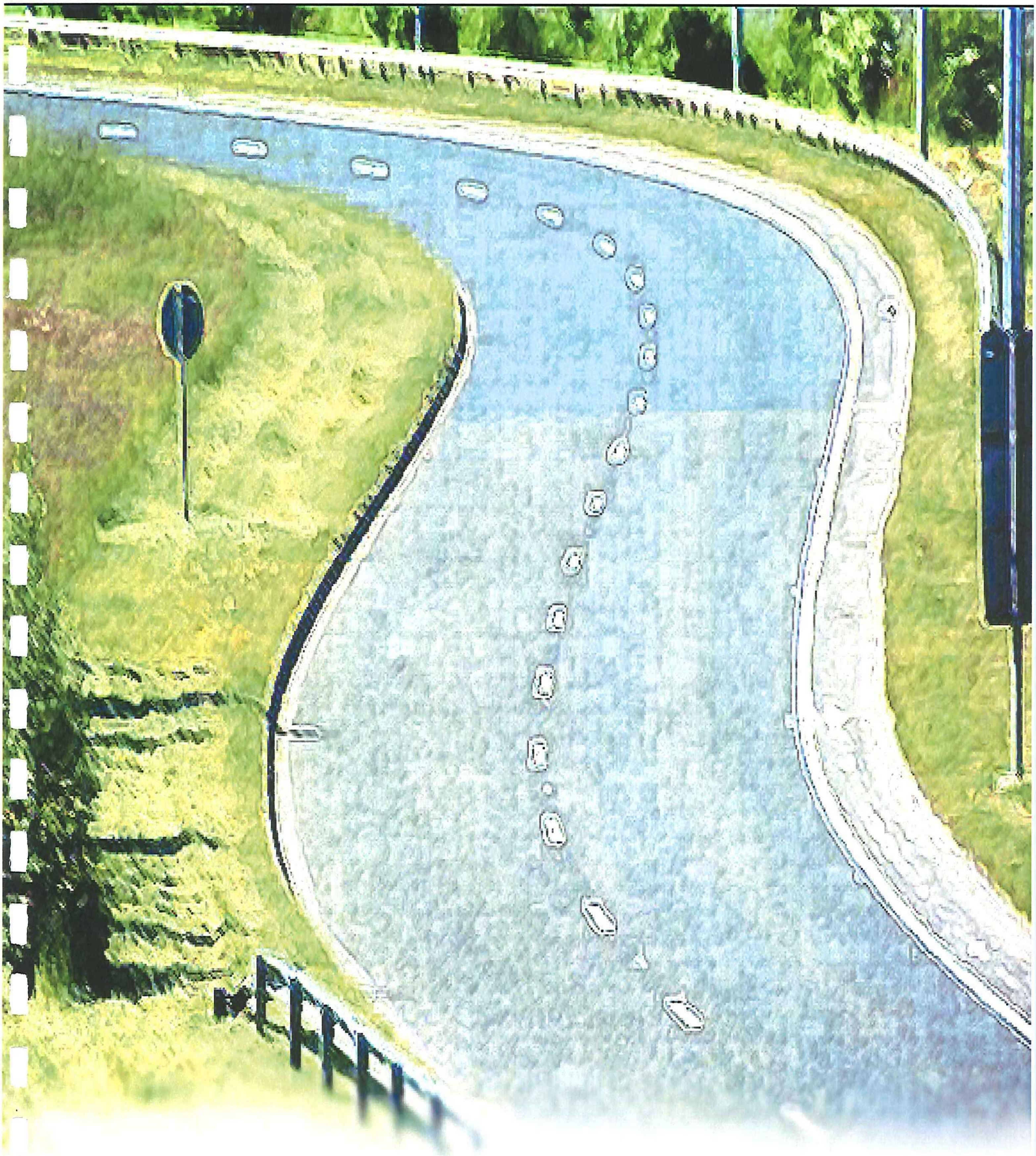

MUC-SUE with Heterogeneous VOT Traffic Assignment Model Using a Variational Inequality Approach

Application to Dutch KMP System

Lan Jiang

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This thesis is a result of the master study performed from 2006 to 2008 at Delft University of Technology, Faculty of Civil Engineering and Geosciences, Transport, Infrastructure & Logistics Program.

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Dedicated to my parents and my dear TUN

Contents

Preface

Mobility is an integral part of modern society. Over the last few years, mobility has been continuously increased and this situation is not to change. Accessibility, safety and quality of the living environment are consequently and increasingly under pressure. Road pricing has been advocated as an efficient transport policy in solving the aforementioned problems for many years. To be able to make forecasts about future traffic conditions and to estimate the effects of road pricing policies, there is a need to develop a traffic assignment model capable of capturing heterogeneous users' responses to road pricing for policy design and evaluation.

In this thesis, we formulate, implement, and apply a traffic assignment model describing heterogeneous VOT between and within multiple user-classes. The thesis project is supported by the Delft University of Technology and **4***cast* B.V. The time I spent at both Delft University of Technology and **4***cast* B.V. has been very beneficial in many ways, and I would like to thank all my professors and colleagues from whom I have learned and who have contributed to this thesis.

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My love and deepest appreciation to my parents and my dear TUN, to whom I dedicate this thesis.

-Lan Jiang

Notations

Sets

A	Set of links in the network
$R \subseteq N$	Subset of origin nodes
$S \subseteq N$	Subset of destination nodes
M	Set of user-classes
N	Set of nodes in the network
P_m^{rs}	Set of available alternative routes from origin r to destination s

Indices

$a \subseteq A$	Link index
$m \subseteq M$	User class index
r	Origin
s	Destination
p	route

Link Variables

τ_{am}	Travel time on link *a* of users of class *m* [min]
τ_a^0	Free flow travel time on link *a* [min]
c_{am}	Generalized cost on link *a* for users of class *m* [€]
u_{am}	Link flow on link *a* for users of class *m* [veh/h]
v_a	Link volume on link *a*
CAP_a	Capacity of link *a* [veh/h]

Route variables

c_m^{prs}	Generalized travel cost users of class *m* using route p from origin *r* to destination *s*
τ_m^{prs}	Travel time on route *p* from origin *r* to destination *s* for users of class *m* [min]
Ψ_m^{prs}	Probability that users of class *m* choose route *p* from origin *r* to destination *s*
f_m^{prs}	Route flow of route *p* from origin *r* to destination *s* for users of class *m* [veh/h]

Demand variable

D_m^{rs}	Travel demand from origin *r* to destination *s* of users of class *m* [veh/h]

Link-Route Variable

δ_{am}^{prs}	Link-route incidence indicator for users of class *m* traveling from origin *r* to destination *s*, equaling one if these can flow into link *a*, or zero otherwise.

Toll Variable

θ_{am}	Toll on link *a* for users of class *m* [€]
θ_m^{prs}	Total toll on route p from origin *r* to destination *s* for users of class *m* [€]

Parameters

pce_m	Passenger car unit (pcu) value of users of class *m*
β	Value of time [€/min]
μ_m	Mean value of a certain distribution for users of class *m*

σ_m	Standard deviation of a certain distribution for users of class m
ω	Scale parameter

Acronyms

VOT	Value Of Time
MUC	multi user-class
DUE	deterministic user equilibrium
SUE	stochastic user equilibrium
VI	variational inequality
QM	Quasi-Monte Carlo technique
KMP	kilometer price
AON	all-or-nothing
PCE	passenger car equivalent
VTT	vehicle traveled time
TOD	time-of-day

1 Introduction

1.1 Context and Background

Road pricing has been advocated as an efficient transport policy in solving the problems of congestion, environmental impacts, safety and the like, for many years. Road pricing can directly benefit road users through reduced congestion or improved roadways. Nevertheless, the ultimate goal of road pricing policy is to guide users' decisions and to achieve rational outcomes.

By implementing road pricing policy, policy makers want to achieve two general objectives: congestion relief and/or revenue generation. The direct outcomes of road pricing policy might be changes of demand patterns and traffic conditions, which will result in multiple impacts at the same time. Due to the change of travel costs, travelers may change to another transport mode, such as public transportation, which may bring a burden to the current public transportation service to accommodate extra demands. For the same reason, people might re-locate their home or work locations to reduce the increased transport expenditures. Similarly, firms may move to ensure a lower transportation costs for their employees and clients. These processes, in turn, will bring impacts to economic conditions and to spatial patterns.

By asking how to allocate revenue and for what purpose, the impacts of using revenue are important for the overall efficiency of the pricing scheme and influence the public acceptability as well. Obviously, the revenue use is just one issue of all the public's concerns. Actually, the mass public also pays much attention to the privacy, the equity and effectiveness of the road pricing policy etc.

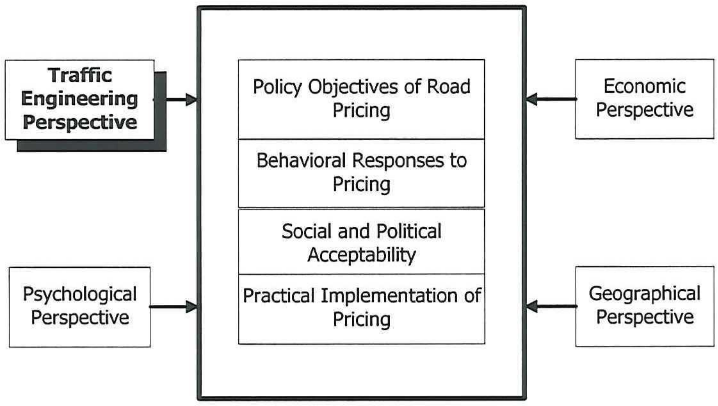

Figure 1.1: Road Pricing from Different Perspectives[1]

The road pricing policy is a complicated problem concerning so many aspects of society, instead of only limited its roots in the field of transportation. Figure 1.1 gives an overview of all the interrelated road pricing issues.

Among various perspectives in the issue of road pricing, this thesis starts from the traffic engineering perspective. Traffic engineers are interested in the effects of transport prices will have on the use of the transportation system. The network effects of future transport policies are often evaluated with mathematical network transportation models (Barry and Erik, 2003). Traffic assignment models are one of the most effective qualitatively analytic methods to assess users' responses to pricing policy.

1. 2 Traffic Assignment Model

To be able to make forecasts about future traffic conditions and to estimate the effects of road pricing plans, traffic assignment models are often considered as an efficient and effective tool. In capacity-limited transportation networks, the planning and operations of various road pricing policies, such as road tolls, cordon (area) tolls, and congestions tolls, require a traffic assignment model that takes into account two essential decision attributes: travel time and out-of-pocket cost. Road users will make trade-off between these two attributes when choose a certain route during their travels. To link the time terms and monetary terms, we need to introduce value of time (VOT). The VOT relative to each trip represents how much money the road user is willing to pay for a unit time saving. In a utility maximization framework, each road user can be assumed to select a route that minimizes a generalized cost function where travel time is weighted by that road user's particular VOT. Various empirical studies (Small, 1983 and 2004) have suggested that VOT varies significantly across individuals because of different socioeconomic characteristics, trip purposes, attitudes, and inherent preferences. The inclusion of heterogeneous VOT in traffic assignment model is therefore of fundamental importance.

More and more efforts have been put into establishing those traffic assignment models to expanding their capabilities and prediction power to provide better predictions of the

[1] Source: from MD-PIT project

network performance under a given road pricing strategy. In literature, previous studies that address user heterogeneity are dominated by two approach categories, the discrete VOT among multiple user-classes approach and the continuously distributed VOT across the whole population of road users (see also Chapter 2). In this thesis, we proposed a traffic assignment model to incorporate heterogeneous VOT both between and within multiple user-classes. The inclusion of greater behavior realism results in more realistic traffic forecasts and enables policy makers and planners to make better decisions concerning the designs of road pricing strategies.

1.3 Research Framework and Research Objectives

The research presented in this thesis is facilitated by **4***cast* B.V. with their specific interest on the possible improvements of the traffic assignment model to provide better support tool for evaluating Dutch KMP system. In this respect, the proposed model implemented in Cube planning system (see also Chapter 6) is dedicated to the data source provided by the company, followed by a case study on the Dutch KMP system.

The objective of the thesis is twofold,

(i) Development of a traffic assignment model capturing heterogeneous VOT between and within multiple user-classes

(ii) Implementation of the proposed model in Cube planning system for large-scale network applications.

It should be stressed that the proposed model to be developed in Chapter 4, is not restricted to applications of Dutch KMP system. It can be applied to assessing most road pricing strategies. Nevertheless, for the sake of simplicity and the interest of the sponsor, the implementation and application of such a model will be restricted to the Dutch kilometer price (KMP) system.

1.4 Research Issues

One of the main research issues of the thesis is to provide sound mathematical formulation and efficient solution algorithm for the development of the proposed traffic assignment model. Recognizing the heterogeneity between and within multiple user-classes on the network considerably complicates the problem. This will have an impact on two parts of the model. First of all, the consideration of multiple user-classes implies that the traffic flow conditions on the network will be affected by interactions among the user classes, thus leading to asymmetric cost functions. The asymmetric cost functions require the model to be formulated as a variational inequality (VI) problem. The VI approach is especially appropriate in modeling traffic assignment problems in which asymmetric interactions exist, and no corresponding optimization problem can be formulated. Secondly, advanced numerical techniques are needed when designing solution algorithm to solve the continuously distributed VOT within multiple user-classes.

A challenge in the development task is a consistent mathematical formulation of this VI model and a design of the solution algorithm subsequently. The following questions should be taken into account:

- Why is the variational inequality approach chosen for formulating the assignment model?

- How to mathematically formulate the proposed traffic assignment model using a VI approach?
- How to design a solution algorithm for the to-be developed VI model?

A second issue to be dealt with concerns the implementation and application of the developed model in Cube planning system. The questions related with this research issue are:

- Whether this VI model and its solution algorithm can be applied to realistic large-scale network;
- Whether by explicitly considering heterogeneous VOT between and within multiple user-classes will affect the estimations of the network performance compared with the conventional assignment approaches.

1.5 Theoretical and Practical Contributions of the Thesis

The research reported in this thesis contributes to both theoretical and practical aspects of the transportation field. These contributions may be summarized as follows:

From theoretical aspects:

(a) Combination of the two conventional approaches that address the user heterogeneity to realize greater behavioral realism in the traffic assignment model;

(b) Formulation of the model using the mathematical approach of variational inequalities. It offers a much more general modeling framework to encompass asymmetric cost functions;

(c) Establishment of a link-based algorithm for solving the proposed assignment model, with elaborations of specific numerical techniques as an inner iterative procedure of the algorithm;

From practical aspects:

(d) Implementation of the developed model in Cube planning system;

(e) Demonstration of the improvement on the estimation of network performance by the developed model onto a large-scale network.

1.6 Outline of the Thesis

The structure of the thesis is as follows. In Chapter 2, an introduction of the categories of the existing approaches dealing with the heterogeneous VOT is literarily reviewed first. The modeling challenges with respect to heterogeneous VOT are presented separately. Subsequently, an overview of assignment models with user heterogeneity is discussed.

Chapter 3 explores basic concepts and formulations for traffic assignment models which incorporates multiple user-classes using variational inequality approach. This chapter begins with a brief discussion on how multiple user-classes presenting on the network will cause possible asymmetric cost function. In order to derive solution algorithms for assignment models with asymmetric cost functions, two static traffic assignment models DUE and SUE are formulated as route-based VI problem. From a practical point of view, the route-based VI model is rewritten into a link-based one for its computational attractiveness.

Chapter 4 mathematically formulates the proposed model based on the knowledge gained from Chapter 3. Assumptions for the proposed model are established. The relationship between DUE model and SUE model is illustrated and additive link functions are assumed, resulting in a link-based plus implicit deterministic route choice behavior formulation for the proposed model. The inclusion of certain numerical techniques further simplifies the problem. Finally, a nested iterative algorithm is presented.

Chapter 5 examines the pre-design algorithm on a small hypothetical network. Insights of the properties of the proposed model and its solution algorithm are obtained. Numerical techniques are tested separately for their computational efficiency. Final decision is made to select Gauss quadrature method for the model implementation and application in the subsequent chapters.

Chapter 6 illustrates the modeling process in Cube planning system. The model is developed for pre-determined toll scheme taken from Dutch KMP system.

Chapter 7 describes the application of the Cube model. In order to demonstrate how heterogeneous VOT between and within multiple user-classes affect the network performance, two scenarios are built within the framework of Dutch KMP system. In addition, elaborations of potential application of the model to support the planning, operation and evaluation of Dutch KMP systems are presented in this chapter.

Chapter 8 states the main conclusions and outlines possible directions for future research.

2

Literature Review

The majority of past literatures on road pricing have ascertained the importance of the value of time (VOT) to traffic assignment models. As a consequence of this, many researchers commit themselves to improving the application of heterogeneous VOT. In this chapter, we will discuss two methodological categories of heterogeneous VOT, and give comprehensive insights into the complexity and challenges when including either of these heterogeneous VOT in the traffic assignment models.

Section 2.1 discusses the classifications of the heterogeneous VOT proposed in the literature, namely discrete VOT among multi user-classes and continuously distributed VOT over whole populations. Special attention is paid to possible asymmetric cost functions when multiple user-classes are presented on the network, therefore, the classic optimization problem can not be used to formulate the assignment model as discussed in Section 2.2. Debates on how to formulate continuously distributed VOT are given in Section 2.3. Section 2.4 gives a brief overview of different approaches to traffic assignment model.

2.1 Classifications of Heterogeneous VOT

To support planning, operation, and evaluation of various road pricing schemes, a traffic assignment model is often applied to predict path choices and the resulting network flow patterns, which in turn form the basis for assessing the economic and financial impacts or benefits of proposed toll schemes. Traditional practices limited their analysis with a single (average) value of time (VOT) for all road users leading to less accurate traffic forecasts than those of heterogeneous ones. It is now widely accepted that each user has a different VOT, depending on one's social-economic status, time of the day, and other factors (Small and Winston (1999), Wardman (2001)). Some studies also suggest that travel time is highly valued by motorists and that there is significant heterogeneity in these values. These findings influence the design and assessment of road pricing schemes a lot. By differentiating VOT in the traffic assignment models, more efficient and realistic forecasts of traffic conditions can be gained compared with traditional analysis (Small, Winston, and Yan, 2005).

Generally, in the literature, there are two approaches to address the heterogeneous VOT problem. The first one is the multi-class approach, which means the entire population of road users is divided into a number of classes according to a discrete VOT distribution. Yang et al. (2002) proposed an elastic demand multi-class network equilibrium model, which divided the whole road users' population into a number of classes, each class was assumed to have an average VOT. The model was developed to investigate how VOT distribution affects traffic flow and profit forecasts of private toll roads. Later, Han and Yang (2006) suggested a multi-class, multi-criterion traffic equilibrium assignment model using the same logic of discrete VOT to analyze the effects of second-best tolls and derive bound for the toll levies. Recently, Zhang et al. (2008), developed a unified framework of multi-class, multi-criteria UE-CN (user-equilibrium and Cournot-Nash principle) mixed equilibrium and found that uniform link tolls supporting such equilibrium not only exist but also lead to a system optimum.

The second category of heterogeneous VOT considers them to be continuously distributed across whole road user population. Leurent (1993) was among the first to propose theoretical framework for a cost versus time equilibrium model with variable demand, continuous distribution of VOT, and elastic travel time functions. Cantarella and Binetti (1998) extended existing fixed-point models for stochastic equilibrium assignment model to deal with the heterogeneous VOT among users. Neilsen (2002) also combined stochastic user equilibrium with continuous distributed VOT functions estimated on SP-data to analyze the traffic condition for Copenhagen Region. Recently Lu et al. (2007) presented a bi-criterion dynamic user equilibrium (BDUE) model, aiming to reflect road users' path choices in response to time-varying toll charges using a simulation based algorithm.

It should be emphasized that it is VOT that bridges the gap between time and money which enables the assignment models to provide policy makers with useful information for assessing the economic and welfare impacts of proposed road pricing facilities or schemes.

2.2 Multiple user-classes in Assignment Models

The inclusion of multiple user-classes in the traffic assignment model (see also Section 2.1) would thus lead to more complexity into the assignment model. The problem roots in the link travel time function. When we consider a multi-user class case the link travel time function now is a function of multiple flows instead of only a single flow. Users interact with each other, the link travel time for user class not only depends on the flow of user class 1, but also on the flow of user class 2. Mathematically,

$$\tau_a = \Gamma(u_a), \quad \forall a \tag{2.1}$$

where $\boldsymbol{u}_a \equiv [u_{am}]$ denotes the vector of all flows of all user classes on link a.

Back to our road pricing issue, as discussed in Section 2.1, the generalized travel cost function is our main concern. Generally, the generalized travel cost is a function of travel times, i.e.

$$c_{am} = c_{am}(\tau_{am}) \tag{2.2}$$

If we assume that the Jacobian matrix of link travel cost function holds for symmetry condition, the multiple user-classes social optimal assignment models can be formulated as optimization problems to find a user equilibrium (UE). Yang et al. (2002) used a summation of user class flows to compute total link flow. By doing this, they obtained symmetric link travel time functions. Thus they formulated a multi-class network equilibrium model with elastic demand as a convex minimization problem to investigate the affects brought by discrete VOT among multi-user classes and forecast the revenue from the tolled roads.

However, this condition is more restrictive in the case of multiclass traffic, since it means that user 1 from a certain class interacts user 2 from another class in the same way that user 2 bothers user 1. In most of the cases this will not be true, for example, fast drivers are hindered by slow drivers but not the reverse. (See also Bliemer (2001)).

Trying to find a solution set where multiple user-classes and heterogeneous VOT are presented on the network, variational inequality (VI) problem can be used to formulate the model, which is more general than an optimization problem in the sense that it can handle asymmetric cost functions. Basic formulations of DUE and SUE models using VI approach will be presented in Chapter 3.

2.3 Continuously Distributed VOT

The estimation of continuous distributed VOT patterns in the disutility function (generalized travel cost) is usually gained from stated preference (SP) surveys. Since little empirical experience existed concerning the distribution of VOT, a number of alternative formulations were proposed in various literatures. Two most commonly used formulations are explored here, called as normal and log-normal distributions.

There was some a prior preference for the non-negative log-normal distributed VOT, therefore eases application of the assignment model. In Leurent's (1993) small numerical experiment, he assumed that VOT was distributed according to a log-normal probability density function with mean value of 10$/h and the standard deviation of its log that was set equal to 0.6. Staring from the same logic, Cantarella and Binetti (1998) also assumed a log-normal distributed VOT in their numerical experiment. Two scenarios for VOT have been analyzed: low VOT, with mean set to 5 €/h, and high VOT, with mean equals to 10 € /h. Six values of standard deviation to mean ration have been considered, with a rate of deviation/mean= 0, 0.1, 0.25, 0.5, 0.75, 1.

On the other hand, there are some supportive voices from normal distributed VOT's side, for the reason that more advanced estimation techniques were available. Lu et al. (2007) assume a normal distribution of VOT with (mean, standard deviation) = (20, 10), which has the unit $/h in their bi-criterion dynamic user equilibrium (BDUE) model. Nielsen (2002) conducted researches based on SP data to gain estimation of continuous distributed patterns for VOT. Both normal and log-normal distributions were tested. Results showed that the normal distribution performed better than theoretically more attractive log-normal distribution (for its non negative variables). In Nielsen's research, a number of tests of log-normally distributed VOT were conducted. But these were not successful, since the variances estimated for the log-normal distribution were implausibly large and not well estimated.

Regardless of the density function chosen for continuous distributed VOT adopted the traffic assignment model, once adding such a component into the model, numerical approximation techniques are required. The consideration of numerical approximation techniques will have some implications for constructing a solution algorithm (See Chapter 4).

2.4 Model Implementations

A concise overview of some implementations of theoretical traffic assignment models proposed in the literature for the heterogeneous VOT problem will be presented in this section.

We will restrict ourselves to the following issues when describing the differences between the traffic assignment models:

A) Categories of Heterogeneous VOT

In transportation modeling with the consideration heterogeneous VOT, traffic assignment models are developed by either a discrete set of VOT for several distinct user classes or continuous distributed VOT across the whole population.

B) Route Choice Behavior

The route choice is based on generalized travel cost either in time-term or monetary-term in all models. The different perceptions can be distinguished. For deterministic route choice models, actual generalized travel costs are used. On the other hand, the perceived generalized travel costs are adopted in stochastic route choice model. The route choice behavior in turn determines the characteristics of the traffic assignment model and its solution algorithm.

C) Model Formulation

Models can be mathematically formulated in many different ways. One simple and convenient formulation is through a (nonlinear) optimization problem. By introducing asymmetric cost functions, models need to be expressed as a variational inequality problem, a fixed-point problem or a complementarity problem. The way of model formulation will influence solution algorithms of the model.

D) Solution Algorithms

Two categories of solution algorithms can be distinguished: a route-based algorithm and a link-based algorithm. The route-based algorithms have intuitive interpretations which are developed directly from route choice behavior. However, these algorithms require explicit route enumeration, a computationally intractable problem for realistic networks. Therefore, link-based algorithms are more practical.

E) Travel Demand Patterns

Two travel demand patterns adopted in the listed traffic assignment models are distinguished. The first one is inelastic demand, i.e. the travel demand is fixed during the whole process of the assignment models. The second one is elastic demand, while assuming that the demand is dependent of prevailing network conditions of the assignment model.

All models described so far considered two categories of VOT separately. Leurent (1993) was one of the pioneers to propose cost versus time network equilibrium and assumed a continuously log-normal distribution of VOT in deterministic user equilibrium algorithm. Yang et al. (2002) developed a discrete multi-class approach dividing the whole population into a number of classes, each holding an average VOT with some intervals. One favorable advantage of their model is its capability to assess the distribution of the benefits generated by the toll road across different classes of user between OD pair. This treatment in certain cases can be regarded as a discrete approximation of the model using continuously distributed VOT. Nonetheless, there exist critical questions to verify this discrete approximation, for instance, how many number of user classes in the equilibrium model are needed to give a good approximation, how to assign VOT to these user class, etc.

The definitions of the models listed in Table 2.1 are derived from extending Wardrop's principle. However, the models differ in formulations of the generalized travel cost or generalized travel time. The assumptions for route choice behavior are distinct as well. Basic methodological frameworks in Cantarella and Binetti (1998) and Nielsen's (2002) assignment models followed the stochastic user equilibrium (SUE). There are other randomly distributed coefficients in their generalized travel time/cost function. In addition, they adopted a probabilistic path choice function in their papers. On the other hand, the remaining models rely on deterministic route choice behavior framework, i.e. all users were assumed to have perfect information on the traffic condition and their path choice behavior is to minimize their generalized travel time/cost. The latter property is reflected from the direction finding step in the algorithms of the remaining models.

Table 2.1: An Overview of Models in Dealing with Heterogeneous VOT

	A	B	C	D	E
Yang et. al (2002)	DM	D	OPT	LB	ED
Yang and Han 2006	DM	D	VI	LB	IE
Zhang et. Al (2008)	DM	D	VI	LB	IE
Leurent (1993)	CD	D	OPT	LB	IE&ED
Cantarella and Binetti (1998)	CD	S	FP	RB	IE
Nielsen (2002)	CD	S	N/A	RB	IE
Lu et. al (2007)	CD	D	VI	RB	IE

Legend

A) DM = discrete multi-class VOT, CD = continuous distributed VOT

B) D = deterministic route choice behavior, S = stochastic route choice behavior

C) OPT = optimization problem, VI = variational inequality problem, FP = fixed-point problem

D) LB = link-based, RB = route-based

E) IE = inelastic demand, ED = elastic demand

Cantarella and Binetti (1998), Nielson et al. (2002) and Lu et al. (2007) presented simulation-route-based algorithms for solving their equilibrium assignments. Developed from a static basis, Cantarell and Binetti's model was applied to a medium-size urban network with 145 nodes and 390 links. Nielson's model was used on a full-scale network of Copenhagen with 2,369 nodes and 3,462 links. The whole model can be run in 24 hours on a 180 MHz Pentium Pro PC and ensures a convergence. On the other hand, Lu's model was embedded in the dynamic assignment modeling, which requires a more complex procedure and a larger set of time-dependent efficient route. When generating this kind of grand route set, the algorithm failed in the large-scale network case. The remaining models in Table 2.1 adopted link-based algorithms, and gave relatively better performance than the route-based one.

The only two models in our list can include elastic demand into their algorithm. Leurent's (1993) assignment model accommodated elastic demand. He defined elastic demand in his paper as follows: the total trip rate *(r to s)* depends on an average generalized travel time *(r to s)*. Yang et al (2002) incorporated the elastic demand by assuming that the demand of user class between each OD pair is a function of generalized travel time. One favorable advantage of their models is its capability of capturing possible demand changes after implementation of a road pricing strategy.

2.5 Summary

The importance of inclusion heterogeneous VOT into traffic assignment models has been extensively discussed in literature. More and more efforts have been put into improving the traffic assignment models to incorporate with heterogeneous VOT, resulting in a wide variety of different approaches to this kind of traffic assignment models. Although a lot of effort has been put into this topic, most of approaches follow two general lines, either considering a discrete VOT among multiple user-classes or a continuously distributed VOT across the whole road user population. Either of these two approaches has it own advantages in predicting a more realistic network performance. Nevertheless, a general traffic assignment model describing further heterogeneities among multiple user-classes in route choice and traffic flow operation is still lacking. An evolution of approaches to heterogeneous VOT is illustrated in Figure 2.1.

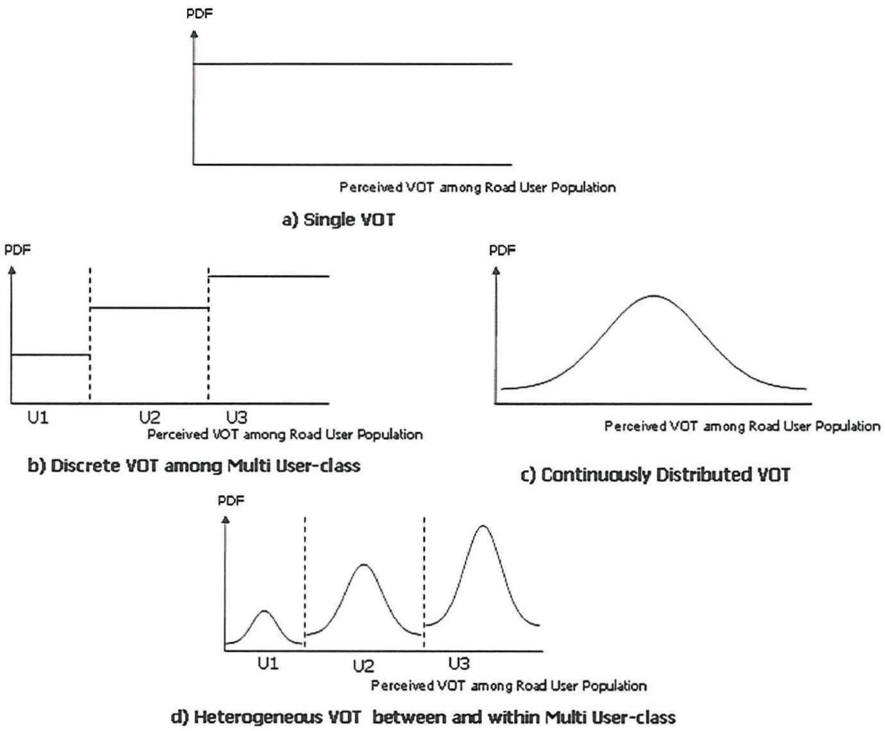

Figure 2.1: An Evolution of Approaches to Heterogeneous VOT

In this thesis, we propose a multiclass user equilibrium traffic assignment model with continuously distributed VOT within each user class for estimating network performance under a given road pricing scheme. The extension of the proposed traffic assignment model to include heterogeneous VOT among multiple user-classes will challenge the conventional approaches from the model formulation aspect and design of the algorithms for these models. A strong emphasis will lie on the mathematical formulation and design of the solution algorithm of the proposed model. To gain insights into the complexity of the mathematical formulation, several types of variational inequality applications for static assignment models are discussed in Chapter 3. This background knowledge provides a basis for the formulation and solution algorithm of the proposed model presented in Chapter 4.

3

Mathematical Foundations: Variational inequality Problems

The variational inequality (VI) problem is a general problem formulation that encompasses a set of mathematical problems, including nonlinear equations, optimization problems, complementarity problems and fixed point problems. Because of its general capability to formulate transportation problems, VI has received increasing attention from transportation field. In this chapter we will present a set of formulations for static assignment models using variational inequality approach.

Section 3.1 describes asymmetric generalized route cost functions caused by interactions among user classes on the road network. Section 3.2 formulates a route-based MUC-DUE VI model. In Section 3.3, the route-based MUC-DUE model is rewritten into a link-based one by assuming additive cost functions. A route-based MUC-SUE VI model is presented in Section 3.4.

3.1 Asymmetries in Multiclass Generalized Route Cost Functions

As discussed in Section 2.2, only by assuming symmetric Jacobian matrix of generalized route cost functions, no interactions among user-classes, an optimization problem can be formulated to find a user equilibrium (UE). However, this assumption is too restrictive to reflect a realistic traffic condition. To improve the performance of the assignment model, in this paper we assume that the generalized route cost function of a certain user-class is assumed to be a function of all vehicles from all user-classes with interactions over the network without loss of generality.

We now start to analyze this type of generalized route travel cost from link travel time function, in terms of link volume v_a,

$$\tau_a = \Gamma(v_a) \tag{3.1}$$

$$v_a = \sum_m pce_m u_{am} \tag{3.2}$$

Then the generalized link travel cost function can be expressed as follows:

$$c_{am} = c_{am}(u_{a1}, u_{a2}, \ldots, u_{am}) \tag{3.3}$$

Therefore, generalized route travel cost function can be formulated as,

$$c_m^{prs} = c_m^{prs}(u_{a1}, u_{a2}, \ldots, u_{am}) \tag{3.4}$$

It is important to note that the generalized route cost function can be written in terms of user-specific link flows by internal relationships between equation (3.1) – (3.4) (See also Figure 3.1).

$$\begin{tikzcd}
u_{am} \arrow[r, "pce_m"] \arrow[rrrrr, dashed, bend right] & v_a \arrow[r, "\Gamma(\cdot)"] & \tau_a \arrow[r, "\beta_m"] & c_{am} \arrow[r] & \delta_{am}^{prs} \arrow[r] & c_m^{prs}
\end{tikzcd}$$

Figure 3.1: Interrelationship between Generalized Route Cost and Link Flow

By introducing passenger car unit, pce^m, in computing total link volume on link *a*, we now have interactions among user-classes over the network, in a coarse way (linear interactions). In mathematical terms, these interactions are asymmetric and therefore there does not exist a corresponding optimization model for this kind of asymmetric cost functions (See also Bliemer 2001). In contrast, the variational inequality approach is necessary due to its capability to deal with asymmetric cost functions.

Proof

We can prove that equation (3.4) is asymmetric using Jacobian matrix of the link generalized travel cost. Consider a certain link *a*, the Jacobian matrix of the link generalized travel cost function can be expressed as follows:

$$\nabla c_a = \begin{bmatrix} \frac{\partial c_{a1}}{\partial u_{a1}}, \frac{\partial c_{a2}}{\partial u_{a1}}, \ldots, \frac{\partial c_{am}}{\partial u_{am}} \\ \frac{\partial c_{a1}}{\partial u_{a2}}, \frac{\partial c_{a2}}{\partial u_{a2}}, \ldots, \frac{\partial c_{am}}{\partial u_{am}} \\ \cdot \;,\; \cdot \;,\cdot\;,\; \cdot \\ \cdot \;,\; \cdot \;,\cdot\;,\; \cdot \\ \cdot \;,\; \cdot \;,\cdot\;,\; \cdot \\ \frac{\partial c_{a1}}{\partial u_{am}}, \frac{\partial c_{a2}}{\partial u_{am}}, \ldots, \frac{\partial c_{am}}{\partial u_{am}} \end{bmatrix} \tag{3.5}$$

By equation (3.1) and (3.2), it holds that

$$\frac{\partial c_{am}}{\partial u_{am'}} \neq \frac{\partial c_{am'}}{\partial u_{am}} \tag{3.6}$$

It is obvious that ∇c_a is asymmetric. Since every link on the network has a similar Jacobian matrix, from(3.3), we can derive that

$$\frac{\partial c_{am}}{\partial u_{a'm}} \neq \frac{\partial c_{a'm}}{\partial u_{am}} \tag{3.7}$$

Equations (3.6) and (3.7) imply that $\nabla_{u_{am}} c_m^{prs}(u_{am})$ is an asymmetric matrix. ■

3.2 A Route-Based MUC-DUE VI Model

3.2.1 Equilibrium Condition

We will present the extension of Wardrop's first principle to include multiple user-classes into DUE traffic assignment model as follows.

Definition 3.1: *For each user-class and for each origin-destination pair, the actual generalized route travel cost for all users traveling between a specific OD pair are equal, and less than the actual generalized route travel cost which would be experienced by a single user on any unused feasible route for that user-class.*

3.2.2 Mathematical Conditions

Let $\Omega = \{N, A\}$ denote a road network consisting of a set of nodes N and a set of links A. Each origin-destination pair (r, s) is defined by an origin $r \in R \subseteq N$ and a destination $s \in S \subseteq N$, while P_m^{rs} presents set of route between origin nodes *r* and destination nodes *s* for user-class $m \in M$. An origin-destination pair can have one or more routes $p \in P_m^{rs}$ connecting the origin and the destination. Every route $p \in P_m^{rs}$ from *r* to *s* is comprised of one or more links.

Recalling definition 3.1 in mathematical terms, it states that all class *m* users from origin *r* to destination *s* experience the same generalized route cost and this generalized route cost is minimal. Let us denote the actual generalized route cost function for class *m* users over route $p \in P_m^{rs}$ by c_m^{prs}, and the minimal generalized route cost for class *m* users from origin *r* to destination *s* by π_m^{rs}, i.e.

$$\pi_m^{rs} = min_p c_m^{prs}, \quad \forall r,s,m,p \tag{3.8}$$

Let f_m^{prs} denote the route flow of class *m* users using route *p* from origin *r* to destination *s*. Suppose that these route flows are feasible. Then the condition for a multiclass user-optimal state – implied by definition 3.1 – can be written in mathematical terms as follows:

$$c_m^{prs*} - \pi_m^{rs*} \geq 0 \tag{3.9}$$

$$f_m^{prs*}(c_m^{prs*} - \pi_m^{rs*}) = 0 \tag{3.10}$$

$$f_m^{prs*} \geq 0 \tag{3.11}$$

$$\forall p,r,s,m$$

The asterisk in the above equations denotes that the flow variables are the optimal solutions under definition 3.1. For any OD pair *(r, s)*, if there is a positive route flow over route *p*, i.e. $f_m^{prs*} > 0$, according to (3.10):

$$c_m^{prs*} = \pi_m^{prs*} \qquad \forall p,r,s,m \tag{3.12}$$

Thus, route flow f_m^{prs*} uses the minimal actual generalized route cost π_m^{rs}. If the flow over route *p* is zero, i.e. $f_m^{prs*} = 0$, equation (3.10) requires that $(c_m^{prs*} - \pi_m^{rs*})$ be either zero or positive (ensured by equation(3.9)). On the other hand, if route *p* has higher generalized route travel cost, i.e. $c_m^{prs*} > \pi_m^{rs*}$, according to (3.10):

$$f_m^{prs*} = 0 \tag{3.13}$$

3.2.3 Constraint Set

The constraint set for this route-based model is summarized as follows.

Flow conservation constraints:

$$\sum_{p} f_m^{prs} = D_m^{rs}, \quad \forall r,s,m \tag{3.14}$$

Nonnegativity constraints:

$$f_m^{prs} \geq 0, \quad \forall r,s,m,p \in P_m^{rs} \tag{3.15}$$

3.2.4 Variational Inequality Problem

In order to find user-equilibrium route flows that satisfy equilibrium conditions (3.9)-(3.11) and consists of mathematical constraints formulated in last section, a variational inequality (VI) problem is formulated:

To find an $\boldsymbol{f}^* \in \Omega$ such that

$$\sum_{r,s}\sum_{m}\sum_{p} c_m^{prs*}(f_m^{prs} - f_m^{prs*}) \geq 0 \tag{3.16}$$

Where Ω is defined as the set of all **f** satisfying the following constraints:

$$\sum_{p} f_m^{prs} = D_m^{rs}, \quad \forall r,s,m \tag{3.17}$$

$$f_m^{prs} \geq 0, \quad \forall r,s,m,p \in P_m^{rs} \tag{3.18}$$

Proof:

Proof of Necessity

We need to prove that the equilibrium conditions (3.9)-(3.11) imply VI problem formulated in (3.16)-(3.18). For any route *p*, let f_m^{prs*} be a feasible route flow which satisfies the condition (3.10). The conditions can be written as:

$$f_m^{prs*}(c_m^{prs*} - \pi_m^{rs*}) = 0 \quad \forall p,r,s,m \tag{3.19}$$

By definitions of π_m^{rs} in (4.1) and nononegativity (3.15), which hold for all $r,s,m,p \in P_m^{rs}$. We have $c_m^{prs*} - \pi_m^{rs*} \geq 0$ and $f_m^{prs} \geq 0$. Hence, the following inequality should hold:

$$f_m^{prs}(c_m^{prs*} - \pi_m^{rs*}) \geq 0 \quad \forall p,r,s,m \tag{3.20}$$

We subtract (3.19) from (3.20) yields

$$(c_m^{prs*} - \pi_m^{rs*})(f_m^{prs} - f_m^{prs*}) \geq 0 \quad \forall p,r,s,m \tag{3.21}$$

Summing (3.21) for all routes *p*, all OD pairs *(r, s)* and all user classes *m*, we obtain:

$$\sum_{r,s}\sum_{m}\sum_{p}(c_m^{prs*}-\pi_m^{rs*})(f_m^{prs}-f_m^{prs*})$$

$$=\sum_{r,s}\sum_{m}\sum_{p}(f_m^{prs}-f_m^{prs*})c_m^{prs*}-\sum_{r,s}\sum_{m}\pi_m^{rs*}\sum_{p}(f_m^{prs}-f_m^{prs*})\geq 0 \quad (3.22)$$

From flow conservation constraint (3.14), it follows that:

$$\sum_{p} f_m^{prs}=\sum_{p} f_m^{prs*}=D_m^{rs} \quad (3.23)$$

Using (3.23), the second term in (3.22) vanishes and implies that (3.22) results in the VI problem (3.16).

$$\sum_{r,s}\sum_{m}\sum_{p} c_m^{prs*}(f_m^{prs}-f_m^{prs*})\geq 0, \quad \forall f\in\Omega \quad (3.24)$$

Proof of Sufficiency

We need to prove that any solution f_m^{prs*} of the VI problem (3..16) satisfies the equilibrium conditions (3.9)-(3.11). The first and third equilibrium conditions (3.9) and (3.11) hold by definition. Thus, we only need to prove that the second equilibrium condition (3.10) also holds.

Assuming that equilibrium condition (3.10) does not hold only for a route *j* for user class *m* travelling between OD pair *(x, y)*, i.e.,

$$f_m^{jxy*}>0 \;\; and \;\; c_m^{jxy*}-\pi_m^{xy*}>0 \quad (3.25)$$

Since (3.10) holds for all routes other than route *j* for OD pair *(x, y)*, it follows that

$$\sum_{r,s}\sum_{m}\sum_{p}(c_m^{prs*}-\pi_m^{rs*})f_m^{prs*}$$

$$=f_m^{jxy*}(c_m^{jxy*}-\pi_m^{xy*})>0 \quad (3.26)$$

Note that all other terms in the above equation vanish due to equilibrium condition (3.10).

For each OD pair *(r, s)*, we can always find one minimal travel cost route *l* for user class *m* travelling between OD pair *(r, s)*, where route *l* was evaluated under the optimal route flow $\{f_m^{prs*}\}$. For this route *l*, equilibrium condition (3.9) becomes equality by definition:

$$c_m^{lrs*}-\pi_m^{rs*}=0 \quad \forall l,r,s,m \quad (3.27)$$

Next we need to find a set of feasible route flow f_m^{prs} so that the following equations

$$f_m^{prs}(c_m^{prs*}-\pi_m^{rs*})=0 \quad \forall p,r,s,m \quad (3.28)$$

Always hold. We consider total travel demand D^{rs} for all OD pairs. For each OD pair *(r, s)* and for user class m, we assign travel demand D_m^{rs} to the minimal actual travel cost route l, which was evaluated under the optimal route flow pattern $\{f_m^{prs*}\}$. This generates a set of feasible route flow patterns $\{f_m^{prs}\}$. Summing up equations for all indices yields

$$\sum_{r,s}\sum_{p}\sum_{m} f_m^{prs}(c_m^{prs*} - \pi_m^{rs*}) = 0 \tag{3.29}$$

We subtract equation (3.27) from equation (3.29) and obtain

$$\begin{aligned}
&\sum_{r,s}\sum_{m}\sum_{p}(c_m^{prs*} - \pi_m^{rs*})(f_m^{prs} - f_m^{prs*}) \\
&= \sum_{r,s}\sum_{m}\sum_{p}(f_m^{prs} - f_m^{prs*})c_m^{prs*} - \sum_{r,s}\sum_{m}\pi_m^{rs*}\sum_{p}(f_m^{prs} - f_m^{prs*}) \\
&= \sum_{r,s}\sum_{m}\sum_{p}c_m^{prs*}(f_m^{prs} - f_m^{prs*}) < 0
\end{aligned} \tag{3.30}$$

Where the flow conservation

$$\sum_{p} f_m^{prs} = \sum_{p} f_m^{prs*} = D_m^{rs}$$

holds for each OD pair (r, s) so that second term vanishes. (3.30) contradict inequation (3.26). Therefore, any optimal solution $\{f_m^{prs*}\}$ to VI problem satisfies equilibrium condition (3.10).

This is the end of the proof. ■

3.3 A Link-Based MUC-DUE VI Model

The route-based VI model presented in Section 3.2 requires a prior enumeration of route sets P_m^{rs} in the solution algorithms. Nonetheless, route enumeration is a great burden if the network is large, thus making the route-based model non-attractive when applied to a realistic transportation network. Therefore, we will rewrite the route-based model into a link-based on to overcome this problem.

3.3.1 Variational Inequality Problem

The equilibrium condition for the link-based VI model is identical to that of the route-based VI model. The basic difference between the two models is that the link-based model is formulated on link-based flow variables, u_{am}, instead of route-based variables, f_m^{prs}, as in the route-based model. If the actual generalized route cost function is additive, i.e. the sum of the actual generalized link cost c_{am} experienced along the route,

$$c_m^{prs} = \sum_a \delta_{am}^{prs} c_{am} \qquad (3.31)$$

then we can formulate a link-based model as a variational inequality problem as follows:

Find a $\boldsymbol{u}^* \in \Omega$ such that

$$\sum_a \sum_m c_{am}^* (u_{am} - u_{am}^*) \geq 0, \quad \forall u \in \Omega \qquad (3.32)$$

where Ω is defined as the set of all **u** satisfying the following constraints:

$$\sum_{p \in P_m^{rs}} f_m^{prs} = D_m^{rs}, \quad \forall r,s,m \qquad (3.33)$$

$$u_{am} = \sum_{r,s} \sum_p \delta_{am}^{prs} f_m^{prs}, \quad \forall a,m \qquad (3.34)$$

$$f_m^{prs} \geq 0, \quad \forall r,s,m,p \in P_m^{rs} \qquad (3.35)$$

Proof

We will show how a route-based VI model can be rewritten into a link-based model by assuming additive generalized route cost function (3.31). In Section 3.2, we have formulated the route-based VI model as follows:

$$\sum_{r,s} \sum_m \sum_p c_m^{prs*} (f_m^{prs} - f_m^{prs*}) \geq 0 \qquad (3.36)$$

Assuming additive generalized route cost function, i.e.,

$$c_m^{prs*} = \sum_a \delta_{am}^{prs} c_{am}^* \qquad (3.37)$$

Substitute equation (3.37) for actual generalized function c_m^{prs*} in (3.36), yielding:

$$\sum_{r,s} \sum_m \sum_p \sum_a \delta_{am}^{prs} c_{am}^* (f_m^{prs} - f_m^{prs*}) \geq 0 \qquad (3.38)$$

Rearranging the summation yields:

$$\sum_a \sum_m c_{am}^* [\sum_{r,s} \sum_p \delta_{am}^{prs} (f_m^{prs} - f_m^{prs*})] \geq 0 \qquad (3.39)$$

Recall constraint condition (3.34):

$$u_{am} = \sum_{r,s} \sum_p \delta_{am}^{prs} f_m^{prs}, \quad \forall a,m \qquad (3.40)$$

means that we can rewrite inequality (3.39) as

$$\sum_a \sum_m c_{am}^* (u_{am} - u_{am}^*) \geq 0 \tag{3.41}$$

Note that we only used substitution and rearranging terms, hence we can reverse the process and go from the link-based formulation to a route-based formulation as well. ■

3.4 A Route-Based MUC-SUE VI Model

3.4.1 Equilibrium Condition

Again the stochastic and multiclass user equilibrium condition is defined by extending Wardrop's first principle as follows:

Definition 3.2: *For each user-class and for each origin-destination pair, the perceived generalized route travel cost for all users traveling between a specific OD pair are equal, and less than the generalized route travel cost which would be experienced by a single user on any unused feasible route for that user-class.*

3.4.2 Mathematical Condition

The above definition can be written mathematically as:

$$f_m^{prs} = D_m^{rs} \Psi_m^{prs}, \quad \forall p, r, s, m \tag{3.42}$$

where Ψ_m^{prs} denotes the probability that users of class *m* choose route *p* from origin *r* to destination *s*.

$$f_m^{prs*} \geq 0 \tag{3.43}$$

We also assume that the perceived generalized route function is increasing with the route flow f_m^{prs}, i.e.,

$$\frac{\partial c_m^{prs}}{\partial f_m^{prs}} > 0 \tag{3.44}$$

3.4.3 Variational Inequality Problem

The above multiclass SUE model can be formulated as a variational inequality problem below:

Find a $\boldsymbol{f}^* \in \Omega$ such that

$$G_m^{prs}(f_m^{prs} - f_m^{prs*}) \geq 0 \qquad (3.45)$$

Where Ω is defined as the set of all **f** satisfying the following constraints:

$$\sum_{p \in P_m^{rs}} f_m^{prs} = D_m^{rs}, \quad \forall r, s, m \qquad (3.46)$$

$$f_m^{prs} \geq 0, \quad \forall r, s, m, p \in P_m^{rs} \qquad (3.47)$$

In (4.38), the cost function G_m^{prs} is defined as follows:

$$G_m^{prs} = (f_m^{prs*} - D_m^{rs}\Psi_m^{prs})\frac{\partial c_m^{prs}}{\partial f_m^{prs}} \qquad (3.48)$$

For more detailed explanation, please refer to Nagurney (1993) and Ran and Boyce (1996). They proved that the variational inequality holds in stochastic dynamic models, but the mathematical proof of this kind of static stochastic user equilibrium model is similar to the dynamic ones, if we omit the time indices in their models.

3.5 Summary

In this chapter we have formulated two static, MUC-DUE and MUC-SUE, assignment models as variational inequality problems. An advantage of these VI models is that they can deal with asymmetric cost functions if interactions among user classes present on the network.

These two models are formulated from their equilibrium conditions and they provide an intuitive interpretation. However, their solution algorithm requires explicit route enumeration, a computationally intractable problem for realistic networks. Therefore, a link-based VI model is proposed for MUC-DUE assignment model. Next chapter will extend these basic concepts to formulate an assignment model to incorporate distributed VOT among multiple user-classes as a variational inequality problem.

4

MUC-SUE with Heterogeneous VOT Model Formulation for Road Pricing Issues and Its Solution Algorithm

This chapter mathematically formulates the proposed MUC-SUE with heterogeneous VOT traffic assignment model. The model to be formulated describes user-optimal flows of different user-classes on a network. Each user chooses his or her preferred route from origin to destination according to that user's particular VOT under a certain road pricing scheme. The different user-classes experience different generalized route costs, and interact with each other in an asymmetric fashion. Formulation for this specific type of assignment model is based on the variational inequality problem, which is generally presented in Chapter 3.

Section 4.1 establishes the assumptions for the MUC-SUE with heterogeneous VOT model. Section 4.2 defines the heterogeneous VOT within multiple user-classes traffic equilibrium. Model constraints are established in Section 4.3. The relationship of the stochastic route choice model and the deterministic route choice model is presented in Section 4.4, followed by a link-based variational inequality model formulation. In order to include continuously distributed VOT within multiple user-classes in the assignment model, numerical techniques are required and discussed briefly in Section 4.5. Section 4.6 designs an algorithm to solve the MUC-SUE with stochastic VOT assignment model.

4.1 Model Assumptions

The study focuses on the flow distribution patterns under a certain road pricing scheme and it is assumed that the road users are informed of the network status and have changed their behavior accordingly. There are a number of behavioral mechanisms through which travelers adapt to new network conditions. Having a road pricing scheme over a network, changing trip frequency, shifting to travel at another time of day, choosing another route or an another mode of transport might be considered by road users. Long-term changes such as changing car-owner ship and reallocation of home and/or place of work could happen as well. In order to simplify the model and highlight the model properties and results in a more distinct way, the proposed model merely considers adaptations in route choice under equilibrium condition. It hence concerns a fixed travel demand during the whole traffic assignment procedure, not allowing for changes in time period, trip frequency and/or transfers to other modes. This may seem inappropriate, since not allowing possible reduction and or time shift in vehicle traffic could lead to a biased assessment of traffic conditions after implementing a road pricing scheme. However, studies show that people in general do not abandon their private car so easily (Transek 1999). Home/work relocations are not so easy and even it is the case this will not change the travel demand to a large extend. Therefore, the exclusion of changes in departure time is more questionable. Nonetheless, if a flat toll scheme is considered and if we only predict a short-term traffic condition, the fixed travel demand model is sufficient, in terms of less complex model structure which in turn resulting a quicker run time needed to reach an equilibrium.

To be more precise, the assumptions are established before we mathematically formulate the proposed model. These assumptions provide insights into the properties of proposed model and simplify the model to some extends so that it can tackle realistic networks.

Assumption 1: *For each OD pair, the travel demand is fixed.*

Fixed demand means that the departure choice is not included in the model. Although we restrict ourselves to consider fixed demand only, it is possible to relax this assumption and include elastic demand in the model by some modification.

Assumption 2: *A logit route choice function with infinite scale parameter,* $\omega \to \infty$, *is used to describe route choice behaviors for all users.*

By this assumption, the route choice behavior collapsed from a stochastic one to a deterministic one. Mathematical proof is given in Section 4.4.

Assumption 3: *A normally distributed VOT among each user class.*

The normally distributed VOT patterns determine certain numerical techniques in the algorithm. More discussions can be found in Section 4.5 and Appendix A.

4.2 MUC-SUE with Heterogeneous VOT Equilibrium Conditions

The equilibrium condition is defined below by extending Wardrop's first principle. Note that, the stochastic VOT is included in the assignment model by re-formulating the perceived generalized route costs.

Definition 4.1: *For each user in any user class and for each origin-destination pair, every* *road user cannot decrease generalized trip travel cost with respect to that user's particular VOT by unilaterally changing paths.*

This implies that each user is assigned to a route having the minimal generalized route cost with respect to his/her own VOT.

4.3 Route and Link flow Constraints

In this section, the constraint set for the proposed model with respect to the assumptions and definition are present here to guarantee that route and link flow describe a feasible pattern.

Flow conservation constraint

$$\sum_{p \in P_m^{rs}} f_m^{prs} = D_m^{rs}, \quad \forall r,s,m \tag{4.1}$$

Nonnegativity constraints

$$f_m^{prs} \geq 0, \quad u_{am} \geq 0, \quad \forall p,r,s,a,m \tag{4.2}$$

Relationship between route and link variables

$$u_{am} = \sum_{r,s} \sum_{p} \delta_{am}^{prs} f_m^{prs} \quad \forall p,r,s,a,m \tag{4.3}$$

$$\tau_m^{prs} = \sum_{a} \delta_{am}^{prs} \tau_{am}, \quad \forall p,r,s,m \tag{4.4}$$

Definitional Constraints

$$c_m^{prs} = \beta_m \tau_m^{prs} + \theta_m^{prs}, \quad \forall p,r,s,m \tag{4.5}$$

$$f_m^{prs} = D_m^{rs} \Psi_m^{prs}, \quad \forall p,r,s,m \tag{4.6}$$

Assumable Conditions

$$\beta_m \sim N(\mu_m, \sigma_m) \tag{4.7}$$

$$\theta_m^{prs} = \sum_{a} \delta_m^{prs} \theta_{am} \tag{4.8}$$

$$\Psi_m^{prs} = \frac{exp(-\omega c_m^{prs})}{\sum_{i \in P_m^{rs}} exp(-\omega c_m^{irs})}, \quad \omega \to \infty \tag{4.9}$$

4.4 Route Choice Behavior Assumptions and Variational Inequality Problem

In Chapter 3, we formulated a general variational inequality problem for multiclass stochastic user equilibrium model, as shown in (3.45)-(3.48). In this section, we will simplify the general VI problem into a link-based model under deterministic route choice with continuous normally distributed VOT as follows:

Find a $\boldsymbol{u}^* \in \Omega$ such that

$$\sum_a \sum_m \int_{\beta_m} c_{am}^*(u_{am} - u_{am}^*) g(\beta_m) d\beta_m \geq 0, \quad \forall u \in \Omega \tag{4.10}$$

where Ω is defined as the set of all **u** satisfying the following constraints:

$$\sum_{p \in P_m^{rs}} f_m^{prs} = D_m^{rs}, \quad \forall r,s,m \tag{4.11}$$

$$u_{am} = \sum_{rs} \sum_p \delta_{am}^{prs} f_m^{prs}, \quad \forall a,m \tag{4.12}$$

$$f_m^{prs} \geq 0, \quad \forall r,s,m,p \in P_m^{rs} \tag{4.13}$$

Proof

The proof of transform a route-based VI problem into a link-based one is analogous to the proof provided in Section 3.3.1. Here we merely present by assuming the scale parameter, $\omega \to \infty$, the logit-based route choice model will collapse to a deterministic one.

The logit-based stochastic route choice function is

$$f_m^{prs} = D_m^{rs} \frac{exp(-\omega c_m^{prs})}{\sum_{i \in P_m^{rs}} exp(-\omega c_m^{irs})}, \quad \forall r,s,m,p \tag{4.14}$$

Rewriting equation (4.14) yields

$$\frac{f_m^{prs}}{exp(-\omega c_m^{prs})} = \frac{D_m^{rs}}{\sum_{i \in P_m^{rs}} exp(-\omega c_m^{irs})}, \quad \forall r,s,m,p \tag{4.15}$$

Then we assume that there is another route l from OD pair (r, s) which has the route flow f_m^{lrs}. It follows that

$$\frac{f_m^{lrs}}{exp(-\omega c_m^{lrs})} = \frac{D_m^{rs}}{\sum_{i \in P_m^{rs}} exp(-\omega c_m^{irs})}, \quad \forall r,s,m,l \tag{4.16}$$

Comparing equations (4.15) and (4.16), we have

$$\frac{f_m^{prs}}{exp(-\omega c_m^{prs})} = \frac{f_m^{lrs}}{exp(-\omega c_m^{lrs})}, \quad \forall r,s,m,p,l \tag{4.17}$$

The route flows are nonnegative, we take the logarithms of equation (4.17), we obtain

$$ln\, f_m^{prs} + \omega c_m^{prs} = ln\, f_m^{lrs} + \omega c_m^{lrs} \tag{4.18}$$

Dividing the above equation by ω yielding

$$\frac{1}{\omega} ln\, f_m^{prs} + c_m^{prs} = \frac{1}{\omega} ln\, f_m^{lrs} + c_m^{lrs} \tag{4.19}$$

As $\omega \to \infty$,

$$\frac{1}{\omega} ln\, f_m^{prs}, \frac{1}{\omega} ln\, f_m^{lrs} \to 0 \tag{4.20}$$

thus we have

$$c_m^{prs} = c_m^{lrs} \tag{4.21}$$

The above equation demonstrates that for any OD pair *(r, s)*, any feasible route flow has equal perceived generalized route cost. ■

4.5 Solutions to Continuously Distributed VOT

We have formulated a link-based VI problem (4.10)-(4.13) for the proposed model. Compared to a route-based model, the link-based one is easier to solve for the reason that the solution algorithm for such a model do not need route enumeration. Nevertheless, the continuously distributed VOT in (4.10) is difficult to solve. Here we propose two numerical techniques to overcome this problem, Quasi-Monte Carlo method with Halton Draws and Gauss-Hermite approximation method.

By introducing these two numerical methods, the integral in (4.10) can be approximated as following:

- Quasi-Monte Carlo method with Halton Draws

$$\sum_a \sum_m \int_{\beta_m} c_{am}^*(u_{am} - u_{am}^*) g(\beta_m) d\beta_m \approx \frac{1}{K} \sum_a \sum_m \sum_k c_{amk}^*(u_{amk} - u_{amk}^*) \tag{4.22}$$

- Gauss-Hermite Approximation

$$\sum_a \sum_m \int_{\beta_m} c^*_{am}(u_{am} - u^*_{am}) g(\beta_m) d\beta_m \approx \sum_a \sum_m \sum_k w_{mk} c^*_{amk}(u_{amk} - u^*_{amk}) \quad (4.23)$$

This yields that it is merely a repeated deterministic user-equilibrium (DUE) problem over different draws of the value-of-time. The inner problem can be solved using an iterative all-or-nothing assignment, which is usual for solving a DUE problem. However, now it needs to be repeated for each draw of the value-of-time and then averaged (Quasi-Monte Carlo) or weighted averaged (Gauss-Hermite).

Quasi-Monte Carlo method is similar to traditional Monte Carlo simulation but using quasi-random sequences instead of (pseudo) random numbers, which requires significantly less draws **K** to get a good approximation of the integral (typically, if 500 pseudo-random draws are need for a good approximation, then about 50 quasi-random draws can be used instead for the same precision, hence 10x less). Even faster would be Gaussian quadrate, which does not use a simple average as in (4.22), but uses a weighted average, in which the weights and the draws depend on the distribution (the assumption of normally distributed VOT leads to Gauss-Hermite approximation). About 3 to 10 Gaussian draws would be sufficient to get a good approximation of the integral. It has been proven that Gaussian quadrature is extremely efficient in low dimensions. The integral in (4.10) is only over one random parameter (therefore only one dimension), it is very suitable of Gaussian quadrature approximation method. More insights into the efficiency of these two numerical techniques will be gained in Chapter 5.

4.6 Solution Algorithm

We proposed a nested iterative algorithm for the MUC-SUE with stochastic VOT model. Note that, the only difference between a Quasi-Monte Carlo simulation and Gauss-Hermite approximation in the algorithms lies in Step 3.2. The algorithm is outlined as follows and the flow chart is presented in Figure 4.1.

<u>Step 1: Initialize</u>

Set *i*:= 1 and set $u^{(i)}_{am} := 0$

Begin Outer Loop (iteration index: *i*)

<u>Step 2: Update link travel times</u>

$$\tau^{(i)}_a = \Gamma(v^{(i)}_a)$$

where

$\Gamma(\cdot)$ link performance function.

$$v^{(i)}_a = \sum_{m \in M} pce_m \cdot u^{(i)}_{am}$$

Begin Inner Loop (iteration index: *k*)

Step 3: Determine descent direction

Step 3.1: Compute generalized link costs

For each user-class and each draw k, $c_{am}^{(i,k)} = \beta_m^{(k)} \tau_{am}^{(i)} + \theta_{am}$

Step 3.2: Determine kth descent direction

For each draw k, perform an AON assignment based on $c_{am}^{(i,k)}$. This yields the auxiliary flow vector:

$$W_{am}^{(i,k)} = \mu \sum_r \sum_s \alpha_{am}^{rs} D_m^{rs}$$

where

$$\mu = \frac{1}{K} \quad \text{in Monte Carlo Simulation}$$

$$\mu = w^{(k)} \quad \text{in Gaussian Quadrature Approximation}$$

Step 3.3: Inner Loop end check

If $k = K$, continue to Step 3.4, otherwise set $k = k+1$ and return to Step 3.1.

End Inner Loop

Step 3.4: Determine descent direction for i^{th} iteration

$$W_a^{(i)} = \sum_{m \in M} \sum_{k=1}^{K} W_{am}^{(i,k)}$$

Step 4: Update link flows

$$u_a^{(i+1)} = u_a^{(i)} + \gamma^{(i)} (W_a^{(i)} - u_a^{(i)}), \text{ where } \gamma^{(i)} = 1/i \text{ (MSA)}$$

Step 5: Convergence Test

If convergence criterion is met, then stop. Otherwise, set $i = i+1$ and return to Step 2.

End Outer Loop

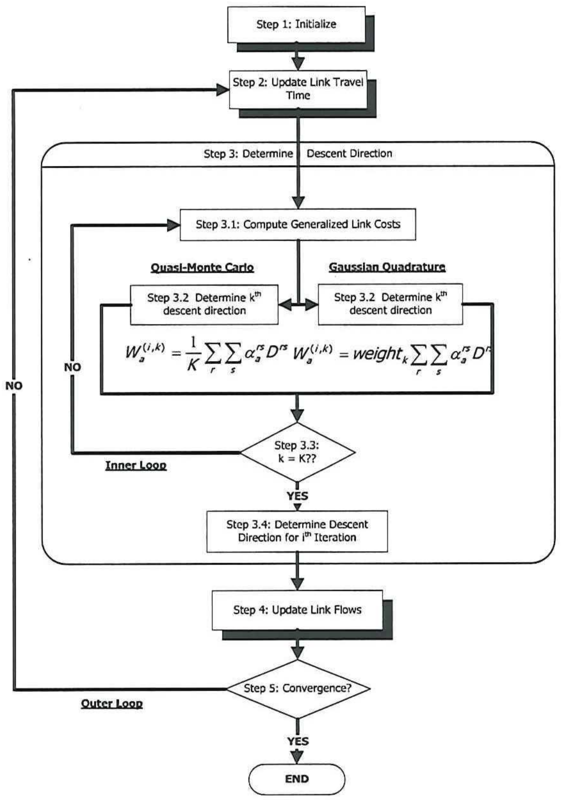

Figure 4.1: General algorithm for MUC-SUE with stochastic VOT model

4.7 Summary

In this chapter, we have mathematically formulated our MUC-SUE with heterogeneous VOT assignment model as a variational inequality problem. Generally, the route-based SUE VI model can be formulated in a straightforward way from its definition as present in Chapter 3. When we assume an infinite scale parameter in the logit-based route choice function, the stochastic route choice behavior collapses to a deterministic one. At this phase, the model is still a route-based one. Although route-based models are fairly intuitive and straightforward, route enumeration is a great burden if the network is large. This issue is the most critical constraint for applying a route-based VI model to realistic networks. To tackle this problem, we have rewritten the route-based VI model into a link-based VI model by introducing addictive link functions. An advantage of the proposed model is that the solution algorithm for such a VI problem will not require explicit route enumeration.

Efforts are made to solve the normally distributed VOT among multiple user-classes. Two numerical techniques, quasi-Monte Carlo technique and Gauss-Hermite approximation method are proposed to be further investigated. By introducing numerical techniques, the distributed VOT problem can be solved through an iterative procedure over different draws of VOT.

A nested iterative all-or-nothing (AON) algorithm is proposed to solve the link-based VI model. In the next chapter, this algorithm will be investigated through a series of experiments that provide more insights into the properties of the algorithm. In addition to that, the efficiency of two numerical techniques will also be studied and final selection will be made for the model implementation presented in Chapter 6.

5

Numerical Tests

In the previous chapter, we have introduced two numerical techniques to deal with the continuously distributed VOT in proposed assignment model, i.e. quasi-Monte Carlo technique and Gauss-Hermite approximation. Here, numerical experiments are set up in order to: (i) examine the MUC-SUE with stochastic VOT algorithm, (ii) examine the efficiency of the methodologies and (iii) make it clear that how inclusion of the heterogeneous VOT will affect the traffic flow and the toll road usage. The tests were coded in Matlab 7.1 on a basis of the algorithms presented in chapter 4 and operated under a computer environment of Windows XP, Intel Pentium 1.60GHz CPU and 1GB RAM.

Six experiments are setup and explored in this chapter to gain insights into the proposed algorithms from different values and functions assigned to travel demands, VOT distribution patterns, and toll rates. Because of the increasing modeling and computational complexity, all experiments are preformed on a simple hypothetical transportation network.

The following questions can be answered in this chapter:

- Which numerical technique is more efficient in dealing with the continuous distributed VOT?
- How do travel demands affect outputs of the algorithms?
- How do VOT distribution patterns influence the outputs of the algorithms?
- How do toll rates make changes in the outputs of the algorithms?

We start with a definition of the assumptions and specifications used in this chapter, based on which various experiments are established in Section 5.1. A general description of the design of the tests is given in Section 5.2. In Section 5.3, results and cross comparisons are presented.

5.1 Experimental Setups

In this section the experimental setups are established as well as assumptions and specifications. Different experiments are performed with different criteria aiming to show the adequacy and the correctness of the proposed model and its algorithm. Six experiments are set up based on the following assumptions and specifications.

Assumptions for the experiments are formulated as follows:

- Behavioral assumption made for route choice decision is in disutility minimization framework, measured by generalized travel cost.
- Single user-class with normally distributed VOT on the network is assumed in all experiments, except in SV experiments (single VOT).
- Travel demands are assumed to be given and fixed.
- Toll location, scheme and rates are pre-determined.

To test the efficiency of the numerical techniques and influencing factor(s) which lead to differences in link flows when we have heterogeneous VOT, specifications are made for each experiments:

- Numerical Techniques
  - Quasi-Monte Carlo: number of draws from Halton sequence.
  - Gauss-Hermite Approximation: number of points to be used in the algorithm.

Table 5.1: Differentiated Cases for Numerical Techniques Setups

	VOT Pattern	Numerical Technique	
SV	SV	N/A	N/A
H15	ND	QM	15 draws
H50			50 draws
H500			500 draws
GH3	ND	GH	3 points
GH6			6 points

Legend:

SV	single VOT
ND	normal distribution pattern
QM	quasi-Monte Carlo using Halton sequence, base 2.
GH#	number of points used in Gauss-Hermite approximation

- Travel Demand
- Mean and Standard Deviation parameters for VOT Distribution Patterns

- Toll Rates

The setups of our experiments are outlined in Table 5.2. We start with reasonable assumptions for model inputs (moderate congestion level and toll rate) in Experiment 1. The aim of Experiment 1 is to prove the correctness of the algorithms described in Chapter 4. Hence it serves as a reference case and all comparisons are made based on the results of Experiment 1.

Table 5.2: Experiments Setups

	Experiments						**D/NC** [veh/h]	**(μ,σ)** [€/min]	**Toll** [€/pass]
	SV	H15	H50	H500	GH3	GH6			
E1	x	x	x	x	x	X	6000/5000	(0.5, 0.15)	3
E2	x	x	x	x	x	X	4000/5000	(0.5, 0.15)	3
E3	x	x	x	x	x	X	8000/5000	(0.5, 0.15)	3
E4	x	x	x	x	x	X	6000/5000	(0.5, 0.05)	3
E5	x	-	x	-	-	X	6000/5000	(0.2, 0.06)	3
E6	x	-	x	-	-	X	6000/5000	(0.5, 0.15)	0,6,9,15,21,30

Legend:

E#	experiment #
x	specific numerical technique contains this experiment
-	specific numerical technique does not contain this experiment
D/NC	travel demand/total network capacity (summation of capacities on link 1 and link 2)

In order to assess the impacts of different numerical techniques and link flows, E2 and E3 are designed in two different traffic conditions respectively. Two extreme ratios of travel demand and network capacity are made in E2 and E3. In the former one, the travel demand is less than the network capacity (4000/5000), while the travel demand far exceeds the network capacity in E3 (8000/5000). E4 and E5 are formulated to explore the influence of the introduction of different VOT distribution patterns.

Note that setups in E6 are different from other experiments. A set of toll rates are tested in E6 to explore the change of link flows in conventional model (single VOT) and normally distributed VOT when we increase the toll rates.

5.2 Description of General Design

5.2.1 Network Specification

One of the simplest networks to consider road pricing problem is a dual link network, which contains two links from a single origin r to a single destination s as illustrated in Figure 5.1. The first link has a shorter free-flow travel time (e.g. freeway) but with a toll on that link, whereas the second link has a longer free-flow travel time (e.g. urban road) without toll. Regardless of the simple layout of the network, it provides insights into the model properties. All experiments are applied to this dual link network but with differentiated input parameters and functions mentioned above.

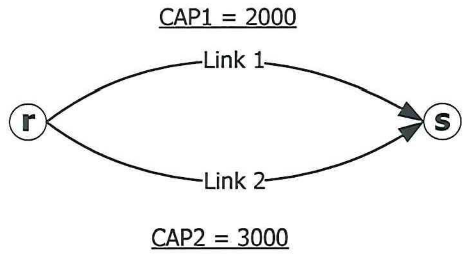

Figure 5.1: Dual Link Network

A same layout of the network shown as Figure 5.1 is applied to all of the six experiments setup in Section 5.1. Also, the identical link travel time function and convergence criterion, exhibited as follows, are applicable for all of them.

- Link Travel Time Function

The BPR (Bureau of Public Roads) link travel time function denoted by τ_a, is used in all case studies. We assign usual values for α, β, 0.15 and 4.0 respectively.

$$\tau_a = \tau_0(1 + \alpha(\frac{u_a}{CAP_a})^{\beta}) \tag{5.1}$$

where:

τ_0	free flow travel time on link *a*
u_a	flow on link *a*
CAP_a	capacity of link *a*
α, β	parameters

5.2.2 Convergence Criterion for the Algorithms

Our proposed algorithms consist of iterative scheme. As with all iterative procedures, the convergence criterion may have a considerable impact on the results. One makes a trade off between computation time and accuracy when choosing the convergence criterion. In the remaining of this chapter, we adopt absolute volume difference as our convergence criterion and the value for δ is set to 1% of the travel demand for the sake of simplicity. It is based upon successive iteration and current iteration, and can be formulated as follows:

$$\left\| u_a^{(i+1)} - u_a^{(i)} \right\| \leq \delta \tag{5.2}$$

5.2.3 Parameters for the VOT Distribution

A normal distribution has been assumed for VOT. The mean of the distribution function are assigned to value, high VOT, μ = 0.5 €/pass, and low VOT, μ = 0.2 €/pass.

The standard deviations are taken as a function of the mean,

$$\sigma = \alpha\mu \tag{5.3}$$

We will consider two situations, a small deviation, using α =0.1, and a relatively large deviation, using α =0.3.

5.3 Discussions of Results

In line with our aims and research objectives aforementioned, results from different experiments will be cross compared.

5.3.1 Correctness and Efficiency of Selected Numerical Techniques (E1 - E4)

As shown in Figure 5.2, all experiments generate stable results of link flow. We can roughly say that all numerical techniques can produce or reflect the traffic condition.

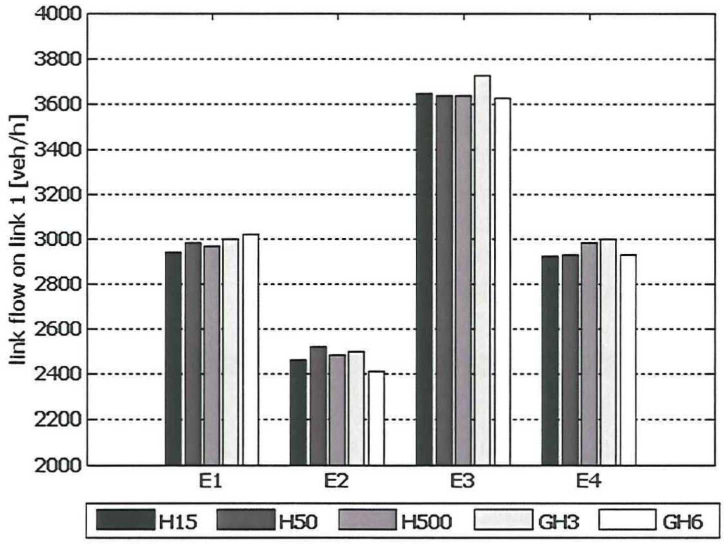

Figure 5.2: Comparison of Link Flows on Link 1

It is assumed that the stability of link flows prediction can be attained in all experiments and turn to be at the approximate same degree. The efficiency of the numerical techniques is, then, of our main concern. In the algorithm demonstrated in Chapter 4, there are two loops in the traffic assignment model, i.e. one outer loop and one inner loop. Inclusions of normally distributed VOT in E1 to E4 do not increase the number of iterations in the outer loop, as can be observed from Figure 5.3. However, the number of iterations needed does vary as we change the travel demand and parameters for the normally distributed VOT. In a non-congestion situation (E2), the number of outer loop needed is less than all other experiments. On the other hand, we found that in the heavy congestion case (E3) and a lower value of standard deviation case (E4), there are more iterations.

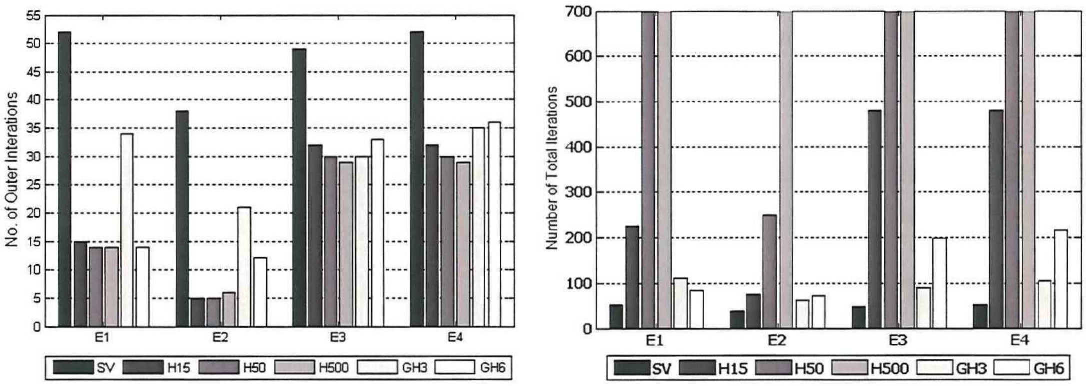

Figure 5.3: Comparison of Number of Outer (left)/Total (right) Iterations (E1-E4)

The total number of iterations is calculated by multiplying the number of iterations for outer loop and the number of draws/points used in different experiments. For example, if the number of iterations for outer loop is 15 in a H15 case, then the total number of iteration is 225 (15*15). Almost in each experiment, total number of iterations rockets in H15 and H500 case, thus making Quasi-Monte Carlo technique unacceptable and unattractive when we considering a large scale network with huge travel demands. Nonetheless, two Gauss-Hermite approximations give satisfied performances in all experiments in terms of number of total iterations.

In our later analysis on other properties of the solution methodologies and algorithms, we will confine ourselves to the cases of SV, H50, and GH6. Case SV is chosen as a baseline experiment for our further analysis. We take case H50 into consideration, for the reason that in our simple network, the computation time needed for H50 only takes for seconds and we need a Quasi-Monte Carlo technique to make reliable analysis. The last 'candidate', GH6 is selected due to its theoretical attractiveness, the ability to make more precise predictions within reasonable computation time.

5.3.2 Travel Demands and Link Flows Patterns (E1- E5)

The extensive contributions in literatures suggested that the introduction of the heterogeneous VOT to assignment models results in a change of traffic distribution (link flow patterns), and better estimations of the value of time should lead to a more realistic forecast of the traffic condition than conventional models in terms of link flow. In the previous studies, only a few discussions are found either on the key influencing factors or upon the extent of the varying of link flows in the assignment models. Cantarella and Binetti (1998) found that the effectiveness of VOT distribution is greater for low demand and low VOT in their SUE with VOT distributed among users. Leurent (1993) proved there is a drawback of the conventional models, when the toll rate is high enough in his two-route network experiments.

In practice, it seems a question of the research objective of this thesis: is it necessary to include heterogeneous VOT in the assignment model even if there is little change in link flows? Or we can doubt ourselves that under which situations it will cause an obvious difference in link flows between conventional assignment models and heterogeneous ones?

Therefore, we try to investigate the influencing factor(s) by adjusting travel demands and VOT distribution patterns. Link flows from case SV are taken as a baseline for the subsequent comparison. The change of link flow in percentage in a certain experiment is calculated using the equation below:

$$\Delta\% = (\frac{u_1^e - u_1^{SV}}{u_1^{SV}}) \times 100\% \tag{5.4}$$

where

u_1^e link flow for link 1 in a certain case (either C50 or GH6)

u_1^{SV} link flow for link 1 in case SV

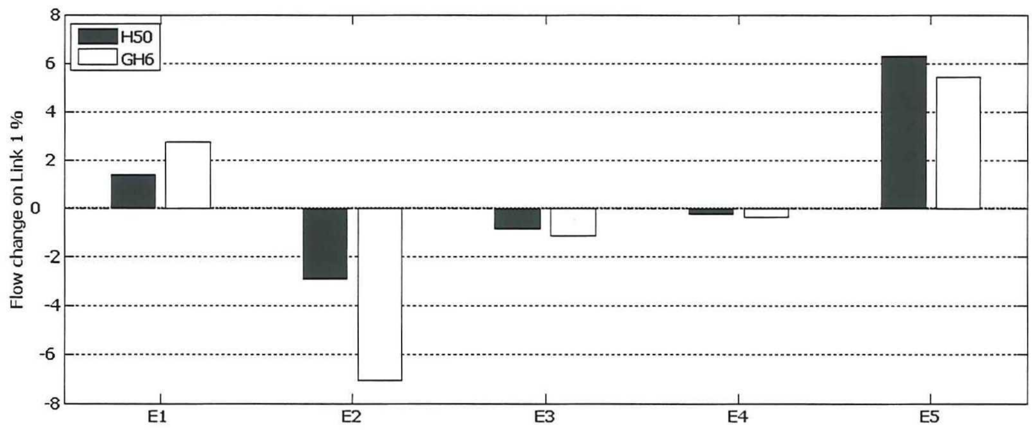

Figure 5.4: Relative Changes in Link Flow Patterns on Link 1

Figure 5.4 shows the relative changes in link flow on Link 1. Only in E2 and E5, the biggest changes are observed, i.e. an approximate 7% change in link flow compared to the one derived from SV. A higher change in link flow can be found in our baseline experiment, E1 as well. In E4, we have a smaller value for standard deviation, which makes our normal distributed VOT behaves more like a single VOT, which changes least in link flow.

E2 suggested an emphasis on the VOT distribution parameters, such as mean and deviation especially under the circumstance of non-congestion. Although the travel demand is smaller than network capacity, mean of VOT may result in a significant variation in link flow.

5.3.3 Toll Rate and Link Flow Patterns (E6)

This section explores the relationship between toll rate and link flow. Again, we use link flow on link 1 from three pre-set cases for numerical techniques. A toll rate of 3€/pass is assumed in all previous experiments, now we start to rise of this toll rate to 6€/pass, 9€

/pass, 15€/pass, 21€/pass and 30€/pass subsequently. Note that, a NON toll case is also tested in E6. The link flow patterns on link 1 under different toll rates are shown in Figure 5.5.

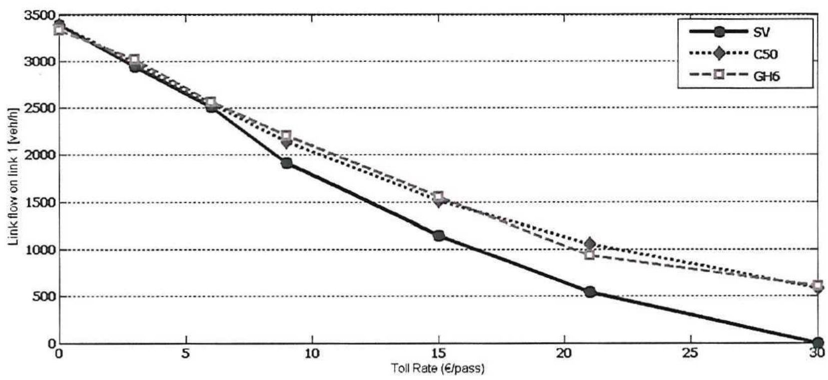

Figure 5.5: Link Flow Patterns on Link 1: Low Toll Rates Vs. High Toll Rates

The link flows under lower toll rates (0-5€/pass) do not be changed a lot even with a heterogeneous VOT in the assignment model. However, when the toll rate continues to increase, considerable disparity between heterogeneous VOT and a single VOT can be observed. When the toll rate is set to 30€/pass as a much higher level, no one will use link 1 in the single VOT case, which is an obvious drawback of conventional model. At this point, we can say that the toll rate is a primary influencing factor on link flows.

5.4 Summary

In this chapter we have presented some numerical experiments to explore the proposed algorithms and numerical techniques. The following experiments were performed with different purposes:

- E1: common traffic density (6000/5000). Its purpose is to serve as a reference case and to test the correctness of the algorithms proposed in Chapter 4.
- E2: a low traffic density, where the network capacity is greater than the travel demand (4000/6000). It aims to show how a decline of travel demand will affect the link flow pattern.
- E3: a high traffic density, where the travel demand exceeds the network capacity a lot. In this case study we try to present whether an increase in travel demand will lead to greater difference in link flow.
- E4: a small standard deviation in VOT distribution function. Its purpose is to depict the model's sensitivity to the standard deviation in distribution function.
- E5: a lower mean in VOT distribution function. Its purpose is to depict the model's sensitivity to the mean value in distribution function.
- E6: a test of changing in toll rates is conducted here. Rates from 0 to 30€/pass are investigated on the network. It intends to reveal how changes in toll rates generate variations in link flows.

E1 considers a common traffic density and VOT distribution pattern, which we mainly tested the correctness of the algorithms and numerical techniques. We further explored the efficiency of the numerical techniques in E2 to E4, and found Gauss-Hermite approximation out performances quasi-Monte Carlo technique in terms of number of total iterations needed.

A set of tests are conducted to investigate normally distributed VOT model's sensitivity to parameters like travel demand, mean and standard deviation of VOT distribution functions and toll rates. Results generated suggested that the link flows are more sensitive to low traffic density, while in a high level of traffic density, the heterogeneous VOT does not induce substantial change in link flow compared to single VOT model. On the other hand, in E4, we observed a significant change in link flow with a low value of mean in VOT distribution function. And in E5, we found that standard deviation of VOT distribution function brings a diversion on link flows as well. The drawbacks of the conventional model appear when the toll rate increases and it fails to produce realistic link flow predictions in these situations, which can be concluded from E5.

A list of influencing factors and their influencing power on the change of link flows are shown in Table 5.3:

Table 5.3: Influencing Factors and Their Influencing Power

Influencing Factors	Influencing Power
Toll Rate	+++
Extremely Low Travel Demand	++
Mean of VOT (μ)	++
VOT Distribution Pattern (σ)	+
Extremely High Travel Demand	0/+

Legend

\+ denotes the level of one parameter's influencing power

6

Model Implementation in Cube

The Dutch KMP is a system change from fixed taxes to payment for use. The starting point is a fair system: not paying more, but paying differently through the variabilisation of the current car taxes. The final goal is a national wide price per kilometer for all kilometers driven in the Netherlands, differentiated by environmental characteristics, time and price. To support the evaluation of Dutch KMP system strategies in a network context, the proposed model is developed in Cube planning system, which aims to capture users' route choices in response to a pre-designed toll scheme, and hence explicitly considers heterogeneous VOT between and within multiple user-classes in the underlying route choice decision framework. This chapter integrates the theoretical prescriptions of a traffic assignment model as developed in Chapter 3, 4 and 5 into a computer implementation for predicting traffic conditions and estimating the effects of KMP scheme in the Netherlands.

Section 6.1 gives a brief introduction of Cube Voyager. Section 6.2 explains the general modeling process consisting of 4 main modules before drawing the practical results which will be discussed in Chapter 7.

6.1 Cube Voyager

Cube Voyager is designed to be an integrated modeling system for transportation planning applications. It is a library of program modules that employs a language allowing the users to write the script to provide instructions for performing all types of typical planning operations. Each module is designed to perform certain operations, but only as specified by the users. At the heart of the Cube Voyager system is a flexible control language referred to as a scripting language. This provides a flexible environment and grants control over all aspects of the modeling process.

Among a comprehensive library of functions, a list of main functions used in this paper is given below:

- Equilibrium feedback on part or all of model chain using fixed number of iterations or user-controlled criterion;
- Module for supply checking, calculation, path building, assignment and skimming;
- Multi user-class assignment process;
- All-or-nothing traffic assignment.

Through its flexible scripting system and its highly flexible network and matrix calculations for the calculation of traffic flow and for the detailed comparison of scenarios, Cube Voyager allows for very advanced model implementation of MUC-SUE with heterogeneous VOT assignment model. In the next few sections, we will present the modeling process in Cube Voyager.

6.2 Modeling Process in Cube Voyager

6.2.1 Overall Structure of Cube Model

The core of the model is the traffic assignment modules which represent the Dutch mobility transport market. It describes, for a given period, a given tolling network and a given pricing scheme, an equilibrium between supply and demand based on a continuous normally distributed VOT among multiple user-classes.

To incorporate with the Dutch KMP's toll scheme, we divide the 24-hour day into three periods, namely AM Peak, PM peak, and off peak. While congestion rates are applicable only during AM peak and PM peak on designated roads, the basic rate applies throughout the Netherlands and for all periods. The implementation of the model comprises the following modules:

- AM peak assignment module (7.00-9.00)
- PM peak assignment module (16.00-18.00)
- Off peak assignment module (other)
- Flow integration module

The assignment procedures are almost identical among all modules as presented in Chapter 5, while the inputs are varying. The network input files is defined as Dutch national road network. Physical attributes such as capacity, free flow speeds and link types etc. of these network inputs files are the same, but they contain information of whether or not a road is tolled during the peak periods. All road users are categorized into four classes (commuters, business, freight, and others) according to their travel purposes. The OD matrix inputs files by travel purposes are provided to each assignment module separately.

The outputs of each assignment module are travel purpose specific link flows which will be integrated by time-of-day (TOD) factors in the flow integration module. Therefore, the final output from the Cube model is a daily link flow pattern over the network. General layout of the Cube model is given in Figure 6.1.

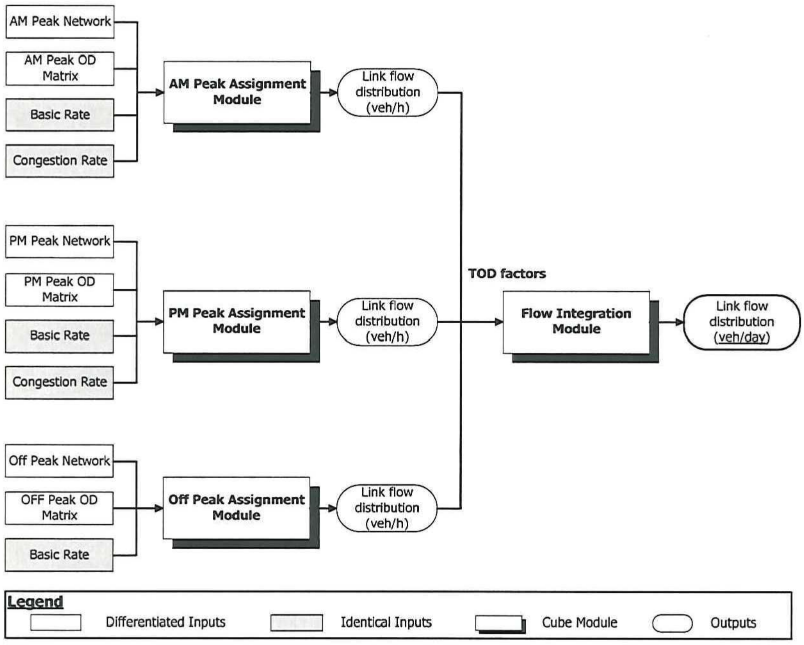

Figure 6.1: General Layout of the Cube Model

6.2.2 Assignment Module

As mentioned before, all assignment procedures are identical. Without loss of generality, in this section, we merely present the AM peak to cover all the assignment modules in the Cube model.

Three main components comprise the AM peak assignment module. They are initial assignment, main assignment and convergence calculation. A general scheme is shown in Figure 6.2.

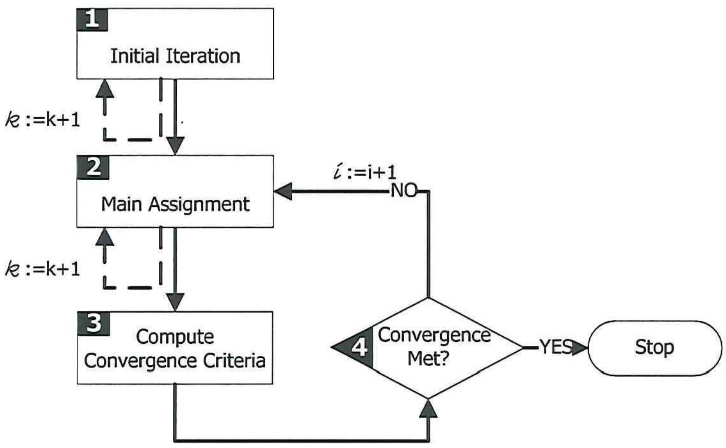

Figure 6.2: Flowchart of the Assignment Module

6.2.2.1 Step 1: Initial Iteration

In Step 1, an initial feasible link flow pattern is determined on an empty network with free-flow link travel times for all user classes. Every trip between any OD pair is assigned to the route which has the minimal generalized route travel cost with respect to that user's particular VOT. After this initial assignment, the road network is loaded with traffic flows. The link travel times are no longer free-flow travel times but congested travel times. We will first compute link volume using passenger car equivalent (PCE) from link flows determined by the assignment process. Then the congested link travel times are calculated according to given link travel time functions. See Figure 6.3.

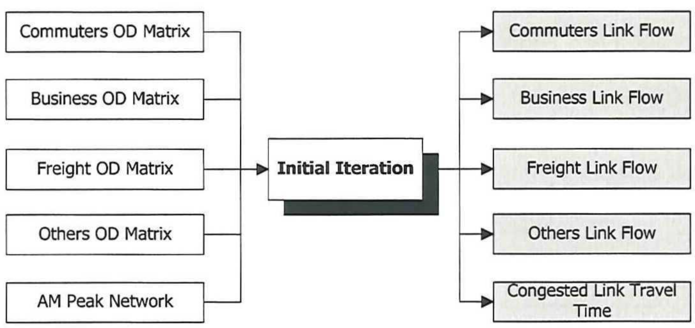

Figure 6.3: Step 1 Initial Iteration

Intermezzo: Modeling Normally Distributed VOT among Multi User-class in Assignment Module

Special attention should be paid to solve the normally distributed VOT among user-class. We will give detailed description on how our Cube model realizes the algorithm presented in Chapter 5. The procedures are the same among user-classes and in all assignment modules contained in the Cube model, so we will only take commuters as an example, the left are the same.

A step-wised procedure is developed to realize Gauss-Hermite approximation method in the assignment process, see Figure 6.4.

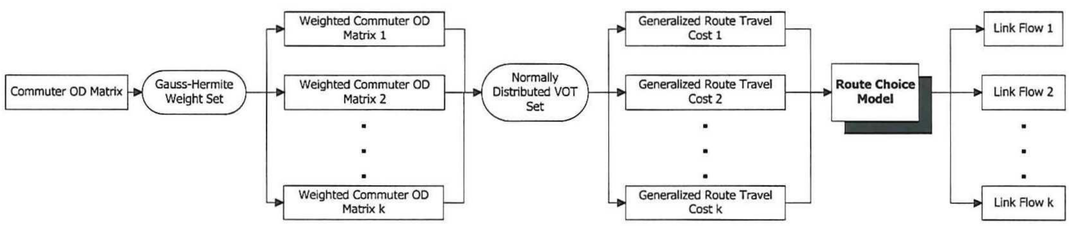

Figure 6.4: Gauss-Hermite Approximation in Assignment Module

<u>Outline of this procedure</u>

Step 1: Given **K** number of points used in Gauss-Hermite approximation method, a set of weights and a set of normally distributed VOT are computed first. Detailed mathematical description on how to compute these sets is presented in Appendix A.

Step 2: Split Commuter OD matrix into **K** submatrices,

$$D_k^{rs,com} = w_k D^{rs,com} \tag{6.1}$$

where

$D_k^{rs,com}$ the weighted demand cell in k^th commuter OD matrix

$D^{rs,com}$ total demand in original commuter OD matrix

w_k k^th weight in Gauss-Hermite weight set

Step 3: Compute generalized route travel cost for k^th weighted commuter users using k^th normally distributed VOT in VOT set,

$$c_k^{prs} = \beta_k \tau^{prs} + \theta^{prs} \tag{6.2}$$

Step 4: Assign all trips from any OD pair in k^th matrix to the route which has the minimal generalized route travel cost, see equation (6.2). This yields link flows for each subclass among commuters.

6.2.2.2 Step 2: Main Assignment

In Step 2, a feasible link flow pattern is determined on loaded network with congested link travel times from previous iteration. Again, every trip between any OD pair is assigned to the route which has the minimal generalized route travel cost with respect to that user's particular VOT and congested travel time calculated from previous iteration, yielding auxiliary link flows for current iteration. To compute the actual link flow over the network, we use the flow average method, which combines auxiliary link flows in current iteration and those from previous iteration. Then new congested link travel times are computed for the next iteration (if necessary) through the same procedure as mentioned in step 1. One main difference in outputs from the initial iteration step is the recorded zone-to-zone based generalized route costs for each user-class with specific VOT. These data will be used to compute duality gap in the next step. See Figure 6.5.

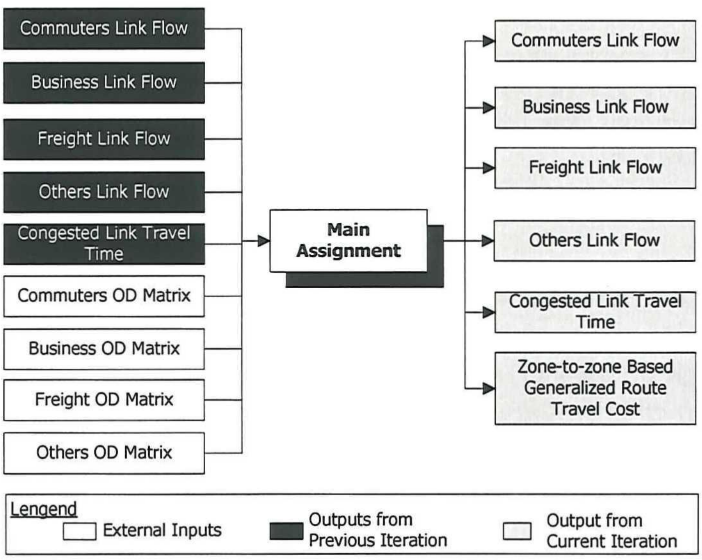

Figure 6.5: Step 2 Main Assignment

6.2.2.3 Step 3: Compute Duality Gap

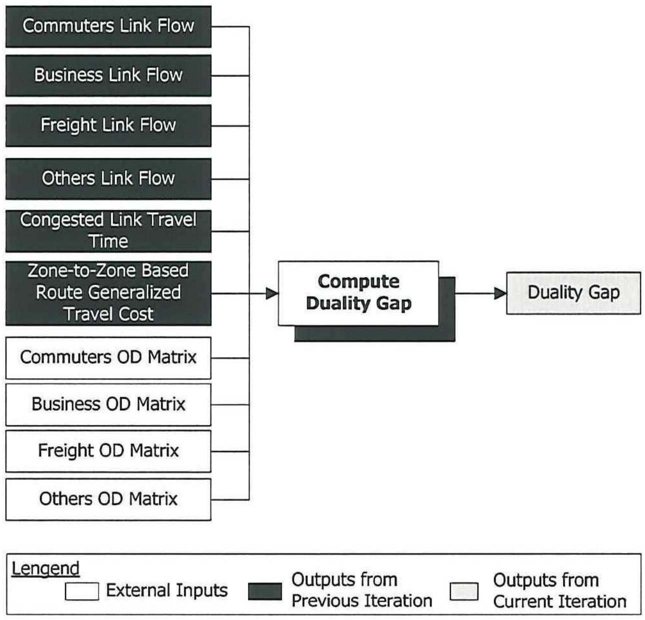

Figure 6.6: Step 3 Compute Duality Gap

In step 3, duality gap is adopted as the convergence criterion. Duality gap is defined as excess travel time relative to the minimum possible zone-to-zone generalized route cost computed in current iteration,

$$DG = \frac{\sum_{k\in K}\sum_{m\in M}\sum_{a\in A} u_{am}^{(i,k)} \bullet c_{am}^{(i,k)} - \sum_{k\in K}\sum_{m\in M}\sum_{r\in R}\sum_{s\in S} D_{m,k}^{rs} c_{m,k}^{rs,(i)}}{\sum_{k\in K}\sum_{m\in M}\sum_{r\in R}\sum_{s\in S} D_{m,k}^{rs} c_{m,k}^{rs,(i)}} \tag{6.3}$$

Because the duality gap takes into the account the differences between generalized route costs and weights these with the route flows, it is an effective criterion to measure whether the Wardrop conditions are met or not (See Definition 4.1). Outline of this step is presented in Figure 6.6.

6.2.2.4 Step 4: Check Duality Gap

Duality gap in current iteration is computed in step 3, in step 4 we examine whether it matches the stop criterion pre-set by model user. If the condition is met, the AM Peak assignment module will end, otherwise, the whole process goes back to step 2.

6.2.3 Flow Integration Module

Travel demands are assigned in three different TOD periods and link flow distributions are calculated by three assignment modules. These will be weighted together in the flow integration module by TOD factors, yielding link flow distributions on a 24-hour basis. See Figure 6.7.

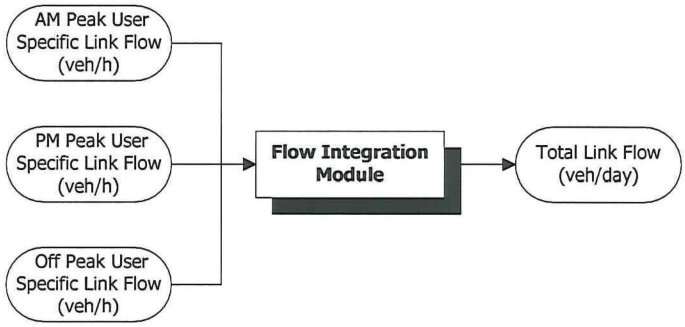

Figure 6.7: Flow Integration Module

6.3 Summary

This chapter has provided a concise overview of modeling process in Cube for developing the proposed assignment model for assessing Dutch KMP systems (more information is given in Appendix B and C). By providing a fully integrated transport planning system, Cube is very suited to the multi-layer approach to modeling where data are passed between different modules in the process. In line with the Dutch KMP toll scheme, the 24-hour day is divided into three discrete time periods. Each of these periods contains a separate route assignment module. Special attention should be paid that travel time shifts between or within these periods can not be captured by the Cube model. On the other hand, the model provides full flexibility to the users in terms of setting input files and parameters for the model. Users can define any combination of possible rate sets and heterogeneous VOT sets, link travel time functions for the network, cutoff point for the convergence criterion, and so the forth, through the common interface of Cube. This full control of the model pre-settings gives model users maximum freedom in the model application phase.

Recognizing the heterogeneity between and within different user classes on the network, the prediction of the network performance can be improved by Cube model. In the next chapter, we will conduct a case study on Dutch KMP system to show the capabilities and the performance of the Cube model. More relatively accurate predictions/estimation of the effects under the KMP system can be obtained. The estimation of the effects is extremely important that planners and policy makers are keenly interested in analyzing in a network setting.

7

Model Application: Dutch KMP System

The traffic assignment model presented before could be used to describe how the road pricing policy affect the road transport network performance for a given infrastructure and a given toll scheme. Here we interpret the model depending on the case of Dutch KMP system. By analyzing the model outputs, we try to verify whether or not the new approach of the heterogeneous VOT between and within multiple user-classes could improve to predict the network conditions more realistically. And the model is also considered as a useful tool for the potential model-users to make strategies upon their specific incentives of the KMP policy.

Section 7.1 introduces the background of KMP rate and what questions could be solved by the interpretation of the Cube model results both for the traffic engineers and potential model-users. As being discussed in Section 7.2 and 7.3, we apply the model into the case of Dutch KMP system with two scenarios and make a brief result analysis. And we present a possible application of the Cube model in Dutch KMP policy design problem in Section 7.4.

7.1 Introduction

Dutch KMP[2] system is a new pricing policy to make road users more fully aware of the impacts they impose to the transport system and vice versa. KMP system aims to increase equity of road charge payment, whilst at the same time improves accessibility, safety and quality of the living environment by the pricing mechanism upon the dimensions of location, time, vehicle category. The price per kilometer is a system change from fixed taxes to payment for use. It comes from a starting point of 'FAIR system': not paying more, but paying differently through the variabilisation of the current fixed car taxes.

The structure of the kilometer charge system can be outlined as: user pays, polluter pays and scarcity has its price. There are two categories of toll rates, the basic rate and the additional rate. The basic rate applies throughout the Netherlands and is differentiated by vehicle characteristics on an environmental basis (by fuel type; by Euro environmental tax level). This is in line with the current differentiations in the motor vehicle tax (MRB) and car and motorcycle tax (BPM). As to the additional rate, it is differentiated by time and place over and above the basic rate, applicable only on designated roads, to improve accessibility. This was investigated to get an impression of the effectiveness of a congestion charge. The surcharges were applied during the morning and evening rush for road sections whose intensity/capacity ratios are more than 0.8.

Whatsoever, main objective of this thesis is to develop a traffic assignment model based on sound theoretical foundation to assist the design and evaluation of road pricing policy. In this chapter, we therefore consider to situations:

(i) for the traffic engineers focusing on the traffic assignment model, what and how can the proposed traffic assignment model presented in the previous chapters contribute to improve the prediction of the network performance under a given road pricing scheme; and

(ii) for the potential users of the developed model, what indicators could they derive from the model outputs and how these indicators provide information to various potential users.

7.2 Case Study: Dutch KMP System

A comparative approach is better suited to derive model implication to the Dutch KMP system. Due to the fixed structure of data source, we merely compare heterogeneous VOT between and within multiple-user classes with the discrete VOT among user-classes under a set of given toll rates taken from the Dutch KMP system. Therefore two scenarios are built in this section. Particular interest of this case study is to investigate how explicit consideration of heterogeneous VOT between and within multiple-user classes affects the revenue generations and traffic conditions under Dutch KMP system.

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[2] KMP: KiloMeter Price

7.2.1 Data Source

In order to utilize the Cube model described in Chapter 6, data are required on OD demands, road network, toll rates etc. Data described in this section are used as inputs, inner functions or parameters for the Cube model and they are provided by the sponsor of this thesis project, **4***cast* B.V.

The Dutch national network in Figure 7.1 is comprised of 17,936 nodes, 39,041 links and 400 zones. The study time span is a 24-hour day which is divided into AM peak (7.00-9.00), PM peak (16.00-18.00) and off peak (others). Four user-classes are presented on the network: commuters, business, freight, and others during all three periods. Users in each class behave differently (due to normally distributed VOT) and each user-class has different average VOT. Interactions among user classes are expressed by passenger car equivalent (PCE). Specific values are given in Table 7.1.

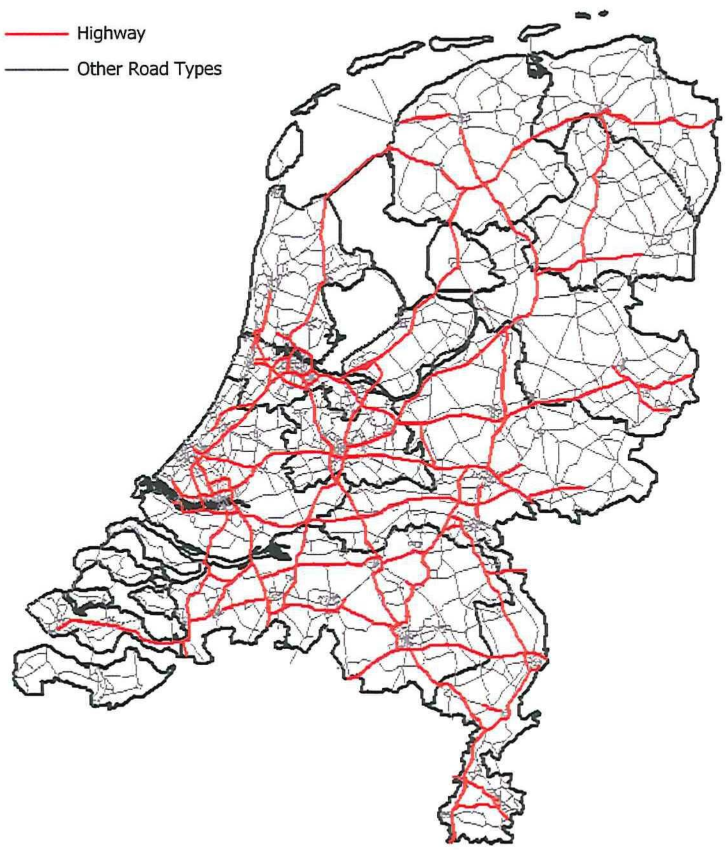

Figure 7.1: Dutch National Road Network

Table 7.1: OD Demand, Average VOT and PCE Values

	OD Demand (veh/h)			Average VOT		PCE
	AM Peak	PM Peak	Off Peak	(€/min)	(€/h)	
Commuters	489,781	336,855	116,804	0.174297	10.46	1
Business	100,006	159,246	127,245	0.605071	36.30	1
Freight	101,516	93,443	112,499	0.426553	25.59	1.9
others	149,282	327,992	311,561	0.151109	9.07	1

In line with Dutch KMP scheme, we apply two categories of rates in the case study,

- Basic rate: the basic rate applies throughout the Netherlands on a daily basis and is differentiated by vehicle characteristics and road type;
- Additional rate: is differentiated by time and vehicle characteristics and only applied to designated road and peak periods to improve accessibility.

The following Table 7.2 gives detailed data of KMP rates adopted in this case study.

Table 7.2: Roll Rates (€/km)

	Basic Rate			Additional Rate
	Link type 1-4	Link type 7*	others	Peak Toll Flag[3] = 1
Car	0.077	5	0.1	0.11
Freight	0.185	5	0.241	0.33

*: unit: €/pass

The speed-density functions $\Gamma(\cdot)$ that will be used in the case study are taken from studies conducted by **4***cast* B.V. These speed-density functions are differentiated by I/C[4] ratios and link types. Mathematically, they can be expressed as follows and parameters of these functions are given in Table 7.3.

- If I/C ratio $\frac{V_a}{CAP_a}$ <0.75,

$$\tau_a = \tau_0(1 + \alpha \cdot \frac{V_a}{CAP_a}) \tag{7.1}$$

- If I/C ratio $\frac{V_a}{CAP_a} \geq \sigma$

$$\tau_a = \tau_0(1 + \alpha \cdot \frac{V_a}{CAP_a} + \beta \cdot (\frac{V_a}{CAP_a} - 0.75)^{\gamma}) \tag{7.2}$$

where,

$$V_a = \sum_{m \in M} pce_m \cdot u_{am} \tag{7.3}$$

Table 7.3: Parameters of Speed-density Functions

	Highway (link type =1)	Others
α	0.22222	0.5
β	8	8
γ	1.5	1.5

[3] Peak toll flag equals to 1 if certain link is applied to a congestion rate, or 0 otherwise

[4] I/C: intensity/capacity

TOD factors[5] used in the flow integration functions and results analysis section by vehicle categories and time-of-day periods are presented in Table 7.4.

Table 7.4: TOD Factors in Different Time Periods

	AM Peak	PM Peak	Off Peak
Car	2	2	12.19
Freight	2	2	10.42

7.2.2 Building Scenarios

Following a series of experiments using the Cube model (See Appendix D), we reduced these to two key scenarios, S1 and S2, as described in Table 7.5.

Table 7.5: Scenario Description

	S.D.	GH Points	Definition
S1	-	1	Discrete VOT among user-classes
S2	$\sigma_m = 0.3 \times \mu$	5	Normally distributed VOT among user-classes

Legend
S.D. standard deviation
GH Gauss-Hermite approximation

S1 is a baseline scenario, which only considers the conventional discrete VOT among multiple user-classes. Average VOTs listed in Table 7.1 are assigned to four different user classes. By using average VOT for each user-class, it can be taken as Gauss-Hermite approximation with one point.

S2 is designed to show the difference in toll revenues and traffic conditions by considering heterogeneous VOT between and within multiple-user classes. The standard deviations are assumed to be taken as a function of the means (average VOTs), $\sigma_m = 0.3 \times \mu_m$. To approximate the normally distribution functions, we use Gauss-Hermite five points in this scenario. Specific weights associated with certain Gauss-Hermite points are also derived to be used in the Cube model. Additional information for these values is given in Appendix A.

There is a trade-off between model computation time and accuracy on determine the equilibrium traffic condition. After several experiments conducted by Cube model, we decided to set the cutoff point for duality gap as 0.001. For more detailed description on the convergence property of the Cube model, see Appendix D.

7.3 Results Analysis

The impact of heterogeneous VOT between and within multiple-user classes under Dutch KMP system has not, so far been discussed. In this section, we will investigate the general

[5] TOD factors: the ratio of trips made in a time period to those made in one day.

traffic demand pattern first and then present results computed for two scenarios to illustrate the improvement of the model performance. Note that TOD factors listed in Table 7.4 are used to scale hourly link flow distributions into period based ones in order to derive various indicators presented in this section.

7.3.1 Travel Demand Patterns

A brief analysis on travel demand patterns is necessary since it can provide insights into the indicators analysis later. OD demands from the inputs files are first weighted into a period basis by TOD factors given in Table 7.4.

Table 7.6: OD Demand on a Daily Basis

	OD Demand (veh/day)			Total
	AM Peak	PM Peak	Off Peak	
Commuters	979,562	673,710	1,423,846	3,077,119
Business	200,012	318,491	1,551,115	2,069,618
Freight	203,032	186,886	1,172,237	1,562,154
others	298,565	655,984	3,797,932	4,752,481
Total	1,681,170	1,835,072	7,945,130	11,461,372

A graphical description of the OD demands by user class and TOD periods applied to the network is given in the following two figures together with a comparison of those illustrated in terms of per hour. Although we only use the OD demand by TOD periods to make further analysis, the pictures in terms of per hour are still considered as necessary because the TOD factor of off peak is much larger than the peak ones, which makes the off peak volume seem to be the most in a 24-hour day.

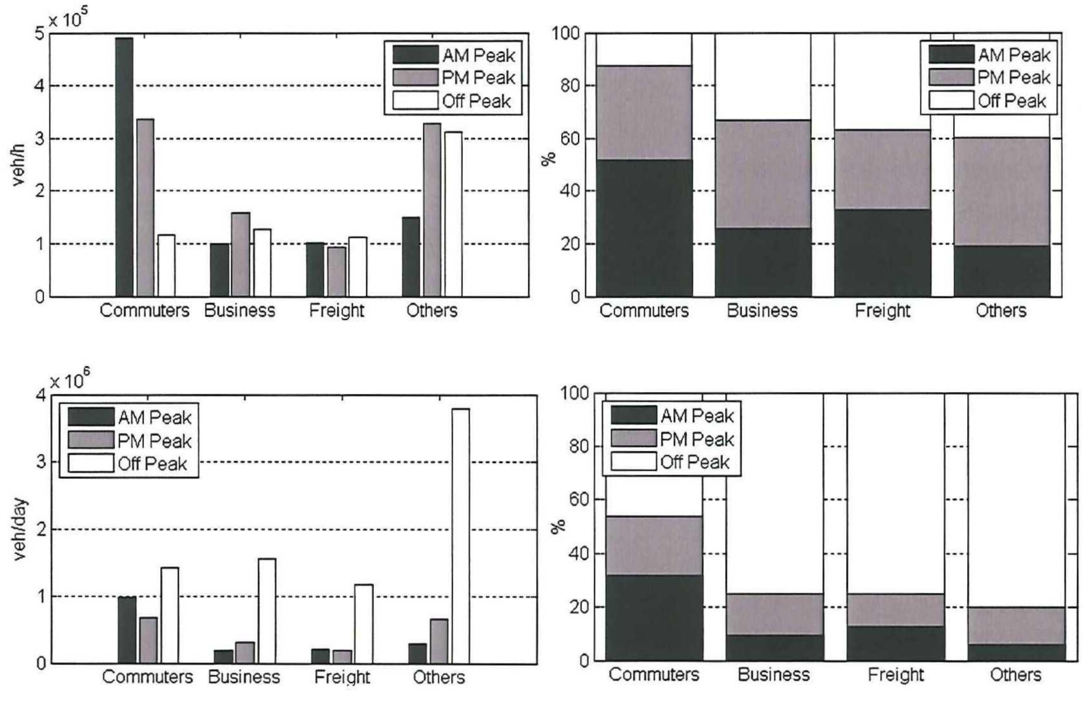

Figure 7.2: OD Demand on User-class Basis (veh/h Vs. veh/day)

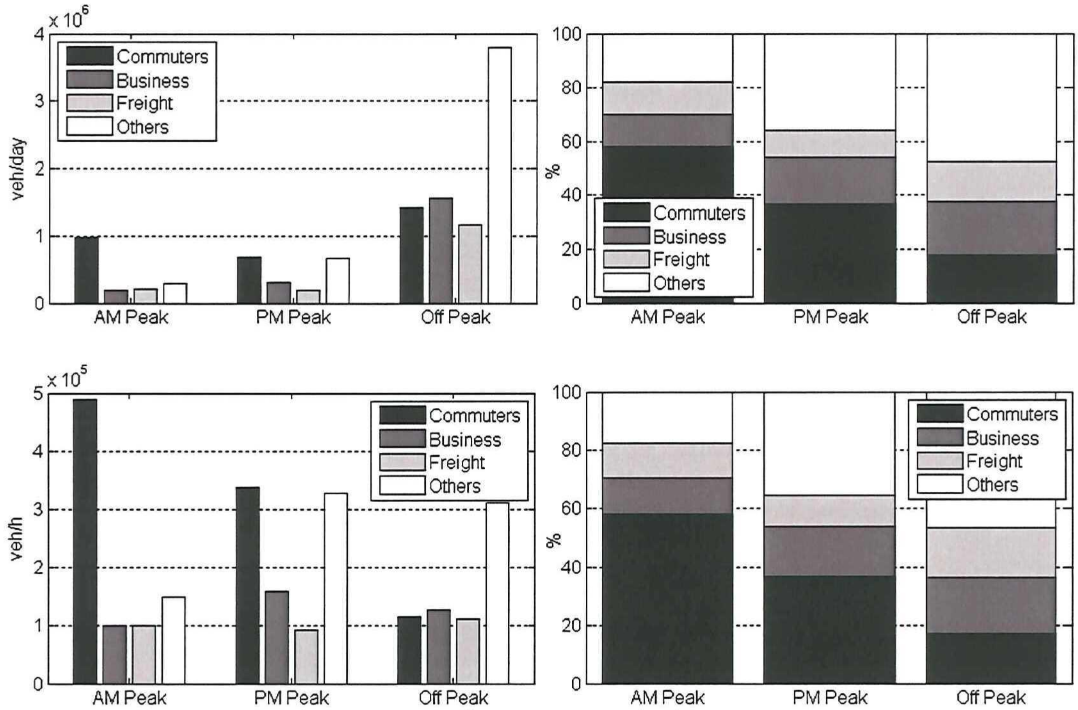

Figure 7.3: OD Demand on Period Basis (veh/h Vs. veh/day)

It could be investigated that different user class have implicitly different travel behaviors according to their specific traveling purpose. As shown in Figure 7.2, most commuters are travelling in the peak time especially in the morning. Compared to this, the other three kinds of users may choose to avoid congestion more flexibly. From Figure 7.3, we can investigate that commuters represent the biggest user-class among the people who will travel during the peak periods even if it is highly charged. And users of others class take the biggest proportion of the ones who will travel during off-peak period.

7.3.2 Total Revenues

For an assessment of the effects in economic terms, it is of interest to consider total revenue collected by the Dutch KMP system from all road users over the Dutch national road network. Table 7.7 presents relative change in total revenue generation by TOD periods and user-classes in Scenario 1 and Scenario 2. Both basic rates and additional rates are considered here.

Table 7.7: Relative Change in Total Revenue (%)

	AM	PM	Off	Total
Commuters	-0.130	-0.167	-0.034	-0.331
Business	-0.074	-0.080	-0.016	-0.170
Freight	0.068	0.012	-0.004	0.076
Others	0.299	0.201	0.087	0.587
Total	0.164	-0.034	0.033	0.162

As shown in the above table, total revenue generation on a daily basis in S2 only increased 0.162% compared to the baseline scenario, S1. This may seem quite small. The reason for this small difference in total revenue is that: the basic rates are also applied over the network during all TOD periods. Even if road users choose to avoid congestion tolled roads during peak periods and take another longer route, they are deemed to pay for basic rates for using non-congestion tolled roads. Therefore, although by assuming normally distributed VOT among multiple user-classes, no big difference will occur in total revenue generation.

Nonetheless, bigger relative changes are observed by TOD periods and user-classes, which imply difference in route choice behaviors between two scenarios, considering heterogeneous VOT between and within multiple-user classes. As can be seen from Figure 7.4, toll paid by users from commuters and business classes is overestimated during all TOD periods in S1. On the other hand, S1 tends to give underestimated revenue generation from users in freight and others classes. For the reason that all road users have to pay basic rates throughout the Netherlands and during all periods, it will offset the changes in total revenue generation in S2. If we only consider additional rates during peak periods, the drawback of the discrete VOT among multiple user classes will become more distinct. We will present this in later analysis.

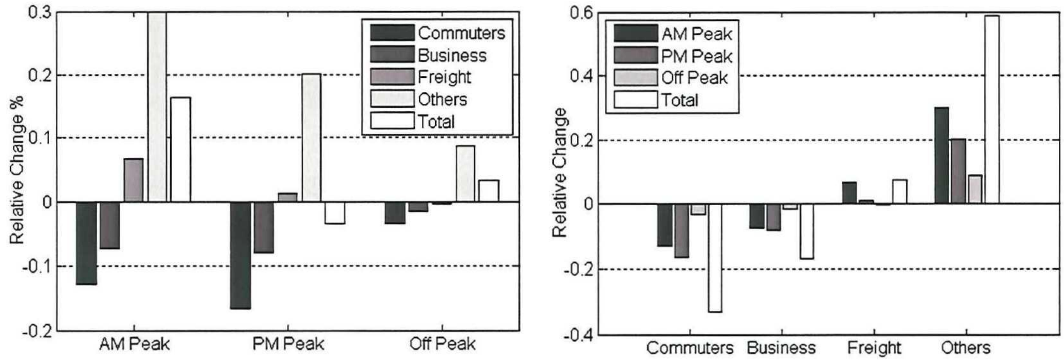

Figure 7.4: Relative Changes of Total Revenue in Two Scenarios

7.3.3 Traffic Condition

Although only small changes are found in total revenue generation on a daily basis, big differences occurred when considering traffic condition in terms of vehicle traveled time (VTT). A comparison of VTT experienced by all road users is given in Table 7.8.

Table 7.8: Relative Change in Vehicle Traveled Time (%)

	AM	PM	Off	Total
Commuters	-10.341	-10.026	2.361	-18.005
Business	4.190	2.393	9.908	16.490
Freight	6.316	5.554	12.606	24.476
Others	-5.941	-9.043	-0.515	-15.499
Total	-5.776	-11.123	24.360	7.461

The failure of depicting heterogeneities within user-classes (S1) gives optimistic predictions of the traffic condition under the given set of rates from Dutch KMP system. The actual VTT experienced by the road users could be 7.461% more as computed in S2. Figure 7.5 illustrates even larger biased predictions by user-classes and TOD periods.

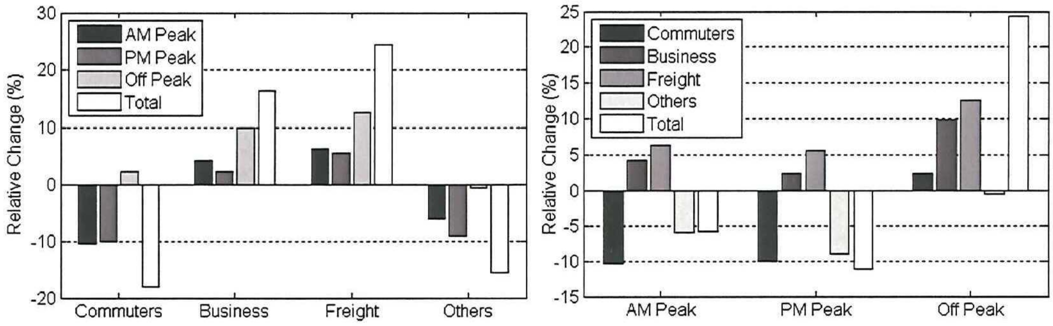

Figure 7.5: Comparisons of VTT in Two Scenarios

An important clarification is that normally distributed VOTs within user-classes, business and freight user classes, with higher average VOT experience longer travel time over the road network compared with those estimated in S1. Fluctuations in VOT among these two user classes result in route choice changes. In addition, road users from business class and freight class normally make longer trips than those from the other two user classes. The average vehicle traveled times in AM peak of business class and freight user classes are 45.58min/veh and 68.86min/veh compared with 27.74min/veh and 30.14min/veh for commuters and others respectively (calculated from S1). An increase in travel time for business and freight users will not bother them too much compared with the users who make shorter trips. Therefore, they might prefer to take a cheaper route with some congestion instead of additional tolled ones. For the same reason, there are a proportion of road users in the lower average VOT class, commuters and others classes have relative higher VOT compared to their counterparts. They are willing to pay for a less congested trip.

Biggest relative changes are observed for commuter class during peak hour, because they are the biggest user classes during these peak periods subsequently leading to a lager total change in vehicle traveled time. Business and freight trips are mostly presented on the network during off peak hours, thus bigger difference in VTT of these users is found during off peak hours. Similar trends for these relative changes can be explained by travel demand patterns during the day for most of the road users.

From the above analysis on relative change in VTTs in two scenarios, we can learn that the inclusion of normally distributed VOTs within multiple user class does lead to considerable changes in traffic condition in terms of VTT. Travel situations for different user classes during different TOD periods will be either overestimated or underestimated. The capability to depict the differences in route choice behaviors among a certain user class is an empirically realistic property of the model.

7.3.4 Equity Regulation

Whereas the aforementioned two indicators aim to illustrate the performance of the Cube model over the whole network and take both basic rates and additional rates in to account, here we will focus on the possible effects only with additional rates during peak periods. As stated before, the basic rates and off peak period trend to mitigate the differences of the model outputs from two designed scenarios. If we only consider additional rates on designated roads during peak hours, more route choice divergences are thought to be disclosed.

7.3.4.1 Flow on Additional Tolled roads

Link flows on the designated roads with additional rates during peak periods are used to explore the impact of VOT distributions among multiple user classes on network. Table 7.9 provides the additional tolled road usage over the Dutch national network.

Table 7.9: Relative Change in Link Flow on Additional Tolled roads (%)

	AM	PM	Total
Commuters	-1.719	-2.470	-4.189
Business	-0.376	-0.381	-0.758
Freight	1.995	1.859	3.854
Others	3.704	2.170	5.875
Total	3.604	1.178	4.782

As listed in this table, the additional tolled road usage predicted by discrete VOTs among multiple user classes (S1) is higher than that forecasted by distributed ones in S2. Since the discrete VOTs scenario assumes homogeneous users in any user classes, all users with lowest average VOTs (others user class) are not willing to use the additional tolled roads. However, there are in fact a certain number of users in this class may want to use these roads. Vice versa, there are a group of users (business class) with highest average VOT may not prefer to use the additional tolled roads due to the distribution VOTs among business class. If we follow this logic line, the flow decrease in commuter class and flow decrease in freight class may seem strange. Possible explanation could be the trip properties of these two user classes. Normally the commuters have trips calculated as 27.74 min/veh, therefore the total lengths of their trips are short. They may want to take a short congested road with less toll charge. On the other hand, users from freight class have an average trip for 68.86 min/veh. That is to say, if they want to avoid the additional tolled roads, they may have a long detour route, which they also face to basic rates. Figure 7.6 gives a clearer overview of link flows on the additional tolled roads by user-classes and TOD periods.

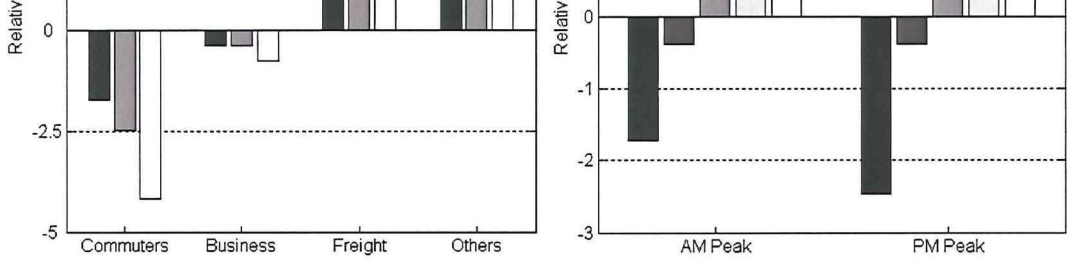

Figure 7.6: Relative Change in Link Flow on Additional Tolled Roads

7.3.4.2 Revenue on Additional Rates

Revenue generated by additional rates on designated roads by user classes and TOD periods are presented in Table 7.10 by user-classes and TOD periods. Again, we found biased estimations in the discrete VOT scenario (S1).

Table 7.10: Relative Change in Revenue from Additional Rates (%)

	AM	PM	Total
Commuters	-1.869	-1.858	-3.727
Business	-0.385	-0.484	-0.869
Freight	1.139	0.062	1.201
Others	4.025	1.059	5.084
Total	2.911	-1.221	1.690

Consisted with the overestimated link flows on these roads in discrete VOT scenario, the revenue generated by additional rates are higher as well. Relative changes in revenue are approximately in proportions of the flow changes as illustrated in Figure 7.7.

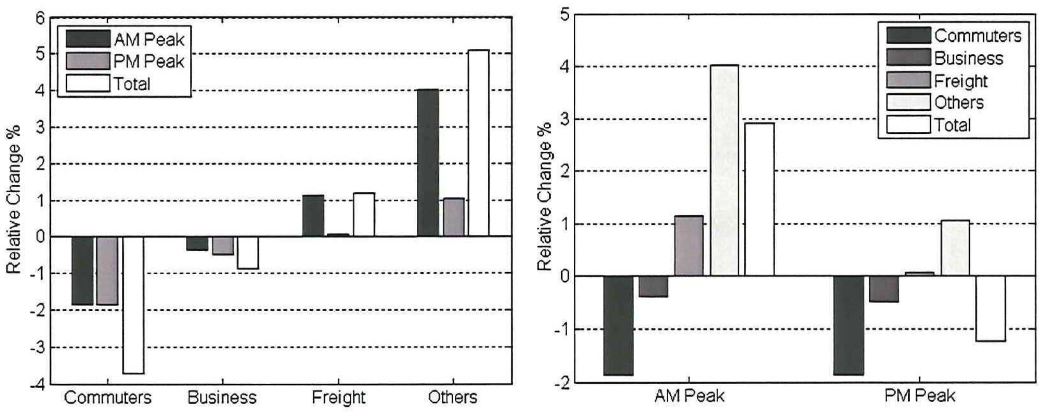

Figure 7.7: Relative Changes of Revenue Generated from Additional Rates

7.3.4.3 VTT on tolled roads

As presented previously, there is a difference in terms of the additional tolled road usage in the two designed scenarios, which in turn leading to a difference in the traffic conditions experienced by all road users measured by vehicle traveled time. This is outlined in Table 7.11.

Table 7.11: Relative Change in VTT on Additional Tolled Links (%)

	AM	PM	Total
COM	-2.27	-2.52	-4.79
BUS	-1.31	-1.28	-2.58
FRT	-0.24	-0.77	-1.01
OTH	1.71	0.30	2.01
Total	-2.11	-4.26	-6.37

Due to the decrease in total link flows over the additional tolled roads, all road users experience a better traffic condition when using these roads. Figure 7.8 shows the relative changes in VTT by user classes and TOD periods. On first sight, the positive values for users from others class may seem unrealistic. But noting that the vehicle traveled time is calculated on a total user class basis and there is a sharp increase in link flows from others class. Therefore, the observed increase is here reasonable when we calculate a total vehicle traveled time for this class. The average vehicle traveled times experienced by users from this class decrease from 1.50 min/veh to 1.47 min/veh.

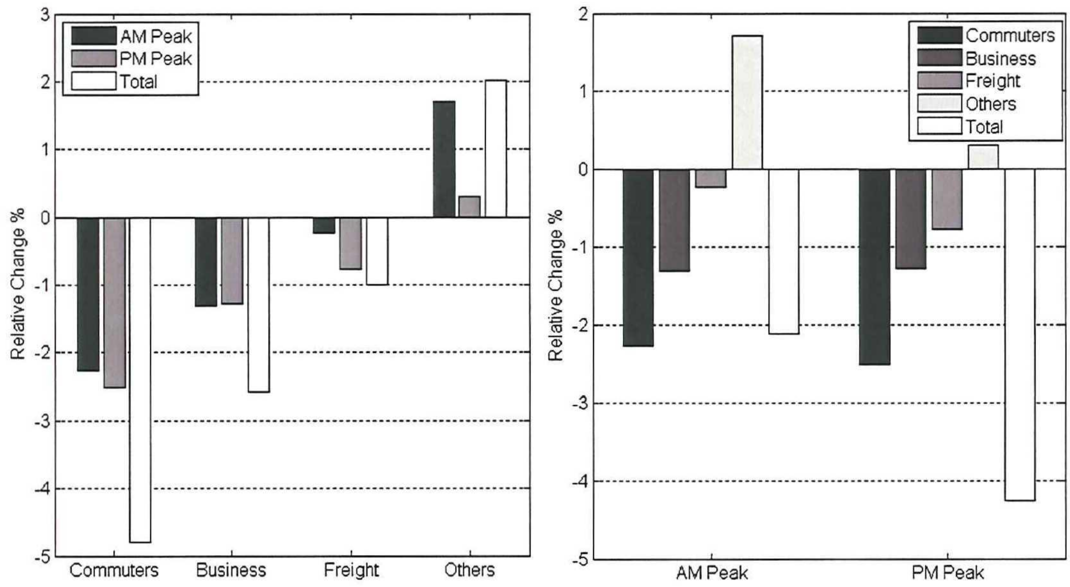

Figure 7.8: Relative Changes of VTT on Additional Tolled Roads

7.3.5 Conclusion

Different indicators are analyzed in this section for the designed scenarios in order to demonstrate the impact of heterogeneous VOT between and within multiple user-classes. Since road users in different user-classes will react differently on a given set of toll rates, they change their travel behaviors according to their particular VOTs. Consequently, the network performances in terms of vehicle traveled time, link flows and the revenue generation will vary compared with a discrete VOT scenario.

This finding provides toll operators with the useful information that when the rate changes, user reactions may not be as drastic as predicted by a discrete VOT assumption. The prediction of network performance under a given road pricing scheme, which is obtained from discrete VOT model, could be biased if user heterogeneity is not captured. With involvement of user heterogeneities in multiple user classes, the MUC-SUE with heterogeneous model can give more realistic predictions of the network performance, under a given road pricing scheme.

7.4 A Guide to Model Application

The second part of our analysis is focused on the inspiration to the potential model users. Based on the outputs generated from the Cube model, both the public and private sectors could check whether their specific expectations of KMP policy have been achieved or not. It is obvious that pricing objectives differ from the actors in KMP system. They come with diverging interests in a series of key issues and try to influence the outcomes towards their personal preferences. This section provides a brief introduction on how the Cube model could be implemented in an extensive scope of potential model-users.

7.4.1 Role of Traffic Assignment Model in Road Pricing Policy Design Problem

A road pricing policy, which can be formulated as a bi-level problem, is not designed in isolation but will consider intensively the possible responses of road users and the society. A graphical illustration of a bi-level problem is given in Figure 7.9. The designing process of road pricing policy starts from a set of objectives and then approaches to predicate and assess all effects throughout the society, which is the core part to determine if the objective had been met. If the objectives were not met, the decision makers would adjust the policy and the whole process would restart again. In other words, it is never a one-time process.

At the upper level, the road authority is the decision maker in practice who tries to solve problems related to road users and society by formulating attainable objectives and by determining instruments (type of tolling) with which to solve the problem. Given the objectives and instruments, the best set of measures (toll levels at links and periods) is to be found and implemented.

At the lower level, the road users are the originators of the problems to be solved with road pricing, while at the same time they influence policy makers to take initiatives to remedy these problems. In addition, the road pricing policy is meant to be applied to influence the travel behaviors of the road users. Indeed, the road users will not simply accept the higher travel costs due to the newly-set tolls, but will somehow tend to adapt their behaviors in order to minimize the burden induced by the road pricing policy. In general, road users have a gamut of options available to achieve this:

- travel decision making (even might decide to give up the travel plan);
- change the destination;
- change the mode of travelling;
- change the departure time;
- change the route.

All these behavioral changes will lead to a consequent change in distribution of traffic flows over time and space. This traffic flow distribution may lead to changes in road safety and living environment. To predict these indirect effects requires other measuring tools and/or models.

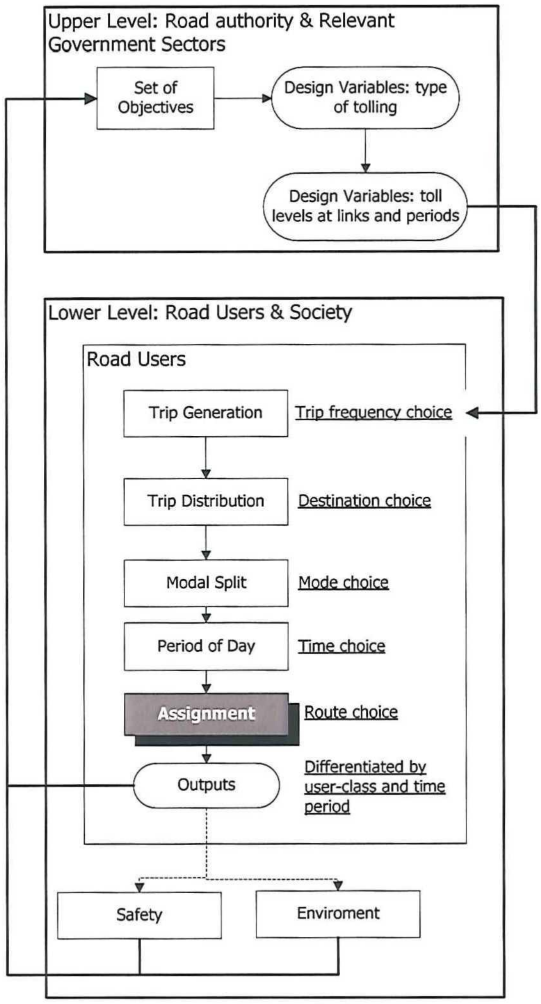

Figure 7.9: The Bi-level Framework of Road pricing Design Problem

In the context of this thesis, we confine ourselves to traffic assignment model, which is capable to capture route choice behavior in terms of heterogeneous VOT. In short, the outputs of the developed model present a picture of possible impacts of the Dutch KMP policy, especially from the road users' perspective. While it does not tell the whole story, it does provide some directions for determining whether the objective(s) of the Dutch KMP policy has (have) been achieved and how Dutch KMP policy affects the outcomes of interest of any of the relevant actors throughout the society. It should be emphasized that the outputs of the model does not automatically provide simple policy analysis to the complex Dutch KMP policy design problem. They can merely provide better predictions of traffic conditions that allow us to consciously translate them into feed-back information which is useful to decision makers and relevant actors. Nonetheless, this process is rarely as simple or straightforward as the model makes it seem. It will depend on who will use these outputs and their approaches of the translation process, and the specific interests for which the translation will takes place.

7.4.2 A Multi-actor Perspectives on the Dutch KMP Policy

Recognizing the role of the traffic assignment model in the road pricing design problem and its application depends on the purposes and approaches of its potential users. This section will identify main actors and their interests under the framework of Dutch KMP policy.

We could figure out two main types of the primary actors from public sector (upper level) and private sector (lower level). Other minor actors such as consulting companies, knowledge organizations, market parties etc. are not discussed here.

Table 7.12 presents these actors and their interests in Dutch KMP system. Firstly, the National Government Bodies may include:

- Ministry of Transport, Public Works and Water Management

  It is the ministry responsible for the Dutch system of water management, public and private transport and infrastructure. In the KMP project, the Ministry is responsible for the research, regulation and management of the policy in the field of transportation.

- Ministry of Finance

  This ministry is occupied with the national budget, taxation and financial economic policy, including supervision of financial markets. It is responsible for the regulation and collection of the charges and taxes under KMP.

- Ministry of Housing, Spatial Planning and the Environment (VROM)

  It is primarily a policy making body that stimulates to the inhabitants and companies in the Netherlands to approach issues on nature, environment and raw materials in a responsible way.

- Ministry of Economic Affairs

  It defines, implements, and regulates the enforcement of economic policy in the Netherlands.

And then, the regional and local government bodies include:

- Municipality of The Hague

- Municipality of Amsterdam

- The City Region of Haaglanden (Stadsregio Haaglanden)

Finally, we also need a social approval for the introduction of variabilisation of vehicle tax. Voices from private interest groups, such as the *ANWB* (car owners club), *VNO-NCW* (employers and employees organizations), the *Stichting Natuur en Milieu* (environmental conservation foundation) and *Transport en Logistiek Nederland* (organization of freight transport and logistical companies), should not be ignored.

As shown in the Table 7.12, actors involved in the transportation system influence the successful implementation of the pricing scheme. The governments take the responsibility to determine the variation of the current taxes, to clarify the principles of the rate structure and to set up specifications for the required equipment. For example, there will be a joint responsibility for the Minister of Finance and the Minister of Transport, Public Works and Water Management to determine rates and charge structures. The collection of tax is the responsibility of the Minister of Finance. The local or regional governments then take the responsibility of regulating the realistic implementation of the KMP policy and providing local feedbacks to the central government. And many examples have proved that without extensive acceptance in the travelers would induce a fatal destruction to the transportation policy.

As aforementioned, the implementation of Dutch KMP policy may result in a number of behavioral responses by road users. Also these behavioral changes may lead to a change in road safety. On the other hand, transport in general, and road transport in particular, are widely recognized as an important source of pollution which threatens environmental sustainability.

A successful implementation of the Dutch KMP policy requires coordination between the public and private sectors. The developed traffic assignment model is designed to facilitate this coordination process by providing information of possible effects of the KMP policy to actors from both sectors.

Table 7.12: Identification of Main Actors and their Interests

	Actor	Interests	Function
Public Sectors (Upper Level)	National Government Bodies		
	1. Ministry of Transport, Public Works and Water Management	Improve Accessibility	Design & Legalization of the Policy
	2. Ministry of Finance	Taxation	
	3. Ministry of Housing, Spatial Planning and the Environment	Environmental Positive (polluter pays)	
	4. Ministry of Economic Affairs	Market Functioning	
	Local and Regional Government Bodies		
	5. Municipality of The Hague	Improve Accessibility	Smooth Implementation of the Policy and Supervising
	6. Municipality of Amsterdam	Improve Accessibility	
	7. The City Region of Haaglanden	Improve Accessibility	
Private Sectors (Lower Level)	8. Dutch Automobile Association (ANWB)	Relief Congestions, Affordable of the System/Safety	Represents the Interests of Car users
	9. Transport and Logistics Organization of the Netherlands (Transport en Logistiek Nederland)	Relief Congestions	Represents the Interests of Freight
	10. The Confederation of Netherlands Industry and Employers(VNO-NCW)	Facilitate Mobility & Safety	Represents the Common Interests of Dutch Business
	11. The Netherlands Society for Nature and Environment (Stichting Natuur en Milieu)	Healthy Nature	Influencing Policy Towards to a Better Living Environment

7.4.3 Preliminary Explorations of Model Applications

Utilization of the model outputs can happen in many direct and/or indirect ways, often interacting with many other factors within a complex environment. In reality, outputs are only able to produce correct explanations if they are applied in the right way to the right situation. No single model is going to be appropriate for all relevant actors and/or for all objectives.

Although simple answers may be beyond our grasp, the quest for understanding will contribute to the development of the field. To stimulate further thinking and determine if this developed traffic assignment model is worthy of further study, we have conducted some preliminary explorations of utilization of the model outputs for relevant actors. While our explorations are at this point at an exploratory stage, they have resulted in additional insights.

- For Actors from Upper Level

Generally speaking, actors from upper level are comprised of various government bodies. They are responsible for the design and implementing the Dutch KMP policy. The objectives of the Dutch KMP are the desire to (1) increase equity, (2) improve accessibility, (3) avoid accidents and (4) protect the environment. Using the model outputs, relevant actors can determine if the policy, as designed, is likely to meet its objectives and whether or not the policy is being implemented as intended regarding to their specific interests.

Table 7.13: Model Outputs and Actors from Upper Level

			Actors from Upper Level						
			1	2	3	4	5	6	7
Direct Outputs	Equality	Revenue Distribution by User-Classes and Periods		√		√			
Direct Outputs	Accessibility	VTT by User-Classes and Periods	√				√	√	√
Direct Outputs	Accessibility	Average Travel Time by User-Classes and Periods	√				√	√	√
Direct Outputs	Accessibility	Average Speed per link by User-Classes and Periods, ect.	√				√	√	√
Indirect Outputs	Safety	Accidents	√						
Indirect Outputs	Safety	Casualty and Death Rate	√						
Indirect Outputs	Environment	Pollution			√				
Indirect Outputs	Environment	Noise			√				
Indirect Outputs	Environment	Fuel Consumption			√				

Table 7.13 illustrates preliminary relationships between model outputs (both direct and indirect) and actors from upper level.

- For Actors from Lower Level

Actors from lower level are mainly road users and environmental groups. They have a right access the information about the Dutch KMP policy they encounter and/or support. The model outputs provide them with information about the policy performance, thereby opening the policy to public scrutiny and judgment. The model outputs can help to answer questions such as whether the designed policy are implemented as intended and how do benefits/losses distribute among actors etc. Table 7.14 illustrates preliminary relationships between model outputs (both direct and indirect) and actors from lower level.

Table 7.14: Model Outputs and Actors from Lower Level

<table>
<tr><th colspan="3"></th><th colspan="4">Actors from Upper Level</th></tr>
<tr><th colspan="3"></th><th>8</th><th>9</th><th>10</th><th>11</th></tr>
<tr><td rowspan="4">Direct Outputs</td><td>Equality</td><td>Revenue Distribution by User-Classes and Periods</td><td>√</td><td>√</td><td>√</td><td></td></tr>
<tr><td rowspan="3">Accessibility</td><td>VTT by User-Classes and Periods</td><td>√</td><td>√</td><td>√</td><td></td></tr>
<tr><td>Average Travel Time by User-Classes and Periods</td><td>√</td><td>√</td><td>√</td><td></td></tr>
<tr><td>Average Speed per link by User-Classes and Periods, ect.</td><td>√</td><td>√</td><td>√</td><td></td></tr>
<tr><td rowspan="5">Indirect Outputs</td><td rowspan="2">Safety</td><td>Accidents</td><td>√</td><td></td><td>√</td><td></td></tr>
<tr><td>Casualty and Death Rate</td><td>√</td><td></td><td>√</td><td></td></tr>
<tr><td rowspan="3">Environment</td><td>Pollution</td><td></td><td></td><td></td><td>√</td></tr>
<tr><td>Noise</td><td></td><td></td><td></td><td>√</td></tr>
<tr><td>Fuel Consumption</td><td></td><td></td><td></td><td>√</td></tr>
</table>

7.5 Summary

This chapter discussed an application of the model with heterogeneous VOT between and within multiple user-classes under Dutch KMP scheme. In applying this model, several indicators could be obtained to evaluate whether the prediction of network performance has been improved in the aspects of revenue, travel conditions and equity regulation. We set up another scenario in discrete VOT to be compared. And we investigated that the prediction of network performance, under a given road pricing scheme, obtained from discrete VOT model could be biased if user heterogeneity is not captured. Finally, a brief introduction has been introduced on how the Cube model could be implemented in an extensive scope of potential model-users. Based on the results generated from the Cube model, both the public and private sectors could check whether their specific expectations of KMP policy have been achieved according to their own interests.

8

Conclusions and Further Research

With increasing interest in road pricing strategies to alleviate congestion and improve network performance, there is a need to develop a traffic assignment model capable of capturing heterogeneous users' responses to toll charges for design and evaluation of toll schemes. Following this motivation, we have proposed a multi uses class traffic assignment model with greater behavior realism in terms of heterogeneous VOT perceived by road users.

A summary of research done in this thesis is given in Section 8.1. Conclusions drawn from the research developed and presented throughout the thesis are presented in Section 8.2. In Section 8.3 we propose some scientifically challenging directions for further research.

8.1 Brief Summary

In this thesis a macroscopic traffic assignment model has been developed to solve the traffic equilibrium problem where there are multiple users classes presenting on the network. More specifically, heterogeneous VOT between and within multiple user-classes are considered. With the developed traffic assignment model, we can predict route flows, link flows, link travel times, route travel costs, and revenue generation, all varying over user classes and time periods.

Interactions among user classes are taken into account. In order to capture these asymmetric interactions the model has been mathematically formulated as a variational inequality problem. It cannot be formulated as an optimization problem, since the latter problem formulation can only be used for models with a symmetric Jacobian of the cost function. Gauss-Hemite approximation method has been adopted to solve the normally distributed VOT within multiple user-classes and a nested iterative solution algorithm has been designed for the model. The developed model has been implemented in Cube planning system, and then applied to a case study on Dutch KMP system, a large-scale network. With this case study, the theoretical correctness and empirical plausibility of the model have been demonstrated. Moreover, the capability of the model to provide policy makers more accurate predictions of the impacts of road pricing strategies has been shown.

8.2 Conclusions

This section will summarize the main achievements and findings established in the previous chapters.

Main achievements for traffic assignment modeling:

- The traffic assignment model has been extended to capture greater heterogeneity in users' route choice behavior by explicitly consideration of heterogeneous VOT between and within multiple user-classes.
- A MUC-SUE with heterogeneous VOT assignment model has been established, and formulated as a variational inequality problem (as the more general route-based model). For practical reasons, it has been further reformulated into a link-based model with implicit deterministic route choice behavior with some additional assumptions.
- It has been proved that Gaussian quadrature is extremely efficient when the integral to be approximated is only over one random parameter.
- A nested iterative solution algorithm has been developed for MUC-SUE with stochastic VOT assignment model. In all experiments and case studies presented, convergence was reached using such algorithm.

Main findings for model applications:

- The structure of the developed model is suitable to be easily applied to assess most of the existing toll schemes.
- The designed algorithm can be easily implemented in Cube planning system and applied to large-scale networks.
- The impacts of network performance estimation biases can be reduced by the developed model.
- It gives model users full flexibilities for further specifications on user heterogeneities, in terms of VOT. Besides average VOT can be assigned per user class, standard deviation can be user class specific. Moreover, all these values can be period specific as well.
- It enables policy makers to develop user-class specific policy instruments. Furthermore, the effects of the road pricing policy can be analyzed for each user-class.

Besides the main achievements in both theoretical and practical aspects, there remain however problems to be addressed in the future. The most important one is the omission of taking elastic demand into account. Furthermore, in model formulation process, a deterministic route choice behavior was assumed. This results in a partial loss of capability of the developed to reflect the real world situation. Finally, there is a lack of empirical data to calibrate/validate the developed model. These problems provide topics for further research, discussed in the next section.

8.3 Further Research

To overcome aforementioned problems, further research in the following directions is necessary:

- Inclusion of the developed assignment model with Dutch national model system (LMS)

  The developed traffic assignment model and its solution algorithm are not confined to road pricing issues. It is interesting to investigate whether the developed assignment model can be applied to more general situation and provide more realistic predictions.

- Consideration of elastic demand

  This will lead to a better assessment of the road pricing strategies. Although various studies have been conducted on this topic, adding elastic demand into the assignment model is not straightforward. Whether the users shift to non-congestion periods, whether they change their travelling modes, or whether they give up making a trip and stay at home. How to modeling elastic demand remains questionable, but it will definitely improve the accuracy of the model outputs.

- Consideration of stochastic route choice

  It will provide more realistic results. Further research would be to incorporate a more general stochastic path choice approach which assumes that users have inaccurate estimations, due to perception errors, of travel times and generalized route costs. However, the motivation for such an approach would diminish due to failure of developing an efficient solution to address the expected longer computational time from a practical point of view.

- Collection of more real life data

  The parameter values in case study are either taken from recent travel behavior studies (provided by **4***cast* B.V.) or by hypothetical assumptions (for example the standard deviations for VOT distribution patterns), while the outputs of the model are checked for plausibility. However, the soundness of the model approach presented in this thesis may improve considerably if compared with real life data.

- Improvement of more efficient solution algorithm

  Efficiency can be increased by searching for different numerical techniques or clever modification of the solution scheme. The current process can be speeded up if the link costs for a specific user class are not only based on the flows of the previous iteration, but also on the link flows of previous user classes in the same iteration.

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APPENDIX

A
Gauss-Hermite Approximation Method

This appendix aims to explore how to generate normally distributed VOT using Gauss-Hermite approximation method for the designed algorithm. It has been proved that Gaussian quadrature is extremely efficient in low dimensions. Since in this thesis, the integral in the algorithm is only over one random parameter (therefore only one dimension), it is very suitable of Gaussian quadrature approximation method. Please note that Gauss-Hermite approximation is a special case for approximating normally distributed random variables among the board family of Gaussian quadrature approximation method. For approximation of uniformly distributed random parameters, Gauss-Legendre approximation can be adopted.

Specific VOTs used in numerical tests (Chapter 5), Cube model applications (Chapter 7) and Cube model experiments (Appendix D) are given in this appendix as well.

A.1 Introduction of Hermite Polynomial

- Hermite Polynomials ($H_n(x)$) are in the first place, an orthogonal polynomial sequence. It can be defined by:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2}) \tag{A.1}$$

The definition of Hermite polynomials stated above is preferred by probabilitists. For the reason that Hermite polynomials behave like probability density function in standard normal distribution:

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2} \tag{A.2}$$

Please recall that our VOT follows a normal distribution that is why Hermite polynomials are chosen to be applied in the Gaussian quadrature in the main algorithm.

- The recurrence relation is:

$$H_{n+1}(x)=2xH_n(x)-2nH_{n-1}(x) \tag{A.3}$$

The first few Hermite polynomials are:

$H_0(x)=1$

$H_1(x)=2x$

$H_2(x)=4x^2-2$

$H_3(x)=8x^3-12x$

A.2 Relationship between Gauss-Hermite Approximation and Solution Algorithm

Our focus is to solve the problem that, the value of time, β, is not fixed but follows a certain given probability distribution. In this thesis, β is assumed to follow a normal distribution:

$$g(\beta)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(\beta-\mu)^2}{2\sigma^2}} \sim N(\mu,\sigma) \tag{A.4}$$

In the developed model, we need to solve the VI problem that (See also Chapter 4):

Find a $\boldsymbol{u}^* \in \Omega$ such that

$$\sum_a \sum_m \int_{\beta_m} c^*_{am}(u_{am} - u^*_{am}) g(\beta_m) d\beta_m \geq 0, \quad \forall u \in \Omega \tag{A.5}$$

where Ω is defined as the set of all **u** satisfying the following constraints:

$$\sum_{p \in P^{rs}_m} f^{prs}_m = D^{rs}_m, \quad \forall r,s,m \tag{A.6}$$

$$u_{am} = \sum_{rs} \sum_p \delta^{prs}_{am} f^{prs}_m, \quad \forall a,m \tag{A.7}$$

$$f^{prs}_m \geq 0, \quad \forall r,s,m,p \in P^{rs}_m \tag{A.8}$$

Note that we now have an integration over a function of β. This integral cannot be computed analytically, therefore has to be approximated numerically using Gauss-Hermite approximation.

Then what is Gauss-Hermite approximation? In short, 'Gauss' stands for Gaussian Quadrature, which is a method to define an approximation of the definite integral of a function in numerical analysis. 'Hermite' means Hermite polynomials which are used to define values to w_i and x_i .It can be stated as:

$$\int g(x)dx = \sum_{i=1}^{n} w_i g(x_i) \tag{A.9}$$

The equation stated above leads our VI model to,

$$\sum_a \sum_m \int_{\beta_m} c^*_{am}(u_{am} - u^*_{am}) g(\beta_m) d\beta_m \approx \sum_a \sum_m \sum_k w_{mk} c^*_{amk}(u_{amk} - u^*_{amk}) \tag{A.10}$$

Note that, c^*_{amk} is a function of VOT, β^k_m and link travel time τ_a.

A.3 Construction of Normally Distributed VOT

Step 1: Determine how many points should be used in the equation:

$$\int g(x)dx = \sum_{i=1}^{n} w_i g(x_i) \tag{A.11}$$

In this step, you actually choose the nth Hermite polynomial $H_n(x)$ in the next step. It should be determined subjectively, but different tests can be conducted to get a better estimation.

Step 2: Compute abscissas of the chosen Hermite polynomial:

By setting $H_n(x)$ =0, and compute the roots of $H_n(x)$, we can get our abscissas→ x_n^k

Step 3: Compute weights for each abscissa:

The weights associated with abscissas can be computed by:

$$weight_n^k = e^{-(x_n^k)^2} \tag{A.12}$$

The weights computed here are just intermediate variables in the whole process!

Step 4: Compute weights which will be used in the algorithm for the model:

$$w_n^k = \frac{weight_n^k}{\sqrt{\pi}} \tag{A.13}$$

Step 5: Construct VOT $\beta \sim N(\mu, \sigma)$

For each abscissa k, $\beta^k = \mu + \sqrt{2}\sigma x_n^k$ (A.14)

For the sake of simplicity, we now present abscissas and weights for Gauss-Hermite approximation in Table A.1 for up to ten points. The weights always sum up to one, i.e.,

$$\sum_k w_k^n = 1$$

for each n. For each on the n parameters, the number of points used, x_k^n can be different.

Table A.1: Abscissas and Weights for Gauss-Hemite Approximation[6]

n	x_n	w_n
1	0.0000000000	1.0000000000
2	± 0.7071067812	0.5000000000
3	0.0000000000 ± 1.2247448714	0.6666666667 0.1666666667
4	± 1.6506801239 ± 0.5246476233	0.0458758548 0.4541241452
5	0.0000000000 ± 2.0201828705 ± 0.9585724646	0.5333333333 0.0112574113 0.2220759220
6	± 2.3506049737 ± 1.3358490740 ± 0.4360774119	0.0025557844 0.0886157460 0.4088284696
7	0.0000000000 ± 2.6519613568 ± 1.6735516288 ± 0.8162878829	0.4571428571 0.0005482689 0.0307571240 0.2401231786
8	± 2.9306374203 ± 1.9816567567 ± 1.1571937125 ± 0.3811869902	0.0001126145 0.0096352201 0.1172399077 0.3730122577
9	0.0000000000 ± 3.1909932018 ± 2.2667805845 ± 1.4685532892 ± 0.7235510188	0.4063492063 0.0000223458 0.0027891413 0.0499164068 0.2440975029
10	± 3.4361591188 ± 2.5327316742 ± 1.7566836493 ± 1.0366108298 ± 0.3429013272	0.0000043107 0.0007580709 0.0191115805 0.1354837030 0.3446423349

A.4 Specific VOT used in Numerical Tests (Chapter 5)

[6] Source: Bliemer et al., *Approximation of Bayesian Efficiency in Experimental Choice Designs*

Table A.2: Normally Distributed VOT used in Numerical Tests

Numerical Setups	NO. of Points	μ	σ	VOT β	Weights w
GH3	3	0.5	0.15	0.7598076211 0.5000000000 0.2401923789	0.1666666667 0.6666666667 0.1666666667
GH3	3	0.5	0.05	0.5866025404 0.5000000000 0.4133974596	0.1666666667 0.6666666667 0.1666666667
GH6	6	0.5	0.15	0.9986386150 0.7833763817 0.5925059885 0.4074940115 0.2166236183 0.0013613850	0.0025557844 0.0886157460 0.4088284696 0.4088284696 0.0886157460 0.0025557844
GH6	6	0.5	0.05	0.6662128717 0.5944587939 0.5308353295 0.4691646705 0.4055412061 0.3337871283	0.0025557844 0.0886157460 0.4088284696 0.4088284696 0.0886157460 0.0025557844
GH6	6	0.2	0.06	0.3994554460 0.3133505527 0.2370023954 0.1629976046 0.0866494473 0.0005445540	0.0025557844 0.0886157460 0.4088284696 0.4088284696 0.0886157460 0.0025557844

A.5 Specific VOT used in Case Study (Chapter 7)

Table A.3: Normally Distributed VOT used in Case Study (S2) ($\sigma = 0.3\mu$)

User-Class	μ	σ	VOT β	Weights w
Commuters	0.174297	0.052289	0.323685 0.245181 0.174297 0.103413 0.024909	0.011257 0.222076 0.533334 0.222076 0.011257
Business	0.605071	0.181521	1.123671 0.851145 0.605071 0.358996 0.08647	0.011257 0.222076 0.533334 0.222076 0.011257
Freight	0.426553	0.127966	0.792149 0.600027 0.426553 0.253079 0.060958	0.011257 0.222076 0.533334 0.222076 0.011257
Others	0.151109	0.045333	0.280623 0.212563 0.151109 0.089655 0.021595	0.011257 0.222076 0.533334 0.222076 0.011257

A.6 Specific VOT used in Cube Model Experiments (Appendix D)

1. Gauss-Hermite 3 points with small deviation ($\sigma = 0.1\mu$)

Table A.4: Normally Distributed VOT used in Cube Model Experiments (E4)

User-Class	μ	σ	VOT β	Weights w
Commuters	0.174297	0.0174297	0.204486 0.174297 0.144108	0.1666665 0.666667 0.1666665
Business	0.605071	0.0605071	0.709872 0.605071 0.500269	0.1666665 0.666667 0.1666665
Freight	0.426553	0.0426553	0.500435 0.426553 0.352672	0.1666665 0.666667 0.1666665
Others	0.151109	0.0151109	0.177282 0.151109 0.124936	0.1666665 0.666667 0.1666665

2. Gauss-Hermite 5 points with small deviation ($\sigma = 0.1\mu$)

Table A.5: Normally Distributed VOT used in Cube Model Experiments (E5)

User-Class	μ	σ	VOT β	Weights w
Commuters	0.174297	0.0174297	0.224093 0.197925 0.174297 0.150669 0.124501	0.011257 0.222076 0.533334 0.222076 0.011257
Business	0.605071	0.0605071	0.777937 0.687095 0.605071 0.523046 0.432204	0.011257 0.222076 0.533334 0.222076 0.011257
Freight	0.426553	0.0426553	0.548418 0.484378 0.426553 0.368729 0.304688	0.011257 0.222076 0.533334 0.222076 0.011257
Others	0.151109	0.0151109	0.19428 0.171594 0.151109 0.130624 0.107937	0.011257 0.222076 0.533334 0.222076 0.011257

3. Gauss-Hermite 5 points with big standard deviation ($\sigma = 0.3\mu$)

All values used are identical as presented in Table A.3 above.

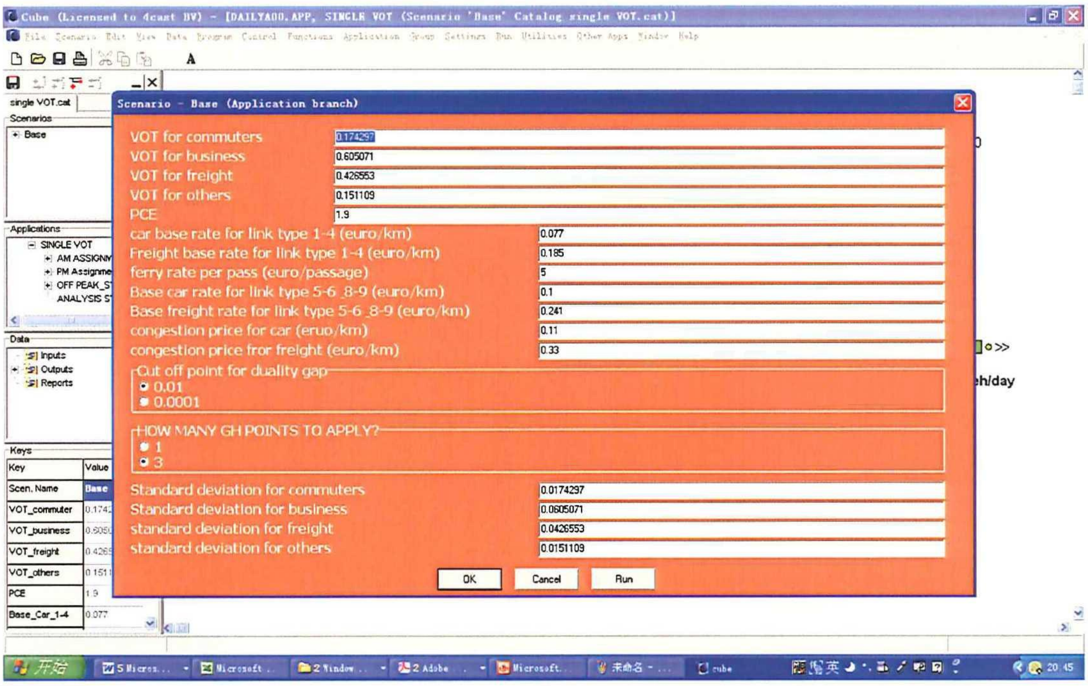
APPENDIX B

Cube Model Layout

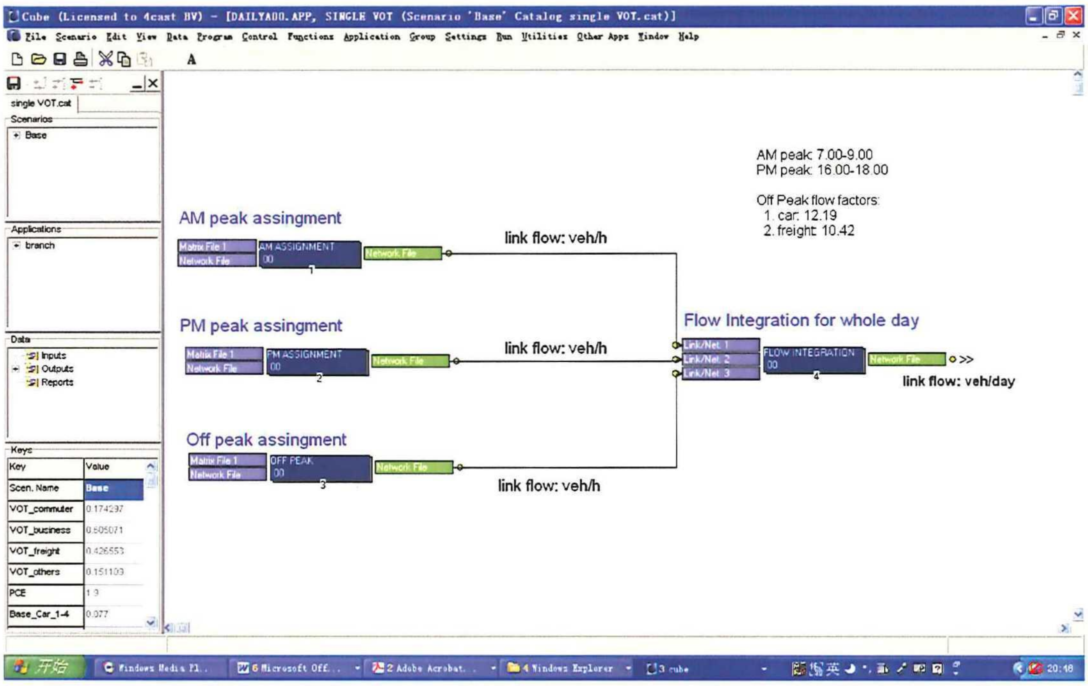
This appendix gives graphical description of the Cube model in 'Cube Language'. All figures in this appendix are copied from Cube Voyager. They are in line with modeling process described in Chapter 6. In addition, it provides supplemental information for Appendix C.

B.1 Input Screen

Here the model user can define all parameters used in Cube model. (See also Chapter 6 and Appendix C).

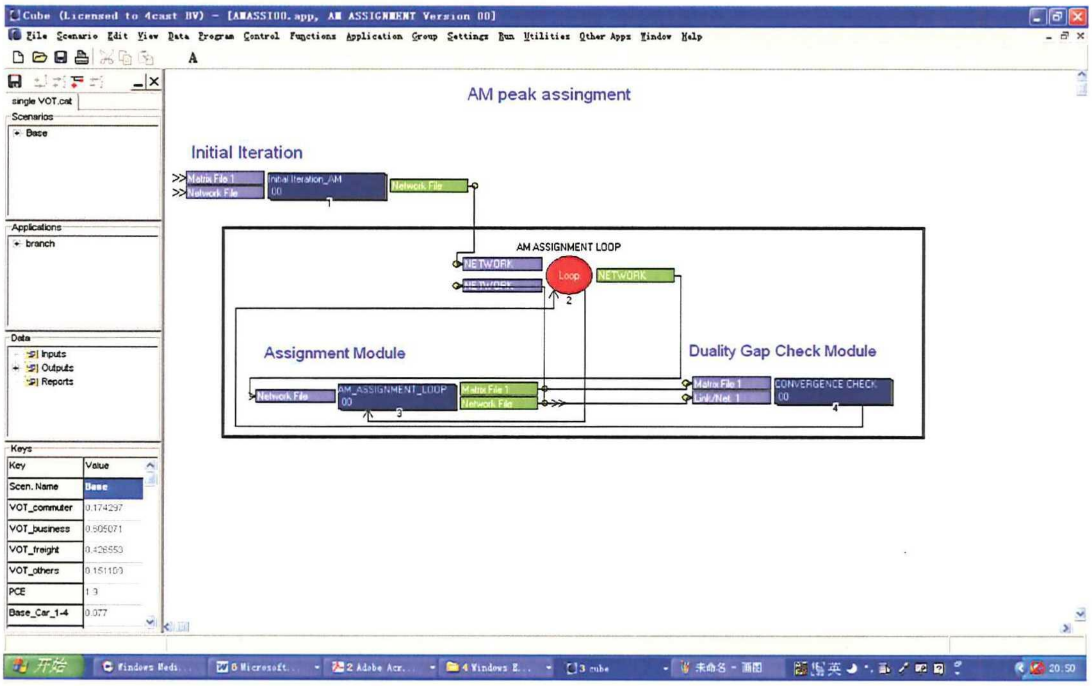


B.2 General Layout

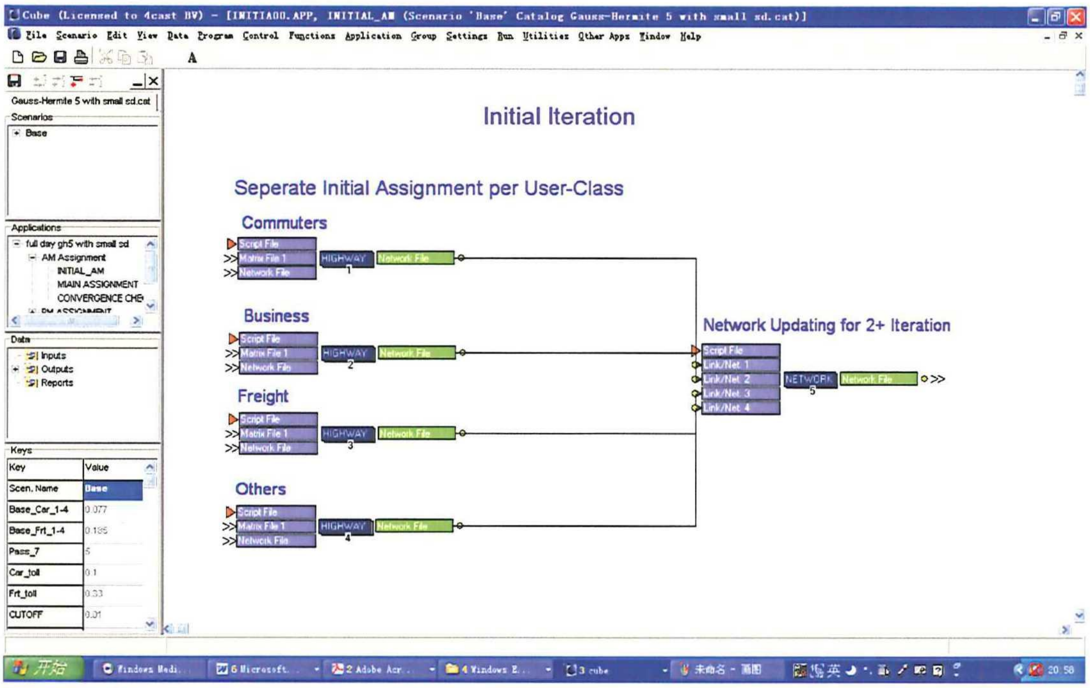


B.3 Assignment Layout

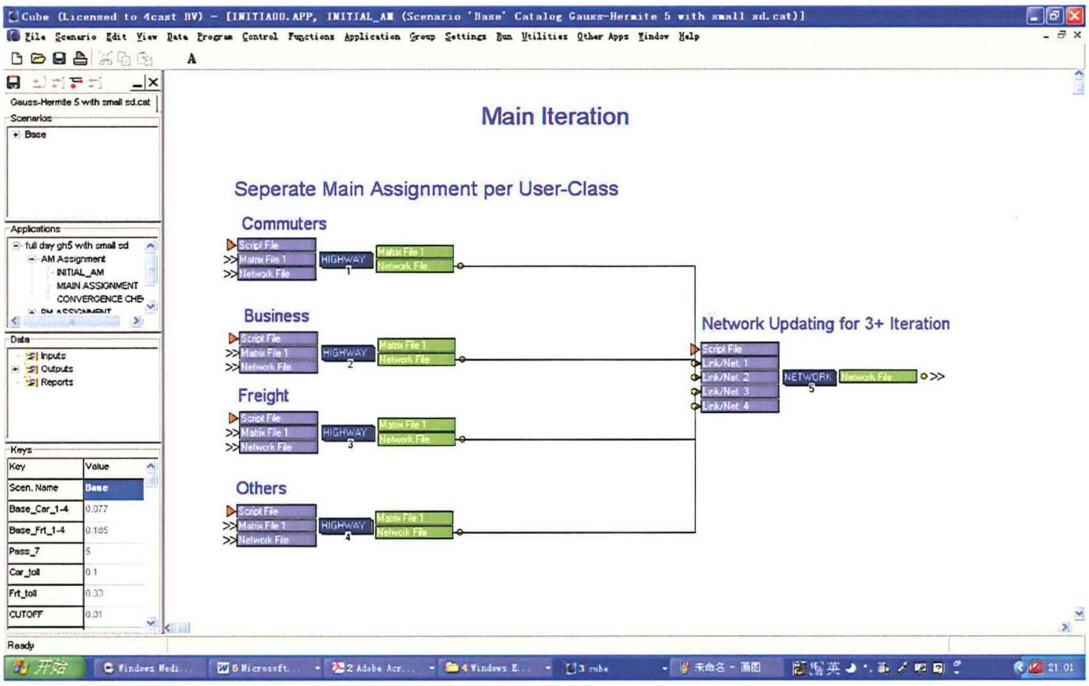
As described in Chapter 6, every assignment model has identical layout. Therefore, we only present here AM assignment module in detail in the following descriptions.



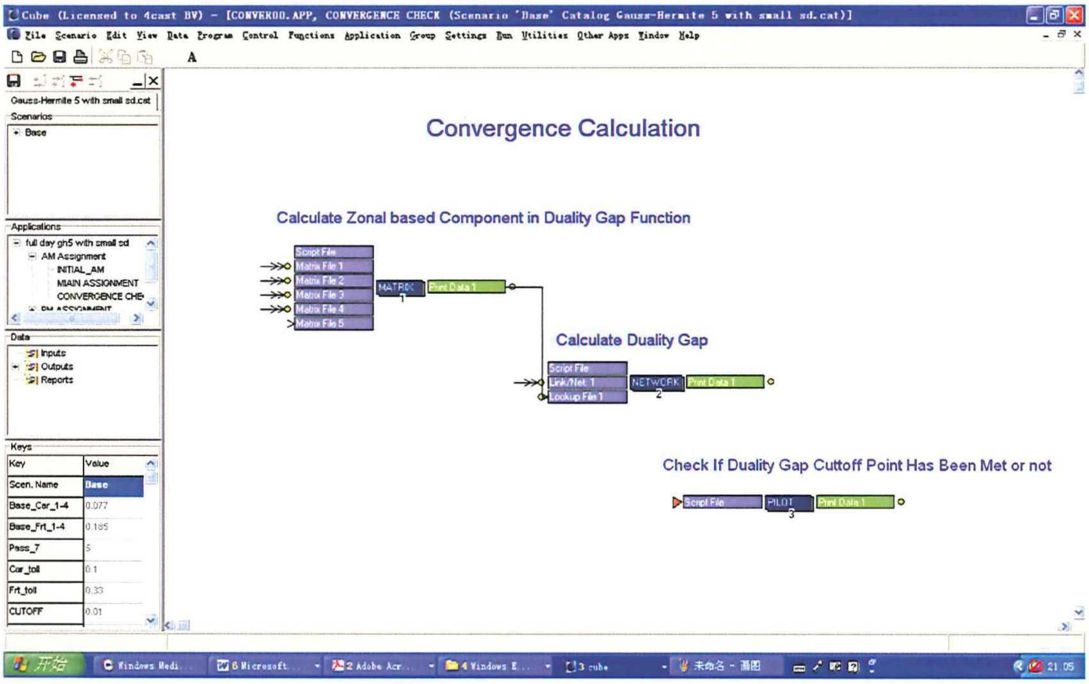
B.4 Initial Iteration



B.5 Main Assignment



B.6 Convergence Calculation



B.7 Flow Integration

APPENDIX

C

Cube Model Data Dictionary

The purpose of this appendix is to describe the Cube model developed in Chapter 6 by Cube/Voyager data dictionary. This data dictionary identifies each input file and parameters related with model steps and programs, file functions, and data formats for core attributes and optional attributes. Separate sections are provided for input files, output files, and file functions. Note that all data described in this appendix are provided by **4***cast* B.V.

A number of different file type extensions are used in the developed Cube model:

- MAT Cube-Voyager matrix format
- NET Cube-Voyager binary network database
- PRN Cube-Voyager output files describing model execution and results
- S Cube-Voyager model script (not described in this appendix)

There is a list of Cube-Voyager modules used during the development of the Cube model (See also Chapter 6):

- **PILOT:** is the basic driver for Cube Voyager application modules. It is used to control main assignment loop in our Cube model.
- **HIGHWAY:** is a module whose primary function is assign trips to highway network

links. It builds routes based upon link cost (defined by model developers) and assign trips to those routes for each origin zone. In addition, generalized route costs are computed and recorded in this module as well.

- **NETWORK:** is a utility program for processing highway networks. In our model, this module is used to update link flow, link travel time, and calculating the convergence criterion.
- **MATRIX:** processes zonal data and matrices according to user specified expressions. We use it to compute minimal generalized route travel cost with OD demand for the convergence criterion.
- **LOOP CONTROL:** is defined to control the program terminate on a condition. We use it to control the main assignment loop on the condition in terms of duality gap.

C.1 Input Files

This section will describe input files for the Cube model.

C.1.1 Network Files

File Type:	NET
Specification:	Specific for TOD periods
Model Step:	Assignment
File used by:	HIGHWAY
Primary Function:	Highway network database for assignment and updating in the initial iteration phase

Data Format:

Variable Name	Description	Usage
A	A node (total number of 17,936)	-
B	B node (total number of 17,936)	-
DISTANCE	Link distance in kilometer	X
TSVA	Free flow speed in kilometer/hour	X
LANES	Directional No. of lanes	-
TYPE	Indicates link type, 9 different types in total	X
HWN	Indicates road network properties	X
CAP	Link capacity in vehicle/hour	X
HEF	Indicates additional tolled links during peak periods	X

Special Note: only variables with 'X' are used in the Cube model. HEF is differentiated by TOD periods. These network files will be only used during the initial assignment module for each TOD periods.

C.1.2 Matrix Files

File Type: MAT

Specification: Specific for TOD periods and user-classes

Model Step: Assignment/Compute for duality gap

File used by: HIGHWAY/MATRIX

Primary Function: OD matrix for Highway assignment and compute duality gap

Data Format:

Table Name	Description	Value (vehicle/hour)		
		AM Peak	PM Peak	Off Peak
1	Demand for COMMUTERS users	489781.12	336855.08	116804.45
2	Demand for BUSINESS users	100005.90	159245.57	127244.88
3	Demand FREIGHT users	101515.83	93443.02	112498.73
4	Demand for OTHERS users	149282.37	327992.12	311561.28

Special Note: OD matrixes are used for the whole process of Cube model when they are needed.

C.2 Output Files

This section will describe output files by Cube model. We will distinguish the output files for the iterative process and final output files for analysis. The iterative output files are generated for each iterative step a time and will be overwritten during the next iterative step for the sake of program efficiency.

C.2.2 Iterative Output Files

There are various output files generated during the iterative process. They are listed as follows:

Step	Module	Output files
Initial Iteration	HIGHWAY	*.NET
	NETWORK	*.NET
Main Assignment	HIGHWAY	*.NET, *.MAT
	NETWORK	*.NET
Convergence Calculation	MATRIX	*.PRN
	NETWORK	*.PRN
	PILOT	*.PRN

C.2.2.1 Initial Step

1. HIGHWAY: Network Files

File Type: NET

Specification: Specific for TOD periods/iterative steps

File used by: NETWORK

Primary Function: Loaded highway network database for updating congested link travel time and link flow

Data Format: (only newly generated by HIGHWAY and useful variables are listed)

Variable Name	Description	Specification
V#_1	Auxiliary link flow for this iteration (veh/h)	VOT draws and user-class

Special Notes: the only useful output data are assigned link flow per user-class and per VOT draws. They will be used in the subsequent NETWORK module. The number sign, #, in the table represents variables related with specific VOT draws. A NET file will be generated for every user-class presented on the network.

2. NEWORK: Network Files

File Type: NET

Specification: Specific for TOD periods/iterative steps

File used by: HIGHWAY

Primary Function: Loaded and updated highway network database for assignment and updating and calculation of duality gap

Data Format: (only newly generated variables by NETWORK are listed)

Variable Name	Description	Specification
BASECAR	Basic rate for car users (€/km)	Link type
BASEFRT	Basic rate for truck users (€/km)	Link type
PASS	Ferry rate for all users (€/passage)	Link type
TOLL_FR	Additional rate for freight users (€/km)	HEF
TOLL_CAR	Additional rate for car users (€/km)	HEF
FC#	Combined link flow for COMMUTERS users (veh/h)	VOT draws
FB#	Combined link flow for BUSINESS users (veh/h)	VOT draws
FF#	Combined link flow for FREIGHT users (veh/h)	VOT draws
FO#	Combined link flow for OTHERS users (veh/h)	VOT draws
F	Combined link flow for all users (veh/h)	-
V	Combined link volume for all flows (veh/h)	-

T0	Calculated free flow link travel time (km/min)	-
T1	Calculated congested link travel time (km/min)	Link type

Special Notes: Toll rates are stick to each link depending on their link type and TOD periods for speeding up the later iterative assignment procedures. Adding rates to links will not be repeated in the remaining process.

C.2.2.2 Main Assignment Step

1. HIGHWAY: Network Files

File Type: NET

Specification: Specific for TOD periods/iterative steps

File used by: NETWORK

Primary Function: Loaded highway network database for updating congested link travel time and link flow

Data Format: (only newly generated by HIGHWAY and useful variables are listed)

Variable Name	Description	Specification
V#_9	Auxiliary link flow for this iteration (veh/h)	VOT draws
LW_GENC_*_9	Generalized link cost in this iteration (€)	VOT draws

Special Notes: to distinguish the NET files with those generated in initial assignment step, here we also record generalized link costs in this iteration and they will be used in the subsequent process. The number sign, #, in the table represents variables related with specific VOT draws. The asterisk, *, in the table represents variables related with specific user-class, possible values could be: COM, BUS, FRT, and OTH. A NET file will be generated for every user-class presented on the network.

2. HIHWAY: Matrix Files

File Type: MAT

Specification: Specific for TOD periods/iterative steps

File used by: MATRIX

Primary Function: Calculated zone-to-zone generalized route cost for calculation of duality gap

Data Format:

Table Name	Description	Specification
#	Zonal based Generalized link cost in this iteration (€)	VOT draws and user-class

Special Notes: the number sign, #, in the table represents variables related with specific VOT draws. A MAT file will be generated for every user-class presented on the network.

3. NETWORK: Network Files

File Type: NET

Specification: Specific for TOD periods/iterative steps

File used by: HIGHWAY/NETWORK

Primary Function: Updated highway network database for assignment and calculation of duality gap

Data Format: (only newly generated variables by NETWORK are listed)

Variable Name	Description	Specification
FC#	Combined link flow for COMMUTERS users (veh/h)	VOT draws
FB#	Combined link flow for BUSINESS users (veh/h)	VOT draws
FF#	Combined link flow for FREIGHT users (veh/h)	VOT draws
FO#	Combined link flow for OTHERS users (veh/h)	VOT draws
F	Combined link flow for all users (veh/h)	-
V	Combined link volume for all flows (veh/h)	-
T1	Calculated congested link travel time (km/min)	Link type
GC_C#	Generalized cost for COMMUTERS users (€)	VOT draws
GC_B#	Generalized cost for BUSINESS users (€)	VOT draws
GC_F#	Generalized cost for FREIGHT users (€)	VOT draws
GC_O#	Generalized cost for OTHERS users (€)	VOT draws

Special Notes: the number sign, #, in the table represents variables related with specific VOT draws.

C.2.2.3 Convergence Calculation

1. MATRIX: PRN files

File Type: PRN

Specification: Specific for TOD periods/iterative steps

File used by: NETWORK

Primary Function: Calculated minimal possible zone-to-zone generalized route cost which is weighted by the demands

Data Format:

Variable Name	Description
_SUMMAT	See Primary Function above

2. NETWORK: PRN files

File Type: PRN

Specification: Specific for TOD periods/iterative steps

File used by: PILOT

Primary Function: Calculated duality gap

Data Format:

Variable Name	Description
*_LOOP	Loop control variable (indicate i^{th} iteration step)
TUN*	See Primary Function above

Special Notes: the asterisks in the table denotes specific TOD periods, possible values could be: AM, PM, OFF.

3. PILOT: PRN files

File Type: PRN

Specification: Specific for TOD periods/iterative steps

File used by: LOOP CONTROL

Primary Function: Calculated duality gap

Data Format:

Variable Name	Description
*_LOOP	Loop control variable (indicate i^{th} iteration step)
LAN*.TUN*	See Primary Function above

Special Notes: the asterisks in the table denotes specific TOD periods, possible values could be: AM, PM, OFF.

C.2.3 Final Output files

If the convergence criterion is met, the assignment process will be stopped and the final outputs are generated by NETWORK module in the main assignment step and by PILOT in the convergence calculation step. After all assignment procedures for every TOD periods are finished, the network files from different TOD periods will be integrated in flow integration module.

1. NETWORK: Network Files

File Type: NET

Specification: NONE

Primary Function: Integrated link flow on a daily basis

Data Format:

Variable Name	Description
TOTAL_FLOW	See Primary Function (veh/day)

Special Notes: possible analysis could be done in this step depending on model users choice.

C.3 File Functions

Essential file functions used in Cube model will be presented in this section.

C.3.1 Generalized Link Travel Cost in HIGHWAY

Generalized link travel cost is calculated in terms of money and takes into consideration of both basic rates and additional rates, as formulated as follows:

$$c_{am}^{(i,k)} = \beta_m^k \cdot \tau_a^{(i)} + (\theta_{a,m}^{con} + \theta_{a,m}^{base,t}) \cdot L_a + \theta_a^{ferry} \qquad \text{(C.1)}$$

where

$t \subseteq T$ link type index

$m \subseteq M$ user class index

$a \subseteq A$ link index

β_m^k k^{th} VOT draw for users of class m [€/min]

$\tau_a^{(i)}$ link travel time on link a in i^{th} iteration [min]

$\theta_{a,m}^{con}$ congestion rate on link a for users of class m [€/km]

$\theta_{a,m}^{base,t}$ basic rates on link a with link type t for users of class m [€/km]

θ_a^{ferry} ferry rate on link a [€/passage]

L_a length of link *a* [km]

	Basic Rate			Congestion Rate
	$\theta_{a,m}^{base,t}$	θ_a^{ferry}	$\theta_{a,m}^{base,t}$	$\theta_{a,m}^{con}$
	Link type 1-4	Link type 7	others	
Car	0.077	5	0.1	0.11
Freight	0.185	5	0.241	0.33

C.3.2 Link Travel Time Function in NETWORK

Link travel time functions are differentiated on link type and I/C ratios as formulated below,

- If I/C ratios $\frac{V_a}{CAP_a}$ <0.75,

$$\tau_a = \tau_0(1 + \alpha \bullet \frac{V_a}{CAP_a}) \qquad \text{(C.2)}$$

- If I/C ratios $\frac{V_a}{CAP_a} \geq 0$,

$$\tau_a = \tau_0(1 + \alpha \bullet \frac{V_a}{CAP_a} + \beta \bullet (\frac{V_a}{CAP_a} - 0.75)^{\gamma}) \qquad \text{(C.3)}$$

where,

$$V_a^{(i)} = \sum_{m \in M} pce_m \cdot u_{am}^{(i)}$$

τ_0 free flow travel time on link a [min]

u_a flow on link a [veh/h]

CAP_a capacity of link a [veh/h]

α, β, γ link type differentiated parameters

	Highway (link type =1)	Others
α	0.22222	0.5
β	8	8
γ	1.5	1.5

User class	PCE
Commuters	1
Business	1
Freight	1.9
others	1

C.3.3 Duality Gap in MARTIX and NETWORK

Duality gap is calculated in present iteration and can be formulated as follows:

$$DG = \frac{\sum_{k\in K}\sum_{m\in M}\sum_{a\in A} u_{am}^{(i,k)} \cdot c_{am}^{(i,k)} - \sum_{k\in K}\sum_{m\in M}\sum_{r\in R}\sum_{s\in S} D_{m,k}^{rs} c_{m,k}^{rs,(i)}}{\sum_{k\in K}\sum_{m\in M}\sum_{r\in R}\sum_{s\in S} D_{m,k}^{rs} c_{m,k}^{rs,(i)}} \tag{C.4}$$

where

$k \subseteq K$ Gauss-Hermite point index

$a \subseteq A$ link index

$m \subseteq M$ user class index

$r \subseteq R$ origin index

$s \subseteq S$ destination index

i iteration index (only check for current iteration!!!)

$u_{a,m}^{(i,k)}$ link flow on link a under k^{th} VOT for user-class m in i^{th} iteration [veh/h]

$c_{am}^{(i,k)}$ generalized link travel cost on link a under k^{th} VOT for user-class m in i^{th} iteration [€]

$D_{m,k}^{rs}$ travel demand from origin r to destination s of users of class m under k^{th} VOT [veh/h]

$c_{m,k}^{rs,(i)}$ generalized route travel cost from origin r to destination s under k^{th} VOT for user-class m in i^{th} iteration [€]

C.3.4 Flow Integration Formula in NETWORK

In scaling of the link flows from three periods of the day, we use the link flow scale formula as follows:

$$f_a = \alpha \cdot (u_a^{AM} + u_a^{PM}) + \beta_{car} \cdot u_{a,car}^{off} + \beta_{freight} u_{a,freight}^{off} \tag{C.5}$$

where

u_a total link flow of the day [veh/day]

u_a^{AM} total link flow during AM peak period [veh/h]

u_a^{PM} total link flow during PM peak period [veh/h]

$u_{a,car}^{off}$ car link flow during off peak period [veh/h]

$u^{off}_{a,freight}$ freight link flow during off peak period [veh/h]

α TOD factor during peak period

β_{car} TOD factor for car users during off peak period

$\beta_{freight}$ TOD factor for freight users during off peak period

α	β_{car}	$\beta_{freight}$
2	12.19	10.42

APPENDIX D

Cube Model Experiments

In this appendix, two sets of experiments are conducted to examine the Cube model developed in Chapter 6. The following questions are addressed in this appendix:

- Set 1 (E1-E3):
  - How to determine the value for cutoff point for the Cube model?
- Set 2 (E4-E6):
  - How will normally distributed VOT affect the convergence pattern of the proposed algorithm?
  - How many Gauss-Hermite points should be used for case study (Chapter 7)?

The data input to Cube model are exactly the same as described in Chapter 7, i.e. all experiments are conducted on Dutch national network.

D.1 Experiment Set 1 (E1-E3)

D.1.1 Experiment Setups

There is always a trade-off between computational time and the accuracy on determine the equilibrium traffic condition, especial considering a relative advanced assignment model on a large-scale network. Before we conduct normally distributed VOT within multiple user-classes, the cutoff point for the duality gap needs to be determined. Without loss of generality, three experiments are setup as follows:

Table D.1: Experiment Set 1 Setups

Experiments	Description	Cutoff Point
E1	Discrete VOT among multiple user-classes	0.01
E2	Discrete VOT among multiple user-classes	0.001
E3	Discrete VOT among multiple user-classes	0.0001

The purpose of this set of experiments is for selecting duality gap for the later experiments and case study presented in Chapter 7. Two attributes are of our main concern,

- Computational time
- Absolute change in total vehicle traveled time

The later attribute can be expressed as,

$$\Delta VTT = \frac{\sum_a \left(\left| u_{a1}^{am} \tau_{a1}^{am} - u_{a2}^{am} \tau_{a2}^{am} \right| + \left| u_{a1}^{pm} \tau_{a1}^{pm} - u_{a2}^{pm} \tau_{a2}^{pm} \right| + \left| u_{a1}^{off} \tau_{a1}^{off} - u_{a2}^{off} \tau_{a2}^{off} \right| \right)}{\sum_a \left(u_{a1}^{am} \tau_{a1}^{am} + u_{a1}^{pm} \tau_{a1}^{pm} + u_{a1}^{off} \tau_{a1}^{off} \right)} \times 100\%$$

(D.1)

D.1.2 Discussions

The experimental results are presented in Table D.2. It can be found that as we began to decrease the cutoff point value, the computational time needed to reach a convergence state significantly increases. A more than thirteen hours run for a discrete VOT among multiple user-classes is needed for a 0.0001 duality gap. We can imagine that when we perform a normally distributed VOT among multiple user-classes for the same cutoff point value, the computational time could be even much longer (See also Chapter 5).

Although the small value set for cutoff point in E3 indicates that E3 is able to find a closer solution to true equilibrium condition, it is not efficient from the practical point of view. We now start to use our second attributes to make trade-off between a closer solution to true equilibrium condition and computational time. Cross comparisons are made for all three experiments. We assume that E3 is an approximate final equilibrium condition. Big violations in relative changes in VTT are observed in E1. On the other hand, the VTT calculated from E2 is approximately the same with that from E3, only 0.989% relative change is observed. Together with it lower and acceptable computational time, we will use value equals to 0.001 for the cutoff point the remaining experiments in this appendix and in the case study (Chapter 7).

Table D.2: Cube Model Results

Experiments	#. Iterations			Computational Time	Relative Change in VTT	
	AM	PM	Off		E2	E3
E1(DG=0.01)	9	9	5	12min	4.956%	5.633%
E2(DG=0.001)	71	70	27	1h26min	-	**0.989%**
E3(DG=0.0001)	630	676	245	13h21min	-	-

D.2 Experiment Set 2 (E4-E6)

D.2.1 Experiment Setups

This set of experiments intends to investigate the impacts of user heterogeneity in terms of normally distributed VOT among multiple user-classes on the convergence pattern of the proposed algorithm. Moreover, with various setups, it is possible to check whether the properties of the proposed algorithm applied to a large-scale network consist with those explored in Chapter 5. Note that the cutoff points for this set of experiments are set to 0.001.

Table D.3: Experiment Set 2 Setups

Experiments	#. Of points for Gauss-Hermite	σ	Description
E4	3	$\sigma = 0.1\mu$	3 points with small s.d.
E5	5	$\sigma = 0.1\mu$	5 points with small s.d.
E6	5	$\sigma = 0.3\mu$	5 points with big s.d.

D.2.2 Convergence Patterns

The convergence patterns of proposed algorithm in different experiments are illustrated in Figure D.1. Note that, these convergence patterns in each experiment are taken from their assignment modules in AM peak periods. As shown in Figure D.1, all the values of duality gap decrease rapidly in the first few iterations and then slowly afterwards, this indicates the similar property of the classical Frank-Wolfe algorithm. Moreover, the result shows that the convergence patterns of the algorithm with different VOT distribution and number of points used for Gauss-Hermite approximation methods are similar, suggesting that they would not affect the convergence performance of the algorithm.

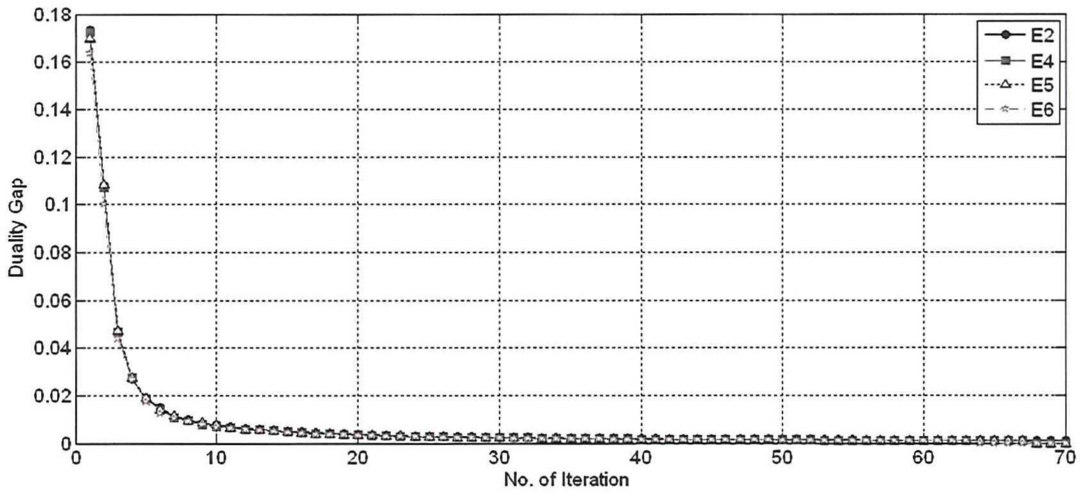


Figure D.1: Convergence Patterns in Different Experiments

D.2.3 Determine Number of Gauss-Hermite Points

Table D.4 lists computational time in different experiments.

Table D.4: Computational Time

Experiments	Computational Time
E4	10h41min
E5	7h3min
E6	6h28min

It is observed that more Gauss-Hemite points used in the Cube model leads to a shorter computational time needed to reach a convergence. For the sake of efficiency, we will use five Gauss-Hermite points in our case study. In addition, the objective of the case study is to illustrate the impact of considering a normally distributed VOT among multiple user-classes in the traffic assignment model. To highlight possible effects, we will use relative bigger standard deviations for the VOT distribution patterns, i.e. $\sigma = 0.3\mu$.

Summary

Motivation

With increasing interest in road pricing strategies to alleviate congestion and improve network performance, there is a need to develop a traffic assignment model capable of capturing heterogeneous users' responses to toll charges for design and evaluation of toll schemes.

More and more efforts are put into establishing those traffic assignment models to expanding their capabilities and prediction power to provide better predictions of the network performance under a given road pricing strategy. In this thesis an MUC-SUE with heterogeneous VOT traffic assignment model has been proposed. The inclusion of greater behavior realism results in more realistic traffic forecasts and enables policy makers and planners to make better decisions concerning the designs of road pricing strategies.

Heterogeneous VOT

In capacity-limited transportation networks, the planning and operations of various road pricing policies, such as road tolls, cordon (area) toll, and congestions tolls, require a traffic assignment model that takes into account two essential decision attributes: travel time and out-of-pocket cost. Road users will make trade-off between these two attributes when choose a certain route during their travels. To link the time terms and monetary terms, we need to introduce value of time (VOT). The VOT relative to each trip represents how much money the road user is willing to pay for a unit time saving. In a utility maximization framework, each road user can be assumed to select a route that minimizes a generalized cost function where travel time is weighted by that road user's particular VOT. Various empirical studies have suggested that VOT varies significantly across individuals because of different socioeconomic characteristics, trip purposes, attitudes, and inherent preferences. The inclusion of heterogeneous VOT in traffic assignment model is therefore of fundamental importance.

In literature, previous studies that address user heterogeneity are dominated by two approach categories, the discrete VOT among multiple user-classes approach and the continuously distributed VOT across the whole population of road users. Either of these two approaches has its own advantages in predicting a more realistic network performance.

Nevertheless, a general traffic assignment model describing further heterogeneities among multiple user-classes in route choice and traffic flow operation is still lacking. In this thesis, we proposed a traffic assignment model to incorporate heterogeneous VOT both between and within multiple user-classes.

Mathematical Formulation and Solution Algorithm

One of the main research issues of the thesis is to provide sound mathematical formulation for the development of the proposed traffic assignment model. Recognizing the heterogeneity between and within multiple user-classes on the network considerably complicates the problem. The consideration of multiple user-classes implies that the traffic flow conditions on the network will be affected by interactions among the user classes, thus leading to asymmetric cost functions. The presence of asymmetric cost functions requires the assignment model to be formulated as a variational inequality (VI) problem. The VI approach is especially appropriate in modeling traffic assignment problems in which asymmetric interactions exist, and no corresponding optimization problem can be formulated.

In order to obtain an assignment model, which is applicable to deal with large scale network, an infinite scale parameter in the logit-based route choice function and addictive costs were assumed. In the thesis, a link-based variational (VI) problem has been formulated. An advantage of the link-based VI model is that the solution algorithm for such a model will not require explicit route enumeration.

Efforts have been made to solve continuously distributed VOT among multiple user-classes. By introducing numerical techniques, distributed VOT problem can be solved through an iterative procedure over different draws of VOT. Finally, a nested iterative all-or-nothing (AON) algorithm has been proposed to solve the link-based VI model.

Applications

The applicability of the model has been demonstrated for the Dutch national road network under the Dutch KMP system. To support the evaluation of Dutch KMP system strategies in a network context, the proposed model is developed in Cube planning system, which aims to capture users' route choices in response to a pre-designed toll scheme, and hence explicitly considers heterogeneous VOT between and within multiple user-classes in the underlying route choice decision framework. Two scenarios, namely discrete VOT among multiple user-classes and heterogeneous VOT between and within multiple user-classes, have been built in the Dutch KMP case study. Results indicate that estimation of network performance, obtained from the discrete VOT scenario, would be biased if user heterogeneity within multiple user-classes is not captured.

A wide variety of model applications can be realized based on the rich modeling capabilities of the proposed model in capturing greater user heterogeneity in terms of VOT. In short, the outputs of the developed model present a picture of possible impacts of the Dutch KMP policy, especially from the road users' perspective. While it does not tell the whole story, it does provide some directions for determining whether the objective(s) of the Dutch KMP policy has (have) been achieved and how Dutch KMP policy affects the outcomes of interest of any of the relevant actors throughout the society. It should be emphasized that the outputs of the model does not automatically provide simple policy analysis to the complex Dutch KMP policy design problem. They can merely provide better predictions of traffic conditions that allow us to consciously translate them into something which feed back into decision making and provide relevant actors with information. Nonetheless, this process is rarely as simple or straightforward as the model makes it seem. It will depend on who will use these outputs and their approaches of the translation process, and the specific interests for which the translation will takes place.

Conclusion

We conclude that the MUC-SUE with heterogeneous traffic assignment model developed in this thesis is capable to capture greater behavioral realism. In addition, the model is computationally efficient for large-scale network applications. A variety of research directions can be continued to further extend the model. One important direction for further research is consideration of the elastic demand to improve the prediction power of the assignment model.