FORM-FINDING OF BRANCHING STRUCTURES SUPPORTING FREEFORM ARCHITECTURAL SURFACES

Alex Kouwenhoven

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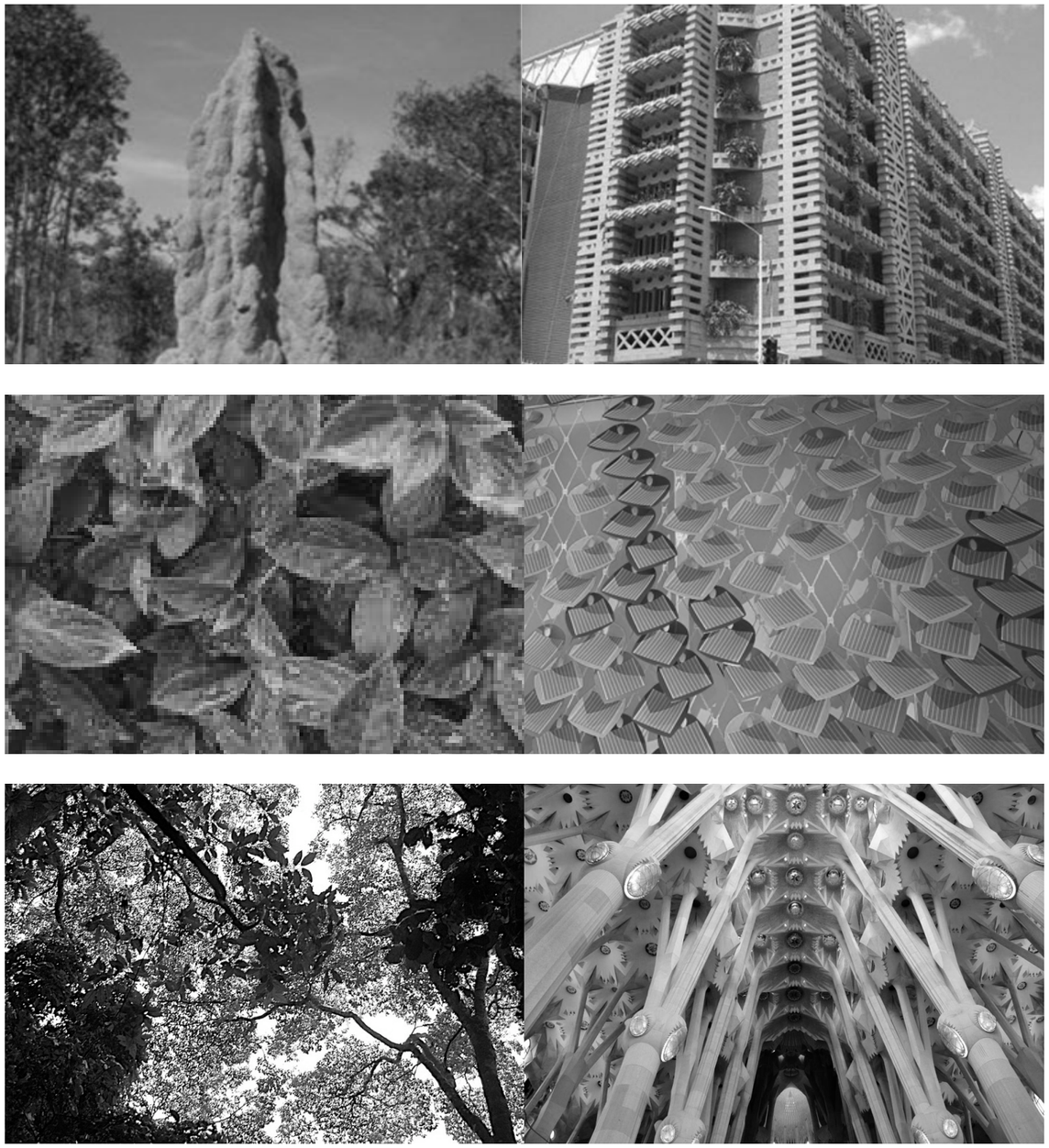
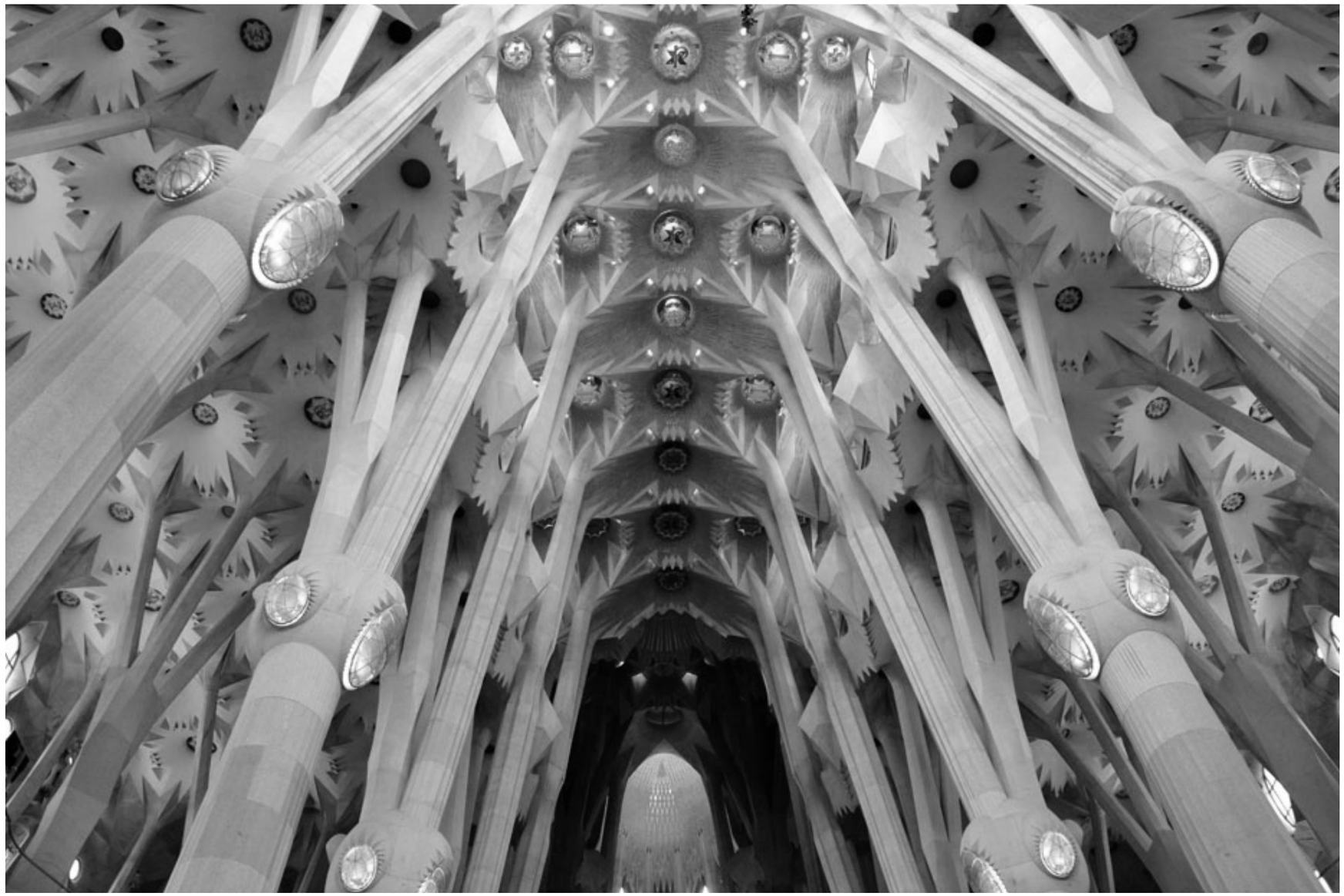
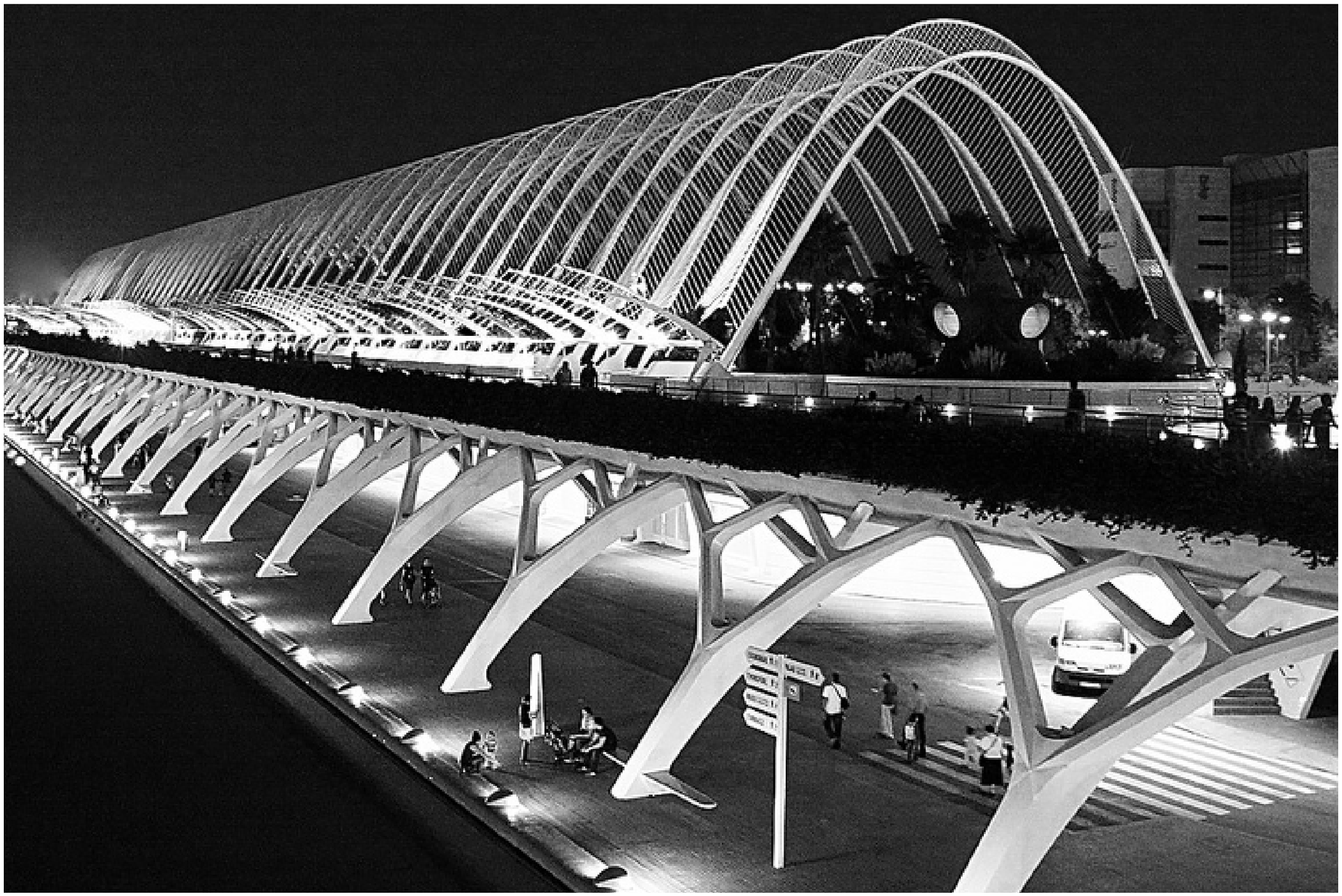
"You could look at nature as being like a catalog of products,
and all of those have benefited from a 3.8 billion year research and development period.
And given that level of investment, it makes sense to use it."

Michael Pawlyn

Background



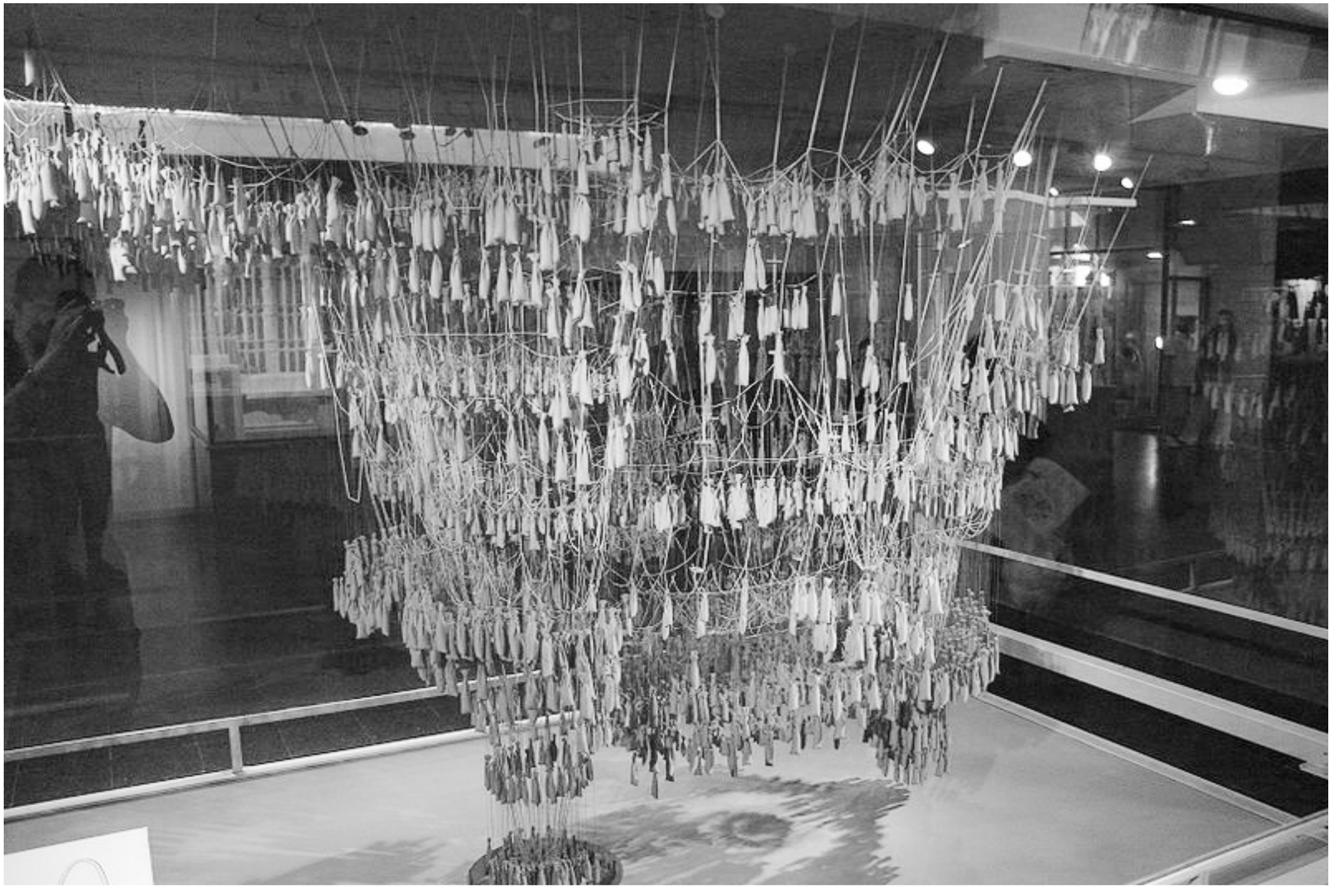


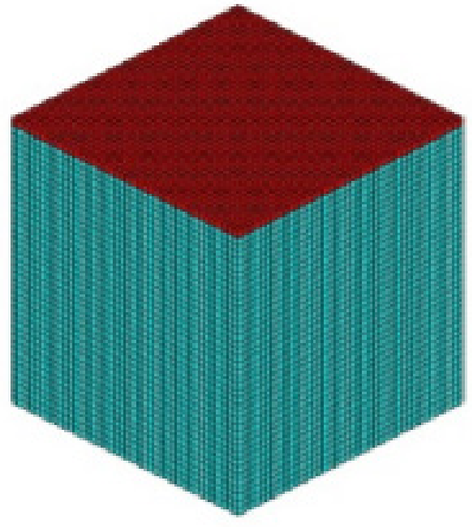


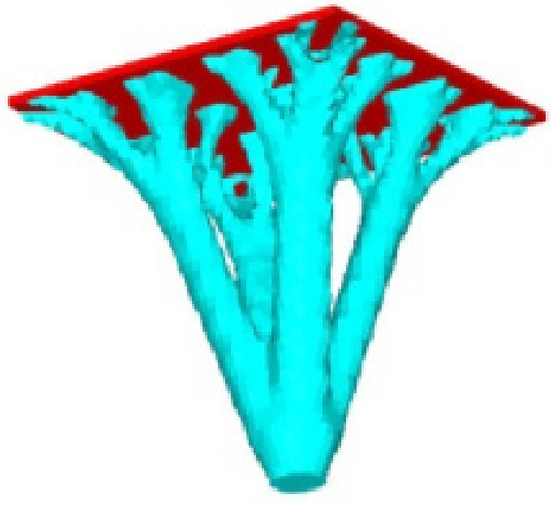
'There is no better structure than the trunk of a tree or a human skeleton'

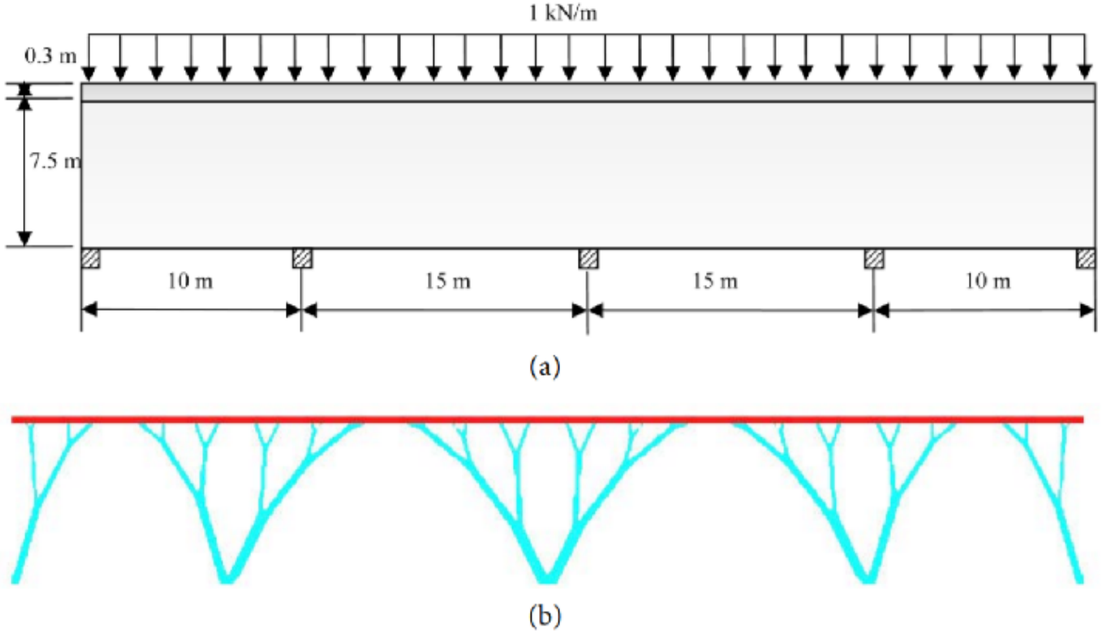
Antonio Gaudí

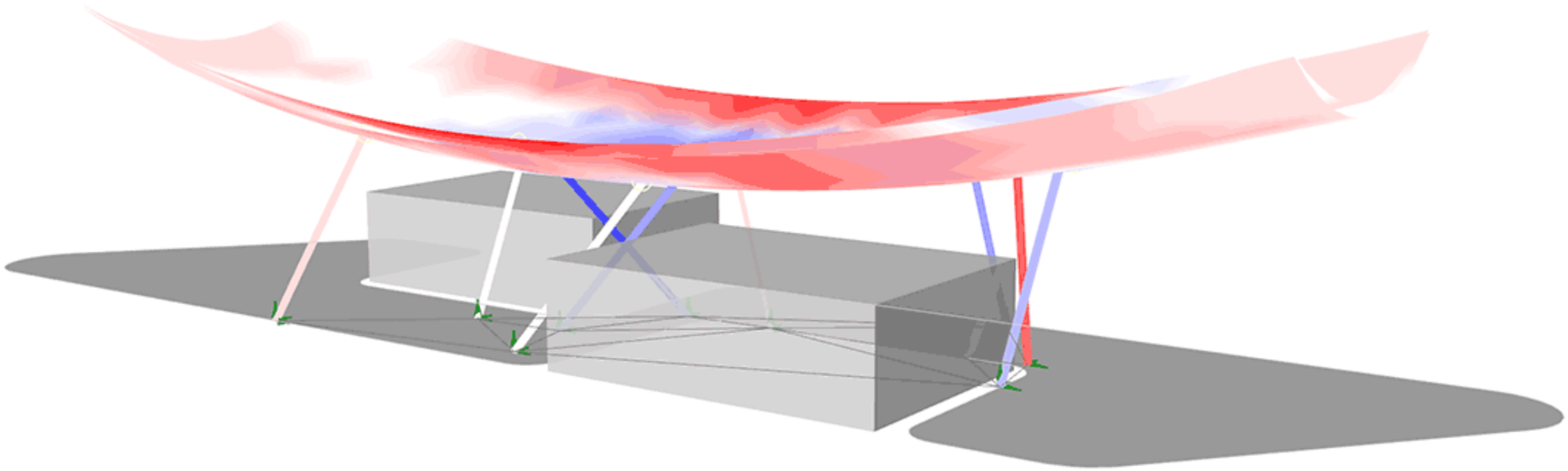
The main issue about these structures is finding the most reasonable form to solve the problem of actual project.








1 kN/m
0.3 m
7.5 m
10 m
15 m
15 m
10 m
(a)
(b)



Problem statement

Problem statement

There is no analytical method of form-finding the optimal branching structure as a support of freeform architectural expressions.

Research question

"How can we design structurally efficient three-dimensional branching structures as a support of freeform architectural surfaces?"

I Research Framework | II Theoretical Framework | III Calculation of structures | IV Design problem | V Design solution | VI Design | VII Conclusion

II

Research question

"How can we design structurally efficient three-dimensional branching structures as a support of freeform architectural surfaces?"

Research question

"How can we design ***structurally efficient*** *three-dimensional branching structures as a support of freeform architectural surfaces?"*

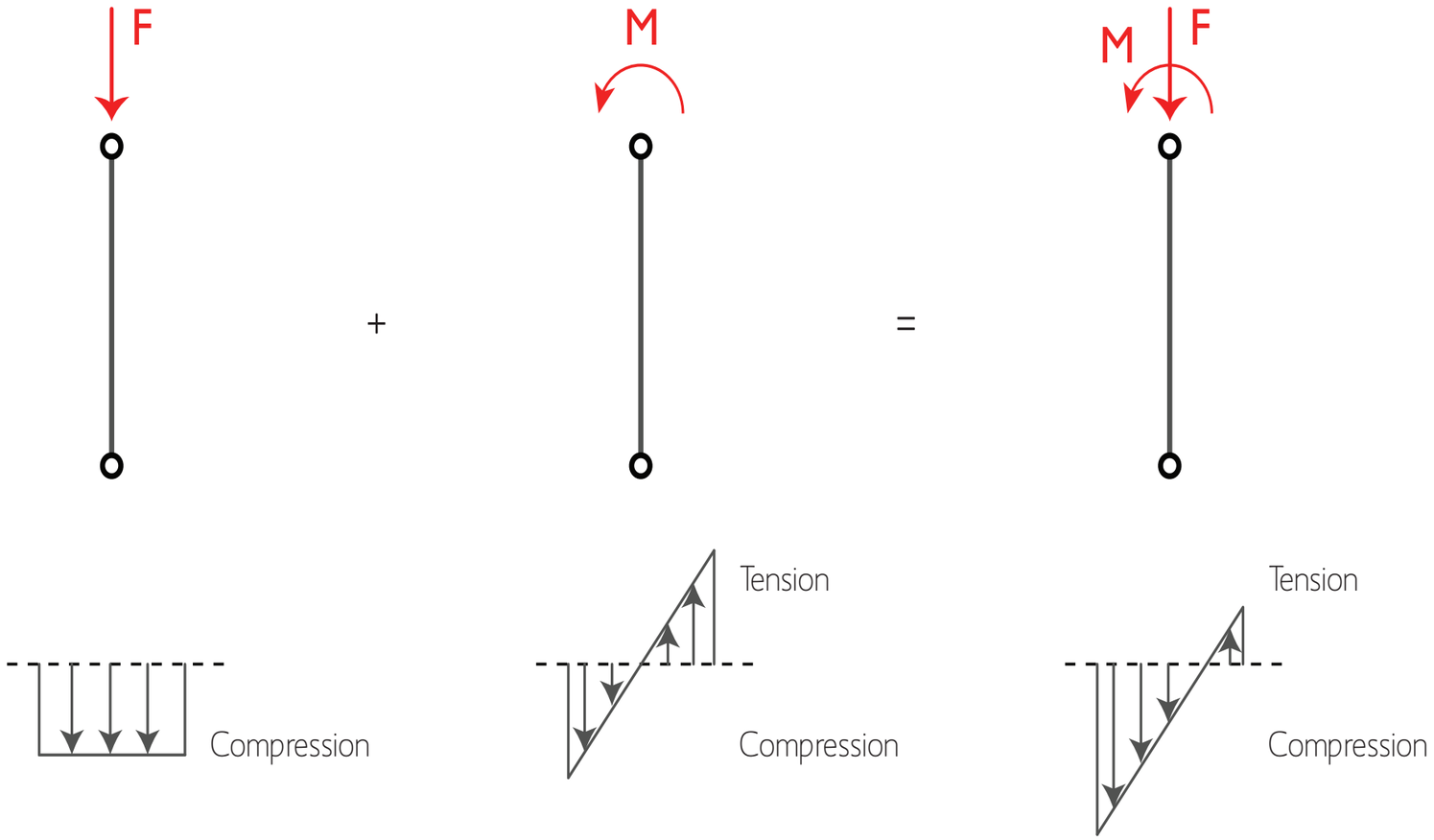
Research question

"How can we design structurally efficient ***three-dimensional branching structures*** *as a support of freeform architectural surfaces?"*

Research question

"How can we design structurally efficient three-dimensional branching structures as a support of freeform architectural surfaces?"


PKP.PL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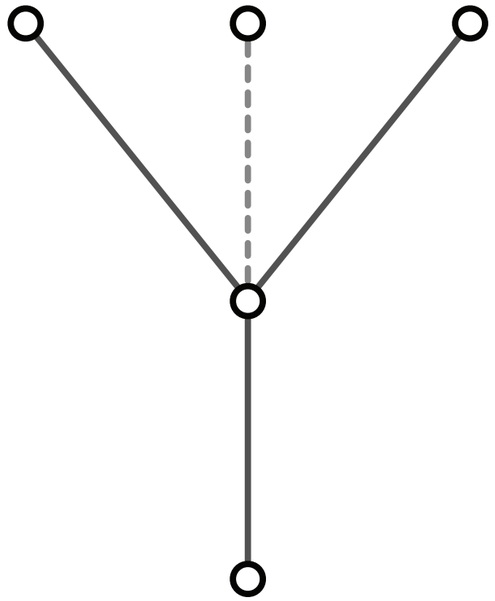
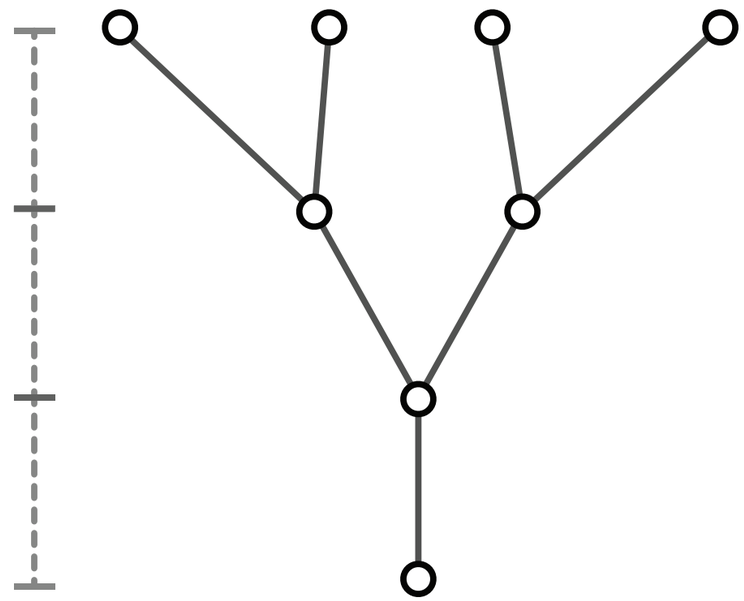
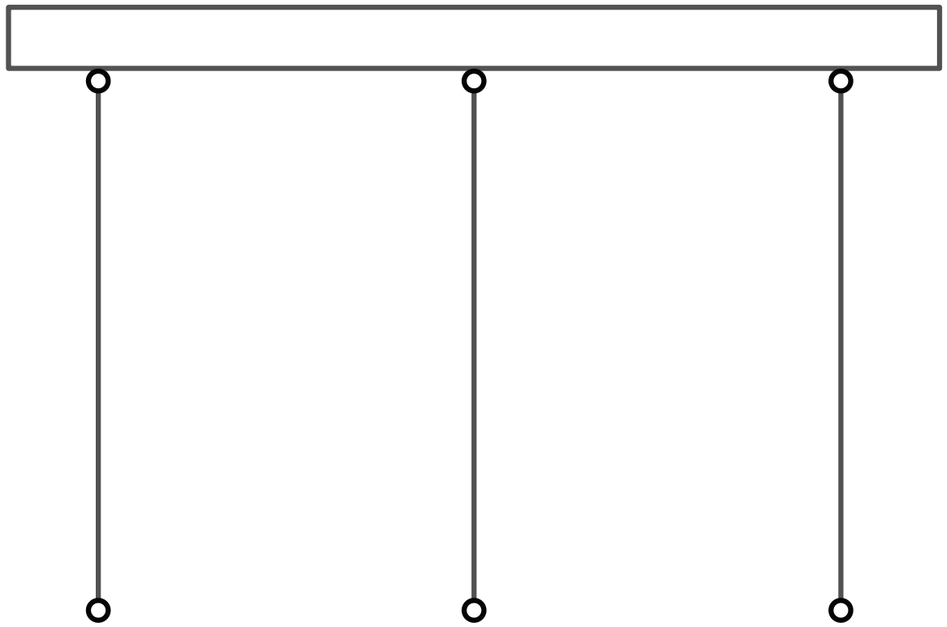


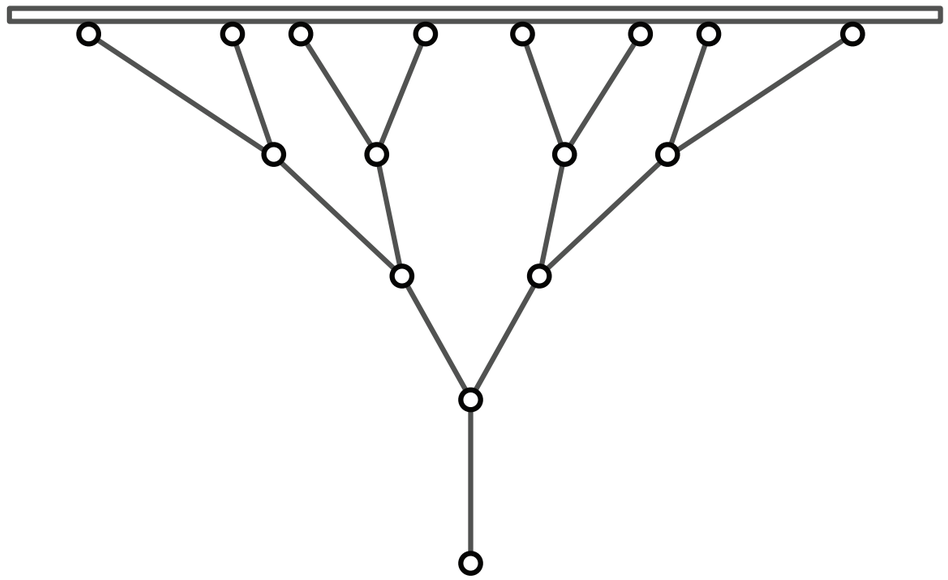
branches



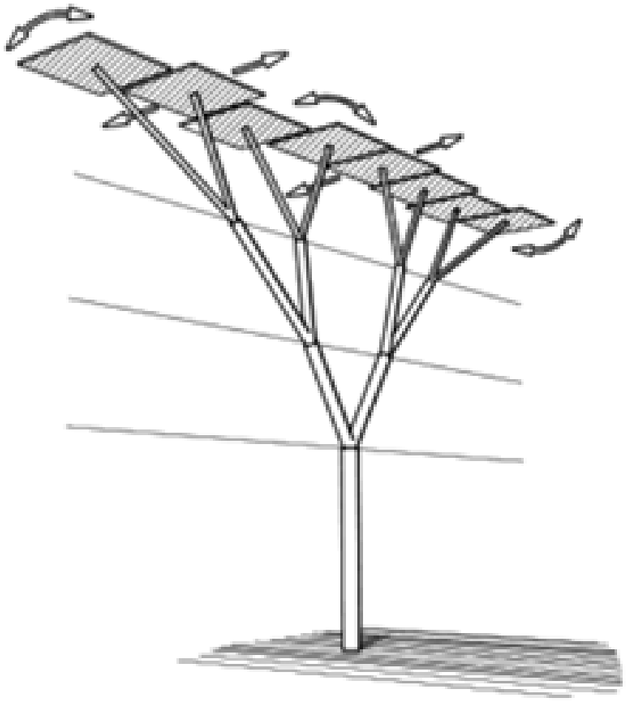
iterations

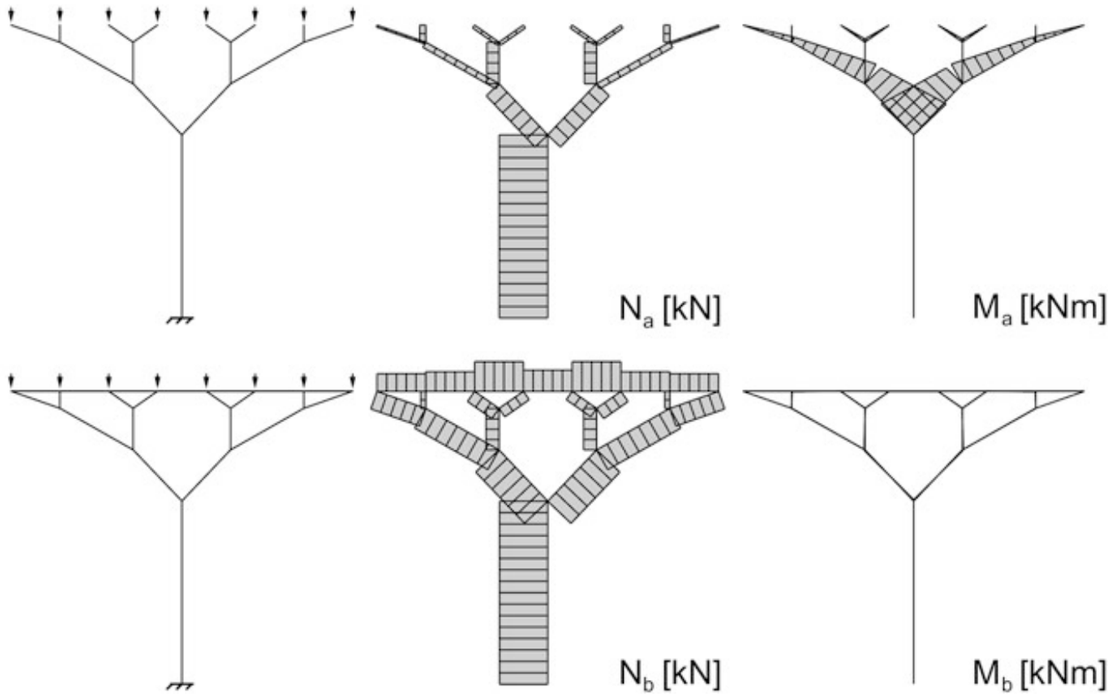


columns and beams

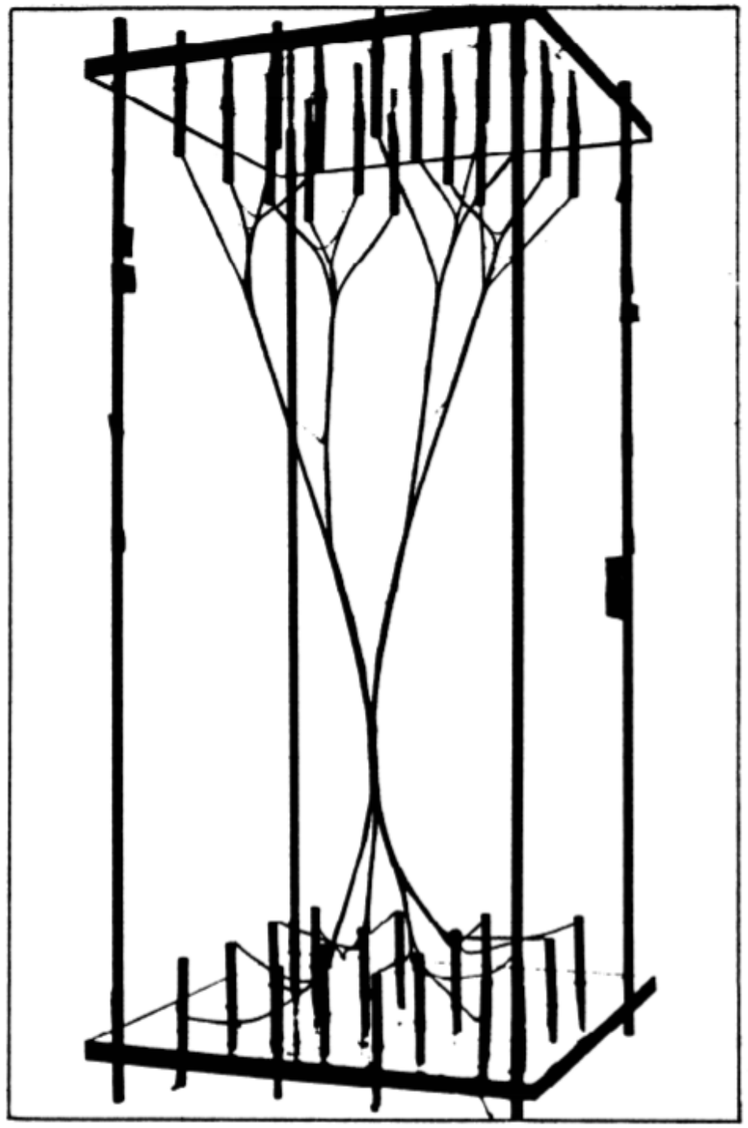


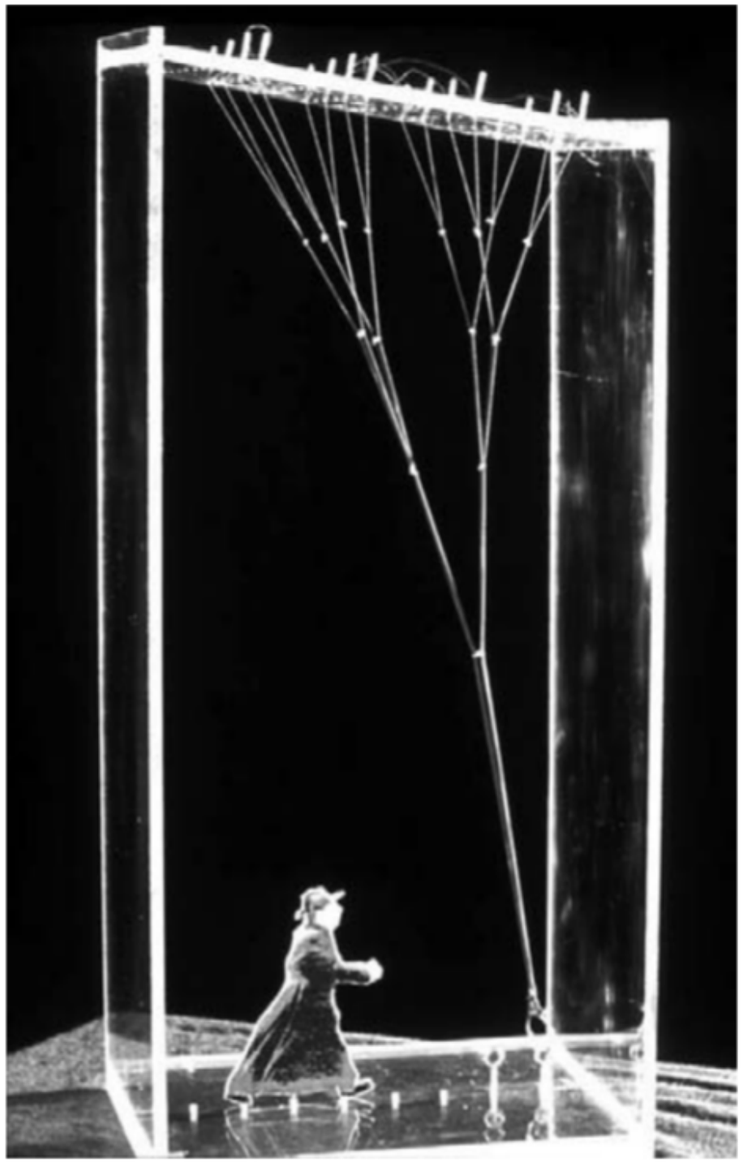
branching column

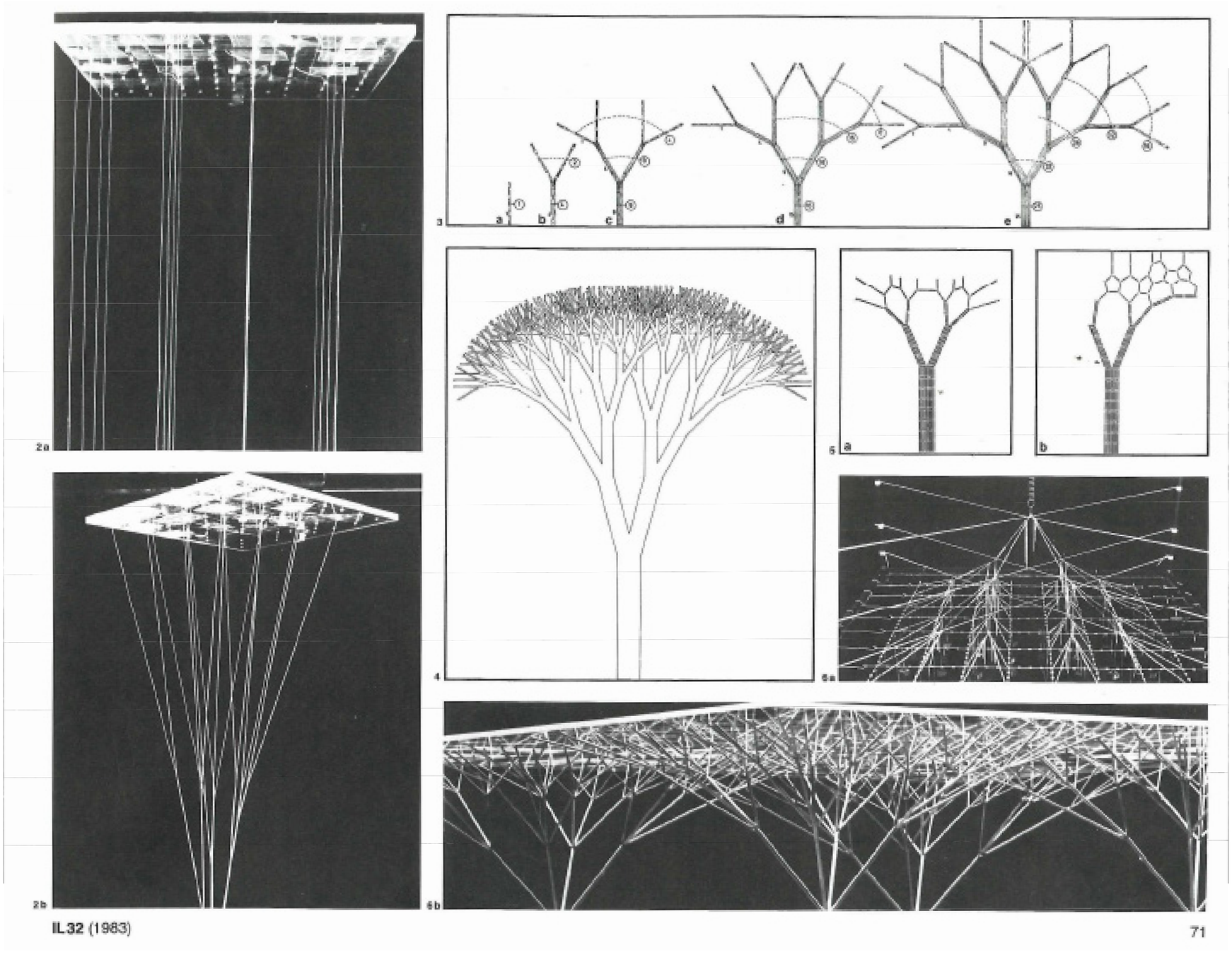



N_a [kN]
M_a [kNm]
N_b [kN]
M_b [kNm]

The main issue about these structures is finding the most reasonable form to solve the problem of actual project.







IL32 (1983)

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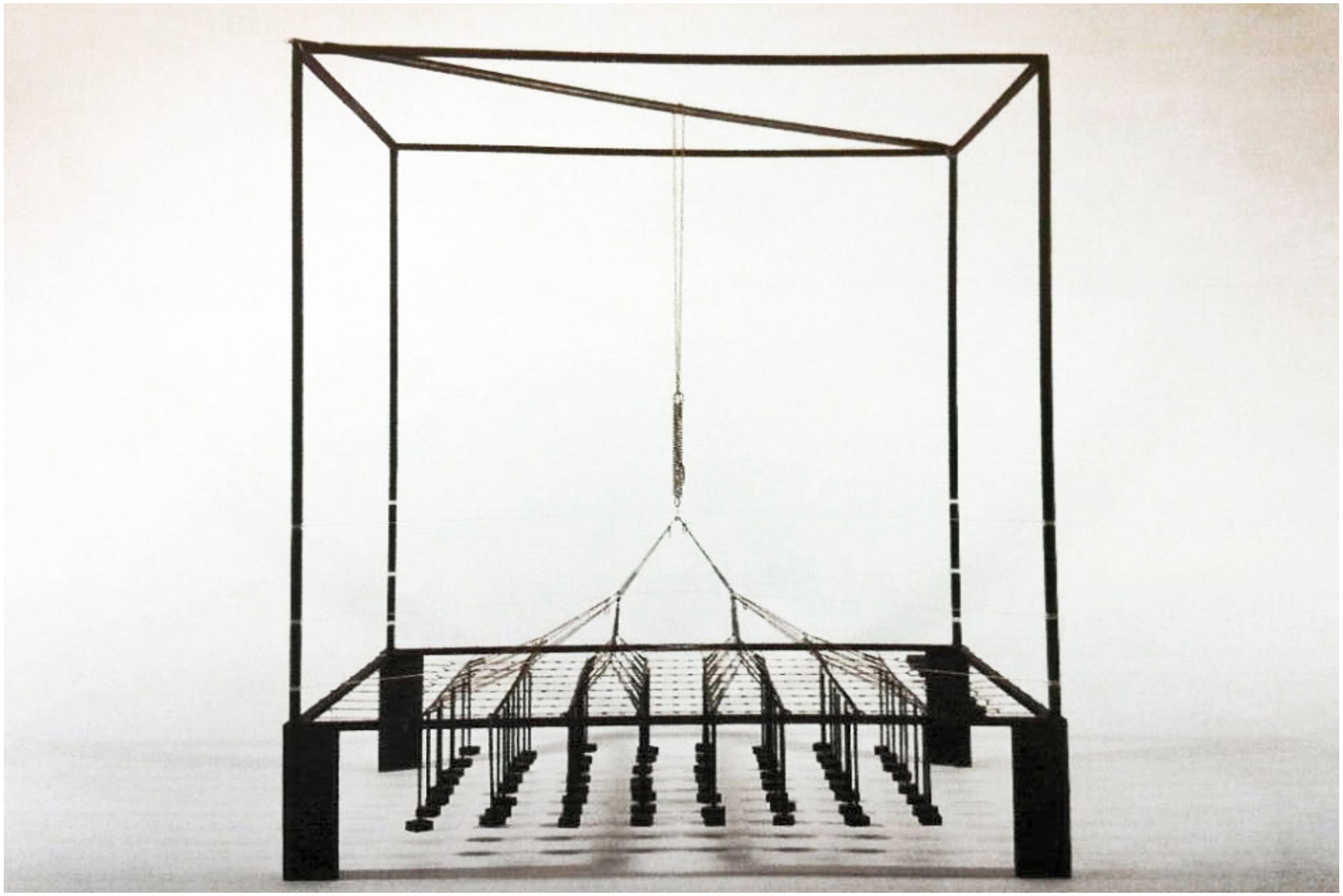
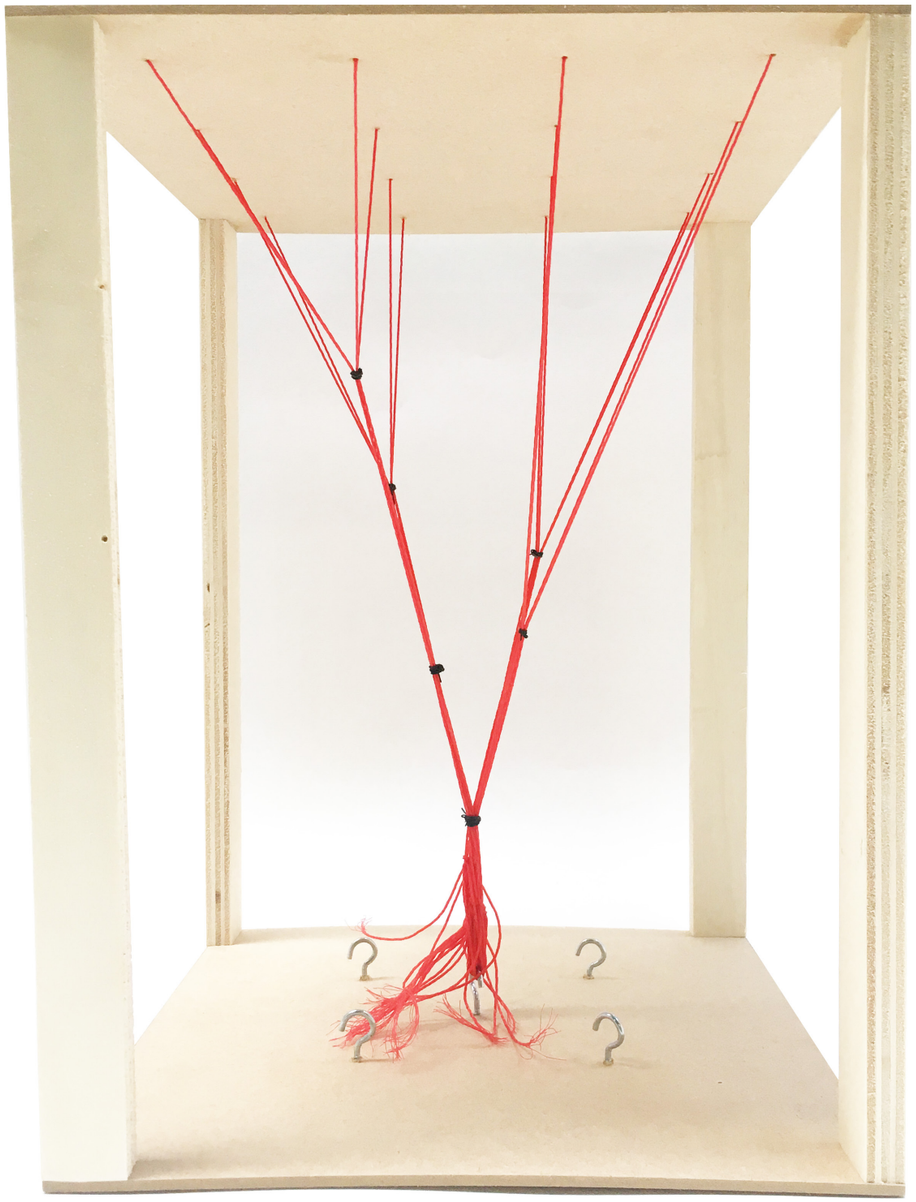
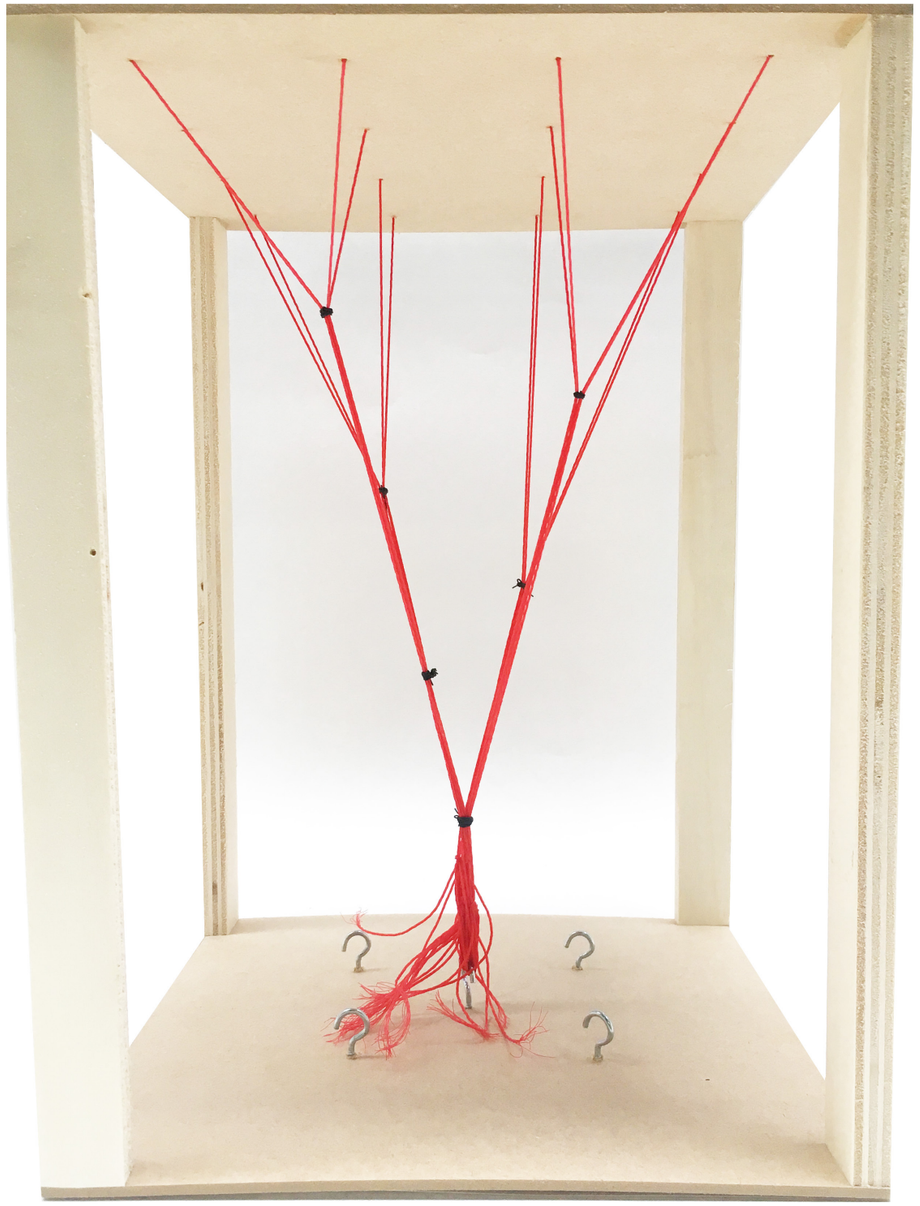
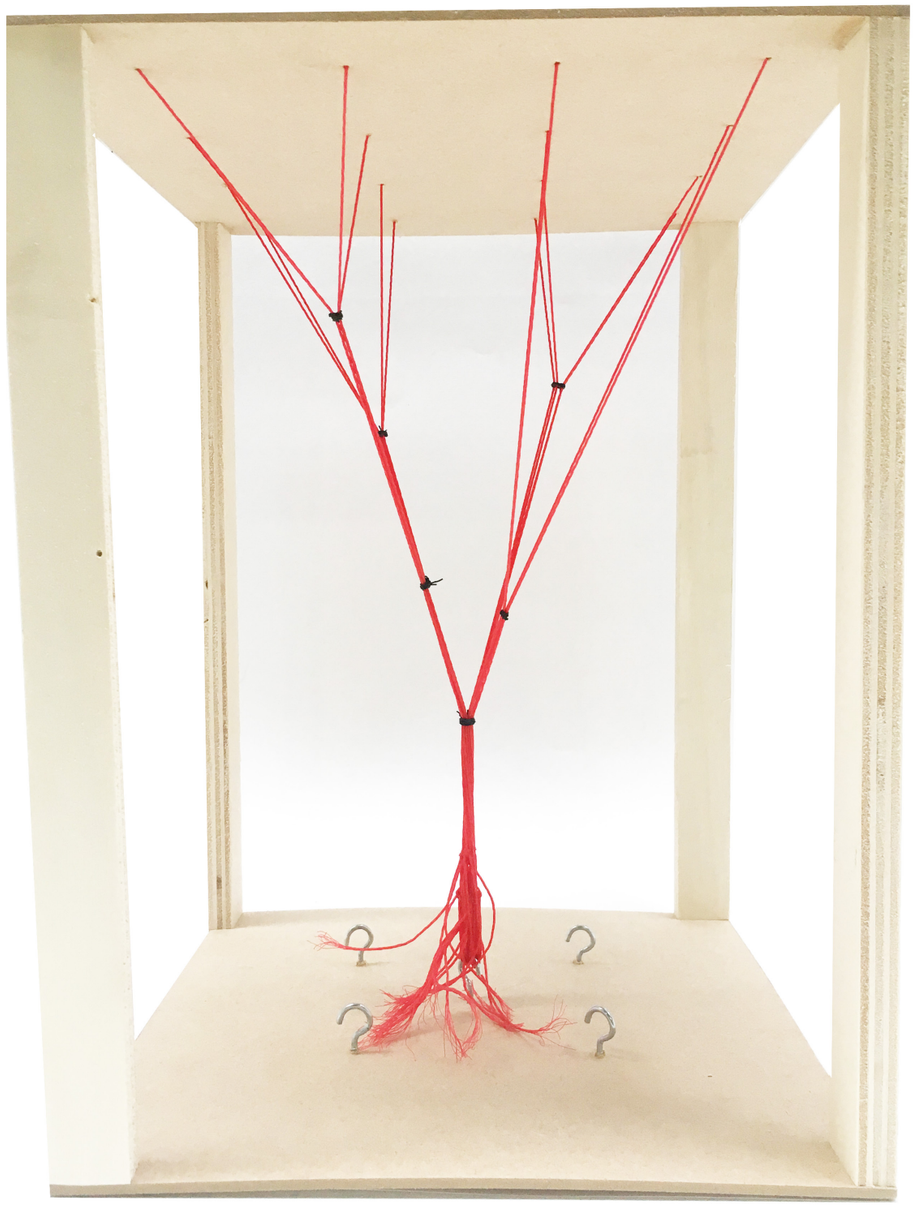


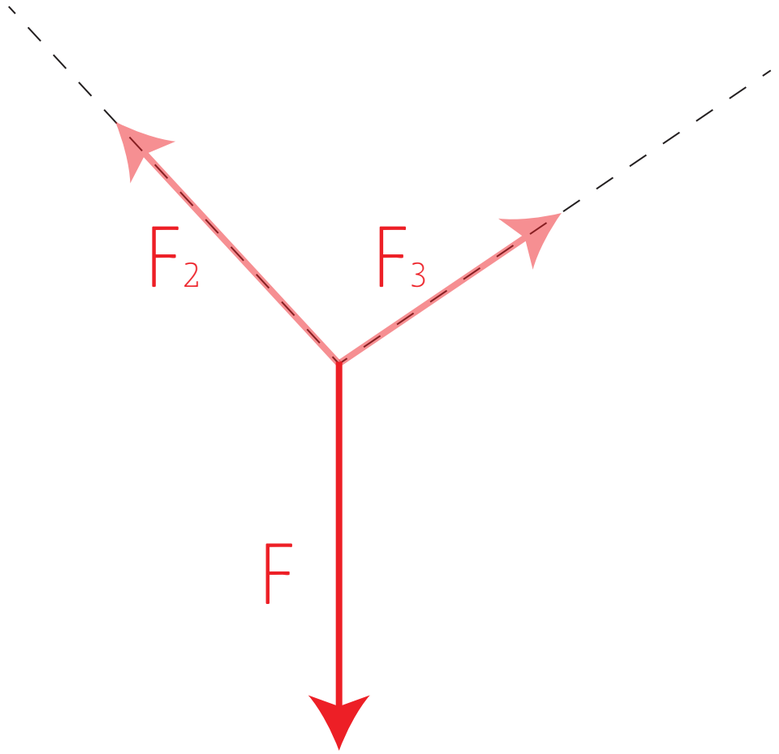






Finding an (optimal) shape of a [form-active structure] that is in (or approximates) a state of static equilibrium.

J. Lewis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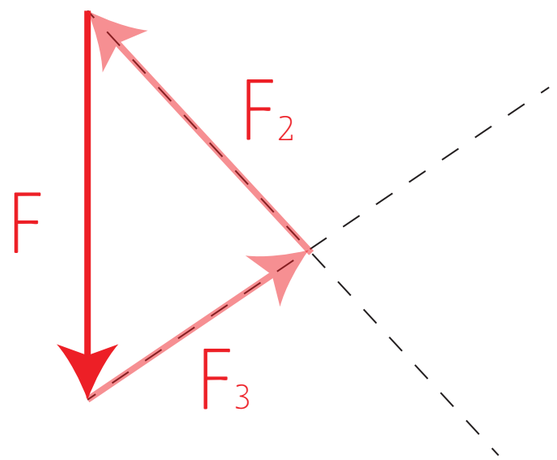
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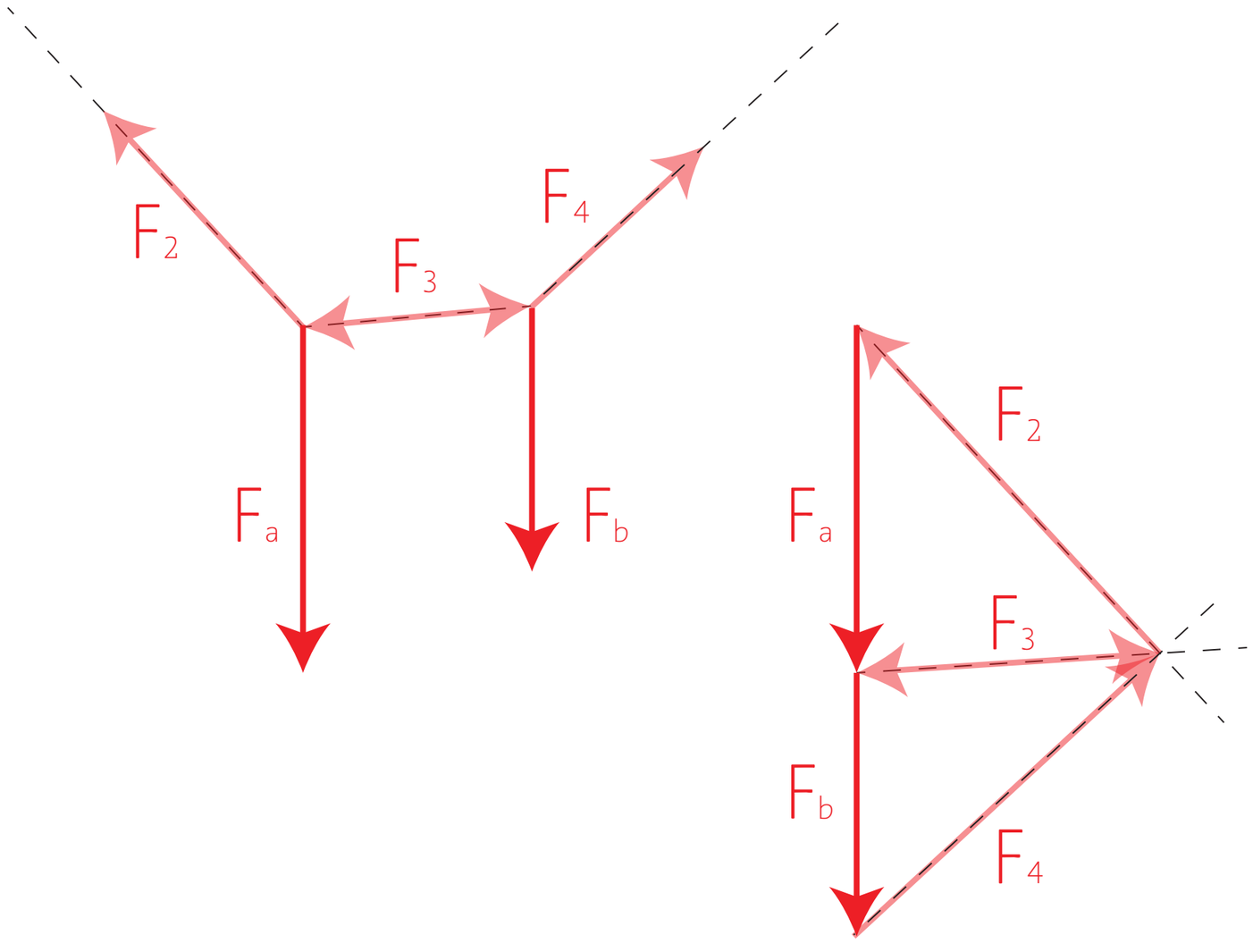
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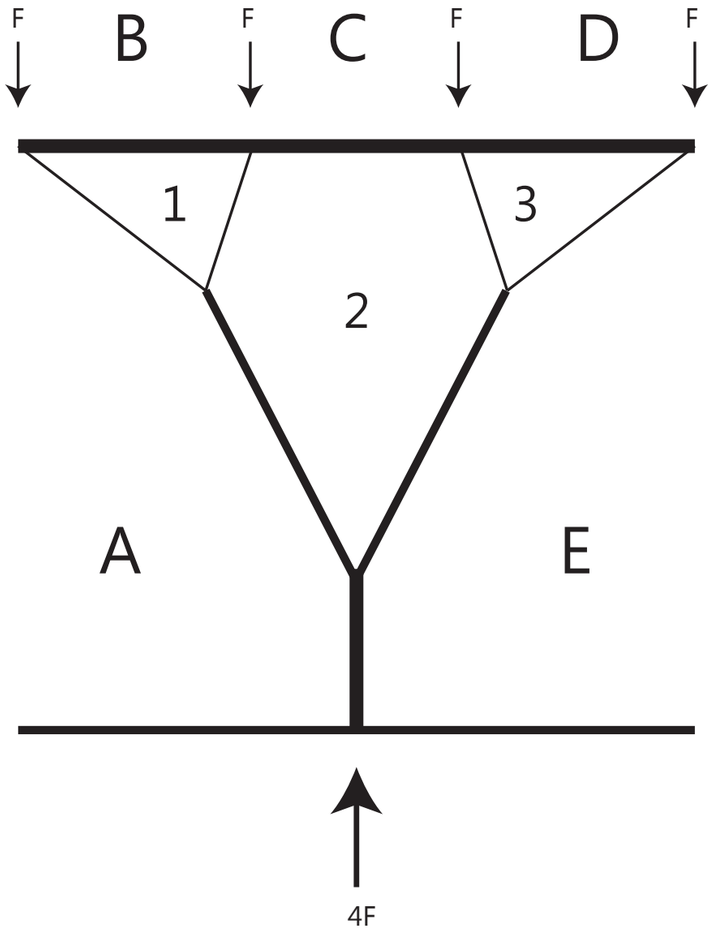
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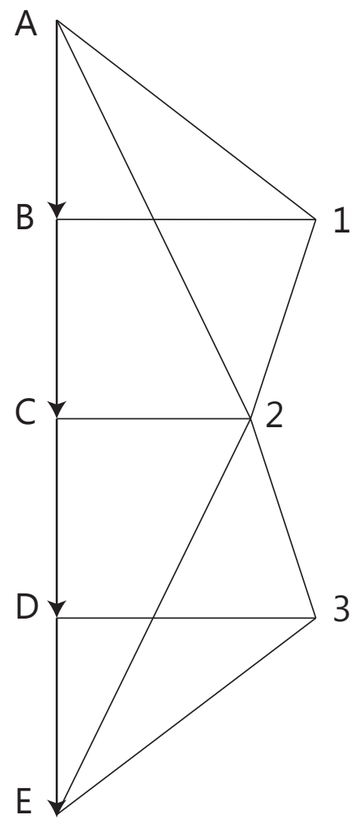
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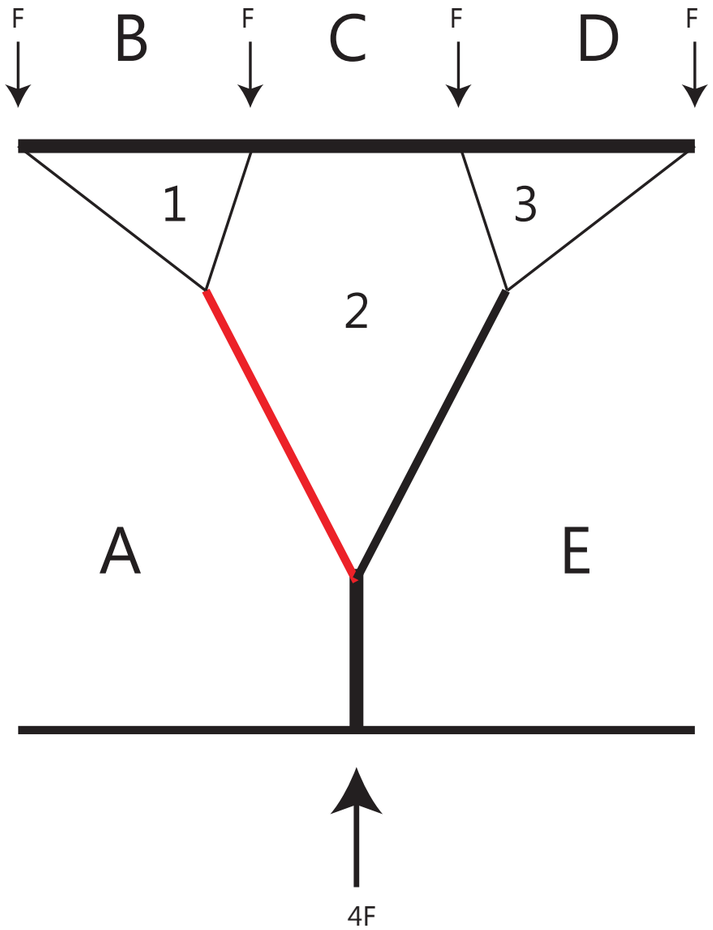
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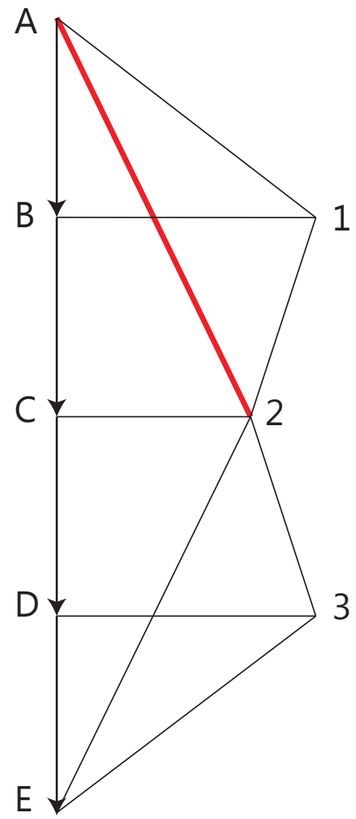
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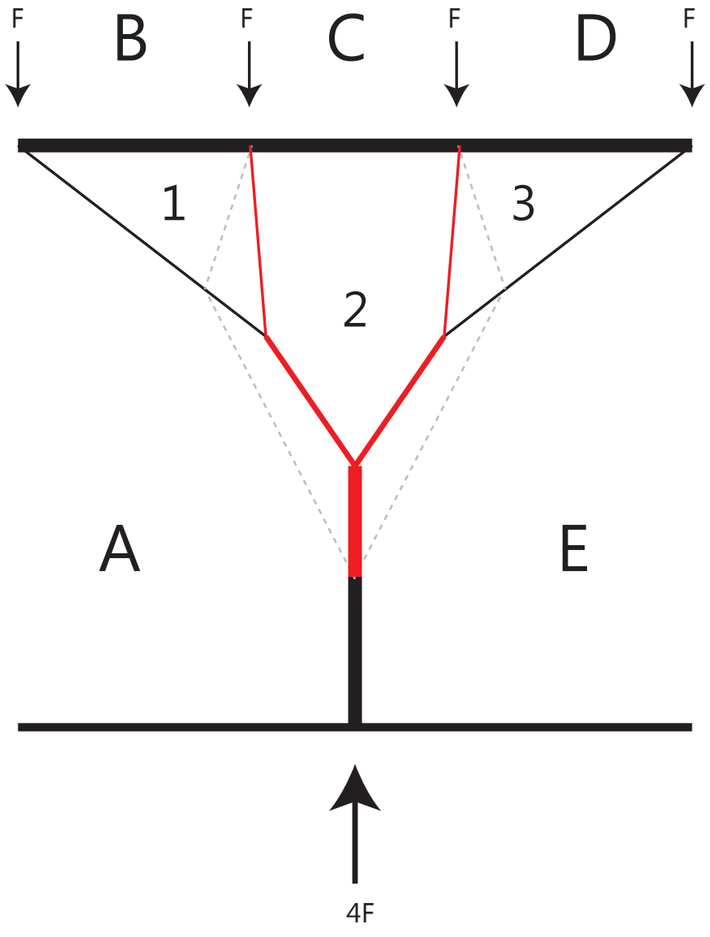
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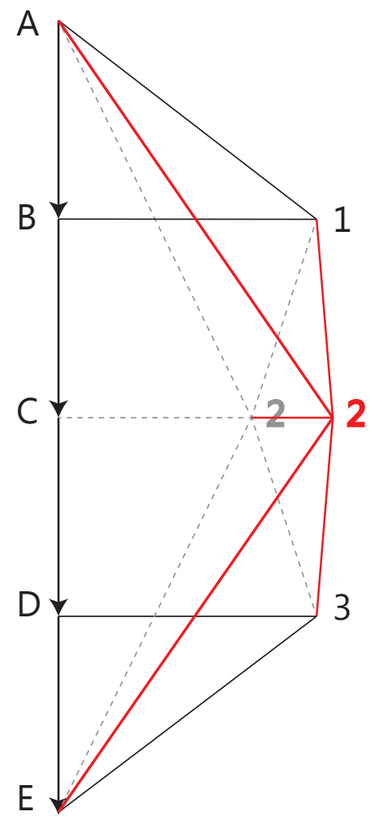
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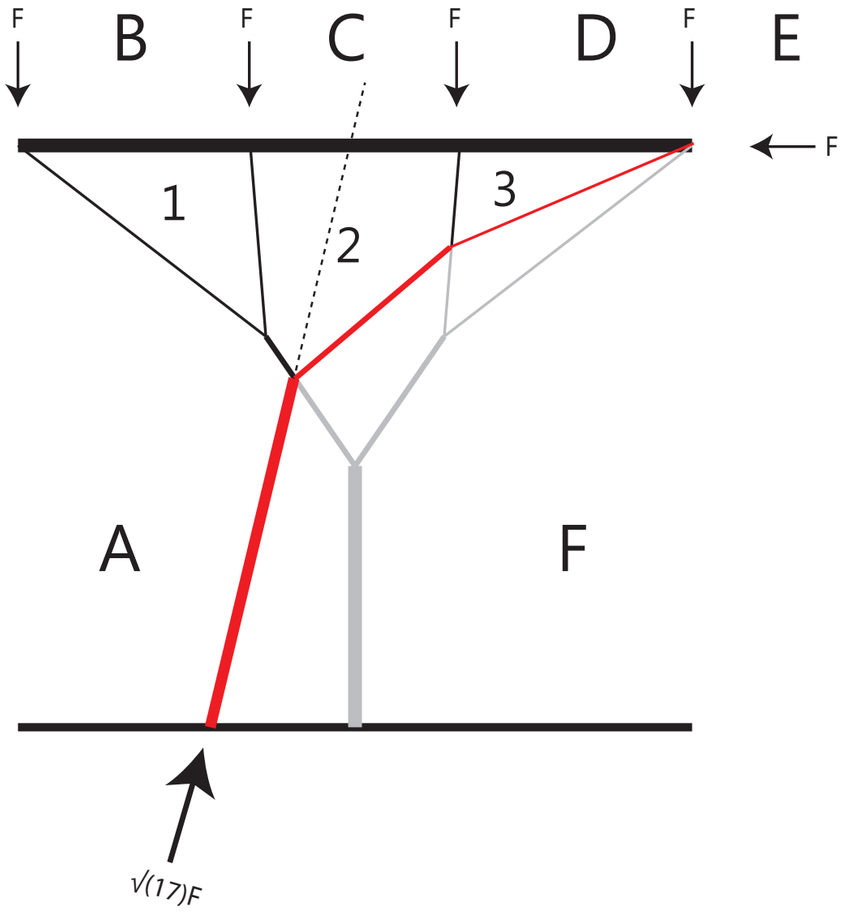
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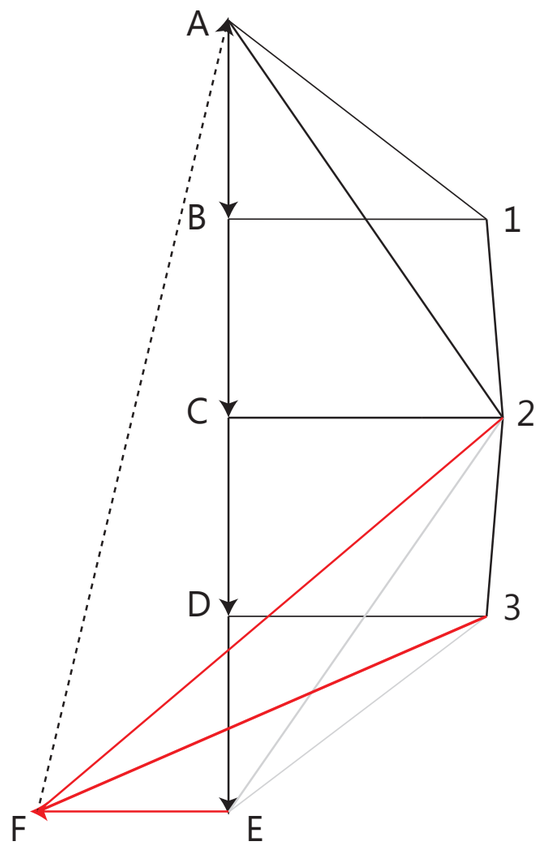
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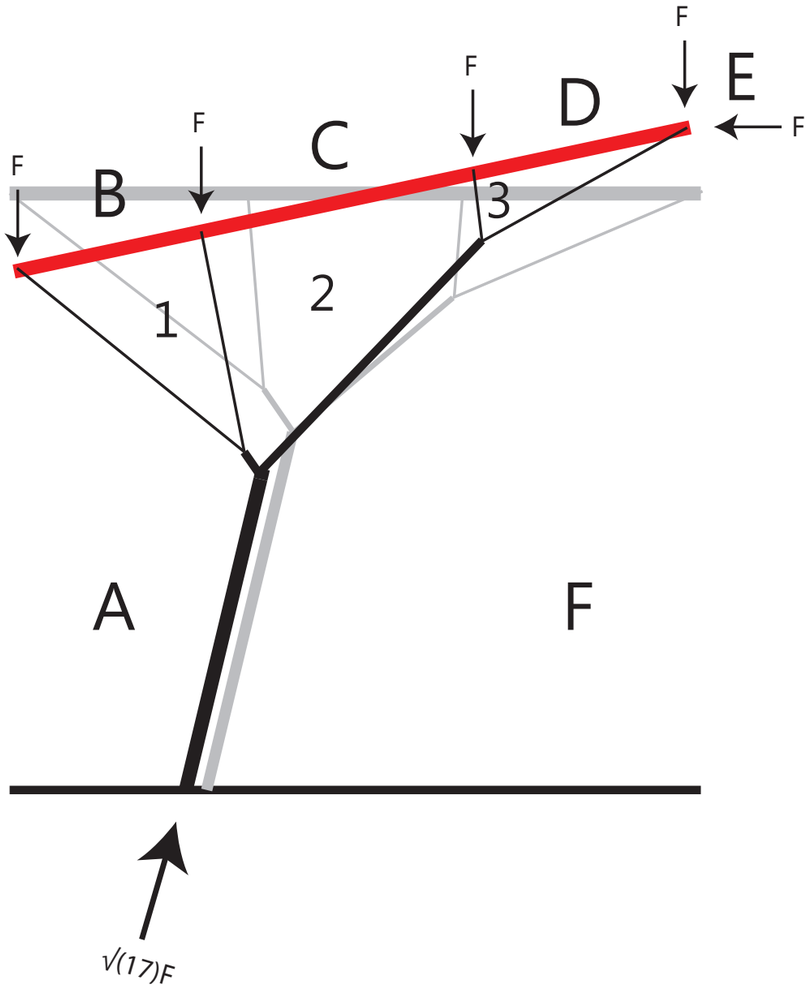
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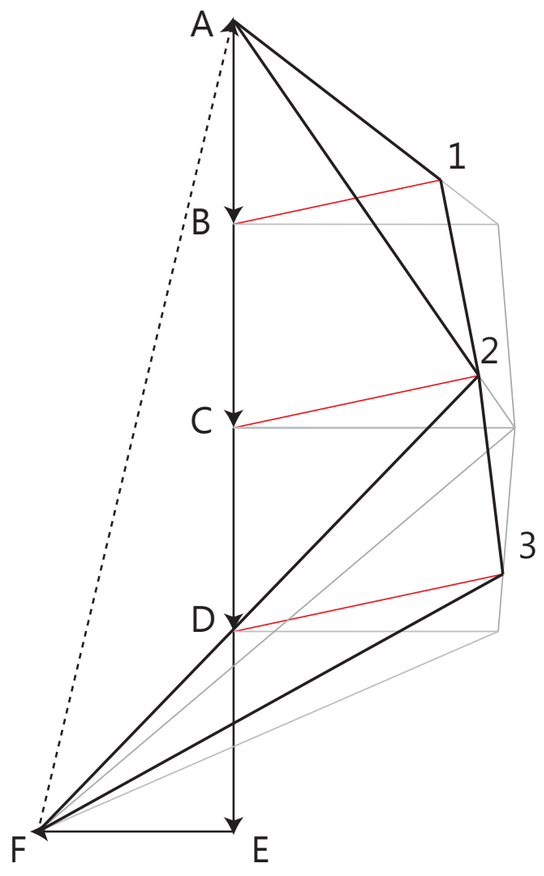
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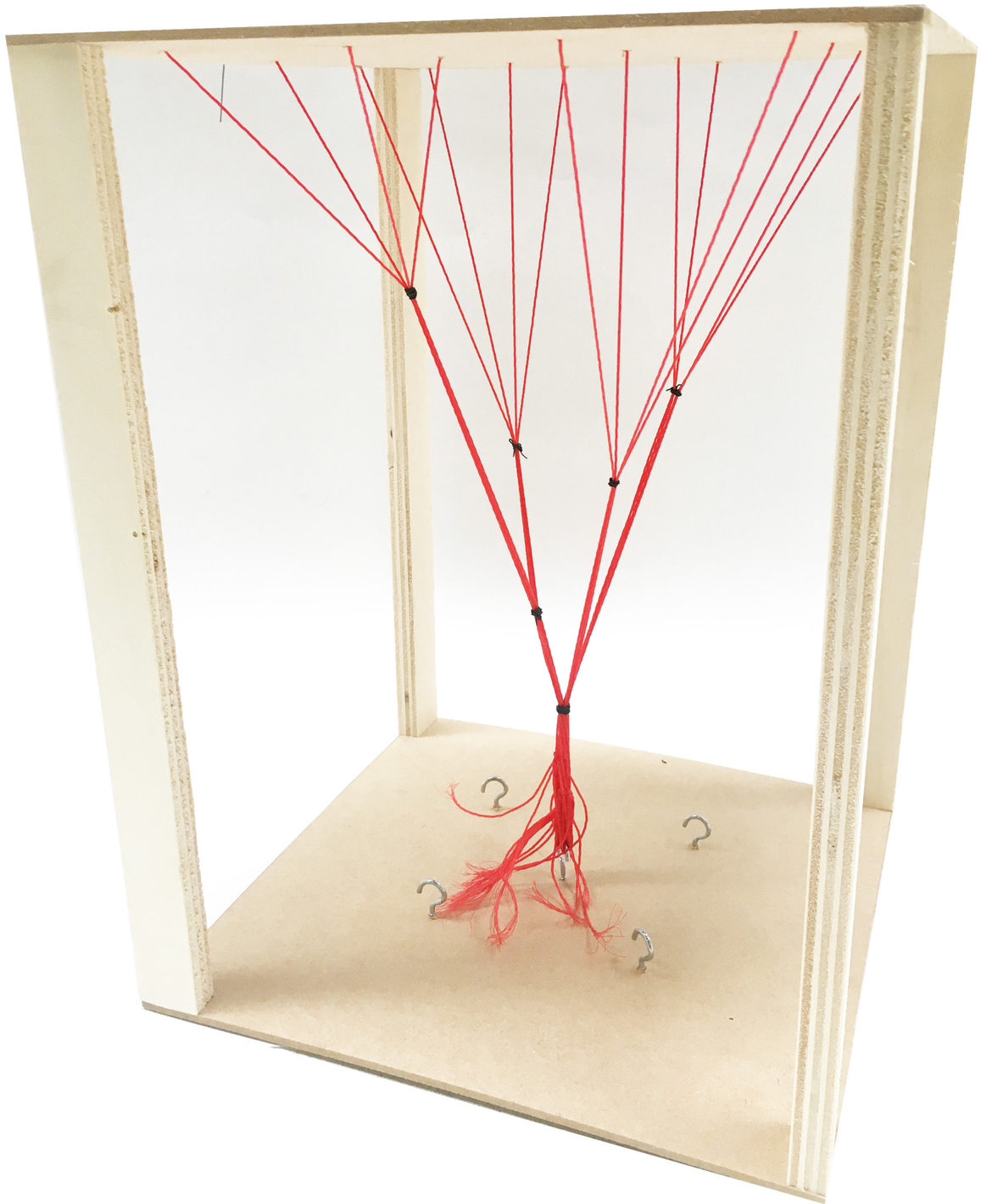
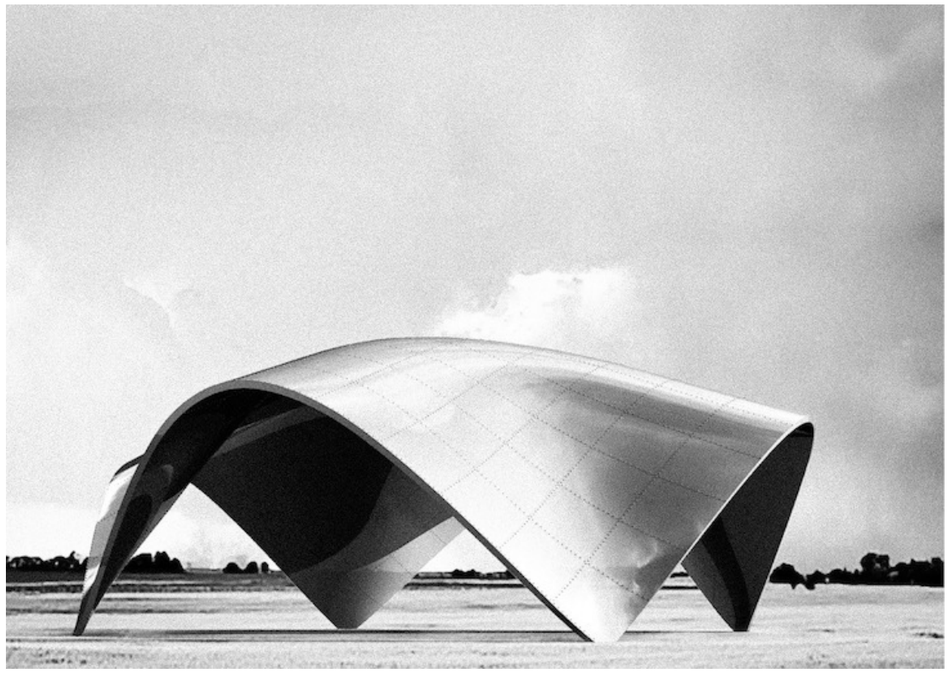
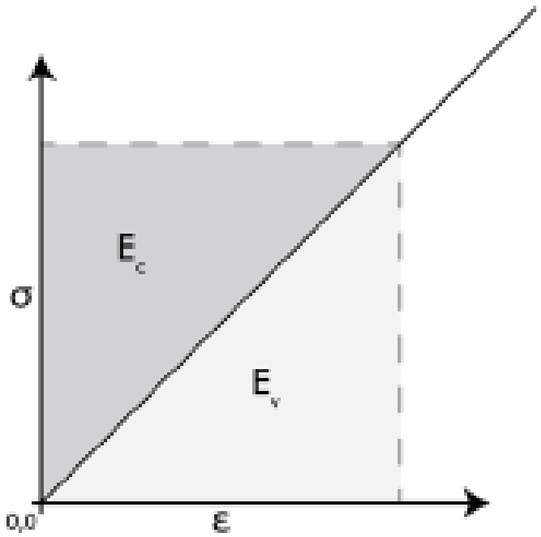


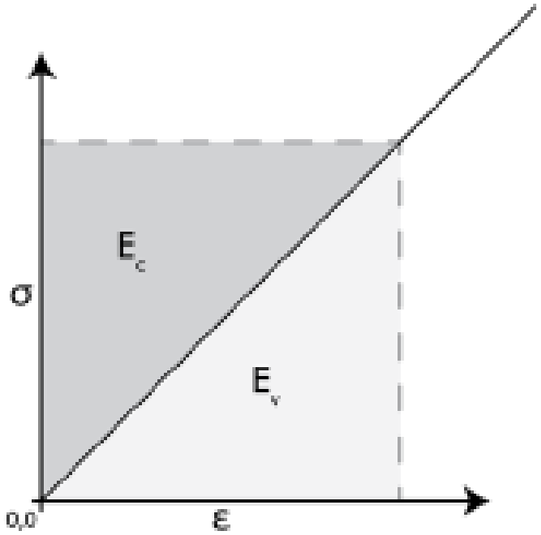




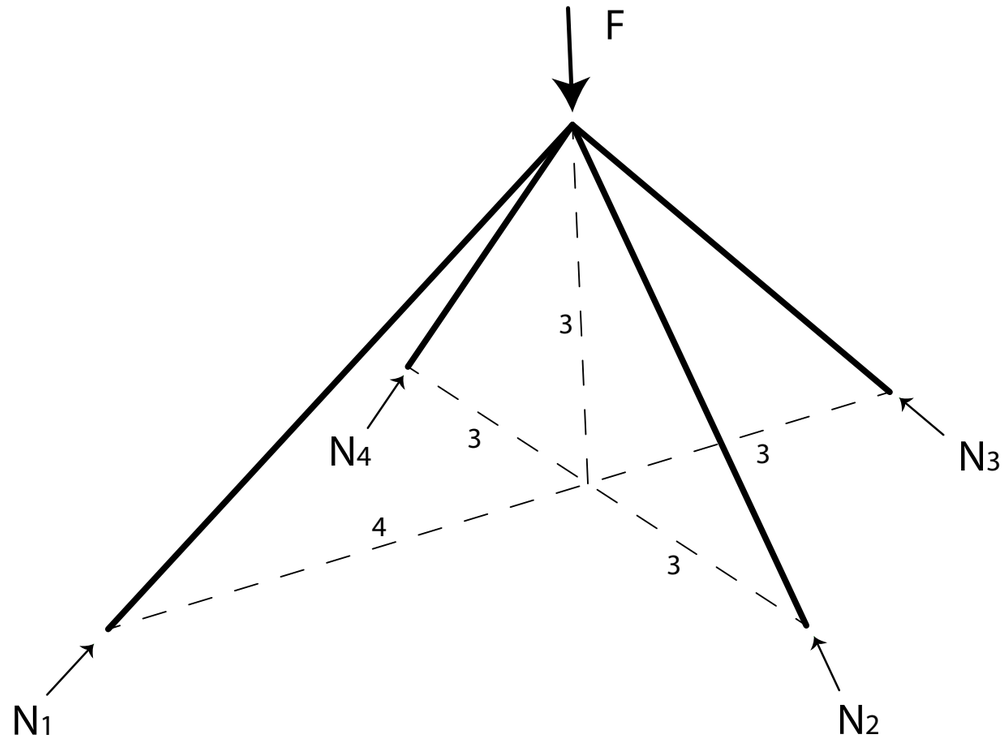
$$E_{compl} = \frac{1}{2}\sigma\varepsilon = E_v$$

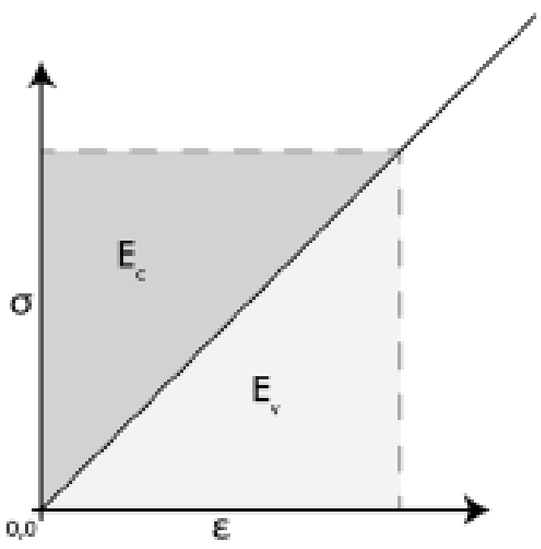
$$E_{compl,total} = \sum_{i=1}^{n} F_i^2 l_i$$

$$E_{compl,total} = \sum_{i=1}^{n} F_i^2 l_i = minimum$$

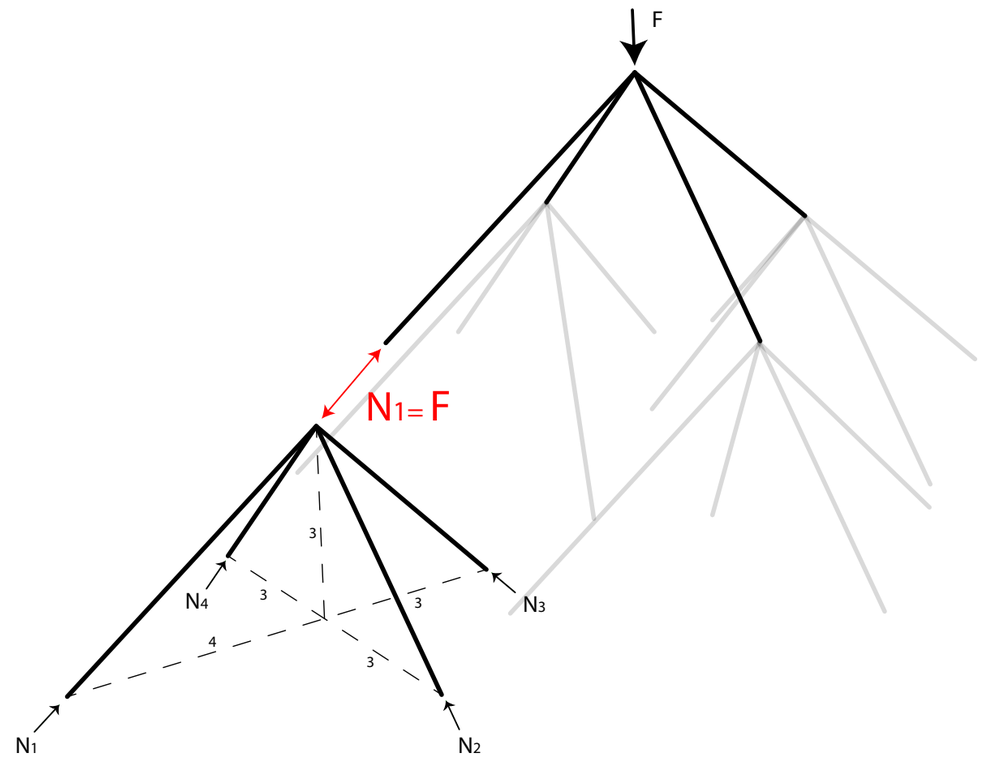


$$E_{compl,total} = \sum_{i=1}^{n} F_i^2 l_i = minimum$$





$$E_{compl,total} = \sum_{i=1}^{n} F_i^2 l_i = minimum$$

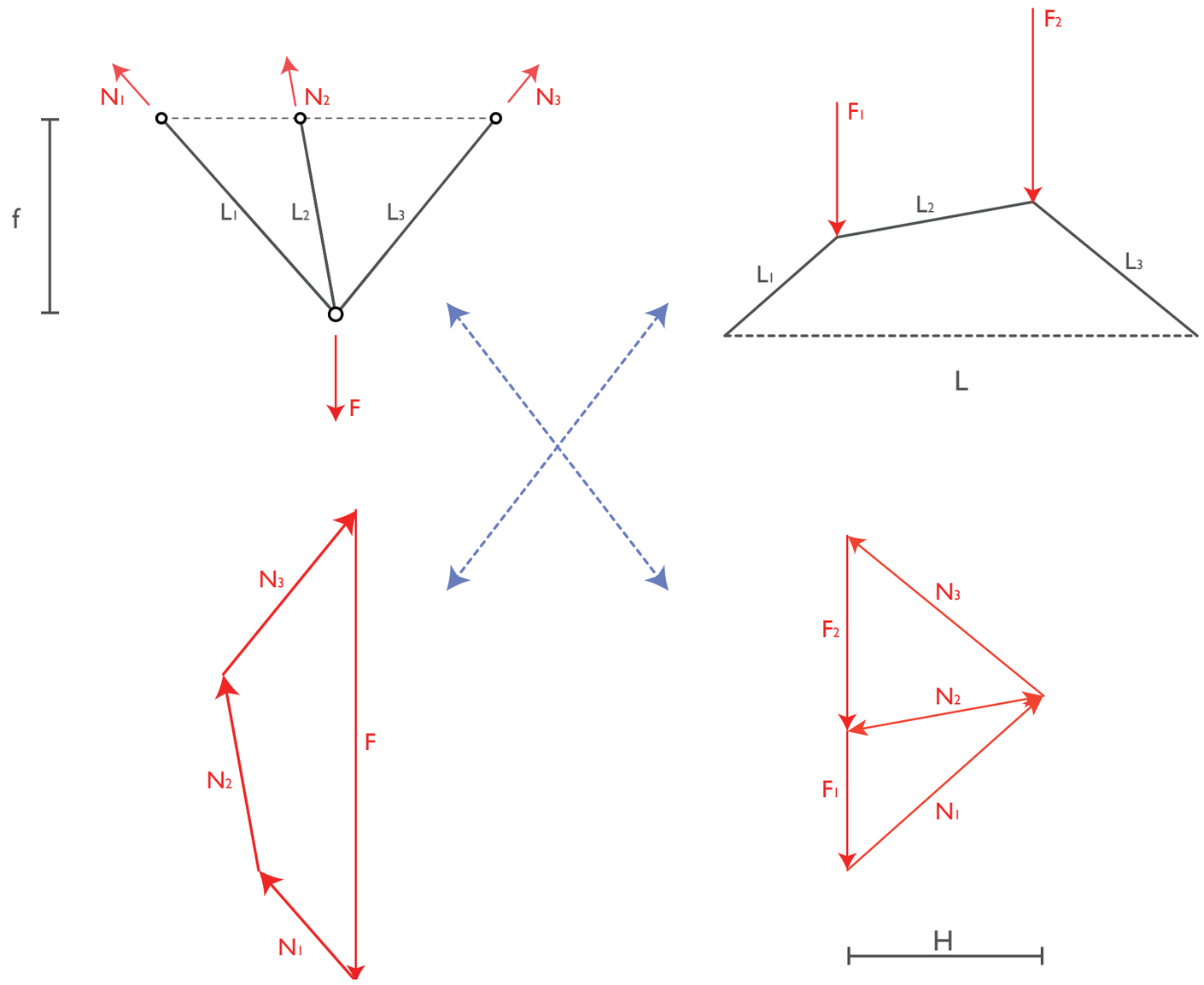


MAXWELL'S THEORY

MAXWELL'S THEORY

$$\min \sum V_i = \min \sum A_i l_i = \min \frac{1}{\sigma} \sum |F_i| l_i$$

$$\left(\sum F_i \cdot l_i\right)_{compression} + \left(\sum F_i \cdot l_i\right)_{tension} = \left(\sum \vec{\mathbf{P}} \cdot \vec{r}\right)_{loads} + \left(\sum \vec{\mathbf{P}} \cdot \vec{r}\right)_{reactions}$$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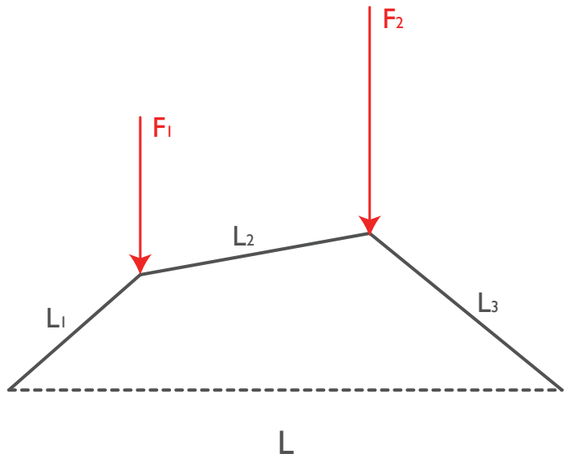
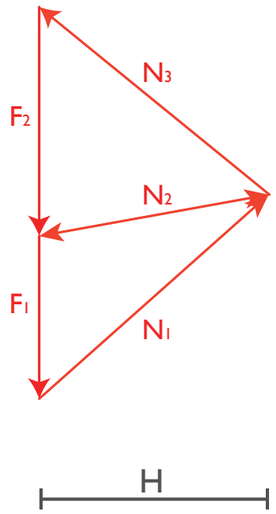
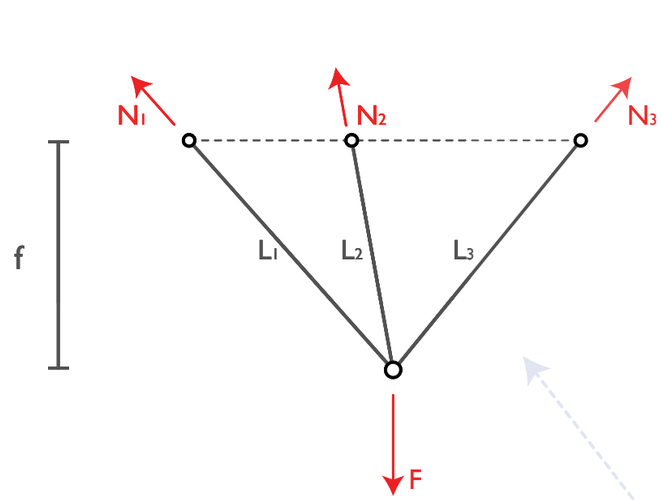
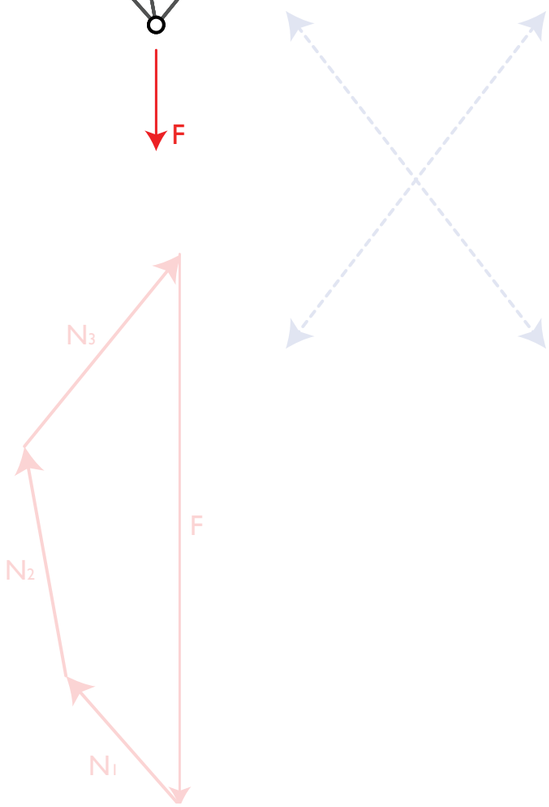
Reciprocal figures: length and force

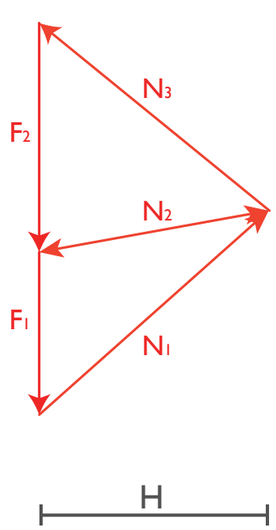




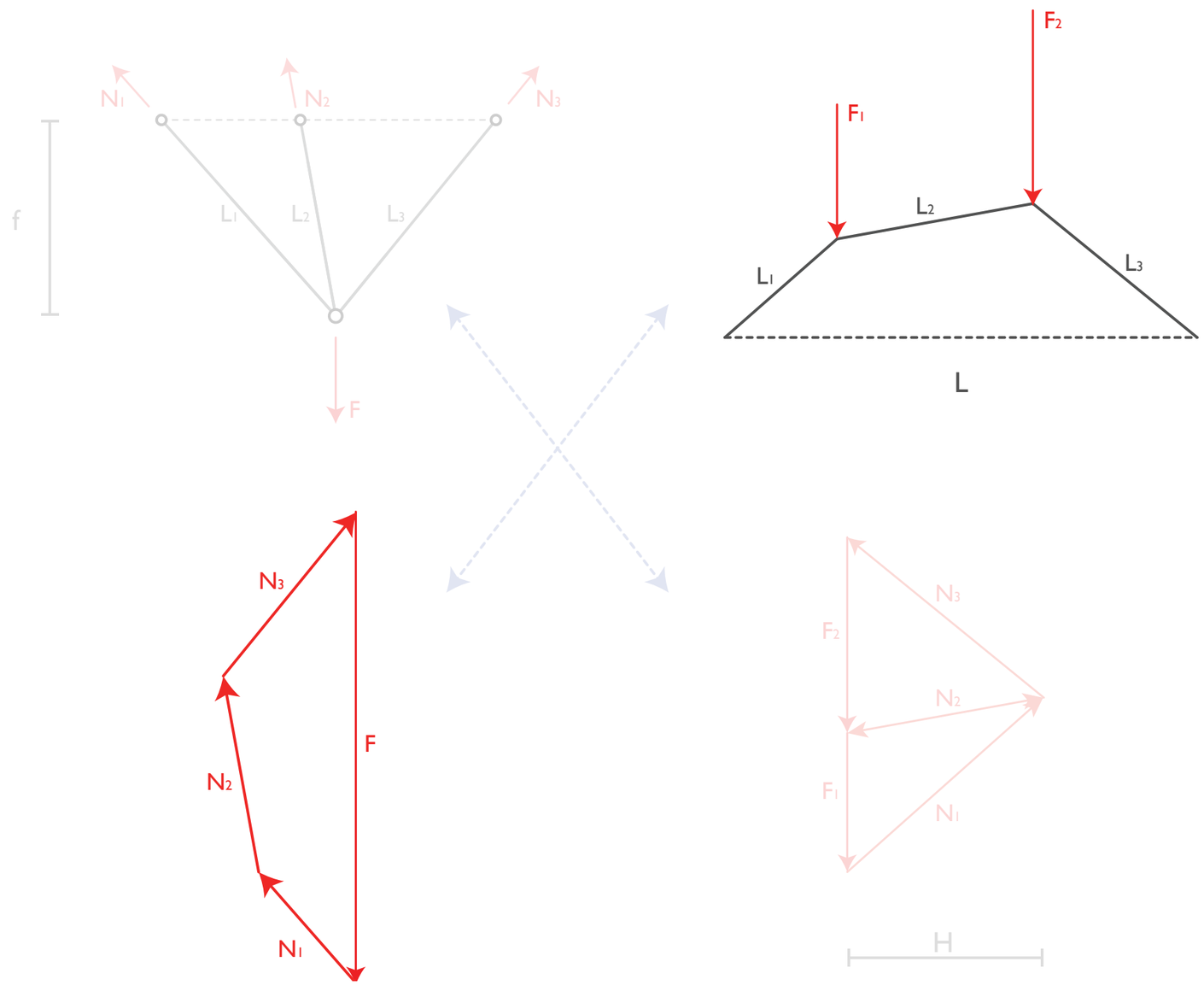
Reciprocal figures: length and force





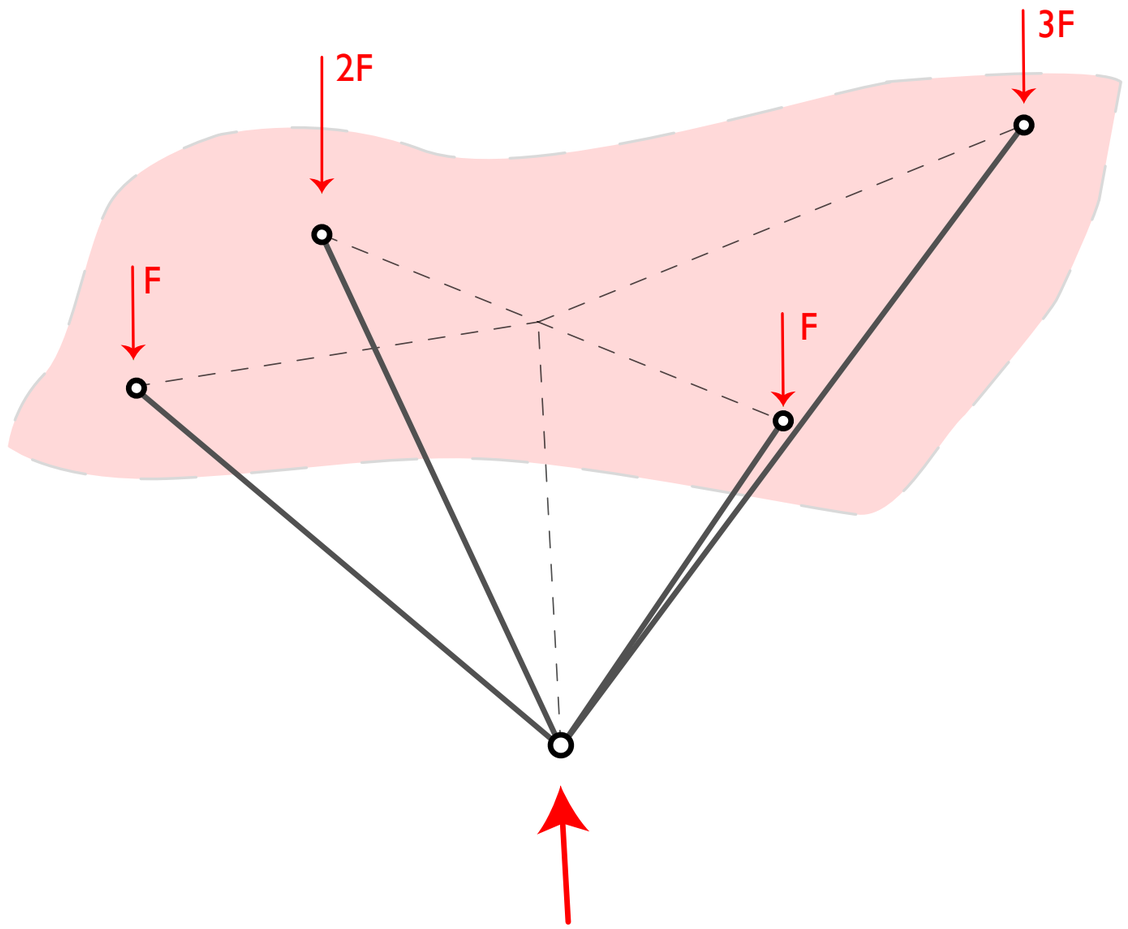


Reciprocal figures: length and force

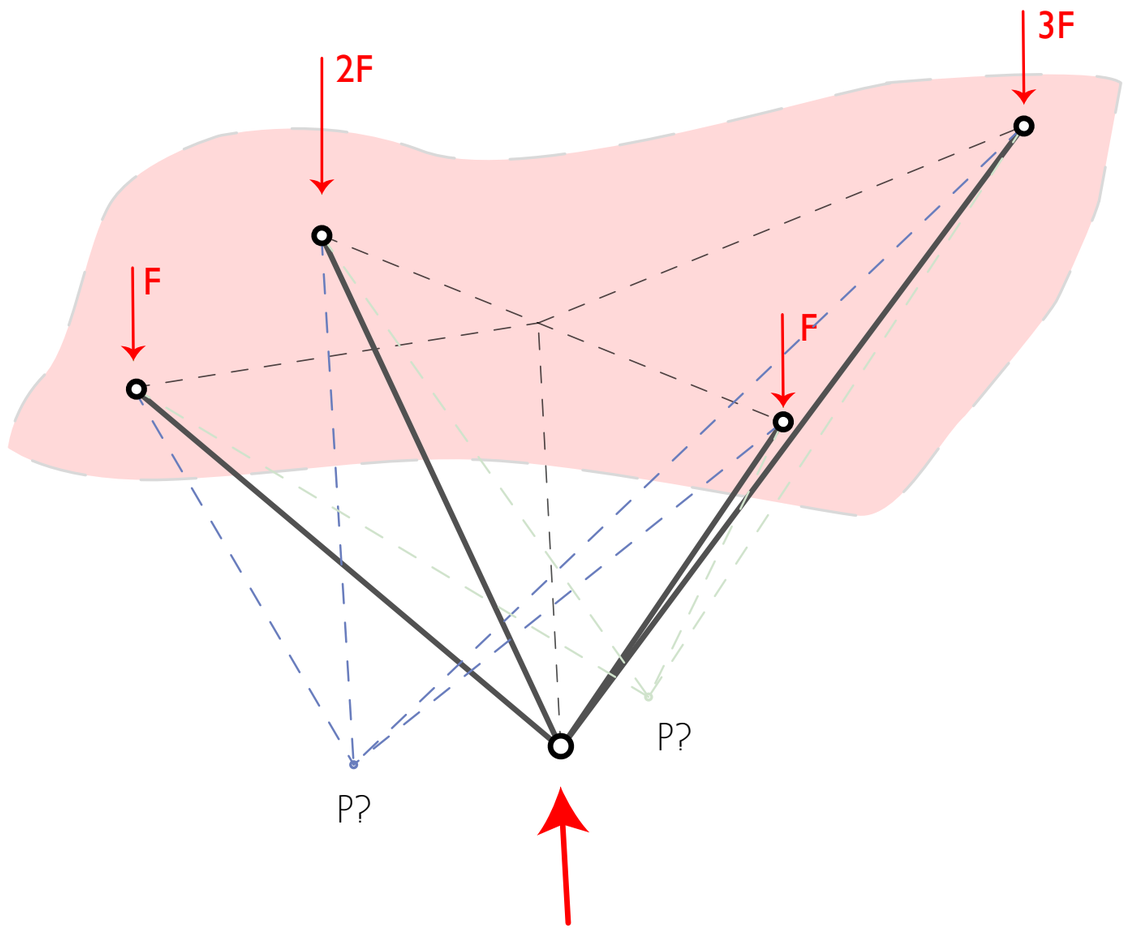


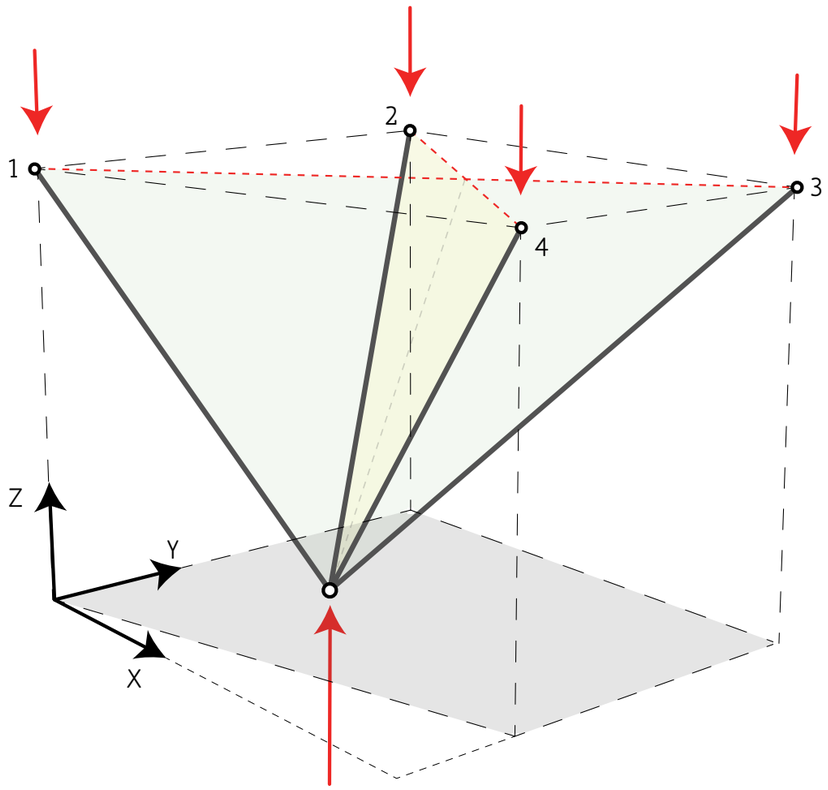
IV

Reversed problem



Reversed problem

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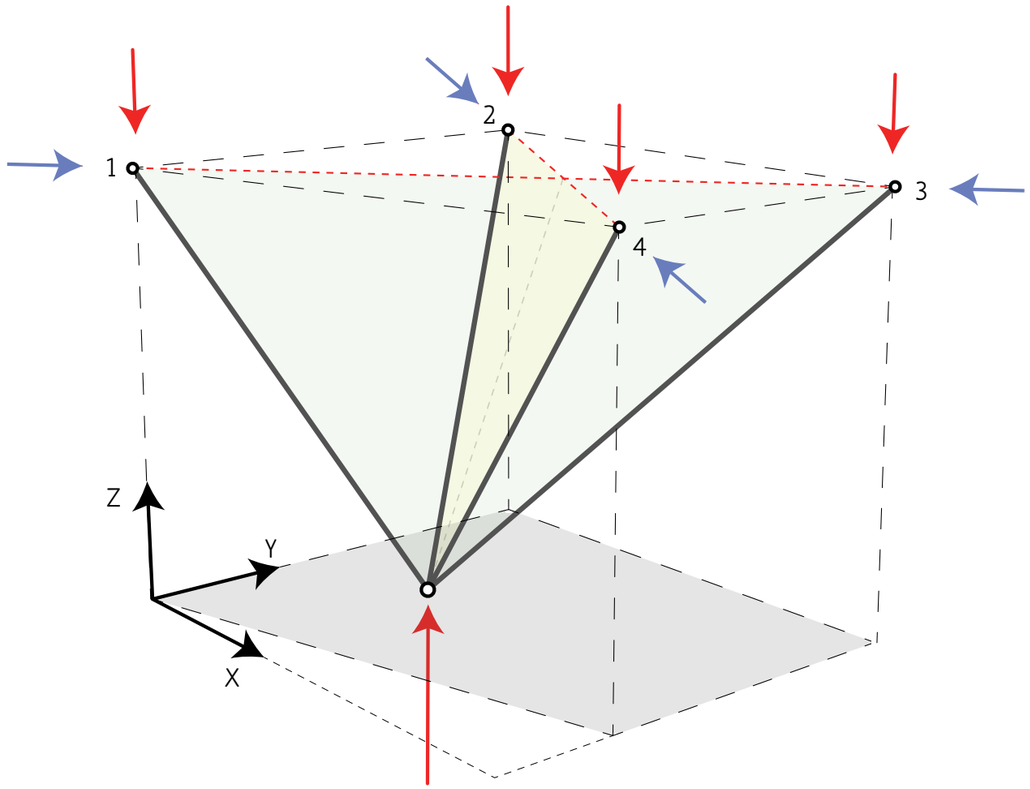
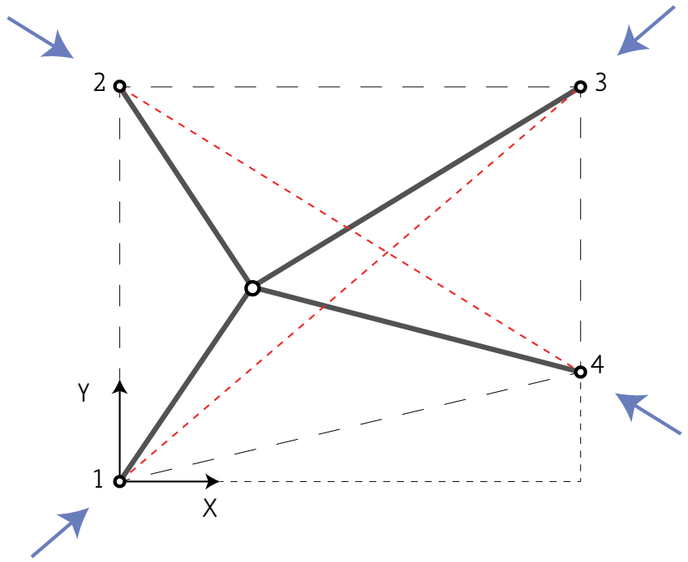
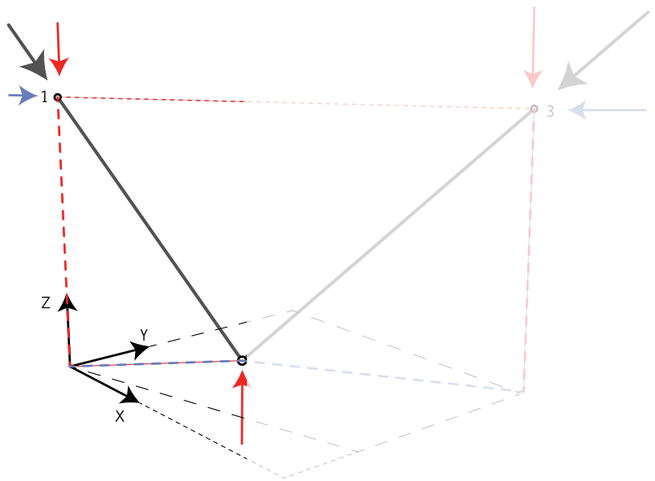
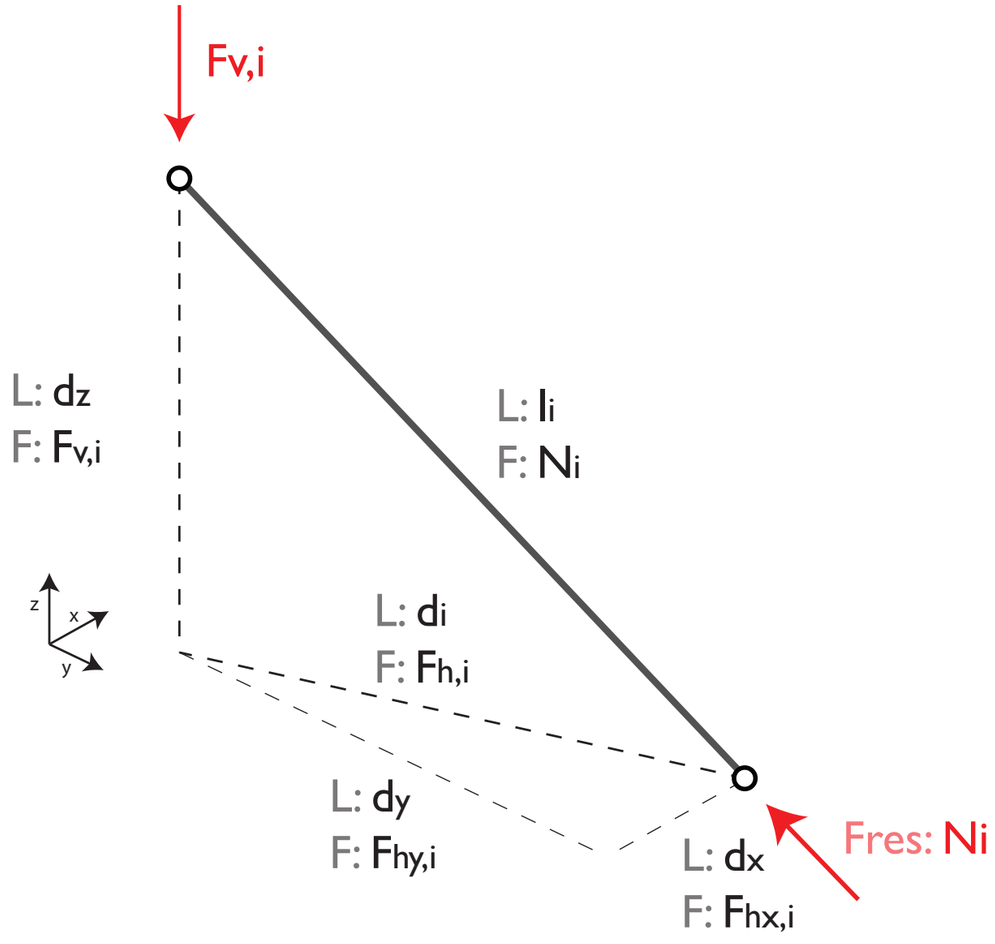
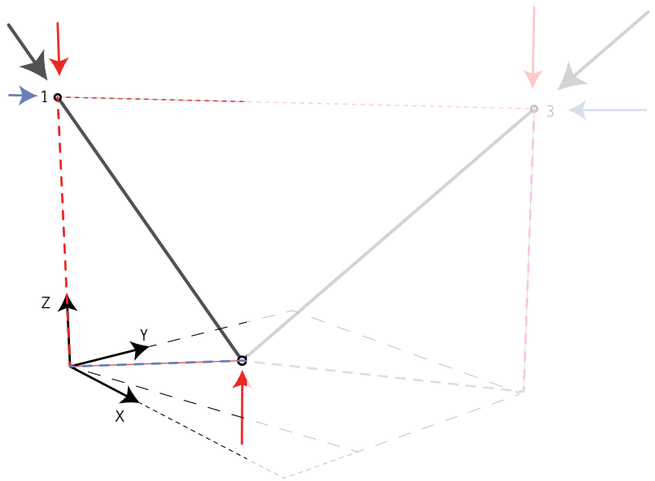
1
X


1
2
3
4
Z
Y
X


2
3
4
Y
1
X

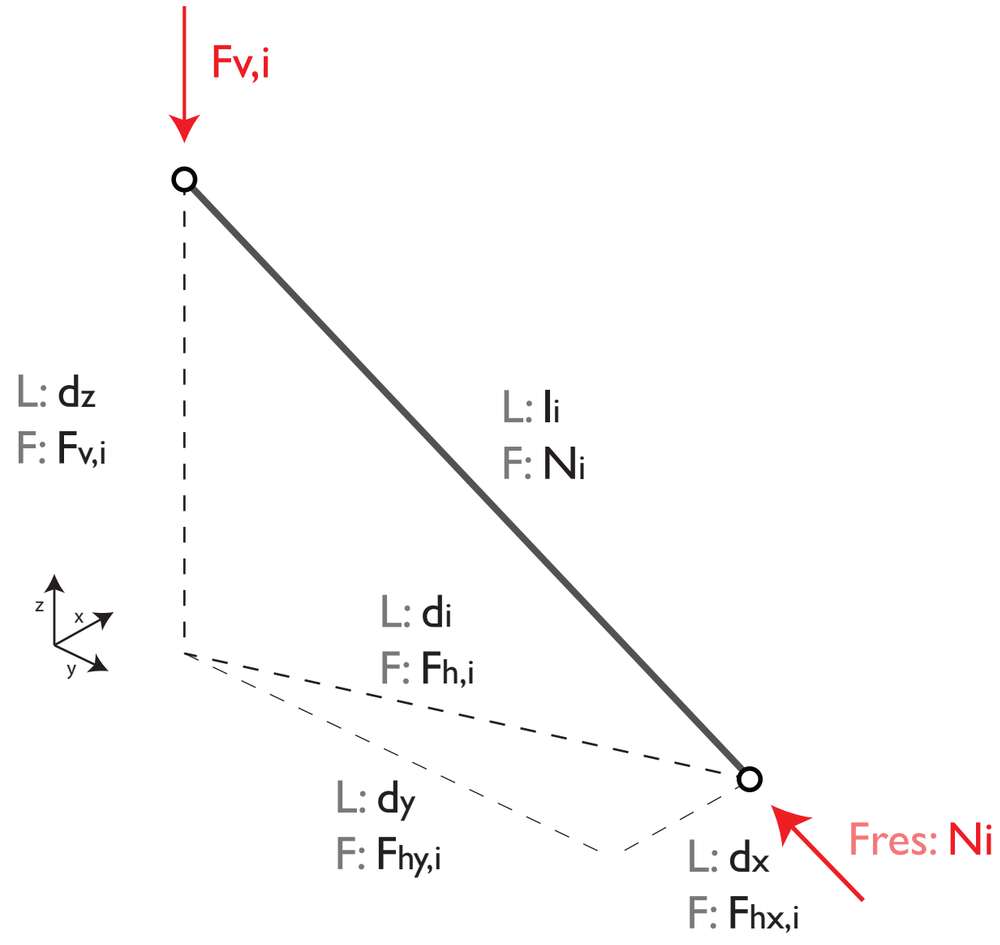

1
3
Z
Y
X

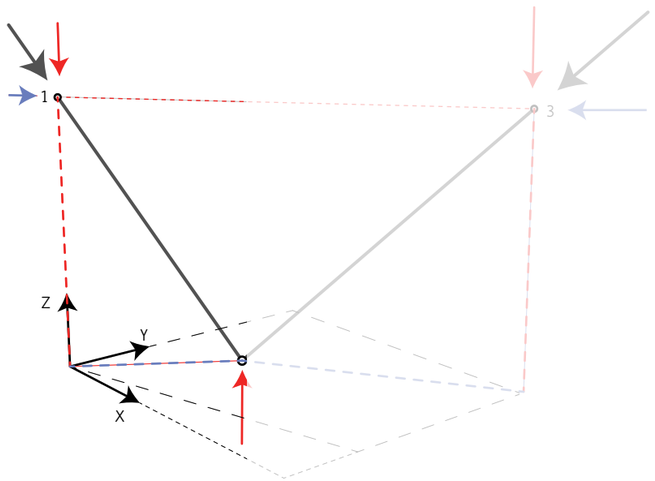

Fv,i
L: dz
F: Fv,i
L: li
F: Ni
z
x
y
L: di
F: Fh,i
L: dy
F: Fhy,i
L: dx
F: Fhx,i
Fres: Ni

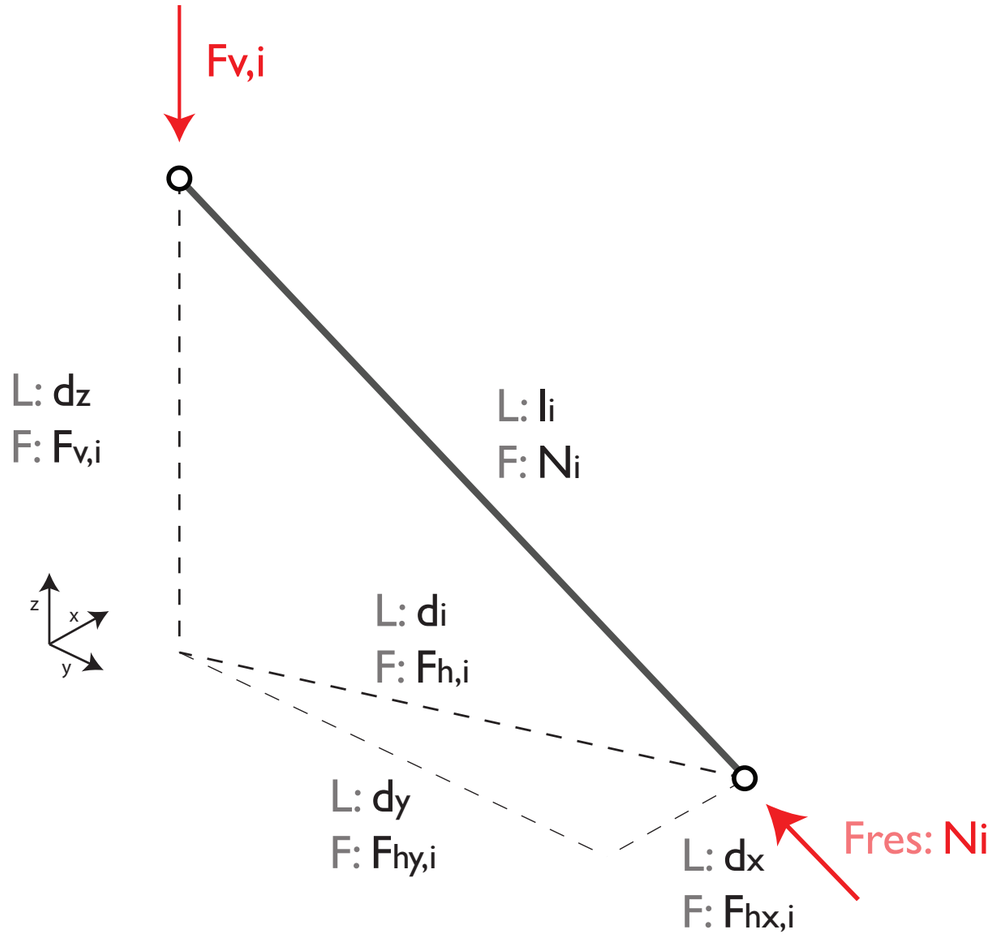


$$\frac{F_{v,i}}{d_z} = \frac{N_i}{l_i}$$

$$N_i = \frac{l_i \cdot F_{v,i}}{d_z}$$

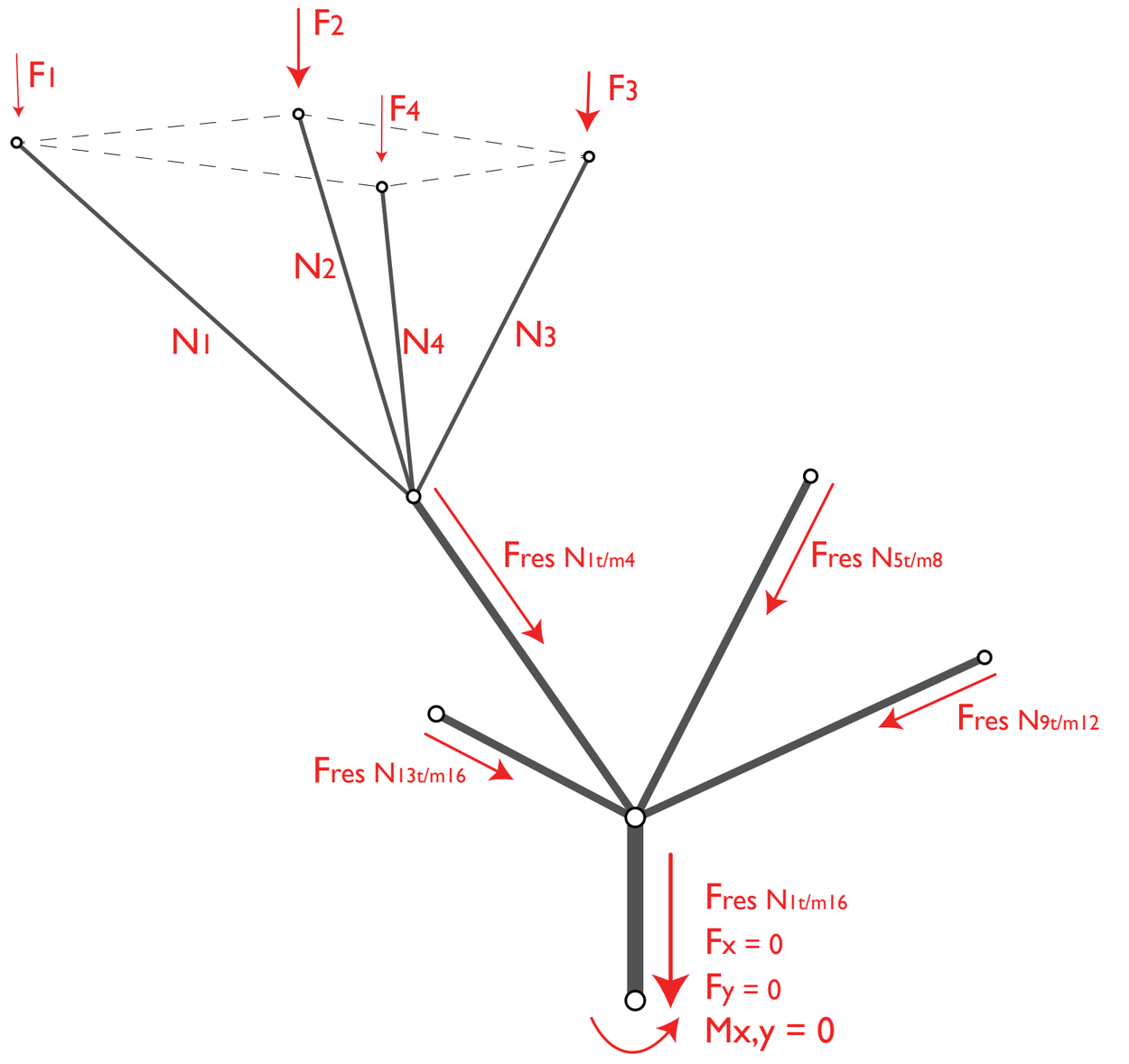


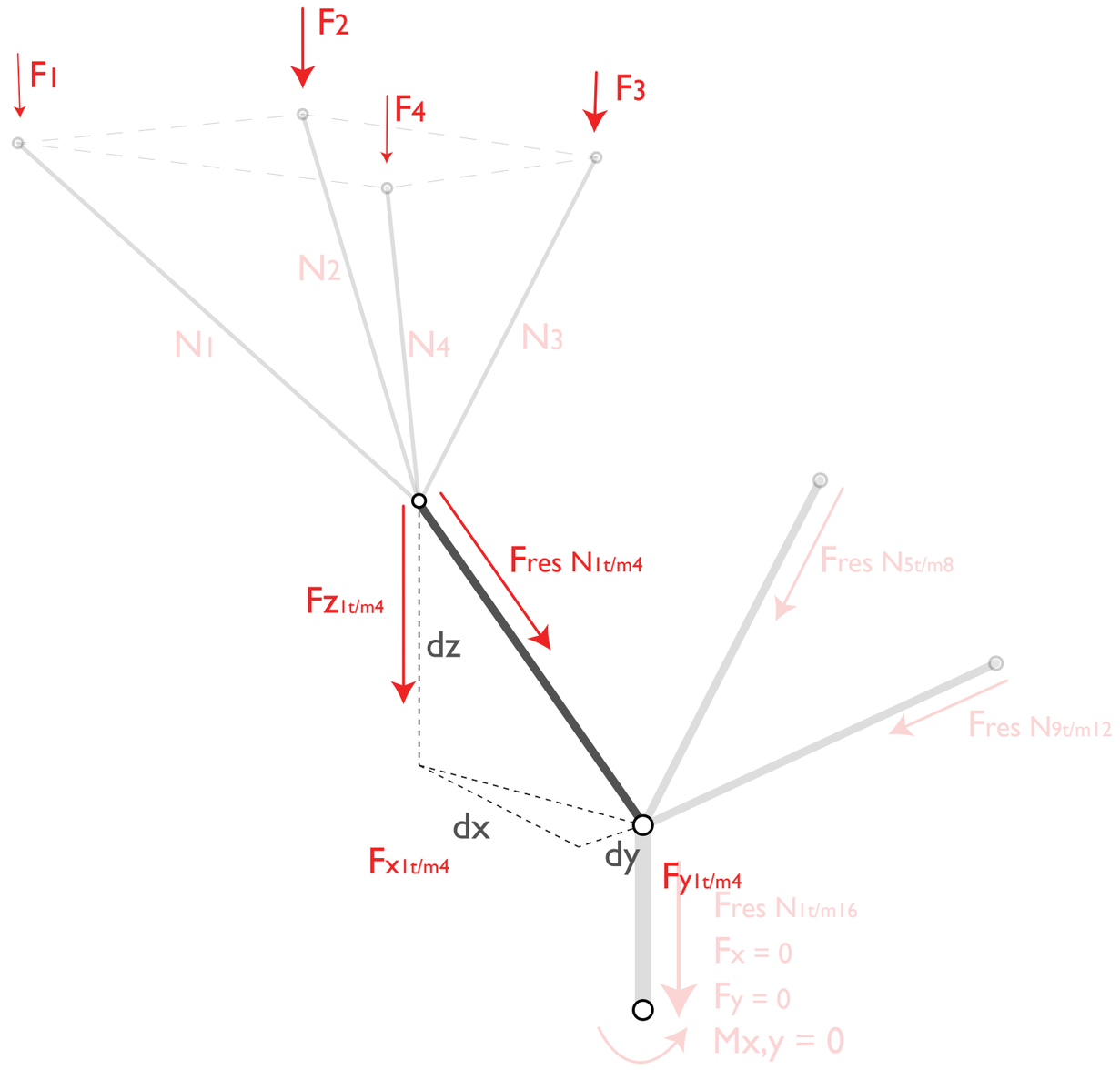


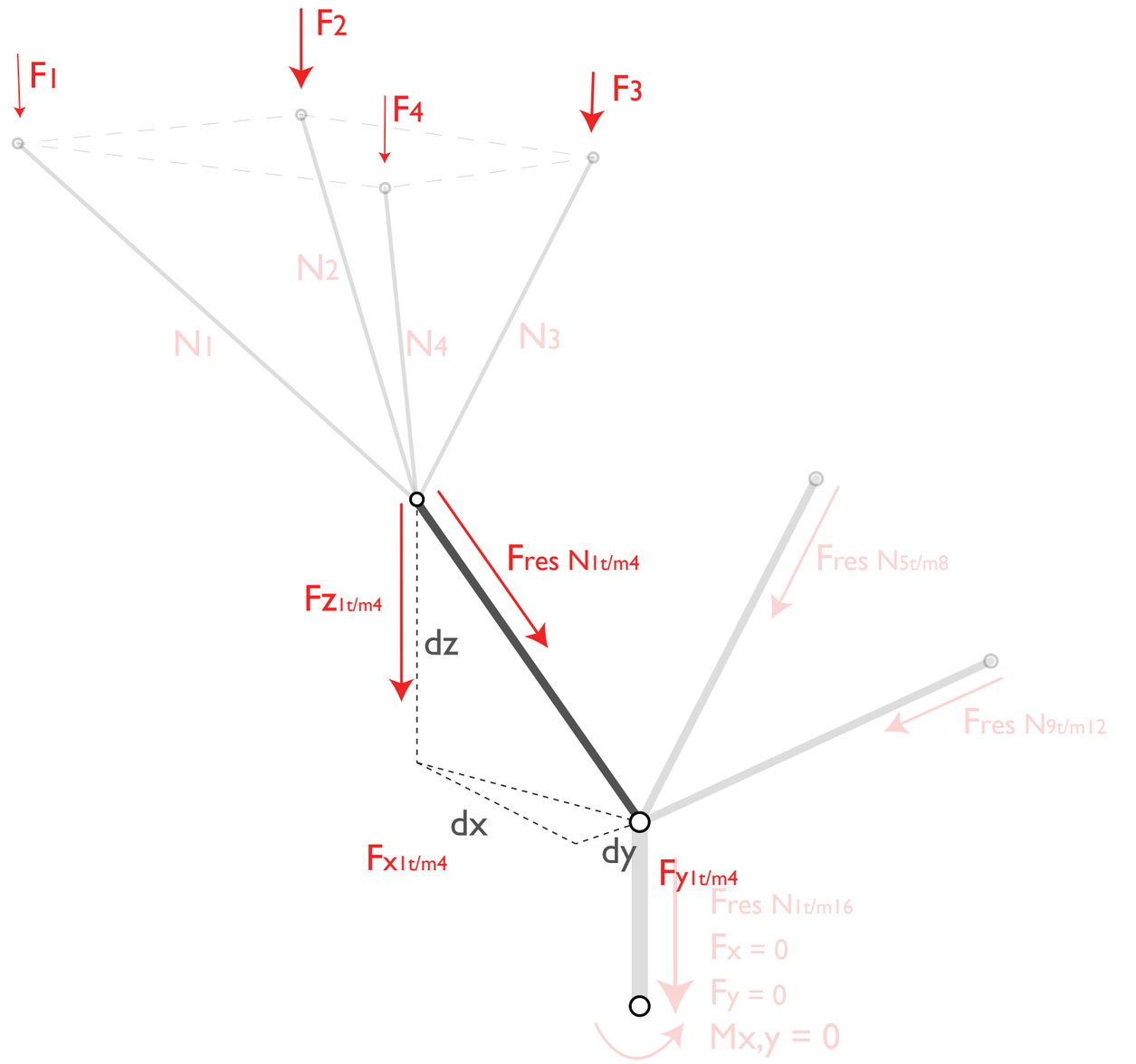


$$\frac{F_{v,i}}{d_z} = \frac{F_{h,i}}{d_i} \qquad \frac{F_{v,i}}{d_z} = \frac{F_{hx,i}}{d_x} \qquad \frac{F_{v,i}}{d_z} = \frac{F_{hy,i}}{d_y}$$

$$F_{h,i} = \frac{d_i \cdot F_{v,i}}{d_z} \qquad F_{hx,i} = \frac{d_x \cdot F_{v,i}}{d_z} \qquad F_{hy,i} = \frac{d_y \cdot F_{v,i}}{d_z}$$


F1
F2
F3
F4
N1
N2
N3
N4
Fres N1t/m4
Fres N5t/m8
Fres N9t/m12
Fres N13t/m16
Fres N1t/m16
Fx = 0
Fy = 0
Mx,y = 0

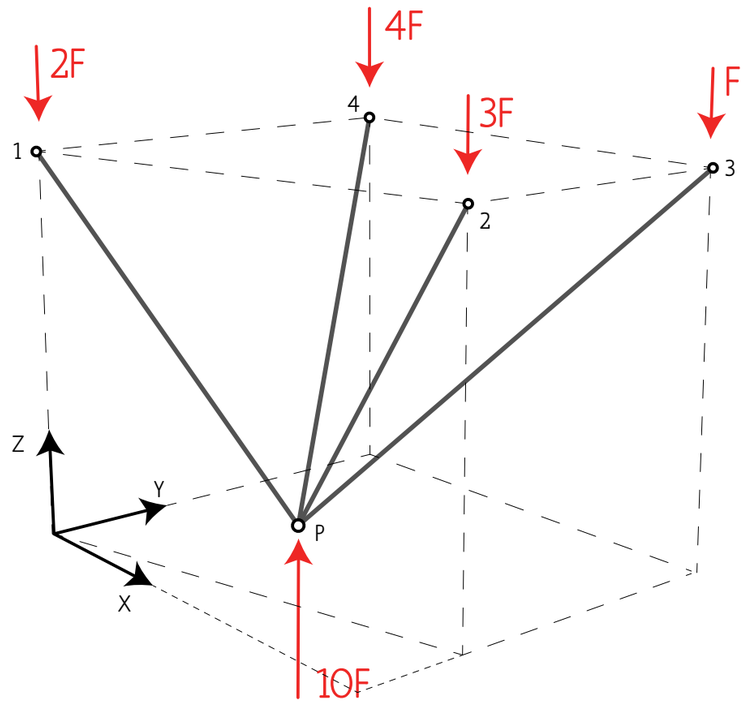

F1
F2
F3
F4
N1
N2
N3
N4
Fres N1t/m4
Fz1t/m4
dz
dx
dy
Fx1t/m4
Fy1t/m4
Fres N5t/m8
Fres N9t/m12
Fres N1t/m16
Fx = 0
Fy = 0
Mx,y = 0

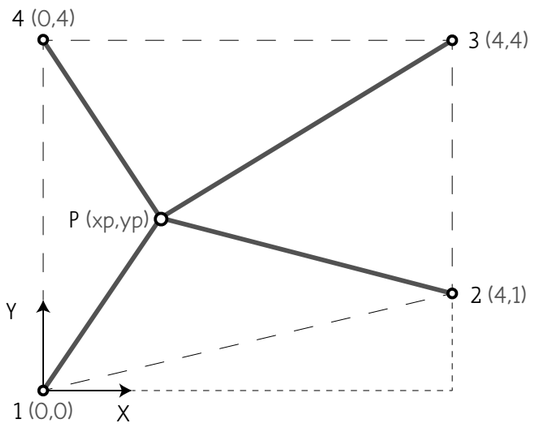


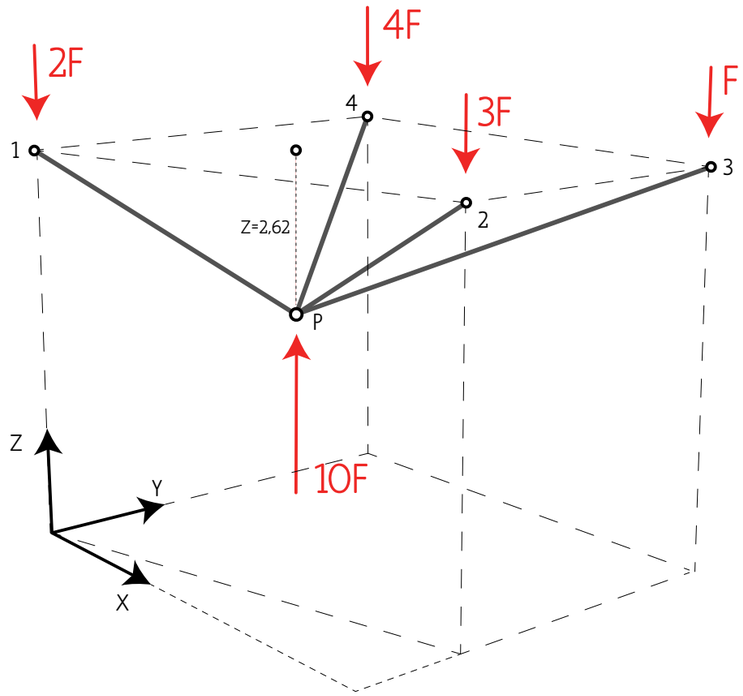
$$\sum_{r=1}^{n} Fx_r = \frac{dx_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$

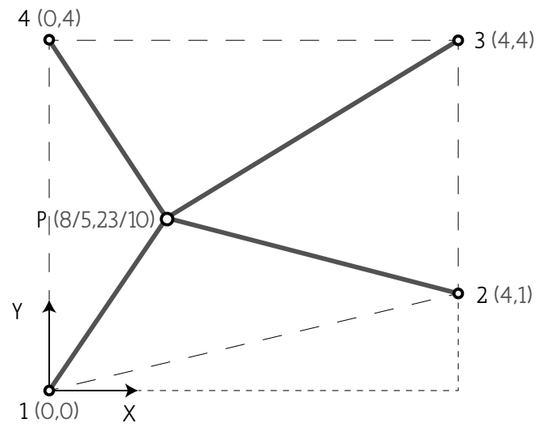
$$\sum_{r=1}^{n} Fy_r = \frac{dy_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$

Example

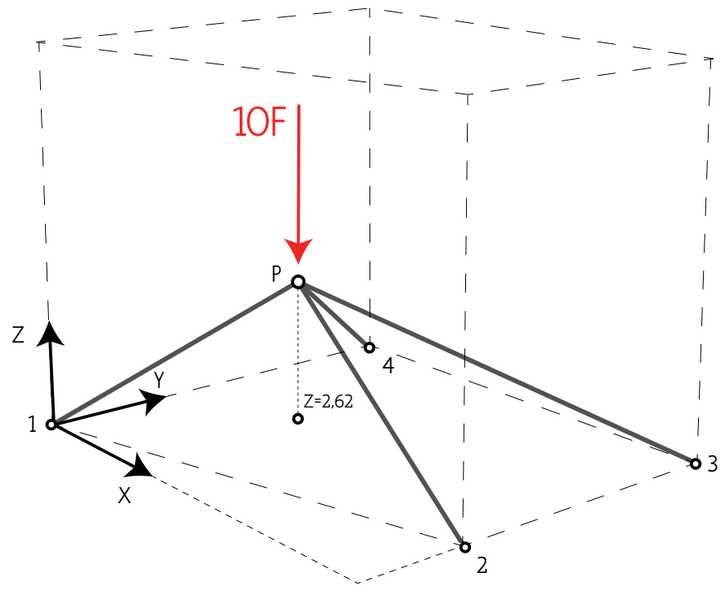


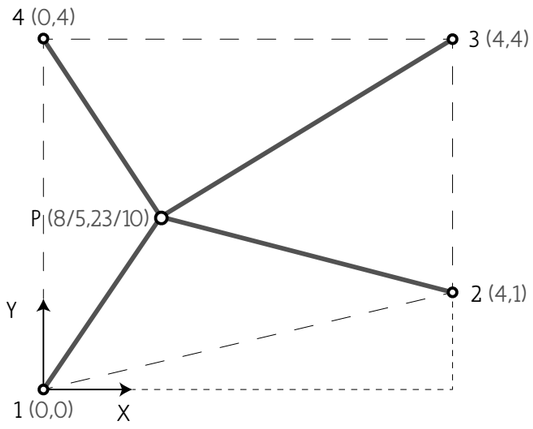


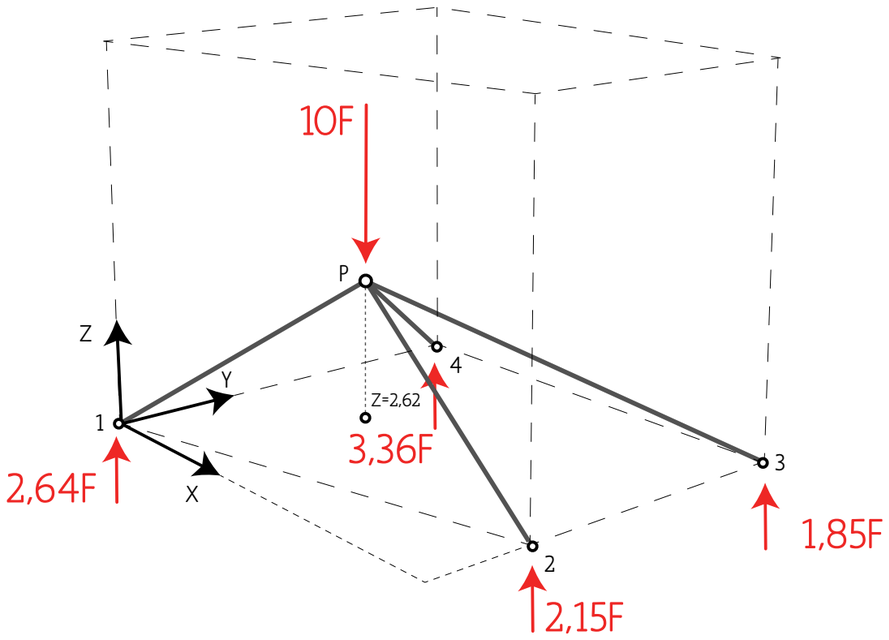

2F
4F
3F
F
1
2
3
4
P
Z=2,62
10F
Z
Y
X

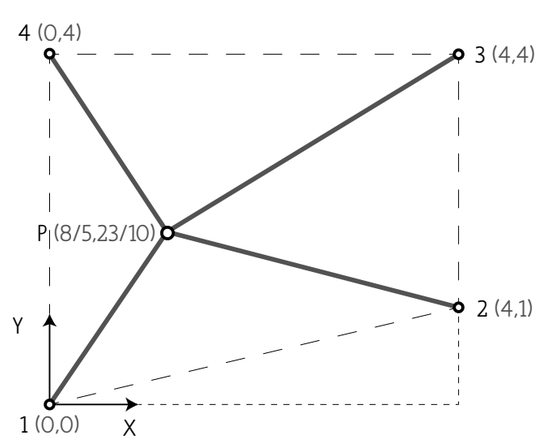

4 (0,4)
3 (4,4)
P (8/5,23/10)
2 (4,1)
1 (0,0)
Y
X

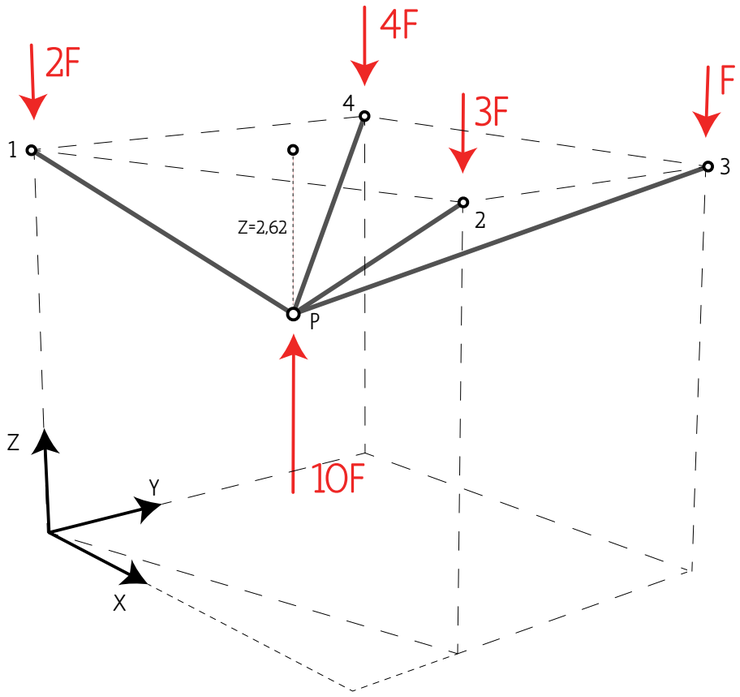
Example

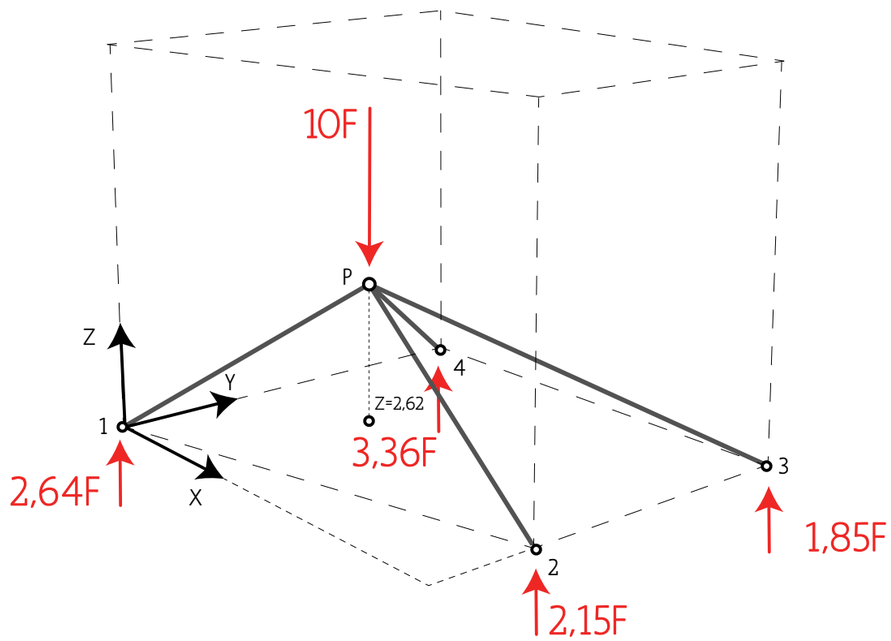


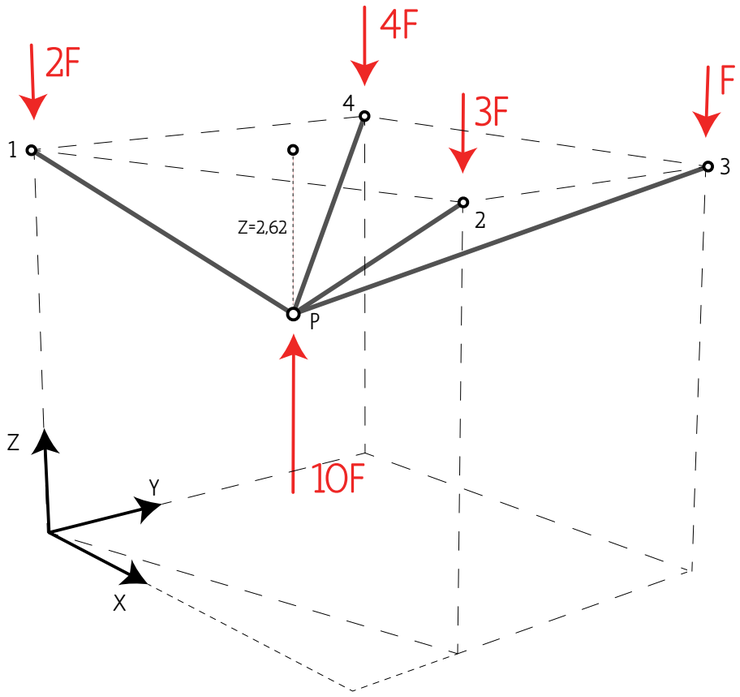



10F
P
Z
Y
X
1
4
Z=2,62
3,36F
2,64F
3
1,85F
2
2,15F

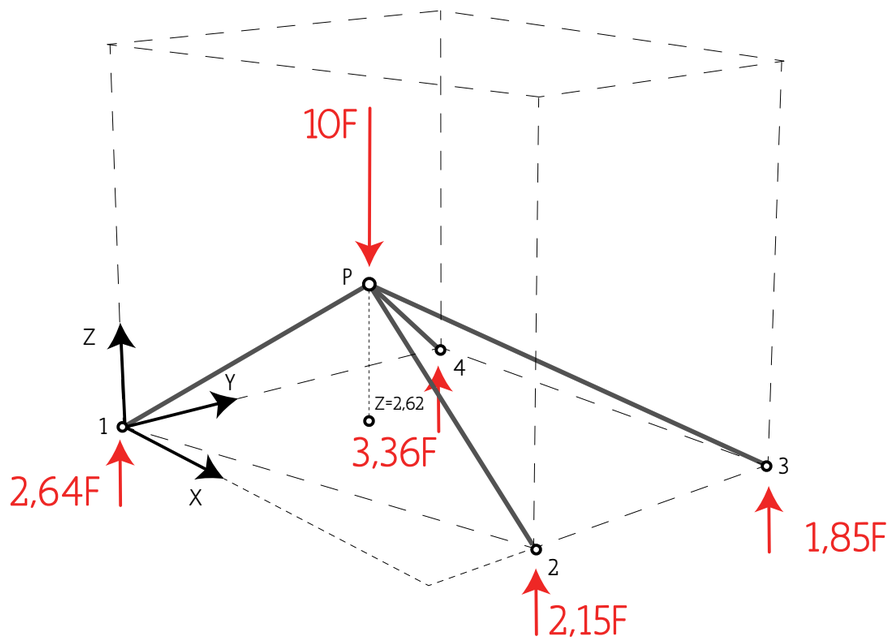

4 (0,4)
3 (4,4)
P (8/5,23/10)
2 (4,1)
Y
1 (0,0)
X


2F
4F
3F
F
1
2
3
4
P
Z=2,62
10F
Z
Y
X

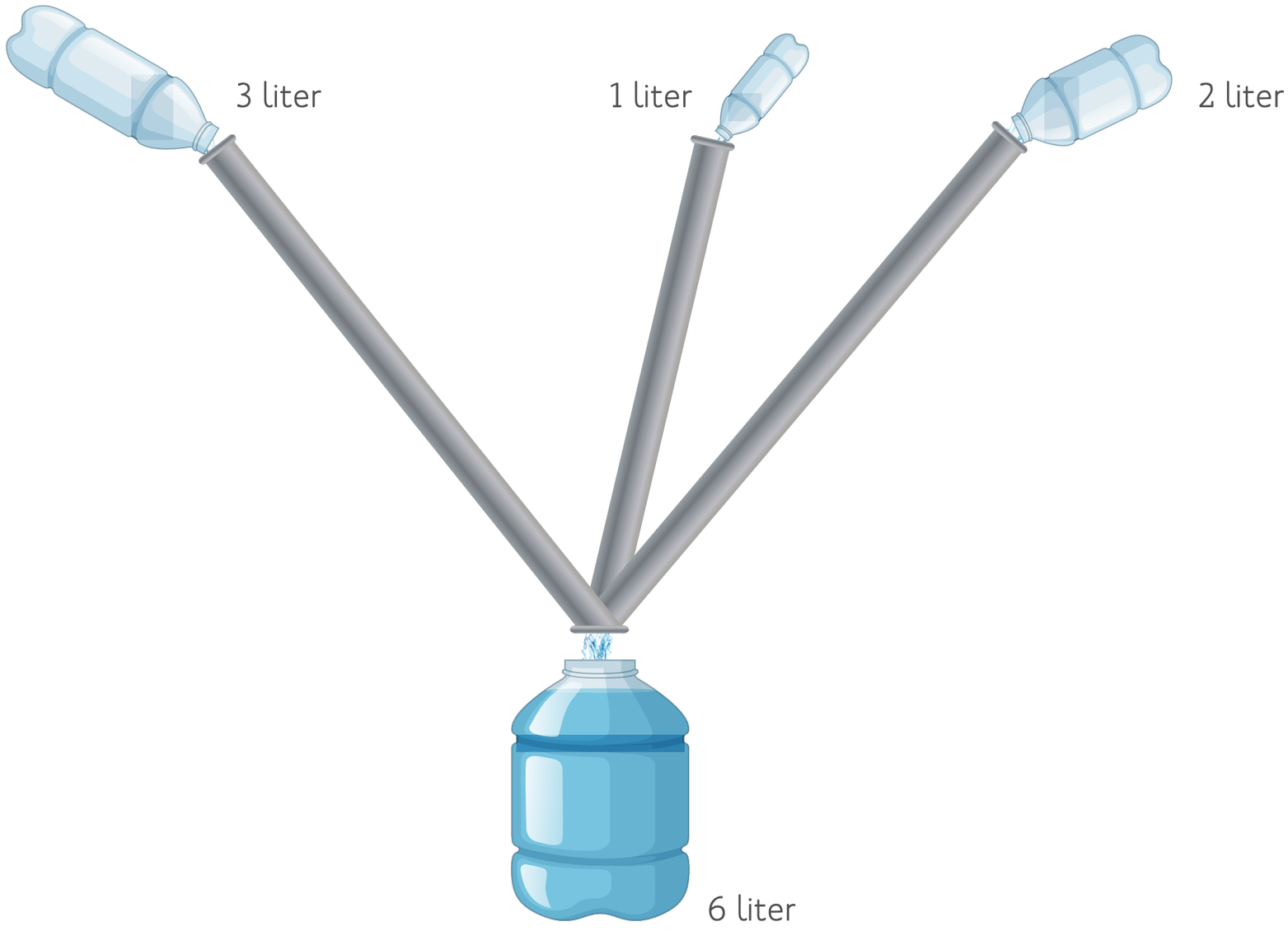

10F
P
Z
Y
X
1
2
3
4
Z=2,62
3,36F
2,64F
1,85F
2,15F

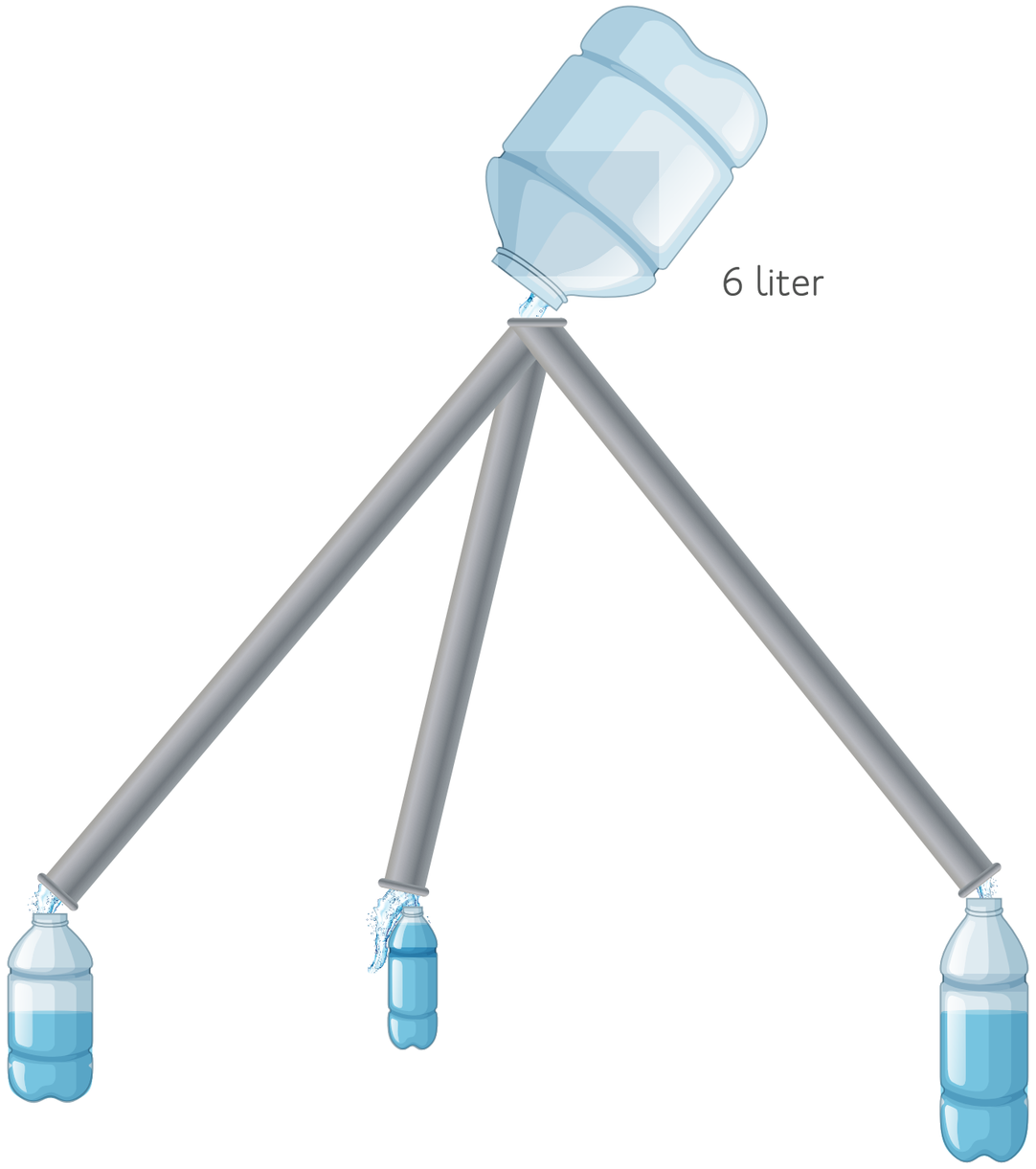


Optimal load path



Optimal distribution of forces


3 liter
1 liter
2 liter
6 liter

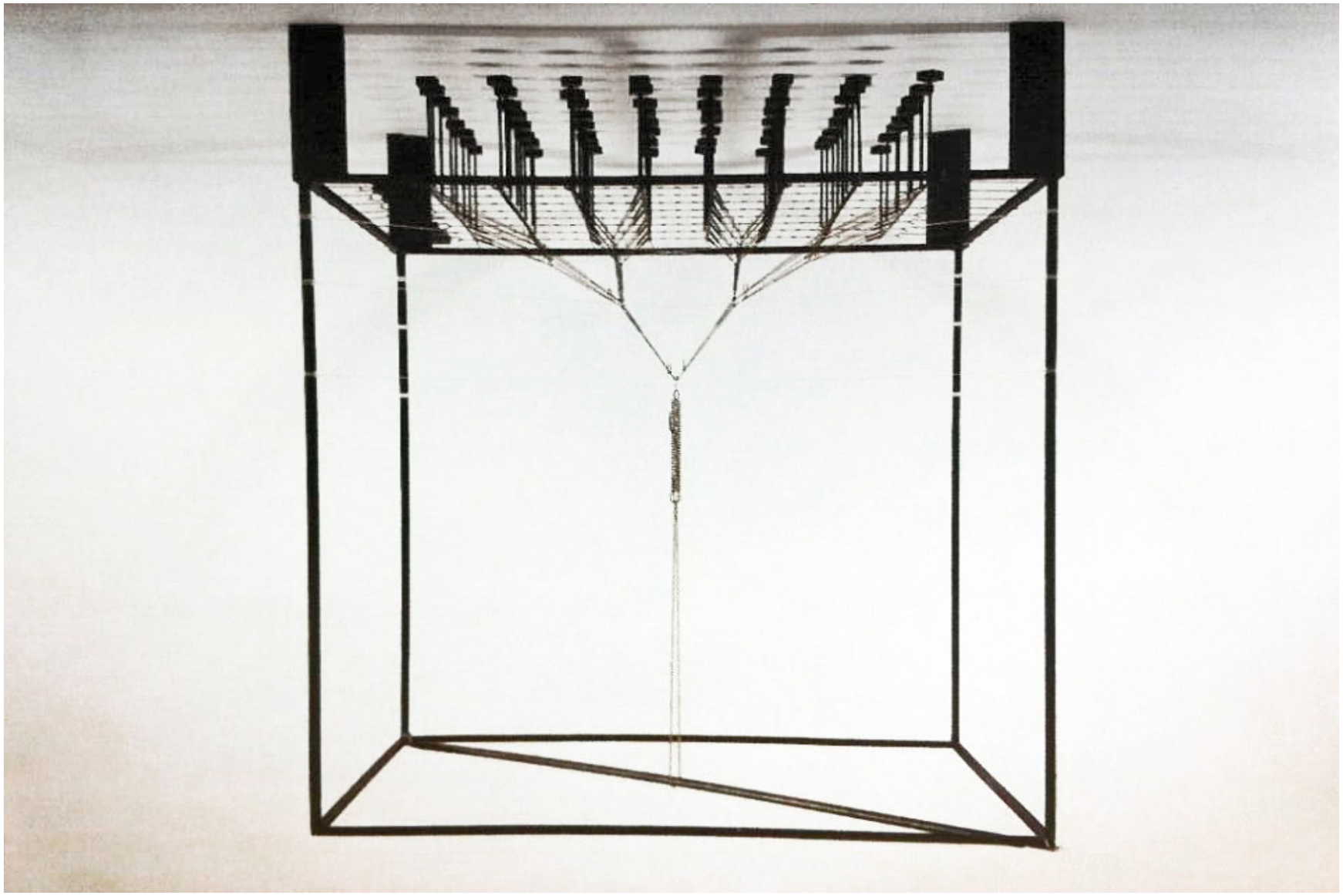

6 liter

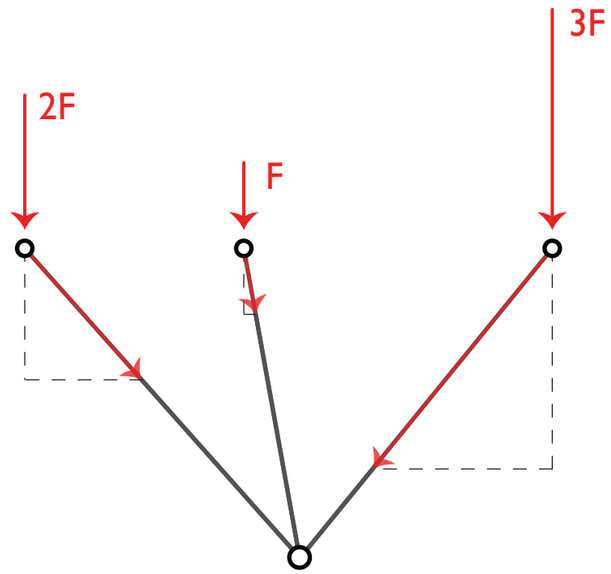
Optimal configuration can be found for a given load case.

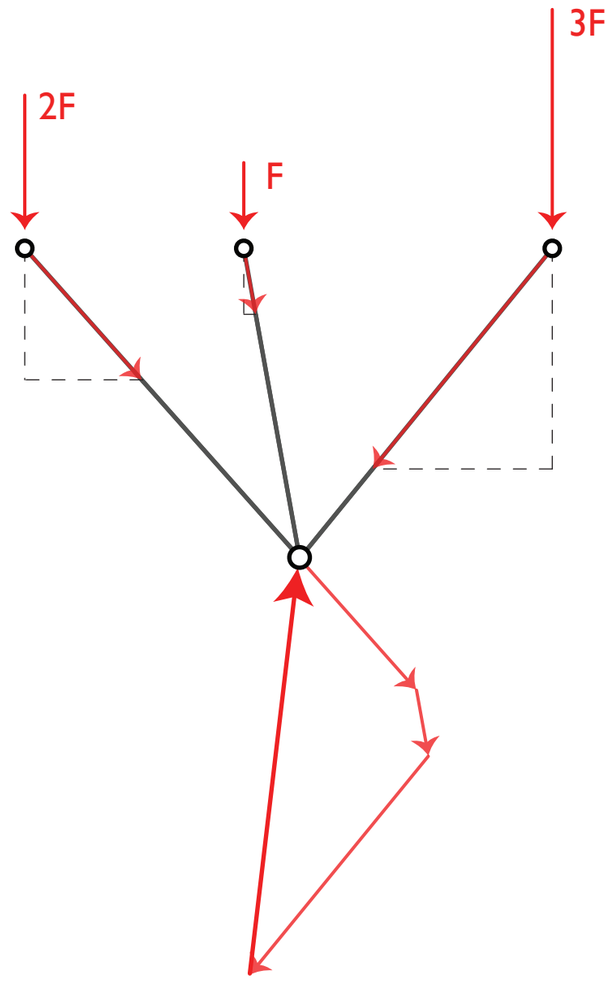
This load case, however, is not the most efficient distribution of forces in the found configuration.

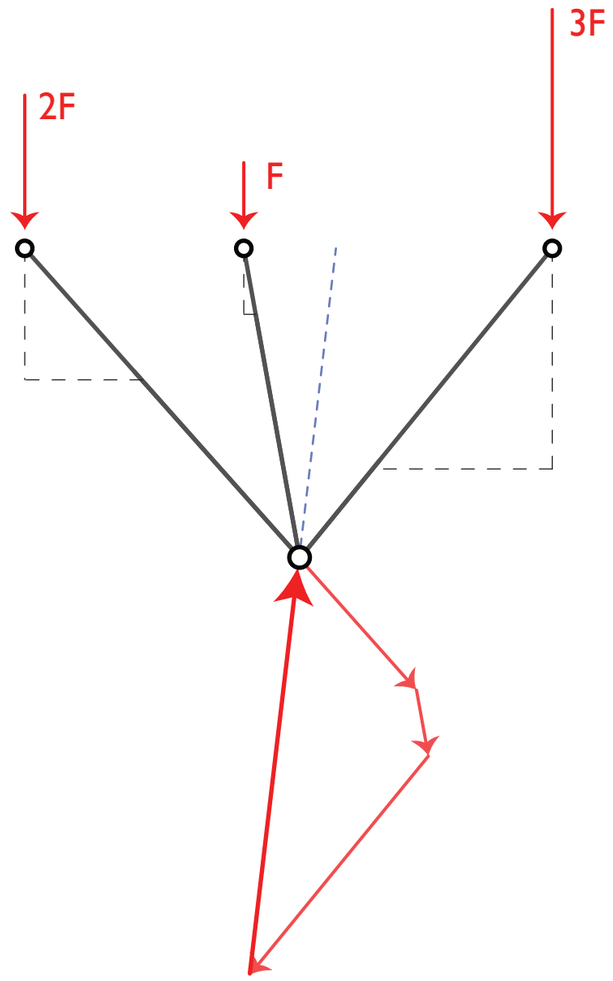
Optimal configuration can be found for a given load case.

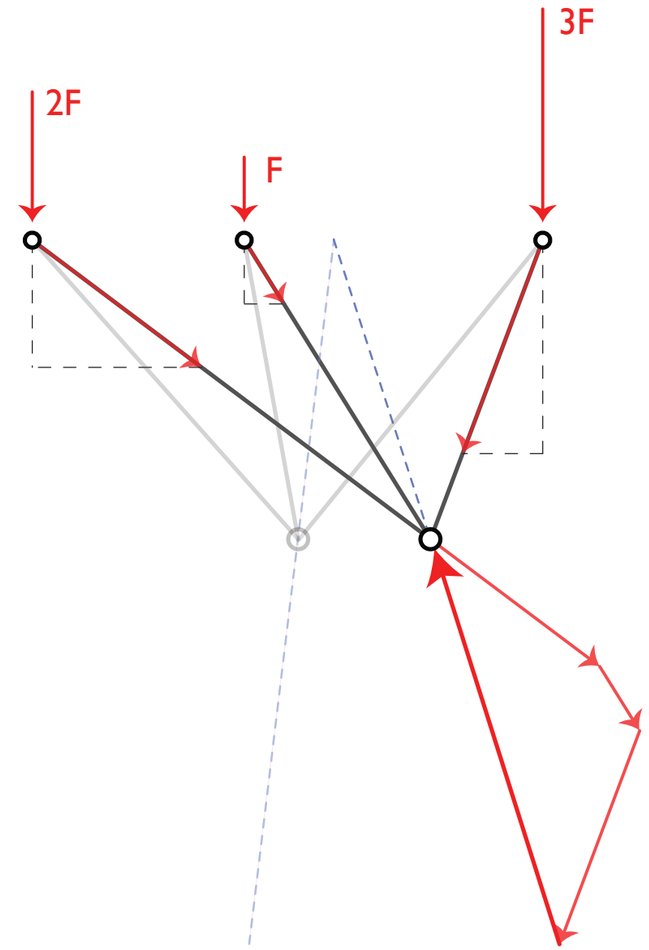
This load case, however, is not the most efficient distribution of forces in the found configuration.

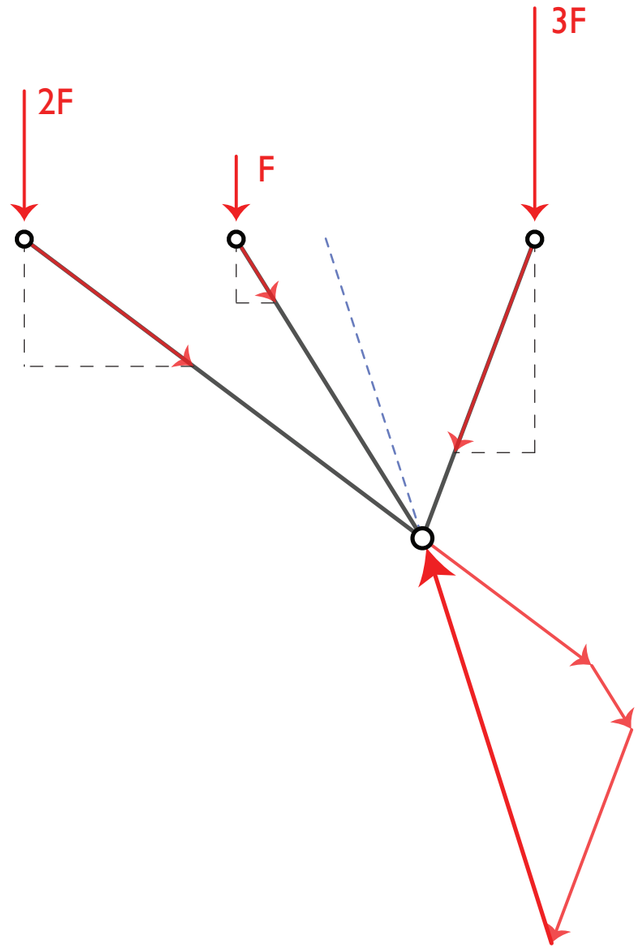



2F
F
3F

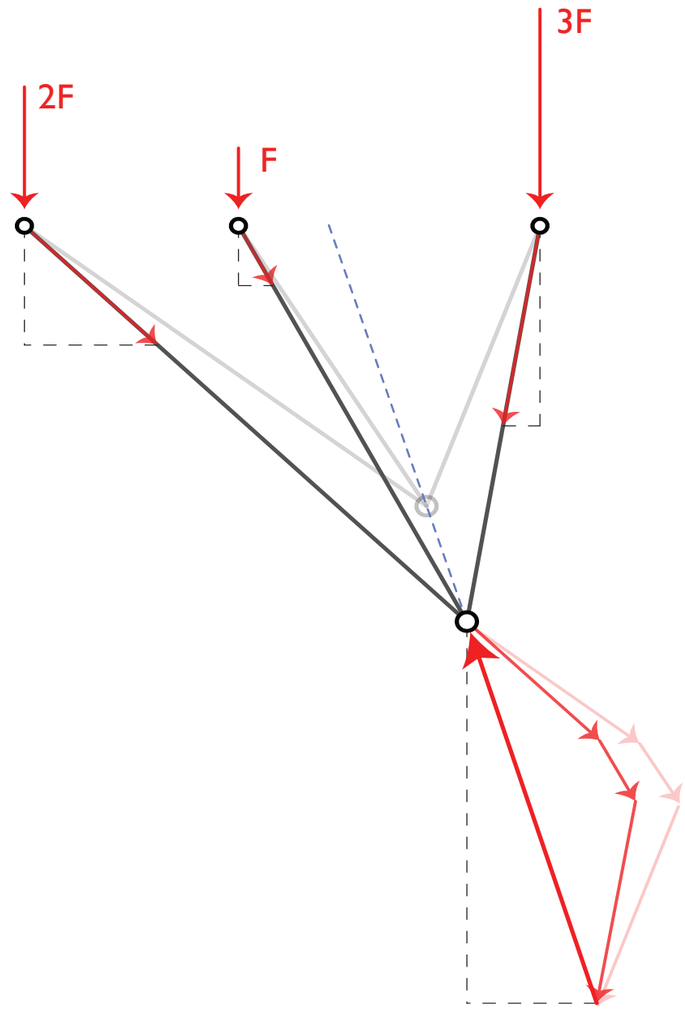

2F
F
3F

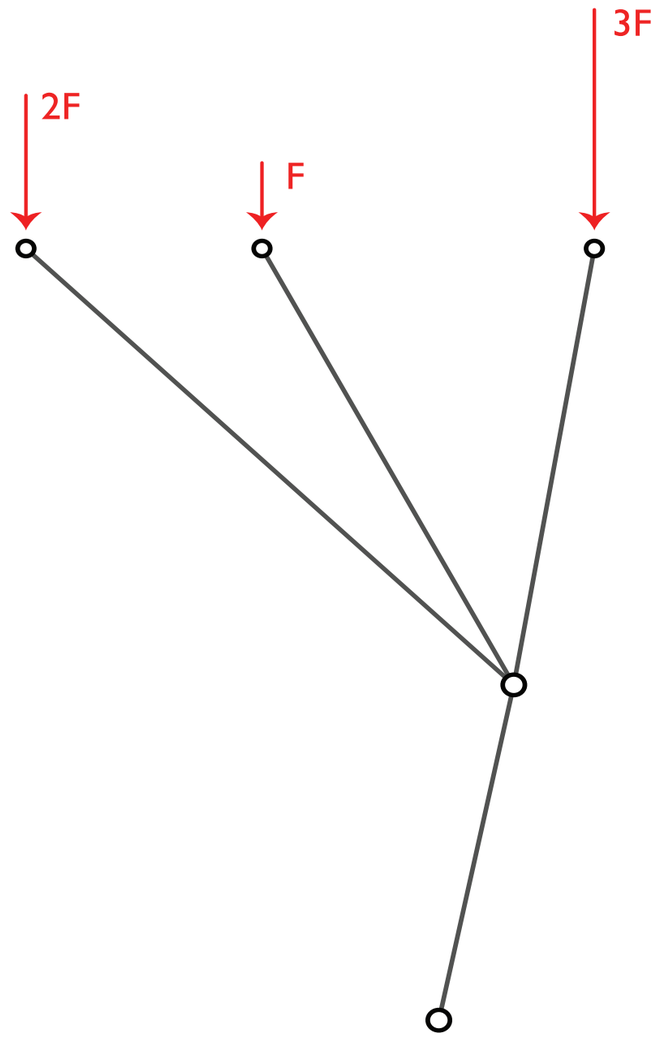

2F
F
3F

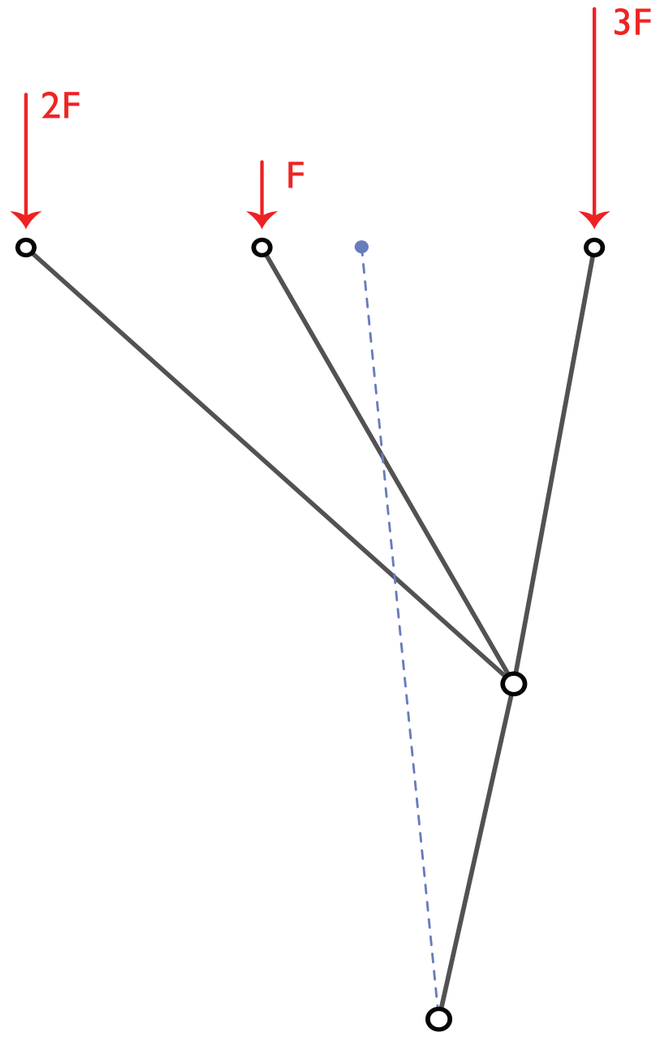

2F
F
3F


2F
F
3F

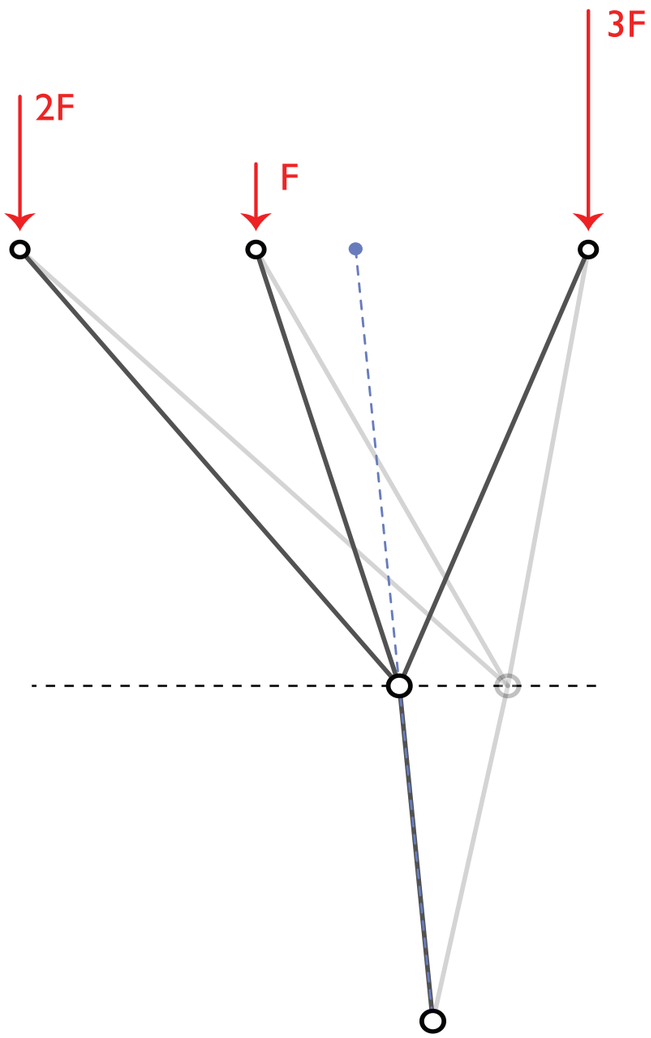
2D working lines: horizontal



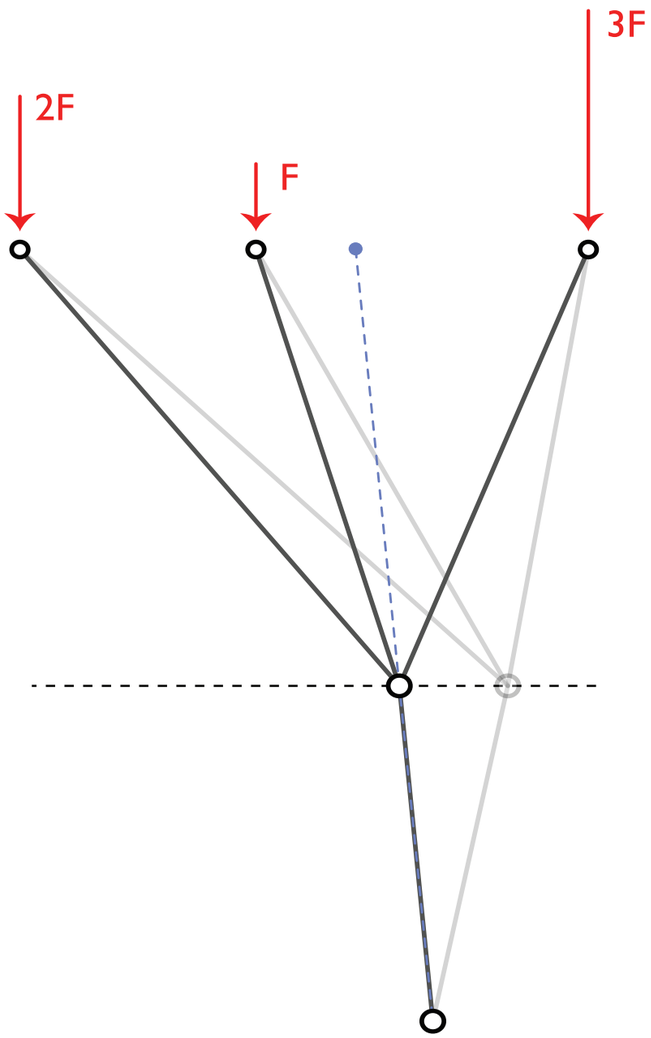

2F
F
3F


2F
F
3F

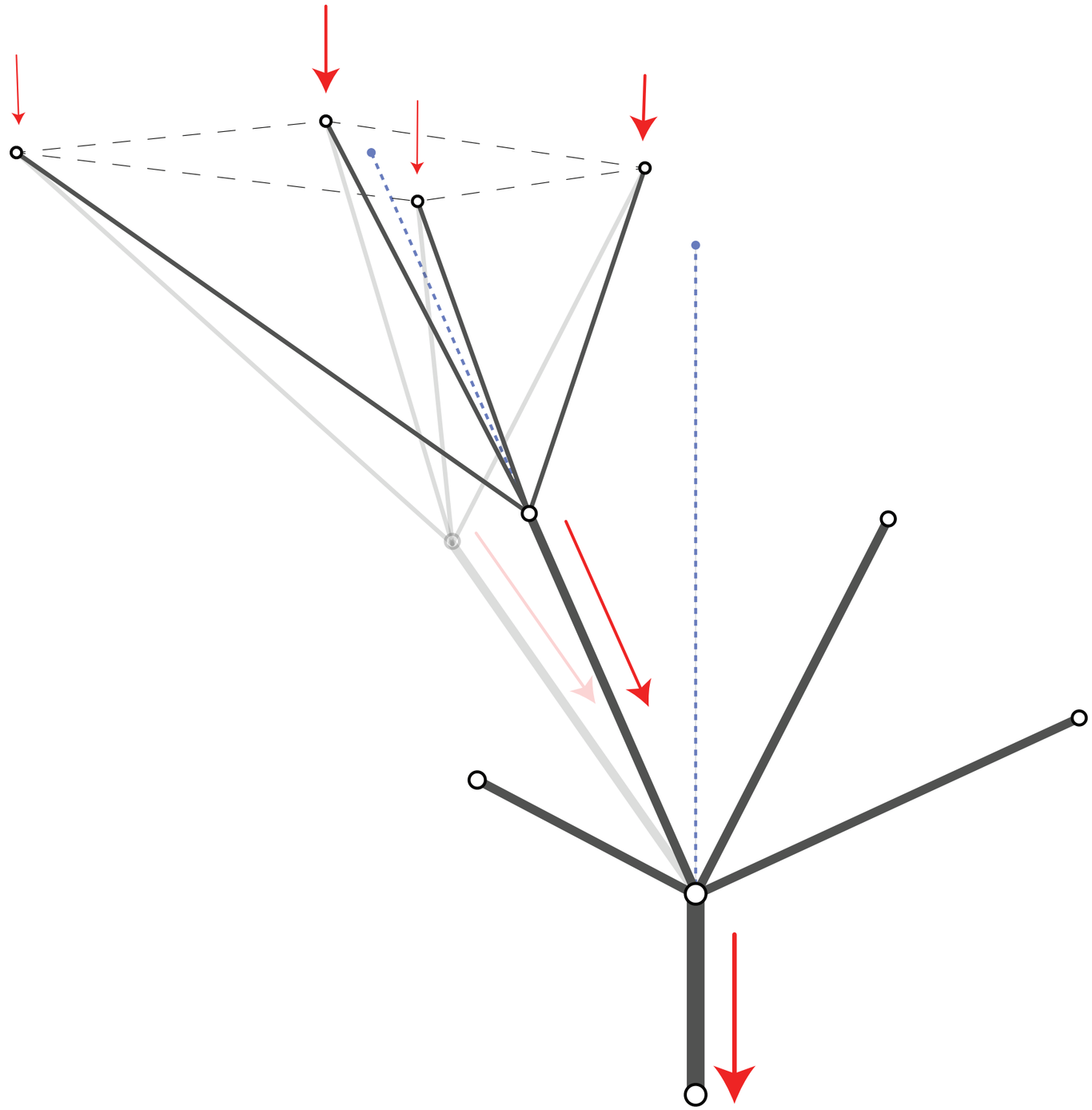
2D working lines: horizontal



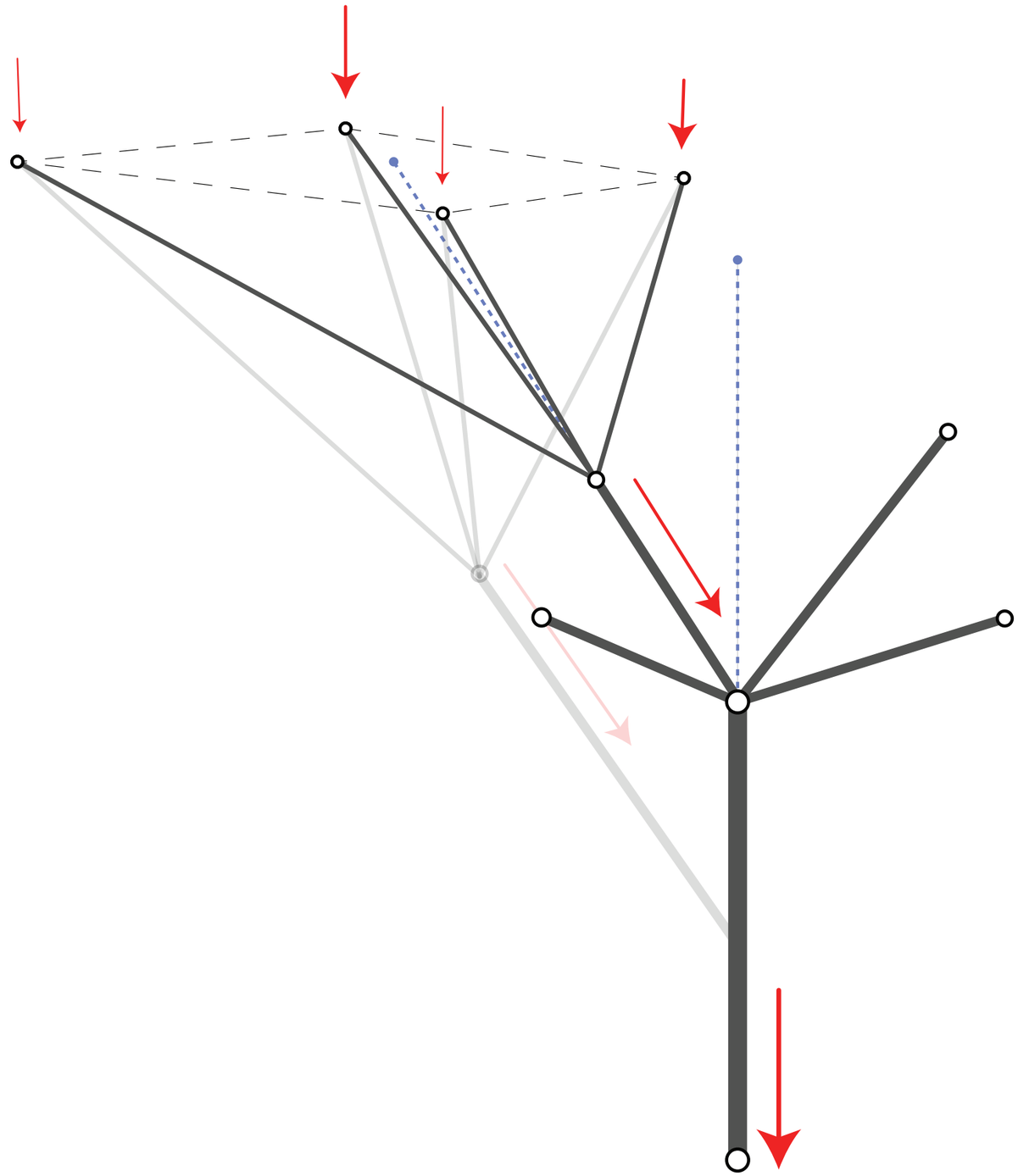
Describe working line



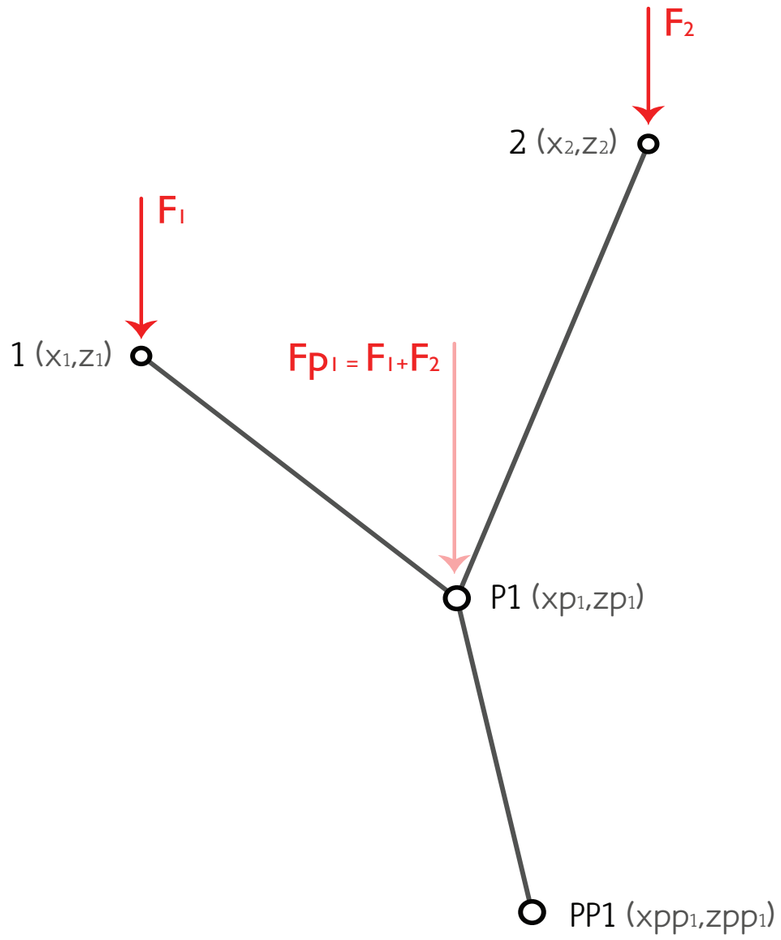
3D working lines: horizontal



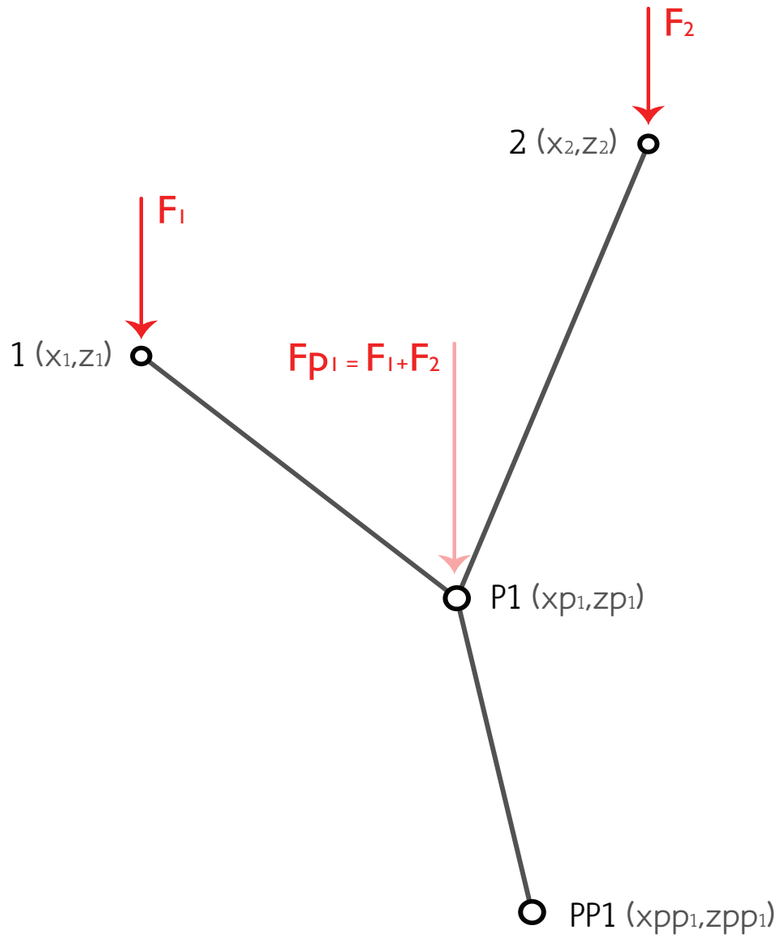
3D working lines: horizontal



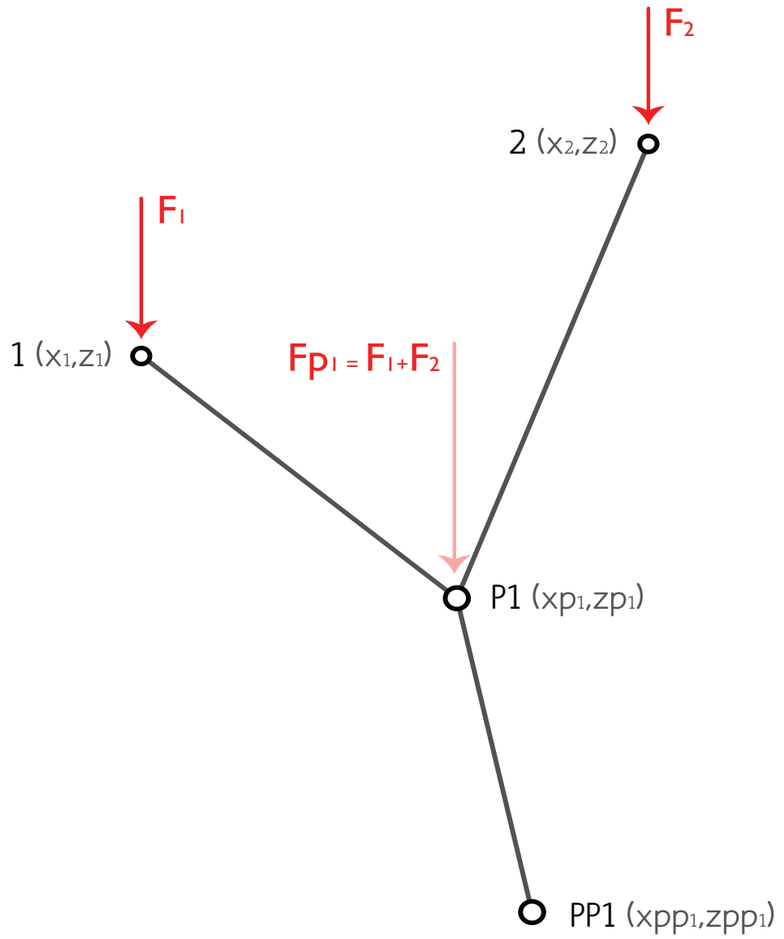
2D working line: non-lineair



2D working line: non-lineair

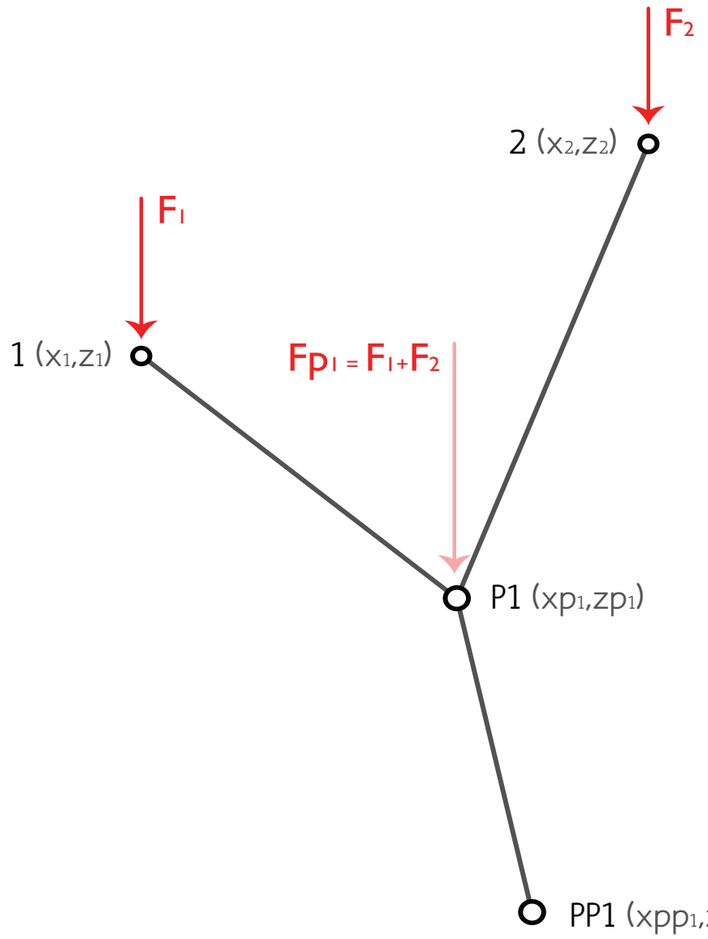


$$\sum_{r=1}^{n} Fx_r = \frac{dx_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$

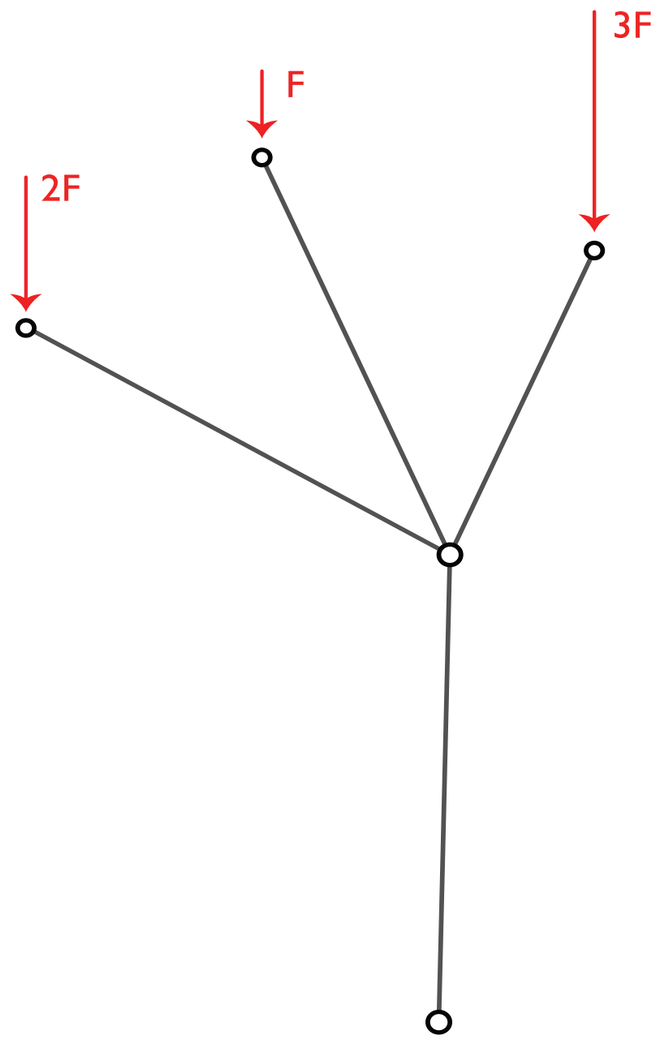


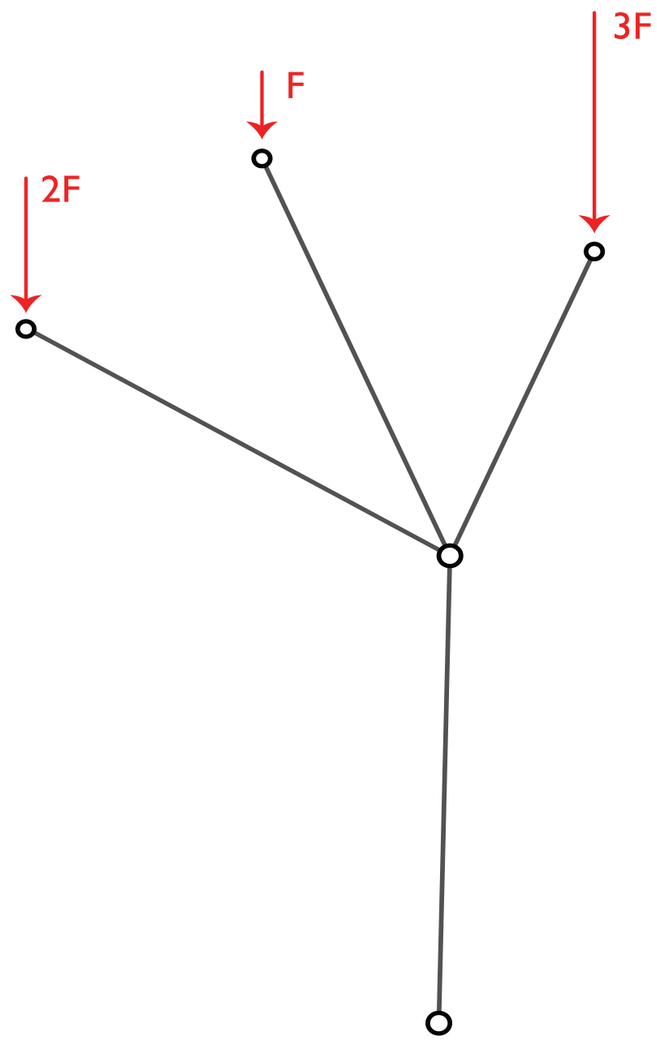
$$\sum_{r=1}^{n} Fx_r = \frac{dx_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$

$$\frac{(x_1 - xp_1) \cdot F_1}{zp_1 - z_1} + \frac{(x_2 - xp_1) \cdot F_2}{zp_1 - z_2} = \frac{(xp_1 - xpp_1) \cdot Fp_1}{zpp_1 - zp_1}$$

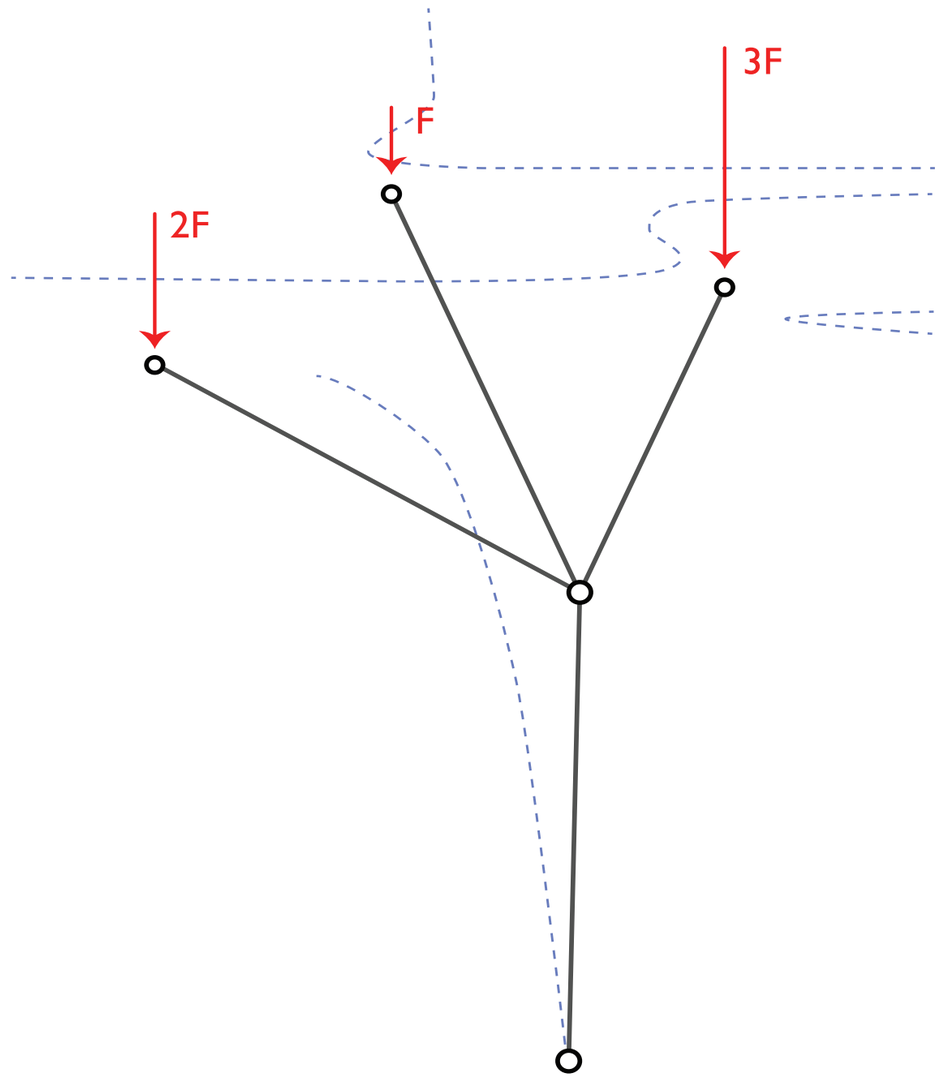


$$xp_1 = \frac{F_1 \cdot x_1 \cdot z_2 \cdot zp_1 - F_1 \cdot x_1 \cdot z_2 \cdot zpp_1 - F_1 \cdot x_1 \cdot zp_1^2 + F_1 \cdot x_1 \cdot zp_1 \cdot zpp_1 + F_2 \cdot x_2 \cdot z_1 \cdot zp_1 - F_2 \cdot x_2 \cdot z_1 \cdot zpp_1 - F_2 \cdot x_2 \cdot zp_1^2 + F_2 \cdot x_2 \cdot zp_1 \cdot zpp_1 + Fp_1 \cdot xpp_1 \cdot z_1 \cdot z_2 - Fp_1 \cdot xpp_1 \cdot z_1 \cdot zp_1 - Fp_1 \cdot xpp_1 \cdot z_2 \cdot zp_1 + Fp_1 \cdot xpp_1 \cdot zp_1^2}{F_1 \cdot z_2 \cdot zp_1 - F_1 \cdot z_2 \cdot zpp_1 - F_1 \cdot zp_1^2 + F_1 \cdot zp_1 \cdot zpp_1 + F_2 \cdot z_1 \cdot zp_1 - F_2 \cdot z_1 \cdot zpp_1 - F_2 \cdot zp_1^2 + F_2 \cdot zp_1 \cdot zpp_1 + Fp_1 \cdot z_1 \cdot z_2 - Fp_1 \cdot z_1 \cdot zp_1 - Fp_1 \cdot z_2 \cdot zp_1 + Fp_1 \cdot zp_1^2}$$

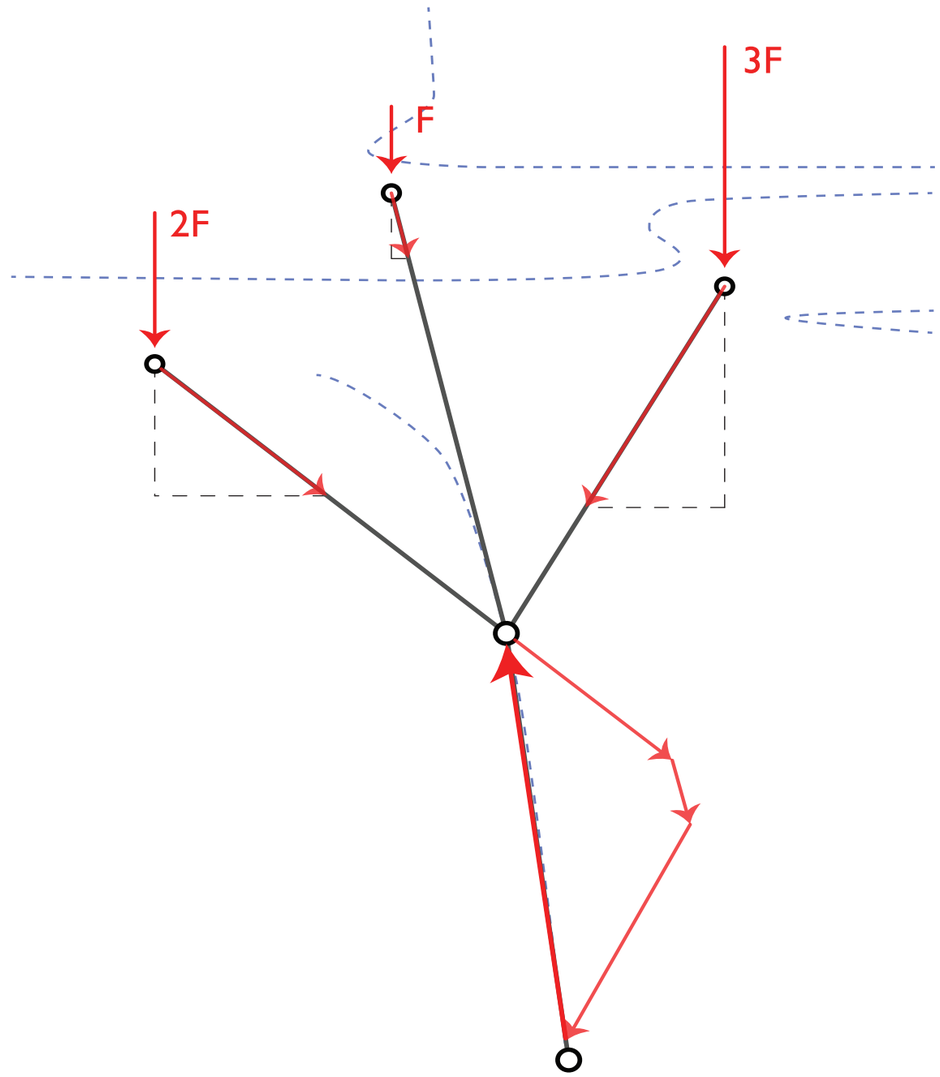

3F
F
2F



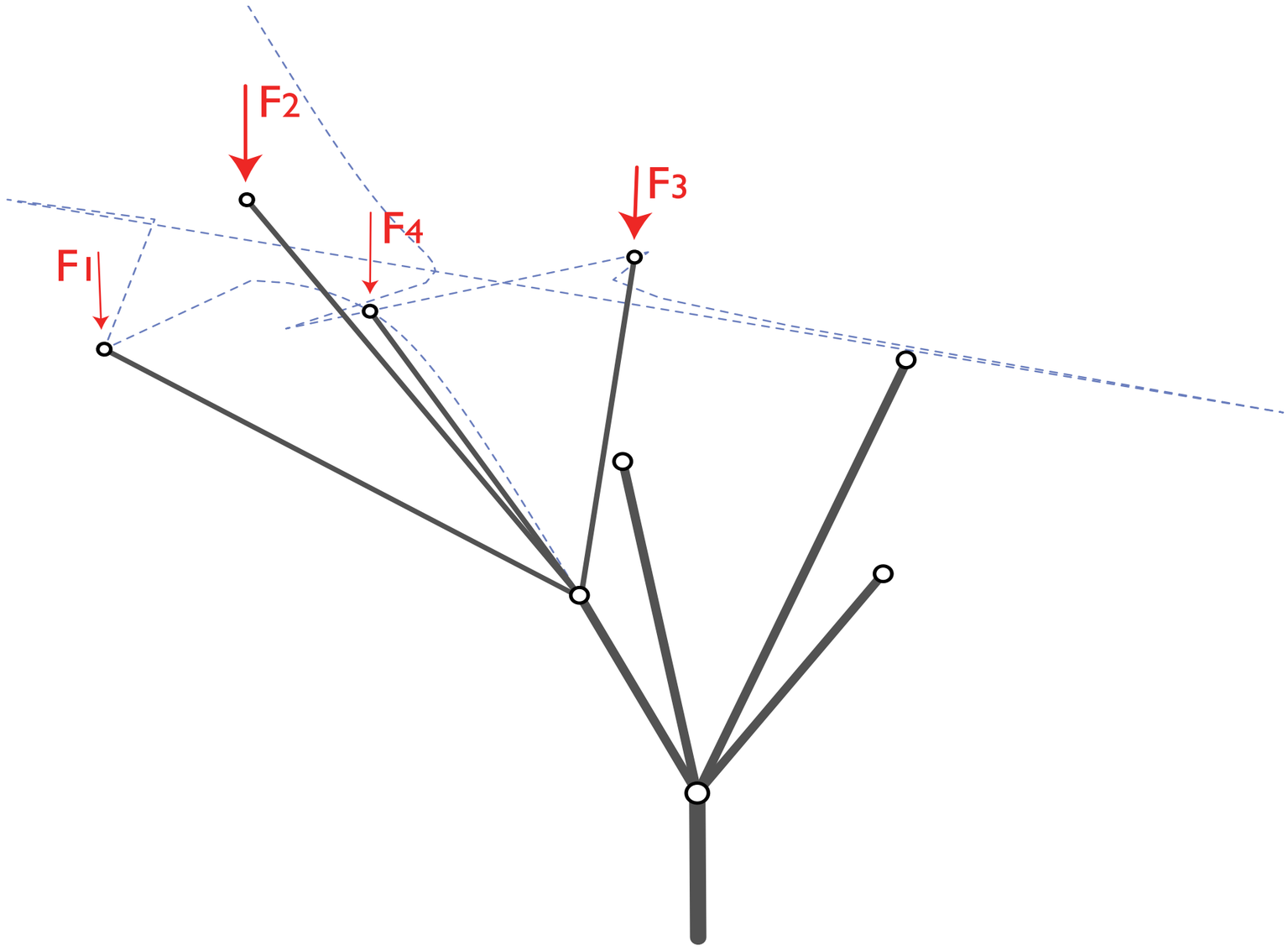
$$\sum_{r=1}^{n} Fx_r = \frac{dx_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$



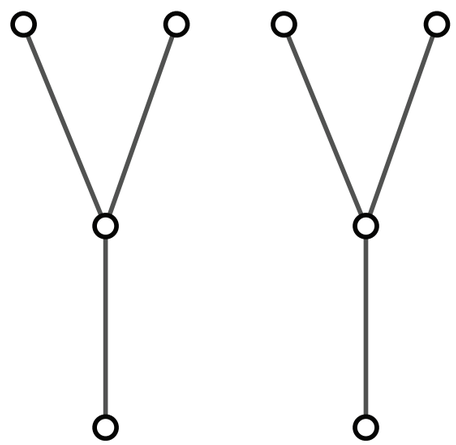
$$\sum_{r=1}^{n} Fx_r = \frac{dx_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$



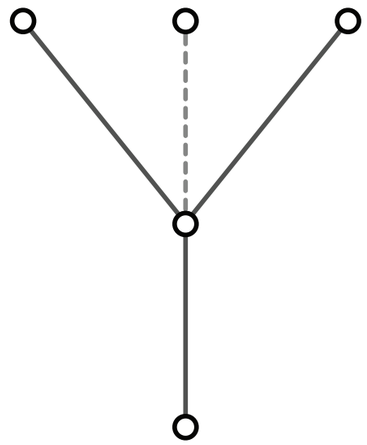
$$\sum_{r=1}^{n} Fx_r = \frac{dx_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$



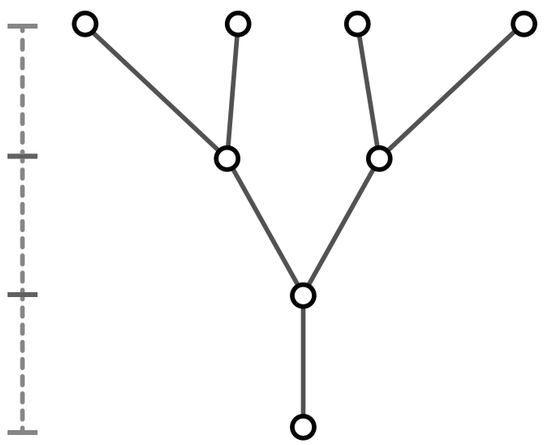
$$\sum_{r=1}^{n} Fx_r = \frac{dx_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}} \qquad \sum_{r=1}^{n} Fy_r = \frac{dy_{next\ member} \cdot \sum_{r=1}^{n} Fz_r}{dz_{next\ member}}$$



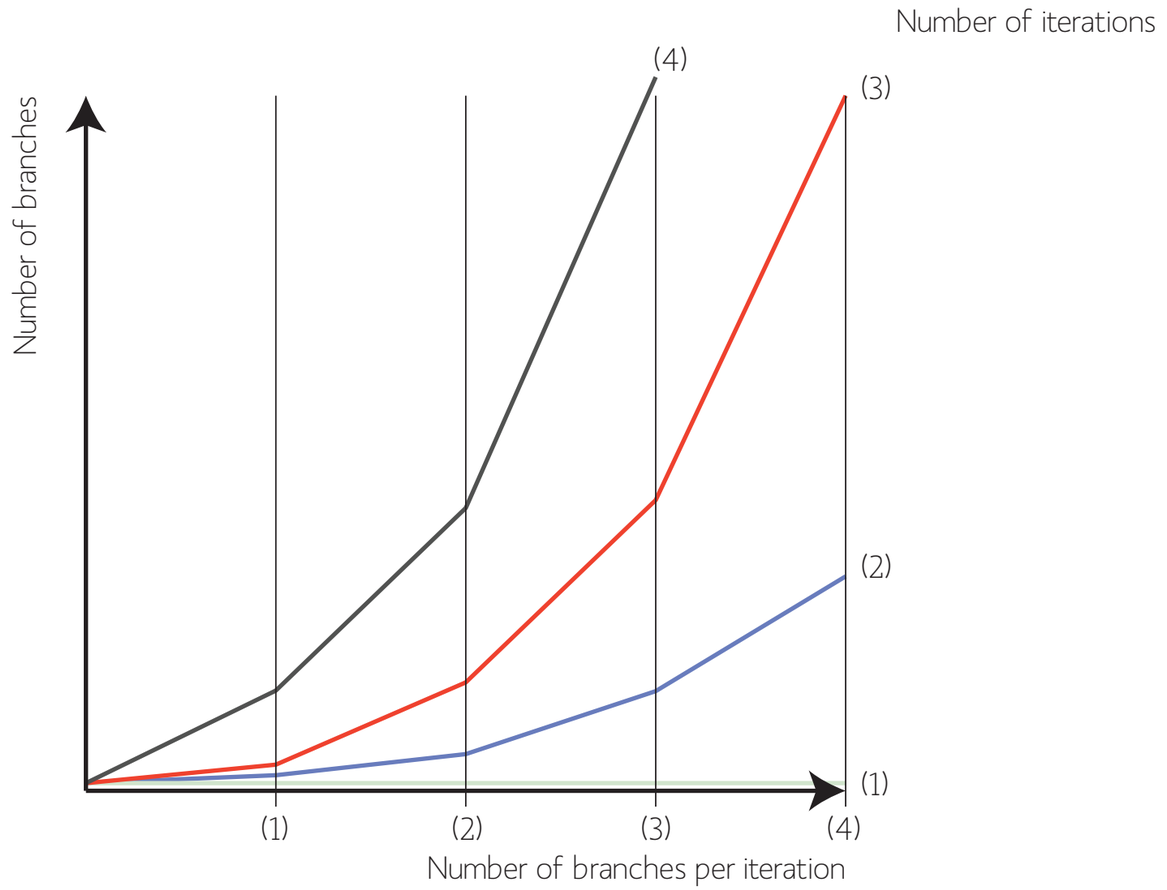
number of trees

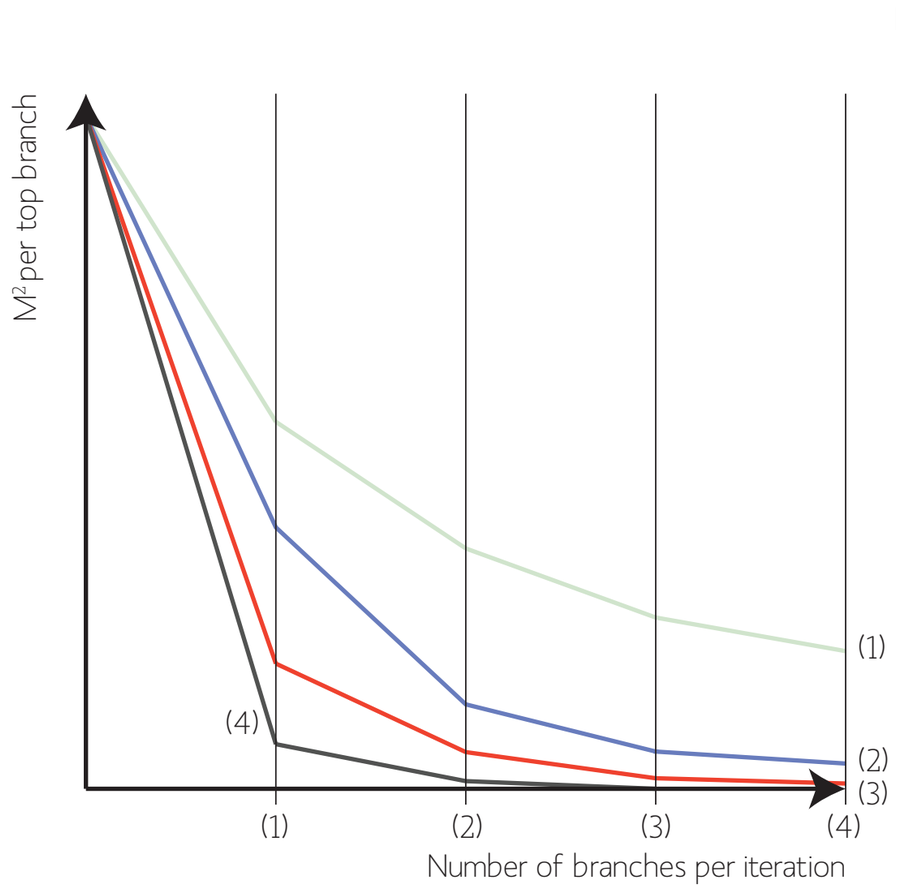


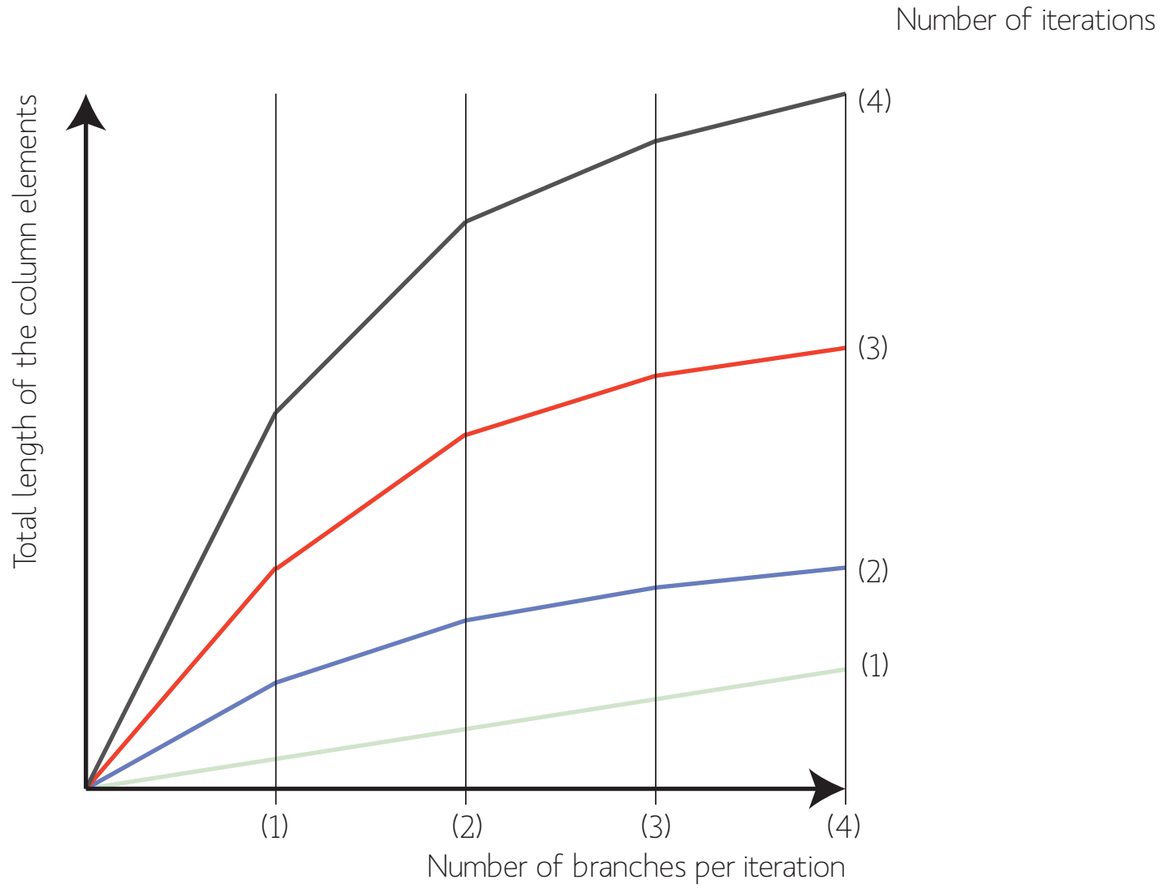
number of branches

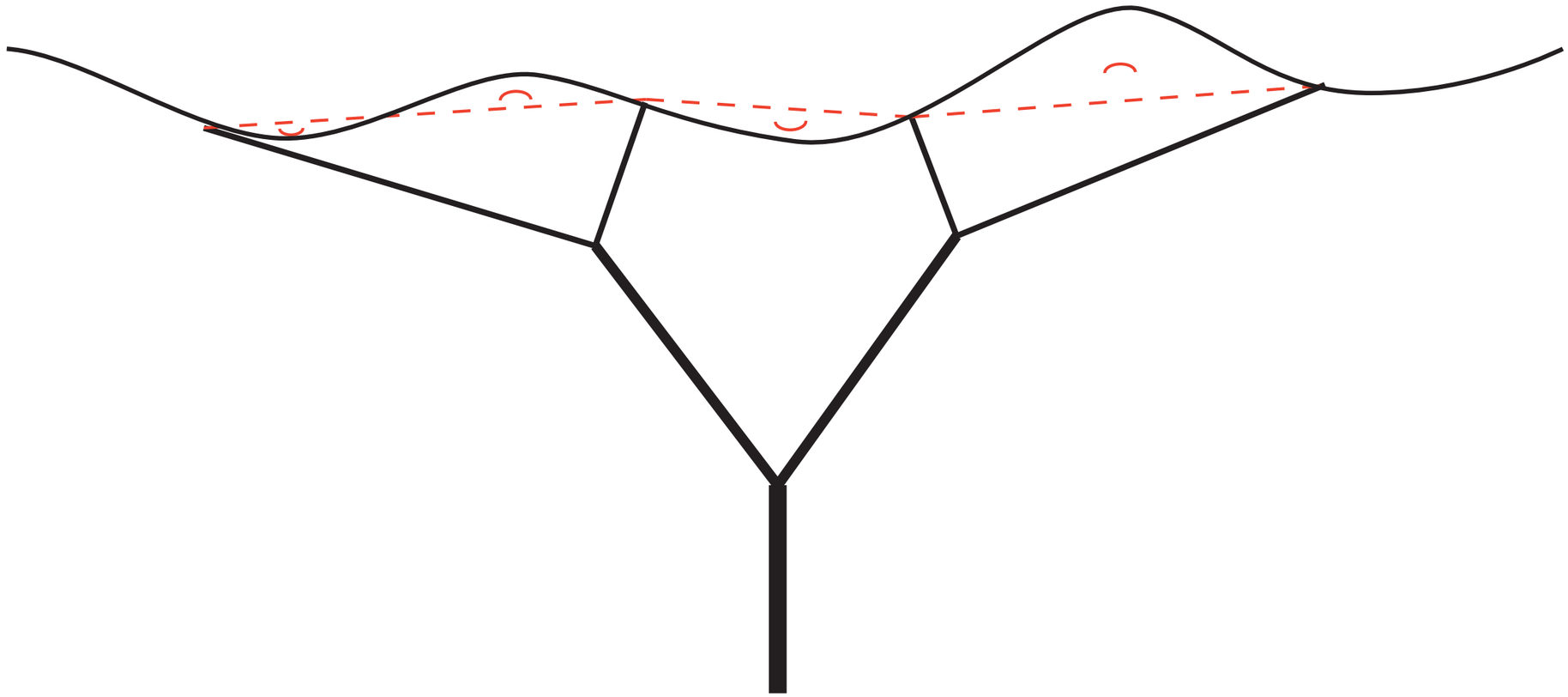


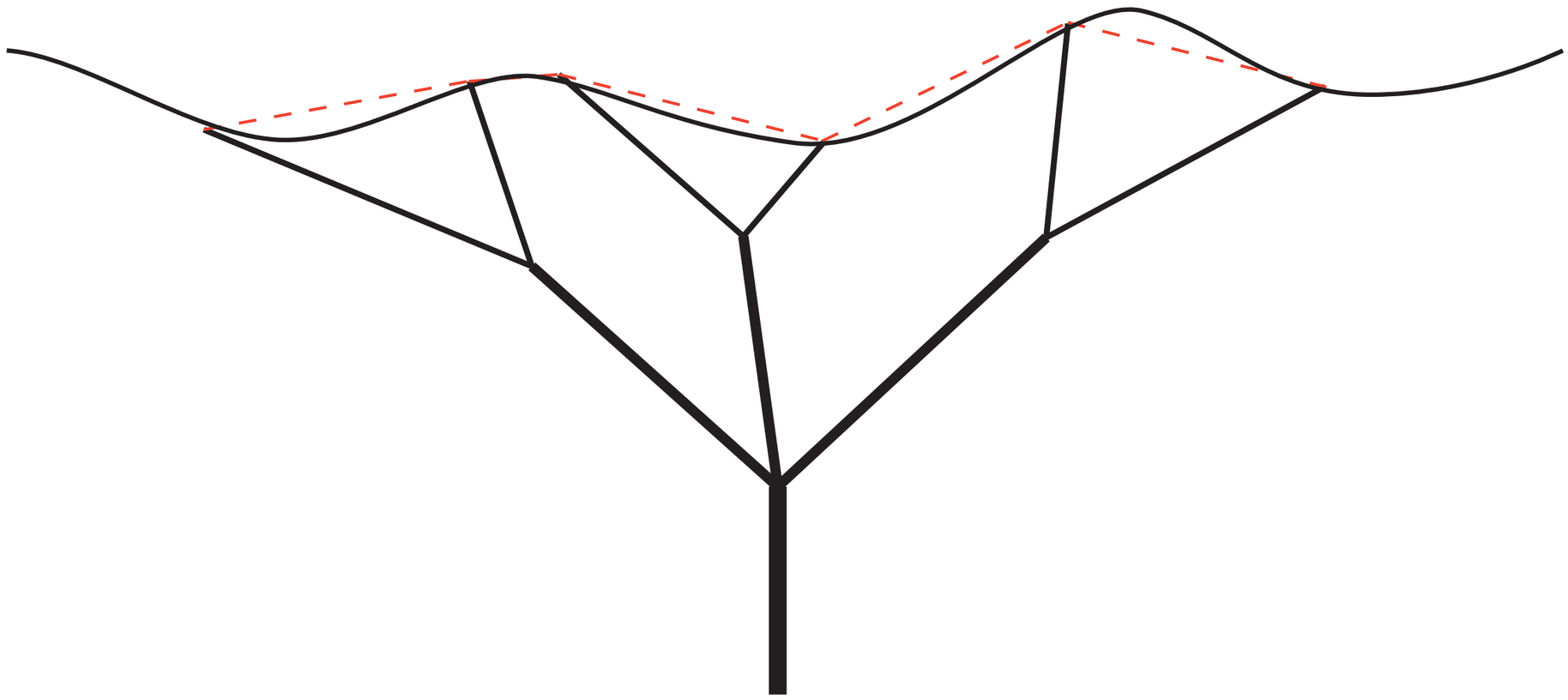
number of iterations

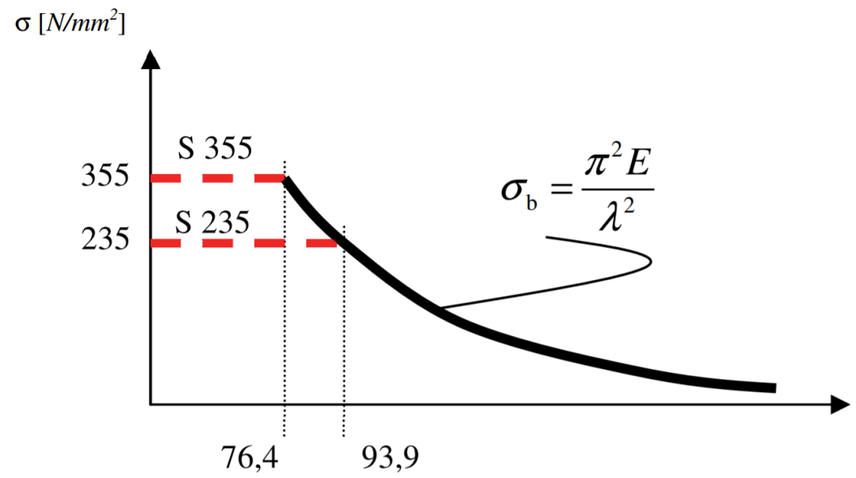

Number of iterations
(4)
(3)
(2)
(1)
Number of branches
(1)
(2)
(3)
(4)
Number of branches per iteration

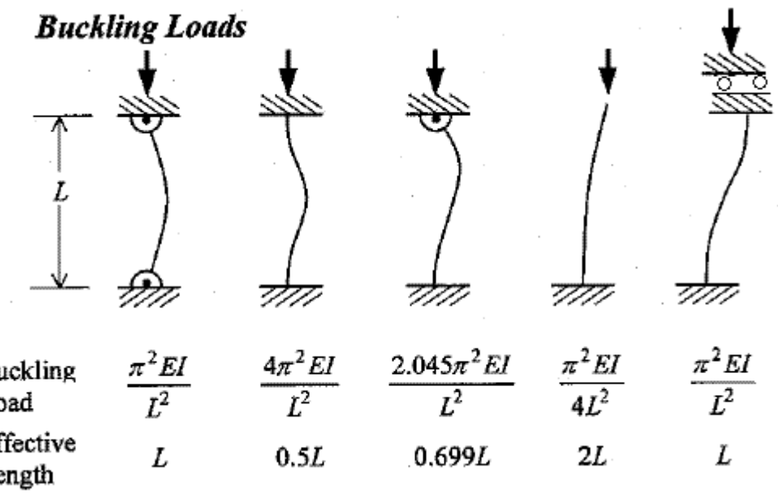

Number of iterations
M² per top branch
(1)
(2)
(3)
(4)
(1)
(2)
(3)
(4)
Number of branches per iteration


Number of iterations
Total length of the column elements
(4)
(3)
(2)
(1)
(1)
(2)
(3)
(4)
Number of branches per iteration






σ [N/mm²]
355
S 355
235
S 235
$\sigma_b = \frac{\pi^2 E}{\lambda^2}$
76,4
93,9


Buckling Loads
L
Buckling Load
$\frac{\pi^2 EI}{L^2}$
$\frac{4\pi^2 EI}{L^2}$
$\frac{2.045\pi^2 EI}{L^2}$
$\frac{\pi^2 EI}{4L^2}$
$\frac{\pi^2 EI}{L^2}$
Effective Length
L
0.5L
0.699L
2L
L

Optimization:

Minimum weight

$$mass = A \cdot l \cdot \rho = \min$$

Optimization:

Minimum weight

$$mass = A \cdot l \cdot \rho = \min$$

Same profiles

$$\Delta A = (A_{\max} - A_{\min}) = \min$$

Optimization:

Minimum weight

$$mass = A \cdot l \cdot \rho = \min$$

Same profiles

$$\Delta A = (A_{\max} - A_{\min}) = \min$$

Maximum material use

$$lowest\ \sigma_b = \max$$

Optimization:

Minimum weight

$$mass = A \cdot l \cdot \rho = \min$$

Same profiles

$$\Delta A = (A_{\max} - A_{\min}) = \min$$

Maximum material use

$$lowest\ \sigma_b = \max$$

Minimum costs

$$\cos t = € \cdot profile \cdot l = \min$$

Optimization:

Minimum weight

$$mass = A \cdot l \cdot \rho = \min$$

Same profiles

$$\Delta A = (A_{\max} - A_{\min}) = \min$$

Maximum material use

$$lowest\ \sigma_b = \max$$

Minimum costs

$$\cos t = € \cdot profile \cdot l = \min$$

Minumum embodied energy

$$E_{mbodied} = E \cdot profile \cdot l = \min$$

Optimization:

Minimum weight

$$mass = A \cdot l \cdot \rho = \min$$

Same profiles

$$\Delta A = (A_{\max} - A_{\min}) = \min$$

Maximum material use

$$lowest\ \sigma_b = \max$$

Minimum costs

$$\cos t = € \cdot profile \cdot l = \min$$

Minumum embodied energy

$$E_{mbodied} = E \cdot profile \cdot l = \min$$

Minimize forces in the roof

Optimization:

Minimum weight $mass = A \cdot l \cdot \rho = \min$

Same profiles $\Delta A = (A_{\max} - A_{\min}) = \min$

Maximum material use $lowest\ \sigma_b = \max$

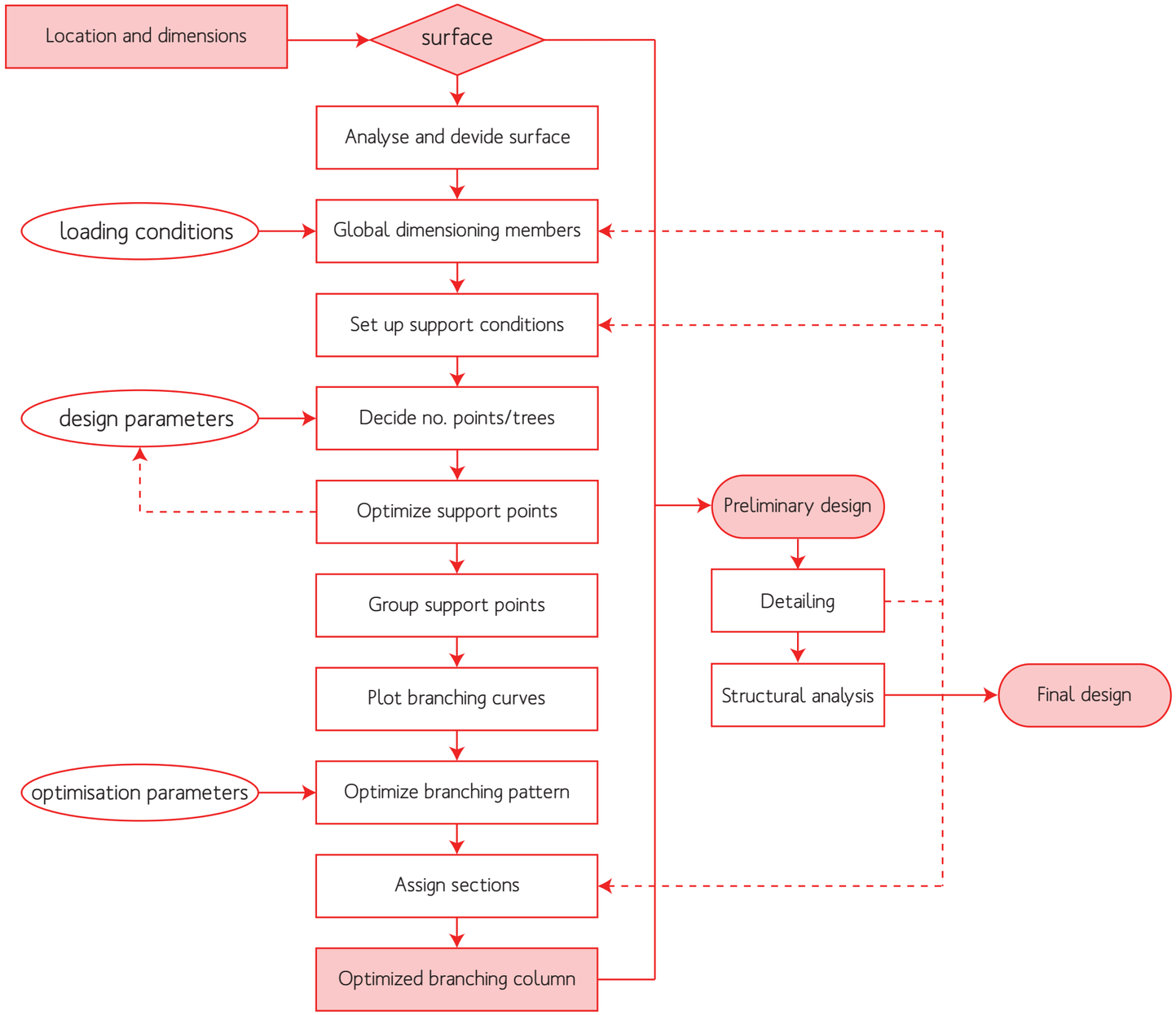
Minimum costs $\cos t = € \cdot profile \cdot l = \min$

Minumum embodied energy $E_{mbodied} = E \cdot profile \cdot l = \min$

Minimize forces in the roof

Other design conditions

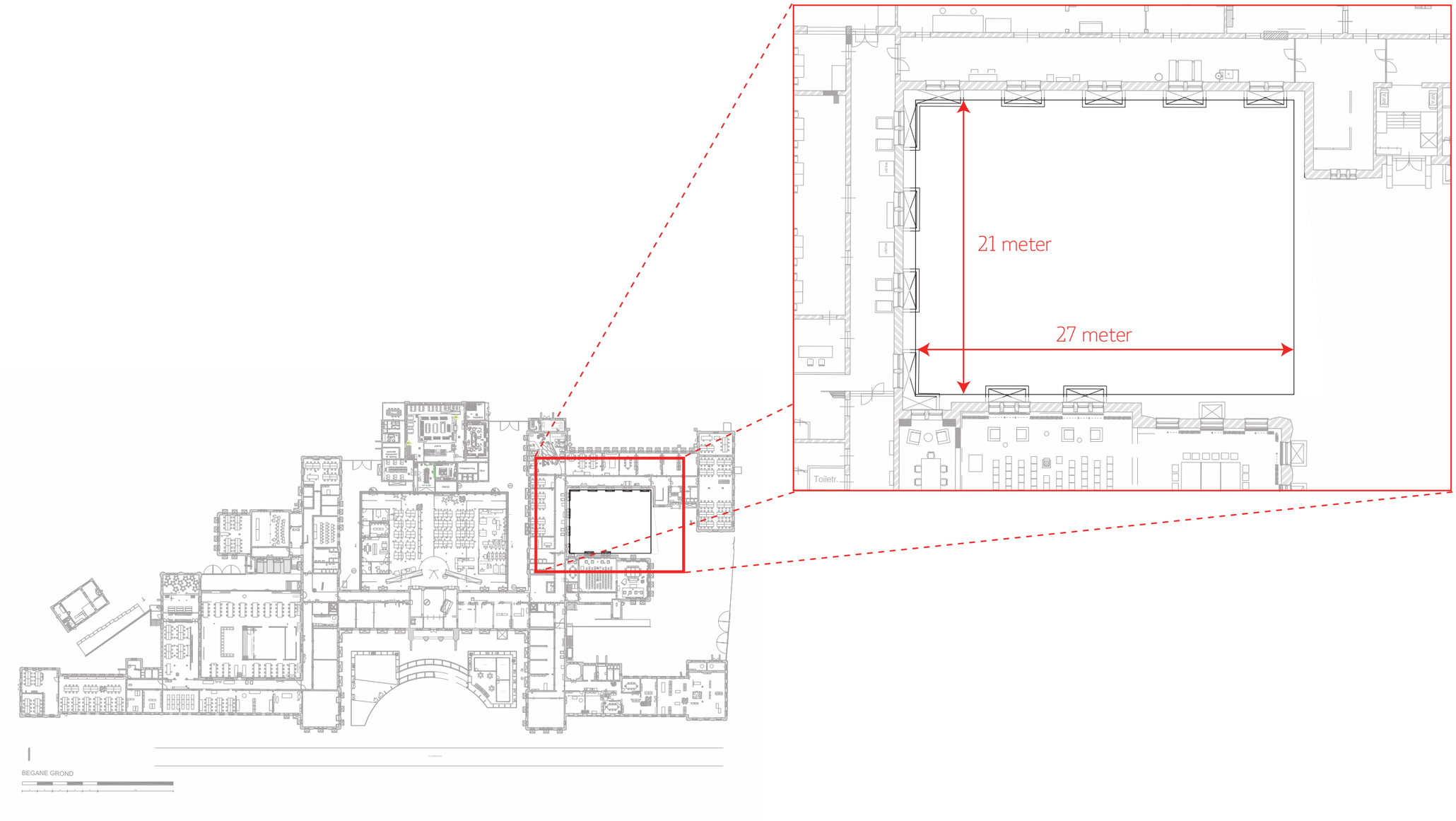
Optimisation	Factor of importance
Minimal weight	4
Same profiles	2
Maximal material use	3
Minimal costs	1
Minimal Embodied Energy	5
Reduce forces roofstructure	2
Other design conditions	3

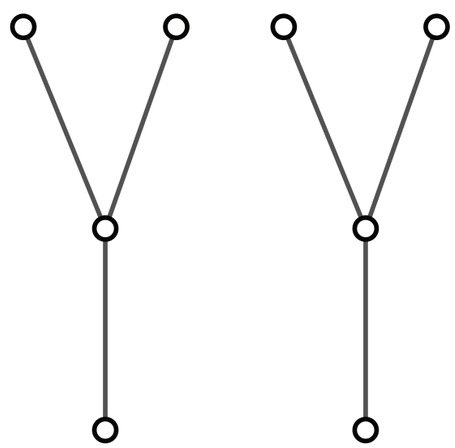

Location and dimensions
surface
Analyse and devide surface
loading conditions
Global dimensioning members
Set up support conditions
design parameters
Decide no. points/trees
Optimize support points
Group support points
Plot branching curves
optimisation parameters
Optimize branching pattern
Assign sections
Optimized branching column
Preliminary design
Detailing
Structural analysis
Final design

VI

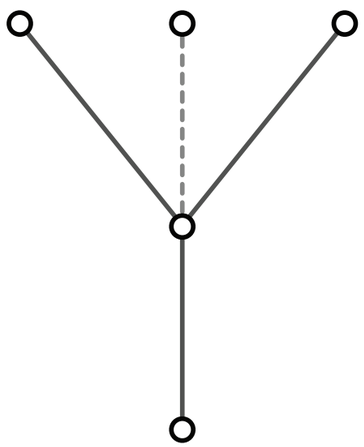
Design: location

Design: location

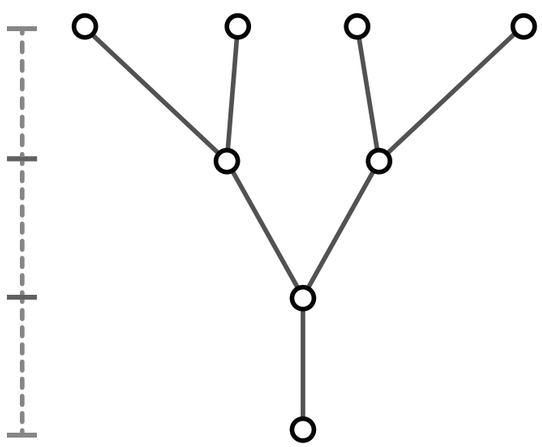




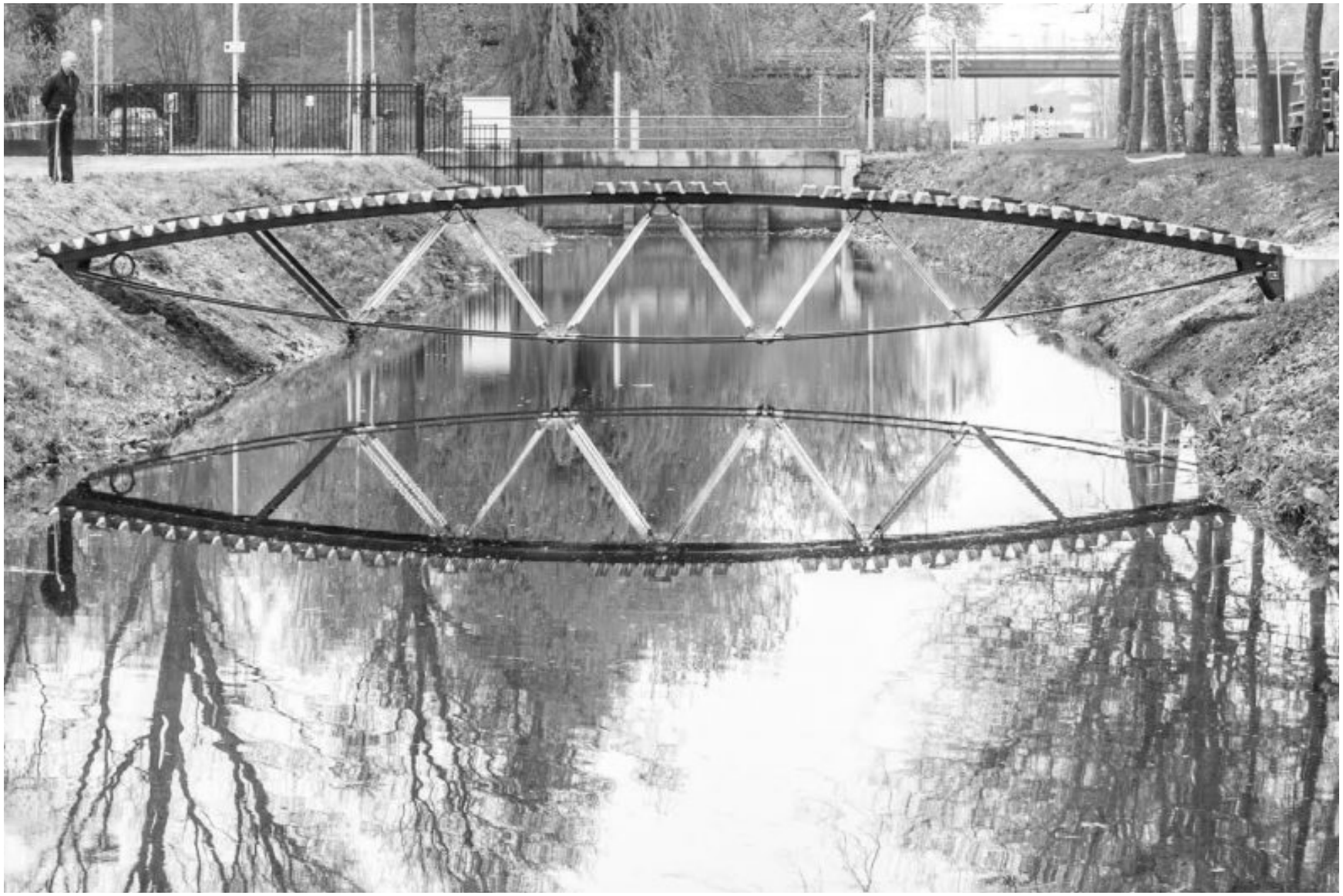
4 trees

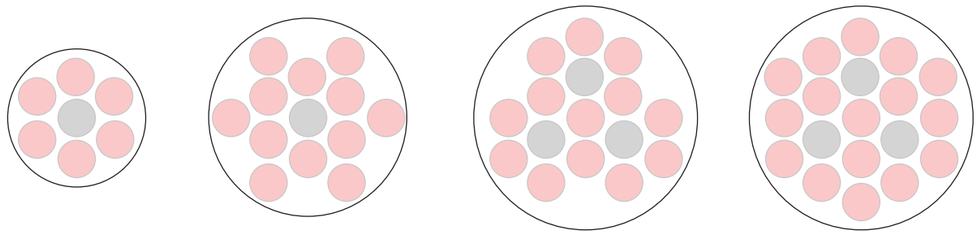


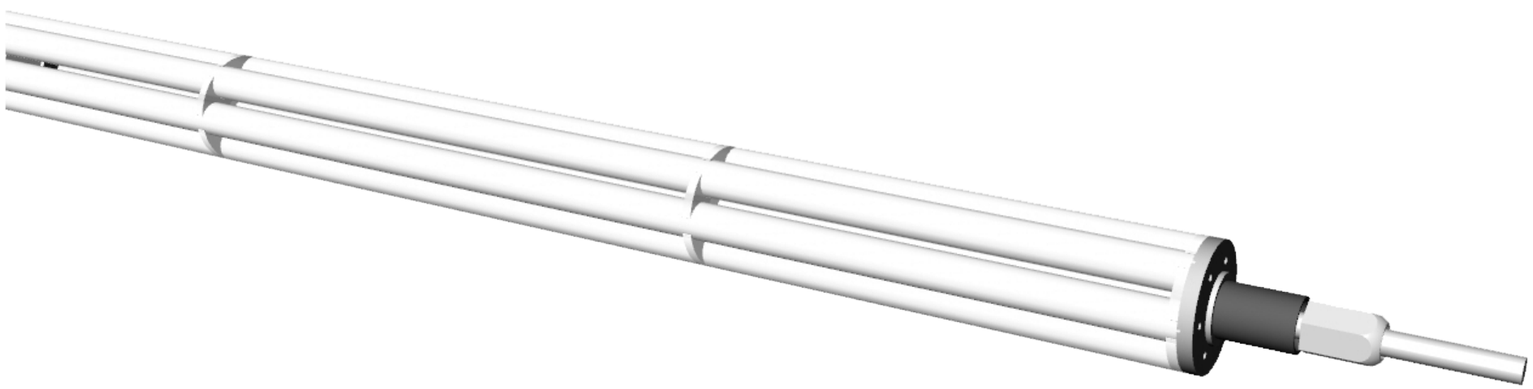
4 branches

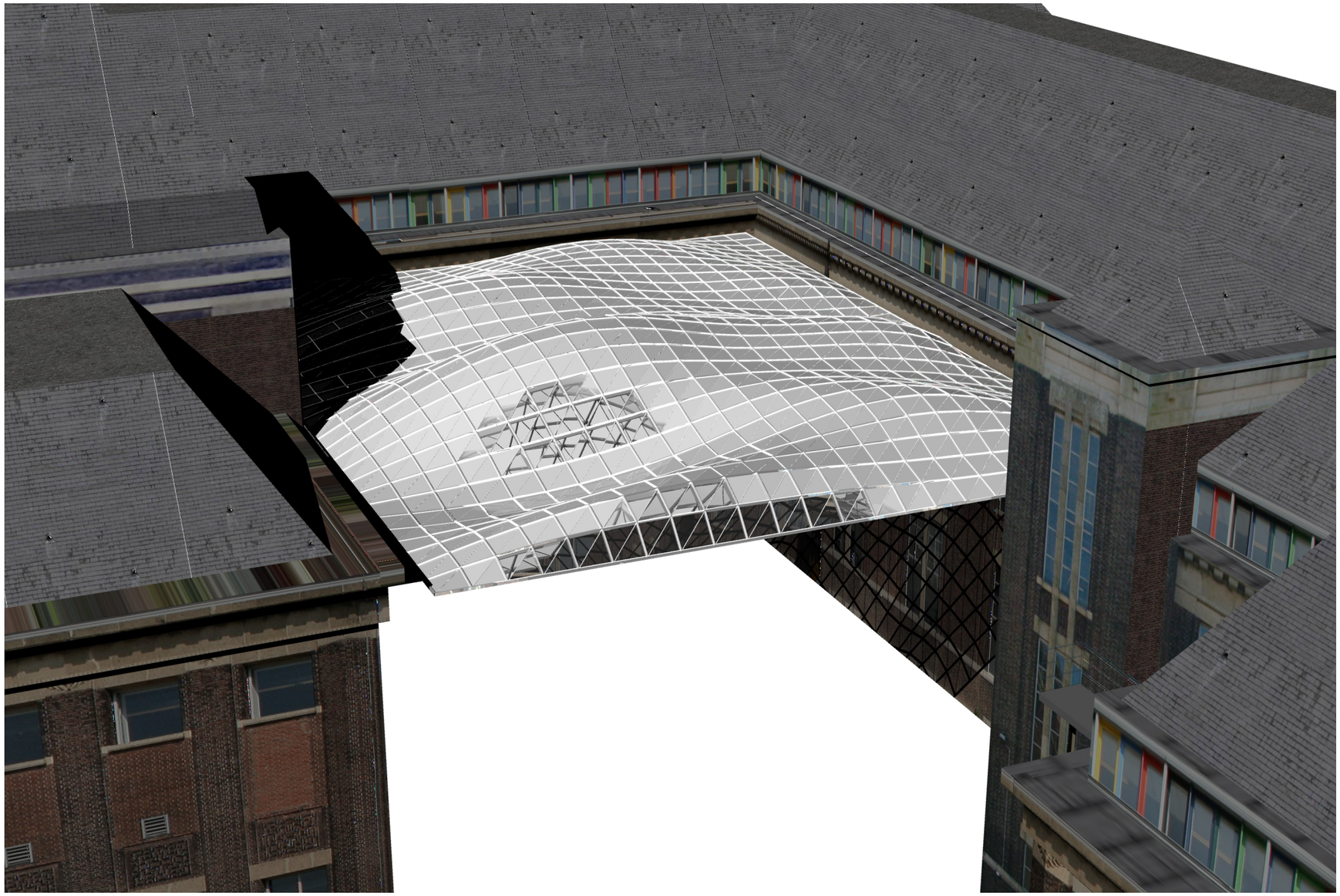


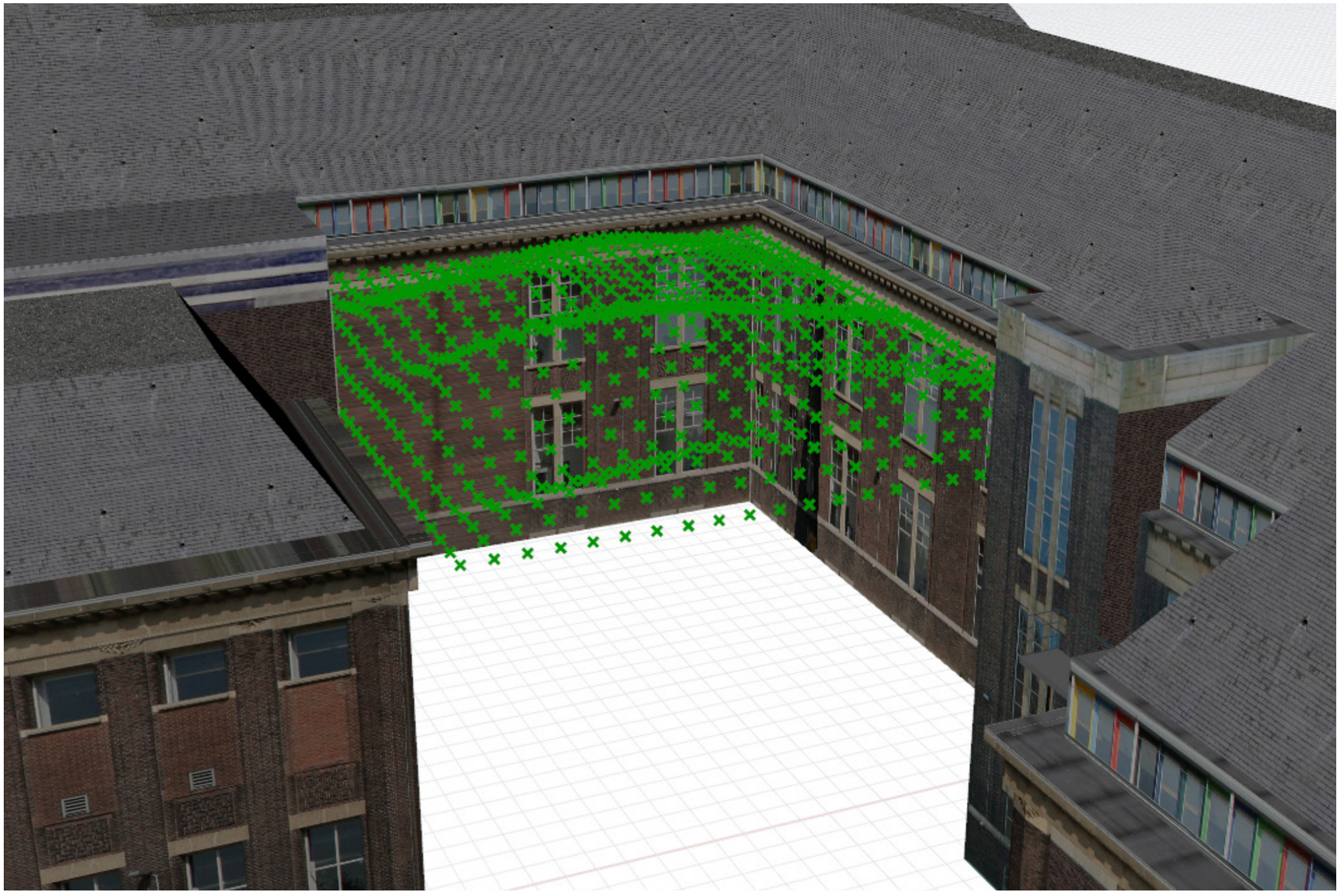
2 iterations

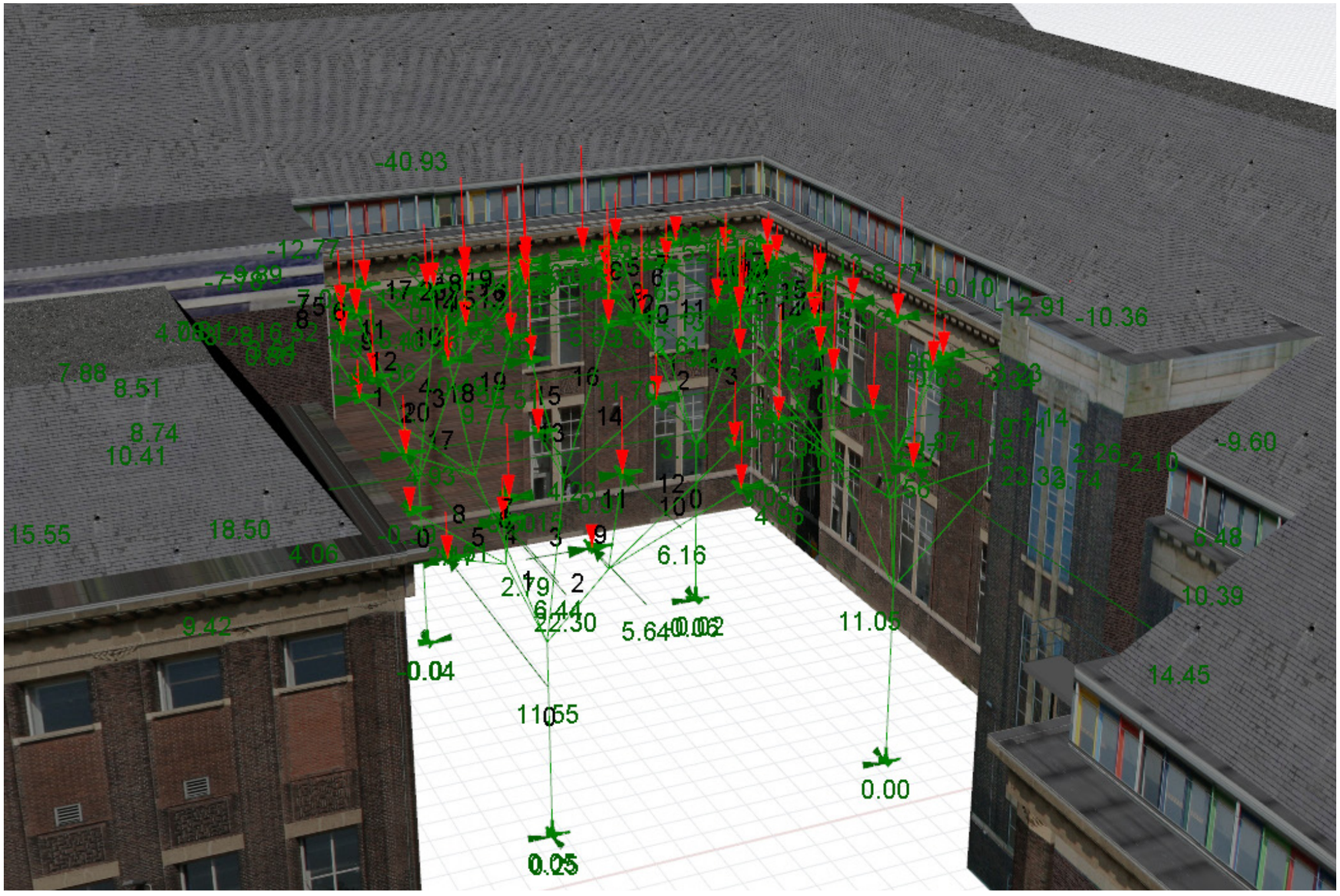


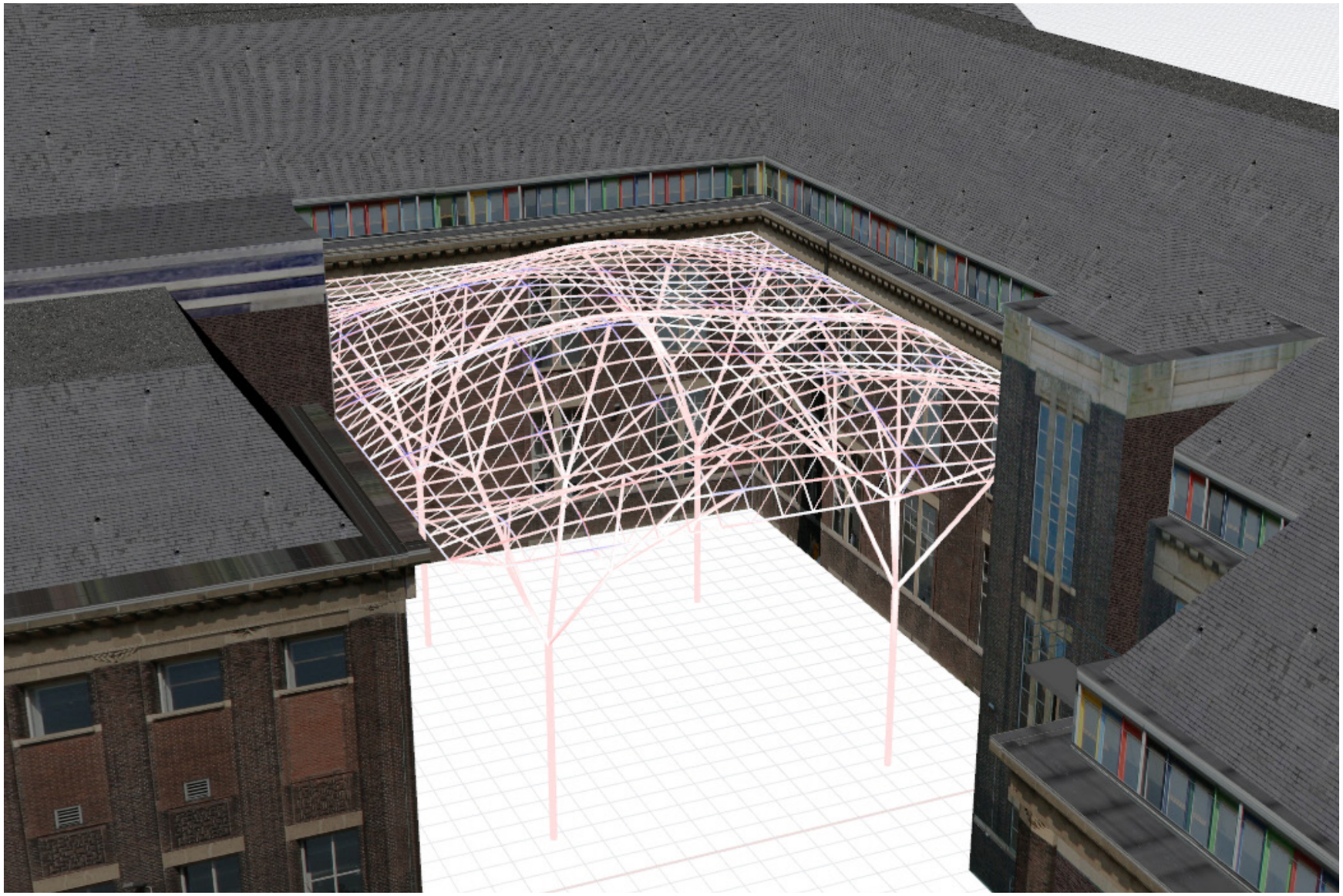


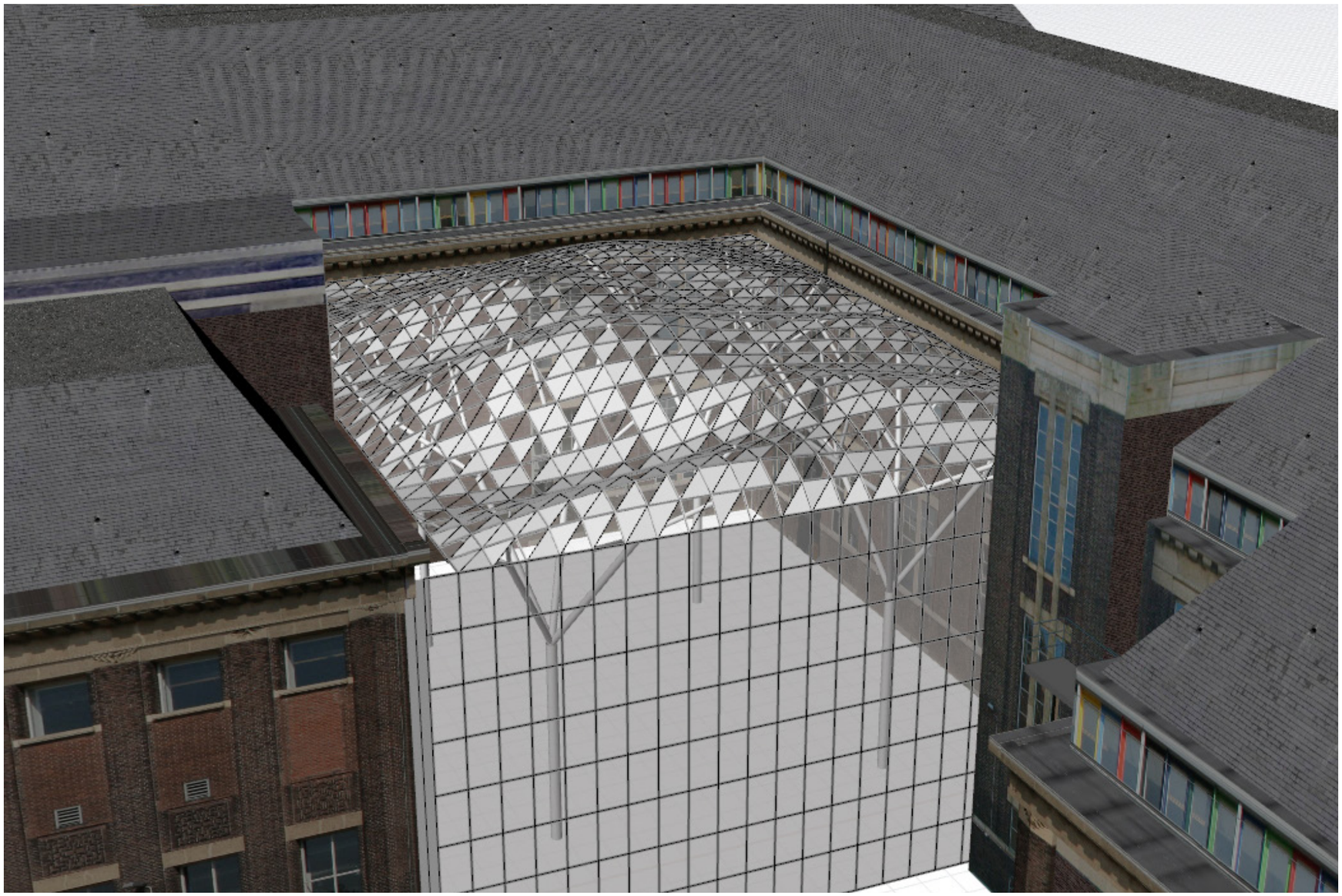




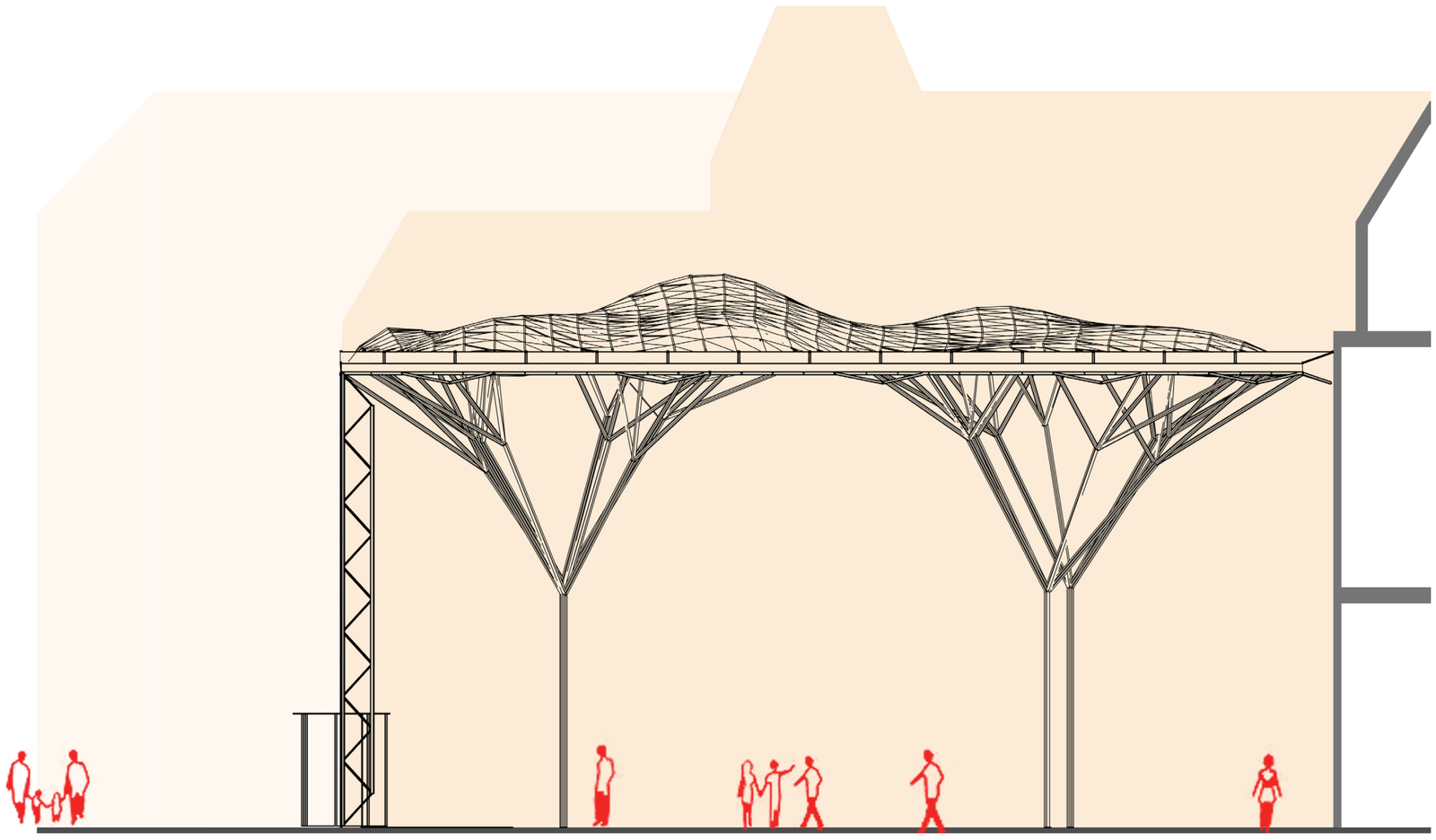


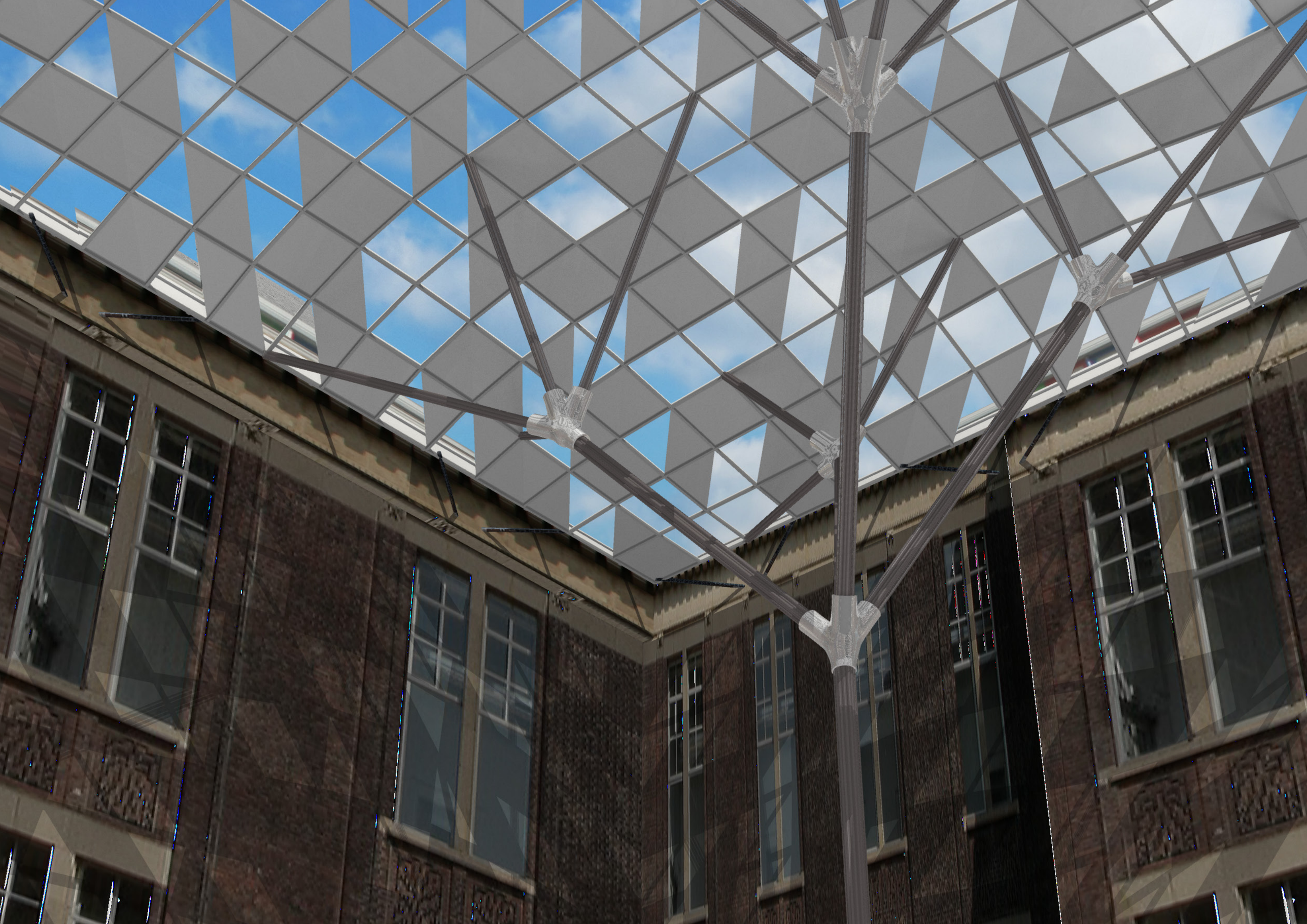

-40.93
-12.77
-10.10
-12.91
-10.36
7.88
8.51
8.74
10.41
15.55
18.50
4.06
9.42
6.16
5.64
11.05
0.00
-9.60
6.48
10.39
14.45
-0.04

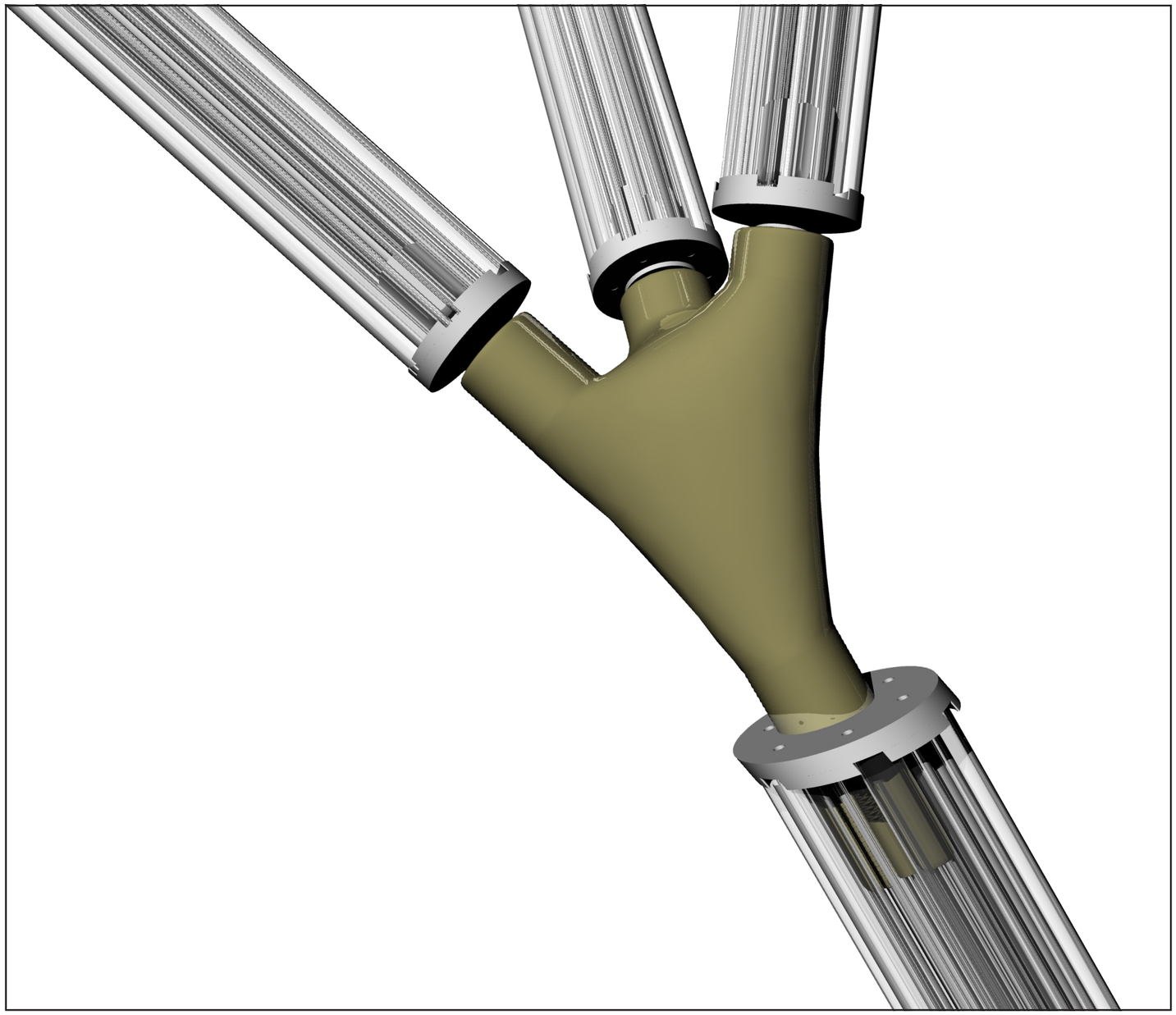


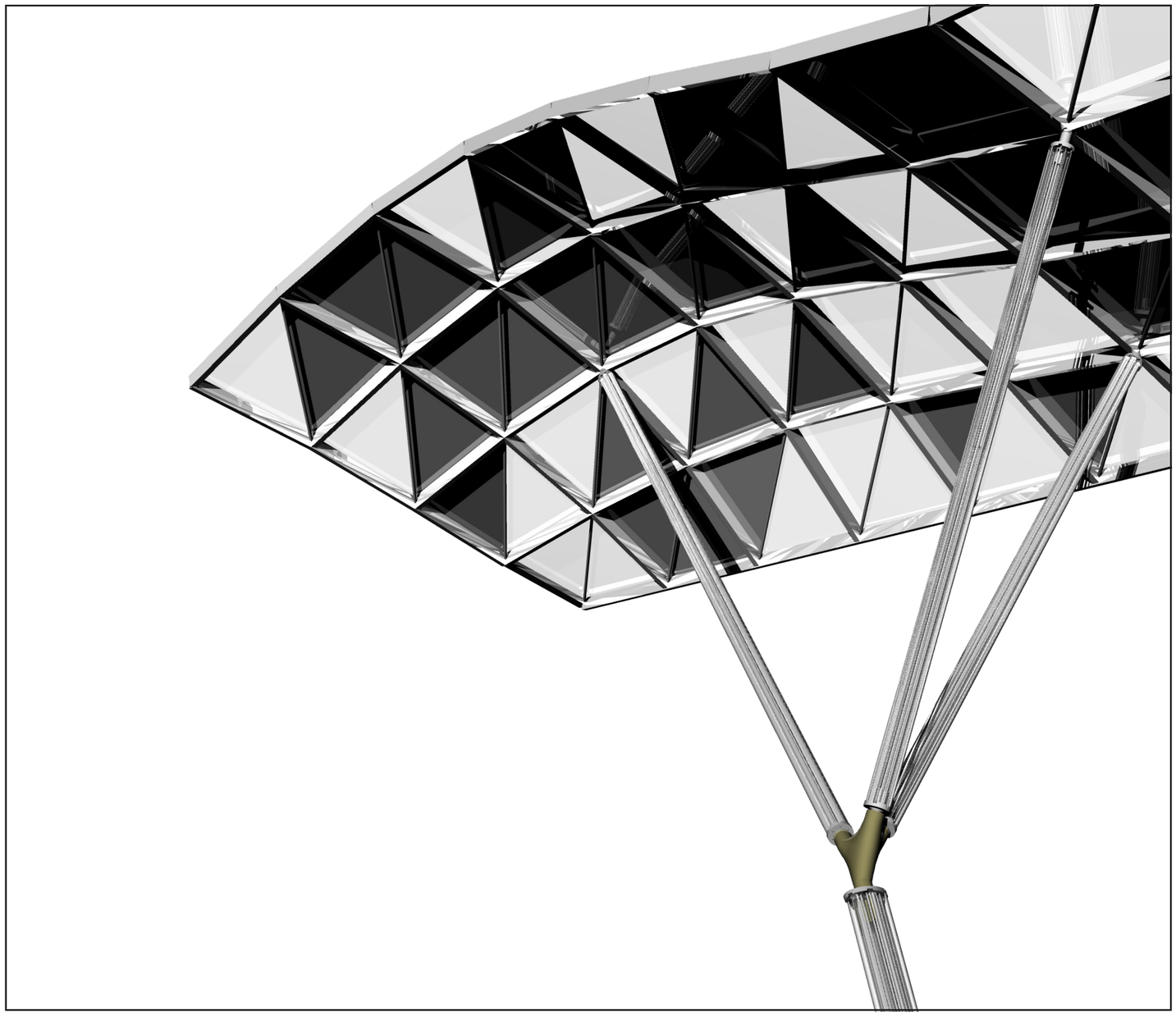


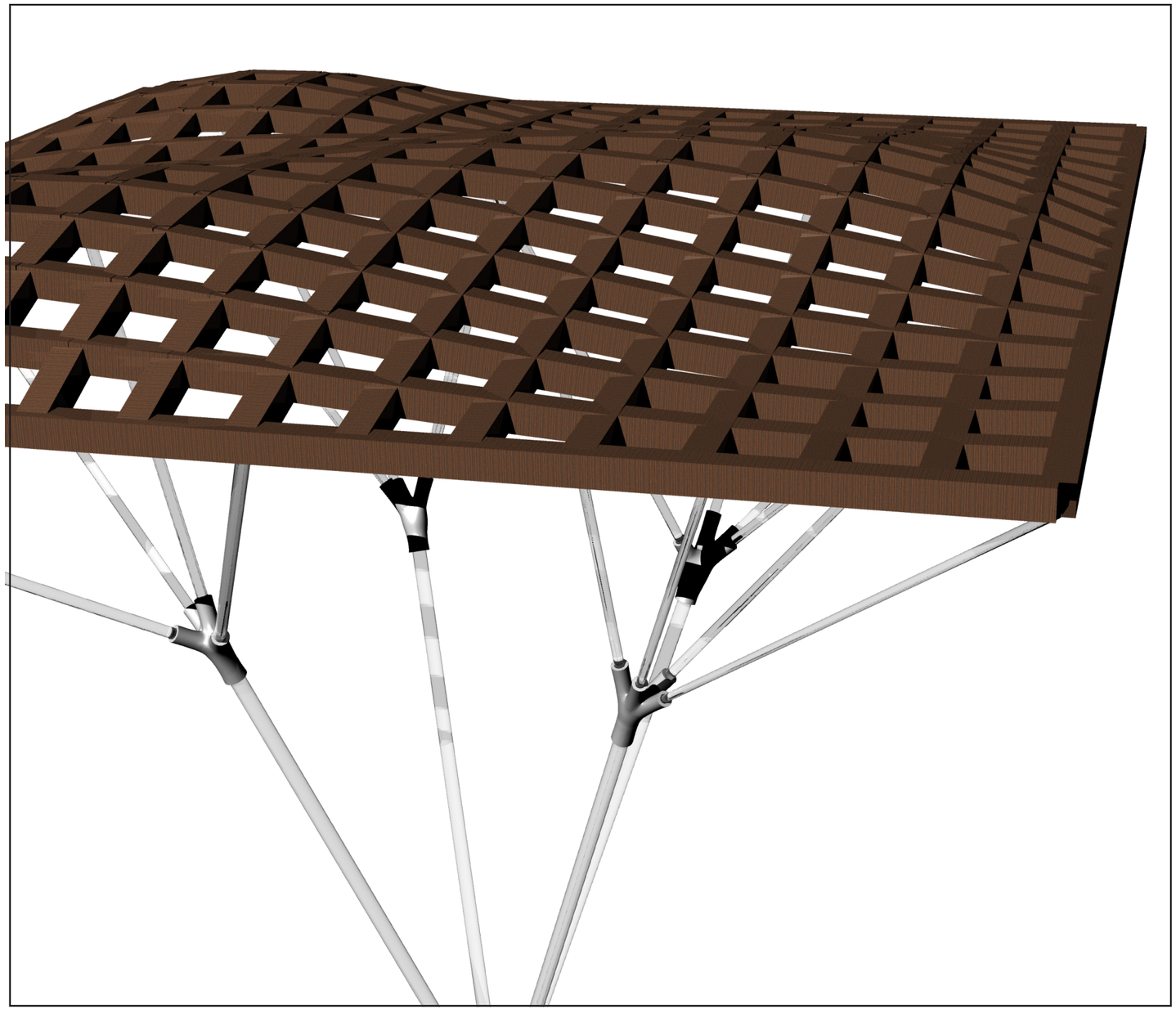
Section

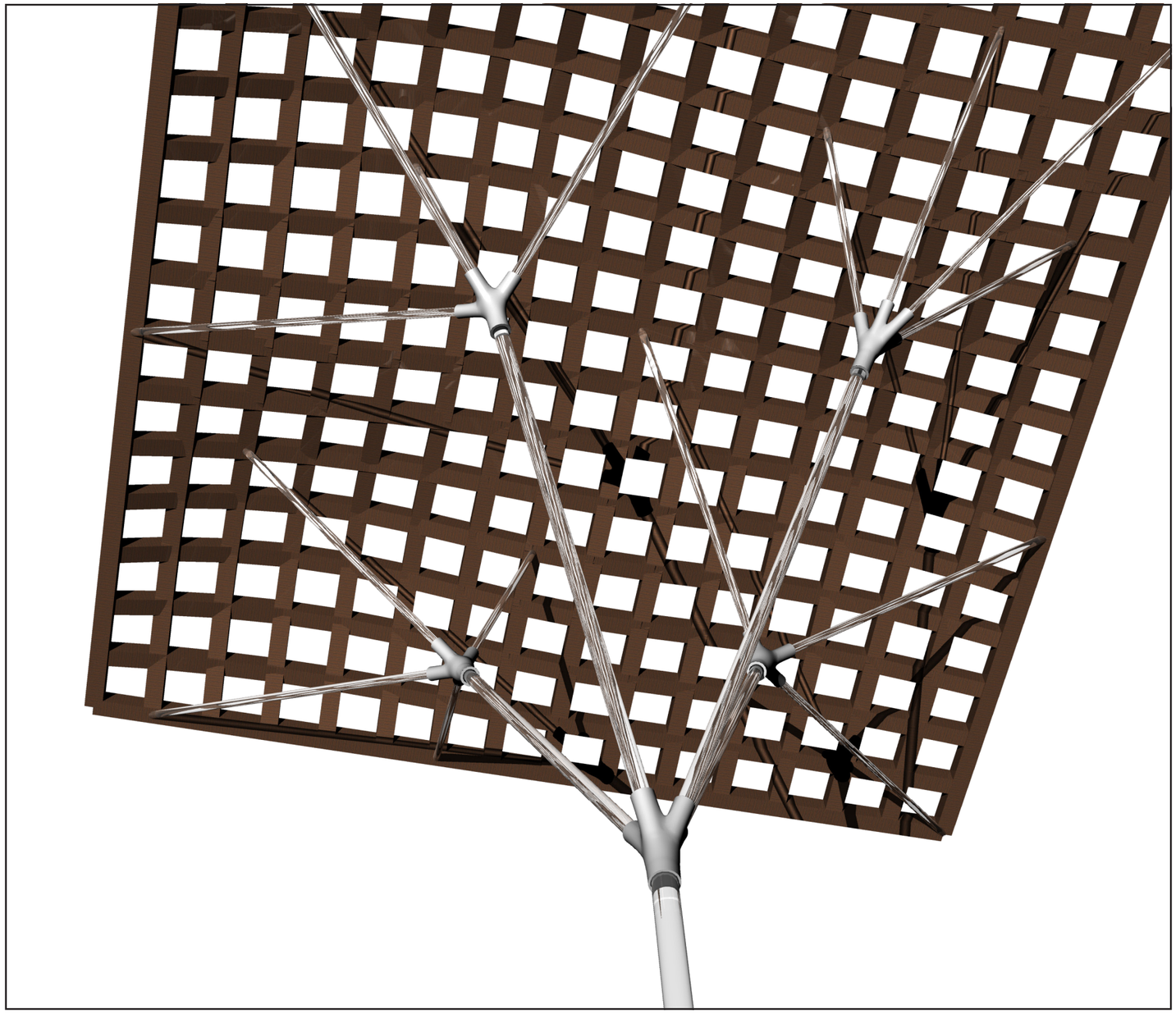












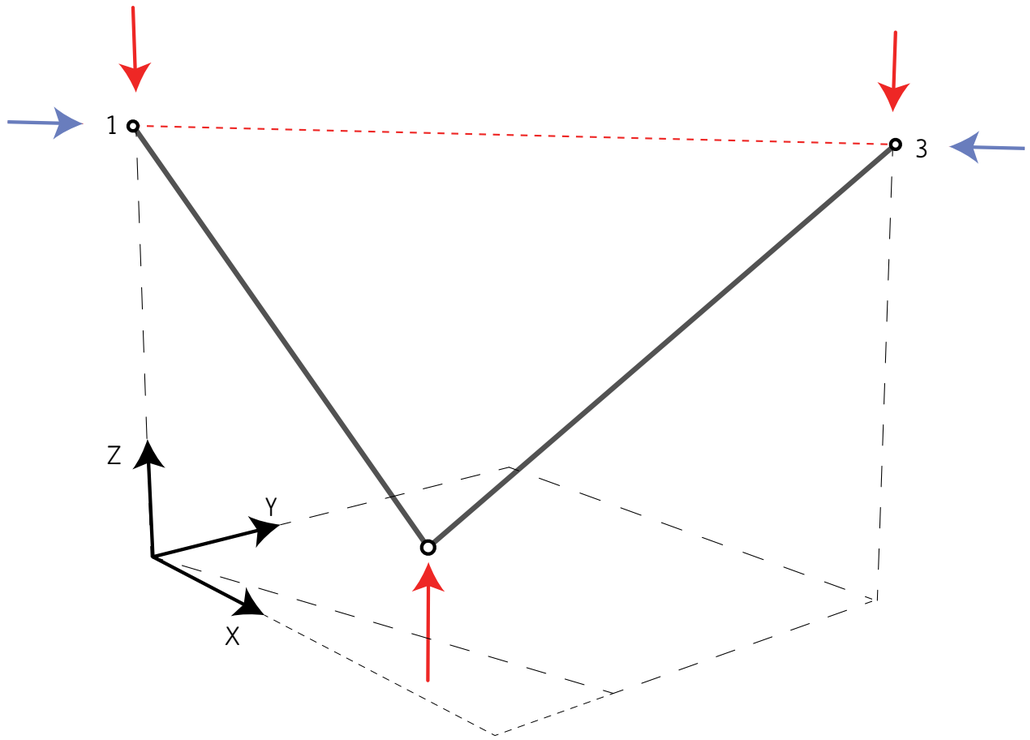
VII

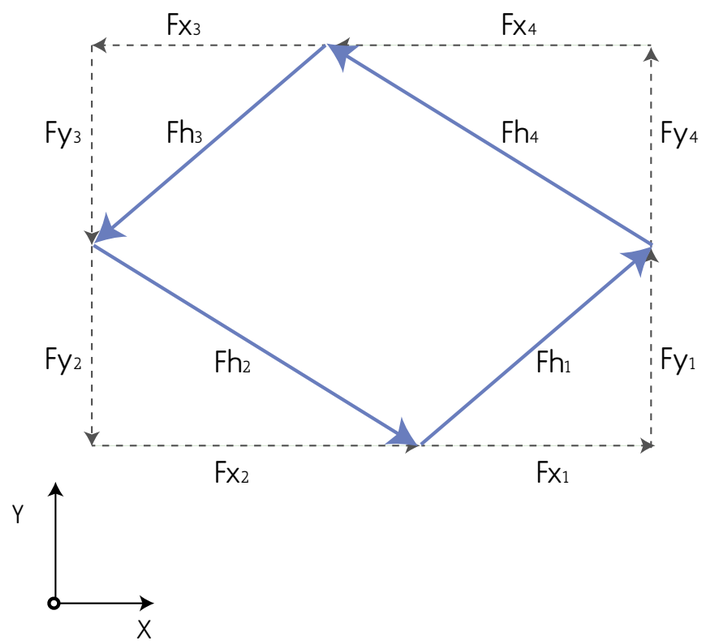
Conclusion

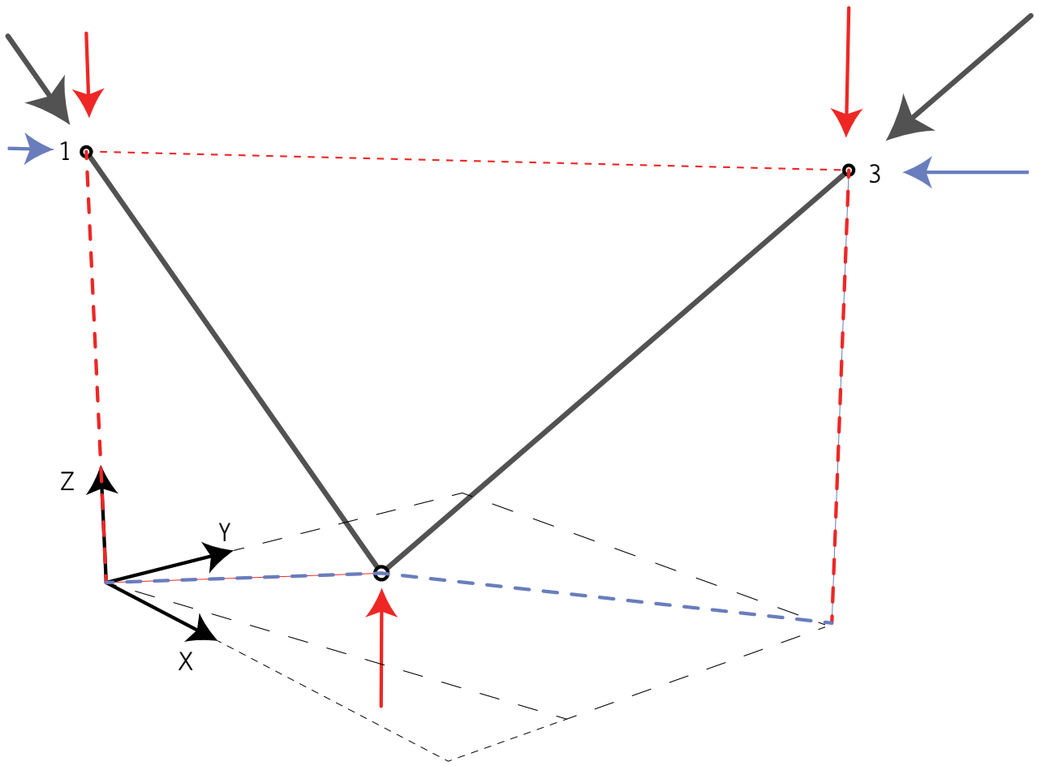
"How can we design structurally efficient three-dimensional branching structures as a support of freeform architectural surfaces?"

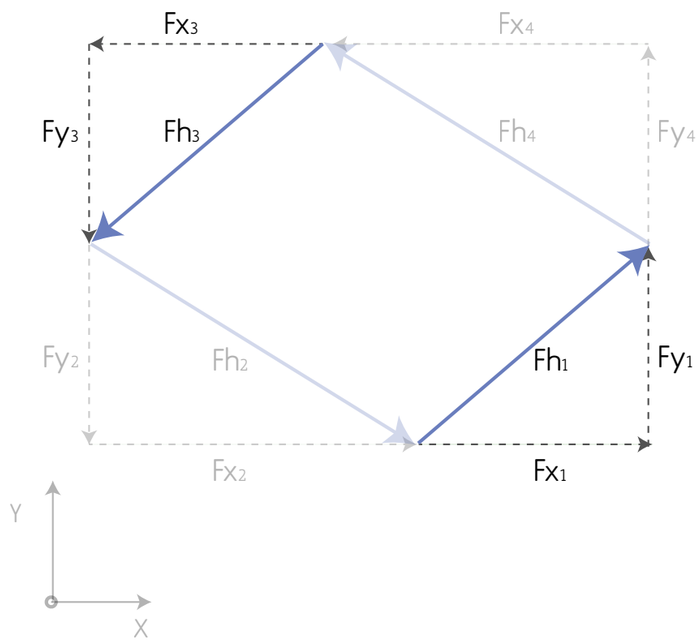
Thank you

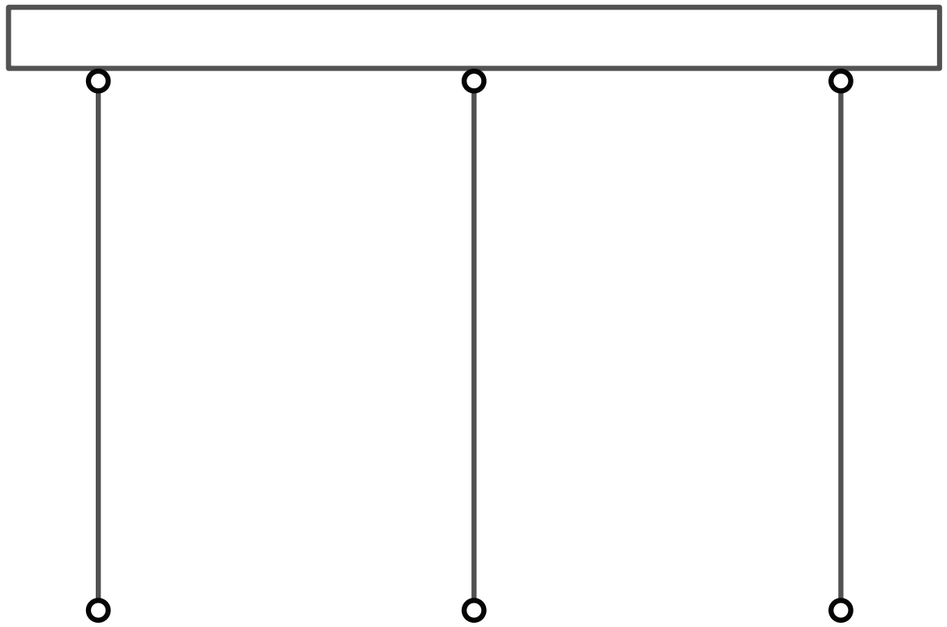
Reversed problem



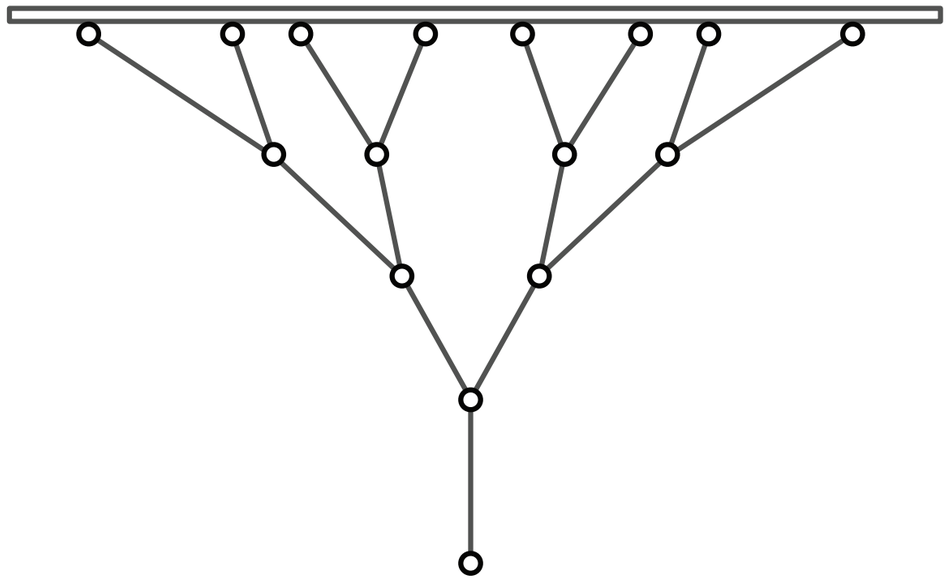



1
3
Z
Y
X


Fx3
Fx4
Fy3
Fh3
Fh4
Fy4
Fy2
Fh2
Fh1
Fy1
Fx2
Fx1
Y
X



columns and beams



branching column