# UNSTRUCTURED PERIODIC GRID GENERATION AROUND 2D TURBINE CASCADES

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**Abstract.** A novel grid generation algorithm for periodic unstructured grid around twodimensional highly-staggered turbine cascades, including multiply connected blade geometries, is presented. The idea is to generate the mesh in a transformed space in which the periodic boundaries are coincident and internal to the computational domain so that no special treatment along these curves is required. The mesh in the transformed u-v space is computed by means of a front-advancing/Delaunay technique and the resulting grid is transformed back into the x-y space of physical variables after introducing a suitable cut, which translates to periodic boundaries in the x-y plane. The cut is not arbitrary and it is automatically managed by the algorithm. The proposed transformation is conformal and therefore no elements are distorted in the process. In this way, the prescribed element size and mesh quality are easily attained and the uniqueness of the periodic nodes is guaranteed by the fact that they are indeed coincident in the space u-v. Numerical simulations of a VKI LS-89 turbine blade are presented to support the present approach.

### **1** INTRODUCTION

Computational Fluid Dynamics (CFD) codes are currently used to study the internal flow in many turbomachinery applications, including three-dimensional simulations of stator-rotor interaction, the study of blade cooling and the design of the geometry of the blade itself, see e.g [7]. State of the art CFD codes include turbulence modeling and also the possibility of relaxing the ideal gas hypothesis to study the internal flow of non-ideal or so-called real gases [5, 7, 2, 3].

A main issue in turbomachinery simulations is that of mesh generation, which is often complicated by the non simple geometry of the blade, possibly including the internal duct system for blade cooling, and by the need of introducing suitable periodic boundaries to reduce the computational effort. In particular, for highly-staggered two-dimensional turbine cascades, the determination of the (fictitious) periodic boundary is not a trivial



Figure 1: Left: exemplary turbine cascade configuration in two-spatial dimensions (stator). The blades are separated by a constant pitch. Right: equivalent computational domain, bounded by the inflow, the outflow and the periodic boundaries.

task. In fact, if the periodic boundaries are chosen to be straight lines aligned with the axial coordinate, then the intersections of these lines with the blade geometry are to be computed explicitly and their determination requires the blade geometry to be described by a continuous function. Alternatively, a cutting curve that does not intersect the blade can be drawn. The latter is usually a complicated task, in that, depending on the blade geometry and the pitch, the periodic boundary can be a highly curved line (2D) or surface (3D).

Grid generation around turbine cascade can be obtained by resorting to both structured and unstructured grid generation approaches. However, structured periodic grids around turbine cascades are usually characterized by high skewness in the case of highly staggered cascades [10]. The quality of unstructured grids can be improved by resorting to multiple-zone mesh generation techniques [6]. In this case, a smooth transition of the element size across fictitious boundaries internal to the domain is to explicitly imposed. Instead, if non periodic structured grids are used, a suitable interpolation technique is to be introduced at the solver level to treat the periodic boundary condition correctly. Alternatively, unstructured grids have been used [6, 7, 10] in the computation of compressible flows around highly staggered cascades, including hybrid grids made of both tetrahedron and prisms clustered in the boundary layer region for viscous flow computations [6]. The main advantage of unstructured grids is related to the high flexibility and grid quality around virtually any geometry. In this case, periodic nodes are then usually obtained by duplicating the boundary nodes on periodic boundaries, since the connectivity between the boundary nodes and the internal ones is not required to satisfy the topological constraints that are typical to structured grids. However, the simple operation of copying periodic boundary nodes does not guarantees the fulfillment of the constraints on the element size or the element quality [10]. Moreover, an arbitrary cut of the periodic geometry is to be introduced explicitly, an operation that is not easy to automate in the case of highly staggered cascades.

In the present paper, a novel algorithm for unstructured grid generation around highlystaggered turbine cascades is presented and applied to two-dimensional geometries. A description of the proposed approach is given in section 2. In section 3, numerical results for the VKI LS-89 turbine blade are computed and compared to available experimental data. Final remarks and comments are given in section 4, where the possibility of extending the present approach to three-dimensional geometries is also discussed.

# 2 PERIODIC GRID GENERATION

In figure 1 (left), an exemplary two-dimensional turbine cascade configuration is shown, together with the lines indicating the inflow and outflow boundaries of the computational domain. The turbine blades are separated by a constant pitch and therefore the computational geometry can be simplified by introducing suitable periodic boundaries as in figure 1 (right). The computational domain is therefore bounded by the two portion of the inflow and outflow boundaries with length b (pitch) and by the two periodic boundaries with length a.

In the present section, the proposed grid generation procedure to discretize the computational domain is described. In particular, the coordinate transformation from the physical space x-y to a suitable u-v space is first introduced to "cancel out" periodic boundaries (section 2.1). Afterward, a front-advancing/Delaunay code is used to generate the grid in the u-v space; the prescribed element size at the domain boundaries and at specified internal points is translated from the x-y representation and enforced during the grid generation process. The grid is transformed back into the x-y plane after introducing a suitable cut, which translates to periodic boundaries in the x-y space. Examples on single- and multiply-connected geometries are also given (section 2.2).

#### 2.1 Geometry transformation

A suitable coordinate transformation is now introduced to simplify the task of generating the computational mesh inside the domain shown in figure 1 (right). The new spatial coordinates u and v in the transformed space are related to the physical coordinates x-yby the following definitions

$$\begin{cases} u(x,y) = r(x) \cos \theta(y) \\ v(x,y) = r(x) \sin \theta(y) \end{cases}$$
(1)



Figure 2: Transformation from the physical space of x-y coordinates (left) into the u-v space (right) where the grid is generated. The inflow and outflow boundaries are transformed into the inner and outer circles, respectively. The periodic boundaries AB and DC collapses into a single line in the u-v plane which is internal to the domain itself, namely, they don't represent anymore a portion of the domain boundaries.

where r and  $\theta$  are the radial and angular coordinate, respectively, of a polar coordinate system in the u-v plane, see figure 2 (right), which are arbitrary assumed here to depend on the x and y coordinate only, respectively.

The functions r = r(x) and  $\theta = \theta(y)$  are now determined to guarantee that the above transformation is conformal and therefore the standard (isotropic) Delaunay triangulation technique can be used in the transformed space as well. In other words, we're looking for a transformation which changes the size of the triangles but not their shape, namely, the angles formed by the edges of each triangle. The Jacobian of the transformation is easily computed from (1) as follows

$$J(r,\theta) = \begin{pmatrix} r'\cos\theta & -r\theta'\sin\theta\\ r'\sin\theta & r\theta'\cos\theta \end{pmatrix}$$
(2)

where r' and  $\theta'$  are the first order derivatives of the functions r and  $\theta$ , respectively. To guarantee that transformation (1) is conformal, the metric  $M = J^{T}J$  in the transformed space should be such that  $M = R^{T}\lambda^{2}R$ , or  $J = \lambda R$ , where  $\lambda$  is a scalar quantity and R is an orthogonal matrix, namely,  $R^{T} = R^{-1}$ . By noticing that the matrix

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{pmatrix}$$

is indeed orthogonal, representing in fact a rotation in the two-dimensional space, the coordinate transformation (1) is conformal provided that  $r'(x) = r(x)\theta'(y) = \lambda$ , cf. relation (2) which immediately gives  $\lambda = \lambda(x)$  and  $\theta'(y) = k$ , or

$$\begin{cases} r(x) &= e^{k(x-x_0)} \\ \theta(y) &= k(y-y_0) \end{cases}$$
(3)

where the value of the constant k is obtained by applying the condition  $\theta(y_1) = 2\pi$  and hence

$$k = \frac{2\pi}{y_1 - y_0} = \frac{2\pi}{b}$$

The result of the above coordinate transformation is shown in figure 2 (right). The ABCD quadrilater, in which the boundary AD and BC correspond to the inflow and outflow boundaries of figure 1, are transformed into two coaxial circles. The inner circle, with radius  $r_0 = 1$ , corresponds to the inflow boundary whereas the outer one, with radius

$$r_1 = e^{k(x_1 - x_0)} = e^{2\pi a/b},\tag{4}$$

is the outflow boundary. Points A and D and points C and D are coincident in the u-v space and therefore the periodic boundaries collapse into a single segment which is internal to the computational domain. Therefore, in the transformed space u-v, periodic boundaries are no longer boundaries of the computational domain and do not require any special treatment as far the grid generation procedure is concerned, as detailed in the following section.

#### 2.2 Grid generation

The generation of the computational grid in the u-v domain is now detailed. The grid is generated by means of the advanced-front/Delaunay technique of Rebay [8], which allows for generating grids made of isotropic triangular elements. The user is required to provide the domain geometry and to specify the size h of the elements along the boundary curves and at selected points internal to the computational domain.

The prescribed value of the element size is therefore first translated from the physical space x-y into the transformed space u-v by multiplying the local element size  $h_{x,y}$  by the determinant of the Jacobian (2) of the transformation, namely,

$$h_{u,v} = \left[\det J(x,y)\right] h_{x,y} = \lambda(x) h_{x,y} = k e^{k(x-x_0)} h_{x,y},\tag{5}$$

where  $h_{u,v}$  is the required element size in the *u*-*v* plane. The resulting triangulation in the *u*-*v* space for the geometry in figure 2 is shown in figure 3 (left), which corresponds to a grid of made of triangles with uniform size  $h_{x,y} = \text{const.}$  in the *x*-*y* space (figure 3, right). Note that an uniform mesh size in the *x*-*y* space translates into a nonuniform mesh size in the transformed space, in accordance with the amplification factor in relation (5).

A suitable cut along the grid is to be introduced before transforming the grid from the *u*-*v* space, where the geometry is periodic with respect to the  $\theta$  variable (with period equal to  $2\pi$ ), to the physical one, where the cut will represent the periodic boundaries. The desired cutting line is easily obtained as a sequence  $i_n$ ,  $n = 1, \dots, N_c$  of grid nodes connected by the edges of the triangles composing the mesh. Along the inner circle corresponding to the inflow boundary in the *x*-*y* space, the starting node  $i_1$  is chosen as that closest to the  $\theta = \pm \pi$  line. From a given node  $i_{n-1}$ , node  $i_n$  is found by selecting the



Figure 3: Grid generation around the ABCD box of figure 2. The cutting line separates element on the  $\theta = \pm \pi$  line, starting from the inflow boundary (inner circle) and moving towards the outflow one (outer circle).



Figure 4: Left: computational grid around the VKI LS-89 blade in the u-v transformed space. Right: enlarged view of the region around the inflow (not visible) and the blade. As the cutting line hits the blade boundary, it continues along the blade itself until a point with  $\theta = \pm \pi$  is reached. From there, it "leaves" the blade to move towards the outflow boundary.



Figure 5: Computational grid around the VKI LS-89 blade in the physical space x-y.

node closest to the  $\theta = \pm \pi$  line among those belonging to the bubble of elements of node  $i_{n-1}$ , namely, to the set of elements sharing node  $i_{n-1}$ . The process is repeated until the outer circle, namely, the outflow boundary, is reached, see figure 3 (left).

If internal geometries are present, such as for example the blade geometry in figures 4 and 5, the above procedure is followed until the cutting line (possibly) intersects an internal boundary. If this is the case, the cutting line follows the internal geometry until the cutting line intersects the  $\theta = \pm \pi$  line again. Starting from the grid node closest to the next intersection of the boundary with the  $\theta = \pm \pi$  line, internal (domain) nodes are considered again according to the procedure detailed above, until either the outer boundary or an internal geometry is reached, see figure 4. Multiple intersections may possibly occur, depending on the pitch of the configuration and on the blade size. Such a situation is sketched in figure 6, where a highly staggered configuration is considered. The geometry in figure 6 has been also used to investigate the performances of the present method with respect to multiply-connected internal geometries, as it is the case for example of turbine blade including a cooling system. Note that the geometry of the cooling lines represented in figure 6 is quite unrealistic and it has been used here only to assess the grid quality.

### 3 VKI LS-89 TURBINE CASCADE

Numerical results of the inviscid flow around a VKI-LS89 cascade configuration are now presented to support the proposed grid generation technique. The flowfield Mach number and pressure isolines are shown in figures 7 and 8 for two exemplary cases taken from [1], together with a comparison of the numerical results with the isentropic Mach number  $M_s$  distribution computed from the experimental data. Under the assumption of polytropic—



Figure 6: Grid generation in the u-v space (top) and corresponding grid in the physical space (bottom) for a multiply-connected blade geometry. The plots on the right are enlargements of the grid near the leading edge.

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Test Number	$M_{2,s}$	$P_1^{\rm t}$ [bar]	$T_1$ [K]	$P_2$ [bar]	$P_2/P_1^{\rm t}$
MUR43 MUR49	$0.84 \\ 1.02$	$1.435 \\ 1.608$	$\begin{array}{c} 420\\ 420\end{array}$	$0.840 \\ 0.750$	$0.630 \\ 0.516$

Table 1: Test cases MUR43 and MUR49 for the VKI-LS89 blade from [1]. The subscripts 1 and 2 indicate inflow and outflow conditions, respectively.



Figure 7: Numerical results for the MUR43 test case in [1]. Mach isolines (left), pressure isolines (center) and isentropic Mach number at measurement station along the blade (right).



Figure 8: Numerical results for the MUR49 test case in [1]. Mach isolines (left), pressure isolines (center) and isentropic Mach number at measurement station along the blade (right).

a gas with constant specific heat at constant temperature—ideal gas considered here, the isentropic Mach number is computed from the value of the pressure at each measurement station as follows

$$M_s(P, P^{\rm t}) = \sqrt{\frac{2}{\gamma - 1} \left[ \left(\frac{P^{\rm t}}{P}\right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]},\tag{6}$$

where  $\gamma$  is the ratio of the specific heat at constant pressure and volume, respectively,  $(\gamma = 1.4 \text{ for air})$ ,  $P^{t}$  is the total pressure and P is the measured pressure. The two cases considered here, labeled MUR43 and MUR49 in [1], are characterized as in table 1.

Numerical results are found to agree fairly well to the experimental data in both the considered cases. All computations have been performed on an unstructured grid made of 4009 nodes (7564 triangles) using the real-gas solver zFlow [3] under the polytropic ideal gas model. zFlow is an unstructured grid solver based on the node-pair data structure of Selmin [9] for the finite volume and finite element method that has been develop to study real gas flows in turbomachinery application using state-of-the-art thermodynamic models. Boundary conditions at periodic boundaries have been enforced by allowing for the number of degrees of freedom to differ from that of boundary nodes. A grid node lying on the top boundary is associated to the corresponding node on the bottom boundary by linking the two nodes to one and the same degree of freedom or nodal unknown. In this way, no periodic boundary conditions are to be imposed explicitly. The pressure boundary condition at the outflow has been imposed according to the nonreflecting strategy proposed by Giles, see e.g. [4], in which only the pressure average value is prescribed. This boundary condition treatment greatly reduces spurious reflections at the outflow boundary which, for a supersonic flow with shock waves crossing the outflow boundary such as that depicted in figure 8, would seriously pollute the solution.

# 4 CONCLUSIONS

A novel approach to periodic unstructured grid generation around highly-staggered turbine cascade has been presented and applied to exemplary geometries for turbomachinery applications in two-spatial dimensions, including multiply-connected geometry such as those describing blade cooling systems. The grid is generated in a suitable transformed space in which periodic boundaries are in fact internal to the domain and therefore do not require any special treatment. The proposed metric transformation is conformal and hence an existing isotropic unstructured grid generation software can be used in the transformed space of u-v variables, without the need of introducing an anisotropic metric. A suitable cut in the u-v space in automatically constructed before transforming the grid back into the physical space, which translates to the two required periodic boundaries. The grid generation scheme proved successfully in all cases resulting in high-quality unstructured grid for turbomachinery computations. Two standard test cases for the compressible flow around the VKI LS-89 blade have been simulated and a fairly good agreement has been found with the experimental data, thus proving the consistency of the proposed approach. Further work in ongoing to extend the proposed technique to hybrid two-dimensional grids including both triangles and quadrilaters for viscous flows computations and to three-dimensional grids. Preliminary studies on the three-dimensional case pointed out two main difficulties. First of all a suitable conformal transformation in three space dimensions has not yet been found. Moreover, even with such a transformation at hand, the metric amplification factor in (5) can be very large, thus leading to large differences in the element sizes in the u-v plane, a situation that can possibly conflict with the use of infinite accuracy algebra—an almost mandatory requirement for Delaunay grid generation in three spatial dimensions.

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