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The effect of velodrome geometry on determining cyclists CdA: an explorative study



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Abstract

Aerodynamics are an important factor in competitive cycling and easy and realistic aerodynamic measurement methods are helpful for the riders and coaches. An indoor velodrome, minimizing the environmental effects, could be a suitable location for quick, reliable, and easy measurements on cyclist CdA. However, the effect of banked straights and bends on the CdA determination remains unclear. The goal of this study is to investigate how the track geometry impacts the CdA measurements.

Two methods for the calculation of CdA where compared, one that does and one that does not take track characteristics into account. For the measured CdA, no difference between the two methods was found. Additionally, a moderate intraclass correlation was found between the two methods and the drag measured in a wind tunnel (method 1: ICC = 0.76, method 2: ICC = 0.77). A minimal difference in drag of 7.6 Watts at a speed of 50 km/h was found to be accurately measured by both methods. In the bends, a significantly lower power output was found, resulting in a different CdA value between the bends and straights. This difference remains after taking the track characteristics into account.

It can be concluded that for velodrome aerodynamic drag measurements, the banking and bends do not have to be considered. In this study, no conclusion can be drawn on the found differences in CdA between the bends and the straights. Future research is needed to determine whether these differences are due to incomplete dynamics or a real difference in aerodynamics.

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1. Introduction

In elite cycling, speeds above 50 km/h are typically reached. At these velocities, aerodynamic drag contributes to approximately 90% of the cycling resistance forces (Oggiano et al., 2008). Aerodynamic drag is a product of air density, airspeed, and the drag area (CdA). Since the drag area can be influenced by the rider, the CdA is an important factor for improving a cyclist performance. Wind tunnels and computational simulations are known to accurately determine the aerodynamic drag. Both have their disadvantages and limitations. Suitable wind tunnels are scarce and expensive to use. Computational fluid dynamics require a high level of knowledge on fluid dynamics and need the right modelling, often validated with wind tunnel measurements, for accurate results (Crouch et al., 2017).

A more accessible method to determine aerodynamic drag is with the use of an on-site power meter. The power meter can measure the input force of the cyclist and after subtracting all forces from the input power, the drag force can be determined as the residual force. By determining the drag while cycling, this method is considered a true to nature measurement. This true to nature environment also comes with the downside of uncontrollable circumstances, like weather conditions (Barry, 2018). To minimalize the effects of environmental conditions, aerodynamic testing can be done on an indoor velodrome. However, on a velodrome the cyclist must ride on a banked surface and through bends, which can complicate the mathematics needed to calculate the CdA.

To the best of the authors knowledge, five mathematical models of track cycling are found that can give insight in the effects of the track geometry on the cyclist (Caddy et al., 2015; Fitton & Symons, 2018; Lukes et al., 2006; 2012; Underwood & Jermy, 2010, 2014). To ride on a banked surface, the cyclists must steer into the slope inducing a scrubbing effect on the tires and increasing the rolling resistance (Lukes et al., 2006). In the bends the banking angle increases, resulting in a larger scrubbing effect compared to the straight. Although with difference in magnitude, the increase of rolling resistance in the bends is mentioned by multiple mathematical models (Caddy et al., 2015; Lukes et al., 2012; Underwood & Jermy, 2010 & 2014). Additionally, in the bends the cyclist needs centripetal force to change the direction of the centre of mass. This centripetal force increases the resulting force on the tires, thereby increasing the rolling resistance (Grappe et al., 1999).

At last, to balance the centripetal force, the cyclist must move their centre of mass into the turn. Due to this inward lean, the centre of mass travels a shorter distance than the wheels. Consequently, the speed of the centre of mass is lower than the speed of the wheels. Because it can be assumed that the centre of drag, the application point of the resulting drag forces, has an equal position to the centre of mass it can be important to use the centre of mass speed to calculate the cyclist CdA (Caddy et al., 2015).

Two CdA determination protocols especially designed for track testing are found. To address the differences between the bends and the straights, Martin et al. chooses to model the track as a circle with the track length as circumference (Martin et al., 2006). Although a clear explanation on the effects of modelling the track as a circle is missing, this protocol is used by multiple other researches (García-López et al., 2014; Valenzuela et al., 2020). The second model by Fitton et al. takes the real track dimensions to determine the CdA (Fitton et al., 2018). No explanation on the differences between the bends and straights is given.

Both models correct for the speed differences between the bends and the straight, but no attention is given to a possible power difference. According to the (unpublished) observations of Craig et al., the power output in the bends is lower compared to the power output on the straights (Craig & Norton, 2001). A lower power output with an equal speed would suggest a

decrease in either potential energy or a decrease of drag. None of the CdA determining protocols takes potential energy or a different CdA value into account.

The aim of this research is to investigate the effects of the velodrome track geometry (banked surfaces and bends) on the determination of CdA. Therefore, the effect of dynamical differences between the straights and the bends on the CdA value will be investigated. The results of this study can reveal if track specific mathematic calculations are essential for determining a reliable CdA value.

2. Method

The track measurements were performed in an indoor 250-meter velodrome (Omnisport, Apeldoorn, The Netherlands). The tests were conducted on two separate days but within three days of each other. To minimize the influences of the environment, the climate control system of the velodrome was shut off and all doors were closed. The wind effect of riders on the track was minimalized by only letting one rider on the track during the measurements. All measurements were carried out by an elite male cyclist (age 31, mass rider and bike 91 kg) who is a multiple Dutch champion on the track. Before the testing started the rider was informed about the procedure and the possible dangers and has signed an informed consent.

To determine the effects of the environment, a kestrel 5100 racing (Kestrel, Boothwyn PA, United States) was used to measure the temperature, humidity, air-density, and airflow inside the velodrome during the runs. The Kestrel was placed at two different locations, i.e. halfway into the turn and halfway on the straight.

The athlete performed the measurements on his own track bike, a KOGA KINSEI (Koga, Heerenveen, The Netherlands) fixed gear bike with a 54/15 gear ratio. The bike was equipped with a statically calibrated SRM (SRM, Jülich, Welldorf, Germany) crank located power meter and a SRM magnet-based speed sensor on the wheel to measure the power output and speed of the bike during the runs. The front wheel of the bike was a Pro five-spoke carbon wheel (Pro, Eindhoven, The Netherlands). The rear wheel was a Pro carbon disc wheel. Both wheels were equipped with tubular tires inflated to 11*10⁵ Pa (Figure 1).

A Mylaps Prochip (MYLAPS, Haarlem, The Netherlands) was used to record the passing time of seven different locations during a lap (Figure 2).

The lean angle of the bicycle was measured by a Shimmer inertia measurement unit, (Shimmer, Dublin, Ireland)

The rider was wearing a race suit from the KNWU (BioRacer, Tessenderlo, Belgium) and an Lazer Victor helmet (Lazer, Antwerpen, Belgium). Because gear and equipment can have a large influence on the aerodynamics of a cyclist (Crouch et al., 2017), the same gear and equipment was used during the measurements.



Figure 1 Koga bike with rod and discs

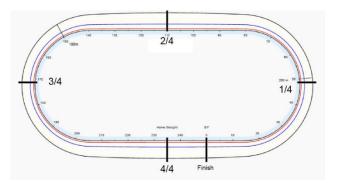


Figure 2 Locations of Mylaps passing points

A total of seventeen runs of six laps over two days were performed. Before each day, a checklist was used to make sure everything was set up in the same way (appendix 7.1). The tire circumference was measured after the tires were inflated with the rider on the bike.

To minimalize the change in CdA by the rider position, the rider was asked to take his usual aerodynamic position during all the runs (Figure 3). The cyclist was asked to cycle at a speed he can easily maintain for 6 laps (14 m/s), to avoid position changes due to fatigue. It is assumed that an elite cyclist has a better than average ability to maintain and retake a certain cycling position. All runs were filmed to be able to perform post testing checks on the position of the rider.



Figure 3 Cyclist position for the measurements

For reliable aerodynamic measurements it is important that

the standard deviation between multiple equal measurements is as low as possible. To measure the repeatability, all five runs on day one were performed without adding drag. At day two, again two runs in this aerodynamic position were performed to evaluate the effect of a different testing day on the repeatability of measurements.

To determine the minimal change in aerodynamic drag that can be measured by a method, drag was added to the bike. Six configurations were tested with an increasing amount of aerodynamic drag. For each configuration, two runs were performed. To alter the aerodynamic drag of the bike a rod was placed to the steering unit of the bike, discs of multiple sizes (50,60, 80 and 100 mm) were attached to this rod. An overview of the different configurations is shown in the appendix (appendix 7.2, Table 4). The rod is a wing shaped profile to minimize the added drag of the rod itself. These discs were placed at one meter from the centerline of the bike to not be influenced by the wake of the cyclist

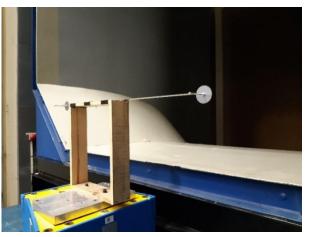


Figure 4 Wind tunnel setup for rod and discs aerodynamic drag

(Spoelstra et al., 2019). Because the maximum weight of the aluminium rod and the two 100 mm discs was only 350 grams, adding the rod and discs to the system will only alter the aerodynamic drag and not create extra rolling resistance

To measure the amount of added drag, the rod and the six discs were measured in a closed loop open jet wind tunnel (TU Delft, The Netherlands) with an octagonal exit of 2.85x2.85 m² (Figure 4). The discs were placed in a room of 13 meters in width by 8 meters of height. To keep the environment steady during the measurements a cooling installation was used to avoid rising temperatures. All configurations were tested at a speed of 14 m/s. The results of the wind tunnel measurements are shown in a table (Table 1).

Table 1 Results of wind tunnel measurements on the different discs

Configuration	Balance	Rod	50 mm	60 mm	80 mm	100 mm
CdA m ²	0.013	0.014	0.018	0.020	0.025	0.032

2.1. Data processing

A total of five different data signals was used during the measurements, namely the power output on the crank, the cadence of the crank, the speed of the wheels, the passing times of seven different transponders, and the roll angle of the Shimmer IMU sensors. All measured data were saved in a datetime format to link them to the transponder passing times. The speed, power and cadence data have a frequency of one Hz. The IMU data measured at a frequency of 51.2 Hz. After evaluating the IMU data, it was dismissed due to unreliable outcomes and not further used in calculations in this experiment.

During the third, fourth and fifth, runs, the speed sensor signal produced incorrect values. For those runs the speed was calculated with the cadence from the power meter, the gear-ratio, and the wheel circumference. Missing speed and power datapoints were interpolated with a cubic interpolation.

The bends and straights were separated with the help of the Mylaps passing times. A custom Matlab function was used to estimate the starting and end time of the bends and straights by calculating a predefined distance from the halfway transponder passing time (appendix 7.4.2). The start of the bend was defined to be 24 meters from the halfway point. This point was taken because the bend resembles a circle with a radius of 23 meters from this point. The length of the straight was set at 34.65 meter. The radius of curvature between the end of the straight and start of the bend was unknown, therefore these transition parts on the track of 21.23 meter were not considered.

Because the IMU sensor is dismissed, the speed of the centre of mass was calculated from the wheel speed. For this calculation it was assumed that the rider is in an equilibrium in the bends and has no instantaneous change in roll angle. Then the force of the weight of the cyclist and the centripetal force must create a zero-net moment at the contact point of the tire with the track (point A), (Figure 5). In Figure 5, φ equals the roll angle, v the measured wheel speed, R the radius of curvature and g the gravitational constant. Note that both the roll angle and the centripetal force

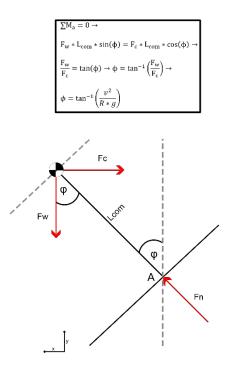


Figure 5 Free body diagram of cyclist in the bend, with the calculation of the roll angle from wheel speed

contain the roll angle variable, therefore an iterative process was used to determine the roll angle from a starting value of 30 degrees, see the appendix (appendix 7.4.3) for the Matlab function used to calculate roll angle.

The speed of the centre of mass was calculated with equation 1.

$$v_{cm} = v_{wh} - \frac{v_{wh}}{R} * L_{cm} * \sin\left(\phi\right) \tag{1}$$

To get to a smooth transition between no roll angle on the straight and the maximum roll angle in the bends, only one point halfway into the bend was used to calculate the speed of the centre of mass. With a cubic interpolation the speed profile of the rest of the bend was determined. This method yields high correlation (>0.95) between the measured and calculated wheel speeds and was therefore assumed to have equal correlation with the actual speed of the centre of mass. A plot of the measured and calculated speed profile is found in the appendix (appendix 7.3).

Two different mathematical calculations were performed on the data, namely method 1 (M1) and method 2 (M2). Both methods are based on the same energy principle, but M1 does not take any track geometry into account. Equation 2 shows the general formula for both methods.

$$\eta P_{in} = \frac{\Delta EK}{\Delta t} + \frac{\Delta EP}{\Delta t} + \frac{\Delta E_{losses}}{\Delta t}$$
(2)

The delivered power by the cyclist was multiplied by the efficiency of 0.98, accounting for the losses of the drivetrain, the flexing of the frame and the bearings (Kyle, 2003). Any build-up of heat during the measurements was neglected and therefore all input power that was not changing the kinetic or potential energy was lost due to rolling or drag resistance. As the rider starts and finishes his run at the same position, the net change in potential energy during the measurements was assumed to be zero. The power needed to overcome the rolling resistance was calculated by equation 3. The coefficient or rolling (C_{rr}) of 0.002 was used for calculating the rolling resistance (Kyle, 2003).

$$P_{crr} = F_n * c_{rr} * v_{wh} = (m * g * c_{rr}) * v_{wh}$$
(3)

The power needed to overcome the aerodynamic drag force was determined with equation 4.

$$P_{drag} = 0.5 * \rho * C dA * v_{air}^2 * v_{cm}$$
⁽⁴⁾

During all measurements, no air flow was measured. Therefore, the velocity of the air around the cyclist is equal to the velocity of the centre of mass.

2.1.1. Method M1

Combining equation 2,3 and 4, the CdA with M1 was determined by equation 5. The variables used for the CdA calculation are found in the appendix (appendix 7.2).

$$CdA = \frac{\left(\eta \overline{P_{ln}} - \frac{\frac{1}{2}m\left(v_{wh_{end}}^2 - v_{wh_{start}}^2\right)}{duration_run} - (m * g * c_{rr}) * \overline{v_{wh}}\right)}{\frac{1}{2} * \rho * \overline{v_{wh}}^3}$$
(5)

2.1.2. Method M2

For method 2 the track geometry was considered. On the straights, this resulted in a different rolling resistance due to scrubbing of the tire on the banked surface. In literature it is found that scrubbing increases the rolling resistance on the straight by 9.7% corresponding with a scrubbing coefficient of Cs = 1.097 (Kyle, 2003). In the bends, the rider therefore also experienced an increase in rolling resistance due to scrubbing. The scrubbing coefficient for the bends was calculated with the following equation, using $\mu_s = 0.0072$ and a camber angle $\sigma = 43$ degrees (Lukes et al., 2012).

$$C_s = 1 + \sigma \mu_s \tag{6}$$

This resulted in an added rolling resistance of 30.96% in the bends due to scrubbing. Apart from the added rolling resistance due to scrubbing, there is also an increase in rolling resistance due to the centripetal force, creating a larger normal force in the bends. As already discussed, the cyclist has a lean angle towards the centre of the turn. Hence, the centre of mass of the cyclist covered less distance than the wheels, thus the centre of mass had a lower speed then the measured wheel speed. When taking the track characteristics into the mathematical equation, it was important to correct the speed of the wheels to the speed of the centre of mass of the cyclist. Equation 7 shows method M2.

$$CdA = \frac{\left(\eta \overline{P_{in}} - \frac{\frac{1}{2}m(v_{cm_{end}}^2 - v_{cm_{start}}^2)}{duration_{run}} - \left(m * \frac{\overline{v_{cm}}^2}{R} * \sin(\phi) + m * g * \cos(\phi)\right)c_{rr} * c_s * \overline{v_{wh}}\right)}{\frac{1}{2} * \rho * \overline{v_{cm}}^3}$$
(7)

2.2. Statistics

The data are presented as mean \pm SD. The repeatability is assessed by the coefficient of variation, which displays the percentage range of datapoints around the mean (Hopkins, 2000). The lower the coefficient of variation, the closer all measurements lie around the mean. The intraclass correlation coefficient (ICC) is used to determine how well the track measurements are correlated with the wind tunnel measurement. The ICC gives a value between zero and one, where one would suggest a perfect correlation (Hopkins, 2000). A one-way ANOVA between subjects with a post-hoc Tukey is performed to compare the means of the runs with different discs (Atkinson & Nevill, 1998). A p-value < 0.05 is considered significant. The means of the two methods are compared with a t-test, where a p-value < 0.05 is considered significant (Atkinson & Nevill, 1998). A Pearson's correlation coefficient (r) is used to determine the agreement between the bend or straight and the whole lap CdA value (Hopkins, 2000).

3. Results

The five repeated trials of the first day were performed at a speed of 14.2 ± 0.35 m/s with a power output of 365 ± 28 W. The two runs without added drag on day two were performed at a speed of 14 ± 0.24 m/s and a power output of 338 ± 30 W. The twelve trials on day two, with the six different drag configurations, were performed at a speed of 13.8 ± 0.31 m/s and a power output of 343 ± 30 W.

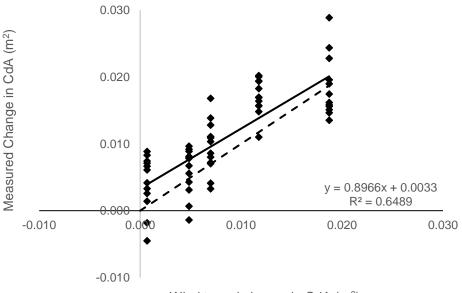
3.1. Simple CdA calculation method

Without the consideration of track geometry, using equation 5, the repeated measurements on day one resulted in a CdA of $0.196 \pm 0.005 \text{ m}^2$. The same configuration on day two gave a CdA of $0.192 \pm 0.005 \text{ m}^2$. The intraday coefficient of variation (CV) on day one was 2.5%. A t-test between the CdA of day one and day two showed a significant difference between the means of the trials with the same configuration (*t*(38) = 2.52, *p* = 0.02). After the repeated measurements drag was added to the bike, resulting in increasing CdA values (Table 2).

Table 2 Results of track measurement M1 CdA values of the cyclist with different drag configurations

Configuratio	on Drops	Rod	50 mm	60 mm	80 mm	100 mm
CdA m ²	0.192 ± 0.005	0.196 ± 0.004	0.197 ± 0.004	0.201 ± 0.004	0.209 ± 0.003	0.210 ± 0.005

Each drag configuration was also tested in a wind tunnel and the increase in CdA after adding the discs was used to compare the results between the wind tunnel and the track test. A scatterplot of the track measured CdA values (y-axis) and the corresponding wind tunnel values (x-axis) gives insight in the spread of measured CdA around the expected CdA value (Figure 6). A moderate degree of reliability (ICC = 0.76) was found between M1 and the wind tunnel measurements.



Wind tunnel change in CdA (m²)

Figure 6 Track measured CdA values M1 (y-axis) against the wind tunnel CdA values (x-axis). The CdA value is the amount of added CdA between the configurations. The solid line represents the regression line of the track measurements. The dotted line represents the identity line.

3.2. Measured differences straights and bends

The measured speed in all runs showed an increase in wheel speed during cornering compared to the straight (Figure 7). After calculating the centre of mass (CM) speed, by correcting for the lean angle, the speed in the bend decreased and resembled the speed on the straight (Figure 7).

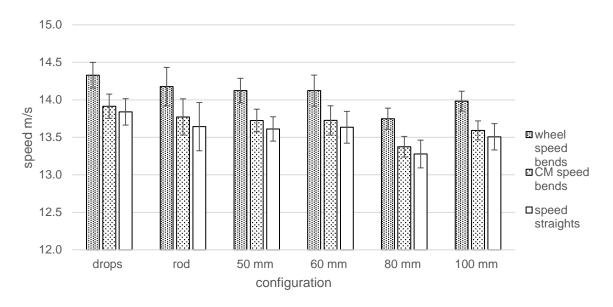


Figure 7 Mean ± SD speed in the bend, on the straight and after adjustment to centre of mass speed. The speed in the bend was high compared to the straight, this effect decreased after adjustment to CM speed.

Along with the speed, the results of the power output in the bends and on the straights showed a difference (Figure 8). A significantly lower power output (t(257) = -8.7, p < 0.005) was found in the bends (M = 335 Watt, SD = 26) compared to the power output on the straights (M = 354 Watt, SD = 28).

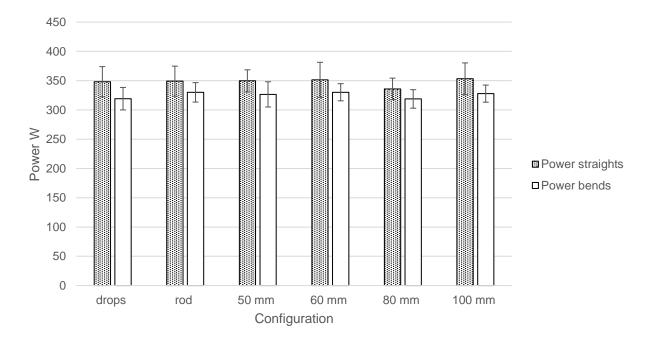
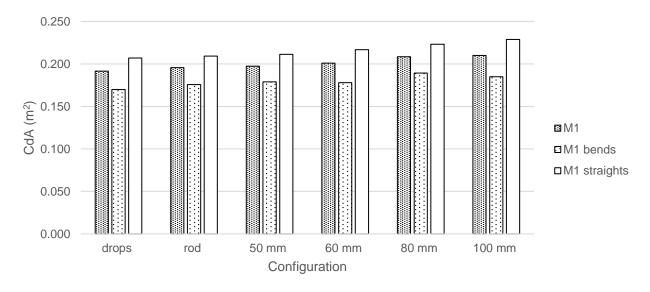
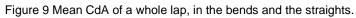


Figure 8 Mean \pm SD of power output in the bends and in the straight for each configuration.

The lower power output in the bends, combined with an equal centre of mass speed resulted in a different CdA value (Figure 9). With a significantly lower Cda (t(20) = -10.5, p < 0.005) in the bends (M = 0.180 m², SD = 0.007) than on the straights (M = 0.216 m², SD = 0.01).





3.3. Differences after incorporation of velodrome characteristics

With the consideration of track geometry, using equation 6, CdA values for method M2 were calculated (Table 3). The CdA calculated with method 2 ($M = 0.197 \text{ m}^2 SD = 0.005$) was found to be comparable to the CdA calculated with method 1 ($M = 0.196 \text{ m}^2$, SD = 0.005), with no significant difference found (t(54) = -1.44, p = 0.08).

Table 3 Results of track measurement M2 CdA values of the cyclist with different drag configurations

Configuration	Drops	Rod	50 mm	60 mm	80 mm	100 mm
CdA m ² M1	0.192 ± 0.005	0.196 ± 0.004	0.197 ± 0.004	0.201 ± 0.004	0.209 ± 0.003	0.210 ± 0.005
M2	0.193 ± 0.004	0.196 ± 0.004	0.199 ± 0.005	0.203 ± 0.004	0.210 ± 0.003	0.212 ± 0.005

A scatterplot shows the comparison of M2 with the wind tunnel measurements (Figure 10). A moderate degree of reliability (ICC = 0.77) was found between M2 and the wind tunnel measurements. A high degree of reliability (ICC = 0.97) was found between M1 and M2.

A one-way between subject ANOVA was performed to compare the CdA values from the different disc configurations. For both methods a significant effect of adding discs was detected on the CdA (M1: F(5, 66) = 43.17, p < 0.005, M2: F(5, 66) = 40.76, p < 0.005). Both methods were able to distinguish the same discs. With a post-hoc Tukey indicating that the mean CdA of the dropped position (M = 0.192 SD = 0.005) differed significantly to the configuration with 50 mm discs (M = 0.197 SD = 0.004). At 14 m/s the wind tunnel measurements show a difference of 7.6 W between those configurations. Smaller differences were not detected.

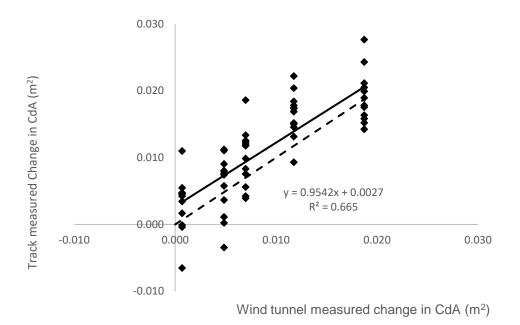


Figure 10 Track measured CdA values M2 (y-axis) against the wind tunnel CdA values (x-axis). The dotted line represents the identity line.

The regression analysis shows that after taking account for the speed of the centre of mass and the centripetal force, the difference in CdA between the corner and straight remains (Figure 11). The CdA in the bend shows a weak Pearson correlation coefficient with the CdA calculated for one lap, this was found for both M1 (r(10) = 0.63) and M2 (r(10) = 0.56). For the straights the Pearson correlation coefficients showed a strong positive correlation (M1 r(10) = 0.91, M2 r(10) = 0.93) with the mean CdA of a lap.

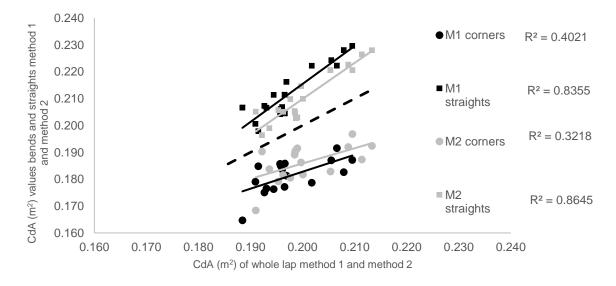


Figure 11 Regression analysis of M1 and M2 on the difference between the straights and bends.

4. Discussion

The aim of the study was to investigate the effect of velodrome bends on the determination of CdA. Therefore, two different methods that were used to calculate the CdA values from the same data. The first method (M1) does not take any track geometry (bends and banking) into account. Method 2 (M2) does include track geometry by correcting the wheel speed with the rolling angle to get the speed of the centre of mass and include extra rolling resistance due to the centripetal force and scrubbing. The results of the study show no difference (p = 0.08) in final CdA value between the two methods.

The small coefficient of variation (M1: CV = 2.5 M2: CV = 2.3) between five repeated measurements suggests a high repeatability of CdA values. The ICC between both M1 and M2 methods with the wind tunnel measurements show a moderate relationship (M1: ICC = 0.74, M2: ICC = 0.77), suggesting that both methods are capable for measuring differences in drag. The ANOVA test shows that both M1 and M2 can measure differences in drag of 7.6 W and more at a speed of 50 km/h.

The consequence of this finding is that track characteristics are not important to measure differences in aerodynamic drag on a velodrome. This makes it relatively easy for riders and coaches to perform aerodynamic measurements on a track with only a power meter and without a thorough understanding of the physics of track cycling. Between day one and day two the means of the runs with the same configuration differ significantly, possibly due to the use of different devices for date storage. Therefore, it is advised to only compare aerodynamic differences found on the same day.

It is also found that the power output in the bends is significantly lower compared to the straights, resulting in a significantly lower CdA value in the bends with the used methods. However, M2 seems to be able to decrease the mean difference between the measured CdA in the bends and the straights suggesting that the centripetal force and scrubbing effects explain a part of the initial CdA fluctuation between the bend and the straight. A regression analysis shows that the CdA values of the bends alone are poorly correlated with the mean CdA of a lap. On the contrary, the CdA values of the straights show a high correlation. These results suggest that method M1 and M2 are not reliable methods to calculate the CdA value while cornering.

The lower power output in the bends confirms the observations made by Craig & Norton (Craig & Norton, 2001). However, no literature on mathematical models for track cycling mentions a difference in power output with track position (Caddy et al., 2015; Fitton & Symons, 2018; Lukes et al., 2006 & 2012; Underwood & Jermy, 2010 & 2014). Lukes et al. however does mention an interview with a former world champion who says cycling on the straights feels like going uphill compared to the bends (Lukes et al., 2012). They suggest this feeling is caused by the difference in speed between the wheels and the centre of mass due to the rolling angle. In contrast, our study showed that the higher wheel speed cannot solely explain the differences between cornering and riding on the straights.

The study has at least three major limitations. First, the frequency of the measured datapoints is 1 Hz. At speeds of 14 m/s this means that in bends, being 48 meters, only a maximum of three datapoints describe the effects of the whole bend. A higher frequency of data capturing, ideally at least a datapoint every meter (frequency of 14 Hz or more), could give a more insightful picture of the bend. Second, another limitation is the determination of the roll angle and consequently the speed of the centre of mass. Due to the dismissing of the IMU data, assumptions were used to come up with a continuous speed of the centre of mass. Therefore, it is possible that interesting data got lost and some care should be given to the results found

with this centre of mass speed. The last limitation is the method of measuring the added drag with a wind tunnel. As discussed in the introduction, wind tunnels are highly accurate but are not as realistic as cycling on a track. The measured added drag of the discs was determined with a very low level of turbulence in the wind tunnel. On the track higher levels of turbulence are expected, possibly changing the aerodynamic drag. In the wind tunnel the discs were measured perpendicular to the airstream. On the track, especially in the bends, the incoming airflow might have a different angle of attack to the discs. Additionally, the speed of the left disc will be different than the speed of the right disc when the cyclist is cornering. This might also change the value of the expected drag increase.

In addition to the last limitation, it is known from literature that the effects of cornering a velodrome have a major influence on the direction of the airflow on the cyclist (Fitzgerald et al., 2019). As change in the direction of the airflow will change the CdA value, a difference in CdA between the bends and straights is likely (Barry, 2018). Although both measurement methods show a difference in CdA values between the bends and the straights, no real conclusions can be drawn from this study. As the CdA is not measured directly but is the residual term when all known effects are taken account of, there is a possibility of missing terms in the equation. Also, there is no explanation on the lower power output on the bends and straights could have multiple benefits. The knowledge of a difference in aerodynamic drag could improve the mathematical models on track cycling and thereby improve pacing strategies. Furthermore, the design and measuring techniques of new aerodynamic equipment could be improved. The understanding of difference in power output can also improve pacing strategies but can also help the coach with more specific trainings.

A future aerodynamic study on a velodrome with the 'Ring of Fire' system could answer the question on difference in CdA on the straights and into the bends (Spoelstra et al., 2019). To know whether potential energy effects can be neglected a study on the possible potential effects can be conducted. With a modified bike, which has wheels spinning in the opposite direction, the gyroscopic effects on the cornering of the bike might be investigated. A bike with a movable mass, which keeps the height of the centre of mass equal during cornering, can be used to investigate whether the lower position of the centre of mass in the bends has any effect on the speed of the bicycle.

In summary, the implementation of track characteristics does not significantly change the results of aerodynamic testing on a velodrome. However, the findings of this study raise interesting questions on the aerodynamics and dynamics of track cycling and future studies can help answer these questions to gain more complete knowledge on the effects a velodrome opposes on a cyclist.

5. Conclusion

The aim of this study was to evaluate the effects of velodrome characteristics on the CdA determination. Therefore, two methods were compared. The first method did not take any track characteristics into account. The second method corrected the speed of the centre of mass and assumed extra rolling resistance due to the centripetal force and scrubbing. It was found that if the CdA value of the cyclist was determined over a whole lap, no differences between the methods were found. Both methods showed the same moderate correlation with wind tunnel measurements. Therefore, it can be concluded that the track characteristics did not have to be included when testing the aerodynamic drag of different components or positions on a track.

Although the track characteristics did not influence the final CdA value, differences in power output between the bends and straights were found. Consequently, the measured CdA value between the bends and straights differed significantly. No conclusions can be drawn yet whether the CdA value differed or the calculation method was not complete. The fact that the mean difference in aerodynamic drag between the bends and straights became less after adding centripetal forces and scrubbing to the equation, showed the importance on having a complete understanding of the forces during track cycling.

Future research could improve the understanding of track cycling. With the improved understanding of aerodynamic and dynamic difference between the straights and the bends the research question of this study could be answered with more certainty.

The results of this study can help cyclist and cycling coaches as no difficult calculations are needed for determining aerodynamic drag on a velodrome. On the other hand, it provides new questions for researcher on the differences of aerodynamics and dynamics between the straight and bends on a velodrome.

6. References

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7. Appendix

7.1. Checklist

Tasks

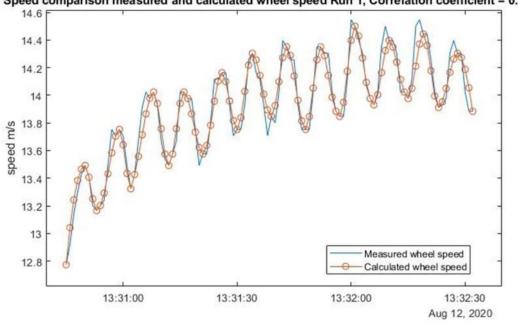
- Climate system Velodrome shut off
- 360 video capture on
- Inflate tires to 11 bar
- Attach speed sensor and magnet to bike
- Attach Notio Konect to bike
- connect power meter, speed sensor and cadance to notio and head unit
- Calibrate power meter
- Attach mylaps prochip to bike
- Attach IMU to saddle
- Attach IMU to steering cockpit
- measure weight of cyclist + bike
- measure wheel circumference of bike + cyclist
- Measure seat height from ground
- Install kestrel wind meter on the track

7.2. Variables calculation

Table 4 Table with variables and configurations used per run as well as the CdA value determined with the two different methods

Run		L_cm m gear ratio		Radius of curvature m	C_rr	Wheel circumference m			
all		1.055	54/15	23	0.002	2.055			
						_			
Run		Density kg*m^3	Mass bike + rider	Coniguration	Date	Start time	CdA M1	CdA M2	
	1	1.166	90.5	Bike + rider	12-Aug	13:29:00	0.196	0.199	
	2	1.165	90.5	Bike + rider	12-Aug	13:45:00	0.196	0.199	
	3	1.165	90.5	Bike + rider	12-Aug	13:57:00	0.192	0.192	
	4	1.164	90.5	Bike + rider	12-Aug	14:21:00	0.196	0.198	
	5	1.163	90.5	Bike + rider	12-Aug	14:36:00	0.197	0.199	
	6	1.164	89.6	bike+rider+rod	14-Aug	13:51:00	0.194	0.198	
	7	1.164	89.6	Bike + rider + rod + disc 60 mm	14-Aug	14:00:00	0.197	0.200	
	8	1.164	89.6	Bike + rider + rod + disc 60 mm	14-Aug	14:10:00	0.202	0.205	
	9	1.164	89.6	Bike + rider + rod + disc 50 mm	14-Aug	14:19:00	0.193	0.196	
	10	1.164	89.6	Bike + rider + rod + disc 50 mm	14-Aug	14:29:00	0.197	0.200	
	11	1.164	89.6	Bike + rider + rod + disc 100 mm	14-Aug	14:40:00	0.208	0.211	
	12	1.164	89.6	Bike + rider + rod + disc 100 mm	14-Aug	14:57:00	0.210	0.213	
	13	1.164	89.6	Bike + rider + rod + disc 80 mm	14-Aug	15:09:00	0.206	0.209	
	14	1.164	89.6	Bike + rider + rod + disc 80 mm	14-Aug	15:21:00	0.207	0.210	
	15	1.164	89.6	bike+rider+rod	14-Aug	15:32:00	0.191	0.194	
	16	1.163	89.6	Bike + rider	14-Aug	15:45:00	0.188	0.191	
	17	1.163	89.6	Bike + rider	14-Aug	15:56:00	0.193	0.195	

7.3. Supplementary figures



Speed comparison measured and calculated wheel speed Run 1, Correlation coefficient = 0.9752

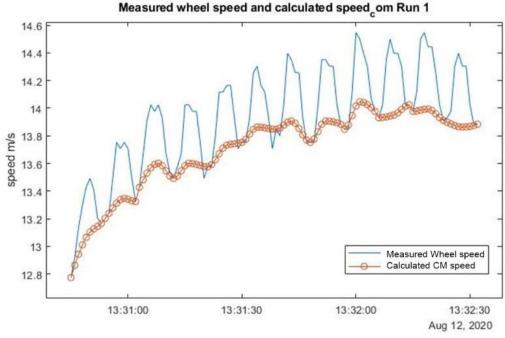


Figure 12 Comparison of measured wheel speed and calculated wheel speed and speed of the centre of mass

7.4. Matlab code

```
7.4.1. Main code
```

```
clc; clear all; close all
% Script to calculate the CdA from the different runs
m_12 = 90.5; % mass of bike and rider kg 12-08-2020
m_14 = 89.6; % mass of bike and rider kg 14-08-2020
com = 1.055; % height of saddle in m
w_c = 2.105; % wheel circumference in m
r_wh = w_c/(2*pi); %radius of wheel
gr = 54/15; % gear ratio
c_rr = 0.002; % rolling resistance
radius = 23; % radius of bends
rho = [1.166;1.165;1.165;1.164;1.163;1.164;1.164;1.164;1.164;...
  1.164;1.164;1.164;1.164;1.164;1.164;1.163;1.163];
for i = 1:length(rho)
  if i<7
    main_var(:,i) = [m_12,com,c_rr,radius,rho(i)];
  else
    main_var(:,i) = [m_14,com,c_rr,radius,rho(i)];
  end
end
% Mylapse data reader
% 12-08-2020
fileloc = 'C:\Users\gertg\Dropbox\Afstuderen\Mylapse\20200812\20200812 Notio';
dat1 = readtable([fileloc,'/1 4/1 4.txt']); % read data from .txt file
ts1_12 = datetime(2020,08,12)+dat1.Time; % transfrom duration to timestamp
ts1_12.Format = 'HH:mm:ss.SSS';
                                       % format for timestamp
dat2 = readtable([fileloc, '/2 4/2 4.txt']);
ts2_12 = datetime(2020,08,12)+dat2.Time;
ts2_12.Format = 'HH:mm:ss.SSS';
dat3 = readtable([fileloc,'/3 4/3 4.txt']);
ts3_12 = datetime(2020,08,12)+dat3.Time;
ts3_12.Format = 'HH:mm:ss.SSS';
dat4 = readtable([fileloc,'/4 4/4 4.txt']);
ts4_12 = datetime(2020,08,12)+dat4.Time;
ts4 12.Format = 'HH:mm:ss.SSS';
dat100m = readtable([fileloc,'/100m/100m.txt']);
ts100m_12 = datetime(2020,08,12)+dat100m.Time;
```

ts100m_12.Format = 'HH:mm:ss.SSS';

dat200m = readtable([fileloc,'/200m/200m.txt']); ts200m_12 = datetime(2020,08,12)+dat200m.Time; ts200m_12.Format = 'HH:mm:ss.SSS';

datfinish = readtable([fileloc,'/Finish/Finish.txt']); tsfinish_12 = datetime(2020,08,12)+datfinish.Time; tsfinish_12.Format = 'HH:mm:ss.SSS';

% Mylapse data reader 14-09-2020

fileloc = 'C:\Users\gertg\Dropbox\Afstuderen\Mylapse\20200814';

dat1 = readtable([fileloc,'\1 4\1 4.txt']); % read data from .txt file ts1_14 = datetime(2020,08,14)+dat1.Time; % transfrom duration to timestamp ts1_14.Format = 'HH:mm:ss.SSS'; % format for timestamp

dat2 = readtable([fileloc, \2 4\2 4.txt']); ts2_14 = datetime(2020,08,14)+dat2.Time; ts2_14.Format = 'HH:mm:ss.SSS';

dat3 = readtable([fileloc,'\3 4\3 4.txt']); ts3_14 = datetime(2020,08,14)+dat3.Time; ts3_14.Format = 'HH:mm:ss.SSS';

dat4 = readtable([fileloc,'\4 4\4 4.txt']); ts4_14 = datetime(2020,08,14)+dat4.Time; ts4_14.Format = 'HH:mm:ss.SSS';

dat100m = readtable([fileloc,'\100m\100m.txt']); ts100m_14 = datetime(2020,08,14)+dat100m.Time; ts100m_14.Format = 'HH:mm:ss.SSS';

dat200m = readtable([fileloc, \200m\200m.txt']); ts200m_14 = datetime(2020,08,14)+dat200m.Time; ts200m_14.Format = 'HH:mm:ss.SSS';

datfinish = readtable([fileloc, '\Finish\Finish.txt']); tsfinish_14 = datetime(2020,08,14)+datfinish.Time; tsfinish_14.Format = 'HH:mm:ss.SSS';

% Load run data

load('srm_12.mat') load('srm_14.mat')

total = [srm;data_notio]; % joins the data of both days

data_notio = retime(data_notio,'regular','makima','SampleRate',1);

total = [srm;data_notio];

% Time of runs

$$\label{eq:run1} \begin{split} & \text{run1} = \text{timerange}(\text{ts4}_12(39),\text{ts4}_12(45)); \\ & \text{run2} = \text{timerange}(\text{ts4}_12(50),\text{ts4}_12(56)); \\ & \text{run3} = \text{timerange}(\text{ts4}_12(61),\text{ts4}_12(67)); \\ & \text{run3corr} = \text{timerange}(\text{ts3}_12(61),\text{ts1}_12(66)); \\ & \text{run4} = \text{timerange}(\text{ts4}_12(72),\text{ts4}_12(78)); \\ & \text{run4corr} = \text{timerange}(\text{ts3}_12(72),\text{ts1}_12(75)); \\ & \text{run5} = \text{timerange}(\text{ts4}_12(83),\text{ts4}_12(87)); \\ & \text{run5corr} = \text{timerange}(\text{ts3}_12(83),\text{ts1}_12(85)); \end{split}$$

$$\label{eq:run6} \begin{split} & \text{run6} = \text{timerange}(\text{ts4}_14(49),\text{ts4}_14(55));\\ & \text{run7} = \text{timerange}(\text{ts4}_14(59),\text{ts4}_14(65));\\ & \text{run8} = \text{timerange}(\text{ts4}_14(71),\text{ts4}_14(77));\\ & \text{run9} = \text{timerange}(\text{ts4}_14(84),\text{ts4}_14(90));\\ & \text{run10} = \text{timerange}(\text{ts4}_14(97),\text{ts4}_14(103));\\ & \text{run11} = \text{timerange}(\text{ts4}_14(110),\text{ts4}_14(103));\\ & \text{run12} = \text{timerange}(\text{ts4}_14(127),\text{ts4}_14(116));\\ & \text{run13} = \text{timerange}(\text{ts4}_14(140),\text{ts4}_14(146));\\ & \text{run14} = \text{timerange}(\text{ts4}_14(140),\text{ts4}_14(146));\\ & \text{run15} = \text{timerange}(\text{ts4}_14(165),\text{ts4}_14(171));\\ & \text{run16} = \text{timerange}(\text{ts4}_14(178),\text{ts4}_14(184));\\ & \text{run17} = \text{timerange}(\text{ts4}_14(191),\text{ts4}_14(197)); \end{split}$$

runs = [ts4_12(39),ts4_12(45);... ts4_12(50),ts4_12(56);... ts4_12(61),ts4_12(67);... ts4_12(72),ts4_12(78);... ts4_12(83),ts4_12(89);... ts4_14(49),ts4_14(55);... ts4_14(59),ts4_14(65);... ts4_14(71),ts4_14(77);... ts4_14(84),ts4_14(90);... ts4_14(97),ts4_14(103);... ts4_14(110),ts4_14(116);... ts4_14(127),ts4_14(133);... ts4_14(140),ts4_14(146);... ts4_14(152),ts4_14(158);... ts4_14(165),ts4_14(171);... ts4_14(178),ts4_14(184);... ts4_14(191),ts4_14(197)];

ML_idx4 = [39;50;61;72;83;49;59;71;84;97;110;127;140;152;165;178;191]; % index of starttime runs transponder 4

% Replace wrong speed data with data from cadence run 3,4,5 and 6

total.speed(run3corr) = total.speed_cadence(run3corr);

total.speed(run4corr) = total.speed_cadence(run4corr);

total.speed(run5corr) = total.speed_cadence(run5corr);

% Comparison of measured speed and transponder speed

for i = 1:5

tr = timerange(runs(i,1),runs(i,2));
speed_meas(i) = mean(total.speed(tr));

 $temp = find(ts4_12 == runs(i,1));$

speed_trans(:,i) = milliseconds(diff(ts4_12(temp:(temp+6))))./1000;

plot(total.speed(tr)); hold on end

for i = 6:17

tr = timerange(runs(i,1),runs(i,2));
speed_meas(i) = mean(total.speed(tr));

```
temp = find(ts4_14 == runs(i,1));
```

speed_trans(:,i) = milliseconds(diff(ts4_14(temp:(temp+6))))./1000;

plot(total.speed(tr)); hold on

end

% com calculation

%

for i = 1:5

<mark>if</mark> i == 1

 $v_com = speed_com(total,runs(i,:),ts1_12,ts2_12,ts3_12,ts4_12,com);$

else

```
v_com = [v_com;speed_com(total,runs(i,:),ts1_12,ts2_12,ts3_12,ts4_12,com)];
```

end

end

for i = 6:17

v_com = [v_com;speed_com(total,runs(i,:),ts1_14,ts2_14,ts3_14,ts4_14,com)];

end

total = v_com(:,[4:7]); % replace the speed per second to speed per mylaps point interpolated

total.Properties.VariableNames = {'power','speed_,'speed_cm','roll'};

for i = 1:5

[speed_bends,speed_straights] = bends_straight(total,runs(i,:),ts1_12,ts2_12,ts3_12,ts4_12,main_var(:,i),'speed'); [speed_bends_cm,~] = bends_straight(total,runs(i,:),ts1_12,ts2_12,ts3_12,ts4_12,main_var(:,i),'speed_cm'); [power_bends,power_straights] = bends_straight(total,runs(i,:),ts1_12,ts2_12,ts3_12,ts4_12,main_var(:,i),'power');

if i ==1

speed_data = [speed_bends,speed_bends_cm, speed_straights];
mean_speed = [mean(speed_bends),mean(speed_bends_cm),mean(speed_straights)];

power_data = [power_bends,power_straights]; mean_power = [mean(power_bends),mean(power_straights)];

else

speed_data = [speed_data,speed_bends,speed_bends_cm, speed_straights];
mean_speed = [mean_speed;mean(speed_bends),mean(speed_bends_cm),mean(speed_straights)];

power_data = [power_data;power_bends, power_straights]; mean_power = [mean_power;mean(power_bends),mean(power_straights)];

end

end

% day 2

for i = 6:17

[speed_bends,speed_straights] = bends_straight(total,runs(i,:),ts1_14,ts2_14,ts3_14,ts4_14,main_var(:,i),'speed'); [speed_bends_cm,~] = bends_straight(total,runs(i,:),ts1_14,ts2_14,ts3_14,ts4_14,main_var(:,i),'speed_cm'); [power_bends,power_straights] = bends_straight(total,runs(i,:),ts1_14,ts2_14,ts3_14,ts4_14,main_var(:,i),'power');

speed_data = [speed_data,speed_bends,speed_bends_cm, speed_straights];
mean_speed = [mean_speed;mean(speed_bends),mean(speed_bends_cm),mean(speed_straights)];

power_data = [power_data;power_bends, power_straights]; mean_power = [mean_power;mean(power_bends),mean(power_straights)];

end

% all runs CdA + CdA bends/straight

CdA_m1_6L = nan(17,1); CdA_m1_1L = nan(17,6); CdA_m2_6L = nan(17,3);

```
for i = 1:5
  if i == 5
     laps = 4;
  else
     laps = 6;
  end
  CdA_m1_6L(i,:) = CDA_M1(total,runs(i,:),main_var(:,i),'mean'); % mean result of 6 laps (1 value)
  CdA_m2(i,:) = CDA_M2(total,runs(i,:),main_var(:,i)); % mean result of 6 laps M2 (1 value)
  [CdAc, CdAs] = CdA_track1(total,runs(i,:),main_var(:,i),ts1_12,ts2_12,ts3_12,ts4_12,laps,'M2'); % bends CdA and straight
CdA M2
  CdA_m2_6L(i,:) = [CdAc, CdAs, mean([CdAc,CdAs])];
  for ii = 1:6 % mean results of CdA m1 per lap (6 values)
     tr = [ts4_12(ML_idx4(i)), ts4_12(ML_idx4(i)+1)];
     CdA_m1_1L(i,ii) = CDA_M1(total,tr,main_var(:,i),'mean');
     CdA_m2_1L(i,ii) = CDA_M2(total,tr,main_var(:,i));
     ML_idx4(i) = ML_idx4(i) + 1;
  end
end
CdA_m1_1L(5,[5:6]) = nan; % delete last two laps of run 5 due to missing data
CdA_m2_1L(5,[5:6]) = nan;
% day 2
for i = 6:17
  laps = 6;
  CdA_m1_6L(i,:) = CDA_M1(total,runs(i,:),main_var(:,i),'mean'); % mean result of 6 laps (1 value)
  CdA_m2(i,:) = CDA_M2(total,runs(i,:),main_var(:,i)); % mean result of 6 laps M2 (1 value)
  [CdAc, CdAs] = CdA_track1(total,runs(i,:),main_var(:,i),ts1_14,ts2_14,ts3_14,ts4_14,laps,'M2'); % bends CdA and straight
CdA M2
  CdA_m2_6L(i,:) = [CdAc, CdAs, mean([CdAc,CdAs])];
  for ii = 1:6
                      % mean results of CdA m1 per lap
     tr = [ts4_14(ML_idx4(i)), ts4_14(ML_idx4(i)+1)];
     CdA_m1_1L(i,ii) = CDA_M1(total,tr,main_var(:,i),'mean');
     CdA_m2_1L(i,ii) = CDA_M2(total,tr,main_var(:,i));
     ML_idx4(i) = ML_idx4(i) + 1;
```

```
end
```

```
end
```

```
% Results

%Day 1

for i = 1:5

if i == 1

M1_day1 = CdA_m1_1L(1:6,i);

M2_day1 = CdA_m2_1L(1:6,i);

elseif i == 5

M1_day1 = [M1_day1;CdA_m1_1L(1:4,i)];

M2_day1 = [M2_day1;CdA_m2_1L(1:4,i)];

else

M1_day1 = [M1_day1;CdA_m1_1L(1:6,i)];

M2_day1 = [M2_day1;CdA_m2_1L(1:6,i)];

end

end
```

%Day 2

```
\label{eq:constraint} \begin{array}{l} dropsM1 = [transpose(CdA_m1_1L(16,:));transpose(CdA_m1_1L(17,:))];\\ rodM1 = [transpose(CdA_m1_1L(6,:));transpose(CdA_m1_1L(15,:))];\\ cm5M1 = [transpose(CdA_m1_1L(9,:));transpose(CdA_m1_1L(10,:))];\\ cm6M1 = [transpose(CdA_m1_1L(7,:));transpose(CdA_m1_1L(8,:))];\\ cm8M1 = [transpose(CdA_m1_1L(13,:));transpose(CdA_m1_1L(14,:))];\\ cm10M1 = [transpose(CdA_m1_1L(11,:));transpose(CdA_m1_1L(12,:))];\\ \end{array}
```

M1_day2 = table(dropsM1,rodM1,cm5M1,cm6M1,cm8M1,cm10M1);

```
\label{eq:constraint} \begin{array}{l} dropsM2 = [transpose(CdA_m2_1L(16,:));transpose(CdA_m2_1L(17,:))];\\ rodM2 = [transpose(CdA_m2_1L(6,:));transpose(CdA_m2_1L(15,:))];\\ cm5M2 = [transpose(CdA_m2_1L(9,:));transpose(CdA_m2_1L(10,:))];\\ cm6M2 = [transpose(CdA_m2_1L(7,:));transpose(CdA_m2_1L(14,:))];\\ cm8M2 = [transpose(CdA_m2_1L(13,:));transpose(CdA_m2_1L(14,:))];\\ cm10M2 = [transpose(CdA_m2_1L(11,:));transpose(CdA_m2_1L(12,:))];\\ \end{array}
```

M2_day2 = table(dropsM2,rodM2,cm5M2,cm6M2,cm8M2,cm10M2);

7.4.2. Start and end time of bends and straights

```
function [starttime,endtime] = timestamp_meters(transpondertime,dist,total,tails)
%This function calculates the timestamp before and after an transponder
% time input with x amount of meters
distance = dist;
dist = 0;
indexcs = find((transpondertime - total.time)< 0,1);
indexcs = indexcs -1;
if nargin < 4
    tails = 'two';</pre>
```

```
end
```

```
if contains(tails,'one')
  dist = 0;
  indexce = find((transpondertime - total.time)< 0,1);
  diff = milliseconds(total.time(indexce) - transpondertime)/1000;
  dist = dist + total.speed(indexce)*diff;
  while dist < (distance)
   dd = total.speed(indexce);
   dist = dist + dd;
   if dist < distance
     indexce = indexce + 1;
   end
  end
  cs = transpondertime;
  ce = total.time(indexce);
end
if contains(tails,'two')
  dist = 0;
  diff = milliseconds(total.time(indexcs) - transpondertime)/1000;
  dist = dist + total.speed(indexcs)*abs(diff);
  while dist < (distance)
   dd = total.speed(indexcs);
    dist = dist + dd;
   if dist < distance
     indexcs = indexcs - 1;
   end
  end
  if indexcs == 0
     indexcs = 1;
  end
```

```
cs = total.time(indexcs);
dist = 0;
indexce = find((transpondertime - total.time)< 0,1);
diff = milliseconds(total.time(indexce) - transpondertime)/1000;
```

```
dist = dist + total.speed(indexce)*abs(diff);
```

```
while dist < (distance)
dd = total.speed(indexce);
dist = dist + dd;
if dist < distance
indexce = indexce + 1;
end
end
if indexce == 0
indexce = 1;
end
ce = total.time(indexce);
end
starttime = cs;
endtime = ce;
end
```

7.4.3. Roll angle function code

```
function [roll] = roll_angle(speedip, radius, com)
%Function that calculates the roll angle from the speed on the track
g = 9.81;
roll = nan(length(speedip),1);
for ii = 1:length(speedip)
speed = speedip(ii);
  for i = 1:10
     if i == 1
       alpha = deg2rad(30);
       rcom = radius-com*sin(alpha);
       expalpha(i) = rad2deg(atan(((speed-(speed/radius*com*sin(alpha)))^2)/(g*rcom)));
     else
       alpha = deg2rad(expalpha(i-1));
       rcom = radius-com*sin(alpha);
       expalpha(i) = atan(((speed-(speed/radius*com*sin(alpha)))^2)/(g*rcom));
     end
  end
roll = expalpha(end);
end
```

```
function [output_bends,output_straight] = bends_straight(total,runs,ts1,ts2,ts3,ts4,main_var,output)
%This function is build to create outputs of data specifically for a bends
%or a straight
if nargin < 8
  output = 'speed';
end
m = main_var(1);
com = main_var(2);
c_rr = main_var(3);
radius = main_var(4);
rho = main_var(5);
g = 9.81;
E = 0.98;
starttime = runs(1,1);
endtime = runs(1,2);
[idx1,~] = find((ts1-starttime) > 0,1);
[idx2,~] = find((ts2-starttime) > 0,1);
[idx3,~] = find((ts3-starttime) > 0,1);
idx4 = find(ts4 == starttime);
mylaps = [idx1, idx2, idx3, idx4];
laps = 6;
for i = 1:laps
  if i == 1
     st1 = datetime(starttime,'Format','HH:mm:ss');
     [~,et1] = timestamp_meters(ts4(mylaps(4)),(17.325),total,'one'); % first half of straight till bends 1
     [st2,et2] = timestamp_meters(ts1(mylaps(1)),24,total); % bends 1
     [st3,et3] = timestamp_meters(ts2(mylaps(2)),17.325,total); % straight 2
     [st4,et4] = timestamp_meters(ts3(mylaps(3)),24,total); % bends 2
```

else

[st1,et1] = timestamp_meters(ts4(mylaps(4)),(17.325),total); % first straight till bends 1

[st2,et2] = timestamp_meters(ts1(mylaps(1)),24,total); % bends 1

[st3,et3] = timestamp_meters(ts2(mylaps(2)),17.325,total); % straight 2

[st4,et4] = timestamp_meters(ts3(mylaps(3)),24,total); % bends 2 17.32

end

% straights

straight1 = timerange(st1,et1); bends1 = timerange(st2,et2); straight2 = timerange(st3,et3); bends2 = timerange(st4,et4);

speeds1_cm = total.speed_cm(straight1); speeds1 = total.speed(straight1); powers1 = total.power(straight1); times1 = total.time(straight1);

```
if length(speeds1) == 0
```

speeds1_cm = total.speed_cm(timerange(st1,et1+seconds(1))); speeds1 = total.speed(timerange(st1,et1+seconds(1))); powers1 = total.power(timerange(st1,et1+seconds(1))); times1 = total.time(timerange(st1,et1+seconds(1)));

end

```
speeds2_cm = total.speed_cm(straight2);
speeds2 = total.speed(straight2);
powers2 = total.power(straight2);
times2 = total.time(straight2);
```

% bends

speedc1_cm = total.speed_cm(bends1); speedc1 = total.speed(bends1); powerc1 = total.power(bends1); timec1 = total.time(bends1);

```
speedc2_cm = total.speed_cm(bends2);
speedc2 = total.speed(bends2);
powerc2 = total.power(bends2);
timec2 = total.time(bends2);
```

if i == 1

speed_bends = [mean(speedc1);mean(speedc2)];
speed_bends_cm = [mean(speedc1_cm);mean(speedc2_cm)];
speed_straights = [mean(speeds1);mean(speeds2)];

power_bends = [mean(powerc1);mean(powerc2)];
power_straights = [mean(powers1);mean(powers2)];

else

```
speed_bends = [speed_bends;mean(speedc1);mean(speedc2)];
speed_bends_cm = [speed_bends_cm;mean(speedc1_cm);mean(speedc2_cm)];
speed_straights = [speed_straights;mean(speeds1);mean(speeds2)];
```

power_bends = [power_bends;mean(powerc1);mean(powerc2)]; power_straights = [power_straights;mean(powers1);mean(powers2)];

end

```
mylaps = mylaps + 1;
```

end

```
if matches(output,'speed')
```

output_bends = speed_bends; output_straight = speed_straights;

```
elseif matches(output,'speed_cm')
```

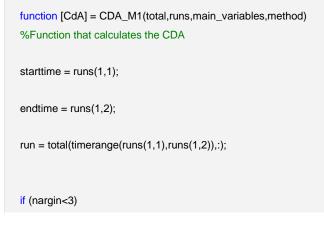
```
output_bends = speed_bends_cm;
output_straight = speed_straights;
```

```
elseif matches(output,'power')
```

output_bends = power_bends; output_straight = power_straights;

else error('wrong output, chose speed or power') end end

7.4.5. Function for CdA M1



```
method = 'stepwise';
end
g=9.81;
E = 0.98;
m = main_variables(1);
c_rr = main_variables(3);
rho = main_variables(5);
time = seconds(run.time(end)-run.time(1));
  if contains(method,'stepwise')
       Ekin = 0.5*m.*(run.speed(2:end).^2 - run.speed(1:(end-1)).^2);
       Ekin = [0;Ekin];
       CdA = (E.*run.power- Ekin - m*g*c_rr.*run.speed)./(0.5*rho.*run.speed.^3);
       CdA = mean(CdA);
  elseif contains(method,'mean')
       mEkin = (0.5*m*(run.speed(end)^2 - run.speed(1)^2))/time;
       CdA = (E*mean(run.power) - m*g*c_rr*mean(run.speed)- mEkin)./(0.5*rho*mean(run.speed).^3);
  else
       error('wrong type of method, choose stepwise or mean')
```

```
end
end
```

radius = vars(4); rho = vars(5);

7.4.6. Function for CdA M2

```
function [CdA_m2] = CDA_M2(total,runs,vars)
%This function calculates the CdA based with bends and difference in
%kinetic energy
starttime = runs(1,1);
endtime = runs(1,2);
tr = timerange(starttime,endtime);
g=9.81;
E = 0.98;
m = vars(1);
com = vars(2);
c_rr = vars(3);
```

```
speed = total.speed(tr);
speed_cm = total.speed_cm(tr);
power = total.power(tr);
roll = total.roll(tr);
Ekin = [0;(0.5*m.*(speed_cm(2:end).^2 - speed_cm(1:(end-1)).^2))];
Epot = m*g*com.*cos(roll);
Epot = [0; Epot(2:end) - Epot(1:(end-1))];
Fc = m.*(speed\_cm.^2)./((speed\_cm.^2)./(tan(roll)*g));
for i = 1:length(roll)
  if rad2deg(roll(i)) < 10
     c_s(i) = 1.097;
  else
     c_s(i) = 1.3096;
  end
end
CdA = ((E.*power) - ...
       (Fc.*sin(roll)+m*g.*cos(roll))*c_rr.*transpose(c_s).*speed...
       -Ekin)./(0.5*rho.*speed_cm.^3);
CdA_m2 = mean(CdA);
end
```

7.4.7. Function that calculates CdA in bends and straights M1& M2

```
function [CdAc,CdAs] = CdA_track1(total,runs,main_var,ts1,ts2,ts3,ts4,laps,method)
%Function

if nargin<9
    method = 'M2';
end

m = main_var(1);
com = main_var(2);
c_rr = main_var(3);
radius = main_var(4);
rho = main_var(5);
g=9.81;
E = 0.98;
camber = 43;
starttime = runs(1,1);</pre>
```

endtime = runs(1,2);

```
[idx1,~] = find((ts1-starttime) > 0,1);
[idx2,~] = find((ts2-starttime) > 0,1);
[idx3,~] = find((ts3-starttime) > 0,1);
idx4 = find(ts4 == starttime);
```

mylaps = [idx1, idx2, idx3, idx4];

for i = 1:laps

if i == 1

st1 = starttime;

[~,et1] = timestamp_meters(ts4(mylaps(4)),(32.5),total,'one'); % first half of straight till bends 1

[st2,et2] = timestamp_meters(ts1(mylaps(1)),24,total,'two'); % bends 1

[st3,et3] = timestamp_meters(ts2(mylaps(2)),32.5,total,'two'); % straight 2

[st4,et4] = timestamp_meters(ts3(mylaps(3)),24,total,'two'); % bends 2

else

```
[st1,et1] = timestamp_meters(ts4(mylaps(4)),(32.5),total,'two'); %straight 1
```

[st2,et2] = timestamp_meters(ts1(mylaps(1)),24,total,'two'); % bends 1

[st3,et3] = timestamp_meters(ts2(mylaps(2)),32.5,total,'two'); % straight 2

[st4,et4] = timestamp_meters(ts3(mylaps(3)),24,total,'two'); % bends 2 17.32

end

```
straight1 = timerange(st1,et1);
bends1 = timerange(st2,et2);
straight2 = timerange(st3,et3);
bends2 = timerange(st4,et4);
if i == laps
    idxend = find(total.time - endtime > 0,1);
    idxend = idxend - 1;
```

```
speed_ls = total.speed(idxend);
power_ls = total.power(idxend);
time_ls = total.time(idxend);
v_com_ls = total.speed_cm(idxend);
```

end

% straights

```
c_ss = 1.097;
dEpot = 0;
speeds1 = total.speed(straight1);
powers1 = total.power(straight1);
times1 = total.time(straight1);
if length(speeds1) == 0
  speeds1 = total.speed(timerange(st1,et1+seconds(1)));
  powers1 = total.power(timerange(st1,et1+seconds(1)));
  times1 = total.time(timerange(st1,et1+seconds(1)));
end
speeds2 = total.speed(straight2);
powers2 = total.power(straight2);
times2 = total.time(straight2);
v_coms1 = total.speed_cm(straight1);
v_coms2 = total.speed_cm(straight2);
  % bends
speedc1 = total.speed(bends1);
powerc1 = total.power(bends1);
radiusc1 = 23;
roll1 = roll_angle(speedc1,radiusc1,com);
timec1 = total.time(bends1);
speedc2 = total.speed(bends2);
powerc2 = total.power(bends2);
radiusc2 = 23;
roll2 = roll_angle(speedc2,radiusc2,com);
timec2 = total.time(bends2);
c_sc = 1 + camber *0.0072;
v_comc1 = total.speed_cm(bends1);
v_comc2 = total.speed_cm(bends2);
```

if i == 1

speed_bends = [speedc1;speedc2]; % speed of bends wheel
speed_bends_cm = [v_comc1;v_comc2]; % speed of bends wheel
speed_straights = [speeds1;speeds2]; % speed of straights

power_bends = [powerc1;powerc2]; % power of bends
power_straights = [powers1;powers2]; % power of straights

times = [times1; times2]; timec = [timec1;timec2];

elseif i == laps

speed_bends = [speed_bends;speedc1;speedc2]; % speed of bends speed_bends_cm = [speed_bends_cm;v_comc1;v_comc2]; % speed of bends wheel speed_straights = [speed_straights; speeds1;speeds2;speed_ls]; % speed of straights

power_bends = [power_bends;powerc1;powerc2]; % power of bends
power_straights = [power_straights;powers1;powers2;power_ls]; % power of straights

times = [times;times1; times2;time_ls]; timec = [timec;timec1;timec2];

else

speed_bends = [speed_bends;speedc1;speedc2]; % speed of bends
speed_bends_cm = [speed_bends_cm;v_comc1;v_comc2]; % speed of bends wheel
speed_straights = [speed_straights; speeds1;speeds2]; % speed of straights

power_bends = [power_bends;powerc1;powerc2]; % power of bends
power_straights = [power_straights;powers1;powers2]; % power of straights

times = [times;times1; times2]; timec = [timec;timec1;timec2];

end

mylaps = mylaps + 1;

end

speed = total.speed(timerange(runs(1,1),runs(1,2))); time = total.time(timerange(runs(1,1),runs(1,2))); dt = seconds(time(end)-time(1)); Ekin = 0.5*m*(speed(end)^2 - speed(1)^2)/dt;

```
dt_bends = seconds(timec(end)- timec(1));
dt_straights = seconds(times(end)- times(1));
```

$$\label{eq:expectation} \begin{split} \mathsf{Ekinc} &= 0.5^* \mathsf{m}^*(\mathsf{speed_bends_cm}(\mathsf{end})^2 - \mathsf{speed_bends_cm}(1)^2)/\mathsf{dt_bends}; \\ \mathsf{Ekins} &= 0.5^* \mathsf{m}^*(\mathsf{speed_straights}(\mathsf{end})^2 - \mathsf{speed_straights}(1)^2)/\mathsf{dt_straights}; \end{split}$$

```
if contains(method,'M2')
```

CdAc = ((E*mean(power_bends)) - ...

((m*(mean(speed_bends_cm)^2)/radiusc1)*sin(roll1)+m*g*cos(roll1))*c_rr*c_sc*mean(speed_bends)... -Ekinc)/(0.5*rho*(mean(speed_bends_cm)^3));

CdAs = (E*mean(power_straights) - m*g*c_rr*c_ss*mean(speed_straights)-Ekins)/(0.5*rho*(mean(speed_straights)^3)); elseif contains(method,'M1')

CdAc = (E*mean(power_bends) - m*g*c_rr*mean(speed_bends)- Ekinc)/(0.5*rho*(mean(speed_bends)^3));

CdAs = (E*mean(power_straights) - m*g*c_rr*mean(speed_straights) - Ekins)/(0.5*rho*(mean(speed_straights)^3)); end

end