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DOI
10.1109/CDC.2018.8619078

Publication date
2018

Document Version
Final published version

Published in
Proceedings of the 57th IEEE Conference on Decision and Control (CDC 2018)

2018 IEEE Conference on Decision and Control (CDC)
Miami Beach, FL, USA, Dec. 17-19, 2018

A Parametrized Model Predictive Control Approach for Microgrids

Tomás Pippia, Joris Sijs, and Bart De Schutter

***Abstract*— We propose a parametrized Model Predictive Control (MPC) approach for optimal operation of microgrids. The parametrization expresses the control input as a function of the states, variables, and parameters. In this way, it is possible to apply an MPC approach by optimizing only the parameters and not the inputs. Moreover, the value of the binary control variables in the model is assigned according to parametrized heuristic rules, thus obtaining a formulation for the optimization problem that is more scalable compared to standard approaches in the literature. Furthermore, we propose a control scheme based on one single controller that uses two different sampling times and prediction models. By doing so, we can include both fast and slow dynamics of the system at the same level. This control approach is applied to an operational control problem of a microgrid, which includes local loads, local production units, and local energy storage systems and results show the effectiveness of the proposed approach.**

***Index Terms*— Model predictive control, Microgrids, Smart Grids**

I. INTRODUCTION

Microgrids are seen as an innovative tool in power networks that can lead the old power networks towards new Smart Grids [1]–[3]. Microgrids can offer significant benefits, e.g. improved reliability, higher power quality by managing local loads, and increased efficiency due to the fact that energy produced locally is also locally consumed. They are also interesting from a control point of view, since, in order to achieve economic profitability of their operation, control actions that minimize an economical cost by optimally managing the power flows inside the microgrid should be considered.

Some Model Predictive Control (MPC) algorithms have been proposed for this purpose [4]–[9]. These works consider microgrids in which local loads and local generators are present and in some cases the microgrids are even in an islanded mode [7], [8], i.e. the microgrid is disconnected from the main grid. Moreover, some of these papers consider controllers with a hierarchical structure [4]–[7], where a higher-level controller computes the optimal setpoints for a lower-level controller, which is in turn responsible for driving the states and inputs of the system to the desired setpoints. Since this approach introduces an extra level in the controller structure, it requires more computational power and amount of communication. Moreover, different controller devices are required to implement this solution.

This work has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 675318 (INCITE).

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In this work, we propose a single-level controller that uses two different models, with different dynamics. One model including a higher number of states is used for predictions close to the current sampling time, while another model with a smaller number of states is used for predictions that are far in time from the current sampling time. Moreover, the rate of dynamics of the two models is different. The benefits of this approach are twofold. In the first place, with this approach we only need one controller and therefore the communication required and the amount of controller devices are reduced. Secondly, by using two different models, we are able to capture the 'fast' dynamics of the system and, when the system has reached the steady state condition, we use the other model that can provide a less complex but still representative model. This is important since the problem that we consider is computationally complex due to the presence of binary decision variables.

In order to reduce the computational complexity of the problem, we consider a parametrized MPC controller, in which the input law is parametrized according to certain parameters. Our approach is inspired by [10], where the parametrized MPC controller was introduced. We define the control law as a parametrized expression of the parameters, the states, the previously computed inputs, and some variables. The resulting parametric law for the continuous control variables is a summation of functions weighted by the parameters. The functions represent different control objectives, e.g. to keep the values of some states close to a given value or to reduce the value of some inputs to reduce the cost, and the parameters represent the importance of each objective. Furthermore, we assign the value to the discrete control variables according to if-then-else rules. The optimization problem is then carried out over the parameters and not over the actual inputs. This approach allows us to increase the scalability of the overall problem, since the number of optimization variables is considerably reduced.

The microgrid that we consider consists of local production units, local loads, and energy storage systems. Moreover, the microgrid is connected to the main grid, so that energy can be bought or sold, if necessary. We consider also two different kinds of energy storage systems, i.e. a battery and an ultracapacitor. The battery is used to store larger amounts of energy while the ultracapacitor is used for cases in which a fast response is needed. Furthermore, the ultracapacitor is used only in predictions for time instants close to the current time step, and therefore it is used only in the 'fast' model. The 'slow' model includes only the dynamics of the battery.

The main contribution of this article is therefore related to the development of a parametrized controller for the

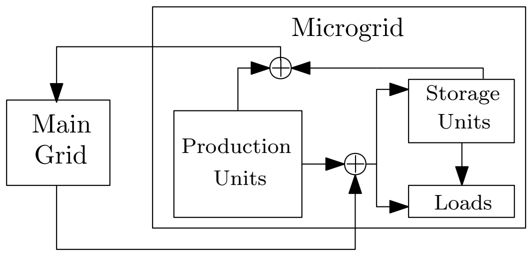

Fig. 1. Microgrid scheme considered in this article. Arrows represent power flows. The microgrid is connected to the main grid.

optimization of the energy management in a microgrid. By parametrizing the inputs of the system, we can reduce the number of optimization variables and thus the computational complexity of the problem. We also propose a parametrization of the integer variables in the model to further decrease the computational complexity. Moreover, by including two different storage devices and providing two different models, we are able to increase the flexibility of the system.

The outline of the article is as follows. We describe the model of the microgrid under control in Section II. In Section III the main features of standard MPC and our proposed single-level parametrized MPC controller are explained. In Section IV we apply our proposed approach to a simulation of the operation of a microgrid, comparing it to an existing controller in the literature, and lastly we present some conclusions in Section V.

Notation: We denote vectors with bold style, e.g. $\boldsymbol{x}$, $\boldsymbol{P}$, and scalars with non-bold text style. We indicate with $\mathbb{R}^+$ the set of positive real numbers, i.e. the set $\{x \in \mathbb{R} | x > 0\}$.

II. MODEL DESCRIPTION

We consider a microgrid, composed of several elements, e.g. storage units, loads, production units, and its operational costs. We want to optimize the operational costs of the microgrid, by implementing a control action on it. The microgrid that we consider is not in an islanded mode and can, therefore, exchange energy with the main grid. The energy that is produced in the microgrid can then be either used locally, stored in storage systems, or sold to the main grid. A scheme is shown in Figure 1.

A. Microgrid Description

We consider a model similar to the one presented in [4], with some modifications and simplifications.

a) Loads: We consider only critical loads, i.e. loads that must always be satisfied. We denote by P_l the total power required by the loads.

b) Dynamics of the energy storage systems: The dynamics of the Energy Storage Systems (ESSs) are expressed with a simplified formulation [11] with respect to [4], i.e.

$$x_\mathrm{st}(k+1) = \begin{cases} x_\mathrm{st}(k) + \frac{T}{\eta_\mathrm{d,st}} P_\mathrm{st}(k), & P_\mathrm{st}(k) < 0 \\ x_\mathrm{st}(k) + T\eta_\mathrm{c,st} P_\mathrm{st}(k), & P_\mathrm{st}(k) \geq 0 \end{cases}, \quad (1)$$

where $x_\mathrm{st}(k)$ indicates the level of energy stored at the ESS at time step k, $\eta_\mathrm{c,st}$ and $\eta_\mathrm{d,st}$ are the charging and discharging efficiencies, respectively, $P_\mathrm{st}(k)$ is the power exchanged with the ESS at time step k, and T is the sampling interval of the discrete-time system. The ESS can only be in one of the two modes, i.e. charging or discharging, at any time step. Following the same modeling approach as in [4], we use a Mixed Logical Dynamical (MLD) model [12] to model the two different modes of the batteries. We denote by δ_st the boolean variable that indicates whether the battery is in the charging or discharging mode, i.e. $\delta_\mathrm{st}(k) = 1 \iff P_\mathrm{st}(k) \geq 0$, and $\delta_\mathrm{st}(k) = 0 \iff P_\mathrm{st}(k) < 0$. Then we define a new auxiliary variable z_st as $z_\mathrm{st}(k) = \delta_\mathrm{st}(k) P_\mathrm{st}(k)$ and we can write (1) more compactly as

$$x_\mathrm{st}(k+1) = x_\mathrm{st}(k) + T\left(\eta_\mathrm{c,st} - \frac{1}{\eta_\mathrm{d,st}}\right) z_\mathrm{st}(k) + \frac{T}{\eta_\mathrm{d,st}} P_\mathrm{st}(k). \quad (2)$$

We consider two ESSs: an ultracapacitor used for fast response and a battery for storing larger amounts of energy.

c) Generators: We consider two different kinds of generators, i.e. dispatchable generators, whose output power can be controlled, and non-dispatchable generators, whose output power cannot be controlled. Renewable sources are considered as non-dispatchable and their output is considered as a known disturbance. We denote the vector of the variables representing the power produced by the generators by $\boldsymbol{P}_\mathrm{p} = \begin{bmatrix} \boldsymbol{P}_\mathrm{dis}^\top & P_\mathrm{res} \end{bmatrix}^\top$, where $\boldsymbol{P}_\mathrm{dis} = \begin{bmatrix} P_1^\mathrm{dis}, \ldots, P_{N_\mathrm{gen}}^\mathrm{dis} \end{bmatrix}^\top$ indicates the vector of the variables representing the power produced by the dispatchable units and P_res denotes the total power produced by renewable energy sources. Furthermore, P_i^dis denotes the power produced by generator i and N_gen denotes the total number of generators. Moreover, we use a variable $\delta_i^\mathrm{on}(k)$ to indicate whether dispatchable generator i is active at time step k, i.e. $\delta_i^\mathrm{on}(k) = 1$, or not, i.e. $\delta_i^\mathrm{on}(k) = 0$.

d) Main grid: The interaction with the main grid is modeled using again a binary variable δ_g, which indicates whether energy is being bought or sold to the main grid. If P_g is the power exchanged with the main grid, then we have

$$\begin{cases} \delta_\mathrm{g}(k) = 0 \iff P_\mathrm{g}(k) < 0, \text{ (exporting case)} \\ \delta_\mathrm{g}(k) = 1 \iff P_\mathrm{g}(k) \geq 0, \text{ (importing case)} \end{cases} \quad (3)$$

e) Energy prices: We assume that the prices of electricity are time-varying and that prices for purchase and sale of electricity are different. We denote with c_s and c_b the price for selling and buying electricity to and from the main grid, respectively. We also consider a fixed tariff c_p for producing electricity with the local dispatchable units.

We can then define a variable C_g as $C_\mathrm{g}(k) = c_\mathrm{s}(k) P_\mathrm{g}(k)$, if $\delta_\mathrm{g}(k) = 0$ and $C_\mathrm{g}(k) = c_\mathrm{b}(k) P_\mathrm{g}(k)$, if $\delta_\mathrm{g}(k) = 1$.

Remark 1: The number of storage devices here is kept limited for simplicity of expression but our approach can also be applied to systems with a higher number of ESSs.

B. Fast and Slow Model

We consider two different models in this work, namely a 'fast' one and a 'slow' one. The 'fast' model is used for predictions that are close to the current sampling time, while the 'slow' one is used for predictions that are farther away in time. Moreover, we consider that the ultracapacitor is available for usage only at time instants close to the current sampling time, in order to provide a fast response to the system. Therefore, it is not used in the 'slow' model, which implies that the number of the state components and of input components in the two models are different, since the 'slow' model does not have the dynamics of the ultracapacitor.

In Figure 2 we show the different sampling times and the time intervals in which each model is used. We denote by T_{f} and T_{s} the sampling interval of the 'fast' and 'slow' model, respectively, and we denote by h and k the time steps of the 'fast' and 'slow' model, respectively. Moreover, we suppose that from time step $N_{\mathrm{f,s}}$ of the 'fast' model we start using the 'slow' model for predictions. Lastly, the step $N_{\mathrm{f,s}}$ of the 'fast' model coincides with time step 0 of the 'slow' model.

For the fast model, by following (2), the equations are

$$\boldsymbol{x}_{\mathrm{f}}(h+1) = \boldsymbol{x}_{\mathrm{f}}(h) + B_1^{\mathrm{f}}\boldsymbol{z}_{\mathrm{f}}(h) + B_2^{\mathrm{f}}\boldsymbol{u}_{\mathrm{f}}(h), \tag{4}$$

where $\boldsymbol{x}_{\mathrm{f}}(h) = \begin{bmatrix} x_{\mathrm{f,b}}(h) & x_{\mathrm{f,uc}}(h) \end{bmatrix}^\top$, with $x_{\mathrm{f,b}}$ and $x_{\mathrm{f,uc}}$ being the storage level of the battery and of the ultracapacitor, respectively, $\boldsymbol{z}_{\mathrm{f}}$ is the auxiliary variable for the fast model, and $B_1^{\mathrm{f}} \in \mathbb{R}^{2\times 2}$, $B_2^{\mathrm{f}} \in \mathbb{R}^{2\times m_{\mathrm{f}}}$. We define the input vector as $\boldsymbol{u}_{\mathrm{f}}(h) = \begin{bmatrix} P_{\mathrm{f,b}}(h) & P_{\mathrm{f,uc}}(h) \end{bmatrix}^\top$, $\boldsymbol{u}_{\mathrm{f}}(h) \in \mathbb{R}^{m_{\mathrm{f}}}$, which represents respectively the power exchanged with the battery and the power exchanged with the ultracapacitor. The slow model is defined in a similar way, i.e.

$$x_{\mathrm{s}}(k+1) = x_{\mathrm{s}}(k) + B_1^{\mathrm{s}} z_{\mathrm{s}}(k) + B_2^{\mathrm{s}} u_{\mathrm{s}}(k), \tag{5}$$

where $x_{\mathrm{s}}(k) = x_{\mathrm{s,b}}(k)$ is the storage level of the battery, z_{s} is the auxiliary variable for the 'slow' model, and $B_1^{\mathrm{s}}, B_2^{\mathrm{s}} \in \mathbb{R}$. The input vector is defined as $u_{\mathrm{s}}(k) = P_{\mathrm{s,b}}(k)$ and it represents the power exchanged with the battery.

By following [4], we can consider the power balance in the microgrid,

$$P_{\mathrm{f,b}}(h) = \sum_{i=1}^{N_{\mathrm{gen}}} P_i^{\mathrm{dis}}(h) + P_{\mathrm{res}}(h) + P_{\mathrm{g}}(h) - P_{\mathrm{f,uc}}(h) - P_{\mathrm{l}}(h), \tag{6}$$

$\forall h \geq 0$, and apply it to (4) to write the expression of the dynamics of the storages as a function of P_{g}, P_{l}, P_{p}. Then, by introducing matrices M_u, M_w and defining $\boldsymbol{u}_{\mathrm{f}}(h) = M_u \overline{\boldsymbol{u}}_{\mathrm{f}}(h) + M_w \boldsymbol{w}_{\mathrm{f}}(h)$, with $\overline{\boldsymbol{u}}_{\mathrm{f}}(h) = \begin{bmatrix} \boldsymbol{P}_{\mathrm{dis}}^\top(h) & P_{\mathrm{g}}(h) & P_{\mathrm{f,uc}}(h) & (\boldsymbol{\delta}^{\mathrm{on}}(h))^\top \end{bmatrix}^\top$, $\boldsymbol{w}_{\mathrm{f}}(h) = \begin{bmatrix} P_{\mathrm{l}}(h) & P_{\mathrm{res}}(h) \end{bmatrix}^\top$, we can link (4) and (6) as

$$\boldsymbol{x}_{\mathrm{f}}(h+1) = \boldsymbol{x}_{\mathrm{f}}(h) + B_1^{\mathrm{f}}\boldsymbol{z}_{\mathrm{f}}(h) + B_2^{\mathrm{f}}\left(M_u \overline{\boldsymbol{u}}_{\mathrm{f}}(h) + M_w \boldsymbol{w}_{\mathrm{f}}(h)\right). \tag{7}$$

A similar expression is obtained for (5).

Since the 'slow' model does not have the dynamics or the inputs related to the ultracapacitor, the number of states and inputs is different in the two models. Therefore, it is necessary to define a way to link the two models. We assume that the 'fast' model is used only until the time instant $T_{\mathrm{f}} N_{\mathrm{f,s}}$, i.e. time step $N_{\mathrm{f,s}}$ of the 'fast' model and after that the 'slow' model is used. We can link the two models as

$$x_{\mathrm{s}}(0) = x_{\mathrm{f,b}}(N_{\mathrm{f,s}}), \tag{8}$$

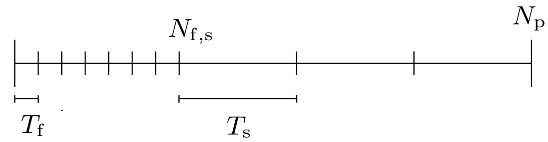

Fig. 2. Scheme adopted for the time step of the two different models.

which means that we can define a matrix $M_{\mathrm{f,s}} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ to link the two models as $x_{\mathrm{s}}(0) = M_{\mathrm{f,s}} \boldsymbol{x}_{\mathrm{f}}(N_{\mathrm{f,s}})$.

C. Constraints

The dynamics and the power flows are subject to constraints. We use an MLD model for the storages and the power exchanged with the main grid and therefore we define the constraints as in [4], [12] by defining matrices E_1, E_2, E_3, E_4. Since we have two different models, we define two different sets of constraints for the two models, denoted with a superscript 'f' and 's' the constraints for the 'fast' and 'slow' model, respectively. We can then write compactly the constraints as

$$E_1^{\mathrm{f}} \boldsymbol{\delta}_{\mathrm{st}}(k) + E_2^{\mathrm{f}} \boldsymbol{z}_{\mathrm{f}}(k) \leq E_3^{\mathrm{f}} \boldsymbol{u}_{\mathrm{f}}(k) + E_4^{\mathrm{f}}. \tag{9}$$

where $\boldsymbol{\delta}_{\mathrm{st}} = \begin{bmatrix} \delta_{\mathrm{st}}^{\mathrm{b}} & \delta_{\mathrm{st}}^{\mathrm{uc}} \end{bmatrix}$ and $\delta_{\mathrm{st}}^{\mathrm{b}}, \delta_{\mathrm{st}}^{\mathrm{uc}}$ are the binary variables associated with the battery and the ultracapacitor, respectively. A similar equation holds for the 'slow' model, where we replace the matrices E_i^{f} with matrices E_i^{s}, $i \in \{1, \cdots, 4\}$ and we replace the variables of the 'fast' model with the ones of the 'slow' model.

We also define the bounds for the states and the inputs,

$$\underline{P}_{\mathrm{b}} \leq P_{\mathrm{b}}(h) \leq \overline{P}_{\mathrm{b}} \tag{10}$$

$$\underline{P}_{\mathrm{uc}} \leq P_{\mathrm{uc}}(h) \leq \overline{P}_{\mathrm{uc}} \tag{11}$$

$$\underline{P}_{\mathrm{g}} \leq P_{\mathrm{g}}(h) \leq \overline{P}_{\mathrm{g}} \tag{12}$$

$$\delta_i^{\mathrm{on}}(h) \underline{P}_{\mathrm{dis}} \leq P_i^{\mathrm{dis}}(h) \leq \delta_i^{\mathrm{on}}(h) \overline{P}_{\mathrm{dis}} \tag{13}$$

$$\underline{x}_{\mathrm{st}} \leq \boldsymbol{x}_{\mathrm{st}}(h) \leq \overline{x}_{\mathrm{st}} \tag{14}$$

for $i \in \{1, \cdots, N_{\mathrm{gen}}\}$. The constraints (10)-(14) are used to model the physical bounds on, respectively, the power exchanged with the battery, the power exchanged with the ultracapacitor, the power exchanged with the main grid, the power produced by the production units, and the level of charge in the storages. These constraints can be applied to the quantities of both the 'slow' and 'fast' model.

III. CONTROLLER SCHEME

A. Standard MPC

MPC is a well-known, established control approach that has been extensively studied in the last forty years [13], [14]. The control action is computed by solving an online optimal control problem, using a model of the plant under control for computing predictions of the future states up to a certain prediction horizon N_{p}. The optimization problem results in a sequence of optimal inputs, but only the first of them is applied to the system. At the next control time step, the system state is measured and a new optimization problem is solved. With this strategy, MPC controllers are able to handle to a certain extent model mismatches, uncertainties, and disturbances [13], [14]. Moreover, the MPC strategy transforms the control problem into an optimization one, and thus constraints can be included into the control problem.

B. Parametrized MPC

In parametrized MPC (PMPC) [10], the inputs are parametrized as a function of parameters θ, variables y, and states x, i.e. $u(k) = f(x(k), y(k), \theta(k))$. The optimization is then carried out over the parameters, instead of over the inputs. The advantage of PMPC is that the computational complexity can be reduced since the number of decision variables is reduced, if the number of components of $\theta(k)$ is less than the number of components of $u(k)$, or if $\theta(k)$ is taken constant over the prediction window. Moreover, while in strategies in which the input is blocked the value of the input remains constant [13], in PMPC the inputs can change since they depend not only on the parameters but also on the states or other variables.

Different numbers of parameters can be used in PMPC, as explained in [10]. For instance, it is possible to allow the parameters to vary at every time step to increase the performance, or to block the value of the parameters so that they cannot vary over the prediction window, yielding a faster solution. Therefore, the number of parameters acts as variable that can be tuned and it provides a trade-off between performance and computational complexity. In this work, we consider only one set of parameters, i.e. the same parameters are kept for the whole prediction horizon. In this way, we are able to reduce the computational complexity, since we reduce the number of decision variables.

C. Single-level Two-model Controller

We consider a single-level centralized controller that uses the two different models of the microgrid defined before, i.e. a 'fast' one and a 'slow' one, to compute the optimal inputs. As explained before, the 'fast' model is used until time step $N_{\mathrm{f,s}}$ and thereafter the 'slow' model is used.

At time step $\bar{h}$ the optimization problem is solved until the time step $\bar{h} + N_{\mathrm{P}} - 1$. Next, only the first input of the optimal sequence is applied, and then the problem is solved again from $\bar{h} + T_{\mathrm{f}}$ until $\bar{h} + T_{\mathrm{f}} + N_{\mathrm{P}} - 1$, and so on so forth. This is the standard approach for MPC controllers.

D. Parametrized Input Laws

The PMPC law of each input is defined as a weighted sum of functions that depend on the states, on the previous continuous control inputs, and on some quantities, e.g. price of electricity. Moreover, we fix the value of the parameters for the whole prediction horizon, in order to reduce the computational complexity of the problem. The discrete control inputs are instead assigned according to if-then-else rules.

The continuous components of $\bar{\boldsymbol{u}}_{\mathrm{f}}$ are parametrized as

$$\sum_{i=1}^{3} \theta_i \frac{f_i\left(x(h), w(h), w(h-1), c_{\mathrm{s}}(h), c_{\mathrm{b}}(h), \bar{\boldsymbol{u}}(h-1)\right)}{f_i^{\max}}, \tag{15}$$

where parameters θ_i and functions f_i are different for each component of $\bar{\boldsymbol{u}}_{\mathrm{f}}$. The value $f_i^{\max}$ corresponds to the maximum of the function f_i and it is used to normalize the term corresponding to the parameter θ_i. Since the parameters θ_i are constant, $\overline{\boldsymbol{u}}_{\mathrm{f}}$ has 3 components, and $\overline{\boldsymbol{u}}_{\mathrm{s}}$ has 2 components, we have in total 15 parameters.

The functions f_i depend either on the states or on variables such as P_{l}, c_{s}, or c_{b}. The idea behind the design of these functions is to assign more or less importance to certain objectives. Following (15), we propose in total 9 different functions, 3 for each component of $\overline{\boldsymbol{u}}_{\mathrm{f}}$; we also denote them with the superscripts 'uc', 'g', 'p', which denote respectively the ultracapacitor, the main grid, the produced power. The functions are defined as follows:

- $f_1^{\mathrm{uc}} = 0.5\left(\overline{x}_{\mathrm{uc}} - \underline{x}_{\mathrm{uc}}\right) - x_{\mathrm{uc}}(h)$, in order to keep the value of the storage of the ultracapacitor close to its medium value, so that the ultracapacitor can react to a change in power by providing power or absorbing it;
- $f_2^{\mathrm{uc}}(h) = -P_{\mathrm{l}}(h-1) + P_{\mathrm{res}}(h-1) + \sum_{i=1}^{N_{\mathrm{gen}}} P_i^{\mathrm{dis}}(h-1)$, so that more power is stored in the ultracapacitor if at the previous time step there was more power produced than consumed locally, and vice versa;
- $f_3^{\mathrm{uc}}(h) = -c_{\mathrm{b}}(h)$, to take more power from the ultracapacitor when the price for buying electricity is high;
- $f_1^{\mathrm{g}}(h) = -c_{\mathrm{b}}(h)$, so that less power is bought from the main grid if the price for buying electricity is high;
- $f_2^{\mathrm{g}}(h) = -c_{\mathrm{s}}(h)$, in order to sell more electricity to the main grid if the price for selling electricity is high (recall (3));
- $f_3^{\mathrm{g}}(h) = -f_2^{\mathrm{uc}}(h)$, so that more power is bought if at the previous time step the local consumption was higher than the local production, and vice versa;
- $f_1^{\mathrm{p}}(h) = P_{\mathrm{l}}(h)$, in order to produce more power when the local demand is high;
- $f_2^{\mathrm{p}}(h) = c_{\mathrm{b}}(h)$, so that more power is produced locally when the price for buying electricity is high;
- $f_3^{\mathrm{p}}(h) = \overline{x}_{\mathrm{b}} - x_{\mathrm{f,b}}(h)$, with the idea that more power is produced proportionally to the level of charge of the battery, i.e. more power is produced if there is not too much 'reserve power' in the battery.

The functions f_i^{uc}, f_i^{p}, f_i^{g}, $i \in \{1, 2, 3\}$ are used in the control law associated to the 'fast' model, while all the functions except for the functions f_i^{uc} are used in the control law associated to the 'slow' model.

Besides functions f_i^{uc}, f_i^{p}, f_i^{g}, $i \in \{1, 2, 3\}$, we also propose a heuristic assignment of the boolean control variables $\boldsymbol{\delta}^{\mathrm{on}}$, δ^{st}, and δ^{g} in order to reduce the computational complexity of the control problem. More specifically, we define a set of if-then-else rules to assign the values 0 or 1 to the boolean values:

- the generators are turned on, i.e. $\boldsymbol{\delta}^{\mathrm{on}}(h) = 1$, if $P_{\mathrm{res}}(h) < P_{\mathrm{l}}(h)$, so that the required power can be provided (at least partially) by the generators;
- power is bought from the main grid, i.e. $\delta^{\mathrm{g}}(h) = 1$, if $P_{\mathrm{res}}(h) - P_{\mathrm{l}}(h) < -\alpha$, where $\alpha \in \mathbb{R}^+$ is a threshold that can be defined by the user. The idea here is that

first we try to satisfy the local loads using the local production units, but if the power required by the loads is quite high, then we also allow the controller to buy energy from the main grid;
- at the same way, we allow the controller to use the energy stored in the battery, i.e. $\delta^{\mathrm{b}}(h) = 0$, if $P_{\mathrm{res}}(h) - P_{\mathrm{l}}(h) < -\alpha$, since the power balance (6) must be always satisfied. Energy can be stored in the battery in the opposite case. Moreover, due to the smaller capacity of the ultracapacitor with respect to the battery, and in order to add more flexibility to achieve the power balance (6), the ultracapacitor is allowed to store energy when the battery is being drained and vice versa, i.e. $\delta^{\mathrm{uc}}(h) = 1$, if $P_{\mathrm{res}}(h) - P_{\mathrm{l}}(h) < -\alpha$.

The threshold α can be defined by the user with some insight in the problem. Note that due to the power balance constraint (6), an upper bound to α must be imposed, which results in $\alpha \leq N_{\mathrm{gen}}\overline{P}_{\mathrm{dis}}$. Otherwise, in the worst case scenario, the power balance (6) cannot be satisfied.

Remark 2: Due to f_1^{uc}, f_3^{p} and to (15), the optimization problem becomes nonlinear. Since we also parametrize the integer values, the problem does not have integer variables. Note that the standard approach for MPC control of MLD systems in the literature results in a Mixed Integer Linear Programming (MILP) problem. While for small-sized problems the MILP approach could be faster, its complexity is exponential in the number of integer optimization variables in the worst case [15], [16]. Although our approach results in a nonlinear programming problem, it will be more scalable, since it does not suffer the exponential increase complexity related to the number of binary variables.

E. Cost Function and Optimization Problem

The cost function of the PMPC problem is a sum of the economical costs related to buying or selling electricity from or to the main grid, and consuming fuel for producing power locally. Moreover, since two different models are used, the cost function consists on the sum of two different terms. The cost function is then defined as

$$\begin{aligned} J(\boldsymbol{P}_{\mathrm{dis}}(h), C_{\mathrm{g}}(h)) = \\ \sum_{j=0}^{N_{\mathrm{f,s}}-1} \left(C_{\mathrm{g}}(h+j) + c_{\mathrm{p}} \sum_{i=1}^{N_{\mathrm{gen}}} P_i^{\mathrm{dis}}(h+j) \right) \\ + \sum_{l=0}^{N_{\mathrm{p}}-1} \left(C_{\mathrm{g}}(h+N_{\mathrm{f,s}}+l) + c_{\mathrm{p}} \sum_{i=1}^{N_{\mathrm{gen}}} P_i^{\mathrm{dis}}(h+N_{\mathrm{f,s}}+l) \right) \end{aligned} \tag{16}$$

Following (16), we define the optimization problem of the MPC controller as

$$\min_{\boldsymbol{\theta}} J(\boldsymbol{x}_{\mathrm{f}}(h), x_{\mathrm{s}}(k)) \tag{17}$$

subject to

$$\begin{aligned} &\text{dynamics } (7), (8), \text{ constraints } (6), \ (9)-(14), \\ &\text{parametrized input } (15) \end{aligned}$$

and $\boldsymbol{x}_{\mathrm{f}}(h)$ is initialized to the current state. As standard in MPC controllers, we compute the optimal parameters θ and thus the optimal inputs from the current time step h until time step $h + N_{\mathrm{p}} - 1$. We apply only the first element of the optimal input sequence and at the next sampling time we solve problem (17) once again.

IV. SIMULATION

We consider a case study similar to the one in [4]. We simulate the behavior of a microgrid that has local production units (both renewable sources and dispatchable generators), local loads, and two energy storage systems, i.e. a battery and an ultracapacitor. The values that we consider for the parameters of the microgrid are: $N_{\mathrm{gen}} = 4$, $\overline{x}_{\mathrm{uc}} = 40$ kWh, $\overline{x}_{\mathrm{b}} = 250$ kWh, $\overline{P}_{\mathrm{dis}} = 120$ kW. Moreover, $T_{\mathrm{f}} = 5$ min, $T_{\mathrm{s}} = 30$ min, $N_{\mathrm{f,s}} = 6$, $N_{\mathrm{p}} = 24$, $\alpha = 200$. We simulate the control problem of the microgrid for a simulation time of 24 h, comparing the results of a centralized MPC MILP algorithm controller (as presented in [4]) with our proposed approach. The variable energy prices are shown in Figure 3.

Figure 4 shows the power exchanged in the microgrid both for our proposed approach and for the MILP approach. Note that there are some differences in the solutions. During the peak hours, i.e. from 9 h until 20 h, the two controllers propose two different solutions: the MILP controller decides to produce power at the maximum capacity and sell all the exceeding one to the main grid, while the PMPC controller produces less power locally and sells less power to the main grid. Moreover, the usage of the storage devices is slightly different: the MILP controller drains almost immediately the power from the ESSs and uses them for mainly for balancing the power, while the PMPC controller keeps charged the ultracapacitor for a longer time. This is also depicted in Figure 5, where we show the evolution of the states x_{st} both for the MILP and the PMPC.

However, the total cost related to the two controllers is similar. A comparison is shown in Table I, where $P_{\mathrm{dis}}^{\mathrm{TOT}}$, $P_{\mathrm{g,s}}^{\mathrm{TOT}}$, and $P_{\mathrm{g,b}}^{\mathrm{TOT}}$ denote respectively the total power produced, the total power sold to the main grid and the total power bought from the main grid. It is possible to observe that the total cost associated to the PMPC controller is very close the one of the MILP controller, although the PMPC controller decides to sell less energy and buy more energy from the main grid, compared to the MILP controller. Therefore, the two controllers have a comparable performance. However, with our proposed approach we get rid of the integer variables and we parametrize the continuous control inputs, therefore we can provide a scalable algorithm compared to the standard MILP approach.

V. CONCLUSIONS

We have presented a parametric MPC approach for the optimization of the operation of a microgrid. Our controller is based on a parametrized control law and also assigns the values to the binary decision variables using parametrized heuristic rules. This approach increases the scalability of the control algorithm and reduces its complexity. Simulations

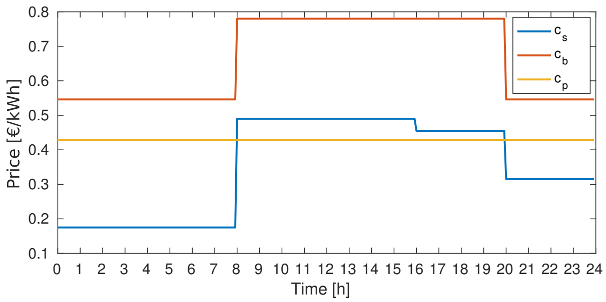

Fig. 3. Electricity purchase (c_{b}), sale (c_{s}), and production (c_{p}) prices in the considered simulation.

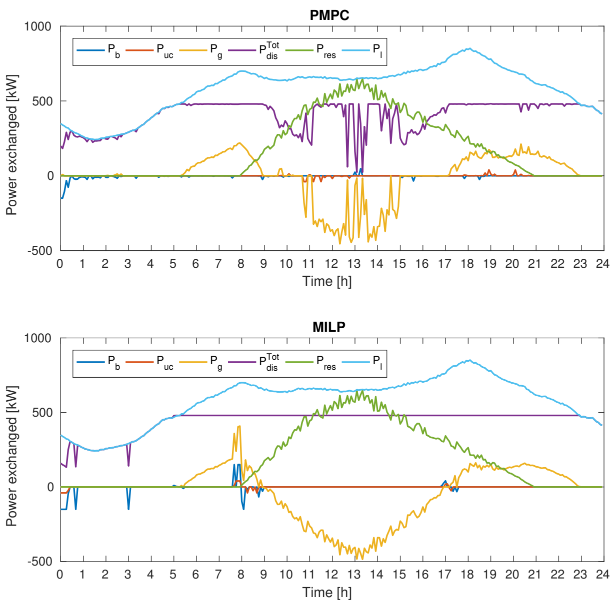

Fig. 4. Power flows in the microgrid during the considered simulation, when the PMPC controller is used (top) and when an MILP controller is applied (bottom).

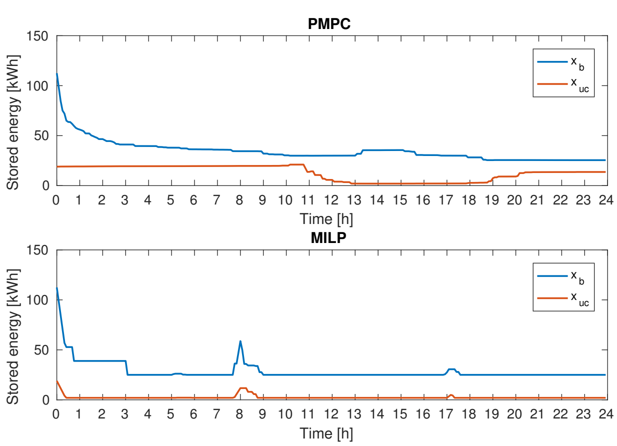

Fig. 5. Stored energy in the storage devices when the PMPC controller is used (top) and when a MILP controller is applied (bottom).

TABLE I

COMPARISON BETWEEN PMPC AND MILP SIMULATION RESULTS

	$P_{\mathrm{dis}}^{\mathrm{TOT}}$	$P_{\mathrm{g,s}}^{\mathrm{TOT}}$	$P_{\mathrm{g,b}}^{\mathrm{TOT}}$	Total cost
PMPC	116440 kW	14210 kW	13312 kW	4294.97 €
MILP	127020 kW	24434 kW	12881 kW	4225.40 €

show that our proposed approach is able to achieve a comparable performance with respect to the standard approach in the literature [4].

Future research includes defining alternative parametric input functions exploiting the dynamics of the system under control and performing a thorough comparison between our approach and the standard one in the literature in terms of scalability of the two algorithms.

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