

## Time-dependent reliability assessment of existing concrete bridges with varying knowledge levels by proof load testing

de Vries, Rein; Lantsoght, Eva O.L.; Steenbergen, Raphaël D.J.M.; Fennis, Sonja A.A.M.

**DOI**

[10.1080/15732479.2023.2280712](https://doi.org/10.1080/15732479.2023.2280712)

**Publication date**

2023

**Document Version**

Final published version

**Published in**

Structure and Infrastructure Engineering

**Citation (APA)**

de Vries, R., Lantsoght, E. O. L., Steenbergen, R. D. J. M., & Fennis, S. A. A. M. (2023). Time-dependent reliability assessment of existing concrete bridges with varying knowledge levels by proof load testing. *Structure and Infrastructure Engineering*, 20(7-8), 1053-1067.  
<https://doi.org/10.1080/15732479.2023.2280712>

**Important note**

To cite this publication, please use the final published version (if applicable).  
Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

**Takedown policy**

Please contact us and provide details if you believe this document breaches copyrights.  
We will remove access to the work immediately and investigate your claim.

# Time-dependent reliability assessment of existing concrete bridges with varying knowledge levels by proof load testing

Rein de Vries, Eva O. L. Lantsoght, Raphaël D. J. M. Steenbergen & Sonja A. A. M. Fennis

**To cite this article:** Rein de Vries, Eva O. L. Lantsoght, Raphaël D. J. M. Steenbergen & Sonja A. A. M. Fennis (24 Nov 2023): Time-dependent reliability assessment of existing concrete bridges with varying knowledge levels by proof load testing, Structure and Infrastructure Engineering, DOI: [10.1080/15732479.2023.2280712](https://doi.org/10.1080/15732479.2023.2280712)

**To link to this article:** <https://doi.org/10.1080/15732479.2023.2280712>



© 2023 Technische Universiteit Delft.  
Published by Informa UK Limited, trading as  
Taylor & Francis Group



Published online: 24 Nov 2023.



Submit your article to this journal [↗](#)



Article views: 372



View related articles [↗](#)



View Crossmark data [↗](#)

# Time-dependent reliability assessment of existing concrete bridges with varying knowledge levels by proof load testing

Rein de Vries<sup>a,b</sup>, Eva O. L. Lantsoght<sup>a,c</sup>, Raphaël D. J. M. Steenbergen<sup>b,d</sup> and Sonja A. A. M. Fennis<sup>e</sup>

<sup>a</sup>Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands; <sup>b</sup>Reliable Structures, Netherlands Organisation for Applied Scientific Research (TNO), Delft, The Netherlands; <sup>c</sup>College of Sciences and Engineering, Universidad San Francisco de Quito, Quito, Ecuador; <sup>d</sup>Faculty of Engineering and Architecture, Ghent University, Ghent, Belgium; <sup>e</sup>Rijkswaterstaat, Ministry of Infrastructure and Water Management, Utrecht, The Netherlands

## ABSTRACT

In the evaluation of existing bridges and viaducts, relying solely on a desk study is often inadequate for determining their structural reliability. Performing a proof load test provides valuable field data that offers detailed information about the structural integrity. However, the relation between the magnitude of the load and the structural reliability is not immediately clear. This study addresses the challenges associated with determining the target load and highlights the uncertainties that play a key role. A case study is presented that shows the time-dependent character of the structural reliability and the influence of an informative and a weakly informative prior distribution in a Bayesian context. It is shown how both past traffic loads and a proof load test may contribute to the proven strength of a structure. The described method provides a starting point towards a flexible approach for proof load testing in which structure-specific knowledge levels and requirements are considered.

## ARTICLE HISTORY

Received 15 March 2023  
Revised 8 August 2023  
Accepted 12 September 2023

## KEYWORDS

Existing structures;  
incomplete knowledge; load  
testing; proof loading;  
reliability updating; time-  
dependence; traffic load

## 1. Introduction

Due to the constant aging of infrastructure, increased traffic load and traffic intensities, methods are explored by which the reliability of existing road bridges and viaducts can be assessed. In case limited information of the structure is available or its condition is of concern, load testing may be used to obtain additional information about the structure. Historically, before complex structural analysis was commonplace, load testing was regularly performed prior to opening a bridge to the public. In a number of countries performing a load test before use is still required (Lantsoght, van der Veen, de Boer, & Hordijk, 2017b).

Three types of load testing may be distinguished: a diagnostic test, a proof load test and a collapse test. A diagnostic load test is performed at moderate load levels to gain improved understanding of the distribution of forces, stiffness of materials or structural components, fixity of connections, composite action, etc. The measurements are typically used to adjust the structural (finite element) model and/or its input parameters. During a proof load test the level of load is typically much higher. The intent of this test is to prove that a bridge or viaduct can satisfactorily carry the traffic live loads. The method of load application varies from heavy vehicles to loading frames with ballast blocks. A load test in which the load is continuously increased until failure occurs is referred to as a collapse test. This type of

test is used to determine the capacity of the bridge and study the mechanisms leading to failure (Lantsoght, 2019a).

Examples of proof load tests in the USA, Denmark and the Netherlands are provided in Zarate Garnica, Lantsoght, and Yang (2022). In the Netherlands, the tests were carried out as a precursor to proof load tests demonstrating sufficient structural reliability. In these pilot tests, the loads were continually increased until collapse was established. One of the pilot tests was performed on the prestressed concrete T-beam bridge Vechtbrug (Ensink et al., 2018). In the test, use was made of a hydraulic jack installed in the loading frame to apply a concentrated load (Figure 1).

In proof load testing the magnitude of the load to be applied, or target load, is of particular importance. If the, relatively large, target load is successfully carried by the structure then it has proven to be sufficiently structurally reliable for future use. In contrast to desk studies and numerical verification methods, different uncertainties play a role during proof load testing. In addition, the condition of the structure may be of particular concern due to the effect of deterioration or other time-dependent processes (Ellingwood, 1996).

In this article, which is an extension of De Vries et al. (2022), it is examined how the structural reliability of reinforced concrete bridges and viaducts can be established by proof load testing. From the literature study, the challenges



Figure 1. Collapse test being performed in October, 2016 on the Vechtbrug in The Netherlands (De Vries, Lantsoght, Steenbergen, & Fennis, 2022).

in determining the target proof load and the associated uncertainties are highlighted. An approach to address the challenges is suggested and illustrated by a case study.

## 2. Literature review

### 2.1. International standards

Proof load testing is not a standardised assessment procedure in many countries. If national guidance is lacking, standards or guidelines from other countries can provide useful insight into accepted practices. In the USA, the Manual for Bridge Evaluation (MBE) (AASHTO, 2018) is used as a guideline for diagnostic and proof load testing. The target proof load is expressed in terms of the regular load model and is magnified by a proof load factor ( $X_p$ ). Its default value (1.4) was derived in a basic probabilistic analysis (Lichtenstein, 1993) that did not address the challenges described in this article. Suggested improvements to the probabilistic background are provided in De Vries, Lantsoght, Steenbergen, and Naaktgeboren (2023). Another relevant American standard is the ACI 437.2M (ACI, 2013a) which describes the requirements for proof load testing of existing concrete buildings including loading protocols and acceptance criteria.

Recently the German committee for reinforced concrete published a new version of its guideline for proof load tests on concrete structures (DAfStb, 2020). The guideline is intended for buildings, but refers in more general terms such to structures and components. The magnitude of the proof load is expressed in a format that resembles the load effect in Equation (6.10) of EN 1990:2019 (CEN, 2019). An interesting aspect of the guideline is the consideration of multiple similar components. It is recognised that two or more components of a structure may not be exactly the same but may be very similar. Similarity is to be expected when a component occurs multiple times and the same design applies (e.g. the floors in a building). The additional

uncertainty introduced by not testing every component is compensated by increasing the test load slightly compared to the case where only each component is tested. The increase depends on the total number of components (elements), the number of elements tested (sample size) and the coefficient of variation (COV) associated with the material that governs failure (Marx, 2019).

### 2.2. State-of-the-art

#### 2.2.1. Proof load testing

Proof load testing is still an active field of research and continues to gain attention due to the growing need for versatile assessment methods for existing structures (Lantsoght et al., 2017b). It is desirable that the assessment of infrastructure is not overly conservative because that may lead to the replacement or upgrading of bridges that are actually satisfactory. Proof load magnitudes can vary depending on the load rating, dead/live load ratios, degradation, bridge age, reference period and prior service loads (Faber, Val, & Stewart, 2000). Recent advances in measurement techniques and the treatment of proof load testing within a reliability-based decision-making context are described in Lantsoght (2019b). In Casas and Gómez (2013) proof load factors are presented that were developed as part of the large scale ARCHES (Assessment and Rehabilitation of Central European Highway Structures) project. The study presents a sophistication with respect to current code-based approaches by making use of recent traffic load data and differentiating the case where bridge documentation is available and the case where it is not.

Because the desired remaining life of existing structures is often less than the normative design life (e.g. 50–100 years), flexibility in choosing an appropriate reference period is needed (Vrouwenvelder & Scholten, 2010). The reference period holds significance in the context of structural reliability as it considers the time-dependent nature of

reliability. Using a time-dependent reliability analysis, it is possible to directly determine if the structural reliability is sufficient for the desired remaining lifespan. An early description of the time-dependence in relation to proof load testing is found in Spaethe (1994). During the proof load test the reliability of the structure is low, due to the relatively large load that is applied, but afterwards reliability increases – in case of a successful test. In more recent works by Schacht, Bolle, and Marx (2019) and Frangopol, Yang, Lantsoght, and Steenbergen (2019), the decrease of reliability with time in case of deterioration is also recognised. In the recent developments on proof load testing the link with structural reliability is recognised. However, the aspects in which the proof load testing situation is markedly different from the design situation are not sufficiently recognised and addressed. The incomplete understanding may lead to over-conservative assessment methods or potentially unsafe situations.

### 2.2.2. System reliability

The assessment of structures by proof load testing is markedly different from the conventional design procedure for new structures. In proof load testing, only the entire system's performance can be observed, whereas in the design process, verifications are typically conducted at the component level rather than the system level. Therefore, system reliability is of particular interest to proof load testing. A system may be thought to be comprised of multiple components. In this scheme, the components may act in parallel or in series. In addition, the combined performance of a group of elements may interact with one component, or another group. In the context of system failure a diagram of the interaction is called a fault tree (Fussell, 1975).

Various methods may be used to calculate the failure probability of a system. The Monte Carlo Simulation (MCS) is a straightforward method that is always applicable, but it is computationally expensive (Metropolis & Ulam, 1949). For better computational efficiency, the equivalent planes method (Roscoe, Diermanse, & Vrouwenvelder, 2015) is used in this article. The method is based on the equivalent component method (Gollwitzer & Rackwitz, 1983) and the first-order system reliability method described by Hohenbichler and Rackwitz (1982). The reliability of the individual components may be determined using the first-order reliability method (FORM) (Hasofer & Lind, 1974), the second-order reliability method (SORM) (Breitung, 1984) or any other method that also provides the influence coefficients of the random variables.

### 2.2.3. Reliability updating

Proof load testing as a means to assess the performance of a structure in relation to its structural reliability was recognised in the 1980s, with pioneering work by Grigoriu and Hall (1984), Lin and Nowak (1984), and Rackwitz and Schrupp (1985). Proof load testing is starting to be considered in the light of maintenance and durability. In particular, the so-called 'updating' of structural reliability as

performed on the basis of Bayesian theory provides the opportunity to incorporate various sources of information. The theory can provide a mathematical basis for the updated distributions of the reliability (Yuefei, Dagang, & Xueping, 2014).

The more generally applicable Bayesian decision theory is also used in the context of proof load testing. It can provide decision support and the identification of information to aid in modelling and monitoring of structures (Schmidt et al., 2020). In Bayesian decision theory, today often mentioned in the context of value of information, the state of information about a structure at a given point in time results in three possible types of analysis: prior analysis, posterior analysis and pre-posterior analysis (Zhang, Lu, Qin, Thöns, & Faber, 2021). Each stage in the analysis has its own set of possibilities ( $E$ ,  $X$ ,  $A$ ,  $\Theta$ ), dependent on earlier choices or outcomes (Figure 2).

The collecting strategy ( $E$ ) involves selecting informative observations or experiments ( $X$ ) to enhance the accuracy of the posterior analysis. Decision alternatives ( $A$ ) represent available choices prior to obtaining new information, while random outcomes ( $\Theta$ ) depict uncertain events associated with each chosen alternative. All possible paths lead to certain consequences or costs ( $C$ ), which may also include the risk of losing human life. Proof load testing may be viewed as a source of information and a pre-posterior analysis can be used to determine its value (Nishijima & Faber, 2007). A decision analytic approach was developed for reclassifying bridges using proof load testing information and pre-posterior decision analysis (Kapoor, Christensen, Schmidt, Sørensen, & Thöns, 2023).

## 3. Challenges and suggested approach

The literature at the interface between proof load testing and structural reliability has been briefly described. In relation to a full probabilistic treatment of proof load testing, it is believed that a number of aspects deserve further attention. These aspects, and their combined usage, give rise to the challenges and the suggested approach described in the following subsections.

### 3.1. Time-dependence

The time-dependent nature of structural reliability may be addressed by using an annual reliability safety format. In this way, flexibility with regard to the remaining functional life span is obtained (De Vries et al., 2022). Considering the time dependence is also beneficial in relation to the proven strength by past traffic loads. In a sense, every truck passing a bridge may be viewed as a test, contributing to the service-proven strength of the bridge (Wang, Ellingwood, & Zureick, 2011). Standard texts on reliability theory describe the concept of proven strength and degradation (or wear out) via the 'bathtub curve' of the failure probability (Smith, 2005).



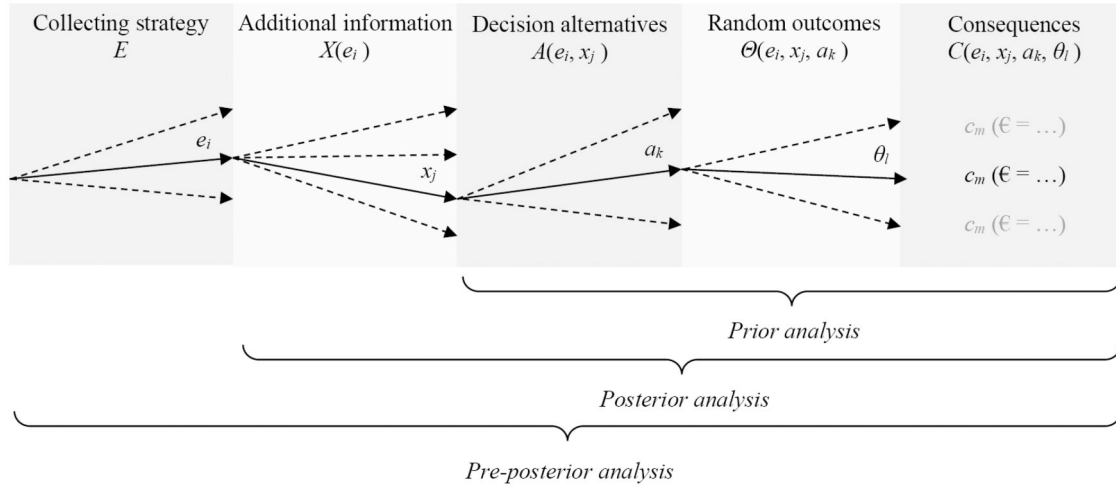


Figure 2. Analysis type depending on the state of information (De Vries et al., 2022).

### 3.2. Stop criteria and structural reliability

During the proof load test signs of distress may appear when the load is gradually increased towards the target load. To test whether distress occurs, stop criteria may be evaluated. Stop criteria typically address unwanted structural behaviour, but act on a measurable property (indicator). For example, excessive strains indicate that the reinforcement is yielding, then a stop criteria on the strains is formulated. In the German guideline for proof load testing (DAfStb, 2020) various criteria are provided. Also in the Czech Republic, Slovakia, Spain, Switzerland, Poland and Hungary guidelines with stop criteria are available (Lantsoght, Yang, Van der Veen, Hordijk, & De Boer, 2019). The effectiveness of various stop criteria for proof load tests is studied in Zarate Garnica and Lantsoght (2021). In the United States of America acceptance criteria apply and may be found in ACI 437.2 M-13 (ACI, 2013b). Acceptance criteria are used to evaluate the state of the structure or component after the test.

The link between structural reliability, after and during the load test, and the formulation of stop criteria including safety margins needs to be explored. If the safety margins of stop criteria are too stringent, the proof load test may be aborted long before the structure is actually near its maximum capacity. Theoretically, the capacity of a structure may be extrapolated from the observed behaviour (e.g. location and width of cracks, deflection, etc.) at low levels of loading. However, this approach has not been applied in practice or thoroughly studied yet.

### 3.3. Knowledge level

A flexible method is needed that can utilise various types of information. Various data sources and their influence on the state of information are collected in Figure 3. A balance should be sought between how much information is collected and analysed prior a proof load test and regarding the proof load test itself as the primary source of information (Kapoor, Schmidt, Sørensen, & Thöns, 2019). It is suggested to follow a Bayesian approach in which the state of

information plays a key role in structural reliability predictions (De Vries et al., 2022). In this approach, large uncertainties may be introduced purposely as ‘objective’ low informative priors (Ditlevsen & Vrouwenvelder, 1994). If available, other broad prior distributions following from basic information (bridge span, traffic type, etc.) may be included.

### 3.4. System-level assessment

In a system-level assessment, the performance of multiple components and spatial variability is incorporated. In addition to the physical components of a bridge, its cross-sections may also be regarded as components. By modelling the bridge as a system including correlations the reliability analysis can address the associated uncertainties directly (De Vries et al., 2022). Also here Bayesian analysis can be utilised to update the system reliability (joint PDF) with incomplete and uncertain information about a limited number of parameters (Schneider, 2020).

An example of a simplified bridge with two spans is provided in Figure 4. In this case only load and spatial variation in the longitudinal direction is considered (and not over the bridge width). The structural schematisation with a distributed load indicates three common design checks: bending moment at midspan (blue), support moment (green) and shear force near the support (orange). The corresponding cross-sections are indicated in the lower part of the figure. Because of spatially varying material properties and execution details other cross-sections may be critical. In Figure 4, these cross-sections have been drawn with the same colour, but transparently.

## 4. Methods

To address the challenges and implement the suggested approach discussed in Section 3, a number of probabilistic methods are required. The methods described in the following subsections will be utilised in the case study (Section 5).

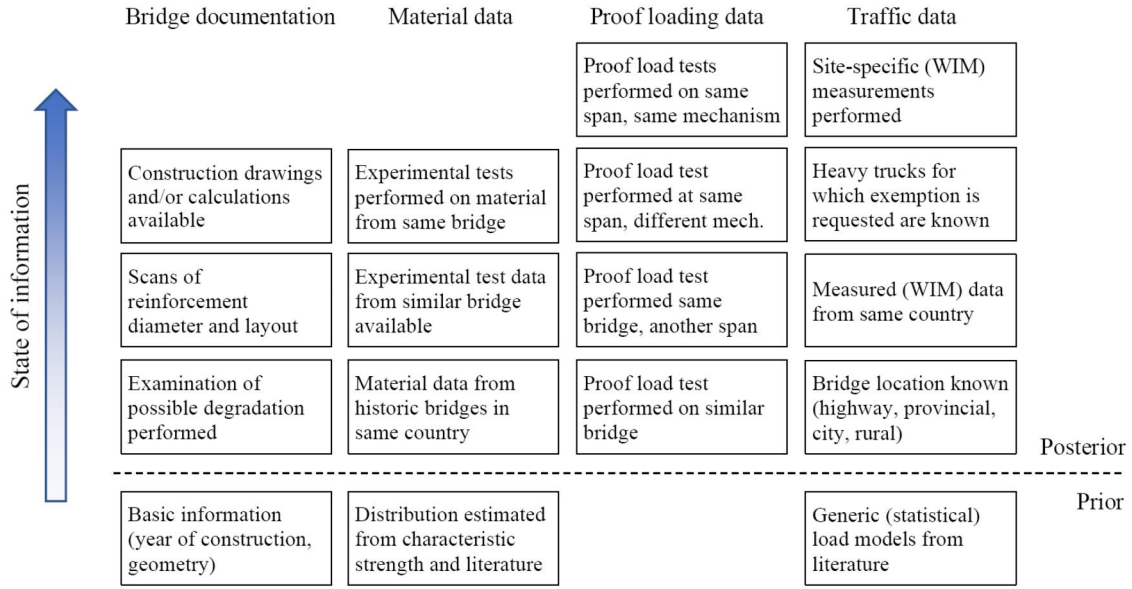


Figure 3. State of information considering various information sources (De Vries et al., 2022).

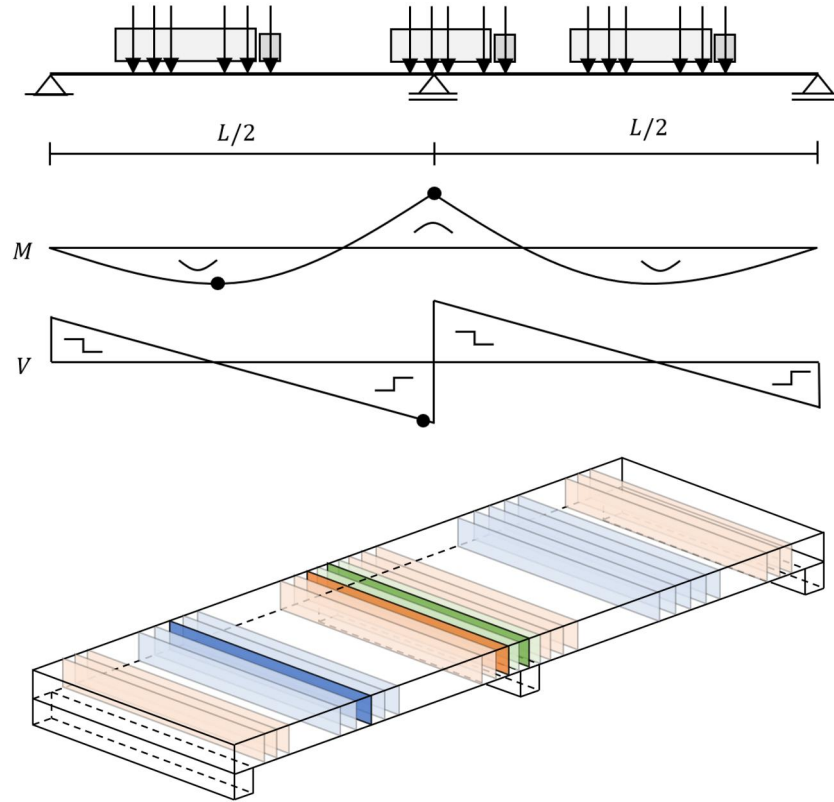


Figure 4. Visualisation of the cross-sections to be assessed in a system-level assessment (De Vries et al., 2022).

#### 4.1. System reliability

Calculating the system reliability using the equivalent planes method works on the basis of two components. The (linearised) limit state functions  $Z_i$  of two components may be written using the reliability index ( $\beta_i$ ) of the component and the influence coefficients ( $\alpha_{ij}$ ) of all random variables present in the system (Roscoe et al., 2015):

$$Z_1 = \beta_1 + \alpha_{11}U_{11} + \alpha_{12}U_{12} + \cdots + \alpha_{1n}U_{1n} \quad (1a)$$

$$Z_2 = \beta_2 + \alpha_{21}U_{21} + \alpha_{22}U_{22} + \cdots + \alpha_{2n}U_{2n} \quad (1b)$$

In this equation,  $U_{ij}$  are standard normally distributed random variables that are statistically independent (i.e. uncorrelated) within the component. However, auto-correlation  $\rho_j = \rho(U_{1j}, U_{2j})$  may exist. In case one or more random variables are used in both components, correlation

between the two limit state functions exists and it is calculated via:

$$\rho = \rho(Z_1, Z_2) = \sum_{j=1}^n \alpha_{1j} \alpha_{2j} \rho_j \quad (2)$$

Using the component correlation coefficient  $\rho$ , the limit state functions of the two components may be expressed using just two independent standard normally distributed random variables ( $U_1$  and  $U_2$ ):

$$Z_1 = \beta_1 + U_1 \quad (3a)$$

$$Z_2 = \beta_2 + \rho U_1 + \sqrt{1 - \rho^2} U_2 \quad (3b)$$

It should be noted that standard normal random variables  $U_1$  and  $U_2$  are unrelated to the random variables  $U_{ij}$  in Equation (1a) and (1b). The same holds for correlation coefficient  $\rho$  used in Equation (3) and the correlation coefficients denoted as  $\rho_j$  in Equation (2).

In case of a parallel system, failure occurs if components 1 and 2 fail. In a series system, failure occurs if component 1 or 2 fails. In the latter case the system failure probability (of only these two components)  $P_{f,or}$  is calculated via:

$$P_{f,or} = P_{f,1} + P_{f,2} - P_{f,and} \quad (4)$$

where  $P_{f,1} = P(Z_1 < 0)$ ,  $P_{f,2} = P(Z_2 < 0)$  and  $P_{f,and} = P(Z_1 < 0 \cap Z_2 < 0)$ . The last probability may be rewritten to a conditional probability such that it may be easily computed using a standard reliability method such as FORM (Hasofer & Lind, 1974).

In case of more components, the combination process needs to be repeated several times until just one component remains. Each time two components are combined to give a new component that replaces the two original components. The most accurate results are obtained when the components with the highest correlation between the limit state functions are combined first in every step (Gong & Zhou, 2017).

## 4.2. Time-dependent reliability analysis

### 4.2.1. Limit state functions

The limit state function used in a time-dependent analysis is the same as used for a regular probabilistic analysis. The main difference is in the reference period used for the variable loads. In a regular probabilistic analysis, the reference period of the load effect will be large, commonly 50–100 years, but if the time-dependence is explicitly studied it will be small. As discussed in Section 3.1, the reference period is chosen as one year, resulting in annual reliability values. Because of the time-dependence auto-correlation of the random variable becomes important. Normally only the variable loads will be uncorrelated in time.

The limit state function for the probabilistic analysis of structural failure may be formulated in terms of resistance ( $R$ ) and the load effect  $\text{fib}$  (2016). The load effect is split into the contributions from the dead load ( $G_{DL}$ ), the superimposed dead load ( $G_{SDL}$ ) and the variable load ( $Q$ ). Both the resistance and load effect are associated with model uncertainty ( $\theta_R$  and  $\theta_E$ ). Specific to the variable load is the

time-invariant part of the variability ( $C_{0Q}$ ). In addition, two random variables are added that account for the deterioration of the resistance ( $c_R$ ) and trend in traffic load ( $c_Q$ ):

$$Z = \theta_R c_R R - \theta_E (G_{DL} + G_{SDL} + c_Q C_{0Q} Q) \quad (5)$$

The limit state function is subsequently adjusted to incorporate the proof load test event. When a proof load test is performed an additional term ( $Q_{PL}$ ) is included for the proof load effect in the limit state function:

$$Z = \theta_R c_R R - \theta_E [G_{DL} + G_{SDL} + \max(c_Q C_{0Q} Q, Q_{PL})] \quad (6)$$

where the max-function is used to ensure that the regular traffic load is also considered for the year in which a proof load test is conducted. If a very low target load is used for proof load testing, it will have no effect. The adoption of the same model uncertainty for the load effect of traffic action and proof load testing is discussed in Section 6.

### 4.2.2. Conditional annual reliability

The annual reliability is calculated under the condition that no failure occurs in any of the years before the year under consideration. Using the following events:

- A failure in the year  $i$ ;
- B failure in the years 1 to  $i - 1$ ;
- B' no failure in the years 1 to  $i - 1$  (complement).

the conditional annual probability of failure can be written as:

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A \cup B) - P(B)}{1 - P(B)} \quad (7)$$

The probability  $P(A \cup B)$  may be read as the cumulative failure probability up to and including the year  $i$ , whereas  $P(B)$  is the cumulative failure probability up to, but not including, the year  $i$ . In the case of a proof load test, the failure probability (in the year) after the test will be significantly less since  $P(B)$  includes the relatively large failure probability associated with proof load testing.

To calculate the conditional annual reliability using the system reliability method, first the reliability index and influence coefficients of each year need to be calculated, e.g. using FORM. The individual years are the system components in this calculation. Next, the cumulative probability of failure can be calculated using the equivalent planes method (OR-combination). Then, the conditional probability of failure in year  $i$  is:

$$P_{f,cond,i} = \frac{P_{f,i} - P_{f,i-1}}{1 - P_{f,i-1}} \quad (8)$$

where  $P_{f,i}$  is the cumulative failure probability up to and including the year  $i$ . In the first year no conditionality holds and thus  $P_{f,cond,1} = P_{f,1}$ .

The conditional probability calculation via Equations (7) or (8), in combination with the limit state functions in Section 4.2.1, effectively performs the update of structural reliability. In this updating process all random variables that are correlated in time will be updated each year. They



include, the resistance, permanent loads, model uncertainties, the deterioration of the resistance and the trend in traffic load – but not the traffic load effect ( $Q$ ). When moving to a context in which subjective knowledge about the resistance plays a significant role, the updating process is often referred to as Bayesian updating.

### 4.3. Bayesian reliability updating

#### 4.3.1. Updating the resistance distribution

At the heart of the probabilistic treatment of proof load testing is the expectation that after a successful test, the load effect produced during a proof load test  $E_{PL}$  can be considered as a lower bound for the resistance ( $R$ ) Lin and Nowak (1984):

$$R \geq E_{PL} \quad (9)$$

Truncating the left tail of the random variable for the resistance ( $R$ ) leads to the posterior distribution for the resistance and may be expressed as:

$$f_R^*(r) = \begin{cases} \frac{f_R(r)}{1 - F_R(E_{PL})} & \text{for } r \geq E_{PL} \\ 0 & \text{for } r < E_{PL} \end{cases} \quad (10)$$

where  $f_R(\cdot)$  is the probability density function and  $F_R(\cdot)$  is the cumulative density function of the prior distribution for  $R$ . It may also be obtained via the application of Bayes' theorem together with a likelihood function providing the value 0 when  $r < E_{PL}$  and 1 otherwise. As noted by Ditlevsen and Madsen (1996), the proof loading must be made at rather high levels in order to achieve a high reliability. If  $E_{PL}$  is a deterministic value, the probabilistic calculation may be performed directly using the updated distribution of resistance  $R$ , i.e. Equation (10). However, commonly the load effect achieved within a proof load test is not precisely known and is better described using random variables (see Section 4.3.2). It should be noted that Equation (10) is not needed when the update is performed using the conditional probability calculation in Section 4.2.2.

As an assessment method for existing structures, proof load testing is typically applied when large doubts exist about the resistance of the structure. Even if drawings and original calculations are available, there may be such significant evidence of deterioration that they become irrelevant. In this context, often a weakly or low-informative prior distribution is desired (Ditlevsen & Vrouwenvelder, 1994). A reliability analysis will typically result in an unacceptably low reliability index when using a prior distribution for  $R$  with such large uncertainty. But, after a successful proof

load test the truncation, Equation (10), will lead to an increase of the reliability index. The prior distribution does not necessarily need to be a (log)normal distribution. In Kapoor, Sørensen, Ghosh, and Thøns (2021), a uniform distribution was chosen to reflect the lack of knowledge about the resistance (Figure 5).

#### 4.3.2. Limit state function

When transitioning from a context where the resistance is based on reliable information (informative) to a context with significant uncertainty surrounding this parameter (weakly informative), it is necessary to adjust the limit state function to reflect the situation. As in the time-dependent analysis, the limit function for a specific failure mechanism is considered (e.g. bending or shear). The corresponding limit state function is expressed as:

$$Z = R - \theta_E(G_{DL} + G_{SDL} + C_{0Q}Q) \quad (11)$$

where the definition of the random variables is the same as in Section 4.2. Here the time-dependent coefficients  $c_R$  and  $c_Q$  have been excluded for simplicity. In this context, no mechanical model will be used to calculate the resistance from a set of basic parameters (such as geometry and material properties). Therefore,  $\theta_R$  is not explicitly included in the limit state function, the remaining random variable  $R$  may be regarded as the resistance including any probabilistic uncertainty. As it is already known that the structure can carry the permanent loads ( $G_{DL}$  and  $G_{SDL}$ ), they may be eliminated from the limit state equation:

$$Z = (\theta_E G_{DL} + \theta_E G_{SDL} + \hat{R}) - \theta_E (G_{DL} + G_{SDL} + C_{0Q}Q) \quad (12) \\ = \hat{R} - \theta_E C_{0Q}Q$$

where  $\hat{R}$  represents the remainder of the capacity available to resist the variable traffic load. Note that this principle is similar to the rating factor used in the MBE (AASHTO, 2018).

At the moment of proof load testing, the load effect following from the traffic is replaced by the load effect achieved during the proof load test (assuming no traffic is allowed onto the bridge during the test):

$$Z_{PL} = \hat{R} - \theta_{E,PL} Q_{PL} \quad (13)$$

where  $\theta_{E,PL}$  is the model uncertainty of the load effect specific to the proof load testing situation. This model uncertainty may be smaller than the one applied in the traffic load situation because it only needs to account for a load or vehicle placed at a known location. Normally the same structural schematisation or finite element model will be used to calculate the load effect in both the regular traffic

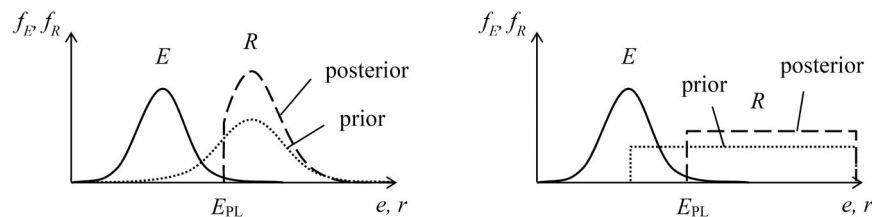


Figure 5. Schematic representation of the Bayesian update of the reliability distribution following a successful proof load test.

load and proof load testing situation. This implies a reasonably strong correlation between  $\theta_E$  and  $\theta_{E,PL}$ .

#### 4.3.3. Lower bound

Although customary, it is not necessary to treat the Bayesian (or probabilistic model) uncertainty of the resistance  $R$  in the same way as the other random variables. Instead of calculating the predictive posterior (integrating the posterior distributions), the failure probability may also be calculated using other methods such as point estimates, nested reliability analysis and confidence bounds (Der Kiureghian, 2022). To reduce complexity, specific attention can be paid to the first ‘slice’ of the posterior distribution – essentially providing the most conservative reliability estimate.

Conceptually, making this slice increasingly smaller results in the resistance being equal to the proof load effect (Figure 6). The limit state function in Equation (13) may then be written as (including model uncertainties):

$$Z = \theta_{E,PL} Q_{PL} - \theta_E C_{0Q} Q \quad (14)$$

This limit state function may also be directly obtained by assuming that the resistance is at least equal to the load effect caused by the permanent loads and the proof load ( $R \geq G + Q_{PL}$ ) as shown in De Vries et al. (2023). This assumption lies at the basis of the probabilistic background for the proof load testing method in the MBE (AASHTO, 2018) as described by Lichtenstein (1993).

## 5. Case study

### 5.1. Description

A case study was performed to explore the implications of the challenges described in Section 3 using the methods described in Section 4. The hypothetical structure under consideration is a concrete slab bridge with a relatively short span of  $L = 10$ . This type of bridge is very common in the Netherlands and also in many other countries (Christensen et al., 2022). Most of the bridges that are in service today were built during the 1960s and 1970s. Due to the continuously increasing traffic intensity and loads, bridges situated in the highway network are of primary interest in relation to their (load-carrying) capacity. If assessed via today’s Eurocode standards these bridges and viaducts often do not fulfil capacity requirements (Lantsoght, Van der Veen, De Boer, & Hordijk, 2017a).

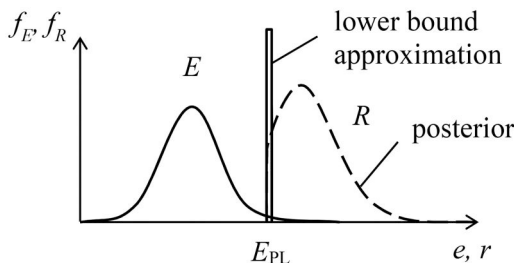


Figure 6. Schematic representation of the lower bound approximation when compared to a posterior distribution of the resistance.

## 5.2. Time-dependent analysis

### 5.2.1. Assumed design

For this example, it is assumed that the structure was built in 1960 and designed according to the prevailing standards of that time (KIVI, 1938, 1950). The traffic load used in its dated design is inappropriate when compared to today’s high traffic intensity. But, the design values of material properties (e.g. steel and concrete strength) were quite conservative. As a result, old bridges and viaducts can still possess adequate structural strength to resist today’s higher loads. In this case study, only the bending moment at mid-span will be considered. In reality, the shear capacity of the slab near the supports and the capacity of other bridge components will require assessment as well.

In case the original bridge documentation such as drawings and calculations are still available, they may be used to infer the (prior) probabilistic description of the resistance of the structure. In this case, the bridge documentation is not available. Therefore, its design was ‘reverse engineered’ by using historic standards (Harrewijn, Vergoossen, & Lantsoght, 2021). For simplicity, only the slab bridge’s right-most lane is considered, primarily used by trucks. This conservative approach does not take into account the distribution of forces that typically occurs across multiple lanes. A top view and cross-section with the inferred bottom reinforcement layout from the historic standards is provided in Figure 7 (De Vries, Lantsoght, & Steenberg, 2021).

The bending moment resistance ( $R$ ) is calculated from the balance of normal forces in the cross-section when the reinforcement yields (Figure 8). Equating the force in the concrete ( $F_c$ ) with the force in the reinforcing steel ( $F_s$ ) and solving for the location of the neutral axis ( $x$ ) gives:

$$F_c = F_s \iff \alpha_{cc} f_c b \lambda x = A_s f_y \quad (15)$$

$$x = \frac{A_s f_y}{\alpha_{cc} f_c b \lambda}$$

Using the effective depth  $d = h - a$  the moment arm is calculated as  $z = d - \lambda x / 2$ . Note that the compressive stress block reduction factor ( $\lambda$ ) disappears in the expression moment arm when neutral axis location  $x$ , Equation (15), is inserted. Finally, the moment resistance is obtained as:

$$R = F_c z = F_s z = A_s f_y \left( d - \frac{A_s f_y}{2 \alpha_{cc} f_c b} \right) \quad (16)$$

where  $A_s$  is the cross-sectional area of the reinforcement over cross-section width  $b = 3$  m,  $f_y$  is the yield strength of the reinforcement,  $f_c$  is the concrete compressive strength. The concrete compressive strength is reduced with factor  $\alpha_{cc} = 0.85$  to account for long-term effects and possible unfavourable effects from the way the load is applied.

### 5.2.2. Probabilistic model

The limit state equations in Section 4.2 are used in the following to perform the time-dependent reliability analysis. The random variables are described in Table 1, where each variable is characterised by a distribution, the mean value, the COV and the auto-correlation coefficient. The auto-

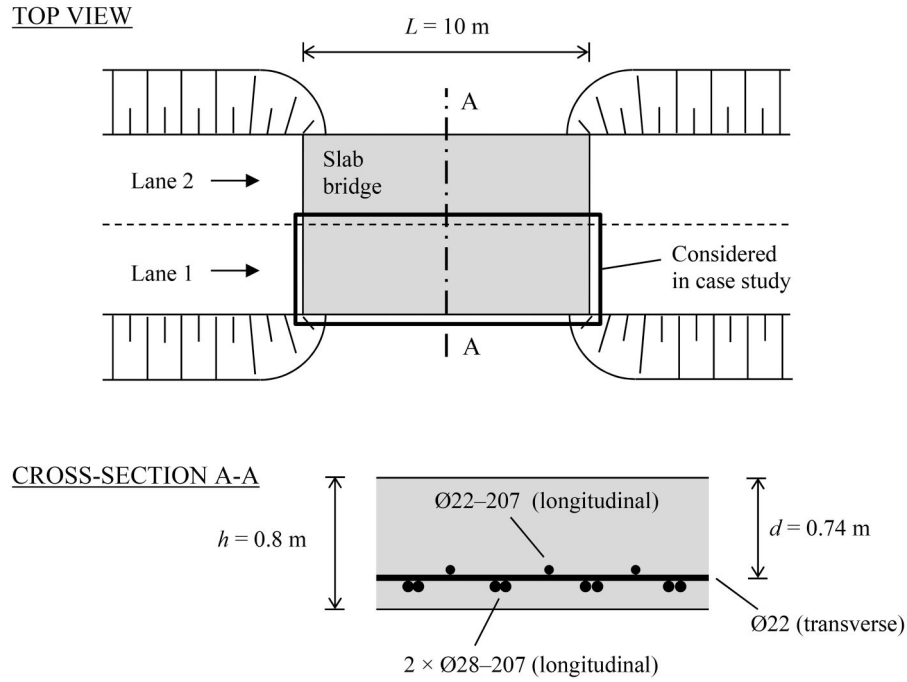


Figure 7. Layout of the reverse-engineered bottom reinforcement of the slab. Rebar size and spacing in mm.

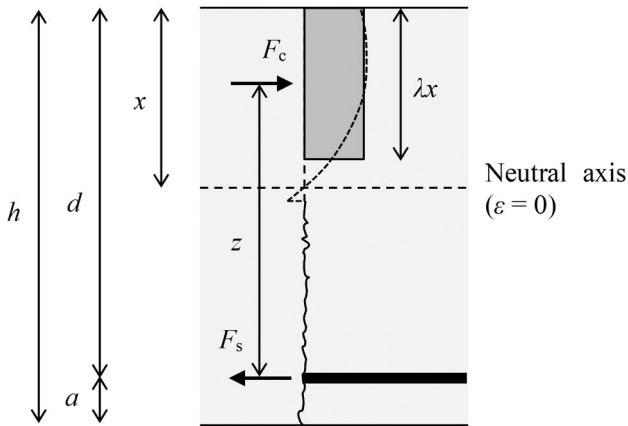


Figure 8. Calculation of moment resistance capacity.

correlation coefficient describes the correlation of the random variable in time. The chosen distribution types and parameter values are based on fib (2016) and JCSS (2015).

The now obsolete concrete type K250 (referring to a compressive strength of 250 kg/cm<sup>2</sup>) was commonly used in the Netherlands in the 1960s. The COV of 0.15, a value commonly used for concrete, is increased to 0.20 to describe the uncertainty associated with historic concrete. Back then, smooth (i.e. not ribbed) steel rebars were extensively applied. The characteristic value (5th percentile) of the QR24 reinforcing steel yield strength corresponds to 2400 kg/cm<sup>2</sup>  $\approx$  235 N/mm<sup>2</sup> when a lognormal distribution is assumed with a COV of 0.05 (RWS, 2013). A mean value of 1.1 is used for the time-independent uncertainty of the traffic load ( $C_{0Q}$ ) to include dynamic effects.

WIM data from 2015 was analysed to determine the load effect ( $Q$ ), expressed as the largest bending moment at midspan within a certain period of time. Only the traffic in the right-most lane, where the trucks drive, has been analysed.

Vehicles with a length smaller than 7 m, such as cars and small vans, have been excluded from the dataset. The combined effect of all vehicle axle loads located on the bridge is calculated via the linear superposition. Effectively a 'train' of axles is moved gradually over the bridge to record the largest moment at midspan within a certain period of time. Over a period of one year the weekly maxima have been collected for various highways in the Netherlands (A16, A27, A50 and A67). Depending on the span length, highway location and load effect considered (i.e. bending or shear), different values for the mean and COV of  $Q$  are obtained (De Vries et al., 2023). The values used for this case study represent an average of the considered highway locations.

Following fib (2016), the area of the reinforcing steel ( $A_s$ ) is not included as a random variable because its small variability. Note that the resistance model uncertainty ( $\theta_R$ ) is intended to account for any remaining uncertainty that may exist between modelling and reality. If corrosion of the reinforcement plays a significant role, the decrease of the reinforcement area should not be neglected and may also be measured to update reliability predictions (Jacinto, Neves, & Santos, 2016). Instead of modifying parameter  $A_s$ , a separate and more general deterioration parameter  $c_R$  is included here. Use is made of the following relations for the time-dependent coefficients to include the deterioration of the resistance and a trend in the traffic load:

$$c_R(t) = \begin{cases} 1 & t \leq t_{R0} \\ 1 - \Delta c_R(t - t_{R0}) & t > t_{R0} \end{cases} \quad (17a)$$

$$c_Q(t) = c_{Q0} + \Delta c_Q t \quad (17b)$$

where the parameters are random variables, listed as well in Table 1. The mean value of the parameters was chosen in such a way that the annual reliability is insufficient around the year 2020.

**Table 1.** Random variables used in the limit state function.

Var.	Description	Distribution	Mean	COV	Auto-corr.
$\theta_R$	Model uncertainty of the resistance	Lognormal	1	0.05	1
$f_c$	Concrete compressive strength (K250)	Lognormal	21.1 MPa	0.20	1
$f_y$	Reinforcement steel yield stress (QR24)	Lognormal	261 MPa	0.05	1
$h$	Height of the slab	Normal	0.8 m	0.02	1
$a$	Distance of reinforcement to surface	Gamma	0.057 m	0.17	1
$\theta_E$	Model uncertainty of the load effect	Lognormal	1	0.11	1
$G_{DL}$	Load effect of the dead load	Normal	721 kNm	0.05	1
$G_{SDL}$	Load effect of the superimposed dead load	Normal	101 kNm	0.1	1
$C_{DQ}$	Time-independent uncertainty of the variable load, including bias for dynamic load effect	Lognormal	1.1	0.1	1
$Q$	Load effect of the traffic load, annual maximum	Gumbel	1150 kNm	0.025	0
$t_{R0}$	Initiation time to deterioration	Lognormal	20 years	0.1	1
$\Delta_{cR}$	Degradation per year	Lognormal	0.0025	0.1	1
$C_{Q0}$	Starting value of the trend	Lognormal	0.78	0.1	1
$\Delta_{cQ}$	Increase of traffic load per year	Lognormal	0.004	0.1	1
$Q_{PL}$	Load effect of the proof load	Normal	1800 and 2000 kNm	0.01	1

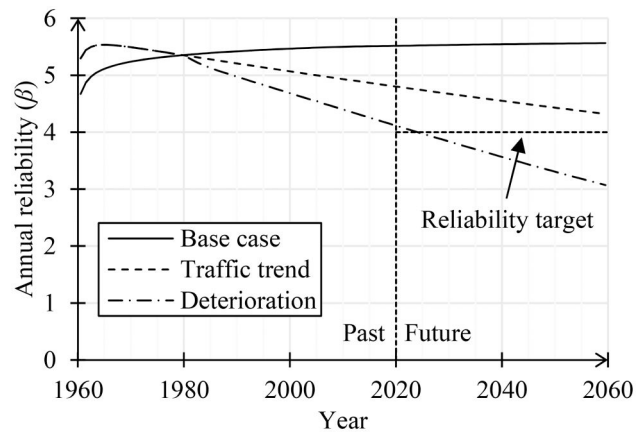
This article aims to provide methods for updating the structural reliability after a proof load test, not to provide an accurate description of resistance degradation. The chosen degradation model includes a time to initiation ( $t_{R0}$ ), followed by a linear reduction of strength (Enright & Frangopol, 1998). Corrosion leading to a reduction of the effective steel area in a cross-section was modelled by a quadratic function in Vu and Stewart (2000). In case of deterioration, a large degree of uncertainty exists with respect to the current capacity of the bridge. In this example only a limited amount of uncertainty is considered for simplicity. It thus represents the rather uncommon scenario where the deterioration process is well-known.

### 5.2.3. Results

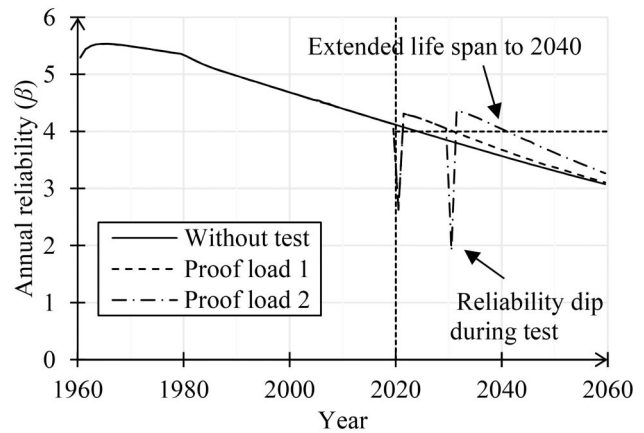
Using the presented probabilistic description, a time-dependent reliability analysis can be performed. The independent reliability analysis for each year (a component) was performed using the improved SORM approximation by Hohenbichler, Gollwitzer, Kruse, and Rackwitz (1987). This approximation provides a good balance between computational effort and accuracy in the presented case study. For the combination of components the FORM (Hasofer & Lind, 1974) is used. Comparisons with MCS indicated an acceptable difference of about a tenth in the annual reliability index. The result of the calculations is displayed in Figure 9.

The base case displays the reliability without traffic trend and degradation. In this case, the annual reliability increases gradually due to proven strength of past traffic loads. The traffic trend and degradation are incorporated subsequently to display their detrimental effect on the evolution of the annual reliability. A higher reliability is attained in the first years when including the traffic trend because the adopted linear trend expresses a reduction before 2015 and an increase afterwards.

Note that in this example the parameters of the degradation and traffic load trend have been tuned to yield a reliability index that drops below the acceptable annual reliability  $\beta = 4$  for CC3 (Steenbergen & Vrouwenvelder, 2010) around 2020. In a real-life situation, the parameters



**Figure 9.** Development of the conditional annual reliability with time, incorporating a traffic load trend and deterioration (De Vries et al., 2022).



**Figure 10.** Effect of proof load testing on the annual reliability (De Vries et al., 2022).

will need to be determined by studying the effect of all possible degradation mechanisms and the actual trend in traffic loads.

Proof load testing is adopted to ensure the bridge meets the required structural reliability. When a proof load test is performed, an additional term is included for the proof load effect in the limit state function, see Equation (6). The first proof load test is performed in the year 2020 and has a



target (mean) value of 1800 kNm. Then, in the year 2030, the second proof load test is performed with a higher target load effect of 2000 kNm (Figure 10). In the year the test is performed, the annual reliability is markedly lower, but as a reward, the reliability in the following years is higher. The low reliability indices during proof load testing are below the acceptable target reliability. Hence, further safety measures are necessary during the load test, such as closing off the area underneath the viaduct or bridge. In addition, instrumenting the structure and evaluating the stop criteria after each load cycle can avert damage and collapse. Incorporating stop criteria results in an increased reliability index during the test. But if the target load is not attained, the post-test reliability is inadequate.

The target loads have been determined in an iterative fashion such that the annual reliability remains above the target in the next 10 years. Alternatively, the higher target load could have been applied directly in 2020, also leading to sufficient reliability until 2040. But, then the probability of failure in the first test in 2020 would be larger.

### 5.3. Bayesian reliability updating

#### 5.3.1. Weakly informative prior

Instead of using a mechanical model, as adopted in the previous time-dependent analysis (Section 5.2), here a weakly informative prior distribution is used. This situation describes the other end of the spectrum; a situation in which very little is known about the resistance. The limit state function of Equation (12) is used to update  $\hat{R}$ , i.e. the remainder of the capacity available to resist the variable traffic load. To enable comparisons, the mean value of the load effect to be attained during the proof load is chosen as  $m_{Q,PL} = 1800$  kNm. This value corresponds to the initial proof load test conducted in the time-dependent analysis of Section 5.2.

The influence of the prior distribution is studied to determine if the distributions are indeed weakly informative. Three different types of prior distributions are employed to compare the reliability calculation outcomes. The parameters of the prior distributions will be chosen such that only little extra, sensible but subjective, information is included (Ditlevsen & Vrouwenvelder, 1994). Information about the traffic load will always be available because its statistical modelling is required to perform the reliability calculation once the posterior of  $\hat{R}$  is obtained. (The accuracy of the traffic load model may be represented in the time-invariant coefficient  $C_{0Q}$ .) Therefore, the mean value of the annual traffic load is used as additional information to determine the prior distribution parameters.

For the normal prior distribution the mean value is equal to the mean value of the annual traffic load effect (1150 kNm, see Table 1). In addition, a normal prior is considered with the mean value equal to 1.5 times the traffic load effect ( $1.5 \times 1150$  kNm = 1725 kNm). The factor 1.5 indicates that one expects a positive outcome – merited by the consideration of performing a proof load test in the first place. To reflect the large uncertainty, the value of the COV is

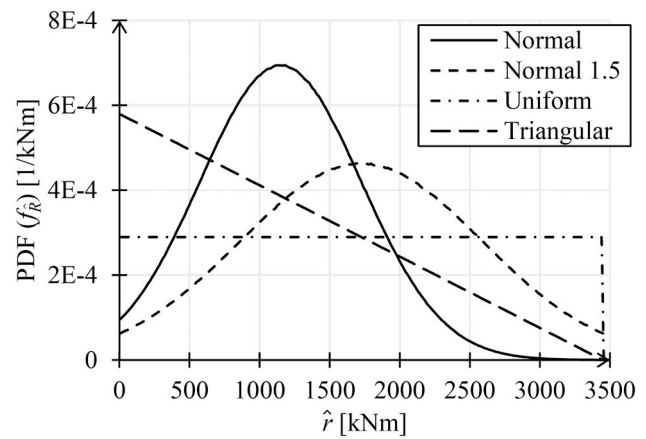


Figure 11. Prior distributions considered for the resistance.

chosen as 0.5. For the uniform prior distribution, the lower bound is not important. For the upper bound, the safety margin associated with new structures may be utilised. Referring to Table 1, the average capacity to resist live loads is  $4100 - 721 - 101 = 3278$  kNm, which is about a factor 3 higher than the average annual traffic load effect. Therefore, the upper bound parameter value is chosen as  $3 \times 1150 = 3450$  kNm. A triangular distribution, with the same bounds as the uniform distribution, is also included to (conservatively) express a stronger belief in lower resistance values (Figure 11).

#### 5.3.2. Results

The Bayesian update is performed in a MCS by removing the samples that do not survive the proof load test. The number of samples in the simulation was  $5 \cdot 10^8$ . This update procedure also accounts for the uncertainty with regard to the proof load effect – in contrast to Equation (10). For simplicity, it is assumed that the same model uncertainty holds for the regular traffic load and the proof load testing situation – in line with the time-dependent example (Section 5.2). By assuming  $\theta_E = \theta_{E,PL}$  the single remaining model uncertainty may be eliminated from the equations (i.e. its contribution is now incorporated in the resistance  $\hat{R}$ ). For each of the prior distribution types (normal, uniform and triangular) the reliability analysis is performed. The posterior distributions obtained for  $\hat{R}$  are plotted in Figure 12. The result of the calculations is expressed as the annual reliability index for the first year after a successful proof load test (Table 2).

There is slight difference between the outcomes, indicating some sensitivity to the chosen prior distribution. The smallest reliability index is obtained using a normal prior distribution with the mean value equal to the mean value of the traffic load effect – which was to be expected with this conservative prior. Additional calculations were performed in which the COV of the load  $Q$  has been increased and the shape of the right tail was varied between light (Weibull) and heavy (Fréchet). The same relative differences between the reliability indices were found, indicating no apparent sensitivity to such alterations.



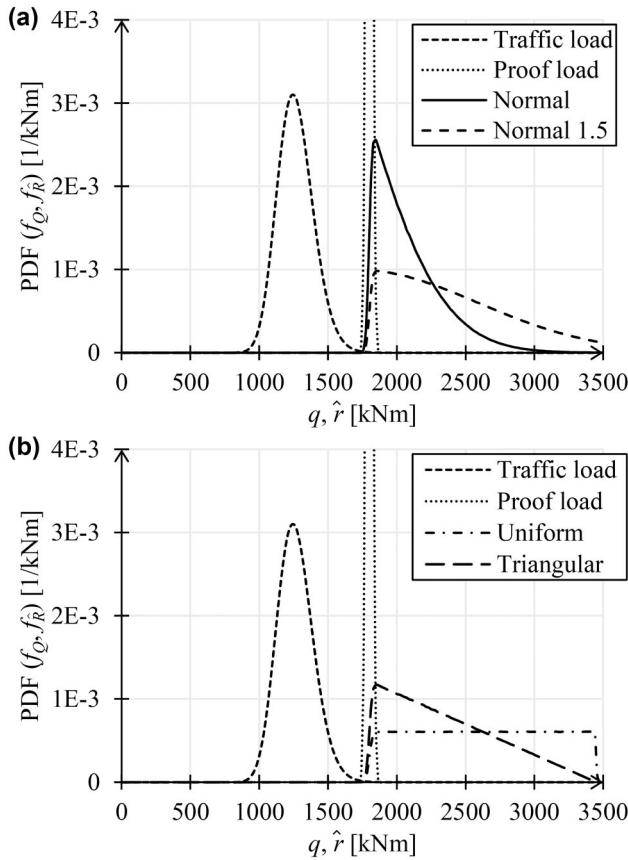


Figure 12. Traffic, proof load and resistance posterior distributions for (a) the normal priors and (b) the uniform and triangular priors.

Table 2. Results of reliability analyses with varying prior distributions.

Prior distribution	Parameter values	Annual reliability index ( $\beta$ ) [–]
Normal	$\mu = 1150$ kNm, $V = 0.5$	3.95
Normal (factor 1.5)	$\mu = 1725$ kNm, $V = 0.5$	4.17
Uniform	$a = 0$ , $b = 3450$ kNm	4.29
Triangular	$a = 0$ , $b = 3450$ kNm, $c = 0$	4.14
Lower bound	–	3.43

The result of the lower bound reliability calculation using the limit state function of Equation (14) has also been included in Table 2. The somewhat lower reliability index is explained by the conservative lower bound assumption. In this light, the values obtained using a prior distribution should be viewed as best estimates of the reliability index. Compared to the time-dependent analysis (Section 5.2), similar reliability indices are found, with the exception of the lower bound calculation. However, the two outcomes are not directly comparable since resistance degradation and the trend in traffic loads is not included.

#### 5.4. Time-dependent Bayesian analysis

The time-dependent and Bayesian analysis considered thus far may also be combined to incorporate (and visualise) time-dependent effects such as resistance deterioration and a trend in traffic loads. To express a lack of knowledge about the resistance the weak or informative normal prior distribution is used, with the mean value equal to 1.5 times the traffic load (Section 5.3.1). The time-dependent

coefficients  $c_R$  and  $c_Q$  for this case study, Equations (17a) and (17b), are inserted in the limit state functions for Bayesian analysis, Equations (12) and (13), to result in:

$$Z = c_R \hat{R} - \max(c_Q C_{0Q} Q, Q_{PL}) \quad (18)$$

where again use is made of the remainder of the resistance available to resist variable loads ( $\hat{R}$ ) and the model uncertainty assumption  $\theta_E = \theta_{E,PL}$  (Section 5.3). For completeness, an overview of the random variables used in the time-dependent Bayesian analysis is provided in Table 3. The description of random variables provided for Table 1 provided in Section 5.2.2 also applies here.

The result of the time-dependent Bayesian analysis with two proof load tests is provided in Figure 13. The previous (informative) time-dependent analysis result (Section 5.2) is included as well for comparison. The conditional annual reliability is markedly lower for the weakly informative case when compared to the informative case – especially before a proof load test is performed. This is because the informative case has high reliability from the start, following from the assumed design.

The annual reliability in the year following the first proof load test is also lower than found in the simplified Bayesian analysis, namely  $\beta = 3.67$  versus  $\beta = 4.17$  (Section 5.3). However, in the simplified analysis the resistance deterioration and the trend in traffic load were not considered. Figure 13 also shows that an annual reliability index of about 2.5 just before the year 2020 is obtained considering proven strength by traffic loads alone – a value far from the reliability target  $\beta = 4.0$ . Also here the same remarks as provided in Section 5.2.3 regarding the unacceptably low reliability during a proof load test hold. To compensate for the lack of knowledge in the weakly informative case, the value of the target load in the proof load tests may be increased. To reach the desired reliability level in the period from 2020 to 2040 the required target loads are 2050 and 2150 kNm (For the informative case 1800 and 2000 kNm were required).

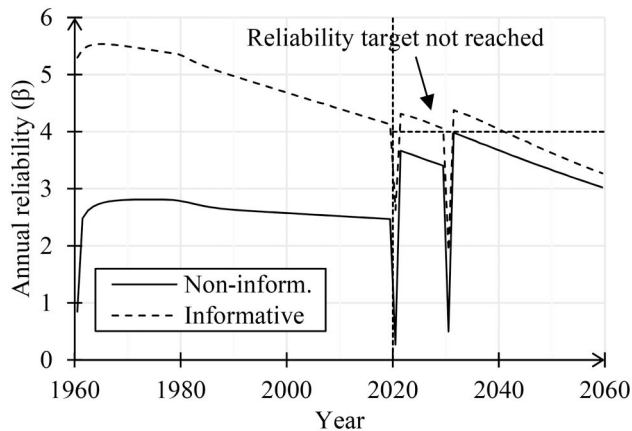
## 6. Discussion

In the literature review, international standards and the state-of-the-art on proof load testing have been briefly described. The body of literature related to proof load testing is much larger. In this article only the most relevant studies with regard to structural reliability have been highlighted and referred to. Although the various subjects identified as challenges are not new, their combined usage in the context of proof load testing deserves extra attention.

To illustrate the challenges, a case study of a hypothetical slab bridge was presented. With regard to the knowledge level, in the time-dependent example a scenario was depicted where the structural properties of the bridge, the traffic trend and the deterioration process are known to a large degree. Normally, this will not be the case. Especially the rate by which deterioration occurs will be difficult to establish. Suitable treatment of these uncertainties is critical. A clear need for stop criteria and their relation to structural

**Table 3.** Random variables used in the time-dependent Bayesian analysis.

Var.	Description	Distribution	Mean	COV	Auto-corr.
$\hat{R}$	Resistance to variable loads	Normal	1725 kNm	0.5	1
$C_{0Q}$	Time-independent uncertainty of the variable load, including bias for dynamic load effect	Lognormal	1.1	0.1	1
$Q$	Load effect of the traffic load, annual maximum	Gumbel	1150 kNm	0.025	0
$t_{R0}$	Initiation time to deterioration	Lognormal	20 years	0.1	1
$\Delta_{cR}$	Degradation per year	Lognormal	0.0025	0.1	1
$c_{Q0}$	Starting value of the trend	Lognormal	0.78	0.1	1
$\Delta_{cQ}$	Increase of traffic load per year	Lognormal	0.004	0.1	1
$Q_{PL}$	Load effect of the proof load	Normal	1800 kNm and 2000 kNm	0.01	1

**Figure 13.** Conditional annual reliability of the weakly informative and the informative time-dependent analysis.

reliability emerges from the relatively large probability of failure during the proof load test.

The mathematical form of the limit state function for the proof load testing situation is not definitive; other formulations are possible as well. In the presented formulation, the model uncertainty of the load effect ( $\theta_E$ ) acts on both the traffic load effect and the load effect produced during the proof load test. In practice, the methods to calculate both effects will likely be similar (e.g. finite element analysis), but it does not guarantee full correlation. In the same way, the choice of the weakly informative prior distribution of the resistance, as used in Bayesian reliability updating, is not absolute. The case study demonstrates that the calculated reliability only differs slightly when the distribution type changes, and the same holds for its parameters. However, slight discrepancies will persist with the weakly informative prior approach.

A future framework for proof load testing should be flexible in such a way that it addresses the needs and reflects the knowledge level. As shown in the case study, the increased knowledge level can influence the level of proof loading needed to reach a certain target reliability. The framework could also consider the method by which a proof load test is performed. For example, less spatial uncertainty remains when driving over a bridge than when a single position is loaded. In the next steps towards a flexible framework for proof load testing, stop criteria in relation to capacity predictions and spatial uncertainty will be addressed.

## 7. Conclusions

With the suggested approach to the reliability assessment of existing reinforced structures through proof load testing a

new framework can be developed that addresses existing challenges. An assessment based on annual reliability highlights the evolution of the structural reliability before, during and after the proof load test. The probabilistic methods that aid in addressing the challenges via time-dependent analysis and Bayesian updating have been described.

By adopting a flexible method, different types of information can be combined to assess the structural reliability through proof load testing. Future research should quantify how much benefit is obtained when considering various kinds of information. The case study of reinforced concrete slab bridge reveals that a significant difference exists in reliability predictions between the use of an informative and weakly informative prior distribution for the resistance. As a result, the required target loads in a proof load test are higher for the latter. Furthermore, uncertainty concerning multiple failure mechanisms and spatial variability can be addressed by judging the reliability on the system level rather than on the component level. This way, reservations regarding the assessment of shear capacity through proof load testing could be lifted.

The Bayesian reliability example considered the case in which very little is known about the resistance. Comparison of time-dependent analysis results between an informative and weakly informative prior distribution reveals that the effort devoted to obtaining an informative prior distribution leads to higher reliability estimates. This result of course depends on whether the original design was adequate and if the resistance deterioration was incorporated properly.

In addition, it was investigated to what extent the historic traffic load influences the reliability at the moment of proof load testing. If the traffic load before the year 2015 (date of measurements) is ignored, comparable outcomes are produced. This result established insensitivity towards historic traffic load (from 1960 to 2015), which is difficult to model. In the weakly informative Bayesian analysis it is found that the reliability before the proof load test is quite low, although the historic traffic load is included. It is concluded that the service-proven strength of past traffic load alone is not enough to reach the desired reliability level for a highway in the Netherlands.

## List of symbols

$a$	Distance of reinforcement to surface
$A_s$	Cross-sectional area of the reinforcement
$b$	Width of the cross-section
$c_R(t)$	Deterioration of the resistance
$c_Q(t)$	Trend in traffic load
$c_{Q0}$	Starting value of the trend

$C_{0Q}$	Time-independent variability of traffic load
$d$	Effective depth of reinforcement
$D$	Data (Bayesian context)
$e$	Realisation of the load effect
$E$	Load effect
$E_{PL}$	Target proof load effect
$f_c$	Concrete compressive strength
$f_y$	Yield strength of the reinforcement
$f_R$	Probability density function of the load effect
$\bar{f}_R$	Probability density function of the resistance
$F_c$	Force in the concrete
$F_s$	Force in the reinforcing steel
$F_R$	Cumulative probability function of the resistance
$G_{DL}$	Dead load
$G_{SDL}$	Super-imposed dead load
$h$	Height of the slab
$H$	Hypothesis (Bayesian context)
$L$	Span length
$P(\cdot)$	Probability of (operator)
$P_f$	Probability of failure
$Q$	Traffic load
$Q_{PL}$	Target proof load
$r$	Realisation of the resistance
$R$	Resistance
$\hat{R}$	Remainder of resistance available to resist variable loads
$t$	Time (year)
$t_{RO}$	Initiation time to deterioration
$U$	Standard normal random variable
$x$	Location of the neutral axis
$Z$	Limit state function
$\alpha$	Influence coefficient
$\alpha_{cc}$	Concrete compressive strength reduction factor
$\beta$	Reliability index
$\Delta_{cQ}$	Starting value of the trend
$\lambda$	Factor for height of the compression block
$\rho$	Pearson correlation coefficient
$\theta_E$	Model uncertainty of the load effect
$\theta_{E,PL}$	Model uncertainty of the target proof load effect
$\theta_R$	Model uncertainty of the resistance

## Acknowledgements

The authors wish to express their gratitude and sincere appreciation to the Dutch Ministry of Infrastructure and Water Management (Rijkswaterstaat) for financing the research work. In addition, the fruitful discussions with M. Naaktgeboren of Rijkswaterstaat and M.A.N. Hendriks of Delft University of Technology have been of great help.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## Funding

This work was supported by the Dutch Ministry of Infrastructure and Water Management (Rijkswaterstaat).

## References

- AASHTO. (2018). *The manual for bridge evaluation*. Standard (3rd ed.). Washington, DC: AASHTO.
- ACI. (2013a). *Code requirements for load testing of existing concrete structures*. Standard, ACI 437.2M. Farmington Hills, MI: American Concrete Institute.
- ACI. (2013b). *Code requirements for load testing of existing concrete structures (ACI 437.2M-13)*. Farmington Hills, MI: American Concrete Institute.
- Breitung, K. (1984). Asymptotic approximations for multinormal integrals. *Journal of Engineering Mechanics*, 110(3), 357–366. doi:10.1061/(ASCE)0733-9399(1984)110:3(357)
- Casas, J. R., & Gómez, J. D. (2013). Load rating of highway bridges by proof-loading. *KSCE Journal of Civil Engineering*, 17(3), 556–567. doi:10.1007/s12205-013-0007-8
- CEN. (2019). Eurocode 0: Basis of structural design. *Standard, EN 1990 + A1 + A1/C2:2019*. Brussels, Belgium: European Committee for Standardization.
- Christensen, C. O., Zhang, F., Zarate Garnica, G. I., Lantsoght, E. O. L., Goltermann, P., & Smith, J. W. (2022). Identification of stop criteria for large-scale laboratory slab tests using digital image correlation and acoustic emission. *Infrastructures*, 7(3), 36. doi:10.3390/infrastructures7030036
- DAFStb. (2020). *DAFStb Guideline: Load tests on concrete structures (DAFStb-Richtlinie: Belastungsversuche an Betonbauwerken)*. Berlin, Germany: Deutscher Ausschuss für Stahlbeton.
- De Vries, R., Lantsoght, E. O. L., & Steenbergen, R. D. J. M. (2021). *Case study proof loading in an annual reliability framework*. *Stevin Report 25.5-21-02*. Delft, The Netherlands: Delft University of Technology.
- De Vries, R., Lantsoght, E. O. L., Steenbergen, R. D. J. M., & Fennis, S. A. A. M. (2022). Reliability assessment of existing reinforced concrete bridges and viaducts through proof load testing. *Proceedings of the 11<sup>th</sup> International Conference on Bridge Maintenance, Safety and Management (IABMAS)*, Barcelona, Spain.
- De Vries, R., Lantsoght, E. O. L., Steenbergen, R. D. J. M., & Naaktgeboren, M. (2023). Proof load testing method by the American association of state highway and transportation officials and suggestions for improvement. *Transportation Research Record*, 2677(2), 036119812311650. doi:10.1177/03611981231165026
- Der Kiureghian, A. (2022). *Structural and system reliability*. Cambridge: Cambridge University Press.
- Ditlevsen, O., & Madsen, H. O. (1996). *Structural reliability methods*. Hoboken, NJ: John Wiley & Sons.
- Ditlevsen, O., & Vrouwenvelder, A. (1994). Objective" low informative priors for Bayesian inference from totally censored Gaussian data. *Structural Safety*, 16(3), 175–188. doi:10.1016/0167-4730(94)00020-Q
- Ellingwood, B. R. (1996). Reliability-based condition assessment and LRFD for existing structures. *Structural Safety*, 18(2–3), 67–80. doi:10.1016/0167-4730(96)00006-9
- Enright, M. P., & Frangopol, D. M. (1998). Probabilistic analysis of resistance degradation of reinforced concrete bridge beams under corrosion. *Engineering Structures*, 20(11), 960–971. doi:10.1016/S0141-0296(97)00190-9
- Ensink, S., Van der Veen, C., Hordijk, D. A., Lantsoght, E. O. L., Van der Ham, H. W. M., & De Boer, A. (2018). Full-size field test of prestressed concrete T-beam bridge. *Proceedings of the Structural Faults & Repair and European Bridge Conference*, Edinburgh.
- Faber, M. H., Val, D. V., & Stewart, M. G. (2000). Proof load testing for bridge assessment and upgrading. *Engineering Structures*, 22(12), 1677–1689. doi:10.1016/S0141-0296(99)00111-X
- fib. (2016). *Partial factor methods for existing concrete structures*. Lausanne, Switzerland: fib.
- Frangopol, D. M., Yang, D. Y., Lantsoght, E. O. L., & Steenbergen, R. D. J. M. (2019). Reliability-based analysis and lifecycle management of load tests. In E.O.L. Lantsoght (Ed.), *Load testing of bridges: Proof load testing and the future of load testing* (pp. 263–292). Boca Raton, FL: CRC Press.
- Fussel, J. B. (1975). A Review of fault tree analysis with emphasis on limitations. *IFAC Proceedings Volumes*, 8(1), 552–557.
- Gollwitzer, S., & Rackwitz, R. (1983). Equivalent components in first-order system reliability. *Reliability Engineering*, 5(2), 99–115. doi:10.1016/0143-8174(83)90024-0
- Gong, C., & Zhou, W. (2017). Improvement of equivalent component approach for reliability analyses of series systems. *Structural Safety*, 68, 65–72. doi:10.1016/j.strusafe.2017.06.001
- Grigoriu, M., & Hall, W. B. (1984). Probabilistic models for proof load testing. *Journal of Structural Engineering*, 110(2), 260–274. doi:10.1061/(ASCE)0733-9445(1984)110:2(260)



- Harrewijn, T. L., Vergoossen, R. P. H., & Lantsoght, E. O. L. (2021). Reverse engineering of existing reinforced concrete slab bridges. *IABSE Congress, Christchurch 2021: Resilient Technologies for Sustainable Infrastructure*. Christchurch, New Zealand.
- Hasofer, A. M., & Lind, N. C. (1974). Exact and invariant second-moment code format. *Journal of the Engineering Mechanics Division*, 100(1), 111–121. doi:10.1061/JMCEA3.0001848
- Hohenbichler, M., Gollwitzer, S., Kruse, W., & Rackwitz, R. (1987). New light on first- and second-order reliability methods. *Structural Safety*, 4(4), 267–284. doi:10.1016/0167-4730(87)90002-6
- Hohenbichler, M., & Rackwitz, R. (1982). First-order concepts in system reliability. *Structural Safety*, 1(3), 177–188. doi:10.1016/0167-4730(82)90024-8
- Jacinto, L., Neves, L. A. C., & Santos, L. O. (2016). Bayesian assessment of an existing bridge: A case study. *Structure and Infrastructure Engineering*, 12(1), 61–77. doi:10.1080/15732479.2014.995105
- JCSS. (2015). *Probabilistic model code*. Retrieved from <https://www.jcss-lc.org/jcss-probabilistic-model-code/>.
- Kapoor, M., Christensen, C. O., Schmidt, J. W., Sørensen, J. D., & Thöns, S. (2023). Decision analytic approach for the reclassification of concrete bridges by using elastic limit information from proof loading. *Reliability Engineering & System Safety*, 232, 109049. doi:10.1016/j.res.2022.109049
- Kapoor, M., Schmidt, J. W., Sørensen, J. D., & Thöns, S. (2019). A decision theoretic approach towards planning of proof load tests. *13th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP)*. Seoul, South Korea.
- Kapoor, M., Sørensen, J. D., Ghosh, S., & Thöns, S. (2021). Decision theoretic approach for identification of optimal proof load with sparse resistance information. *Proceedings of the 10th International Conference on Bridge Maintenance, Safety and Management (IABMAS 2020)*, Sapporo, Japan.
- KIVI. (1938). Regulations for the design and for the manufacture and erection of steel bridges (Voorschrift voor het ontwerpen en voor het vervaardigen en opstellen van stalen bruggen). *Koninklijk Instituut Van Ingenieurs, Standard, N 1008*. The Netherlands: Dutch Standard.
- KIVI. (1950). Reinforced concrete regulations (Gewapend-beton-voorschriften). *Koninklijk Instituut van Ingenieurs, Standard, N 1009*. The Netherlands: Dutch Standard.
- Lantsoght, E. O. L. (2019a). *Load testing of bridges: Current practice and diagnostic load testing*. London: CRC Press/Balkema - Taylor & Francis Group.
- Lantsoght, E. O. L. (2019b). *Load testing of bridges: Proof load testing and the future of load testing*. London: CRC Press/Balkema - Taylor & Francis Group.
- Lantsoght, E. O. L., Van der Veen, C., De Boer, A., & Hordijk, D. A. (2017a). Proof load testing of reinforced concrete slab bridges in the Netherlands. *Proceedings of the 96th TRB Annual Meeting*, Washington, DC.
- Lantsoght, E. O. L., van der Veen, C., de Boer, A., & Hordijk, D. A. (2017b). State-of-the-art on load testing of concrete bridges. *Engineering Structures*, 150, 231–241. doi:10.1016/j.engstruct.2017.07.050
- Lantsoght, E. O. L., Yang, Y., Van der Veen, C., Hordijk, D. A., & De Boer, A. (2019). Stop criteria for flexure for proof load testing of reinforced concrete structures. *Frontiers in Built Environment*, 5, 47. doi:10.3389/fbuil.2019.00047
- Lichtenstein, A. G. (1993). *Bridge rating through nondestructive load testing*. Technical Report, NCHRP Project 12-28(13)A. Washington, DC: TRB, National Research Council.
- Lin, T. S., & Nowak, A. S. (1984). Proof loading and structural reliability. *Reliability Engineering*, 8(2), 85–100. doi:10.1016/0143-8174(84)90057-X
- Marx, S. (2019). Neufassung der DAfStb-Richtlinie: Belastungsversuche an betonbauwerken. *Presentation, 7. Jahrestagung, 60. Forschungskolloquium des DAfStb*.
- Metropolis, N., & Ulam, S. (1949). The Monte Carlo method. *Journal of the American Statistical Association*, 44(247), 335–341. doi:10.1080/01621459.1949.10483310
- Nishijima, K., & Faber, M. H. (2007). Bayesian approach to proof loading of quasi-identical multi-components structural systems. *Civil Engineering and Environmental Systems*, 24(2), 111–121. doi:10.1080/10286600601159172
- Rackwitz, R., & Schrupp, K. (1985). Quality control, proof testing and structural reliability. *Structural Safety*, 2(3), 239–244. doi:10.1016/0167-4730(85)90030-X
- Roscoe, K., Diermanse, F., & Vrouwenvelder, T. (2015). System reliability with correlated components: Accuracy of the equivalent planes method. *Structural Safety*, 57, 53–64. doi:10.1016/j.strusafe.2015.07.006
- RWS. (2013). Guidelines for assessing infrastructure - Assessment of the structural safety of an existing infrastructure for renovation, use and rejection (Richtlijnen beoordeling kunstwerken - Beoordeling van de constructieve veiligheid van een bestaand kunstwerk bij verbouw, gebruik en afkeur). *Rijkswaterstaat, Report, Version 1.1, 27 May*. The Netherlands: Dutch Guideline.
- Schacht, G., Bolle, G., & Marx, S. (2019). Load testing of concrete building constructions. In E.O.L. Lantsoght (Eds.), *Load testing of bridges: Proof load testing and the future of load testing* (pp. 107–138). Boca Raton, FL: CRC Press.
- Schmidt, J. W., Thöns, S., Kapoor, M., Christensen, C. O., Englund, S., & Sørensen, J. D. (2020). Challenges related to probabilistic decision analysis for bridge testing and reclassification. *Frontiers in Built Environment*, 6, 14. doi:10.3389/fbuil.2020.00014
- Schneider, R. (2020). *Time-variant reliability of deteriorating structural systems conditional on inspection and monitoring data*. PhD Thesis, Technical University of Munich, Bundesanstalt für Materialforschung und -prüfung (BAM), München, Germany.
- Smith, D. J. (2005). Understanding terms and jargon. In D. J. Smith (Ed.), *Reliability, maintainability and risk* (7th ed., pp. 11–23). Oxford: Butterworth-Heinemann.
- Spaethe, G. (1994). The influence of load testing on the structural safety (Die Beeinflussung der Sicherheit eines Tragwerks durch Probebelastung). *Bauingenieur*, 69, 459–468.
- Steenbergen, R. D. J. M., & Vrouwenvelder, A. C. W. M. (2010). Safety philosophy for existing structures and partial factors for traffic loads on bridges. *Heron*, 55(2), 123–139.
- Vrouwenvelder, T., & Scholten, N. (2010). Assessment criteria for existing structures. *Structural Engineering International*, 20(1), 62–65. doi:10.2749/101686610791555595
- Vu, K. A. T., & Stewart, M. G. (2000). Structural reliability of concrete bridges including improved chloride-induced corrosion models. *Structural Safety*, 22(4), 313–333. doi:10.1016/S0167-4730(00)00018-7
- Wang, N., Ellingwood, B. R., & Zureick, A. H. (2011). Bridge rating using system reliability assessment. II: Improvements to bridge rating practices. *Journal of Bridge Engineering*, 16(6), 863–871. doi:10.1061/(ASCE)BE.1943-5592.0000171
- Yuefei, L., Dagang, L., & Xueping, F. (2014). Reliability updating and prediction of bridge structures based on proof loads and monitored data. *Construction and Building Materials*, 66, 795–804. doi:10.1016/j.conbuildmat.2014.06.025
- Zarate Garnica, G. I., & Lantsoght, E. O. L. (2021). Stop criteria for proof load testing of reinforced concrete structures. *Proceedings of the 2021 Session of the 13th fib International PhD Symposium in Civil Engineering*, 195–202.
- Zarate Garnica, G. I., Lantsoght, E. O. L., & Yang, Y. (2022). Monitoring structural responses during load testing of reinforced concrete bridges: A review. *Structure and Infrastructure Engineering*, 18(10–11), 1558–1580. doi:10.1080/15732479.2022.2063906
- Zhang, W. H., Lu, D. G., Qin, J., Thöns, S., & Faber, M. H. (2021). Value of information analysis in civil and infrastructure engineering: A review. *Journal of Infrastructure Preservation and Resilience*, 2(1), 16. doi:10.1186/s43065-021-00027-0