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Citation (APA)

Mishra, S., Singh, S., Das, S., & Lekić, A. (2026). Enhanced Dynamic Performance of Modular Multilevel Converters Using Exponential Cost Function-Based Model Predictive Control. *IEEE Access*, 14, 67967-67990. <https://doi.org/10.1109/ACCESS.2026.3689159>

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Received 9 April 2026, accepted 24 April 2026, date of publication 30 April 2026, date of current version 7 May 2026.

Digital Object Identifier 10.1109/ACCESS.2026.3689159

RESEARCH ARTICLE

Enhanced Dynamic Performance of Modular Multilevel Converters Using Exponential Cost Function-Based Model Predictive Control

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The financial support for this research was provided under the project “Smart and Flexible Control for a Power Electronics-based Electrical Grid (SAFE-GRID)”, Project No. 20248.

ABSTRACT This paper proposes the use of Model Predictive Control (MPC) with an exponential cost function for the Modular Multilevel Converter (MMC), which is widely recognized as a preferred converter topology for integrating and converting renewable energy sources into electrical energy. MPC provides a superior control strategy in the presence of system constraints, a straightforward control design, facilitates the inclusion of multiple control objectives through a flexible cost-function formulation, and offers excellent control performance. By formulating an appropriate cost function, an MMC’s operational goals can be effectively achieved through MPC. However, non-exponential MPC approaches typically employ a rectangular moving-horizon window whose length matches the chosen prediction horizon, which can affect closed-loop stability. The results based on the non-exponential cost further reveal that the choice of prediction horizon notably influences the numerical conditioning of MPC algorithms. In particular, as the prediction horizon lengthens, the numerical condition tends to degrade rapidly when a large control horizon is used. This research work uses an exponentially weighted moving horizon window to overcome these issues. Employing the exponential-based cost function further significantly reduces the condition number of the Hessian matrix, thereby improving the numerical properties of the MPC. We further analyze the effects of different constraints, observing that the MPC strictly adheres to them and that the control variable influences the response of the MMC plant’s performance. We further compared our results with those of other controllers and analyzed performance metrics, demonstrating that the exponential MPC is effective in this case. Additionally, the results presented in this paper demonstrate the prescribed degree of stability and highlight the importance of fine-tuning key MPC parameters for the MMC model. The exponential-based MPC is validated for the MMC under scenarios involving small and large active and reactive power disturbances, considering offline simulations.

INDEX TERMS Model predictive control, modular multilevel converter, exponential cost function, Laguerre’s function, active reactive power, prescribed stability.

I. INTRODUCTION

MMCs are an up-and-coming solution among various voltage source converter technologies due to their excellent scalability, modularity, and suitability for high-power and high-voltage applications, particularly in high-voltage direct

current (HVDC) transmission systems [1], [2], [3]. MMC has garnered significant interest recently due to its modular structure and provides several advantages, including superior harmonic performance and improved efficiency, including enhanced power quality, scalability, low harmonic distortion, high efficiency, fast dynamics response, and improved fault tolerance [4], [5], [6], [7], [8]. The MMC can synthesize a high-quality AC output by integrating multiple low-voltage

The associate editor coordinating the review of this manuscript and approving it for publication was Bang L. H. Nguyen.

DC submodules. They offer advantages, including high efficiency, system reliability, fast dynamic response, and modularity [9]. Compared with conventional converter systems, MMCs offer improved reliability, making them well-suited for modern HVDC systems and grid-integration technologies such as wind energy conversion systems and HVDC transmission systems [10], [11], [12]. Similarly, MMC is widely applied in areas such as medium-voltage motor drives, railway traction power supplies, medium-voltage static synchronous compensators, and wind energy conversion systems [13], [14], [15]. Despite the many advantages MPC offers, a significant limitation in controlling MMCs stems from their computational complexity. This challenge escalates rapidly with increasing numbers of submodules (SMs) or output voltage levels. Additionally, maintaining low harmonic distortion, achieving a fast dynamic response, and addressing various design constraints add to the overall complexity of control. HVDC systems typically rely on a centralized power management strategy [16]. In such systems, a centralized entity, dispatch control, determines the operational mode and issues setpoints to the upper-level active/reactive power control loops. However, as multi-terminal DC (MTDC) networks become more widespread, the demand for interconverter communication and coordination will increase. To address these issues, distributed optimal power management algorithms are being explored [17], [18], particularly for estimating and allocating active/reactive power and DC voltage setpoints. Regardless of the specific application, an MMC must employ an appropriate control strategy to fulfill several objectives: regulating output currents and SM capacitor voltages, suppressing circulating currents (CC), and minimizing voltage ripple in the SM capacitors as well as fluctuations in the DC-link current [19].

By formulating an appropriate cost function, the operational goals of an MMC can be effectively achieved through the MPC. Among its various forms, the direct and indirect MPC formulations are particularly systems in which systems where power electronic converters dominate the dynamics [20], [21]. Both approaches have been successfully implemented in MMC systems to achieve rapid mitigation—within just a few milliseconds—of frequency deviations caused by sudden active power imbalances in low-inertia grids [22], [23], [24], [25]. Additional studies have reported MPC applications in VSC units operating within interconnected power networks [26]. Furthermore, the works in [27], [28], [29], and [30] demonstrate how MPC can be used to expand the operational range of single-phase and three-phase MMC while enabling simultaneous active and reactive power control. Again, these works are mainly based on a shorter horizon to investigate the dynamics, and they struggle to use a longer horizon due to computational complexity. In power converter applications, adopting a longer prediction horizon in MPC yields significant advantages, including enhanced closed-loop stability, reduced harmonic distortion, and effective regulation of the switching frequency. Again,

another key reason for their popularity is their ability to handle systems with time delays (or dead time) and those that do not exhibit minimum-phase behavior. These advantages enable these methods to predict how the plant will behave over a future time window rather than relying on a single future point. As a result, they are less sensitive to delays and uncertainties in the system model [31], [32], [33]. Furthermore, our exponential MPC can efficiently handle voltage constraints and the rate of change of voltage constraints on inputs, $u_{\min} \leq u \leq u_{\max}$, $\Delta u_{\min} \leq \Delta u \leq \Delta u_{\max}$, as well as output current constraints on the dq -frame current, $y_{\min} \leq y \leq y_{\max}$. As a result, many studies in the literature have focused on predictive control with only control constraints; however, very few have focused on output constraints [28], [34], [35], [36], [37]. In this work, we are using a variant of implicit MPC that incorporates Laguerre functions into the formulation and solution of the MMC model. Using Laguerre functions reduces the dependence on the control horizon found in most MPC schemes [28], [34], [35], [36], [37], thereby significantly reducing computational complexity. In this approach, the indirect implicit MPC generates modulating signals that are fed to the lower-level control loops of the MMC plant. These lower-level controls convert the modulating signals into gating pulses for the semiconductor switches within the submodules. One notable advantage of the implicit MPC method is that the harmonics it excites remain close to the switching frequency. Moreover, the MPC continuously produces optimized modulating signals within defined safe limits during the online optimization of an exponential-type cost function. The constraints are applied to the modulating signals' rates of change of voltage and amplitude, and to the measured output current in the dq -frame to ensure safe operation. The considered exponential MPC is computationally efficient and closely resembles the Discrete Linear Quadratic Regulator (DLQR) [38] because it can accommodate a sufficiently large prediction horizon and a relatively small number of parameters. Its superiority over the DLQR and non-exponential MPC lies in its ability to efficiently handle constraints on both the control input and the plant output, with a lower Hessian condition number.

Most existing MPC schemes for the power converter employ a quadratic cost function, but they are typically limited to relatively short prediction horizons [28], [39], [40]. As the prediction horizon increases, the associated Hessian matrix of the optimization problem becomes increasingly ill-conditioned; its condition number can grow rapidly with the horizon length, which leads to numerical difficulties and higher computational effort in the online solver. At the same time, for power converter applications, the number of admissible switching combinations grows exponentially with the horizon, further increasing the computational burden and the effective control effort. These issues make it challenging to exploit long prediction horizons in conventional

quadratic-cost MPC formulations. An exhaustive search over all possibilities is no longer feasible as the problem size grows. Nevertheless, increasing N_p extends the prediction horizon, allowing the controller to exploit a longer preview of the system's evolution [31], [33], [41], [42], [43]. This article uses an exponential-based MPC, which can handle a large prediction horizon, requires less control effort, and achieves lower settling time and overshoot than the non-exponential MPC for the MMC plant. Again, in this work, we further extend the controller performance by calculating the performance indices, investigating the integral absolute error (IAE), integral squared error (ISE), and root mean square error (RMSE) for the exponential-based MPC cases and comparing them with those of existing non-exponential approaches, FCS-MPC, conventional MPC, and a conventional PI controller. The results demonstrate that the exponential-based MPC outperforms the conventional controllers across all evaluated performance indices. This analysis shows that the exponential-based MPC is superior to the other non-exponential-based MPCs reported in [16], [28], [34], [35], [36], and [37]. The comparative results demonstrate that exponential-based MPC performs better than non-exponential-based MPC, delivering excellent tuning, reduced overshoot, and a faster dynamic response to both small and large disturbances, with a lower condition number and avoidance of numerical ill-conditioning. Motivated by the preceding discussion, the primary contributions of this work are as follows.

- 1) In this article, we are using an exponential-based MPC for the MMC plant, which is capable of easily demonstrating its advantages in handling large prediction horizons with significantly lower condition number in the Hessian matrix compared to the non-exponential MPC case. So far in this work, we have considered exponential weighting factors in three different ways and implemented them for each case. Moreover, for each case, condition numbers are given in this article with different variations, for example, different N_p and different exponential factors.
- 2) In this article, we present a comparative study of the MMC plant's simulation results using both exponential and non-exponential cost functions. The predicted results clearly show that the exponential-based MPC provides a faster, more stable response, particularly at disturbance points (both small and large).
- 3) In this article, we examine the prescribed degree of stability and the corresponding condition number, showing that the closed-loop eigenvalues can be maintained within certain prescribed bounds inside the unit circle.
- 4) In this article, the output of the MMC plant is investigated using the exponential-based MPC, by considering constraints on the rate, amplitude, and plant output current. The outcomes demonstrate that using an exponential controller outperforms its non-exponential counterpart.

- 5) In this article, the controller incorporates a Laguerre-based MPC formulation, effectively reducing computational burden while maintaining excellent performance due to its tunable parameters. Furthermore, we discuss MPC tuning, focusing on the key tunable parameters (a and N) that enhance the controller's overall performance. This tuning precisely matches the DLQR controller, as discussed in the parameter tuning strategies sections of this article, and the corresponding output exactly matches the reference signal.
- 6) In this article, we compared the performance of exponential-based MPC with the conventional PI, conventional MPC, and FCS-MPC controllers, as well as their computational burdens. The exponential-based MPC outperforms the others, as clearly shown in the simulation, and achieves better performance indices.

The rest of this work is organized as follows. In Section II, the State-Space Model of the MMC. In Section III, the Exponentially Weighted MPC for the MMC is presented along with several case studies. In Section IV, we provide various simulation results to validate the proposed approach. Finally, in Section V, the overall work is concluded, and the future direction of the research is given.

II. STATE-SPACE MODEL OF THE MMC

The MMC is a three-phase converter comprising three legs, each corresponding to a separate phase in the abc -frame. Every leg includes an upper and a lower arm, and each arm is composed of N_{SM} half-bridge SMs. Fig. 1 illustrates the

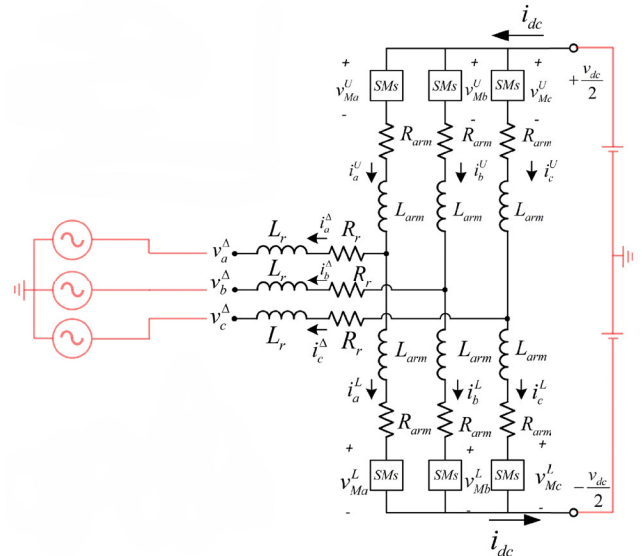


FIGURE 1. Three-phase MMC topology.

MMC layout and the associated variables. For a given phase $j \in \{a, b, c\}$, the voltages and currents in the upper and lower arms can be described by:

$$v_{Mj}^{U,L} = m_j^{U,L} v_{Cj}^{U,L}, \quad i_{Mj}^{U,L} = m_j^{U,L} i_j^{U,L} \quad (1)$$

Here, $m_j^{U,L}$ are the insertion indices for the upper and lower SMs, respectively, and $v_{Cj}^{U,L}$, $v_{Mj}^{U,L}$ represent their respective capacitor voltages. The proposed plant model is developed following the methodology reported in [44]. Using the Σ - Δ nomenclature, the variables in the upper and lower converter arms can be represented as:

$$i_j^\Delta = i_j^U - i_j^L, \quad i_j^\Sigma = \frac{i_j^U + i_j^L}{2}, \quad (2a)$$

$$v_{Cj}^\Delta = \frac{v_{Cj}^U - v_{Cj}^L}{2}, \quad v_{Cj}^\Sigma = \frac{v_{Cj}^U + v_{Cj}^L}{2}, \quad (2b)$$

$$m_j^\Delta = m_j^U - m_j^L, \quad m_j^\Sigma = m_j^U + m_j^L \quad (2c)$$

$$v_{Mj}^\Delta = \frac{-v_{Mj}^U + v_{Mj}^L}{2} = \frac{-m_j^\Delta v_{Cj}^\Sigma + m_j^\Sigma v_{Cj}^\Delta}{2} \quad (2d)$$

$$v_{Mj}^\Sigma = \frac{v_{Mj}^U + v_{Mj}^L}{2} = \frac{m_j^\Sigma v_{Cj}^\Sigma + m_j^\Delta v_{Cj}^\Delta}{2} \quad (2e)$$

where $j \in \{a, b, c\}$ represents the three phases, and the superscripts U and L denote the upper and lower arms respectively.

In dqz -frame [16], [44], the set of differential equations can be written with capacitor voltages and inductor currents as state variables:

$$\frac{d}{dt} (\vec{i}_{dq}^\Delta) = \frac{\vec{v}_{Mdq}^\Delta - (-\omega L_{eq}^{ac} J_2 + R_{eq}^{ac} I_2) \vec{i}_{dq}^\Delta - \vec{v}_{dq}^G}{L_{eq}^{ac}} \quad (3a)$$

$$\frac{d}{dt} (\vec{i}_{dq}^\Sigma) = -\frac{\vec{v}_{Mdq}^\Sigma + (R_{arm} I_2 - 2\omega L_{arm} J_2) \vec{i}_{dq}^\Sigma}{L_{arm}}, \quad (3b)$$

$$\frac{d}{dt} (i_z^\Sigma) = \frac{v_{dc}}{2L_{arm}} - \frac{v_{Mz}^\Sigma + R_{arm} i_z^\Sigma}{L_{arm}}, \quad (3c)$$

$$\frac{d}{dt} (\vec{v}_{CZdq}^\Delta) = \frac{N_{SM}}{2C} \vec{i}_{Mdq}^\Delta - \omega J_2 \vec{v}_{Cdq}^\Delta, \quad (3d)$$

$$\frac{d}{dt} (\vec{v}_{CZdq}^\Sigma) = -\frac{N_{SM}}{8C} \Psi - 3\omega J_2 \vec{v}_{Cdq}^\Sigma, \quad (3e)$$

$$\frac{d}{dt} (\vec{v}_{Cdqz}^\Sigma) = \frac{N_{SM}}{2C} \vec{i}_{Mdqz}^\Sigma + 2\omega J_3 \vec{v}_{Cdqz}^\Sigma, \quad (3f)$$

where N_{SM} is the number of SMS per arm, and ω is the angular frequency, and:

$$\vec{i}_{Mdq}^\Delta = P_\omega(t) \left(P_{-2\omega}^{-1}(t) \vec{m}_{dq}^\Sigma \circ \frac{P_\omega^{-1}(t) \vec{i}_{dq}^\Delta}{2} \right) + P_\omega^{-1}(t) \vec{m}_{dq}^\Delta \circ P_{-2\omega}^{-1}(t) \vec{i}_{dq}^\Sigma, \quad (4a)$$

$$\vec{i}_{Mdq}^\Sigma = P_{-2\omega}(t) \left(P_{-2\omega}^{-1}(t) \vec{m}_{dq}^\Sigma \circ P_\omega^{-1}(t) \vec{i}_{dq}^\Sigma \right) + P_\omega^{-1}(t) \vec{m}_{dq}^\Delta \circ \frac{P_{-2\omega}^{-1}(t) \vec{i}_{dq}^\Delta}{2}, \quad (4b)$$

$$\vec{v}_{Mdq}^\Delta = -\frac{P_\omega(t)}{2} \left(P_\omega^{-1}(t) \vec{m}_{dq}^\Delta \circ P_{-2\omega}(t) \vec{v}_{Cdq}^\Sigma \right) + P_{-2\omega}(t) \vec{m}_{dq}^\Sigma \circ P_\omega^{-1}(t) \vec{v}_{Cdq}^\Delta, \quad (4c)$$

$$\vec{v}_{Mdq}^\Sigma = -\frac{P_{-2\omega}(t)}{2} \left(P_\omega^{-1}(t) \vec{m}_{dq}^\Delta \circ P_{-2\omega}(t) \vec{v}_{Cdq}^\Sigma \right) + P_{-2\omega}(t) \vec{m}_{dq}^\Sigma \circ P_\omega^{-1}(t) \vec{v}_{Cdq}^\Delta, \quad (4d)$$

$$\psi = \begin{bmatrix} i_d^\Delta m_d^\Sigma + 2 i_d^\Sigma m_d^\Delta + i_q^\Delta m_q^\Sigma + 2 i_q^\Sigma m_q^\Delta + 4 i_z^\Sigma m_z^\Delta \\ i_q^\Delta m_d^\Sigma + 2 i_q^\Sigma m_d^\Delta - i_d^\Delta m_q^\Sigma - 2 i_d^\Sigma m_q^\Delta + 4 i_z^\Sigma m_z^\Delta \end{bmatrix}, \quad (4e)$$

where $L_{eq}^{ac} = L_r + \frac{L_{arm}}{2}$, $R_{eq}^{ac} = R_r + \frac{R_{arm}}{2}$. Here, the

$$P_{\omega_0}(t) = \frac{2}{3} \begin{bmatrix} \cos(\omega_0 t) & \cos\left(\omega_0 t - \frac{2\pi}{3}\right) & \cos\left(\omega_0 t - \frac{4\pi}{3}\right) \\ \sin(\omega_0 t) & \sin\left(\omega_0 t - \frac{2\pi}{3}\right) & \sin\left(\omega_0 t - \frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (4f)$$

and $P_{\omega_0}^{-1}(t) = \frac{3}{2} P_{\omega_0}^T(t) + \frac{1}{2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ are the Park's trans-

formation and inverse park's transformation at ω_0 angular frequency and $\omega_0 \in \{-2\omega, \omega, 3\omega\}$. I_n is the identity matrix with an order of $n \times n$, while matrices $J_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $J_3 =$

$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Here, $P_\omega(t)$ represents the Park transformation

matrix, and $P_\omega^{-1}(t)$ its inverse, evaluated at angular frequency ω_0 . The terms $\vec{Z}_{dq}^\Delta$ and $\vec{C}_{dqz}^\Delta$ represent the dq components of the impedance and capacitor dynamics.

The operational dynamics of the MMC can be fully characterized using the AC-side and DC-side modeling framework [45] represented by the following discrete-time differential equation:

$$\dot{\vec{x}}_m = A_m \vec{x}_m + B_m \vec{u}, \quad (5)$$

where $\vec{x}_m = [i_d^\Sigma, i_q^\Sigma, i_z^\Sigma, i_d^\Delta, i_q^\Delta]^T$ is the state vector of the output current in a dq -frame, and $\vec{u} = [v_{Md}^\Sigma, v_{Mq}^\Sigma, v_{Mz}^\Sigma - \frac{v_{dc}}{2}, v_{Md}^\Delta - v_d^\Delta, v_{Mq}^\Delta - v_q^\Delta]^T$ is the voltage difference between in the dq -frame reference. Again, A and B are matrices which are given by:

$$A_m = \begin{bmatrix} \frac{-R_{arm}}{L_{arm}} & 2\omega & 0 & 0 & 0 \\ -2\omega & \frac{-R_{arm}}{L_{arm}} & 0 & 0 & 0 \\ 0 & 0 & \frac{-R_{arm}}{L_{arm}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-R_{eq}^{ac}}{L_{eq}^{ac}} & -\omega \\ 0 & 0 & 0 & \omega & \frac{-R_{eq}^{ac}}{L_{eq}^{ac}} \end{bmatrix}, \quad B_m = \text{diag} \left(-\frac{1}{L_{arm}}, -\frac{1}{L_{arm}}, -\frac{1}{L_{arm}}, \frac{1}{L_{eq}^{ac}}, \frac{1}{L_{eq}^{ac}} \right). \quad (6)$$

III. EXPONENTIALLY WEIGHTED MPC FOR MMC

The primary motivation behind considering the exponential cost function is its multiple advantages for MPC. Firstly, in systems whose design models include embedded integrators, the prediction horizon N_p must usually remain finite because these models possess at least one eigenvalue located on the unit circle. The design model is effectively transformed by applying exponential weighting so that all eigenvalues lie strictly inside the unit circle in the complex plane. This transformation ensures that predictions rely on

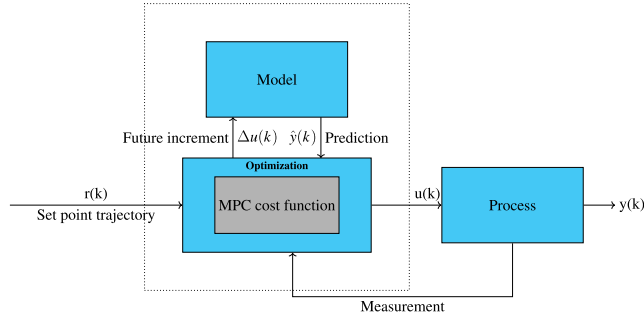


FIGURE 2. Working algorithm of the MPC.

a stable model, which in turn permits the selection of a sufficiently large N_p to approximate the infinite horizon case. Again, using an exponential weighing cost function reduces the numerical ill-conditioning that often arises when large horizons are used without such weighting. This improvement in numerical stability is of significant practical importance, as it enhances both the robustness and computational efficiency of the MPC design. The following diagram depicts the MPC algorithm with an exponential cost function, illustrating how the control input is computed for use in the plant's model, as shown in Fig. 2.

We use the zero-order hold (ZOH) method to discretize the plant model. The ZOH method [16] is ideal for discretizing the exact model of switching power converters, as it maintains harmonic accuracy up to half the switching frequency. For this reason, ZOH is standard in controller design and is applied here to discretize the MMC continuous state-space model [45]. The discrete-time of (5) is then equivalent to:

$$\vec{x}(k+1) = A_p(T_s)\vec{x}(k) + B_p(T_s)\vec{u}(k) \quad (7a)$$

$$\vec{y}(k+1) = C_p(T_s)\vec{x}(k), \quad (7b)$$

where $C_p(T_s)$ is an identity matrix, whereas $A_p(T_s)$ and $B_p(T_s)$ are given by

$$A_p(T_s) = e^{A_m T_s}, \quad B_p(T_s) = \int_0^{T_s} e^{A_m \tilde{T}_s} B_m d\tilde{T}_s. \quad (8)$$

The resulting model provides both AC and DC representations of the MMC, enabling active-reactive power control, circulating-current suppression, and AC grid support. Among several MPC topologies, the indirect implicit MPC approach is most suitable for power converters, for example, in MMC applications. From the AC-side perspective, the discrete-time MMC model can be characterized as a two-input, two-output system. In this representation, $\vec{u}$ denotes the control signals while $\vec{x}_m$ represents the output signals.

To ensure zero steady-state error, an integral term is incorporated into the output formulation, yielding the augmented discrete-time MMC (DMMC) model. The standard MMC model, given by the following equation (9)

$$\underbrace{\begin{bmatrix} \Delta \vec{x}_m(k+1) \\ \vec{y}(k+1) \end{bmatrix}}_{\vec{x}(k+1)} = \underbrace{\begin{bmatrix} A_p(T_s) & \mathbf{0}^T \\ C_p(T_s)A_p(T_s) & \mathbf{1} \end{bmatrix}}_A \underbrace{\begin{bmatrix} \Delta \vec{x}_m(k) \\ \vec{y}(k) \end{bmatrix}}_{\vec{x}(k)}$$

$$+ \underbrace{\begin{bmatrix} B_p(T_s) \\ C_p(T_s)B_p(T_s) \end{bmatrix}}_B \Delta \vec{u}(k), \quad (9a)$$

$$\vec{y}(k) = \underbrace{\begin{bmatrix} \mathbf{0}_m & \mathbf{I} \end{bmatrix}}_C \vec{x}(k). \quad (9b)$$

Here, $\mathbf{0}_m$ is a zero matrix with size $1 \times n_s$, where n_s is the number of state variables. In addition, $\Delta \vec{x}(k+1) = \vec{x}(k+1) - \vec{x}(k)$ and $\Delta \vec{u}(k) = \vec{u}(k) - \vec{u}(k-1)$ are the difference operator in specified time step respectively. The representation of the DMMC model in (9) is the starting point for the formulation of MPC. So that from equation (9), the expression can be reformulated as:

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k), \\ y(k) &= Cx(k) \end{aligned} \quad (10)$$

Then, based on the current state $x(k_i)$, the manipulated variables for the future are calculated over the horizon N_p . These can be computed iteratively as follows:

$$x(k_i + N_p | k_i) = A^{N_p} x(k_i) + \sum_{i=0}^{N_c-1} A^{N_p-i-1} B \Delta u(k_i + i) \quad (11)$$

where k_i is the current time, and N_c is the control horizon. The term $k_i + N_p | k_i$ denotes the prediction of the state at time $k_i + N_p$, given that the current time is k_i . In a similar iterative process, the predicted output variables can be expressed as follows:

$$y(k_i + N_p | k_i) = CA^{N_p} x(k_i) + \sum_{i=0}^{N_c-1} CA^{N_p-i-1} B \Delta u(k_i + i) \quad (12)$$

Define the predicted output as well as the control Y , ΔU , respectively, as follows

$$\begin{aligned} Y &= [y(k_i)^T \cdots y(k_i + N_p - 1)^T]^T \\ \Delta U &= [\Delta u(k_i)^T \Delta u(k_i + 1)^T \cdots \Delta u(k_i + N_c - 1)^T]^T, \end{aligned}$$

Then the output will become:

$$Y = Fx(k+i) + \Phi \Delta U \quad (13)$$

with

$$\begin{aligned} F &= \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}, \\ \Phi &= \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \cdots & CA^{N_p-N_c}B \end{bmatrix}. \end{aligned} \quad (14)$$

For a given time step k_i , we assume the reference value is $r(k_i)$, which we want the system to follow. To achieve this, we must find the optimal control sequence, i.e., ΔU , that

helps minimize the difference between the predicted output and the set-point trajectory.

Due to the complex system dynamics and high demands on closed-loop performance, solving for ΔU may entail a heavy computational burden. Laguerre functions can represent the control input to reduce the computational load in MPC. Laguerre functions form an orthonormal basis and are very frequently used in system modeling and identification. The Δu over time in the MPC context can be approximated using a finite set of discrete-time Laguerre functions. This approach simplifies the optimization problem by reducing the number of decision variables. A set of Laguerre functions can determine the control trajectory as:

$$\Delta u(k_i + k) = \sum_{j=1}^N (k_i) l_j(i) = L(k)^T \eta \quad (15)$$

Here, $L(k) = [l_1(k), l_2(k), \dots, l_N(k)]^T$ is the discrete-time Laguerre functions, and $\eta = [c_1, c_2, \dots, c_N]^T$ is the corresponding coefficient vector. The parameter N refers to the number of terms used in the trajectory approximation, where $l_j(i)$ denotes the j -th Laguerre function and c_j is its associated coefficient, for $j \in \{1, 2, \dots, N\}$. The Laguerre functions have the relation that can be constructed iteratively as

$$L(k+1) = A_L L(k), \quad (16)$$

where

$$A_L = \begin{bmatrix} a & 0 & 0 & \dots & 0 \\ \beta & a & 0 & \dots & 0 \\ -a\beta & \beta & a & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{N-2} a^{N-2} \beta & (-1)^{N-3} a^{N-3} \beta & \dots & \beta & a \end{bmatrix},$$

$$L(0)^T = \sqrt{\beta} [1 - a \ a^2 - a^3 \dots (-1)^{N-1} a^{N-1}]$$

where the factor $\beta = 1 - a^2$ depends on the pole a of the discrete-time Laguerre network, and $0 \leq a < 1$. For the considered systems, each control input Δu_m is linked to its own Laguerre basis L_m and the corresponding coefficient vector η_m . Then, in order to obtain the predicted future state using Laguerre-based, it is obtained by substituting equation (15) into equation (11):

$$x(k_i + N_p | k_i) = A^{N_p} x(k_i) + \sum_{i=0}^{N_p-1} A^{N_p-i-1} \left[B_1 L_1(i)^T + B_2 L_2(i)^T + \dots + B_m L_m(i)^T \right] \eta_p, \quad (17)$$

where $\eta_p^T = [\eta_1^T \ \eta_2^T \ \dots \ \eta_m^T]$; and $B_1, B_2, \dots, B_m$ is the i -column of the B matrix in the incremental model. The variables that need to be optimized are the coefficients of the Laguerre functions. To determine the coefficients in η , a constrained optimization problem must be designed and solved. Accordingly, the control sequence is obtained

by solving the following minimization problem. Moreover, in the case of using Laguerre functions in the cost function, the optimal control problem is considered as:

$$J = \sum_{m=1}^{N_p} \bar{x}(k+m|k)^T Q \bar{x}(k+m|k) + \bar{\eta}^T R \bar{\eta}, \quad (18)$$

using the system from the equation (17) in the cost function (18), then it will become:

$$J = \eta^T H \eta + 2\eta^T \psi x(k_i) + \delta(k_i) \quad (19)$$

where $H = \sum_{i=1}^{N_p} \phi(i) Q \phi(i) + R$, $\psi = \sum_{i=1}^{N_p} \phi(k) Q A^i$, $\delta = \sum_{i=1}^{N_p} x(k_i)^T (A^T)^i Q \phi(i)$.

Accordingly, the control sequence is obtained by solving the minimization problem. A closed-form solution to the unconstrained optimization problem in equation (18) is available under the stated condition.

$$\eta = H^{-1} \psi x(k_i) \quad (20)$$

By computing the optimal parameter vector η , the receding-horizon control law is realized as

$$\Delta u = \begin{bmatrix} L_1(0)^T & \mathbf{o}_2^T & \dots & \mathbf{o}_m^T \\ \mathbf{o}_1^T & L_2(0)^T & \dots & \mathbf{o}_m^T \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{o}_1^T & \mathbf{o}_2^T & \dots & L_m(0)^T \end{bmatrix} \eta,$$

where $\mathbf{o}_k^T, k = 1, 2, \dots, m$ represents a zero matrix whose block is identical to $L_k(0)^T$. and using the formula

$$u(k) = \Delta u + u(k-1)$$

we can easily calculate the u vector for our system.

A. EXPONENTIAL COST

Consider the exponential cost function in the following form:

$$J = \sum_{j=1}^{N_p} \lambda^{-2j} x(k_i + j | k_i)^T Q x(k_i + j | k_i) + \sum_{j=0}^{N_p} \lambda^{-2j} \Delta u(k_i + j)^T R \Delta u(k_i + j). \quad (21)$$

From here, we have three choices for the exponential factor λ . If $\lambda < 1$, the exponential factor λ^{-2j} , for $j \in \{1, 2, \dots, N_p\}$, assigns less importance to the state $x(k_i + j | k_i)$ at current times and progressively higher importance to the future states. Motivated by a similar concept previously introduced by Anderson and Moore [38] in the DLQR design. If $\lambda > 1$, the exponential weighting sequence λ^{-2j} , where $j \in \{1, 2, \dots, N_p\}$, assigns greater importance to the predicted state $x(k_i + j | k_i)$ at the beginning of the prediction horizon and progressively less importance to future states, since the weights decrease as j increases. This weighting strategy is adopted in the present study of predictive control systems. It is well known that when long-horizon predictions are employed in MPC, the Hessian matrix H increases

significantly in magnitude. This growth often leads to numerical ill-conditioning in the MPC optimization problem. The condition number of the Hessian matrix, which is defined as:

$$\kappa(H) = \|H\| \cdot \|H^{-1}\|. \quad (22)$$

The very large condition numbers κ reflect the high sensitivity of the optimization problem to numerical errors. In the context of MPC, a higher condition number indicates an ill-conditioned Hessian matrix, leading to greater sensitivity to disturbances and modeling inaccuracies. Moreover, even with advanced quadratic solvers, a high condition number may yield inaccurate or unreliable solutions. Sometimes, if the condition number becomes too large, the Hessian matrix H can get very close to being singular [46], [47]. One way to handle numerical issues in MPC is to adjust the cost function by adding an exponential term, such as λ^{2j} . Motivated by this approach, we have considered the cost function as presented in equation (15). Based on this cost function, we consider two distinct cases, each discussed separately. In the first case, we use exponential weighting in the cost function without modifying the augmented matrix. In the second scenario, both the augmented matrix and the state variables are modified accordingly. These are discussed in detail as **Case I** and **Case II** in the following sections.

1) CASE I

Let us consider the cost function as:

$$J = \sum_{j=1}^{N_p} \lambda^{-2j} x(k_i + j | k_i)^T Q x(k_i + j | k_i) + \sum_{j=0}^{N_p} \lambda^{-2j} \Delta u(k_i + j)^T R \Delta u(k_i + j). \quad (23)$$

In its original form, the exponentially weighted cost function does not explicitly illustrate its influence on MPC design. This influence becomes apparent when we reformulate the problem using transformed variables of the cost functions.

$$\Delta \hat{U}^\top = [\lambda^{-0} \Delta u^\top(k_i) \cdots \lambda^{-N_p} \Delta u^\top(k_i + N_p)],$$

Moreover, the exponentially weighted predicted state sequence is:

$$\Delta \hat{X}^\top = [\lambda^{-1} x^\top(k_i + 1 | k_i) \cdots \lambda^{-N_p} x^\top(k_i + N_p | k_i)].$$

Because of these new variables, the original cost function (23) can be rewritten in terms of $\Delta \hat{U}$ and $\Delta \hat{X}$, which provides more profound insight into the structure of the predictive control formulation. After applying this transformation, the cost function, constraints, and state variables are reformulated into the following structure:

$$\hat{J} = \sum_{j=1}^{N_p} \hat{x}(k_i + j | k_i)^T Q \hat{x}(k_i + j | k_i)$$

$$+ \sum_{j=0}^{N_p} \Delta \hat{u}(k_i + j)^T R \Delta \hat{u}(k_i + j), \quad (24)$$

$$\text{subject to} \quad M_\lambda \Delta \hat{U} \leq \gamma, \quad (25)$$

where $\hat{x}(k_i + j | k_i)$ and $\Delta \hat{u}(k_i + j)$ follow the transformed dynamics:

$$\hat{x}(k_i + j + 1 | k_i) = \frac{A}{\lambda} \hat{x}(k_i + j | k_i) + \frac{B}{\lambda} \Delta \hat{u}(k_i + j), \quad (26)$$

and M_λ is defined as:

$$M_\lambda = M \begin{bmatrix} I & 0 & \cdots & 0 & 0 \\ 0 & \lambda I & \cdots & 0 & 0 \\ 0 & 0 & \ddots & \lambda^{N_p-1} I & 0 \\ 0 & 0 & \cdots & 0 & \lambda^{N_p} I \end{bmatrix}. \quad (27)$$

The corresponding results are given in Section IV.

2) CASE II

The exponentially decreasing weight $\lambda > 1$ increases the magnitudes of the actual closed-loop eigenvalues by λ . However, if the new weighting matrices Q and R are appropriately chosen such that they reduce the magnitudes of the eigenvalues of the exponentially weighted system by a factor of λ^{-1} , then the overall effect is neutralized, and the magnitudes of the actual closed-loop eigenvalues remain unchanged. Starting from the cost function:

$$J = \sum_{j=1}^{N_p} \lambda^{-2j} x(k_i + j | k_i)^T Q x(k_i + j | k_i) + \sum_{j=0}^{N_p} \lambda^{-2j} \Delta u(k_i + j)^T R \Delta u(k_i + j) \quad (28)$$

by using the transform variables,

$$\Delta \hat{u}(k + j) = \lambda^{-j} \Delta u(k_i + j), \quad (29)$$

$$\hat{x}(k_i + j | k_i) = \lambda^{-j} x(k_i + j | k_i), \quad (30)$$

$$\hat{x}(k_i + j + 1 | k_i) = \hat{A} \hat{x}(k_i + j | k_i) + \hat{B} \Delta \hat{u}(k_i + j). \quad (31)$$

where $\hat{A} = \frac{A}{\lambda}$ and $\hat{B} = \frac{B}{\lambda}$. By using these exponentially weighted variables, the exponentially weighted cost function is expressed in terms of the transformed variables, and the construction is now based on the pair $(\lambda^{-1}A, \lambda^{-1}B)$. Thus, the eigenvalues do not take a large value, which avoids the ill-conditioned matrix problem. Suppose the optimal solution of the cost function is obtained under certain constraints on the control inputs. $u_{\min} \leq M\eta + u(k-1) \leq u_{\max}$.

$$M = \begin{bmatrix} \sum_{i=0}^{k-1} L_1(i)^T & \mathbf{0}_2^T & \cdots & \mathbf{0}_m^T \\ \mathbf{0}_1^T & \sum_{i=0}^{k-1} L_2(i)^T & \cdots & \mathbf{0}_m^T \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_1^T & \mathbf{0}_2^T & \cdots & \sum_{i=0}^{k-1} L_m(i)^T \end{bmatrix},$$

Then the above cost function is further rewritten along with the constraints, and the optimal value of the control input can

TABLE 1. System Parameters of MMC Used in MPC Validation simulations.

Name	Symbol	Value
Arm inductance	L_{arm}	0.15 [p.u.]
Arm resistance	R_{arm}	0.0015 [p.u.]
AC filter inductance	L_f	0.12 [p.u.]
AC filter resistance	R_f	0.003 [p.u.]
Sample time	T_s	2 [ms]

be further evaluated by minimizing this cost function:

$$J_\lambda = \sum_{j=1}^{N_p} \hat{x}_m(k_i + j | k_i)^T Q_\lambda \hat{x}_m(k_i + j | k_i) + \sum_{j=0}^{N_c} \Delta \hat{u}(k_i + j)^T R_\lambda \Delta \hat{u}(k_i + j), \quad (32)$$

subject to

$$\hat{x}_m(k_i + j + 1 | k_i) = \lambda^{-1} A \hat{x}_m(k_i + j | k_i) + \lambda^{-1} B \Delta \hat{u}(k_i + j). \quad (33)$$

$$u_{\min} \leq M \eta + u(k-1) \leq u_{\max}, \quad (34)$$

and

$$\gamma = \frac{1}{\lambda}, \quad Q_\lambda = \gamma^2 Q + (1 - \gamma^2) P_\infty, \quad R_\lambda = \gamma^2 R,$$

where $Q_\lambda \geq 0, R_\lambda > 0$ are matrices of appropriate dimensions; $\mathbf{0}_k, k \in \{1, 2, \dots, m\}$ is a zero row vector of $L(0)^T$ dimensions and $u_{\min}, u_{\max}$ are the lower and upper limit in the control input, and P_∞ is the solution of the algebraic Riccati equation:

$$A^T \left[P_\infty - P_\infty B (R + B^T P_\infty B)^{-1} B^T P_\infty \right] A + Q - P_\infty = 0.$$

The effectiveness of the exponential MPC is further analyzed by the appropriate selection of important parameters, namely a, N, λ, Q, R , and N_p . In particular, the parameters associated with the Laguerre functions play a critical role in accurately approximating the control input trajectory over the prediction horizon.

IV. SIMULATION RESULTS AND DISCUSSION

To evaluate the performance of the MMC controlled by exponential-based MPC, we have considered the parameters of the plant, which are given in the Table 1, and to observe the output current in dq frame, we have considered several disturbances to test the performance of the controller which are given in the Table 2. The active and reactive power setpoints were intentionally varied to evaluate the dynamic performance of the MMC under MPC. These variations were clearly observed through the output currents in the dq reference frame. The study examines two main scenarios: small and large disturbances. For each scenario, simulations of two events were performed at different time intervals. Both active and reactive power variables were evaluated, each changing at specific time instances, as detailed in Table 2.

TABLE 2. Simulation scenarios for MPC under small and large disturbances.

Changes	Variable	1st Change	2nd Change
Small	Active power	0.5 p.u. (10)	1.0 p.u. (30)
	Reactive Power	0.2 p.u. (20)	1.0 p.u. (30)
Large	Active power	-1.0 p.u. (40)	1.0 p.u. (80)
	Reactive Power	-1.0 p.u. (50)	0.5 p.u. (80)

A. SIMULATION FOR THE CASE-I

First, we analyze the unconstrained simulation results and use exponential weighting with different values of the exponential parameter λ . Initially, both active and reactive power are set to zero, and the closed-loop response of the exponential-based MPC is illustrated in Fig. 3. When there are changes in the reference signal at various sampling intervals, the output current demonstrates only a slight deviation from the reference signal for small disturbances. However, for larger disturbances, the mismatch becomes more significant compared to that observed with smaller disturbances.

The output current is in the dq frame for different horizons N_p , and it is evident that when $\lambda = 1$, the first two subplots in Fig. 3 show noticeable variations in the output depending on the prediction horizon. Moreover, the different outputs are visible for the different prediction horizons N_p . In contrast, when exponential data $\lambda = 1.3$ is used, the MMC output current in dq -frame becomes practically invariant with respect to the different prediction horizons, demonstrating that system performance is no longer sensitive to this parameter. This implies that a longer prediction horizon can be selected without influencing the output current. For small setpoint changes, the output current with $\lambda = 1.3$ shows only a slight overshoot and faster dynamic response, and less mismatch from the set point trajectories. Again, the corresponding control inputs u_d and u_q depicted the sharp peak and show maximum amplitude corresponding to the significant change in the reference set point, a change in the active power from a positive unit step to a negative unit step, and are presented in Figs. 4–5. Here, it is evident that, with the exponential weighting factor, the control trajectories in u_d and u_q are consistent across different prediction horizons. Again, the incremental control input Δu_d and Δu_q trajectories remain consistent for the exponential data. For different prediction horizons, the incremental control trajectories are distinct, as shown in Fig. 5, indicating that the non-exponential case is more sensitive to variations in the prediction horizon. Another important observation from the exponential-weighting data concerns the prediction horizon. Even for large horizons, the exponential cost function results in a significantly lower condition number of the Hessian matrix than the non-exponential cost function, which is clearly shown in the Table 3. We observe that after using the exponential factor with varying prediction horizons, the condition number converges to a finite value. This suggests that a sufficiently large prediction horizon can be used with

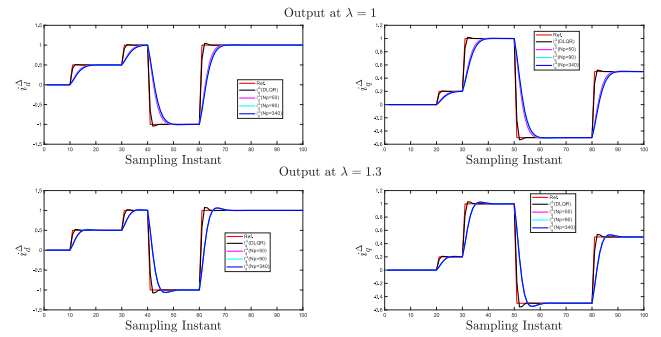
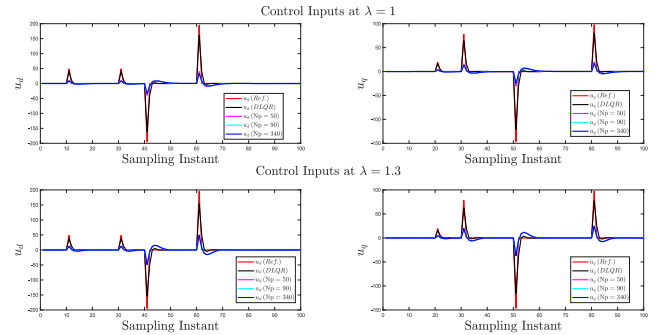
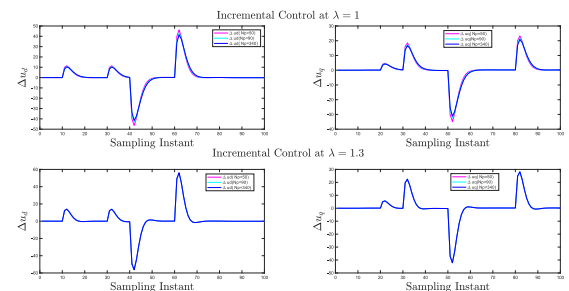
TABLE 3. Condition numbers of the Hessian matrix H and eigenvalues with different and sufficiently large values of the prediction horizon.

λ	N_p	$\tilde{\lambda}_{\max}(H)$ (max eigen value)	$\tilde{\lambda}_{\min}(H)$ (min eigen value)	$\kappa(H)$ condition number
0.8	40	6.1354×10^7	7.1651×10^{-4}	8.5629×10^{10}
	80	1.5145×10^{16}	-1.992×10^{-1}	1.3669×10^{19}
	180	1.9181×10^{36}	-4.1578×10^{20}	6.50377×10^{38}
	300	9.5855×10^{59}	-3.1676×10^{44}	1.7118×10^{39}
1	40	2.3996×10^1	5.9804×10^{-4}	4.0667×10^4
	80	3.6226×10^2	1.1×10^{-3}	3.1838×10^5
	180	7.0127×10^3	1.3×10^{-3}	5.3038×10^6
	300	3.8329×10^4	1.4×10^{-4}	2.8287×10^7
1.3	40	1.880×10^{-2}	2.5179×10^{-4}	7.4845×10^1
	80	2.05×10^{-2}	3.2133×10^{-4}	6.3642×10^1
	180	2.050×10^{-2}	3.2143×10^{-4}	6.3626×10^1
	300	2.051×10^{-2}	3.2143×10^{-4}	6.3626×10^1

this MPC. Additionally, the condition number of the Hessian matrix is shown in Fig. 6. The graphical representation shows that the condition number decreases as the prediction horizon increases, a trend that is particularly evident with exponential weights. For different values of λ , the parameter effect on closed-loop stability is also highlighted. Specifically, when $\lambda < 1$, the closed-loop eigenvalues shift outside the unit circle, indicating instability under the same parameter conditions. In contrast, for $\lambda = 1$ and $\lambda = 1.3$, the eigenvalues remain inside the unit circle, ensuring stability. Moreover, in the case of $\lambda = 1.3$, the closed-loop eigenvalues align more closely with those obtained from the DLQR, showing substantial similarity across different choices of Laguerre poles, which are depicted in Fig. 9. Again, the simulation results depicted in the Fig. 7 clearly depict that the gain element of the DLQR and exponential-based MPC convergence, and the sum of the square error between the two incremental trajectories of the DLQR and MPC is depicted in the graph 7, which indicates the error is small with sampling instants. To investigate the performance of the exponential MPC, the control effort has been evaluated for different N_p . For the non-exponential MPC, the control efforts are 0.20699 and 0.19080 corresponding to $N_p = 50$ and 90, respectively. Again, the control effort for the conventional MPC are 0.50087 and 0.3128 respectively for the same N_p . In comparison, for the exponential-based MPC with the exponential weighting factor $\lambda = 1.1$, the control efforts are 0.17316 and 0.17113 for the same N_p . This represents reductions of 16.34% and 10.31% in control effort, respectively, using the exponential-based MPC. These results indicate that using the exponential MPC achieves lower control effort than the conventional MPC. Therefore, the proposed controller is more efficient and reliable, offering reduced control effort and smaller condition numbers in the corresponding Hessian matrix. Again, the performance indices for exponential-based MPC with $\lambda = 1.1$ and for the larger values of the $N_p = 170$ are as follows: IAE = 18.065905, ISE = 11.908, RMSE = 0.266520.

B. TUNING FOR THE IMPORTANT MPC PARAMETERS

Variations in the Laguerre parameters a and N have opposite effects on the system's output and the closed-loop poles.

**FIGURE 3.** Case-study with unconstrained scenario showing i_d^* current, i_q^* with different value of λ and N_p .**FIGURE 4.** Case-study with unconstrained scenario showing u_d voltage, and u_q voltage with different value of λ and N_p .**FIGURE 5.** Case-study with unconstrained scenario showing Δu_d voltage, and Δu_q voltage with different value of λ and N_p .

Specifically, as N increases, the poles gradually converge toward those of the origin and DLQR eigenvalues, indicating improved damping and stability characteristics. Conversely,

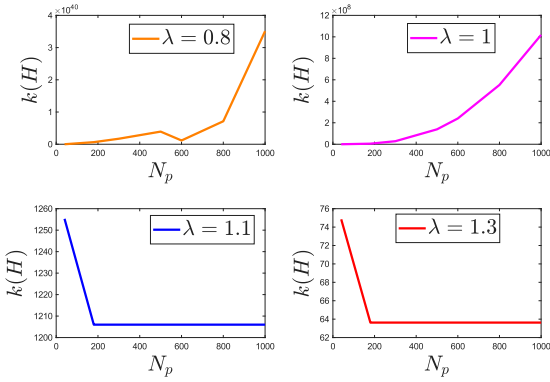


FIGURE 6. Condition numbers with different values of λ and N_p .

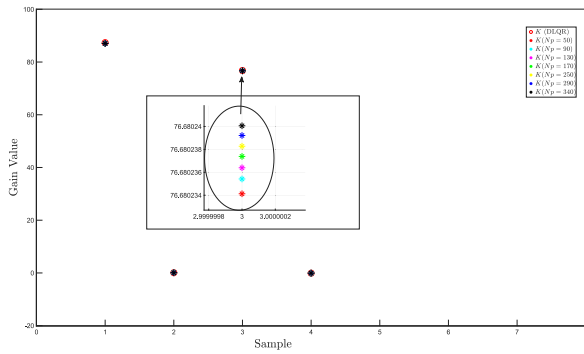


FIGURE 7. Gain element of the DLQR and MPC with the different values of λ and N_p .

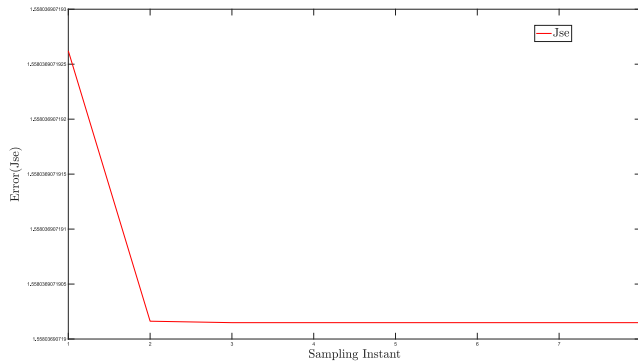


FIGURE 8. Error of the incremental control between the DLQR and MPC with exponential factors λ and along with sampling instant.

increasing the value of a causes the dominant poles to move closer to the unit circle. Moreover, for the same parameters, with nonexponential MPC, as we increase a , the pole's closed-loop eigenvalue moves toward the boundary of the circle, and it is on the boundary for $a = 1$. The boundary of the eigenvalues eventually leads to instability when a becomes excessively large, as illustrated in Figs. 10-11. It is also important to note that when $a = 0$, and the exponential factor is zero, then the employing exponential weighting in the cost function disappears. The control law effectively reduces to the conventional MPC formulation, where the incremental control input serves as the optimization

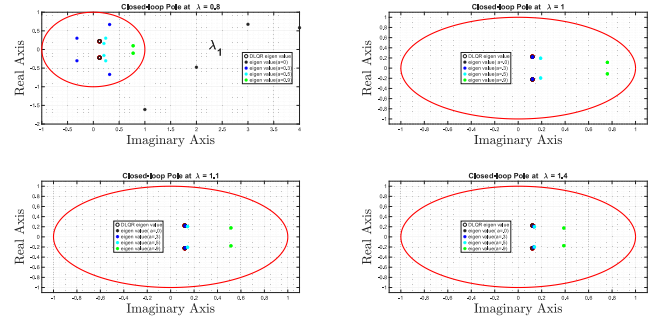


FIGURE 9. Closed-loop poles for different values of the exponential factors λ along with the sampling instant.

variable. Moreover, using the employing exponential MPC, the unstable system remains stable over the same parameter range, meaning that the proposed system remains more robust to tuning important parameters and maintains stability, as clearly shown in the Figs. 10-11. Furthermore, comparative simulations were conducted using both conventional PI control and FCS-MPC. The results clearly show that the employing exponential-based MPC outperforms both controllers, particularly for larger prediction horizons N_p . The employing exponential approach achieves more accurate output-current tracking with significantly lower overshoot and reduced control effort for both small and large set-point changes.

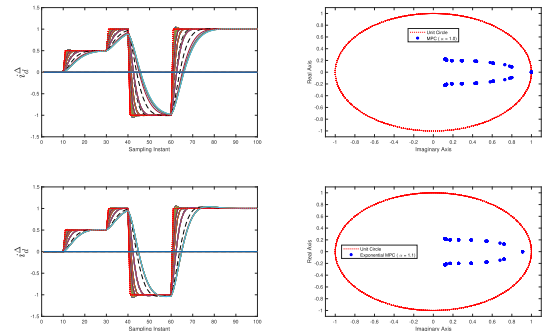


FIGURE 10. Effect of a on i_d^* and closed loop eigenvalues with and without exponential factor for the fixed $N = [10, 10]$ and variation.

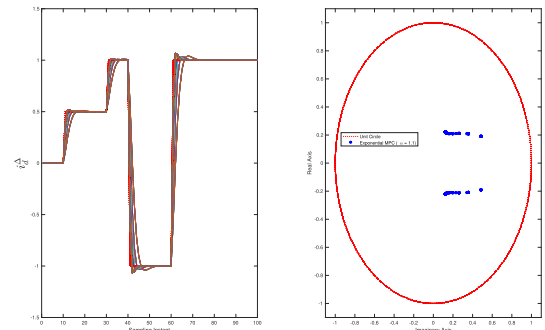


FIGURE 11. Effect of N on i_d^* and closed loop eigenvalues with and without exponential factor for the fixed $a = [0, 1]$ and variation.

These attributes make the exponential MPC a more reliable, general, and scalable control strategy for MMC applications, capable of delivering improved steady-state accuracy and enhanced long-term dynamic performance. The simulation results shown in Fig. 12 support these observations. Moreover, the performance indices for the FCS-MPC and PI are evaluated. For the FCS-MPC, IAE, ISE, and RMSE are 40.0267, 22.920, and 0.3227, respectively, and the control effort is 15.8210, and for the PI, IAE, ISE, and RMSE are 23.6960, 22.71220, and 0.4766, respectively, which are higher than those of the exponential-based MPC. Again, the average overshoots are 10.1245, 43.501, and 17.5175 for the controllers PI, FCS-MPC, and Conventional MPC, respectively. Further, the average settling times are 0.45 and 0.136 for the PI and conventional MPC, respectively. In contrast, for our exponential-based MPC, the average overshoot and average settling time are 4.1810 and 0.0170, respectively, outperforming the other mentioned controllers (PI, FCS-MPC, and conventional MPC).

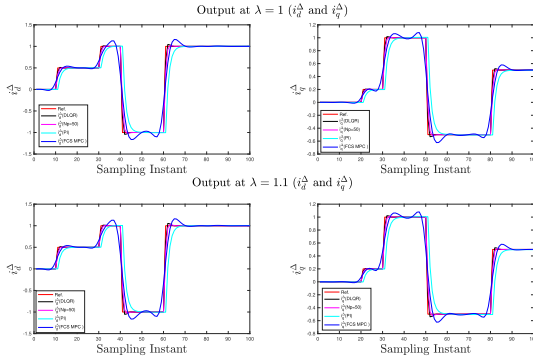


FIGURE 12. Case-study showing i_d^* current, i_q^* with different value of λ and different controller.

C. MODEL UNCERTAINTIES AND ROBUSTNESS ANALYSIS

To see the effects of parameter variations and uncertainties, closed-loop robustness simulations were conducted by perturbing the system matrix A as

$$A_{\Delta} = A + \Delta A,$$

Let $\Delta A \in \mathbb{R}^{n \times n}$ be a random perturbation matrix whose entries are independent, zero-mean, unit-variance Gaussian variables, scaled by a factor $a_{\ell} < 1$, i.e.,

$$\Delta A = a_{\ell} W,$$

where W has i.i.d. $\mathcal{N}(0, 1)$ entries and ΔA has the same dimensions as A represents the uncertainty level. The perturbation magnitude a_{ℓ} is selected such that the considered system remains within practical operating limits and the control inputs remain bounded. Closed-loop performance and stability are verified by evaluating the plant output and its control input. The simulation results shown in Fig. 13 validate the claims for the considered MMC plant for $a_{\ell} = 0.01$, whereas in the non-exponential case, the system is

unstable, which is clearly visible from the first two graphs of Fig. 13. The graphical results clearly illustrate the improved robustness of the exponential MPC over conventional MPC. Again, the performance metrics for the noise disturbance for the $N_p = 50$ and $\lambda = 1.1$ are as follows:

With noise:

$$\text{IAE} = 18.358, \quad \text{ISE} = 13.533, \quad \text{RMSE} = 0.2555.$$

Without noise:

$$\text{IAE} = 17.273, \quad \text{ISE} = 13.466, \quad \text{RMSE} = 0.25487.$$

This confirms that the exponential-based MPC is robust against realistic sensor and process noise levels.

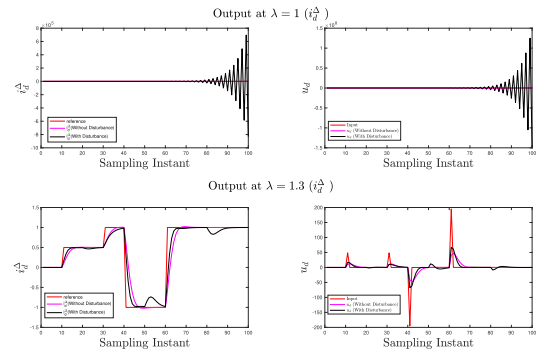


FIGURE 13. Output and input with and without exponential cases.

D. SIMULATION FOR CASE-II

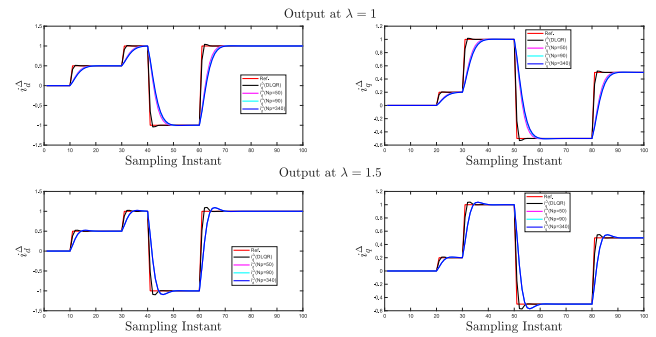
In the previous case, sufficiently large values of λ from unity cannot be considered because they cannot guarantee the closed-loop stability. We introduce an alternative formulation by defining a new factor, $\gamma = \frac{1}{\lambda}$, to overcome this limitation. In the present case, there is flexibility to select λ as a sufficiently large value from unity, depending on the desired closed-loop performance. Due to the exponentially decreasing weight factor $\lambda > 1$, it effectively increases the magnitude of the actual closed-loop eigenvalues by a factor of λ . However, the weighting matrices Q and R are chosen in such a way that they reduce the eigenvalue magnitudes of the exponentially weighted system by a factor of λ^{-1} . In that case, the actual closed-loop eigenvalues remain unaffected. Again, it can be observed that the condition numbers of the Hessian matrix are significantly lower in the exponential weighting case and converge to finite values. For reference, Table 4 presents the values of the condition number, along with the maximum and minimum eigenvalues of the Hessian matrix, for different values of the exponential factor λ along with the prediction horizon N_p .

From this, it clearly shows the dependence of the condition number on both the prediction horizon and the choice of λ . It is demonstrated that increasing the prediction horizon and the value of λ effectively reduces the condition number, thereby improving numerical ill conditioning for the exponential cases. Furthermore, the simulation results for the Hessian matrix, illustrated in Fig. 17, provide additional

TABLE 4. Condition number of the Hessian matrix H and eigenvalues with different and sufficiently large values of the prediction horizon.

λ	N_p	$\tilde{\lambda}_{\max}(H)$ (max eigen value)	$\tilde{\lambda}_{\min}(H)$ (min eigen value)	$\kappa(H)$ condition number
0.8	40	5.797×10^{-1}	3.9×10^{-3}	1.4994×10^2
	80	9.887×10^{-1}	2.24×10^{-2}	4.4157×10^1
	180	1.0630×10^0	2.56×10^{-2}	4.1506×10^1
	300	1.0630×10^0	2.56×10^{-2}	4.1505×10^1
1	40	7.42×10^{-2}	2.3×10^{-4}	3.2455×10^1
	80	9.06×10^{-2}	7.9×10^{-3}	1.1537×10^1
	180	9.21×10^{-2}	8.1×10^{-3}	1.1321×10^1
	300	9.21×10^{-2}	8.1×10^{-3}	1.1321×10^1
1.1	40	7.9×10^{-3}	8.0×10^{-4}	9.8174×10^0
	80	8.5×10^{-3}	1.9×10^{-2}	4.4791×10^0
	180	8.6×10^{-3}	1.9×10^{-2}	4.4570×10^0
	300	8.6×10^{-3}	1.9×10^{-2}	4.4570×10^0
1.3	40	3.6873×10^{-4}	8.7503×10^{-5}	4.2139×10^0
	80	3.7551×10^{-4}	1.8576×10^{-4}	2.0224×10^0
	180	3.7662×10^{-4}	1.8652×10^{-4}	2.0191×10^0
	300	3.7662×10^{-4}	1.8652×10^{-4}	2.0191×10^0
1.4	40	4.2248×10^{-5}	1.3006×10^{-5}	3.2484×10^0
	80	4.2575×10^{-5}	2.9314×10^{-5}	1.4524×10^0
	180	4.2646×10^{-5}	2.9408×10^{-5}	1.4501×10^0
	300	4.2646×10^{-5}	2.9408×10^{-5}	1.4501×10^0

verification of the author's claim across different parameter values. Further, the analysis is divided into two categories: first, the unconstrained case, and then, subsequently, the constrained case. We begin with the unconstrained scenario, meaning no constraints are imposed on the state variables of the input and output variables. For comparison purposes, the output of the MPC is plotted for different N_p , alongside the corresponding DLQR response. As shown in Fig. 14, the closed-loop responses with both the exponential and non-exponential weighting factors are presented. It can be clearly observed that the use of the exponential weighting costs in this case yields a significantly improved closed-loop response compared to the non-exponential case, i.e., the prediction horizons are not much affected for the closed loop response for this case and the output current in the dq frame are coincide with each other even for the different values of the N_p and the response is more closer and faster toward the set point rather than the original non-exponential cases which are visible in the Fig. 14. Moreover, the control and incremental control for different prediction horizons are depicted in Figs. 15–16. For reference, the DLQR control input and step response are also plotted, and it can be seen that its response closely matches that of the exponential-based MPC solution. Again, it is observed that for a small change in the set point signal, there is a small change in the control input. Corresponding to a significant change in the set point, there is a prominent peak in the control input, which is clearly visible, and for different prediction horizons, in the case of the exponential, it is almost the same response; however, for non-exponential cases, it seems different for different prediction horizons, and again further validating the consistency of the approach. Again, the closed-loop eigenvalues are depicted in Fig. 19 with the different exponential weighting factors. The closed-loop eigenvalues are depicted in Fig. 19 with the different exponential weighting factors. To investigate the

**FIGURE 14.** Case-study with unconstrained scenario showing i_d^{Δ} current, i_q^{Δ} with different value of λ .

performance of the exponential MPC, the control effort is evaluated for different N_p . For the non-exponential MPC, the control efforts are 0.20699 and 0.19080 corresponding to $N_p = 50$ and $N_p = 90$, respectively. Whereas, for the exponential-based MPC with the exponential weighting factor $\lambda = 1.3$, the control efforts are 0.17795 and 0.17762 for the same N_p . This represents reductions of 14.032% and 6.9055% in control effort, respectively, through the exponential-based MPC in Case-II. These results indicate that the exponential-based MPC achieves lower control effort than the conventional MPC. Therefore, the exponential-based controller is more efficient and reliable, offering reduced control effort and smaller condition numbers in the corresponding Hessian matrix. Again, the performance indices for exponential-based MPC with the larger values of the $N_p = 170$ and a weighting factor of $\lambda = 1.5$ are as follows: IAE = 12.113879, ISE = 8.394854, RMSE = 0.204876.

E. CONSTRAINT HANDLING PROBLEM

One of the primary motivations for the MPC approach is its ability to handle constrained control problems effectively,

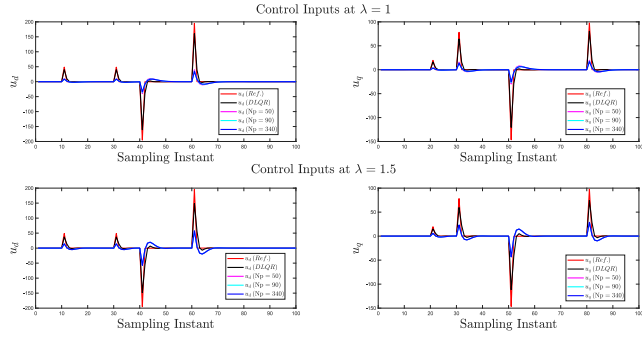


FIGURE 15. Case-study with unconstrained scenario showing u_d voltage, and u_q voltage with different value of λ .

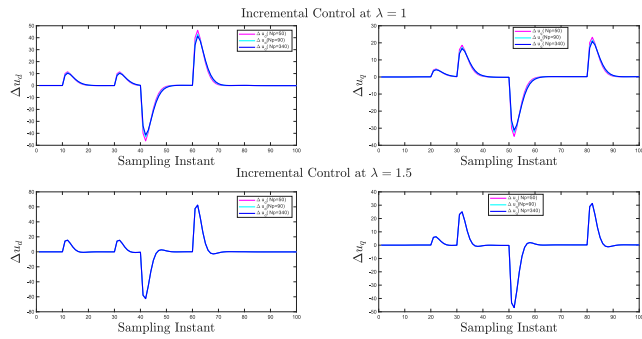


FIGURE 16. Case-study with unconstrained scenario showing Δu_d voltage, and Δu_q voltage with different value of λ .

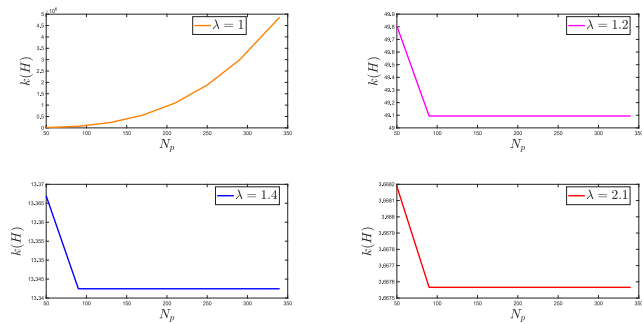


FIGURE 17. Condition number with different values of λ .

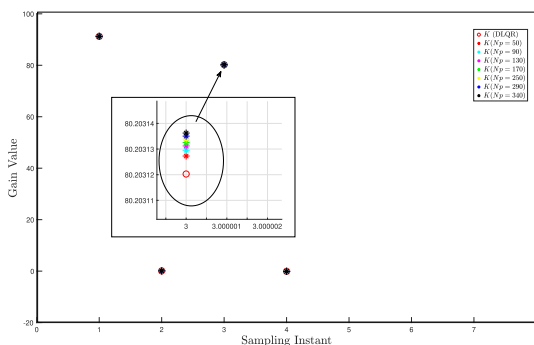


FIGURE 18. Gain element of the DLQR and MPC with the different values of λ .

which clearly differentiates it from the well-known DLQR method. To demonstrate this advantage, we consider three

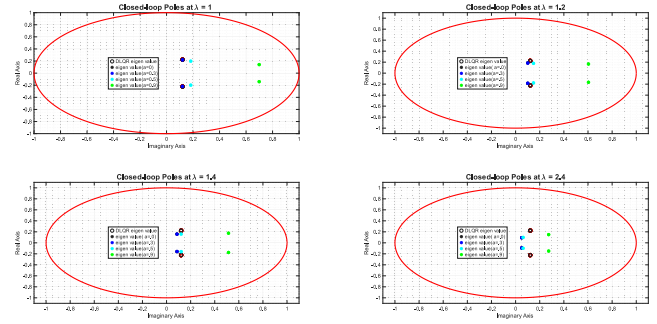


FIGURE 19. Closed loop poles at the different values of the exponential factors λ along with sampling instant.

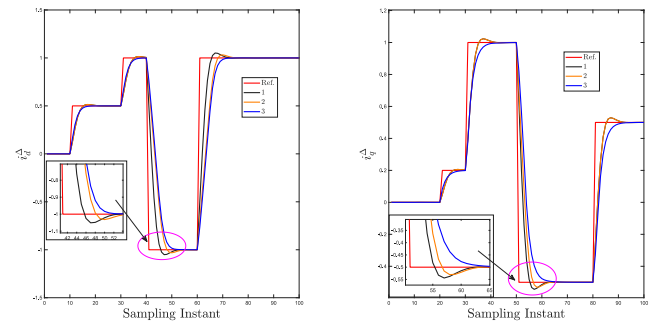


FIGURE 20. Plant output under 1 – unconstrained, 2 – constrained, exponential, and 3 – exponential constrained MPC.

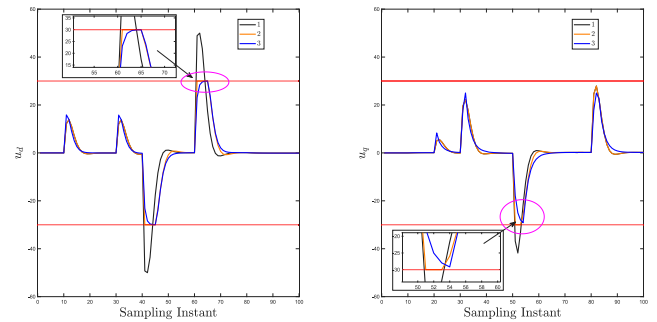


FIGURE 21. Control input under 1 – unconstrained, 2 – constrained, exponential, and 3 – exponential constrained over the sampling instants.

different constraint scenarios in this section: rate, amplitude, and output constraints, to assess the benefits of handling these constraints. Again, instead of discussing the constraint rate and the constraints separately in this instance, we have analyzed them together, and the performance of the exponential-based MPC is discussed below within the constraints scenario.

1) AMPLITUDE AND THE RATE CONSTRAINTS

To see the effect of the exponential-based MPC for the MMC plant, here we impose two different constraints

$$(a) |u_d| = |u_q| = 30, \quad (b) |\Delta u_d| = |\Delta u_q| = 30$$

respectively. When these control constraints are imposed individually on the control. In the case of the MMC plant, The

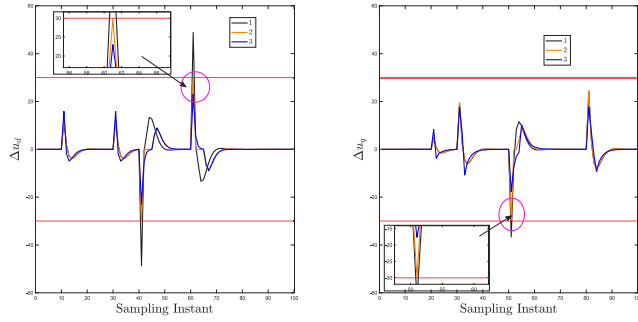


FIGURE 22. Incremental control under 1 – unconstrained, 2 – constrained, exponential, and 3 – exponential constrained over the sampling instants.

corresponding MPC output response is depicted in Fig. 20. From this output, it is clear that using exponential constraints results in a faster, less-overshoot dynamic response for the MPC compared to simple constraints, indicating that the limitations directly affect the MPC's output. Again, from the output, the current in the dq -frame of reference shows higher overshoot in the unconstrained case and lower overshoot in the simple-constraint case. For the exponential constraints, it has less overshoot, which is clearly depicted by the numbers 1, 2, and 3, respectively, in Fig. 20.

Again, the amplitude control behavior is depicted in Fig. 21. It can be observed that under small disturbances, the imposed limits are not violated. However, during large disturbances, the control inputs tend to approach the constraint boundaries. Nevertheless, due to the Hildreth online optimization algorithm, the inputs are effectively constrained, preventing them from exceeding the prescribed limits. Again, the incremental control graph indicates that the control input initially tends to exceed the imposed limits during large disturbances. The online optimization mechanism of the MPC successfully limits rate variations, ensuring that constraints are not exceeded, as shown in Fig. 22. Again, due to these constraints, the dynamic responses of i_d^Δ and i_q^Δ are directly affected, thereby influencing the overall output response. For small disturbances, the impact of these constraints is relatively minor. However, i_d^Δ exhibits a more pronounced undershoot during larger disturbances, indicating the constraints' more substantial influence on the output current. As a result, we imposed both conventional and exponential weighting constraints. With this modification, the exponential-based response exhibits faster dynamics and a smaller overshoot than the cases with only conventional constraints and the unconstrained case, denoted by 1, 2, and 3, respectively. This improvement is clearly illustrated in Fig. 22. The corresponding control input trajectories under these constraints are shown in Fig. 22.

2) OUTPUT CONSTRAINTS

To further investigate the effect of output constraints, we successfully implemented the following limits on the

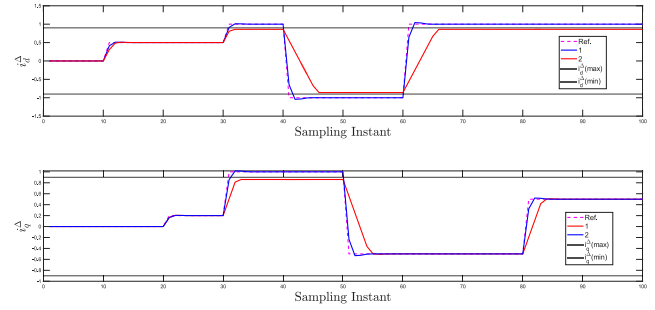


FIGURE 23. Plant output under 1 – constrained, and 2 – unconstrained over the sampling instants.

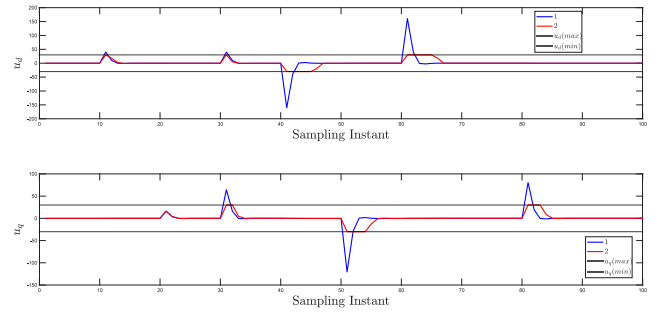


FIGURE 24. Control input under 1 – unconstrained, and 2 – constrained, over the sampling instants.

output.

$$|i_d^\Delta|_{\min, \max} = 0.9, \quad |i_q^\Delta|_{\min, \max} = 0.9.$$

The corresponding output responses, as well as the amplitude responses, are illustrated in Figs. 23–24. From these figures, it is evident that the output response strictly follows the imposed constraint limits, and in this case, no overshoot is depicted from the given set point due to the presence of the constraints on the output limit. For reference, the unconstrained responses have also been plotted in each case to highlight the improvement achieved by applying constraints; they are denoted as 1 and 2 in the corresponding figures. For the constrained case, two approaches are considered: the conventional constraints and the exponential-based constraint formulation. Based on the observations, we indicate that the exponential-based constrained MPC outperforms the conventional constrained MPC. Specifically, the exponential formulation results in lower overshoot and a faster dynamic response, thereby improving the overall transient behavior of the system. These advantages are consistently observed in both small- and large-disturbance cases.

F. TUNING FOR THE IMPORTANT MPC PARAMETERS

Variations in a and N show opposite effects on the system's output and closed-loop poles. Specifically, as N increases, the poles gradually converge toward those of the origin and DLQR eigenvalues, which are very close to the origin, indicating improved damping and stability characteristics. Conversely, increasing the value of a causes the dominant

poles to move closer to the unit circle. Moreover, for the same parameters, with nonexponential MPC, as we increase a , the pole's closed-loop eigenvalue moves toward the boundary of the circle, and it is on the boundary for $a = 1$. The boundary of the eigenvalues eventually leads to instability when a becomes excessively large, as illustrated in Figs. 25-26 of the revised article. This indicates that excessive weighting on the exponential basis functions (through a) can degrade robustness. It is also important to note that when $a = 0$, and the exponential factor is zero, then the given exponential weighting in the cost function disappears. The control law effectively reduces to the conventional MPC formulation, where the incremental control input (Δu) serves as the optimization variable. In this limiting case, the controller behaves equivalently to a standard MPC without exponential shaping, confirming the generalization capability of the exponential MPC approach. Moreover, using the exponential MPC, the unstable system remains stable over the same parameter range, meaning that the proposed system remains more robust to parameter tuning and maintains stability, as clearly shown in Figs. 25-26.

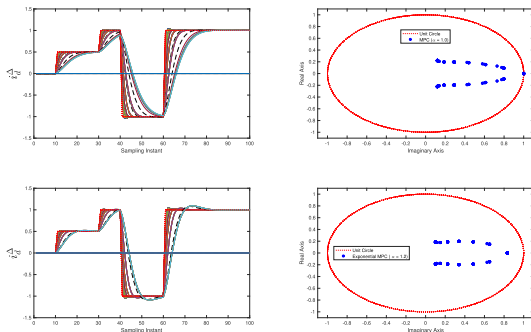


FIGURE 25. Effect of a on i_d^{Δ} and closed loop eigenvalues with and without exponential factor for the fixed $N = [10, 10]$ and variation.

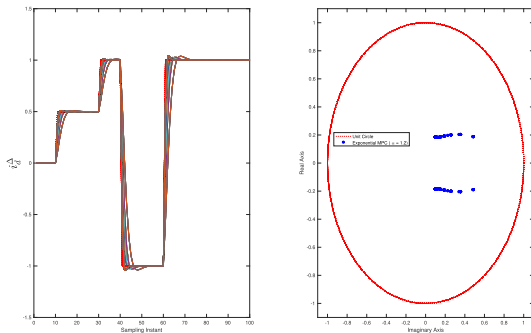


FIGURE 26. Effect of N on i_d^{Δ} and closed loop eigenvalues with and without exponential factor for the fixed $a = [0, 1]$ and variation.

Furthermore, comparative simulations were conducted using both conventional PI control and FCS-MPC. The results clearly show that the exponential-based MPC outperforms other controllers, particularly for larger prediction horizons N_p and sufficiently large exponential factors. The proposed approach achieves more accurate output-current tracking with significantly lower overshoot and reduced control effort

for both small and large set-point changes. The output current tracking performance of the exponential-based MPC in the dq -reference frame is consistently better than that of the PI and FCS-MPC controllers for all examined exponential weighting factors, including both short and large disturbance set-point changes. Moreover, the exponential MPC scheme provides flexible tuning capability by decreasing the R matrix weights and increasing the Q matrix weights. Together with proper selection of the exponential and Laguerre parameters, the exponential-based MPC controller can be tuned to achieve a faster dynamic response without sacrificing stability. The simulation results shown in Fig. 27 further support these observations.

Moreover, the performance indices for the FCS-MPC, conventional MPC, and PI are evaluated. For the FCS-MPC, IAE, ISE, and RMSE are 40.0267, 22.920, and 0.3227, respectively, and the control effort is 15.8210, which is higher than those of the proposed exponential in this case. Again, and for the PI controller, IAE, ISE, and RMSE are 23.6960, 22.71220, and 0.4766, respectively, and control effort is 0.80892. Moreover, the average overshoots are 10.1245, 43.501, and 17.5175 for the controllers PI, FCS-MPC, and Conventional MPC, respectively. Further, the average settling times are 0.45 and 0.136 for the PI and conventional MPC, respectively. In contrast, for our exponential-based MPC, the average overshoot and average settling time are 5.47 and 0.0170, respectively, outperforming those of other controllers (PI, FCS-MPC, and conventional MPC).

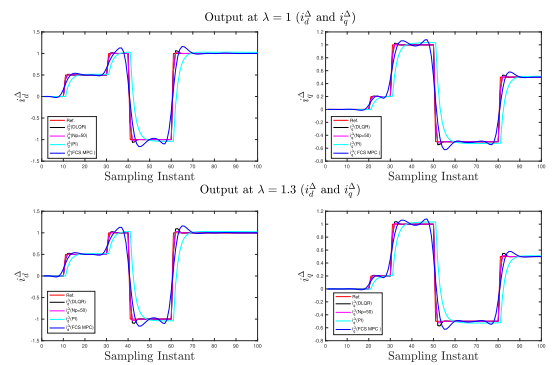


FIGURE 27. Case-study showing i_d^{Δ} current, i_q^{Δ} with different value of λ and different controller.

To see the effects of parameter variations and uncertainties. The simulation results shown in Fig. 28 also validate the claims which are discussed in the subsection IV-C for $a_l = 0.01$. In contrast, in the non-exponential case, the system is unstable, which is clearly visible from the first two subplots of Fig. 28. The graphical results clearly illustrate the improved robustness of exponential MPC compared to conventional MPC. Again, the performance indices for the noise disturbance:

With noise:

$$\text{IAE} = 16.700, \quad \text{ISE} = 11.908, \quad \text{RMSE} = 0.24017.$$

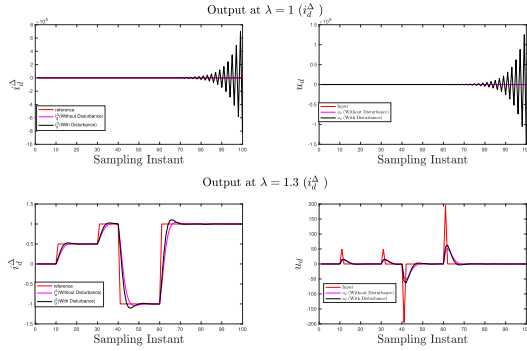


FIGURE 28. Output and input with and without exponential cases.

Without noise:

$$\text{IAE} = 15.607, \quad \text{ISE} = 11.908, \quad \text{RMSE} = 0.2399.$$

For this case, the performance indices under noisy conditions are very close to those obtained in the noiseless case, showing only slight degradation. This confirms that the exponential MPC is robust against realistic sensor and process noise levels.

G. SIMULATION RESULTS WITH PRESCRIBED DEGREE OF STABILITY USING THE FACTOR $\gamma = \frac{\mu}{\lambda}$

The design objective is to achieve the desired closed-loop response determined by a decay rate parameter μ , where $0 < \mu < 1$. The predictive controller is configured such that all closed-loop eigenvalues of the system are located within a disk of radius μ in the complex plane.

This condition is defined as for any initial state $x(k_i)$, the predicted future state will satisfy the following:

$$\|x(k_i + j | k_i)\| \leq \mu^j \|x(k_i)\|, \quad (35)$$

where j is the prediction step, this guarantees that the state norm decays at a rate of at least μ per step, imposing a specified level of stability and responsiveness in the closed-loop system. So far, in this case, the cost function:

$$J = \sum_{j=1}^{N_p} \lambda^{-2j} \mathbf{x}(k_i + j | k_i)^T Q_{\lambda} \mathbf{x}(k_i + j | k_i) + \sum_{j=0}^{N_p} \lambda^{-2j} \Delta u(k_i + j)^T R_{\lambda} \Delta u(k_i + j) \quad (36)$$

$\lambda > 1$, $0 < \mu < 1$, $Q \geq 0$, $R > 0$ are given, and is identical to the solution found by minimizing the original cost.

$$J = \sum_{j=1}^{N_p} \mu^{-2j} \mathbf{x}(k_i + j | k_i)^T Q \mathbf{x}(k_i + j | k_i) + \sum_{j=0}^{N_p} \mu^{-2j} \Delta u(k_i + j)^T R \Delta u(k_i + j)$$

$$\gamma = \frac{\mu}{\lambda}, \quad Q_{\lambda} = \gamma^2 Q + (1 - \gamma^2) P_{\infty}, \quad R_{\lambda} = \gamma^2 R.$$

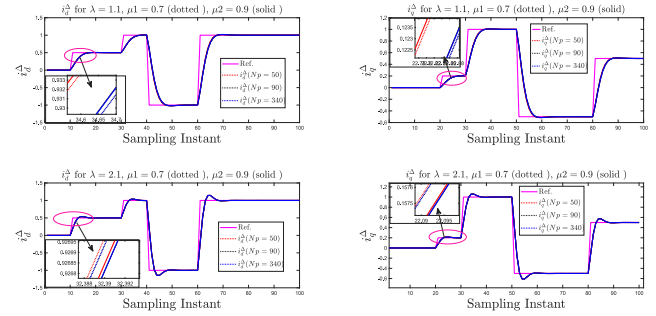


FIGURE 29. Case-study with unconstrained scenario showing i_q^{Δ} current, i_d^{Δ} with different value of λ .

Thus, the optimal parameters η can be determined by minimizing the following cost function, and the control law is obtained by minimizing the following exponential cost function:

$$\min_{\Delta \hat{u}} J_{\lambda} = \sum_{j=1}^{N_p} \hat{x}_m(k_i + j | k_i)^T Q_{\lambda} \hat{x}_m(k_i + j | k_i) + \sum_{j=0}^{N_c} \Delta \hat{u}(k_i + j)^T R_{\lambda} \Delta \hat{u}(k_i + j), \quad (37)$$

subject to

$$\hat{x}_m(k_i + j | k_i) = \lambda^{-1} A x_m(k_i) + \lambda^{-1} B \Delta \hat{u}(k_i), \quad (38)$$

$$u_{\min} \leq M \eta + u(k-1) \leq u_{\max}, \quad (39)$$

and P_{∞} is the solution of the algebraic Riccati equation:

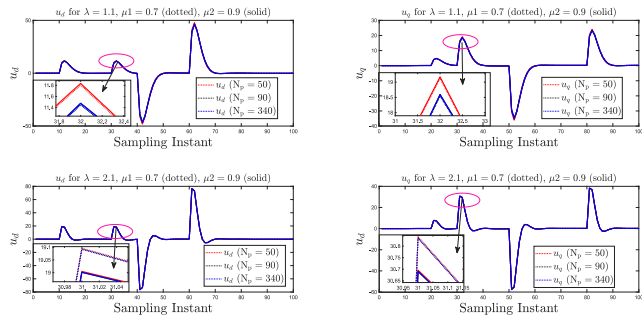
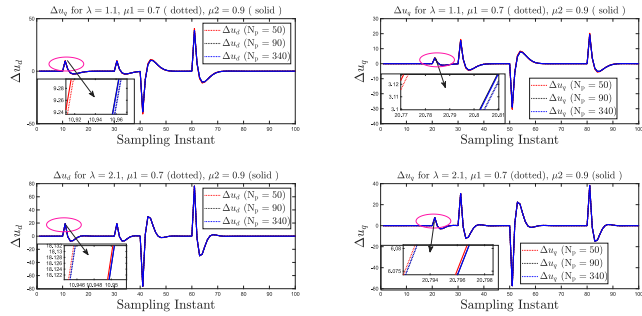
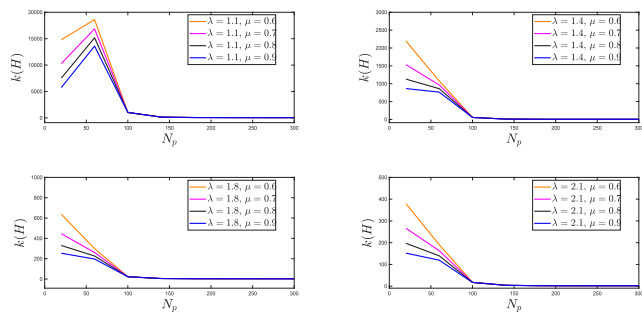
$$\frac{\hat{A}^T}{\gamma} \left[P_{\infty} - P_{\infty} \frac{\hat{B}}{\gamma} \left(R + \frac{\hat{B}^T}{\gamma} P_{\infty} \frac{\hat{B}}{\gamma} \right)^{-1} \frac{\hat{B}^T}{\gamma} P_{\infty} \right] \frac{\hat{A}}{\gamma} + Q - P_{\infty} = 0,$$

where the matrices $\hat{A} = \frac{A}{\lambda}$ and $\hat{B} = \frac{B}{\lambda}$. In this case, the condition number, the prescribed degree of stability, and the prediction horizon are presented in the Table 5.

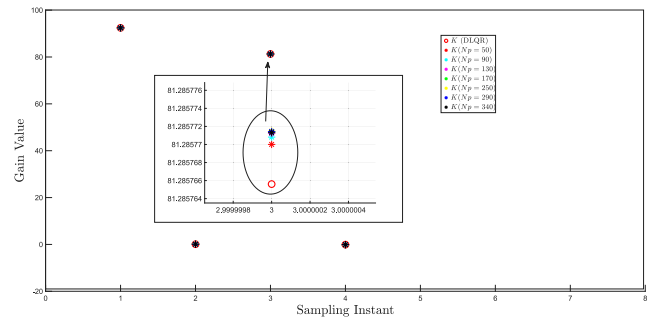
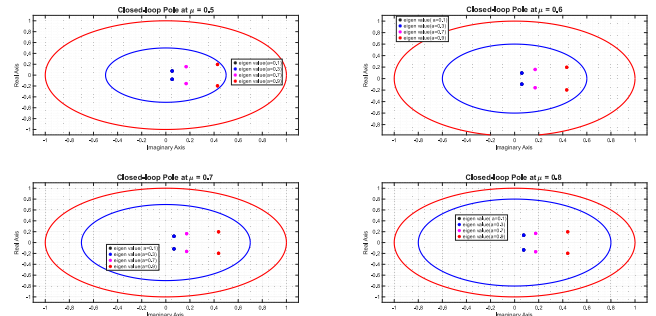
From Table 5, it is very clear that the condition number converges to some finite values with different exponential exponent factors and different μ values as the prediction horizon increases. Even for small values of λ , the condition number remains within two to three digits. The complete values of the condition number, κ of the Hessian matrix, are provided in Table 5. The validity of this claim is further confirmed in Fig. 32, which illustrates the behavior of the condition number for different combinations of the parameters λ and μ along with the large horizon. This modification is motivated by the observation that, in certain scenarios, the closed-loop eigenvalues tend to approach the unit circle boundary. To ensure the prescribed degree of stability, we impose that all eigenvalues remain strictly inside the circle of radius μ . Therefore, the scaling introduced through γ plays an important role in enforcing this stability margin.

TABLE 5. Eigenvalues and condition number of the Hessian matrix H .

λ	N_p	μ	$\tilde{\lambda}_{\max}(H)$ (max eigen value)	$\tilde{\lambda}_{\min}(H)$ (min eigen value)	$\kappa(H)$ condition number
0.8	40	0.8	6.175×10^{-1}	4.1×10^{-2}	1.4925×10^2
	80	0.9	1.0177×10^0	2.31×10^{-2}	4.4126×10^1
	300	0.8	1.1315×10^0	2.73×10^{-2}	4.1459×10^1
1	40	0.8	5.969×10^{-1}	8.5648×10^{-4}	9.8144×10^0
	80	0.8	1.0177×10^0	0.0020×10^{-4}	4.4762×10^0
	300	0.8	1.0942×10^0	0.0020×10^{-4}	4.4541×10^0
1.1	40	0.8	9.2×10^{-3}	9.9306×10^{-4}	9.8154×10^0
	80	0.8	9.9×10^{-3}	0.0022×10^{-4}	4.4755×10^0
	300	0.8	1.00×10^{-2}	0.0022×10^{-4}	4.4534×10^0

**FIGURE 30.** Case-study with unconstrained scenario showing u_d voltage, and u_q voltage with different value of λ .**FIGURE 31.** Case-study with unconstrained scenario showing Δu_d voltage, and Δu_q voltage with different value of λ .**FIGURE 32.** Condition number with different values of λ .

First, we present the results for unconstrained cases, and then we move to the constrained instances. From Fig. 29, it is evident that several set-point changes, both small and large disturbances, were applied under different parameter

**FIGURE 33.** Gain element of the DLQR and MPC with the different values of λ .**FIGURE 34.** Closed loop poles at the different values of the exponential factors λ along with sampling instant.

combinations of λ and μ , with a sufficiently large prediction horizon N_p . The results show that, for each value of N_p , the output responses in the dq frame closely coincide for different combinations of the exponent. This indicates that the prediction horizon is not a sensitive parameter in this case, and the output response remains consistent. However, a noticeable overshoot occurs during the large set-point disturbances, as shown in the last two subplots of Fig. 29, which shows that when we select a large value of the exponential factor, the response is aggressive and faster, and follows the set point more closely and has better performance than the non-exponential cases. The corresponding control and incremental control inputs for different values of the prediction horizon are illustrated in Figs. 30–31. It is clear that, for each prediction horizon, the control input and the incremental control input exhibit nearly identical

responses, with negligible differences. However, during the large disturbances, a significant peak appears in both the control and incremental input, clearly visible in Figs. 30–31. A clear representation of the closed-loop eigenvalues under different prescribed degrees of stability is also depicted in Fig. 34. The results show that the closed-loop eigenvalues adhere to the prescribed stability region. Specifically, when a sufficiently small prescribed degree of stability is selected, the closed-loop eigenvalues remain well inside that circle. This demonstrates a key advantage of the proposed approach: by specifying the degree of stability in advance, the system can maintain stable closed-loop performance. The closed-loop eigenvalues in Fig. 34, under different parameter combinations and prescribed degrees of stability, clearly validate our proposed theoretical claim.

Next, we proceed with simulations under constrained scenarios to evaluate the effects of constraints on both the output and the control input. In this case, we successfully address the rate, control input, and output constraints, as well as their prescribed limits. In addition, we claim that within this constrained framework, a sufficiently large prediction horizon can be employed without numerical issues, as the numerically improved conditioning of the Hessian matrix ensures robust performance.

1) CONSTRAINTS ON THE RATE AND AMPLITUDE

To analyze the effect of the constraints, we consider both the rate and amplitude constraints. So in this case, first of all, we consider the constraints limit on the incremental control given as $|\Delta u_d| = |\Delta u_q| = 30$ $|u_d| = |u_q| = 30$

and constraints on the amplitude limit are also the same as In this case, we observe that the rate of increase of the input voltages is lower than in the unconstrained scenario. Without constraints, both u_d and u_q tend to exceed acceptable limits. However, online optimization (Hildreth's method) strictly enforces the constraints, ensuring that the control parameters (rate and amplitude) do not exceed the imposed limits 30. The corresponding simulation results are illustrated in Figs. 34–35, showing both the output responses and the incremental control and amplitude trajectories. The results clearly demonstrate that both constraints affect the output, confirming that the imposed limits are satisfied while maintaining acceptable closed-loop performance. The effect of these constraints is that results in less overshoot than the unconstrained cases, and the figures clearly show that the unconstrained, constrained, and exponential-constrained cases give less overshoot than the other two cases in the dq current. Additionally, the corresponding rate and amplitude trajectories clearly follow the imposed limit.

2) OUTPUT CONSTRAINTS

Imposing constraints on the output is essential to enforce bounds on the current in the dq frame, ensuring it remains within the limits. This means that the current does not exceed the allowable maximum (or drop below the minimum), thereby preserving safety, preventing component stress,

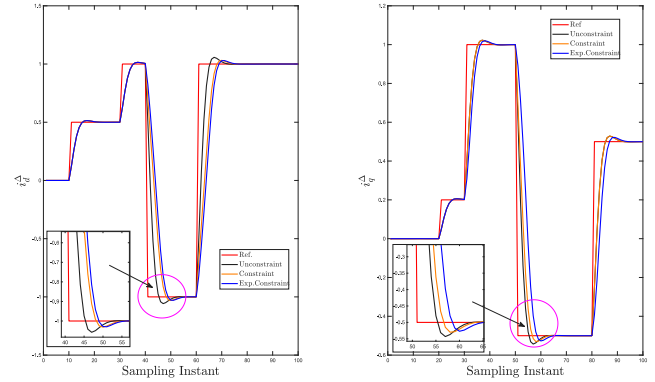


FIGURE 35. Plant output under exponential and non-exponential constrained MPC for different prediction horizons over the sampling instants.

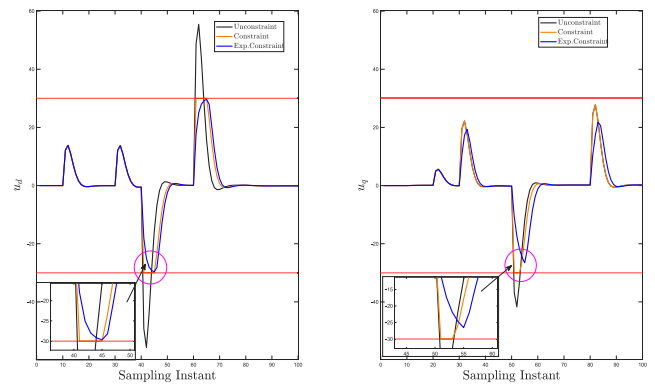


FIGURE 36. Control input under exponential and non-exponential constrained MPC for different prediction horizons over the sampling instants.

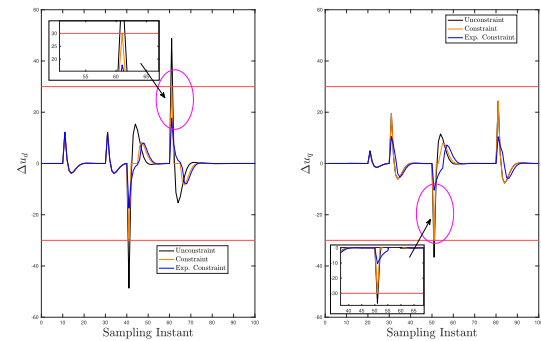


FIGURE 37. Incremental control under exponential and non-exponential constrained MPC for different prediction horizons over the sampling instants.

and maintaining regulatory compliance. From a practical perspective, imposing these constraints in the controller formulation causes the optimizer to reject any solution that violates the current bounds, so the implemented control action respects the admissible operating range. To examine their impact, we imposed the limits.

$$|i_d^{\Delta}|_{\min, \max} = 0.9, \quad |i_q^{\Delta}|_{\min, \max} = 0.9.$$

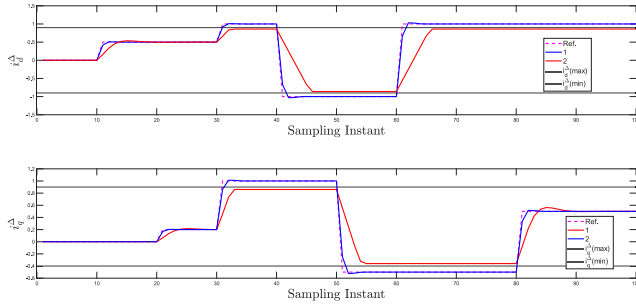


FIGURE 38. Plant output under 1 – unconstrained, and 2 – constrained over the sampling instants.

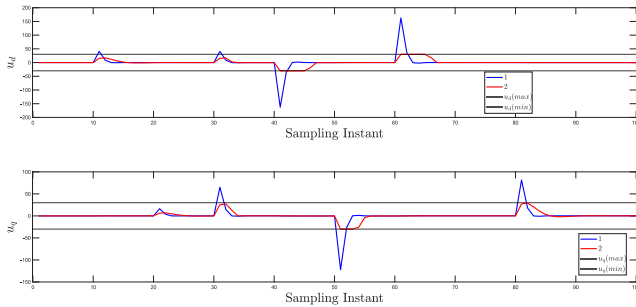


FIGURE 39. Control input under 1 – unconstrained, and 2 – constrained over the sampling instants.

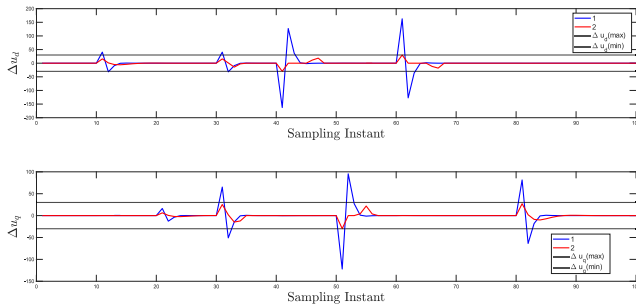


FIGURE 40. Incremental control under 1 – unconstrained, 2 – constrained over the sampling instants.

The output current trajectories, along with their corresponding rate and amplitude responses, are illustrated in Figs.38–40. These figures show that the outputs closely follow the specified bounds due to online optimization and exhibit less overshoot compared to the unconstrained scenario. For comparison, the unconstrained responses are included in each plot to emphasize the improvements in constraint satisfaction achieved by enforcing output constraints.

Again, the performance indices analysis for the exponential MPC with a different large N_p is shown in Table 6. This table presents the IAE, ISE, and RMSE for distinct N_p under varying exponential weighting factors λ and μ . Again, the control effort for this case is also evaluated for different N_p and combinations of μ and λ . For the conventional non-exponential MPC, the control efforts are 0.20699 and 0.19080 corresponding to $N_p = 50$ and $N_p = 90$, respectively. In comparison, for the proposed exponential in

TABLE 6. Performance indices for MPC with $\lambda = 1.1$, $\mu = 0.9$.

N_p	IAE	ISE	RMSE
50	17.360763	13.537608	0.260169
170	18.144776	14.274849	0.267160
340	18.144776	14.274849	0.267160

this case, with the exponential weighting factor $\lambda = 1.1$, $\mu = 0.7$, the control efforts are 0.17247 and 0.17050 for the same N_p . This represents reductions of 16.68% and 10.64% in control effort, respectively, through the exponential-based MPC. These results indicate that the proposed exponential achieves lower control effort than the conventional MPC. Therefore, the proposed controller is more efficient and reliable, offering reduced control effort and smaller condition numbers in the corresponding Hessian matrix. Again, the performance indices of exponential-based MPC with larger values of N_p are given in Table 6.

H. TUNING FOR THE IMPORTANT MPC PARAMETERS

Variations in a and N show opposite effects on the system's output and closed-loop poles. Specifically, as N increases, the poles gradually converge toward those of the origin and DLQR eigenvalues, which are very close to the origin, indicating improved damping and stability characteristics. Conversely, increasing the value of a causes the dominant poles to move closer to the unit circle. Moreover, for the same parameters, with nonexponential MPC, as we increase a , the pole's closed-loop eigenvalue moves toward the boundary of the circle, and it is on the boundary for $a = 1$. The boundary of the eigenvalues eventually leads to instability when a becomes excessively large, as illustrated in Figs.41–42 of the revised article. This indicates that excessive weighting on the exponential basis functions (through a) can degrade robustness. Moreover, using the exponential MPC, the unstable system remains stable over the same parameter range, indicating that the proposed system is more robust to parameter tuning and maintains stability, as clearly shown in Figs. 41-42.

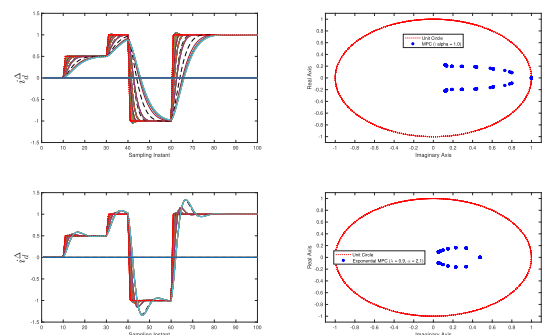


FIGURE 41. Effect of a on $i_d^Δ$ and closed loop eigenvalues with and without exponential factor for the fixed $N = [10, 10]$ and variation.

In this case, we also compared exponential MPC performance with the FCS-MPC conventional PI controller and

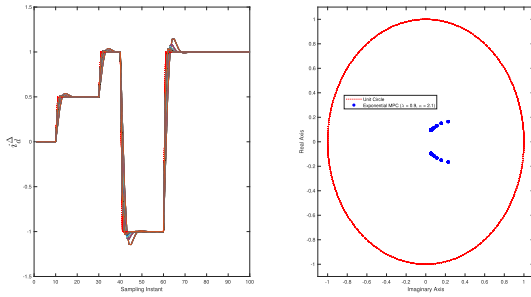


FIGURE 42. Effect of N on i_d^{Δ} and closed loop eigenvalues with and without exponential factor for the fixed $a = [0, 1]$ and variation.

the conventional MPC, using a sufficiently large prediction horizon. Furthermore, comparative simulations are presented for both conventional PI control and FCS-MPC to achieve better output performance and the prescribed degree of stability. The results clearly show that the considered exponential-based MPC outperforms other controllers in this case, particularly for larger prediction horizons N_p and sufficiently large exponential factors. The proposed approach achieves more accurate output-current tracking with significantly lower overshoot and reduced control effort for both small and large set-point changes. The output current tracking performance of the exponential-based MPC in the dq -reference frame is consistently better than that of the PI and FCS-MPC controllers for all examined exponential weighting factors, including both short and large disturbance set-point changes. Moreover, the exponential-based MPC scheme provides flexible tuning by considering several combinations of the exponential factors λ and μ . The simulation results shown in Fig. 43 further support these observations.

Moreover, the performance indices for the FCS-MPC and PI are evaluated. For the FCS-MPC, IAE, ISE, and RMSE are 40.0267, 22.920, and 0.3227, respectively, and the control effort is 15.8210, which is higher than those of the exponential-based MPC in this case. Again, and for the PI controller IAE, ISE, and RMSE are 23.6960, 22.71220, and 0.4766, respectively, and control effort is 0.80892. Again, the average overshoots are 10.1245, 43.501, and 17.5175 for the controllers PI, FCS-MPC, and Conventional MPC, respectively. Further, the average settling times are 0.45 and 0.136 for the PI and conventional MPC, respectively. In contrast, for exponential-based MPC, the average overshoot and average settling time are 6.43 and 0.0526, respectively, outperforming those of other controllers (PI, FCS-MPC, and conventional MPC).

I. MPC CLOSED-LOOP PERFORMANCE IMPROVEMENT VIA TUNING STRATEGIES

Closed-loop performance of exponential MPC is significantly influenced by the selection of the appropriate weighting matrices, Q and R , and Laguerre parameters, particularly when a sufficiently long N_p is used in the MPC selection.

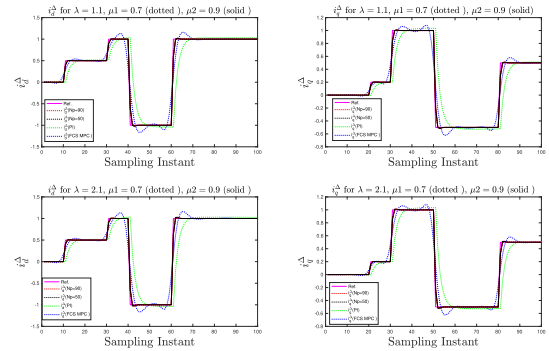


FIGURE 43. Case-study showing i_d^{Δ} current, i_q^{Δ} with different value of λ and different controller.

We have implemented two different tuning procedures given as follows.

1) TUNING STRATEGIES USING AN APPROPRIATE WEIGHTING MATRIX

Since the DLQR is well known as the optimal controller derived under the infinite-horizon framework, our exponential-based MPC can also handle arbitrarily large prediction horizons, as discussed in the previous sections. The objective here is to tune the MPC weighting matrices Q and R so that the closed-loop performance matches the DLQR and the MPC response. Specifically, we consider

$$Q = C^T Q_y C, \quad R > 0,$$

where Q_y is a diagonal matrix. A wider choice of Q_y penalizes deviations in the state variables more strongly, while a smaller value of R corresponds to a faster control response from the MPC. For this case study, we select

$$Q_y = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}, \quad R = 1 \times e^{-5}.$$

The corresponding output response, control input, and incremental control input are shown in Figs. 44-46. Additionally, the error between the DLQR and MPC outputs is plotted, demonstrating that the MPC response is almost identical to that of the DLQR, since the error between the two controllers is minimal in this configuration. These results demonstrate that the exponential-based MPC achieves the same level of performance and stability as the DLQR, validating the effectiveness of the constructed controller.

2) TUNING STRATEGIES USING LAGUERRE PARAMETERS

In this case, we aim to tune the Laguerre parameters so that the MPC performance matches the DLQR performance and tracking curve. By suitable choices of the a and N , the MPC response becomes identical to that of the DLQR and reference trajectories. The corresponding Figs. 47-49 validate the claim, which shows that the MPC, DLQR, and reference trajectories are almost similar. Additionally, we have included error plots for both controllers, indicating that the errors are

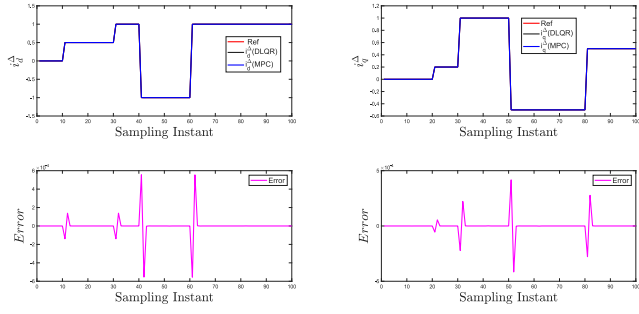


FIGURE 44. Case-study with unconstrained scenario showing i_d^{Δ} current, i_q^{Δ} with different value of λ .

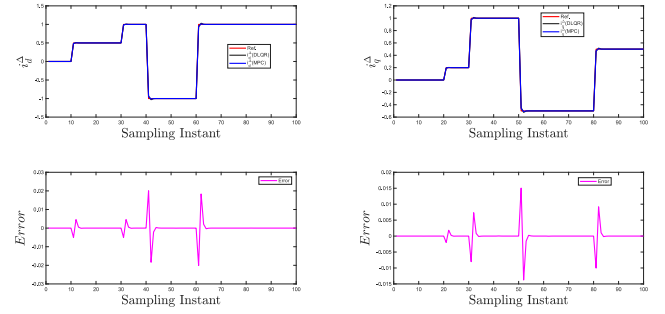


FIGURE 47. Case-study with unconstrained scenario showing i_d^{Δ} current, i_q^{Δ} with different value of λ .

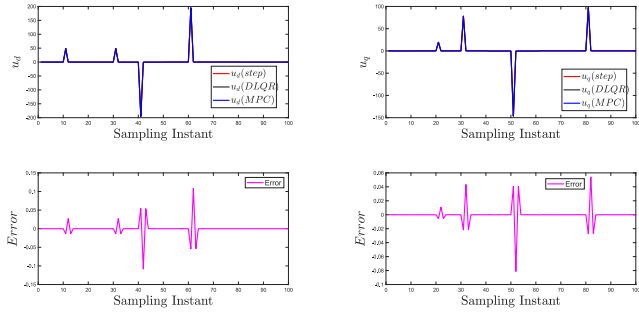


FIGURE 45. Case-study with unconstrained scenario showing u_d voltage, and u_q voltage with different value of λ .

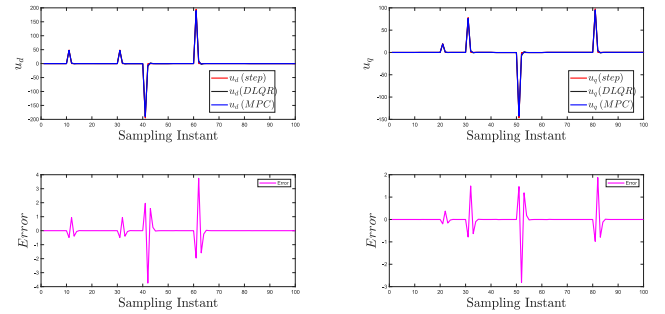


FIGURE 48. Case-study with unconstrained scenario showing u_d voltage, and u_q voltage with different value of λ .

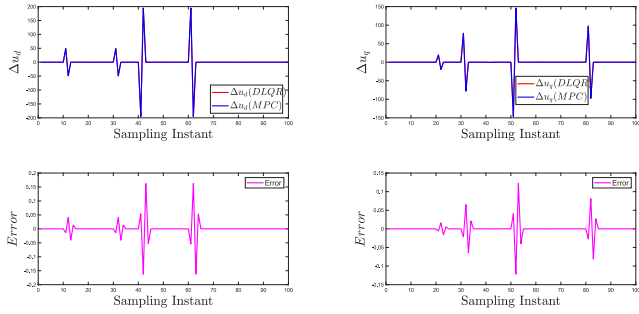


FIGURE 46. Case-study with unconstrained scenario showing Δu_d voltage, and Δu_q voltage with different value of λ .

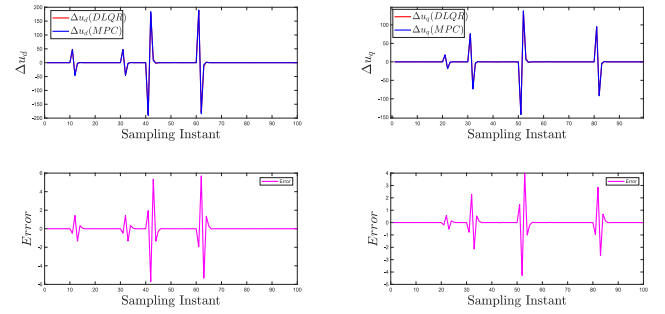


FIGURE 49. Case-study with unconstrained scenario showing Δu_d voltage, and Δu_q voltage with different value of λ .

minimal and can be practically disregarded at this stage. The optimal parameters for this configuration are as follows: $a = [0.3, 0.3]$, $N = 90$, $R = 1 \times 10^{-6}$, $Q = C^T C$, with the prediction horizon chosen from $N_p \in \{50, 90, 130, 170, 210, 290, 340\}$. These parameter values are used to validate the performance of the exponential-based MPC against the DLQR benchmark.

Remark 1: In this work, we analyze an MPC formulation with an exponentially weighted cost, in contrast to existing studies that adopt a time-invariant (non-exponential) quadratic cost. The exponential weighting reduces numerical ill-conditioning in the Hessian. It improves closed-loop performance, particularly for long horizons up to the DLQR horizon, whereas the non-exponential form typically yields larger condition numbers and numerical ill-conditioning.

Moreover, the proposed formulation strictly contains the conventional one as a special case. By selecting the factor $\lambda = 1$, the exponential cost reduces to the standard quadratic cost used in existing work, thereby establishing a direct equivalence.

Remark 2: In this article, we examine several exponential weighting scenarios, and the corresponding Hessian condition numbers are reported in several tables, demonstrating a consistent advantage over the non-exponential cost formulation across all tested scenarios.

V. CONCLUSION

In this study, we investigated the performance of an MMC plant model controlled by an MPC with an exponential-based cost function under both small and large disturbances.

More specifically, an exponential-based MPC algorithm is implemented in different scenarios. In each scenario, we tested the controller's performance and subjected it to various constraints in our systems. We observed that using an exponential-type cost function significantly reduces the computational burden and alleviates numerical ill-conditioning issues in the Hessian matrix, particularly for large prediction horizons. This advantage allows the prediction horizon to be extended to infinity, similar to DLQR controllers with a minimal Hessian condition number; it also handles different constraint cases. The simulation results show that the exponential cost function-based MPC achieves a faster dynamic response than the non-exponential counterpart. The system adapts rapidly to both small and large disturbances in the set point. Again, in this study, we investigated the performance of the designed MPC controller and tuned its essential parameters so that its behavior is almost identical to that of the optimal controller, DLQR. This article also examines the MMC plant's output of active/reactive power, output currents, and voltages at various rates, amplitudes, and output constraints, and confirms these findings through simulation results. The results confirm that the proposed controller maintains strong performance even under constrained conditions, further demonstrating the superiority of the proposed exponential-based MPC over non-exponential-based MPC and well-known DLQR controllers. Again, in this study, the prescribed degree of stability is also achieved, as demonstrated by simulation, in which the closed-loop eigenvalues are confined within a prespecified region of the unit circle. Furthermore, we tuned our MPC controller in two different ways to ensure that the predicted output exactly matches both the DLQR and the reference trajectories. In these cases, there is no overshoot or undershoot in the output responses, even during large disturbances in the set point. Again, we have obtained a faster response, with a significantly shorter settling time.

In our future work, we will focus on real-time implementation and hardware-in-the-loop testing of the exponential-based MPC. Again, we plan to develop a radial basis function neural network framework to address this type of cost function. As part of our upcoming research, we will implement and validate the neural network-based controller in a hardware-in-the-loop setup.

APPENDIX

In this section, we compared the designed cost function with the non-exponential cost function, and the detailed analysis is discussed as follows:

$$J_1 = \sum_{j=1}^{N_p} \lambda^{-2j} x(k_i + j | k_i)^T Q x(k_i + j | k_i) + \sum_{j=0}^{N_p} \lambda^{-2j} \times \Delta u(k_i + j)^T R \Delta u(k_i + j), \quad (40)$$

where the meaning of each term is already defined. Now, if you consider the cost function with $\lambda = 1$, then the above

equation becomes

$$J_2 = \sum_{j=1}^{N_p} x(k_i + j | k_i)^T Q x(k_i + j | k_i) + \sum_{j=0}^{N_p} \Delta u(k_i + j)^T \times R \Delta u(k_i + j), \quad (41)$$

Now, taking the difference between these two cost functions as

$$J_3 = J_1 - J_2 = \sum_{j=1}^{N_p} x(k_i + j | k_i)^T Q x(k_i + j | k_i) (\lambda^{-2j} - 1) + \sum_{j=0}^{N_p} \Delta u(k_i + j)^T R \Delta u(k_i + j) (\lambda^{-2j} - 1) \quad (42)$$

From here, the three cases will arise:

- Case-1: $\lambda = 1$ is corrected, then equations (40) and (41) are identical and there are no changes.
- Case-2: $\lambda \leq 1$ is corrected, this means that the exponential weighing factor λ^{-2j} , for $j = 1, 2, \dots, N_p$, has less effect on the current state of the MMC and has more effect on the future time due to increasing the weight as the sample increases.
- Case-3: $\lambda \geq 1$ is corrected, then in this case the exponential weighing factor λ^{-2j} , for $j = 1, 2, \dots, N_p$ places more weight on the current state at the current time and less effect on those at the future time due to the decrease in weight as the future sample increases. So this case is more reliable and more convenient for us to deal with in the whole theoretical part of the exponential-based cost function in this article.

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