Introducing Process Induced Anisotropy in Topology Optimization

Leveraging Topology Optimization Algorithms to Address Anisotropic Behavior Introduced by Additive Manufacturing Methods in Glass Structures

June 23, 2025 – P5 Presentation

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Why

Glass

Topology
Optimization

Additive
Manufacturing

Why

Glass

Topology Optimization

Additive Manufacturing

Why

Glass

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Topology Optimization

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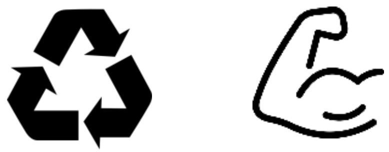
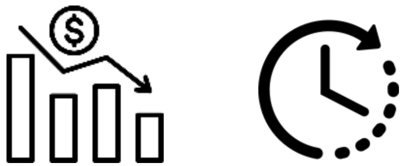
Additive Manufacturing

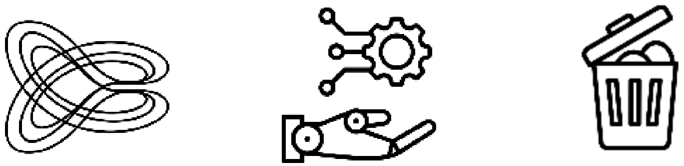
- Transparent
- Recyclable
- Durable
- Strong



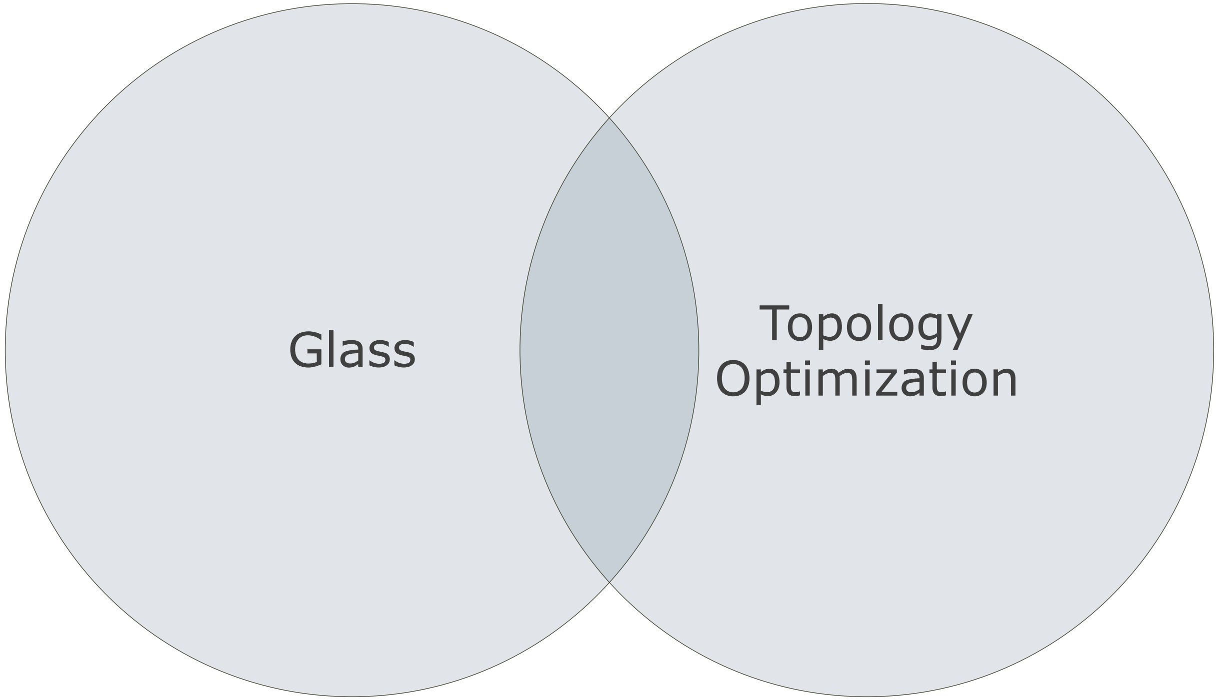
- Efficient use of material
- Reduced costs



- Complex geometries
- Reduced material waste
- Process automation



Koniari + Schoenmaker

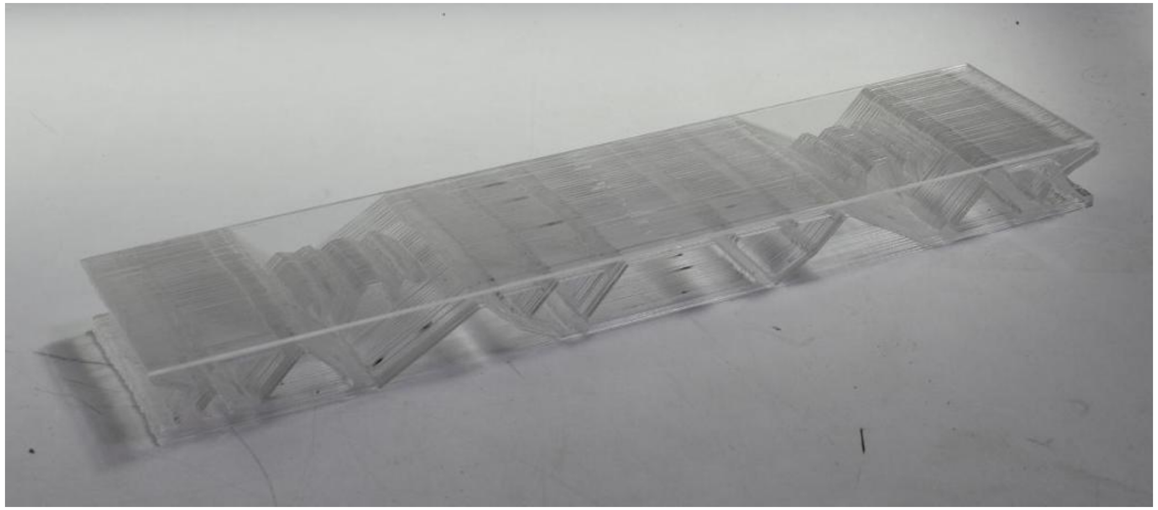


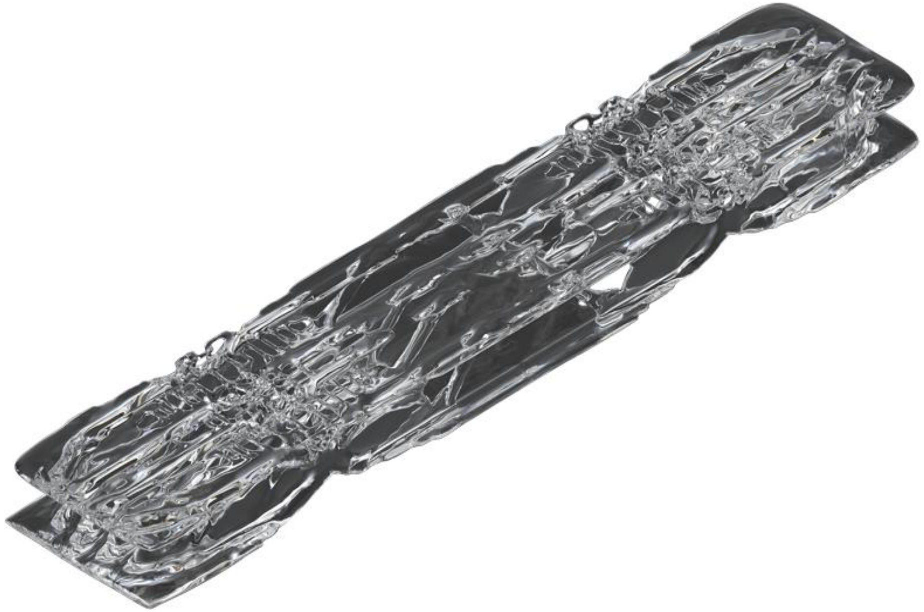
Koniari

- 2D algorithm
- With focus on integrating annealing times into the optimization process

Schoenmaker

- 3D algorithm
- With focus on volume and compliance optimization





Koniari

- 2D algorithm
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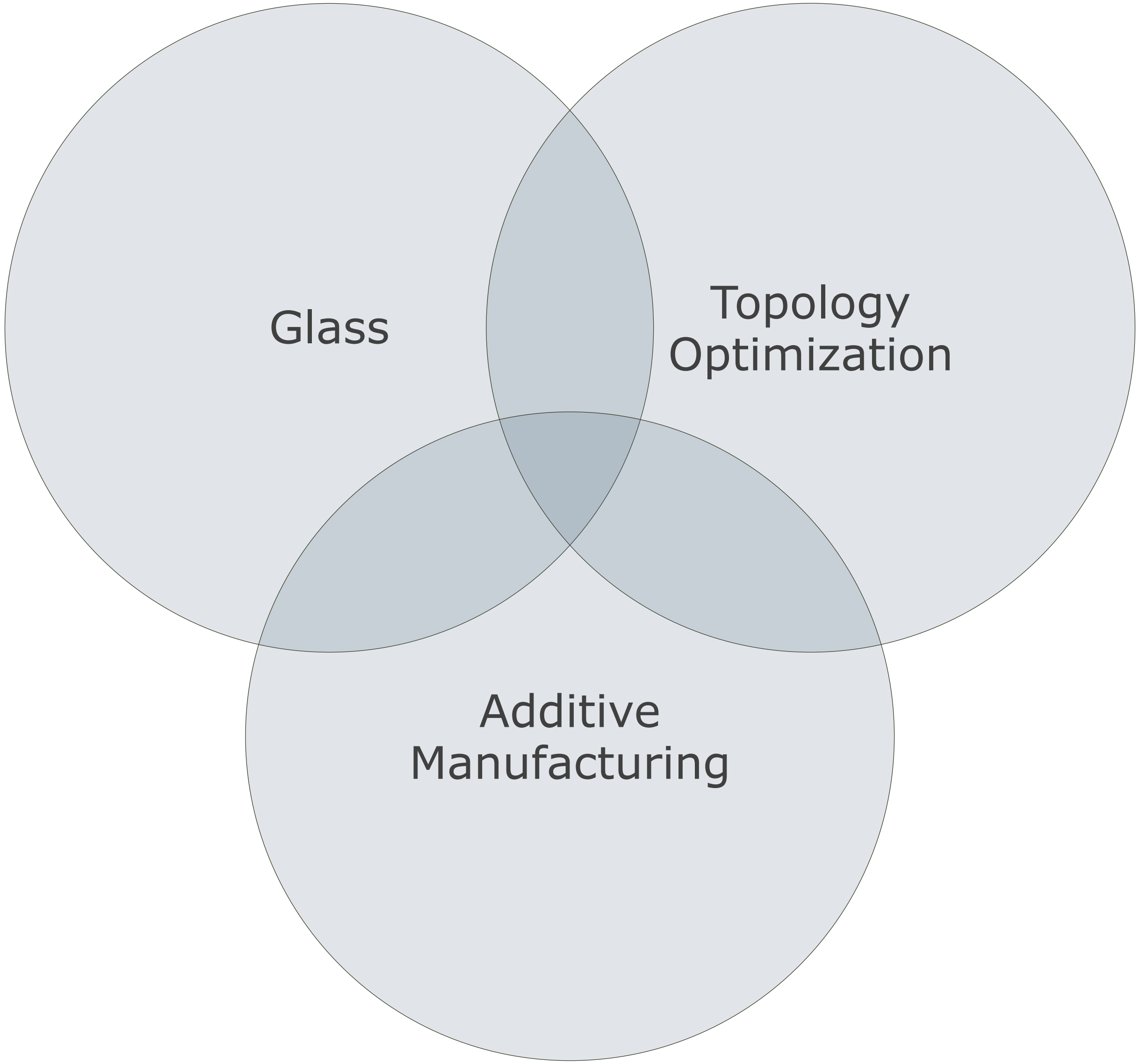
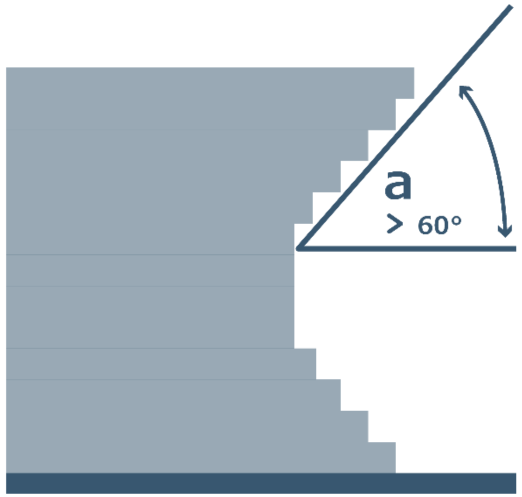

Glass
Topology Optimization
Additive Manufacturing

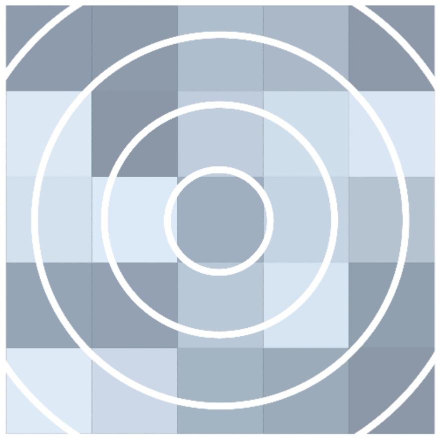
Brueren

Overhang



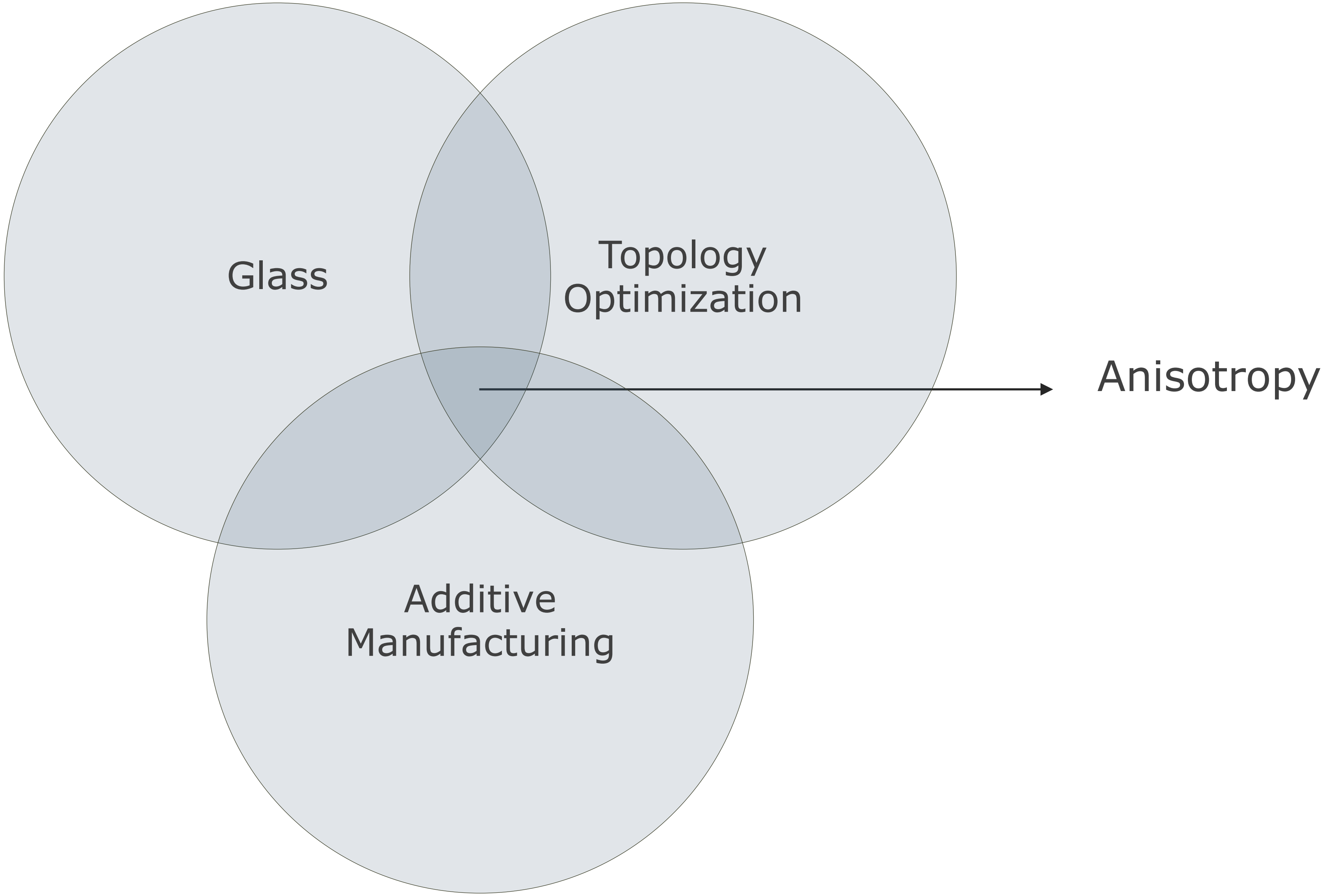
Path Continuation

Nozzle Size



What is left?

Mananas (me)

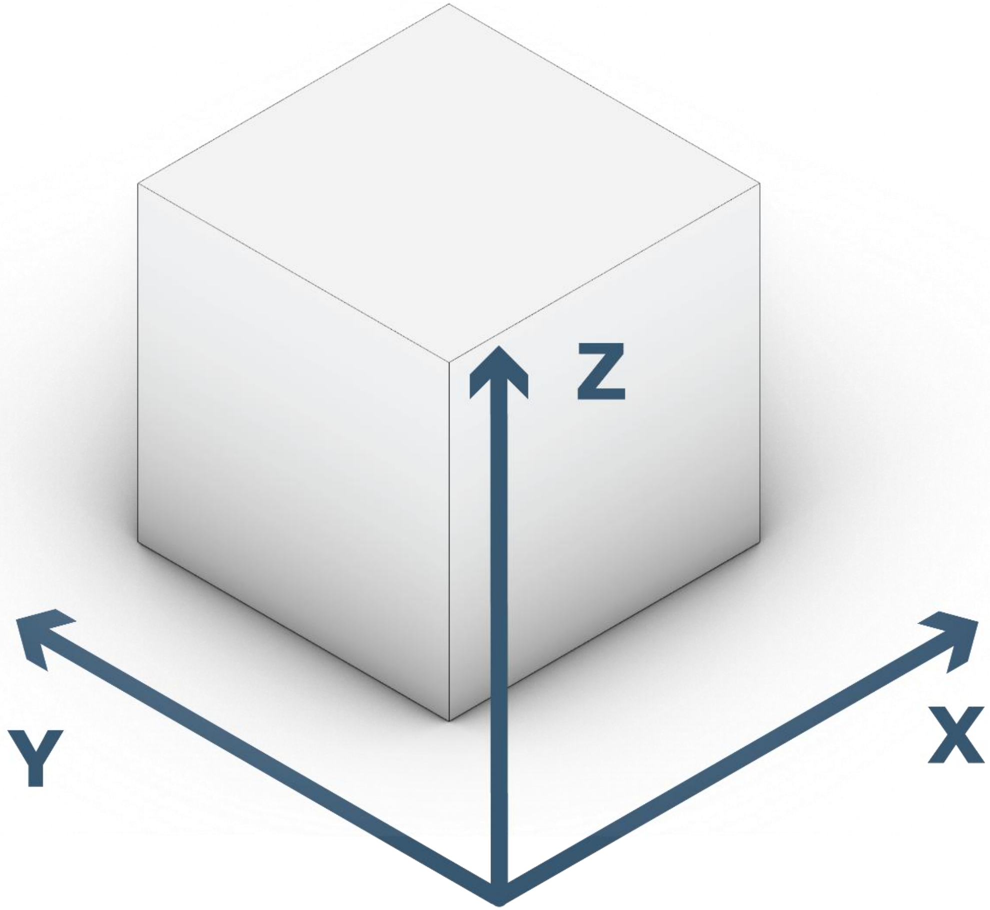

Glass
Topology Optimization
Additive Manufacturing
Anisotropy

Anisotropy

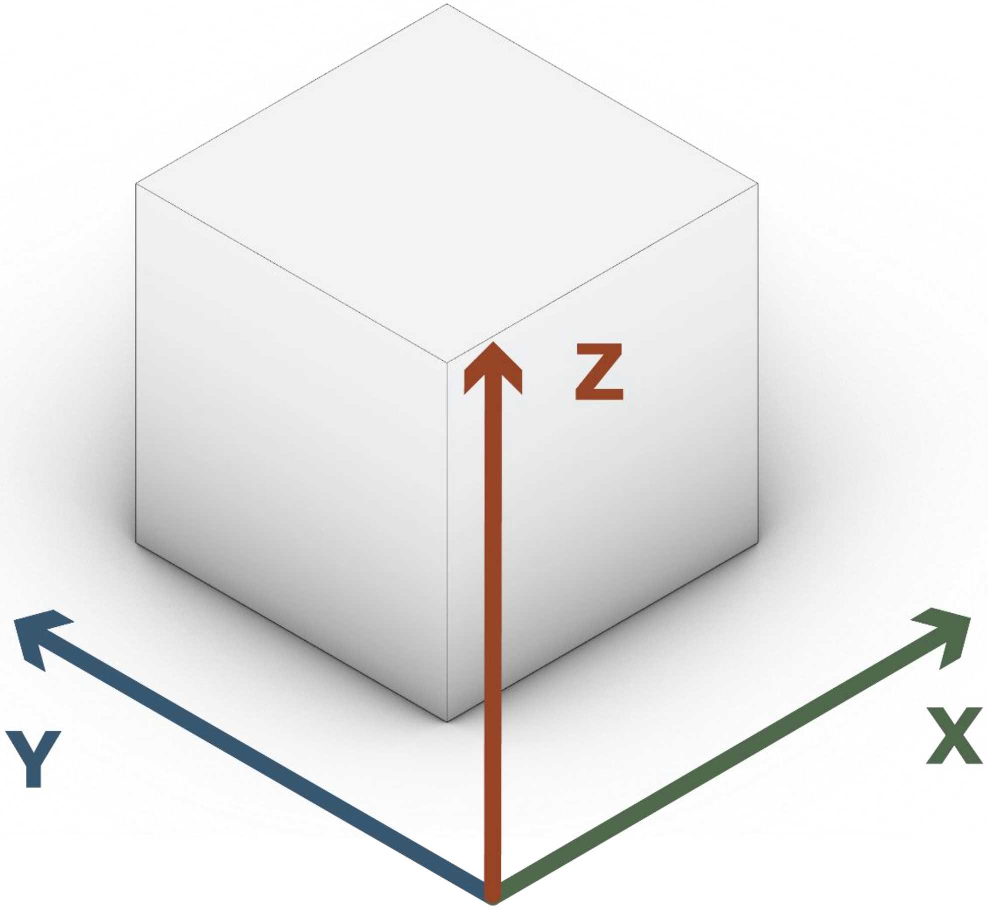
“Anisotropy refers to the **directional dependence of a material's properties**. Unlike isotropic materials, which exhibit the same characteristics in all directions, anisotropic materials display different behaviors when measured along different axes.”

Anisotropy

Isotropy



Anisotropy



Anisotropy

Is glass anisotropic?

No.

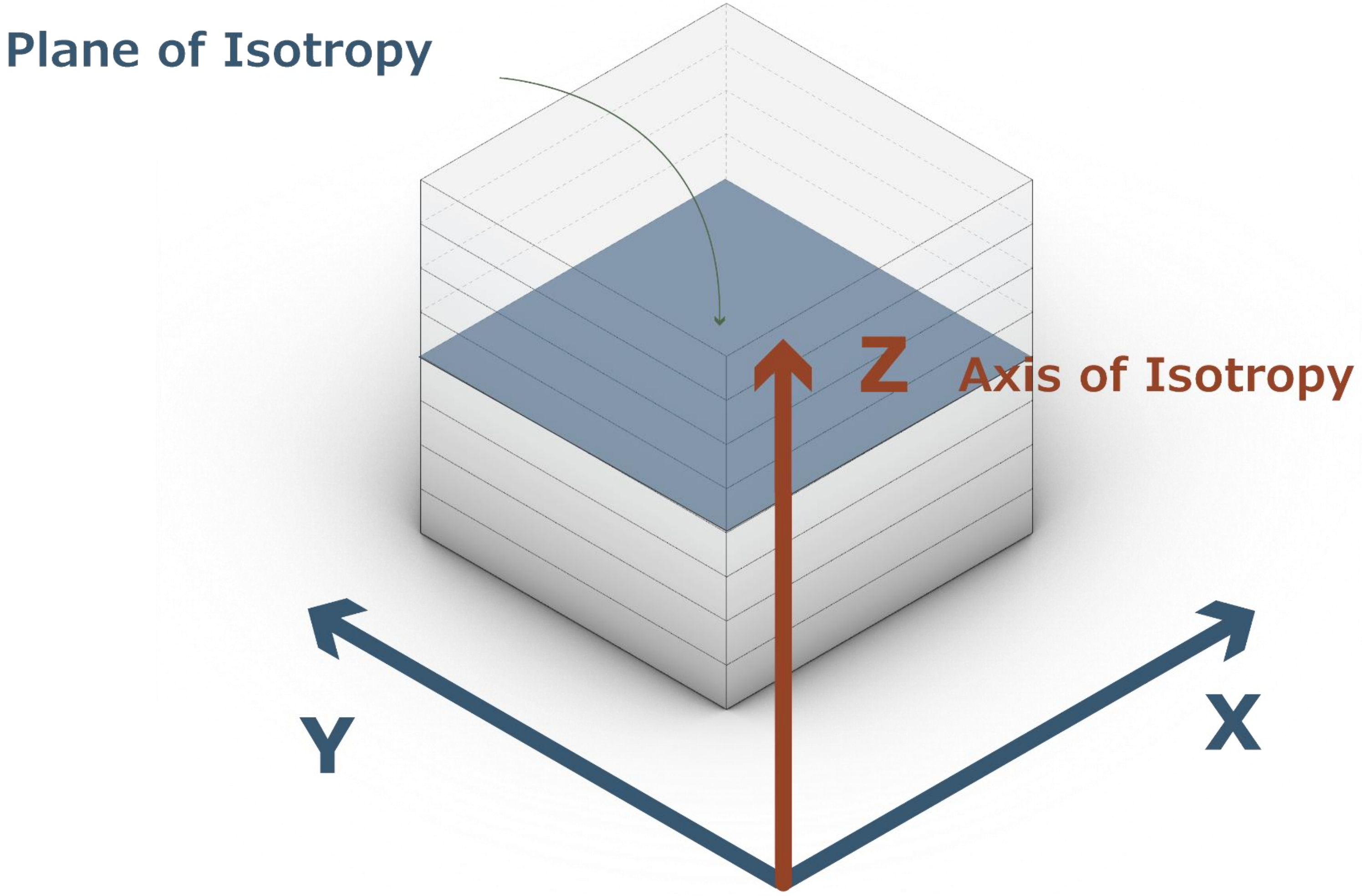
Anisotropy

Process Induced Anisotropy

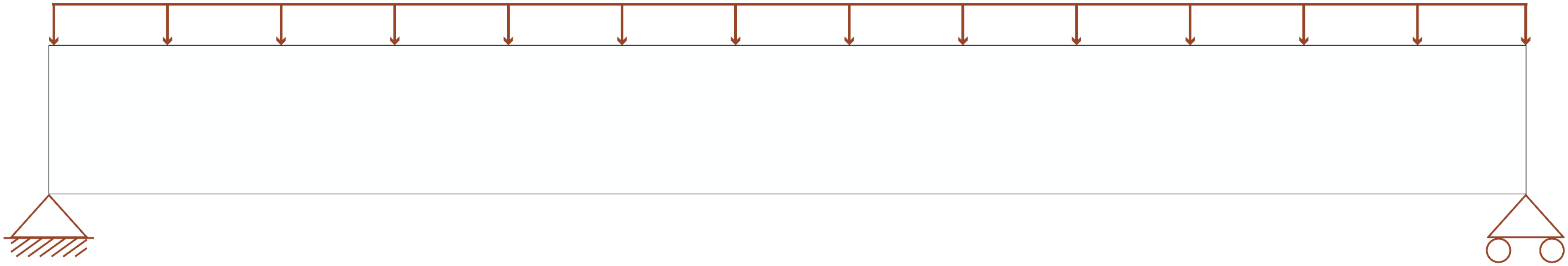
Process-induced anisotropy usually occurs due to the **manufacturing process**, primarily because of **insufficient** or **non uniform inter-layer bonding** owing to layer-by-layer manufacturing strategy for most additive manufacturing processes.

Kundu et. al 2023

Transverse Isotropy



Problem Statement



Problem Statement

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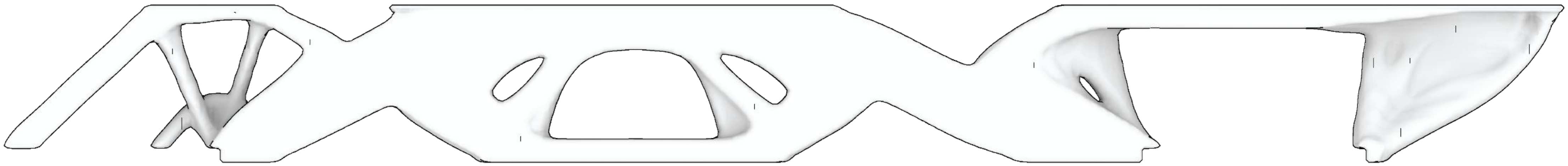
Problem Statement



Problem Statement

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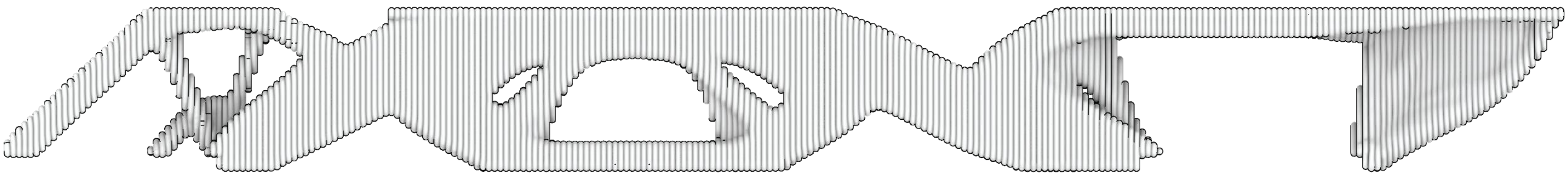
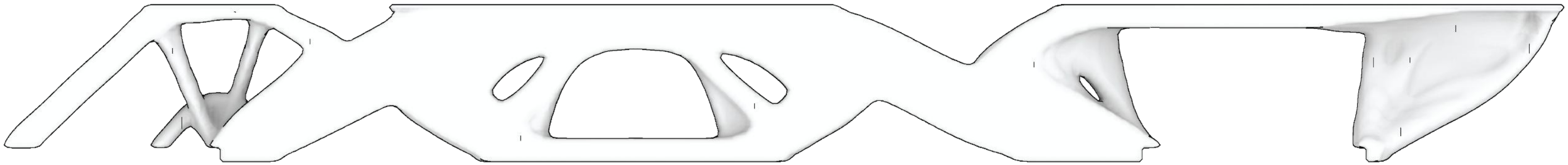
Problem Statement



Problem Statement



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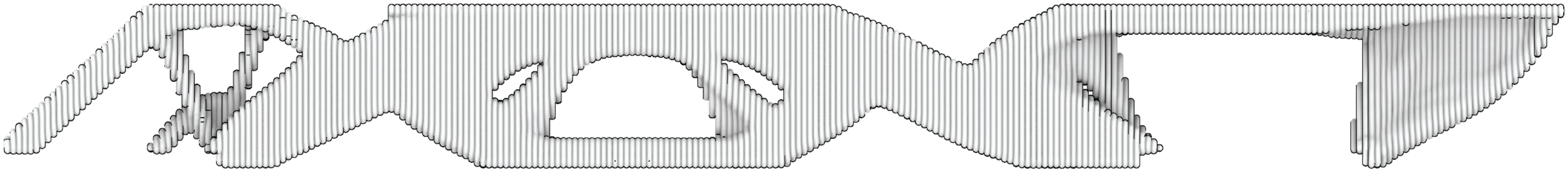
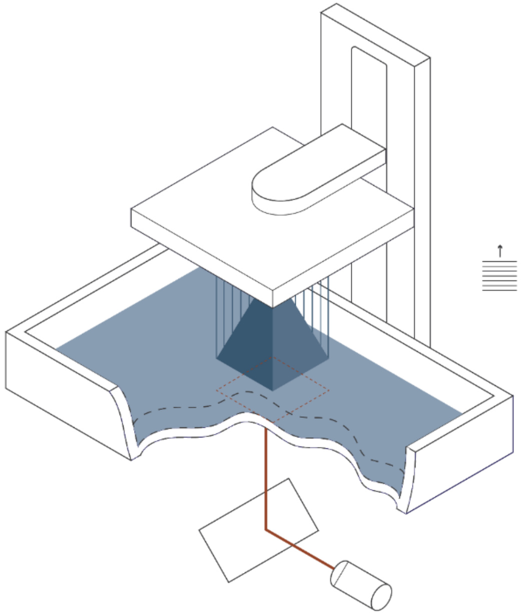
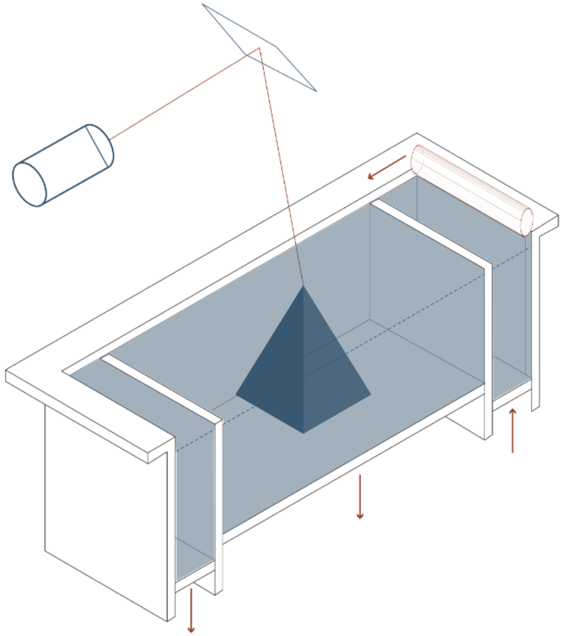


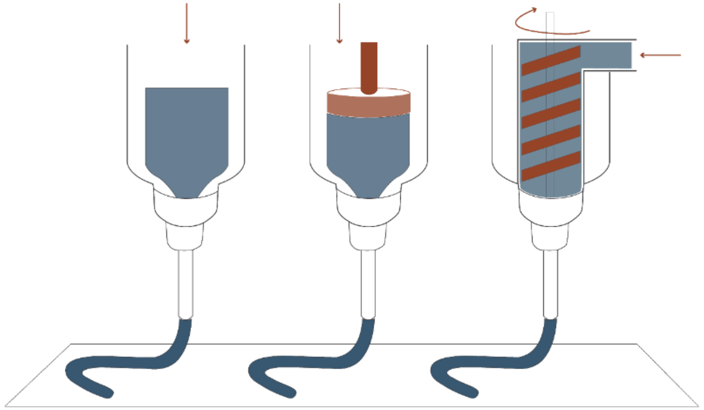
Additive Manufacturing



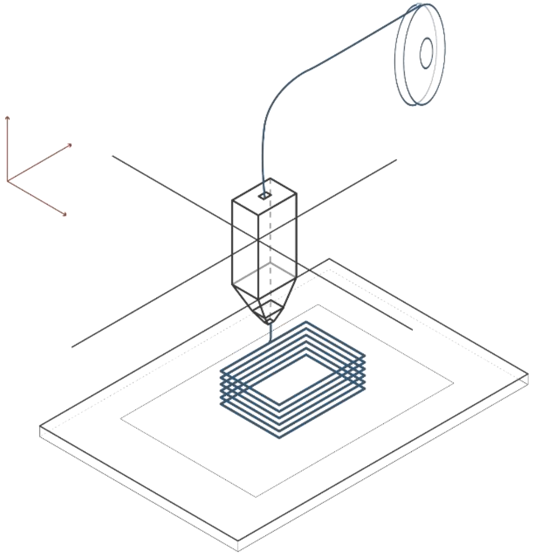
SLA



SLS

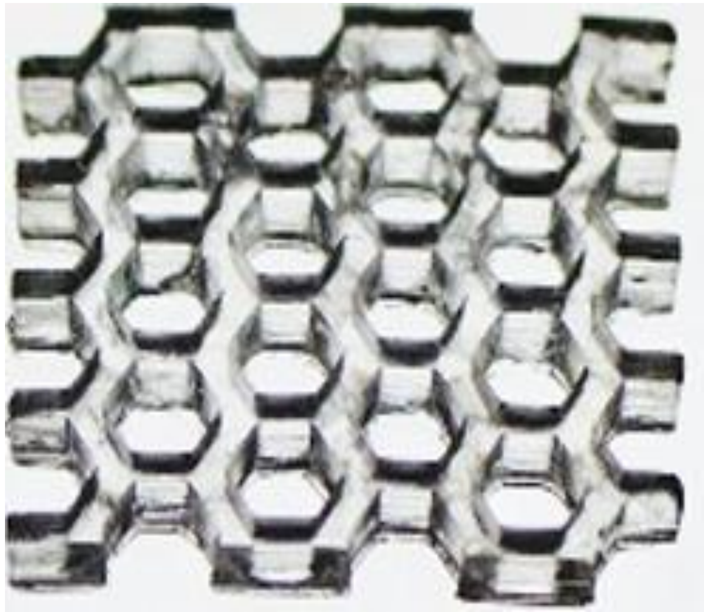


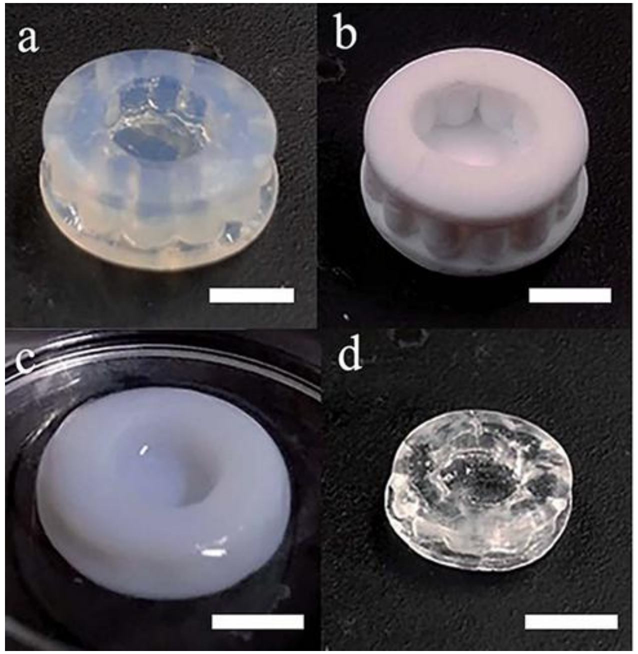
DIW

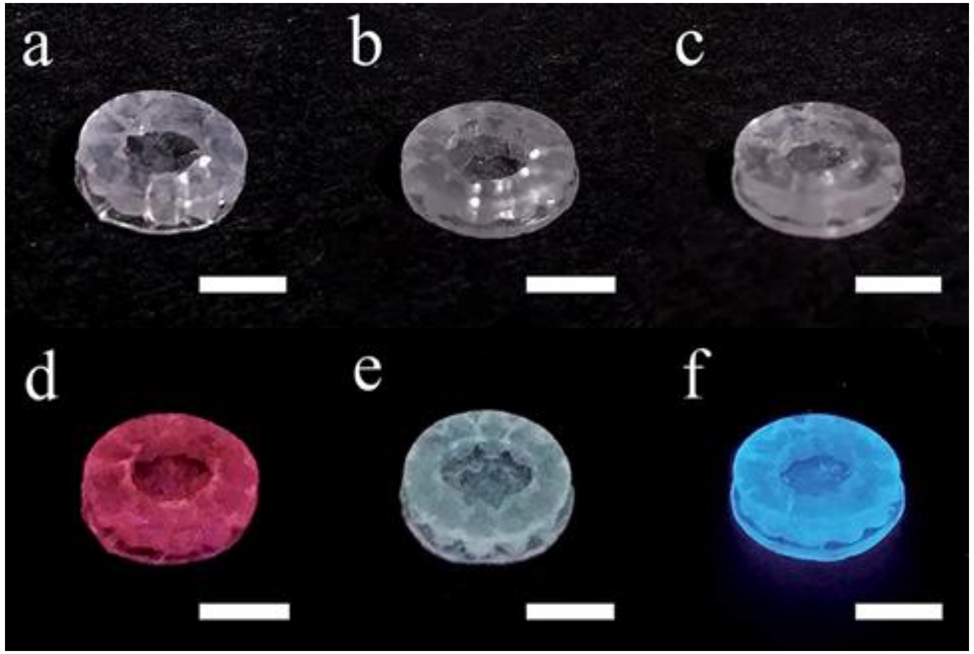


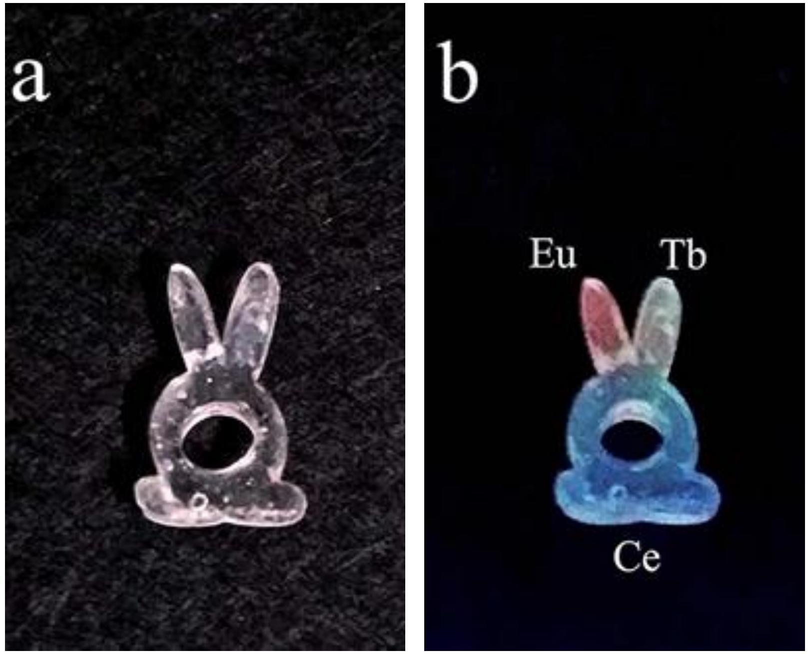
FDM

Additive Manufacturing - Stereolithography

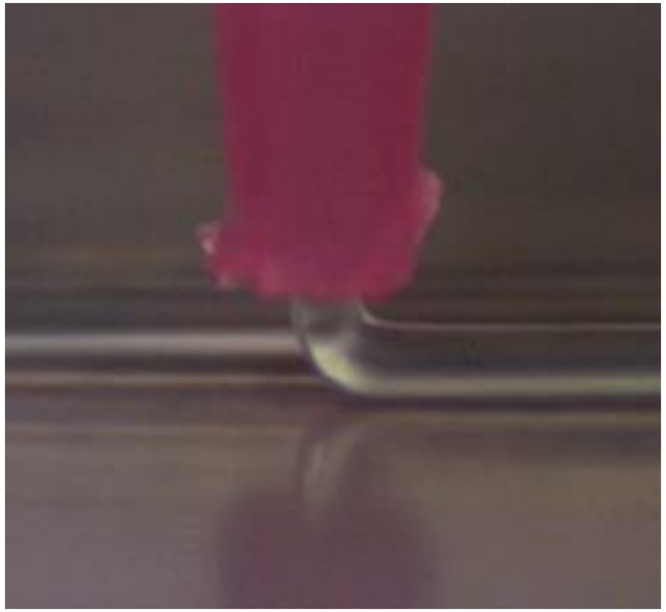
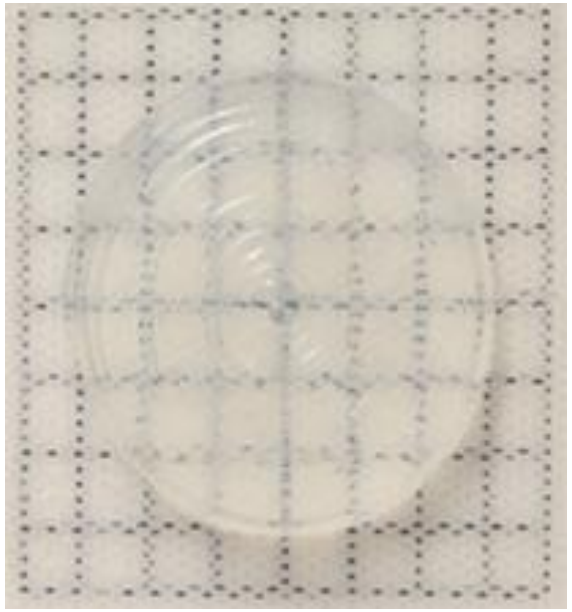


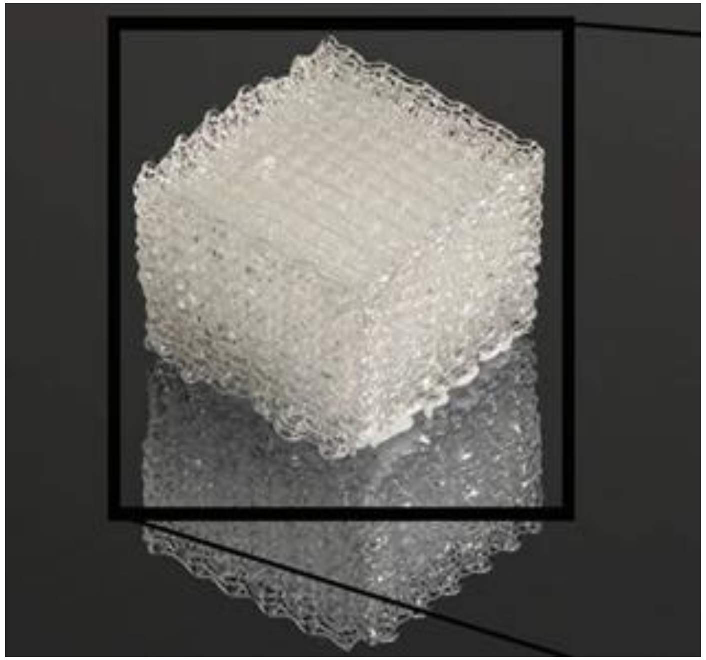






Additive Manufacturing – SLS & DIW

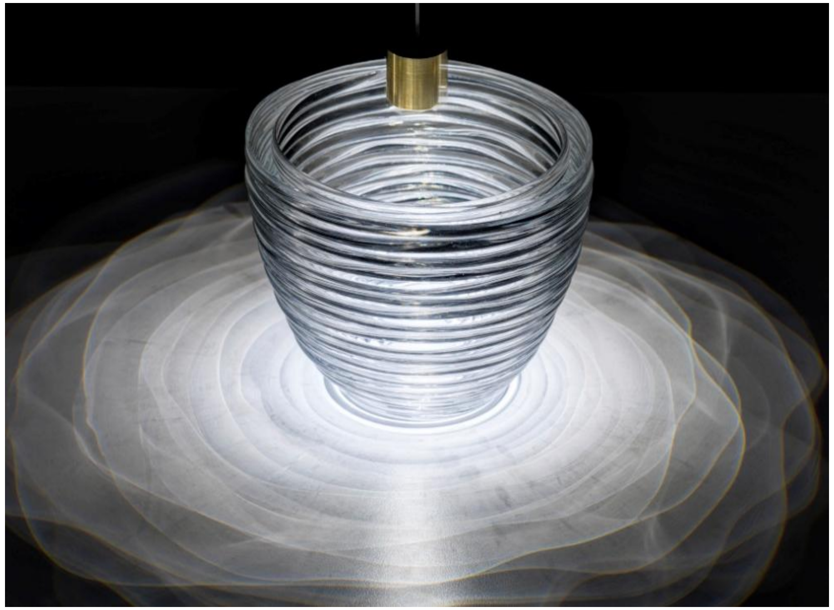



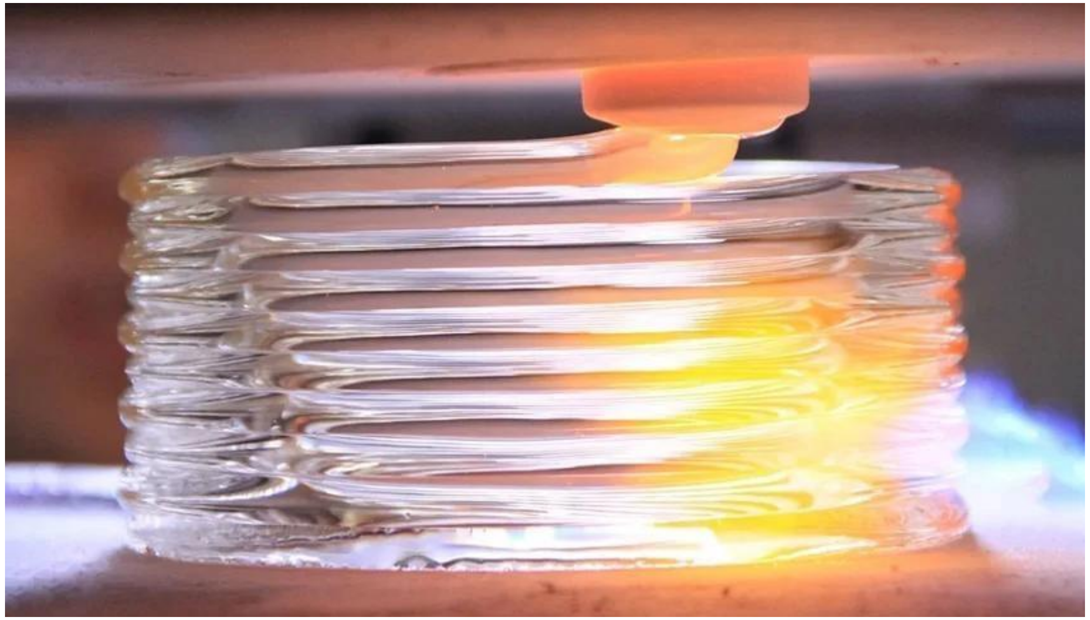



Additive Manufacturing

G3DP Platform

Additive Manufacturing




Research Question

"How can **process induced anisotropy** be integrated into **topology optimization**, using a **SIMP** algorithm, to ensure both manufacturability and the expected structural integrity of glass components?"

Overview

Case Study

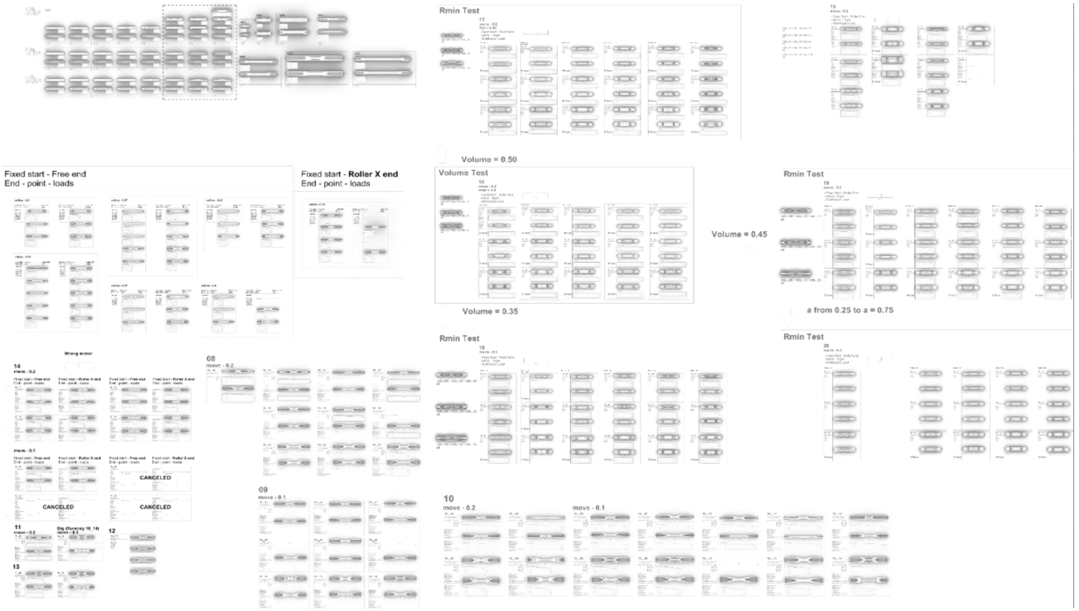
Topology Optimization

FEA

Flexural Testing

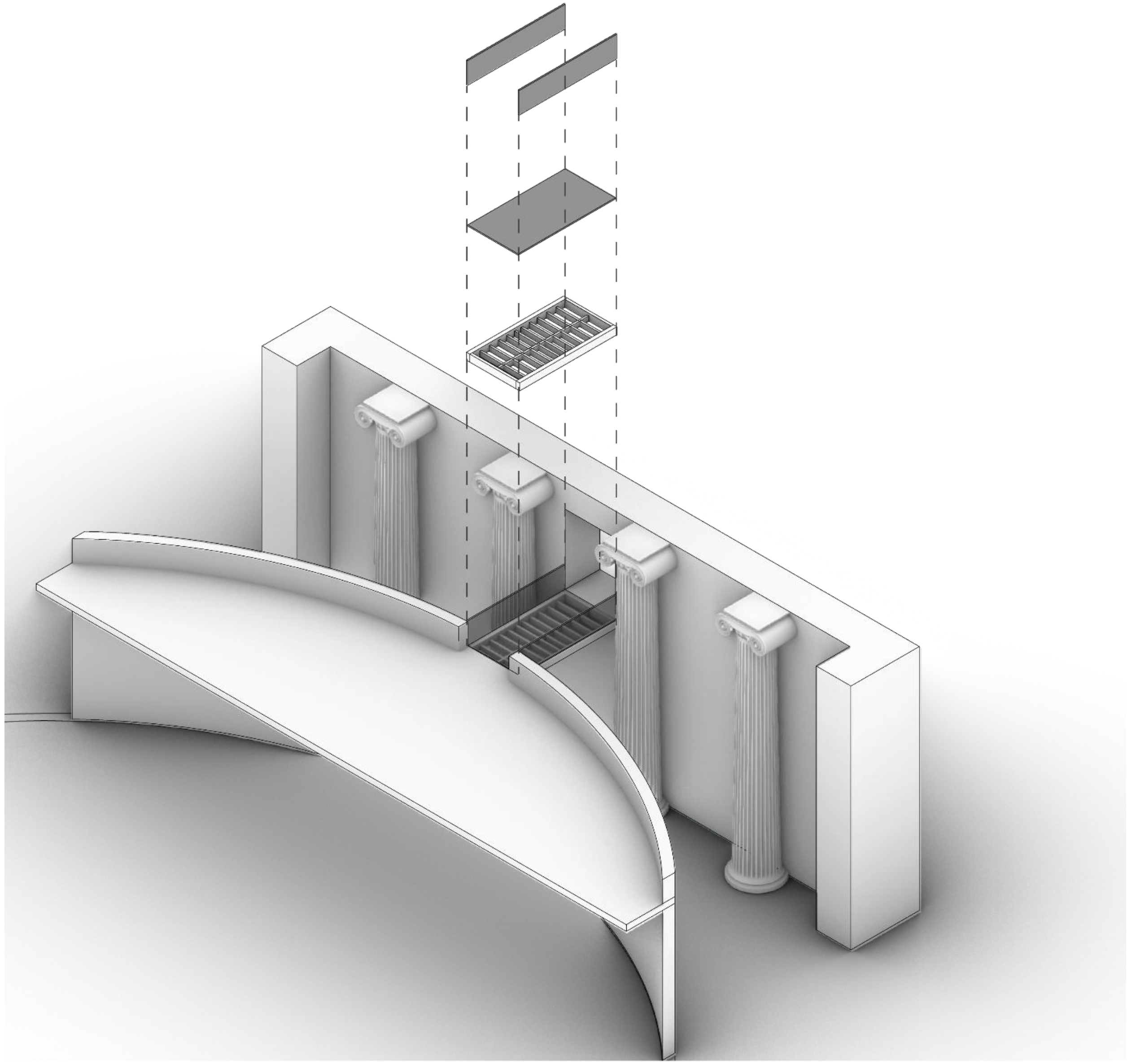
Algorithm Development

Testing & Results



Case Study

Case Study



Structural System Old

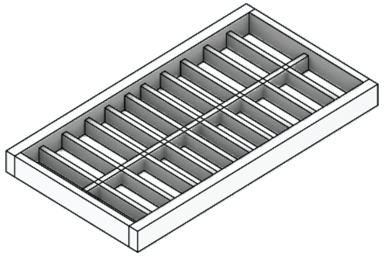
1) Glass Railing

2) Laminated Glass Sheets

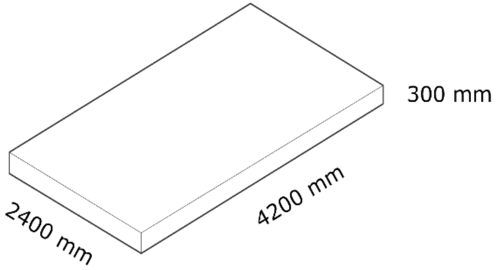
3) Steel Frame

Structural System **New**

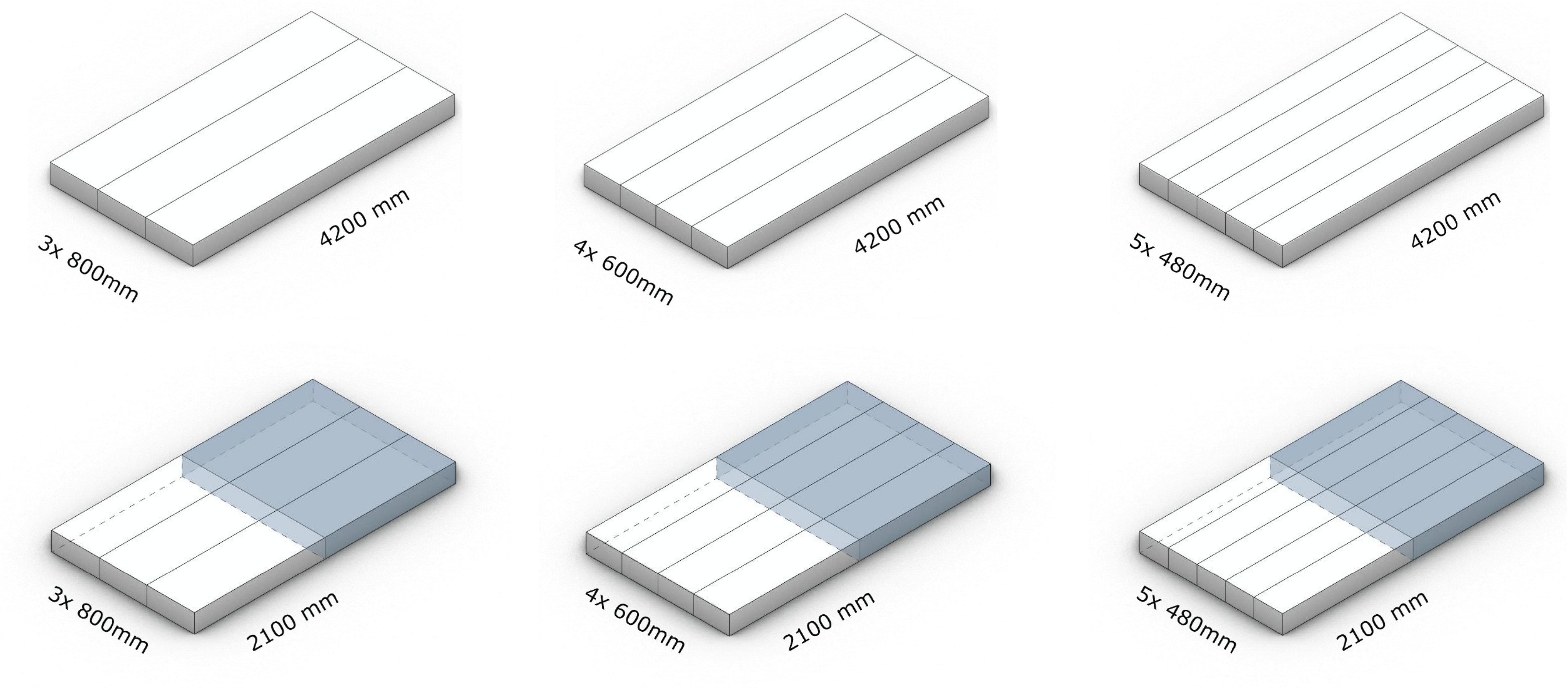
Steel Frame



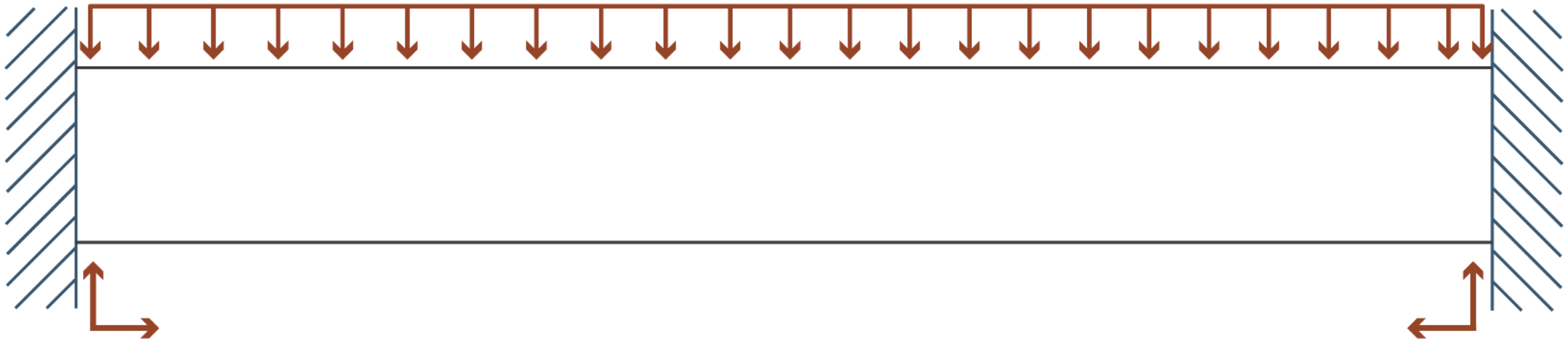
Glass Beam(s)



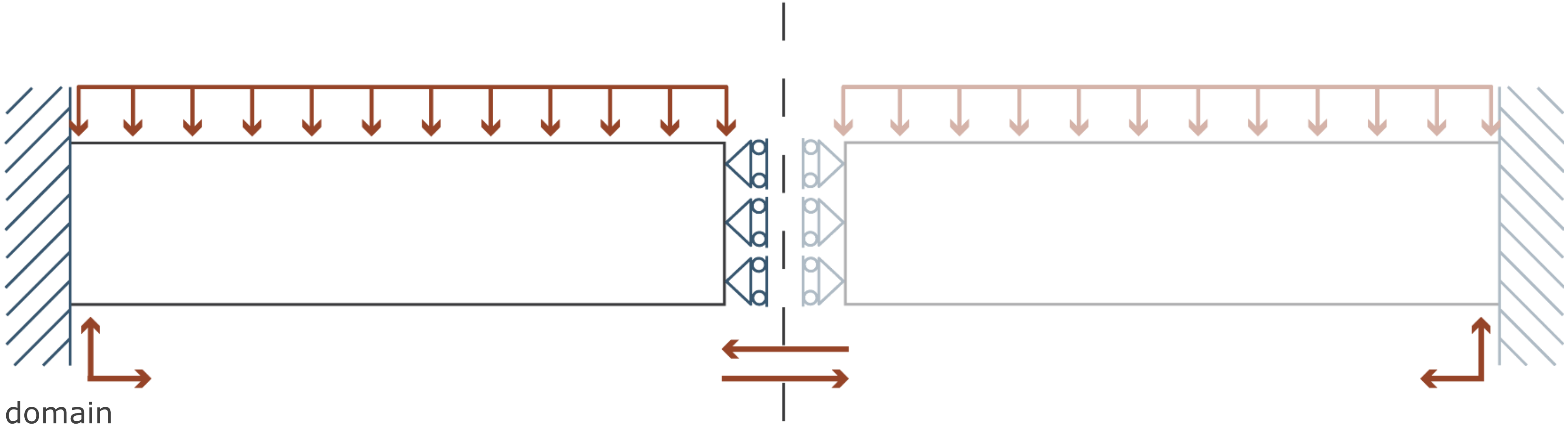
beam division



boundary conditions



Full domain

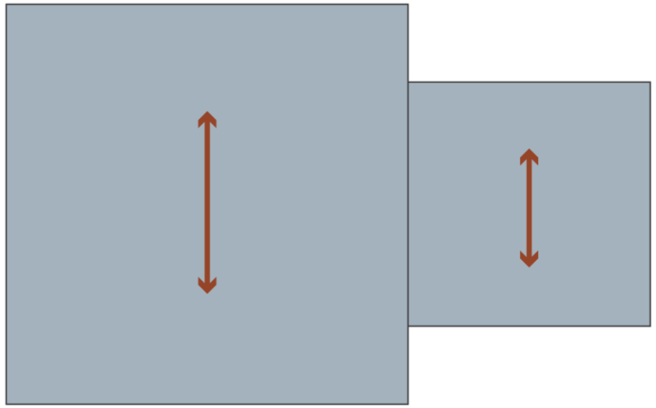


Halved domain

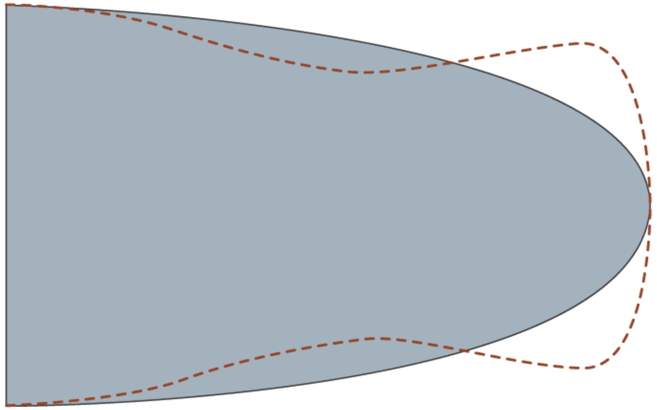
Topology Optimization

Topology Optimization

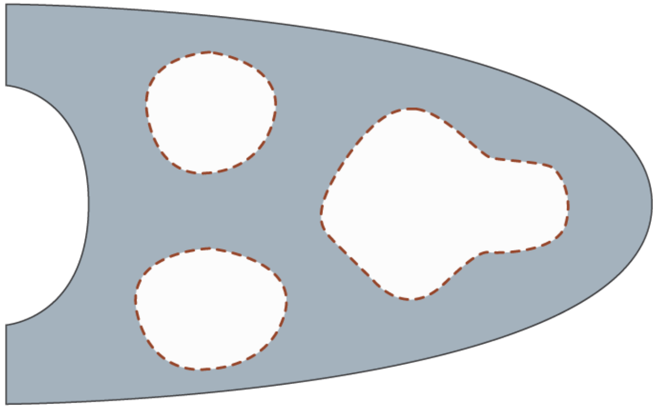
Size Optimization



Shape Optimization



Topology Optimization



SIMP algorithm

Topology Optimization (Solid Isotropic Material with Penalization)

Based on :

$$E_e = \rho_e^p E_e^0$$

Minimize :

$$C(\rho) = U^T K\, U$$

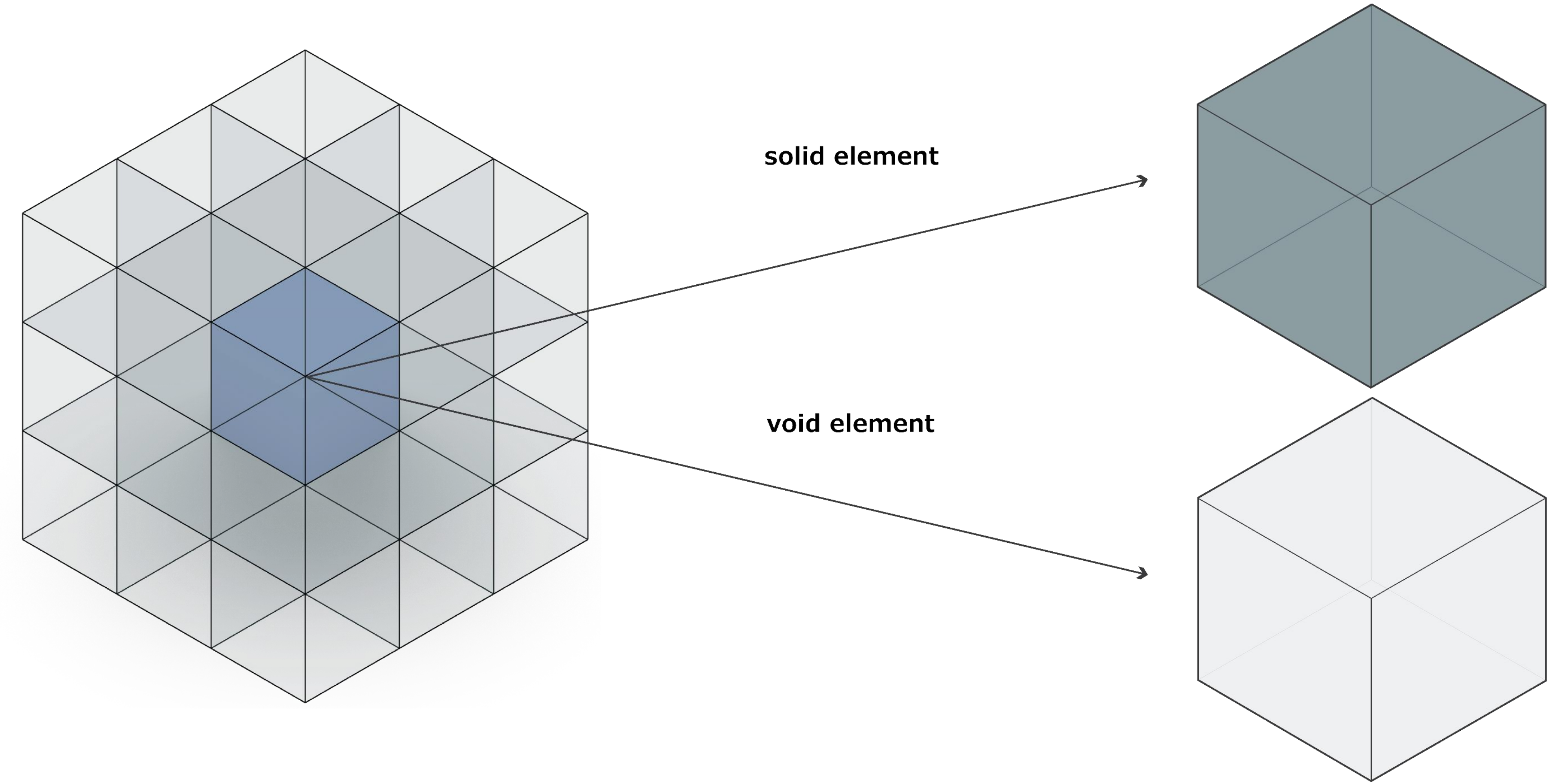
Subject to:

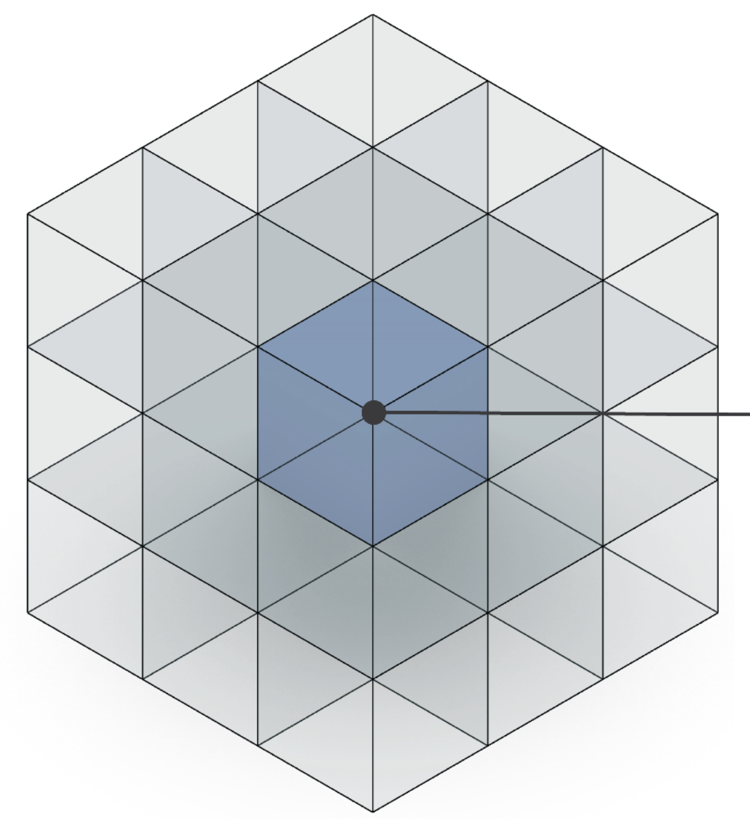
$$\frac{V_{(x)}}{V_0} = f$$

$$[K]\{U\} = \{F\}$$

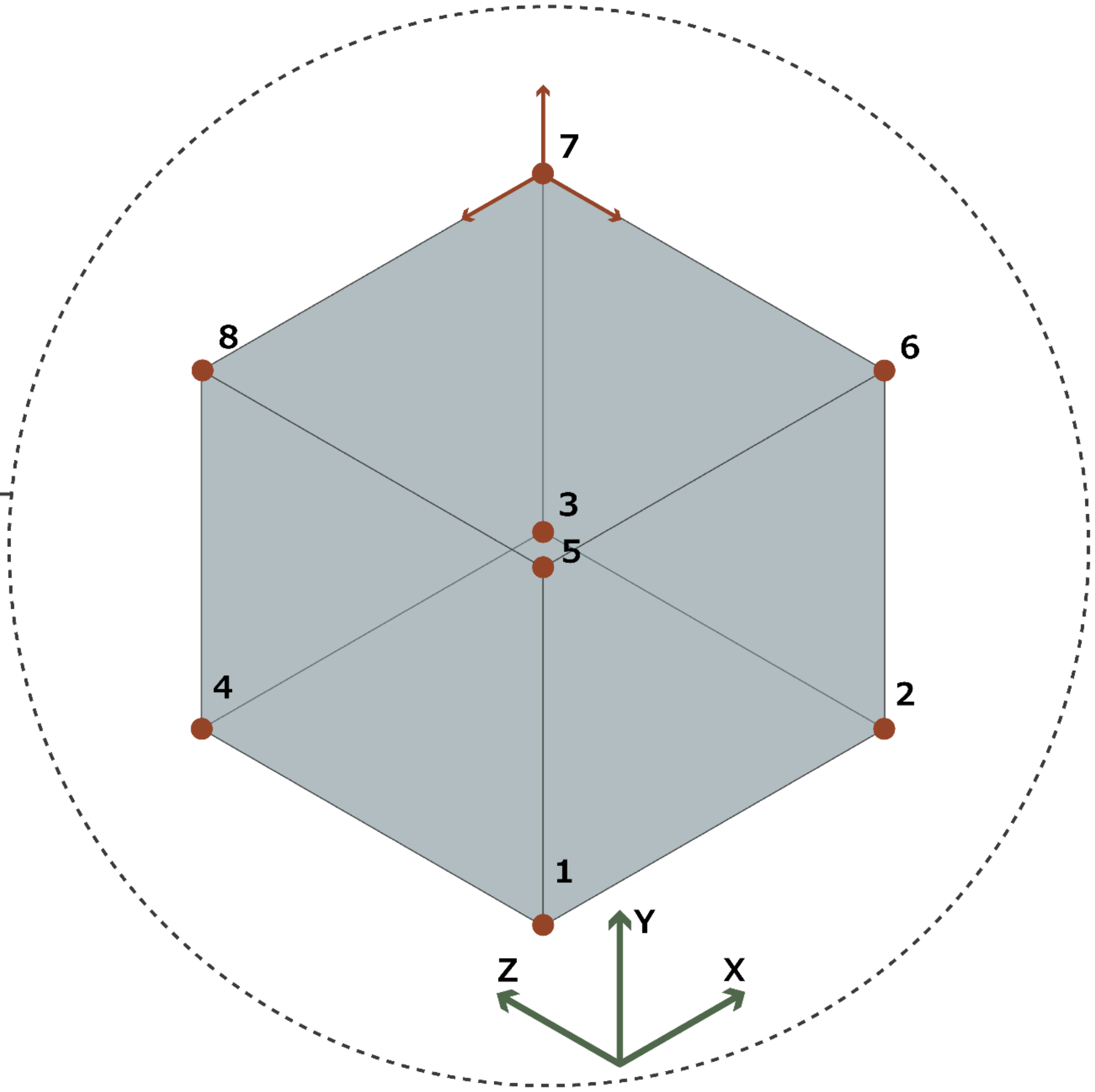
$$0 < \rho_{min} \le \rho \le 1$$

Topology Optimization



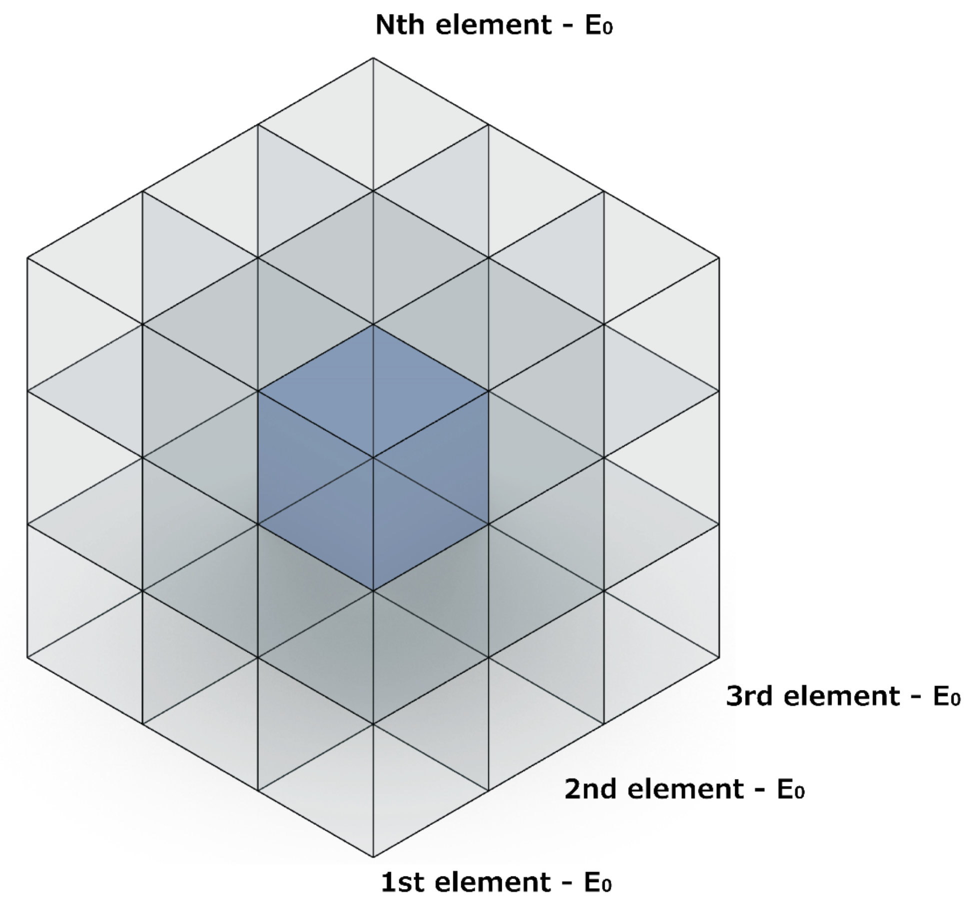


mesh of 8-node hexahedral elements



8-node hexahedral element (3-DOFs)

Topology Optimization



mesh - all elements

0 - 1

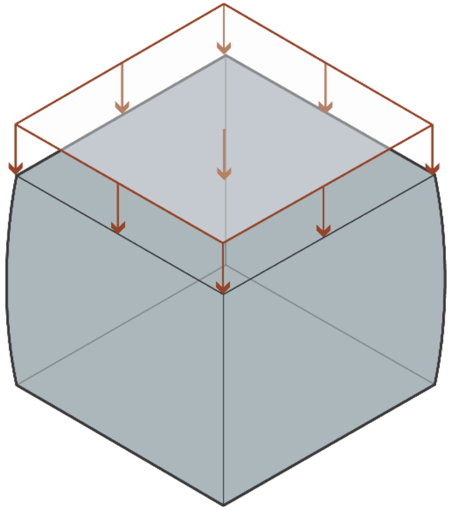
$$E_e = \rho^p * E_0$$

change in density

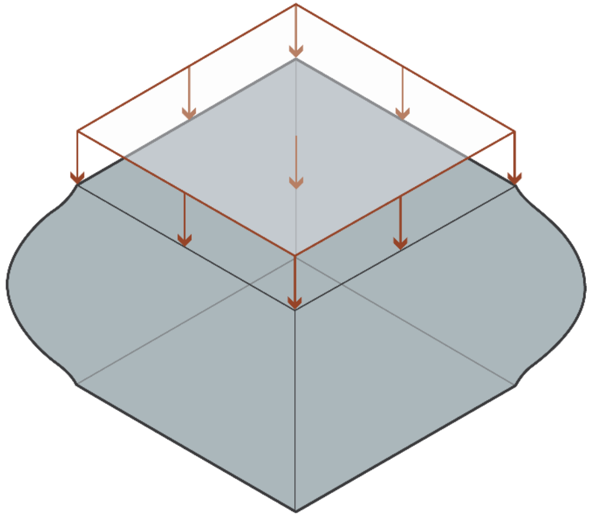
1st element - E_{1_new}

•
•
•
•
•
•
•
•

Nth element - E_{n_new}

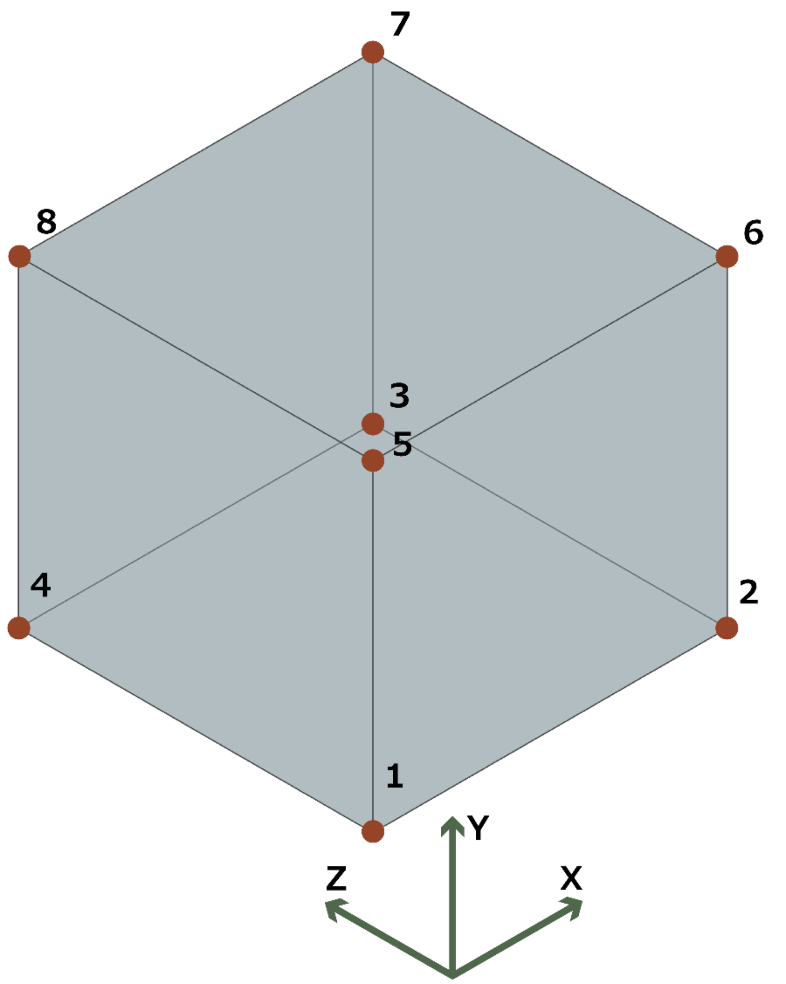


$\rho^p = 1$ - stiff element



$\rho^p = 0$ - less stiff element

Topology Optimization

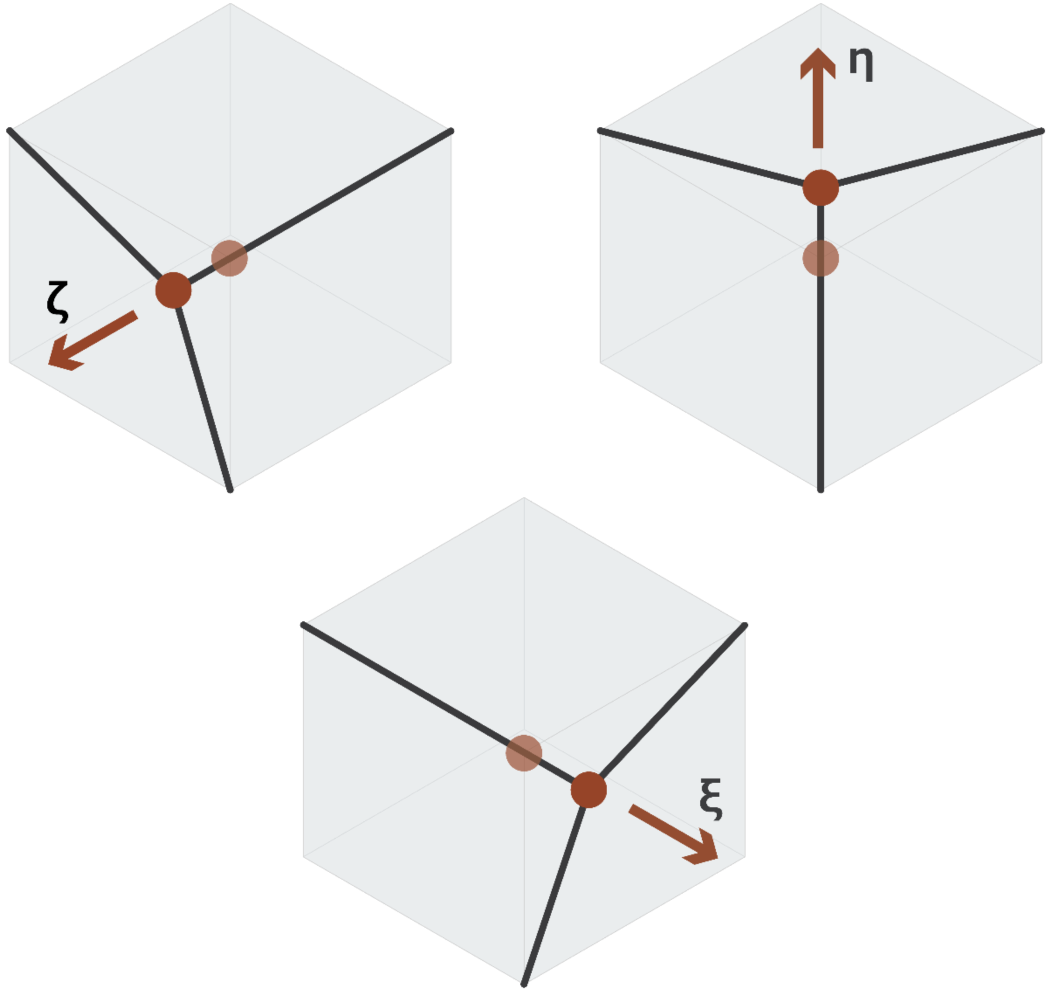


undeformed element

$[K]\{U\} = \{F\}$

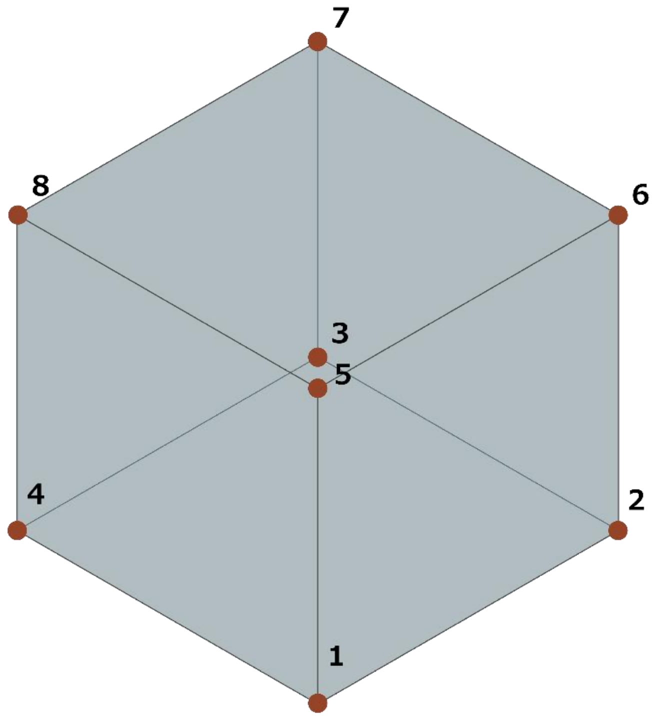
$$\{U\} = [K]^{-1}\{F\}$$

solve equilibrium w.r.t nodal displacements

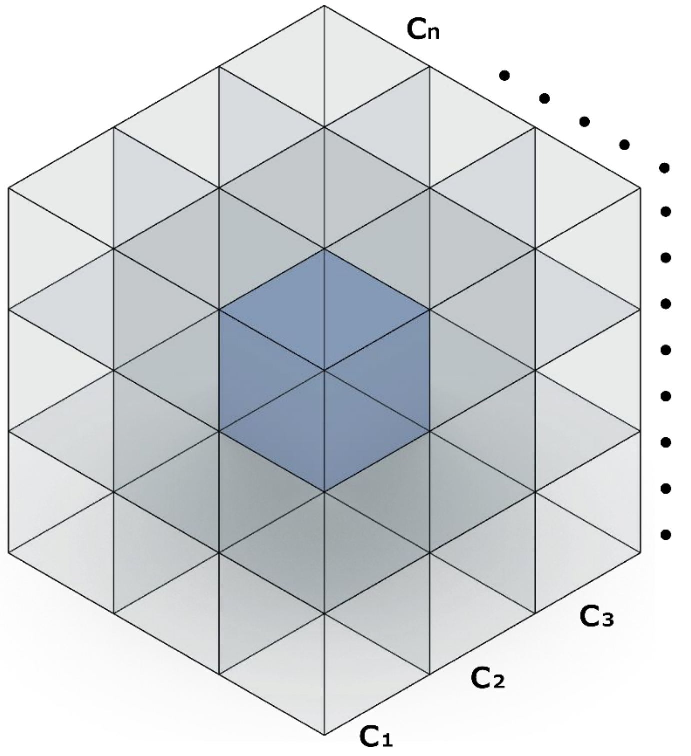


individual deformations of {U}

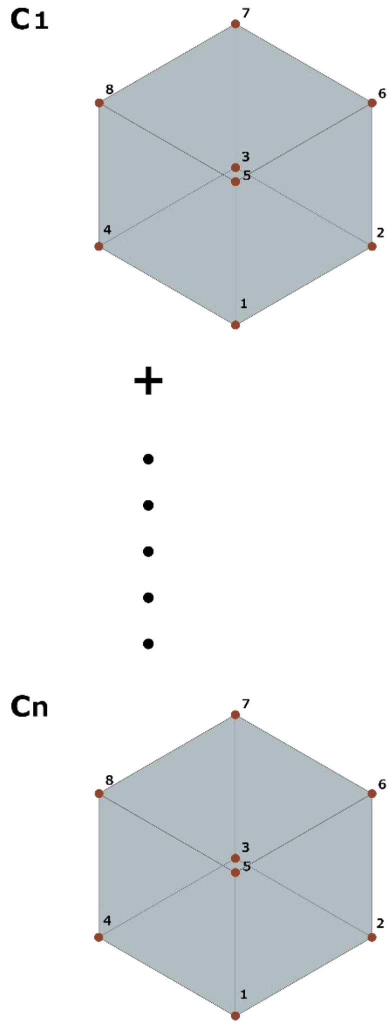
Topology Optimization



$c_e = u_e^T K_e u_e$



$C = \Sigma c_e$

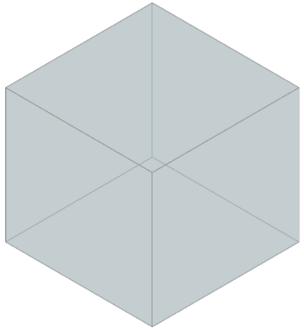


element - calculated nodal displacements

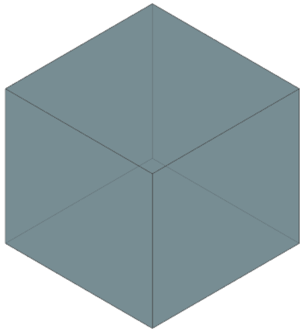
calculate compliance per element

Global compliance C

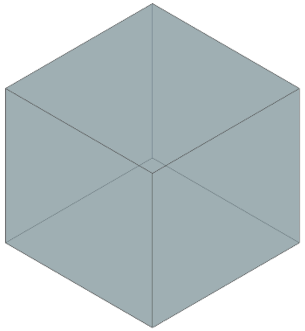
Topology Optimization



$\rho_1 = 0.23$
$C_1 = 6.46$

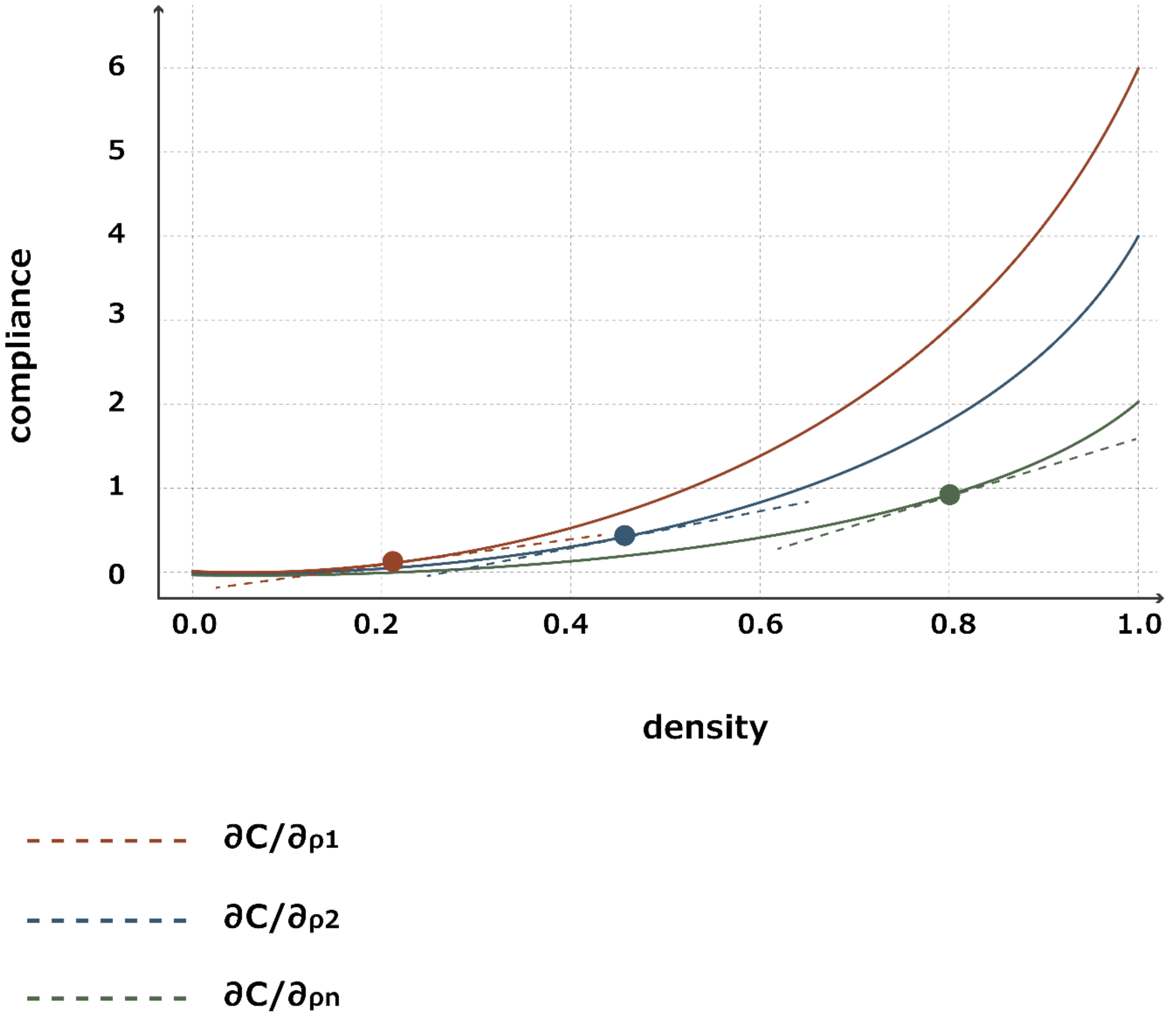


$\rho_2 = 0.75$
$C_2 = 2.84$



$\rho_n = 0.46$
$C_n = 5.73$

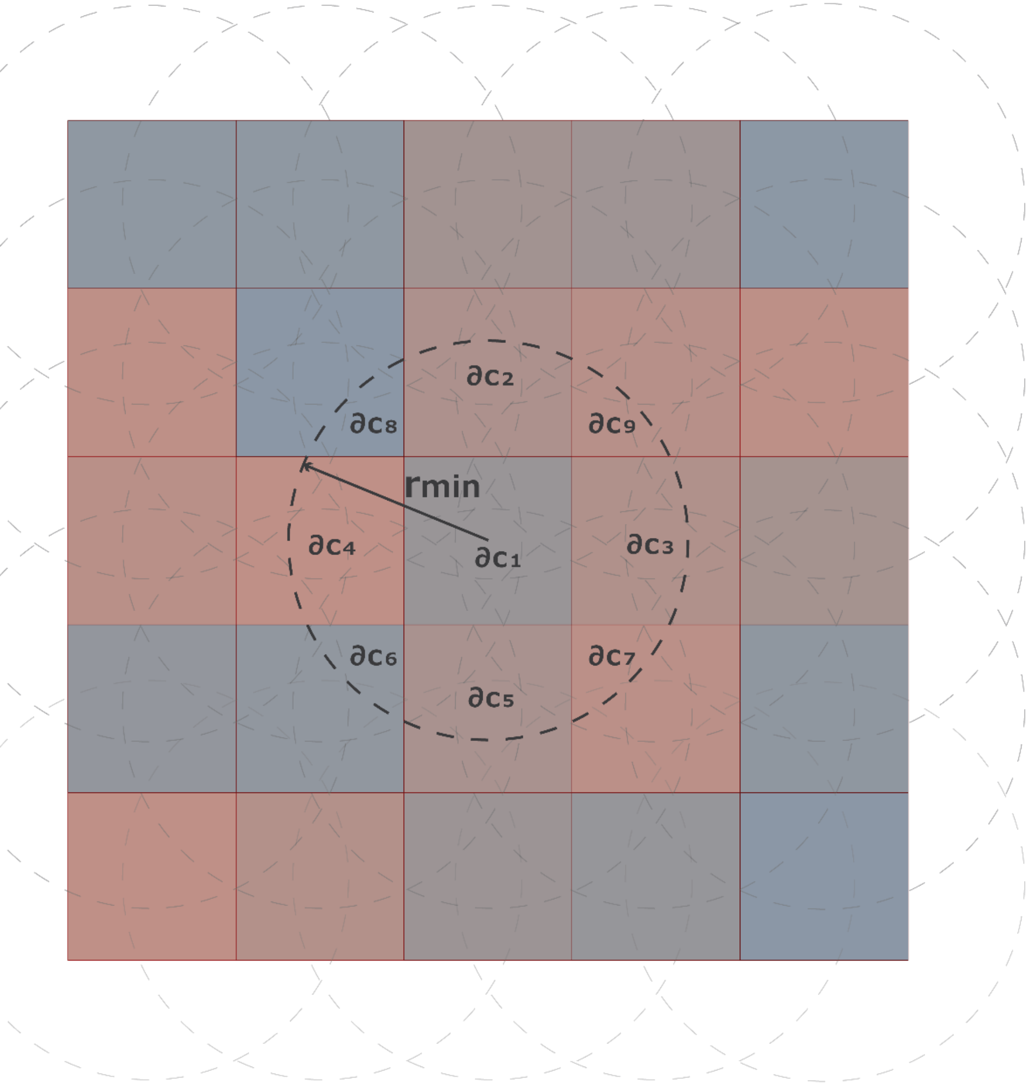
$\partial C/\partial \rho_e$ ⟶



elements with their *x* and *c* values

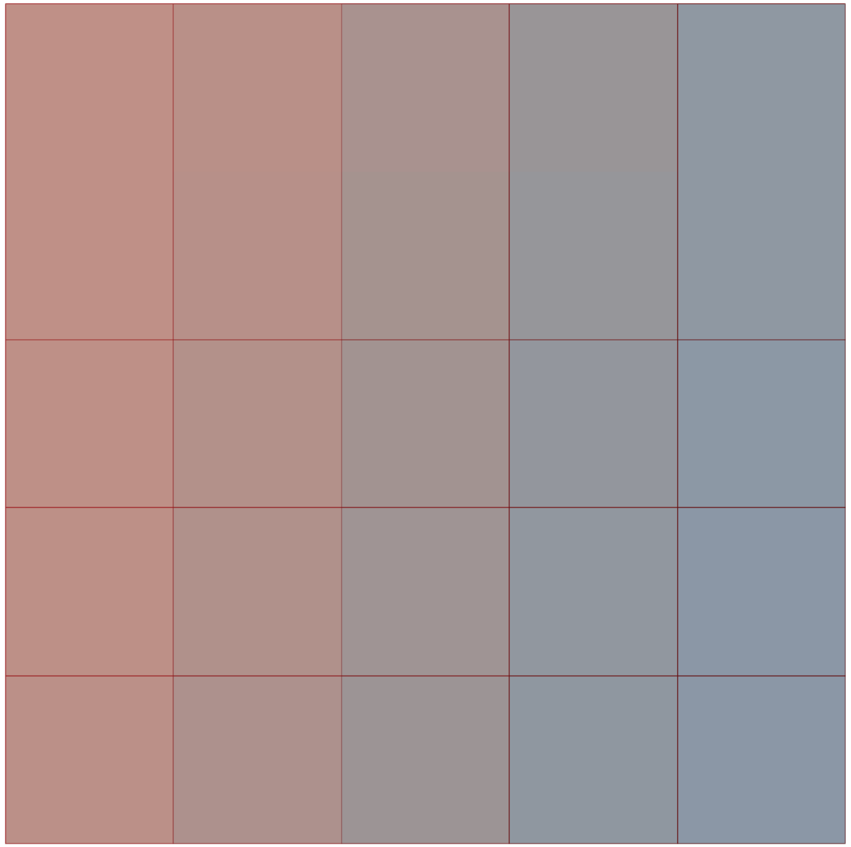
indication of how change in x_e is going to affect c_e

Topology Optimization



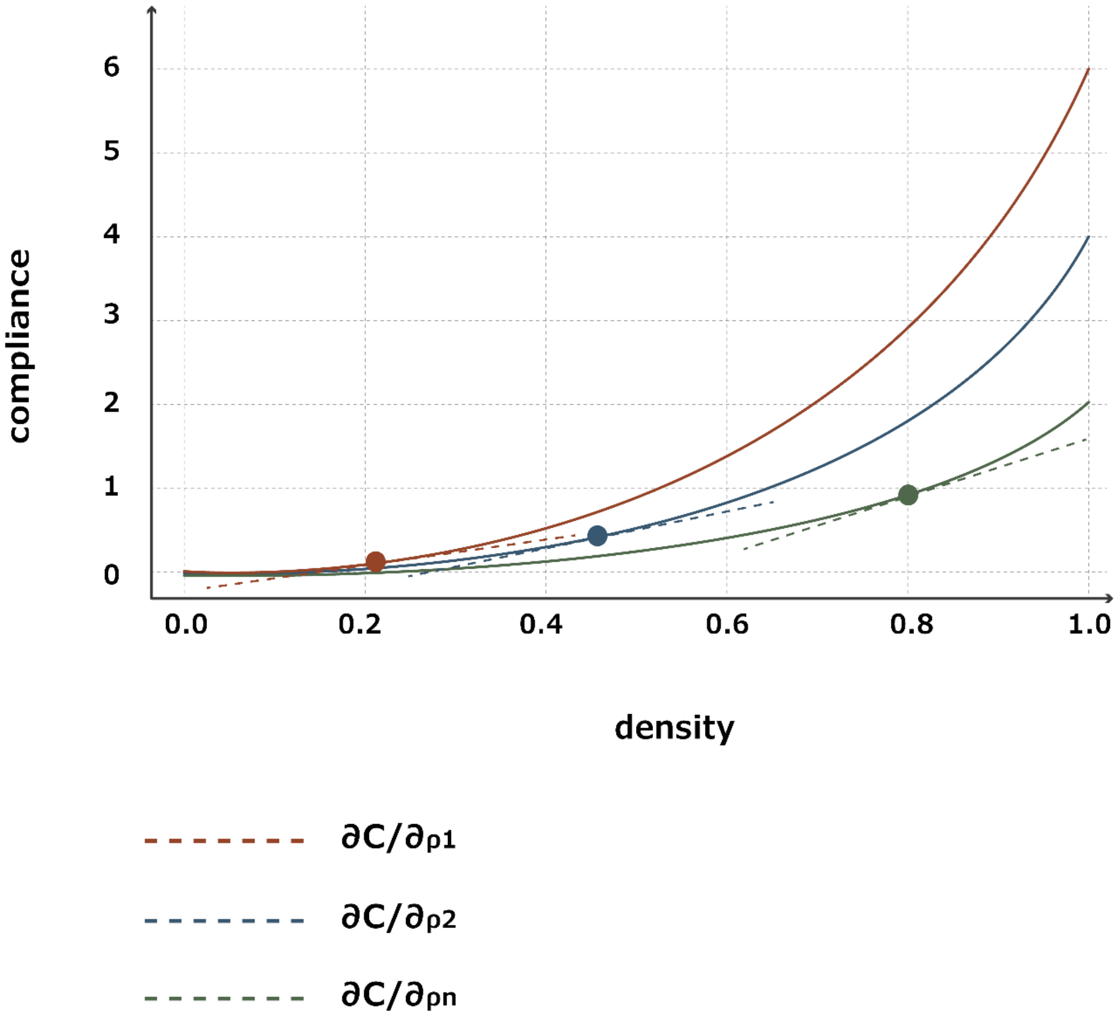
$$\partial c_e = (1/\Sigma w_{ej}) * \Sigma w_{ej} * \partial c_j$$

weighted average between neighbouring elements



unfiltered sensitivities

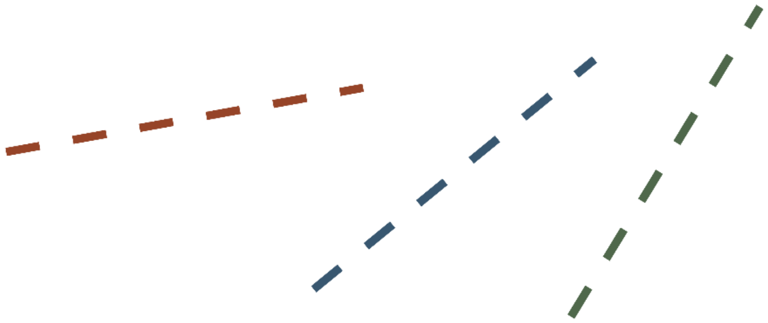
filtered sensitivities without irregularities



filtered sensitivities and their slopes

$$\rho_e^{new} = \rho_e \sqrt{D_e}$$

$$D_e = \lambda * (\partial C/\partial \rho_e)/(\partial V/\partial \rho_e)$$



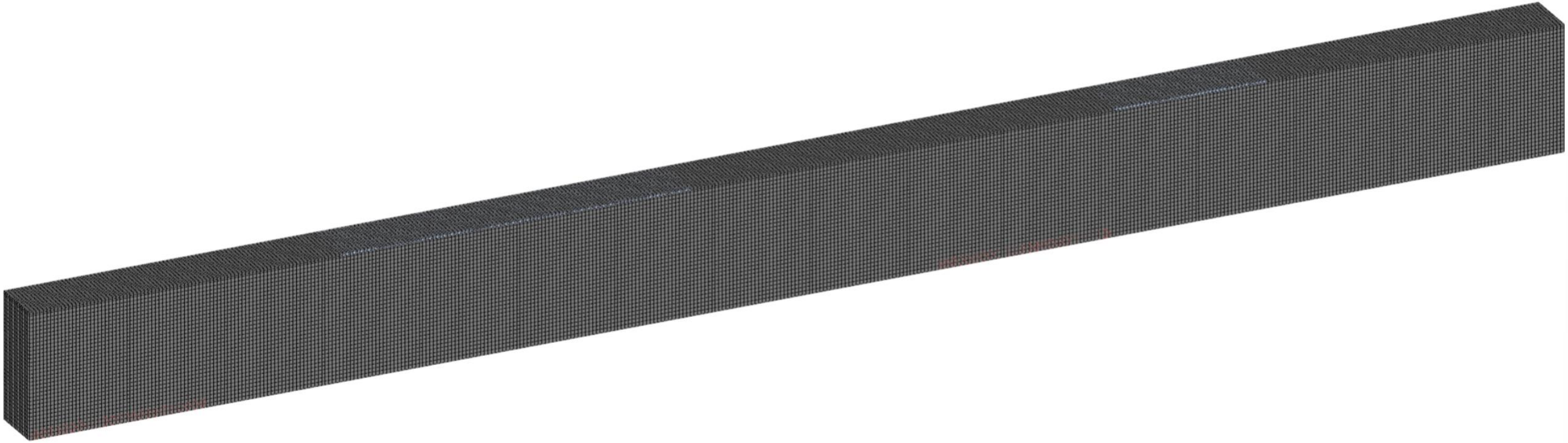
low slope ⟶ reduce density

medium slope ⟶ keep same density

high slope ⟶ add density

update rule

Topology Optimization



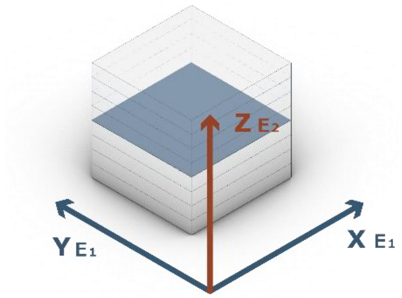
$$E_e = \rho_e^p E_e^0$$

$$E_e = \rho_e^p E_e^0 ???$$

Topology Optimization

E_e^0

$\boldsymbol{E_e^1}$

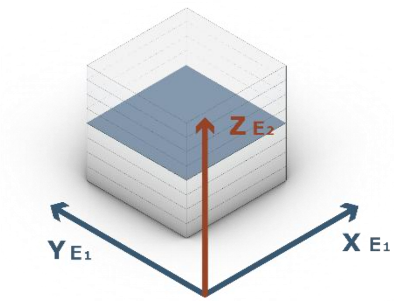


$\boldsymbol{E_e^2}$

$$E_e = \rho_e^p E_e^0 ???$$

E_e^1

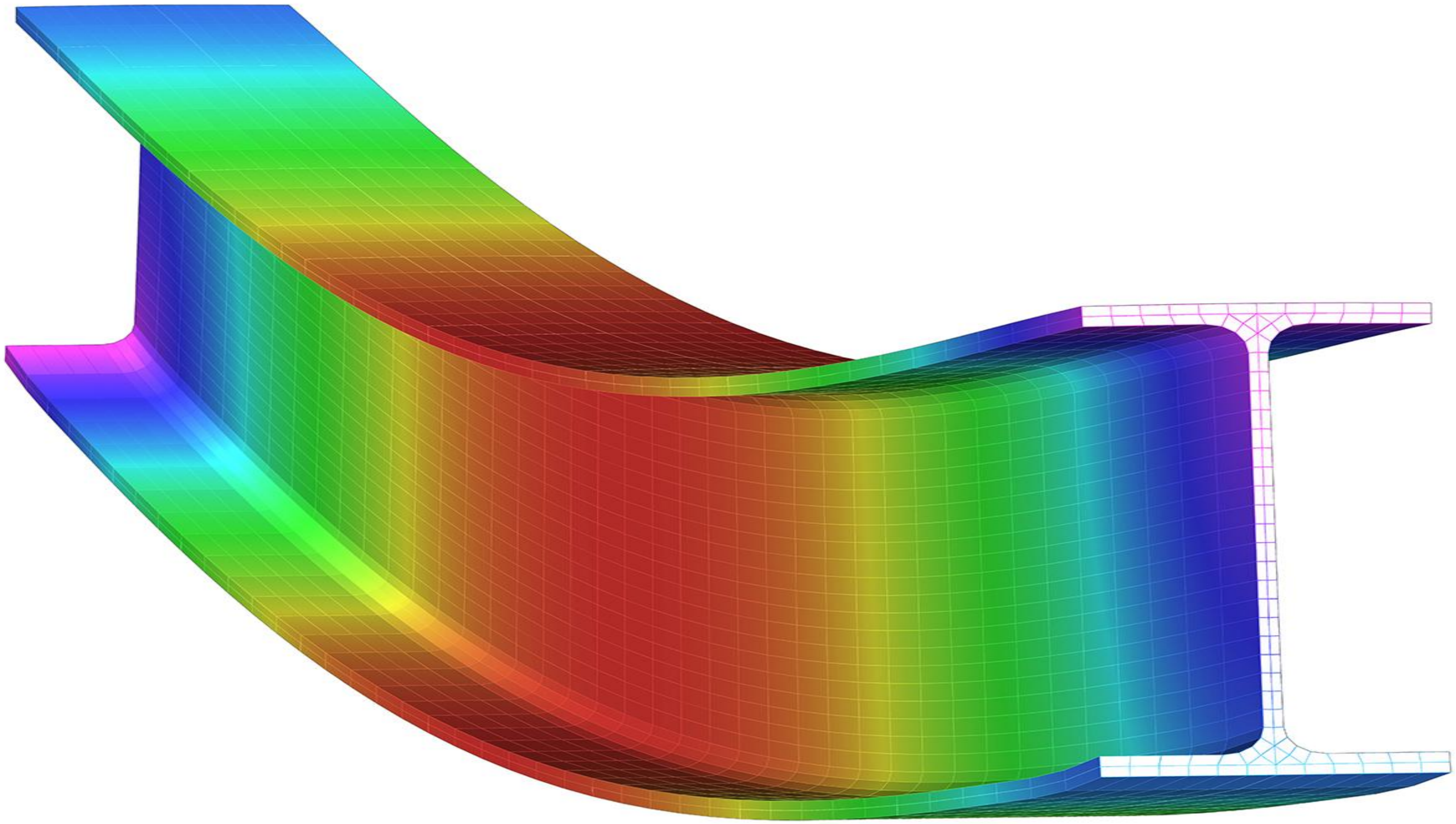
E_e^0

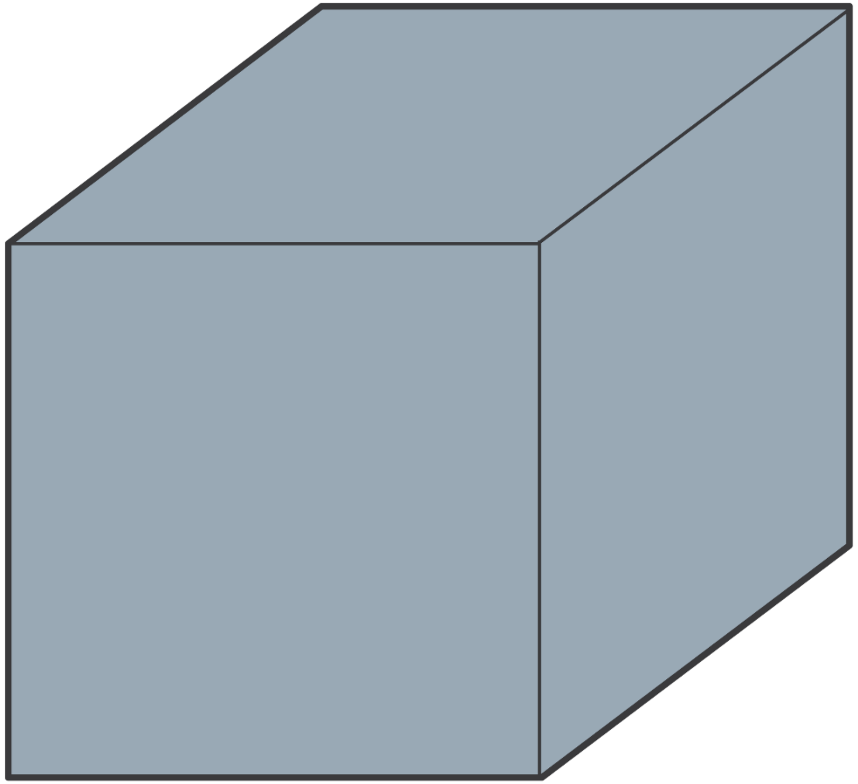


E_e^2

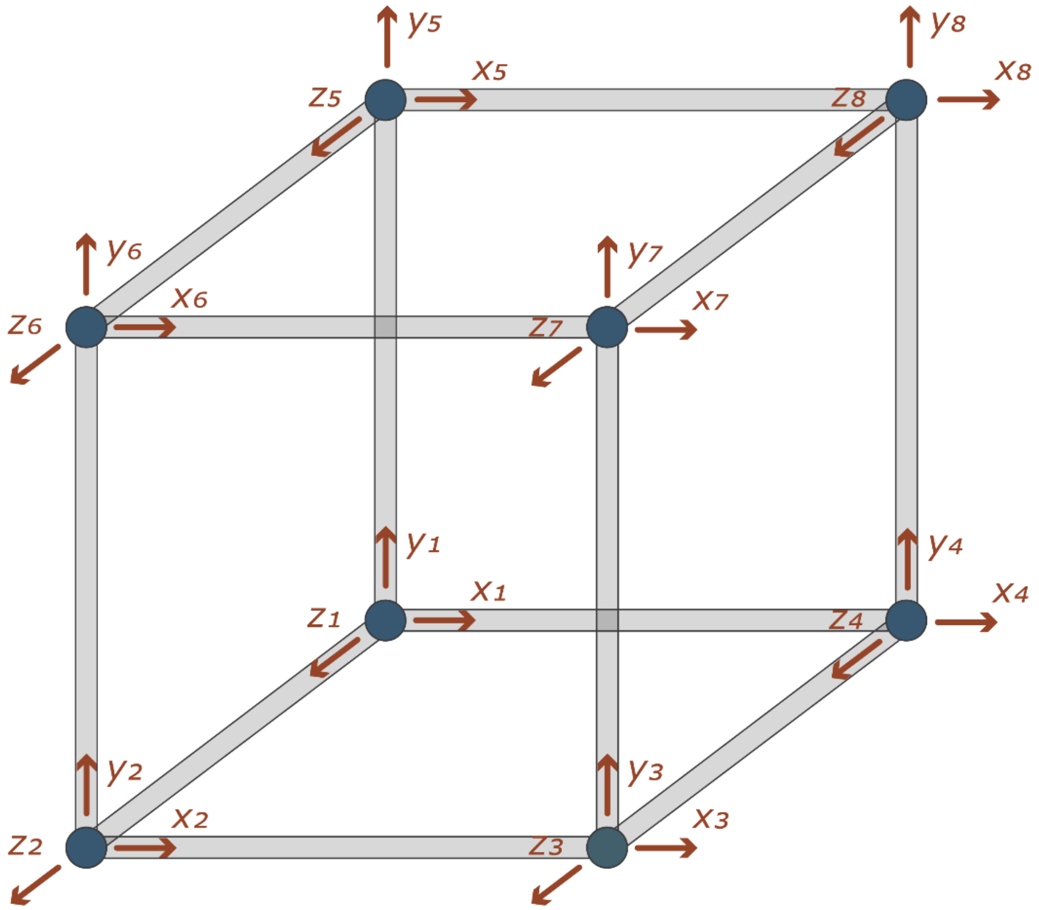
$$E_2 = a * E_1$$

Finite Element Method





8-node hexahedral element (brick)



8-node hexahedral element (brick)

Finite Element Method

Solving for that

Defined

$$[K] = \{u\} \{F\}$$

Finite Element Method

$$[K] = \int_{\infty}^{\infty} [B]^T [D][B] dV$$

[B] = Strain-Displacement matrix

[D] = Elasticity Matrix

Finite Element Method

B = Strain-Displacement matrix

$$[B] = \begin{bmatrix} \frac{\partial Ni}{\partial x} & 0 & 0 \\ 0 & \frac{\partial Ni}{\partial y} & 0 \\ 0 & 0 & \frac{\partial Ni}{\partial z} \\ \frac{\partial Ni}{\partial y} & \frac{\partial Ni}{\partial x} & 0 \\ \frac{\partial Ni}{\partial z} & 0 & \frac{\partial Ni}{\partial x} \\ 0 & \frac{\partial Ni}{\partial z} & \frac{\partial Ni}{\partial y} \end{bmatrix}$$

$$\{\varepsilon\} = [B]\{\delta\}$$

ε = strain
δ = nodal displacements
Ni = shape functions

D = Elasticity Matrix

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{12} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{13} & D_{23} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix}$$

$$[\sigma] = [D][\varepsilon]$$

$$D_{11} = \frac{E_x^2\left(E_z v_{yz}^2 - E_y\right)}{D_{denom}} \qquad D_{12} = -\frac{E_x E_y\left(E_z v_{yz} v_{xz} + E_y v_{xy}\right)}{D_{denom}}$$

$$D_{13} = -\frac{E_x E_y E_z\left(v_{xy} v_{yz} + v_{xz}\right)}{D_{denom}} \qquad D_{22} = \frac{E_y^2\ \left(E_z v_{xz}^2 - E_x\right)}{D_{denom}}$$

$$D_{33} = \frac{E_y E_z\ \left(E_y v_{xy}^2 - E_x\right)}{D_{denom}} \qquad D_{44} = G_{xy}\ , D_{55} = G_{yz}\ , D_{66} = G_{xz}$$

$$D_{denom} = E_y E_z v_{xz}^2 - E_x E_y + 2 v_{xy} v_{yz} v_{xz} E_y E_z + E_x E_z v_{yz}^2 + E_y^2 v_{xy}^2$$

D = Elasticity Matrix Orthotropic material

S = Compliance Matrix Orthotropic material

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{12} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{13} & D_{23} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix}$$

$[\sigma] = [D][\varepsilon]$

inverse

$[\varepsilon] = [D]^{-1}[\sigma]$,

$[D]^{-1} = [S]$

$$S = \begin{bmatrix} \frac{1}{E_1} & \frac{-v_{21}}{E_2} & \frac{-v_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-v_{12}}{E_1} & \frac{1}{E_2} & \frac{-v_{32}}{E_3} & 0 & 0 & 0 \\ \frac{-v_{13}}{E_1} & \frac{-v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix}$$

$$D_{11} = \frac{E_x^2(E_z v_{yz}^2 - E_y)}{D_{denom}} \qquad D_{12} = -\frac{E_x E_y(E_z v_{yz} v_{xz} + E_y v_{xy})}{D_{denom}}$$

$$D_{13} = -\frac{E_x E_y E_z(v_{xy} v_{yz} + v_{xz})}{D_{denom}} \qquad D_{22} = \frac{E_y^2\ (E_z v_{xz}^2 - E_x)}{D_{denom}}$$

$$D_{33} = \frac{E_y E_z\ (E_y v_{xy}^2 - E_x)}{D_{denom}} \qquad D_{44} = G_{xy}\ , D_{55} = G_{yz}\ \ , D_{66} = G_{xz}$$

$$D_{denom} = E_y E_z v_{xz}^2 - E_x E_y + 2 v_{xy} v_{yz} v_{xz} E_y E_z + E_x E_z v_{yz}^2 + E_y^2 v_{xy}^2$$

$$S = \begin{bmatrix} \frac{1}{E_1} & \frac{-v_{21}}{E_2} & \frac{-v_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-v_{12}}{E_1} & \frac{1}{E_2} & \frac{-v_{32}}{E_3} & 0 & 0 & 0 \\ \frac{-v_{13}}{E_1} & \frac{-v_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}} \end{bmatrix}$$

12 independent variables

Transverse Isotropy Properties 1:

$$E_y = Ez \longrightarrow \mathbf{E_2 = E_3}$$

Transverse Isotropy Properties 2:

$$v_{xy} = vxz \longrightarrow \mathbf{v_{12} = v_{13}}$$

Symmetric Tensors $\sigma_{ij} = \sigma_{ji}$ and $\varepsilon_{ij} = \varepsilon_{ji}$:

$$\frac{-v_{21}}{E_2} = \frac{-v_{12}}{E_1}, \quad \frac{-v_{31}}{E_2} = \frac{-v_{12}}{E_1}, \quad \frac{-v_{32}}{E_2} = \frac{-v_{23}}{E_2}$$

Shear modulus formulation:

$$G = \frac{E}{2(1+v)} \longrightarrow G_{23} = \frac{E_2}{2(1+v_{23})}$$

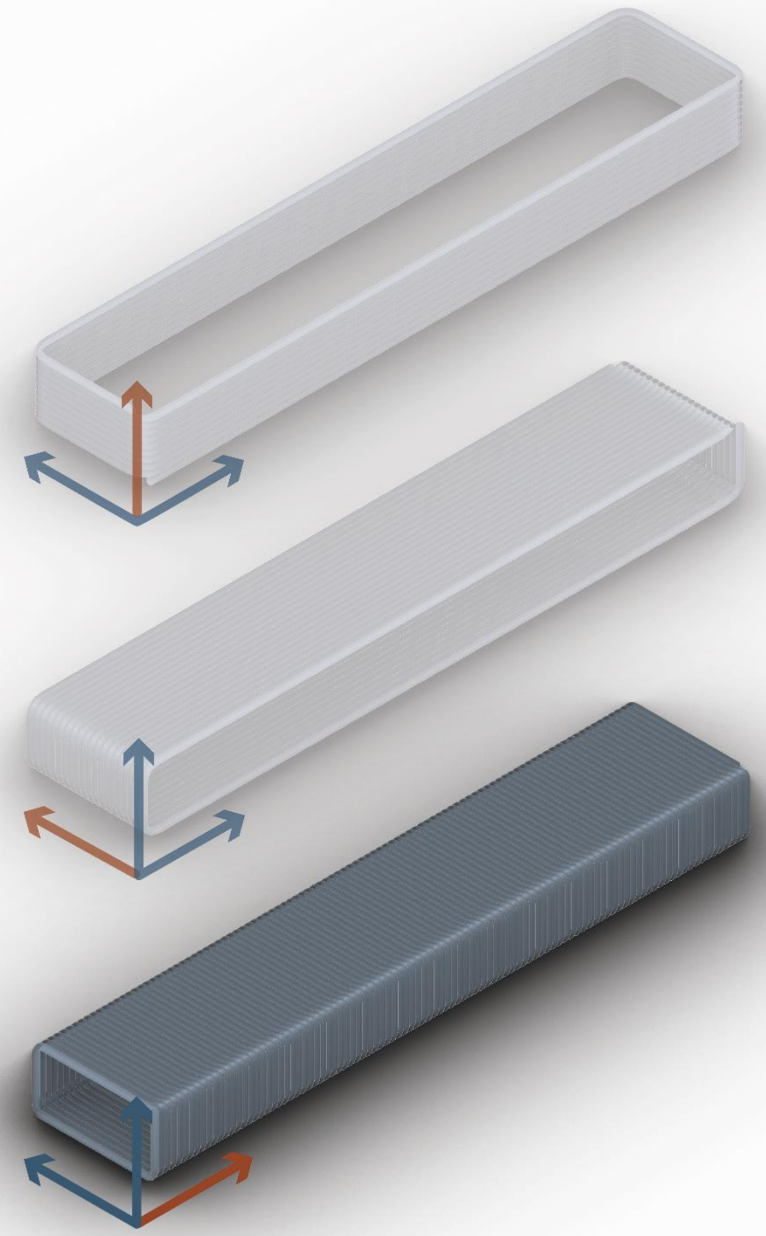
$$G_{12} = \frac{E_1}{2(1+v_{12})} \underset{v_{12} = v_{13}}{=} G_{13} = \frac{E_1}{2(1+v_{13})}$$

Shear modulus formulation:

$$E_2 = a * E_1$$

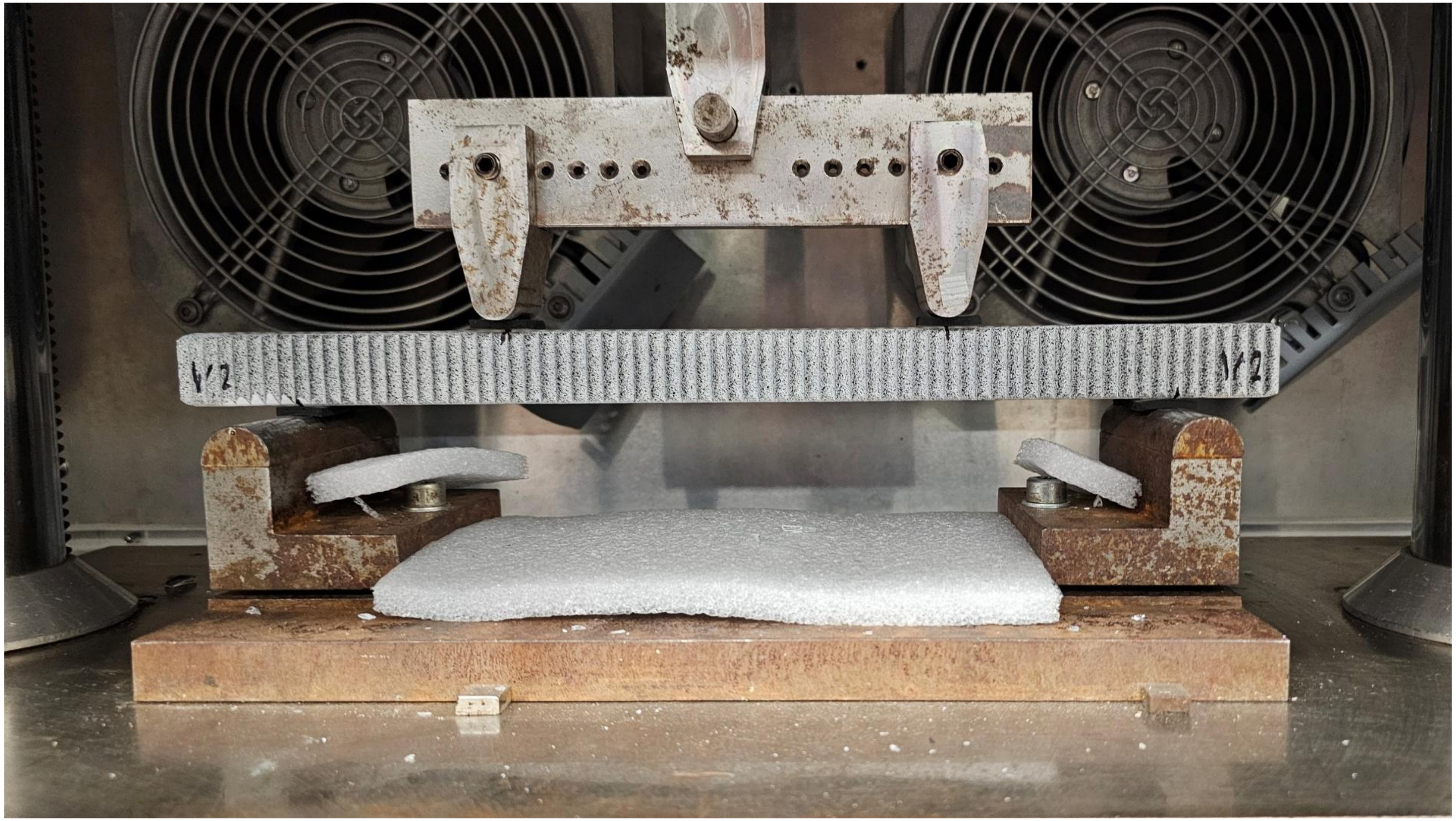
Finite Element Method

$$S = \begin{bmatrix} \frac{1}{a * E_2} & \frac{-v_{12}}{a * E_2} & \frac{-v_{12}}{a * E_2} & 0 & 0 & 0 \\ \frac{-v_{12}}{a * E_2} & \frac{1}{E_2} & \frac{-v_{23}}{E_2} & 0 & 0 & 0 \\ \frac{-v_{12}}{a * E_2} & \frac{-v_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\frac{a * E_2}{2(1+v_{12})}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\frac{E_2}{2(1+v_{23})}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\frac{a * E_2}{2(1+v_{12})}} \end{bmatrix}$$

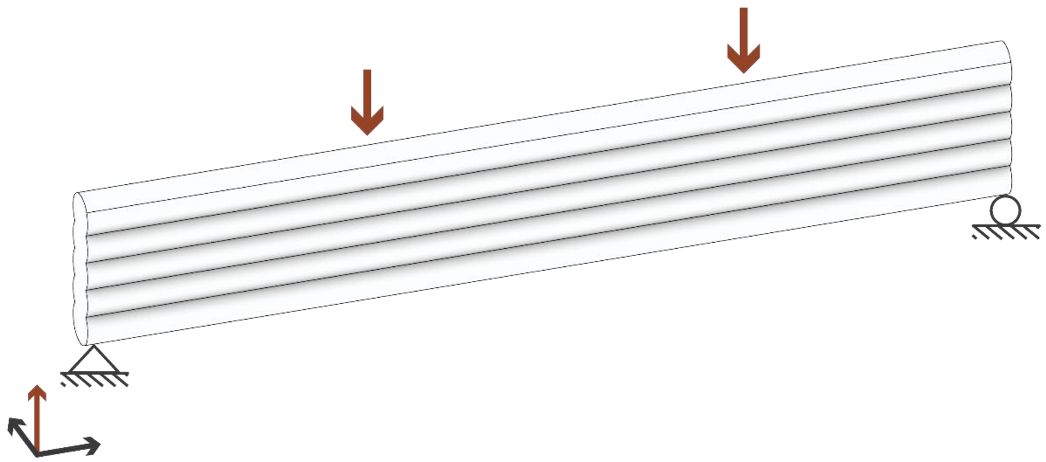


3 independent variables

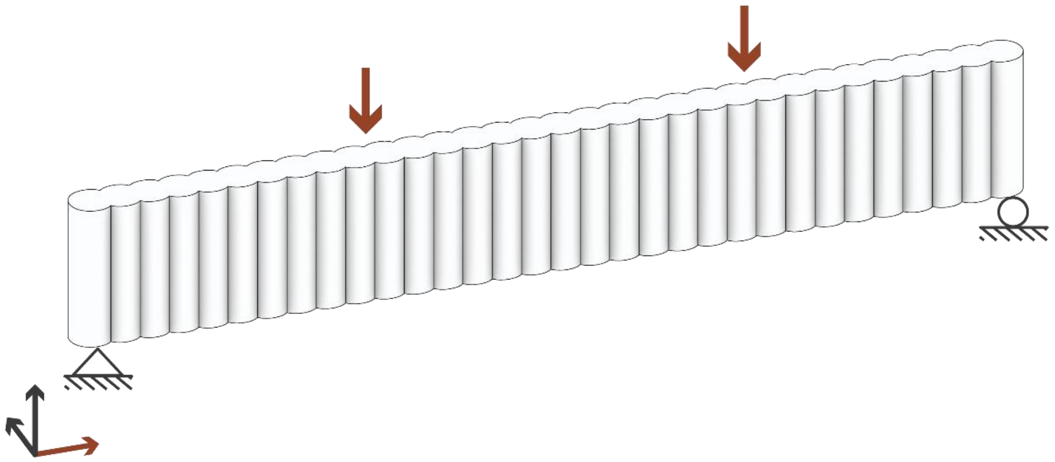
Flexural Testing



beam types – 4 point bending



Longitudinal



Vertical

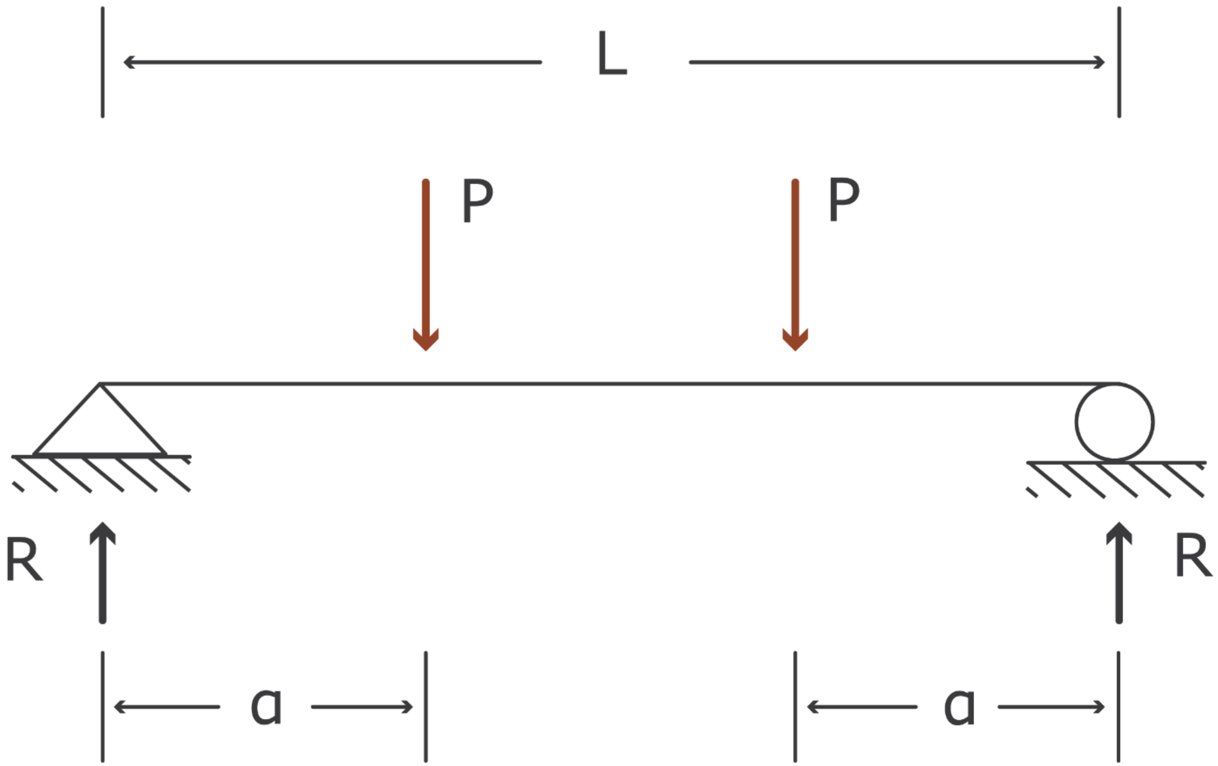
E_L = ?

E_V = ?

$E_V / E_L = a = ?$

Flexural Testing

equations



at center:

$$\Delta_{max} = \frac{P\alpha}{24EI}(3L^2 - 4a^2)$$

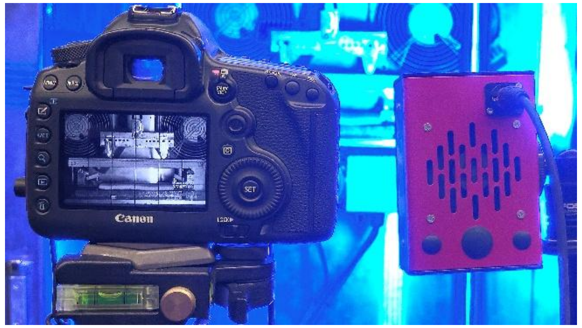
w.r.t Young's modulus:

$$E = \frac{P\alpha(3L^2 - 4\alpha^2)}{24I\Delta max}$$

Flexural Testing

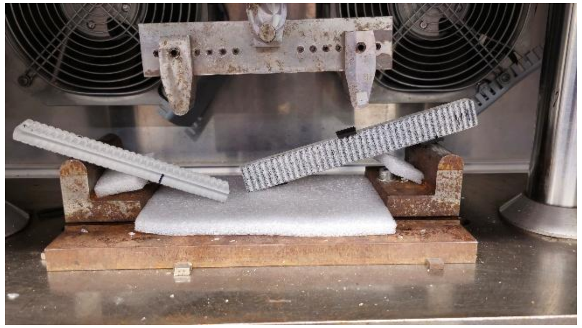
testing

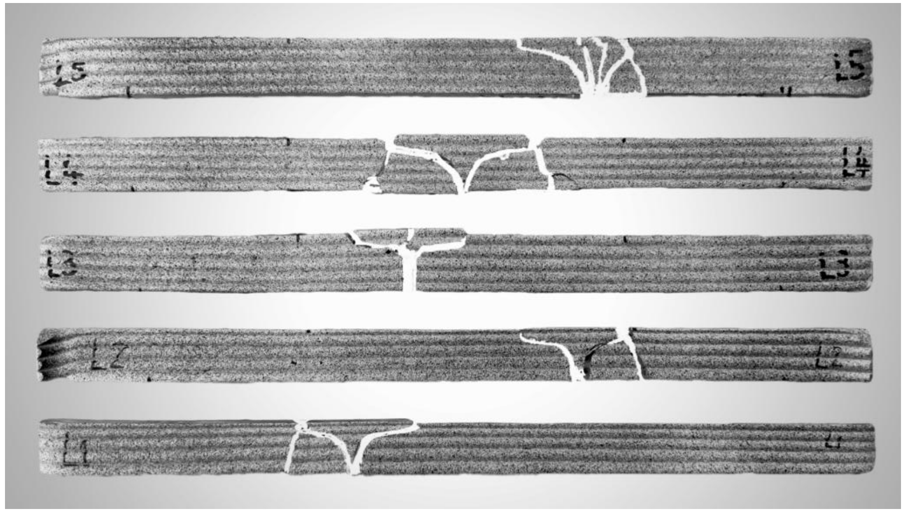
DIC set-up



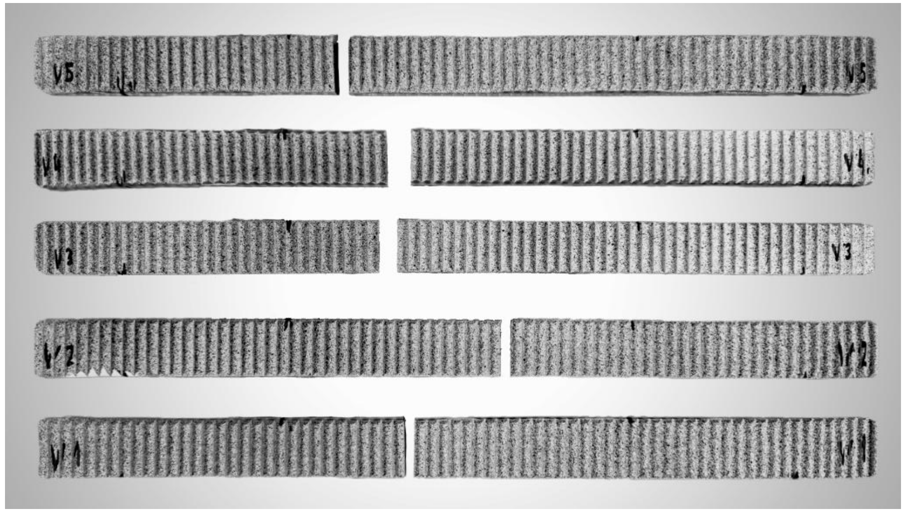
Longitudinal samples

Vertical samples



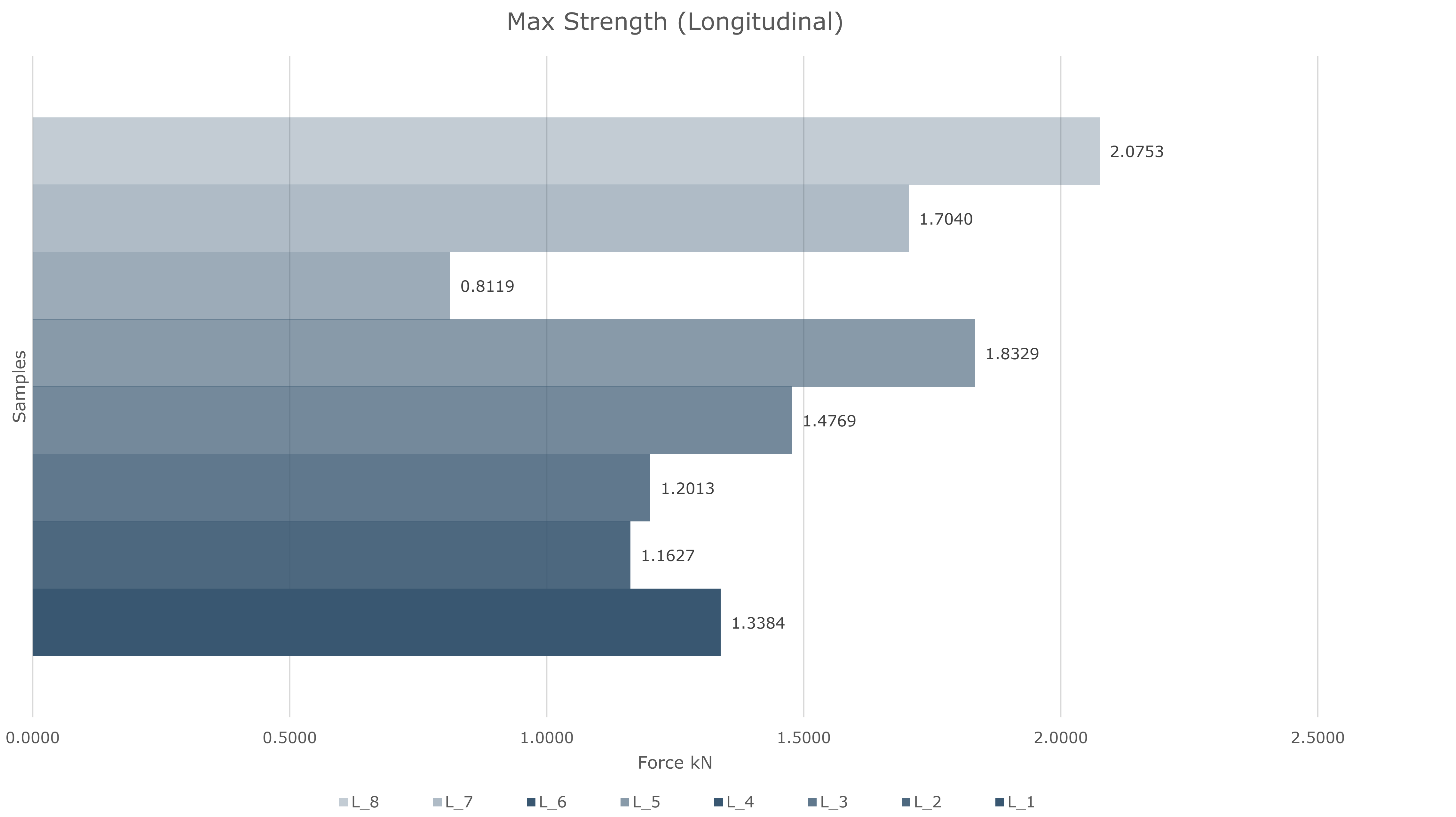


Broken longitudinal samples L1-L5

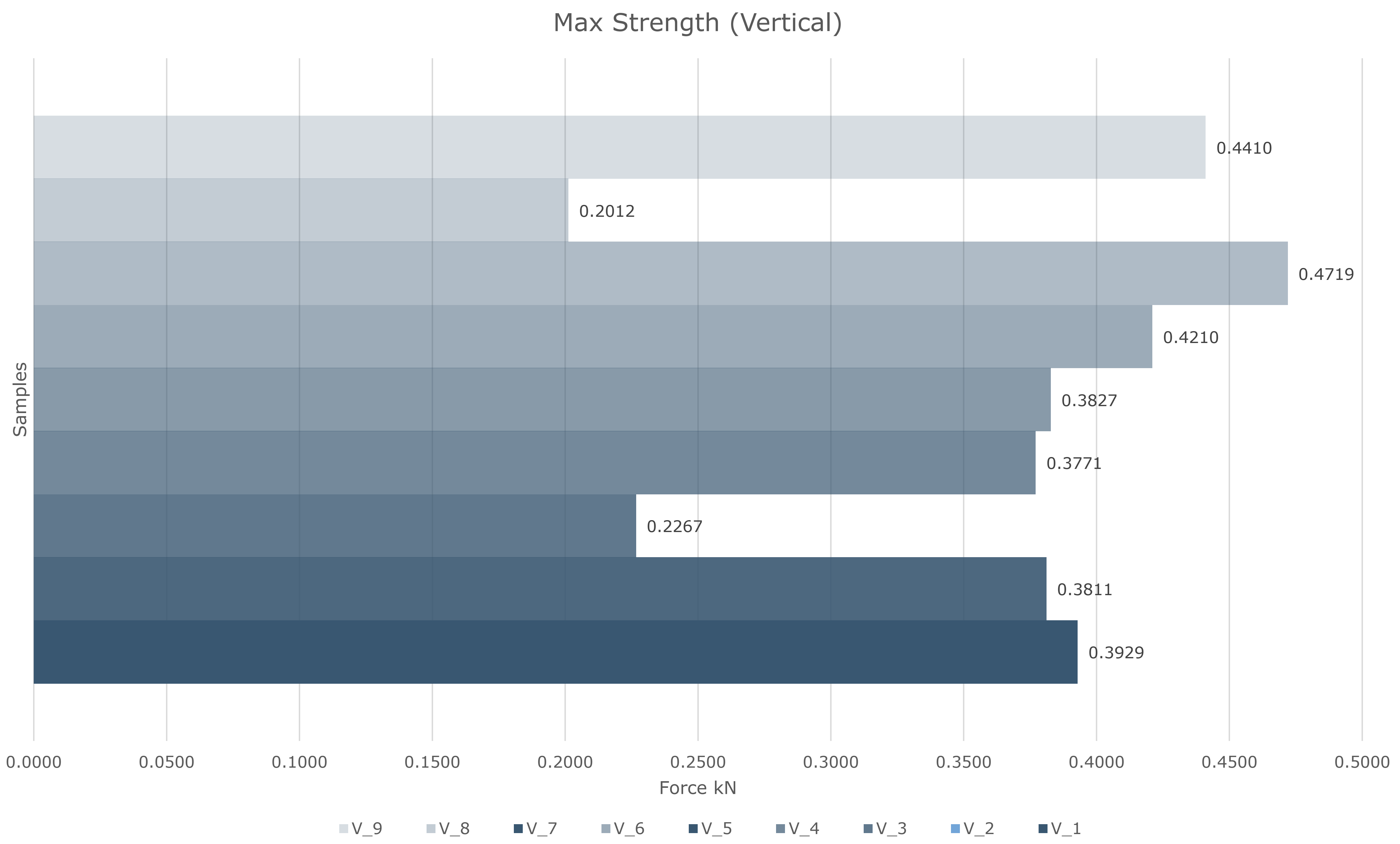


Broken vertical samples V1-V5

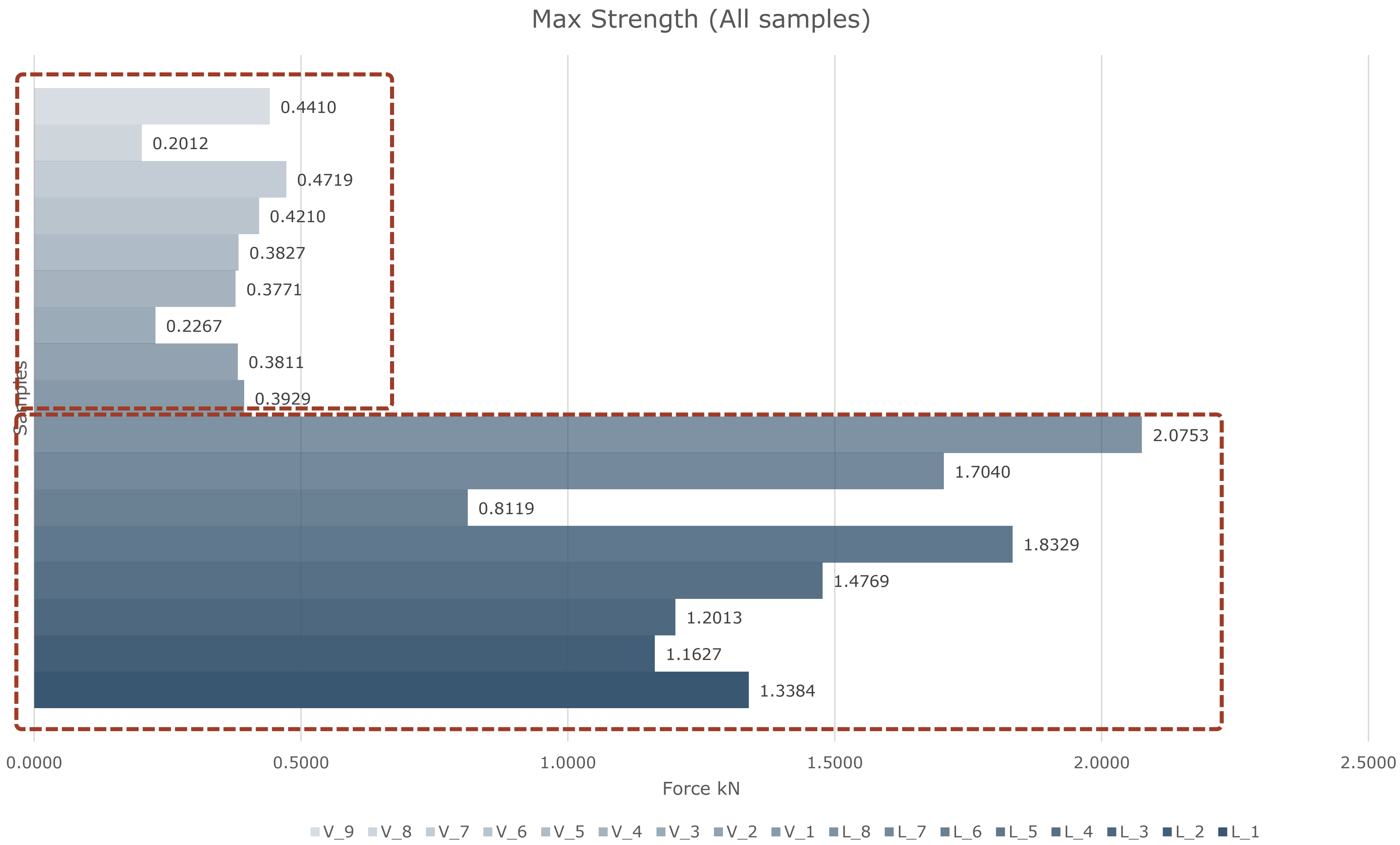
Flexural Testing



Flexural Testing

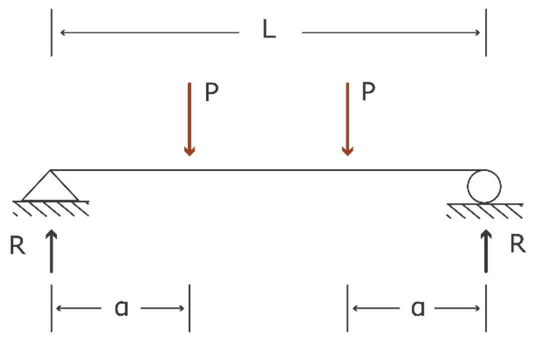


Flexural Testing



Flexural Testing

calculations



$$E = \frac{P\alpha(3L^2 - 4\alpha^2)}{24I\Delta max}$$

Specimen Orientation	E_1	E_2
	Gpa	Gpa
Longitudinal_1	67.36	67.26
Longitudinal_2	62.45	62.42
Longitudinal_3	61.09	61.01
Longitudinal_4	61.52	61.54
Longitudinal_5	36.77	64.12
Longitudinal_6	63.80	62.39
Longitudinal_7	65.90	65.86
Longitudinal_8	63.21	62.92
Vertical_1	53.85	60.55
Vertical_2	55.87	64.62
Vertical_3	55.53	75.53
Vertical_4	60.43	69.97
Vertical_5	55.67	63.99
Vertical_6	59.04	66.25
Vertical_7	60.64	68.89
Vertical_8	57.78	80.03
Vertical_9	54.61	62.90

P = total concentrated load — P = df/2 N

ɑ = distance to point load — ɑ = 280 – 140 mm

L = span length — 280 mm

I = second moment of inertia — $I = b*h^3 / 12\ mm^4$

Δmax = deflection — calculated with DIC

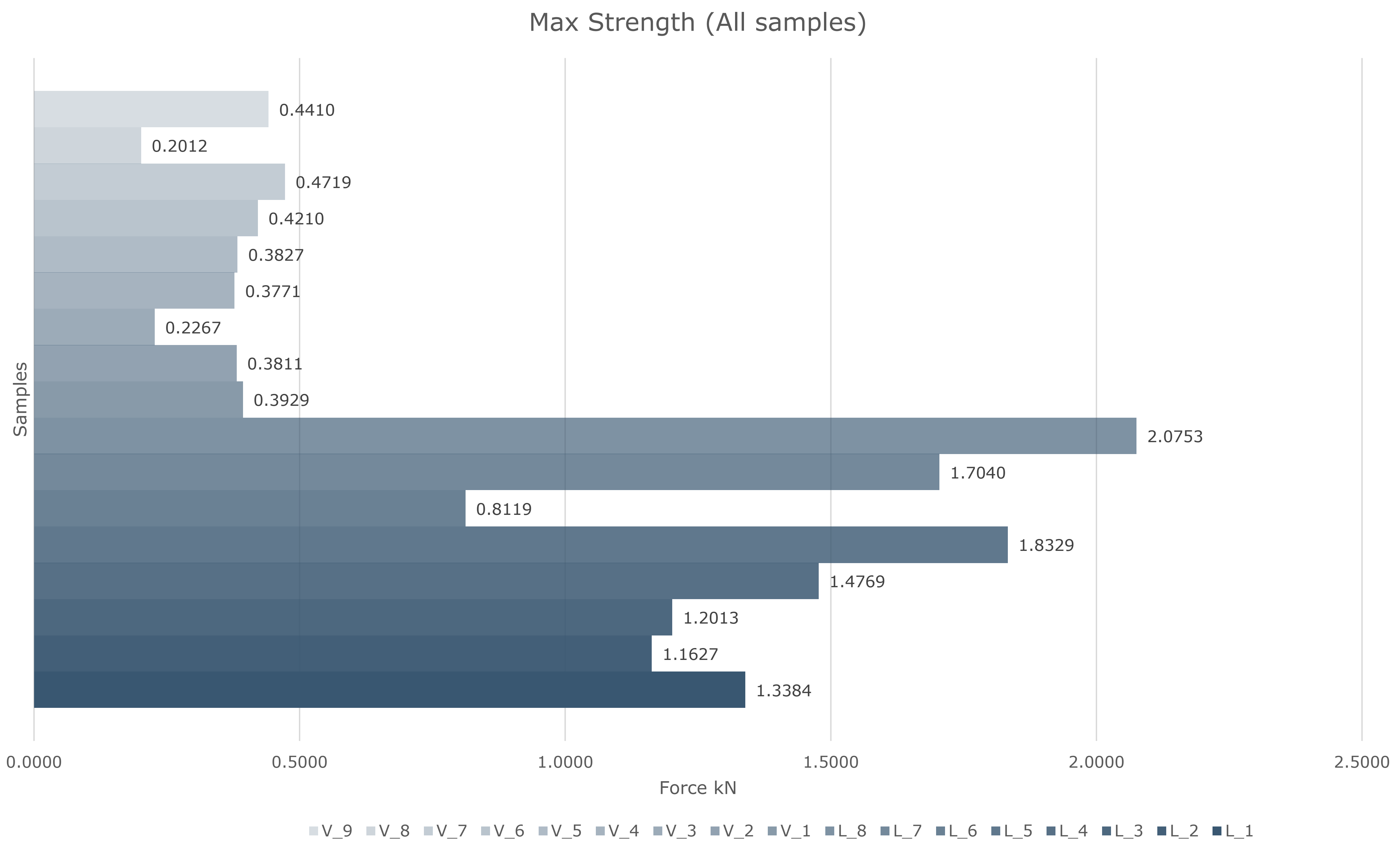
Flexural Testing

Specimen Orientation	E_1	E_2
	Gpa	Gpa
Longitudinal_1	67.36	67.26
Longitudinal_2	62.45	62.42
Longitudinal_3	61.09	61.01
Longitudinal_4	61.52	61.54
Longitudinal_5	36.77	64.12
Longitudinal_6	63.80	62.39
Longitudinal_7	65.90	65.86
Longitudinal_8	63.21	62.92
Vertical_1	53.85	60.55
Vertical_2	55.87	64.62
Vertical_3	55.53	75.53
Vertical_4	60.43	69.97
Vertical_5	55.67	63.99
Vertical_6	59.04	66.25
Vertical_7	60.64	68.89
Vertical_8	57.78	80.03
Vertical_9	54.61	62.90

E_1 (Gpa)	L. Beams	V. Beams	All Beams	**Ratio (a)**
Mean	60.26	57.05	58.56	**0.95**
SD	9.10	2.37	6.67	
CV	0.15	0.04	0.11	

E_2 (Gpa)	L. Beams	V. Beams	All Beams	**Ratio (a)**
Mean	63.44	68.08	65.90	**1.07**
SD	2.03	5.95	5.10	
CV	0.03	0.09	0.08	

Flexural Testing

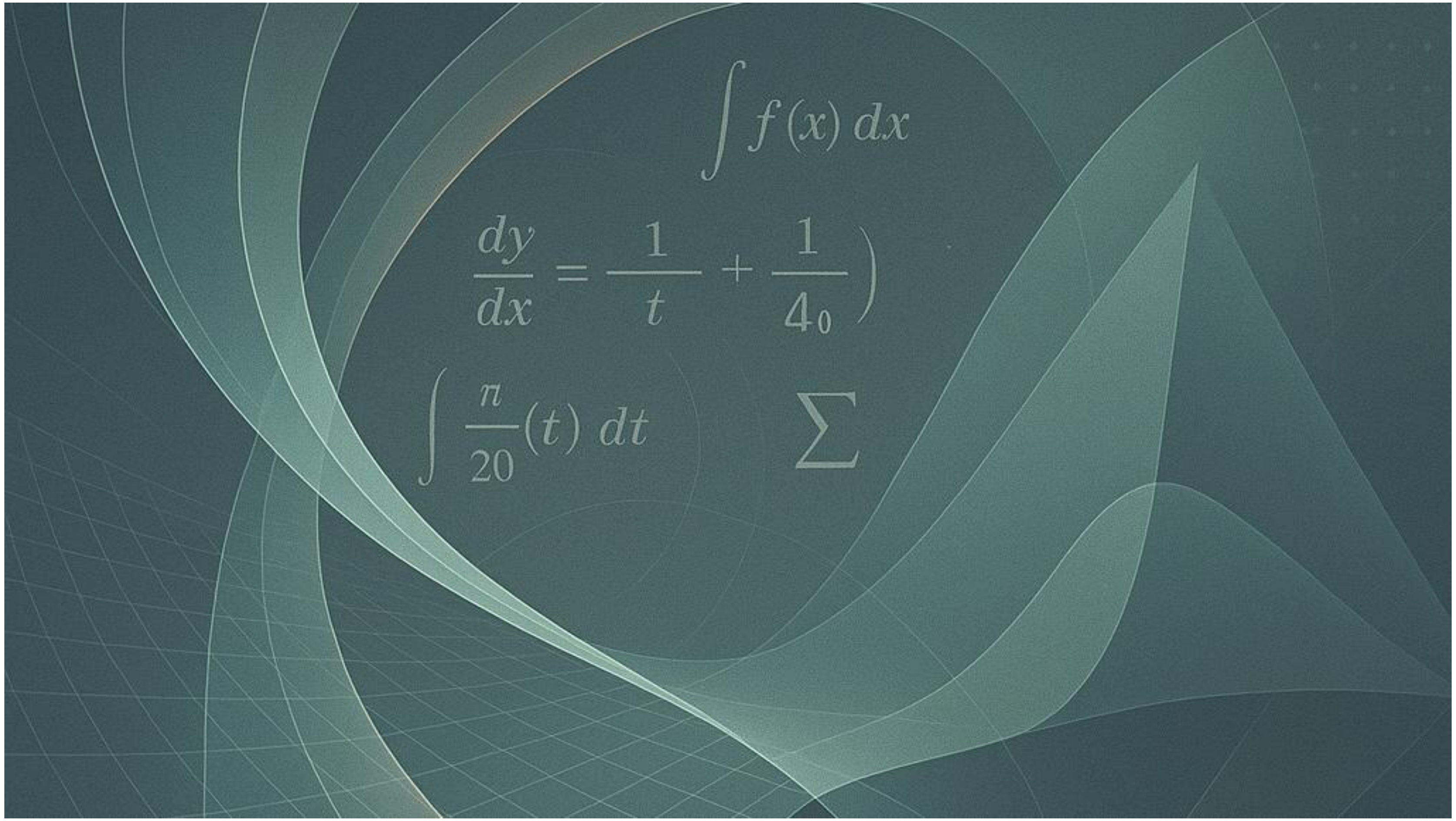


$$E_2 = a * E_1$$

$$E_2 = 0.25 * E_1$$

$$E_2 = 0.75 * E_1$$

Algorithm Development



Algorithm Development

flowchart

Initialization →

Domain Set-up

- number of elements
- overhang setting

Optimization Settings

- volume fraction
- penalization
- filter radius
- number of iterations
- convergence criterion

Boundary Conditions

- load conditions
- support conditions

Preprocessing →

Stiffness Matrix

- material properties of elements
- compute Ke

Connectivity Matrix/DOF Assembly

- 3D mesh construction and nodal relationships
- mapping of DOFs into global coordinates

Filtering

- filterring matrix (H) construction
- identify neighboring elements
- compute weights based on distance

Optimization Loop →

SIMP Interpolation

- $E_e = \rho_e^p * E_e^0$

Objective Function / Sensitivities

- min $C = F^T U$
- partial derivatives w.r.t to density/volume

Optimality Criteria Update

- $x_e^{new}=x_e^{old} * \surd(D_e$

Converged?

↓ ***Yes***

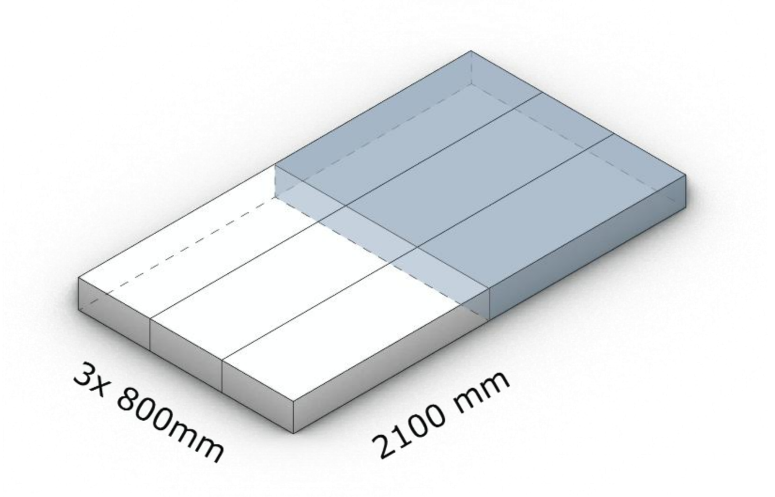
↓ ***No*** (→ back to SIMP Interpolation)

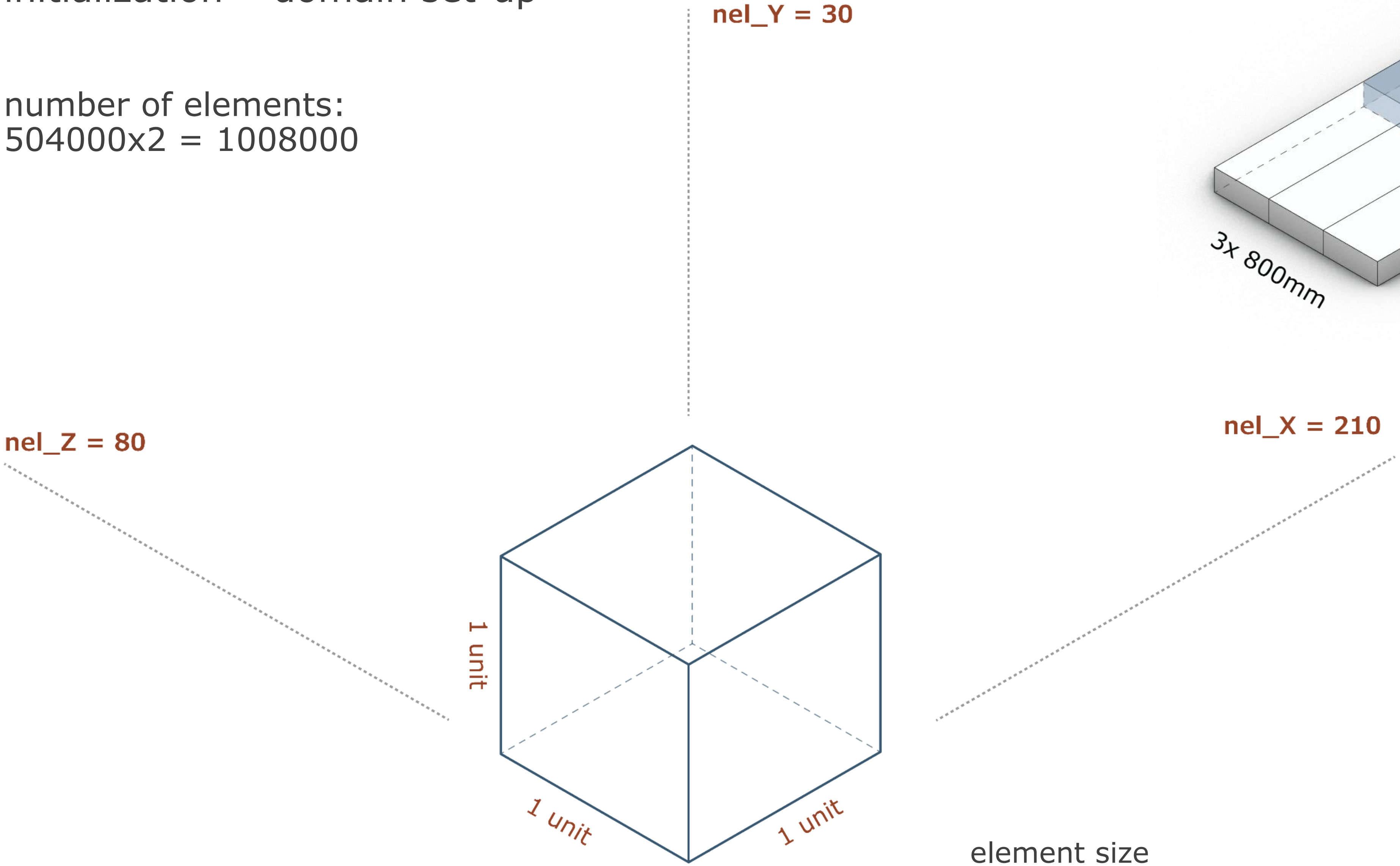
Post-Processing

- plot objective function
- save STL geometries

initialization – domain set-up

number of elements:
504000x2 = 1008000

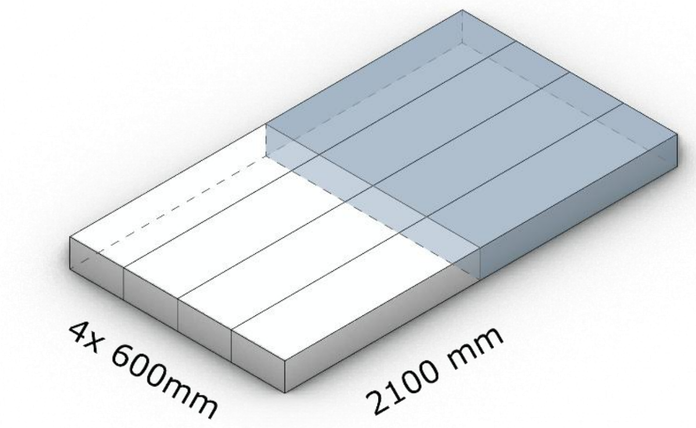


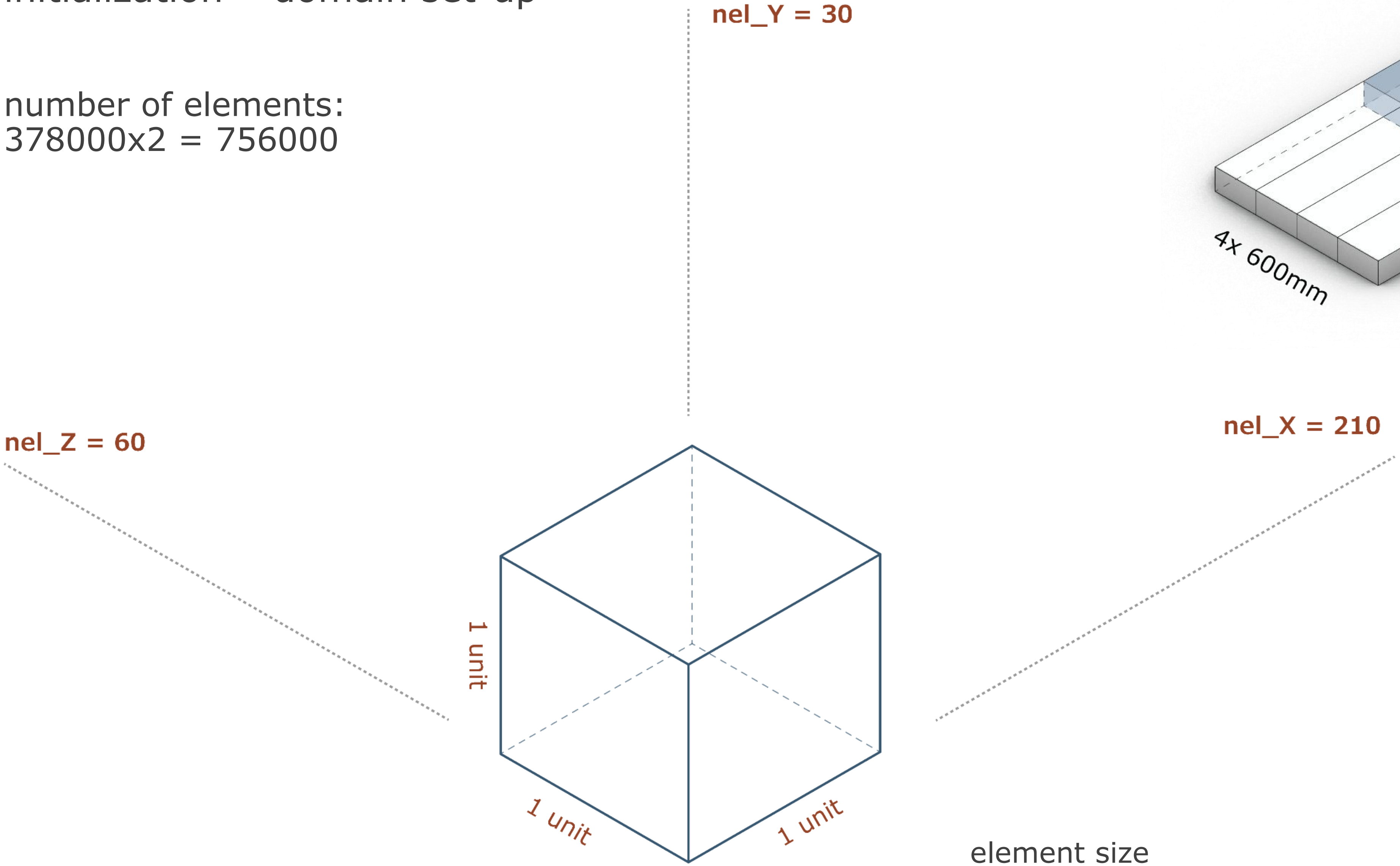


element size

initialization – domain set-up

number of elements:
378000x2 = 756000

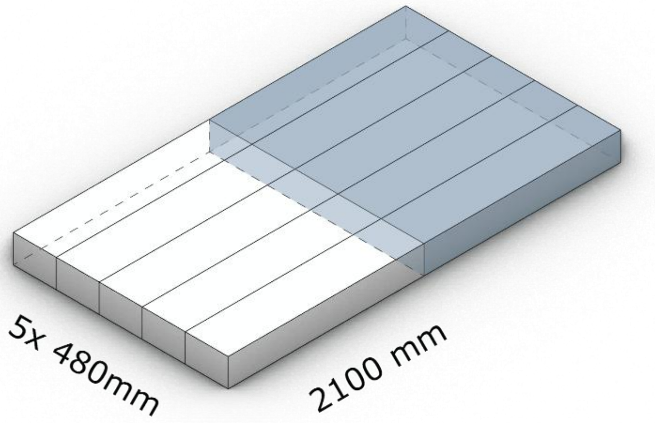




element size

initialization – domain set-up

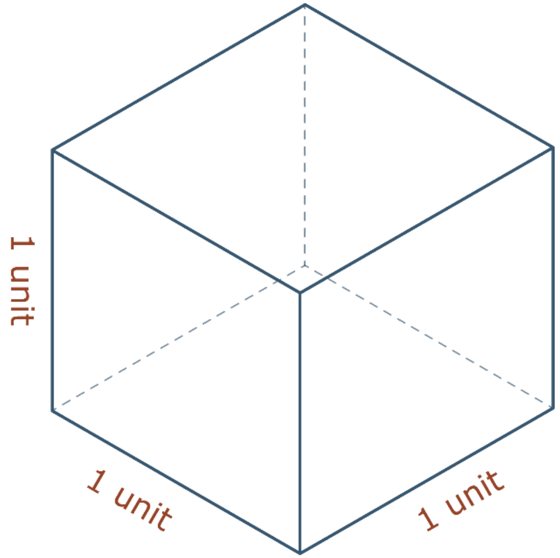
number of elements:
302400x2 = 604800



nel_Y = 30

nel_Z = 48

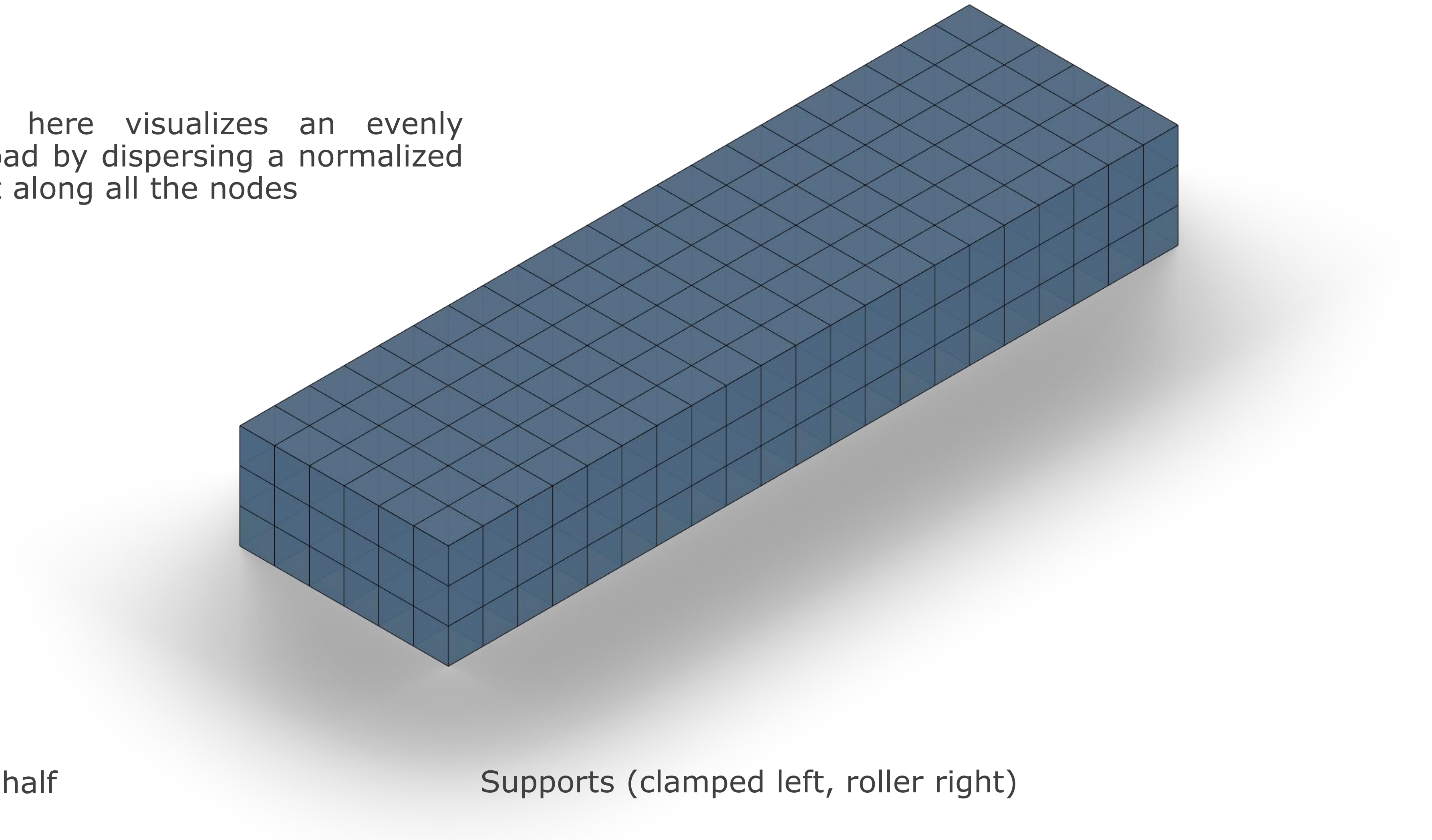
nel_X = 210



element size

initialization – boundary conditions

The loading here visualizes an evenly distributed load by dispersing a normalized load of 1 unit along all the nodes



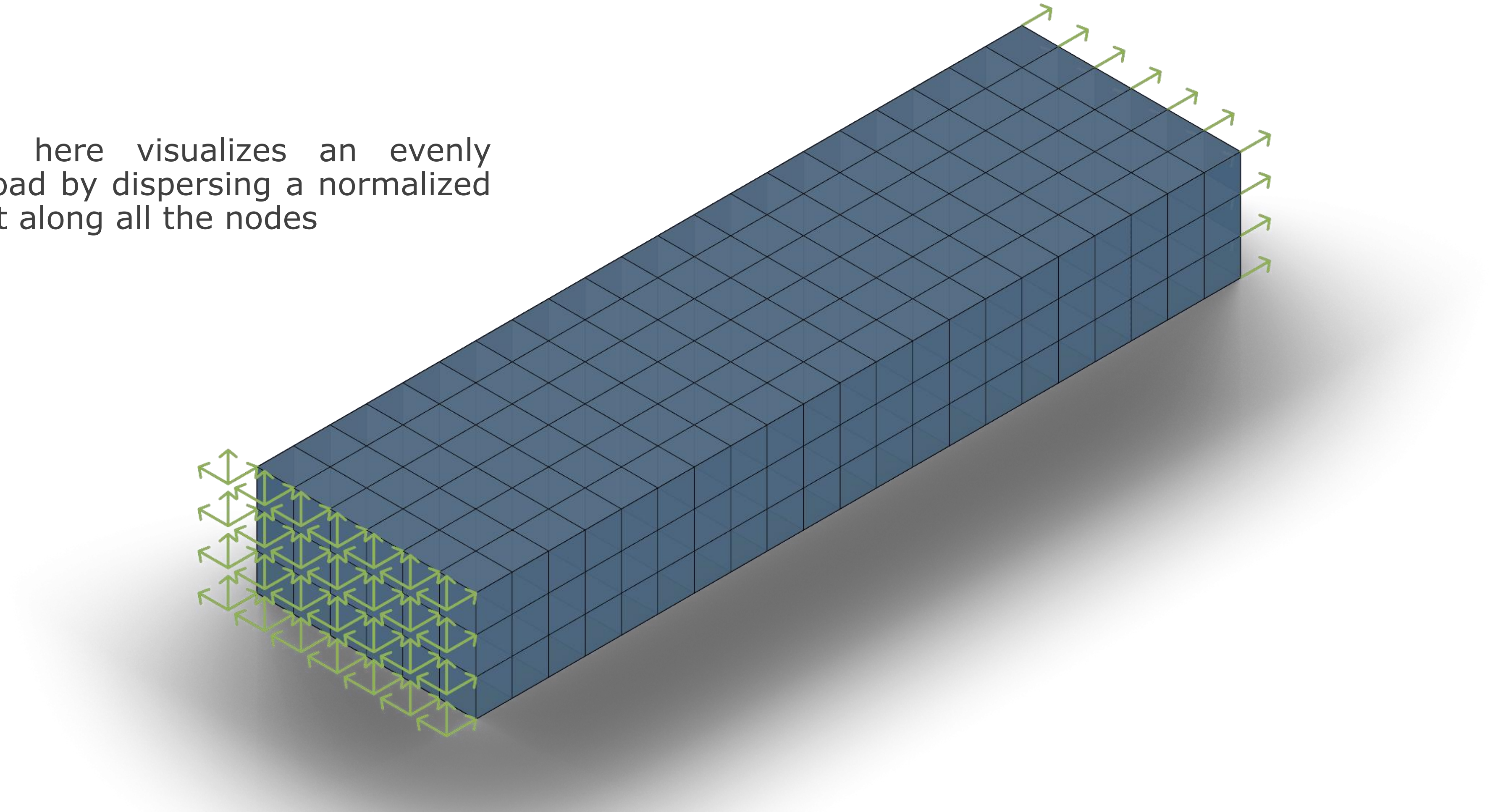
Mesh domain – half

Supports (clamped left, roller right)

Loads

initialization – boundary conditions

The loading here visualizes an evenly distributed load by dispersing a normalized load of 1 unit along all the nodes



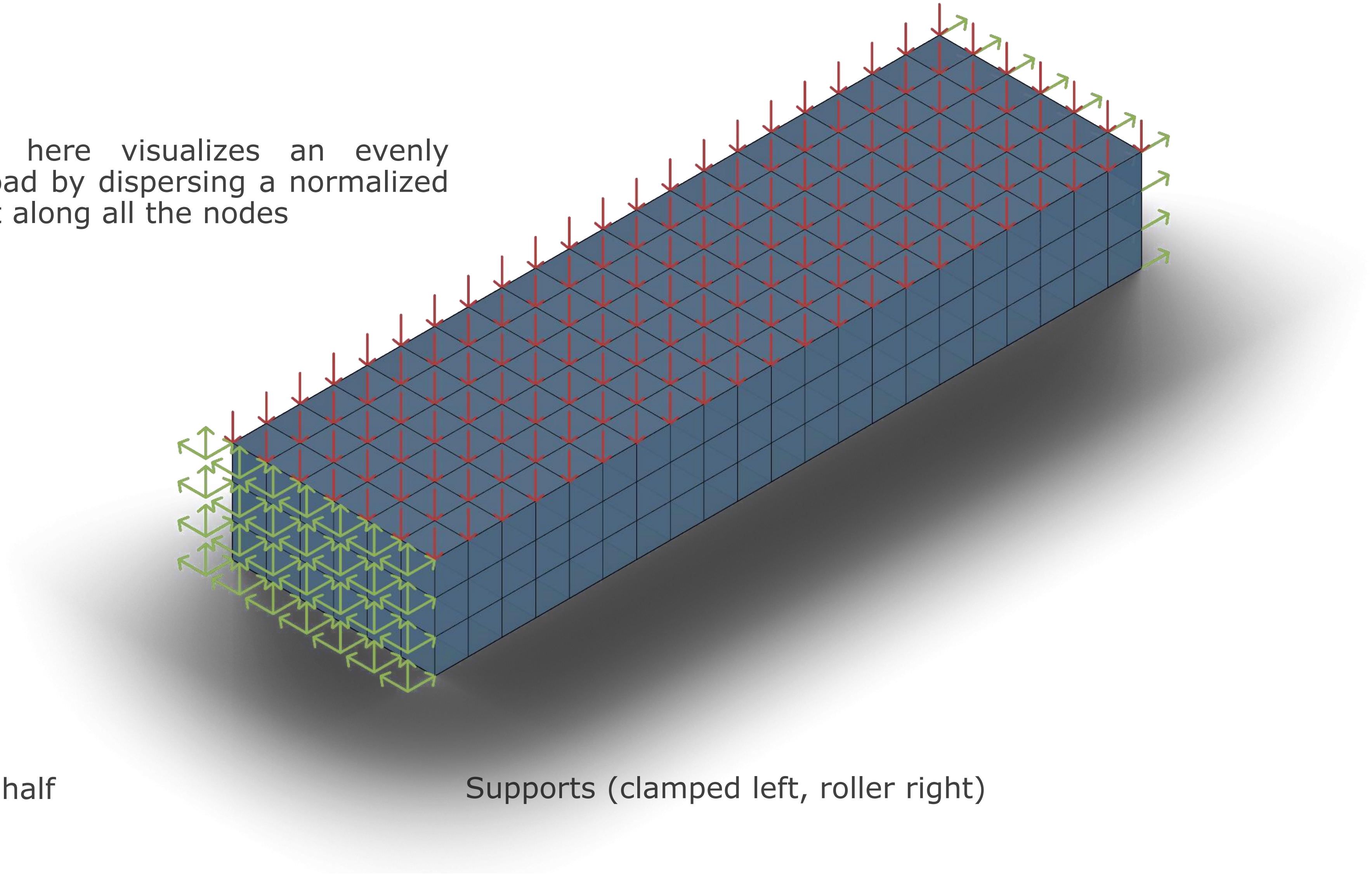
Mesh domain – half

Supports (clamped left, roller right)

Loads

initialization – boundary conditions

The loading here visualizes an evenly distributed load by dispersing a normalized load of 1 unit along all the nodes



Mesh domain – half

Supports (clamped left, roller right)

Loads

Algorithm Development

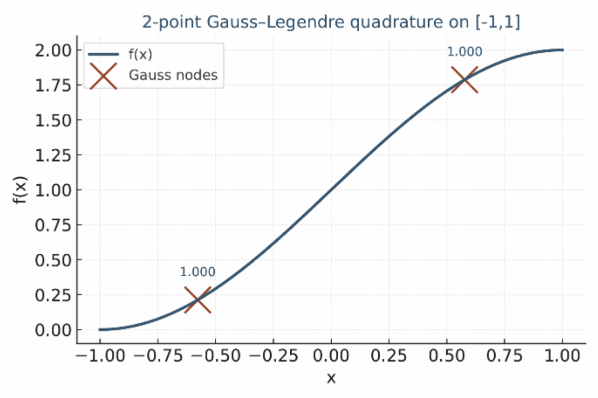
$$[Ke] = \int_{\infty}^{\infty} [B]^T [D][B] dV$$

$$[S]^{-1} = D$$

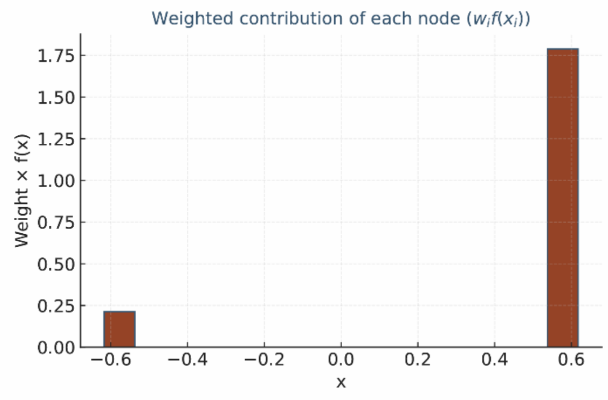
$$[S]^{-1} = \frac{1}{detS} \cdot Adj\ S = D, detS \neq 0$$

Algorithm Development

Gaussian Quadrature



No. of points, n (order of integration	Location, ξi	Weights, Wi
n = 1	$\xi_1 = 0.0$	$W_1 = 2.0$
n = 2	$\xi_1 = +\sqrt{1/3}$ $\xi_2 = -\sqrt{1/3}$	$W_1 = 1.0$ $W_2 = 1.0$
n = 3	$\xi_1 = -\sqrt{3/5}$ $\xi_2 = 0.0$ $\xi_3 = +\sqrt{3/5}$	$W_1 = 5/9$ $W_2 = 8/9$ $W_3 = 5/9$

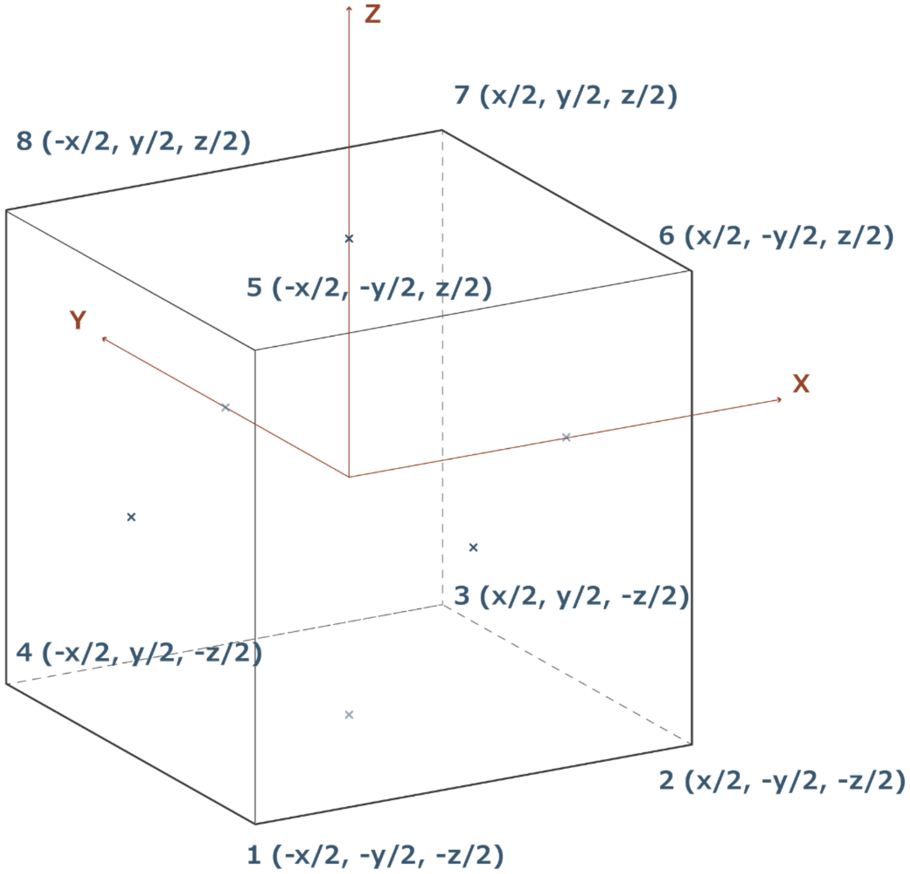


Algorithm Development

$$[Ke] = \int_{\infty}^{\infty} [B]^T [D][B] dV$$

$$[Ke] \approx \sum_{i,j,k} w_i w_j w_k \cdot [B]^T [D][B] |detJ|$$

Element in Cartesian coordinates

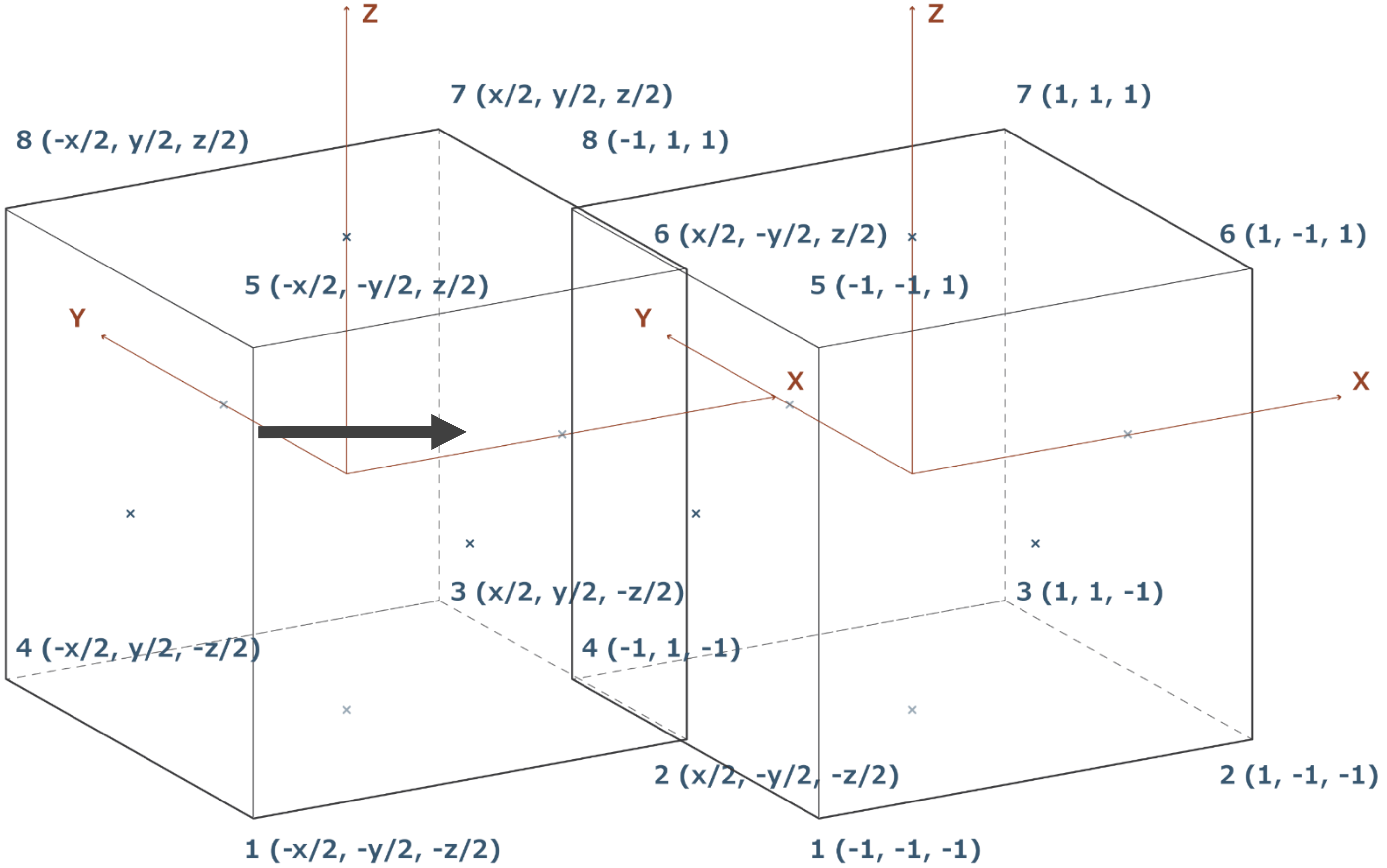


Partial derivatives w.r.t natural coordinate system

$$Ni(\xi,\eta,\zeta) = \frac{1}{8}(1+\xi_i\xi)(1+\eta_i\eta)(1+\zeta_i\zeta)$$

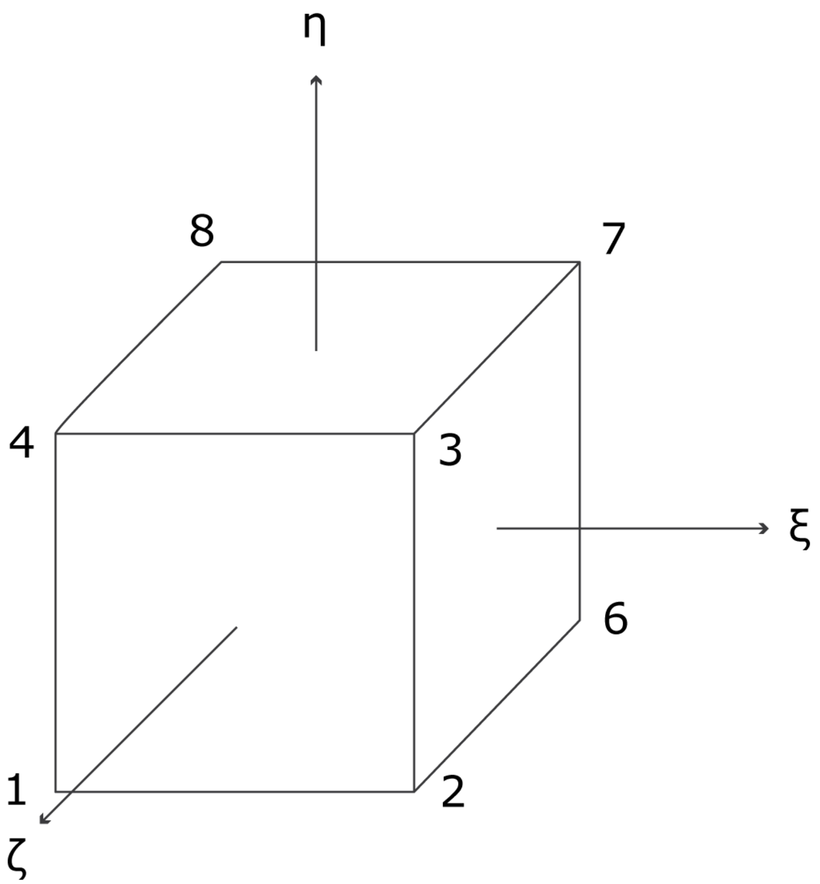
	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
$\frac{\partial N_i}{\partial \xi}$	$-\frac{1}{8}(1-\eta)(1-\zeta)$	$\frac{1}{8}(1-\eta)(1-\zeta)$	$\frac{1}{8}(1+\eta)(1-\zeta)$	$-\frac{1}{8}(1+\eta)(1-\zeta)$	$-\frac{1}{8}(1-\eta)(1+\zeta)$	$\frac{1}{8}(1-\eta)(1+\zeta)$	$\frac{1}{8}(1+\eta)(1+\zeta)$	$-\frac{1}{8}(1+\eta)(1+\zeta)$
$\frac{\partial N_i}{\partial \eta}$	$-\frac{1}{8}(1-\xi)(1-\zeta)$	$-\frac{1}{8}(1+\xi)(1-\zeta)$	$\frac{1}{8}(1+\xi)(1-\zeta)$	$\frac{1}{8}(1-\xi)(1-\zeta)$	$-\frac{1}{8}(1-\xi)(1+\zeta)$	$-\frac{1}{8}(1+\xi)(1+\zeta)$	$\frac{1}{8}(1+\xi)(1+\zeta)$	$\frac{1}{8}(1-\xi)(1+\zeta)$
$\frac{\partial N_i}{\partial \zeta}$	$-\frac{1}{8}(1-\xi)(1-\eta)$	$-\frac{1}{8}(1+\xi)(1-\eta)$	$-\frac{1}{8}(1+\xi)(1+\eta)$	$-\frac{1}{8}(1-\xi)(1+\eta)$	$\frac{1}{8}(1-\xi)(1-\eta)$	$\frac{1}{8}(1+\xi)(1-\eta)$	$\frac{1}{8}(1+\xi)(1+\eta)$	$\frac{1}{8}(1-\xi)(1+\eta)$

Cartesian - Natural coordinates

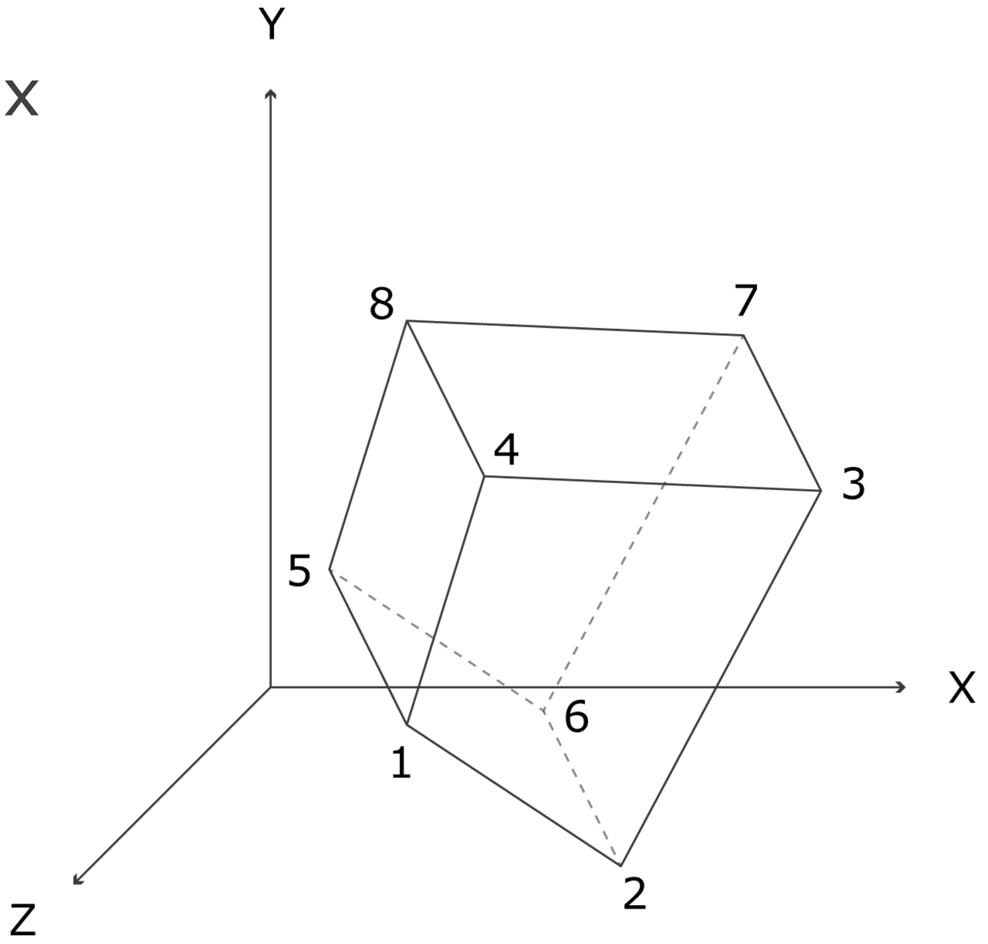


Jacobian matrix

$$[J] = \frac{\partial(x,y,z)}{\partial(\xi,\eta,\zeta)} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix}$$



Natural Coordinates



Cartesian Coordinates

Strain – displacement matrix assembly

$$[J]^{-1} \cdot \nabla N_i^{(\xi,\eta,\zeta)} \longrightarrow \begin{bmatrix} \frac{\partial N_1}{\partial x} \\ \frac{\partial N_1}{\partial y} \\ \frac{\partial N_1}{\partial z} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial N_8}{\partial x} \\ \frac{\partial N_8}{\partial y} \\ \frac{\partial N_8}{\partial z} \end{bmatrix} \longrightarrow [B] = \begin{bmatrix} \frac{\partial Ni}{\partial x} & 0 & 0 \\ 0 & \frac{\partial Ni}{\partial y} & 0 \\ 0 & 0 & \frac{\partial Ni}{\partial z} \\ \frac{\partial Ni}{\partial y} & \frac{\partial Ni}{\partial x} & 0 \\ \frac{\partial Ni}{\partial z} & 0 & \frac{\partial Ni}{\partial x} \\ 0 & \frac{\partial Ni}{\partial z} & \frac{\partial Ni}{\partial y} \end{bmatrix}$$

Algorithm Development

$$[Ke] = \int_{\infty}^{\infty} [B]^T [D][B] dV$$

Algorithm Development

Initialization →

Domain Set-up

- number of elements
- overhang setting

Optimization Settings

- volume fraction
- penalization
- filter radius
- number of iterations
- convergence criterion

Boundary Conditions

- load conditions
- support conditions

Preprocessing →

Stiffness Matrix

- material properties of elements
- compute Ke

Connectivity Matrix/DOF Assembly

- 3D mesh construction and nodal relationships
- mapping of DOFs into global coordinates

Filtering

- filterring matrix (H) construction
- identify neighboring elements
- compute weights based on distance

Optimization Loop →

SIMP Interpolation

- $E_e = \rho_e^p * E_e^0$

Objective Function / Sensitivities

- min $C = F^T U$
- partial derivatives w.r.t to density/volume

Optimality Criteria Update

- $x_e^{new}=x_e^{old} * \sqrt{(D_e}$

Converged?

↓ ***Yes***   ↓ ***No*** (→ back to SIMP Interpolation)

Post-Processing

- plot objective function
- save STL geometries

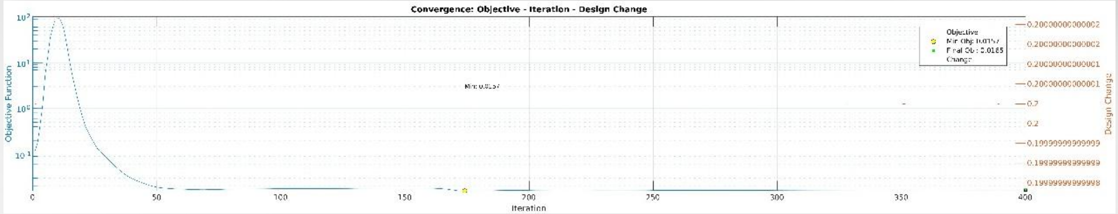
Testing & Results

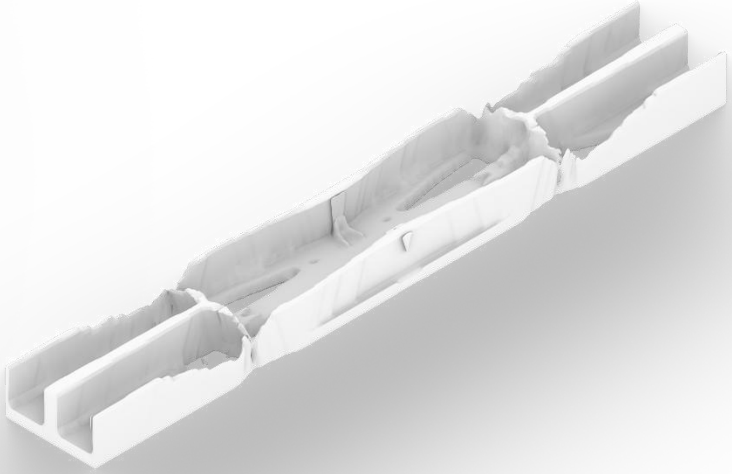
Testing & Results

Test_ID	Mesh_Set-Up		Algorithms			Tested_Variable	Variable_Start	Variable_Step	No. Steps	No. Scripts	Constant_Variables
			No. Algorithms	Standard_SIMP	Anisotropy-Aware_SIMP						
Test_16	3	Mesh_1 210x30x48	2	TRUE	TRUE	Volfrac	0.3	0.05	5	30	rmin = 2.0 penal = 3.0 move = 0.2 a = 0.25
		Mesh_2 210x30x60		TRUE	TRUE						
		Mesh_3 210x30x80		TRUE	TRUE						
Test_17	3	Mesh_1 210x30x48	2	TRUE	TRUE	rmin	1.5	0.5	6	36	volfrac = 0.5 penal = 3.0 move = 0.2 a = 0.25
		Mesh_2 210x30x60		TRUE	TRUE						
		Mesh_3 210x30x80		TRUE	TRUE						
Test_18	3	Mesh_1 210x30x48	2	TRUE	TRUE	rmin	1.5	0.5	6	36	volfrac = 0.35 penal = 3.0 move = 0.2 a = 0.25
		Mesh_2 210x30x60		TRUE	TRUE						
		Mesh_3 210x30x80		TRUE	TRUE						
Test_19	3	Mesh_1 210x30x48	2	TRUE	TRUE	rmin	1.5	0.5	6	36	volfrac = 0.45 penal = 3.0 move = 0.2 a = 0.25
		Mesh_2 210x30x60		TRUE	TRUE						
		Mesh_3 210x30x80		TRUE	TRUE						
Test_20	3	Mesh_1 210x30x48	1	FALSE	TRUE	rmin	0.3	0.05	5	15	volfrac = 0.5 penal = 3.0 move = 0.2 a = 0.75
		Mesh_2 210x30x60		FALSE	TRUE						
		Mesh_3 210x30x80		FALSE	TRUE						
Test_21	3	Mesh_1 210x30x48	2	TRUE	TRUE	Density_Check	-	-	-	6	volfrac = 0.3 rmin = 4.0 penal = 3.0 move = 0.2 a = 0.25
		Mesh_2 210x30x60		TRUE	TRUE						
		Mesh_3 210x30x80		TRUE	TRUE						
Test_22	3	Mesh_1 210x30x48	2	TRUE	TRUE	Density_Check	-	-	-	6	volfrac = 0.35 rmin = 4.0 penal = 3.0 move = 0.2 a = 0.25
		Mesh_2 210x30x60		TRUE	TRUE						
		Mesh_3 210x30x80		TRUE	TRUE						

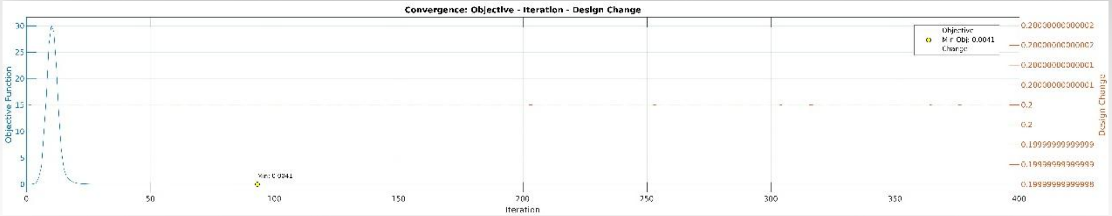
Testing & Results

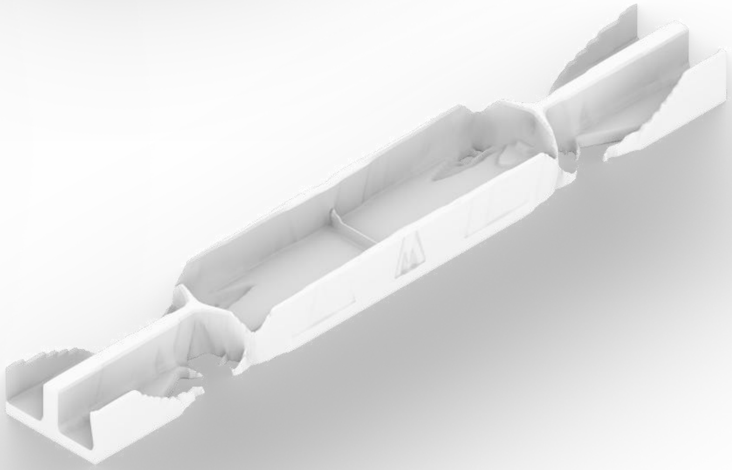
16_02





Transverse Isotropic

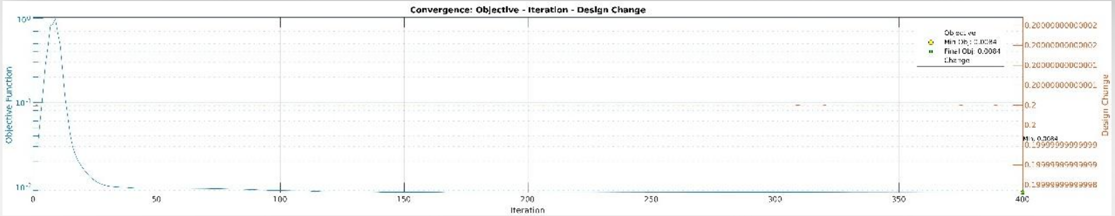


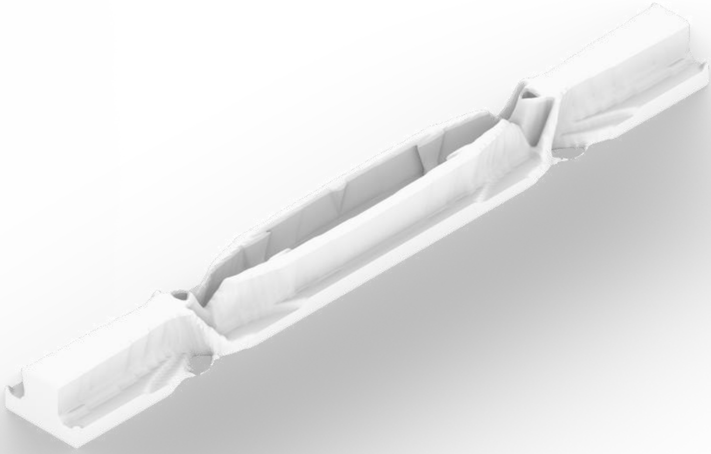


Isotropic

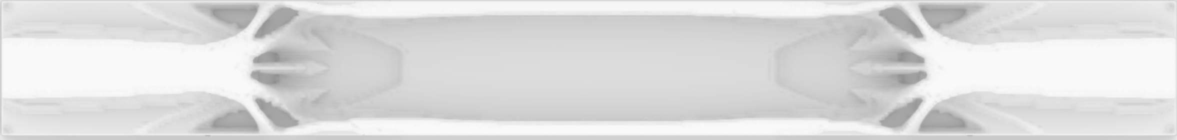
Testing & Results

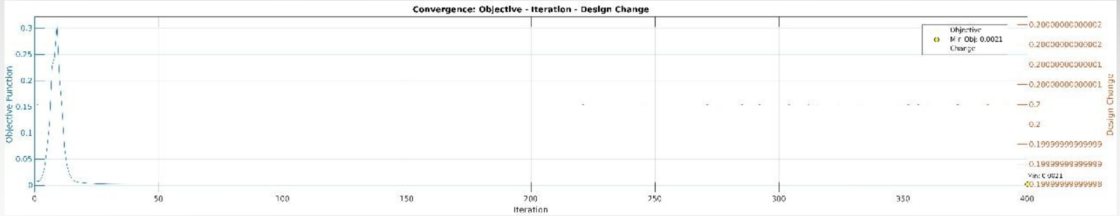
16_13





Transverse Isotropic

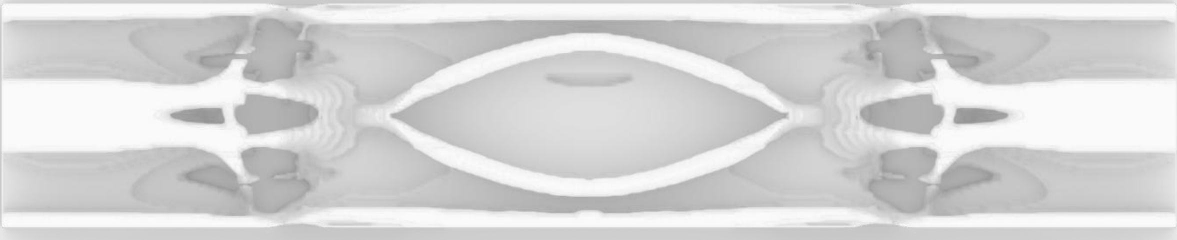


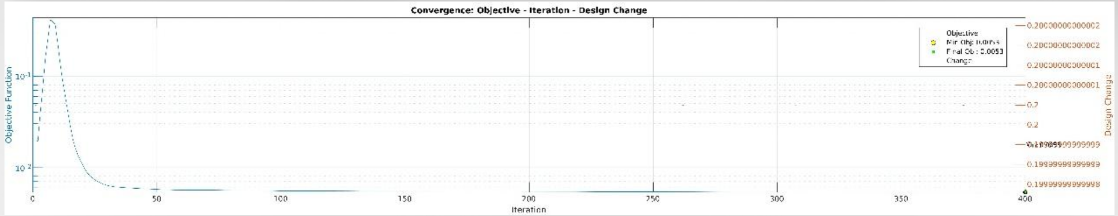


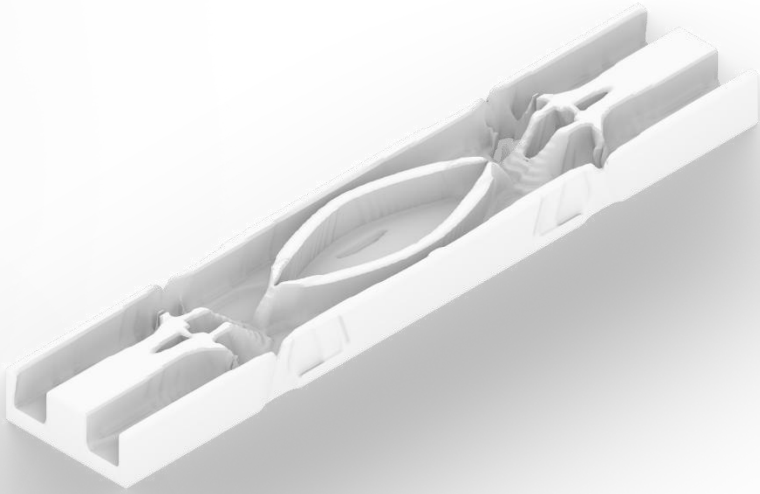
Isotropic

Testing & Results

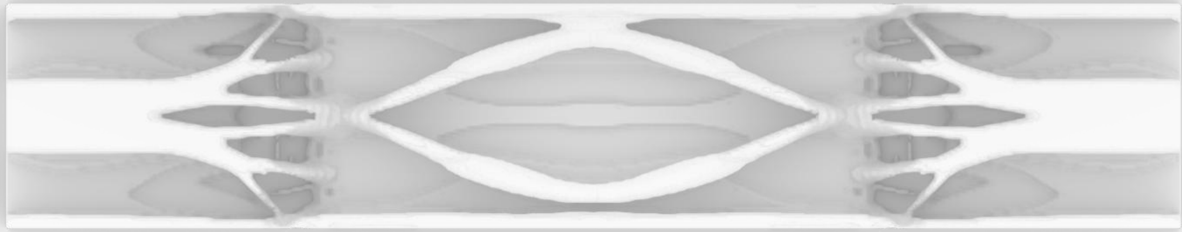
16_15

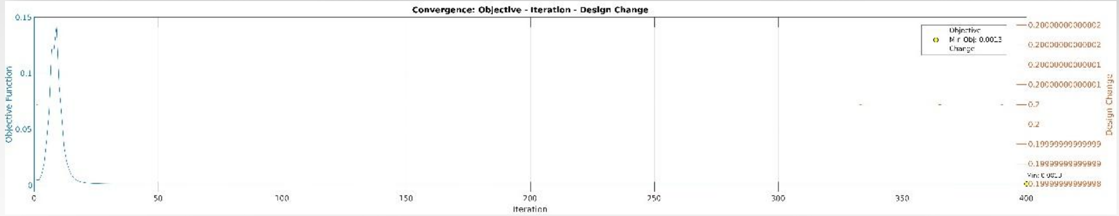


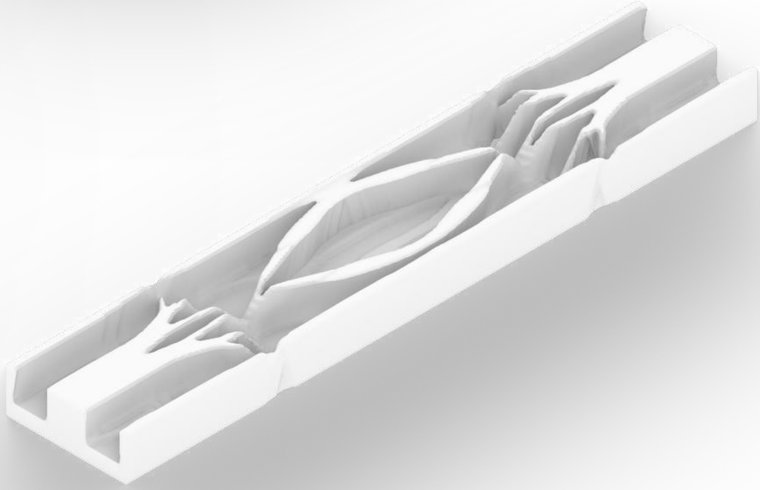




Transverse Isotropic



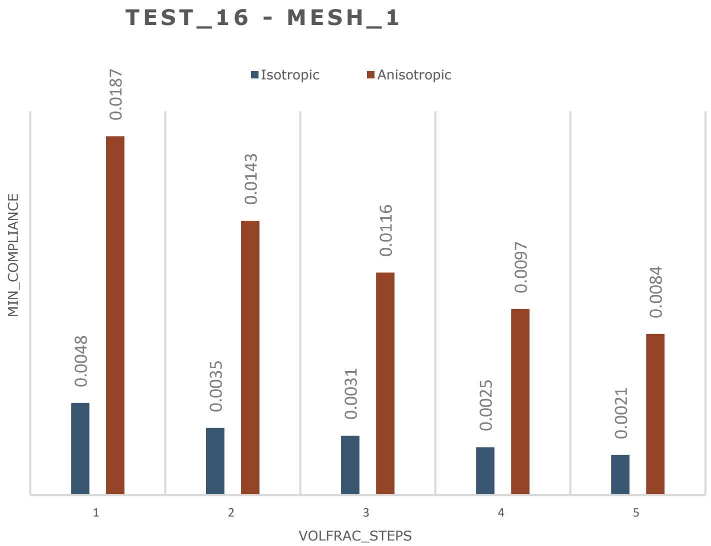


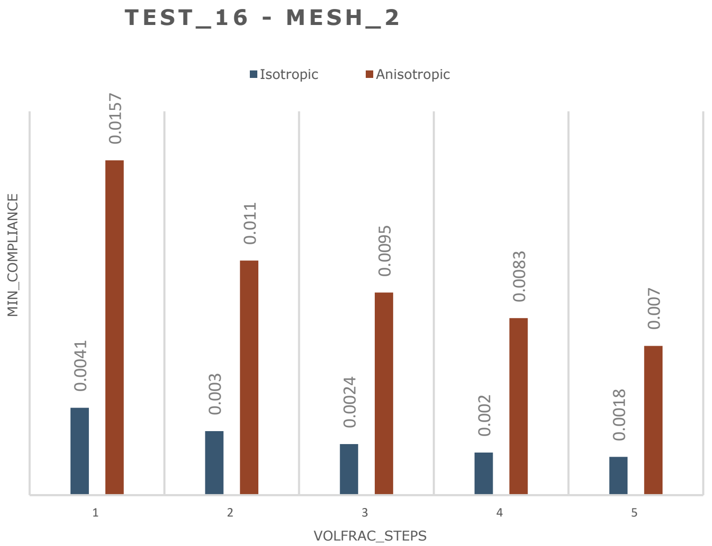


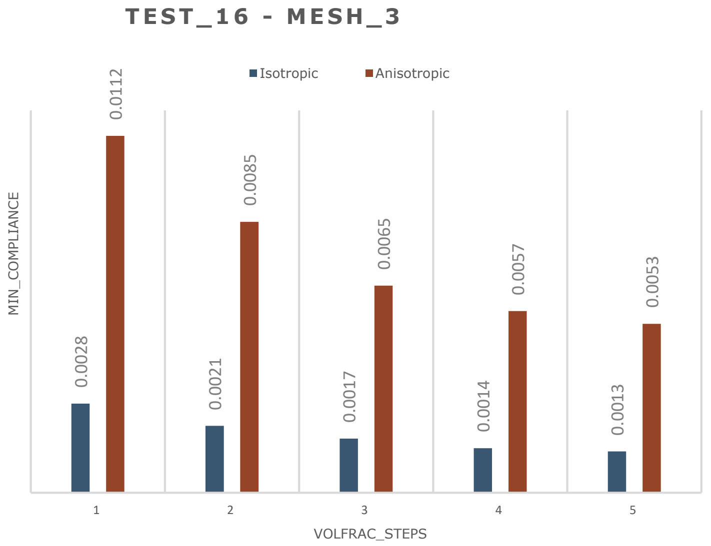
Isotropic

Testing & Results

Test_16 **a = 0.25** Rmin = 2.0 tolx = 0.01 penal 3.0	Mesh_1 210x30x48	Mesh_2 210x30x60	Mesh_3 210x30x80
Volfrac = 0,3 (01 - 03)			
Min_Compliance_Isotropic	0.0048	0.0041	0.0028
Min_Compliance_Transverse	0.0187	0.0157	0.0112
Ratio	**0.2567**	**0.2611**	**0.25**
Volfrac = 0,35 (04 - 06)			
Min_Compliance_Isotropic	0.0035	0.003	0.0021
Min_Compliance_Transverse	0.0143	0.011	0.0085
Ratio	**0.2448**	**0.2727**	**0.2471**
Volfrac = 0,40 (07 - 09)			
Min_Compliance_Isotropic	0.0031	0.0024	0.0017
Min_Compliance_Transverse	0.0116	0.0095	0.0065
Ratio	**0.2672**	**0.2526**	**0.2615**
Volfrac = 0,45 (10 - 12)			
Min_Compliance_Isotropic	0.0025	0.002	0.0014
Min_Compliance_Transverse	0.0097	0.0083	0.0057
Ratio	**0.2577**	**0.2410**	**0.2456**
Volfrac = 0,50 (13 - 15)			
Min_Compliance_Isotropic	0.0021	0.0018	0.0013
Min_Compliance_Transverse	0.0084	0.007	0.0053
Ratio	**0.2500**	**0.2571**	**0.2453**
Mean	**0.2553**	**0.2569**	**0.2499**
SD	0.008	0.010	0.006
CV	0.030	0.040	0.024

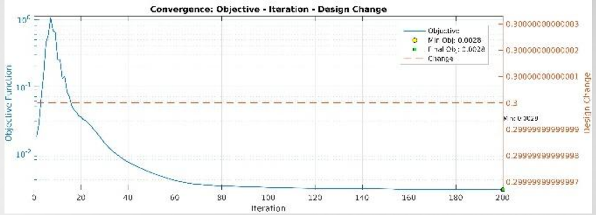


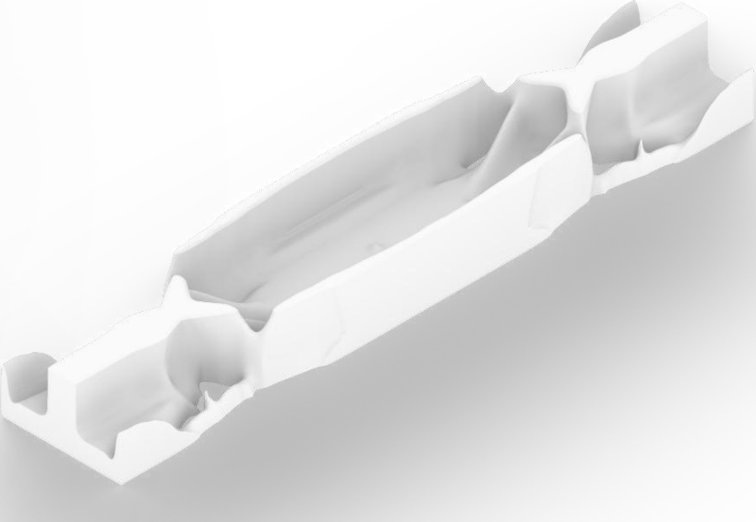




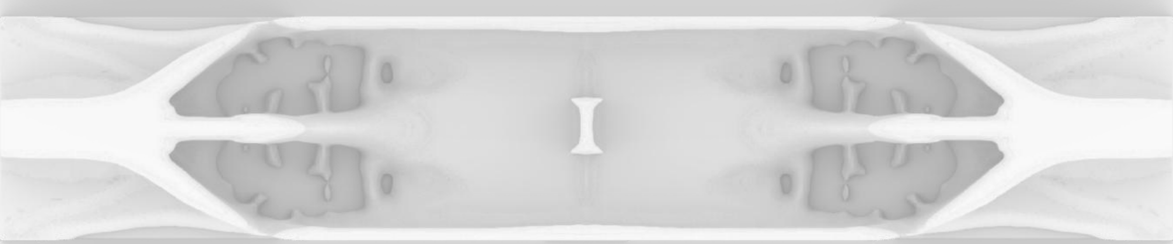
Testing & Results

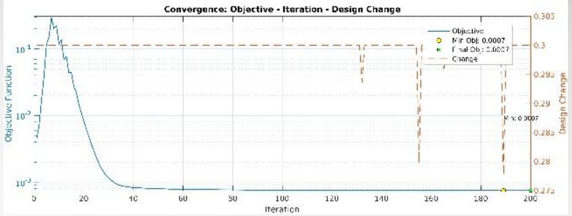
18_15

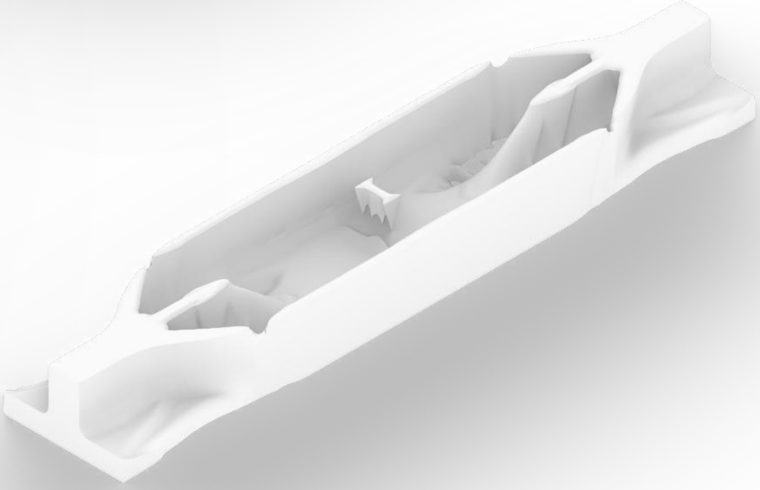




Transverse Isotropic



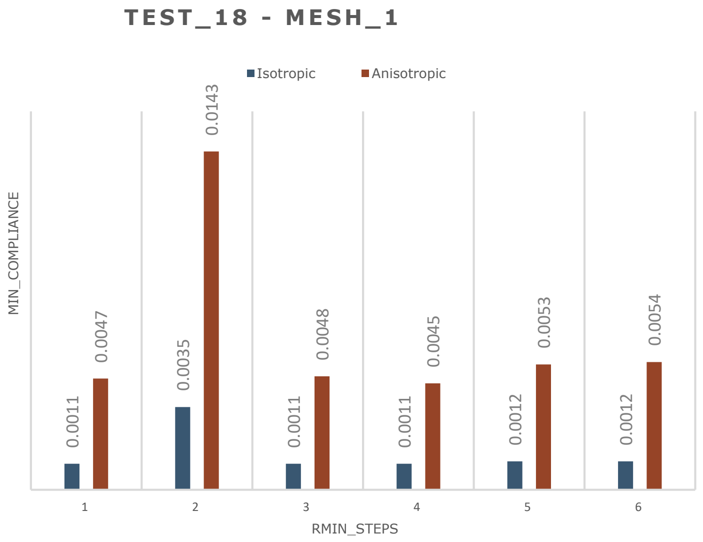


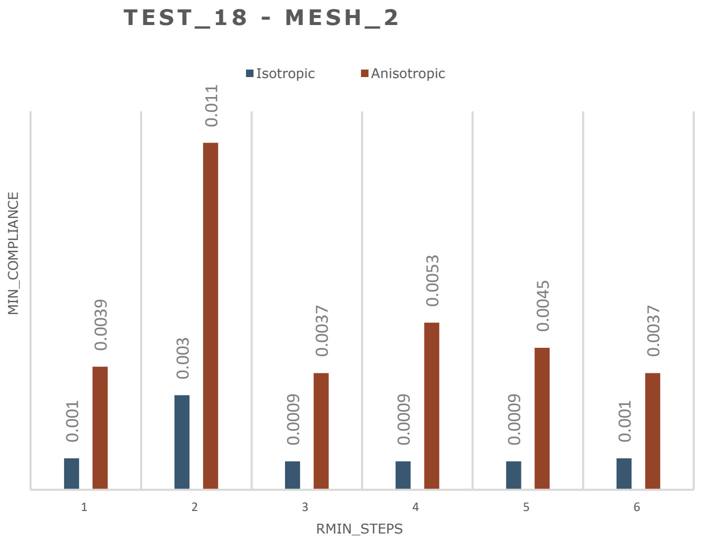


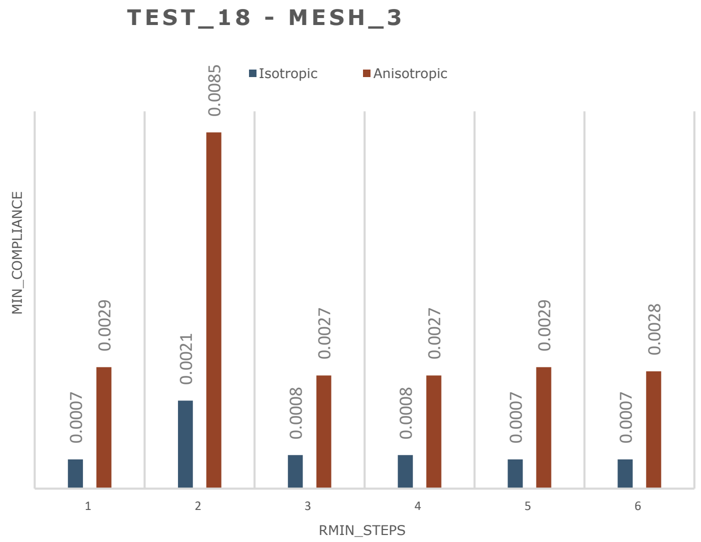
Isotropic

Testing & Results

Test_18 **a = 0.25** VolFrac = 0,35 tolx = 0.01 penal 3.0	Mesh_1 210x30x48	Mesh_2 210x30x60	Mesh_3 210x30x80
Rmin = 1,5 (01 - 03)			
Min_Compliance_Isotropic	0.0011	0.001	0.0007
Min_Compliance_Transverse	0.0047	0.0039	0.0029
Ratio	**0.2340**	**0.2564**	**0.2414**
Rmin = 2,0 (04 -06 Test_16)			
Min_Compliance_Isotropic	0.0035	0.003	0.0021
Min_Compliance_Transverse	0.0143	0.011	0.0085
Ratio	**0.2448**	**0.2727**	**0.2471**
Rmin = 2,5 (04 - 06)			
Min_Compliance_Isotropic	0.0011	0.0009	0.0008
Min_Compliance_Transverse	0.0048	0.0037	0.0027
Ratio	**0.2292**	**0.2432**	**0.2963**
Rmin = 3,0 (07 - 09)			
Min_Compliance_Isotropic	0.0011	0.0009	0.0008
Min_Compliance_Transverse	0.0045	0.0053	0.0027
Ratio	**0.2444**	**0.1698**	**0.2963**
Rmin = 3,5 (10 - 12)			
Min_Compliance_Isotropic	0.0012	0.0009	0.0007
Min_Compliance_Transverse	0.0053	0.0045	0.0029
Ratio	**0.2264**	**0.2000**	**0.2414**
Rmin = 4,0 (13 - 15)			
Min_Compliance_Isotropic	0.0012	0.001	0.0007
Min_Compliance_Transverse	0.0054	0.0037	0.0028
Ratio	**0.2222**	**0.2703**	**0.2500**
Mean	**0.2335**	**0.2354**	**0.2621**
SD	0.009	0.038	0.024
CV	0.037	0.161	0.093

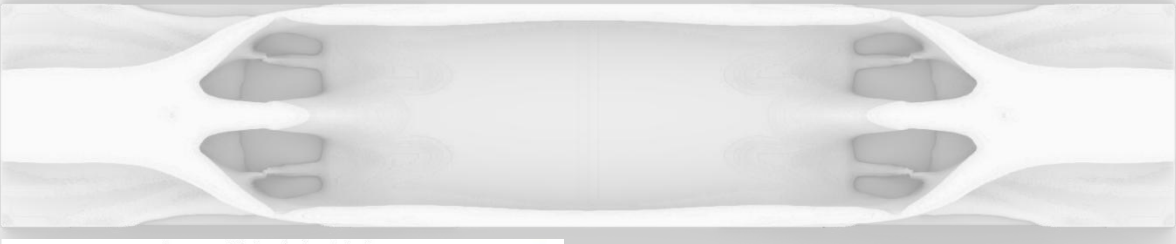


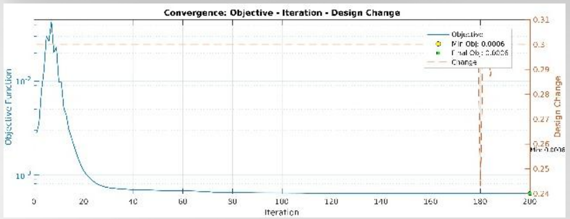


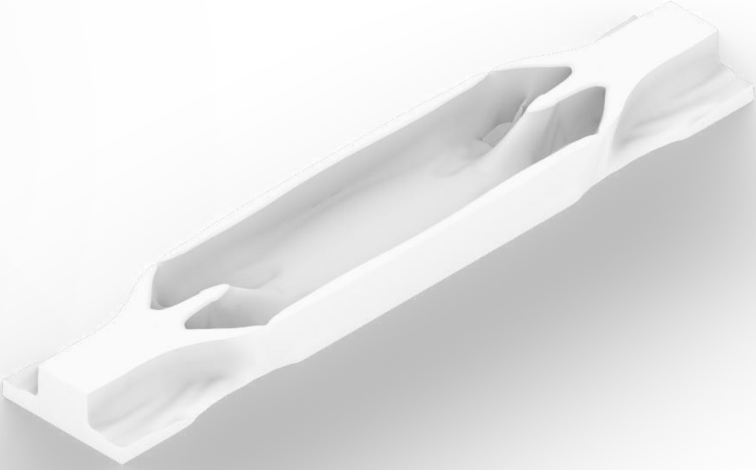


Testing & Results

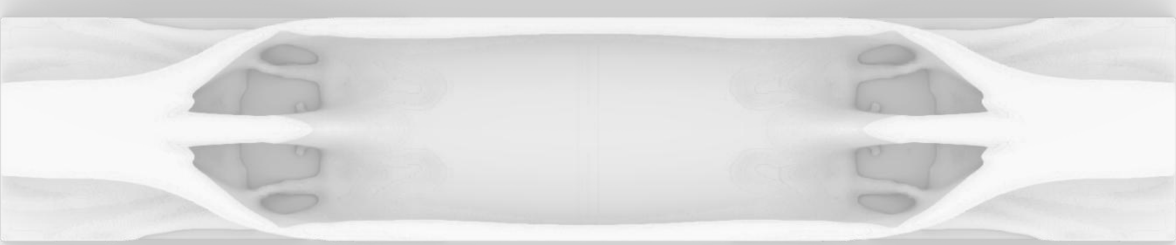
20_15

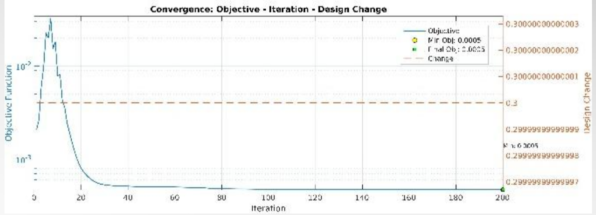


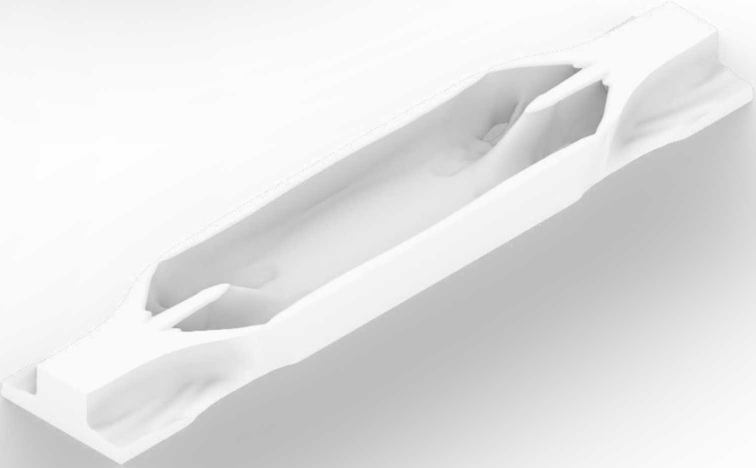




Transverse Isotropic



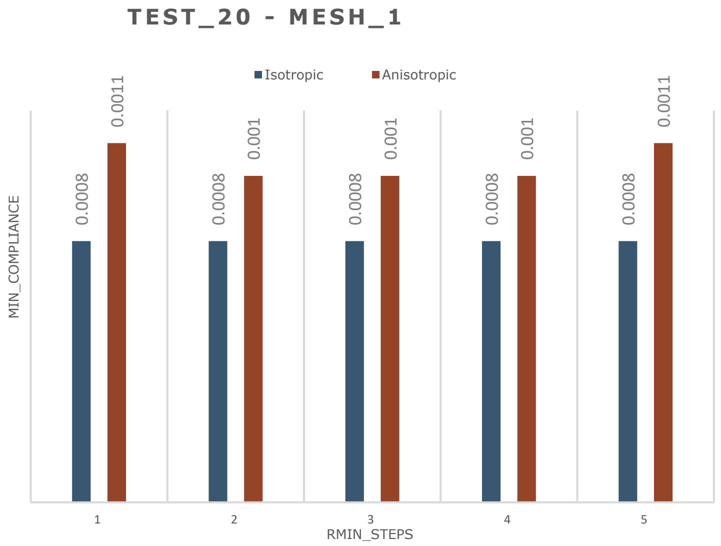


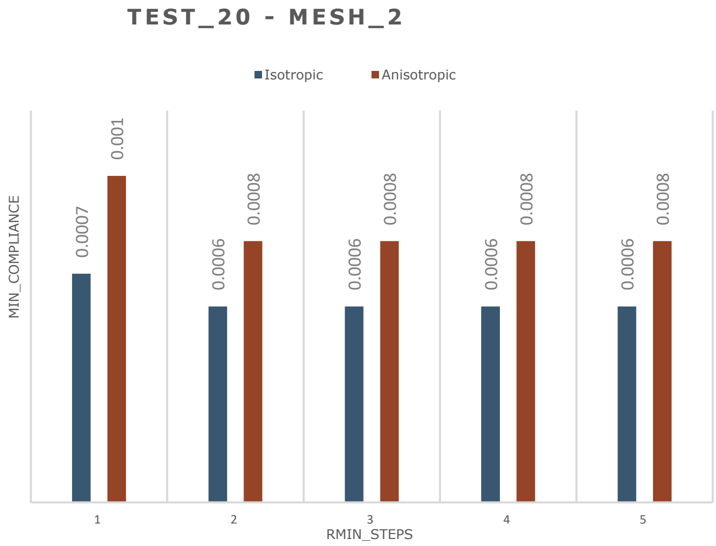


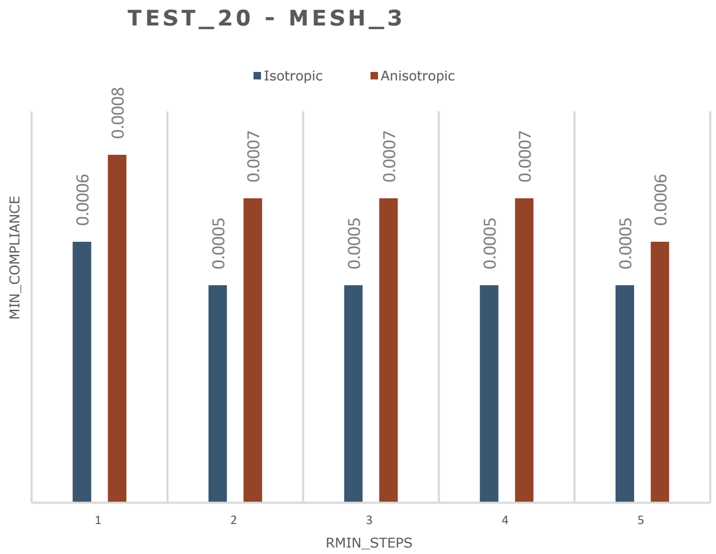
Isotropic

Testing & Results

Test_20 **a = 0.75** Rmin = 2.0 tolx = 0.01 penal 3.0	Mesh_1 210x30x48	Mesh_2 210x30x60	Mesh_3 210x30x80
Rmin = 1,5 (01 - 03)			
Min_Compliance_Isotropic	0.0008	0.0007	0.0006
Min_Compliance_Transverse	0.0011	0.001	0.0008
Ratio	**0.7273**	**0.7000**	**0.75**
Rmin = 2,5 (04 - 06)			
Min_Compliance_Isotropic	0.0008	0.0006	0.0005
Min_Compliance_Transverse	0.001	0.0008	0.0007
Ratio	**0.8000**	**0.7500**	**0.7143**
Rmin = 3,0 (07 - 09)			
Min_Compliance_Isotropic	0.0008	0.0006	0.0005
Min_Compliance_Transverse	0.001	0.0008	0.0007
Ratio	**0.8000**	**0.7500**	**0.7143**
Rmin = 3,5 (10 - 12)			
Min_Compliance_Isotropic	0.0008	0.0006	0.0005
Min_Compliance_Transverse	0.001	0.0008	0.0007
Ratio	**0.8000**	**0.7500**	**0.7143**
Rmin = 4,0 (13 - 15)			
Min_Compliance_Isotropic	0.0008	0.0006	0.0005
Min_Compliance_Transverse	0.0011	0.0008	0.0006
Ratio	**0.7273**	**0.7500**	**0.8333**
Mean	**0.7709**	**0.7400**	**0.7452**
SD	0.036	0.020	0.046
CV	0.046	0.027	0.062

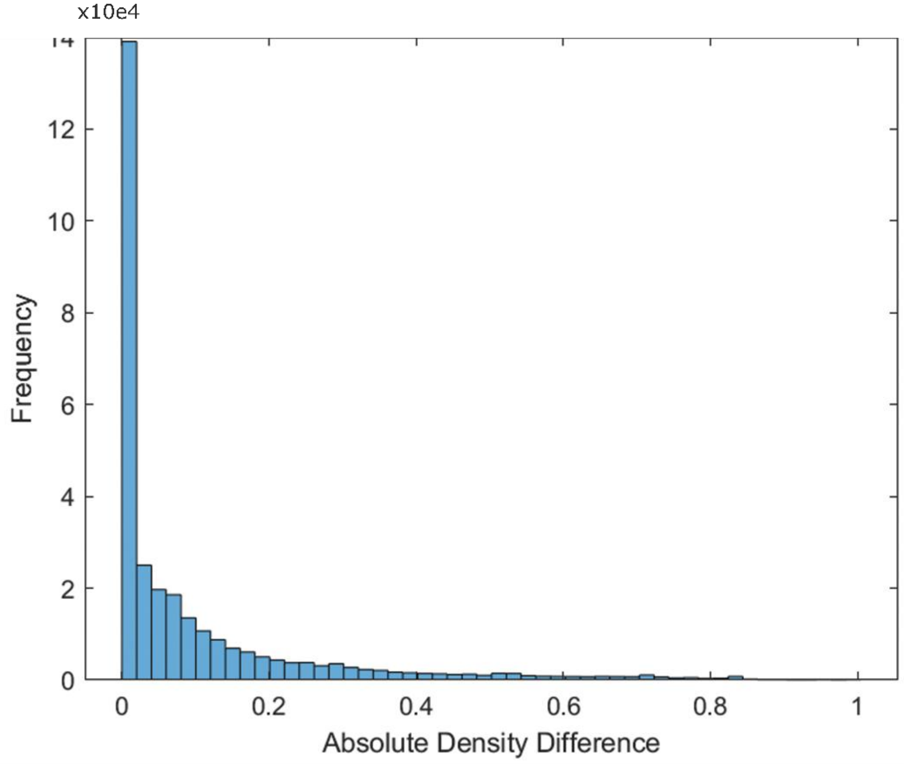




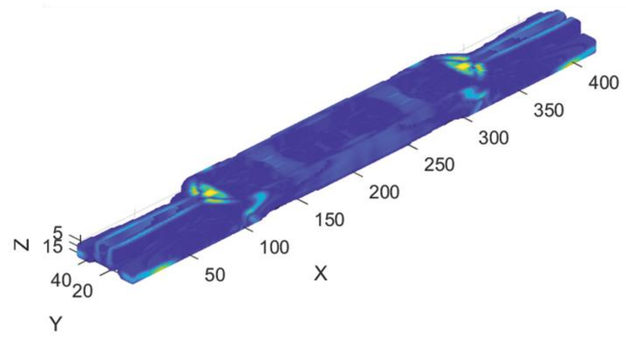


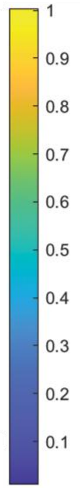
Testing & Results

Histogram - Similarity 40.21%

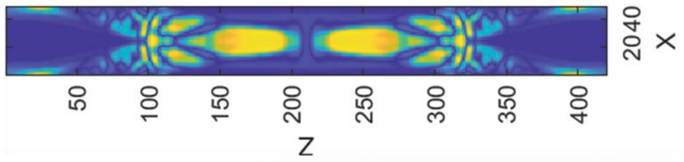


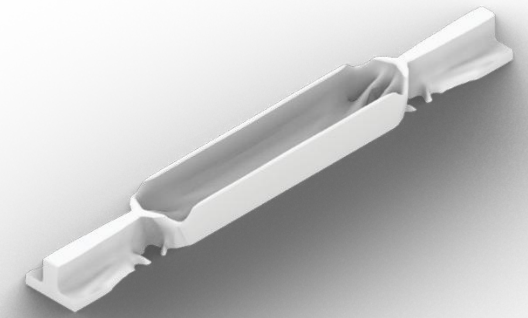
Isometric View



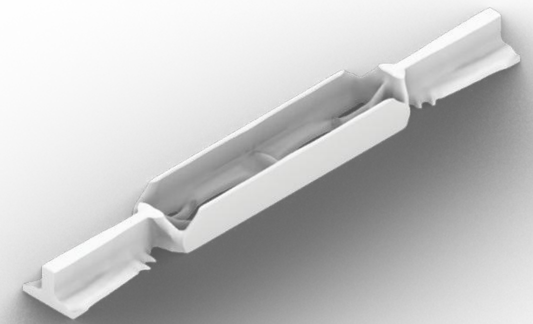


Section (max difference)





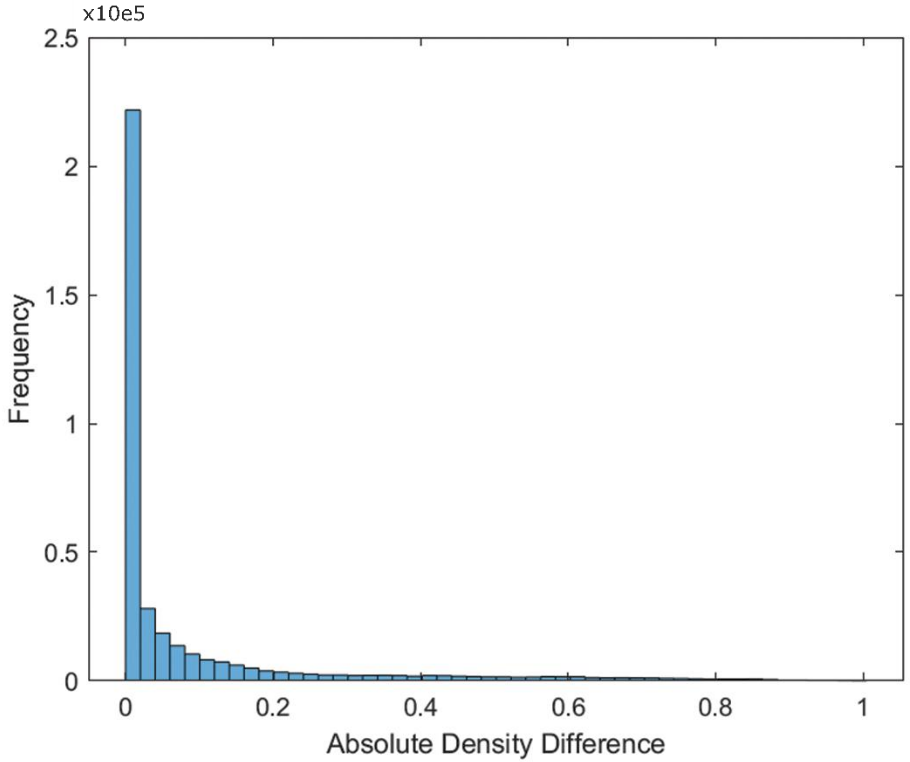
Isotropic



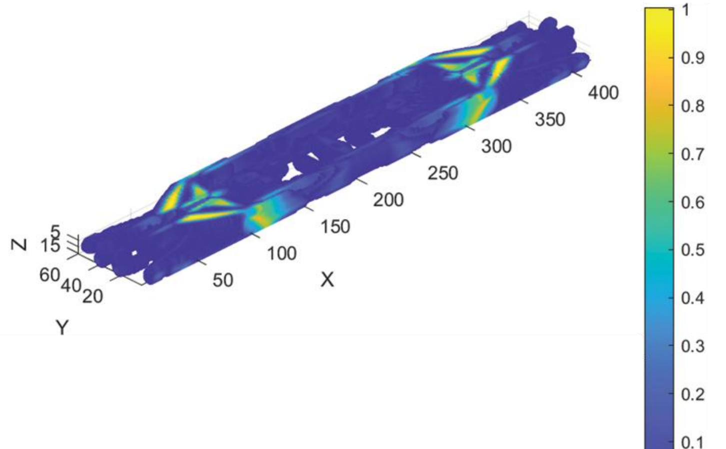
Transversely Isotropic

Testing & Results

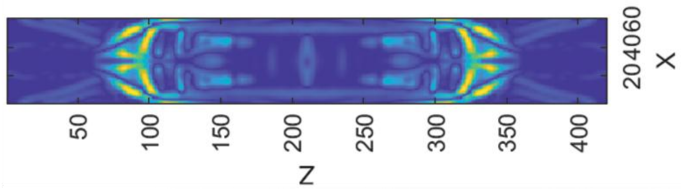
Histogram - Similarity 53.30%

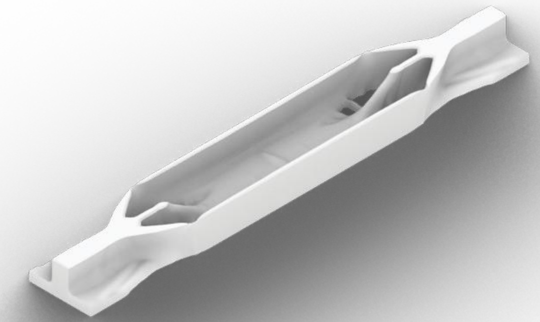


Isometric View

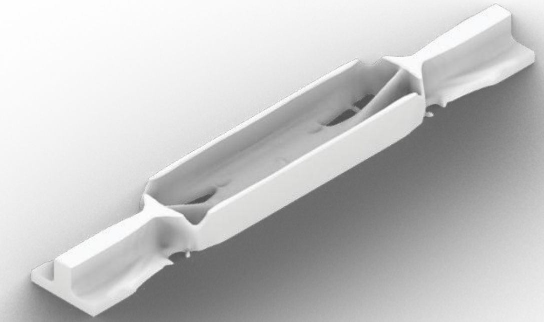


Section (max difference)





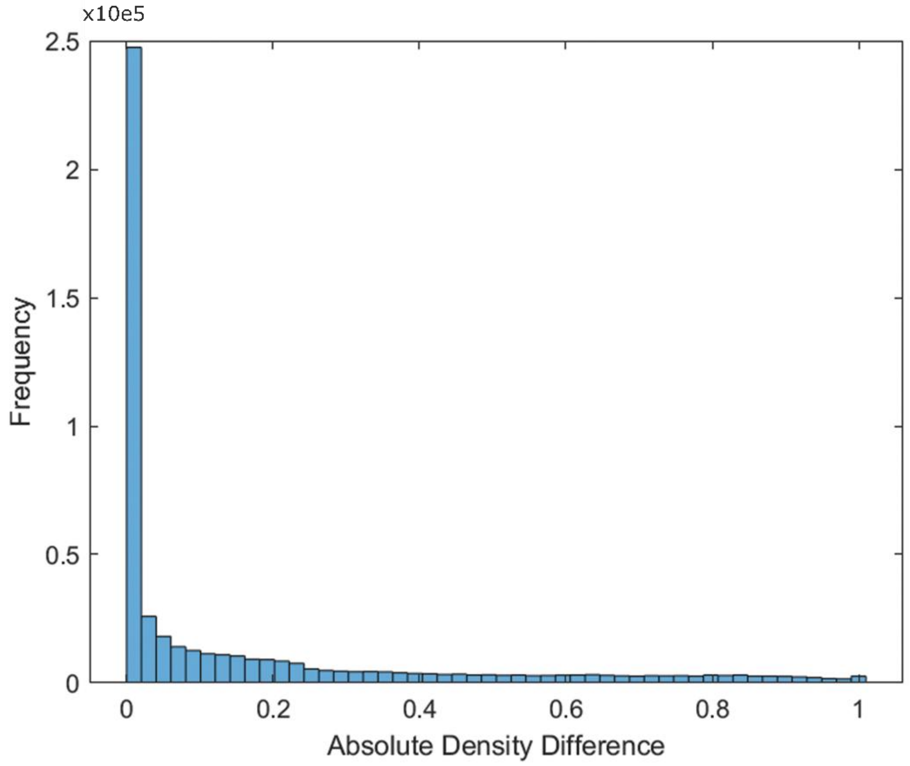
Isotropic



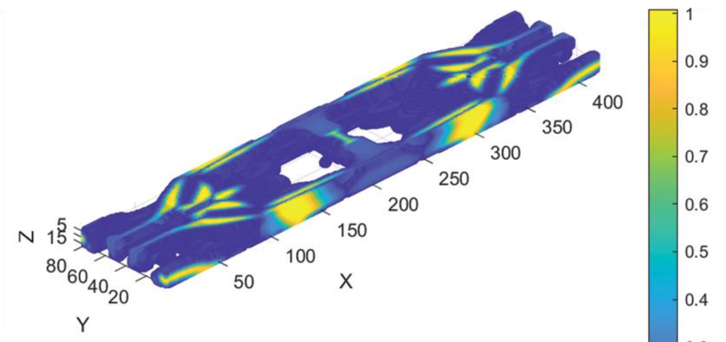
Transversely Isotropic

Testing & Results

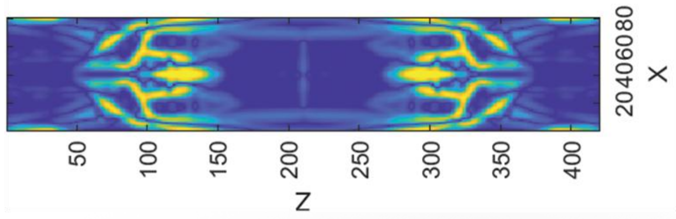
Histogram - Similarity 45.31%

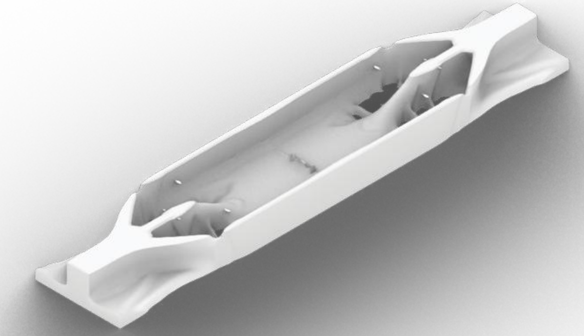


Isometric View

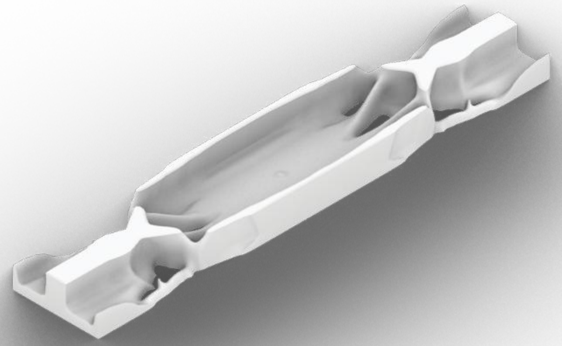


Section (max difference)



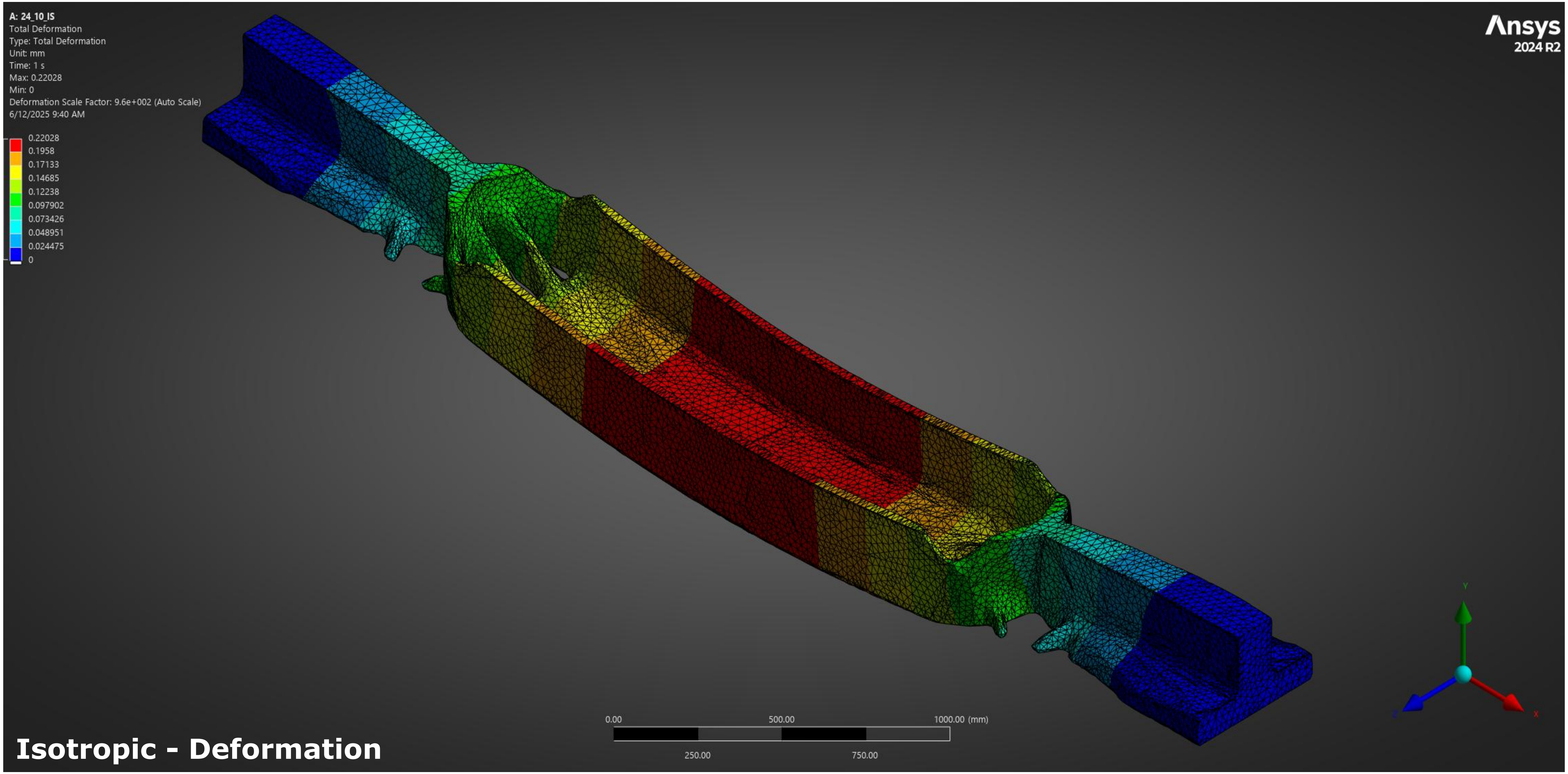


Isotropic

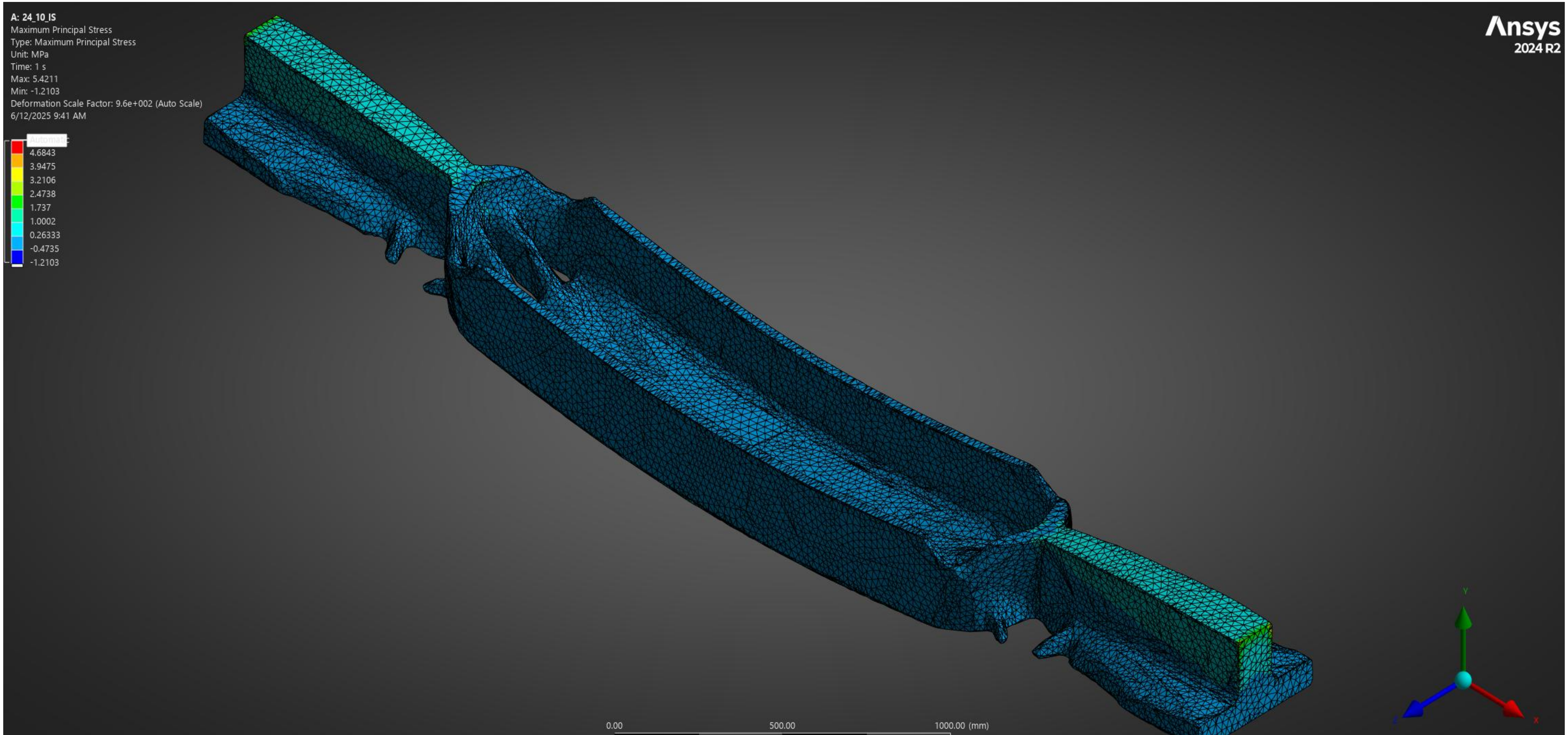


Transversely Isotropic

Testing & Results



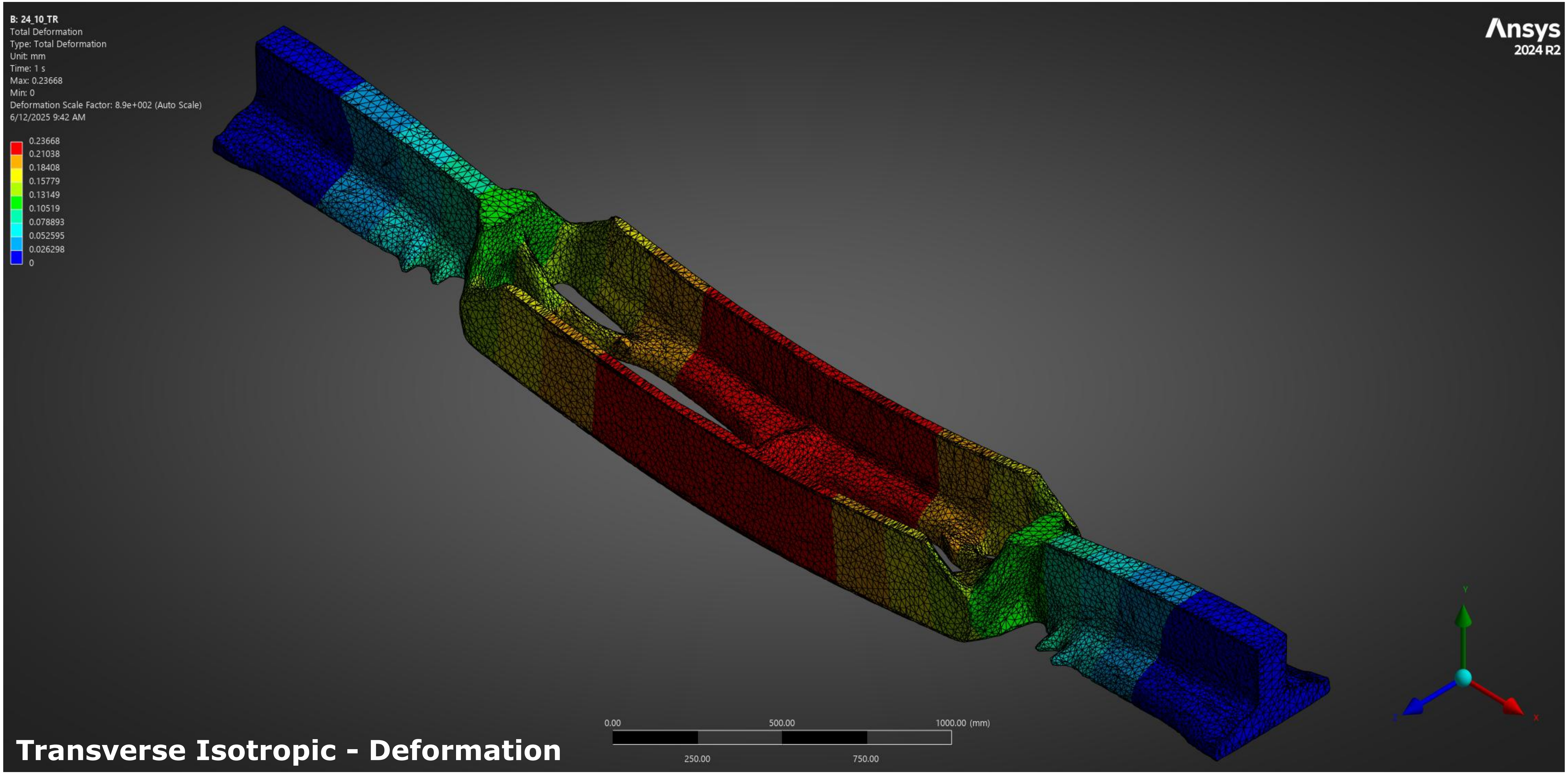
Testing & Results



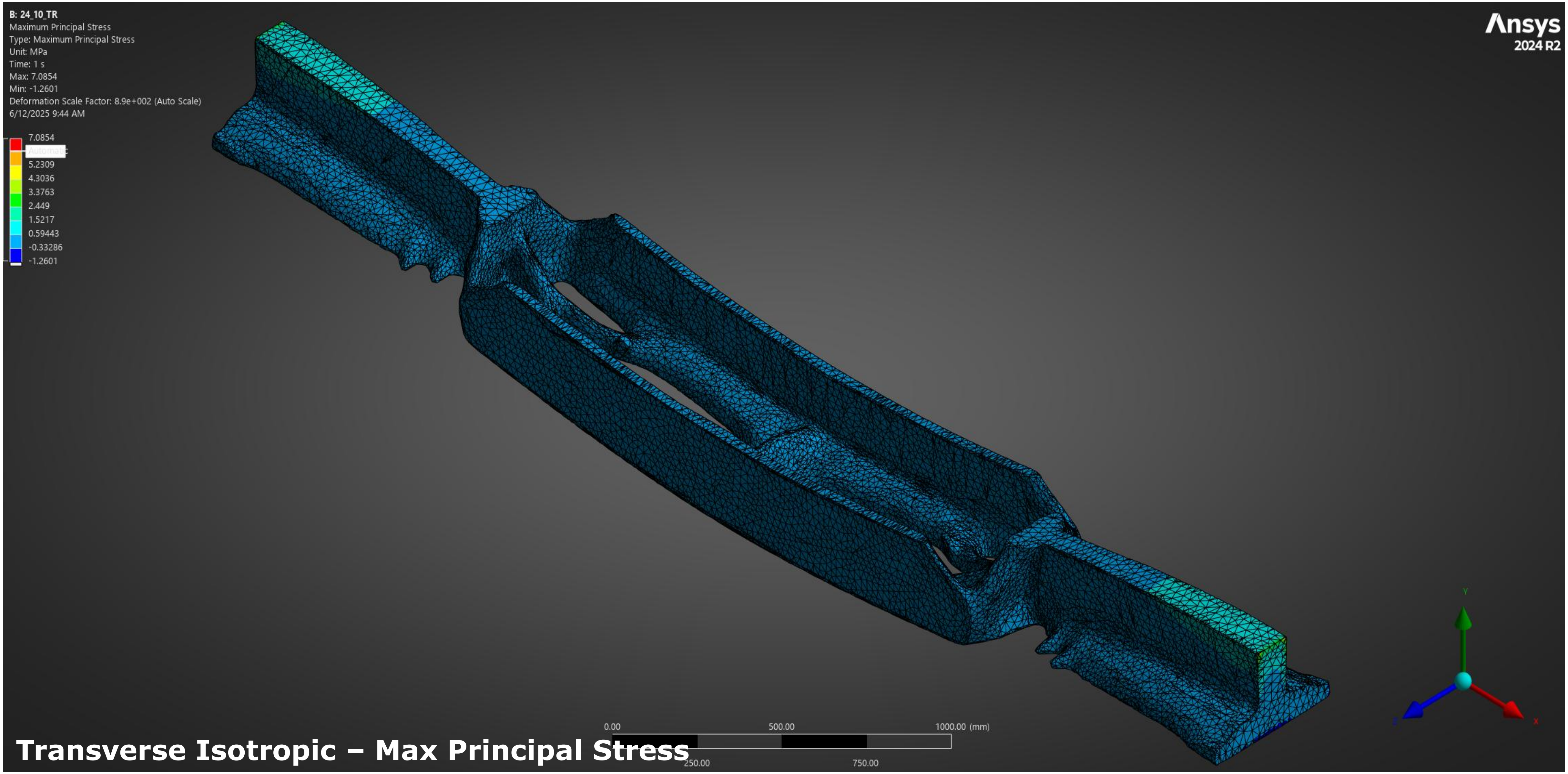
Isotropic – Max Principal Stress

Testing & Results

Testing & Results



Testing & Results



Transverse Isotropic – Max Principal Stress

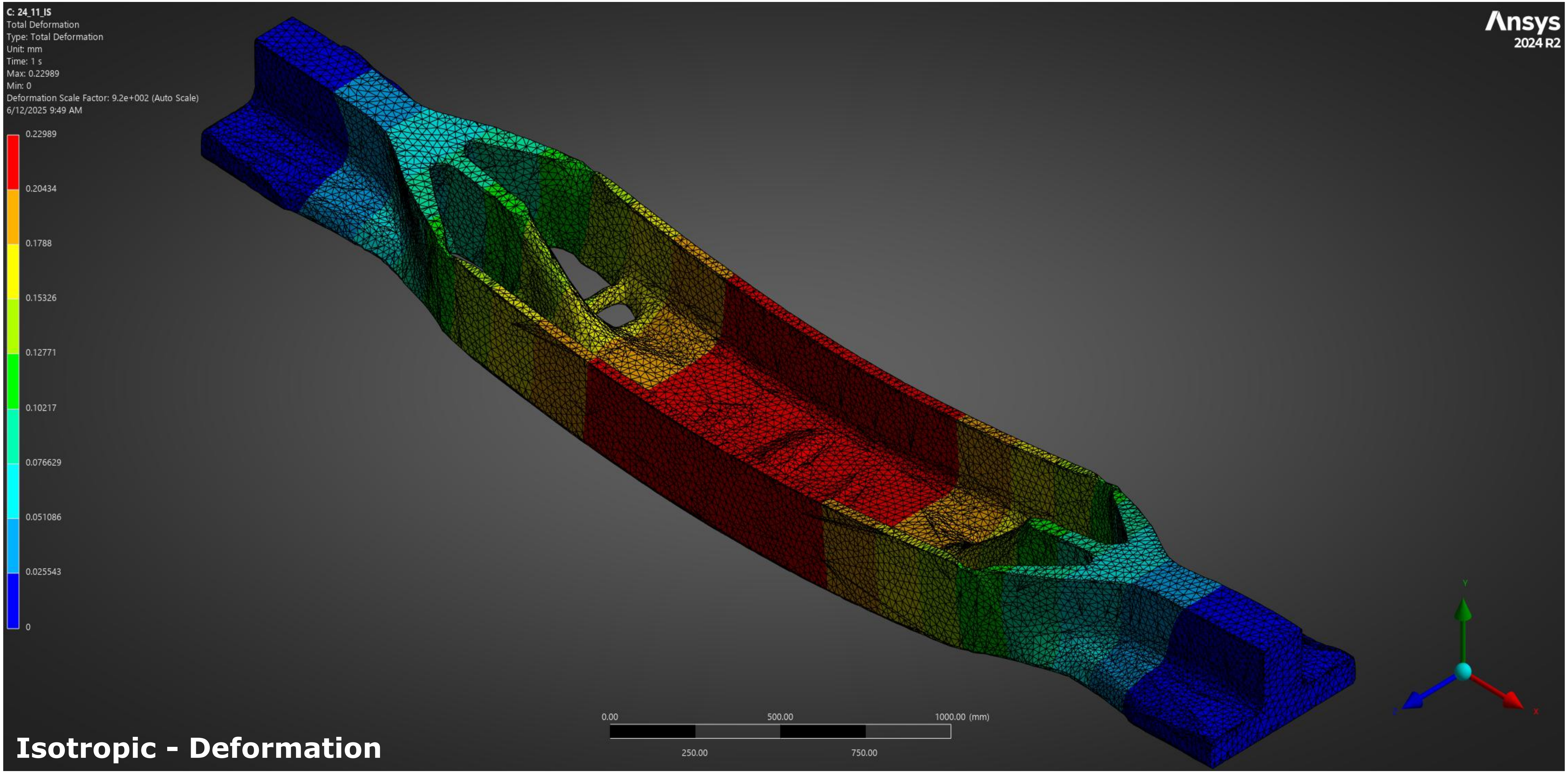
Testing & Results

Transverse Isotropic– Min Principal Stress

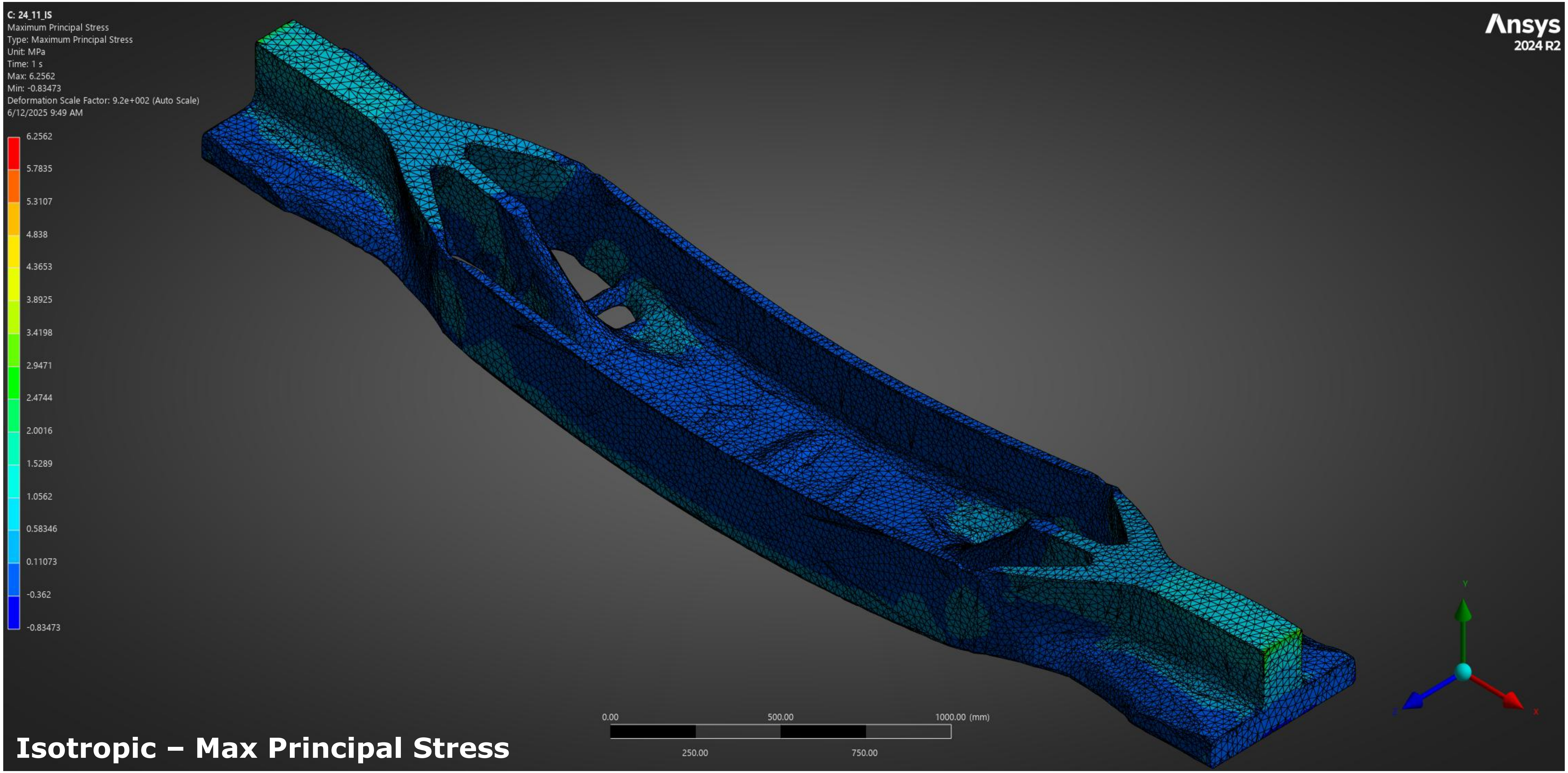
Testing & Results

Specimen	Deformation		Iso/Tr Δ	Max. Princ.		Iso/Tr Δ	Min Princ.		Iso/Tr Δ
	min	max		min	max		min	max	
	mm			Mpa			Mpa		
24_10_IS	0	0.2203	**-0.0164**	-1.210	5.421	**-1.664**	-3.738	1.128	**-0.103**
24_10_TR	0	0.2367		-1.260	7.085		-3.815	1.230	

Testing & Results



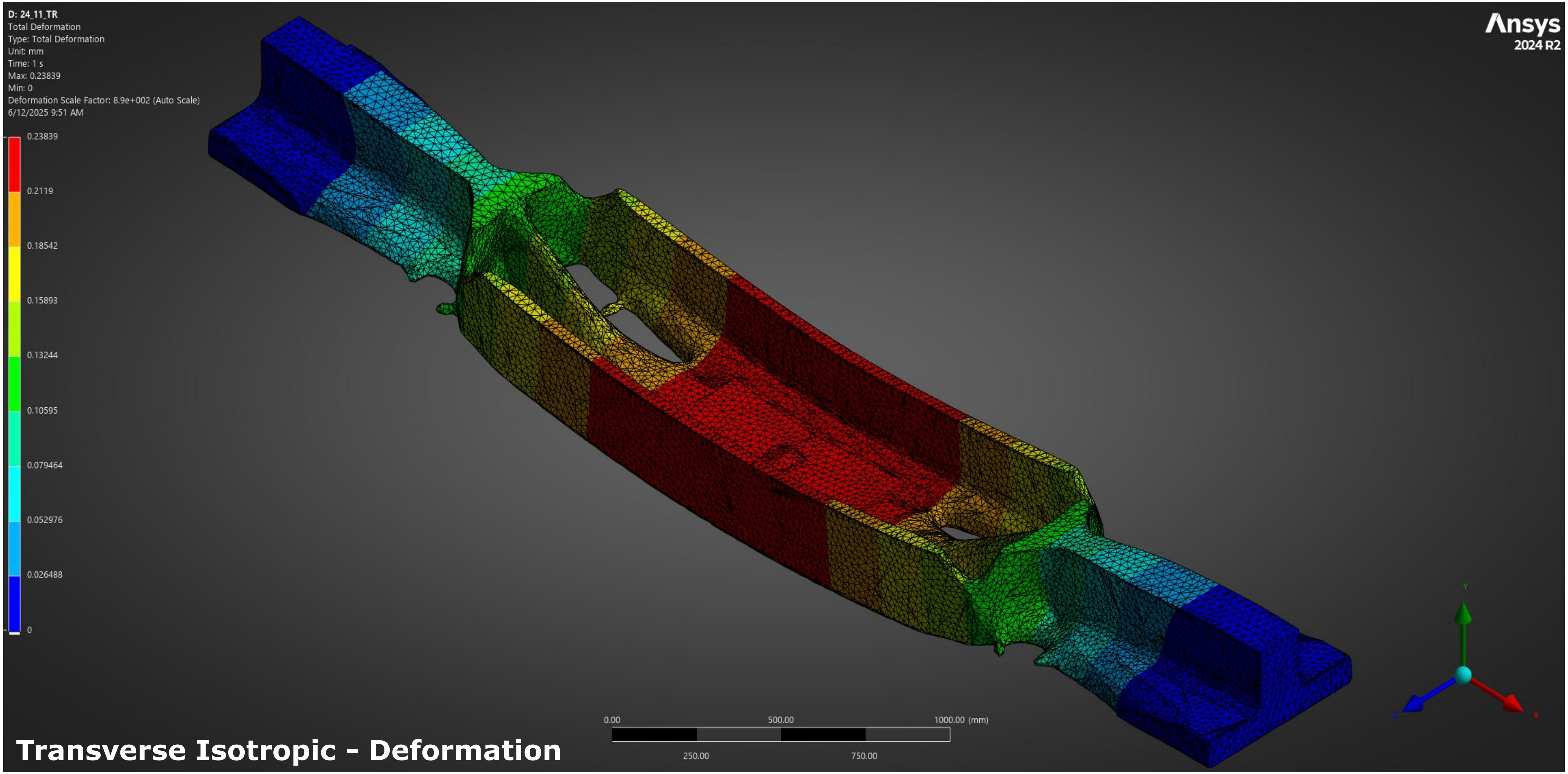
Testing & Results



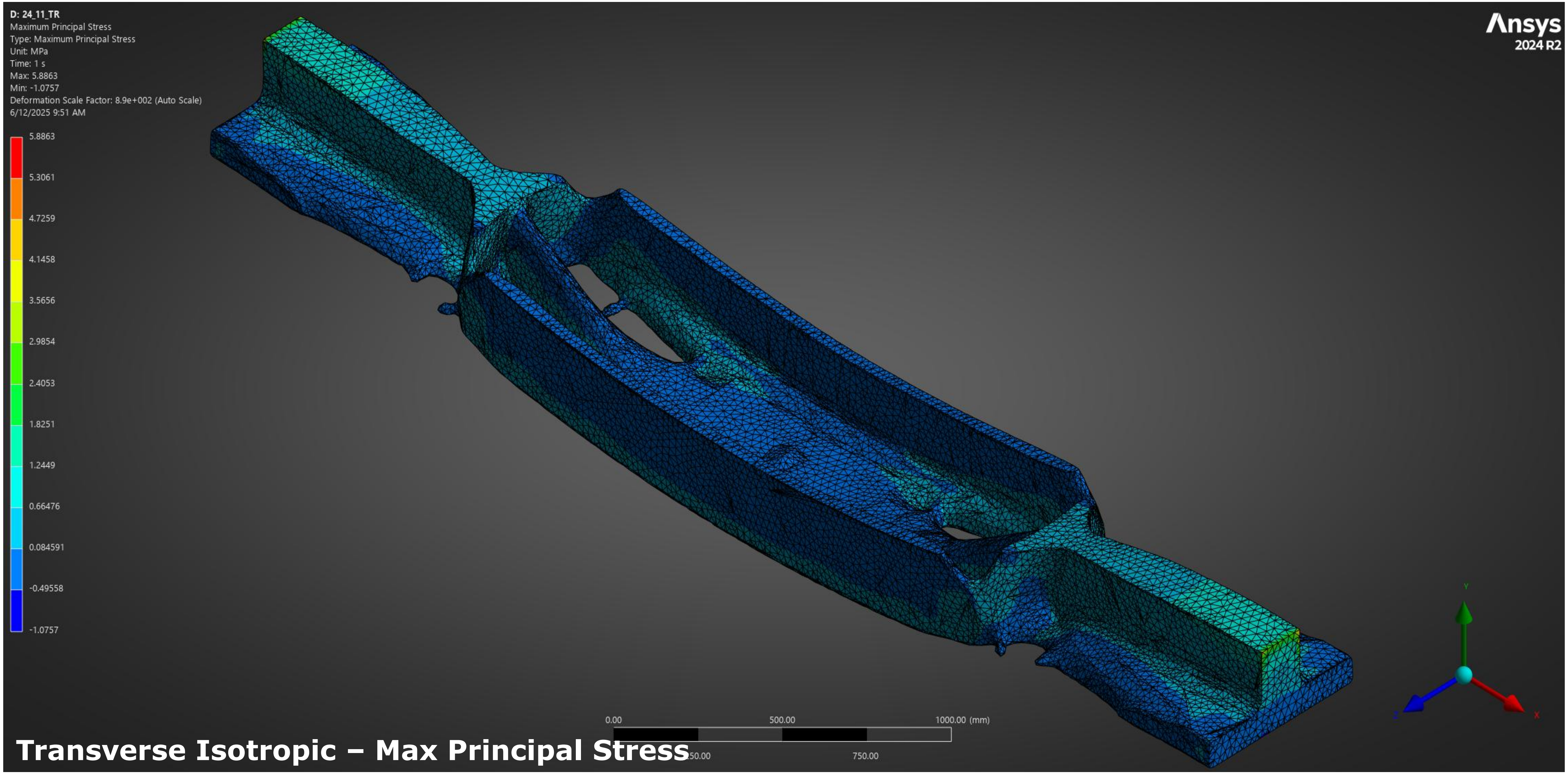
Isotropic – Max Principal Stress

Testing & Results

Testing & Results

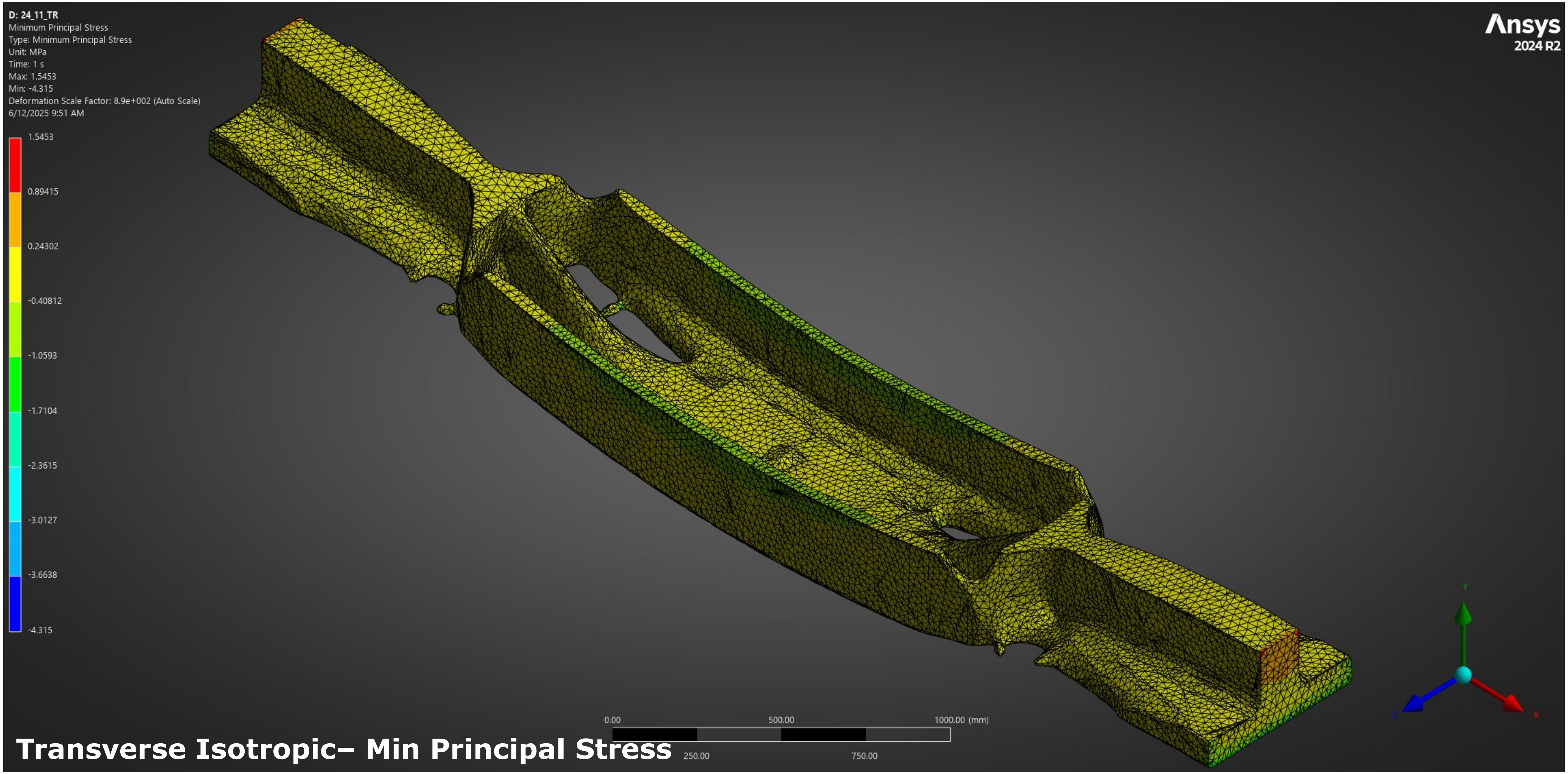


Testing & Results



Transverse Isotropic – Max Principal Stress

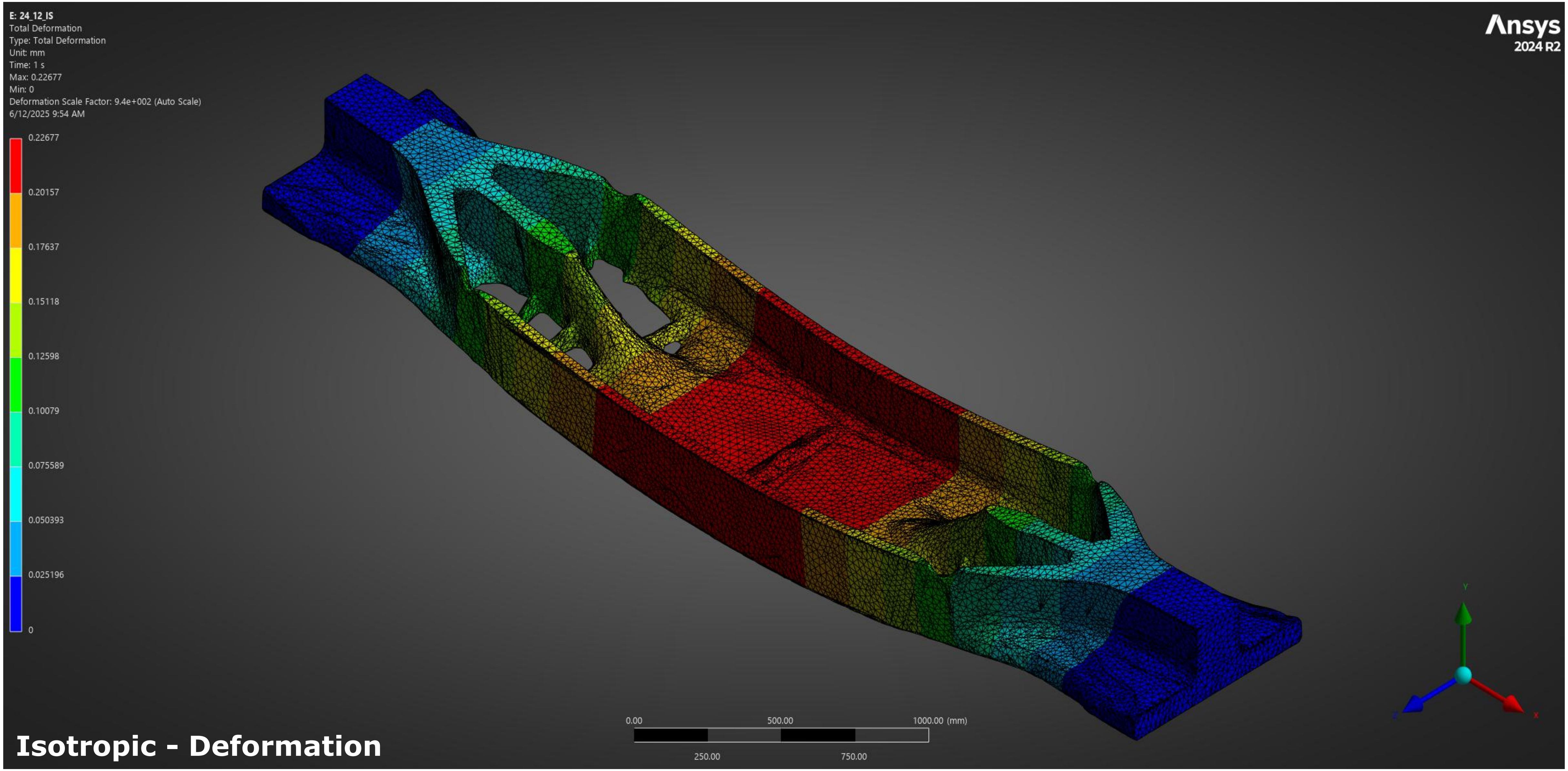
Testing & Results



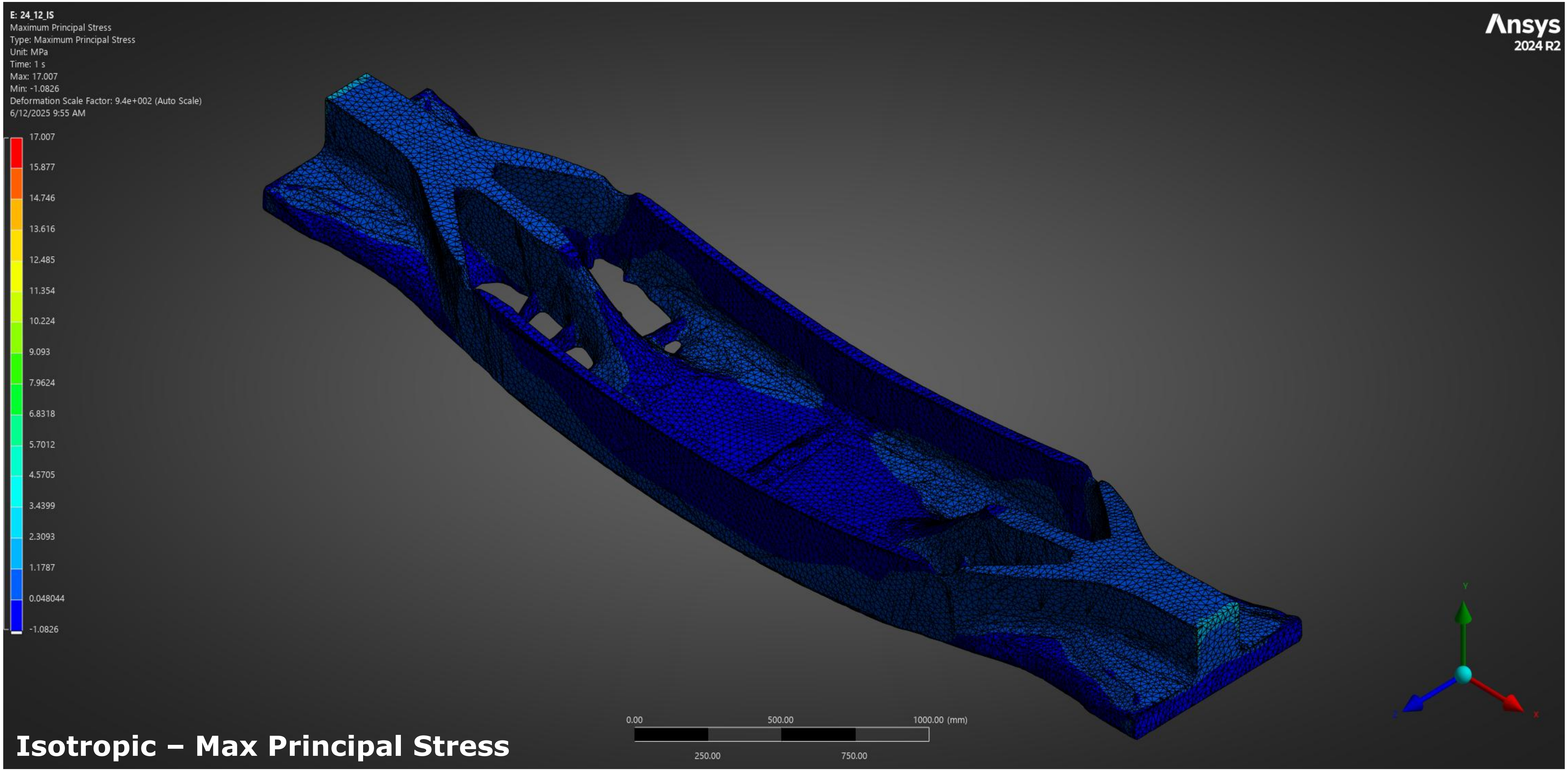
Testing & Results

Specimen	Deformation		Iso/Tr Δ	Max. Princ.		Iso/Tr Δ	Min Princ.		Iso/Tr Δ
	min	max		min	max		min	max	
	mm			Mpa			Mpa		
24_11_IS	0	0.2299	**-0.0085**	-0.835	6.256	**0.370**	-2.877	1.843	**0.298**
24_11_TR	0	0.2384		-1.076	5.886		-4.315	1.545	

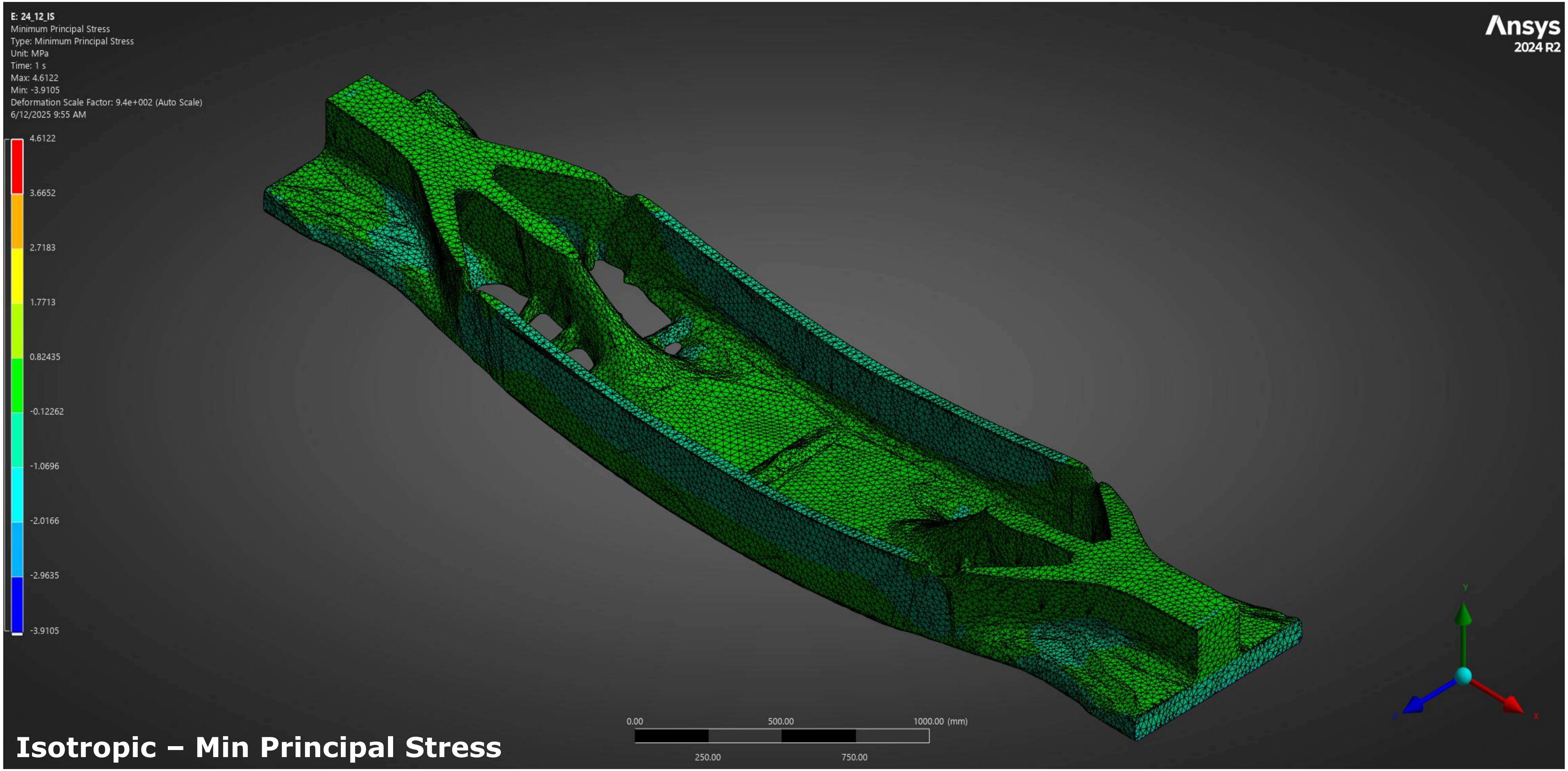
Testing & Results



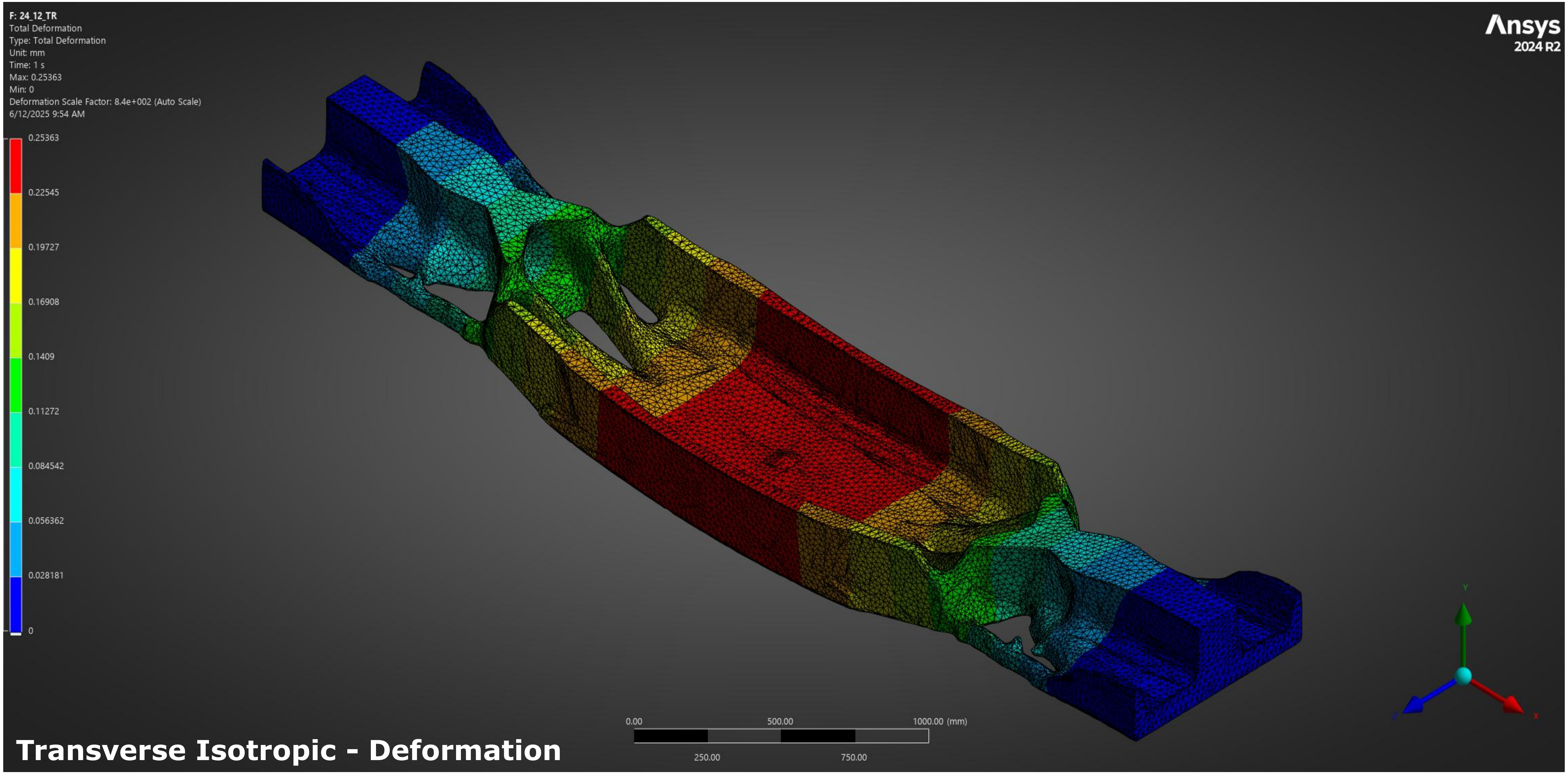
Testing & Results



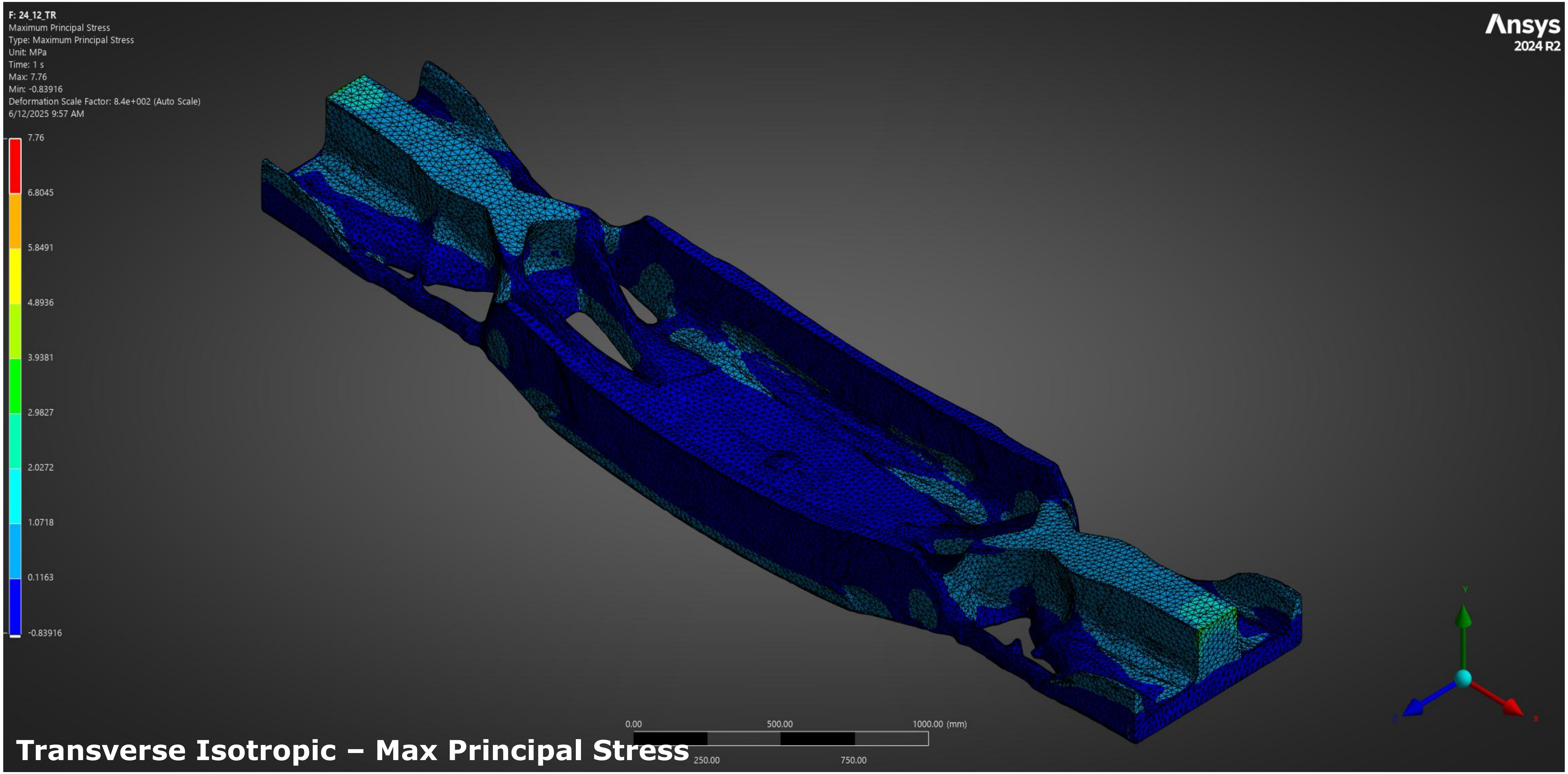
Testing & Results



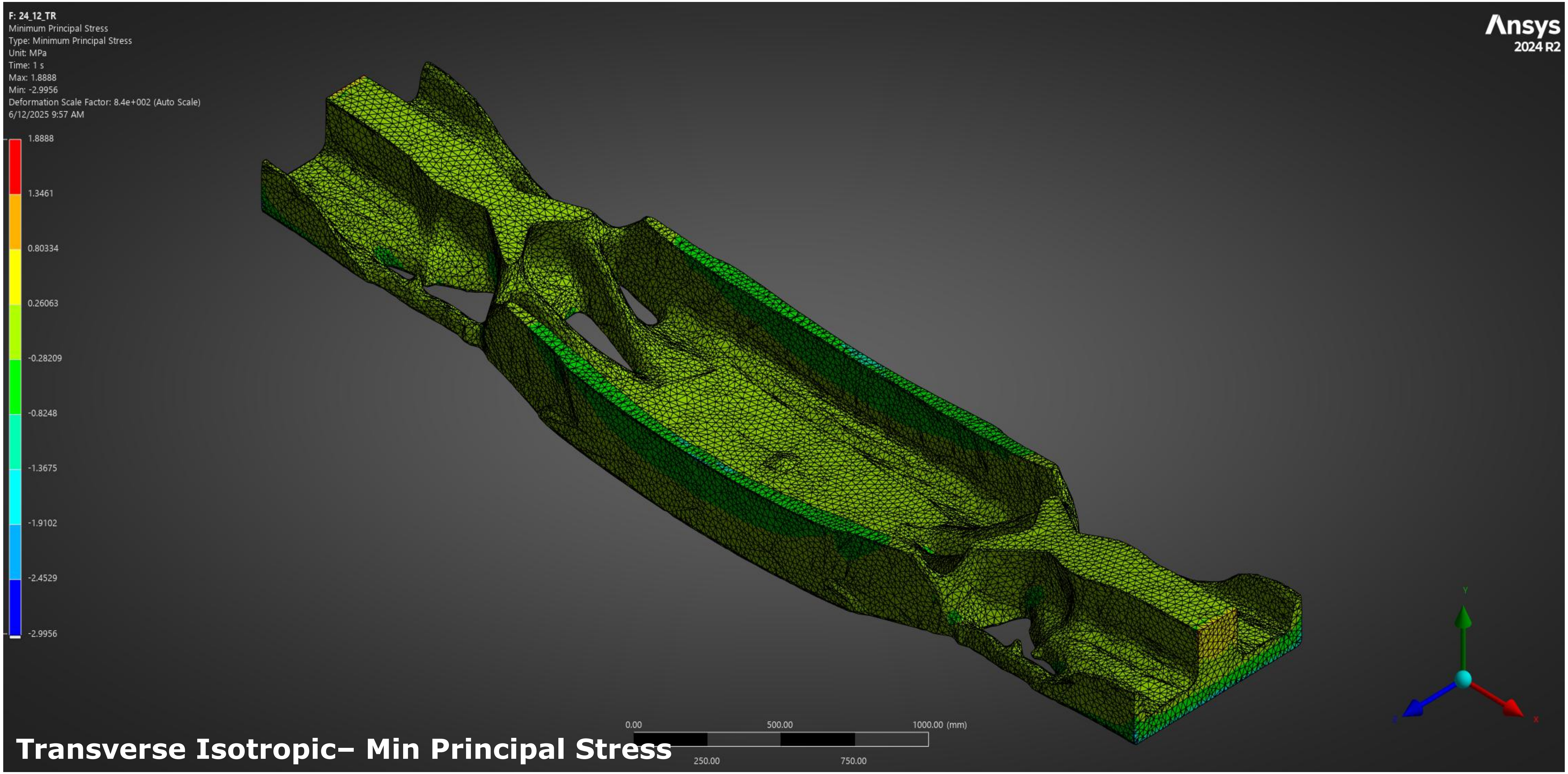
Testing & Results



Testing & Results



Testing & Results



Transverse Isotropic– Min Principal Stress

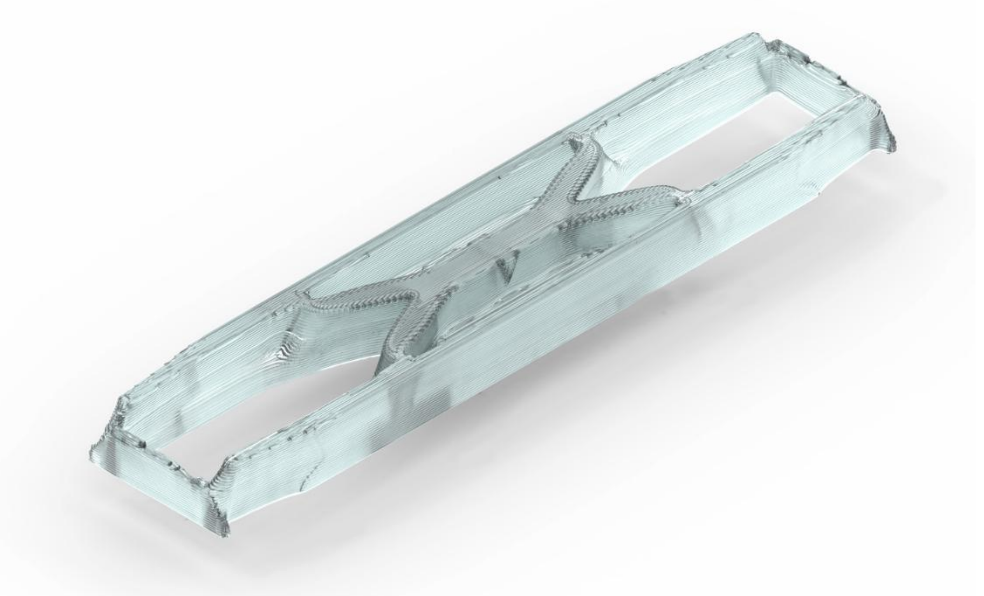
Testing & Results

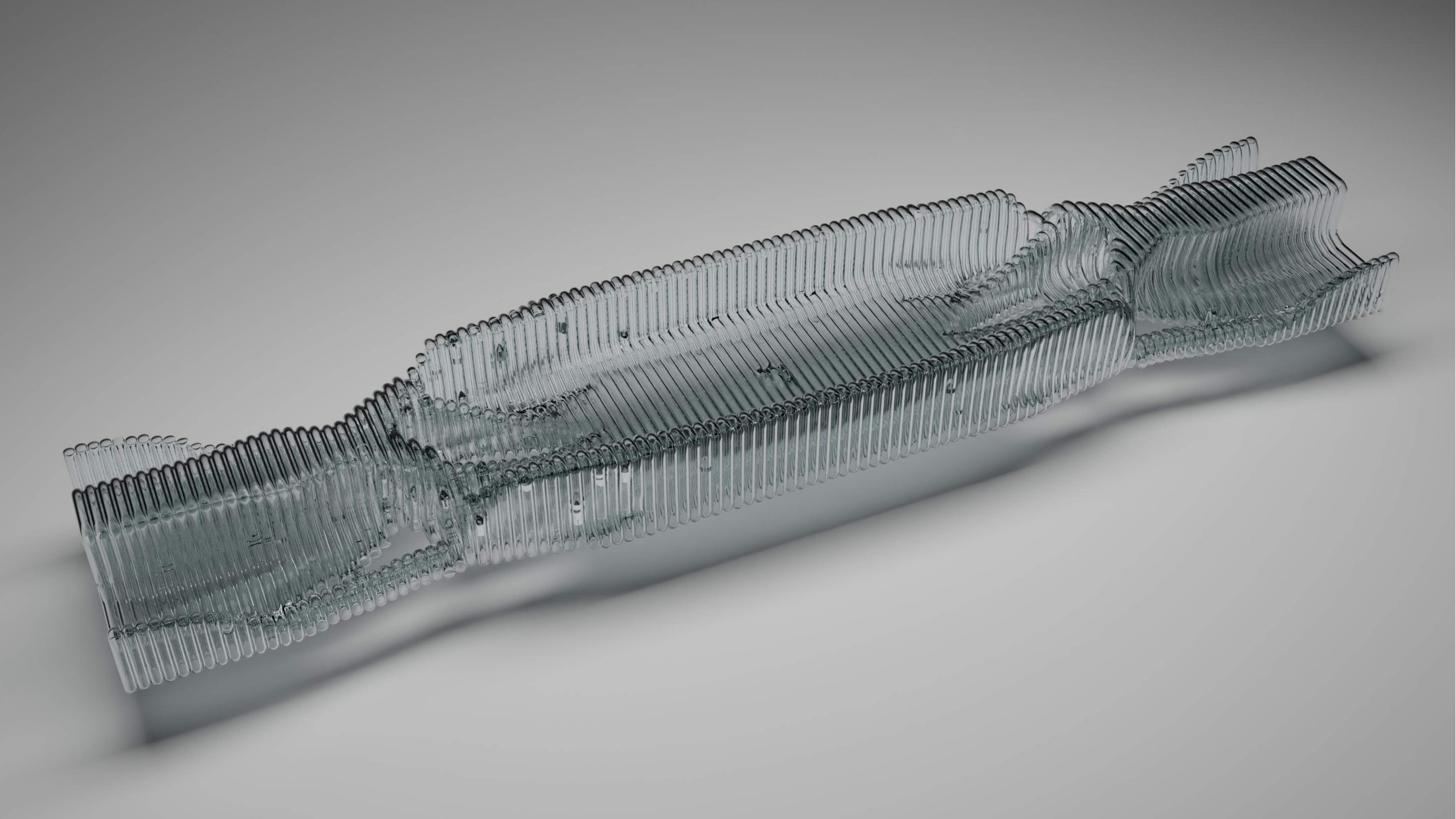
Specimen	Deformation		Iso/Tr Δ	Max. Princ.		Iso/Tr Δ	Min Princ.		Iso/Tr Δ
	min	max		min	max		min	max	
	mm			Mpa			Mpa		
24_12_IS	0	0.2268	**-0.0269**	-1.083	17.007	**9.247**	-3.911	4.612	**2.723**
24_12_TR	0	0.2536		-0.839	7.760		-2.996	1.889	

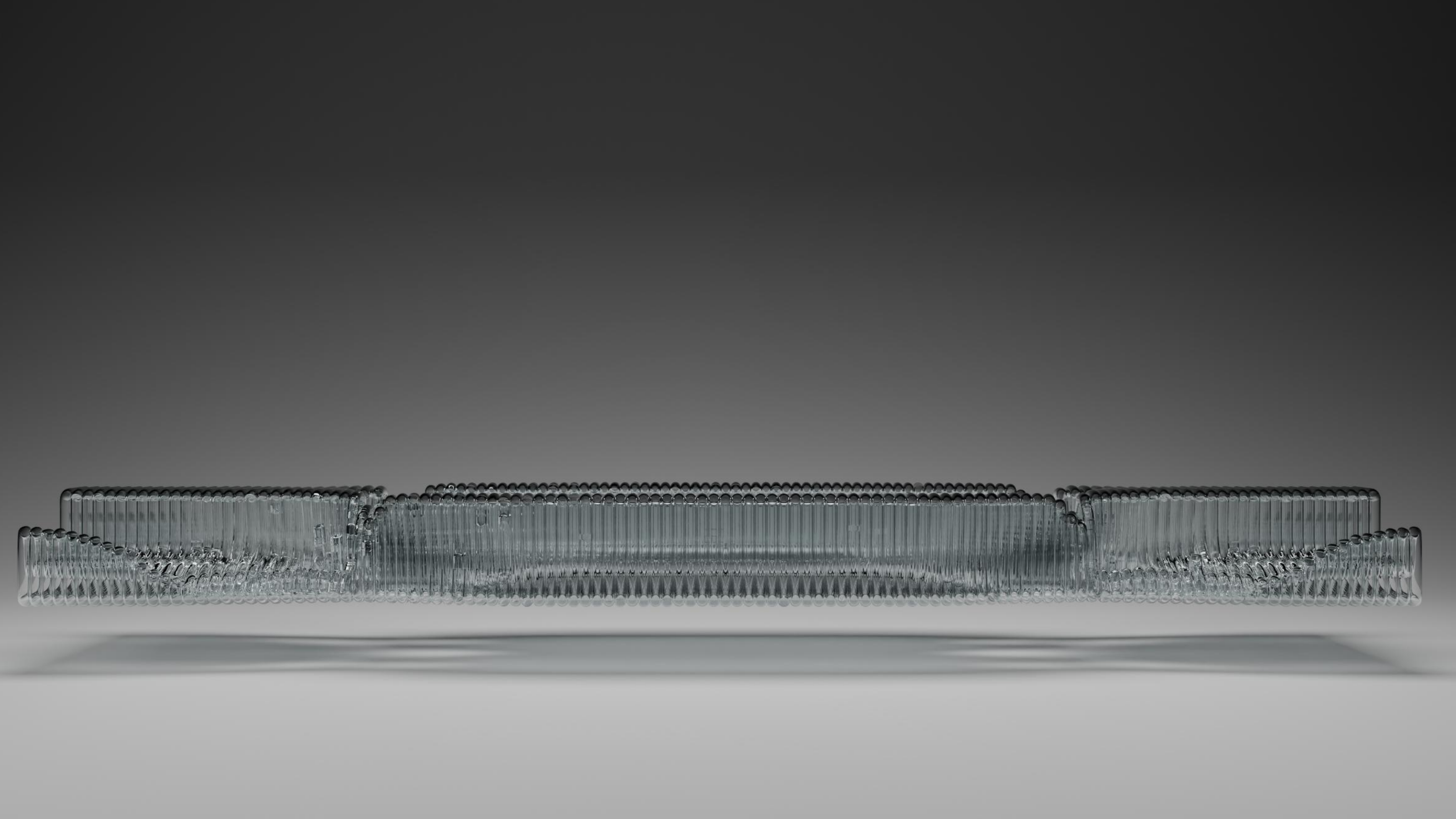
Testing & Results

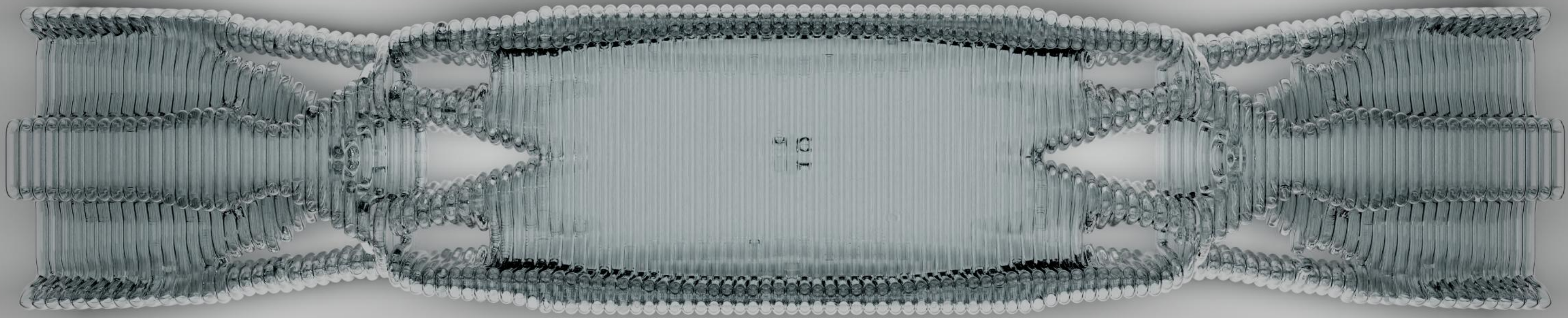
Specimen	Deformation min	Deformation max	Iso/Tr Δ	Max. Princ. min	Max. Princ. max	Iso/Tr Δ	Min Princ. min	Min Princ. max	Iso/Tr Δ
	mm	mm		Mpa	Mpa		Mpa	Mpa	
24_10_IS	0	0.2203	**-0.0164**	-1.210	5.421	**-1.664**	-3.738	1.128	**-0.103**
24_10_TR	0	0.2367		-1.260	7.085		-3.815	1.230	
24_11_IS	0	0.2299	**-0.0085**	-0.835	6.256	**0.370**	-2.877	1.843	**0.298**
24_11_TR	0	0.2384		-1.076	5.886		-4.315	1.545	
24_12_IS	0	0.2268	**-0.0269**	-1.083	17.007	**9.247**	-3.911	4.612	**2.723**
24_12_TR	0	0.2536		-0.839	7.760		-2.996	1.889	

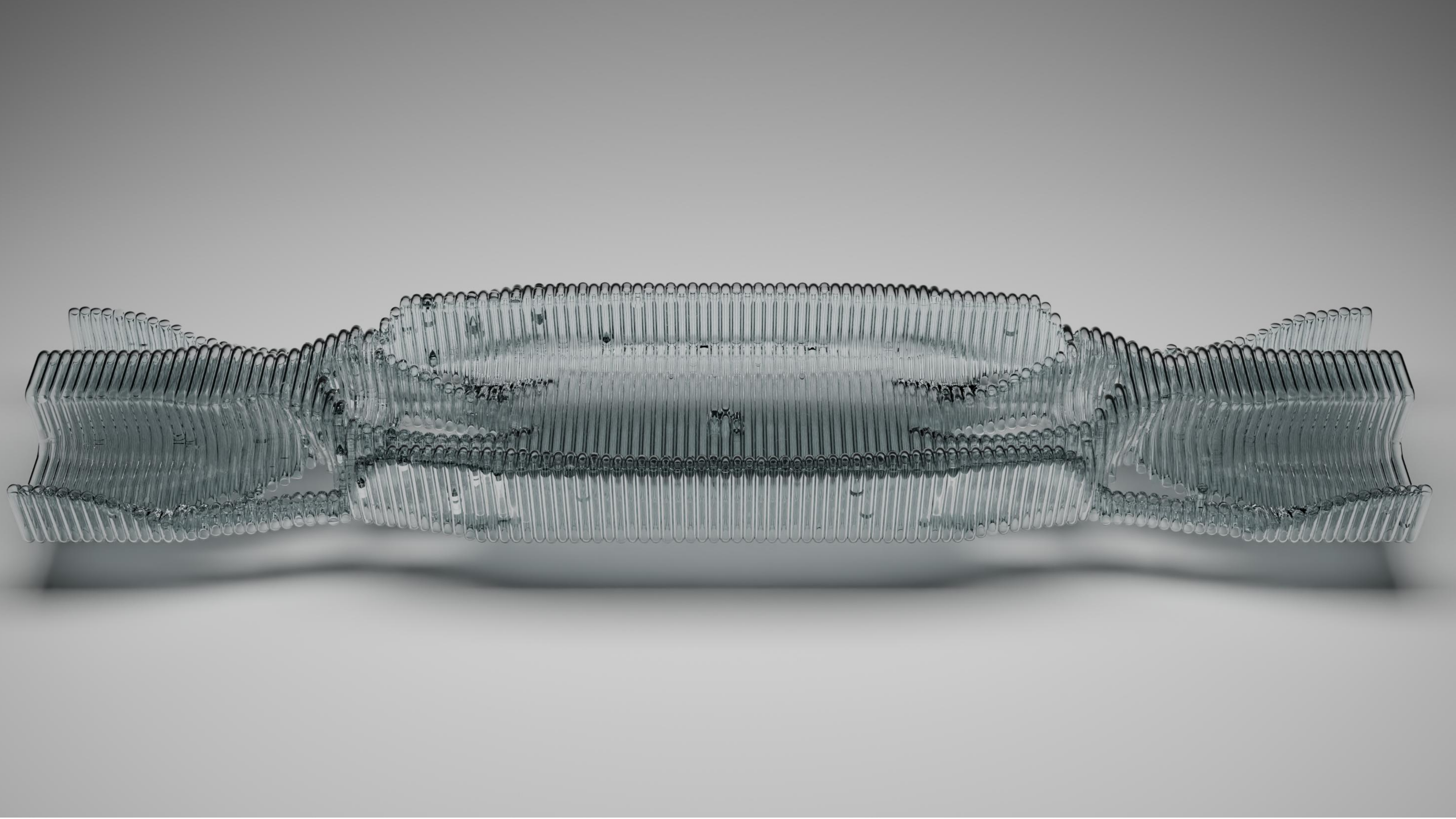
Researcher	Deformation	Max Principal	Min Principal
	mm	Mpa	Mpa
Koniari	0.120	2.960	-5.560
Schoenmaker	0.237	5.400	-4.900
Brueren	0.253	3.700	-4.800
Mananas_Isotropic	0.226	17.007	-3.911
Mananas_Anisotropic	0.253	7.760	-2.996

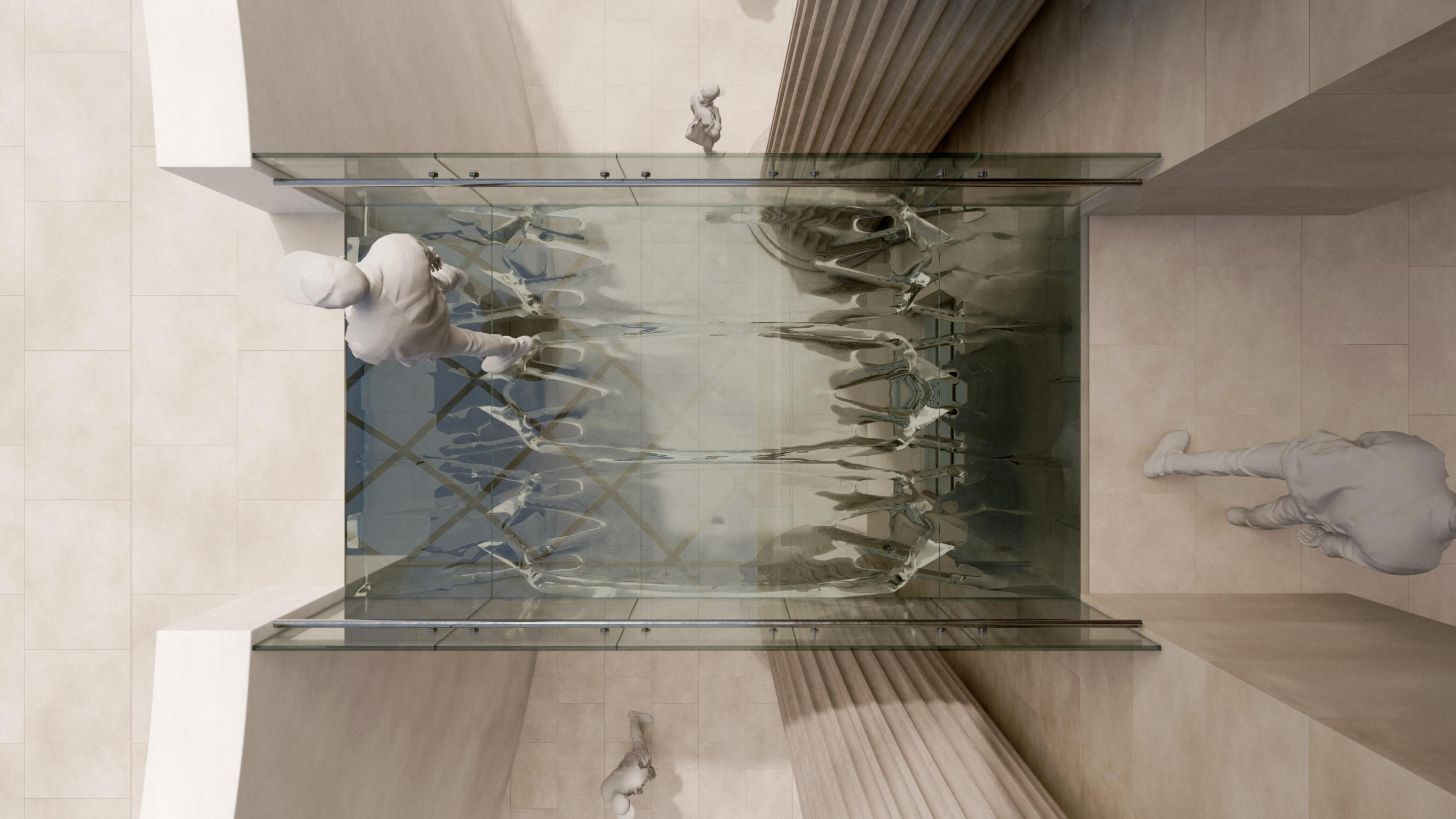




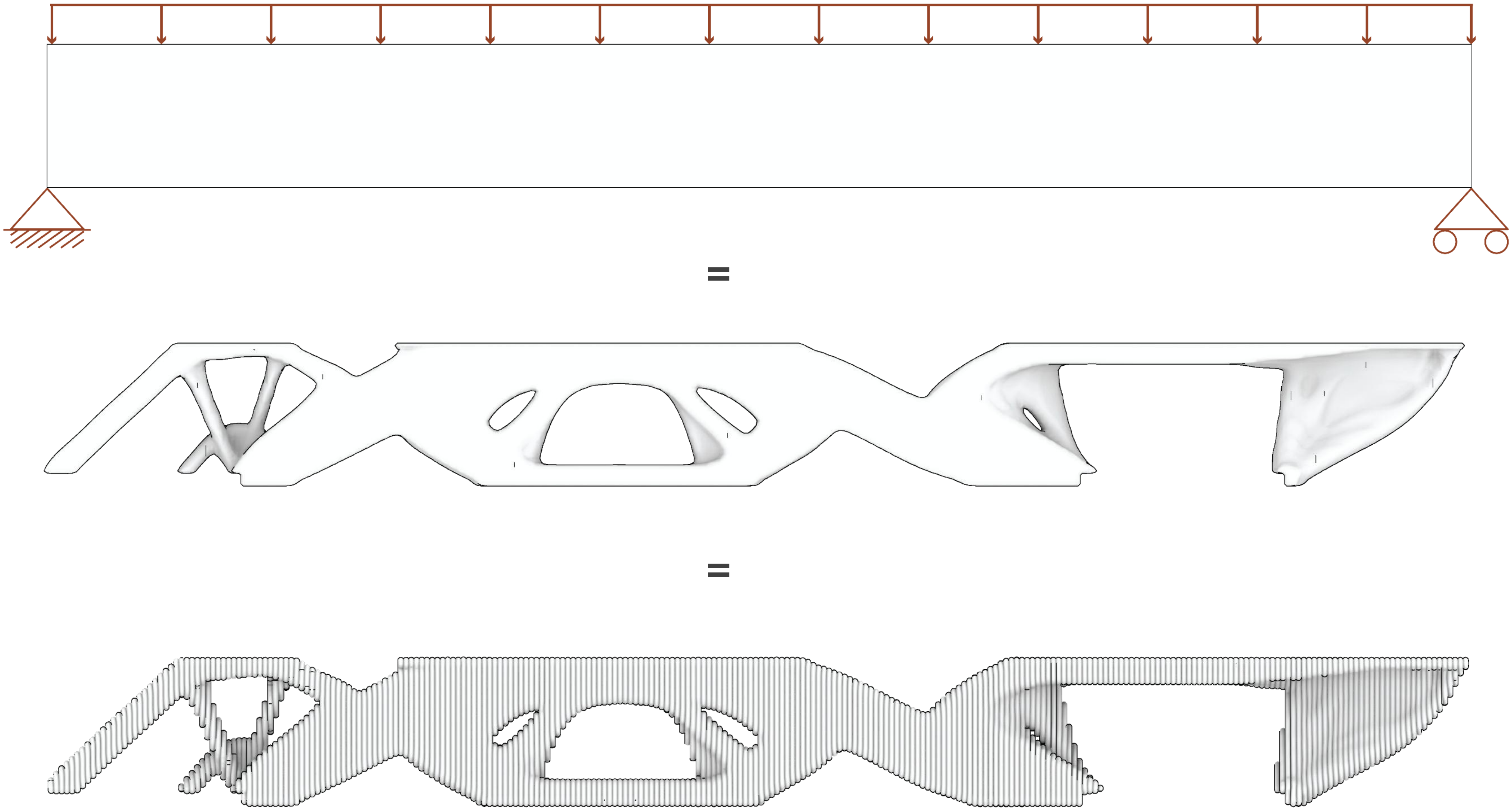


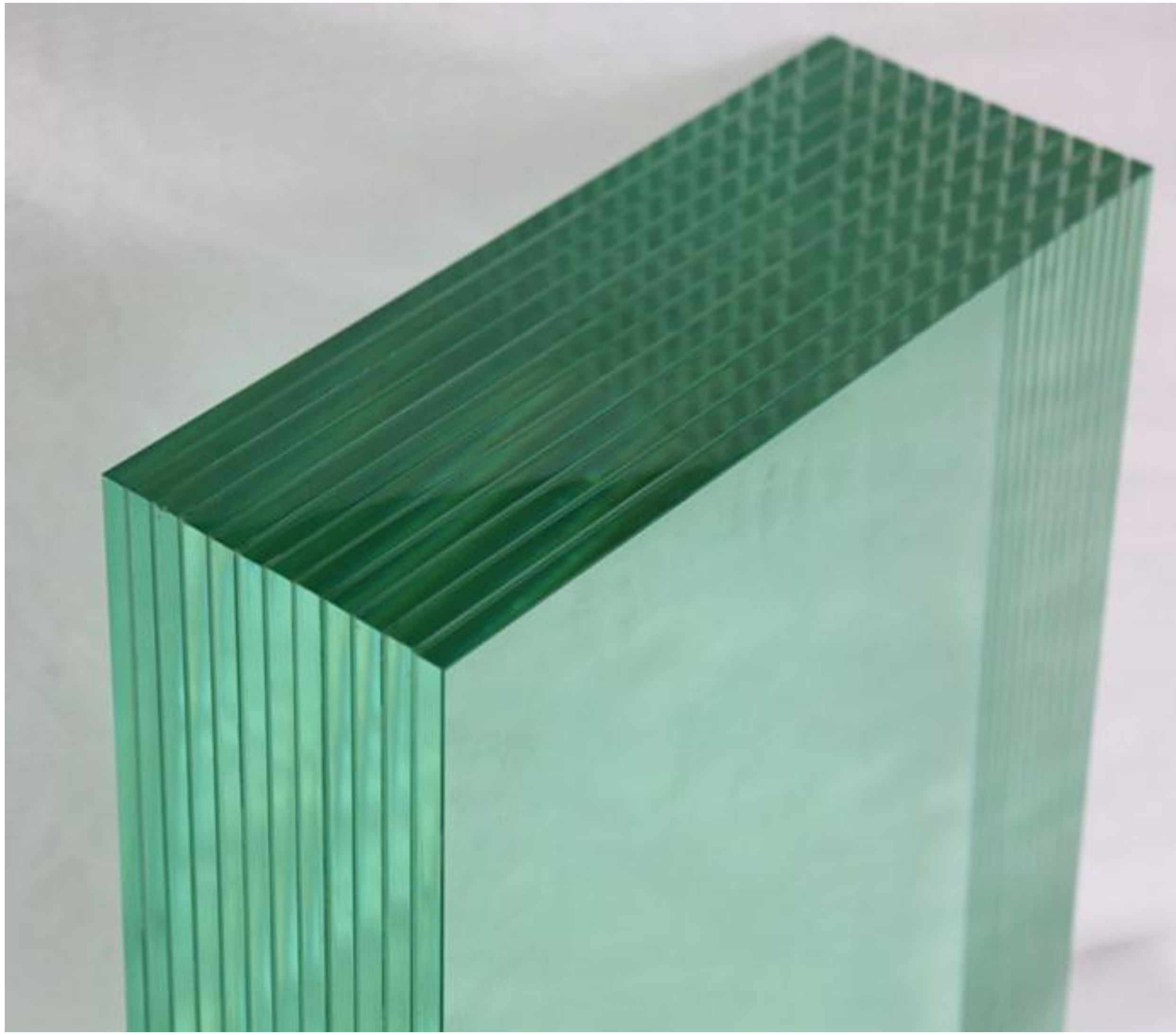






So...




Thank you! ☺