Probabilistic calibration procedure for the derivation of partial safety factors for the Netherlands building codes

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1 Introduction

In the design of structures a margin of safety is introduced between the design value of the strength adopted in the calculations, on the one hand, and the reference value of the load, on the other. Over the years, this principle has been embodied in the structural design codes in various ways.

Formerly, the usual approach was to accommodate this margin entirely within the available strength by requiring that the allowable stress in the structure must not be exceeded. After 1970, in most countries there occurred a change-over to "limit state design", in which the safety margin was accommodated entirely within the loading. In this way the possibility of applying limit state design and plastic theory was opened up, and a better balanced design procedure as regards structural safety was achieved.

In the present decade most codes of practice for structural design introduce partial safety factors which ensure that there is a safety margin both on the strength side and on the load side. This margin is conceived as being proportional to the uncertainty and influence of the relevant quantity and to the desired level of safety. According to this conception the self-weight has a smaller safety factor than wind load has, and larger factors should be adopted for timber structures than for steel structures.

The magnitude of the partial factors is preferably established with the aid of probabilistic calculations. The basis for this procedure is contained in some important international publications [1], [2], [3] and [4]. In preparing the new version of the Netherlands code "Technical principles for structures - General part and loads" (TGB-Algemeen) it was decided, in 1982, to fall in with this development and, if possible, to contribute to it.

The following fundamental points were formulated:

- design calculations should be based on limit states;
- different safety classes with a clearly defined degree of reliability should be introduced for different situations;
- calculations should be based on statistically supported characteristic values and partial safety factors, both for each source of loading and for each material;
- material factors should be independent of load factors, and vice versa;
- rules for load combinations should likewise be statistically supported.

The foregoing considerations led to initiating the project "Safety of structures" [6].

2 Set-up of the project

In carrying out the project the underlying conception was that the level of safety as embodied in the existing building codes is, on the whole, acceptable. The new safety code could therefore be calibrated against this. To this end, in the project, a number of structural elements such as beams, columns and joints of concrete, steel and timber were designed in accordance with the existing codes. Next, these elements were subjected to a FOSM (First Order Second Moment) reliability analysis [4] in order to determine the safety level. In this way an idea was obtained of the average safety level and of the scatter with respect to it. The purpose of the project was so to derive partial safety factors that this scatter would, in the future code, be reduced while the average remained approximately unchanged. The calibration procedure is illustrated in Fig. 1. Implementing this aim is, however, subject to two restrictions:

- for the sake of clarity and convenience the number of factors that can be introduced must be limited; for example, one of the requirements was more particularly (see Introduction) that load factors should be material-independent and material factors load-independent;
- changes in the safety level involve changes in the design, i.e., the dimensioning or proportioning, of the structural elements; changes that are too great are not accepted in practice.

Because of these two restrictions there is still considerable scatter in the safety levels as conceived in the new code. This aspect will be further considered later on.

The relationship between the partial safety factors and the safety level can be explained with reference to a simple basic case. Starting from one resistance parameter R and one load parameter S, the calculation can, on the basis of partial safety factors (so-called level I analysis), be represented as follows:

$$\frac{R_k}{\gamma_R} \geq \gamma_S \cdot S_k \qquad (1)$$

where:

R_k and S_k = the characteristic values of the load-carrying capacity and the load respectively

γ_R and γ_S = the partial safety factors for the load-carrying capacity and the load respectively

In a (level II) FOSM analysis, in addition to the probability of failure the so-called design point ($R = R^*, S = S^*$) is determined. This denotes the most probable of all combinations (R, S) for which failure occurs (see Fig. 2). The formula for the design point is:

$$\begin{aligned} S^* &= \mu(S) - \alpha_S \cdot \beta \cdot \sigma(S) \\ R^* &= \mu(R) - \alpha_R \cdot \beta \cdot \sigma(R) \end{aligned} \qquad (2)$$

where:

$\mu(R)$ and $\mu(S)$ = the mean values of R and S respectively

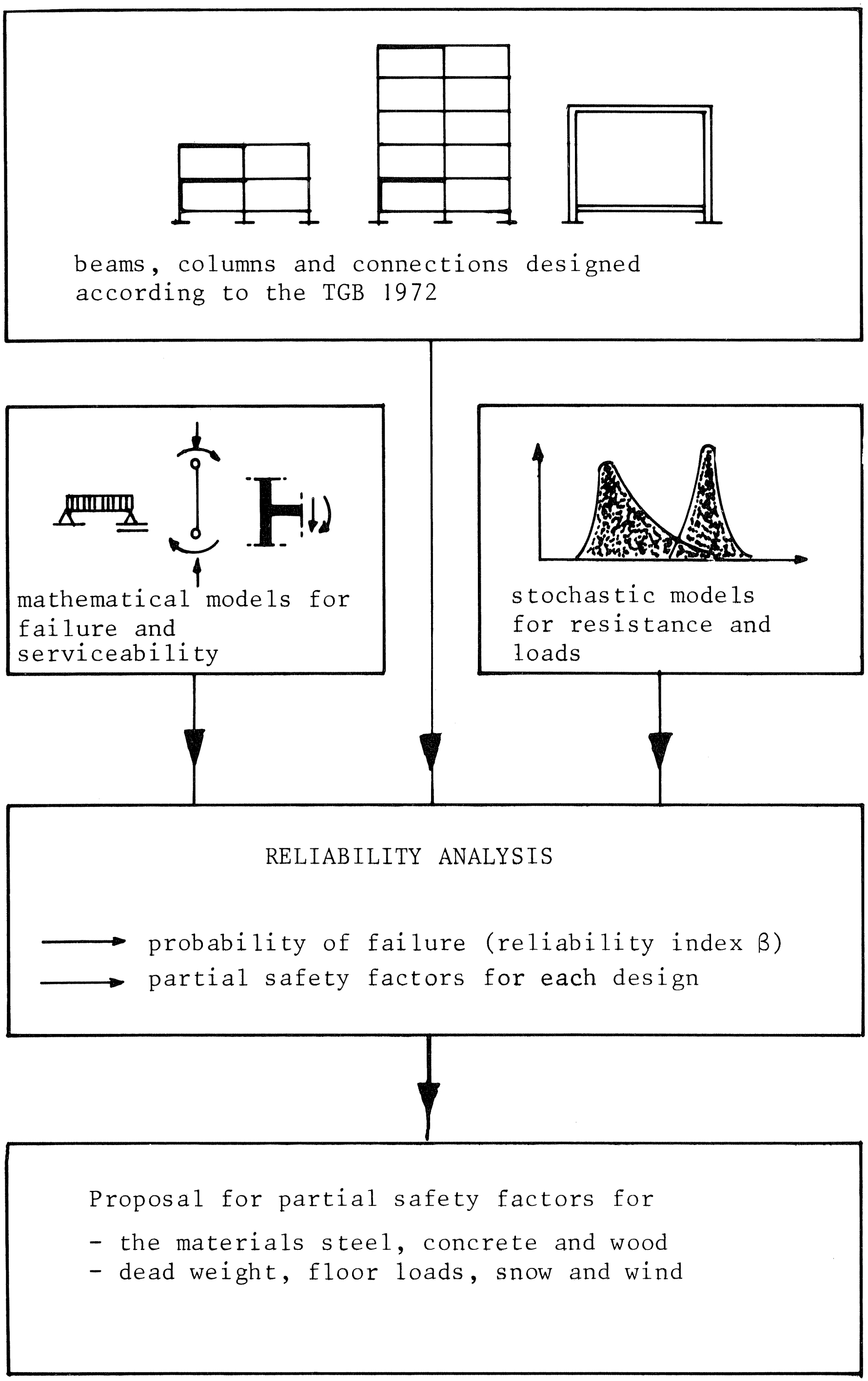

Fig. 1. Diagram of the calibration procedure employed.

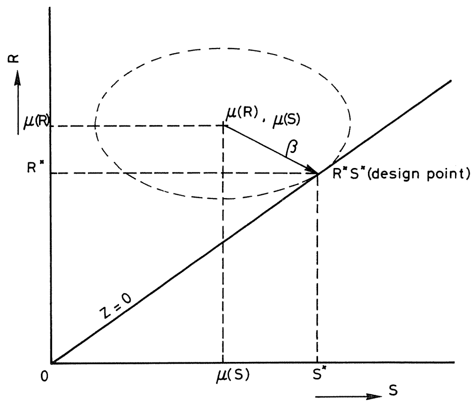

Fig. 2. Definition of the design point in the FOSM analysis.

$\sigma(R)$ and $\sigma(S)$ = the standard deviations of R and S
β = the reliability index as a measure for the level of safety: $\beta = [\mu(R) - \mu(S)]/\sqrt{[\sigma^2(R) + \sigma^2(S)]}$; the probability of failure is then equal to $\Phi(-\beta) \approx 10^{-\beta}$
α_R and α_S = coefficients which both depend on $\sigma(R)$ and $\sigma(S)$: $\alpha_R = \sigma_R/\sqrt{[\sigma^2(R) + \sigma^2(S)]}$ and $\alpha_S = \sigma_S/\sqrt{[\sigma^2(R) + \sigma^2(S)]}$

The relation between the level I practical analysis and the level II analysis is established by equating R_k/γ_R to the design point for the strength R^* and similarly equating $\gamma_S S_k$ to the design point for S^*. This gives:

$$\gamma_S = \frac{S^*}{S_k} \qquad \gamma_R = \frac{R_k}{R^*} \tag{3}$$

By first carrying out a FOSM analysis and then applying (3) it is possible to "calculate" partial safety factors. If it is assumed that the α coefficients remain unchanged, it is also possible to make a simple assessment of the effect of a higher or lower reliability level β. Further consideration shows that this is admittedly not exact, but does usually provide a serviceable approximation. Conversely, the effect of the choice of the partial safety factors on the reliability index can be investigated. In Appendix 3, based on this, a procedure is derived with which the scatter around reliability levels designated as "ideal" can be minimized.

In the foregoing derivation a simple relationship has been adopted for describing the limit state. In general, this relationship - called the limit state function or reliability function - is more complicated. One part of the present project was therefore devoted to establishing these functions, while also paying attention to the uncertainty that may be inherent in these models. The reason for such uncertainty is that, in consequence of schematization, deviations from realistic behaviour are liable to occur. These can be taken into account by the introduction of a model factor. Another part of the project related to collecting statistical information [5]. A summary is contained in Appendix 1.

3 Concrete beam example

The calibration procedure will be explained with reference to a relatively simple but realistic example, adopted from the project. For this purpose a reinforced concrete beam loaded in bending has been chosen. The principle of the beam and of the governing cross-section are shown in Fig. 3. The ultimate moment M_u that the concrete section can resist is:

$$M_u = m_b \cdot A_a \cdot f_a \cdot z \tag{4}$$

where:

$$z = h\left(1 - 0.55 \cdot \frac{A_a}{b \cdot h} \cdot \frac{f_a}{f_c'}\right) \tag{5}$$

$$h = h_t - \bar{c} - \phi/2 \tag{6}$$

The symbols employed here are explained in Table 1. Furthermore the following are to be noted:

z = lever arm
h = effective depth

If this beam is loaded by self-weight and a live load, a bending moment M_0 will occur in it:

$$M_0 = 1/8(m_{eg} \cdot q_{eg} + m_q \cdot q_{ll} + m_q \cdot q_{dl})l^2 \tag{7}$$

The symbols are explained in Table 1. Model factors have been introduced into (4) and (7) in order, inter alia, to correct schematization deviations which are due to:
- the non-bilinear material behaviour of concrete;
- the fact that the bearings are not true hinges;
- the non-uniform distribution of the live load;
- etc.

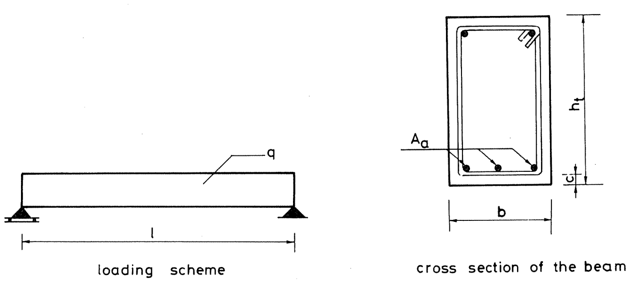

Fig. 3. Loading arrangement and cross-section of the concrete beam.

Table 1. Overview of the basic variables of the concrete beam

variable	description	unit	charac-teristic/ nominal value	distribution	μ	σ
q_{eg}	self-weight	N/mm	27.6	normal	29.0	2.02
q_{ll}	live load*	N/mm	10.0	Gumbel	7.5	3.0
q_{dl}	dead load	N/mm	2.5	normal	1.5	0.6
m_{eg}	modelfactor for self-weight	-	-	log-normal	1.0	0.05
m_q	modelfactor for live load	-	-	log-normal	1.0	0.05
b	width of beam	mm	300	normal	300.0	4.0
h_t	depth of beam	mm	500	normal	500.0	4.0
$\bar{c}$	concrete cover	mm	33	log-normal	33.0	7.0
f_c'	compressive strength of concrete	N/mm^2	18	log-normal	22.0	3.3
f_a	yield stress of reinforcement	N/mm^2	400	log-normal	460.0	46.0
m_b	model factor for ultimate load	-	-	log-normal	1.1	0.11
ϕ	diameter of main reinforcing bars	mm	20	deterministic	20.0	-
A_a	cross-sectional area of reinforcement	mm^2	1220	deterministic	1257.0	-
l	length of span	mm	4800	deterministic	4800.0	-

* maximum in 50 years

The difference between the ultimate moment that the beam can resist and the moment due to the load determines whether or not the beam will fail. Hence this difference is called the reliability function or limit state function and is usually denoted by the symbol Z:

$$Z = M_u - M_0 \tag{8}$$

Table 1 gives an overview of all the basic variables with their characteristic or nominal values; furthermore it gives the statistical parameters as obtained from Appendix 1: type of distribution, mean value and standard deviation. If the characteristic values are substituted into the relations (4) and (7), it is found that $M_u = 196$ kNm and $M_0 = 116$ kNm, i.e., $M_u = 1.7 M_0$. The margin of safety of $\gamma = 1.7$ required by the present Netherlands code of practice for concrete (Voorschriften Beton 1974, NEN 3880 [7]) is thus exactly complied with.

If a probabilistic level II analysis is performed on the basis of the statistical parameters, a reliability index $\beta = 4.7$ is obtained as the result. This corresponds to a failure probability of about 10^{-6}. The other results of the probabilistic analysis, the design point $\underline{X}^*$ and the parameters α are given in Table 2.

Table 2. Results of the level II analysis

variable	description	unit	design point $\underline{X}^*$	α^2
q_{eg}	self-weight	N/mm	30.7	0.030
q_{ll}	live load*	N/mm	24.6	0.506
q_{dl}	dead load	N/mm	1.6	0.003
m_{eg}	modelfactor for self-weight	-	1.03	0.017
m_q	modelfactor for live load	-	1.03	0.012
b	width of beam	mm	300.0	0.000
h_t	depth of beam	mm	499.3	0.002
$\bar{c}$	concrete cover	mm	35	0.007
f_c'	compressive strength of concrete	N/mm^2	20.7	0.005
f_a	yield stress of reinforcement	N/mm^2	373	0.187
m_b	model factor for ultimate load	-	0.872	0.231
ϕ	diameter of main reinforcing bars	mm	20	-
A_a	cross-sectional area of reinforcement	mm^2	1257	-
l	length of span	mm	4800	-
				$\Sigma \alpha^2 = 1.000$

* maximum in 50 years

Partial safety factors can, as indicated in Chapter 2, be derived from these results. If separate factors are to be determined for self-weight, live load, dead load and the material, then the calculation is as follows:

$$\gamma_{eg} = m_{eg}^* \cdot q_{eg}^* / q_{eg,kar} = 1.03 \cdot 30.7/27.6 = 1.1$$
$$\gamma_r = m_q^* \cdot q_{dl}^* / q_{dl,kar} = 1.03 \cdot 1.6/2.5 = 0.6$$
$$\gamma_{ll} = m_q^* \cdot q_{ll}^* / q_{ll,kar} = 1.03 \cdot 24.6/10.0 = 2.5$$
$$\gamma_m = M_{u,kar}/M_u^* = 196/163 = 1.2$$

The factor for the self-weight is indeed found to be much lower than that for the live load. The γ for dead load is so low because (in the opinion of most designers) the average dead load is considerably less than the design value.

In the above expressions γ_m refers to the ultimate moment. Another possibility would be to derive separate coefficients for the reinforcing steel and the compressive strength of the concrete. This is a matter of individual preference.

If the beam were redesigned with the above-mentioned factors, obviously the result would be the same beam again (and thus the same level of safety). The factors obtained, however, depend on the case under consideration. A different beam (e.g., with a different length of span or consisting of a different material) yields different factors.

4 Results of reliability analyses

To obtain a good idea of the level of safety embodied in the existing building codes it is necessary to analyse a large number of structural elements, comprising:

- various relevant components (beams, columns, joints);
- various construction materials (steel, reinforced concrete, laminated wood);
- various loads (self-weight, live load, snow load, wind load);
- several limit states;
- various types and dimensions of buildings.

On the basis of all the possibilities that follow from these criteria a selection has been made, of which an overview is presented in Table 3. So far as is relevant a distinction is drawn between arbitrary point in time loads (more particularly, those which are generally present) and maximum loads (more particularly, the largest in a certain period).

Table 3. Overview of the structural elements considered

	beam	column	joint
material	steel Fe 360 concrete B 22.5/FeB 400 standard timber	steel Fe 360 concrete B 22.5/FeB 400 standard timber	steel precast concrete wood
element	floor beam (simply supported) roof beam (simply supported)	pin-ended column column with end moments	T-joint cross joint
loading	self-weight live load snow	self-weight live load snow wind	self-weight live load snow wind
limit states	ultimate load deflection crack width	ultimate load deflection	ultimate load
type of building	office building house sports hall warehouse theatre	office building high-rise office building shed-type industrial building	office building high-rise office building

For all the structural elements comprised in the investigation, reliability analyses were carried out in the manner described in the previous chapter. The reliability indices β thus found, and the associated partial safety factors, are assembled in Appendix 2. The mean values of β are given in Table 4, itemized according to the type of structural element, the material employed, and the ultimate and the serviceability limit state. It appears from the table that the value β for the ultimate load ranges from 2.2 to 6.1. Higher values are also given in Appendix 1, but these relate to mechanisms which are found not to be decisive. On average, β was equal to 3.8 with a standard deviation of 1.4. For the serviceability limit states these values were 1.7 and 1.2.

Table 4. Average value for the reliability index β (reference period 50 years)

limit state	structural element	average β steel	concrete	timber
ultimate load	floor beam	4.2	4.7	3.9
	roof beam	2.7	4.5	3.1
	pin-ended column	5.0	6.1	4.8
	unbraced column	2.2	2.5	2.3
	joint	4.0	4.7	3.0
serviceability	floor beam	2.8	−0.6	3.0
	roof beam	1.6	1.2	2.2

For the ultimate limit states the difference between the respective materials are found to be inconsiderable. The general trend to emerge is that for all materials a high reliability index is obtained for pin-ended columns and for loading cases comprising a relatively high proportion of self-weight. The low values are found more particularly for the unbraced columns (i.e., with sidesway) for which the variable loads dominate. The negative value −0.6 found for the serviceability limit states of concrete beams is intriguing. It indicates that the average crack width and deflection exceed the standard values.

For most structures the actual reliability index will in practice be greater than these calculated values. There are various reasons for this:

- practical rounding of values in determining the dimensions has not been applied;
- only failure-governing loading cases have been considered in the project;
- the co-operation of various elements as a structural system has not been taken into account;
- hidden reserves of safety have not been considered.

However, this is no objection with regard to the calibration procedure to be applied. These positive effects can be expected to be similarly present in future structures.

5 Determination of partial safety factors

Starting from the calculated reliability indices, recommendations for a set of safety factors for the new building codes had to be established. Several calibration scenarios were considered for the purpose, two of which will be further elaborated:

I. The safety factors are so chosen that the scatter with regard to the target value β is as small as possible; as an example it will be indicated what factors are found if $\beta = 3.8$ is adopted for the ultimate load and $\beta = 1.7$ for the serviceability, in conformity with the average levels found for the existing design procedure as were determined in the project.

II. The safety factors are so chosen that the reliability levels differ as little as possible from the historically evolved and accepted levels; such an approach results in a minimum of changes to the design.

For the elaboration of these scenarios a computer program has been developed which, for a given set of partial safety factors, recalculates the value of the reliability index for all the structural elements under consideration. The program is based on an approximation because, to obtain an exact result, it would be necessary to do the design in accordance with the existing code over again and also to repeat the level II analysis (see Chapter 2 and Appendix 3). For a given set of safety factors the program calculates the average safety level and the scatter around this. It is also able so to select a particular sub-set of factors that the scatter in relation to a target value of the safety level is a minimum.

Since the program optimizes in a strictly mathematical way, it ignores the aims of the partial safety factor, namely, the proportionality to the uncertainty and influence of the relevant parameter. Besides, all deviations from the target value are rated as of equal weight and are conceived as completely interchangeable. Therefore corrections have been made independently of the program, while at the same time it has been endeavoured to achieve user-friendliness of the set of safety factors ultimately obtained.

The results of the calculations that have been performed are presented in Table 5. The values are defined in relation to the reference values of the loads and the characteristic values of the material strengths in accordance with the existing building codes. For timber, however, the new code has been adopted because the existing code is based on allowable stresses. The values given in the table relate to short-term loads and to the highest safety class. The material factors for the ultimate limit states (ULS) relate to ultimate stresses and yield stresses. For the serviceability limit states (SLS) the material factors relate to the modulus of elasticity. The combination factors indicated in the

Table 5. Proposals for partial safety factors:
scenario I (aiming at one reliability level for all materials and structural elements)
scenario II (aiming at minimum changes in design)

	scerario I			scenario II		
quantity	ULS	combina-tion-factors	SLS	ULS	combina-tion-factors	SLS
loads						
self-weight	1.20	-	1.00	1.20	-	1.00
live load	2.00	0.90	0.90	1.70	0.90	0.90
snow load	2.13	0.46	0.46	1.70	0.50	0.50
wind load	2.22	≥ 0.22*	≥ 0.22*	1.50	≥ 0.20*	≥ 0.20*
materials						
structural steel	1.00	-	1.00	1.10	-	1.10
reinforcing steel	1.10	-	1.00	1.30	-	1.00
compressive strength of concrete	1.40	-	1.00	1.70	-	1.00
standard timber	1.30	-	1.00	1.20	-	1.10
laminated wood	1.30	-	1.00	1.20	-	1.10

* Combination factors with wind were found not to be decisive in the project; hence the value obtained is a lower bound. It has been recommended that the same values be adopted for wind as for snow.

table are reduction factors for combinations of two or more variable loads (in Eurocode notation this is represented by $\psi_0 \times \gamma_S$). The procedure is that one of the variable loads is multiplied by the load coefficient and the other by the combination factor(s).
For guidance it can be noted that in the existing codes for reinforced concrete structures the load factors applied are 1.7 (ULS) and 1.0 (SLS). The corresponding values for steel structures are 1.5 (ULS) and 1.0 (SLS). For timber the values can be taken as approximately equal to 1.5 (ULS) and 1.0 (SLS) if the allowable stresses are converted so as to obtain load factors. The material factors in the present code are 1.0.

6 Summary and conclusions

For the purpose of determining partial safety factors in the new Netherlands building codes, a study of the safety margins embodied in the present codes has been carried out. For this purpose a large number of different structural elements of various materials - reinforced concrete, steel and timber - have been designed, comprising ultimate limit states and serviceability limit states. The type of element ranges from beams and columns to joints (structural connections) employed in buildings of various types. The loads taken into account are self-weight, live load, dead load, snow and wind.
The safety levels of these structural elements have been calculated by means of a level II analysis, the statistical information required for the purpose have been collected on the basis of a comprehensive study of the literature [5].
Proposals for partial safety factors have been made. The basis for this has been derived from the (weighted) average level of safety deduced from the above-mentioned calculations. These proposals have been submitted for discussion to the Standards Committee which is to take charge of revising the building code in question. These matters are still under discussion.
Besides the activities reported in this article, attention has also been paid to long-term loads, safety classes, loading patterns, prestressing of concrete structures, and favourable effect of permanent loads. Further information on the project as a whole is given in the research report [6].
The principal conclusions to emerge from the project are:

1. The safety level of structural elements designed in accordance with the present codes and analysed in accordance with the existing conceptions is $\beta = 3.8$ (failure probability $\approx 10^{-4}$) for the ultimate limit state and $\beta = 1.7$ (failure probability $\approx 10^{-2}$) for the serviceability limit state (probabilities relate to a service life of 50 years).
2. There is considerable scatter around these averages. It was found possible substantially to reduce this scatter by means of partial safety factors determined with an optimization program. The changes that this brought about in the design of various structural elements proved to be unacceptably great for practical purposes. However, it was possible also to work out other scenarios with the aid of the data that had become available and the optimization program. On the basis of the results thus obtained a choice is now being made for the safety factors finally to be adopted in the new Netherlands building codes.

7 Acknowledgements

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APPENDIX 1

Overview of statistical parameters

Explanation of the symbols

V = coefficient of variation
σ = standard deviation
A = floor area
A_0 = reference area
X_{nom} = nominal value
N = normal distribution
LN = log-normal distribution
G = Gumbel distribution
Ex I = extreme-value I-distribution
W = Weibul distribution
TN = truncated normal

loads in kN/m²

quantity		type	average	scatter
self-weight		N	$1.05N_{nom}$	$V = 0.07$
partitions		LN	0.30	0.40
live load ($A_0 = 4$ m²)				
office	- arbitrary point in time	G	0.50	$0.4 + A_0/A$
	- maximum ($A < 100$ m)	Ex I	1.50	0.4
house	- arbitrary point in time	G	0.8	$0.4 + A_0/A$
	- maximum ($A < 100$ m)	Ex I	1.00	0.4
shed-type	- arbitrary point in time	G	0.8	$0.4 + A_0/A$
	- maximum	Ex I	2.5	0.4
theatre	- arbitrary point in time	G	1.0	$0.4 + A_0/A$
	- maximum	Ex I	2.5	0.4
warehouse	- arbitrary point in time	G	1.0	$0.4 + A_0/A$
	- maximum	Ex I	2.5	0.4
wind load (10 m height)				
arbitrary point in time		W	0.1	1.00
maximum		Ex I	1.0	0.30
snow load				
arbitrary point in time		LN	0.2	1.00
maximum		Ex I	0.7	0.30

material properties

quantity	type	average	scatter
reinforced concrete			
ultimate stress B 22.5 (short term)	LN	22 MPa	$V = 0.15$
tensile strength B 22.5 (short term)	LN	2.5 MPa	0.20
modulus of elasticity B 22.5 (short term)	LN	28 GPa	0.10
long-term factor for compression	LN	0.9	0.10
creep factor B 22.5	LN	3.0	0.20
yield stress of reinforcing steel FeB 400	LN	460 MPa	0.10
structural steel			
yield stress of steel Fe 360	LN	280 MPa	0.08
ultimate stress of steel Fe 360	LN	430 MPa	0.08
modulus of elasticity	N	210 GPa	0.04
standard timber			
compressive strength ($u = 16\%$)	W	28 MPa	0.15
tensile strength	W	26 MPa	0.30
flexural strength	W	36 MPa	0.25
modulus of elasticity	W	12 GPa	0.20
structural timber			
compressive strength	W	32 MPa	0.15
tensile strength	W	32 MPa	0.30
flexural strength	W	45 MPa	0.25
modulus of elasticity	W	13 GPa	0.20
laminated wood			
compressive strength	W	32 MPa	0.15
tensile strength	W	32 MPa	0.20
flexural strength	W	45 MPa	0.15
modulus of elasticity	W	13 GPa	0.15
long term factor for strength ($t = \infty$)	LN	0.53	0.10
creep factor	LN	1.0	0.30
joint (ring connector)	W	approx. 3 × permissible value	0.15

dimensional deviations

quantity	type	average	scatter
rolled steel sections A, W, I	N	X_{nom}	$V = 0.04$
timber dimensions b, h	N	X_{nom}	$V = 0.04$
dimensions of concrete beam/column/slab	N	X_{nom}	$\sigma = 4$ mm
cover to top reinforcement	LN	$X_{nom} + 5$ mm	$\sigma = 10$ mm
cover to bottom reinforcement	LN	X_{nom}	$\sigma = 7$ mm
cover in columns/walls	LN	$X_{nom} + 5$ mm	$\sigma = 7$ mm

eccentricities

quantity	type	average	scatter
concrete			
braced $\lvert e \rvert$	TN	$0.0020l$	$V = 0.8$
unbraced $\lvert e \rvert$	TN	$0.0060l$	0.8
steel			
braced $\lvert e \rvert$	TN	$0.0010l$	0.8
unbraced $\lvert e \rvert$	TN	$0.0015l$	0.8
timber			
braced $\lvert e \rvert$	TN	$0.0010l$	0.8
unbraced $\lvert e \rvert$	TN	$0.0015l$	0.8

model-factors

quantity	type	average	scatter
load-effects			
self-weight	LN	1.00	$V = 0.05$
live load	LN	1.00	0.05
wind	LN	1.00	0.20
snow	LN	1.00	0.20
structural steel			
ultimate load of beam	LN	1.10	0.07
deflection of beam	LN	1.10	0.07
pin-ended column	LN	1.30	0.10
column	LN	1.30	0.10
welded-joint	LN	1.20	0.20
bolted joint - failure of flange	LN	1.07	0.11
- bolt failure/ yielding of flange	LN	1.05	0.06
- bolt failure	LN	1.11	0.05
structural concrete			
ultimate load of beam in bending	LN	1.10	$V = 0.10$
ultimate load of beam in shear	LN	1.00	0.18
deflection of beam	LN	1.10	0.10
crack width in beam	LN	1.00	0.30
pin-ended column	LN	1.10 à 1.30	0.15
column	LN	1.10	0.15
joint	LN	1.10	0.10
timber			
ultimate load of beam	LN	1.10	0.07
deflection of beam	LN	1.10	0.07
pin-ended column	LN	1.30	0.10
column	LN	1.10	0.10
joint	LN	1.20	0.10

APPENDIX 2

Overview of calculations performed

The following abbreviations are used in the tables presented in this appendix:

sw = self-weight
d*l* = dead load
ll = live load
sn = snow load
wi = wind load
m = material
e = eccentricity
s = (reinforcing) steel
b = compressive strength of concrete
aip = arbitrary point in time
max = maximum (in a period of 50 years)
st = short-term
lt = long-term
jnt = joint
sbh = standard timber
lam = laminated timber

The code of the calculation compressive five digits, the meanings of which are as follows:

first digit: material type: 1 = structural steel, 2 = concrete, 3 = timber
second digit: type of element: 1 = beam, 2 = roof beam, 3 = pin-ended column, 4 = column, 5 = joint
third digit: load combination: 1 = sw + *ll*, 2 = sw + sn, 3 = sw + *ll* + sn, 4 = sw + sn + wi, 5 = sw + *ll* + sn + wi
fourth digit: limit state: 1 = ULS, 2 = SLS
fifth digit: type of building

A number of loading cases have been analysed more than once, taking account of differences in conception existing between designers.

Table 2.1. Results of probabilistic analysis and partial safety factors for steel structures

		limit				γ_{ll}		γ_{sn}		γ_{wi}		
code	element	state	β	γ_{eg}	γ_{dl}	aip	max	aip	max	aip	max	γ_m
11111	floor beam	strenght	4.5	1.2	0.6	–	2.3	–	–	–	–	1.1
11111	floor beam	strenght	4.4	1.1	0.6	–	2.3	–	–	–	–	1.2
11111	floor beam	strenght	3.8	1.1	0.6	–	2.4	–	–	–	–	1.0
11113	floor beam	strenght	4.8	1.2	0.6	–	1.6	–	–	–	–	1.2
11113	floor beam	strenght	4.4	1.1	0.6	–	2.4	–	–	–	–	1.0
11114	floor beam	strenght	4.2	1.1	–	–	2.2	–	–	–	–	1.0
11114	floor beam	strenght	3.6	1.1	–	–	2.0	–	–	–	–	0.9
11115	floor beam	strenght	4.2	1.2	–	–	2.2	–	–	–	–	1.0
11116	floor beam	strenght	4.2	1.1	–	–	2.1	–	–	–	–	1.0
11121	floor beam	deflection	2.1	1.2	0.6	0.3	–	–	–	–	–	1.0
11121	floor beam	deflection	3.3	1.2	0.7	0.7	–	–	–	–	–	1.1
11121	floor beam	deflection	2.7	1.2	0.6	0.6	–	–	–	–	–	1.1
11123	floor beam	deflection	2.5	1.2	0.7	0.3	–	–	–	–	–	1.1
11124	floor beam	deflection	3.5	1.2	–	0.5	–	–	–	–	–	1.1
11125	floor beam	deflection	2.8	1.2	–	0.6	–	–	–	–	–	1.1
11126	floor beam	deflection	2.8	1.2	–	0.4	–	–	–	–	–	1.1
12211	roof beam	strenght	3.2	1.1	–	–	–	–	2.2	–	–	0.9
12214	roof beam	strenght	2.4	1.1	–	–	–	–	2.8	–	–	0.9
12216	roof beam	strenght	2.6	1.1	–	–	–	–	2.9	–	–	0.9
12221	roof beam	deflection	2.1	1.1	–	–	–	0.8	–	–	–	1.0
12224	roof beam	deflection	1.5	1.1	–	–	–	1.1	–	–	–	0.9
12226	roof beam	deflection	1.2	1.1	–	–	–	0.6	–	–	–	1.0
13214	pin-ended column	strength	4.8	1.1	–	–	–	–	5.7	–	–	0.7
13311	pin-ended column	strength	5.3	1.2	0.6	–	2.4	0.3	–	–	–	1.2
13312	pin-ended column	strength	4.8	1.2	0.6	0.5	1.9	0.2	–	–	–	1.4
14414	column	strength	1.5	1.0	–	–	–	0.4	–	–	2.0	0.9
14511	column	strength	2.4	1.1	0.6	0.2	–	0.2	–	–	2.9	0.9
14512	column	strength	2.7	1.1	0.6	0.3	–	0.2	–	–	2.6	0.9
15111	T-joint	web-buckling	2.5	1.1	–	–	1.1	–	–	–	–	1.4
15111	T-joint	shear	3.1	1.1	–	–	1.4	–	–	–	–	1.2
15111	T-joint	strength	5.4	1.1	–	–	3.8	–	–	–	–	0.7
15111	T-joint	strength	4.8	1.1	–	–	2.8	–	–	–	–	0.9
15113	T-joint	web-buckling	2.6	1.1	–	–	1.0	–	–	–	–	1.4
15113	T-joint	shear	3.3	1.1	–	–	1.2	–	–	–	–	1.3
15113	T-joint	strength	5.4	1.1	–	–	3.7	–	–	–	–	0.7
15113	T-joint	strength	5.0	1.1	–	–	2.6	–	–	–	–	0.9

Table 2.2. Results of probabilistic analyses and partial safety factors for concrete structure

code	element	limit state	β	γ_{eg}	γ_{dl}	γ_{ll} aip	γ_{ll} max	γ_{sn} aip	γ_{sn} max	γ_{wi} aip	γ_{wi} max	γ_m e	γ_m s	γ_m b
21111	floor beam	bending	4.7	1.2	0.6	-	2.3	-	-	-	-	-	1.2	-
21111	floor beam	bending	4.9	1.1	0.6	-	2.2	-	-	-	-	-	1.2	-
21111	floor beam	bending	4.7	1.1	0.7	-	2.6	-	-	-	-	-	1.2	-
21113	floor beam	bending	5.2	1.2	0.7	-	1.9	-	-	-	-	-	1.3	-
21114	floor beam	bending	4.5	1.1	0.7	-	2.3	-	-	-	-	-	1.1	-
21115	floor beam	bending	4.5	1.1	0.6	-	2.2	-	-	-	-	-	1.3	-
21116	floor beam	bending	4.5	1.1	0.6	-	2.0	-	-	-	-	-	1.2	-
21121	floor beam	deflection	-1.7	1.0	0.5	0.6	-	-	-	-	-	-	1.1	-
21121	floor beam	deflection	-0.9	1.0	0.5	0.2	-	-	-	-	-	-	0.8	-
21123	floor beam	deflection	-1.1	1.2	0.5	0.2	-	-	-	-	-	-	0.8	-
21124	floor beam	deflection	1.8	1.1	-	0,4	-	-	-	-	-	-	1.0	-
21125	floor beam	deflection	-1.1	1.0	0.5	0.5	-	-	-	-	-	-	1.2	-
21126	floor beam	deflection	-1.4	1.0	0.5	0.5	-	-	-	-	-	-	1.2	-
21131	floor beam	crack width	-0.7	1.0	0.6	0.7	-	-	-	-	-	-	1.1	-
21134	floor beam	crack width	0.4	1.0	0.9	0.9	-	-	-	-	-	-	0.9	-
22211	roof beam	bending	4.4	1.1	-	-	-	-	2.4	-	-	-	1.2	-
22214	roof beam	bending	4.5	1.1	-	-	-	-	2.4	-	-	-	1.4	-
22216	roof beam	bending	4.7	1.2	-	-	-	-	1.5	-	-	-	1.3	-
22221	roof beam	deflection	1.8	1.1	-	-	-	0.2	-	-	-	-	1.1	-
22224	roof beam	deflection	0.8	1.0	-	-	-	0.3	-	-	-	-	0.9	-
22226	roof beam	deflection	1.2	1.1	-	-	-	0.3	-	-	-	-	1.0	-
23214	pin-ended column	strength st	7.6	1.2	-	-	-	-	1.1	-	-	1.0	1.2	2.7
23214	pin-ended column	strength st	6.7	1.2	-	-	-	-	1.0	-	-	0.9	1.2	1.9
23214	pin ended column	strength lt	7.6	1.2	-	-	-	0.3	-	-	-	1.0	1.2	2.4
23214	pin-ended column	strength lt	7.1	1.2	-	-	-	0.3	-	-	-	0.9	1.2	1.8
23311	pin-ended column	strength st	6.7	1.3	0.6	-	2.5	0.3	-	-	-	0.7	1.2	1.7
23311	pin-ended column	strength st	6.9	1.2	0.6	-	1.0	0.2	-	-	-	1.6	1.2	1.9
23311	pin-ended column	strength lt	6.1	1.3	0.6	0.3	-	0.3	-	-	-	0.6	1.2	1.6
23311	pin-ended column	strength lt	6.8	1.2	0.6	0.3	-	0.2	-	-	-	1.5	1.2	1.8
23311	pin-ended column	strength st	6.3	1.3	0.6	-	1.9	0.2	-	-	-	0.6	1.2	1.7
23311	pin-ended column	strength st	6.7	1.2	0.6	-	1.0	0.2	-	-	-	1.4	1.2	2.2
23311	pin-ended column	strength lt	5.6	1.3	0.6	0.3	-	0.2	-	-	-	0.5	1.2	1.5
23311	pin-ended column	strength lt	6.8	1.3	0.6	0.3	-	0.2	-	-	-	1.4	1.2	2.1
24414	column	strength st	2.2	1.1	-	-	-	-	1.5	-	1.3	0.3	1.2	2.3
24414	column	strength st	4.2	1.2	-	-	-	-	1.9	-	1.5	0.4	1.2	1.9
24414	column	strength lt	3.2	1.2	-	-	-	0.6	-	1.0	-	0.3	1.2	2.5
24414	column	strength lt	6.1	1.2	-	-	-	2.7	-	1.1	-	0.4	1.2	2.1
24511	column	strength st	3.0	1.1	-	0.2	-	0.2	-	-	2.7	0.2	1.2	1.4
24511	column	strength st	3.2	1.1	-	0.2	-	0.2	-	-	2.9	0.2	1.2	1.1
24511	column	strength lt	5.5	1.1	-	0.2	-	0.2	-	1.8	-	0.4	1.2	5.8
24511	column	strength lt	6.2	1.1	-	0.3	-	0.2	-	2.2	-	0.4	1.2	1.8
25511	T-joint	bending	4.7	1.2	0.6	-	2.3	-	-	-	-	-	2.5	-

Tabel 2.3. Results of probabilistic analyses and partial safety factors for timber structures

code	element	limit state	β	γ_{eg}	γ_{dl}	$\gamma_{//}$ aip	$\gamma_{//}$ max	γ_{sb} aip	γ_{sb} max	γ_{wi} aip	γ_{wi} max	γ_m	mat
31113	joist	bending st	3.7	1.1	-	-	1.0	-	-	-	-	2.2	sbh
31113	joist	bending lt	3.9	1.1	-	0.6	-	-	-	-	-	2.9	sbh
31113	(main) beam	bending st	3.8	1.1	-	-	1.4	-	-	-	-	1.1	lam
31113	(main) beam	bending lt	4.7	1.1	-	0.3	-	-	-	-	-	3.3	lam
31115	joist	bending st	4.5	1.1	-	-	1.5	-	-	-	-	1.6	lam
31115	joist	bending lt	4.2	1.1	-	0.2	-	-	-	-	-	1.3	lam
31123	joist	deflection	2.9	1.1	-	0.8	-	-	-	-	-	1.1	sbh
31123	(man) beam	deflection	3.9	1.1	-	0.3	-	-	-	-	-	1.8	lam
31125	joist	deflection	2.9	1.1	-	1.3	-	-	-	-	-	0.8	lam
32213	roof beam	bending st	2.7	1.1	-	-	-	-	2.1	-	-	1.0	lam
32213	roof beam	bending lt	3.4	1.0	-	-	-	1.9	-	-	-	1.1	lam
32213	purlin	bending st	3.5	1.1	-	-	-	-	0.7	-	-	1.9	sbh
32213	purlin	bending lt	3.7	1.1	-	-	-	0.5	-	-	-	2.4	sbh
32214	(man) beam	bending st	3.2	1.1	-	-	-	-	2.6	-	-	1.0	lam
32214	(man) beam	bending lt	3.3	1.1	-	-	-	2.4	-	-	-	1.0	lam
32216	roof beam	bending st	3.3	1.1	-	-	-	-	1.7	-	-	1.3	lam
32216	roof beam	bending lt	3.2	1.1	-	-	-	0.8	-	-	-	1.5	lam
32221	purlin	deflection	1.3	1.1	-	-	-	2.3	-	-	-	0.6	sbh
32223	roof beam	deflection	2.0	1.1	-	-	-	1.0	-	-	-	0.8	lam
32224	(main) beam	deflection	1.7	1.1	-	-	-	1.0	-	-	-	0.7	lam
32226	roof beam	deflection	2.7	1.1	-	-	-	1.9	-	-	-	0.8	lam
32413	roof beam	bending st	2.7	1.1	-	-	-	-	-	-	2.1	1.0	lam
32413	roof beam	bending lt	4.0	1.1	-	-	-	-	-	0.2	-	2.1	lam
32413	purlin	bending st	3.5	1.1	-	-	-	-	-	-	0.7	1.9	sbh
32413	purlin	bending lt	4.4	1.1	-	-	-	-	-	0.1	-	5.2	sbh
32414	(main) beam	bending st	3.2	1.1	-	-	-	-	-	-	2.6	1.0	lam
32414	(main) beam	bending lt	4.6	1.1	-	-	-	-	-	0.5	-	2.4	lam
32416	roof beam	bending st	3.3	1.1	-	-	-	-	-	-	1.7	1.3	lam
32416	roof beam	bending lt	3.7	1.1	-	-	-	-	-	0.1	-	1.8	lam
33313	pin-ended column	strength st	5.1	1.1	-	-	1.2	0.5	-	-	-	2.9	lam
33313	pin-ended column	strength st	5.3	1.1	-	0.3	-	-	1.9	-	-	3.8	lam
33313	pin-ended column	strength lt	5.4	1.1	-	0.3	-	1.0	-	-	-	4.6	lam
33313	pin-ended column	strength st	5.9	1.1	-	-	1.3	0.9	-	-	-	1.9	lam
34414	column	strength st	4.3	1.1	-	-	-	-	1.4	1.3	-	0.9	lam
34414	column	strength lt	4.1	1.1	-	-	-	0.3	-	1.3	-	0.9	lam
34414	column	strength st	2.3	1.1	-	-	-	0.3	-	-	2.6	0.5	lam
35210	ring connector	strength	3.0	1.1	-	-	-	-	2.4	-	-	1.0	-
35210	ring connector	strength	3.1	1.1	-	-	-	1.6	-	-	-	1.2	-
35410	ring connector	strength	3.8	1.1	-	-	-	-	-	-	2.4	1.0	-
35410	ring connector	strength	3.0	1.1	-	-	-	-	-	0.2	-	1.8	-

APPENDIX 3

Estimation of partial safety factors with the aid of the method of least squares

In the vicinity of the design point $\underline{X}_k^*$ the reliability function Z_k is given by:

$$Z_k = \sum_{i=1}^{n} \frac{\partial Z_k}{\partial X_i} \{X_i - X_{ki}^*\} \tag{3.1}$$

The partial safety factors γ_i should be so chosen that for $X_i = \gamma_i X_i^N$ (where X_i^N denotes the nominal or characteristic value) the value of Z_k is equal to zero. For convenience, here the same definition of partial safety factors is adopted for strength parameters as for load parameters. In that case, however, the reciprocal value of the material factors are obtained. If only one Z_k function need to be considered, then:

$$\gamma_i = X_i^* / X_i^N \tag{3.2}$$

If several Z_k functions have to be considered, then γ_i can be determined from the condition:

$$\sum_{k=1}^{m} \{Z_k\}^2 \text{ must be a minimum} \tag{3.3}$$

Equation (3.3) suffers from the drawback that not all the Z_k functions are equivalent, if only on account of a difference in the units. Therefore it is better to normalize the Z_k values by dividing them by the standard deviation of Z_k at the design point $\sigma^*(Z_k)$:

$$\min \sum_{k=1}^{m} \left\{ \frac{Z_k}{\sigma^*(Z_k)} \right\}^2 \tag{3.4}$$

In this expression $Z_k/\sigma^*(Z_k)$ is given by (see 3.1):

$$\frac{Z_k}{\sigma^*(Z_k)} = \sum_{i=1}^{n} \frac{1}{\sigma^*(Z_k)} \frac{\partial Z_k}{\partial X_i} (\gamma_i X_i^N - X_{ki}^*) \tag{3.5}$$

Via $\alpha_{ki} = \{\partial Z_k / \partial X_i\} \sigma_k(X_i) / \sigma^*(Z_k)$ (by definition) this can be worked out to:

$$\frac{Z_k}{\sigma^*(Z_k)} = \sum_{i=1}^{n} \alpha_{ki} \left\{ \frac{\gamma_i X_i^N - X_{ki}^*}{\sigma_k(X_i)} \right\} \tag{3.6}$$

For the value of the design point X_{ki}^* the following is obtained, in accordance with the relationships (3) and (4) presented earlier on:

$$X_{ki}^* = \mu_k(X_i) - \alpha_{ki} \beta_k \sigma_k(X_i) \tag{3.7}$$

where β_k is the value for the reliability index actually found in the level II analysis. However, in deriving the new factors it is in many cases desirable to adopt a target value $\bar{\beta}_k$ for the reliability index which differs from the β_k that has been found. For this reason the following expression is employed:

$$X_{ki}^* = \mu_k(X_i) - \alpha_{ki}\bar{\beta}_k\sigma_k(X_i) \tag{3.8}$$

It is assumed that the stochastic parameters μ, σ and α can permissibly be kept constant, both on passing to a new target value and with regard to deviations therefrom. Keeping these coefficients constant forms the central feature of the approximation procedure:

$$\frac{Z_k}{\sigma^*(Z_k)} = \sum_{i=1}^{n} \alpha_{ki} \left\{ \frac{\gamma_i X_i^N - \mu_k(X_i)}{\sigma_k(X_i)} + (\alpha_{ki}\bar{\beta}_k) \right\} \tag{3.9}$$

Because $\Sigma\alpha_{ki}^2 = 1$, the following holds:

$$\frac{Z_k}{\sigma^*(Z_k)} = \bar{\beta}_k + \Sigma\alpha_{ki} \left\{ \frac{\gamma_i X_i^N - \mu_k(X_i)}{\sigma_k(X_i)} \right\} \tag{3.10}$$

for the ideal γ values $Z_k = 0$, and therefore $-\Sigma\alpha_{ki}\{...\} = \bar{\beta}_k$. Hence $-\Sigma\alpha_{ki}\{...\}$ can be interpreted as the reliability index which is associated with a particular set of γ values. The quotient $-Z_k/\sigma^*(Z_k)$ then represents the difference $\Delta\beta_k$ between the actually obtained and the desired (target) values of the reliability index.

The required values for γ_i are found on differentiating

$$\sum_{k=1}^{m} \{Z_k/\sigma^*(Z_k)\}^2$$

with respect to γ_j; m is the number of reliability functions considered. Starting from (3.9), it then follows that:

$$\frac{\partial}{\partial\gamma_j} \sum_{k=1}^{m} \left\{ \sum_{i=1}^{n} \alpha_{ki} \left\{ \frac{\gamma_i X_i^N - \mu_k(X_i) + \alpha_{ki}\bar{\beta}_k\sigma_k(X_i)}{\sigma_k(X_i)} \right\}^2 \right\} = 1 \qquad (j = 1 \dots n) \tag{3.11}$$

In many cases it is desirable to establish particular values, e.g., γ(self-weight) = 1.2 or γ (model factor) = 1.0 or γ (geometric parameter) = 1.0.

Suppose that $\gamma_1, \gamma_2, \dots \gamma_n$ are the free parameters and $\gamma_{n+1}, \dots \gamma_{nt}$ the prescribed parameters. Equation (3.11) can then be worked out to give:

$$\sum_{k=1}^{m} \left\{ \sum_{i=1}^{nt} \alpha_{ki} \left\{ \frac{\gamma_i X_i^N - \mu_k(X_i) + \alpha_{ki}\beta_k\sigma_k(X_i)}{\sigma_k(X_i)} \right\} \right\} \frac{\alpha_{kj}}{\sigma_k(X_j)} X_j^N = 0 \quad (j = 1 \dots n) \tag{3.12}$$

or:

$$\sum_{i=1}^{n} A_{ij}\gamma_i = b_j \tag{3.13}$$

where:

$$A_{ij} = \sum_{k=1}^{m} a_{ki}a_{kj} \quad \text{with} \quad a_{ki} = [\alpha_{ki}X_i^N]/\sigma_k(X_i)$$

$$b_j = \sum_{k=1}^{m} \left\{ \sum_{i=1}^{nt} \{\mu(X_i) - \alpha_{ki}\bar{\beta}_k\sigma(X_i)\} a_{ki}a_{kj}/X_i^N - \sum_{i=n+1}^{nt} a_{ki}a_{kj}\gamma_i \right\} \tag{3.14}$$

The problem has thus been reduced to solving at set of "n by n" equations.