# SKIN FRICTION FACTOR AND MEAN VELOCITY PROFILE MEASURED IN HIGH-REYNOLDS-NUMBER TURBULENT PIPE FLOW

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<u>Abstract</u> A friction factor in a fully developed pipe flow is measured very accurately over wide range Reynolds number from  $10^3$  to  $10^7$  at the high Reynolds number facility in AIST,NMIJ. Also a velocity profile is measured by using a LDV up to  $10^6$ . From the comparison between a velocity profile and the friction factor, we found that a log-law profile is well observed and Kármán constant is estimated to be 0.385. Based on the measurements, we present a new functional form of the friction factor which is consistent with mean velocity profile.

### **INTRODUCTION**

The equation of the friction factor for fully developed flow in a smooth pipe is derived by the integration of the velocity profile and it is the function of Reynolds number. Both equations are semi-empirical and several constants in the equations such as Kármán constant are defined by experimentally. There are many previous reports for these constants and the exact values are still not fixed. One of the reasons is the Reynolds number dependency. Since these constants have Reynolds number dependency in low Reynolds number, the experiment in higher Reynolds number region is required to obtain the exact constants. Zagarola and Smith [1] has published the friction factor and velocity profile for fully developed pipe flow in high Reynolds number region over  $10^7$  as bulk and they reported the values of the constants based on their result. This report leads many discussions especially about the Karman constant and it still continues due to the lack of other experimental results in high Reynolds number region.

In this experiment, the measurement of the friction factor and the velocity profile is performed with high accuracy using "High Reynolds number actual flow facility" which serves as the national measurement standard for water flowrate in Japan. The Reynolds number range for the measurement of the friction factor is very wide from order of  $10^3$  to  $10^7$ . The feature of this experiment is the highly accurate measurement of a bulk velocity in the pipe. In this paper, the bulk velocity is calculated by flowrate given by the national standard of flowrate directly with very small uncertainty. Then, the measurement uncertainty of the friction factor is achieved approximately 0.9% in this experiment. In this report, the highly accurate measurement result of the friction factor at high Reynolds number up to  $10^7$  is presented. Based on the constants such as Kármán one obtained here, new relation between the friction factor and the velocity profile is proposed.

## EXPERIMENT

The experiment is performed in wide Reynolds number range from  $7.7 \times 10^3$  to  $1.8 \times 10^7$  using water flow facilities in AIST,NMIJ. Especially, for higher Reynolds number region, the experiment is performed using High Reynolds number actual flow facility as shown in Fig.1 [2]. These facilities are the national measurement standard of water flowrate in Japan. The uncertainty of the flowrate is very low ranging from 0.04% to 0.10% with coverage factor 2. The pipe diameters used in the experiment are 387 mm (P1) and 100 mm (P2). The surface roughness is  $Ra=0.2 \mu m$  for P1 and 0.1  $\mu m$  for P2. The bulk velocity in the pipes ranges from 0.07 m/s to 18.9 m/s. Temperature of water is 20 °C and 70 °C for P1 and 20 °C for P2. The temperature fluctuation during the measurement is within ±0.2 °C. The measurement uncertainty of the friction factor is estimated to be from 0.86% to 0.90% with several exceptions. The measurement of the velocity profile is done for P2 by using a LDV. For this purpose, a window chamber is installed at downstream of the test section of P2. Reynolds number range for the velocity profile measurement is from  $3.8 \times 10^4$  to  $1.0 \times 10^6$ . The measurement volume of the LDV is  $48 \times 48 \times 400 \mu m$ .

#### RESULT

The experimental result is shown in Fig.2. Fig.2(a) shows the relation between the measured friction factor and Reynolds number. The curve of the present friction factor shows very smooth with very little scattering of the plotted data although the data is obtained for different two pipes with different two upstream flow conditions, and for P1, at two different water temperatures. The good agreement between the results obtained for these different conditions proves the reliability of the experiment. Comparing with the previous reports by McKeon et al.[3] and Swanson et al.[4], the present experimental result and their one show good agreement in lower Reynolds number region  $Re_D>10^5$ , but deviate each other in higher Reynolds number region. The present experimental result is approximately 3 % lower than Prandtl equation and 6% lower than McKeon's at  $Re_D=1.0\times10^7$ .

Figure 2(b) shows the measurement result of the velocity profiles normalized by inner scaling values. The friction factor used to estimate the wall shear stress is given by the result of Fig.2(a). The overlap region is clearly observed in this result. The solid line shows the equation by log-law using the constants given by the following analysis.

Based on McKeon's analysis [5], the velocity profile and the friction factor including a non-dimensional offset  $a^+$  are given by the following equations with some assumptions.

$$u^{+} = \frac{1}{\kappa} \ln\left(y^{+} + a^{+}\right) + B$$
$$\frac{1}{\sqrt{\lambda}} = \frac{1}{2\kappa\sqrt{2}\log(e)} \left\{ \log\left(Re_{\rm D}\sqrt{\lambda}\right) - 8\sqrt{2} \frac{a^{+}}{Re_{\rm D}\sqrt{\lambda}} \left(\log\frac{a^{+}}{Re_{\rm D}\sqrt{\lambda}} + \log\left(4\sqrt{2}\right)\right) \right\} + C$$

where,  $\kappa$  is Kármán constant.  $a^+$  and  $\kappa$  are determined to give the optimum value for each equation. In the equation of the velocity profile,  $a^+$  is determined so that the constant *B* dose not vary in the overlap region  $y^+>30$ . In the equation of the friction factor, the both constants are obtained from the best fitting. Finally,  $a^+$  and  $\kappa$  which satisfy both equations are determined. The constant values are obtained as follows;  $\kappa=0.385$ ,  $a^+=4.5$ , B=4.59 and C=-1.283 for  $Re_D>2.0\times10^5$ .

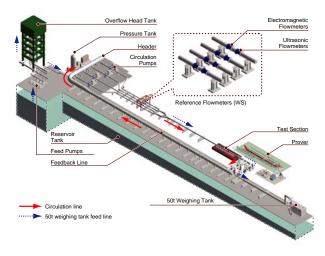


Figure 1. High Reynolds number actual flow facility in AIST, NMIJ.

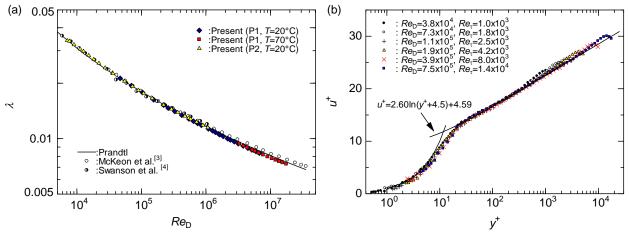


Figure 2. Experimental result (a) friction factor (b) velocity profiles.

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