

# Probabilistic Characterization of a Soft Scandinavian Clay Supporting a Light Quay Structure

Anders Beijer LUNDBERG<sup>a</sup> and Yajun LI<sup>b</sup>

<sup>a</sup>*ELU Konsult, Sweden*

<sup>b</sup>*Department of Geoscience and Engineering, Delft University of Technology, The Netherlands*

**Abstract.** A full geotechnical risk analysis includes site investigation, model simplification and analysis. The site characterization forms the basis for the choice of the soil parameters used in the subsequent analysis, but the soil data is normally analyzed with crude methods that do not include the effect of the soil variability. A case study is here presented of soil characterization for later analysis of the stability of a light quay structure constructed on very soft Scandinavian clay. The field data consists of undrained shear strength measurements from field vane tests (FVT) and cone penetration tests (CPT), carried out at different locations along the structure. The measurements show a relatively large variation of the undrained shear strength, making a statistical characterization of the soil properties helpful for a later analysis of the reliability of the structure. In the current paper, the field measurement data (i.e. the tip resistance from CPT tests) are analyzed to obtain the correlation distance of the soil strength. The two types of measurements are combined to get an average of the undrained shear strength, while the spatial statistics are obtained from CPT measurements.

**Keywords.** probabilistic site characterization, regression analysis, scale of fluctuation, spatial variability, site investigation

## 1. Introduction

Geotechnical design includes assessment of soil parameters, simplifications of field conditions, and modelling of the soil structure by analytical or numerical methods (Wroth, 1984). In the case of impermeable clay slopes during short-term loading, the undrained shear strength forms the basis for the analysis. Calculations methods include analytical methods, e.g. Bishop (1955), and advanced numerical methods, e.g. Griffiths and Lane (1999). More advanced reliability-based analyses are currently used, and the trend of taking account of soil spatial variability is likely to continue, e.g. Li et al. (2013), Li and Hicks (2014), Low and Phoon (2015). This will ensure increased reliability of geotechnical structures, but it is therefore important to simultaneously develop methods to characterize the variability of the soil properties, and to assess these methods on different cases of natural soil. Improved methods for parameter selection need to be tested on a representative sample of different cases of natural soils for the methods to gain acceptance, and should preferably be related

to empirical experience in site investigation and parameters choice, which gives improved confidence in the method (Schnaid, 2009).

In the current paper, the variability of soft Scandinavian clay is characterized from FVT and CPT measurements conducted in soft clay covered with fill. The average undrained shear strength is evaluated from field vane tests (FVT) and the CPT tests, and the variability of the undrained shear strength is subsequently estimated from the CPT measurements. The advantage of the current methodology is that the average value of the undrained shear strength is measured by a locally accepted method which has been proved to give accurate results for soft soils, and that the variability is analyzed from independent measurements carried out at the same location, with CPTs, due to the lower measurement density in the FVTs, compared to the CPT measurements.

## 2. Shear Strength of Natural Clay

Analysis of the stability of a clay slope is normally based on the undrained shear strength of the natural clay, e.g. Griffiths and Lane (1999). In varved clays this is a large simplification, since thin seams of silt or more permeable layers could result in locally drained conditions. But even this simplification results in a complex soil response, depending on the loading conditions (Schnaid, 2009, Wroth, 1984). Natural clay displays a very intricate behavior, including viscous and strain-rate dependent response, and both the soil variability and the anisotropy of natural soil are often not regarded in most simplified models to ensure that the model is manageable in routine design (Schnaid, 2009).

This is especially true for the soft Scandinavian clays. The undrained shear strength of natural Scandinavian soft clay is anisotropic and strain-rate dependent, and therefore depends on the testing methods (Wroth, 1984). A full analysis of the soil behavior requires triaxial tests in compression and extension, as well as direct shear tests to present the full behavior. Special tests are needed to analyze the effect of strain rate, Lunne et al. (1997).

The advanced models of the soil behavior represent the natural soil behavior; however, for highly variable soils, it is not obvious how the undrained shear strength in respective shear mode should be chosen. In-situ measurements display a considerable scatter owing to these effects, (Wroth, 1984).

The current paper discusses routine in-situ measurements where measured value of the natural clay shows large scatter, and where a statistical characterization clarifies the variability of measured soil parameters.

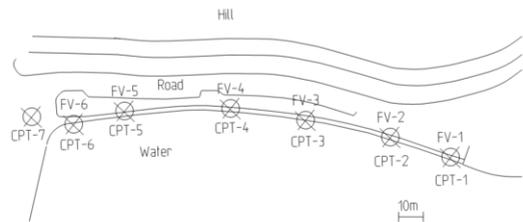
## 3. Field Measurements

### 3.1. Vane Shear Test

Field measurements were carried out for the assessment of the stability of a clay slope in the vicinity of Stockholm. A large number of such measurements are collected in the municipal geo-archive (Stockholm Geo-archive, 2014). The site consisted of an older quay where an 8 m clay

layer was covered with around 1 m fill. On the fill layer, a road was located, shown in Figure 1. The quayside was stabilized with various older wooden and concrete structures. The site is typical for the soil conditions where Scandinavian clays occur, as specified in Bjerrum (1967), but in this urban area the soil was altered from the natural state. The soil conditions exhibited disturbance from earlier fill and construction in the area, as well as the vicinity to the sea, which meant that layers of various soils had been deposited in the soil.

The aim of the site investigation was to evaluate the soil profile and obtain strength characteristics of the soil for a subsequent stability analysis. The measurements consisted of soil sampling for routine tests, rock sounding to establish the rock floor beneath, and field vane tests (FVT) and cone penetration tests (CPT) for establishing a design value for the undrained shear strength. The field vane tests followed the field test standard of Eurocode 7 (Eurocode 7, 1997; Schnaid, 2009). The locations for the measurements are shown in Figure 1. The field vane tests are denoted FV-1 to FV-6.



**Figure 1.** Locations of the field vane tests (FV) and cone penetration tests (CPT) along the quay.

The water content at liquid limit (WL) and the natural water content (WN) were analysed from samples retrieved at locations FV in Figure 1, following EC 7 and are shown in Figure 2a. The natural water content is typically higher than that at liquid limit in these clays, and the soil response is dependent on the structure, or fabric, of the undisturbed soil (Bjerrum, 1967).

Figure 2a shows a considerable variation of the water content at liquid limit (WL) and the natural water content (WN) of the soil. This is believed to be a result of the varved structure of the clay observed in the samples, and the disturbance from the fill, which had been assembled over a longer time period. Especially

large difference between the liquid limit (WL) and the natural water content (WN) in FV-3 is probably the result of the varved clay structure, in which silt and sulphuric soil were observed in the lower soil samples. The sample is specifically shown here to exhibit the large sample variety that is typically not found in more homogenous clay depositions (Schnaid, 2009; Wroth, 1984).

The field vane test is the preferred site investigation method in soft clay in Sweden, since a considerable experience has been built up around the method (Kjellman, 1951). A detailed description of the method is found in Chandler (1988). Among the reasons for the reliability of the FVT is that the strain rate has a large effect on the undrained shear strength of the soft Scandinavian clays, and the relatively controlled test conditions assure that the rate effects are considered in a controlled and repeatable fashion (Schnaid, 2009).

The low undrained shear strength and the high excess pore water pressures that arise during cone penetration make the analysis of the undrained shear strength relatively more sensitive to variability in equipment and calibration (Lunne et al, 1997). The field vane test is therefore among the primary methods to assess the undrained shear strength.

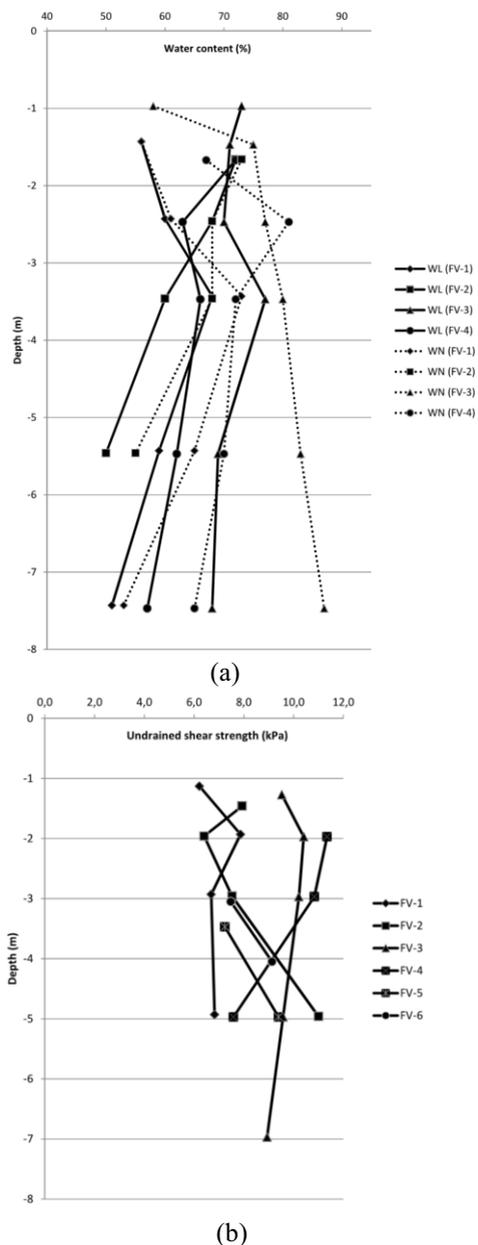
Measurements of the undrained shear strength from the site (Figure 1) are shown in Figure 2b. The measurements were corrected for the liquid limit (WL) according to Eurocode 7 (Eurocode 7, 1997). The measurements in Figure 2a and Figure 2b show similar variability.

The average value of the undrained shear strength is 8.6 kPa, but there is a significant variation along the depth of the soil stratum, which means that the exact average value should be augmented with the variability. The large variability of the measurements makes a reliability analysis sensible, since the lower values of the undrained shear strength are considerably lower than the average value.

### 3.2. Cone Penetration Test

7 CPT tests were carried out on the site (Fig. 1). The soundings ranged from -2 m to -9 m for the clay. The CPT data gathered consists of cone tip resistance ( $q_c$ , MPa), sleeve friction ( $f_s$ , kPa) and pore-water pressure ( $u$ , kPa). The

measurements were recorded at vertical intervals of 0.02 m. Despite the aforementioned issues when conducting a CPT test, the relatively intensive testing data points make a statistical analysis possible.



**Figure 2.** (a) water content at liquid limit (WL) and natural water content (WN); (b) Measurements of the undrained shear strength from FVT.

### 4. Statistical Characterization

#### 4.1. Regression Analysis

The  $q_c$  readings are analyzed in this paper, following standard scientific practice (e.g. Fenton, 1999b). As this paper mainly focuses on analyzing the variability in the clay layer, the CTP recordings are shortened by removing the top fill layer recordings and/or the bottom till layer recordings.

In this section, a regression analysis is first carried out to determine whether or not a statistically significant trend with depth is present. As for random field simulation, it is required that the data are stationary. This is usually achieved by removing the trend from the non-stationary data (Wickremesinghe and Campanella, 1993).

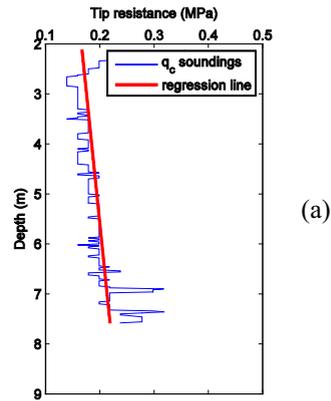
The regression analysis is restricted to a linear trend due partly to the observation that cone bearing exhibit linear trends (Wickremesinghe and Campanella, 1993) and partly to its simplicity and sound physical basis (Fenton, 1999b).

For each of the individual CPT profile, an intercept and a slope is identified,

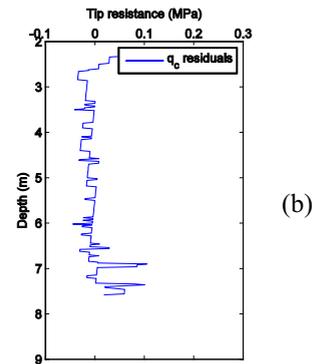
$$q_c = a + bz + \varepsilon \tag{1}$$

Where  $a$  is the mean value of  $q_c$  at depth  $z = 0$ ,  $b$  is the slope,  $\varepsilon$  is a residual random component with zero mean. One typical regression analysis result is shown in Fig. 3a. It is seen that a linear trend exists in the clay layer. Fig. 3b shows the residuals by detrending the original data. Fig. 3c shows the histogram of the detrended data, it is seen that a normal distribution fits the data reasonably well.

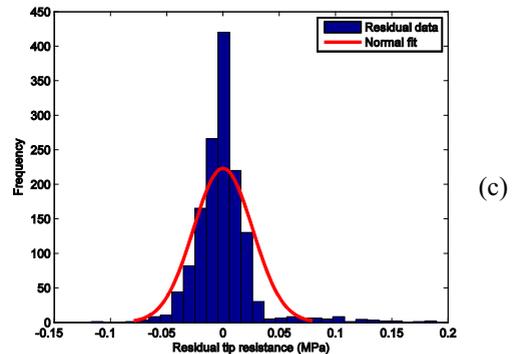
The significance of the slope is tested against the null hypothesis  $b = 0$  at  $\alpha = 5\%$  significance level. For all the 7 cases, a depth trend is present. After detrending the data, the correlation structure is analyzed to find out the scale of fluctuation, which is defined as the distance within which soil properties show significant correlation and beyond which little correlation exists (Vanmarcke, 1983).



(a)



(b)



(c)

**Figure 3.** (a)  $q_c$  variation with depth and linear trend; (b)  $q_c$  residuals with depth; and (c) histogram of and normal fit to the residuals for CPT 5.

#### 4.2. Correlation Structural Analysis

The sample covariance is defined by:

$$\hat{C}(\tau_j = j\Delta\tau) = \frac{1}{n-j} \sum_{i=1}^{n-j} (x_i - \hat{\mu}_X)(x_{i+j} - \hat{\mu}_X) \quad (2)$$

where  $j = 0, 1, \dots, n-1$ ,  $\tau_j$  is the lag distance between data points  $x_i$  and  $x_{i+j}$ ,  $\hat{\mu}_X$  is the estimated mean,  $n$  is the number of data points.  $\Delta\tau = 0.02$  m in this case.

The correlation function is then

$$\hat{\rho}(\tau_j) = \hat{C}(\tau_j) / \hat{C}(0) \quad (3)$$

where  $\hat{C}(0) = \hat{\sigma}_X^2$  and  $\hat{\sigma}_X^2$  is the estimated variance (Fenton, 1999b). Note that a biased covariance estimator (in equation 2, use  $1/n$  outside the sum sign instead of  $1/(n-j)$ ) could have been used to calculate the sample correlation as in Fenton (1999a) and Jaksa et al. (1999). However, this has little effect on the estimated scales of fluctuation in this study. Therefore, the unbiased estimator (equation 2) is used despite the popularity of the biased estimator in time series analysis (Priestley, 1981).

The correlation function used to fit the sample correlation is of an exponential form,

$$\rho = e^{-\left(\frac{2\tau}{\theta}\right)} \quad (4)$$

where  $\tau$  is lag distance and  $\theta$  is the scale of fluctuation.

A typical fit corresponding to Figure 3b is shown in Fig. 4 with the corresponding sample correlation. The scale of fluctuation in this case was found to be 0.3 m.

The large fluctuation at the larger lags is apparent in Figure 4. This is because the pair numbers are smaller at large lags, therefore the accuracy for larger lags is not as good as that for smaller lags. In fact, the maximum number of lags is suggested to be  $n/4$  in equation (2) ( $\max(j) = n/4$ , Box and Jenkins, 1970). When one looks at these lags that are less than 2.0 m, the exponential function with  $\theta = 0.3$  m fits the sample correlation function quite well.

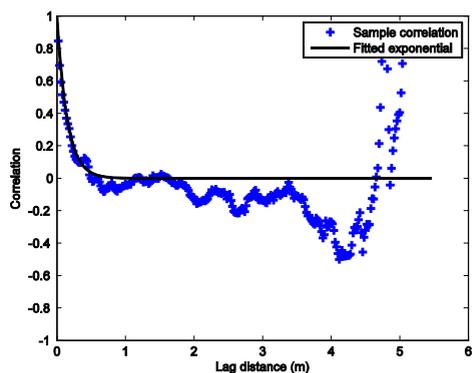


Figure 4. Correlation functions for CPT 5.

The results for all the CPT profiles are listed in Table 1. The scale of fluctuation is in the range of 0.1-0.3 m, with a mean of 0.2 m and a coefficient of variation of 0.48. The scales of fluctuation lie in the range of 0.13-8.6 m (Li and Lee, 1991) and is consistent with Jaksa et al. (1999) and Wickremesinghe and Campanella, (1993).

Table 1. Results of analyses on detrended residuals of  $q_c$  measurements

CPT No.	Depth (m)		Number of data	Scale of fluctuation (m)
	Min	Max		
1	2.02-8.14		307	0.3
2	2.28-5.16		145	0.15
3	2.10-5.18		155	0.15
4	2.04-6.16		207	0.1
5	2.12-7.58		274	0.3
6	2.02-7.56		278	0.3
7	2.02-3.46		73	0.1
Mean				0.2
Standard deviation				0.096
Coefficient of Variation				0.48*

\* units not applicable

It should be noted that the spatial correlation structure of the undrained shear strength and  $q_c$  is identical due to the commonly used linear relationships suggested by Lunne et al. (1997). Therefore, the results are also representative of the undrained shear strength.

#### 4.3. Statistics of Undrained Shear Strength

The undrained shear strength calculated from the tip resistance from CPT data (based on the transformation model suggested by Lunne et al. (1997)) has a mean of 8.05 kPa (a bit smaller

than 8.60 kPa from FVT) and a standard deviation of 2.58 kPa, which corresponds to a coefficient of variation of 0.32, which lies in the range of 0.1–0.5 as reported by Lee et al. (1983), Phoon and Kulhawy (1999) and Hicks and Samy (2002). Note that the statistical analysis in this study does not include the uncertainties involved in the transformation model and in random measurement error.

## 5. Conclusion

Advanced reliability analysis methods require extensive input data. This is currently not available from routine site investigations in Scandinavia, which prevents adaption of improvements of the reliability analysis. In the current paper field measurements from a site investigation at an urban location in the eastern part of Sweden has been analyzed, based on empirical experience, to characterize the variability of the undrained shear strength. Characterization of the soil variability improves the possibility to analyze the structure reliability, since undrained shear strength was low and displayed a large variation. The scale of fluctuation is found to be 0.2 m on average. The current paper presents a modest step towards the use of more advanced reliability analysis methods, and the results presented will be used in a subsequent slope stability analysis taking account of the spatial variability, the result of which will be used to find a characteristic value for the undrained shear strength satisfying the requirement of Eurocode 7.

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