.14	-4.14000	0	0
7	1950	20.25	-22	-5.28	-5.28000	0	0
8	1940	16	-26.25	-6.3	-6.30000	0	0
9	1930	12.25	-30	-7.2	-7.20000	0	0
10	1920	9	-33.25	-7.98	-7.98000	0	0
11	1910	6.25	-36	-8.64	-8.64000	0	0
12	1900	4	-38.25	-9.1800	-9.18000	0	0
13	1890	2.25	-40	-9.6	-9.60000	0	0
14	1880	1	-41.25	-9.9	-9.90000	0	0
15	1870	0.25	-42	-10.08	-10.08000	0	0

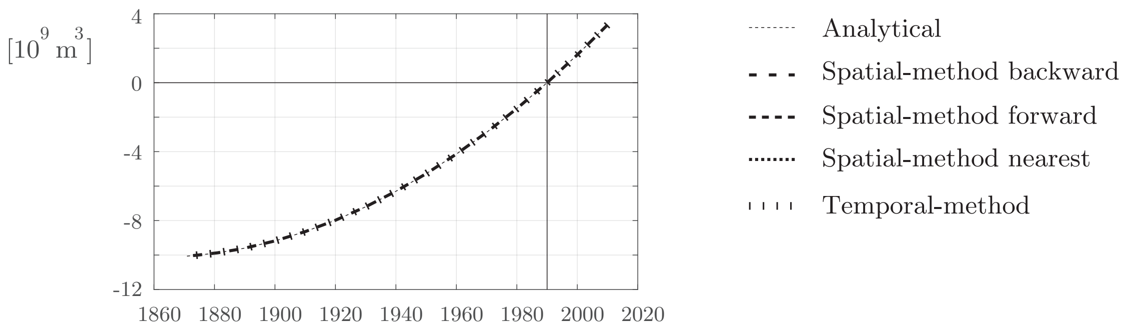

Figure A-27: Unit Test 12: Synthetic volume difference calculation relative to 1990

A-13 Unit Test 13

Table A-73: Unit Test 13

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
13	Diamond Small	Linear	10	No	1

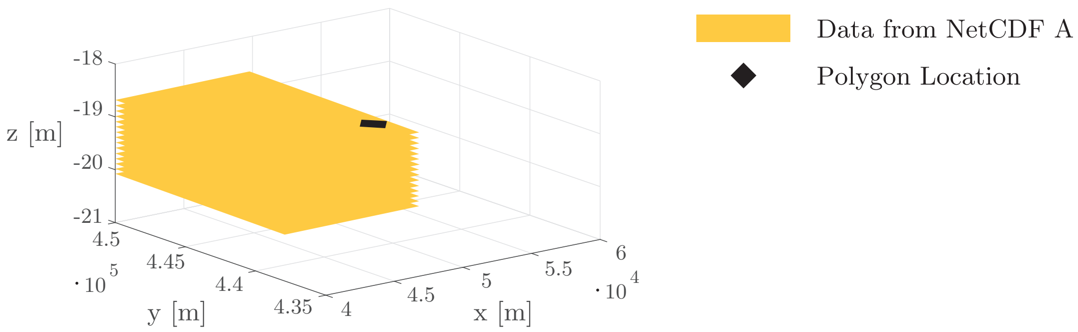

Figure A-28: Unit Test 13

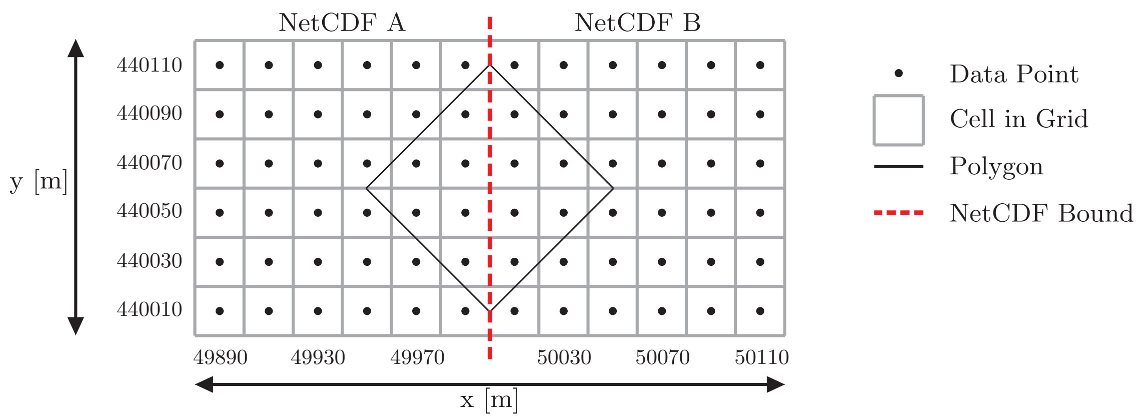

Figure A-29: Unit Test 13: Region of Interest double netCDF diamond

Table A-74: Unit Test 13: Synthetic dataset values

	NetCDF File	**Range** [m]		**Difference** [m]	**Analytical Surface** [m^2]	**Computed Surface** [m^2]
Region of Interest x		49890	49990		5,000	5,000
Region of Interest y		440010	440110			
NetCDF x	A1	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					5,000	5,000

Table A-75: Unit Test 13: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^3\ m^3]$	$[10^3\ m^3]$	$[10^{-3}\ m^3]$	$[-]$
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.80 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.80 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.80 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.70 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.9	$2.60 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	2.0	$1.00 \cdot 10^{-6}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.7	$5.67 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.4	$1.54 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$3.90 \cdot 10^{-6}$
12	1900	-19.0	0.9	4.5	4.50000	2.0	$4.44 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	2.0	$4.00 \cdot 10^{-6}$
15	1870	-18.7	1.2	6	6.00000	5.2	$8.75 \cdot 10^{-7}$

Table A-76: Unit Test 13: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^3\ m^3]$	$[10^3\ m^3]$	$[10^{-3}\ m^3]$	$[-]$
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.80 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.80 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.80 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.70 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.9	$2.60 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	2.0	$1.00 \cdot 10^{-6}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.7	$5.67 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.4	$1.54 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$3.90 \cdot 10^{-6}$
12	1900	-19.0	0.9	4.5	4.50000	2.0	$4.44 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	2.0	$4.00 \cdot 10^{-6}$
15	1870	-18.7	1.2	6	6.00000	5.2	$8.75 \cdot 10^{-7}$

Table A-77: Unit Test 13: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^3\ m^3]$	**Computed Volume** $[10^3\ m^3]$	**Error** $[10^{-3}\ m^3]$	$[-]$
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.80 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.80 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.80 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.70 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.9	$2.60 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	2.0	$1.00 \cdot 10^{-6}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.7	$5.67 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.4	$1.54 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$3.90 \cdot 10^{-6}$
12	1900	-19.0	0.9	4.5	4.50000	2.0	$4.44 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	2.0	$4.00 \cdot 10^{-6}$
15	1870	-18.7	1.2	6	6.00000	5.2	$8.75 \cdot 10^{-7}$

Table A-78: Unit Test 13: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^3\ m^3]$	**Computed Volume** $[10^3\ m^3]$	**Error** $[10^{-3}\ m^3]$	$[-]$
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.81 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.81 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.81 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.69 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.8	$2.54 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	1.9	$9.69 \cdot 10^{-7}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.9	$6.46 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.7	$1.63 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$9.69 \cdot 10^{-7}$
12	1900	-19.0	0.9	4.5	4.50000	1.8	$4.04 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	1.8	$3.30 \cdot 10^{-7}$
15	1870	-18.7	1.2	6	5.99999	5.7	$9.49 \cdot 10^{-7}$

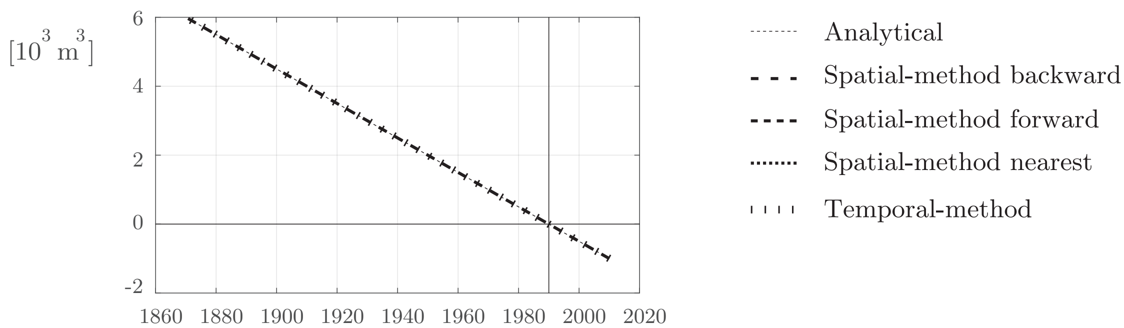

Figure A-30: Unit Test 13: Synthetic volume difference calculation relative to 1990

A-14 Unit Test 14

Table A-79: Unit Test 14

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
14	Diamond Small	Linear	10	No	2

Table A-80: Unit Test 14: Synthetic dataset values

	NetCDF File	Range [m]		Difference [m]	Analytical Surface [m^2]	Computed Surface [m^2]
Region of Interest x		49950	50050		5,000	5,000
Region of Interest y		440010	440110			
NetCDF x	A1	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B1	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					5,000	5,000

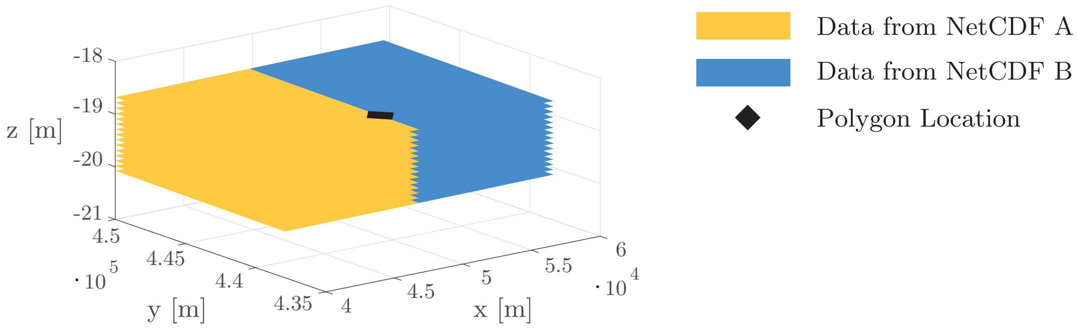

Figure A-31: Unit Test 14

See Figure A-29 for the used polygon.

Table A-81: Unit Test 14: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [$10^3\ m^3$]	**Computed Volume** [$10^3\ m^3$]	**Error** [$10^{-3}\ m^3$]	[$-$]
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.80 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.80 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.80 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.70 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.9	$2.60 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	2.0	$1.00 \cdot 10^{-6}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.7	$5.67 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.4	$1.54 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$3.90 \cdot 10^{-6}$
12	1900	-19.0	0.9	4.5	4.50000	2.0	$4.44 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	2.0	$4.00 \cdot 10^{-6}$
15	1870	-18.7	1.2	6	6.00000	5.2	$8.75 \cdot 10^{-7}$

Table A-82: Unit Test 14: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^3\ m^3]$	$[10^3\ m^3]$	$[10^{-3}\ m^3]$	$[-]$
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.80 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.80 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.80 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.70 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.9	$2.60 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	2.0	$1.00 \cdot 10^{-6}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.7	$5.67 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.4	$1.54 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$3.90 \cdot 10^{-6}$
12	1900	-19.0	0.9	4.5	4.50000	2.0	$4.44 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	2.0	$4.00 \cdot 10^{-6}$
15	1870	-18.7	1.2	6	6.00000	5.2	$8.75 \cdot 10^{-7}$

Table A-83: Unit Test 14: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^3\ m^3]$	$[10^3\ m^3]$	$[10^{-3}\ m^3]$	$[-]$
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.80 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.80 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.80 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.70 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.9	$2.60 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	2.0	$1.00 \cdot 10^{-6}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.7	$5.67 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.4	$1.54 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$3.90 \cdot 10^{-6}$
12	1900	-19.0	0.9	4.5	4.50000	2.0	$4.44 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	2.0	$4.00 \cdot 10^{-6}$
15	1870	-18.7	1.2	6	6.00000	5.2	$8.75 \cdot 10^{-7}$

Table A-84: Unit Test 14: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^3\ m^3]$	**Computed Volume** $[10^3\ m^3]$	**Error** $[10^{-3}\ m^3]$	$[-]$
1	2010	-20.1	-0.2	-1	-1.00000	3.8	$3.81 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-0.5	-0.50000	1.9	$3.81 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	0.5	0.50000	1.9	$3.81 \cdot 10^{-6}$
5	1970	-19.7	0.2	1	0.99999	5.7	$5.69 \cdot 10^{-6}$
6	1960	-19.6	0.3	1.5	1.50000	3.8	$2.54 \cdot 10^{-6}$
7	1950	-19.5	0.4	2	2.00000	1.9	$9.69 \cdot 10^{-7}$
8	1940	-19.4	0.5	2.5	2.50000	0	0
9	1930	-19.3	0.6	3	3.00000	1.9	$6.46 \cdot 10^{-7}$
10	1920	-19.2	0.7	3.5	3.50000	5.7	$1.63 \cdot 10^{-6}$
11	1910	-19.1	0.8	4	4.00000	3.9	$9.69 \cdot 10^{-7}$
12	1900	-19.0	0.9	4.5	4.50000	1.8	$4.04 \cdot 10^{-7}$
13	1890	-18.9	1.0	5	5.00000	0	0
14	1880	-18.8	1.1	5.5	5.50000	1.8	$3.30 \cdot 10^{-7}$
15	1870	-18.7	1.2	6	5.99999	5.7	$9.49 \cdot 10^{-7}$

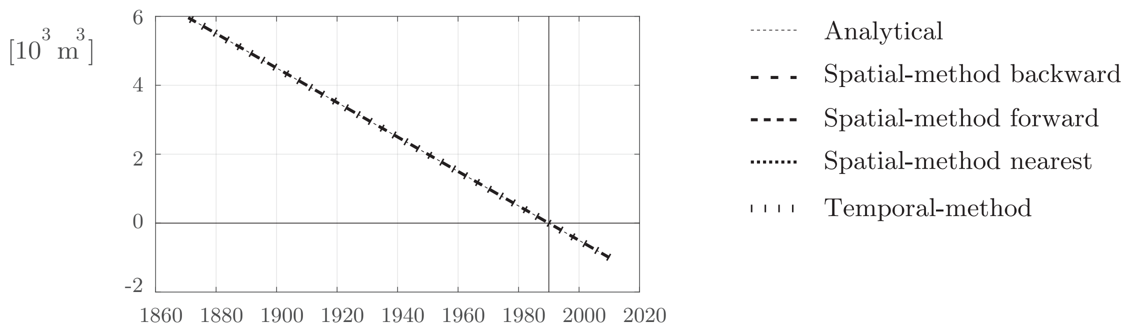

Figure A-32: Unit Test 14: Synthetic volume difference calculation relative to 1990

A-15 Unit Test 15

Table A-85: Unit Test 15

Unit Test	**Region of Interest**	**Trend**	**Time Step** $[y]$	**Gap**	**Amount of NetCDF Files**
15	Triangle Large	Linear	10	No	1

Table A-86: Unit Test 15: Synthetic dataset values

	NetCDF File	**Range** $[m]$		**Difference** $[m]$	**Analytical Surface** $[m^2]$	**Computed Surface** $[m^2]$
Region of Interest x		40,000	60,000		125,000,000	125,000,000
Region of Interest y		437,500	450,000			
NetCDF x	A1	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					62,500,000	62,500,000

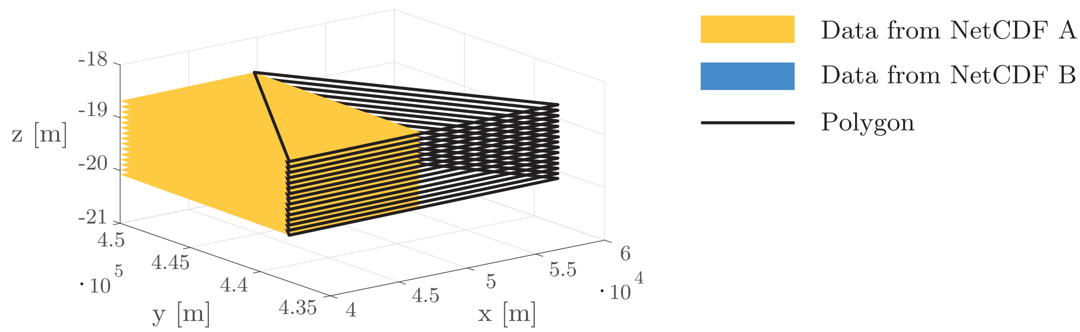

Figure A-33: Unit Test 15

Table A-87: Unit Test 15: S-approach backward volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^6\ m^3]$	**Computed Volume** $[10^6\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	-20.1	-0.2	-12.5	-12.50005	5.0	$4.00 \cdot 10^{-7}$
2	2000	-20.0	-0.1	-6.25	-6.25003	2.5	$4.00 \cdot 10^{-7}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	6.25	6.25003	27.0	$4.32 \cdot 10^{-6}$
5	1970	-19.7	0.2	12.5	12.49999	7.0	$5.60 \cdot 10^{-7}$
6	1960	-19.6	0.3	18.75	18.75000	5.0	$2.67 \cdot 10^{-7}$
7	1950	-19.5	0.4	25	25.00005	49.0	$1.96 \cdot 10^{-6}$
8	1940	-19.4	0.5	31.25	31.25012	23.0	$7.36 \cdot 10^{-7}$
9	1930	-19.3	0.6	37.5	37.50000	1.0	$2.67 \cdot 10^{-8}$
10	1920	-19.2	0.7	43.75	43.74983	17.0	$3.89 \cdot 10^{-7}$
11	1910	-19.1	0.8	50	49.99995	5.0	$1.00 \cdot 10^{-7}$
12	1900	-19.0	0.9	56.25	56.25011	7.0	$1.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	62.5	62.50011	7.0	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	68.75	68.75009	9.0	$1.31 \cdot 10^{-7}$
14	1870	-18.7	1.2	75	74.99974	39.0	$5.20 \cdot 10^{-7}$

Table A-88: Unit Test 15: S-approach forward volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z** $[m]$	**z** $[m]$	**Analytical Volume** $[10^6\ m^3]$	**Computed Volume** $[10^6\ m^3]$	**Error** $[m^3]$	$[-]$
1	2010	-20.1	-0.2	-12.5	-12.50005	5.0	$4.00 \cdot 10^{-7}$
2	2000	-20.0	-0.1	-6.25	-6.25003	2.5	$4.00 \cdot 10^{-7}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	6.25	6.25003	27.0	$4.32 \cdot 10^{-6}$
5	1970	-19.7	0.2	12.5	12.49999	7.0	$5.60 \cdot 10^{-7}$
6	1960	-19.6	0.3	18.75	18.75000	5.0	$2.67 \cdot 10^{-7}$
7	1950	-19.5	0.4	25	25.00005	49.0	$1.96 \cdot 10^{-6}$
8	1940	-19.4	0.5	31.25	31.25012	23.0	$7.36 \cdot 10^{-7}$
9	1930	-19.3	0.6	37.5	37.50000	1.0	$2.67 \cdot 10^{-8}$
10	1920	-19.2	0.7	43.75	43.74983	17.0	$3.89 \cdot 10^{-7}$
11	1910	-19.1	0.8	50	49.99995	5.0	$1.00 \cdot 10^{-7}$
12	1900	-19.0	0.9	56.25	56.25011	7.0	$1.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	62.5	62.50011	7.0	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	68.75	68.75009	9.0	$1.31 \cdot 10^{-7}$
14	1870	-18.7	1.2	75	74.99974	39.0	$5.20 \cdot 10^{-7}$

Table A-89: Unit Test 15: S-approach nearest volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-12.5	-12.50005	5.0	$4.00 \cdot 10^{-7}$
2	2000	-20.0	-0.1	-6.25	-6.25003	2.5	$4.00 \cdot 10^{-7}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	6.25	6.25003	27.0	$4.32 \cdot 10^{-6}$
5	1970	-19.7	0.2	12.5	12.49999	7.0	$5.60 \cdot 10^{-7}$
6	1960	-19.6	0.3	18.75	18.75000	5.0	$2.67 \cdot 10^{-7}$
7	1950	-19.5	0.4	25	25.00005	49.0	$1.96 \cdot 10^{-6}$
8	1940	-19.4	0.5	31.25	31.25012	23.0	$7.36 \cdot 10^{-7}$
9	1930	-19.3	0.6	37.5	37.50000	1.0	$2.67 \cdot 10^{-8}$
10	1920	-19.2	0.7	43.75	43.74983	17.0	$3.89 \cdot 10^{-7}$
11	1910	-19.1	0.8	50	49.99995	5.0	$1.00 \cdot 10^{-7}$
12	1900	-19.0	0.9	56.25	56.25011	7.0	$1.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	62.5	62.50011	7.0	$1.12 \cdot 10^{-7}$
14	1880	-18.8	1.1	68.75	68.75009	9.0	$1.31 \cdot 10^{-7}$
14	1870	-18.7	1.2	75	74.99974	39.0	$5.20 \cdot 10^{-7}$

Table A-90: Unit Test 15: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-12.5	-12.50005	47.7	$3.81 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-6.25	-6.25002	23.8	$3.81 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	6.25	6.25002	23.8	$3.81 \cdot 10^{-6}$
5	1970	-19.7	0.2	12.5	12.49993	71.5	$5.72 \cdot 10^{-6}$
6	1960	-19.6	0.3	18.75	18.74995	47.7	$2.54 \cdot 10^{-6}$
7	1950	-19.5	0.4	25	24.99998	23.8	$9.54 \cdot 10^{-7}$
8	1940	-19.4	0.5	31.25	31.25000	0	0
9	1930	-19.3	0.6	37.5	37.50002	23.8	$6.36 \cdot 10^{-7}$
10	1920	-19.2	0.7	43.75	43.74993	71.5	$1.63 \cdot 10^{-6}$
11	1910	-19.1	0.8	50	49.99995	47.7	$9.54 \cdot 10^{-7}$
12	1900	-19.0	0.9	56.25	56.24998	23.8	$4.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	62.5	62.50000	0	0
14	1880	-18.8	1.1	68.75	68.75002	23.8	$3.47 \cdot 10^{-7}$
15	1870	-18.7	1.2	75	74.99993	71.5	$9.54 \cdot 10^{-7}$

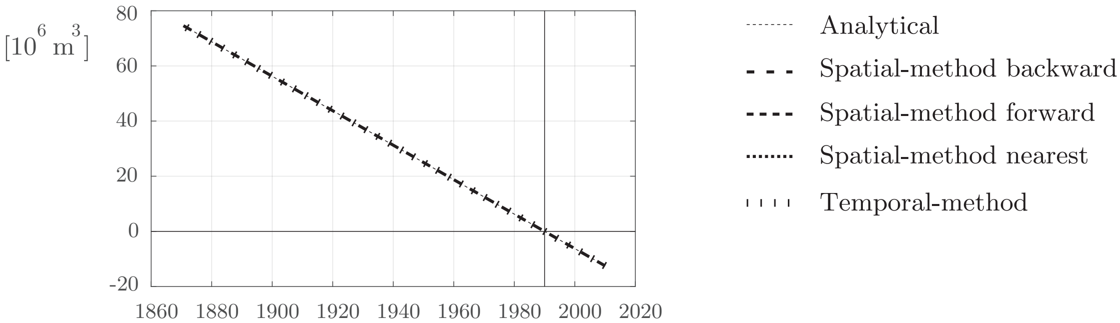

Figure A-34: Unit Test 15: Synthetic volume difference calculation relative to 1990

A-16 Unit Test 16

Table A-91: Unit Test 16

Unit Test	Region of Interest	Trend	Time Step [y]	Gap	Amount of NetCDF Files
16	Triangle Large	Linear	10	No	2

Table A-92: Unit Test 16: Synthetic dataset values

	NetCDF File	Range [m]		Difference [m]	Analytical Surface [m^2]	Computed Surface [m^2]
Region of Interest x		40,000	60,000		125,000,000	125,000,000
Region of Interest y		437,500	450,000			
NetCDF x	A1	40,000	50,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
NetCDF x	B1	50,000	60,000	10,000	125,000,000	125,000,000
NetCDF y		437,500	450,000	12,500		
Net. Surface					125,000,000	125,000,000

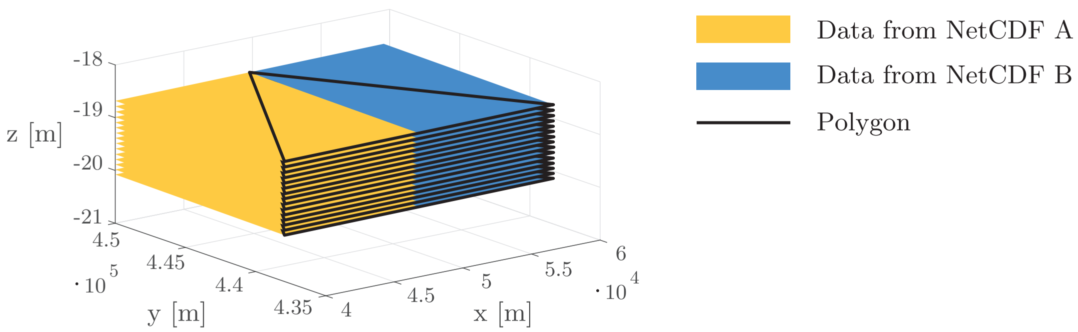

Figure A-35: Unit Test 16

Table A-93: Unit Test 16: S-approach backward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			Calculation relative to 1990				
n	**Year, yr**	**z** [m]	**z** [m]	**Analytical Volume** [10^6 m^3]	**Computed Volume** [10^6 m^3]	**Error** [m^3]	[$-$]
1	2010	-20.1	-0.2	-25	-25.00006	6.0	$2.40 \cdot 10^{-7}$
2	2000	-20.0	-0.1	-12.5	-12.50000	3.0	$2.40 \cdot 10^{-7}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50019	10.0	$8.00 \cdot 10^{-7}$
5	1970	-19.7	0.2	25	25.00000	10.0	$4.00 \cdot 10^{-7}$
6	1960	-19.6	0.3	37.5	37.49999	6.0	$1.60 \cdot 10^{-7}$
7	1950	-19.5	0.4	50	50.00038	22.0	$4.40 \cdot 10^{-7}$
8	1940	-19.4	0.5	62.5	62.50027	26.0	$4.16 \cdot 10^{-7}$
9	1930	-19.3	0.6	75	75.00000	2.0	$2.67 \cdot 10^{-8}$
10	1920	-19.2	0.7	87.5	87.49978	22.0	$2.51 \cdot 10^{-7}$
11	1910	-19.1	0.8	100	100.00000	6.0	$6.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	112.5	112.50019	14.0	$1.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	125	125.00076	38.0	$3.04 \cdot 10^{-7}$
14	1880	-18.8	1.1	137.5	137.50015	46.0	$3.35 \cdot 10^{-7}$
15	1870	-18.7	1.2	150	149.99943	34.0	$2.27 \cdot 10^{-7}$

Table A-94: Unit Test 16: S-approach forward synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-25	-25.00006	6.0	$2.40 \cdot 10^{-7}$
2	2000	-20.0	-0.1	-12.5	-12.50000	3.0	$2.40 \cdot 10^{-7}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50019	10.0	$8.00 \cdot 10^{-7}$
5	1970	-19.7	0.2	25	25.00000	10.0	$4.00 \cdot 10^{-7}$
6	1960	-19.6	0.3	37.5	37.49999	6.0	$1.60 \cdot 10^{-7}$
7	1950	-19.5	0.4	50	50.00038	22.0	$4.40 \cdot 10^{-7}$
8	1940	-19.4	0.5	62.5	62.50027	26.0	$4.16 \cdot 10^{-7}$
9	1930	-19.3	0.6	75	75.00000	2.0	$2.67 \cdot 10^{-8}$
10	1920	-19.2	0.7	87.5	87.49978	22.0	$2.51 \cdot 10^{-7}$
11	1910	-19.1	0.8	100	100.00000	6.0	$6.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	112.5	112.50019	14.0	$1.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	125	125.00076	38.0	$3.04 \cdot 10^{-7}$
14	1880	-18.8	1.1	137.5	137.50015	46.0	$3.35 \cdot 10^{-7}$
15	1870	-18.7	1.2	150	149.99943	34.0	$2.27 \cdot 10^{-7}$

Table A-95: Unit Test 16: S-approach nearest synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-25	-25.00006	6.0	$2.40 \cdot 10^{-7}$
2	2000	-20.0	-0.1	-12.5	-12.50000	3.0	$2.40 \cdot 10^{-7}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50019	10.0	$8.00 \cdot 10^{-7}$
5	1970	-19.7	0.2	25	25.00000	10.0	$4.00 \cdot 10^{-7}$
6	1960	-19.6	0.3	37.5	37.49999	6.0	$1.60 \cdot 10^{-7}$
7	1950	-19.5	0.4	50	50.00038	22.0	$4.40 \cdot 10^{-7}$
8	1940	-19.4	0.5	62.5	62.50027	26.0	$4.16 \cdot 10^{-7}$
9	1930	-19.3	0.6	75	75.00000	2.0	$2.67 \cdot 10^{-8}$
10	1920	-19.2	0.7	87.5	87.49978	22.0	$2.51 \cdot 10^{-7}$
11	1910	-19.1	0.8	100	100.00000	6.0	$6.00 \cdot 10^{-8}$
12	1900	-19.0	0.9	112.5	112.50019	14.0	$1.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	125	125.00076	38.0	$3.04 \cdot 10^{-7}$
14	1880	-18.8	1.1	137.5	137.50015	46.0	$3.35 \cdot 10^{-7}$
15	1870	-18.7	1.2	150	149.99943	34.0	$2.27 \cdot 10^{-7}$

Table A-96: Unit Test 16: T-approach synthetic volume difference calculation by using Eq. 4.1

NetCDF Data			**Calculation relative to 1990**				
n	**Year, yr**	**z**	**z**	**Analytical Volume**	**Computed Volume**	**Error**	
		$[m]$	$[m]$	$[10^6\ m^3]$	$[10^6\ m^3]$	$[m^3]$	$[-]$
1	2010	-20.1	-0.2	-25	-25.00010	95.4	$3.81 \cdot 10^{-6}$
2	2000	-20.0	-0.1	-12.5	-12.50005	47.7	$3.81 \cdot 10^{-6}$
3	1990	-19.9	0	0	0.00000	0	0
4	1980	-19.8	0.1	12.5	12.50005	47.7	$3.81 \cdot 10^{-6}$
5	1970	-19.7	0.2	25	24.99986	95.4	$5.72 \cdot 10^{-6}$
6	1960	-19.6	0.3	37.5	37.49991	143.1	$2.54 \cdot 10^{-6}$
7	1950	-19.5	0.4	50	49.99995	47.7	$9.54 \cdot 10^{-7}$
8	1940	-19.4	0.5	62.5	62.50000	0	0
9	1930	-19.3	0.6	75	75.00005	47.7	$6.36 \cdot 10^{-7}$
10	1920	-19.2	0.7	87.5	87.49986	143.1	$1.63 \cdot 10^{-6}$
11	1910	-19.1	0.8	100	99.9991	95.4	$9.54 \cdot 10^{-7}$
12	1900	-19.0	0.9	112.5	112.49995	47.7	$4.24 \cdot 10^{-7}$
13	1890	-18.9	1.0	125	125.00000	0	0
14	1880	-18.8	1.1	137.5	137.50005	47.7	$3.47 \cdot 10^{-7}$
15	1870	-18.7	1.2	150	149.99986	143.1	$9.54 \cdot 10^{-7}$

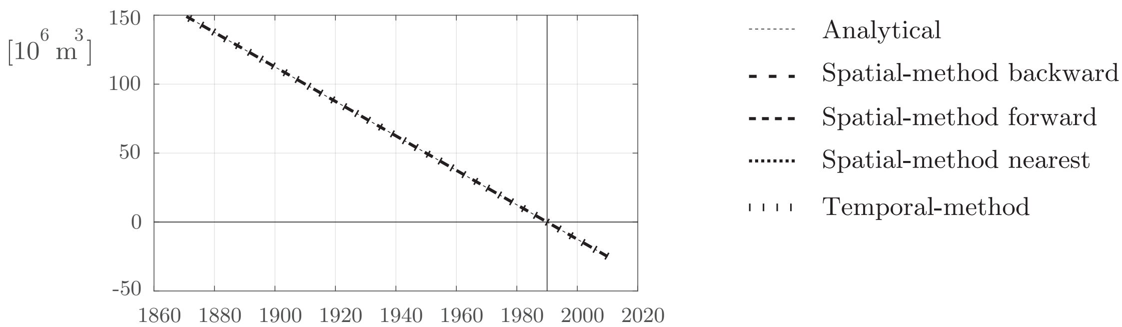

Figure A-36: Unit Test 16: Synthetic volume difference calculation relative to 1990

Appendix 3: Unit Test Matlab Code

Listing A-2: Unit Test Creator *unittestcreator.m*

```
clearvars; close all; clc

%% Input
tn = 10;          % Unit test number
%I0.Filename = 'vaklodingenKB115_3938.nc';    % Filename of to-be-used file
I0.Filename = 'vaklodingenKB116_3938.nc';

%%
yr = 15;         % Amount of different years

I0 = ncinfo(I0.Filename);

I = I0;
I.Filename = strrep(I.Filename,'.nc',['_unit',num2str(tn),'.nc']);

%[I.Dimensions.Name]
I.Dimensions(3).Length = yr;

%{I.Variables.Name}
I.Variables(6).Size(3) = yr;
I.Variables(6).Dimensions(3).Length = yr;

I.Variables(7).Size = yr;
I.Variables(7).Dimensions.Length = yr;

I.Variables(8).Size(3) = yr;
I.Variables(8).Dimensions(3).Length = yr;

ncwriteschema(I.Filename,I);

%%
     if tn == 1 || tn == 2
        z_final = -20.1;
        z_start = -18.7;
        d = linspace(z_start,z_final,yr);
     elseif tn == 3 || tn == 4
        d(1:3) = [-18.7 -18.71 -18.72];
        d(4:6) = [-19.0 -19.01 -19.02];
        d(7:9) = [-19.3 -19.31 -19.32];
        d(10:12) = [-19.6 -19.61 -19.62];
        d(13:15) = [-19.9 -19.91 -19.92];
     elseif tn == 9 || tn == 10
        t_final = 2010;
        t_start = 1870;
        d=ones(yr,1);
        yrs = linspace(t_start,t_final,yr);
        for m=1:yr
%             d(m) = ((yrs(m)-1700)./-100).^3;
              d(m) = (m./2).^2;
        end
     end
```

```
f = ones(yr,1);
g = ones(yr,1);

z = ncread(I.Filename,'z');
time = ncread(I.Filename,'time');

ncwrite(I.Filename,'z', repmat(1,[I.Variables(6).Size])); % overwrite 3D
ncwrite(I.Filename,'time', ones(1,yr)); % overwrite 3D

lat = ncread(I0.Filename,'lat');
lon = ncread(I0.Filename,'lon');
ncwrite(I.Filename,'lat', lat); % overwrite 3D
ncwrite(I.Filename,'lon', lon); % overwrite 3D

x = ncread(I0.Filename,'x');
y = ncread(I0.Filename,'y');
ncwrite(I.Filename,'x', x); % overwrite 3D
ncwrite(I.Filename,'y', y); % overwrite 3D

crs = ncread(I0.Filename,'crs');
ncwrite(I.Filename,'crs', crs); % overwrite 3D

    for n=1:yr
        ncwrite(I.Filename,'z', repmat(d(n),[I.Variables(6).Size(1:2) 1]),[1 1 n])

        % Gat in midden tussen x=[200:300] y=[250:375]
        if tn == 10
            ncwrite(I.Filename,'z', repmat(nan,[100 125 1]),[200 250 n])
        end

%         f(n,1) = round(datenum((yrs(n)*-100),1,1));
        f(n,1) = datenum(yrs(n),1,1);
        g(n,1) = datenum(f(n,1)-719529);
        ncwrite(I.Filename,'time', g, 1)
        ncwrite(I.Filename,'isource', n, [1 1 n])
    end

if tn == 1 || tn == 2 || tn == 3 || tn == 4
%     for n=1:yr
%         ncwrite(I.Filename,'z', repmat(d(n),[I.Variables(6).Size(1:2) 1]),[1 1 n])
%
%         Gat in midden tussen x=[200:300] y=[250:375]
%         if tn == 2 || tn == 4
%             ncwrite(I.Filename,'z', repmat(nan,[100 125 1]),[200 250 n])
%         end
%
%         f(n,1) = round(datenum((d(n)*-100),1,1));
%         g(n,1) = datenum(f(n,1)-719529);
%         ncwrite(I.Filename,'time', g, 1)
%         ncwrite(I.Filename,'isource', n, [1 1 n])
%     end
elseif tn == 7 || tn == 8

    mm = 625;
    qq = 500;
%
% for n=1:yr
%   for m=1:mm
%      ncwrite(I.Filename,'z', repmat(((-(y-1870).^3)./10.^5).*m./mm,[1 1 1]),[11 m n])
%   end
% end
%
    % m loopt van 1 t/m 625
    % dd moet 625 lang zijn en gevuld worden door (n./m).*625

    if tn == 8
```

```
        ncwrite(I.Filename,'z', repmat(nan,[100 125 1]),[200 250 n])
    end

    f(n,1) = round(datenum((d(n)*-100),1,1));
    g(n,1) = datenum(f(n,1)-719529);
    ncwrite(I.Filename,'time', g, 1)
    ncwrite(I.Filename,'isource', n, [1 1 n])
end

z = ncread(I.Filename,'z');whos z
crs = ncread(I.Filename,'crs');whos crs
x = ncread(I.Filename,'x');whos x
time = ncread(I.Filename,'time');whos time

% zI0 = ncread(I0.Filename,'z');whos z
% timeI0 = ncread(I0.Filename,'time');whos time
crsI0 = ncread(I0.Filename,'crs');whos crs
```