Improving the Hydrodynamic Performance of a Combined Wells-Darrieus Rotor

Master of Science Thesis by E.J. Soons

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ECOFYS

Preface

Under construction

Abstract

Under construction

Nomenclature

Variables

A_r	Blade planform area $l \times c$	(m^2)
A_S	Cross-sectional area of the streamtube	(m^2)
a	Axial induction factor	$(-)$
a	Wave amplitude	(m)
a_x	Acceleration component in x-direction	(m/s^2)
a_z	Acceleration component in z-direction	(m/s^2)
B	Number of blades	$(-)$
C_d	Two-dimensional drag coefficient	$(-)$
C_l	Two-dimensional lift coefficient	$(-)$
c	Chord length	(m)
c	Wave phase velocity	(m/s)
D	Mean depth sea bed from SWL	(m)
E	Total energy per unit crest width	(J)
E_k	Kinetic energy per unit crest width	(J)
E_p	Potential energy per unit crest width	(J)
$\overline{E}$	Average wave energy per unit surface area	(J)
F_L	Lift force	(N)
F_D	Drag force	(N)
F_x	Streamwise force	(N)
$\overline{F_x}$	Average streamwise force	(N)
F_x^*	Non-dimensinal streamwise force	$(-)$
g	Gravity constant on earth's surface	(m/s^2)
H	Wave height	(m)
k	Wave number	(rad/m)
L	Lift force	(N)
r	Radius	(m)
p	Pressure	(N/m^2)
S_ζ	Wave spectrum	$(m^2 s/rad)$
T	Thrust	(N)
T	Wave period	(s)

U	Flow velocity	(m/s)
U_1	Free stream flow velocity	(m/s)
U_2	Flow velocity at rotor disk area	(m/s)
U_{rel}	Relative flow velocity	(m/s)
u	Velocity component in x-direction	(m/s)
V	Velocity	(m/s)
v	Velocity component in y-direction	(m/s)
w	Velocity component in z-direction	(m/s)
$\widetilde{T}$	Apparent or virtual wave period	(s)
t	Time	(s)

Greek symbols

α	Blade incident flow angle	(rad)
ζ_a	Distance from the still water level to the crest or through	(m)
η	Wave surface height	(m)
λ	Wavelength	(m)
$\widetilde{\lambda}$	Apparent or virtual wave length	(m)
ρ	Density	(kg/m^3)
Φ	Velocity potential	$(-)$
φ	Angle of relative flow	(rad)
ω	Circular wave velocity	(rad/s)
ω	Angular rotor frequency	(rad/s)

Sub- & Superscripts

av	Time-averaged value in oscillating flow

Abbreviations

BBC	Bottom Boundary Condition
BEM	Blade Element Momentum
BVP	Boundary Value Problem
DSBC	Dynamic Surface Boundary Condition
IFREMER	Institut franais de recherche pour l'exploitation de la mer
KSBC	Kinematic Surface Boundary Condition
NaREC	New and Renewable Energy Centre
OWC	Oscillating Water Column
SWL	Still Water Level
TUD	Technical University Delft
UK	United Kingdom
VAWT	Vertical Axis Wind Turbine

Contents

CHAPTER 1

Introduction

The always rising world energy consumption, the increasing price of fossil fuels and the global environmental problems have resulted in new developments for alternative ways of energy generation. In this new developments tidal- and wave energy is emerging. Tides and waves form a massive reservoir of kinetic and potential energy and waves have the highest energy density among renewable energy sources.
A unique and relatively new concept, which combines the current of tides and waves, is the Wave Rotor. This design is under development by Ecofys Netherlands B.V. in cooperation with Engineering Firm Eric A. Rossen, Denmark. The design of the Wave Rotor is depicted in Figure 1.1. The concept combines two kind of rotors which are connected to the same vertical axis, a Wells rotor and a Darrieus rotor. Both rotors generate hydrodynamic lift by moving water particles over their hydrofoils, comparable with wind turbines in air. The Wells rotor is positioned horizontally in the waves and extracts energy from the vertical component of the circulating motion of water particles in waves. The Darrieus rotor is able to extract energy from the tides and the horizontal movement of water particles in waves. The main energy supplier are the waves, because the intention is to place the rotor in the most optimal wave conditions. In the Wave Rotor design (small) currents from tides will have a positive effect on the overall performance of the rotor, in contrast with most other wave energy generators.

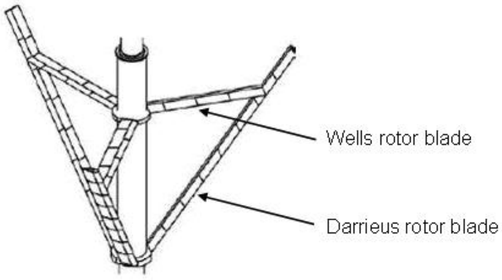

Figure 1.1: Wave Rotor.

1.1 Problem Description

Because of the combination of a turbine in wave flow in the Wave Rotor design, no standard models or tools are available to describe the hydrodynamic performance. Its is therefore hard to make design choices and to optimize the geometry and configuration of the Wave Rotor for different sea states.

Earlier scale tests proved the working principle of the Wave Rotor, but further development needs to be done by simulations. These simulations are than used to make preliminary design choices and to optimize the design. To be able to run these simulations a tool is desired which can describe the hydrodynamic performance of the Wave Rotor in a wave. This Master's thesis report explains the development of such a tool and the optimization trajectory.

1.2 Thesis Objective

The first objective of this research is to develop and validate a model which describes the hydrodynamic performance of the Wave Rotor for different geometries and configurations in different sea states.

The second objective is to use the tool to make design modifications and to optimize the rotor design. Therefore the model should meet the following requirements:

- Relatively simple and convenient model that can be understood by multiple end-users
- The tool should be built in such a way that the following input parameters are variable and free to choose: sea state, Wave Rotor geometry and its configuration, Wave Rotor position in the wave, Wave Rotors revolutions per minute.
- Acceptable calculation times for designing purposes
- Accurate description of test results
- The coding programm should be available for Ecofys

1.3 Thesis Outline

Under construction

CHAPTER 2

The Wave Rotor

2.1 Working Principle

As earlier mentioned, the Wave Rotor is a combination of two different rotors, a Wells- and a Darrieus rotor, which turn around the same vertical axis. Both rotors are lift driven, meaning that instead of drag lift is the operating force to create torque. The Wave Rotor is also an omni directional rotor, indicating that the performance of the rotor is irrespective of the direction of the upcoming wave.

To get an understanding of the working principle of the Wave Rotor a basic understanding of the flow in a water wave is required. In a water wave there is no net displacement of water particles. For a deep water wave, which is assumed in this thesis, a water particle makes a circular motion as depicted in Figure 2.1.

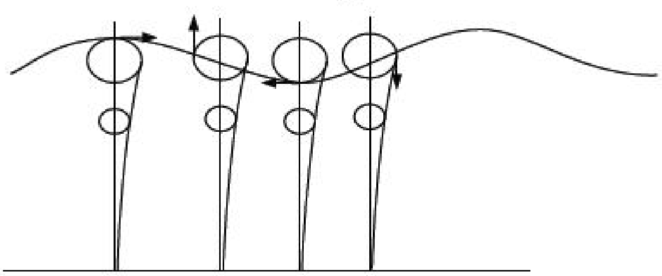

Figure 2.1: Water particle motion in a wave.[13]

The particle describes a circular path and has a vertical- and horizontal component at each point of the circle. This water particle property in a wave is used to generate lift over the hydrofoils of the Wave Rotor. The horizontal component of the water particle velocity is used to generate lift over the hydrofoils of the Darrieus rotor and the vertical component of the water particle velocity is used to generate lift over the hydrofoils of the Wells rotor.

2.1.1 Wells Rotor

The Wells rotor is a relatively new design developed for Oscillating Water Column (OWC) device. In this device energy is extracted from a reciprocal air flow, which is generated by water waves. See for more information about the Wells turbines in these application [8] or [9].

The hydrofoils of the Wells rotor are symmetrical and positioned close to the water flow as depicted in Figure 1.1. The lift force over the horizontal and thus zero pitch placed hydrofoils is perpendicular to the incident flow. As shown in Figure **??** the lift vector over the hydrofoil is always directed forward and is independent of the phase of the wave.

INSERT FIGURE

Figure 2.2: Lift vector over a Wells rotor blade.

This always forward directed lift vector will thus generate a torque around the shaft in a direction that is independent of the incoming flow.

2.1.2 Darrieus Rotor

The Darrieus rotor is a well known turbine used for Vertical Axis Wind Turbine (VAWT) applications. The obligue blades (V-shaped), shown in Figure 1.1, use the horizontal velocity of the water particles to generate lift over the hydrofoils of Darrieus rotor. A top cross-section of the Darrieus rotor blades is depicted in Figure

The direction of the torque over the shaft, generated by the lift forces, is independent of the inflow direction and thus will always be rotating in the same direction.

2.1.3 Combined Wells-Darriues Rotor in a Water Wave

The Wells rotor blades are propelled by vertical flow and the Darrieus blades by the horizontal flow. Both rotors are omnidirectional and use lift to generate a torque around a shaft.

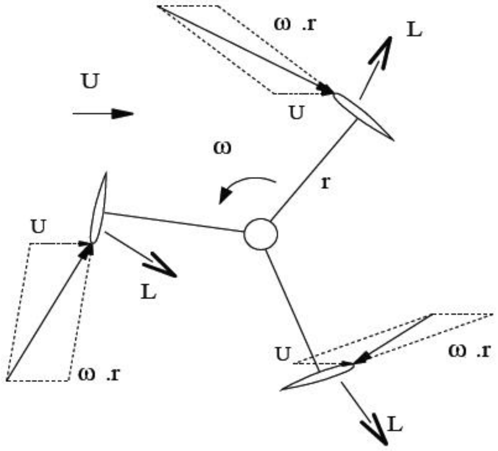

Figure 2.3: Lift vectors over a Darriues rotor.

The reciprocating flow in a water wave causes a very complex flow pattern around a rotor. As a result of earlier tests some general observations can be made [13]:

- The water particles lose speed because of the rotor blades blocking their path.
- The free surface is deformed by the presence of the rotor, because of the pressures that it induces.
- Turbulence generated by the blades does not wash downstream, but remain in the rotor area; the rotor blades pass each others wake all the time.
- The blades piercing the surface cause surface waves and spray.

2.2 History & Development

To get a better insight in thesis objectives mentioned in Section 1.2 is sensible to look at the Wave Rotors history and its development trajectory. First a concise overview of the Wave Rotors development trajectory till now is given below:

- **1998** Peter Scheijgrond and Eric Rossen meet each other at a conference and invent the Wave Rotor principle.
- **1999** An 1 meter diameter Darrieus rotor, with fixed airfoils, is tested in periodical waves.
- **2000** A vertical axis Wells rotor is tested in regular and irregular waves to optimize the overall performance.
- **2001** The power output of the 1999 Darrieus rotor is tested in periodical waves.

- **2002** The V-shaped Wave Rotor ($r = 2m$) is tested at Nissum Bredning in the Limfjord, Denmark, with wave heights up to 33 centimeter. The number of blades and pitch angle of the model could be adapted. The power was transported to the national grid. The test demonstrated that the principle of operation works well.
- **2003** The Carbon Trust Marine Energy Challenge assessed the Wave Rotor to see its potential contribution to the development of wave energy in the UK. Two analysis were executed; Current Technology Cost Analysis and Current Technology Performance Analysis. There was a large variance between the initial performance prediction, based on spreadsheet calculations, presented in the second analysis report compared to the earlier scale test results. It therefore was agreed that physical model testing had to be carried out in 2004.
- **2004** Seperate and combined Wells- and Darrieus rotor, with a diameter of 2 meter, test at the NaREC, Blyth, UK. The test showed that the individual efficiencies of the Wells- and Darrieus rotor may be added to get the Wave Rotors efficiency.
- **2005** To further develop the Wave Rotor a Hybrid Vortex Model was development by J.R. Versteegh as his final thesis project.

For further development of the Wave Rotor a simulation tool is desired, since scale tests are too cost- and time expensive. The tool, based on the Hybrid Vortex Model, can predict the performance, but is quite complicated, time expensive and not in every region as accurate. For this reason a more straightforward and less time expensive tool is desired that can be used by the Ecofys developers in the preliminary and optimization phase.

2.3 Future

To prove the concept and show that it is economically viable a prototype is planned to be built in 2008. Before this prototype building there are first new (1:5) scale tests planned at the deep water basin at IFREMER, France, in the spring of 2007. After these basin test the same model will be installed to a jetty to test its performance in currents. These tests should help to finalize the prototype design for 2008.

CHAPTER 3

Hydrodynamics of Ocean Energy Devices

3.1 Wave Energy

Wave energy is a form of ocean energy. The term ocean energy is an encompassing name for the following means of extracting energy from the vast amount of energy stored in the oceans: wave power, wind power, tidal power, marine current power and thermal energy conversion.
The Wave Rotor is meant to operate in waves and thus the focus will be on wave energy.

Waves are formed by the wind, as a result of the friction between the wind and the water. The wind transfers a part of its energy to the water. There a two kind of wind generated waves [7]:

- **Sea**
  These waves are driven by the prevailing local wind field. The lengths of the crests are 2-3 times the apparent wave length. These waves are very irregular and the apparent or virtual wave period, $\widetilde{T}$ and wave length, $\widetilde{\lambda}$ varies continuously.
- **Swell**
  These waves have propagated out of the area and local and local wind in which they were generated. These waves are independent upon the wind and can propagate for hundreds of kilometers in areas with calm winds. Here the lengths of the crests are 6-7 times the apparent wave length. Individual swell waves are more regular than sea waves and the wave height is more predictable.

In this research regular waves are considered as driving force for the Wave Rotor, meaning that swell waves are the dominant waves.
The power in a wave is proportional to the square of the amplitude and to the period of the motion. Energy fluxes of a wave are measured in kW per meter width of oncoming wave. In deep water sea waves, with long periods (10s) and large amplitudes (2m), very large energy

fluxes averaging between 50 and 70 kW per meter are common. In Figures 3.1 & 3.2 the average wave energy in the world seas and in North West Europe are depicted.

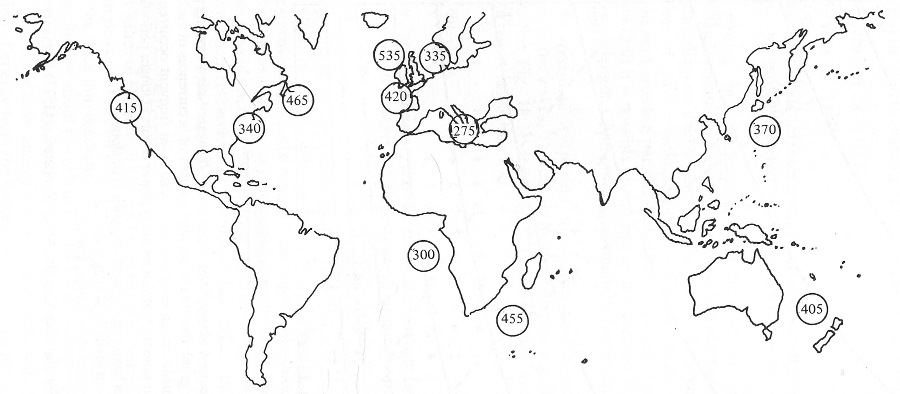

Figure 3.1: Annual average wave energy (MWh per m) in certain sea areas of the world. After NEL (1976) [5]

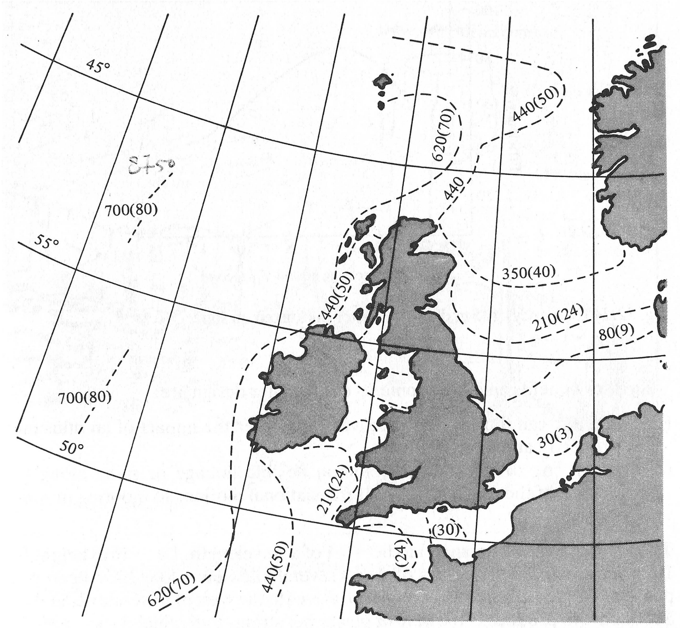

Figure 3.2: Contours of average wave energy of North West Europe. Numbers indicate annual energy in the unit of MWh, and bracketed the power intensity in the unit of kW/m. Note: local effects are not indicated [5]

Waves form a massive reservoir of kinetic and potential energy and the energy density of waves is the highest among renewable energy sources. Despite this encouraging fact the wave energy industry has not accomplished to become as economically competitive as the wind energy industry at this stage. The last 25 years a lot of R&D, testing and improved performances have caused that the wave energy industry is becoming closer to commercial exploitation. Especially in the United Kingdom and Portugal there are a lot of developments in wave energy technology. For example the first commercial wave generators, with a total rated power of 2.25 MW, are being build and will be placed this year in the seas of Portugal.

3.2 Wave flow motion

A deep water wave is the most common form of wave. The elementary theory of waves starts by considering a single regular wave. This wave is found when the mean depth of the sea bed D is more than half the wavelength λ. In Figure 3.3 the water particle motion in a wave is depicted. Figure 3.3(a) shows the motion in a deep water wave. The amplitude of the circular motion in deep water waves decreases exponentially with depth and becomes negligible for $D > \frac{\lambda}{2}$. Figure 3.3(b) shows the elliptical motion in shallow water where energy dissipation occurs with the sea bottom.

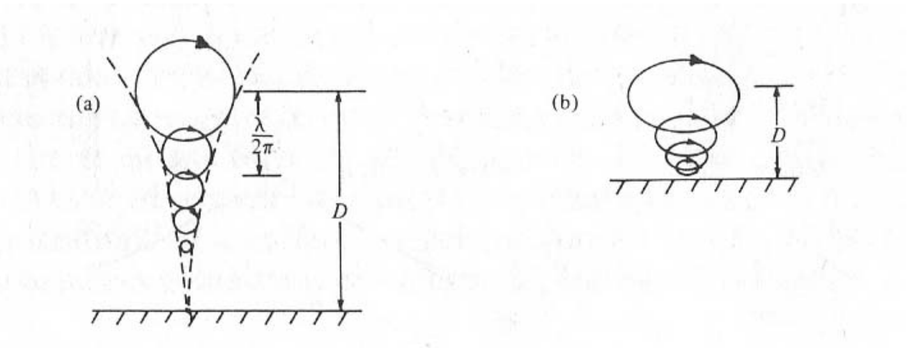

Figure 3.3: (a) circular motion of water particles in deep water, (b) elliptical motion of water particles in shallow water [5]

The two dominant forces for deep water waves are gravity and circular motion. The frictional-, surface tension- and inertial forces are all negligible. The result of these two dominant forces is that the water is shaped so that the tangent of the force is perpendicular to the resultant of the two forces.

The amplitude a of a wave is the distance from the still water level to the crest or trough, depicted in Figure 4.1. The wave height H is the vertical distance from the top of a crest to the bottom of a through, this is twice the amplitude $H = 2a$. The angular velocity of the water particles is ω (rad/s). The horizontal distance between two successive wave crests or wave troughs is called the wavelength λ (measured in the direction of wave propagation). The moving shape of a wave is the result of the phase differences in the motion of successive water particles.

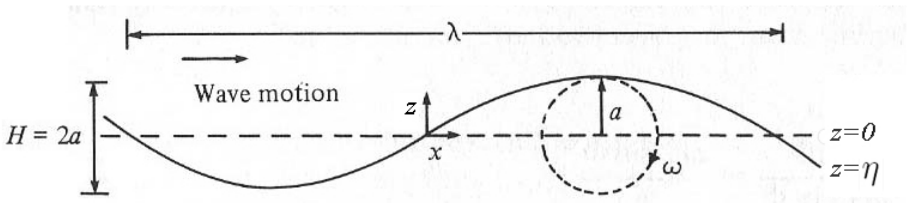

Figure 3.4: Definition of wave parameters

3.3 Regular Waves

Regular waves can be assumed as sine or cosine functions. Since sine and cosine are expressed in terms of angular arguments, the wave length and -period are converted to angles using [7]

$$k\lambda = 2\pi \qquad or \quad k = \frac{2\pi}{\lambda} \tag{3.1}$$

$$\omega T = 2\pi \qquad or \quad \omega = \frac{2\pi}{T} \tag{3.2}$$

Where k is the wave number (rad/m), which refers to the number of waves per unit distance in a series of waves in a given wavelength. And ω is the circular wave frequency (rad/s). During one wave period T the waves moves one wave length, indicating that its speed or phase velocity c is given by

$$c = \frac{\lambda}{T} = \frac{\omega}{k} \tag{3.3}$$

The form of the water, called the wave profile, can be expressed as a function of time t and position x

$$\eta = a\cos(kx - \omega t) \tag{3.4}$$

3.4 Irregular Waves

As earlier mentioned in Subsection 3.1 waves generated by local wind fields are very irregular. But they can be considered as a superposition of many simple, regular harmonic wave components, each with its own amplitude, length, period and propagation direction. This superposition principle is illustrated in Figure 3.5.

Real sea states are irregular and can be quantified in a wave energy spectrum. The first step to create such a spectrum is by decomposing the irregular surface into singular and thus regular components by making use of Fourier transformation [10]. The wave energy spectrum removes phase information by transforming the time signal to the frequency domain. These wave spectra are actually variance density spectra and are generally accepted to specify sea areas with specific weather conditions.

Two fully developed, unidirectional and applicable for deep oceanic waters are the *Pierson-Moskowitz* spectrum and the *JONSWAP* spectrum, see Figure 3.6. Fully developed sea state

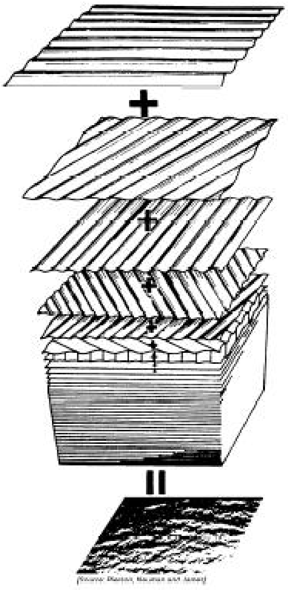

Figure 3.5: A sum of many simple sine waves makes an irregular sea [7]

means that the generation of the waves by the wind is in equilibrium with the dissipation, and the wind travels at the same velocity as the waves and thus the waves cannot get any higher. Unidirectional means that all the waves travel in the same direction.

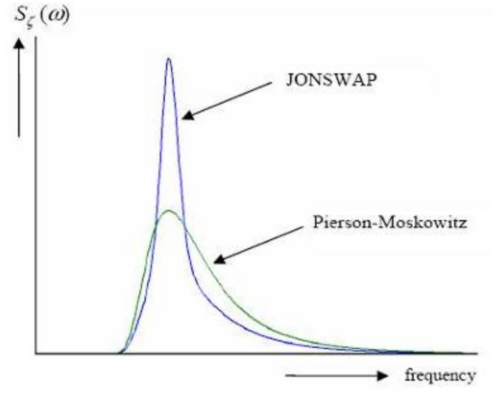

Figure 3.6: Pierson-Moskowitz spectrum and JONSWAP spectrum [4]

Irregular waves are not the main focus for this research and are thus of minor importance. For more information about irregular waves and wave spectra literature [7] can be consulted.

CHAPTER 4

Computational Model

Since the Wave Rotor is a turbine that operates in a water wave, two important choices need to be made for a suitable computational model. First of all which hydrodynamic performance model is chosen and secondly which wave theory is chosen. Two surveys have been executed to get a better insight in the available possibilities for both choices, see literature [2] & [3]. For the wave theory the decisive factor to chose for the *Small Amplitude Wave Theory* is based on the simplicity and comprehensibility of the theory and the region of applicability. For a suitable performance model there are in principle two kind of models for rotors: momentum- and wake models. Because a wake model is relative complicated compared with a momentum model, and already developed by J. Versteegh [13], the latter is chosen. Among the momentum models a *Multiple Streamtube method* is often applied in VAWT and shows fairly good results for its calculation time. For these reasons this method is chosen.

The *Small Amplitude Wave Theory* and the *Multiple Streamtube method* are both explained in more detail in separate Sections in this Chapter. The last Section explains the merge of the wave theory and the performance method in the computational model to describe the performance of the Wave Rotor.

4.1 Small Amplitude Wave Theory

The small amplitude wave theory is based on potential flow, which will be explained in the first subsection. This is followed by another important factor in the theory, which is the description of the two-dimensional surface wave. In the last two subsections the governing assumptions, equations, boundary conditions and derivation, and some main small amplitude wave characteristics will be discussed respectively.

4.1.1 Potential flow

Potential flow is flow where the velocity of the flow is the gradient of a scalar function, which is known as the velocity potential Φ. This single velocity potential describes the three unknown scalar components in a velocity field.
A potential flow field is an irrotational velocity field, implying that there is no shear stress at the air-water interface or at the bottom. Thus

$$\nabla \times V = 0 \tag{4.1}$$

Since the velocity V is the gradient of the scalar function Φ it can be written as

$$V = \nabla\Phi \qquad \text{or} \quad (u, v, w) = (\tfrac{\partial\Phi}{\partial x}, \tfrac{\partial\Phi}{\partial y}, \tfrac{\partial\Phi}{\partial z}) \tag{4.2}$$

Laplace's equation

For a potential flow the continuity equation needs to be satisfied, because the flow is assumed to be incompressible. This indicates that no mass is created or destroyed.
This means that the divergence, indicating if a vector field is originates or converges, of the gradient of V should be zero, in mathematical formulation

$$\nabla \cdot V = \nabla \cdot (\nabla\Phi) = \nabla^2\Phi = 0 \tag{4.3}$$

In three dimensions this expression can also be written as follows

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \qquad \text{or} \quad \triangle\Phi = 0 \tag{4.4}$$

This expression is called Laplace's equation and is an expression of continuity for an irrotational flow. Note that this equation is linear.

4.1.2 Two-Dimensional Surface Waves

In the small amplitude wave theory the waves are described by running sine shaped waves. The main characteristics of this kind of wave are discussed in Section 3.3. The definition of wave parameters are once more depicted in Figure 4.1

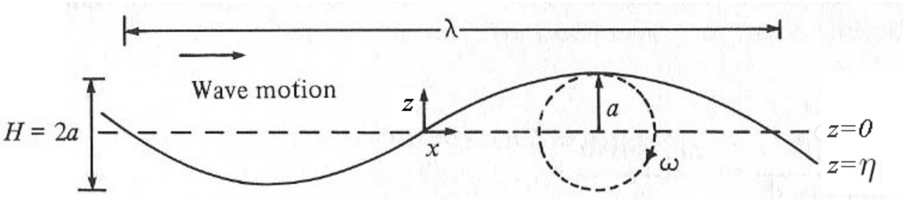

Figure 4.1: Definition of wave parameters

4.1.3 Governing Equations, Assumptions, Boundary Conditions and Derivation

The periodic velocity potential that can describe the three unknown scalar components of the velocity should satisfy the Laplace equation. The Laplace equation for two-dimensional flow, which is being considered in this research, is

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{4.5}$$

Assumptions

The assumptions made in the small amplitude wave theory are summarized below:

- The flow is continuous, incompressible and irrotational.
- Surface tension forces are negligible.
- The pressure along the air-water interface is constant
- The water surface sloped is assumed very small. This assumptions allows the free surface boundary conditions to be linearized by dropping wave heights terms which are beyond the first order.
- The sea bed, which is another boundary, is assumed horizontal, impermeable and stationary.

Boundary Conditions

There are three boundary conditions applied to this boundary value problem (BVP), two at the free surface and one at the bottom.
The bottom boundary condition (BBC) states that there is no flow normal to the bottom. At the free surface there is a dynamic boundary condition (DSBC). This condition states that the pressure at the surface of the fluid is equal to the atmospheric pressure. The second boundary at the free surface is the kinetic boundary condition (KSBC), which means that the water particle in the surface of the fluid remains in that surface. So the water surface is impervious too.

Derivation

A solution is developed for the BVP described by the Laplace equation which should satisfy the BBC, DSBC and the KSBC. Because the water surface slope is assumed very small, the two non-linear boundary conditions (DSBC & KSBC) can be linearized and apply them at the still water level instead of at the water surface. Because of this linearization the velocity potential for small amplitude waves can be derived. The velocity potential becomes

$$\Phi = \frac{gH}{2\omega} \frac{\cosh k(D+z)}{\cosh kD} \sin(kx - \omega t) \tag{4.6}$$

where g is the gravity constant on the earth's surface, H is the wave height, ω the circular wave frequency, k the wave number and D the water depth.

4.1.4 Small Amplitude Wave Characteristics

The derived velocity potential gives a complete definition of the flow field for a small amplitude progressive wave. From this definition most of the important wave characteristics can be developed. The most important characteristics for this research are the particle velocities and accelerations, the pressure field and the wave energy.

Particle velocity and acceleration

The particle velocities in the x and z direction can be found by differentiating the velocity potential to the desired direction. The horizontal- and vertical velocity can be obtained by

$$u = \frac{\partial \Phi}{\partial x} = \frac{kgH}{2\omega} \frac{\cosh k(D+z)}{\cosh kD} \cos(kx - \omega t) \tag{4.7}$$

$$w = \frac{\partial \Phi}{\partial z} = \frac{kgH}{2\omega} \frac{\sinh k(D+z)}{\cosh kD} \sin(kx - \omega t) \tag{4.8}$$

The horizontal and vertical components of acceleration have convective and local components. Convective acceleration is due to changes in flow geometry. The local acceleration is the local rate of change of the magnitude of the velocity. Both acceleration components are given by

$$a_x = u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \tag{4.9}$$

$$a_z = u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \tag{4.10}$$

The first two terms on the right are the convective acceleration terms. The magnitude of these convective terms is of the order of the wave steepness H/λ squared. Since the water surface slope is assumed very small, the convective terms are normally neglected when determining the wave acceleration. This results in

$$a_x \approx \frac{\partial u}{\partial t} = \frac{kgH}{2} \frac{\cosh k(D+z)}{\cosh kD} \sin(kx - \omega t) \tag{4.11}$$

$$a_z \approx \frac{\partial w}{\partial t} = -\frac{kgH}{2} \frac{\sinh k(D+z)}{\cosh kD} \cos(kx - \omega t) \tag{4.12}$$

When the velocities and accelerations are plotted for one wave length, as in Figure 4.2, a 90 degrees phase difference can be seen between respective velocity and acceleration terms. This phase difference can be explained by considering a water particle orbit. A water particle velocity is tangent to the orbit and the particle acceleration is at right angles to the velocity, and thus directed to the center of the orbit.

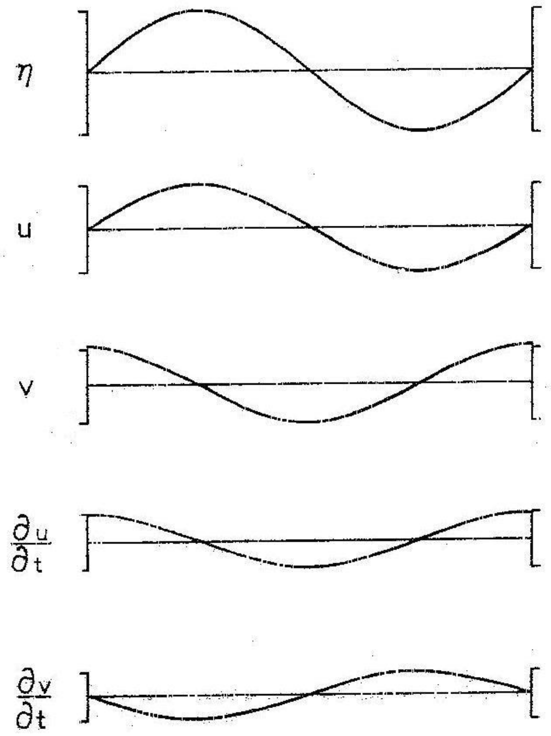

Figure 4.2: Example wave profile, -velocities & -accelerations [1]

Pressure field

The pressure field in a wave can be obtained by substituting the velocity potential into the linearized form of the unsteady Bernoulli equation. This unsteady Bernoulli equation, derived from the Euler equations, for two dimensional flows is

$$\frac{1}{2}(u^2 + w^2) + gz + \frac{p}{\rho} + \frac{\partial \Phi}{\partial t} = 0 \tag{4.13}$$

This equation is linearized because second order terms are not taken into account in the small amplitude wave theory, since it is a linear theory. The absolute values of the second order term in this equation are negligible compared to the other terms. The linearized unsteady Bernoulli equation can be rewritten to find an expression for the pressure p

$$p = -\rho g z - \rho \frac{\partial \Phi}{\partial t} = -\rho g z + \frac{\rho g H}{2} \frac{\cosh k(D+z)}{\cosh kD} \cos(kx - \omega t) \tag{4.14}$$

This expression for the pressure fields in only valid below the still water level and contains a normal hydrostatic pressure term, which increases with distance below the still water level. The second term on the right is the dynamic pressure caused by fluid particle acceleration in the wave motion. Both components are plotted for vertical sections under the wave crest and trough of a deep water wave in Figure 4.3

Downward acceleration of water particles takes place below the wave crest, see Figure 4.2. This downward acceleration results in a downward pressure gradient below a wave crest and should be added to the static pressure. Under a wave trough the opposite is the case and a upward pressure gradient that can be subtracted from the static pressure.

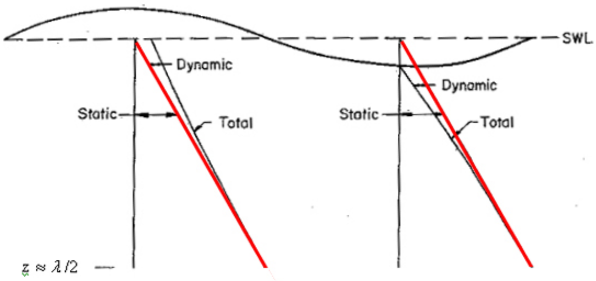

Figure 4.3: Pressure field, deep water wave [11]

Energy

The total energy in a wave is the sum of its kinetic energy and its potential energy. The kinetic energy is a result of the water particle motion and the potential energy is caused by the water surface displacement from the still water level (SWL) condition. Both energy components are expressed per unit wave of wave crest and for one wave length and result in equal expression

$$E_k = E_p = \frac{\rho g H^2 \lambda}{16} \tag{4.15}$$

The total energy E is the sum of the kinetic- and potential energy and thus becomes

$$E = \frac{\rho g H^2 \lambda}{8} \tag{4.16}$$

Within a wave length the energy content is variable, caused by the fact that the potential energy is maximal under a wave crest and minimal at the wave trough. For this reason the average energy per unit surface area, also known as the specific energy or energy density of a wave can be expressed as

$$\overline{E} = \frac{E}{(1)\lambda} = \frac{\rho g H^2}{8} \tag{4.17}$$

4.2 Single Multiple Streamtube Theory

For non-uniform flow velocities in a rotor the *single multiple streamtube theory* has been developed. This model can not predict the flow field very adequate, but does predict the performance very well, which is desired for the Wave Rotor. In this model a series of streamtubes is assumed to pass through the rotor instead of a single streamtube. For each streamtube a computation is made which results in a velocity distribution through the rotor that

is a function of two spacial coordinates perpendicular to the stream wise direction. This is thus a description for the non-uniform velocity distribution through the rotor.
The performance analysis for each singular streamtube is based on the blade element momentum (BEM) theory, which will be discussed in the first subsection. The governing assumptions, equations and derivation of the single multiple streamtube theory are explained in the second subsection.

4.2.1 Blade Element Momentum Theory

In this theory the momentum principles are applied on a small blade element. The momentum principles, derived from the *linear momentum theory*, and *blade element theory* both have an expression for the trust T. These expressions must be equal and will lead to an axial induction factor a, which is the fractional decrease in flow velocity between the free stream and the rotor plane. This induction factor can be expressed as

$$a = \frac{U_1 - U_2}{U_1} = 1 - \frac{U_2}{U_1} \tag{4.18}$$

Linear momentum theory

In the linear momentum theory a control volume is defined, see Figure 4.4, in which two boundaries are the surface of the stream tube and its two cross-sections.

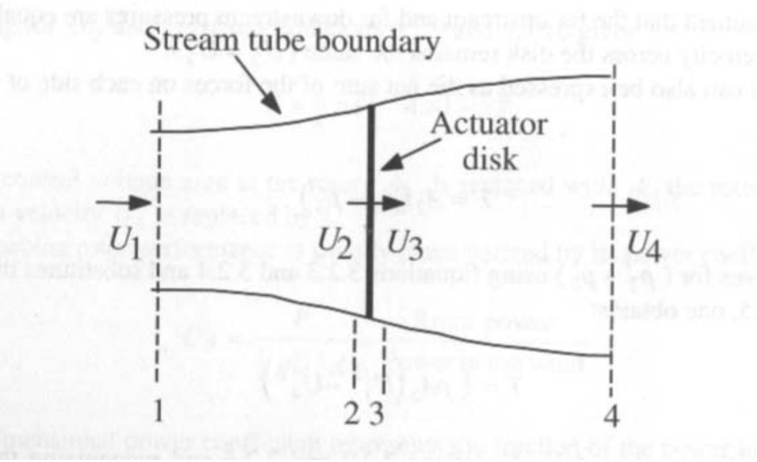

Figure 4.4: Actuator disk model of a wind turbine, U means flow velocity, 1,2,3 and 4 indicate locations [6]

The change in momentum of the flow in the control volume is equal and opposite to the thrust

$$T = U_1(\rho AU)_1 - U_4(\rho AU)_4 = \dot{m}(U_1 - U_4) \tag{4.19}$$

Blade element theory

The blade element theory analysis the forces of a blade section as a function of blade geometry.

INSERT FIGURE

The following equation, where the thrust of an element at a distance r from the center, is expressed as a function of the flow angles and airfoil characteristics

$$T = F_L \cos\varphi + F_D \sin\varphi = \frac{1}{2}\rho U_{rel}^2 B(C_l \cos\varphi + C_d \sin\varphi)cdr \tag{4.20}$$

4.2.2 Governing Assumptions, Equations and Derivation

Assumptions

To have a good understanding when the *single multiple streamtube theory* can be applied, knowledge of the assumptions is required. The following assumptions are used [6]

- Homogeneous, incompressible, steady state fluid flow
- A non-rotating wake
- The static pressure far upstream and far downstream of the rotor is equal to the undisturbed ambient static pressure
- There is no hydrodynamic interaction between elements
- The forces on the blades are determined solely by the lift and drag characteristics of the hydrofoil shape of the blades

Equations

A typical streamtube used in the *single multiple streamtube theory* is presented in Figure 4.5. The height of the streamtube is Δh and the width is $r\Delta\theta\sin\theta$, where r is the local radius and θ the azimuthal angle giving the streamtube position on the blade element flight path.

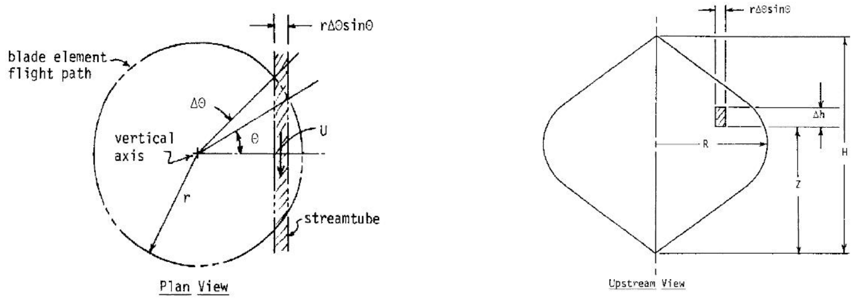

Figure 4.5: Typical Streamtube, the stream tube cross-sectional area is assumed to be constant as it passes through the rotor.[12]

Applying the *blade element momentum theory* on each blade element and making it a time averaged results in an expression for the average streamwise force $\overline{F_x}$ as they pass through a streamtube

$$\overline{F_x} = 2\rho A_S U_2 (U_1 - U_2) \tag{4.21}$$

where $A_S = \Delta h r \Delta\theta \sin\theta$, which is the cross-sectional area of the streamtube, U_1 is the free stream flow velocity and U_2 is the flow velocity at the rotor disk.

The average streamwise force can be related to the streamwise force F_x exerted by an individual blade element as it passes through a streamtube

$$\overline{F_x} = N F_x \frac{\Delta\theta}{\pi} \tag{4.22}$$

where N denotes the number of blades and each blade spends $\frac{\Delta\theta}{\pi}$ percent of time in the streamtube.
Relating to this the average stream wise momentum equation in combination with Bernoulli's equation, the following can be derived for the non-dimensional streamwise force F_X^*

$$F_X^* = \frac{N F_X}{2\pi\rho r \triangle h \sin\theta U_1^2} = \frac{U_2}{U_1}(1 - \frac{U_2}{U_1}) \tag{4.23}$$

4.3 Computer Implementation Model

4.3.1 Program Layout

4.3.2 Input and Output

CHAPTER 5

Validation

Under construction

CHAPTER 6

Optimization

Under construction

CHAPTER 7

Conclusions and Recommendations

Under construction

Definitions

Celerity	Velocity of wave propagation
Crest	Lowest point on a wave surface
Dissipation	Loss of energy over time, due to friction or turbulence
Trough	Highest point on a wave surface
Wave height	The vertical distance between a crest and the adjacent trough
Wave length	The horizontal distance between similar points on two successive waves measured in the direction of propagation
Wave period	The time required for a crest to travel a distance of one wave length
Wave steepness	H/λ

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Appendix A

A-1 Title appendix

A-2 Title appendix