WAVE RESISTANCE

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THE STATE OF THE ART

by J. N. Newman

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INTRODUCTION

The past three years have witnessed two major events in the subject area of wave resistance -- the widespread research projects aimed at direct measurement of wave resistance, and the prolific "International Seminar on Theoretical Wave-Resistance" which took place at Ann Arbor in August 1963. Interest in wave resistance has therefore been at an all-time high. However, while we have learned a great deal from this activity, much of our new knowledge is destructive, insofar as it suggests more-and-more the importance of interactions between viscous and wavemaking phenomena. Indeed, we have seen (Wu, 1963) a theoretical prediction of the wave resistance of a flat plate, moving in its own plane. Under these circumstances it may seem especially inappropriate for us to have divided the subject of resistance into separate reports on wavemaking and viscous resistance, but this has been dictated by practical considerations and by the increased level of activity which must be covered.

Responsibility for the preparation of this report was given to Panel H-5 (Analytical Ship-Wave Relations) of the Society of Naval Architects and Marine Engineers. Current membership of this panel is as follows:

J. N. Newman, Chairman	R. C. MacCamy
M. A. Abkowitz	F. C. Michelsen
J. P. Breslin	H. L. Pond
W. E. Cummins	E. O. Tuck
P. Kaplan	L. W. Ward
B. V. Korvin-Kroukovsky	J. V. Wehausen
J. Kotik	T. Y. Wu

The author acknowledges the assistance of this group in preparing the present report, but at the same time claims sole responsibility for its omissions, errors, and statements of opinion.

ANALYTICAL PREDICTIONS OF WAVE RESISTANCE

Michell's Integral

There is recent evidence to support the classical Michell theory, especially for fine ship forms at high Froude numbers, say above .35 or beyond the last "hollow" of the theoretical wave resistance curve. The best example of this is shown in the recent paper by Lackenby (1965), based upon the very extensive research conducted by Shearer and Cross (1965), but that work is based upon a very fine hull form. A similar comparison for the Series 60 (.60 block) model has been made by Webster at Berkeley, and the results are shown by Ward (1964); here the Michell resistance is higher than the residual resistance by a nearly constant difference, with the relative error decreasing, as the resistance rises, from about 30% at .35 to 10% at the peak value near .5 Froude number. Graff, Kracht, and Weinblum (1964) compared the theoretical wave resistance and measured residual resistance of two Taylor-Series models, with similar results; in fact the discrepancy between theory and experiments was no larger than the discrepancy between their 3 meter and 6 meter model results.

Michell's integral becomes more reliable, especially in the lower speed range, when empirical corrections are made. Inui (1962) and Wigley (1962) have both shown good agreement between theory and experiments in this way. Wigley requires only two empirical constants, which are claimed to represent viscous interaction effects. It remains to be shown that these constants can be estimated satisfactorily, for a given hull form, before making the experiments.

Computer Studies and Second-Order Theories

In this age of large scale computers it may seem inexcusable that we are not able to make substantially better predictions of wave resistance than were the courageous workers of the first half of this century. This lack of progress must be attributed jointly to viscous effects and non-linear free surface effects, neither of which can be completely analysed, to say nothing of the combination of the two simultaneously.

Restricting ourselves to non-viscous fluids, some efforts have been made to improve upon Michell's integral or to free ourselves from the limitations of thin ship theory. Here we must distinguish between a consistent mathematical approach (usually attributed to Peters and Stoker, cf. Stoker 1963) and a more expedient engineering approach. In Michell's classical theory two approximations are made, viz., the free surface waves are small in amplitude so that linearized water wave theory can be applied, and the ship is thin so that singularities and boundary conditions can be moved from the ship's surface to the centerline plane. From the mathematician's viewpoint these two approximations are mutually dependent and it is not legitimate to improve upon one (usually the assumption that the ship is thin) while retaining the other. From the engineering viewpoint, on the other hand, the more important assumption is that pertaining to the hull; typical ships are not thin, but they do make waves which for the most part are not too steep. This controversy is now about ten years old, and neither side shows signs of weakening. Clearly if the engineering approach can be satisfactorily computerized, and if this gives agreement with practical experience, then it will certainly be used. (The mathematician regards this as permissible if it is recognized as a "numerical experiment.")

The ultimate goal in the engineering approach is to satisfy the exact boundary condition on the ship hull and the linearized free surface condition. In principle this is possible, and at least two major efforts have been underway for some time, the first by Hershey at the Naval Weapons

Laboratory and the second by Hess and Smith at the Douglas Aircraft Co. Neither of these efforts shows immediate signs of success. In practice it has not been possible to compute the free-surface source function with sufficient speed to use it as the kernel function of the appropriate integral equation. Breslin and Eng (1963) attempted to short-cut this difficulty by using the source strength for the ship with a rigid free surface and then computing the wave resistance of the free-surface source distribution with the same strength*; the results were disappointing in that they exaggerated the wave resistance even more than Michell's integral. On the other hand essentially the same principles have been successfully applied by Inui and his colleagues in Japan, and Pien (1963, 1964) at DTMB, to the problem of finding optimum ship forms with low wave resistance. One might conclude that the engineering approach described above will be useful for certain purposes, such as the determination of optimum forms, but not for the actual prediction of wave resistance on a given ship.

Progress can in fact be made from the mathematical viewpoint, proceeding iteratively to correct both the hull and free surface conditions simultaneously. The second-order corrections to Michell's integral were outlined ten years ago by Wehausen, and presented to the H-5 Panel, but the results were considered at that time to be too complicated and of too little interest to publish. Subsequently Sisov (1961) published the second-order equations and these are now being programmed by Eggers at Hamburg. For the idealized case of a two-dimensional circular cylinder Tuck (1965) has

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* It should be emphasized that the resulting potential is a hybrid which does not satisfy the hull boundary condition except in the limit of zero speed. Moreover the wave resistance thus obtained is not unique insofar as it depends on the manner inwhich the ship is represented with the rigid free surface condition; different values of the wave resistance would obtain from, e.g., a surface distribution of normal dipoles, or an internal distribution of sources.

presented the complete second-order theory including calculations, and has shown that the second-order effects from the free surface dominate these from the body surface; if this conclusion were valid also for ship forms it would effectively refute the engineering approach.

Slender-Body Theory

Michell's integral, the cornerstone of wave-resistance theory, is limited to ships which are geometrically thin, or of small beam compared to their length and draft. Since ships are typically "slender" but not "thin", there was hope for many years that a wave-resistance theory analogous to the slender-body theory of aerodynamics might provide engineering estimates of practical ships' wave resistance. Sadly, this hope has proved to be in vain. A theory for the wave resistance of slender ships was obtained independently by several workers in 1962-3, including as the final result an integral analogous to Michell's integral. Unfortunately this integral can be looked upon as a special case of Michell's integral, for small draft; optimistically we might say that this proves that in Michell's integral the beam/draft ratio need no longer be small, but the fact remains that the new slender-body result is of no greater practical value than Michell's integral. In fact Lewison (1963) has shown that the agreement of experiments with the slender-body integral is worse than with Michell's integral.

Nevertheless work in this area has not been given up entirely. Tuck (1965) has derived the second-order contribution to the wave resistance of a slender ship and this may prove to be practically useful. Secondly, there remains the possibility of a "short-wavelength" or low-speed slender-body theory which should be more accurate than any existing theories in the low Froude number range. Finally we may note that in work which is as yet unpublished, Tuck has successfully used the slender-body theory to predict sinkage and trim in shallow water; an outline is presented as discussion to the paper by Graff, Kracht, and Weinblum (1964).

Boundary-Layer Interaction with Surface Waves

For suitably high Reynolds numbers the effects of viscosity can be concentrated inside a thin boundary layer along the ship's surface and the fluid outside of this layer can be analysed as a potential flow. Since the characteristics of the boundary layer depend not only on the body shape, but also on the pressure gradient of the external potential flow, and, conversely, the displacement thickness of the boundary layer will affect the external flow, it follows that there will be a two-way interaction between the boundary layer and the free surface wave field. This seemingly intractable problem has been studied by Wu (1963), using two-dimensional semi-empirical methods to approximate the boundary layer motion and thin-ship methods to analyse the exterior flow. As a consequence of the interaction, the results appear formally as an integral equation for the boundary-layer thickness. The assumptions involved are rather drastic (notably, two-dimensional boundary-layer flow, no separation, and no treatment of the wake) but a start has been made towards a rational understanding of the limitations of Froude's hypothesis.

EXPERIMENTAL MEASUREMENTS OF WAVE RESISTANCE

Since viscous drag is the dominant portion of most ships' resistance, it is somewhat unfortunate that we use model tests to determine the wave resistance and assume that we can predict the viscous resistance, rather than vice-versa; this complaint is not new, but it emphasizes the fact that in measurements or predictions of wave resistance we are looking for the needle in the haystack.

In order to measure wave resistance directly and thus avoid the possible errors inherent in equating wave resistance and residual resistance, various schemes have been proposed which have in common the measurement and analysis of the energy in the wave system generated by the ship. In a real fluid this is not a unique quantity, since there will always be a certain amount of interchange of energy between the viscous and wave wakes, but at least outside the (relatively narrow) region of the viscous wake itself, it would seem reasonable to neglect viscous dissipation.

Inui must be credited with instigating most of this research, when he addressed the H-5 Panel in July 1960 and showed his beautiful stereo photographs of model wave systems. This led Professor Korvin-Kroukovsky to suggest the use of a wave probe to measure wave heights along a line across the wake, in the hopes that this could be related to the wave resistance. Eggers and Ward both took up this idea and others entered subsequently. Two variations are common: the "transverse cut," which gives a wave record across the wake, beginning and ending outside the region of significant wave heights (in principle, outside the angle 19½ degrees) or, in a towing tank traversing the tank from wall-to-wall or centerline to wall; and the "longitudinal cut" which gives a record of wave height along a line parallel to the ship's course, beginning upstream of the wave system and terminating downstream either when the wave height has diminished to zero or, more practically, when the wave has become a fairly regular transverse wave system.

The longitudinal cut has the advantage that the wave probe can be fixed while the model passes by, whereas for the transverse cut a moving probe or multi-probe rake must be used. A second advantage claimed for the longitudinal cut (to some extent with tongue-in-cheek) is the fact that it can be kept outside of the viscous wake, whereas the transverse cut must cross the viscous wake and presumably is thereby more subject to viscous interactions.* On the other hand, the longitudinal cut must be very long downstream, and in conventional (narrow) towing tanks this forces a compromise between a short record which is free of wall reflections and a long record which has "converged." Some analyses, notably Eggers (1962), account for wall reflections from perfectly reflecting walls of the tank, but this makes the convergence problem worse (since the resulting waves propagate downstream without attenuation) and introduces another source of viscous interaction (in the multiple reflection process at the wall).

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* In order to better understand this interaction process Lurye (TRG) is currently conducting theoretical research on the propagation of plane waves through a viscous wake. Preliminary results with an idealized wake show that an extra term must be added to the equations relating the wave resistance to the transverse cut wave record. Lurye is also investigating the possible effect of the wake on the wave resistance itself.

Ward (Webb Institute)

While Ward has developed other methods as well, his forte is the "XY Method" (Ward, 1964). In this method the wave is "measured" by a stationary vertical circular cylinder which passes down through the free surface to a large depth; as the waves pass by the cylinder they give rise to a horizontal hydrodynamic force on the cylinder, which is measurable. If X(t) and Y(t) are the longitudinal and transverse components of this force, as functions of time, then it can be shown that the wave resistance of the model, as it passes by the cylinder, is proportional (through a known simple constant involving the fluid density and the cylinder radius) to the integral, with respect to time, of the product of these two forces. This method had been used to measure the wave resistance of five foot models of the Series 60 (.60 block), A.T.T.C. Standard Model, and an "optimum symmetric ship" developed at Berkeley. These tests covered the Froude number range from .2 to .5 . In all cases the measured wave resistance is substantially lower (20 - 50%) than the residual resistance, and in most cases it is lower than the predicted Michell resistance. The results are encouraging in regard to the optimum symmetric ship, which has a very low theoretical minimum wave resistance at a Froude number of .316; no indication of this minimum is apparent from the residual resistance, but the XY test shows a very low wave resistance (although not so low as the theory) in the range .29 to .32 . But unfortunately at a Froude number of .4, where the residual and Michell resistances are nearly equal (and thus, presumably, reliable) the XY test is again about 50% low.

Eggers and Sharma (Institut für Schiffbau)

Eggers (1962, 1963) has performed an elaborate analysis of the wave system behind a model which is moving with constant velocity down the center of a tank. The tank is assumed to have a rectangular cross section with perfectly reflecting walls. Eggers shows how this wave system may be measured and related to the wave resistance through an infinite series over all of the "free wave" components of the system. Sharma (1963, 1964) has extended this theory and has applied it in an exceedingly thorough manner, including extensive theoretical and experimental comparisons of the wave resistance of one model 4m long. The results are similar to Ward's at lower Froude numbers, but in the range .35 to .7 there is good agreement between residual and measured wave resistance, and fair agreement of both with the theory. Sharma also shows that the results at low speeds can be dramatically improved through the use of Inui's (1957) semi-empirical correction factors, but this comparison is less meaningful since the correction factors involve a three- or four-parameter fit which should allow sufficient flexibility to improve any comparison dramatically.

Pien and Moore (DTMB)

Pien and Moore (1963) measured the wave height along two longitudinal tracks, parallel to the model's course, and used these measurements to compute the wave resistance. Measurements were made with a sonic wave probe, the tests being carried out with an Inui U-series model in the Maneuvering and Seakeeping Basin at DTMB, in order to avoid wall reflections. The analysis technique employed Fourier analysis to first find the spectrum of the free waves, before integrating to get the wave resistance. The values obtained in this way were approximately twice as large as the residuary resistance, based upon the "adjusted" (plus 14% form factor) Schoenherr friction line.

Kobus (Iowa Institute of Hydraulic Research)

In a recently published thesis Kobus (1965) has made an extensive investigation of the use of Eggers' method (based upon transverse wave profile acrosss the tank) in conjunction with a vertical strut of modified ogive section and beam/length ratio of about .10. This form was chosen because of its narrow viscous wake and freedom from bilge vortices. (An end plate was placed at the bottom to minimize three-dimensional effects.) Tests were restricted to a Froude number of 0.36, but at this one speed various comparisons were made:

(1) The measured and computed wave heights along transverse cuts at various distances downstream: the agreement was not very good

at any distance downstream, and bad in the viscous wake; relatively small differences were also demonstrated between the "exact" (including local effects) and asymptotic (i.e. "free wave") theoretical predictions.

(2) The wave resistance as predicted from theory (Michell's integral but with the exact zero-Froude-number source distribution) with (a) Eggers' method using theoretical values of the wave height, and (b) Eggers' method using experimental values of the wave height; the results were, respectively, 20% and 40-50% lower than Michell's integral and this error was not sensitive to distance downstream.

Kobus concludes that viscous interactions are important, and moreover that local effects persist downstream to an extent which must be accounted for in any successful wave analysis. (In regard to the persistence of local effects downstream it should be noted that the tank was only 10' wide, and the model 6' long.)

Taniguchi (Mitsubishi - Nagasaki)

In recent tests Taniguchi has used three geosim models of a tanker, ranging in length from 4.2m to 10m, to determine the wave resistance (obtained by considering the total resistance as a function of the flat plate viscous resistance and extrapolating back to zero, with Froude number constant). Measurements were made of the wave height along a longitudinal cut, these were fitted by least squares to a Fourier series, and the wave resistance computed therefrom. Comparison was made in the range of Froude numbers between .15 and .22, and the wave resistance obtained from wave analysis was generally about 60% of the wave resistance obtained from geosim analysis.

Gadd and Hogben (N.P.L.)

Gadd and Hogben (1963) have been developing techniques for measuring both the viscous and wave resistance, by momentum surveys and wave height measurements respectively. These have been applied to one large (20') and one small (4') mathematical model, the larger of the two being the same model as tested by Shearer (1965). Several longitudinal wave records are taken and a least squares analysis used to give the wave resistance. Early results (1963) at Froude numbers between .24 and .35 gave values of the measured wave resistance which were below the residual reristance by about 40% , but more recent data which covers eight Froude numbers between .24 and .5 , for the 20' model, shows excellent agreement (± 10%) with Shearer's residual resistance throughout the whole range.

Gadd and Hogben (1963) have also applied their wave analysis technique to the drag of a circular hovercraft at a Froude number of .55 , and found good agreement both with directly measured drag and with theoretical calculations. (Note that in this case viscous effects should be nil, and moreover the Froude number is quite high.)

Summary

It will be seen, from the above abstracts of the individual investigations in this field, that the results fall primarily into two groups: (1) the "negative" group, in which the measured wave resistance differs substantially from, and is usually less than, the residual resistance and the theoretical predictions from Michell's integral; and (2) the "positive" investigations of Sharma in Germany and Gadd and Hogben at N.P.L. wherein, especially at higher Froude numbers, there is substantial agreement between the three values of resistance. (In this context, the overall accuracy of Michell's integral does not concern us so much as the fact that in those instances where it does agree with the residual resistance, added validity must be attached to the residual resistance, as opposed to some different measured wave resistance.)

While firm conclusions would be premature, two items may be noted. Firstly that the method is more likely to be accurate at high Froude numbers than at low ones (cf. Sharma, where the absolute error is essentially constant, or Pien, where the error is of the same order of magnitude as the form factor contribution to the viscous resistance). Secondly, the "positive" investigators have only succeeded after much effort, and moreover their experiments were performed with fairly large models in large towing tanks. In this respect one may question Inui's (1962) suggestion that very small tanks would be most suitable for wave measurements and analysis. It remains to be shown whether the outstanding agreement obtained by Gadd and Hogben with their 20' model can be reproduced with smaller models, or whether in fact there is still a significant viscous interaction with the wave resistance at the lower Reynolds numbers associated with the smaller models.

Of still greater importance is the question of whether the favorable results obtained by Gadd and Hogben can be repeated with practical merchant ship forms.

HULL FORMS OF LOW WAVE RESISTANCE

Certainly the raison d'être of research on wave resistance is the development of ship forms with reduced resistance. Efforts directed toward this end cover the broadest possible spectrum, from mathematical proofs of existence and uniqueness to systematic series tests of a purely empirical nature. The results range from mathematical proofs that the wave resistance is non-zero to derivations of ship "forms" with zero wave resistance, and from experimental claims of "waveless" ships to experimental studies showing that the addition of a bulbous bow decreases the viscous resistance rather than the wave resistance!

From the practical viewpoint, it can be stated that the work reported by Inui (1962), Pien (1964), and Graff, Kracht, and Weinblum (1964) provides the design naval architect with ample food for thought. To say more than this at the present time can only lead to controversy, as will be clear from the diversity of the discussion of Inui's paper.

From the scientific viewpoint, the work of Lin, Paulling, and Wehausen (1964) at Berkeley is at a convenient mid-point of the above spectrum and it shows substantial promise for practical application in the near future. The approach is semi-empirical: the total resistance is set equal to the Michell resistance plus frictional resistance, where the latter is the product of wetted surface area and a suitable flat plate coefficient. This total resistance is minimized on a digital computer, subject to initial constraints on the length, beam, draft, and displacement, and a given velocity. (Since the resistance depends on both Reynolds' and Froude numbers, the absolute length and velocity must be given.) This program is now working, and generates fair hull forms with fore-and-aft symmetry and with bulbs! It has also been used to show the small differences between a 5' model and a 500' ship, each of which is optimized for the same Froude number. Comparison with experiments has been made, using a conventional residual resistance experiment, and this did not confirm the good wave-resistance properties of the symmetric hull. However, Ward tested the same model with his "XY" method at Webb and showed dramatic reduction of the measured wave resistance near the design speed. The ultimate aim of this work is to prescribe the afterbody, in order to ensure good propulsive and viscous separation properties, and then optimize the forebody as above; initial attempts in this direction have led to negative offsets in the bow and further refinement of the program is underway to prevent this.

CONCLUSIONS AND RECOMMENDATIONS

1. The classical Michell integral for calculating wave resistance has not been significantly improved upon. It gives good predictions at high speeds, particularly in the range between the last hollow and the final hump (say for Froude numbers between .35 and .50). At lower speeds it is not reliable except with empirical corrections.

2. Two experimental attempts to measure wave resistance directly from wave records appear to have been successful, in regard to correlation with the residual resistance, and one of these (Gadd and Hogben) gives engineering accuracy over the speed range from .24 to .5 for the one model tested. This successful correlation with the residual resistance suggests that predictions of interference between viscosity and wave effects may be exaggerated, and experimental results which demonstrate serious interference should be re-examined with skepticism.

3. The direct measurement of wave resistance should be pursued in a full-scale test. No special dynamometry or towing arrangements are required; only a good wave height buoy and recording apparatus.

4. There has been substantial and interesting work done on new ship forms of reduced wave resistance. Some of this work is still underway and the final results are yet to appear. On the other hand the work reported by others is essentially complete, and also controversial. It would seem constructive to evaluate this work on an unbiased basis, including comparisons with conventional Series 60, Taylor series, and other successful ship forms, and illuminating the various ambiguities which may arise: the different results obtained for full scale total resistance depending on the extrapolation method employed, the difference in effective Froude number due to the increased length of a bulbous bow, etc.

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