



**Numerical Analysis for Fatigue  
Life Prediction on Railroad RCF  
Crack Initiation**

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# Outline

- ✓ Research Motivation
- ✓ Sub-modelling & Results
- ✓ Crack initiation analysis
- ✓ Conclusions & Outlooks





# Research Motivation

## ➤ Background

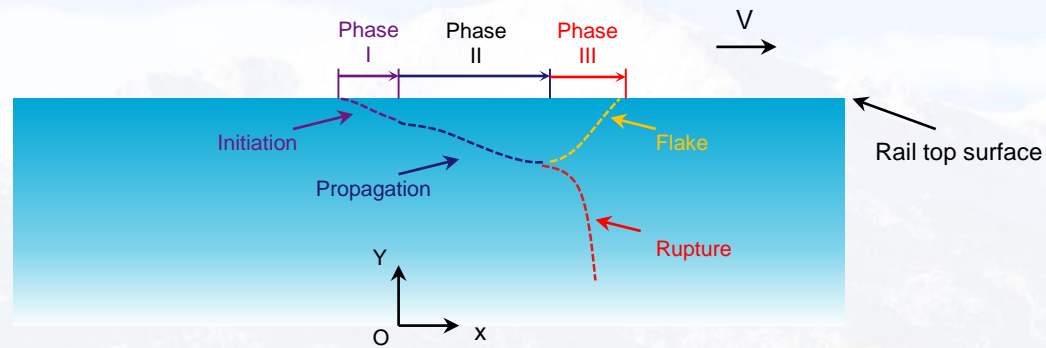


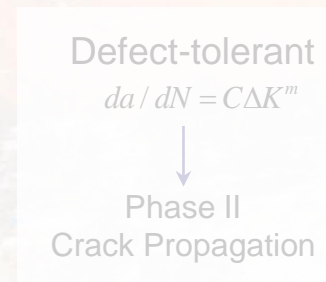
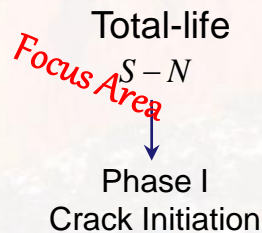
Fig 1. Rail surface fatigue crack growing process on longitudinal-vertical plane<sup>[1]</sup>.

Phase I: Micro-crack length of about 3-5 grains (0.1-0.5mm).

Phase II: Stable macro crack growth rate.

Phase III: Structural instability.

## ➤ Classical approaches to fatigue<sup>[2]</sup>



Ref. [1] On propagation of short rolling contact fatigue cracks. J. W. Ringsberg, Fatigue & Fracture of Engineering Materials, 2003.

[2] Fatigue of materials, S. Suresh, 1998.

**Question:** FE tool >> crack initiation analysis in the context of crack length less than 0.5mm ?



# Research Motivation

## ➤ Sub-modelling Technique

Sub-modelling technique known is based on St. Venant's principle, which states: the difference in effects due to two statically equivalent loadings becomes insignificant as the distance from the load application increases. **(Static analysis approach)**

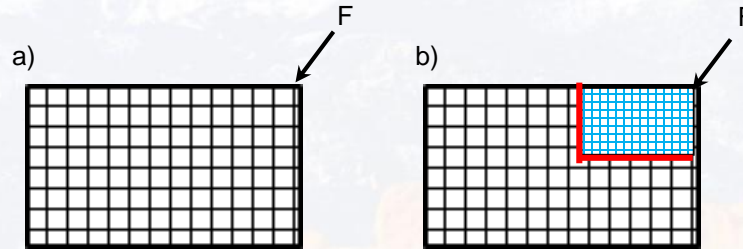






Fig. 2. schematic graph of sub-modelling problem. a)  coarse meshed full-model ; b)  refined sub-model

**Feature:** Element size can be reduced to less than 0.5mm.

## ➤ Challenges

- ❖ Submodeling approach **Static**   **Dynamic loads**
- ❖ **Multiple** crack initiation analysis approaches.

## ➤ Aim

Overcome the incompatibility and choose the appropriate crack analysis approaches.

# Outline

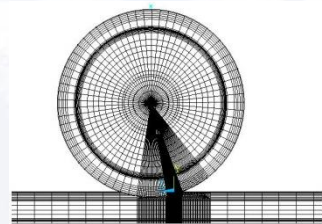
- ✓ Research Motivation
  
- ✓ Sub-modelling & Results
  - ❖ Sub-modelling analysis procedure
  - ❖ Comparison and Boundary verification
  - ❖ Stress/strain response



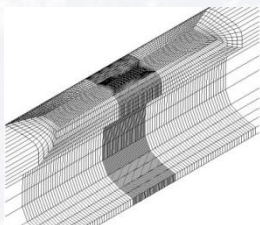


# Sub-modelling & Results

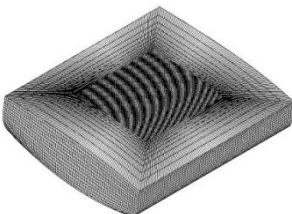
- Sub-modelling analysis procedure:



Dynamic model



Full model



Sub model

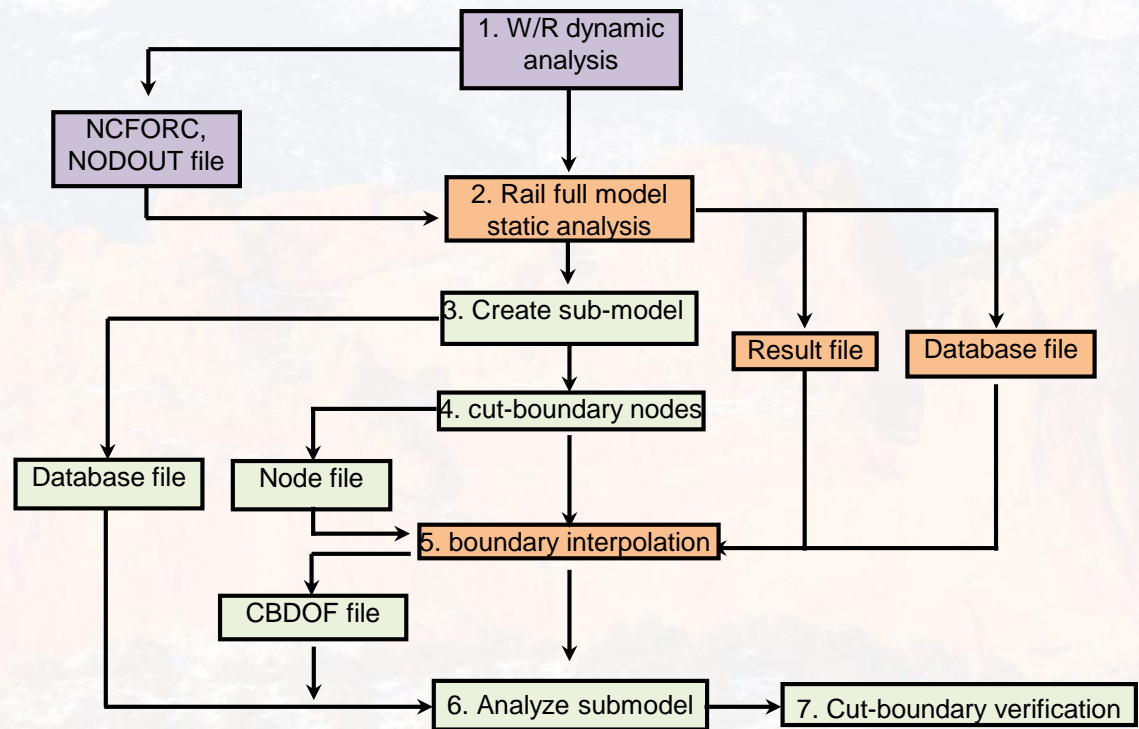
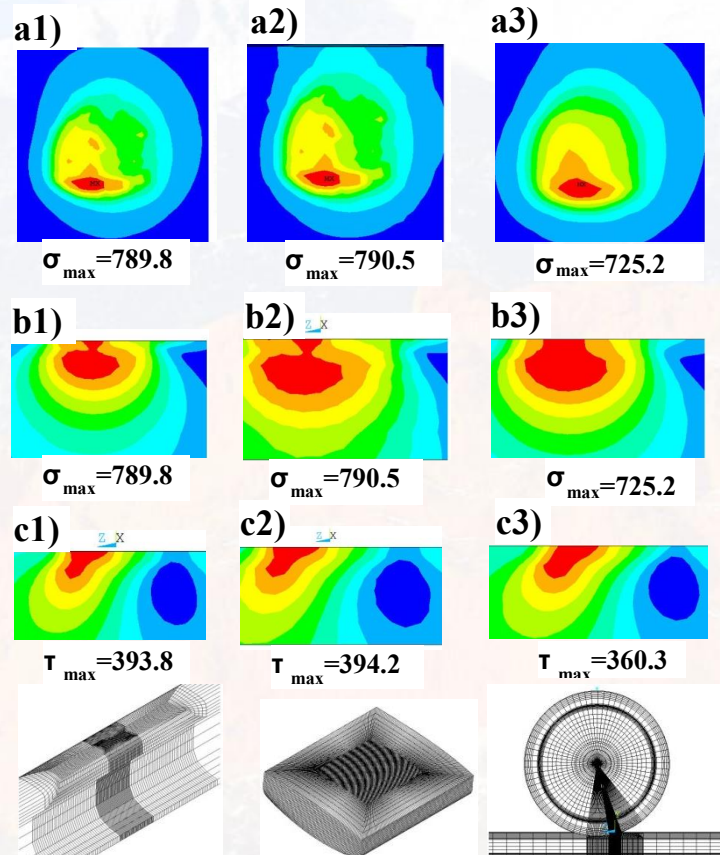


Fig.3 Flow diagram of sub-modelling analysis procedure.

# Sub-modelling & Results

## Comparison and Boundary verification



### Notation:

“a”: Von Mises stress on rail surface >> top view;  
 “b”: Von Mises stress >> longitudinal-vertical plane;  
 “c”: shear stress >> longitudinal-vertical plane;  
 “1”, “2” & “3”: results from full-, sub- and dynamic model respectively.

**Consistent** results between full- and sub-model;

**Slight deviation** with dynamic-model;

Fig. 4. Stress distribution (Unit: MPa) from full-, sub- and dynamic-model.



# Sub-modelling & Results

- ❑ Comparison and Boundary verification

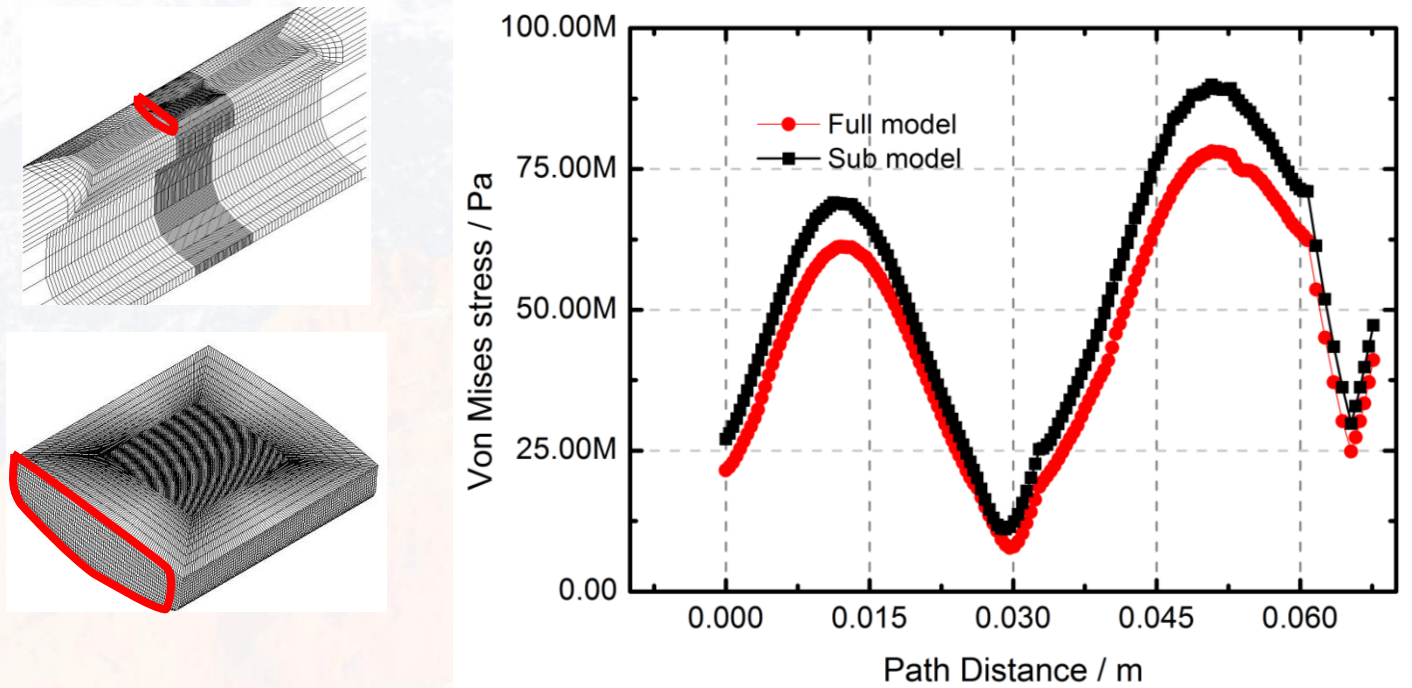
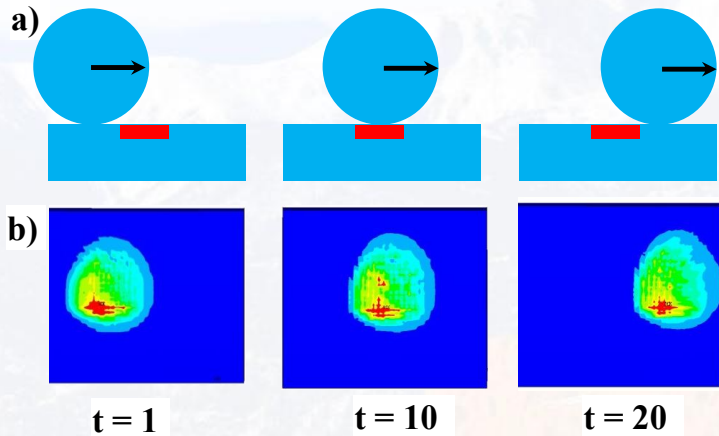


Fig.5 Von-Mises stress variation comparison on the cross sectional boundaries of both sub-model and coarse model.

**Proper Sub-model dimension (selected)**

# Sub-modelling & Results

□ Stress & strain response:



Moving contact loads on the sub-model;

Jump from dynamic to static analysis;

Fig. 6. a) Schematic of wheel-rail rolling contact; b) Von Mises stress distribution on the rail top surface under moving contact loads. (Notation: "t" means time step in sub-modelling analysis).

Potential crack initiation site (Most stressed element according to Von Mises yield criterion)

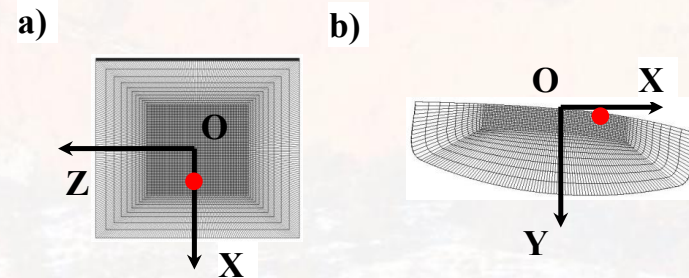


Fig. 7. location of the material point (denoted by red solid circle) on the rail top surface. a) top view; b) cross sectional view;

# Sub-modelling & Results

## □ Stress & strain response:

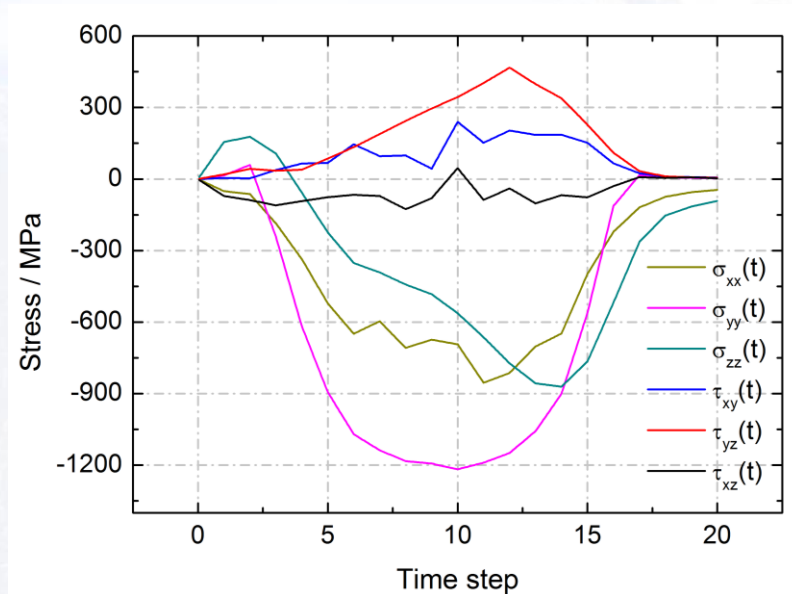


Fig. 8. stress-time history of material point.

Comparable with stress history;

Out-of-phase, multi-axial >>  
Potential crack initiation plane ? ;

Maximum normal stress > 1200MPa (Yield stress 480MPa);

Non-proportional (Out-of-phase), multi-axial;

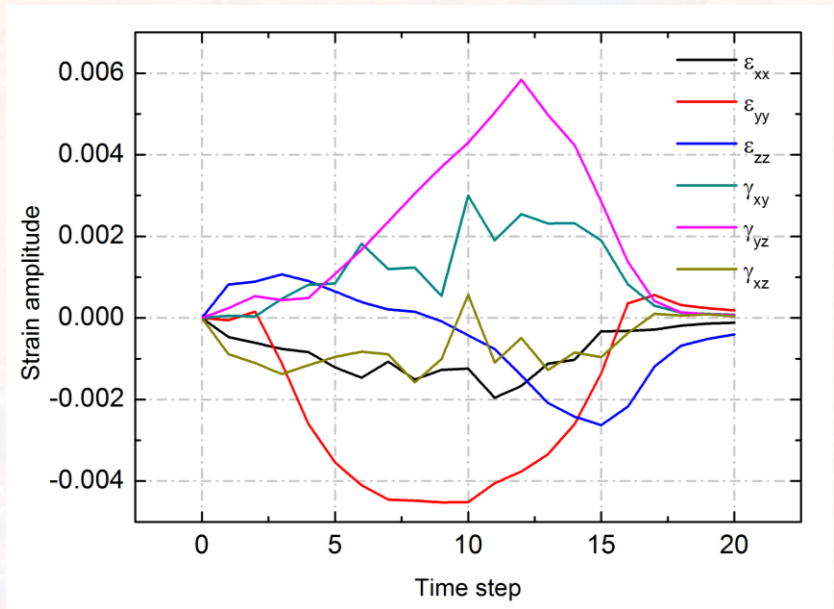
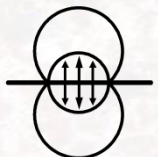


Fig. 9 strain-time history of material point.





# Outline

- ✓ Research Motivation
- ✓ Sub-modelling & Results
- ✓ Crack initiation analysis
  - ❖ Critical plane approach
  - ❖ Crack orientation



# Crack initiation analysis

- Critical plane approach:

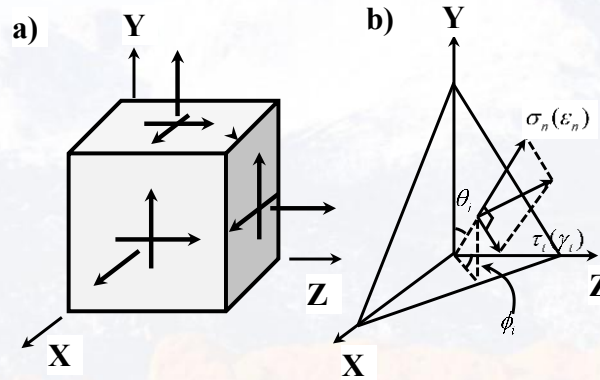


Fig. 10 a) an arbitrary 3D stress/strain state of a point on the rail surface; b) an arbitrary material plane  $\Delta$

Smith, Watson and Topper(SWT):

$$\text{Maximum normal strain energy: } FP_{\max} = \max_{\Delta} \left( \sigma_n \frac{\Delta \epsilon}{2} \right)$$

Fatemi - Soice (FS):

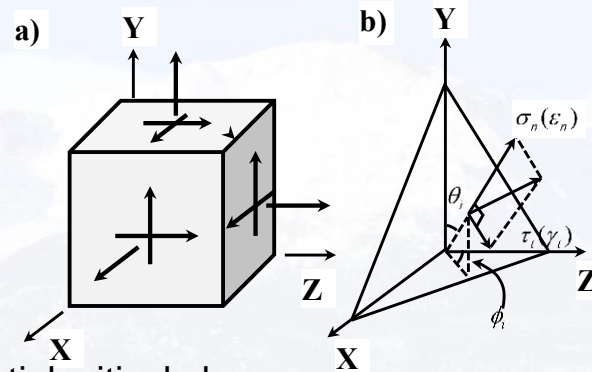
$$\text{Maximum shear strain: } FP_{\max} = \max_{\Delta} \left[ \frac{\Delta \gamma_{\max}}{2} \left( 1 + k \frac{\sigma_{n,\max}}{\sigma_y} \right) \right]$$

Jiang - Sehitoglu (JS):

$$\text{Maximum shear and normal strain energy: } FP_{\max} = \max_{\Delta} \left( \langle \sigma_{\max} \rangle \frac{\Delta \epsilon}{2} + J \Delta \gamma \Delta \tau \right)$$

# Crack initiation analysis

- Solving procedure:



Step 1: Normal vector of the potential critical plane:

$$\mathbf{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \sin(\theta_i) \cos(\phi_i) \\ \cos(\theta_i) \\ \sin(\theta_i) \sin(\phi_i) \end{bmatrix}$$

Step 2: Stress/strain components on the critical plane:

$$\sigma_n = \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}, \quad \tau_t = \|\boldsymbol{\sigma} \cdot \mathbf{n} - \sigma_n \mathbf{n}\|$$

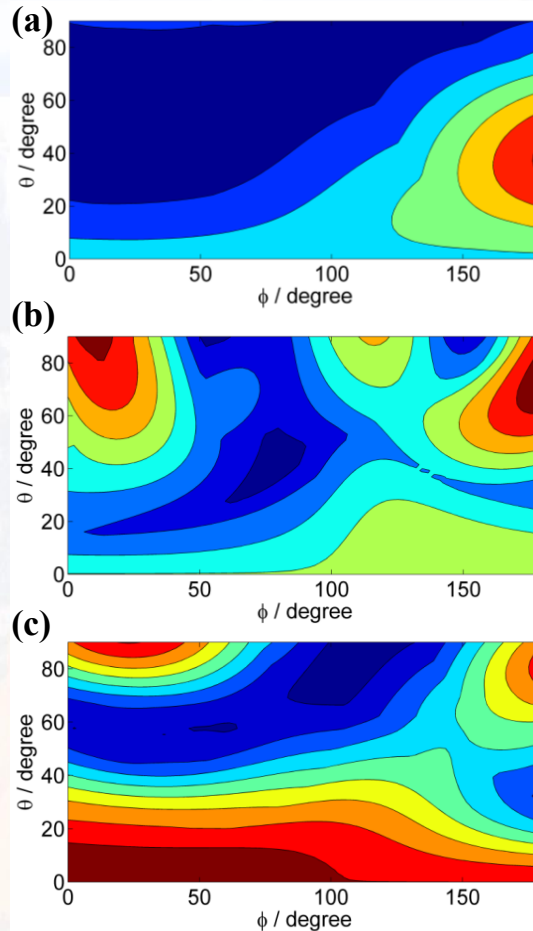
$$\epsilon_n = \mathbf{n} \cdot \boldsymbol{\epsilon} \cdot \mathbf{n}, \quad \gamma_t = \|\boldsymbol{\epsilon} \cdot \mathbf{n} - \epsilon_n \mathbf{n}\|$$

Step 3: Identify the critical plane with maximum fatigue parameters.



# Crack initiation analysis

## □ Crack orientation:



SWT model are completely different from the other two models.

1): Compressive normal stress/strain state of W/R interaction will obstruct the crack to grow, the feasibility of SWT criteria on crack initiation analysis is doubted.

2): The other two models, the fatigue parameter distribution is similar but still slight difference exists. That is because that FS model is based on **maximum shear strain theory**, while JS model is depending on **superposition of positive normal strain energy and shear strain energy**.

Fig. 11. Fatigue parameter variation with plane orientation during one load-cycle. a) SWT model; b) FS model; c) JS model. (Color notation: from dark red to dark blue, the possibility of crack initiation is gradually decreasing).

# Crack initiation analysis

□ Crack orientation:

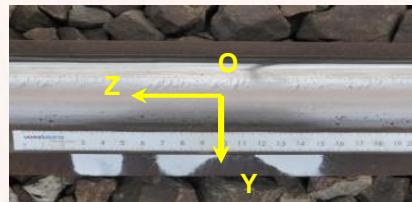
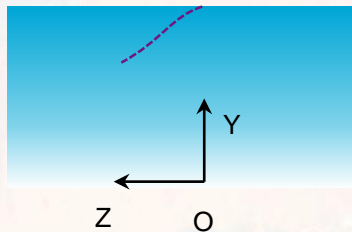
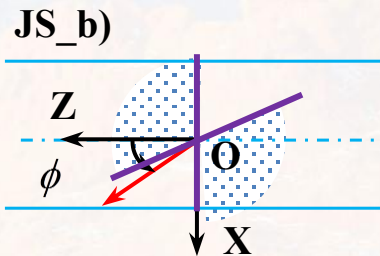
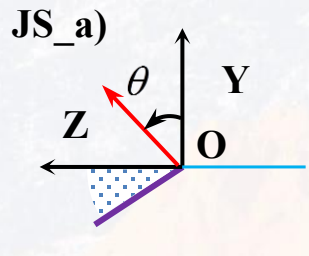
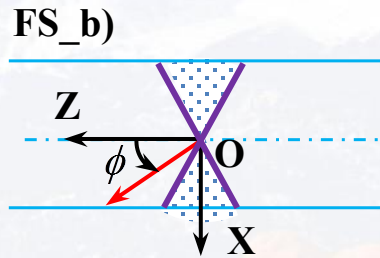
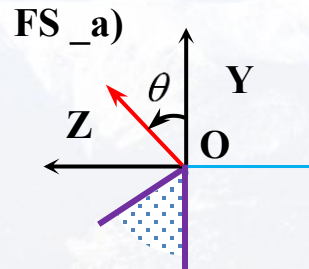



Fig. 12. Crack initiation on the rail surface. “a” represents crack orientation on rail longitudinal cross section; blue line is rail top surface; “b” refers to crack orientation on rail surface.  denotes the potential crack initiation region.

FS model:  $\phi \in [0, 20] \& [160, 180]$

$\theta \in [60, 90]$

JS model:  $\phi \in [0, 100]$

$\theta \in [0, 20]$

Initiated crack orientation are likely to fall in the **calculated domain**;

More supports from **field observations or lab tests** are required;

# Conclusions

- With the application of sub-modelling technique, **Refined element size =  $0.25\text{mm} < 0.5\text{mm}$** , which is suitable for crack initiation analysis.
- **Dynamic contact loads** instead of static one are employed for crack initiation analysis.
- **Multi-axial stress/strain response** can be captured through sub-model analysis; Based on the results, **crack orientation** can be predicted using critical plane approach.

# Outlooks

- Cyclic hysteresis loop >> fatigue life prediction. (**One load cycle limitation**)
- Parametric studies on different loading conditions on crack orientation. (**different friction coefficient, braking/ accelerating, contact point**)
- Crack **propagation** in sub-modelling analysis.
- Field observation or lab test to **verify** the obtained results.





Thank you for listening

