Ultra-high performance fiber-reinforced concrete in incremental bridge launching

A study on the application of UHPFRC in the superstructure of an incrementally launched box girder bridge in The Netherlands

Master's Thesis – Appendices

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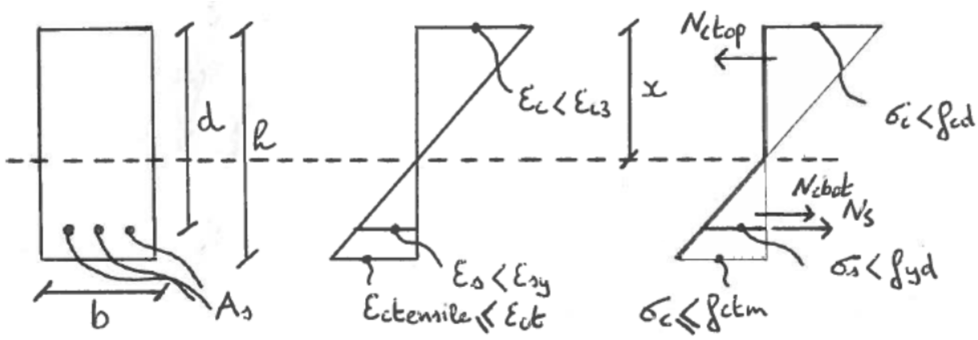


Figure 1.1: deformation and stress diagram when $\varepsilon_{ctensile} \leq \varepsilon_{ct}$.

1.1 Uncracked beam

The beam will remain uncracked as long as the tensile stress doesn't exceed the tensile strength: $\varepsilon_{ctensile} < \varepsilon_{ct}$.

With respect to the deformation and stress diagram of [Figure 1.1] the strain in the steel reinforcement bars ε_s and the concrete tensile strain $\varepsilon_{ctensile}$ can be expressed as:

$$\varepsilon_s = \frac{d-x}{x}\varepsilon_c$$

$$\varepsilon_{ctensile} = \frac{h-x}{x}\varepsilon_c$$

With above two equations and ε_c as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{ctop} = N_s + N_{cbot} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\varepsilon_s + \frac{1}{2}b(h-x)E_c\varepsilon_{ctensile} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\frac{d-x}{x}\varepsilon_c + \frac{1}{2}b(h-x)E_c\frac{h-x}{x}\varepsilon_c \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_s(d-x) + \frac{1}{2}b(h-x)^2E_c \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_sd - A_sE_sx + \frac{1}{2}bh^2E_c + \frac{1}{2}bx^2E_c - bhxE_c \rightarrow$$

$$A_sE_sx + bhxE_c = A_sE_sd + \frac{1}{2}bh^2E_c \rightarrow$$

$$x = \frac{A_sE_sd + \frac{1}{2}bh^2E_c}{A_sE_s + bhE_c}$$

The corresponding bending moment capacity and curvature:

$$M = N_s\left(d - \frac{1}{3}x\right) + N_{cbot} \cdot \frac{2}{3}h = A_sE_s\varepsilon_s\left(d - \frac{1}{3}x\right) + \frac{1}{2}b(h-x)E_c\varepsilon_{ctensile} \cdot \frac{2}{3}h$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d}$$

1.2 The cracking moment

The cracking moment M_{cr} is reached at a concrete tensile strain $\varepsilon_{ctensile} = \varepsilon_{ct}$ and a curvature κ_{cr}. The strain in the concrete ε_c and steel ε_s can be derived from [Figure 1.1].

$$\varepsilon_{ct} = \frac{f_{ctm}}{E_c}$$

$$\varepsilon_s = \frac{d-x}{h-x}\varepsilon_{ct}$$

$$\varepsilon_c = \frac{x}{h-x}\varepsilon_{ct}$$

With the equations above and ε_{ct} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{ctop} = N_s + N_{cbot} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\varepsilon_s + \frac{1}{2}b(h-x)E_c\varepsilon_{ct} \rightarrow$$

$$\frac{1}{2}bxE_c\frac{x}{h-x}\varepsilon_{ct} = A_sE_s\frac{d-x}{h-x}\varepsilon_{ct} + \frac{1}{2}b(h-x)E_c\varepsilon_{ct} \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_s(d-x) + \frac{1}{2}b(h-x)^2E_c \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_sd - A_sE_sx + \frac{1}{2}bh^2E_c + \frac{1}{2}bx^2E_c - bhxE_c \rightarrow$$

$$(A_sE_s + bhE_c)x = A_sE_sd + \frac{1}{2}bh^2E_c \rightarrow$$

$$x = \frac{A_sE_sd + \frac{1}{2}bh^2E_c}{A_sE_s + bhE_c}$$

The cracking moment and the corresponding curvature:

$$M_{cr} = N_s\left(d - \frac{1}{3}x\right) + N_{cbot}\cdot\frac{2}{3}h = A_sE_s\varepsilon_s\left(d - \frac{1}{3}x\right) + \frac{1}{2}b(h-x)E_c\varepsilon_{ct}\cdot\frac{2}{3}h$$

$$\kappa_{cr} = \frac{\varepsilon_c + \varepsilon_s}{d}$$

1.3 Cracked beam

When the load is further increased, concrete cracking starts to occur and the steel reinforcing bars will provide tensile capacity to restore equilibrium. $\varepsilon_{ctensile}$ becomes zero.

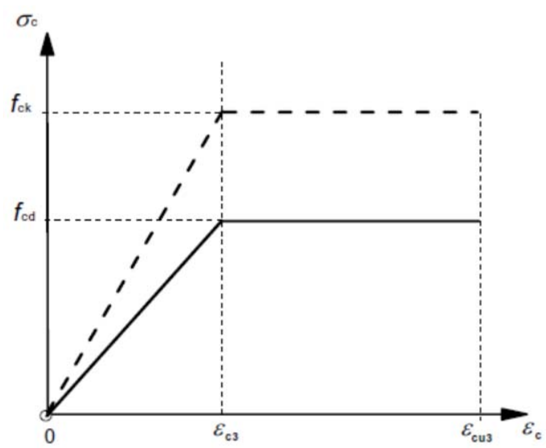


Figure 1.2: bi-linear stress-strain relation for NSC and HSC in compression (NEN-EN 1992-1-1).

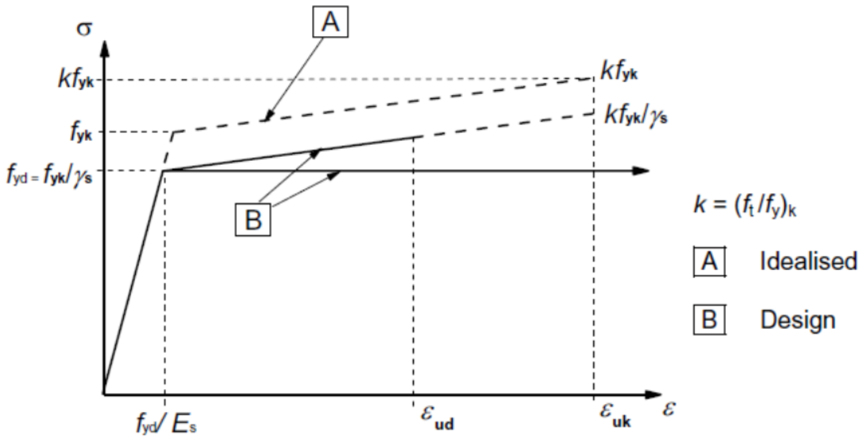


Figure 1.3: stress-strain diagram for reinforcing steel (NEN-EN 1992-1-1).

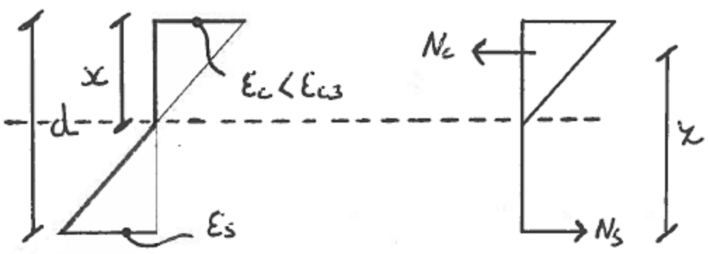


Figure 1.4: deformation and stress diagram for elastic material behavior ($\varepsilon_c < \varepsilon_{c3}$) and $\left(\varepsilon_s < \varepsilon_{sy}\right)$.

With respect to the deformation and stress diagram of [Figure 1.4] the strain in the reinforcement bars can be expressed as:

$$\varepsilon_s = \frac{d-x}{x}\varepsilon_c$$

With the equation above and ε_c as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\varepsilon_s \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\frac{d-x}{x}\varepsilon_c \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_s(d-x) \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_sd - A_sE_sx \rightarrow$$

$$\frac{1}{2}bx^2E_c + A_sE_sx - A_sE_sd = 0 \rightarrow$$

$$x = \frac{-A_sE_s + \sqrt{(A_sE_s)^2 - 4 \cdot \frac{1}{2}bE_c \cdot -A_sE_sd}}{bE_c}$$

The corresponding bending moment capacity and curvature:

$$M = N_s\left(d - \frac{1}{3}x\right) = A_sE_s\varepsilon_s\left(d - \frac{1}{3}x\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d}$$

1.4 The yield moment $(M_y < M_{c,pl})$

The concrete strength class and the amount of steel reinforcing bars determine whether concrete or steel will reach the plastic phase first. The steel reinforcing bars will start to yield at a yield strain $\varepsilon_{sy} = 2{,}17‰$. Concrete starts to become plastic when $\varepsilon_c = \varepsilon_{c3}$. **It is now assumed that the reinforcement ratio is not too high and the steel yields first**.

$$\varepsilon_s = \varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{d - x}\varepsilon_s = \frac{x}{d - x}\varepsilon_{sy}$$

With the equation above and ε_{sy} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\varepsilon_s \rightarrow$$

$$\frac{1}{2}bxE_c\frac{x}{d - x}\varepsilon_{sy} = A_sE_s\varepsilon_{sy} \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_s(d-x) \rightarrow$$

$$\frac{1}{2}bx^2E_c + A_sE_sx - A_sE_sd = 0 \rightarrow$$

$$x = \frac{-A_sE_s + \sqrt{(A_sE_s)^2 - 4 \cdot \frac{1}{2}bE_c \cdot -A_sE_sd}}{bE_c}$$

The yield moment can be expressed by:

$$M_y = N_s\left(d - \frac{1}{3}x\right) = A_sf_{yd}\left(d - \frac{1}{3}x\right)$$

The corresponding curvature:

$$\kappa_y = \frac{\varepsilon_c + \varepsilon_{sy}}{d}$$

After the yield moment is reached the moment capacity further increases until the concrete compressive strain reaches ε_{c3}. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\varepsilon_s > \varepsilon_{sy}$$

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\varepsilon_s \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\varepsilon_{sy} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sf_{yd} \rightarrow$$

$$x = \frac{2A_sf_{yd}}{bE_c\varepsilon_c}$$

The corresponding bending moment and curvature:

$$M = A_s f_{yd}\left(d - \frac{1}{3}x\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d}$$

1.5 The plastic moment $\left(M_y < M_{c,pl}\right)$

The plastic moment occurs when $\varepsilon_c = \varepsilon_{c3}$. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\varepsilon_s > \varepsilon_{sy}$$

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_c = A_s E_s \varepsilon_s \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_{c3} = A_s E_s \varepsilon_{sy} \rightarrow$$

$$\frac{1}{2} b x f_{cd} = A_s f_{yd} \rightarrow$$

$$x = \frac{2 A_s f_{yd}}{b f_{cd}}$$

The plastic moment can be expressed by:

$$M_{c,pl} = A_s f_{yd}\left(d - \frac{1}{3}x\right)$$

The corresponding curvature:

$$\kappa_{c,pl} = \frac{\varepsilon_{c3} + \varepsilon_s}{d}$$

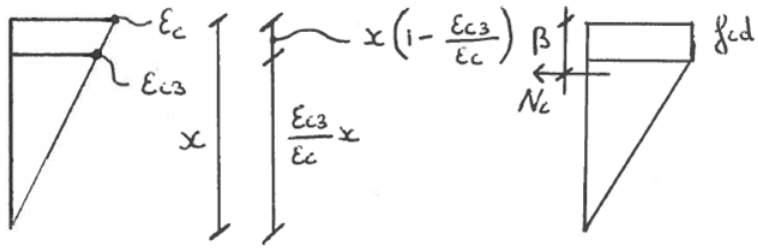


Figure 1.5: deformation and stress diagram when $\varepsilon_c > \varepsilon_{c3}$.

When $\varepsilon_c > \varepsilon_{c3}$ [Figure 1.5] is valid. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$N_c = \frac{1}{2}\frac{\varepsilon_{c3}}{\varepsilon_c} x f_{cd} + x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) f_{cd} = bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)$$

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_s f_{yd} \rightarrow$$

$$x = \frac{A_s f_{yd}}{bf_{ck}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)}$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta = \frac{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) \cdot \frac{x}{2}\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{b}{2}\frac{\varepsilon_{c3}}{\varepsilon_c} x \cdot \left(x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{\varepsilon_{c3}}{\varepsilon_c}\frac{x}{3}\right)}{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{bx}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}}$$

1.6 The ultimate bending moment resistance

The ultimate bending moment resistance can be derived when $\varepsilon_c = \varepsilon_{cu3}$:

$$M_{Rd} = A_s f_{yd}(d - \beta)$$

The corresponding curvature:

$$\kappa_{Rd} = \frac{\varepsilon_{cu3} + \varepsilon_s}{d}$$

1.7 The plastic moment $\left(M_{c,pl} < M_y\right)$

The concrete strength class and the amount of steel reinforcing bars determine whether concrete or steel will reach the plastic phase first. The steel reinforcing bars will start to yield at a yield strain $\varepsilon_{sy} = 2{,}17‰$. Concrete starts to become plastic from $\varepsilon_c = \varepsilon_{c3}$. **It is now assumed that the reinforcement ratio is high and the concrete will reach the plastic phase first**.

$$\varepsilon_c = \varepsilon_{c3}$$

$$\varepsilon_s = \frac{d-x}{x}\varepsilon_c = \frac{d-x}{x}\varepsilon_{c3}$$

With the equation above and ε_{c3} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c = A_sE_s\varepsilon_s \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_{c3} = A_sE_s\frac{d-x}{x}\varepsilon_{c3} \rightarrow$$

$$\frac{1}{2}bx^2E_c = A_sE_s(d-x) \rightarrow$$

$$\frac{1}{2}bx^2E_c + A_sE_sx - A_sE_sd = 0 \rightarrow$$

$$x = \frac{-A_sE_s + \sqrt{(A_sE_s)^2 - 4\cdot\frac{1}{2}bE_c\cdot -A_sE_sd}}{bE_c}$$

The plastic moment can be expressed by:

$$M_{c,pl} = N_s\left(d - \frac{1}{3}x\right) = A_s E_s \varepsilon_s \left(d - \frac{1}{3}x\right)$$

The corresponding curvature:

$$\kappa_{c,pl} = \frac{\varepsilon_{c3} + \varepsilon_s}{d}$$

1.8 The yield moment $\left(M_{c,pl} < M_y\right)$

After the plastic moment is reached the moment capacity further increases until the steel strain ε_s reaches ε_{sy}.

$$\varepsilon_s = \varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{d - x}\varepsilon_s = \frac{x}{d - x}\varepsilon_{sy}$$

When $\varepsilon_c > \varepsilon_{c3}$ [Figure 1.5] is valid. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$N_c = \frac{1}{2}\frac{\varepsilon_{c3}}{\varepsilon_c} x f_{cd} + x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) f_{cd} = b x f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)$$

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$b x f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_s E_s \varepsilon_s \rightarrow$$

$$b x f_{cd} - b x f_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} = A_s E_s \varepsilon_{sy} \rightarrow$$

$$2 b x f_{cd} \varepsilon_c - b x f_{cd} \varepsilon_{c3} = 2 A_s f_{yd} \varepsilon_c \rightarrow$$

$$2 b x f_{cd} \frac{x}{d - x}\varepsilon_{sy} - b x f_{cd} \varepsilon_{c3} = 2 A_s f_{yd} \frac{x}{d - x}\varepsilon_{sy} \rightarrow$$

$$2 b x f_{cd} \varepsilon_{sy} - b(d - x) f_{cd} \varepsilon_{c3} = 2 A_s f_{yd} \varepsilon_{sy} \rightarrow$$

$$bf_{cd}(2\varepsilon_{sy} + \varepsilon_{c3})x = 2A_s f_{yd}\varepsilon_{sy} + bdf_{cd}\varepsilon_{c3} \rightarrow$$

$$x = \frac{2A_s f_{yd}\varepsilon_{sy} + bdf_{cd}\varepsilon_{c3}}{bf_{cd}(2\varepsilon_{sy} + \varepsilon_{c3})}$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta = \frac{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) \cdot \frac{x}{2}\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{b}{2}\frac{\varepsilon_{c3}}{\varepsilon_c} x \cdot \left(x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{\varepsilon_{c3}}{\varepsilon_c}\frac{x}{3}\right)}{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{bx}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}}$$

The yield moment can be expressed by:

$$M_y = N_s(d - \beta) = A_s f_{yd}(d - \beta)$$

The corresponding curvature:

$$\kappa_y = \frac{\varepsilon_c + \varepsilon_{sy}}{d}$$

When $\boldsymbol{\varepsilon_s < \varepsilon_{sy}}$:

$$\varepsilon_s = \frac{d - x}{x}\varepsilon_c$$

The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_s E_s \varepsilon_s \rightarrow$$

$$bxf_{cd} - bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} = A_s E_s \frac{d - x}{x}\varepsilon_c \rightarrow$$

$$2bxf_{cd}\varepsilon_c - bxf_{cd}\varepsilon_{c3} = 2A_s E_s \frac{d - x}{x}\varepsilon_c^2 \rightarrow$$

$$2bx^2 f_{cd}\varepsilon_c - bx^2 f_{cd}\varepsilon_{c3} = 2A_s E_s(d - x)\varepsilon_c^2 \rightarrow$$

$$bf_{cd}(2\varepsilon_c - \varepsilon_{c3})x^2 + 2A_sE_s\varepsilon_c^2 x - 2A_sE_s\varepsilon_c^2 d = 0 \rightarrow$$

$$x = \frac{-2A_sE_s\varepsilon_c^2 + \sqrt{(2A_sE_s\varepsilon_c^2)^2 - 4 \cdot bf_{cd}(2\varepsilon_c - \varepsilon_{c3}) \cdot -2A_sE_s\varepsilon_c^2 d}}{2bf_{cd}(2\varepsilon_c - \varepsilon_{c3})}$$

The corresponding bending moment and curvature:

$$M = A_sE_s\varepsilon_s(d - \beta)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d}$$

When $\boldsymbol{\varepsilon_s \geq \varepsilon_{sy}}$:

The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c = N_s \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_sE_s\varepsilon_s \rightarrow$$

$$bxf_{cd} - bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} = A_sE_s\varepsilon_{sy} \rightarrow$$

$$2bxf_{cd}\varepsilon_c - bxf_{cd}\varepsilon_{c3} = 2A_sf_{yd}\varepsilon_c \rightarrow$$

$$bf_{cd}(2\varepsilon_c - \varepsilon_{c3})x = 2A_sf_{yd}\varepsilon_c \rightarrow$$

$$x = \frac{2A_sf_{yd}\varepsilon_c}{bf_{cd}(2\varepsilon_c - \varepsilon_{c3})}$$

The corresponding bending moment and curvature:

$$M = A_sf_{yd}(d - \beta)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d}$$

1.9 Shear

The design shear resistance of the member without shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$v_{min} = 0{,}035k^{\frac{3}{2}}\sqrt{f_{ck}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$V_{Rd,cmin} = (v_{min} + k_1\sigma_{cp})b_w d$$

$$k_1 = 0{,}15$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0{,}2f_{cd}$$

$$V_{Rd,c} = \left[C_{Rd,c}k\sqrt[3]{100\rho_l f_{ck}} + k_1\sigma_{cp}\right]b_w d$$

$$C_{Rd,c} = 0{,}18/\gamma_c$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$$

The design value of the shear force, which can be sustained by the yielding shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta$$

$$z = d - \beta$$

$$f_{ywd} = 0{,}8f_{yk}$$

$$1 \leq \cot\theta \leq 2{,}5$$

The design value of the maximum shear force, which can be sustained by the member, limited by crushing of the compression struts [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,max} = \frac{\alpha_{cw} b_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta}$$

For non-prestressed structures: $\alpha_{cw} = 1$

For: $f_{ck} \leq 60\ N/mm^2$: $\nu_1 = 0{,}6$

For: $f_{ck} \geq 90\ N/mm^2$: $\nu_1 = 0{,}9 - \frac{f_{ck}}{200}$

1.10 Crack width

Determine w_{max} from [NEN-EN 1992-1-1: Table 7.1N]:

$$w_{max} = w_k$$

$$k_1 = 0{,}8$$

$$k_2 = 0{,}5$$

$$k_3 = 3{,}4$$

$$k_4 = 0{,}425$$

For short term loading: $k_t = 0{,}6$

For long term loading: $k_t = 0{,}4$

$$\alpha_e = \frac{E_s}{E_{cm}}$$

$$f_{ct,eff} = f_{ctm}(t)$$

The beam is in the cracked phase.

For a rectangular reinforced beam that is cracked, the concrete compressive zone height x remains the same under increasing load until the yield moment is reached.

$$x = \frac{-A_s E_s + \sqrt{(A_s E_s)^2 - 4 \cdot \frac{1}{2} b E_c \cdot -A_s E_s d}}{b E_c}$$

With x the maximum allowable steel stress σ_s can be determined:

$$h_{c,eff} = \min\left\{2{,}5(h-d); \frac{h-x}{3}; 0{,}5h\right\}$$

$$A_{c,eff} = b h_{c,eff}$$

$$\rho_{p,eff} = \frac{A_s}{A_{c,eff}}$$

$$s_{r,max} = k_3 c + \frac{k_1 k_2 k_4 \phi}{\rho_{p,eff}}$$

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \rightarrow \varepsilon_{sm} - \varepsilon_{cm} = \frac{w_k}{s_{r,max}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \rightarrow$$

$$\sigma_s = E_s(\varepsilon_{sm} - \varepsilon_{cm}) + k_t \frac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)$$

With the maximum allowable steel stress σ_s and [Figure 1.4] the maximum moment in SLS can be calculated.

$$\varepsilon_s = \frac{\sigma_s}{E_s}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d-x} \rightarrow \varepsilon_c = \varepsilon_s \frac{x}{d-x}$$

$$N_c = \frac{1}{2} b x E_c \varepsilon_c$$

$$N_s = A_s E_s \varepsilon_s$$

$$M_{qp} = A_s E_s \varepsilon_s \left(d - \frac{1}{3}x\right)$$

1.11 Concrete compressive zone height

This paragraph is based on [NEN-EN 1992-1-1+C2/NB: 6.1].

For: $f_{ck} \leq 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{500}{500 + f}$$

For: $f_{ck} > 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{\varepsilon_{cu} \cdot 10^6}{\varepsilon_{cu} \cdot 10^6 + 7f}$$

$$f = \frac{\left(\frac{f_{pk}}{\gamma_s} - \sigma_{pm\infty}\right) A_p + f_{yd} A_s}{A_p + A_s}$$

2 Rectangular doubly reinforced NSC beam

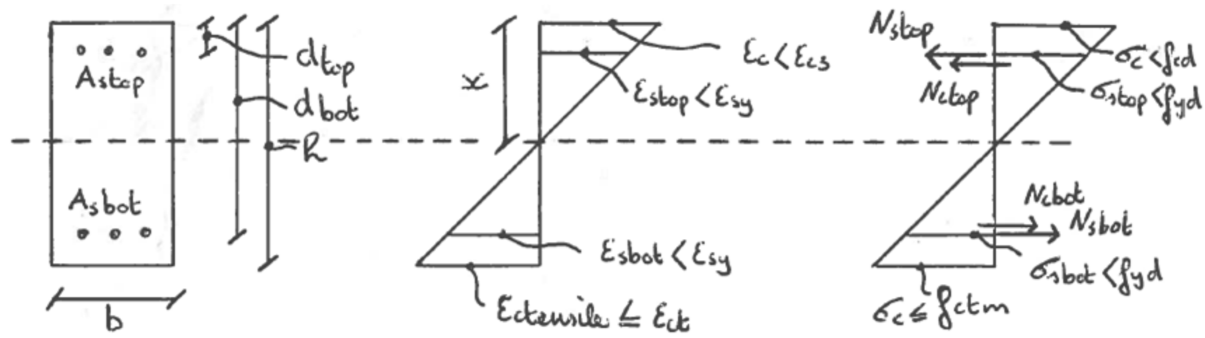


Figure 2.1: deformation and stress diagram when $\varepsilon_{ctensile} \leq \varepsilon_{ct}$.

2.1 Uncracked beam

The beam will remain uncracked as long as the concrete tensile strain $\varepsilon_{ctensile}$ doesn't exceed ε_{ct}.

With respect to the deformation and stress diagram of [Figure 2.1] the strain in the top and bottom steel reinforcement bars and the concrete tensile strain can be expressed as:

$$\varepsilon_{stop} = \frac{x - d_{top}}{x} \varepsilon_c$$

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x} \varepsilon_c$$

$$\varepsilon_{ctensile} = \frac{h - x}{x} \varepsilon_c$$

With above three equations and ε_c as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{ctop} + N_{stop} = N_{sbot} + N_{cbot} \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_c + A_{stop} E_s \varepsilon_{stop} = A_{sbot} E_s \varepsilon_{sbot} + \frac{1}{2} b(h - x) E_c \varepsilon_{ctensile} \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_c + A_{stop} E_s \frac{x - d_{top}}{x} \varepsilon_c = A_{sbot} E_s \frac{d_{bot} - x}{x} \varepsilon_c + \frac{1}{2} b(h - x) E_c \frac{h - x}{x} \varepsilon_c \rightarrow$$

$$\frac{1}{2}bx^2E_c + A_{stop}E_s(x - d_{top}) = A_{sbot}E_s(d_{bot} - x) + \frac{1}{2}b(h-x)^2E_c \rightarrow$$

$$\frac{1}{2}bx^2E_c + A_{stop}E_sx - A_{stop}E_sd_{top} = A_{sbot}E_sd_{bot} - A_{sbot}E_sx + \frac{1}{2}bh^2E_c + \frac{1}{2}bx^2E_c - bhxE_c \rightarrow$$

$$A_{stop}E_sx + A_{sbot}E_sx + bhxE_c = A_{stop}E_sd_{top} + A_{sbot}E_sd_{bot} + \frac{1}{2}bh^2E_c \rightarrow$$

$$x\left(E_s\left(A_{stop} + A_{sbot}\right) + bhE_c\right) = E_s\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) + \frac{1}{2}bh^2E_c \rightarrow$$

$$x = \frac{E_s\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) + \frac{1}{2}bh^2E_c}{E_s\left(A_{stop} + A_{sbot}\right) + bhE_c}$$

The corresponding bending moment capacity and curvature:

$$M = N_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + N_{cbot} \cdot \frac{2}{3}h + N_{stop}\left(\frac{1}{3}x - d_{top}\right) =$$
$$A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + \frac{1}{2}b(h-x)E_c\varepsilon_{ctensile} \cdot \frac{2}{3}h + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

2.2 The cracking moment

The cracking moment M_{cr} is reached at a concrete tensile strain ε_{ct} and a curvature κ_{cr}. The strain in the concrete and steel reinforcing bars can be derived using the strain diagram drawn in [Figure 2.1].

$$\varepsilon_{ct} = \frac{f_{ctm}}{E_c}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{h - x}\varepsilon_{ct}$$

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{h - x}\varepsilon_{ct}$$

$$\varepsilon_c = \frac{x}{h - x}\varepsilon_{ct}$$

With the previous equations and ε_{ct} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{ctop} + N_{stop} = N_{sbot} + N_{cbot} \rightarrow$$

$$\frac{1}{2} bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + \frac{1}{2} b(h-x)E_c\varepsilon_{ct} \rightarrow$$

$$\frac{1}{2} bxE_c \frac{x}{h-x}\varepsilon_{ct} + A_{stop}E_s \frac{x-d_{top}}{h-x}\varepsilon_{ct} = A_{sbot}E_s \frac{d_{bot}-x}{h-x}\varepsilon_{ct} + \frac{1}{2} b(h-x)E_c\varepsilon_{ct} \rightarrow$$

$$\frac{1}{2} bx^2E_c + A_{stop}E_s\left(x-d_{top}\right) = A_{sbot}E_s(d_{bot}-x) + \frac{1}{2} b(h-x)^2E_c \rightarrow$$

$$\frac{1}{2} bx^2E_c + A_{stop}E_sx - A_{stop}E_sd_{top} = A_{sbot}E_sd_{bot} - A_{sbot}E_sx + \frac{1}{2} bh^2E_c + \frac{1}{2} bx^2E_c - bhxE_c \rightarrow$$

$$A_{stop}E_sx + A_{sbot}E_sx + bhxE_c = A_{stop}E_sd_{top} + A_{sbot}E_sd_{bot} + \frac{1}{2} bh^2E_c \rightarrow$$

$$x\left(E_s\left(A_{stop} + A_{sbot}\right) + bhE_c\right) = E_s\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) + \frac{1}{2} bh^2E_c \rightarrow$$

$$x = \frac{E_s\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) + \frac{1}{2} bh^2E_c}{E_s\left(A_{stop} + A_{sbot}\right) + bhE_c}$$

The cracking moment and the corresponding curvature:

$$M_{cr} = N_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + N_{cbot}\cdot\frac{2}{3}h + N_{stop}\left(\frac{1}{3}x - d_{top}\right) =$$

$$A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + \frac{1}{2} b(h-x)E_c\varepsilon_{ct}\cdot\frac{2}{3}h + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right)$$

$$\kappa_{cr} = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

2.3 Cracked beam

When the load is further increased, concrete cracking starts to occur and the steel reinforcing bars will provide tensile capacity to restore equilibrium. $\varepsilon_{ctensile}$ becomes zero.

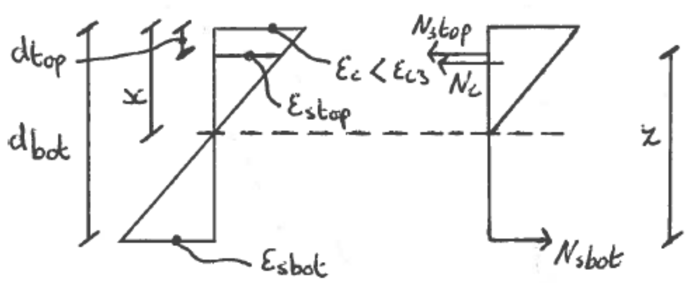


Figure 2.2: deformation and stress for elastic material behavior $(\varepsilon_c < \varepsilon_{c3})$ and $(\varepsilon_s < \varepsilon_{sy})$.

With respect to the deformation and stress diagram of [Figure 2.2] the next three equations are valid:

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x} \varepsilon_c$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{x} \varepsilon_c$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot}$$

With these three equations and ε_c as input, the concrete compressive zone height x can be found.

$$\frac{1}{2} b x E_c \varepsilon_c + A_{stop} E_s \varepsilon_{stop} = A_{sbot} E_s \varepsilon_{sbot} \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_c + A_{stop} E_s \frac{x - d_{top}}{x} \varepsilon_c = A_{sbot} E_s \frac{d_{bot} - x}{x} \varepsilon_c \rightarrow$$

$$\frac{1}{2} b x^2 E_c + A_{stop} E_s (x - d_{top}) = A_{sbot} E_s (d_{bot} - x) \rightarrow$$

$$\frac{1}{2} b x^2 E_c + A_{stop} E_s x - A_{stop} E_s d_{top} = A_{sbot} E_s d_{bot} - A_{sbot} E_s x \rightarrow$$

$$\frac{1}{2} b x^2 E_c + (A_{stop} + A_{sbot}) E_s x - E_s (A_{stop} d_{top} + A_{sbot} d_{bot}) = 0 \rightarrow$$

$$x = \frac{-(A_{stop} + A_{sbot})E_s + \sqrt{\left((A_{stop} + A_{sbot})E_s\right)^2 - 4 \cdot \frac{1}{2} bE_c \cdot -E_s(A_{stop}d_{top} + A_{sbot}d_{bot})}}{bE_c}$$

The corresponding bending moment capacity and curvature:

$$M = N_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + N_{stop}\left(\frac{1}{3}x - d_{top}\right) =$$

$$A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

2.4 The yield moment $(M_{ybot} < M_{c,pl})$

The concrete strength class and the amount of steel reinforcing bars determine whether concrete or steel will reach the plastic phase first. The steel reinforcing bars will start to yield at a yield strain $\varepsilon_{sy} = 2{,}17‰$. Concrete starts to become plastic from $\varepsilon_c = \varepsilon_{c3}$. **It is now assumed that the reinforcement ratio is not too high and the bottom steel reinforcement yields first**.

$$\varepsilon_{sbot} = \varepsilon_{sy}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sbot} = \frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{d_{bot} - x}\varepsilon_{sbot} = \frac{x}{d_{bot} - x}\varepsilon_{sy}$$

With the equations above and ε_{sy} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} \rightarrow$$

$$\frac{1}{2}bxE_c\frac{x}{d_{bot} - x}\varepsilon_{sy} + A_{stop}E_s\frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sy} = A_{sbot}E_s\varepsilon_{sy} \rightarrow$$

$$\frac{1}{2}bx^2E_c + A_{stop}E_s(x - d_{top}) = A_{sbot}E_s(d_{bot} - x) \rightarrow$$

$$\frac{1}{2}bx^2E_c + A_{stop}E_sx - A_{stop}E_sd_{top} = A_{sbot}E_sd_{bot} - A_{sbot}E_sx \rightarrow$$

$$\frac{1}{2}bx^2E_c + (A_{stop} + A_{sbot})E_sx - E_s(A_{stop}d_{top} + A_{sbot}d_{bot}) = 0 \rightarrow$$

$$x = \frac{-(A_{stop} + A_{sbot})E_s + \sqrt{\left((A_{stop} + A_{sbot})E_s\right)^2 - 4 \cdot \frac{1}{2}bE_c \cdot -E_s(A_{stop}d_{top} + A_{sbot}d_{bot})}}{bE_c}$$

The yield moment can be expressed by:

$$M_{ybot} = N_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + N_{stop}\left(\frac{1}{3}x - d_{top}\right) =$$
$$A_{sbot}f_{yd}\left(d_{bot} - \frac{1}{3}x\right) + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right)$$

The corresponding curvature:

$$\kappa_{ybot} = \frac{\varepsilon_c + \varepsilon_{sy}}{d_{bot}}$$

After the yield moment is reached the moment capacity further increases until the concrete compressive strain reaches ε_{c3}. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\varepsilon_{sbot} > \varepsilon_{sy}$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_c = A_{sbot}E_s\varepsilon_{sy} \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + A_{stop}E_s(x - d_{top})\varepsilon_c = A_{sbot}f_{yd}x \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + (A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd})x - A_{stop}E_s\varepsilon_c d_{top} = 0 \rightarrow$$

$$x = \frac{-(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd}) + \sqrt{(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd})^2 - 4 \cdot \frac{1}{2}bE_c\varepsilon_c \cdot -A_{stop}E_s\varepsilon_c d_{top}}}{bE_c\varepsilon_c}$$

The corresponding bending moment capacity and curvature:

$$M = A_{sbot}f_{yd}\left(d_{bot} - \frac{1}{3}x\right) + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

2.5 The plastic moment $\left(M_{ybot} < M_{c,pl}\right)$

The plastic moment occurs when $\varepsilon_c = \varepsilon_{c3}$. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\varepsilon_{sbot} > \varepsilon_{sy}$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_{c3} + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_{c3} = A_{sbot}E_s\varepsilon_{sy} \rightarrow$$

$$\frac{1}{2}bx^2f_{cd} + A_{stop}E_s(x - d_{top})\varepsilon_{c3} = A_{sbot}f_{yd}x \rightarrow$$

$$\frac{1}{2}bx^2f_{cd} + (A_{stop}E_s\varepsilon_{c3} - A_{sbot}f_{yd})x - A_{stop}E_s d_{top}\varepsilon_{c3} = 0 \rightarrow$$

$$x = \frac{-(A_{stop}E_s\varepsilon_{c3} - A_{sbot}f_{yd}) + \sqrt{(A_{stop}E_s\varepsilon_{c3} - A_{sbot}f_{yd})^2 - 4 \cdot \frac{1}{2}bf_{cd} \cdot -A_{stop}E_s d_{top}\varepsilon_{c3}}}{bf_{cd}}$$

The plastic moment can be expressed by:

$$M_{c,pl} = A_{sbot} f_{yd} \left(d_{bot} - \frac{1}{3} x \right) + A_{stop} E_s \varepsilon_{stop} \left(\frac{1}{3} x - d_{top} \right)$$

The corresponding curvature:

$$\kappa_{c,pl} = \frac{\varepsilon_{c3} + \varepsilon_{sbot}}{d_{bot}}$$

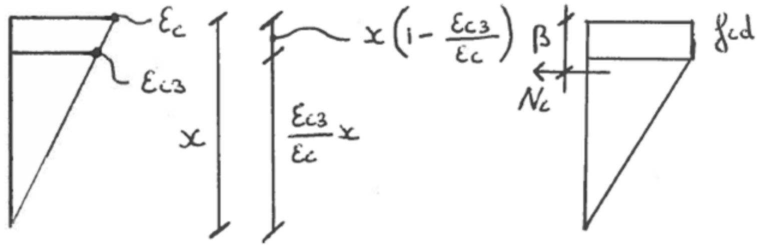


Figure 2.3: deformation and stress diagram when $\varepsilon_c > \varepsilon_{c3}$.

When $\varepsilon_c > \varepsilon_{c3}$ [Figure 2.3] is valid. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$N_c = \frac{1}{2} \frac{\varepsilon_{c3}}{\varepsilon_c} x f_{cd} + x \left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c} \right) f_{cd} = b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c} \right)$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c} \right) + A_{stop} E_s \varepsilon_{stop} = A_{sbot} f_{yd} \rightarrow$$

$$b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c} \right) = A_{sbot} f_{yd} - A_{stop} E_s \varepsilon_c \frac{x - d_{top}}{x} \rightarrow$$

$$b x^2 f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c} \right) = A_{sbot} f_{yd} x - A_{stop} E_s \varepsilon_c \left(x - d_{top} \right) \rightarrow$$

$$b x^2 f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c} \right) + \left(A_{stop} E_s \varepsilon_c - A_{sbot} f_{yd} \right) x - A_{stop} E_s \varepsilon_c d_{top} = 0 \rightarrow$$

$$x = \frac{-\left(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd}\right) + \sqrt{\left(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd}\right)^2 - 4 \cdot bf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) \cdot -A_{stop}E_s\varepsilon_c d_{top}}}{2bf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)}$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta = \frac{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) \cdot \frac{x}{2}\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{b}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}x \cdot \left(x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{\varepsilon_{c3}}{\varepsilon_c}\frac{x}{3}\right)}{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{bx}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}}$$

2.6 The ultimate bending moment resistance

The ultimate bending moment resistance can be derived when $\varepsilon_c = \varepsilon_{cu3}$:

$$M_{Rd} = A_{sbot}f_{yd}(d_{bot} - \beta) + A_{stop}E_s\varepsilon_{stop}\left(\beta - d_{top}\right)$$

The corresponding curvature:

$$\kappa_{Rd} = \frac{\varepsilon_{cu3} + \varepsilon_{sbot}}{d_{bot}}$$

2.7 The plastic moment $\left(M_{c,pl} < M_{ybot}\right)$

The concrete strength class and the amount of steel reinforcing bars determine whether concrete or steel will reach the plastic phase first. The steel reinforcing bars will start to yield at a yield strain $\varepsilon_{sy} = 2{,}17‰$. Concrete starts to become plastic from $\varepsilon_c = \varepsilon_{c3}$. **It is now assumed that the reinforcement ratio is high and the concrete will reach the plastic phase first**.

$$\varepsilon_c = \varepsilon_{c3}$$

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x}\varepsilon_c = \frac{d_{bot} - x}{x}\varepsilon_{c3}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{x}\varepsilon_c = \frac{x - d_{top}}{x}\varepsilon_{c3}$$

With the previous equations and ε_{c3} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_c + A_{stop} E_s \varepsilon_{stop} = A_{sbot} E_s \varepsilon_{sbot} \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_{c3} + A_{stop} E_s \frac{x - d_{top}}{x} \varepsilon_{c3} = A_{sbot} E_s \frac{d_{bot} - x}{x} \varepsilon_{c3} \rightarrow$$

$$\frac{1}{2} b x^2 E_c + A_{stop} E_s (x - d_{top}) = A_{sbot} E_s (d_{bot} - x) \rightarrow$$

$$\frac{1}{2} b x^2 E_c + A_{stop} E_s x - A_{stop} E_s d_{top} = A_{sbot} E_s d_{bot} - A_{sbot} E_s x \rightarrow$$

$$\frac{1}{2} b x^2 E_c + (A_{stop} + A_{sbot}) E_s x - E_s (A_{stop} d_{top} + A_{sbot} d_{bot}) = 0 \rightarrow$$

$$x = \frac{-(A_{stop} + A_{sbot}) E_s + \sqrt{\left((A_{stop} + A_{sbot}) E_s\right)^2 - 4 \cdot \frac{1}{2} b E_c \cdot -E_s (A_{stop} d_{top} + A_{sbot} d_{bot})}}{b E_c}$$

The plastic moment can be expressed by:

$$M_{c,pl} = A_{sbot} E_s \varepsilon_{sbot} \left(d_{bot} - \frac{1}{3} x\right) + A_{stop} E_s \varepsilon_{stop} \left(\frac{1}{3} x - d_{top}\right)$$

The corresponding curvature:

$$\kappa_{c,pl} = \frac{\varepsilon_{c3} + \varepsilon_{sbot}}{d_{bot}}$$

2.8 The yield moment $\left(M_{c,pl} < M_{ybot}\right)$

After the plastic moment is reached the moment capacity further increases until the steel strain ε_s reaches ε_{sy}.

$$\varepsilon_{sbot} = \varepsilon_{sy}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sbot} = \frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{d_{bot} - x}\varepsilon_{sbot} = \frac{x}{d_{bot} - x}\varepsilon_{sy}$$

When $\varepsilon_c > \varepsilon_{c3}$ [Figure 2.3] is valid. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$N_c = \frac{1}{2}\frac{\varepsilon_{c3}}{\varepsilon_c} x f_{cd} + x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) f_{cd} = bx f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$bx f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} \rightarrow$$

$$bx f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop}E_s\frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sy} = A_{sbot}E_s\varepsilon_{sy} \rightarrow$$

$$bx f_{cd} - bx f_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop}\frac{x - d_{top}}{d_{bot} - x} f_{yd} = A_{sbot} f_{yd} \rightarrow$$

$$2bx(d_{bot} - x) f_{cd}\varepsilon_c - bx(d_{bot} - x) f_{cd}\varepsilon_{c3} + 2A_{stop}\left(x - d_{top}\right) f_{yd}\varepsilon_c = 2A_{sbot}(d_{bot} - x) f_{yd}\varepsilon_c \rightarrow$$

$$2bx(d_{bot} - x) f_{cd}\varepsilon_{sy} - b(d_{bot} - x)^2 f_{cd}\varepsilon_{c3} + 2A_{stop}\left(x - d_{top}\right) f_{yd}\varepsilon_{sy} =$$
$$2A_{sbot}(d_{bot} - x) f_{yd}\varepsilon_{sy} \rightarrow$$

$$2bxd_{bot} f_{cd}\varepsilon_{sy} - 2bx^2 f_{cd}\varepsilon_{sy} - bd_{bot}^2 f_{cd}\varepsilon_{c3} - bx^2 f_{cd}\varepsilon_{c3} + 2bd_{bot}x f_{cd}\varepsilon_{c3} + 2A_{stop}x f_{yd}\varepsilon_{sy}$$
$$-2A_{stop}d_{top} f_{yd}\varepsilon_{sy} = 2A_{sbot}d_{bot} f_{yd}\varepsilon_{sy} - 2A_{sbot}x f_{yd}\varepsilon_{sy} \rightarrow$$

$$-bf_{cd}(2\varepsilon_{sy}+\varepsilon_{c3})x^2+2\left(bd_{bot}f_{cd}(\varepsilon_{sy}+\varepsilon_{c3})+f_{yd}\varepsilon_{sy}(A_{stop}+A_{sbot})\right)x-bd_{bot}^2f_{cd}\varepsilon_{c3}$$

$$-2f_{yd}\varepsilon_{sy}(A_{stop}d_{top}+A_{sbot}d_{bot})=0\rightarrow$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$a=-bf_{cd}(2\varepsilon_{sy}+\varepsilon_{c3})$$

$$b=2\left(bd_{bot}f_{cd}(\varepsilon_{sy}+\varepsilon_{c3})+f_{yd}\varepsilon_{sy}(A_{stop}+A_{sbot})\right)$$

$$c=-bd_{bot}^2f_{cd}\varepsilon_{c3}-2f_{yd}\varepsilon_{sy}(A_{stop}d_{top}+A_{sbot}d_{bot})$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta=\frac{bx\left(1-\frac{\varepsilon_{c3}}{\varepsilon_c}\right)\cdot\frac{x}{2}\left(1-\frac{\varepsilon_{c3}}{\varepsilon_c}\right)+\frac{b}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}x\cdot\left(x\left(1-\frac{\varepsilon_{c3}}{\varepsilon_c}\right)+\frac{\varepsilon_{c3}}{\varepsilon_c}\frac{x}{3}\right)}{bx\left(1-\frac{\varepsilon_{c3}}{\varepsilon_c}\right)+\frac{bx}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}}$$

The yield moment can be expressed by:

$$M_{ybot}=N_{sbot}(d_{bot}-\beta)+N_{stop}(\beta-d_{top})=A_{sbot}f_{yd}(d_{bot}-\beta)+A_{stop}E_s\varepsilon_{stop}(\beta-d_{top})$$

The corresponding curvature:

$$\kappa_{ybot}=\frac{\varepsilon_c+\varepsilon_{sy}}{d_{bot}}$$

When $\varepsilon_{sbot} < \varepsilon_{sy}$:

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x} \varepsilon_c$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{x} \varepsilon_c$$

The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} \rightarrow$$

$$bxf_{cd} - bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_c = A_{sbot}E_s\frac{d_{bot} - x}{x}\varepsilon_c \rightarrow$$

$$2bx^2 f_{cd}\varepsilon_c - bx^2 f_{cd}\varepsilon_{c3} + 2A_{stop}E_s(x - d_{top})\varepsilon_c^2 = 2A_{sbot}E_s(d_{bot} - x)\varepsilon_c^2 \rightarrow$$

$$2bx^2 f_{cd}\varepsilon_c - bx^2 f_{cd}\varepsilon_{c3} + 2A_{stop}E_s x\varepsilon_c^2 - 2A_{stop}E_s d_{top}\varepsilon_c^2 - 2A_{sbot}E_s d_{bot}\varepsilon_c^2 + 2A_{sbot}E_s x\varepsilon_c^2 = 0 \rightarrow$$

$$bf_{cd}(2\varepsilon_c - \varepsilon_{c3})x^2 + 2E_s\varepsilon_c^2(A_{stop} + A_{sbot})x - 2E_s\varepsilon_c^2(A_{stop}d_{top} + A_{sbot}d_{bot}) = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = bf_{cd}(2\varepsilon_c - \varepsilon_{c3})$$

$$b = 2E_s\varepsilon_c^2(A_{stop} + A_{sbot})$$

$$c = -2E_s\varepsilon_c^2(A_{stop}d_{top} + A_{sbot}d_{bot})$$

The corresponding bending moment and curvature:

$$M = N_{sbot}(d_{bot} - \beta) + N_{stop}(\beta - d_{top}) = A_{sbot}E_s\varepsilon_{sbot}(d_{bot} - \beta) + A_{stop}E_s\varepsilon_{stop}(\beta - d_{top})$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

When $\boldsymbol{\varepsilon_{sbot}} > \boldsymbol{\varepsilon_{sy}}$:

The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} \rightarrow$$

$$bxf_{cd} - bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_c = A_{sbot}E_s\varepsilon_{sy} \rightarrow$$

$$2bx^2 f_{cd}\varepsilon_c - bx^2 f_{cd}\varepsilon_{c3} + 2A_{stop}E_s\left(x - d_{top}\right)\varepsilon_c^2 = 2A_{sbot}f_{yd}x\varepsilon_c \rightarrow$$

$$bf_{cd}(2\varepsilon_c - \varepsilon_{c3})x^2 + 2\varepsilon_c\left(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd}\right)x - 2A_{stop}E_s d_{top}\varepsilon_c^2 = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = bf_{cd}(2\varepsilon_c - \varepsilon_{c3})$$

$$b = 2\varepsilon_c\left(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd}\right)$$

$$c = -2A_{stop}E_s d_{top}\varepsilon_c^2$$

The corresponding bending moment and curvature:

$$M = N_{sbot}(d_{bot} - \beta) + N_{stop}\left(\beta - d_{top}\right) = A_{sbot}f_{yd}(d_{bot} - \beta) + A_{stop}E_s\varepsilon_{stop}\left(\beta - d_{top}\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

2.9 Shear

The design shear resistance of the member without shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$v_{min} = 0{,}035k^{\frac{3}{2}}\sqrt{f_{ck}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$V_{Rd,cmin} = \left(v_{min} + k_1\sigma_{cp}\right)b_w d$$

$$k_1 = 0{,}15$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0{,}2f_{cd}$$

$$V_{Rd,c} = \left[C_{Rd,c}k\sqrt[3]{100\rho_l f_{ck}} + k_1\sigma_{cp}\right]b_w d$$

$$C_{Rd,c} = 0{,}18/\gamma_c$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$$

The design value of the shear force, which can be sustained by the yielding shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta$$

$$z = d - \beta$$

$$f_{ywd} = 0{,}8f_{yk}$$

$$1 \leq \cot\theta \leq 2{,}5$$

The design value of the maximum shear force which can be sustained by the member, limited by crushing of the compression struts [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,max} = \frac{\alpha_{cw} b_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta}$$

For non-prestressed structures: $\alpha_{cw} = 1$

For: $f_{ck} \leq 60\ N/mm^2$: $\nu_1 = 0{,}6$

For: $f_{ck} \geq 90\ N/mm^2$: $\nu_1 = 0{,}9 - \frac{f_{ck}}{200}$

2.10 Crack width

This paragraph is based on [NEN-EN 1992-1-1: 7.3.4].

Determine w_{max} from [NEN-EN 1992-1-1: Table 7.1N].

$$w_{max} = w_k$$

$$k_1 = 0{,}8$$

$$k_2 = 0{,}5$$

$$k_3 = 3{,}4$$

$$k_4 = 0{,}425$$

For short term loading: $k_t = 0{,}6$

For long term loading: $k_t = 0{,}4$

$$\alpha_e = \frac{E_s}{E_{cm}}$$

$$f_{ct,eff} = f_{ctm}(t)$$

The beam is in the cracked phase.

For a rectangular doubly reinforced beam that is cracked, the concrete compressive zone height x remains the same under increasing load until the yield moment is reached.

$$x = \frac{-\left(A_{stop} + A_{sbot}\right)E_s + \sqrt{\left(\left(A_{stop} + A_{sbot}\right)E_s\right)^2 - 4 \cdot \frac{1}{2} bE_c \cdot -E_s\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right)}}{bE_c}$$

With x, the maximum allowable steel stress σ_s can be determined:

$$h_{c,eff} = \min\left\{2{,}5(h-d); \frac{h-x}{3}; 0{,}5h\right\}$$

$$A_{c,eff} = bh_{c,eff}$$

$$\rho_{p,eff} = \frac{A_s}{A_{c,eff}}$$

$$s_{r,max} = k_3 c + \frac{k_1 k_2 k_4 \phi}{\rho_{p,eff}}$$

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \rightarrow \varepsilon_{sm} - \varepsilon_{cm} = \frac{w_k}{s_{r,max}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \rightarrow$$

$$\sigma_s = E_s(\varepsilon_{sm} - \varepsilon_{cm}) + k_t \frac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)$$

With the maximum allowable steel stress σ_s and [Figure 2.2] the maximum moment in SLS can be calculated.

$$\varepsilon_{sbot} = \frac{\sigma_s}{E_s}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_{sbot}}{d_{bot} - x} \rightarrow \varepsilon_c = \varepsilon_{sbot}\frac{x}{d_{bot} - x}$$

$$\frac{\varepsilon_{stop}}{x - d_{top}} = \frac{\varepsilon_{sbot}}{d_{bot} - x} \rightarrow \varepsilon_{stop} = \varepsilon_{sbot}\frac{x - d_{top}}{d_{bot} - x}$$

$$N_{ctop} = \frac{1}{2} bxE_c\varepsilon_c$$

$$N_{stop} = A_{stop}E_s\varepsilon_{stop}$$

$$N_{sbot} = A_{sbot}E_s\varepsilon_{sbot}$$

$$M_{qp} = A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right)$$

2.11 Concrete compressive zone height

This paragraph is based on [NEN-EN 1992-1-1+C2/NB: 6.1].

For: $f_{ck} \leq 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{500}{500 + f}$$

For: $f_{ck} > 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{\varepsilon_{cu} \cdot 10^6}{\varepsilon_{cu} \cdot 10^6 + 7f}$$

$$f = \frac{\left(\frac{f_{pk}}{\gamma_s} - \sigma_{pm\infty}\right) A_p + f_{yd}A_s}{A_p + A_s}$$

3 Rectangular doubly reinforced NSC beam + normal force

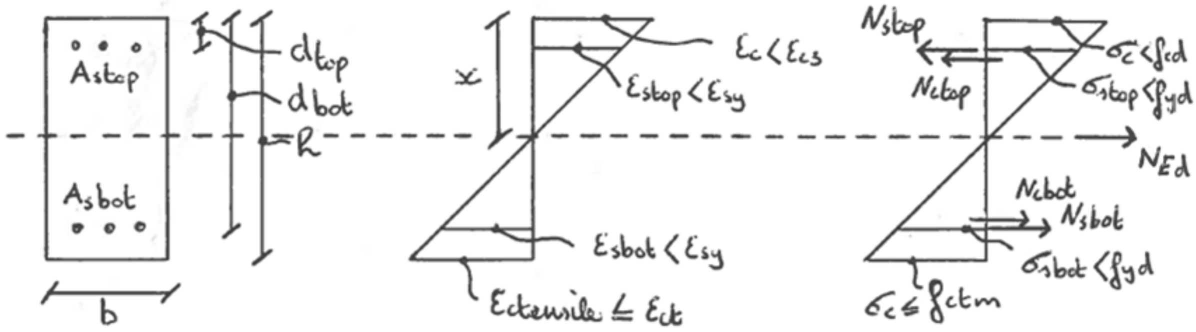


Figure 3.1: deformation and stress diagram when $\varepsilon_{ctensile} \leq \varepsilon_{ct}$.

3.1 Uncracked beam

The beam will remain uncracked as long as the concrete tensile strain $\varepsilon_{ctensile}$ doesn't exceed ε_{ct}.

With respect to the deformation and stress diagram of [Figure 3.1] the strain in the top and bottom steel reinforcement bars and the concrete tensile strain can be expressed as:

$$\varepsilon_{stop} = \frac{x - d_{top}}{x}\varepsilon_c$$

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x}\varepsilon_c$$

$$\varepsilon_{ctensile} = \frac{h - x}{x}\varepsilon_c$$

With above three equations and ε_c as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{ctop} + N_{stop} = N_{sbot} + N_{cbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + \frac{1}{2}b(h-x)E_c\varepsilon_{ctensile} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_c = A_{sbot}E_s\frac{d_{bot} - x}{x}\varepsilon_c + \frac{1}{2}b(h-x)E_c\frac{h-x}{x}\varepsilon_c + N_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + A_{stop}E_s(x - d_{top})\varepsilon_c = A_{sbot}E_s(d_{bot} - x)\varepsilon_c + \frac{1}{2}b(h-x)^2E_c\varepsilon_c + xN_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + A_{stop}E_sx\varepsilon_c - A_{stop}E_sd_{top}\varepsilon_c =$$
$$A_{sbot}E_sd_{bot}\varepsilon_c - A_{sbot}E_sx\varepsilon_c + \frac{1}{2}bh^2E_c\varepsilon_c + \frac{1}{2}bx^2E_c\varepsilon_c - bhxE_c\varepsilon_c + xN_{Ed} \rightarrow$$

$$A_{stop}E_sx\varepsilon_c + A_{sbot}E_sx\varepsilon_c + bhxE_c\varepsilon_c - xN_{Ed} = A_{stop}E_sd_{top}\varepsilon_c + A_{sbot}E_sd_{bot}\varepsilon_c + \frac{1}{2}bh^2E_c\varepsilon_c \rightarrow$$

$$x\left(\varepsilon_c\left(E_s(A_{stop} + A_{sbot}) + E_cbh\right) - N_{Ed}\right) = \varepsilon_c\left(E_s(A_{stop}d_{top} + A_{sbot}d_{bot}) + \frac{1}{2}bh^2E_c\right) \rightarrow$$

$$x = \frac{\varepsilon_c\left(E_s(A_{stop}d_{top} + A_{sbot}d_{bot}) + \frac{1}{2}bh^2E_c\right)}{\varepsilon_c\left(E_s(A_{stop} + A_{sbot}) + E_cbh\right) - N_{Ed}}$$

The corresponding bending moment capacity and curvature:

$$M = N_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + N_{cbot}\cdot\frac{2}{3}h + N_{stop}\left(\frac{1}{3}x - d_{top}\right) + N_{Ed}\left(\frac{1}{2}h - \frac{1}{3}x\right) =$$
$$A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + \frac{1}{2}b(h-x)E_c\varepsilon_{ctensile}\cdot\frac{2}{3}h + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right) +$$
$$N_{Ed}\left(\frac{1}{2}h - \frac{1}{3}x\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

3.2 The cracking moment

The cracking moment M_{cr} is reached at a concrete tensile strain $\varepsilon_{ctensile} = \varepsilon_{ct}$ and a curvature κ_{cr}. The strain in the concrete and steel reinforcing bars can be derived using the strain diagram drawn in [Figure 3.1].

$$\varepsilon_{ct} = \frac{f_{ctm}}{E_c}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{h - x}\varepsilon_{ct}$$

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{h - x}\varepsilon_{ct}$$

$$\varepsilon_c = \frac{x}{h - x}\varepsilon_{ct}$$

With the previous equations and ε_{ct} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{ctop} + N_{stop} = N_{sbot} + N_{cbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + \frac{1}{2}b(h - x)E_c\varepsilon_{ct} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\frac{x}{h - x}\varepsilon_{ct} + A_{stop}E_s\frac{x - d_{top}}{h - x}\varepsilon_{ct} = A_{sbot}E_s\frac{d_{bot} - x}{h - x}\varepsilon_{ct} + \frac{1}{2}b(h - x)E_c\varepsilon_{ct} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_{ct} + A_{stop}E_s(x - d_{top})\varepsilon_{ct} = A_{sbot}E_s(d_{bot} - x)\varepsilon_{ct} + \frac{1}{2}b(h - x)^2E_c\varepsilon_{ct} + (h - x)N_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_{ct} + A_{stop}E_sx\varepsilon_{ct} - A_{stop}E_sd_{top}\varepsilon_{ct} =$$
$$A_{sbot}E_sd_{bot}\varepsilon_{ct} - A_{sbot}E_sx\varepsilon_{ct} + \frac{1}{2}bh^2E_c\varepsilon_{ct} + \frac{1}{2}bx^2E_c\varepsilon_{ct} - bhxE_c\varepsilon_{ct} + hN_{Ed} - xN_{Ed} \rightarrow$$

$$A_{stop}E_sx\varepsilon_{ct} + A_{sbot}E_sx\varepsilon_{ct} + bhxE_c\varepsilon_{ct} + xN_{Ed} =$$
$$A_{stop}E_sd_{top}\varepsilon_{ct} + A_{sbot}E_sd_{bot}\varepsilon_{ct} + \frac{1}{2}bh^2E_c\varepsilon_{ct} + hN_{Ed} \rightarrow$$

$$x\left(\varepsilon_{ct}\left(E_s\left(A_{stop} + A_{sbot}\right) + bhE_c\right) + N_{Ed}\right) = \varepsilon_{ct}\left(E_s\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) + \frac{1}{2}bh^2E_c\right) + hN_{Ed} \rightarrow$$

$$x = \frac{\varepsilon_{ct}\left(E_s\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) + \frac{1}{2}bh^2E_c\right) + hN_{Ed}}{\varepsilon_{ct}\left(E_s\left(A_{stop} + A_{sbot}\right) + bhE_c\right) + N_{Ed}}$$

The cracking moment and the corresponding curvature:

$$M_{cr} = N_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + N_{cbot} \cdot \frac{2}{3}h + N_{stop}\left(\frac{1}{3}x - d_{top}\right) + N_{Ed}\left(\frac{1}{2}h - \frac{1}{3}x\right) =$$
$$A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + \frac{1}{2}b(h-x)E_c\varepsilon_{ct} \cdot \frac{2}{3}h + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right) + N_{Ed}\left(\frac{1}{2}h - \frac{1}{3}x\right)$$

$$\kappa_{cr} = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

3.3 Cracked beam

When the load is further increased, concrete cracking starts to occur and the steel reinforcing bars will provide tensile capacity to restore equilibrium. $\varepsilon_{ctensile}$ becomes zero.

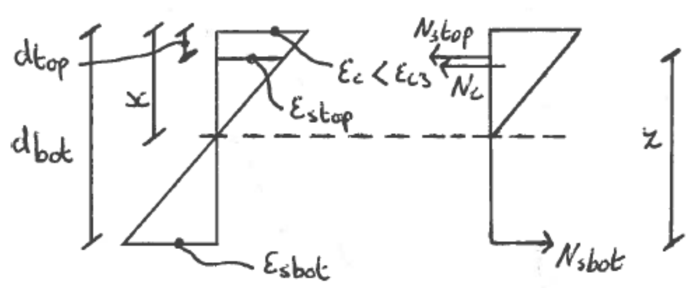


Figure 3.2: deformation and stress for elastic material behavior ($\varepsilon_c < \varepsilon_{c3}$ and $\varepsilon_s < \varepsilon_{sy}$).

With respect to the deformation and stress diagram of [Figure 3.2] the next three equations are valid:

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x}\varepsilon_c$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{x}\varepsilon_c$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed}$$

With these three equations and ε_c as input, the concrete compressive zone height x can be found.

When $\boldsymbol{\varepsilon_c < \varepsilon_{c3}}$:

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_c = A_{sbot}E_s\frac{d_{bot} - x}{x}\varepsilon_c + N_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + A_{stop}E_s(x - d_{top})\varepsilon_c = A_{sbot}E_s(d_{bot} - x)\varepsilon_c + N_{Ed}x \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + A_{stop}E_sx\varepsilon_c - A_{stop}E_sd_{top}\varepsilon_c = A_{sbot}E_sd_{bot}\varepsilon_c - A_{sbot}E_sx\varepsilon_c + N_{Ed}x \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + \left(E_s\varepsilon_c\left(A_{stop} + A_{sbot}\right) - N_{Ed}\right)x - E_s\varepsilon_c\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{1}{2}bE_c\varepsilon_c$$

$$b = E_s\varepsilon_c\left(A_{stop} + A_{sbot}\right) - N_{Ed}$$

$$c = -E_s\varepsilon_c\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right)$$

The corresponding bending moment capacity and curvature:

$$M = A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right) + N_{Ed}\left(\frac{1}{2}h - \frac{1}{3}x\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

3.4 The plastic moment $\left(M_{c,pl} < M_y\right)$

Because of the presence of the normal force it is assumed that the plastic moment will occur before the steel reinforcement bars start to yield.

$$\varepsilon_c = \varepsilon_{c3}$$

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x}\varepsilon_c = \frac{d_{bot} - x}{x}\varepsilon_{c3}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{x}\varepsilon_c = \frac{x - d_{top}}{x}\varepsilon_{c3}$$

With the equations above and ε_{c3} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_{c3} + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_{c3} = A_{sbot}E_s\frac{d_{bot} - x}{x}\varepsilon_{c3} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_{c3} + A_{stop}E_s\left(x - d_{top}\right)\varepsilon_{c3} = A_{sbot}E_s(d_{bot} - x)\varepsilon_{c3} + N_{Ed}x \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_{c3} + A_{stop}E_sx\varepsilon_{c3} - A_{stop}E_sd_{top}\varepsilon_{c3} = A_{sbot}E_sd_{bot}\varepsilon_{c3} - A_{sbot}E_sx\varepsilon_{c3} + N_{Ed}x \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_{c3} + A_{stop}E_sx\varepsilon_{c3} + A_{sbot}E_sx\varepsilon_{c3} - A_{stop}E_sd_{top}\varepsilon_{c3} - A_{sbot}E_sd_{bot}\varepsilon_{c3} - N_{Ed}x = 0 \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_{c3} + \left(E_s\varepsilon_{c3}\left(A_{stop} + A_{sbot}\right) - N_{Ed}\right)x - E_s\varepsilon_{c3}\left(A_{stop}d_{top} + A_{sbot}d_{bot}\right) = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{1}{2} b E_c \varepsilon_{c3}$$

$$b = E_s \varepsilon_{c3} \left(A_{stop} + A_{sbot} \right) - N_{Ed}$$

$$c = -E_s \varepsilon_{c3} \left(A_{stop} d_{top} + A_{sbot} d_{bot} \right)$$

The plastic moment can be expressed by:

$$M_{c,pl} = A_{sbot} E_s \varepsilon_{sbot} \left(d_{bot} - \frac{1}{3} x \right) + A_{stop} E_s \varepsilon_{stop} \left(\frac{1}{3} x - d_{top} \right) + N_{Ed} \left(\frac{1}{2} h - \frac{1}{3} x \right)$$

The corresponding curvature:

$$\kappa_{c,pl} = \frac{\varepsilon_{c3} + \varepsilon_{sbot}}{d_{bot}}$$

3.5 The yield moment $\left(M_{c,pl} < M_{ytop} \right)$

After the plastic moment is reached the moment capacity further increases until the steel strain ε_s reaches ε_{sy}.

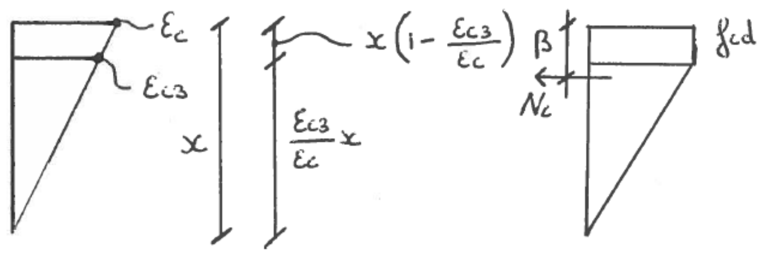


Figure 3.3: deformation and stress diagram when $\varepsilon_c > \varepsilon_{c3}$.

When $\varepsilon_c > \varepsilon_{c3}$ [Figure 3.3] is valid. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

When ε_{stop} reaches ε_{sy} first:

$$\varepsilon_{stop} = \varepsilon_{sy}$$

$$\varepsilon_{sbot} = \frac{d_{bot} - x}{x - d_{top}} \varepsilon_{stop} = \frac{d_{bot} - x}{x - d_{top}} \varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{x - d_{top}} \varepsilon_{stop} = \frac{x}{x - d_{top}} \varepsilon_{sy}$$

$$N_c = \frac{1}{2} \frac{\varepsilon_{c3}}{\varepsilon_c} x f_{cd} + x \left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) f_{cd} = b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop} E_s \varepsilon_{stop} = A_{sbot} E_s \varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$b x f_{cd} - b x f_{cd} \frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop} E_s \varepsilon_{sy} = A_{sbot} E_s \frac{d_{bot} - x}{x - d_{top}} \varepsilon_{sy} + N_{Ed} \rightarrow$$

$$2bx(x - d_{top}) f_{cd} \varepsilon_c - bx(x - d_{top}) f_{cd} \varepsilon_{c3} + 2(x - d_{top}) A_{stop} f_{yd} \varepsilon_c =$$
$$2A_{sbot}(d_{bot} - x) f_{yd} \varepsilon_c + 2(x - d_{top}) N_{Ed} \varepsilon_c \rightarrow$$

$$2bx(x - d_{top}) f_{cd} \varepsilon_{sy} - b(x - d_{top})^2 f_{cd} \varepsilon_{c3} + 2(x - d_{top}) A_{stop} f_{yd} \varepsilon_{sy} =$$
$$2A_{sbot}(d_{bot} - x) f_{yd} \varepsilon_{sy} + 2(x - d_{top}) N_{Ed} \varepsilon_{sy} \rightarrow$$

$$2bx^2 f_{cd} \varepsilon_{sy} - 2bx d_{top} f_{cd} \varepsilon_{sy} - b d_{top}^2 f_{cd} \varepsilon_{c3} - bx^2 f_{cd} \varepsilon_{c3} + 2bx d_{top} f_{cd} \varepsilon_{c3} + 2x A_{stop} f_{yd} \varepsilon_{sy}$$
$$-2 d_{top} A_{stop} f_{yd} \varepsilon_{sy} = 2A_{sbot} d_{bot} f_{yd} \varepsilon_{sy} - 2A_{sbot} x f_{yd} \varepsilon_{sy} + 2x N_{Ed} \varepsilon_{sy} - 2 d_{top} N_{Ed} \varepsilon_{sy} \rightarrow$$

$$2bx^2 f_{cd} \varepsilon_{sy} - bx^2 f_{cd} \varepsilon_{c3} - 2bx d_{top} f_{cd} \varepsilon_{sy} + 2bx d_{top} f_{cd} \varepsilon_{c3} + 2x A_{stop} f_{yd} \varepsilon_{sy} + 2A_{sbot} x f_{yd} \varepsilon_{sy}$$
$$-2x N_{Ed} \varepsilon_{sy} - b d_{top}^2 f_{cd} \varepsilon_{c3} - 2 d_{top} A_{stop} f_{yd} \varepsilon_{sy} - 2A_{sbot} d_{bot} f_{yd} \varepsilon_{sy} + 2 d_{top} N_{Ed} \varepsilon_{sy} = 0 \rightarrow$$

$$b f_{cd}(2\varepsilon_{sy} - \varepsilon_{c3}) x^2 - 2\left(b d_{top} f_{cd}(\varepsilon_{sy} - \varepsilon_{c3}) - \varepsilon_{sy}\left(f_{yd}(A_{stop} + A_{sbot}) - N_{Ed}\right)\right) x$$
$$-b d_{top}^2 f_{cd} \varepsilon_{c3} - 2 f_{yd} \varepsilon_{sy}(d_{top} A_{stop} + A_{sbot} d_{bot}) + 2 d_{top} N_{Ed} \varepsilon_{sy} = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = bf_{cd}\left(2\varepsilon_{sy} - \varepsilon_{c3}\right)$$

$$b = -2\left(bd_{top}f_{cd}\left(\varepsilon_{sy} - \varepsilon_{c3}\right) - \varepsilon_{sy}\left(f_{yd}\left(A_{stop} + A_{sbot}\right) - N_{Ed}\right)\right)$$

$$c = -bd_{top}^2 f_{cd}\varepsilon_{c3} - 2f_{yd}\varepsilon_{sy}(d_{top}A_{stop} + A_{sbot}d_{bot}) + 2d_{top}N_{Ed}\varepsilon_{sy}$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta = \frac{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) \cdot \frac{x}{2}\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{b}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}x \cdot \left(x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{\varepsilon_{c3}}{\varepsilon_c}\frac{x}{3}\right)}{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{bx}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}}$$

The yield moment can be expressed by:

$$M_{ytop} = A_{sbot}E_s\varepsilon_{sbot}(d_{bot} - \beta) + A_{stop}f_{yd}\left(\beta - d_{top}\right) + N_{Ed}\left(\frac{1}{2}h - \beta\right)$$

The corresponding curvature:

$$\kappa_{ytop} = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

When $\boldsymbol{\varepsilon_{sbot}}$ reaches $\boldsymbol{\varepsilon_{sy}}$:

$$\varepsilon_{sbot} = \varepsilon_{sy}$$

$$\varepsilon_{stop} > \varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{d_{bot} - x}\varepsilon_{sbot} = \frac{x}{d_{bot} - x}\varepsilon_{sy}$$

$$N_c = \frac{1}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}xf_{cd} + x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right)f_{cd} = bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$bxf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_c}\right)+A_{stop}E_s\varepsilon_{stop}=A_{sbot}E_s\varepsilon_{sbot}+N_{Ed}\rightarrow$$

$$bxf_{cd}-bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c}+A_{stop}E_s\varepsilon_{sy}=A_{sbot}E_s\varepsilon_{sy}+N_{Ed}\rightarrow$$

$$2bxf_{cd}\varepsilon_c-bxf_{cd}\varepsilon_{c3}+2A_{stop}f_{yd}\varepsilon_c=2A_{sbot}f_{yd}\varepsilon_c+2N_{Ed}\varepsilon_c\rightarrow$$

$$2bxf_{cd}\frac{x}{d_{bot}-x}\varepsilon_{sy}-bxf_{cd}\varepsilon_{c3}+2A_{stop}f_{yd}\frac{x}{d_{bot}-x}\varepsilon_{sy}=$$
$$2A_{sbot}f_{yd}\frac{x}{d_{bot}-x}\varepsilon_{sy}+2N_{Ed}\frac{x}{d_{bot}-x}\varepsilon_{sy}\rightarrow$$

$$2bxf_{cd}\varepsilon_{sy}-b(d_{bot}-x)f_{cd}\varepsilon_{c3}+2A_{stop}f_{yd}\varepsilon_{sy}=2A_{sbot}f_{yd}\varepsilon_{sy}+2N_{Ed}\varepsilon_{sy}\rightarrow$$

$$bf_{cd}(2\varepsilon_{sy}+\varepsilon_{c3})x=2f_{yd}\varepsilon_{sy}(A_{sbot}-A_{stop})+bd_{bot}f_{cd}\varepsilon_{c3}+2N_{Ed}\varepsilon_{sy}\rightarrow$$

$$x=\frac{2f_{yd}\varepsilon_{sy}(A_{sbot}-A_{stop})+bd_{bot}f_{cd}\varepsilon_{c3}+2N_{Ed}\varepsilon_{sy}}{bf_{cd}(2\varepsilon_{sy}+\varepsilon_{c3})}$$

The yield moment can be expressed by:

$$M_{ybot}=A_{sbot}f_{yd}(d_{bot}-\beta)+A_{stop}f_{yd}(\beta-d_{top})+N_{Ed}\left(\frac{1}{2}h-\beta\right)$$

The corresponding curvature:

$$\kappa_{ybot}=\frac{\varepsilon_c+\varepsilon_{sy}}{d_{bot}}$$

When $\varepsilon_{stop}<\varepsilon_{sy}$:

$$\varepsilon_{stop}=\frac{x-d_{top}}{x}\varepsilon_c$$

$$\varepsilon_{sbot}=\frac{d_{bot}-x}{x}\varepsilon_c$$

$$\sum F_H=0\rightarrow N_c+N_{stop}=N_{sbot}+N_{Ed}\rightarrow$$

$$bxf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_c}\right)+A_{stop}E_s\varepsilon_{stop}=A_{sbot}E_s\varepsilon_{sbot}+N_{Ed}\rightarrow$$

$$bxf_{cd}-bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c}+A_{stop}E_s\frac{x-d_{top}}{x}\varepsilon_c=A_{sbot}E_s\frac{d_{bot}-x}{x}\varepsilon_c+N_{Ed}\rightarrow$$

$$2bx^2f_{cd}\varepsilon_c-bx^2f_{cd}\varepsilon_{c3}+2A_{stop}E_s\left(x-d_{top}\right)\varepsilon_c^2=2A_{sbot}E_s(d_{bot}-x)\varepsilon_c^2+2xN_{Ed}\varepsilon_c\rightarrow$$

$$2bx^2f_{cd}\varepsilon_c-bx^2f_{cd}\varepsilon_{c3}+2A_{stop}E_sx\varepsilon_c^2-2A_{stop}E_sd_{top}\varepsilon_c^2=$$
$$2A_{sbot}E_sd_{bot}\varepsilon_c^2-2A_{sbot}E_sx\varepsilon_c^2+2xN_{Ed}\varepsilon_c\rightarrow$$

$$bf_{cd}(2\varepsilon_c-\varepsilon_{c3})x^2+2\left(\varepsilon_c\left(E_s\varepsilon_c\left(A_{stop}+A_{sbot}\right)-N_{Ed}\right)\right)x-2E_s\varepsilon_c^2\left(A_{stop}d_{top}+A_{sbot}d_{bot}\right)=0\rightarrow$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$a=bf_{cd}(2\varepsilon_c-\varepsilon_{c3})$$

$$b=2\left(\varepsilon_c\left(E_s\varepsilon_c\left(A_{stop}+A_{sbot}\right)-N_{Ed}\right)\right)$$

$$c=-2E_s\varepsilon_c^2\left(A_{stop}d_{top}+A_{sbot}d_{bot}\right)$$

The bending moment and corresponding curvature:

$$M=A_{sbot}E_s\varepsilon_{sbot}(d_{bot}-\beta)+A_{stop}E_s\varepsilon_{stop}\left(\beta-d_{top}\right)+N_{Ed}\left(\frac{1}{2}h-\beta\right)$$

$$\kappa=\frac{\varepsilon_c+\varepsilon_{sbot}}{d_{bot}}$$

When $\boldsymbol{\varepsilon_{sbot}<\varepsilon_{sy}<\varepsilon_{stop}}$:

$$\varepsilon_{stop}>\varepsilon_{sy}$$

$$\varepsilon_{sbot}=\frac{d_{bot}-x}{x}\varepsilon_c$$

$$\sum F_H=0\rightarrow N_c+N_{stop}=N_{sbot}+N_{Ed}\rightarrow$$

$$bxf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_c}\right)+A_{stop}E_s\varepsilon_{stop}=A_{sbot}E_s\varepsilon_{sbot}+N_{Ed}\rightarrow$$

$$bxf_{cd}-bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c}+A_{stop}E_s\varepsilon_{sy}=A_{sbot}E_s\frac{d_{bot}-x}{x}\varepsilon_c+N_{Ed}\rightarrow$$

$$2bx^2f_{cd}\varepsilon_c-bx^2f_{cd}\varepsilon_{c3}+2xA_{stop}f_{yd}\varepsilon_c=2A_{sbot}E_s(d_{bot}-x)\varepsilon_c^2+2xN_{Ed}\varepsilon_c\rightarrow$$

$$2bx^2f_{cd}\varepsilon_c-bx^2f_{cd}\varepsilon_{c3}+2xA_{stop}f_{yd}\varepsilon_c+2A_{sbot}E_sx\varepsilon_c^2-2xN_{Ed}\varepsilon_c-2A_{sbot}E_sd_{bot}\varepsilon_c^2=0\rightarrow$$

$$bf_{cd}(2\varepsilon_c-\varepsilon_{c3})x^2+2\varepsilon_c(A_{stop}f_{yd}+A_{sbot}E_s\varepsilon_c-N_{Ed})x-2A_{sbot}E_sd_{bot}\varepsilon_c^2=0\rightarrow$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$a=bf_{cd}(2\varepsilon_c-\varepsilon_{c3})$$

$$b=2\varepsilon_c(A_{stop}f_{yd}+A_{sbot}E_s\varepsilon_c-N_{Ed})$$

$$c=-2A_{sbot}E_sd_{bot}\varepsilon_c^2$$

The bending moment and corresponding curvature:

$$M=A_{sbot}E_s\varepsilon_{sbot}(d_{bot}-\beta)+A_{stop}f_{yd}(\beta-d_{top})+N_{Ed}\left(\frac{1}{2}h-\beta\right)$$

$$\kappa=\frac{\varepsilon_c+\varepsilon_{sbot}}{d_{bot}}$$

When $\boldsymbol{\varepsilon_s}>\boldsymbol{\varepsilon_{sy}}$:

$$\varepsilon_{stop}>\varepsilon_{sy}$$

$$\varepsilon_{sbot}>\varepsilon_{sy}$$

$$\sum F_H=0\rightarrow N_c+N_{stop}=N_{sbot}+N_{Ed}\rightarrow$$

$$bxf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_c}\right)+A_{stop}E_s\varepsilon_{stop}=A_{sbot}E_s\varepsilon_{sbot}+N_{Ed}\rightarrow$$

$$bxf_{cd} - bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop}E_s\varepsilon_{sy} = A_{sbot}E_s\varepsilon_{sy} + N_{Ed} \rightarrow$$

$$2bxf_{cd}\varepsilon_c - bxf_{cd}\varepsilon_{c3} + 2A_{stop}f_{yd}\varepsilon_c = 2A_{sbot}f_{yd}\varepsilon_c + 2N_{Ed}\varepsilon_c \rightarrow$$

$$bf_{cd}(2\varepsilon_c - \varepsilon_{c3})x = 2\varepsilon_c\left(\left(A_{sbot} - A_{stop}\right)f_{yd} + N_{Ed}\right) \rightarrow$$

$$x = \frac{2\varepsilon_c\left(\left(A_{sbot} - A_{stop}\right)f_{yd} + N_{Ed}\right)}{bf_{cd}(2\varepsilon_c - \varepsilon_{c3})}$$

The bending moment and corresponding curvature:

$$M = A_{sbot}f_{yd}(d_{bot} - \beta) + A_{stop}E_s\varepsilon_{stop}\left(\beta - d_{top}\right) + N_{Ed}\left(\frac{1}{2}h - \beta\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

3.6 The yield moment $\left(M_{c,pl} < M_{ybot}\right)$

When ε_{sbot} reaches ε_{sy} first:

$$\varepsilon_{sbot} = \varepsilon_{sy}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sbot} = \frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{d_{bot} - x}\varepsilon_{sbot} = \frac{x}{d_{bot} - x}\varepsilon_{sy}$$

$$N_c = \frac{1}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}xf_{cd} + x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right)f_{cd} = bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$bxf_{cd} - bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop}E_s\frac{x - d_{top}}{d_{bot} - x}\varepsilon_{sy} = A_{sbot}E_s\varepsilon_{sy} + N_{Ed} \rightarrow$$

$$2bx(d_{bot}-x)f_{cd}\varepsilon_c - bx(d_{bot}-x)f_{cd}\varepsilon_{c3} + 2A_{stop}(x-d_{top})f_{yd}\varepsilon_c =$$
$$2A_{sbot}(d_{bot}-x)f_{yd}\varepsilon_c + 2(d_{bot}-x)N_{Ed}\varepsilon_c \rightarrow$$

$$2bx(d_{bot}-x)f_{cd}\varepsilon_{sy} - b(d_{bot}-x)^2 f_{cd}\varepsilon_{c3} + 2A_{stop}(x-d_{top})f_{yd}\varepsilon_{sy} =$$
$$2A_{sbot}(d_{bot}-x)f_{yd}\varepsilon_{sy} + 2(d_{bot}-x)N_{Ed}\varepsilon_{sy} \rightarrow$$

$$-2bx^2 f_{cd}\varepsilon_{sy} - bx^2 f_{cd}\varepsilon_{c3} + 2bxd_{bot}f_{cd}\varepsilon_{sy} + 2bxd_{bot}f_{cd}\varepsilon_{c3} + 2A_{stop}xf_{yd}\varepsilon_{sy} + 2A_{sbot}xf_{yd}\varepsilon_{sy}$$
$$+2xN_{Ed}\varepsilon_{sy} - bd_{bot}^2 f_{cd}\varepsilon_{c3} - 2A_{stop}d_{top}f_{yd}\varepsilon_{sy} - 2A_{sbot}d_{bot}f_{yd}\varepsilon_{sy} - 2d_{bot}N_{Ed}\varepsilon_{sy} = 0 \rightarrow$$

$$-bf_{cd}(2\varepsilon_{sy}+\varepsilon_{c3})x^2 + 2\left(bd_{bot}f_{cd}(\varepsilon_{sy}+\varepsilon_{c3}) + \left((A_{stop}+A_{sbot})f_{yd} + N_{Ed}\right)\varepsilon_{sy}\right)x$$
$$-bd_{bot}^2 f_{cd}\varepsilon_{c3} - 2\left((A_{stop}d_{top} + A_{sbot}d_{bot})f_{yd} + d_{bot}N_{Ed}\right)\varepsilon_{sy} = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -bf_{cd}(2\varepsilon_{sy} + \varepsilon_{c3})$$

$$b = 2\left(bd_{bot}f_{cd}(\varepsilon_{sy}+\varepsilon_{c3}) + \left((A_{stop}+A_{sbot})f_{yd} + N_{Ed}\right)\varepsilon_{sy}\right)$$

$$c = -bd_{bot}^2 f_{cd}\varepsilon_{c3} - 2\left((A_{stop}d_{top} + A_{sbot}d_{bot})f_{yd} + d_{bot}N_{Ed}\right)\varepsilon_{sy}$$

The yield moment can be expressed by:

$$M_{ybot} = A_{sbot}f_{yd}(d_{bot}-\beta) + A_{stop}E_s\varepsilon_{stop}(\beta - d_{top}) + N_{Ed}\left(\frac{1}{2}h - \beta\right)$$

The corresponding curvature:

$$\kappa_{ybot} = \frac{\varepsilon_c + \varepsilon_{sy}}{d_{bot}}$$

When ε_{stop} reaches ε_{sy}:

$$\varepsilon_{stop} = \varepsilon_{sy}$$

$$\varepsilon_{sbot} > \varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{x - d_{top}} \varepsilon_{stop} = \frac{x}{x - d_{top}} \varepsilon_{sy}$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$bxf_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop} E_s \varepsilon_{stop} = A_{sbot} E_s \varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$bxf_{cd} - bxf_{cd} \frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop} E_s \varepsilon_{sy} = A_{sbot} E_s \varepsilon_{sy} + N_{Ed} \rightarrow$$

$$2bxf_{cd}\varepsilon_c - bxf_{cd}\varepsilon_{c3} + 2A_{stop} f_{yd}\varepsilon_c = 2A_{sbot} f_{yd}\varepsilon_c + 2N_{Ed}\varepsilon_c \rightarrow$$

$$2bxf_{cd}\varepsilon_{sy} - b(x - d_{top}) f_{cd}\varepsilon_{c3} + 2A_{stop} f_{yd}\varepsilon_{sy} = 2A_{sbot} f_{yd}\varepsilon_{sy} + 2N_{Ed}\varepsilon_{sy} \rightarrow$$

$$2bxf_{cd}\varepsilon_{sy} - bxf_{cd}\varepsilon_{c3} + bd_{top} f_{cd}\varepsilon_{c3} + 2A_{stop} f_{yd}\varepsilon_{sy} = 2A_{sbot} f_{yd}\varepsilon_{sy} + 2N_{Ed}\varepsilon_{sy} \rightarrow$$

$$bf_{cd}(2\varepsilon_{sy} - \varepsilon_{c3})x = 2(A_{sbot} - A_{stop}) f_{yd}\varepsilon_{sy} - bd_{top} f_{cd}\varepsilon_{c3} + 2N_{Ed}\varepsilon_{sy} \rightarrow$$

$$x = \frac{2(A_{sbot} - A_{stop}) f_{yd}\varepsilon_{sy} - bd_{top} f_{cd}\varepsilon_{c3} + 2N_{Ed}\varepsilon_{sy}}{bf_{cd}(2\varepsilon_{sy} - \varepsilon_{c3})}$$

The yield moment can be expressed by:

$$M_{ytop} = A_{sbot} f_{yd}(d_{bot} - \beta) + A_{stop} f_{yd}(\beta - d_{top}) + N_{Ed}\left(\frac{1}{2}h - \beta\right)$$

The corresponding curvature:

$$\kappa_{ytop} = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

When $\boldsymbol{\varepsilon_{stop} < \varepsilon_{sy} < \varepsilon_{sbot}}$:

$$\varepsilon_{stop} = \frac{x - d_{top}}{x} \varepsilon_c$$

$$\varepsilon_{sbot} > \varepsilon_{sy}$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$bxf_{cd} - bxf_{cd}\frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_c = A_{sbot}E_s\varepsilon_{sy} + N_{Ed} \rightarrow$$

$$2bx^2 f_{cd}\varepsilon_c - bx^2 f_{cd}\varepsilon_{c3} + 2A_{stop}E_s(x - d_{top})\varepsilon_c^2 = 2A_{sbot}f_{yd}x\varepsilon_c + 2x\varepsilon_c N_{Ed} \rightarrow$$

$$2bx^2 f_{cd}\varepsilon_c - bx^2 f_{cd}\varepsilon_{c3} + 2A_{stop}E_s x\varepsilon_c^2 - 2A_{sbot}f_{yd}x\varepsilon_c - 2x\varepsilon_c N_{Ed} - 2A_{stop}E_s d_{top}\varepsilon_c^2 = 0 \rightarrow$$

$$bf_{cd}(2\varepsilon_c - \varepsilon_{c3})x^2 + 2\left(\varepsilon_c\left(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd} - N_{Ed}\right)\right)x - 2A_{stop}E_s d_{top}\varepsilon_c^2 = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = bf_{cd}(2\varepsilon_c - \varepsilon_{c3})$$

$$b = 2\left(\varepsilon_c\left(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd} - N_{Ed}\right)\right)$$

$$c = -2A_{stop}E_s d_{top}\varepsilon_c^2$$

The bending moment and corresponding curvature:

$$M = A_{sbot}f_{yd}(d_{bot} - \beta) + A_{stop}E_s\varepsilon_{stop}(\beta - d_{top}) + N_{Ed}\left(\frac{1}{2}h - \beta\right)$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_{sbot}}{d_{bot}}$$

3.7 The ultimate bending moment resistance

The ultimate bending moment resistance can be derived when $\varepsilon_c = \varepsilon_{cu3}$:

$$M_{Rd} = A_{sbot} f_{yd} (d_{bot} - \beta) + A_{stop} E_s \varepsilon_{stop} \left(\beta - d_{top}\right) + N_{Ed} \left(\frac{1}{2} h - \beta\right)$$

The corresponding curvature:

$$\kappa_{Rd} = \frac{\varepsilon_{cu3} + \varepsilon_{sbot}}{d_{bot}}$$

3.8 The yield moment $\left(\boldsymbol{M_{ybot} < M_{c,pl}}\right)$

It is now assumed that normal force is not too high and the bottom steel reinforcement yields before the concrete reaches the plastic phase.

$$\varepsilon_{sbot} = \varepsilon_{sy}$$

$$\varepsilon_{stop} = \frac{x - d_{top}}{d_{bot} - x} \varepsilon_{sbot} = \frac{x - d_{top}}{d_{bot} - x} \varepsilon_{sy}$$

$$\varepsilon_c = \frac{x}{d_{bot} - x} \varepsilon_{sbot} = \frac{x}{d_{bot} - x} \varepsilon_{sy}$$

With the equations above and ε_{sy} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2} b x E_c \varepsilon_c + A_{stop} E_s \varepsilon_{stop} = A_{sbot} E_s \varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2} b x E_c \frac{x}{d_{bot} - x} \varepsilon_{sy} + A_{stop} E_s \frac{x - d_{top}}{d_{bot} - x} \varepsilon_{sy} = A_{sbot} E_s \varepsilon_{sy} + N_{Ed} \rightarrow$$

$$\frac{1}{2} b x^2 E_c \varepsilon_{sy} + A_{stop} \left(x - d_{top}\right) f_{yd} = A_{sbot} (d_{bot} - x) f_{yd} + (d_{bot} - x) N_{Ed} \rightarrow$$

$$\frac{1}{2} b x^2 E_c \varepsilon_{sy} + A_{stop} x f_{yd} + A_{sbot} x f_{yd} + x N_{Ed} - A_{stop} d_{top} f_{yd} - A_{sbot} d_{bot} f_{yd} - d_{bot} N_{Ed} = 0 \rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_{sy}+\left(f_{yd}\left(A_{stop}+A_{sbot}\right)+N_{Ed}\right)x\ -\left(f_{yd}\left(A_{stop}d_{top}+A_{sbot}d_{bot}\right)\right)-d_{bot}N_{Ed}=0\rightarrow$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$a=\frac{1}{2}bE_c\varepsilon_{sy}$$

$$b=\left(f_{yd}\left(A_{stop}+A_{sbot}\right)+N_{Ed}\right)$$

$$c=-\left(f_{yd}\left(A_{stop}d_{top}+A_{sbot}d_{bot}\right)\right)-d_{bot}N_{Ed}$$

The yield moment can be expressed by:

$$M_{ybot}=A_{sbot}f_{yd}\left(d_{bot}-\frac{1}{3}x\right)+A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x-d_{top}\right)+N_{Ed}\left(\frac{1}{2}h-\frac{1}{3}x\right)$$

The corresponding curvature:

$$\kappa_{ybot}=\frac{\varepsilon_c+\varepsilon_{sbot}}{d_{bot}}$$

When $\boldsymbol{\varepsilon_{sbot}>\varepsilon_{sy}}$ and $\boldsymbol{\varepsilon_c<\varepsilon_{c3}}$:

$$\varepsilon_{stop}=\frac{x-d_{top}}{x}\varepsilon_c$$

$$\sum F_H=0\rightarrow N_c+N_{stop}=N_{sbot}+N_{Ed}\rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c+A_{stop}E_s\varepsilon_{stop}=A_{sbot}E_s\varepsilon_{sy}+N_{Ed}\rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c+A_{stop}E_s\frac{x-d_{top}}{x}\varepsilon_c=A_{sbot}f_{yd}+N_{Ed}\rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c+A_{stop}E_s\varepsilon_c\left(x-d_{top}\right)=A_{sbot}f_{yd}x+N_{Ed}x\rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c+A_{stop}E_s\varepsilon_cx-A_{stop}E_s\varepsilon_cd_{top}-A_{sbot}f_{yd}x-N_{Ed}x=0\rightarrow$$

$$\frac{1}{2}bx^2E_c\varepsilon_c + (A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd} - N_{Ed})x - A_{stop}E_s\varepsilon_c d_{top} = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{1}{2}bE_c\varepsilon_c$$

$$b = A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd} - N_{Ed}$$

$$c = -A_{stop}E_s\varepsilon_c d_{top}$$

3.9 The plastic moment $(M_y < M_{c,pl})$

The plastic moment occurs when $\varepsilon_c = \varepsilon_{c3}$. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\varepsilon_{stop} = \frac{x - d_{top}}{x}\varepsilon_c = \frac{x - d_{top}}{x}\varepsilon_{c3}$$

$$\varepsilon_{sbot} > \varepsilon_{sy}$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_c + A_{stop}E_s\varepsilon_{stop} = A_{sbot}E_s\varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bxE_c\varepsilon_{c3} + A_{stop}E_s\frac{x - d_{top}}{x}\varepsilon_{c3} = A_{sbot}E_s\varepsilon_{sy} + N_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2f_{cd} + A_{stop}E_s(x - d_{top})\varepsilon_{c3} = A_{sbot}f_{yd}x + xN_{Ed} \rightarrow$$

$$\frac{1}{2}bx^2f_{cd} + A_{stop}E_sx\varepsilon_{c3} - A_{stop}E_sd_{top}\varepsilon_{c3} - A_{sbot}f_{yd}x - xN_{Ed} = 0 \rightarrow$$

$$\frac{1}{2}bx^2f_{cd} + (A_{stop}E_s\varepsilon_{c3} - A_{sbot}f_{yd} - N_{Ed})x - A_{stop}E_sd_{top}\varepsilon_{c3} = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{1}{2} b f_{cd}$$

$$b = A_{stop} E_s \varepsilon_{c3} - A_{sbot} f_{yd} - N_{Ed}$$

$$c = -A_{stop} E_s d_{top} \varepsilon_{c3}$$

The plastic moment can be expressed by:

$$M_{c,pl} = A_{sbot} f_{yd} \left(d_{bot} - \frac{1}{3} x \right) + A_{stop} E_s \varepsilon_{stop} \left(\frac{1}{3} x - d_{top} \right) + N_{Ed} \left(\frac{1}{2} h - \frac{1}{3} x \right)$$

The corresponding curvature:

$$\kappa_{c,pl} = \frac{\varepsilon_{c3} + \varepsilon_s}{d_{bot}}$$

When $\varepsilon_c > \varepsilon_{c3}$ [Figure 3.3] is valid. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$N_c = \frac{1}{2} \frac{\varepsilon_{c3}}{\varepsilon_c} x f_{cd} + x \left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c} \right) f_{cd} = b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c} \right)$$

$$\sum F_H = 0 \rightarrow N_c + N_{stop} = N_{sbot} + N_{Ed} \rightarrow$$

$$b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c} \right) + A_{stop} E_s \varepsilon_{stop} = A_{sbot} E_s \varepsilon_{sbot} + N_{Ed} \rightarrow$$

$$b x f_{cd} - b x f_{cd} \frac{\varepsilon_{c3}}{2\varepsilon_c} + A_{stop} E_s \frac{x - d_{top}}{x} \varepsilon_c = A_{sbot} E_s \varepsilon_{sy} + N_{Ed} \rightarrow$$

$$2 b x^2 f_{cd} \varepsilon_c - b x^2 f_{cd} \varepsilon_{c3} + 2 A_{stop} E_s \left(x - d_{top} \right) \varepsilon_c^2 = 2 A_{sbot} x f_{yd} \varepsilon_c + 2 x N_{Ed} \varepsilon_c \rightarrow$$

$$2 b x^2 f_{cd} \varepsilon_c - b x^2 f_{cd} \varepsilon_{c3} + 2 A_{stop} E_s x \varepsilon_c^2 - 2 A_{sbot} x f_{yd} \varepsilon_c - 2 x N_{Ed} \varepsilon_c - 2 A_{stop} E_s d_{top} \varepsilon_c^2 = 0 \rightarrow$$

$$b f_{cd} (2\varepsilon_c - \varepsilon_{c3}) x^2 + 2\varepsilon_c \left(A_{stop} E_s \varepsilon_c - A_{sbot} f_{yd} - N_{Ed} \right) x - 2 A_{stop} E_s d_{top} \varepsilon_c^2 = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = bf_{cd}(2\varepsilon_c - \varepsilon_{c3})$$

$$b = 2\varepsilon_c\left(A_{stop}E_s\varepsilon_c - A_{sbot}f_{yd} - N_{Ed}\right)$$

$$c = -2A_{stop}E_s d_{top}\varepsilon_c^2$$

3.10 Shear

The design shear resistance of the member without shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$v_{min} = 0{,}035k^{\frac{3}{2}}\sqrt{f_{ck}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$V_{Rd,cmin} = \left(v_{min} + k_1\sigma_{cp}\right)b_w d$$

$$k_1 = 0{,}15$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0{,}2f_{cd}$$

$$V_{Rd,c} = \left[C_{Rd,c}k\sqrt[3]{100\rho_l f_{ck}} + k_1\sigma_{cp}\right]b_w d$$

$$C_{Rd,c} = 0{,}18/\gamma_c$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$$

The design value of the shear force, which can be sustained by the yielding shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,s} = \frac{A_{sw}}{s}zf_{ywd}\cot\theta$$

$$z = d - \beta$$

$f_{ywd} = 0{,}8 f_{yk}$

$1 \leq \cot\theta \leq 2{,}5$

The design value of the maximum shear force, which can be sustained by the member, limited by crushing of the compression struts [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,max} = \frac{\alpha_{cw} b_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta}$$

For: $0 < \sigma_{cp} \leq 0{,}25 f_{cd}$: $\alpha_{cw} = \left(1 + \frac{\sigma_{cp}}{f_{cd}}\right)$

For: $0{,}25 f_{cd} < \sigma_{cp} \leq 0{,}5 f_{cd}$: $\alpha_{cw} = 1{,}25$

For: $0{,}5 f_{cd} < \sigma_{cp} \leq 1{,}0 f_{cd}$: $\alpha_{cw} = 2{,}5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right)$

For: $f_{ck} \leq 60\ N/mm^2$: $\nu_1 = 0{,}6$

For: $f_{ck} \geq 90\ N/mm^2$: $\nu_1 = 0{,}9 - \frac{f_{ck}}{200}$

3.11 Crack width

This paragraph is based on [NEN-EN 1992-1-1: 7.3.4].

Determine w_{max} from [NEN-EN 1992-1-1: Table 7.1N].

$w_{max} = w_k$

$k_1 = 0{,}8$

$k_2 = 0{,}5$

$k_3 = 3{,}4$

$k_4 = 0{,}425$

For short term loading: $k_t = 0{,}6$

For long term loading: $k_t = 0{,}4$

$$\alpha_e = \frac{E_s}{E_{cm}}$$

$$f_{ct,eff} = f_{ctm}(t)$$

The beam is in the cracked phase.

When there is a normal force applied to a rectangular doubly reinforced beam, the concrete compressive zone height x, in the cracked stage, does not remain the same under increasing load until the yield moment is reached. It depends on the concrete strain at the top of the beam ε_c.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{1}{2} b E_c \varepsilon_c$$

$$b = E_s \varepsilon_c \left(A_{stop} + A_{sbot}\right) - N_{Ed}$$

$$c = -E_s \varepsilon_c \left(A_{stop} d_{top} + A_{sbot} d_{bot}\right)$$

Therefore it is not known for which x the crack width criterion is met. To find x and ε_c the GOALSEEK function in Excel can be used. All terms in the crack width formula depend on x and thus indirectly on ε_c.

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$s_{r,max} = k_3 c + \frac{k_1 k_2 k_4 \phi}{\rho_{p,eff}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} \left(1 + \alpha_e \rho_{p,eff}\right)}{E_s}$$

$$h_{c,eff} = \min\left\{2{,}5(h-d); \frac{h-x}{3}; 0{,}5h\right\}$$

$$A_{c,eff} = b h_{c,eff}$$

$$\rho_{p,eff} = \frac{A_s}{A_{c,eff}}$$

$$\sigma_s = E_s \varepsilon_{sbot}$$

$$\varepsilon_{sbot} = \varepsilon_c \frac{d_{bot} - x}{x}$$

w_k is known as it is equal to w_{max} . With this GOALSEEK function Excel can vary ε_c until the input value for w_k is found. With ε_c, x is known and the maximum bending moment can be determined.

$$N_{ctop} = \frac{1}{2} bxE_c\varepsilon_c$$

$$N_{stop} = A_{stop}E_s\varepsilon_{stop}$$

$$N_{sbot} = A_{sbot}E_s\varepsilon_{sbot}$$

$$M_{qp} = M = A_{sbot}E_s\varepsilon_{sbot}\left(d_{bot} - \frac{1}{3}x\right) + A_{stop}E_s\varepsilon_{stop}\left(\frac{1}{3}x - d_{top}\right) + N_{Ed}\left(\frac{1}{2}h - \frac{1}{3}x\right)$$

3.12 Concrete compressive zone height

This paragraph is based on [NEN-EN 1992-1-1+C2/NB: 6.1].

For: $f_{ck} \leq 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{500}{500 + f}$$

For: $f_{ck} > 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{\varepsilon_{cu} \cdot 10^6}{\varepsilon_{cu} \cdot 10^6 + 7f}$$

$$f = \frac{\left(\frac{f_{pk}}{\gamma_s} - \sigma_{pm\infty}\right)A_p + f_{yd}A_s}{A_p + A_s}$$

4 Rectangular prestressed NSC beam

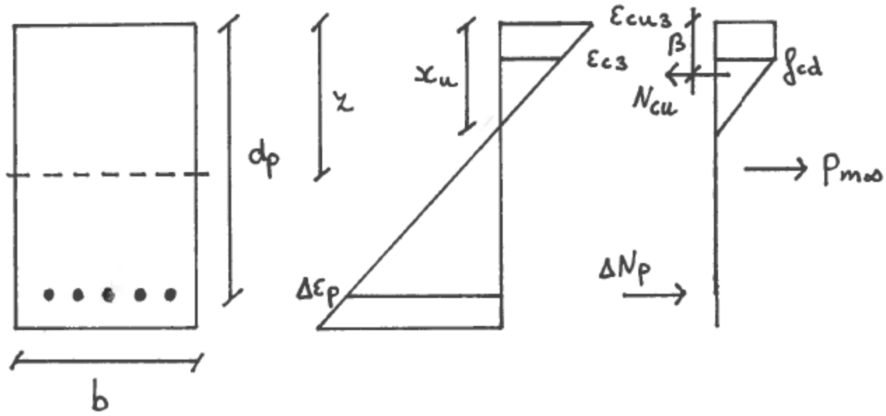


Figure 4.1: deformation and stress diagram when $\varepsilon_c = \varepsilon_{cu3}$.

4.1 The ultimate bending moment resistance

The ultimate bending moment can be found when $\varepsilon_c = \varepsilon_{cu3}$.

$\varepsilon_p > \varepsilon_{py}$.

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

$$\varepsilon_{p\infty} = \frac{\sigma_{pm\infty}}{E_p}$$

$$\frac{\varepsilon_{cu3}}{x_u} = \frac{\Delta\varepsilon_p}{d_p - x_u} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u}\left(d_p - x_u\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u} d_p - \varepsilon_{cu3}$$

The concrete compressive zone height x_u can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{cu} = \Delta N_p \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_p\sigma_p - P_{m\infty} \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right) = A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - A_p\sigma_{pm\infty} \rightarrow$$

$$bx_u f_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)=A_p\left(f_{pd}+\frac{\frac{\varepsilon_{cu3}}{x_u}d_p-\varepsilon_{cu3}+\frac{\sigma_{pm\infty}}{E_p}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)-A_p\sigma_{pm\infty}\rightarrow$$

$$bx_u f_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)=A_p f_{pd}+A_p\frac{\frac{\varepsilon_{cu3}}{x_u}d_p-\varepsilon_{cu3}+\frac{\sigma_{pm\infty}}{E_p}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\sigma_{pm\infty}\rightarrow$$

$$bx_u f_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)(\varepsilon_{uk}-\varepsilon_{py})=A_p f_{pd}(\varepsilon_{uk}-\varepsilon_{py})$$
$$+A_p\left(\frac{\varepsilon_{cu3}}{x_u}d_p-\varepsilon_{cu3}+\frac{\sigma_{pm\infty}}{E_p}-\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\sigma_{pm\infty}(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bx_u f_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)(\varepsilon_{uk}-\varepsilon_{py})=A_p f_{pd}(\varepsilon_{uk}-\varepsilon_{py})+A_p\frac{\varepsilon_{cu3}}{x_u}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$
$$-A_p\varepsilon_{cu3}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+A_p\frac{\sigma_{pm\infty}}{E_p}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\sigma_{pm\infty}(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bx_u^2 f_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)(\varepsilon_{uk}-\varepsilon_{py})=A_p f_{pd}(\varepsilon_{uk}-\varepsilon_{py})x_u+A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$
$$-A_p\varepsilon_{cu3}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)x_u+A_p\frac{\sigma_{pm\infty}}{E_p}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)x_u-A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)x_u$$
$$-A_p\sigma_{pm\infty}(\varepsilon_{uk}-\varepsilon_{py})x_u\rightarrow$$

$$-bf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)(\varepsilon_{uk}-\varepsilon_{py})x_u^2$$
$$+A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})-\left(\varepsilon_{cu3}-\frac{\sigma_{pm\infty}}{E_p}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)x_u$$
$$+A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)=0\rightarrow$$

$$x_u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -bf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)(\varepsilon_{uk} - \varepsilon_{py})$$

$$b = A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) - \left(\varepsilon_{cu3} - \frac{\sigma_{pm\infty}}{E_p} + \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$c = A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta = \frac{bx_u\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) \cdot \frac{1}{2}x_u\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{1}{2}b\frac{\varepsilon_{c3}}{\varepsilon_c}x_u\left(x_u\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{1}{3}\frac{\varepsilon_{c3}}{\varepsilon_c}x_u\right)}{bx_u\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{1}{2}b\frac{\varepsilon_{c3}}{\varepsilon_c}x_u}$$

The ultimate bending moment:

$$M_{Rd} = N_{cu}(x_u - \beta) + P_{m\infty}(z - x_u) + \Delta N_p(d_p - x_u)$$

4.2 Shear

The design shear resistance of the member without shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$v_{min} = 0{,}035k^{\frac{3}{2}}\sqrt{f_{ck}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$V_{Rd,cmin} = (v_{min} + k_1\sigma_{cp})b_wd$$

$$k_1 = 0{,}15$$

$$\sigma_{cp} = \frac{P_{m\infty}}{A_c} < 0{,}2f_{cd}$$

$$V_{Rd,c} = \left[C_{Rd,c} k \sqrt[3]{100 \rho_l f_{ck}} + k_1 \sigma_{cp}\right] b_w d$$

$$C_{Rd,c} = 0{,}18/\gamma_c$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$$

The design value of the shear force, which can be sustained by the yielding shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} (\cot\theta + \cot\alpha) \sin\alpha$$

$$z = d - \beta$$

$$f_{ywd} = 0{,}8 f_{yk}$$

$$1 \leq \cot\theta \leq 2{,}5$$

For vertical stirrups: $\alpha = 90°$.

The design value of the maximum shear force, which can be sustained by the member, limited by crushing of the compression struts [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,max} = b_w z (\cot\theta + \cot\alpha) \sin^2\theta \, \alpha_{cw} \nu_1 f_{cd}$$

For: $0 < \sigma_{cp} \leq 0{,}25 f_{cd}$: $\alpha_{cw} = \left(1 + \frac{\sigma_{cp}}{f_{cd}}\right)$

For: $0{,}25 f_{cd} < \sigma_{cp} \leq 0{,}5 f_{cd}$: $\alpha_{cw} = 1{,}25$

For: $0{,}5 f_{cd} < \sigma_{cp} \leq 1{,}0 f_{cd}$: $\alpha_{cw} = 2{,}5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right)$

For: $f_{ck} \leq 60\ N/mm^2$: $\nu_1 = 0{,}6$

For: $f_{ck} \geq 90\ N/mm^2$: $\nu_1 = 0{,}9 - \frac{f_{ck}}{200}$

4.3 Crack width

Requirement: the beam remains uncracked in the serviceability limit state.

At $t = 0$, no time-dependent losses are present, so the prestressing force will be at its maximum. Because of the positioning of the tendons the beam will be slightly cambered and tensile stresses will occur at the top of the beam. Bending moments cause compressive stresses at the top and tensile stresses at the bottom of the beam.

$t = 0 \rightarrow$ check top fibre:

$$-\frac{P_{m0}}{A_c}+\frac{P_{m0}\cdot e}{W_{top}}-\frac{M}{W_{top}}\leq f_{ctm}\rightarrow\frac{M}{W_{top}}\geq-\frac{P_{m0}}{A_c}+\frac{P_{m0}\cdot e}{W_{top}}-f_{ctm}\rightarrow$$

$$M\geq-\frac{P_{m0}}{A_c}W_{top}+P_{m0}\cdot e-f_{ctm}W_{top}$$

$t = 0 \rightarrow$ check bottom fibre:

$$-\frac{P_{m0}}{A_c}-\frac{P_{m0}\cdot e}{W_{bot}}+\frac{M}{W_{bot}}\leq f_{ctm}\rightarrow\frac{M}{W_{bot}}\leq\frac{P_{m0}}{A_c}+\frac{P_{m0}\cdot e}{W_{bot}}+f_{ctm}\rightarrow$$

$$M\leq\frac{P_{m0}}{A_c}W_{bot}+P_{m0}\cdot e+f_{ctm}W_{bot}$$

At $t = \infty$, the prestressing force has been reduced by time-dependent losses, which means that the compressive stresses working on the cross-section will be limited. Dead and live loads are present and will cause tensile stresses at the bottom fibre in the span. The bending moment caused by these loads should be limited:

$t = \infty \rightarrow$ check top fibre:

$$-\frac{P_{m\infty}}{A_c}+\frac{P_{m\infty}\cdot e}{W_{top}}-\frac{M}{W_{top}}\leq f_{ctm}\rightarrow\frac{M}{W_{top}}\geq-\frac{P_{m\infty}}{A_c}+\frac{P_{m\infty}\cdot e}{W_{top}}-f_{ctm}\rightarrow$$

$$M\geq-\frac{P_{m\infty}}{A_c}W_{top}+P_{m\infty}\cdot e-f_{ctm}W_{top}$$

$t = \infty \rightarrow$ check bottom fibre:

$$-\frac{P_{m\infty}}{A_c} - \frac{P_{m\infty} \cdot e}{W_{bot}} + \frac{M}{W_{bot}} \leq f_{ctm} \rightarrow \frac{M}{W_{bot}} \leq \frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{bot}} + f_{ctm} \rightarrow$$

$$M \leq \frac{P_{m\infty}}{A_c} W_{bot} + P_{m\infty} \cdot e + f_{ctm} W_{bot}$$

4.4 The concrete compressive zone height

This paragraph is based on [NEN-EN 1992-1-1+C2/NB: 6.1].

For: $f_{ck} \leq 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{500}{500 + f}$$

For: $f_{ck} > 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{\varepsilon_{cu} \cdot 10^6}{\varepsilon_{cu} \cdot 10^6 + 7f}$$

$$f = \frac{\left(\frac{f_{pk}}{\gamma_s} - \sigma_{pm\infty}\right) A_p + f_{yd} A_s}{A_p + A_s}$$

5 Reinforced NSC box girder

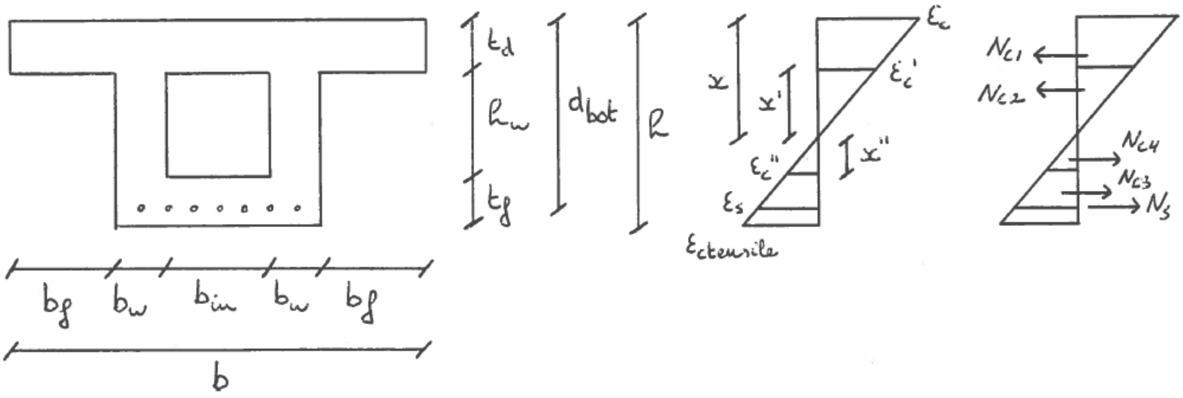


Figure 5.1: deformation and stress diagram when $\varepsilon_{ctensile} \leq \varepsilon_{ct}$.

5.1 Uncracked beam

The beam will remain uncracked as long as $\varepsilon_{ctensile} < \varepsilon_{ct}$.

$x > t_d$.

With respect to the deformation and stress diagram of [Figure 5.1] the following relations are valid:

$$\frac{\varepsilon_s}{d_{bot} - x} = \frac{\varepsilon_c}{x} \rightarrow \varepsilon_s = \frac{d_{bot} - x}{x} \varepsilon_c$$

$$\frac{\varepsilon_c{}'}{x'} = \frac{\varepsilon_c}{x} \rightarrow \varepsilon_c{}' = \frac{x'}{x} \varepsilon_c$$

$$\frac{\varepsilon_c{}''}{x''} = \frac{\varepsilon_c}{x} \rightarrow \varepsilon_c{}'' = \frac{x''}{x} \varepsilon_c$$

$$\frac{\varepsilon_{ctensile}}{h - x} = \frac{\varepsilon_c}{x} \rightarrow \varepsilon_{ctensile} = \frac{h - x}{x} \varepsilon_c$$

$$x' = x - t_d$$

$$x'' = h_w - x'$$

With the previous equations and ε_c as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} + N_{c2} = N_s + N_{c3} + N_{c4} \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_c x - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_c' x' = A_sE_s\varepsilon_s + \frac{1}{2}(2b_w + b_{in})E_c\varepsilon_{ctensile}(h-x) - \frac{1}{2}b_{in}E_c\varepsilon_c"x" \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_c x - b_fE_c\varepsilon_c' x' - \frac{1}{2}b_{in}E_c\varepsilon_c' x' = A_sE_s\varepsilon_s + b_wE_c\varepsilon_{ctensile}h - b_wE_c\varepsilon_{ctensile}x$$
$$+\frac{1}{2}b_{in}E_c\varepsilon_{ctensile}h - \frac{1}{2}b_{in}E_c\varepsilon_{ctensile}x - \frac{1}{2}b_{in}E_c\varepsilon_c"x" \rightarrow$$

$$bE_cx^2 - 2b_fE_cx'^2 - b_{in}E_cx'^2 = 2A_sE_sd_{bot} - 2A_sE_sx + 2b_wE_ch^2 - 4b_wE_chx + 2b_wE_cx^2$$
$$+b_{in}E_ch^2 - 2b_{in}E_chx + b_{in}E_cx^2 - b_{in}E_cx"^2 \rightarrow$$

$$bE_cx^2 - 2b_fE_cx^2 + 4b_fE_ct_dx - 2b_fE_ct_d^2 = 2A_sE_sd_{bot} - 2A_sE_sx + 2b_wE_ch^2 - 4b_wE_chx$$
$$+2b_wE_cx^2 + b_{in}E_ch^2 - 2b_{in}E_chx + b_{in}E_cx^2 - b_{in}E_ch_w^2 + 2b_{in}E_ch_wx - 2b_{in}E_ch_wt_d \rightarrow$$

$$2\left(E_c\left(2b_ft_d + 2b_wh + b_{in}h - b_{in}h_w\right) + A_sE_s\right)x$$
$$= E_c(2b_ft_d^2 + 2b_wh^2 + b_{in}h^2 - b_{in}h_w^2 - 2b_{in}h_wt_d) + 2A_sE_sd_{bot} \rightarrow$$

$$x = \frac{E_c(2b_ft_d^2 + 2b_wh^2 + b_{in}h^2 - b_{in}h_w^2 - 2b_{in}h_wt_d) + 2A_sE_sd_{bot}}{2\left(E_c\left(2b_ft_d + 2b_wh + b_{in}h - b_{in}h_w\right) + A_sE_s\right)}$$

The corresponding bending moment capacity and curvature:

$$M = N_s(d_{bot} - x) + N_{c1}\cdot\frac{2}{3}x + N_{c2}\cdot\frac{2}{3}x' + N_{c3}\cdot\frac{2}{3}(h-x) + N_{c4}\cdot\frac{2}{3}x"$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d_{bot}}$$

5.2 The cracking moment

The cracking moment M_{cr} is reached at a concrete tensile strain $\varepsilon_{ctensile} = \varepsilon_{ct}$ and a curvature κ_{cr}. The strain in the concrete and steel can be derived from [Figure 5.1].

$$\varepsilon_{ct} = \frac{f_{ctm}}{E_c}$$

$$\frac{\varepsilon_s}{d_{bot} - x} = \frac{\varepsilon_{ct}}{h - x} \rightarrow \varepsilon_s = \frac{d_{bot} - x}{h - x}\varepsilon_{ct}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_{ct}}{h - x} \rightarrow \varepsilon_c = \frac{x}{h - x}\varepsilon_{ct}$$

$$\frac{\varepsilon_c{}'}{x'} = \frac{\varepsilon_{ct}}{h - x} \rightarrow \varepsilon_c{}' = \frac{x'}{h - x}\varepsilon_{ct}$$

$$\frac{\varepsilon_c{}''}{x''} = \frac{\varepsilon_{ct}}{h - x} \rightarrow \varepsilon_c{}'' = \frac{x''}{h - x}\varepsilon_{ct}$$

$$x' = x - t_d$$

$$x'' = h_w - x'$$

With the equations above and ε_{ct} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} + N_{c2} = N_s + N_{c3} + N_{c4} \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_c x - \frac{1}{2}(2b_f + b_{in})E_c\varepsilon_c' x' = A_s E_s \varepsilon_s + \frac{1}{2}(2b_w + b_{in})E_c\varepsilon_{ct}(h - x) - \frac{1}{2}b_{in}E_c\varepsilon_c'' x'' \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_c x - b_f E_c\varepsilon_c' x' - \frac{1}{2}b_{in}E_c\varepsilon_c' x' = A_s E_s \varepsilon_s + b_w E_c \varepsilon_{ct} h - b_w E_c \varepsilon_{ct} x + \frac{1}{2}b_{in}E_c\varepsilon_{ct}h - \frac{1}{2}b_{in}E_c\varepsilon_{ct}x$$
$$- \frac{1}{2}b_{in}E_c\varepsilon_c'' x'' \rightarrow$$

$$bE_c x^2 - 2b_f E_c x'^2 - b_{in}E_c x'^2 = 2A_s E_s d_{bot} - 2A_s E_s x + 2b_w E_c h^2 - 4b_w E_c hx + 2b_w E_c x^2$$
$$+ b_{in}E_c h^2 - 2b_{in}E_c hx + b_{in}E_c x^2 - b_{in}E_c x''^2 \rightarrow$$

$$bE_cx^2 - 2b_fE_cx^2 + 4b_fE_ct_dx - 2b_fE_ct_d^2 = 2A_sE_sd_{bot} - 2A_sE_sx + 2b_wE_ch^2 - 4b_wE_chx$$
$$+2b_wE_cx^2 + b_{in}E_ch^2 - 2b_{in}E_chx + b_{in}E_cx^2 - b_{in}E_ch_w^2 + 2b_{in}E_ch_wx - 2b_{in}E_ch_wt_d \rightarrow$$

$$2\big(E_c\big(2b_ft_d + 2b_wh + b_{in}h - b_{in}h_w\big) + A_sE_s\big)x$$
$$= E_c(2b_ft_d^2 + 2b_wh^2 + b_{in}h^2 - b_{in}h_w^2 - 2b_{in}h_wt_d) + 2A_sE_sd_{bot} \rightarrow$$

$$x = \frac{E_c(2b_ft_d^2 + 2b_wh^2 + b_{in}h^2 - b_{in}h_w^2 - 2b_{in}h_wt_d) + 2A_sE_sd_{bot}}{2\big(E_c\big(2b_ft_d + 2b_wh + b_{in}h - b_{in}h_w\big) + A_sE_s\big)}$$

The cracking moment can be expressed by:

$$M_{cr} = N_s(d_{bot} - x) + N_{c1} \cdot \frac{2}{3}x + N_{c2} \cdot \frac{2}{3}x' + N_{c3} \cdot \frac{2}{3}(h - x) + N_{c4} \cdot \frac{2}{3}x"$$

The corresponding curvature:

$$\kappa_{cr} = \frac{\varepsilon_c + \varepsilon_s}{d_{bot}}$$

5.3 Cracked beam

When the load is further increased, concrete cracking starts to occur and the steel reinforcing bars will provide tensile capacity to restore equilibrium. $\varepsilon_{ctensile}$ becomes zero.

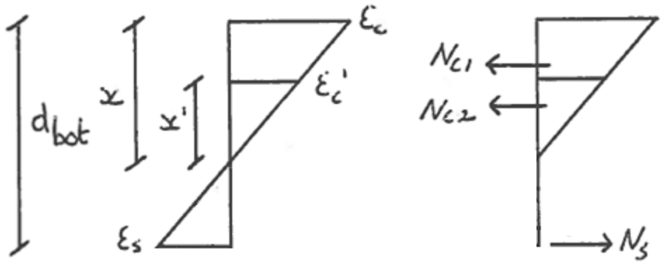


Figure 5.2: deformation and stress for elastic material behavior ($\varepsilon_c < \varepsilon_{c3}$ and $\varepsilon_s < \varepsilon_{sy}$).

$x > t_d$.

When $\boldsymbol{\varepsilon_s < \varepsilon_{sy}}$:

With respect to the deformation and stress diagram of [Figure 5.2] the strain in the steel and concrete can be expressed as:

$$\frac{\varepsilon_s}{d_{bot} - x} = \frac{\varepsilon_c}{x} \rightarrow \varepsilon_s = \frac{d_{bot} - x}{x}\varepsilon_c$$

$$\frac{\varepsilon_c{}'}{x'} = \frac{\varepsilon_c}{x} \rightarrow \varepsilon_c{}' = \frac{x'}{x}\varepsilon_c$$

With the equation above and ε_c as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} + N_{c2} = N_s \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_c x - \frac{1}{2}(2b_f + b_{in})E_c\varepsilon_c' x' = A_s E_s \varepsilon_s \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_c x - \frac{1}{2}(2b_f + b_{in})E_c\frac{x'}{x}\varepsilon_c x' = A_s E_s \frac{d_{bot} - x}{x}\varepsilon_c \rightarrow$$

$$bE_c x^2 - (2b_f + b_{in})E_c x'^2 = 2A_s E_s(d_{bot} - x) \rightarrow$$

$$bE_c x^2 - 2b_f E_c\left(x^2 - 2t_d x + t_d^2\right) - b_{in} E_c\left(x^2 - 2t_d x + t_d^2\right) = 2A_s E_s d_{bot} - 2A_s E_s x \rightarrow$$

$$bE_c x^2 - 2b_f E_c x^2 + 4b_f E_c t_d x - 2b_f E_c t_d^2 - b_{in} E_c x^2 + 2b_{in} E_c t_d x - b_{in} E_c t_d^2$$
$$= 2A_s E_s d_{bot} - 2A_s E_s x \rightarrow$$

$$bE_c x^2 - 2b_f E_c x^2 - b_{in} E_c x^2 + 4b_f E_c t_d x + 2b_{in} E_c t_d x + 2A_s E_s x - 2b_f E_c t_d^2$$
$$-b_{in} E_c t_d^2 - 2A_s E_s d_{bot} = 0 \rightarrow$$

$$E_c\left(b - 2b_f - b_{in}\right)x^2 + 2\left(E_c t_d\left(2b_f + b_{in}\right) + A_s E_s\right)x - E_c t_d^2(2b_f + b_{in}) - 2A_s E_s d_{bot} = 0$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(b - 2b_f - b_{in}\right)$$

$$b = 2\left(E_c t_d\left(2b_f + b_{in}\right) + A_s E_s\right)$$

$$c = -E_c t_d^2(2b_f + b_{in}) - 2A_s E_s d_{bot}$$

The corresponding bending moment capacity and curvature:

$$M = N_s(d_{bot} - x) + N_{c1} \cdot \frac{2}{3} x + N_{c2} \cdot \frac{2}{3} x'$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d_{bot}}$$

5.4 The yield moment

The concrete strength class and the amount of steel reinforcing bars determine whether concrete or steel will reach the plastic phase first. The steel reinforcing bars will start to yield at a yield strain $\varepsilon_{sy} = 2{,}17‰$. Concrete starts to become plastic when $\varepsilon_c = \varepsilon_{c3}$. **It is now assumed that the reinforcement ratio is not too high and the steel yields first**.

$x > t_d$.

With respect to the deformation and stress diagram of [Figure 5.2] the strain in the steel and concrete can be expressed as:

$$\varepsilon_s = \varepsilon_{sy}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d_{bot} - x} \rightarrow \varepsilon_c = \frac{x}{d_{bot} - x}\varepsilon_s \rightarrow \varepsilon_c = \frac{x}{d_{bot} - x}\varepsilon_{sy}$$

$$\frac{\varepsilon_c{}'}{x'} = \frac{\varepsilon_s}{d_{bot} - x} \rightarrow \varepsilon_c{}' = \frac{x'}{d_{bot} - x}\varepsilon_s \rightarrow \varepsilon_c{}' = \frac{x'}{d_{bot} - x}\varepsilon_{sy}$$

With the equations above and ε_{sy} as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} + N_{c2} = N_s \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_c x - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_c' x' = A_s E_s \varepsilon_s \rightarrow$$

$$\frac{1}{2}bE_c\frac{x}{d_{bot} - x}\varepsilon_{sy}x - b_f E_c\frac{x'}{d_{bot} - x}\varepsilon_{sy}x' - \frac{1}{2}b_{in}E_c\frac{x'}{d_{bot} - x}\varepsilon_{sy}x' = A_s E_s \varepsilon_{sy} \rightarrow$$

$$bE_c x^2 - 2b_f E_c {x'}^2 - b_{in}E_c {x'}^2 = 2A_s E_s(d_{bot} - x) \rightarrow$$

$$bE_c x^2 - 2b_f E_c\left(x^2 - 2t_d x + t_d^2\right) - b_{in}E_c\left(x^2 - 2t_d x + t_d^2\right) = 2A_s E_s d_{bot} - 2A_s E_s x \rightarrow$$

$$bE_c x^2 - 2b_f E_c x^2 + 4b_f E_c t_d x - 2b_f E_c t_d^2 - b_{in}E_c x^2 + 2b_{in}E_c t_d x - b_{in}E_c t_d^2 - 2A_s E_s d_{bot}$$
$$+2A_s E_s x = 0 \rightarrow$$

$$E_c(b - 2b_f - b_{in})x^2 + 2(E_c t_d(2b_f + b_{in}) + A_s E_s)x - E_c t_d^2(2b_f + b_{in}) - 2A_s E_s d_{bot} = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c(b - 2b_f - b_{in})$$

$$b = 2(E_c t_d(2b_f + b_{in}) + A_s E_s)$$

$$c = -E_c t_d^2(2b_f + b_{in}) - 2A_s E_s d_{bot}$$

The yield moment can be expressed by:

$$M_y = N_s(d_{bot} - x) + N_{c1} \cdot \frac{2}{3}x + N_{c2} \cdot \frac{2}{3}x'$$

The corresponding curvature:

$$\kappa_y = \frac{\varepsilon_c + \varepsilon_{sy}}{d}$$

After the yield moment is reached the moment capacity further increases until the concrete compressive strain reaches ε_{c3}.

When $\boldsymbol{\varepsilon_s > \varepsilon_{sy}}$ & $\boldsymbol{\varepsilon_c < \varepsilon_{c3}}$:

$x > t_d$.

With respect to the deformation and stress diagram of [Figure 5.2] the strain in the steel and concrete can be expressed as:

$$\varepsilon_s > \varepsilon_{sy}$$

$$\frac{\varepsilon_c{}'}{x'} = \frac{\varepsilon_c}{x} \rightarrow \varepsilon_c{}' = \frac{x'}{x}\varepsilon_c$$

With the previous equations and ε_c as input, the concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} + N_{c2} = N_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_c x - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_c' x' = A_s E_s \varepsilon_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_c x - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \frac{x'}{x} \varepsilon_c x' = A_s E_s \varepsilon_{sy} \rightarrow$$

$$b E_c \varepsilon_c x^2 - 2b_f E_c \varepsilon_c \left(x^2 - 2t_d x + t_d^2\right) - b_{in} E_c \varepsilon_c \left(x^2 - 2t_d x + t_d^2\right) = 2A_s f_{yd} x \rightarrow$$

$$b E_c \varepsilon_c x^2 - 2b_f E_c \varepsilon_c x^2 - b_{in} E_c \varepsilon_c x^2 + 4b_f E_c \varepsilon_c t_d x + 2b_{in} E_c \varepsilon_c t_d x - 2A_s f_{yd} x - 2b_f E_c \varepsilon_c t_d^2$$
$$- b_{in} E_c \varepsilon_c t_d^2 = 0 \rightarrow$$

$$E_c \varepsilon_c \left(b - 2b_f - b_{in}\right) x^2 + 2\left(E_c \varepsilon_c t_d \left(2b_f + b_{in}\right) - A_s f_{yd}\right) x - E_c \varepsilon_c t_d^2 \left(2b_f + b_{in}\right) = 0 \rightarrow$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c \varepsilon_c \left(b - 2b_f - b_{in}\right)$$

$$b = 2\left(E_c \varepsilon_c t_d \left(2b_f + b_{in}\right) - A_s f_{yd}\right)$$

$$c = -E_c \varepsilon_c t_d^2 \left(2b_f + b_{in}\right)$$

The corresponding bending moment capacity and curvature:

$$M = N_s (d_{bot} - x) + N_{c1} \cdot \frac{2}{3} x + N_{c2} \cdot \frac{2}{3} x'$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d_{bot}}$$

When $\boldsymbol{x = t_d}$:

$$\varepsilon_s > \varepsilon_{sy}$$

The compressive stress at the top of the beam ε_c can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} = N_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_c x = A_s E_s \varepsilon_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_c t_d = A_s E_s \varepsilon_{sy} \rightarrow$$

$$\varepsilon_c = \frac{2 A_s f_{yd}}{b E_c t_d}$$

When $\boldsymbol{\varepsilon_s > \varepsilon_{sy}}$ & $\boldsymbol{\varepsilon_c < \varepsilon_{c3}}$:

$x > t_d$.

The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} = N_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_c x = A_s E_s \varepsilon_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_c x = A_s E_s \varepsilon_{sy} \rightarrow$$

$$x = \frac{2 A_s f_{yd}}{b E_c \varepsilon_c}$$

The corresponding bending moment capacity and curvature:

$$M = N_s (d_{bot} - x) + N_{c1} \cdot \frac{2}{3} x$$

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d_{bot}}$$

5.5 The plastic moment

The plastic moment occurs when $\varepsilon_c = \varepsilon_{c3}$. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{c1} = N_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_c x = A_s E_s \varepsilon_s \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_{c3} x = A_s E_s \varepsilon_{sy} \rightarrow$$

$$x = \frac{2 A_s f_{yd}}{b f_{cd}} \rightarrow$$

The plastic moment can be expressed by:

$$M_{c,pl} = A_s f_{yd} \left(d - \frac{1}{3} x \right)$$

The corresponding curvature:

$$\kappa_{c,pl} = \frac{\varepsilon_{c3} + \varepsilon_s}{d}$$

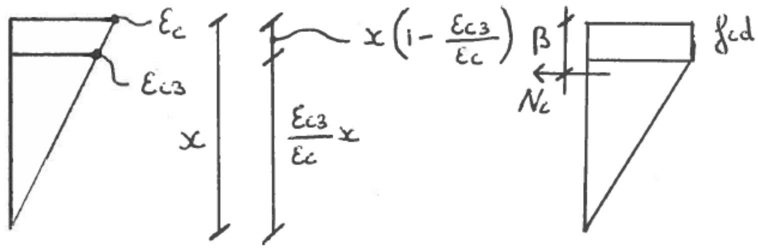


Figure 5.3: deformation and stress diagram when $\varepsilon_c > \varepsilon_{c3}$.

When $\boldsymbol{\varepsilon_c > \varepsilon_{c3}}$ [Figure 5.3] is valid. The concrete compressive zone height x can be derived from equilibrium of horizontal forces:

$$N_{c1} = \frac{1}{2} \frac{\varepsilon_{c3}}{\varepsilon_c} x f_{cd} + x \left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c} \right) f_{cd} = b x f_{cd} \left(1 - \frac{\varepsilon_{c3}}{2 \varepsilon_c} \right)$$

$$\sum F_H = 0 \rightarrow N_{c1} = N_s \rightarrow$$

$$bxf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_s f_{yd} \rightarrow$$

$$x = \frac{A_s f_{yd}}{bf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right)}$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta = \frac{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) \cdot \frac{x}{2}\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{b}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}x \cdot \left(x\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{\varepsilon_{c3}}{\varepsilon_c}\frac{x}{3}\right)}{bx\left(1 - \frac{\varepsilon_{c3}}{\varepsilon_c}\right) + \frac{bx}{2}\frac{\varepsilon_{c3}}{\varepsilon_c}}$$

5.6 The ultimate bending moment resistance

The ultimate bending moment resistance can be derived when $\varepsilon_c = \varepsilon_{cu3}$:

$$M_{Rd} = A_s f_{yd}(d_{bot} - \beta)$$

The corresponding curvature:

$$\kappa_{Rd} = \frac{\varepsilon_{cu3} + \varepsilon_s}{d_{bot}}$$

5.7 Shear

The design shear resistance of the member without shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$v_{min} = 0{,}035k^{\frac{3}{2}}\sqrt{f_{ck}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$V_{Rd,cmin} = \left(v_{min} + k_1\sigma_{cp}\right)b_w d$$

$$k_1 = 0{,}15$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0{,}2f_{cd}$$

$$V_{Rd,c} = \left[C_{Rd,c}k\sqrt[3]{100\rho_l f_{ck}} + k_1\sigma_{cp}\right]b_w d$$

$$C_{Rd,c} = 0{,}18/\gamma_c$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$$

The design value of the shear force, which can be sustained by the yielding shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta$$

$$z = d - \beta$$

$$f_{ywd} = 0{,}8f_{yk}$$

$$1 \leq \cot\theta \leq 2{,}5$$

The design value of the maximum shear force, which can be sustained by the member, limited by crushing of the compression struts [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,max} = \frac{\alpha_{cw} b_w z \nu_1 f_{cd}}{\cot\theta + \tan\theta}$$

For non-prestressed structures: $\alpha_{cw} = 1$

For: $f_{ck} \leq 60\ N/mm^2$: $\nu_1 = 0{,}6$

For: $f_{ck} \geq 90\ N/mm^2$: $\nu_1 = 0{,}9 - \frac{f_{ck}}{200}$

5.8 Crack width

This paragraph is based on [NEN-EN 1992-1-1: 7.3.4].

Determine w_{max} from [NEN-EN 1992-1-1: Table 7.1N].

$$w_{max} = w_k$$

$$k_1 = 0{,}8$$

$$k_2 = 0{,}5$$

$$k_3 = 3{,}4$$

$$k_4 = 0{,}425$$

For short term loading: $k_t = 0{,}6$

For long term loading: $k_t = 0{,}4$

$$\alpha_e = \frac{E_s}{E_{cm}}$$

$$f_{ct,eff} = f_{ctm}(t)$$

The beam is in the cracked phase.

For a reinforced box girder that is cracked, the concrete compressive zone height x remains the same under increasing load until the yield moment is reached.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(b - 2b_f - b_{in}\right)$$

$$b = 2\left(E_c t_d\left(2b_f + b_{in}\right) + A_s E_s\right)$$

$$c = -E_c t_d^2(2b_f + b_{in}) \ - 2A_s E_s d_{bot}$$

With x the maximum allowable steel stress σ_s can be determined:

$$h_{c,eff} = \min\left\{2{,}5(h - d); \frac{h - x}{3}; 0{,}5h\right\}$$

$$A_{c,eff} = b h_{c,eff}$$

$$\rho_{p,eff} = \frac{A_s}{A_{c,eff}}$$

$$s_{r,max} = k_3 c + \frac{k_1 k_2 k_4 \phi}{\rho_{p,eff}}$$

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \rightarrow \varepsilon_{sm} - \varepsilon_{cm} = \frac{w_k}{s_{r,max}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \rightarrow$$

$$\sigma_s = E_s(\varepsilon_{sm} - \varepsilon_{cm}) + k_t \frac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)$$

With the maximum allowable steel stress σ_s and [Figure 5.2] the maximum moment in SLS can be calculated.

$$\varepsilon_s = \frac{\sigma_s}{E_s}$$

$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d - x} \rightarrow \varepsilon_c = \varepsilon_s \frac{x}{d - x}$$

$$\frac{\varepsilon_c{}'}{x'} = \frac{\varepsilon_s}{d-x} \rightarrow \varepsilon_c' = \varepsilon_s \frac{x'}{d-x}$$

$$N_{c1} = \frac{1}{2} bxE_c\varepsilon_c$$

$$N_{c2} = -\frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_c' x'$$

$$N_s = A_sE_s\varepsilon_s$$

$$M_{qp} = N_s(d_{bot} - x) + N_{c1} \cdot \frac{2}{3}x + N_{c2} \cdot \frac{2}{3}x'$$

5.9 Concrete compressive zone height

This paragraph is based on [NEN-EN 1992-1-1+C2/NB: 6.1].

For: $f_{ck} \leq 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{500}{500+f}$$

For: $f_{ck} > 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{\varepsilon_{cu} \cdot 10^6}{\varepsilon_{cu} \cdot 10^6 + 7f}$$

$$f = \frac{\left(\frac{f_{pk}}{\gamma_s} - \sigma_{pm\infty}\right)A_p + f_{yd}A_s}{A_p + A_s}$$

6 Prestressed NSC box girder

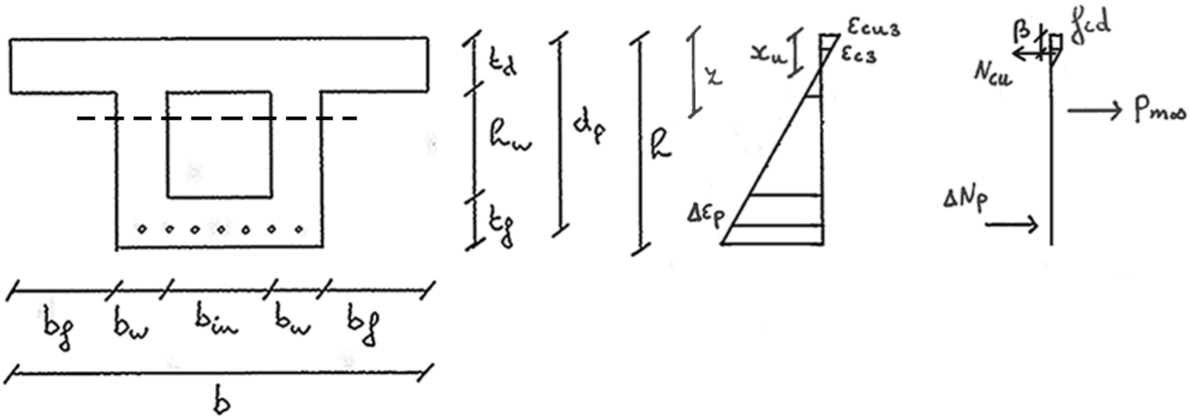


Figure 6.1: deformation and stress diagram when $\varepsilon_c = \varepsilon_{cu3}$ and $x_u < t_d$.

6.1 The ultimate bending moment $(x_u < t_d)$

$x_u < t_d$.

$\varepsilon_p > \varepsilon_{py}$.

The ultimate bending moment can be found when $\varepsilon_c = \varepsilon_{cu3}$ [Figure 6.1].

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

$$\varepsilon_{p\infty} = \frac{\sigma_{pm\infty}}{E_p}$$

$$\frac{\varepsilon_{cu3}}{x_u} = \frac{\Delta\varepsilon_p}{d_p - x_u} \to \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u}\left(d_p - x_u\right) \to \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u} d_p - \varepsilon_{cu3}$$

The concrete compressive zone height x_u can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \to N_{cu} = \Delta N_p \to$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_p \sigma_p - P_{m\infty} \to$$

$$bx_uf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)=A_p\left(f_{pd}+\frac{\varepsilon_p-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)-A_p\sigma_{pm\infty}\rightarrow$$

$$bx_uf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)=A_p\left(f_{pd}+\frac{\frac{\varepsilon_{cu3}}{x_u}d_p-\varepsilon_{cu3}+\frac{\sigma_{pm\infty}}{E_p}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)-A_p\sigma_{pm\infty}\rightarrow$$

$$bx_uf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)=A_pf_{pd}+A_p\frac{\frac{\varepsilon_{cu3}}{x_u}d_p-\varepsilon_{cu3}+\frac{\sigma_{pm\infty}}{E_p}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\sigma_{pm\infty}\rightarrow$$

$$bx_uf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)=A_pf_{pd}\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$+A_p\left(\frac{\varepsilon_{cu3}}{x_u}d_p-\varepsilon_{cu3}+\frac{\sigma_{pm\infty}}{E_p}-\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\sigma_{pm\infty}\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$bx_uf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)=A_pf_{pd}\left(\varepsilon_{uk}-\varepsilon_{py}\right)+A_p\frac{\varepsilon_{cu3}}{x_u}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-A_p\varepsilon_{cu3}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+A_p\frac{\sigma_{pm\infty}}{E_p}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p\sigma_{pm\infty}\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$bx_u^2f_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)=A_pf_{pd}\left(\varepsilon_{uk}-\varepsilon_{py}\right)x_u+A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-A_p\varepsilon_{cu3}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)x_u+A_p\frac{\sigma_{pm\infty}}{E_p}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)x_u-A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)x_u$$

$$-A_p\sigma_{pm\infty}\left(\varepsilon_{uk}-\varepsilon_{py}\right)x_u\rightarrow$$

$$-bf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)x_u^2$$

$$+A_p\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)-\left(\varepsilon_{cu3}-\frac{\sigma_{pm\infty}}{E_p}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)x_u$$

$$+A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)=0\rightarrow$$

$$x_u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -bf_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)$$

$$b = A_p\left(\left(f_{pd} - \sigma_{pm\infty}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) - \left(\varepsilon_{cu3} - \frac{\sigma_{pm\infty}}{E_p} + \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$c = A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

The corresponding bending moment capacity and curvature:

$$M_{Rd} = N_{cu}(x_u - \beta) + P_{m\infty}(z - x_u) + \Delta N_p\left(d_p - x_u\right)$$

$$\kappa_{Rd} = \frac{\varepsilon_{cu3}}{x_u}$$

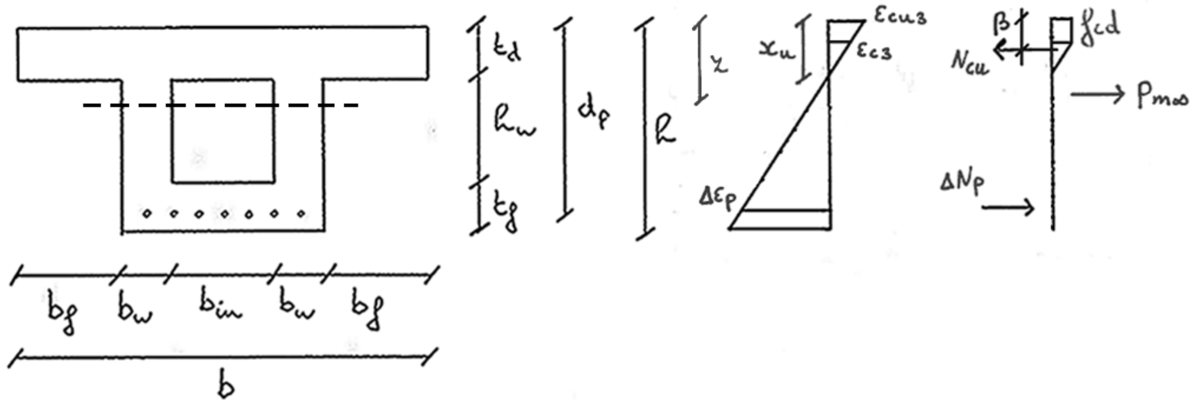


Figure 6.2: deformation and stress diagram when $\varepsilon_c = \varepsilon_{cu3}$ and $x_u = t_d$.

6.2 The ultimate bending moment $(x_u = t_d)$

$x_u = t_d$.

$\varepsilon_p > \varepsilon_{py}$.

The ultimate bending moment can be found when $\varepsilon_c = \varepsilon_{cu3}$ [Figure 6.2].

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

$$\varepsilon_{p\infty} = \frac{\sigma_{pm\infty}}{E_p}$$

$$\frac{\varepsilon_{cu3}}{x_u} = \frac{\Delta\varepsilon_p}{d_p - x_u} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u}\left(d_p - x_u\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u} d_p - \varepsilon_{cu3}$$

The corresponding amount of prestressing steel A_p can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{cu} = \Delta N_p \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) = A_p\sigma_p - P_{m\infty} \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right) = A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - A_p\sigma_{pm\infty} \rightarrow$$

$$b t_d f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right) = A_p\left(f_{pd} + \frac{\frac{\varepsilon_{cu3}}{t_d} d_p - \varepsilon_{cu3} + \frac{\sigma_{pm\infty}}{E_p} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}\right) \rightarrow$$

$$A_p = \frac{b t_d f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)}{f_{pd} + \frac{\frac{\varepsilon_{cu3}}{t_d} d_p - \varepsilon_{cu3} + \frac{\sigma_{pm\infty}}{E_p} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}}$$

6.3 The ultimate bending moment $(x_u > t_d)$

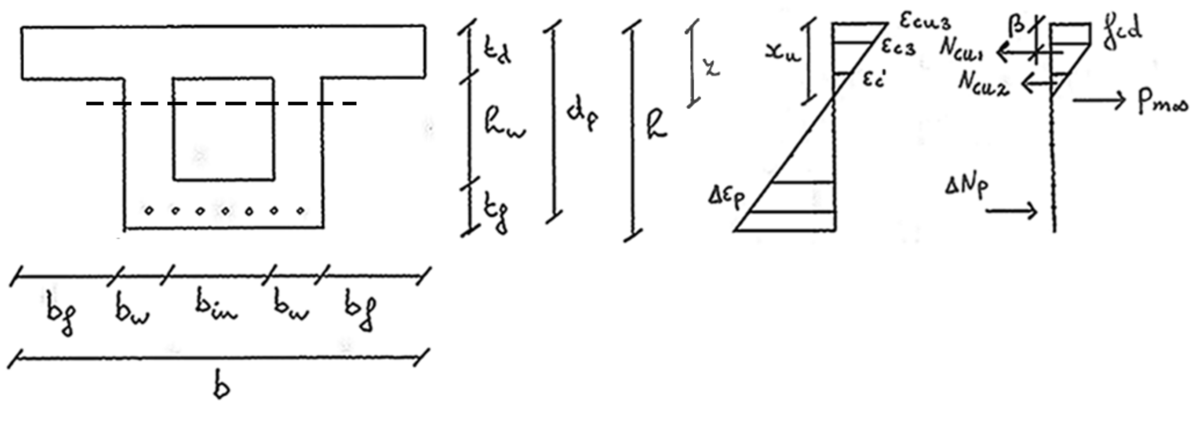


Figure 6.3: deformation and stress diagram when $\varepsilon_c = \varepsilon_{cu3}$ and $x_u > t_d$.

$x_u > t_d$.

$\varepsilon_p > \varepsilon_{py}$.

The ultimate bending moment can be found when $\varepsilon_c = \varepsilon_{cu3}$ [Figure 6.3].

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

$$\varepsilon_{p\infty} = \frac{\sigma_{pm\infty}}{E_p}$$

$$\frac{\varepsilon_{cu3}}{x_u} = \frac{\varepsilon_c'}{x_u - t_d} \rightarrow \varepsilon_c' = \frac{\varepsilon_{cu3}}{x_u}(x_u - t_d) \rightarrow \varepsilon_c' = \varepsilon_{cu3} - \frac{\varepsilon_{cu3}}{x_u} t_d$$

$$\frac{\varepsilon_{cu3}}{x_u} = \frac{\Delta\varepsilon_p}{d_p - x_u} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u}(d_p - x_u) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_{cu3}}{x_u} d_p - \varepsilon_{cu3}$$

The concrete compressive zone height x_u can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow N_{cu1} + N_{cu2} = \Delta N_p \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_c}\right) - \frac{1}{2}(2b_f + b_{in}) E_c \varepsilon_c' (x_u - t_d) = A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right) - \frac{1}{2}(2b_f + b_{in}) E_c \left(\varepsilon_{cu3} - \frac{\varepsilon_{cu3}}{x_u} t_d\right)(x_u - t_d)$$
$$= A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - A_p \sigma_{pm\infty} \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right) - \frac{1}{2}(2b_f + b_{in}) E_c \varepsilon_{cu3} x_u + (2b_f + b_{in}) E_c \varepsilon_{cu3} t_d - \frac{1}{2}(2b_f + b_{in}) E_c \frac{\varepsilon_{cu3}}{x_u} t_d^2$$
$$= A_p\left(f_{pd} + \frac{\Delta\varepsilon_p + \varepsilon_{p\infty} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - A_p \sigma_{pm\infty} \rightarrow$$

$$b x_u f_{cd}\left(1 - \frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right) - \frac{1}{2}(2b_f + b_{in}) E_c \varepsilon_{cu3} x_u + (2b_f + b_{in}) E_c \varepsilon_{cu3} t_d - \frac{1}{2}(2b_f + b_{in}) E_c \frac{\varepsilon_{cu3}}{x_u} t_d^2$$
$$= A_p f_{pd} + A_p \frac{\frac{\varepsilon_{cu3}}{x_u} d_p - \varepsilon_{cu3} + \varepsilon_{p\infty} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - A_p \sigma_{pm\infty} \rightarrow$$

$$bf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)(\varepsilon_{uk}-\varepsilon_{py})x_u^2-\frac{1}{2}(2b_f+b_{in})E_c\varepsilon_{cu3}(\varepsilon_{uk}-\varepsilon_{py})x_u^2$$

$$+(2b_f+b_{in})E_c\varepsilon_{cu3}(\varepsilon_{uk}-\varepsilon_{py})t_d x_u-\frac{1}{2}(2b_f+b_{in})E_c\varepsilon_{cu3}(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$=A_p f_{pd}(\varepsilon_{uk}-\varepsilon_{py})x_u+A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-A_p(\varepsilon_{cu3}-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)x_u$$

$$-A_p\sigma_{pm\infty}(\varepsilon_{uk}-\varepsilon_{py})x_u\rightarrow$$

$$\left(\left(bf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)-\frac{1}{2}(2b_f+b_{in})E_c\varepsilon_{cu3}\right)(\varepsilon_{uk}-\varepsilon_{py})\right)x_u^2$$

$$+\left((2b_f+b_{in})E_c\varepsilon_{cu3}(\varepsilon_{uk}-\varepsilon_{py})t_d-A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})-(\varepsilon_{cu3}-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)x_u$$

$$-\frac{1}{2}(2b_f+b_{in})E_c\varepsilon_{cu3}(\varepsilon_{uk}-\varepsilon_{py})t_d^2-A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)=0\rightarrow$$

$$x_u=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=\left(bf_{cd}\left(1-\frac{\varepsilon_{c3}}{2\varepsilon_{cu3}}\right)-\frac{1}{2}(2b_f+b_{in})E_c\varepsilon_{cu3}\right)(\varepsilon_{uk}-\varepsilon_{py})$$

$$b=(2b_f+b_{in})E_c\varepsilon_{cu3}(\varepsilon_{uk}-\varepsilon_{py})t_d-A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})-(\varepsilon_{cu3}-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)$$

$$c=-\frac{1}{2}(2b_f+b_{in})E_c\varepsilon_{cu3}(\varepsilon_{uk}-\varepsilon_{py})t_d^2-A_p\varepsilon_{cu3}d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

The corresponding bending moment capacity and curvature:

$$M_{Rd}=N_{cu1}(x_u-\beta)+N_{cu2}\cdot\frac{2}{3}(x_u-t_d)+P_{m\infty}(z-x_u)+\Delta N_p(d_p-x_u)$$

$$\kappa_{Rd}=\frac{\varepsilon_{cu3}}{x_u}$$

6.4 Shear

The design shear resistance of the member without shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$v_{min} = 0{,}035k^{\frac{3}{2}}\sqrt{f_{ck}}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2$$

$$V_{Rd,cmin} = \left(v_{min} + k_1\sigma_{cp}\right)b_w d$$

$$k_1 = 0{,}15$$

$$\sigma_{cp} = \frac{P_{m\infty}}{A_c} < 0{,}2f_{cd}$$

$$V_{Rd,c} = \left[C_{Rd,c}k\sqrt[3]{100\rho_l f_{ck}} + k_1\sigma_{cp}\right]b_w d$$

$$C_{Rd,c} = 0{,}18/\gamma_c$$

$$\rho_l = \frac{A_{sl}}{b_w d} \leq 0{,}02$$

The design value of the shear force, which can be sustained by the yielding shear reinforcement [NEN-EN 1992-1-1: 6.2]:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd}(\cot\theta + \cot\alpha)\sin\alpha$$

$$z = d - \beta$$

$$f_{ywd} = 0{,}8f_{yk}$$

$$1 \leq \cot\theta \leq 2{,}5$$

For vertical stirrups: $\alpha = 90°$.

The design value of the maximum shear force, which can be sustained by the member, limited by crushing of the compression struts [NEN-EN 1992-1-1: 6.2].

$V_{Rd,max} = b_w z(\cot\theta + \cot\alpha)\sin^2\theta\,\alpha_{cw}\nu_1 f_{cd}$

For: $0 < \sigma_{cp} \leq 0{,}25f_{cd}$: $\alpha_{cw} = \left(1 + \frac{\sigma_{cp}}{f_{cd}}\right)$

For: $0{,}25f_{cd} < \sigma_{cp} \leq 0{,}5f_{cd}$: $\alpha_{cw} = 1{,}25$

For: $0{,}5f_{cd} < \sigma_{cp} \leq 1{,}0f_{cd}$: $\alpha_{cw} = 2{,}5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right)$

For: $f_{ck} \leq 60\ N/mm^2$: $\nu_1 = 0{,}6$

For: $f_{ck} \geq 90\ N/mm^2$: $\nu_1 = 0{,}9 - \frac{f_{ck}}{200}$

6.5 Crack width

Requirement: the box girder remains uncracked in the serviceability limit state.

At $t = 0$, no time-dependent losses are present, so the prestressing force will be at its maximum. Because of the positioning of the tendons the box girder will be slightly cambered and tensile stresses will occur at the top. Bending moments cause compressive stresses at the top and tensile stresses at the bottom of the beam.

$t = 0 \rightarrow$ check top fibre:

$$-\frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{top}} - \frac{M}{W_{top}} \leq f_{ctm} \rightarrow \frac{M}{W_{top}} \geq -\frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{top}} - f_{ctm} \rightarrow$$

$$M \geq -\frac{P_{m0}}{A_c} W_{top} + P_{m0} \cdot e - f_{ctm} W_{top}$$

$t = 0 \rightarrow$ check bottom fibre:

$$-\frac{P_{m0}}{A_c} - \frac{P_{m0} \cdot e}{W_{bot}} + \frac{M}{W_{bot}} \leq f_{ctm} \rightarrow \frac{M}{W_{bot}} \leq \frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{bot}} + f_{ctm}$$

$$\rightarrow M \leq \frac{P_{m0}}{A_c} W_{bot} + P_{m0} \cdot e + f_{ctm} W_{bot}$$

At $t = \infty$, the prestressing force has been reduced by time-dependent losses, which means that the compressive stresses working on the cross-section will be limited. Dead and live loads are present and will cause tensile stresses at the bottom fibre in the span. The bending moment caused by these loads should be limited:

$t = \infty \rightarrow$ check top fibre:

$$-\frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{top}} - \frac{M}{W_{top}} \leq f_{ctm} \rightarrow \frac{M}{W_{top}} \geq -\frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{top}} - f_{ctm} \rightarrow$$

$$M \geq -\frac{P_{m\infty}}{A_c} W_{top} + P_{m\infty} \cdot e - f_{ctm} W_{top}$$

$t = \infty \rightarrow$ check bottom fibre:

$$-\frac{P_{m\infty}}{A_c} - \frac{P_{m\infty} \cdot e}{W_{bot}} + \frac{M}{W_{bot}} \leq f_{ctm} \rightarrow \frac{M}{W_{bot}} \leq \frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{bot}} + f_{ctm}$$

$$\rightarrow M \leq \frac{P_{m\infty}}{A_c} W_{bot} + P_{m\infty} \cdot e + f_{ctm} W_{bot}$$

6.6 Concrete compressive zone height

This paragraph is based on [NEN-EN 1992-1-1+C2/NB: 6.1].

For: $f_{ck} \leq 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{500}{500 + f}$$

For: $f_{ck} > 50\ N/mm^2$:

$$\frac{x_u}{d} \leq \frac{\varepsilon_{cu} \cdot 10^6}{\varepsilon_{cu} \cdot 10^6 + 7f}$$

$$f = \frac{\left(\frac{f_{pk}}{\gamma_s} - \sigma_{pm\infty}\right) A_p + f_{yd} A_s}{A_p + A_s}$$

7 Rectangular unreinforced UHPC beam

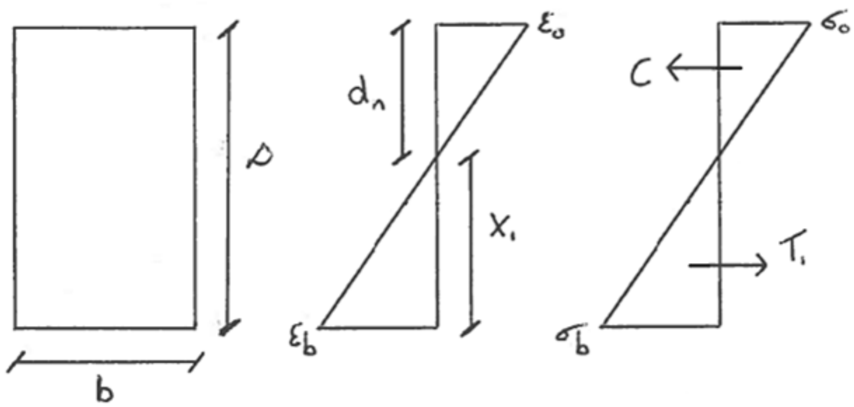


Figure 7.1: deformation and stress diagram when $\varepsilon_b < \varepsilon_{ctmax}$.

7.1 When $\varepsilon_b < \varepsilon_{ctmax}$

With respect to the deformation and stress diagram of [Figure 7.1] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} X_1 \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} D - \varepsilon_0$$

$$X_1 = D - d_n$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b E_c \varepsilon_{ctmax} X_1 \rightarrow$$

$$\varepsilon_0 d_n = \varepsilon_{ctmax}(D - d_n) \rightarrow$$

$$\varepsilon_0 d_n = \varepsilon_{ctmax} D - \varepsilon_{ctmax} d_n \rightarrow$$

$$d_n(\varepsilon_0 + \varepsilon_{ctmax}) = \varepsilon_{ctmax} D \rightarrow$$

$$d_n = \frac{\varepsilon_{ctmax} D}{\varepsilon_0 + \varepsilon_{ctmax}}$$

The corresponding bending moment capacity and curvature:

$$M = C \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

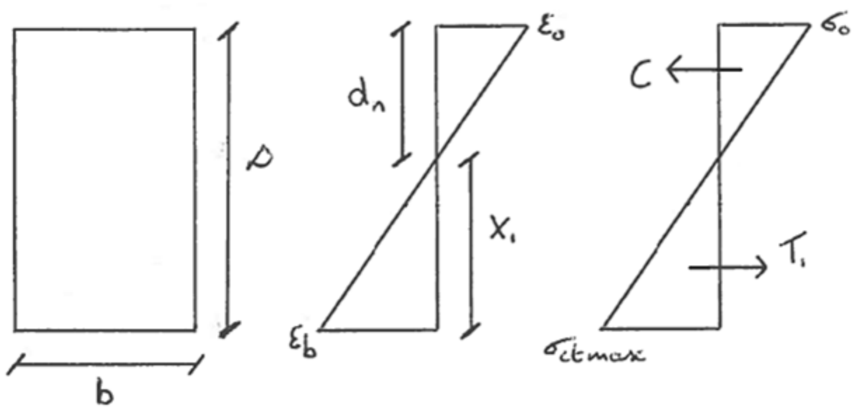


Figure 7.2: deformation and stress diagram when $\varepsilon_b = \varepsilon_{ctmax}$.

7.2 When $\varepsilon_b = \varepsilon_{ctmax}$

With respect to the deformation and stress diagram of [Figure 7.2] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{D - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}(D - d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} D - \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n$$
$$\rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} D \rightarrow d_n \left(1 + \frac{\varepsilon_0}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}} D \rightarrow d_n \left(\frac{\varepsilon_0 + \varepsilon_{ctmax}}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}} D \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}} D$$

$$X_1 = D - d_n$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b E_c \varepsilon_{ctmax} X_1 \rightarrow$$

$$\varepsilon_0 d_n = \varepsilon_{ctmax}(D - d_n) \rightarrow$$

$$\varepsilon_0 d_n = \varepsilon_{ctmax} D - \varepsilon_{ctmax} d_n \rightarrow$$

$$\frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}} D = \varepsilon_{ctmax} D - \varepsilon_{ctmax} \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}} D \rightarrow$$

$$\varepsilon_0^2 D = (\varepsilon_0 + \varepsilon_{ctmax})\varepsilon_{ctmax} D - \varepsilon_{ctmax}\varepsilon_0 D \rightarrow$$

$$\varepsilon_0^2 D = \varepsilon_{ctmax}^2 D \rightarrow$$

$$\varepsilon_0 = \varepsilon_{ctmax}$$

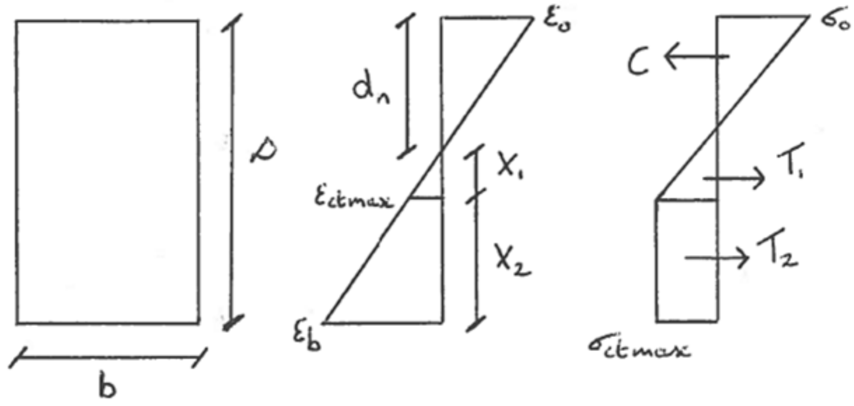


Figure 7.3: deformation and stress diagram when $\varepsilon_{ctmax} < \varepsilon_b < \varepsilon_{t,p}$.

7.3 When $\varepsilon_{ctmax} < \varepsilon_b < \varepsilon_{t,p}$

With respect to the deformation and stress diagram of [Figure 7.3] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$X_2 = D - X_1 - d_n = D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n = D - \left(\frac{\varepsilon_{ctmax}}{\varepsilon_0} + 1\right) d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1 + X_2} \rightarrow \varepsilon_b = \frac{X_1 + X_2}{d_n} \varepsilon_0$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \left(D - \left(\frac{\varepsilon_{ctmax}}{\varepsilon_0} + 1 \right) d_n \right) \rightarrow$$

$$\frac{1}{2} \varepsilon_0 d_n = \frac{1}{2} \frac{\varepsilon_{ctmax}^2}{\varepsilon_0} d_n + \varepsilon_{ctmax} \left(D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n \right) \rightarrow$$

$$\frac{1}{2} \varepsilon_0 d_n = \frac{1}{2} \frac{\varepsilon_{ctmax}^2}{\varepsilon_0} d_n + \varepsilon_{ctmax} D - \frac{\varepsilon_{ctmax}^2}{\varepsilon_0} d_n - \varepsilon_{ctmax} d_n \rightarrow$$

$$\frac{1}{2} \varepsilon_0 d_n + \frac{1}{2} \frac{\varepsilon_{ctmax}^2}{\varepsilon_0} d_n + \varepsilon_{ctmax} d_n = \varepsilon_{ctmax} D \rightarrow$$

$$\left(\frac{1}{2} \varepsilon_0 + \frac{1}{2} \frac{\varepsilon_{ctmax}^2}{\varepsilon_0} + \varepsilon_{ctmax} \right) d_n = \varepsilon_{ctmax} D \rightarrow$$

$$d_n = \frac{\varepsilon_{ctmax} D}{\frac{1}{2} \varepsilon_0 + \frac{1}{2} \frac{\varepsilon_{ctmax}^2}{\varepsilon_0} + \varepsilon_{ctmax}}$$

The corresponding bending moment capacity and curvature:

$$M = C \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

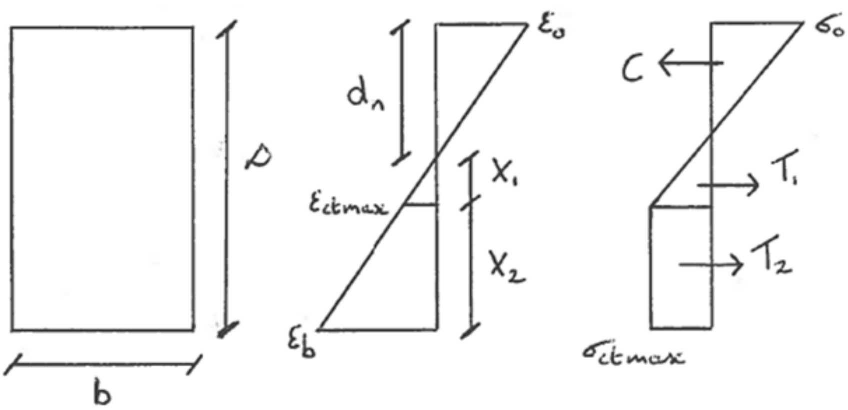


Figure 7.4: Deformation and stress diagram when $\varepsilon_b = \varepsilon_{t,p}$.

7.4 When $\varepsilon_b = \varepsilon_{t,p}$

With respect to the deformation and stress diagram of [Figure 7.4] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \to X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \to X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \to X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \to C = T_1 + T_2 \to$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 \to$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n \right) \to$$

$$\frac{1}{2} E_c \varepsilon_0^2 = \frac{1}{2} \sigma_{ctmax} \varepsilon_{ctmax} + \sigma_{ctmax} \varepsilon_{t,p} - \sigma_{ctmax} \varepsilon_{ctmax} \to$$

$$\frac{1}{2} E_c \varepsilon_0^2 = -\frac{1}{2} \sigma_{ctmax} \varepsilon_{ctmax} + \sigma_{ctmax} \varepsilon_{t,p} \to$$

$$\varepsilon_0^2 = \frac{-\sigma_{ctmax} \varepsilon_{ctmax} + 2 \sigma_{ctmax} \varepsilon_{t,p}}{E_c} \to$$

$$\varepsilon_0 = \sqrt{\frac{-\sigma_{ctmax}\varepsilon_{ctmax} + 2\sigma_{ctmax}\varepsilon_{t,p}}{E_c}}$$

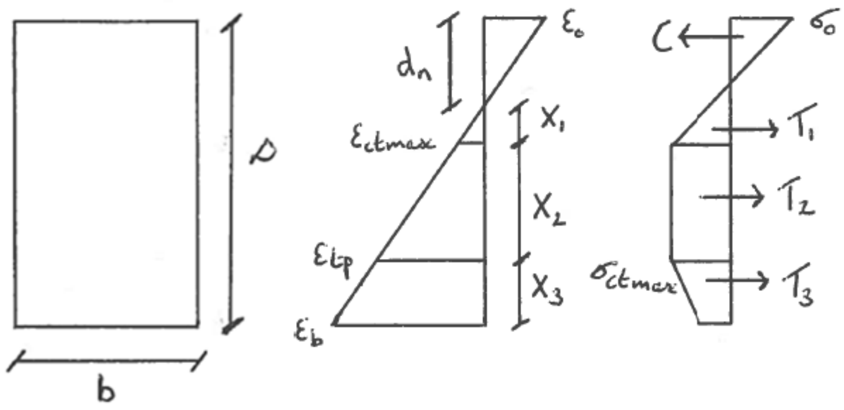


Figure 7.5: deformation and stress diagram when $\varepsilon_{t,p} < \varepsilon_b < \varepsilon_{t,u}$.

7.5 When $\varepsilon_{t,p} < \varepsilon_b < \varepsilon_{t,u}$

With respect to the deformation and stress diagram of [Figure 7.5] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$X_3 = D - (X_1 + X_2) - d_n = D - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{D - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) = \frac{\varepsilon_0}{d_n} D - \varepsilon_0$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + b \sigma_{ctmax} \left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right) X_3 + \frac{1}{2} b \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} X_3 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + b \sigma_{ctmax} X_3 - \frac{1}{2} b \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} X_3 \rightarrow$$

$$E_c\varepsilon_0 d_n = \sigma_{ctmax}X_1 + 2\sigma_{ctmax}X_2 + 2\sigma_{ctmax}X_3 - \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3 \rightarrow$$

$$E_c\varepsilon_0 d_n = \sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2\sigma_{ctmax}\left(\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n\right) + 2\sigma_{ctmax}\left(D - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - d_n\right)$$
$$-\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\left(D - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - d_n\right) \rightarrow$$

$$E_c\varepsilon_0 d_n = -\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2\sigma_{ctmax}D - 2\sigma_{ctmax}d_n - \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}D$$
$$+\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}d_n \rightarrow$$

$$E_c\varepsilon_0^2(\varepsilon_{t,u} - \varepsilon_{t,p})d_n = -\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{t,u} - \varepsilon_{t,p})d_n + 2\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u} - \varepsilon_{t,p})D$$
$$-2\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u} - \varepsilon_{t,p})d_n - \sigma_{ctmax}\varepsilon_0(\varepsilon_b - \varepsilon_{t,p})D + \sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_b - \varepsilon_{t,p})d_n$$
$$+\sigma_{ctmax}\varepsilon_0(\varepsilon_b - \varepsilon_{t,p})d_n \rightarrow$$

$$E_c\varepsilon_0^2\varepsilon_{t,u}d_n - E_c\varepsilon_0^2\varepsilon_{t,p}d_n = -\sigma_{ctmax}\varepsilon_{ctmax}\varepsilon_{t,u}d_n + \sigma_{ctmax}\varepsilon_{ctmax}\varepsilon_{t,p}d_n + 2\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}D$$
$$-\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}D - 2\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}d_n + \sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n - \sigma_{ctmax}\varepsilon_0\left(\frac{\varepsilon_0}{d_n}D - \varepsilon_0\right)D$$
$$+\sigma_{ctmax}\varepsilon_{t,p}\left(\frac{\varepsilon_0}{d_n}D - \varepsilon_0\right)d_n - \sigma_{ctmax}\varepsilon_{t,p}^2 d_n + \sigma_{ctmax}\varepsilon_0\left(\frac{\varepsilon_0}{d_n}D - \varepsilon_0\right)d_n \rightarrow$$

$$E_c\varepsilon_0^2\varepsilon_{t,u}d_n^2 - E_c\varepsilon_0^2\varepsilon_{t,p}d_n^2 + \sigma_{ctmax}\varepsilon_{ctmax}\varepsilon_{t,u}d_n^2 - \sigma_{ctmax}\varepsilon_{ctmax}\varepsilon_{t,p}d_n^2 + 2\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}d_n^2$$
$$+\sigma_{ctmax}\varepsilon_{t,p}^2 d_n^2 + \sigma_{ctmax}\varepsilon_0^2 d_n^2 - 2\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}Dd_n - 2\sigma_{ctmax}\varepsilon_0^2 Dd_n + \sigma_{ctmax}\varepsilon_0^2 D^2 = 0 \rightarrow$$

$$\left(E_c\varepsilon_0^2(\varepsilon_{t,u} - \varepsilon_{t,p}) + \sigma_{ctmax}\left(\varepsilon_{ctmax}(\varepsilon_{t,u} - \varepsilon_{t,p}) + 2\varepsilon_0\varepsilon_{t,u} + \varepsilon_{t,p}^2 + \varepsilon_0^2\right)\right)d_n^2$$
$$-2\sigma_{ctmax}\varepsilon_0 D(\varepsilon_{t,u} + \varepsilon_0)d_n + \sigma_{ctmax}\varepsilon_0^2 D^2 = 0 \rightarrow$$

$$d_n = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\varepsilon_0^2(\varepsilon_{t,u} - \varepsilon_{t,p}) + \sigma_{ctmax}\left(\varepsilon_{ctmax}(\varepsilon_{t,u} - \varepsilon_{t,p}) + 2\varepsilon_0\varepsilon_{t,u} + \varepsilon_{t,p}^2 + \varepsilon_0^2\right)$$

$$b = -2\sigma_{ctmax}\varepsilon_0 D(\varepsilon_{t,u} + \varepsilon_0)$$

$$c = \sigma_{ctmax}\varepsilon_0^2 D^2$$

In order to determine the bending moment capacity the centre of gravity of part X_3 needs to be known:

$$y = \frac{b\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3 \cdot \frac{1}{2}X_3 + \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3 \cdot \frac{1}{3}X_3}{b\sigma_{ctmax}X_3 - \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3} \rightarrow$$

$$y = \frac{b\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right) \cdot \frac{1}{2}X_3^2 + \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} \cdot \frac{1}{3}X_3^2}{b\sigma_{ctmax}X_3\left(1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)} \rightarrow$$

$$y = \frac{\frac{1}{2}b\sigma_{ctmax}X_3^2 - \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3^2 + \frac{1}{6}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3^2}{b\sigma_{ctmax}X_3\left(1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)} \rightarrow$$

$$y = \frac{b\sigma_{ctmax}\left(\frac{1}{2} - \frac{1}{3}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3^2}{b\sigma_{ctmax}X_3\left(1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)} \rightarrow$$

$$y = \frac{\left(\frac{1}{2} - \frac{1}{3}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3}{1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}}$$

The bending moment capacity and curvature can be expressed by:

$$M = C \cdot \frac{2}{3}d_n + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3(X_1 + X_2 + y)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

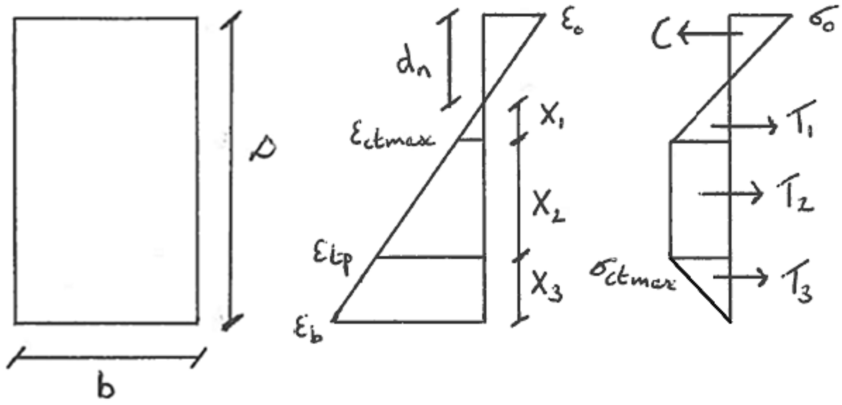


Figure 7.6: deformation and stress diagram when $\varepsilon_b = \varepsilon_{t,u}$.

7.6 When $\varepsilon_b = \varepsilon_{t,u}$

With respect to the deformation and stress diagram of [Figure 7.6] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + \frac{1}{2} b \sigma_{ctmax} X_3 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n \right) + \frac{1}{2} b \sigma_{ctmax} \left(\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \right)$$

$$\frac{1}{2} \varepsilon_0^2 = \frac{1}{2} \varepsilon_{ctmax}^2 + \varepsilon_{ctmax} \varepsilon_{t,p} - \varepsilon_{ctmax}^2 + \frac{1}{2} \varepsilon_{ctmax} \varepsilon_{t,u} - \frac{1}{2} \varepsilon_{ctmax} \varepsilon_{t,p} \rightarrow$$

$$\varepsilon_0^2 = -\varepsilon_{ctmax}^2 + \varepsilon_{ctmax} \varepsilon_{t,p} + \varepsilon_{ctmax} \varepsilon_{t,u} \rightarrow$$

$$\varepsilon_0 = \sqrt{-\varepsilon_{ctmax}^2 + \varepsilon_{ctmax}\varepsilon_{t,p} + \varepsilon_{ctmax}\varepsilon_{t,u}}$$

7.7 Shear

The ultimate shear capacity is given by:

$$V_u = V_{Rb} + V_f + V_s$$

In case of reinforced concrete, the participation of the concrete will be:

$$V_{Rb} = \frac{1}{\gamma_E}\frac{0,21}{\gamma_b}\sqrt{f_c'}bd$$

In case of prestressed concrete, the participation of the concrete will be:

$$V_{Rb} = \frac{1}{\gamma_E}\frac{0,24}{\gamma_b}\sqrt{f_c'}bz$$

The contribution of the fibres can be expressed by:

$$V_f = \frac{S_{eff}\sigma(w0,3)_k}{\gamma_{bf}\tan\beta_u}$$

$$S_{eff} = bz$$

$$z = d - \frac{1}{3}d_n$$

The shear reinforcement:

$$V_s = \frac{A_{sw}}{s}zf_{ywd}\cot\beta_u$$

The angle of the compression struts should be limited to 30° as opposed to 21,8° ,which NEN-EN 1992-1-1 prescribes.

The contribution of all three parts depend on either the effective depth d or the internal lever arm z, which are not present for unreinforced concrete. Therefore, the shear capacity of unreinforced concrete will be set to zero.

7.8 Crack width

In case of a cross-section not containing any bonded tendons in the tensile zone, the design crack width w_{max} at the extreme tensile fibre of the section may be taken as:

$$w_{max} = 1{,}5D(\varepsilon_b - 0{,}00016)$$

With respect to the deformation and stress diagram of [Figure 7.4] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{D - d_n} \rightarrow \varepsilon_0 = \frac{\varepsilon_b}{D - d_n} d_n$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_b}{D - d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_b}(D - d_n)$$

$$X_2 = D - X_1 - d_n = D - \frac{\varepsilon_{ctmax}}{\varepsilon_b}(D - d_n) - d_n$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 \rightarrow$$

$$\frac{1}{2} b E_c \frac{\varepsilon_b}{D - d_n} d_n^2 = \frac{1}{2} b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_b}(D - d_n) + b \sigma_{ctmax} \left(D - \frac{\varepsilon_{ctmax}}{\varepsilon_b}(D - d_n) - d_n\right) \rightarrow$$

$$E_c \frac{\varepsilon_b}{D - d_n} d_n^2 = -\sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_b} D + \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_b} d_n + 2\sigma_{ctmax} D - 2\sigma_{ctmax} d_n \rightarrow$$

$$E_c \varepsilon_b^2 d_n^2 = -\sigma_{ctmax}\varepsilon_{ctmax} D(D - d_n) + \sigma_{ctmax}\varepsilon_{ctmax}(D - d_n)d_n + 2\sigma_{ctmax}\varepsilon_b D(D - d_n)$$
$$-2\sigma_{ctmax}\varepsilon_b(D - d_n)d_n \rightarrow$$

$$E_c \varepsilon_b^2 d_n^2 + \sigma_{ctmax}\varepsilon_{ctmax} d_n^2 - 2\sigma_{ctmax}\varepsilon_b d_n^2 - 2\sigma_{ctmax}\varepsilon_{ctmax} D d_n + 4\sigma_{ctmax}\varepsilon_b D d_n$$
$$+\sigma_{ctmax}\varepsilon_{ctmax} D^2 - 2\sigma_{ctmax}\varepsilon_b D^2 = 0 \rightarrow$$

$$\left(E_c \varepsilon_b^2 + \sigma_{ctmax}(\varepsilon_{ctmax} - 2\varepsilon_b)\right) d_n^2 - 2\sigma_{ctmax} D(\varepsilon_{ctmax} - 2\varepsilon_b) d_n + \sigma_{ctmax} D^2 (\varepsilon_{ctmax} - 2\varepsilon_b) = 0$$
$$\rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c \varepsilon_b^2 + \sigma_{ctmax}(\varepsilon_{ctmax} - 2\varepsilon_b)$$

$$b = -2\sigma_{ctmax} D(\varepsilon_{ctmax} - 2\varepsilon_b)$$

$$c = \sigma_{ctmax} D^2(\varepsilon_{ctmax} - 2\varepsilon_b)$$

The maximum moment in SLS:

$$M_{qp} = C \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right)$$

8 Rectangular prestressed UHPC beam

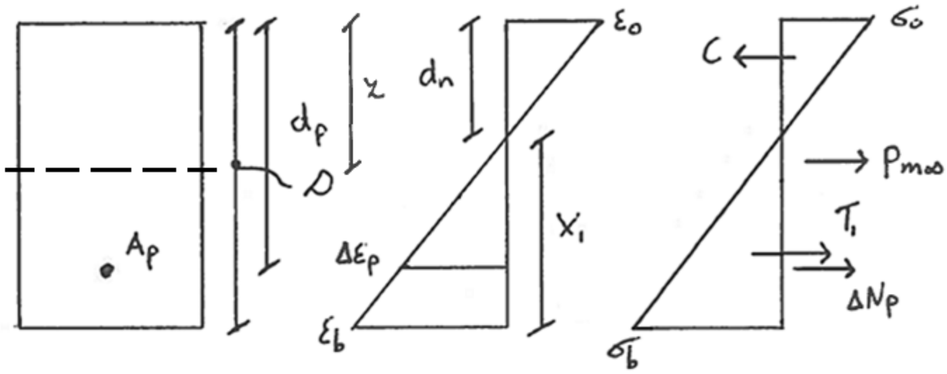


Figure 8.1: deformation and stress diagram when $\varepsilon_b < \varepsilon_{ctmax}$.

8.1 When $\varepsilon_b < \varepsilon_{ctmax}$

$\varepsilon_p < \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.1] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} X_1 \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} D - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$X_1 = D - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b E_c \varepsilon_b X_1 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = b E_c \varepsilon_b X_1 + 2 A_p E_p \Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n = bE_c\varepsilon_b D - bE_c\varepsilon_b d_n + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n = bE_c\frac{\varepsilon_0}{d_n}D^2 - 2bE_c\varepsilon_0 D + bE_c\varepsilon_0 d_n + 2A_pE_p\frac{\varepsilon_0}{d_n}d_p - 2A_pE_p\varepsilon_0 \rightarrow$$

$$bE_cD^2 + 2A_pE_pd_p = 2bE_cDd_n + 2A_pE_pd_n \rightarrow$$

$$d_n = \frac{bE_cD^2 + 2A_pE_pd_p}{2bE_cD + 2A_pE_p}$$

The corresponding bending moment capacity and curvature:

$$M = C \cdot \frac{2}{3}d_n + T_1 \cdot \frac{2}{3}X_1 + P_{m\infty}(z - d_n) + \Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

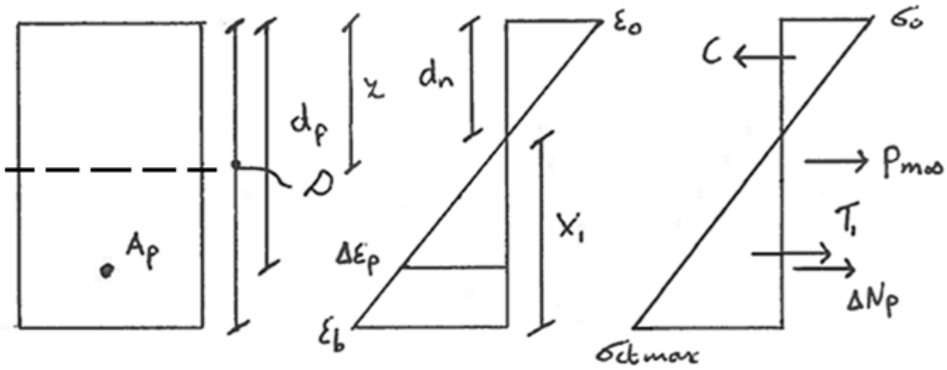


Figure 8.2: deformation and stress diagram when $\varepsilon_b = \varepsilon_{ctmax}$.

8.2 When $\varepsilon_b = \varepsilon_{ctmax}$

$$\varepsilon_p < \varepsilon_{py}$$

With respect to the deformation and stress diagram of [Figure 8.2] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{D - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}(D - d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}D - \frac{\varepsilon_0}{\varepsilon_{ctmax}}d_n$$

$$\rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ctmax}}d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}D \rightarrow d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}D \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{ctmax}}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}D \rightarrow$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}}D$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$X_1 = D - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + \Delta N_p \rightarrow$$

$$\frac{1}{2} bE_c\varepsilon_0 d_n = \frac{1}{2} bE_c\varepsilon_b X_1 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = bE_c\varepsilon_b X_1 + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n = bE_c\varepsilon_b D - bE_c\varepsilon_b d_n + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n = b\sigma_{ctmax}D - b\sigma_{ctmax}d_n + 2A_pE_p\frac{\varepsilon_0}{d_n} d_p - 2A_pE_p\varepsilon_0 \rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}} D = b\sigma_{ctmax}D - b\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}} D + 2A_pE_p\frac{\varepsilon_0 + \varepsilon_{ctmax}}{D} d_p - 2A_pE_p\varepsilon_0 \rightarrow$$

$$bE_c\varepsilon_0^2 D^2 = b\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{ctmax})D^2 - b\sigma_{ctmax}\varepsilon_0 D^2 + 2A_pE_p(\varepsilon_0 + \varepsilon_{ctmax})^2 d_p$$
$$-2A_pE_p\varepsilon_0(\varepsilon_0 + \varepsilon_{ctmax})D \rightarrow$$

$$bE_c\varepsilon_0^2 D^2 = b\sigma_{ctmax}\varepsilon_{ctmax}D^2 + 2A_pE_p\varepsilon_0^2 d_p + 4A_pE_p\varepsilon_0\varepsilon_{ctmax}d_p + 2A_pE_p\varepsilon_{ctmax}^2 d_p - 2A_pE_p\varepsilon_0^2 D$$
$$-2A_pE_p\varepsilon_0\varepsilon_{ctmax}D \rightarrow$$

$$-\left(bE_cD^2 + 2A_pE_p\left(D - d_p\right)\right)\varepsilon_0^2$$

$$+2\left(A_pE_p\varepsilon_{ctmax}\left(2d_p - D\right)\right)\varepsilon_0$$

$$+\varepsilon_{ctmax}\left(b\sigma_{ctmax}D^2 + 2A_pE_p\varepsilon_{ctmax}d_p\right) = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\left(bE_cD^2 + 2A_pE_p(D - d_p)\right)$$

$$b = 2\left(A_pE_p\varepsilon_{ctmax}(2d_p - D)\right)$$

$$c = \varepsilon_{ctmax}\left(b\sigma_{ctmax}D^2 + 2A_pE_p\varepsilon_{ctmax}d_p\right)$$

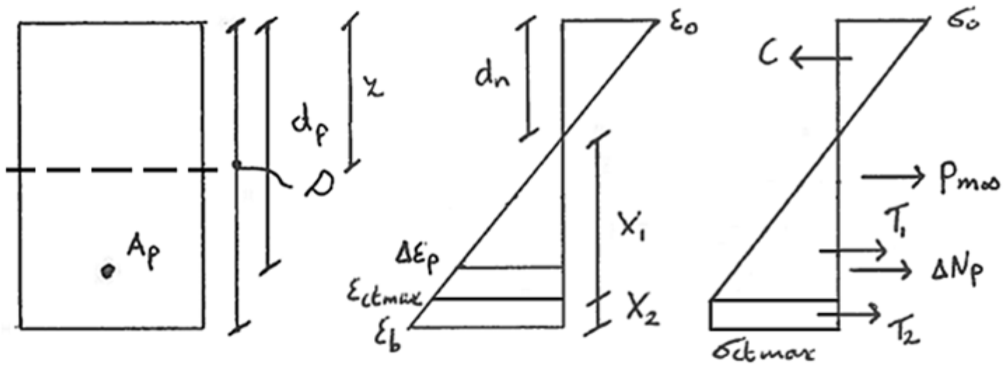


Figure 8.3: deformation and stress diagram when $\Delta\varepsilon_p < \varepsilon_{ctmax} < \varepsilon_b$.

8.3 When $\Delta\varepsilon_p < \varepsilon_{ctmax} < \varepsilon_b$

$\varepsilon_p < \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.3] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}(d_p - d_n) = \frac{\varepsilon_0}{d_n}d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1 + X_2} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}D - \varepsilon_0$$

$$X_1 + X_2 = D - d_n \rightarrow X_2 = D - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n \right) + 2 A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} D - 2 b \sigma_{ctmax} d_n + 2 A_p E_p \frac{\varepsilon_0}{d_n} d_p - 2 A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0^2 d_n^2 = -b \sigma_{ctmax} \varepsilon_{ctmax} d_n^2 + 2 b \sigma_{ctmax} \varepsilon_0 D d_n - 2 b \sigma_{ctmax} \varepsilon_0 d_n^2 + 2 A_p E_p \varepsilon_0^2 d_p - 2 A_p E_p \varepsilon_0^2 d_n \rightarrow$$

$$b \left(E_c \varepsilon_0^2 + \sigma_{ctmax} (\varepsilon_{ctmax} + 2\varepsilon_0) \right) d_n^2 - 2 \left(b \sigma_{ctmax} D - A_p E_p \varepsilon_0 \right) \varepsilon_0 d_n - 2 A_p E_p \varepsilon_0^2 d_p = 0 \rightarrow$$

$$d_n = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = b \left(E_c \varepsilon_0^2 + \sigma_{ctmax} (\varepsilon_{ctmax} + 2\varepsilon_0) \right)$$

$$b = -2 \left(b \sigma_{ctmax} D - A_p E_p \varepsilon_0 \right) \varepsilon_0$$

$$c = -2 A_p E_p \varepsilon_0^2 d_p$$

The corresponding bending moment capacity and curvature:

$$M = C \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right) + P_{m\infty} (z - d_n) + \Delta N_p \left(d_p - d_n \right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

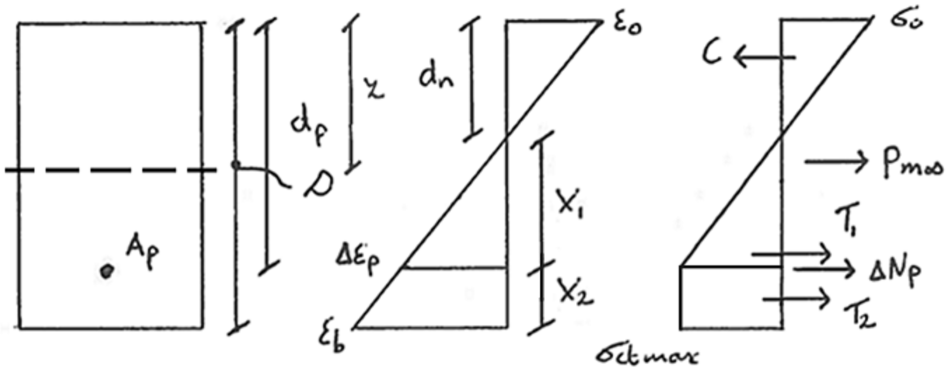


Figure 8.4: deformation and stress diagram when $\Delta\varepsilon_p = \varepsilon_{ctmax}$.

8.4 When $\Delta\varepsilon_p = \varepsilon_{ctmax}$

$\varepsilon_p < \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.4] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{X_1} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{d_p - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}(d_p - d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_p - \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n$$
$$\rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_p \rightarrow d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_p \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{ctmax}}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_p \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}} d_p$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1 + X_2} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} D - \varepsilon_0$$

$$X_1 = d_p - d_n$$

$$X_2 = D - d_p$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = b\sigma_{ctmax}(d_p - d_n) + 2b\sigma_{ctmax}(D - d_p) + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n = -b\sigma_{ctmax}d_p - b\sigma_{ctmax}d_n + 2b\sigma_{ctmax}D + 2A_pE_p\varepsilon_{ctmax} \rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}}d_p = -b\sigma_{ctmax}d_p - b\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}}d_p + 2b\sigma_{ctmax}D + 2A_pE_p\varepsilon_{ctmax} \rightarrow$$

$$bE_c\varepsilon_0^2 d_p = -b\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{ctmax})d_p - b\sigma_{ctmax}\varepsilon_0 d_p + 2b\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{ctmax})D$$
$$+2A_pE_p(\varepsilon_0 + \varepsilon_{ctmax})\varepsilon_{ctmax} \rightarrow$$

$$bE_c\varepsilon_0^2 d_p = -2b\sigma_{ctmax}\varepsilon_0 d_p - b\sigma_{ctmax}\varepsilon_{ctmax}d_p + 2b\sigma_{ctmax}\varepsilon_0 D + 2b\sigma_{ctmax}\varepsilon_{ctmax}D$$
$$+2A_pE_p\varepsilon_0\varepsilon_{ctmax} + 2A_pE_p\varepsilon_{ctmax}^2 \rightarrow$$

$$bE_c\varepsilon_0^2 d_p + 2\left(b\sigma_{ctmax}(d_p - D) - A_pE_p\varepsilon_{ctmax}\right)\varepsilon_0 + \left(b\sigma_{ctmax}(d_p - 2D) - 2A_pE_p\varepsilon_{ctmax}\right)\varepsilon_{ctmax}$$
$$= 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = bE_cd_p$$

$$b = 2\left(b\sigma_{ctmax}(d_p - D) - A_pE_p\varepsilon_{ctmax}\right)$$

$$c = \left(b\sigma_{ctmax}(d_p - 2D) - 2A_pE_p\varepsilon_{ctmax}\right)\varepsilon_{ctmax}$$

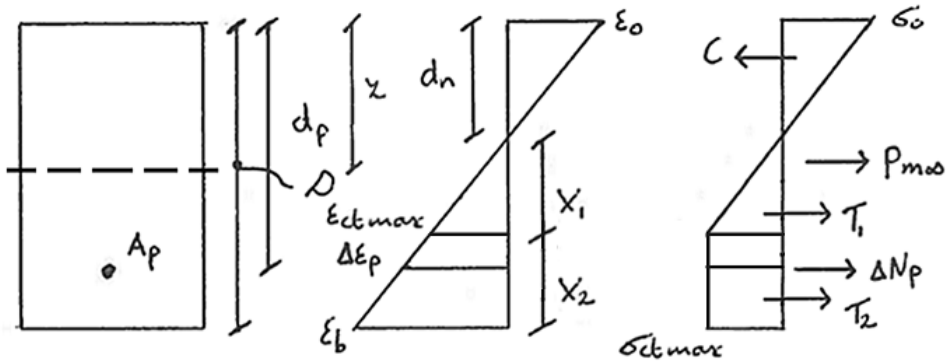


Figure 8.5: deformation and stress diagram when $\Delta\varepsilon_p > \varepsilon_{ctmax}$ & $\varepsilon_b < \varepsilon_{t,p}$.

8.5 When $\Delta\varepsilon_p > \varepsilon_{ctmax}$ & $\varepsilon_b < \varepsilon_{t,p}$

$\varepsilon_p < \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.5] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1 + X_2} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} D - \varepsilon_0$$

$$X_1 + X_2 = D - d_n \rightarrow X_2 = D - X_1 - d_n = D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = b \sigma_{ctmax} X_1 + 2 b \sigma_{ctmax} X_2 + 2 A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n\right) + 2 A_p E_p \left(\frac{\varepsilon_0}{d_n} d_p - \varepsilon_0\right) \rightarrow$$

$$b E_c \varepsilon_0 d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} D - 2 b \sigma_{ctmax} d_n + 2 A_p E_p \frac{\varepsilon_0}{d_n} d_p - 2 A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0^2 d_n^2 = -b \sigma_{ctmax} \varepsilon_{ctmax} d_n^2 + 2 b \sigma_{ctmax} \varepsilon_0 D d_n - 2 b \sigma_{ctmax} \varepsilon_0 d_n^2 + 2 A_p E_p \varepsilon_0^2 d_p - 2 A_p E_p \varepsilon_0^2 d_n \rightarrow$$

$$b\left(E_c \varepsilon_0^2 + \sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)\right) d_n^2$$

$$-2\varepsilon_0\left(b \sigma_{ctmax} D - A_p E_p \varepsilon_0\right) d_n$$

$$-2 A_p E_p \varepsilon_0^2 d_p = 0 \rightarrow$$

$$d_n = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = b\left(E_c\varepsilon_0^2 + \sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)\right)$$

$$b = -2\varepsilon_0\left(b\sigma_{ctmax}D - A_pE_p\varepsilon_0\right)\varepsilon_0$$

$$c = -2A_pE_p\varepsilon_0^2 d_p$$

The corresponding bending moment capacity and curvature:

$$M = C \cdot \frac{2}{3}d_n + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + P_{m\infty}(z - d_n) + \Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

8.6 When $\varepsilon_p = \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.5] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{py} - \varepsilon_{p\infty}}{d_p - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}\left(d_p - d_n\right) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}d_p - \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}d_n = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}d_p \rightarrow$$

$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}d_p \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{\varepsilon_{py} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}d_p \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}d_p$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1 + X_2} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}D - \varepsilon_0$$

$$X_1 = d_p - d_n$$

$$X_1 + X_2 = D - d_n \rightarrow X_2 = D - X_1 - d_n = D - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n - d_n$$

$$\varepsilon_p = \varepsilon_{py} \rightarrow \Delta\varepsilon_p + \varepsilon_{p\infty} = \varepsilon_{py} \rightarrow \Delta\varepsilon_p = \varepsilon_{py} - \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b\sigma_{ctmax} X_2 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = b\sigma_{ctmax} X_1 + 2b\sigma_{ctmax} X_2 + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n = b\sigma_{ctmax}\left(\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n\right) + 2b\sigma_{ctmax}\left(D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n\right) + 2A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty}) \rightarrow$$

$$b E_c \varepsilon_0 d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b\sigma_{ctmax} D - 2b\sigma_{ctmax} d_n + 2A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty}) \rightarrow$$

$$b E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} d_p = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} d_p + 2b\sigma_{ctmax} D - 2b\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} d_p$$
$$+2A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty}) \rightarrow$$

$$b E_c \varepsilon_0^2 d_p = -b\sigma_{ctmax}\varepsilon_{ctmax} d_p + 2b\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}) D - 2b\sigma_{ctmax}\varepsilon_0 d_p$$
$$+2A_p E_p(\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{py} - \varepsilon_{p\infty}) \rightarrow$$

$$b E_c \varepsilon_0^2 d_p = -b\sigma_{ctmax}\varepsilon_{ctmax} d_p + 2b\sigma_{ctmax}\varepsilon_0 D + 2b\sigma_{ctmax}(\varepsilon_{py} - \varepsilon_{p\infty}) D - 2b\sigma_{ctmax}\varepsilon_0 d_p$$
$$+2A_p E_p \varepsilon_0(\varepsilon_{py} - \varepsilon_{p\infty}) + 2A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty})^2 \rightarrow$$

$$-b E_c \varepsilon_0^2 d_p$$

$$+2\left(b\sigma_{ctmax}(D - d_p) + A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty})\right)\varepsilon_0$$

$$-b\sigma_{ctmax}(\varepsilon_{ctmax} d_p - 2(\varepsilon_{py} - \varepsilon_{p\infty}) D) + 2A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty})^2 = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -bE_c d_p$$

$$b = -2\left(b\sigma_{ctmax}\left(d_p - D\right) - A_p f_{pd} + P_{m\infty}\right)$$

$$c = -\left(b\sigma_{ctmax}\left(d_p - 2D\right) - 2\left(A_p f_{pd} - P_{m\infty}\right)\right)\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)$$

8.7 When $\Delta\boldsymbol{\varepsilon_p} > \boldsymbol{\varepsilon_{ctmax}}$ & $\boldsymbol{\varepsilon_b} < \boldsymbol{\varepsilon_{t,p}}$

$$\varepsilon_p > \varepsilon_{py}$$

With respect to the deformation and stress diagram of [Figure 8.5] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{X_1 + X_2} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} D - \varepsilon_0$$

$$X_1 + X_2 = D - d_n \rightarrow X_2 = D - X_1 - d_n = D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + \Delta N_p \rightarrow$$

$$\frac{1}{2} bE_c\varepsilon_0 d_n = \frac{1}{2} b\sigma_{ctmax} X_1 + b\sigma_{ctmax} X_2 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b\sigma_{ctmax}\left(D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n\right) + 2A_p\sigma_p - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b\sigma_{ctmax}D - 2b\sigma_{ctmax}d_n$$

$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_pE_p\varepsilon_{p\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D$$

$$-2b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p\varepsilon_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$-2A_pE_p(\varepsilon_{uk} - \varepsilon_{py})\varepsilon_{p\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D$$

$$-2b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p\frac{\varepsilon_0}{d_n}d_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_pE_p(\varepsilon_{uk} - \varepsilon_{py})\varepsilon_{p\infty} \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk} - \varepsilon_{py})d_n^2 = -b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})d_n^2 + 2b\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})Dd_n$$

$$-2b\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n^2 + 2A_pf_{pd}\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_n + 2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_n - 2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_n$$

$$-2A_pE_p\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})\varepsilon_{p\infty}d_n \rightarrow$$

$$-b\left(E_c\varepsilon_0^2 + \sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)\right)(\varepsilon_{uk} - \varepsilon_{py})d_n^2$$

$$+2\varepsilon_0\left(b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D + A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) - (\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)d_n$$

$$+2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p = 0 \rightarrow$$

$$d_n = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -b\left(E_c \varepsilon_0^2 + \sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)$$

$$b = 2\varepsilon_0 \left(b\sigma_{ctmax}\left(\varepsilon_{uk} - \varepsilon_{py}\right)D + A_p\left(\left(f_{pd} - \sigma_{pm\infty}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) - \left(\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)$$

$$c = 2A_p \varepsilon_0^2 \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_p$$

The corresponding bending moment capacity and curvature:

$$M = C \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1 + T_2\left(X_1 + \frac{1}{2} X_2\right) + P_{m\infty}(z - d_n) + \Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

8.8 When $\varepsilon_b = \varepsilon_{t,p}$

$\varepsilon_p > \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.5] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{D - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}(D - d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}} D - \frac{\varepsilon_0}{\varepsilon_{t,p}} d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,p}} d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}} D \rightarrow$$
$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}} D \rightarrow d_n\left(\frac{\varepsilon_{t,p} + \varepsilon_0}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}} D \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p} + \varepsilon_0} D$$

$$X_1 + X_2 = D - d_n \rightarrow X_2 = D - X_1 - d_n = D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(D - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n \right) + 2 A_p \sigma_p - 2 A_p \sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} D - 2 b \sigma_{ctmax} d_n$$
$$+2 A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) \right) - 2 A_p E_p \varepsilon_{p\infty} \rightarrow$$

$$b E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) D$$
$$-2 b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py}) + 2 A_p \varepsilon_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$
$$-2 A_p E_p (\varepsilon_{uk} - \varepsilon_{py}) \varepsilon_{p\infty} \rightarrow$$

$$b E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) D$$
$$-2 b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py}) + 2 A_p \frac{\varepsilon_0}{d_n} d_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$
$$+2 A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p E_p (\varepsilon_{uk} - \varepsilon_{py}) \varepsilon_{p\infty} \rightarrow$$

$$b E_c \frac{\varepsilon_0^2}{\varepsilon_{t,p} + \varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) D = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_{t,p} + \varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) D + 2 b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) D$$
$$-2 b \sigma_{ctmax} \frac{\varepsilon_0}{\varepsilon_{t,p} + \varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) D + 2 A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py}) + 2 A_p \frac{\varepsilon_{t,p} + \varepsilon_0}{D} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p$$
$$-2 A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) + 2 A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p E_p (\varepsilon_{uk} - \varepsilon_{py}) \varepsilon_{p\infty} \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})D^2 = -b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})D^2 + 2b\sigma_{ctmax}(\varepsilon_{t,p}+\varepsilon_0)(\varepsilon_{uk}-\varepsilon_{py})D^2$$

$$-2b\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})D^2 + 2A_pf_{pd}(\varepsilon_{t,p}+\varepsilon_0)(\varepsilon_{uk}-\varepsilon_{py})D + 2A_p(\varepsilon_{t,p}+\varepsilon_0)^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0(\varepsilon_{t,p}+\varepsilon_0)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D + 2A_p(\varepsilon_{t,p}+\varepsilon_0)\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D$$

$$-2A_p(\varepsilon_{t,p}+\varepsilon_0)\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D - 2A_pE_p(\varepsilon_{t,p}+\varepsilon_0)(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_{p\infty}D \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})D^2 = -b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})D^2 + 2b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})D^2$$

$$+2A_pf_{pd}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})D + 2A_pf_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})D + 2A_p\varepsilon_{t,p}^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$+4A_p\varepsilon_0\varepsilon_{t,p}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p + 2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p - 2A_p\varepsilon_0\varepsilon_{t,p}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D$$

$$-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D + 2A_p\varepsilon_{t,p}\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D + 2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D$$

$$-2A_p\varepsilon_{t,p}\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D - 2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)D - 2A_pE_p\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_{p\infty}D$$

$$-2A_pE_p\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_{p\infty}D \rightarrow$$

$$-\left(bE_c(\varepsilon_{uk}-\varepsilon_{py})D^2 - 2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(d_p-D)\right)\varepsilon_0^2$$

$$+2A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})D + \left(\varepsilon_{t,p}(2d_p-D) + (\varepsilon_{p\infty}-\varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\varepsilon_0$$

$$-(\varepsilon_{ctmax}-2\varepsilon_{t,p})b\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})D^2 + 2A_p\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})D(f_{pd}-\sigma_{pm\infty})$$
$$+ 2A_p\varepsilon_{t,p}\left(\varepsilon_{t,p}d_p + (\varepsilon_{p\infty}-\varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\left(bE_c(\varepsilon_{uk} - \varepsilon_{py})D^2 - 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)(d_p - D)\right)$$

$$b = 2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})D + \left(\varepsilon_{t,p}(2d_p - D) + (\varepsilon_{p\infty} - \varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$c = -(\varepsilon_{ctmax} - 2\varepsilon_{t,p})b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D^2 + 2A_p\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})D(f_{pd} - \sigma_{pm\infty}) + 2A_p\varepsilon_{t,p}\left(\varepsilon_{t,p}d_p + (\varepsilon_{p\infty} - \varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

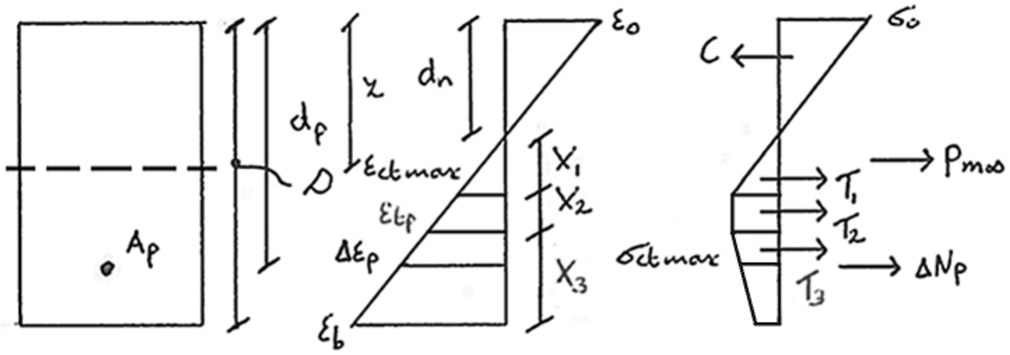


Figure 8.6: deformation and stress diagram when $\varepsilon_{t,p} < \varepsilon_b < \varepsilon_{t,u}$.

8.9 When $\varepsilon_{t,p} < \varepsilon_b < \varepsilon_{t,u}$

$\varepsilon_p > \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.6] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}(d_p - d_n) = \frac{\varepsilon_0}{d_n}d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{D - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(D - d_n) = \frac{\varepsilon_0}{d_n}D - \varepsilon_0$$

$$X_1 + X_2 + X_3 = D - d_n \rightarrow X_3 = D - (X_1 + X_2) - d_n = D - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 + \Delta N_p \rightarrow$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}b\sigma_{ctmax}X_1 + b\sigma_{ctmax}X_2 + b\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3 + \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3$$
$$+A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b\sigma_{ctmax}\left(\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n\right) + 2b\sigma_{ctmax}\left(D - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - d_n\right)$$
$$-b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\left(D - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - d_n\right) + 2A_p\sigma_p - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b\sigma_{ctmax}D - 2b\sigma_{ctmax}d_n - b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}D$$
$$+b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}d_n + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$
$$-2A_pE_p\varepsilon_{p\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D$$
$$-2b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})d_n - b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}(\varepsilon_{uk} - \varepsilon_{py})D$$
$$+b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py})$$
$$+2A_p\varepsilon_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_pE_p\varepsilon_{p\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n + 2b\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})D$$

$$-2b\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_n - b\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{t,u}-\varepsilon_{t,p}}(\varepsilon_{uk}-\varepsilon_{py})D$$

$$+b\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{t,u}-\varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n + b\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{t,u}-\varepsilon_{t,p}}(\varepsilon_{uk}-\varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p\frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p - 2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_pE_p\varepsilon_{p\infty}(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})D - 2b\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$-b\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk}-\varepsilon_{py})D + b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})D + b\sigma_{ctmax}\varepsilon_b\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$-b\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n + b\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk}-\varepsilon_{py})d_n - b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2A_pf_{pd}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) + 2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) + 2A_p\varepsilon_{p\infty}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_{py}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) - 2A_pE_p\varepsilon_{p\infty}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})D - 2b\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$-b\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})D^2 + b\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})D + 2b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})D$$

$$-2b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n - b\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n + b\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})D$$

$$-b\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) + 2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) + 2A_p\varepsilon_{p\infty}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_{py}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) - 2A_pE_p\varepsilon_{p\infty}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2 = -b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+2b\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})Dd_n - 2b\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$-b\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})D^2 + b\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})Dd_n + 2b\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})Dd_n$$
$$-2b\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n^2 - b\sigma_{ctmax}\varepsilon_{t,p}^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + b\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})Dd_n$$
$$-b\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2A_pf_{pd}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+2A_p\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p - 2A_p\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$+2A_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n - 2A_p\varepsilon_0\varepsilon_{py}(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$-2A_pE_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{t,u}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n \rightarrow$$

$$-b\left(E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p}) + \sigma_{ctmax}\left((\varepsilon_{ctmax}+2\varepsilon_0)(\varepsilon_{t,u}-\varepsilon_{t,p}) + (\varepsilon_{t,p}+\varepsilon_0)^2\right)\right)(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$+2\varepsilon_0\left(b\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})D + A_p(\varepsilon_{t,u}-\varepsilon_{t,p})\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py}) - (\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)d_n$$

$$-\varepsilon_0^2\left(b\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})D^2 - 2A_p(\varepsilon_{t,u}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right) = 0 \rightarrow$$

$$d_n = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -b\left(E_c\varepsilon_0^2(\varepsilon_{t,u} - \varepsilon_{t,p}) + \sigma_{ctmax}\left((\varepsilon_{ctmax} + 2\varepsilon_0)(\varepsilon_{t,u} - \varepsilon_{t,p}) + (\varepsilon_{t,p} + \varepsilon_0)^2\right)\right)(\varepsilon_{uk} - \varepsilon_{py})$$

$$b = 2\varepsilon_0\left(b\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py})D + A_p(\varepsilon_{t,u} - \varepsilon_{t,p})\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) - (\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)$$

$$c = -\varepsilon_0^2\left(b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D^2 - 2A_p(\varepsilon_{t,u} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p\right)$$

In order to determine the bending moment capacity the centre of gravity of part X_3 needs to be known:

$$y = \frac{b\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3 \cdot \frac{1}{2}X_3 + \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3 \cdot \frac{1}{3}X_3}{b\sigma_{ctmax}X_3 - \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3}$$

$$y = \frac{\frac{1}{2}b\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3^2 + \frac{1}{6}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3^2}{b\sigma_{ctmax}X_3 - \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3}$$

$$y = \frac{\frac{1}{2}b\sigma_{ctmax}X_3^2 - \frac{1}{3}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3^2}{b\sigma_{ctmax}X_3 - \frac{1}{2}b\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}X_3}$$

$$y = \frac{b\sigma_{ctmax}\left(\frac{1}{2} - \frac{1}{3}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3^2}{b\sigma_{ctmax}\left(1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3}$$

$$y = \frac{\left(\frac{1}{2} - \frac{1}{3}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}\right)X_3}{1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}}$$

The bending moment capacity and corresponding curvature:

$$M = C \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right) + T_3 (X_1 + X_2 + y) + P_{m\infty} (z - d_n) + \Delta N_p \left(d_p - d_n \right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

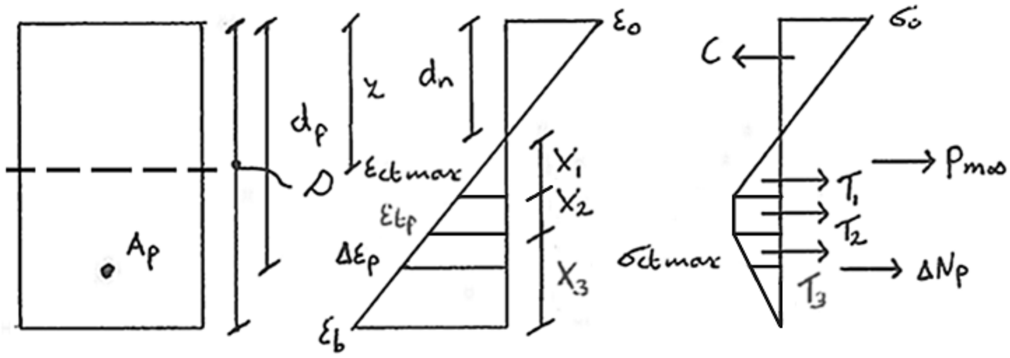


Figure 8.7: deformation and stress diagram when $\varepsilon_b = \varepsilon_{t,u}$.

8.10 When $\varepsilon_b = \varepsilon_{t,u}$

$\varepsilon_p > \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.7] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} \left(d_p - d_n \right) = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,u}}{D - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}} (D - d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}} D - \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{tu}} d_n = \frac{\varepsilon_0}{\varepsilon_{tu}} D \rightarrow$$
$$d_n \left(1 + \frac{\varepsilon_0}{\varepsilon_{t,u}} \right) = \frac{\varepsilon_0}{\varepsilon_{t,u}} D \rightarrow d_n \left(\frac{\varepsilon_{t,u} + \varepsilon_0}{\varepsilon_{t,u}} \right) = \frac{\varepsilon_0}{\varepsilon_{t,u}} D \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,u} + \varepsilon_0} D$$

$$X_1 + X_2 + X_3 = D - d_n \rightarrow X_3 = D - (X_1 + X_2) - d_n = D - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - d_n$$

$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + \frac{1}{2} b \sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n \right) + b \sigma_{ctmax} \left(D - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - d_n \right)$$
$$+ 2 A_p \sigma_p - 2 A_p \sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = - b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b \sigma_{ctmax} D - b \sigma_{ctmax} d_n$$
$$+ 2 A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) \right) - 2 A_p E_p \varepsilon_{p\infty} \rightarrow$$

$$b E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_n = - b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n$$
$$+ b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) D - b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py}) + 2 A_p \varepsilon_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$
$$- 2 A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p E_p \varepsilon_{p\infty} (\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$b E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_n = - b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n$$
$$+ b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) D - b \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py}) + 2 A_p \frac{\varepsilon_0}{d_n} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p$$
$$- 2 A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) + 2 A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p E_p \varepsilon_{p\infty} (\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c \frac{\varepsilon_0^2}{\varepsilon_{t,u} + \varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})D = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_{t,u} + \varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})D + b\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_{t,u} + \varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})D$$

$$+b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D - b\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_{t,u} + \varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})D + 2A_p f_{pd}(\varepsilon_{uk} - \varepsilon_{py})$$

$$+2A_p\frac{\varepsilon_{t,u} + \varepsilon_0}{D}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p - 2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$-2A_p E_p \varepsilon_{p\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk} - \varepsilon_{py})D^2 = -b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D^2 + b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})D^2$$

$$+b\sigma_{ctmax}(\varepsilon_{t,u} + \varepsilon_0)(\varepsilon_{uk} - \varepsilon_{py})D^2 - b\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})D^2 + 2A_p f_{pd}(\varepsilon_{t,u} + \varepsilon_0)(\varepsilon_{uk} - \varepsilon_{py})D$$

$$+2A_p(\varepsilon_{t,u} + \varepsilon_0)^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p - 2A_p\varepsilon_0(\varepsilon_{t,u} + \varepsilon_0)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D$$

$$+2A_p(\varepsilon_{t,u} + \varepsilon_0)\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D - 2A_p(\varepsilon_{t,u} + \varepsilon_0)\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D$$

$$-2A_p E_p(\varepsilon_{t,u} + \varepsilon_0)\varepsilon_{p\infty}(\varepsilon_{uk} - \varepsilon_{py})D \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk} - \varepsilon_{py})D^2 = -b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D^2 + b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})D^2$$

$$+b\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})D^2 + 2A_p f_{pd}\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})D + 2A_p f_{pd}\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})D$$

$$+2A_p\varepsilon_{t,u}^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p + 4A_p\varepsilon_0\varepsilon_{t,u}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p + 2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0\varepsilon_{t,u}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D - 2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D + 2A_p\varepsilon_{t,u}\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D$$

$$+2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D - 2A_p\varepsilon_{t,u}\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D - 2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)D$$

$$-2A_p E_p\varepsilon_{t,u}\varepsilon_{p\infty}(\varepsilon_{uk} - \varepsilon_{py})D - 2A_p E_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{uk} - \varepsilon_{py})D \rightarrow$$

$$-\left(bE_c(\varepsilon_{uk} - \varepsilon_{py})D^2 - 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)(d_p - D)\right)\varepsilon_0^2$$

$$+2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})D + \left(\varepsilon_{t,u}(2d_p - D) + (\varepsilon_{p\infty} - \varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\varepsilon_0$$

$$-b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D^2(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) + 2A_p(f_{pd} - \sigma_{pm\infty})\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})D$$

$$+ 2A_p\varepsilon_{t,u}\left(\varepsilon_{t,u}d_p + (\varepsilon_{p\infty} - \varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\left(bE_c(\varepsilon_{uk} - \varepsilon_{py})D^2 - 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)(d_p - D)\right)$$

$$b = 2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})D + \left(\varepsilon_{t,u}(2d_p - D) + (\varepsilon_{p\infty} - \varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$c = -b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})D^2(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) + 2A_p(f_{pd} - \sigma_{pm\infty})\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})D + 2A_p\varepsilon_{t,u}\left(\varepsilon_{t,u}d_p + (\varepsilon_{p\infty} - \varepsilon_{py})D\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

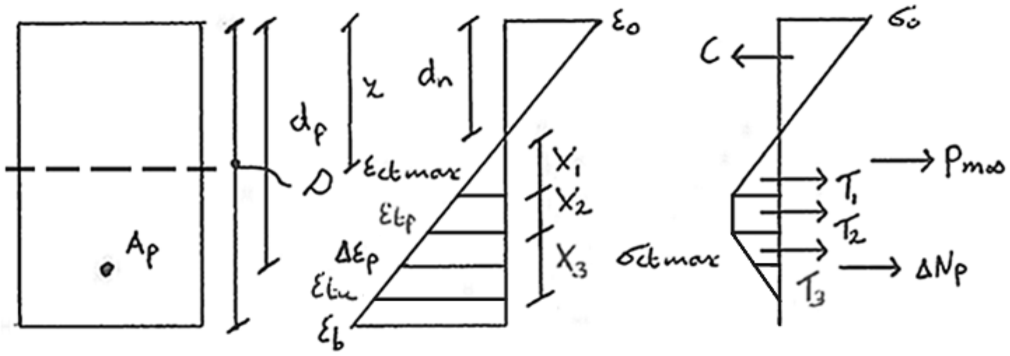


Figure 8.8: deformation and stress diagram when $\varepsilon_b > \varepsilon_{t,u}$.

8.11 When $\varepsilon_b > \varepsilon_{t,u}$

$\varepsilon_p > \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 8.8] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}(d_p - d_n) = \frac{\varepsilon_0}{d_n}d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 + \Delta N_p \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + \frac{1}{2} b \sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n \right) + b \sigma_{ctmax} \left(\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \right)$$
$$+ 2 A_p \sigma_p - 2 A_p \sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n$$
$$+ 2 A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) \right) - 2 A_p E_p \varepsilon_{p\infty} \rightarrow$$

$$b E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n$$
$$+ b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py}) + 2 A_p (\varepsilon_p - \varepsilon_{py}) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$
$$- 2 A_p E_p \varepsilon_{p\infty} (\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$b E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n$$
$$+ b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py}) + 2 A_p \frac{\varepsilon_0}{d_n} d_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$
$$+ 2 A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2 A_p E_p \varepsilon_{p\infty} (\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 = -b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$+b\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2A_pf_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n + 2A_p\varepsilon_0^2d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n + 2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n - 2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$

$$-2A_pE_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{uk}-\varepsilon_{py})d_n \rightarrow$$

$$-b\left(E_c\varepsilon_0^2+\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$+2A_p\varepsilon_0\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})-(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)d_n$$

$$+2A_p\varepsilon_0^2d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)=0 \rightarrow$$

$$d_n = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$a = -b\left(E_c\varepsilon_0^2+\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})$$

$$b = 2A_p\varepsilon_0\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})-(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)$$

$$c = 2A_p\varepsilon_0^2d_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

The corresponding bending moment capacity and curvature:

$$M = C\cdot\frac{2}{3}d_n + T_1\cdot\frac{2}{3}X_1 + T_2\left(X_1+\frac{1}{2}X_2\right) + T_3\left(X_1+X_2+\frac{1}{3}X_3\right) + P_{m\infty}(z-d_n) + \Delta N_p(d_p-d_n)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

8.12 When $\varepsilon_p = \varepsilon_{ud}$

With respect to the deformation and stress diagram of [Figure 8.8] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \to X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \to X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \to X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \to \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ud} - \varepsilon_{p\infty}}{d_p - d_n} \to d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} (d_p - d_n) \to$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p - \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n \to d_n + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \to$$

$$d_n \left(1 + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \to d_n \left(\frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \to$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} \to X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \to X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty} \to \varepsilon_{ud} = \Delta\varepsilon_p + \varepsilon_{p\infty} \to \Delta\varepsilon_p = \varepsilon_{ud} - \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \to C = T_1 + T_2 + T_3 + \Delta N_p \to$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + \frac{1}{2} b \sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \to$$

$$b E_c \varepsilon_0 d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n\right) + b \sigma_{ctmax} \left(\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n\right)$$

$$+ 2 A_p \sigma_p - 2 A_p \sigma_{pm\infty} \to$$

$$b E_c \varepsilon_0 d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n$$

$$+ 2 A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2 A_p E_p \varepsilon_{p\infty} \to$$

$$bE_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n=-b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n+b\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+b\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n+2A_pf_{pd}(\varepsilon_{uk}-\varepsilon_{py})+2A_p(\varepsilon_p-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_pE_p\varepsilon_{p\infty}(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p=-b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+b\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p+b\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+2A_pf_{pd}(\varepsilon_{uk}-\varepsilon_{py})+2A_p(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_pE_p\varepsilon_{p\infty}(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_p=-b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_p+b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+b\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})d_p+2A_pf_{pd}(\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p(\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_pE_p\varepsilon_{p\infty}(\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_p=-b\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_p+b\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+b\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})d_p+2A_pf_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})+2A_pf_{pd}(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p\varepsilon_0(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+2A_p(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_pE_p\varepsilon_{p\infty}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})-2A_pE_p\varepsilon_{p\infty}(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$-bE_c(\varepsilon_{uk}-\varepsilon_{py})d_p\varepsilon_0^2$$

$$+2A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\varepsilon_0$$

$$-b\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_p(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})$$

$$+2A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(\varepsilon_{ud}-\varepsilon_{p\infty})=0\rightarrow$$

$$\varepsilon_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = -bE_c(\varepsilon_{uk} - \varepsilon_{py})d_p$$

$$b = 2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) + (\varepsilon_{ud} - \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$c = -b\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})d_p(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) + 2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) + (\varepsilon_{ud} - \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)(\varepsilon_{ud} - \varepsilon_{p\infty})$$

8.13 When $\varepsilon_p = \varepsilon_{ud}$ & $\varepsilon_0 = \varepsilon_{cmax}$

With respect to the deformation and stress diagram of [Figure 8.8] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ud} - \varepsilon_{p\infty}}{d_p - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}(d_p - d_n) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}d_p - \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}d_p \rightarrow$$

$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}d_p \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}d_p \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}d_p$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0}d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0}d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0}d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \varepsilon_{ud} = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \Delta\varepsilon_p = \varepsilon_{ud} - \varepsilon_{p\infty}$$

The amount of prestressing steel A_p can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 + \Delta N_p \rightarrow$$

$$\frac{1}{2} bE_c\varepsilon_0 d_n = \frac{1}{2} b\sigma_{ctmax}X_1 + b\sigma_{ctmax}X_2 + \frac{1}{2} b\sigma_{ctmax}X_3 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b\sigma_{ctmax}\left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n\right) + b\sigma_{ctmax}\left(\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n\right)$$
$$+2A_p\sigma_p - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n$$
$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_pE_p\varepsilon_{p\infty} \rightarrow$$

$$bE_c\frac{\varepsilon_{cmax}^2}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p = -b\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + b\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$
$$+b\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + 2A_p\left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_pE_p\varepsilon_{p\infty} \rightarrow$$

$$A_p\left(2\left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}\right)\right) = \frac{bd_p\left(E_c\varepsilon_{cmax}^2 + \sigma_{ctmax}(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u})\right)}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}}$$
$$\rightarrow$$

$$A_p = \frac{\dfrac{bd_p\left(E_c\varepsilon_{cmax}^2 + \sigma_{ctmax}(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u})\right)}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}}}{2\left(f_{pd} + \dfrac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\dfrac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}\right)}$$

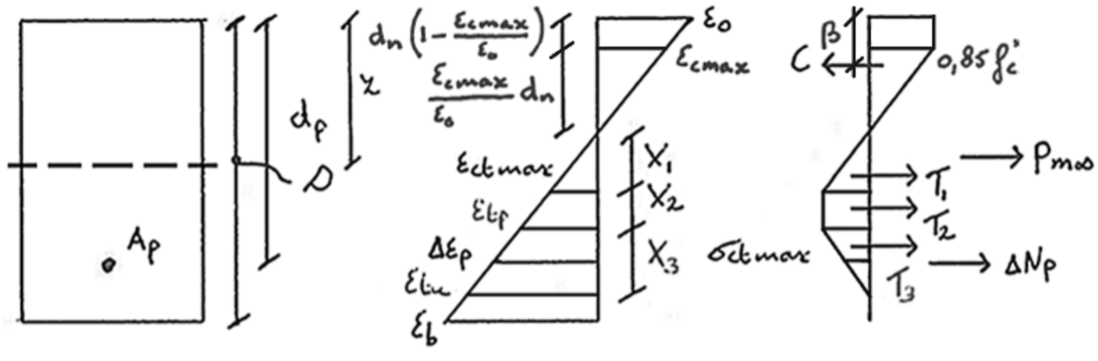


Figure 8.9: deformation and stress diagram when $\varepsilon_0 > \varepsilon_{cmax}$.

8.14 When $\varepsilon_0 > \varepsilon_{cmax}$

With respect to the deformation and stress diagram of [Figure 8.9] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$C = \frac{1}{2} b \frac{\varepsilon_{cmax}}{\varepsilon_0} d_n \cdot 0{,}85 f_c{}' + d_n \left(1 - \frac{\varepsilon_{cmax}}{\varepsilon_0}\right) \cdot 0{,}85 f_c{}' = 0{,}85 f_c{}' \left(1 - \frac{\varepsilon_{cmax}}{2\varepsilon_0}\right) b d_n$$

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 + \Delta N_p \rightarrow$$

$$0{,}85 f_c' \left(1 - \frac{\varepsilon_{cmax}}{2\varepsilon_0}\right) b d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + \frac{1}{2} b \sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' \frac{\varepsilon_{cmax}}{\varepsilon_0} b d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n\right)$$
$$+ b \sigma_{ctmax} \left(\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n\right) + 2 A_p \sigma_p - 2 A_p \sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' \frac{\varepsilon_{cmax}}{\varepsilon_0} b d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n$$
$$+ 2 A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2 A_p E_p \varepsilon_{p\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n$$
$$+ b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2 A_p f_{pd} \left(\varepsilon_{uk} - \varepsilon_{py}\right)$$
$$+ 2 A_p \left(\varepsilon_p - \varepsilon_{py}\right) \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2 A_p E_p \varepsilon_{p\infty} \left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n = -b\sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n$$

$$+b\sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + b\sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2A_p f_{pd} \left(\varepsilon_{uk} - \varepsilon_{py}\right)$$

$$+2A_p \frac{\varepsilon_0}{d_n} d_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$-2A_p E_p \varepsilon_{p\infty} \left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 - 0{,}85 f_c' b \varepsilon_{cmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 = -b\sigma_{ctmax} \varepsilon_{ctmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2$$

$$+b\sigma_{ctmax} \varepsilon_{t,p} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 + b\sigma_{ctmax} \varepsilon_{t,u} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 + 2A_p f_{pd} \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n$$

$$+2A_p \varepsilon_0^2 d_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p \varepsilon_0^2 \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_n + 2A_p \varepsilon_0 \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_n$$

$$-2A_p \varepsilon_0 \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_n - 2A_p E_p \varepsilon_0 \varepsilon_{p\infty} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n \rightarrow$$

$$b \left(0{,}85 f_c' (\varepsilon_{cmax} - 2\varepsilon_0) - \sigma_{ctmax} \left(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}\right)\right) \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2$$

$$+2A_p \varepsilon_0 \left(\left(f_{pd} - \sigma_{pm\infty}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) - \left(\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py}\right) \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) \right) d_n$$

$$+2A_p \varepsilon_0^2 d_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) = 0 \rightarrow$$

$$d_n = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = b \left(0{,}85 f_c' (\varepsilon_{cmax} - 2\varepsilon_0) - \sigma_{ctmax} \left(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}\right)\right) \left(\varepsilon_{uk} - \varepsilon_{py}\right)$$

$$b = 2A_p \varepsilon_0 \left(\left(f_{pd} - \sigma_{pm\infty}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) - \left(\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py}\right) \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) \right)$$

$$c = 2A_p \varepsilon_0^2 d_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta = \frac{bd_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)\cdot\frac{d_n}{2}\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{b}{2}\frac{\varepsilon_{cmax}}{\varepsilon_0}d_n\cdot\left(d_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{\varepsilon_{cmax}}{\varepsilon_0}\frac{d_n}{3}\right)}{bd_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{bd_n}{2}\frac{\varepsilon_{cmax}}{\varepsilon_0}}$$

The corresponding bending moment capacity and curvature:

$$M = C(d_n-\beta)+T_1\cdot\frac{2}{3}X_1+T_2\left(X_1+\frac{1}{2}X_2\right)+T_3\left(X_1+X_2+\frac{1}{3}X_3\right)+P_{m\infty}(z-d_n)$$
$$+\Delta N_p\left(d_p-d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

8.15 When $\varepsilon_p = \varepsilon_{ud}$ & $\varepsilon_0 > \varepsilon_{cmax}$

With respect to the deformation and stress diagram of [Figure 8.9] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1+X_2} \rightarrow X_1+X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p-d_n} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ud}-\varepsilon_{p\infty}}{d_p-d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}\left(d_p-d_n\right) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}d_p - \frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}d_n = \frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}d_p \rightarrow$$

$$d_n\left(1+\frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}d_p \rightarrow d_n\left(\frac{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}{\varepsilon_{ud}-\varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud}-\varepsilon_{p\infty}}d_p \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}d_p$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,u}}{X_1+X_2+X_3} \rightarrow X_1+X_2+X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0}d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0}d_n - (X_1+X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0}d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \varepsilon_{ud} = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \Delta\varepsilon_p = \varepsilon_{ud} - \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$C = \frac{1}{2} b \frac{\varepsilon_{cmax}}{\varepsilon_0} d_n \cdot 0{,}85 f_c{}' + d_n \left(1 - \frac{\varepsilon_{cmax}}{\varepsilon_0}\right) \cdot 0{,}85 f_c{}' = 0{,}85 f_c{}' \left(1 - \frac{\varepsilon_{cmax}}{2\varepsilon_0}\right) b d_n$$

$$\sum F_H = 0 \rightarrow C = T_1 + T_2 + T_3 + \Delta N_p \rightarrow$$

$$0{,}85 f_c' \left(1 - \frac{\varepsilon_{cmax}}{2\varepsilon_0}\right) b d_n = \frac{1}{2} b \sigma_{ctmax} X_1 + b \sigma_{ctmax} X_2 + \frac{1}{2} b \sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' \frac{\varepsilon_{cmax}}{\varepsilon_0} b d_n = b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b \sigma_{ctmax} \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n\right)$$
$$+ b \sigma_{ctmax} \left(\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n\right) + 2 A_p \sigma_p - 2 A_p \sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' \frac{\varepsilon_{cmax}}{\varepsilon_0} b d_n = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n$$
$$+ 2 A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2 A_p E_p \varepsilon_{p\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p = -b \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$
$$+ b \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + b \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + 2 A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$
$$- 2 A_p E_p \varepsilon_{p\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \varepsilon_0 d_p - 0{,}85 f_c' b \varepsilon_{cmax} d_p = -b \sigma_{ctmax} \varepsilon_{ctmax} d_p + b \sigma_{ctmax} \varepsilon_{t,p} d_p + b \sigma_{ctmax} \varepsilon_{t,u} d_p$$
$$+ 2 A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}) - 2 A_p E_p \varepsilon_{p\infty} (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}) \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \varepsilon_0 d_p - 2 A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) \varepsilon_0 + 2 A_p E_p \varepsilon_{p\infty} \varepsilon_0 = 0{,}85 f_c' b \varepsilon_{cmax} d_p$$
$$- b \sigma_{ctmax} \varepsilon_{ctmax} d_p + b \sigma_{ctmax} \varepsilon_{t,p} d_p + b \sigma_{ctmax} \varepsilon_{t,u} d_p$$
$$+ 2 A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) (\varepsilon_{ud} - \varepsilon_{p\infty}) - 2 A_p E_p \varepsilon_{p\infty} (\varepsilon_{ud} - \varepsilon_{p\infty}) \rightarrow$$

$$2\left(0{,}85f_c'bd_p - A_p\left(f_{pd} + \frac{\varepsilon_{ud}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}\right)\right)\varepsilon_0 = 0{,}85f_c'b\varepsilon_{cmax}d_p$$

$$-b\sigma_{ctmax}d_p(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) + 2A_p\left(f_{pd} + \frac{\varepsilon_{ud}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}\right)(\varepsilon_{ud} - \varepsilon_{p\infty}) \rightarrow$$

$$\varepsilon_0 = \frac{\alpha}{\gamma}$$

$$\alpha = 0{,}85f_c'b\varepsilon_{cmax}d_p - b\sigma_{ctmax}d_p(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) + 2A_p\left(f_{pd} + \frac{\varepsilon_{ud}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}\right)(\varepsilon_{ud} - \varepsilon_{p\infty})$$

$$\gamma = 2\left(0{,}85f_c'bd_p - A_p\left(f_{pd} + \frac{\varepsilon_{ud}-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - \sigma_{pm\infty}\right)\right)$$

8.16 Shear

The ultimate shear capacity is given by:

$$V_u = V_{Rb} + V_f + V_s$$

In case of prestressed concrete, the participation of the concrete will be:

$$V_{Rb} = \frac{1}{\gamma_E}\frac{0{,}24}{\gamma_b}\sqrt{f_c'}bz$$

The contribution of the fibres can be expressed by:

$$V_f = \frac{S_{eff}\sigma(w0{,}3)_k}{\gamma_{bf}\tan\beta_u}$$

$$S_{eff} = bz$$

$$z = d_p - \frac{1}{3}d_n$$

The shear reinforcement:

$$V_s = \frac{A_{sw}}{s}zf_{ywd}\cot\beta_u$$

The angle of the compression struts should be limited to 30° as opposed to 21,8° ,which NEN-EN 1992-1-1 prescribes.

8.17 Crack width

Requirement: the beam remains uncracked in the serviceability state.

At $t = 0$, no time-dependent losses are present, so the prestressing force will be at its maximum. Because of the positioning of the tendons the beam will be slightly cambered and tensile stresses will occur at the top of the beam. Bending moments cause compressive stresses at the top and tensile stresses at the bottom of the beam.

$t = 0 \rightarrow$ check top fibre:

$$-\frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{top}} - \frac{M}{W_{top}} \leq f'_{ct} \rightarrow \frac{M}{W_{top}} \geq -\frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{top}} - f'_{ct} \rightarrow$$

$$M \geq -\frac{P_{m0}}{A_c} W_{top} + P_{m0} \cdot e - f'_{ct} W_{top}$$

$t = 0 \rightarrow$ check bottom fibre:

$$-\frac{P_{m0}}{A_c} - \frac{P_{m0} \cdot e}{W_{bot}} + \frac{M}{W_{bot}} \leq f'_{ct} \rightarrow \frac{M}{W_{bot}} \leq \frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{bot}} + f'_{ct}$$

$$\rightarrow M \leq \frac{P_{m0}}{A_c} W_{bot} + P_{m0} \cdot e + f'_{ct} W_{bot}$$

At $t = \infty$, the prestressing force has been reduced by time-dependent losses, which means that the compressive stresses working on the cross-section will be limited. Dead and live loads are present and will cause tensile stresses at the bottom fibre in the span. The bending moment caused by these loads should be limited:

$t = \infty \rightarrow$ check top fibre:

$$-\frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{top}} - \frac{M}{W_{top}} \leq f'_{ct} \rightarrow \frac{M}{W_{top}} \geq -\frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{top}} - f'_{ct} \rightarrow$$

$$M \geq -\frac{P_{m\infty}}{A_c} W_{top} + P_{m\infty} \cdot e - f'_{ct} W_{top}$$

$t = \infty \rightarrow$ check bottom fibre:

$$-\frac{P_{m\infty}}{A_c} - \frac{P_{m\infty} \cdot e}{W_{bot}} + \frac{M}{W_{bot}} \leq f'_{ct} \rightarrow \frac{M}{W_{bot}} \leq \frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{bot}} + f'_{ct}$$

$$\rightarrow M \leq \frac{P_{m\infty}}{A_c} W_{bot} + P_{m\infty} \cdot e + f'_{ct} W_{bot}$$

9 Unreinforced UHPC box girder

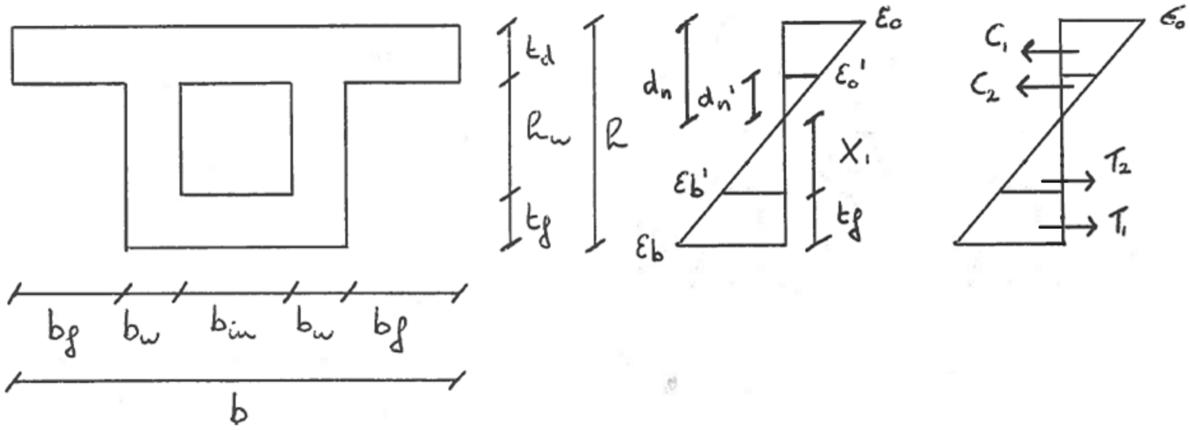


Figure 9.1: deformation and stress diagram when $\varepsilon_b < \varepsilon_{ctmax}$.

9.1 When $\varepsilon_b < \varepsilon_{ctmax}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.1] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0{}'}{d_n{}'} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b{}'}{X_1} \rightarrow \varepsilon_b{}' = \frac{\varepsilon_0}{d_n} X_1$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}(2b_f + b_{in})E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w + b_{in})E_c\varepsilon_b(X_1 + t_f) - \frac{1}{2}b_{in}E_c\varepsilon_b' X_1 \rightarrow$$

$$b\varepsilon_0 d_n - 2b_f\varepsilon_0' d_n' - b_{in}\varepsilon_0' d_n' = 2b_w\varepsilon_b X_1 + 2b_w\varepsilon_b t_f + b_{in}\varepsilon_b X_1 + b_{in}\varepsilon_b t_f - b_{in}\varepsilon_b' X_1 \rightarrow$$

$$b\varepsilon_0 d_n - 2b_f\varepsilon_0' d_n + 2b_f\varepsilon_0' t_d - b_{in}\varepsilon_0' d_n + b_{in}\varepsilon_0' t_d = 2b_w\varepsilon_b h - 2b_w\varepsilon_b d_n + b_{in}\varepsilon_b h - b_{in}\varepsilon_b d_n$$
$$-b_{in}\varepsilon_b' h + b_{in}\varepsilon_b' d_n + b_{in}\varepsilon_b' t_f \rightarrow$$

$$b\varepsilon_0 d_n - 2b_f\varepsilon_0 d_n + 4b_f\varepsilon_0 t_d - 2b_f\frac{\varepsilon_0}{d_n}t_d^2 - b_{in}\varepsilon_0 d_n + 2b_{in}\varepsilon_0 t_d - b_{in}\frac{\varepsilon_0}{d_n}t_d^2 = 2b_w\frac{\varepsilon_0}{d_n}h^2 - 4b_w\varepsilon_0 h$$
$$+2b_w\varepsilon_0 d_n + 2b_{in}\frac{\varepsilon_0}{d_n}t_f h - 2b_{in}\varepsilon_0 t_f - b_{in}\frac{\varepsilon_0}{d_n}t_f^2 \rightarrow$$

$$4b_f t_d d_n + 2b_{in}t_d d_n + 4b_w h d_n + 2b_{in}t_f d_n = 2b_w h^2 + 2b_{in}t_f h - b_{in}t_f^2 + 2b_f t_d^2 + b_{in}t_d^2 \rightarrow$$

$$2\left((2b_f + b_{in})t_d + b_{in}t_f + 2b_w h\right)d_n = 2b_w h^2 + b_{in}t_f(2h - t_f) + (2b_f + b_{in})t_d^2 \rightarrow$$

$$d_n = \frac{2b_w h^2 + b_{in}t_f(2h - t_f) + (2b_f + b_{in})t_d^2}{2\left((2b_f + b_{in})t_d + b_{in}t_f + 2b_w h\right)}$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d_n' + T_1 \cdot \frac{2}{3}(X_1 + t_f) + T_2 \cdot \frac{2}{3}X_1$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

9.2 When $\varepsilon_b = \varepsilon_{ctmax}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.1] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{h-d_n} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}(h-d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}h - \frac{\varepsilon_0}{\varepsilon_{ctmax}}d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ctmax}}d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}h$$

$$\rightarrow d_n\left(1+\frac{\varepsilon_0}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}h \rightarrow d_n\left(\frac{\varepsilon_0+\varepsilon_{ctmax}}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}h \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{ctmax}}h$$

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n}d_n{}'$$

$$\frac{\varepsilon_b{}'}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}X_1$$

$$d_n{}' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}(2b_f+b_{in})E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w+b_{in})\sigma_{ctmax}(X_1+t_f) - \frac{1}{2}b_{in}E_c\varepsilon_b' X_1 \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}t_f + b_{in}\sigma_{ctmax}X_1 + b_{in}\sigma_{ctmax}t_f$$
$$-b_{in}E_c\varepsilon_b' X_1 \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n + 2b_f E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d = 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n$$
$$+b_{in}\sigma_{ctmax}h - b_{in}\sigma_{ctmax}d_n - b_{in}E_c\varepsilon_b' h + b_{in}E_c\varepsilon_b' d_n + b_{in}E_c\varepsilon_b' t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2 = 2b_w\sigma_{ctmax}h$$
$$-2b_w\sigma_{ctmax}d_n + b_{in}\sigma_{ctmax}h - b_{in}\sigma_{ctmax}d_n - b_{in}E_c\frac{\varepsilon_0}{d_n}h^2 + 2b_{in}E_c\varepsilon_0 h + 2b_{in}E_c\frac{\varepsilon_0}{d_n}t_f h$$
$$-2b_{in}E_c\varepsilon_0 t_f - b_{in}E_c\frac{\varepsilon_0}{d_n}t_f^2 \rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ctmax}}h - 2b_f E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ctmax}}h + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0+\varepsilon_{ctmax}}{h}t_d^2 + 2b_{in}E_c\varepsilon_0 t_d$$
$$-b_{in}E_c\frac{\varepsilon_0+\varepsilon_{ctmax}}{h}t_d^2 = 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{ctmax}}h + b_{in}\sigma_{ctmax}h$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{ctmax}}h - b_{in}E_c\frac{\varepsilon_0+\varepsilon_{ctmax}}{h}h^2 + 2b_{in}E_c\varepsilon_0 h + 2b_{in}E_c\frac{\varepsilon_0+\varepsilon_{ctmax}}{h}t_f h$$
$$-2b_{in}E_c\varepsilon_0 t_f - b_{in}E_c\frac{\varepsilon_0+\varepsilon_{ctmax}}{h}t_f^2 \rightarrow$$

$$bE_c\varepsilon_0^2h^2 - 2b_f E_c\varepsilon_0^2h^2 + 4b_f E_c\varepsilon_0(\varepsilon_0+\varepsilon_{ctmax})t_d h - 2b_f E_c(\varepsilon_0+\varepsilon_{ctmax})^2t_d^2$$
$$+2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{ctmax})t_d h - b_{in}E_c(\varepsilon_0+\varepsilon_{ctmax})^2t_d^2 = 2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{ctmax})h^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0 h^2 + b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{ctmax})h^2 - b_{in}\sigma_{ctmax}\varepsilon_0 h^2 - b_{in}E_c(\varepsilon_0+\varepsilon_{ctmax})^2h^2$$
$$+2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{ctmax})h^2 + 2b_{in}E_c(\varepsilon_0+\varepsilon_{ctmax})^2t_f h - 2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{ctmax})t_f h$$
$$-b_{in}E_c(\varepsilon_0+\varepsilon_{ctmax})^2t_f^2 \rightarrow$$

$$bE_c\varepsilon_0^2h^2 - 2b_f E_c\varepsilon_0^2h^2 + 4b_f E_c\varepsilon_0^2t_d h + 4b_f E_c\varepsilon_0\varepsilon_{ctmax}t_d h - 2b_f E_c\varepsilon_0^2t_d^2 - 4b_f E_c\varepsilon_0\varepsilon_{ctmax}t_d^2$$
$$-2b_f E_c\varepsilon_{ctmax}^2t_d^2 + 2b_{in}E_c\varepsilon_0^2t_d h + 2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_d h - b_{in}E_c\varepsilon_0^2t_d^2 - 2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_d^2$$
$$-b_{in}E_c\varepsilon_{ctmax}^2t_d^2 - 2b_w\sigma_{ctmax}\varepsilon_{ctmax}h^2 - b_{in}\sigma_{ctmax}\varepsilon_{ctmax}h^2 - b_{in}E_c\varepsilon_0^2h^2 + b_{in}E_c\varepsilon_{ctmax}^2h^2$$
$$-2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_f h - 2b_{in}E_c\varepsilon_{ctmax}^2t_f h + b_{in}E_c\varepsilon_0^2t_f^2 + 2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_f^2 + b_{in}E_c\varepsilon_{ctmax}^2t_f^2 = 0 \rightarrow$$

$$E_c\left(\left(b - 2b_f - b_{in}\right)h^2 + \left(2b_f + b_{in}\right)(2h - t_d)t_d + b_{in}t_f^2\right)\varepsilon_0^2$$

$$+2E_c\varepsilon_{ctmax}\left(\left(2b_f + b_{in}\right)(h - t_d)t_d - b_{in}t_f\left(h - t_f\right)\right)\varepsilon_0$$

$$-\varepsilon_{ctmax}\left(E_c\varepsilon_{ctmax}\left(\left(2b_f + b_{in}\right)t_d^2 - b_{in}\left(h - t_f\right)^2\right) + (2b_w + b_{in})\sigma_{ctmax}h^2\right) = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(\left(b - 2b_f - b_{in}\right)h^2 + \left(2b_f + b_{in}\right)(2h - t_d)t_d + b_{in}t_f^2\right)$$

$$b = 2E_c\varepsilon_{ctmax}\left(\left(2b_f + b_{in}\right)(h - t_d)t_d - b_{in}t_f\left(h - t_f\right)\right)$$

$$c = -\varepsilon_{ctmax}\left(E_c\varepsilon_{ctmax}\left(\left(2b_f + b_{in}\right)t_d^2 - b_{in}\left(h - t_f\right)^2\right) + (2b_w + b_{in})\sigma_{ctmax}h^2\right)$$

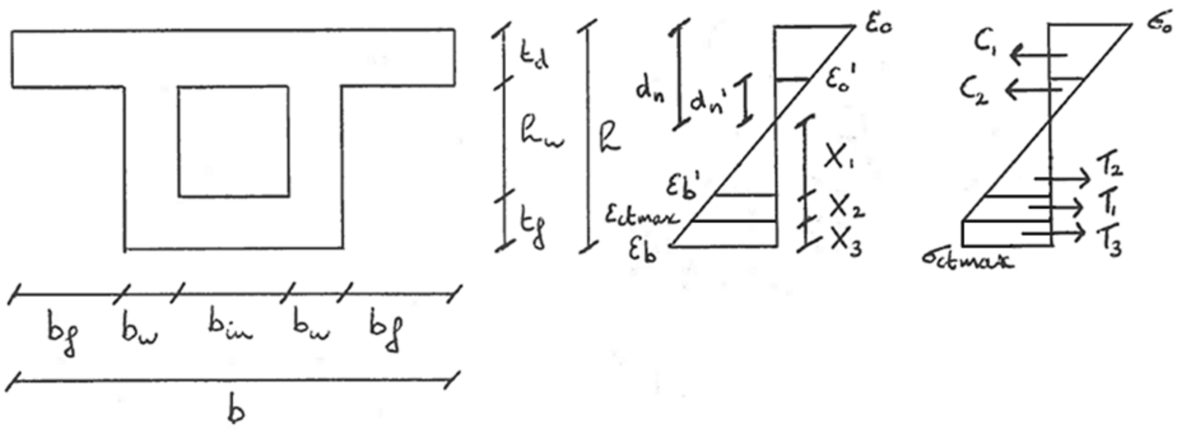


Figure 9.2: deformation and stress diagram when $\varepsilon_b' < \varepsilon_{ctmax} < \varepsilon_b$.

9.3 When $\varepsilon_b' < \varepsilon_{ctmax} < \varepsilon_b$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.2] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0'}{d_n'} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} d_n'$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b'}{X_1} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n} X_1$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - h + d_n + t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

$$X_2 + X_3 = t_f \rightarrow X_3 = t_f - X_2 = h - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}(X_1 + X_2) - \frac{1}{2}b_{in}E_c\varepsilon_b' X_1$$
$$+(2b_w + b_{in})\sigma_{ctmax}X_3 \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + b_{in}\sigma_{ctmax}X_1 + b_{in}\sigma_{ctmax}X_2$$
$$-b_{in}E_c\varepsilon_b' X_1 + 4b_w\sigma_{ctmax}X_3 + 2b_{in}\sigma_{ctmax}X_3 \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n + 2b_f E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n - b_{in}E_c\varepsilon_b' h + b_{in}E_c\varepsilon_b' d_n + b_{in}E_c\varepsilon_b' t_f + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n$$
$$+2b_{in}\sigma_{ctmax}h - 2b_{in}\sigma_{ctmax}d_n \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n - b_{in}\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n - b_{in}E_c\frac{\varepsilon_0}{d_n}h^2 + 2b_{in}E_c\varepsilon_0 h + 2b_{in}E_c\frac{\varepsilon_0}{d_n}t_f h$$
$$-2b_{in}E_c\varepsilon_0 t_f - b_{in}E_c\frac{\varepsilon_0}{d_n}t_f^2 + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n + 2b_{in}\sigma_{ctmax}h - 2b_{in}\sigma_{ctmax}d_n \rightarrow$$

$$bE_c\varepsilon_0^2 d_n^2 - 2b_f E_c\varepsilon_0^2 d_n^2 + 4b_f E_c\varepsilon_0^2 t_d d_n - 2b_f E_c\varepsilon_0^2 t_d^2 + 2b_{in}E_c\varepsilon_0^2 t_d d_n - b_{in}E_c\varepsilon_0^2 t_d^2$$
$$= -2b_w\sigma_{ctmax}\varepsilon_{ctmax}d_n^2 - b_{in}\sigma_{ctmax}\varepsilon_{ctmax}d_n^2 - b_{in}E_c\varepsilon_0^2 h^2 + 2b_{in}E_c\varepsilon_0^2 h d_n + 2b_{in}E_c\varepsilon_0^2 t_f h$$
$$-2b_{in}E_c\varepsilon_0^2 t_f d_n - b_{in}E_c\varepsilon_0^2 t_f^2 + 4b_w\sigma_{ctmax}\varepsilon_0 h d_n - 4b_w\sigma_{ctmax}\varepsilon_0 d_n^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0 h d_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0 d_n^2 \rightarrow$$

$$\left(\left(b - 2b_f\right)E_c\varepsilon_0^2 + (2b_w + b_{in})(\varepsilon_{ctmax} + 2\varepsilon_0)\sigma_{ctmax}\right)d_n^2$$

$$+2\varepsilon_0\left(E_c\varepsilon_0\left(\left(2b_f + b_{in}\right)t_d - b_{in}\left(h - t_f\right)\right) - (2b_w + b_{in})\sigma_{ctmax}h\right)d_n$$

$$-E_c\varepsilon_0^2\left(\left(2b_f + b_{in}\right)t_d^2 - b_{in}\left(h - t_f\right)^2\right) = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \left(\left(b - 2b_f\right)E_c\varepsilon_0^2 + (2b_w + b_{in})(\varepsilon_{ctmax} + 2\varepsilon_0)\sigma_{ctmax}\right)$$

$$b = 2\varepsilon_0\left(E_c\varepsilon_0\left(\left(2b_f + b_{in}\right)t_d - b_{in}\left(h - t_f\right)\right) - (2b_w + b_{in})\sigma_{ctmax}h\right)$$

$$c = -E_c\varepsilon_0^2\left(\left(2b_f + b_{in}\right)t_d^2 - b_{in}\left(h - t_f\right)^2\right)$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d_n' + T_1 \cdot \frac{2}{3}(X_1 + X_2) + T_2 \cdot \frac{2}{3}X_1 + T_3\left(X_1 + X_2 + \frac{1}{2}X_3\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

9.4 When $\varepsilon_b' = \varepsilon_{ctmax}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.2] the following relations are valid:

$$\frac{\varepsilon_b'}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow \frac{\varepsilon_{ctmax}}{h - d_n - t_f} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}\left(h - d_n - t_f\right) \rightarrow$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} h - \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n - \frac{\varepsilon_0}{\varepsilon_{ctmax}} t_f \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} h - \frac{\varepsilon_0}{\varepsilon_{ctmax}} t_f \rightarrow$$

$$d_n \left(1 + \frac{\varepsilon_0}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}\left(h - t_f\right) \rightarrow d_n \left(\frac{\varepsilon_0 + \varepsilon_{ctmax}}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}\left(h - t_f\right) \rightarrow$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}}\left(h - t_f\right)$$

$$\frac{{\varepsilon_0}'}{{d_n}'} = \frac{\varepsilon_0}{d_n} \rightarrow {\varepsilon_0}' = \frac{\varepsilon_0}{d_n} {d_n}'$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$${d_n}' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + (2b_w + b_{in})\sigma_{ctmax} t_f \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} t_f + 2b_{in} \sigma_{ctmax} t_f \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n + 2b_f E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d = 2b_w \sigma_{ctmax} h - 2b_w \sigma_{ctmax} d_n$$
$$+2b_w \sigma_{ctmax} t_f + 2b_{in} \sigma_{ctmax} t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2$$

$$= 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n + 2b_w\sigma_{ctmax}t_f + 2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ctmax}}\left(h-t_f\right) - 2b_fE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ctmax}}\left(h-t_f\right) + 4b_fE_c\varepsilon_0 t_d - 2b_fE_c\frac{\varepsilon_0+\varepsilon_{ctmax}}{h-t_f}t_d^2$$

$$-b_{in}E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ctmax}}\left(h-t_f\right) + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0+\varepsilon_{ctmax}}{h-t_f}t_d^2 = 2b_w\sigma_{ctmax}h$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{ctmax}}\left(h-t_f\right) + 2b_w\sigma_{ctmax}t_f + 2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0^2\left(h-t_f\right)^2 - 2b_fE_c\varepsilon_0^2\left(h-t_f\right)^2 + 4b_fE_c\varepsilon_0(\varepsilon_0+\varepsilon_{ctmax})\left(h-t_f\right)t_d - 2b_fE_c(\varepsilon_0+\varepsilon_{ctmax})^2t_d^2$$

$$-b_{in}E_c\varepsilon_0^2\left(h-t_f\right)^2 + 2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{ctmax})\left(h-t_f\right)t_d - b_{in}E_c(\varepsilon_0+\varepsilon_{ctmax})^2t_d^2$$

$$= 2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{ctmax})\left(h-t_f\right)h - 2b_w\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)^2$$

$$+2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{ctmax})\left(h-t_f\right)t_f + 2b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{ctmax})\left(h-t_f\right)t_f \rightarrow$$

$$bE_c\varepsilon_0^2\left(h-t_f\right)^2 - 2b_fE_c\varepsilon_0^2\left(h-t_f\right)^2 + 4b_fE_c\varepsilon_0^2\left(h-t_f\right)t_d + 4b_fE_c\varepsilon_0\varepsilon_{ctmax}\left(h-t_f\right)t_d$$

$$-2b_fE_c\varepsilon_0^2t_d^2 - 4b_fE_c\varepsilon_0\varepsilon_{ctmax}t_d^2 - 2b_fE_c\varepsilon_{ctmax}^2t_d^2 - b_{in}E_c\varepsilon_0^2\left(h-t_f\right)^2 + 2b_{in}E_c\varepsilon_0^2\left(h-t_f\right)t_d$$

$$+2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}\left(h-t_f\right)t_d - b_{in}E_c\varepsilon_0^2t_d^2 - 2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_d^2 - b_{in}E_c\varepsilon_{ctmax}^2t_d^2$$

$$= 2b_w\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)h + 2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(h-t_f\right)h - 2b_w\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)t_f + 2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(h-t_f\right)t_f + 2b_{in}\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)t_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{ctmax}\left(h-t_f\right)t_f \rightarrow$$

$$E_c\left(\left(b-2b_f-b_{in}\right)\left(h-t_f\right)^2 + \left(2b_f+b_{in}\right)\left(\left(h-t_f\right)2t_d - t_d^2\right)\right)\varepsilon_0^2$$

$$+2\left(E_c\varepsilon_{ctmax}\left(2b_f+b_{in}\right)\left(\left(h-t_f\right)t_d - t_d^2\right) - \sigma_{ctmax}\left(b_w\left(h-t_f\right)\left(h-\left(h-t_f\right)\right) + \left(b_w+b_{in}\right)\left(h-t_f\right)t_f\right)\right)\varepsilon_0$$

$$-\varepsilon_{ctmax}\left(\left(2b_f+b_{in}\right)E_c\varepsilon_{ctmax}t_d^2 + 2\sigma_{ctmax}\left(b_w\left(h-t_f\right)h + \left(b_w+b_{in}\right)\left(h-t_f\right)t_f\right)\right) = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(\left(b - 2b_f - b_{in}\right)\left(h - t_f\right)^2 + \left(2b_f + b_{in}\right)\left(\left(h - t_f\right)2t_d - t_d^2\right)\right)$$

$$b = 2\left(E_c\varepsilon_{ctmax}\left(2b_f + b_{in}\right)\left(\left(h - t_f\right)t_d - t_d^2\right)\right.$$
$$\left. - \sigma_{ctmax}\left(b_w\left(h - t_f\right)\left(h - \left(h - t_f\right)\right) + (b_w + b_{in})\left(h - t_f\right)t_f\right)\right)$$

$$c = -\varepsilon_{ctmax}\left(\left(2b_f + b_{in}\right)E_c\varepsilon_{ctmax}t_d^2 + 2\sigma_{ctmax}\left(b_w\left(h - t_f\right)h + (b_w + b_{in})\left(h - t_f\right)t_f\right)\right)$$

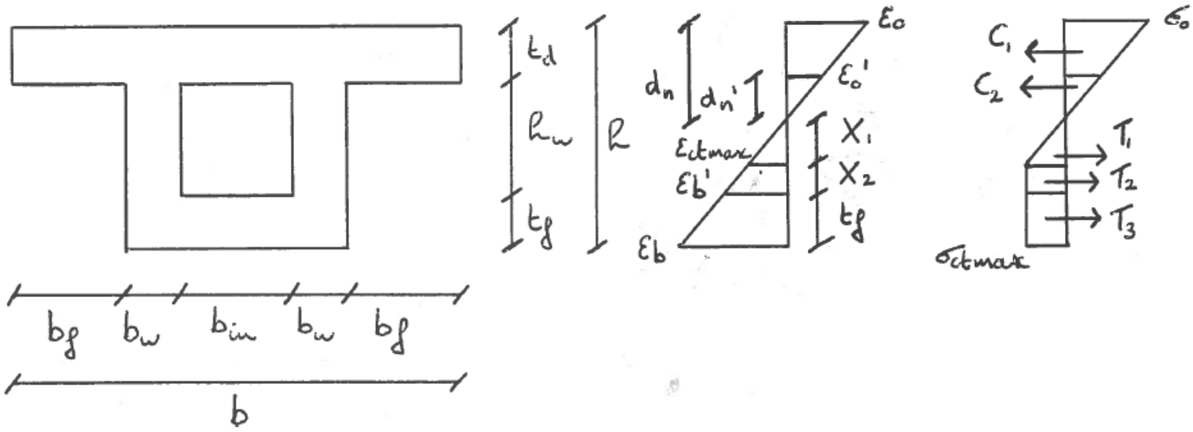


Figure 9.3: deformation and stress diagram when $\varepsilon_{ctmax} < \varepsilon_b{}'$.

9.5 When $\varepsilon_{ctmax} < \varepsilon_b{}'$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.3] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0{}'}{d_n{}'} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b{}'}{X_1 + X_2} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{h - d_n}{d_n} \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + (2b_w + b_{in})\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}t_f$$
$$+2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n + 2b_f E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$+4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n + 2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n + 2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0^2 d_n^2 - 2b_f E_c\varepsilon_0^2 d_n^2 + 4b_f E_c\varepsilon_0^2 t_d d_n - 2b_f E_c\varepsilon_0^2 t_d^2 - b_{in}E_c\varepsilon_0^2 d_n^2 + 2b_{in}E_c\varepsilon_0^2 t_d d_n - b_{in}E_c\varepsilon_0^2 t_d^2$$
$$= -2b_w\sigma_{ctmax}\varepsilon_{ctmax}d_n^2 + 4b_w\sigma_{ctmax}\varepsilon_0 h d_n - 4b_w\sigma_{ctmax}\varepsilon_0 d_n^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0 t_f d_n \rightarrow$$

$$\left(\left(b - 2b_f - b_{in}\right)E_c\varepsilon_0^2 + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)\right)d_n^2$$
$$+2\varepsilon_0\left(\left(2b_f + b_{in}\right)E_c\varepsilon_0 t_d - \left(2b_w h + b_{in}t_f\right)\sigma_{ctmax}\right)d_n - \left(2b_f + b_{in}\right)E_c\varepsilon_0^2 t_d^2 = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \left(b - 2b_f - b_{in}\right)E_c\varepsilon_0^2 + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)$$

$$b = 2\varepsilon_0\left(\left(2b_f + b_{in}\right)E_c\varepsilon_0 t_d - \left(2b_w h + b_{in}t_f\right)\sigma_{ctmax}\right)$$

$$c = -\left(2b_f + b_{in}\right)E_c\varepsilon_0^2 t_d^2$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d_n' + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3\left(X_1 + X_2 + \frac{1}{2}t_f\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

9.6 When $\varepsilon_b = \varepsilon_{t,p}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.3] the following relations are valid:

$$\frac{\varepsilon_{t,p}}{h-d_n} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}(h-d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}h - \frac{\varepsilon_0}{\varepsilon_{t,p}}d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,p}}d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}h \rightarrow d_n\left(1+\frac{\varepsilon_0}{\varepsilon_{t,p}}\right)$$
$$= \frac{\varepsilon_0}{\varepsilon_{t,p}}h \rightarrow d_n\left(\frac{\varepsilon_0+\varepsilon_{t,p}}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}}h \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,p}}h$$

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n}d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_b{}'}{X_1+X_2} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b{}' = \frac{\varepsilon_0}{d_n}(X_1+X_2)$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + (2b_w + b_{in})\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}t_f$$
$$+2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n' + 2b_f E_c\frac{\varepsilon_0}{d_n}d_n' t_d - b_{in}E_c\varepsilon_0 d_n' + b_{in}E_c\frac{\varepsilon_0}{d_n}d_n' t_d = 2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$+4b_w\sigma_{ctmax}\left(h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n\right) + 4b_w\sigma_{ctmax}t_f + 2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c \frac{\varepsilon_0}{d_n} t_d^2$$

$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n + 2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}h - 2b_f E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}h + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0+\varepsilon_{t,p}}{h}t_d^2 - b_{in}E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}h$$

$$+2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0+\varepsilon_{t,p}}{h}t_d^2 = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{t,p}}h + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,p}}h$$

$$+2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_0^2h^2 - 2b_f E_c\varepsilon_0^2h^2 + 4b_f E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})t_d h - 2b_f E_c(\varepsilon_0+\varepsilon_{t,p})^2 t_d^2 - b_{in}E_c\varepsilon_0^2h^2$$

$$+2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})t_d h - b_{in}E_c(\varepsilon_0+\varepsilon_{t,p})^2t_d^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}h^2$$

$$+4b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})h^2 - 4b_w\sigma_{ctmax}\varepsilon_0 h^2 + 2b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})t_f h \rightarrow$$

$$bE_c\varepsilon_0^2h^2 - 2b_fE_c\varepsilon_0^2h^2 + 4b_fE_c\varepsilon_0^2t_dh + 4b_fE_c\varepsilon_0\varepsilon_{t,p}t_dh - 2b_fE_c\varepsilon_0^2t_d^2 - 4b_fE_c\varepsilon_0\varepsilon_{t,p}t_d^2$$

$$-2b_fE_c\varepsilon_{t,p}^2t_d^2 - b_{in}E_c\varepsilon_0^2h^2 + 2b_{in}E_c\varepsilon_0^2t_dh + 2b_{in}E_c\varepsilon_0\varepsilon_{t,p}t_dh - b_{in}E_c\varepsilon_0^2t_d^2 - 2b_{in}E_c\varepsilon_0\varepsilon_{t,p}t_d^2$$

$$-b_{in}E_c\varepsilon_{t,p}^2t_d^2 + 2b_w\sigma_{ctmax}\varepsilon_{ctmax}h^2 - 4b_w\sigma_{ctmax}\varepsilon_{t,p}h^2 - 2b_{in}\sigma_{ctmax}\varepsilon_0t_fh - 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_fh$$

$$= 0 \rightarrow$$

$$E_c\left(bh^2 - (2b_f+b_{in})(h-t_d)^2\right)\varepsilon_0^2 + 2\left(E_c\varepsilon_{t,p}t_d\left(2b_f(h-t_d)+b_{in}(h-t_d)\right) - b_{in}\sigma_{ctmax}t_fh\right)\varepsilon_0$$

$$-E_c\varepsilon_{t,p}^2t_d^2(2b_f+b_{in}) + 2\sigma_{ctmax}h\left(b_wh(\varepsilon_{ctmax}-2\varepsilon_{t,p}) - b_{in}\varepsilon_{t,p}t_f\right) = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$a = E_c\left(bh^2 - (2b_f+b_{in})(h-t_d)^2\right)$$

$$b = 2\left(E_c\varepsilon_{t,p}t_d\left(2b_f(h-t_d)+b_{in}(h-t_d)\right) - b_{in}\sigma_{ctmax}t_fh\right)\varepsilon_0$$

$$c = -E_c\varepsilon_{t,p}^2t_d^2(2b_f+b_{in}) + 2\sigma_{ctmax}h\left(b_wh(\varepsilon_{ctmax}-2\varepsilon_{t,p}) - b_{in}\varepsilon_{t,p}t_f\right)$$

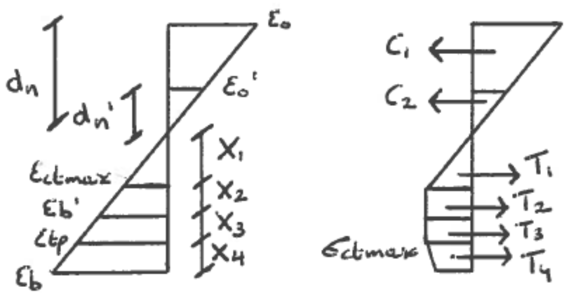


Figure 9.4: deformation and stress diagram when $\varepsilon_b' < \varepsilon_{t,p} < \varepsilon_b$

9.7 When $\boldsymbol{\varepsilon_b' < \varepsilon_{t,p} < \varepsilon_b}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.4] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0{}'}{d_n{}'} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b{}'}{X_1 + X_2} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2 + X_3} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow$$
$$X_3 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \left(h - d_n - t_f\right) = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - h + d_n + t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{h - d_n}{d_n} \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$X_3 + X_4 = t_f \rightarrow X_4 = t_f - X_3 = t_f - \left(\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - h + d_n + t_f\right) = h - d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2$$
$$+(2b_w + b_{in})\sigma_{ctmax} X_3 + (2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4$$
$$+\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4 \rightarrow$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - b_f E_c \varepsilon_0' d_n' - \frac{1}{2} b_{in} E_c \varepsilon_0' d_n' = b_w \sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3$$
$$+b_{in}\sigma_{ctmax} X_3 + 2b_w \sigma_{ctmax} X_4 - b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4 + b_{in}\sigma_{ctmax} X_4 - \frac{1}{2} b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4$$
$$\rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in} E_c \varepsilon_0 d_n + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$= -2b_w \sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_{in}\sigma_{ctmax} t_f + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n - 2b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$
$$+2b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n + 2b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow$$

$$b E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) d_n^2 - 2b_f E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) d_n^2 + 4b_f E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d d_n - 2b_f E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2$$
$$-b_{in} E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) d_n^2 + 2b_{in} E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d d_n - b_{in} E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2$$
$$= -2b_w \sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p}) d_n^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) t_f d_n$$
$$+4b_w \sigma_{ctmax}\varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) h d_n - 4b_w \sigma_{ctmax}\varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) d_n^2 - 2b_w \sigma_{ctmax}\varepsilon_0 (\varepsilon_b - \varepsilon_{t,p}) h d_n$$
$$+2b_w \sigma_{ctmax}\varepsilon_0 (\varepsilon_b - \varepsilon_{t,p}) d_n^2 + 2b_w \sigma_{ctmax}(\varepsilon_b - \varepsilon_{t,p})\varepsilon_{t,p} d_n^2 - b_{in}\sigma_{ctmax}\varepsilon_0 (\varepsilon_b - \varepsilon_{t,p}) h d_n$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0 (\varepsilon_b - \varepsilon_{t,p}) d_n^2 + b_{in}\sigma_{ctmax}(\varepsilon_b - \varepsilon_{t,p})\varepsilon_{t,p} d_n^2 \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_fd_n$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_n+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+2b_w\sigma_{ctmax}\varepsilon_0^2h^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2hd_n-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n-2b_w\sigma_{ctmax}\varepsilon_0^2hd_n+2b_w\sigma_{ctmax}\varepsilon_0^2d_n^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2+2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0^2h^2-b_{in}\sigma_{ctmax}\varepsilon_0^2hd_n-b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n-b_{in}\sigma_{ctmax}\varepsilon_0^2hd_n+b_{in}\sigma_{ctmax}\varepsilon_0^2d_n^2$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2-b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n+b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2+b_{in}\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2=0\rightarrow$$

$$\Big((b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p}) + \left(2b_w(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{ctmax}+2\varepsilon_0)+(2b_w+b_{in})(\varepsilon_0+\varepsilon_{t,p})^2\right)\sigma_{ctmax}\Big)d_n^2$$

$$+\left(\varepsilon_0\left(2(2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d - \left(2(b_{in}t_f+2b_wh)(\varepsilon_{tu}-\varepsilon_{t,p})+2(2b_w+b_{in})\left((\varepsilon_0+\varepsilon_{t,p})h\right)\right)\sigma_{ctmax}\right)\right)d_n$$

$$-(2b_f+b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2+(2b_w+b_{in})\sigma_{ctmax}\varepsilon_0^2h^2=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=(b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p}) + \left(2b_w(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{ctmax}+2\varepsilon_0)+(2b_w+b_{in})(\varepsilon_0+\varepsilon_{t,p})^2\right)\sigma_{ctmax}$$

$$b=\varepsilon_0\left(2(2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d - \left(2(b_{in}t_f+2b_wh)(\varepsilon_{tu}-\varepsilon_{t,p})+2(2b_w+b_{in})\left((\varepsilon_0+\varepsilon_{t,p})h\right)\right)\sigma_{ctmax}\right)$$

$$c=-(2b_f+b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2+(2b_w+b_{in})\sigma_{ctmax}\varepsilon_0^2h^2$$

In order to determine the bending moment capacity the centre of gravity of part X_4 needs to be known:

$$y = \frac{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4 \cdot \frac{1}{2} X_4 + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4 \cdot \frac{1}{3} X_4}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4 + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4} \rightarrow$$

$$y = \frac{\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4^2 + \frac{1}{6}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4^2}{\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax} X_4} \rightarrow$$

$$y = \frac{\left(\frac{1}{2}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax} X_4^2}{\left(1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax} X_4} \rightarrow$$

$$y = \frac{\left(\frac{1}{2} - \frac{1}{3}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4}{1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}}$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3} d_n + C_2 \cdot \frac{2}{3} d_n' + T_1 \cdot \frac{2}{3} X_1 + T_2\left(X_1 + \frac{1}{2} X_2\right) + T_3\left(X_1 + X_2 + \frac{1}{2} X_3\right)$$
$$+ T_4(X_1 + X_2 + X_3 + y)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

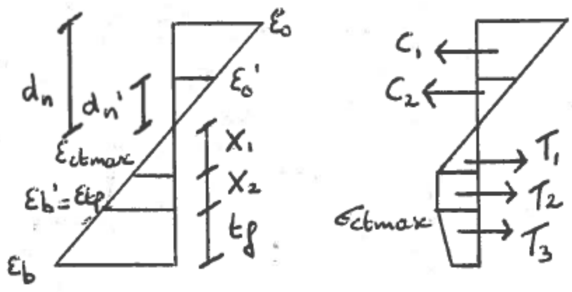


Figure 9.5: deformation and stress diagram when $\varepsilon_b' = \varepsilon_{t,p}$.

9.8 When $\varepsilon_b' = \varepsilon_{t,p}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 9.5] the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{h - d_n}{d_n} \varepsilon_0$$

$$\frac{\varepsilon_{t,p}}{h - d_n - t_f} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}} (h - d_n - t_f) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}} h - \frac{\varepsilon_0}{\varepsilon_{t,p}} d_n - \frac{\varepsilon_0}{\varepsilon_{t,p}} t_f \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,p}} d_n$$
$$= \frac{\varepsilon_0}{\varepsilon_{t,p}} h - \frac{\varepsilon_0}{\varepsilon_{t,p}} t_f \rightarrow d_n \left(1 + \frac{\varepsilon_0}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}} (h - t_f) \rightarrow d_n \left(\frac{\varepsilon_0 + \varepsilon_{t,p}}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}} (h - t_f) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,p}} (h - t_f)$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2$$
$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}t_f$$
$$-2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n + 2b_f E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$+4b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}t_f - 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}t_f - 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$
$$+2b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n - 2b_f E_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n + 4b_f E_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2$$
$$-b_{in}E_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n + 2b_{in}E_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n + 4b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n + 4b_w\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_f$$
$$-2b_w\sigma_{ctmax}\frac{h - d_n}{d_n}\varepsilon_0 t_f + 2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_{in}\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_f - b_{in}\sigma_{ctmax}\frac{h - d_n}{d_n}\varepsilon_0 t_f$$
$$+b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n-2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f$$
$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f+2b_w\sigma_{ctmax}\varepsilon_0t_f+2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f+b_{in}\sigma_{ctmax}\varepsilon_0t_f+b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f\rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)-2b_fE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$-2b_fE_c\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-b_{in}E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$-b_{in}E_c\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)$$
$$+4b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f-2b_w\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}ht_f$$
$$+2b_w\sigma_{ctmax}\varepsilon_0t_f+2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f-b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}ht_f$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0t_f+b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2$$
$$+4b_fE_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_d-2b_fE_c(\varepsilon_0+\varepsilon_{t,p})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2+2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_d$$
$$-b_{in}E_c(\varepsilon_0+\varepsilon_{t,p})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2$$
$$+4b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2+4b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_f$$
$$-2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})^2ht_f+2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(h-t_f)t_f$$
$$+2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})\varepsilon_{t,p}(h-t_f)t_f+2b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_f$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})^2ht_f+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(h-t_f)t_f$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})\varepsilon_{t,p}(h-t_f)t_f\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2+4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_d$$
$$+4b_fE_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_d-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-2b_fE_c\varepsilon_{t,p}^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_d$$
$$+2b_{in}E_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_d-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-b_{in}E_c\varepsilon_{t,p}^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2+2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2$$
$$-4b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_f$$
$$-4b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_f+2b_w\sigma_{ctmax}\varepsilon_0^2ht_f+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_f$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}^2ht_f-2b_w\sigma_{ctmax}\varepsilon_0^2(h-t_f)t_f-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(h-t_f)t_f$$
$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2(h-t_f)t_f-2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_f$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)t_f+b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_f$$
$$+b_{in}\sigma_{ctmax}\varepsilon_{t,p}^2ht_f-b_{in}\sigma_{ctmax}\varepsilon_0^2(h-t_f)t_f-2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(h-t_f)t_f$$
$$-b_{in}\sigma_{ctmax}\varepsilon_{t,p}^2(h-t_f)t_f=0\rightarrow$$

$$\left(E_c(\varepsilon_{tu}-\varepsilon_{t,p})\left((b-2b_f-b_{in})(h-t_f)^2+(2b_f+b_{in})(2(h-t_f)-t_d)t_d\right)\right.$$
$$\left.+\sigma_{ctmax}t_f^2(2b_w+b_{in})\right)\varepsilon_0^2$$

$$+2\left(E_c\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(2b_f+b_{in})\left((h-t_f)-t_d\right)t_d\right.$$
$$\left.-\sigma_{ctmax}(2b_w+b_{in})(\varepsilon_{tu}(h-t_f)t_f-\varepsilon_{t,p}ht_f)\right)\varepsilon_0$$

$$-(2b_f+b_{in})E_c\varepsilon_{t,p}^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$+\sigma_{ctmax}\left(2b_w(\varepsilon_{ctmax}-2\varepsilon_{t,p})(\varepsilon_{tu}-\varepsilon_{t,p})(h-t_f)^2\right.$$
$$\left.-(2b_w+b_{in})(2\varepsilon_{tu}-\varepsilon_{t,p})\varepsilon_{t,p}(h-t_f)t_f+(2b_w+b_{in})\varepsilon_{t,p}^2ht_f\right)=0\rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\left(b - 2b_f - b_{in}\right)\left(h - t_f\right)^2 + \left(2b_f + b_{in}\right)\left(2\left(h - t_f\right) - t_d\right)t_d\right) + \sigma_{ctmax}t_f^2(2b_w + b_{in})$$

$$b = 2\left(E_c\varepsilon_{t,p}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(2b_f + b_{in}\right)\left(\left(h - t_f\right) - t_d\right)t_d - \sigma_{ctmax}(2b_w + b_{in})\left(\varepsilon_{tu}\left(h - t_f\right)t_f - \varepsilon_{t,p}ht_f\right)\right)$$

$$c = -\left(2b_f + b_{in}\right)E_c\varepsilon_{t,p}^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2 + \sigma_{ctmax}\left(2b_w\left(\varepsilon_{ctmax} - 2\varepsilon_{t,p}\right)\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(h - t_f\right)^2 - (2b_w + b_{in})\left(2\varepsilon_{tu} - \varepsilon_{t,p}\right)\varepsilon_{t,p}\left(h - t_f\right)t_f + (2b_w + b_{in})\varepsilon_{t,p}^2ht_f\right)$$

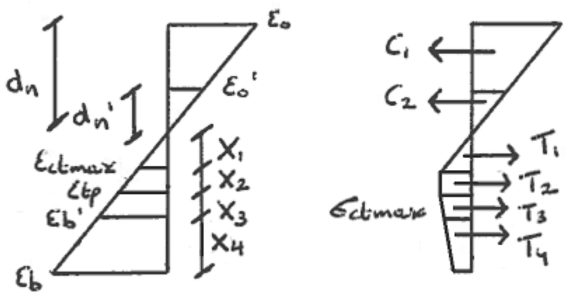


Figure 9.6: Deformation and stress diagram when $\varepsilon_{t,p} < \varepsilon_b'$.

9.9 When $\varepsilon_{t,p} < \varepsilon_b'$

$d_n > t_d$.

With respect to the deformation and stress diagram of figure 4 the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n}\left(h - d_n - t_f\right) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_4 = t_f$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2$$

$$+2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 + \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$

$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_4 + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}X_3$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3 + 4b_w\sigma_{ctmax}X_4 - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 + 2b_{in}\sigma_{ctmax}X_4$$

$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n + 2b_f E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2$$

$$+4b_w\sigma_{ctmax}X_3 - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3 + 4b_w\sigma_{ctmax}X_4 - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4$$

$$+2b_{in}\sigma_{ctmax}X_4 - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2$$

$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$

$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \rightarrow$$

$$bE_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n^2 - 2b_f E_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n^2 + 4b_f E_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d d_n$$

$$-2b_f E_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2 - b_{in}E_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n^2 + 2b_{in}E_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d d_n$$

$$-b_{in}E_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n^2 + 4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)hd_n$$

$$-4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_n^2 - 2b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_b' - \varepsilon_{t,p}\right)hd_n + 2b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_b' - \varepsilon_{t,p}\right)d_n^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_b' - \varepsilon_{t,p}\right)t_f d_n + 2b_w\sigma_{ctmax}\left(\varepsilon_b' - \varepsilon_{t,p}\right)\varepsilon_{t,p}d_n^2 - 4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_b - \varepsilon_{t,p}\right)t_f d_n$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_f d_n - 2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_b - \varepsilon_{t,p}\right)t_f d_n + 2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_b - \varepsilon_b')t_f d_n$$

$$+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_b - \varepsilon_b')t_f d_n \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n$$
$$-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2\ -b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_n$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_b'hd_n+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_b'd_n^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n+2b_w\sigma_{ctmax}\varepsilon_b'\varepsilon_{t,p}d_n^2-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_bt_fd_n+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_fd_n-b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_bt_fd_n$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n-b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_b't_fd_n\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n$$
$$-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2\ -b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2+2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_n$$
$$+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+2b_w\sigma_{ctmax}\varepsilon_0^2h^2-2b_w\sigma_{ctmax}\varepsilon_0^2hd_n-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2hd_n+2b_w\sigma_{ctmax}\varepsilon_0^2d_n^2+2b_w\sigma_{ctmax}\varepsilon_0^2t_fd_n+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n+2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2t_fd_n-2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_fd_n+2b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f-2b_{in}\sigma_{ctmax}\varepsilon_0^2t_fd_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n-b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2=0\rightarrow$$

$$\left((b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})+2b_w\sigma_{ctmax}\left((\varepsilon_{ctmax}+2\varepsilon_0)(\varepsilon_{tu}-\varepsilon_{t,p})+(\varepsilon_0+\varepsilon_{t,p})^2\right)\right)d_n^2$$

$$+2\varepsilon_0\left((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-\left((2b_wh+b_{in}t_f)(\varepsilon_{tu}+\varepsilon_0)\right)\sigma_{ctmax}\right)d_n$$

$$-\left((2b_f+b_{in})E_c(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-\left(2b_wh^2+b_{in}t_f(2h-t_f)\right)\sigma_{ctmax}\right)\varepsilon_0^2=0$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=(b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})+2b_w\sigma_{ctmax}\left((\varepsilon_{ctmax}+2\varepsilon_0)(\varepsilon_{tu}-\varepsilon_{t,p})+(\varepsilon_0+\varepsilon_{t,p})^2\right)$$

$$b=2\varepsilon_0\left((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-\left((2b_wh+b_{in}t_f)(\varepsilon_{tu}+\varepsilon_0)\right)\sigma_{ctmax}\right)$$

$$c=-\left((2b_f+b_{in})E_c(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-\left(2b_wh^2+b_{in}t_f(2h-t_f)\right)\sigma_{ctmax}\right)\varepsilon_0^2$$

In order to determine the bending moment capacity the centres of gravity of part X_3 and X_4 need to be known:

$$y = \frac{2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3\cdot\frac{1}{2}X_3+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3\cdot\frac{1}{3}X_3}{2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3}\rightarrow$$

$$y = \frac{b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3^2+\frac{1}{3}b_w\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3^2}{\left(2-2\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3}\rightarrow$$

$$y = \frac{\left(\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)+\frac{1}{3}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3^2}{\left(2-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3}\rightarrow$$

$$y = \frac{\left(1-\frac{2}{3}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3}{2-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}}$$

$$z = \frac{(2b_w+b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4\cdot\frac{1}{2}X_4+\frac{1}{2}(2b_w+b_{in})\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4\cdot\frac{1}{3}X_4}{(2b_w+b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4+\frac{1}{2}(2b_w+b_{in})\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4}\rightarrow$$

$$z = \frac{\frac{1}{2}(2b_w+b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4^2+\frac{1}{6}(2b_w+b_{in})\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4^2}{\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{2}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)(2b_w+b_{in})\sigma_{ctmax}X_4}\rightarrow$$

$$z = \frac{\left(\frac{1}{2}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)+\frac{1}{6}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)(2b_w+b_{in})\sigma_{ctmax}X_4^2}{\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{2}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)(2b_w+b_{in})\sigma_{ctmax}X_4}\rightarrow$$

$$z = \frac{\left(\frac{1}{2}-\frac{1}{2}\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{6}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4}{1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{2}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}}$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3} d_n + C_2 \cdot \frac{2}{3} d_n' + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right) + T_3 (X_1 + X_2 + y) + T_4 (X_1 + X_2 + X_3 + z)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

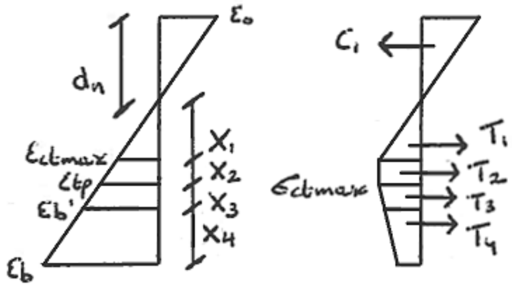


Figure 9.7: Deformation and stress diagram when $\varepsilon_{t,p} < \varepsilon_b' \,\&\, d_n = t_d$.

9.10 When $\varepsilon_{t,p} < \varepsilon_b' \,\&\, d_n = t_d$

With respect to the deformation and stress diagram of [Figure 9.7] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$X_4 = t_f$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3$$

$$+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3 + (2b_w + b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4$$

$$+\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}X_3 - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3$$

$$+4b_w\sigma_{ctmax}X_4 - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 + 2b_{in}\sigma_{ctmax}X_4 - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 \rightarrow$$

$$bE_c\varepsilon_0 t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}t_d + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}t_d - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}t_d + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}t_d$$

$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_d + 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d - 2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_b'-\varepsilon_{t,p})h + 2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_b'-\varepsilon_{t,p})t_d$$

$$+2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_b'-\varepsilon_{t,p})t_f + 2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})\varepsilon_{t,p}t_d - 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_b-\varepsilon_{t,p})t_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_f - 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_b-\varepsilon_{t,p})t_f + 2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_b-\varepsilon_b')t_f$$

$$+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_b-\varepsilon_b')t_f \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_d + 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d - 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_b'h + 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}h + 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_b't_d$$

$$-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_d + 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\varepsilon_b'\varepsilon_{t,p}t_d - 2b_w\sigma_{ctmax}\varepsilon_{t,p}^2t_d$$

$$-2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_b t_f + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_f - b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_b t_f + 2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_f$$

$$-b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_b't_f \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_d + 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d - 2b_w\sigma_{ctmax}\frac{\varepsilon_0^2}{d_n}h^2 + 2b_w\sigma_{ctmax}\varepsilon_0^2 h + 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_0^2}{d_n}ht_d - 2b_w\sigma_{ctmax}\varepsilon_0^2 t_d - 2b_w\sigma_{ctmax}\frac{\varepsilon_0^2}{d_n}t_f t_d - 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_d$$

$$+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\varepsilon_{t,p}ht_d - 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\varepsilon_{t,p}t_f t_d - 2b_w\sigma_{ctmax}\varepsilon_{t,p}^2 t_d$$

$$+2b_w\sigma_{ctmax}\varepsilon_0^2 t_f + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_0^2}{d_n}ht_f + 2b_{in}\sigma_{ctmax}\varepsilon_0^2 t_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0^2}{d_n}t_f^2 \rightarrow$$

$$-bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})ht_d$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_w\sigma_{ctmax}\varepsilon_0^2 h^2 + 2b_w\sigma_{ctmax}\varepsilon_0^2 ht_d + 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_d$$
$$+2b_w\sigma_{ctmax}\varepsilon_0^2 ht_d - 2b_w\sigma_{ctmax}\varepsilon_0^2 t_d^2 - 2b_w\sigma_{ctmax}\varepsilon_0^2 t_f t_d - 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_d^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_f t_d + 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_d - 2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_f t_d - 2b_w\sigma_{ctmax}\varepsilon_{t,p}^2 t_d^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_0^2 t_f t_d + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_f t_d - 2b_{in}\sigma_{ctmax}\varepsilon_0^2 ht_f + 2b_{in}\sigma_{ctmax}\varepsilon_0^2 t_f t_d$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_f t_d + b_{in}\sigma_{ctmax}\varepsilon_0^2 t_f^2 = 0 \rightarrow$$

$$-\left(bE_c(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + \left(2b_w(h-t_d)^2 + b_{in}t_f\left(2(h-t_d)-t_f\right)\right)\sigma_{ctmax}\right)\varepsilon_0^2$$
$$+2\left(2b_w(h-t_d)+b_{in}t_f\right)\sigma_{ctmax}t_d\varepsilon_{tu}\varepsilon_0 - 2b_w\sigma_{ctmax}t_d^2\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+\varepsilon_{t,p}^2\right) = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$a = -\left(bE_c(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + \left(2b_w(h-t_d)^2 + b_{in}t_f\left(2(h-t_d)-t_f\right)\right)\sigma_{ctmax}\right)$$

$$b = 2\left(2b_w(h-t_d)+b_{in}t_f\right)\sigma_{ctmax}t_d\varepsilon_{tu}$$

$$c = -2b_w\sigma_{ctmax}t_d^2\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+\varepsilon_{t,p}^2\right)$$

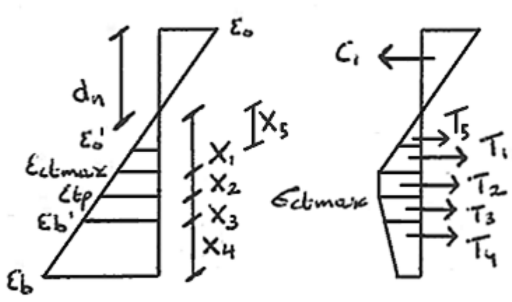


Figure 9.8: deformation and stress diagram when $\boldsymbol{\varepsilon_{t,p}} < \boldsymbol{\varepsilon_b'}$ & $\boldsymbol{\varepsilon_b} < \boldsymbol{\varepsilon_{t,u}}$

9.11 when $\boldsymbol{\varepsilon_{t,p} < \varepsilon_b' \,\&\, \varepsilon_b < \varepsilon_{t,u}}$

$d_n < t_d$.

With respect to the deformation and stress diagram of [Figure 9.8] the following relations are valid:

$$\frac{\varepsilon_0{}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$X_4 = t_f$$

$$X_5 = t_d - d_n$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + T_5$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3$$
$$+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3 + (2b_w + b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4$$
$$+\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 + \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' X_5 \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}X_3 - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3$$
$$+4b_w\sigma_{ctmax}X_4 - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 + 2b_{in}\sigma_{ctmax}X_4 - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4 + 2b_f E_c\varepsilon_0' X_5 + b_{in}E_c\varepsilon_0' X_5 \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}h$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$
$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}t_f + 2b_f E_c\varepsilon_0' t_d - 2b_f E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n + 4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)h$$
$$-4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n - 2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)h + 2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)d_n$$
$$+2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)t_f + 2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - 4b_w\sigma_{ctmax}\left(\varepsilon_b-\varepsilon_{t,p}\right)t_f$$
$$+2b_{in}\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_f - 2b_{in}\sigma_{ctmax}\left(\varepsilon_b-\varepsilon_{t,p}\right)t_f + 2b_w\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')t_f$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')t_f + 2b_f E_c\varepsilon_0'\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d - 2b_f E_c\varepsilon_0'\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n + b_{in}E_c\varepsilon_0'\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d$$
$$-b_{in}E_c\varepsilon_0'\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})h + 2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})d_n$$

$$+2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})t_f + 2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - 4b_w\sigma_{ctmax}(\varepsilon_b-\varepsilon_{t,p})t_f$$

$$+2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f - 2b_{in}\sigma_{ctmax}(\varepsilon_b-\varepsilon_{t,p})t_f + 2b_w\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')t_f$$

$$+b_{in}\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')t_f + 2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$

$$+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$

$$+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}\varepsilon_b'h + 2b_w\sigma_{ctmax}\varepsilon_{t,p}h + 2b_w\sigma_{ctmax}\varepsilon_b'd_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}d_n + 2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\varepsilon_b'\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_bt_f + 2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f - b_{in}\sigma_{ctmax}\varepsilon_bt_f + 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f$$

$$-b_{in}\sigma_{ctmax}\varepsilon_b't_f + 2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$

$$+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$

$$+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2 + 4b_w\sigma_{ctmax}\varepsilon_0h + 4b_w\sigma_{ctmax}\varepsilon_{t,p}h$$

$$-2b_w\sigma_{ctmax}\varepsilon_0d_n - 4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n + 2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f$$

$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f + 2b_{in}\sigma_{ctmax}\varepsilon_0t_f + 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}t_f^2$$

$$+2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d + 2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n$$

$$+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d + b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n \rightarrow$$

$$-bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_n$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_w\sigma_{ctmax}\varepsilon_0^2h^2+4b_w\sigma_{ctmax}\varepsilon_0^2hd_n+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2d_n^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_fd_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0^2t_fd_n+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n+b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2$$
$$+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2$$
$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2=0\rightarrow$$

$$-\Big((b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})$$
$$+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+2\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})+(\varepsilon_0+\varepsilon_{t,p})^2\right)\Big)d_n^2$$

$$+2\varepsilon_0\left(\sigma_{ctmax}\left((2b_wh+b_{in}t_f)(\varepsilon_{tu}+\varepsilon_0)\right)-(2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d\right)d_n$$

$$-\sigma_{ctmax}\varepsilon_0^2\left(2b_wh^2+b_{in}t_f(2h-t_f)\right)+(2b_f+b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=-(b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})$$
$$+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+2\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})+(\varepsilon_0+\varepsilon_{t,p})^2\right)$$

$$b=2\varepsilon_0\left(\sigma_{ctmax}\left((2b_wh+b_{in}t_f)(\varepsilon_{tu}+\varepsilon_0)\right)-(2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d\right)$$

$$c=-\sigma_{ctmax}\varepsilon_0^2\left(2b_wh^2+b_{in}t_f(2h-t_f)\right)+(2b_f+b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$

In order to determine the bending moment capacity the centres of gravity of part X_3 and X_4 need to be known:

$$y=\frac{2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3\cdot\frac{1}{2}X_3+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3\cdot\frac{1}{3}X_3}{2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3}\rightarrow$$

$$y=\frac{b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3^2+\frac{1}{3}b_w\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3^2}{\left(2-2\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3}\rightarrow$$

$$y=\frac{\left(\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)+\frac{1}{3}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3^2}{\left(2-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3}\rightarrow$$

$$y=\frac{\left(1-\frac{2}{3}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3}{2-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}}$$

$$z=\frac{(2b_w+b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4\cdot\frac{1}{2}X_4+\frac{1}{2}(2b_w+b_{in})\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4\cdot\frac{1}{3}X_4}{(2b_w+b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4+\frac{1}{2}(2b_w+b_{in})\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4}\rightarrow$$

$$z=\frac{\frac{1}{2}(2b_w+b_{in})\sigma_{ctmax}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4^2+\frac{1}{6}(2b_w+b_{in})\sigma_{ctmax}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}X_4^2}{\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{2}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)(2b_w+b_{in})\sigma_{ctmax}X_4}\rightarrow$$

$$z=\frac{\left(\frac{1}{2}\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)+\frac{1}{6}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)(2b_w+b_{in})\sigma_{ctmax}X_4^2}{\left(1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{2}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)(2b_w+b_{in})\sigma_{ctmax}X_4}\rightarrow$$

$$z=\frac{\left(\frac{1}{2}-\frac{1}{2}\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{6}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_4}{1-\frac{\varepsilon_b-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}+\frac{1}{2}\frac{\varepsilon_b-\varepsilon_b'}{\varepsilon_{tu}-\varepsilon_{t,p}}}$$

The corresponding bending moment capacity and curvature:

$$M=C_1\cdot\frac{2}{3}d_n+T_1\cdot\frac{2}{3}X_1+T_2\left(X_1+\frac{1}{2}X_2\right)+T_3(X_1+X_2+y)+T_4(X_1+X_2+X_3+z)+T_5\cdot\frac{2}{3}X_5$$

$$\kappa=\frac{\varepsilon_0}{d_n}$$

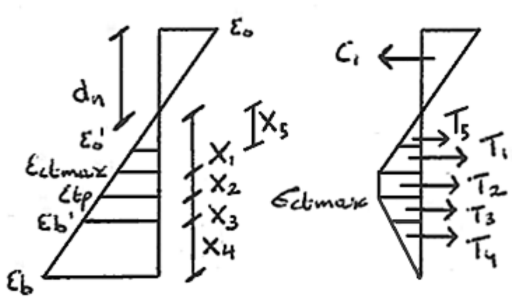


Figure 9.9: deformation and stress diagram when $\varepsilon_b = \varepsilon_{t,u}$.

9.12 When $\varepsilon_b = \varepsilon_{t,u}$

$d_n < t_d$.

With respect to the deformation and stress diagram of [Figure 9.9] the following relations are valid:

$$\frac{{\varepsilon_0}'}{X_5} = \frac{\varepsilon_0}{d_n} \to \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \to X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \to X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \to X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \to \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n}\left(h - d_n - t_f\right) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_{t,u}}{h - d_n} = \frac{\varepsilon_0}{d_n} \to d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}}(h - d_n) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h - \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n \to d_n + \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}} h \to$$
$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h \to d_n\left(\frac{\varepsilon_0 + \varepsilon_{t,u}}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h \to d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,u}} h$$

$${d_n}' = d_n - t_d$$

$$X_4 = t_f$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \to X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$X_5 = t_d - d_n$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + T_5$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}(X_3 + X_4)$$
$$+\frac{1}{2} b_{in}\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4 + \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' X_5 \rightarrow$$

$$b E_c \varepsilon_0 d_n = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax}(X_3 + X_4)$$
$$+b_{in}\sigma_{ctmax} X_4 - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4 + 2b_f E_c \varepsilon_0' X_5 + b_{in} E_c \varepsilon_0' X_5 \rightarrow$$

$$b E_c \varepsilon_0 d_n = -2b_w \sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} h - 2b_w \sigma_{ctmax} d_n$$
$$+b_{in}\sigma_{ctmax} t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_f E_c \varepsilon_0' t_d - 2b_f E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n \rightarrow$$

$$b E_c \varepsilon_0 d_n = -2b_w \sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} h - 2b_w \sigma_{ctmax} d_n$$
$$+b_{in}\sigma_{ctmax} t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - 4b_f E_c \varepsilon_0 t_d + 2b_f E_c \varepsilon_0 d_n + b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$-2b_{in} E_c \varepsilon_0 t_d + b_{in} E_c \varepsilon_0 d_n \rightarrow$$

$$b E_c \varepsilon_0 \left(\varepsilon_{tu} - \varepsilon_{t,p}\right) d_n = -2b_w \sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) d_n$$
$$+2b_w \sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) d_n + 2b_w \sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) h - 2b_w \sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) d_n$$
$$+b_{in}\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n} h t_f + b_{in}\sigma_{ctmax}\varepsilon_0 t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n} t_f^2 + b_{in}\sigma_{ctmax}\varepsilon_{t,p} t_f$$
$$+2b_f E_c \frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) t_d^2 - 4b_f E_c \varepsilon_0 \left(\varepsilon_{tu} - \varepsilon_{t,p}\right) t_d + 2b_f E_c \varepsilon_0 \left(\varepsilon_{tu} - \varepsilon_{t,p}\right) d_n$$
$$+b_{in} E_c \frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) t_d^2 - 2b_{in} E_c \varepsilon_0 \left(\varepsilon_{tu} - \varepsilon_{t,p}\right) t_d + b_{in} E_c \varepsilon_0 \left(\varepsilon_{tu} - \varepsilon_{t,p}\right) d_n \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,u}}h=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})\frac{h}{\varepsilon_0+\varepsilon_{t,u}}$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})\frac{h}{\varepsilon_0+\varepsilon_{t,u}}+2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$
$$-2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,u}}h+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f-b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,u}}{h}ht_f$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,u}}{h}t_f^2+b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_fE_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,u}}h+b_{in}E_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,u}}h\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})h^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})h^2+2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})h^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})h^2+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})ht_f-b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})^2ht_f$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})ht_f+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})^2t_f^2+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})\varepsilon_{t,p}ht_f$$
$$+2b_fE_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})ht_d+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})h^2$$
$$+b_{in}E_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})ht_d+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})h^2\rightarrow$$

$$-bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})h^2-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h^2+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})h^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})h^2+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})ht_f+b_{in}\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})ht_f$$
$$-b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}ht_f-b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2ht_f+b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}t_f^2+b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2t_f^2$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_f+b_{in}\sigma_{ctmax}\varepsilon_{t,u}\varepsilon_{t,p}ht_f+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2+4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$+2b_fE_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})ht_d-4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})ht_d$$
$$+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})h^2+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2+2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$+b_{in}E_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})ht_d-2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})ht_d$$
$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})h^2=0\rightarrow$$

$$-\left(E_c(\varepsilon_{tu}-\varepsilon_{t,p})\left(bh^2-(2b_f+b_{in})(h-t_d)^2\right)-b_{in}\sigma_{ctmax}t_f^2\right)\varepsilon_0^2$$

$$+\left(2b_{in}\sigma_{ctmax}\varepsilon_{t,u}t_f^2+2(2b_f+b_{in})(t_d-h)t_dE_c\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})\right)\varepsilon_0$$

$$-\sigma_{ctmax}\left(2b_w(\varepsilon_{tu}-\varepsilon_{t,p})h^2(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})-b_{in}\varepsilon_{t,u}^2t_f^2\right)+(2b_f+b_{in})E_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$=0\rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\left(E_c\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(bh^2 - \left(2b_f + b_{in}\right)(h - t_d)^2\right) - b_{in}\sigma_{ctmax}t_f^2\right)$$

$$b = 2b_{in}\sigma_{ctmax}\varepsilon_{t,u}t_f^2 + 2\left(2b_f + b_{in}\right)(t_d - h)t_d E_c \varepsilon_{t,u}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)$$

$$c = -\sigma_{ctmax}\left(2b_w\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)h^2\left(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}\right) - b_{in}\varepsilon_{t,u}^2 t_f^2\right)$$
$$+\left(2b_f + b_{in}\right)E_c\varepsilon_{t,u}^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2$$

9.13 Crack width

In case of a cross-section not containing any bonded tendons in the tensile zone, the design crack width w_{max} at the extreme tensile fibre of the section may be taken as:

$$w_{max} = 1{,}5D(\varepsilon_b - 0{,}00016)$$

With respect to the deformation and stress diagram of [Figure 9.3] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_0 = \frac{\varepsilon_b}{h - d_n} d_n$$

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_b}{h - d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_b}{h - d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_b}(h - d_n)$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_b}{h - d_n}(X_1 + X_2)$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_b}(h - d_n)$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + (2b_w + b_{in})\sigma_{ctmax} t_f \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} t_f$$
$$+2b_{in}\sigma_{ctmax} t_f \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n + 2b_f E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_b}(h - d_n)$$
$$+4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n + 2b_{in} \sigma_{ctmax} t_f \rightarrow$$

$$bE_c\frac{\varepsilon_b}{h-d_n}d_n^2 - 2b_fE_c\frac{\varepsilon_b}{h-d_n}d_n^2 + 4b_fE_c\frac{\varepsilon_b}{h-d_n}t_dd_n - 2b_fE_c\frac{\varepsilon_b}{h-d_n}t_d^2 - b_{in}E_c\frac{\varepsilon_b}{h-d_n}d_n^2$$

$$+2b_{in}E_c\frac{\varepsilon_b}{h-d_n}t_dd_n - b_{in}E_c\frac{\varepsilon_b}{h-d_n}t_d^2 = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_b}(h-d_n) + 4b_w\sigma_{ctmax}h$$

$$-4b_w\sigma_{ctmax}d_n + 2b_{in}\sigma_{ctmax}t_f \rightarrow$$

$$bE_c\varepsilon_b^2d_n^2 - 2b_fE_c\varepsilon_b^2d_n^2 + 4b_fE_c\varepsilon_b^2t_dd_n - 2b_fE_c\varepsilon_b^2t_d^2 - b_{in}E_c\varepsilon_b^2d_n^2$$

$$+2b_{in}E_c\varepsilon_b^2t_dd_n - b_{in}E_c\varepsilon_b^2t_d^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(h-d_n)^2 + 4b_w\sigma_{ctmax}\varepsilon_bh(h-d_n)$$

$$-4b_w\sigma_{ctmax}\varepsilon_b(h-d_n)d_n + 2b_{in}\sigma_{ctmax}\varepsilon_bt_f(h-d_n) \rightarrow$$

$$bE_c\varepsilon_b^2d_n^2 - 2b_fE_c\varepsilon_b^2d_n^2 + 4b_fE_c\varepsilon_b^2t_dd_n - 2b_fE_c\varepsilon_b^2t_d^2 - b_{in}E_c\varepsilon_b^2d_n^2 + 2b_{in}E_c\varepsilon_b^2t_dd_n - b_{in}E_c\varepsilon_b^2t_d^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{ctmax}h^2 - 4b_w\sigma_{ctmax}\varepsilon_{ctmax}hd_n + 2b_w\sigma_{ctmax}\varepsilon_{ctmax}d_n^2 - 4b_w\sigma_{ctmax}\varepsilon_bh^2$$

$$+8b_w\sigma_{ctmax}\varepsilon_bhd_n - 4b_w\sigma_{ctmax}\varepsilon_bd_n^2 - 2b_{in}\sigma_{ctmax}\varepsilon_bht_f + 2b_{in}\sigma_{ctmax}\varepsilon_bt_fd_n = 0 \rightarrow$$

$$\left(E_c\varepsilon_b^2(b-2b_f-b_{in}) + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-2\varepsilon_b)\right)d_n^2$$

$$+2\left(E_c\varepsilon_b^2t_d(2b_f+b_{in}) - \sigma_{ctmax}\left(2b_wh(\varepsilon_{ctmax}-2\varepsilon_b) - b_{in}\varepsilon_bt_f\right)\right)d_n$$

$$-E_c\varepsilon_b^2t_d^2(2b_f+b_{in}) + 2\sigma_{ctmax}\left(b_wh^2(\varepsilon_{ctmax}-2\varepsilon_b) - b_{in}\varepsilon_bht_f\right) = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$a = E_c\varepsilon_b^2(b-2b_f-b_{in}) + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-2\varepsilon_b)$$

$$b = 2\left(E_c\varepsilon_b^2t_d(2b_f+b_{in}) - \sigma_{ctmax}\left(2b_wh(\varepsilon_{ctmax}-2\varepsilon_b) - b_{in}\varepsilon_bt_f\right)\right)$$

$$c = -E_c\varepsilon_b^2t_d^2(2b_f+b_{in}) + 2\sigma_{ctmax}\left(b_wh^2(\varepsilon_{ctmax}-2\varepsilon_b) - b_{in}\varepsilon_bht_f\right)$$

The maximum moment in SLS:

$$M_{qp} = C_1\cdot\frac{2}{3}d_n + C_2\cdot\frac{2}{3}d_n' + T_1\cdot\frac{2}{3}X_1 + T_2\left(X_1+\frac{1}{2}X_2\right) + T_3\left(X_1+X_2+\frac{1}{2}t_f\right)$$

10 Prestressed UHPC box girder

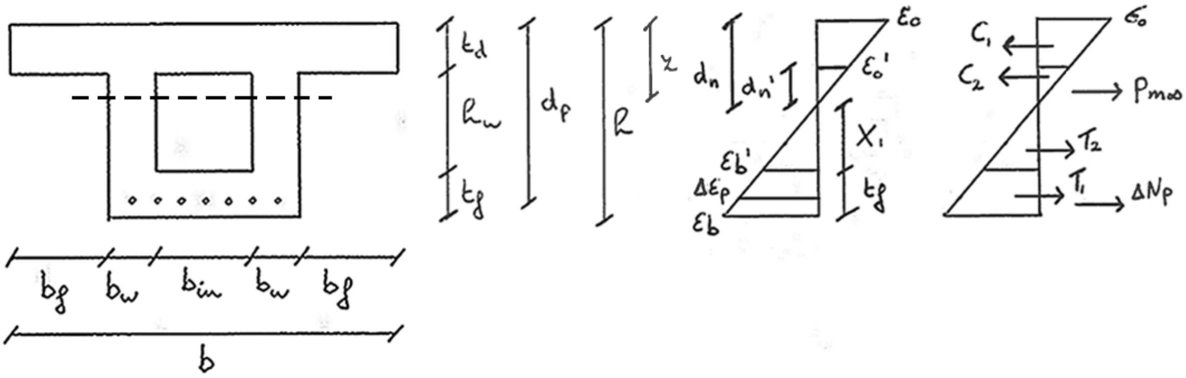


Figure 10.1: deformation and stress diagram when $\varepsilon_b < \varepsilon_{ctmax}$.

10.1 When $\varepsilon_b < \varepsilon_{ctmax}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.1] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0{}'}{d_n{}'} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b{}'}{X_1} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n} X_1$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w + b_{in})E_c\varepsilon_b\left(X_1 + t_f\right) - \frac{1}{2}b_{in}E_c\varepsilon_b' X_1 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w E_c\varepsilon_b X_1 + 2b_w E_c\varepsilon_b t_f + b_{in}E_c\varepsilon_b X_1 + b_{in}E_c\varepsilon_b t_f$$
$$-b_{in}E_c\varepsilon_b' X_1 + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n + 2b_f E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d = 2b_w E_c\varepsilon_b h - 2b_w E_c\varepsilon_b d_n$$
$$+b_{in}E_c\varepsilon_b h - b_{in}E_c\varepsilon_b d_n - b_{in}E_c\varepsilon_b' h + b_{in}E_c\varepsilon_b' d_n + b_{in}E_c\varepsilon_b' t_f + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2 = 2b_w E_c\frac{\varepsilon_0}{d_n}h^2$$
$$-4b_w E_c\varepsilon_0 h + 2b_w E_c\varepsilon_0 d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}h^2 - 2b_{in}E_c\varepsilon_0 h + b_{in}E_c\varepsilon_0 d_n - b_{in}E_c\frac{\varepsilon_0}{d_n}h^2 + 2b_{in}E_c\varepsilon_0 h$$
$$+2b_{in}E_c\frac{\varepsilon_0}{d_n}t_f h - 2b_{in}E_c\varepsilon_0 t_f - b_{in}E_c\frac{\varepsilon_0}{d_n}t_f^2 + 2A_p E_p\frac{\varepsilon_0}{d_n}d_p - 2A_p E_p\varepsilon_0 \rightarrow$$

$$4b_f E_c t_d d_n - 2b_f E_c t_d^2 + 2b_{in}E_c t_d d_n - b_{in}E_c t_d^2 = 2b_w E_c h^2 - 4b_w E_c h d_n + b_{in}E_c h^2 - 2b_{in}E_c h d_n$$
$$-b_{in}E_c h^2 + 2b_{in}E_c h d_n + 2b_{in}E_c t_f h - 2b_{in}E_c t_f d_n - b_{in}E_c t_f^2 + 2A_p E_p d_p - 2A_p E_p d_n \rightarrow$$

$$4b_f E_c t_d d_n + 2b_{in}E_c t_d d_n + 4b_w E_c h d_n + 2b_{in}E_c t_f d_n + 2A_p E_p d_n = 2b_w E_c h^2 + 2b_{in}E_c t_f h$$
$$-b_{in}E_c t_f^2 + 2b_f E_c t_d^2 + b_{in}E_c t_d^2 + 2A_p E_p d_p \rightarrow$$

$$2\left(E_c\left(\left(2b_f + b_{in}\right)t_d + 2b_w h + b_{in}t_f\right) + A_p E_p\right)d_n$$
$$= E_c\left(2b_w h^2 + b_{in}t_f\left(2h - t_f\right) + \left(2b_f + b_{in}\right)t_d^2\right) + 2A_p E_p d_p \rightarrow$$

$$d_n = \frac{E_c\left(2b_w h^2 + b_{in}t_f\left(2h - t_f\right) + \left(2b_f + b_{in}\right)t_d^2\right) + 2A_p E_p d_p}{2\left(E_c\left(\left(2b_f + b_{in}\right)t_d + 2b_w h + b_{in}t_f\right) + A_p E_p\right)}$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d_n' + T_1 \cdot \frac{2}{3}\left(X_1 + t_f\right) + T_2 \cdot \frac{2}{3}X_1 + P_{m\infty}(e - d_n) + \Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

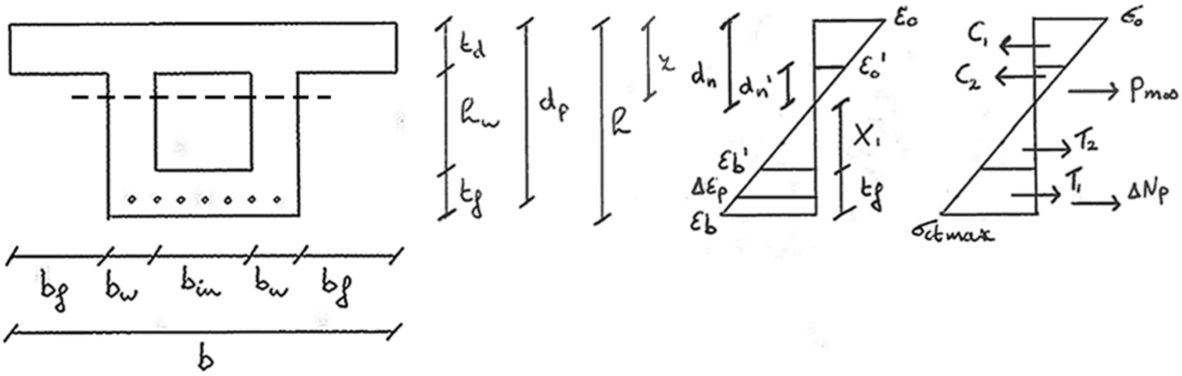


Figure 10.2: deformation and stress diagram when $\varepsilon_b = \varepsilon_{ctmax}$.

10.2 When $\varepsilon_b = \varepsilon_{ctmax}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.2] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{{\varepsilon_0}'}{{d_n}'} \rightarrow {\varepsilon_0}' = \frac{\varepsilon_0}{d_n} {d_n}'$$

$$\frac{\varepsilon_0}{d_n} = \frac{{\varepsilon_b}'}{X_1} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n} X_1$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{h - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}(h - d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} h - \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ctmax}} d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}} h$$
$$\rightarrow d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}} h \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{ctmax}}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}} h \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}} h$$

$${d_n}' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\left(X_1 + t_f\right) - \frac{1}{2}b_{in}E_c\varepsilon_b' X_1 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}t_f + b_{in}\sigma_{ctmax}X_1 + b_{in}\sigma_{ctmax}t_f$$
$$-b_{in}E_c\varepsilon_b' X_1 + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n + 2b_f E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d = 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n$$
$$+b_{in}\sigma_{ctmax}h - b_{in}\sigma_{ctmax}d_n - b_{in}E_c\varepsilon_b' h + b_{in}E_c\varepsilon_b' d_n + b_{in}E_c\varepsilon_b' t_f + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2 = 2b_w\sigma_{ctmax}h$$
$$-2b_w\sigma_{ctmax}d_n + b_{in}\sigma_{ctmax}h - b_{in}\sigma_{ctmax}d_n - b_{in}E_c\frac{\varepsilon_0}{d_n}h^2 + 2b_{in}E_c\varepsilon_0 h + 2b_{in}E_c\frac{\varepsilon_0}{d_n}t_f h$$
$$-2b_{in}E_c\varepsilon_0 t_f - b_{in}E_c\frac{\varepsilon_0}{d_n}t_f^2 + 2A_pE_p\frac{\varepsilon_0}{d_n}d_p - 2A_pE_p\varepsilon_0 \rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}}h - 2b_f E_c\frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}}h + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0 + \varepsilon_{ctmax}}{h}t_d^2 + 2b_{in}E_c\varepsilon_0 t_d$$
$$-b_{in}E_c\frac{\varepsilon_0 + \varepsilon_{ctmax}}{h}t_d^2 = 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}}h + b_{in}\sigma_{ctmax}h$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}}h - b_{in}E_c\frac{\varepsilon_0 + \varepsilon_{ctmax}}{h}h^2 + 2b_{in}E_c\varepsilon_0 h + 2b_{in}E_c\frac{\varepsilon_0 + \varepsilon_{ctmax}}{h}t_f h$$
$$-2b_{in}E_c\varepsilon_0 t_f - b_{in}E_c\frac{\varepsilon_0 + \varepsilon_{ctmax}}{h}t_f^2 + 2A_pE_p\frac{\varepsilon_0 + \varepsilon_{ctmax}}{h}d_p - 2A_pE_p\varepsilon_0 \rightarrow$$

$$bE_c\varepsilon_0^2 h^2 - 2b_f E_c\varepsilon_0^2 h^2 + 4b_f E_c\varepsilon_0(\varepsilon_0 + \varepsilon_{ctmax})t_d h - 2b_f E_c(\varepsilon_0 + \varepsilon_{ctmax})^2 t_d^2$$
$$+2b_{in}E_c\varepsilon_0(\varepsilon_0 + \varepsilon_{ctmax})t_d h - b_{in}E_c(\varepsilon_0 + \varepsilon_{ctmax})^2 t_d^2 = 2b_w\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{ctmax})h^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0 h^2 + b_{in}\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{ctmax})h^2 - b_{in}\sigma_{ctmax}\varepsilon_0 h^2 - b_{in}E_c(\varepsilon_0 + \varepsilon_{ctmax})^2 h^2$$
$$+2b_{in}E_c\varepsilon_0(\varepsilon_0 + \varepsilon_{ctmax})h^2 + 2b_{in}E_c(\varepsilon_0 + \varepsilon_{ctmax})^2 t_f h - 2b_{in}E_c\varepsilon_0(\varepsilon_0 + \varepsilon_{ctmax})t_f h$$
$$-b_{in}E_c(\varepsilon_0 + \varepsilon_{ctmax})^2 t_f^2 + 2A_pE_p(\varepsilon_0 + \varepsilon_{ctmax})^2 d_p - 2A_pE_p\varepsilon_0(\varepsilon_0 + \varepsilon_{ctmax})h \rightarrow$$

$$bE_c\varepsilon_0^2h^2 - 2b_fE_c\varepsilon_0^2h^2 + 4b_fE_c\varepsilon_0^2t_dh + 4b_fE_c\varepsilon_0\varepsilon_{ctmax}t_dh - 2b_fE_c\varepsilon_0^2t_d^2 - 4b_fE_c\varepsilon_0\varepsilon_{ctmax}t_d^2$$
$$-2b_fE_c\varepsilon_{ctmax}^2t_d^2 + 2b_{in}E_c\varepsilon_0^2t_dh + 2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_dh - b_{in}E_c\varepsilon_0^2t_d^2 - 2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_d^2$$
$$-b_{in}E_c\varepsilon_{ctmax}^2t_d^2 - 2b_w\sigma_{ctmax}\varepsilon_{ctmax}h^2 - b_{in}\sigma_{ctmax}\varepsilon_{ctmax}h^2 - b_{in}E_c\varepsilon_0^2h^2 + b_{in}E_c\varepsilon_{ctmax}^2h^2$$
$$-2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_fh - 2b_{in}E_c\varepsilon_{ctmax}^2t_fh + b_{in}E_c\varepsilon_0^2t_f^2 + 2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_f^2 + b_{in}E_c\varepsilon_{ctmax}^2t_f^2$$
$$-2A_pE_p\varepsilon_0^2d_p - 4A_pE_p\varepsilon_0\varepsilon_{ctmax}d_p - 2A_pE_p\varepsilon_{ctmax}^2d_p + 2A_pE_p\varepsilon_0^2h + 2A_pE_p\varepsilon_0\varepsilon_{ctmax}h = 0 \rightarrow$$

$$\left(E_c\left(\left(b - 2b_f - b_{in}\right)h^2 + \left(2b_f + b_{in}\right)(2h - t_d)t_d + b_{in}t_f^2\right) - 2A_pE_p\left(d_p - h\right)\right)\varepsilon_0^2$$

$$+2\varepsilon_{ctmax}\left(E_c\left(\left(2b_f + b_{in}\right)(h - t_d)t_d - b_{in}t_f\left(h - t_f\right)\right) - A_pE_p\left(2d_p - h\right)\right)\varepsilon_0$$

$$-\varepsilon_{ctmax}\left(E_c\varepsilon_{ctmax}\left(\left(2b_f + b_{in}\right)t_d^2 - b_{in}\left(h - t_f\right)^2\right) + (2b_w + b_{in})\sigma_{ctmax}h^2 + 2A_pE_p\varepsilon_{ctmax}d_p\right)$$
$$= 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(\left(b - 2b_f - b_{in}\right)h^2 + \left(2b_f + b_{in}\right)(2h - t_d)t_d + b_{in}t_f^2\right) - 2A_pE_p\left(d_p - h\right)$$

$$b = 2\varepsilon_{ctmax}\left(E_c\left(\left(2b_f + b_{in}\right)(h - t_d)t_d - b_{in}t_f\left(h - t_f\right)\right) - A_pE_p\left(2d_p - h\right)\right)$$

$$c = -\varepsilon_{ctmax}\left(E_c\varepsilon_{ctmax}\left(\left(2b_f + b_{in}\right)t_d^2 - b_{in}\left(h - t_f\right)^2\right) + (2b_w + b_{in})\sigma_{ctmax}h^2 + 2A_pE_p\varepsilon_{ctmax}d_p\right)$$

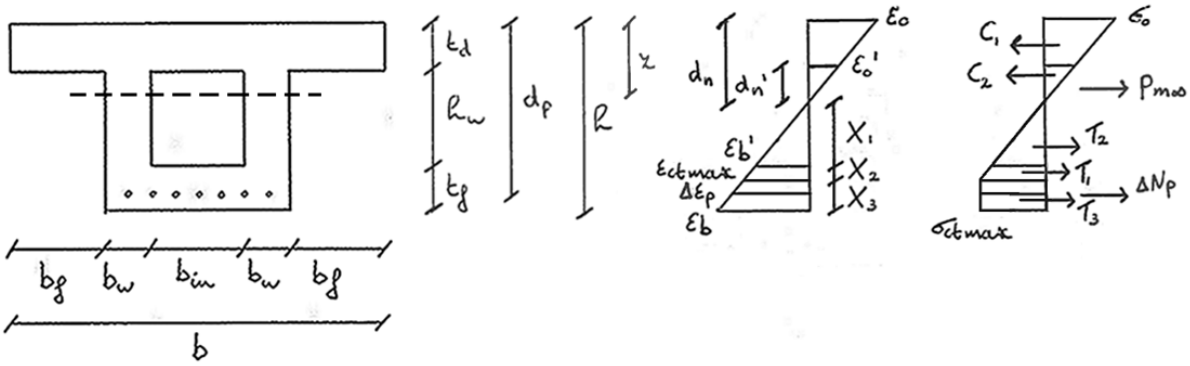


Figure 10.3: deformation and stress diagram when $\varepsilon_b' < \varepsilon_{ctmax} < \varepsilon_b$.

10.3 When $\varepsilon_b' < \varepsilon_{ctmax} < \varepsilon_b$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.3] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0{}'}{d_n{}'} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b{}'}{X_1} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n} X_1$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{X_1 + X_2} \rightarrow X_1 + X_2 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - X_1 \rightarrow X_2 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - h + d_n + t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

$$X_2 + X_3 = t_f \rightarrow X_3 = t_f - X_2 \rightarrow X_3 = h - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}(X_1 + X_2) - \frac{1}{2} b_{in} E_c \varepsilon_b' X_1$$
$$+(2b_w + b_{in})\sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + b_{in} \sigma_{ctmax} X_1 + b_{in} \sigma_{ctmax} X_2$$
$$-b_{in} E_c \varepsilon_b' X_1 + 4b_w \sigma_{ctmax} X_3 + 2b_{in} \sigma_{ctmax} X_3 + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n + 2b_f E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$-b_{in} \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - b_{in} E_c \varepsilon_b' h + b_{in} E_c \varepsilon_b' d_n + b_{in} E_c \varepsilon_b' t_f + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n$$
$$+2b_{in} \sigma_{ctmax} h - 2b_{in} \sigma_{ctmax} d_n + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - b_{in} \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n - b_{in} E_c \frac{\varepsilon_0}{d_n} h^2 + 2b_{in} E_c \varepsilon_0 h + 2b_{in} E_c \frac{\varepsilon_0}{d_n} t_f h$$
$$-2b_{in} E_c \varepsilon_0 t_f - b_{in} E_c \frac{\varepsilon_0}{d_n} t_f^2 + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n + 2b_{in} \sigma_{ctmax} h - 2b_{in} \sigma_{ctmax} d_n$$
$$+2A_p E_p \frac{\varepsilon_0}{d_n} d_p - 2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0^2 d_n^2 - 2b_f E_c \varepsilon_0^2 d_n^2 + 4b_f E_c \varepsilon_0^2 t_d d_n - 2b_f E_c \varepsilon_0^2 t_d^2 + 2b_{in} E_c \varepsilon_0^2 t_d d_n - b_{in} E_c \varepsilon_0^2 t_d^2$$
$$= -2b_w \sigma_{ctmax} \varepsilon_{ctmax} d_n^2 - b_{in} \sigma_{ctmax} \varepsilon_{ctmax} d_n^2 - b_{in} E_c \varepsilon_0^2 h^2 + 2b_{in} E_c \varepsilon_0^2 h d_n + 2b_{in} E_c \varepsilon_0^2 t_f h$$
$$-2b_{in} E_c \varepsilon_0^2 t_f d_n - b_{in} E_c \varepsilon_0^2 t_f^2 + 4b_w \sigma_{ctmax} \varepsilon_0 h d_n - 4b_w \sigma_{ctmax} \varepsilon_0 d_n^2 + 2b_{in} \sigma_{ctmax} \varepsilon_0 h d_n$$
$$-2b_{in} \sigma_{ctmax} \varepsilon_0 d_n^2 + 2A_p E_p \varepsilon_0^2 d_p - 2A_p E_p \varepsilon_0^2 d_n \rightarrow$$

$$\left(\left(b - 2b_f\right) E_c \varepsilon_0^2 + \sigma_{ctmax}(2b_w + b_{in})(\varepsilon_{ctmax} + 2\varepsilon_0)\right) d_n^2$$

$$+2\varepsilon_0 \left(E_c \varepsilon_0 \left(\left(2b_f + b_{in}\right) t_d - b_{in}\left(h - t_f\right)\right) - (2b_w + b_{in})\sigma_{ctmax} h + A_p E_p \varepsilon_0\right) d_n$$

$$-\varepsilon_0^2 \left(E_c \left(\left(2b_f + b_{in}\right) t_d^2 - b_{in}\left(h - t_f\right)^2\right) + 2A_p E_p d_p\right) = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = (b - 2b_f)E_c\varepsilon_0^2 \ + \sigma_{ctmax}(2b_w + b_{in})(\varepsilon_{ctmax} + 2\varepsilon_0)$$

$$b = 2\varepsilon_0 \left(E_c\varepsilon_0 \left((2b_f + b_{in})t_d - b_{in}(h - t_f) \right) - (2b_w + b_{in})\sigma_{ctmax}h + A_pE_p\varepsilon_0 \right)$$

$$c = -\varepsilon_0^2 \left(E_c \left((2b_f + b_{in})t_d^2 - b_{in}(h - t_f)^2 \right) + 2A_pE_pd_p \right)$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d'_n + T_1 \cdot \frac{2}{3}(X_1 + X_2) + T_2 \cdot \frac{2}{3}X_1 + T_3 \left(X_1 + X_2 + \frac{1}{2}X_3 \right) + P_{m\infty}(e - d_n)$$
$$+\Delta N_p(d_p - d_n)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

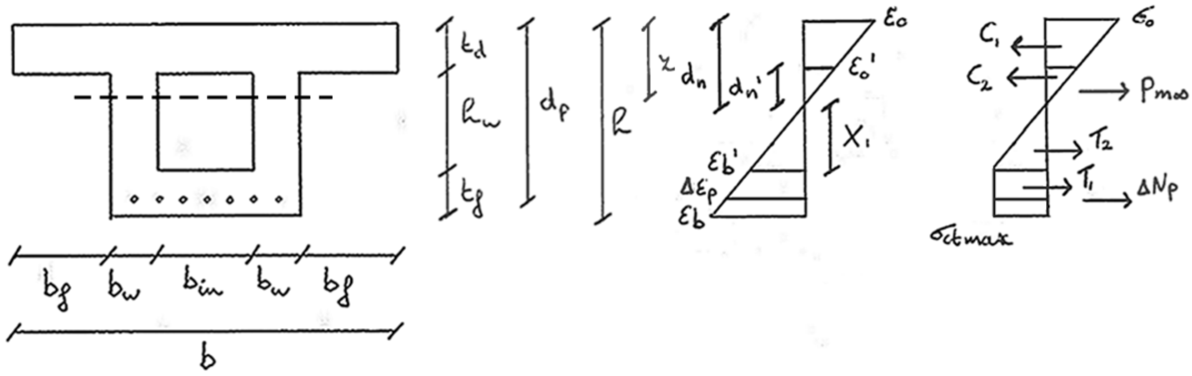


Figure 10.4: deformation and stress diagram when $\varepsilon_b' = \varepsilon_{ctmax}$.

10.4 When $\varepsilon_b' = \varepsilon_{ctmax}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.4] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{{\varepsilon_0}'}{{d_n}'} \rightarrow {\varepsilon_0}' = \frac{\varepsilon_0}{d_n}{d_n}'$$

$$\frac{\varepsilon_0}{d_n} = \frac{{\varepsilon_b}'}{X_1} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ctmax}}{h - d_n - t_f} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}\left(h - d_n - t_f\right) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}h - \frac{\varepsilon_0}{\varepsilon_{ctmax}}d_n - \frac{\varepsilon_0}{\varepsilon_{ctmax}}t_f \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ctmax}}d_n = \frac{\varepsilon_0}{\varepsilon_{ctmax}}h - \frac{\varepsilon_0}{\varepsilon_{ctmax}}t_f \rightarrow$$
$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}\left(h - t_f\right) \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{ctmax}}{\varepsilon_{ctmax}}\right) = \frac{\varepsilon_0}{\varepsilon_{ctmax}}\left(h - t_f\right) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}}\left(h - t_f\right)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}h - \varepsilon_0$$

$${d_n}' = d_n - t_d$$

$$X_1 = h - d_n - t_f$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = (2b_w + b_{in})\sigma_{ctmax} t_f + \frac{1}{2}(2b_w) E_c \varepsilon_b' X_1 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 4b_w \sigma_{ctmax} t_f + 2b_{in}\sigma_{ctmax} t_f + 2b_w E_c \varepsilon_b' X_1 + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n + 2b_f E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d = 4b_w \sigma_{ctmax} t_f + 2b_{in} \sigma_{ctmax} t_f$$
$$+2b_w E_c \varepsilon_b' h - 2b_w E_c \varepsilon_b' d_n - 2b_w E_c \varepsilon_b' t_f + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in} E_c \varepsilon_0 d_n + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$= 2b_w \sigma_{ctmax} t_f + 2b_{in} \sigma_{ctmax} t_f + 2b_w \sigma_{ctmax} h - 2b_w \sigma_{ctmax} d_n + 2A_p E_p \frac{\varepsilon_0}{d_n} d_p - 2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}}\left(h - t_f\right) - 2b_f E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}}\left(h - t_f\right) + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0 + \varepsilon_{ctmax}}{h - t_f} t_d^2$$
$$-b_{in} E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ctmax}}\left(h - t_f\right) + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0 + \varepsilon_{ctmax}}{h - t_f} t_d^2 = 2b_w \sigma_{ctmax} t_f + 2b_{in} \sigma_{ctmax} t_f$$
$$+2b_w \sigma_{ctmax} h - 2b_w \sigma_{ctmax} \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ctmax}}\left(h - t_f\right) + 2A_p E_p \frac{\varepsilon_0 + \varepsilon_{ctmax}}{h - t_f} d_p - 2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0^2 \left(h - t_f\right)^2 - 2b_f E_c \varepsilon_0^2 \left(h - t_f\right)^2 + 4b_f E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{ctmax})\left(h - t_f\right) t_d - 2b_f E_c (\varepsilon_0 + \varepsilon_{ctmax})^2 t_d^2$$
$$-b_{in} E_c \varepsilon_0^2 \left(h - t_f\right)^2 + 2b_{in} E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{ctmax})\left(h - t_f\right) t_d - b_{in} E_c (\varepsilon_0 + \varepsilon_{ctmax})^2 t_d^2$$
$$= 2b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{ctmax})\left(h - t_f\right) t_f + 2b_{in} \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{ctmax})\left(h - t_f\right) t_f$$
$$+2b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{ctmax})\left(h - t_f\right) h - 2b_w \sigma_{ctmax} \varepsilon_0 \left(h - t_f\right)^2 + 2A_p E_p (\varepsilon_0 + \varepsilon_{ctmax})^2 d_p$$
$$-2A_p E_p \varepsilon_0 (\varepsilon_0 + \varepsilon_{ctmax})\left(h - t_f\right) \rightarrow$$

$$bE_c\varepsilon_0^2\left(h-t_f\right)^2-2b_fE_c\varepsilon_0^2\left(h-t_f\right)^2+4b_fE_c\varepsilon_0^2\left(h-t_f\right)t_d+4b_fE_c\varepsilon_0\varepsilon_{ctmax}\left(h-t_f\right)t_d$$
$$-2b_fE_c\varepsilon_0^2t_d^2-4b_fE_c\varepsilon_0\varepsilon_{ctmax}t_d^2-2b_fE_c\varepsilon_{ctmax}^2t_d^2-b_{in}E_c\varepsilon_0^2\left(h-t_f\right)^2+2b_{in}E_c\varepsilon_0^2\left(h-t_f\right)t_d$$
$$+2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}\left(h-t_f\right)t_d-b_{in}E_c\varepsilon_0^2t_d^2-2b_{in}E_c\varepsilon_0\varepsilon_{ctmax}t_d^2-b_{in}E_c\varepsilon_{ctmax}^2t_d^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)t_f-2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(h-t_f\right)t_f-2b_{in}\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)t_f$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_{ctmax}\left(h-t_f\right)t_f-2b_w\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)h-2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(h-t_f\right)h$$
$$+2b_w\sigma_{ctmax}\varepsilon_0\left(h-t_f\right)^2-2A_pE_p\varepsilon_0^2d_p-4A_pE_p\varepsilon_0\varepsilon_{ctmax}d_p-2A_pE_p\varepsilon_{ctmax}^2d_p+2A_pE_p\varepsilon_0^2\left(h-t_f\right)$$
$$+2A_pE_p\varepsilon_0\varepsilon_{ctmax}\left(h-t_f\right)=0\rightarrow$$

$$\left(E_c\left(\left(b-2b_f-b_{in}\right)\left(h-t_f\right)^2+\left(2b_f+b_{in}\right)\left(2\left(h-t_f\right)-t_d\right)t_d\right)+2A_pE_p\left(h-t_f-d_p\right)\right)\varepsilon_0^2$$

$$+2\left(\sigma_{ctmax}\left(\left(\left(2b_f+b_{in}\right)t_d-\left(2b_w+b_{in}\right)t_f\right)\left(h-t_f\right)-\left(2b_f+b_{in}\right)t_d^2\right)\right.$$
$$\left.+A_pE_p\varepsilon_{ctmax}\left(h-t_f-2d_p\right)\right)\varepsilon_0$$

$$-\sigma_{ctmax}\varepsilon_{ctmax}\left(\left(2b_f+b_{in}\right)t_d^2+\left(2b_w+2b_{in}\right)\left(h-t_f\right)t_f+2b_w\left(h-t_f\right)h\right)-2A_pE_p\varepsilon_{ctmax}^2d_p=0$$
$$\rightarrow$$

$$\varepsilon_0=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=E_c\left(\left(b-2b_f-b_{in}\right)\left(h-t_f\right)^2+\left(2b_f+b_{in}\right)\left(2\left(h-t_f\right)-t_d\right)t_d\right)-2A_pE_p\left(d_p-\left(h-t_f\right)\right)$$

$$b=2\left(\sigma_{ctmax}\left(\left(\left(2b_f+b_{in}\right)t_d-\left(2b_w+b_{in}\right)t_f\right)\left(h-t_f\right)-\left(2b_f+b_{in}\right)t_d^2\right)\right.$$
$$\left.+A_pE_p\varepsilon_{ctmax}\left(h-t_f-2d_p\right)\right)$$

$$c=-\sigma_{ctmax}\varepsilon_{ctmax}\left(\left(2b_f+b_{in}\right)t_d^2+\left(2b_w+2b_{in}\right)\left(h-t_f\right)t_f+2b_w\left(h-t_f\right)h\right)-2A_pE_p\varepsilon_{ctmax}^2d_p$$

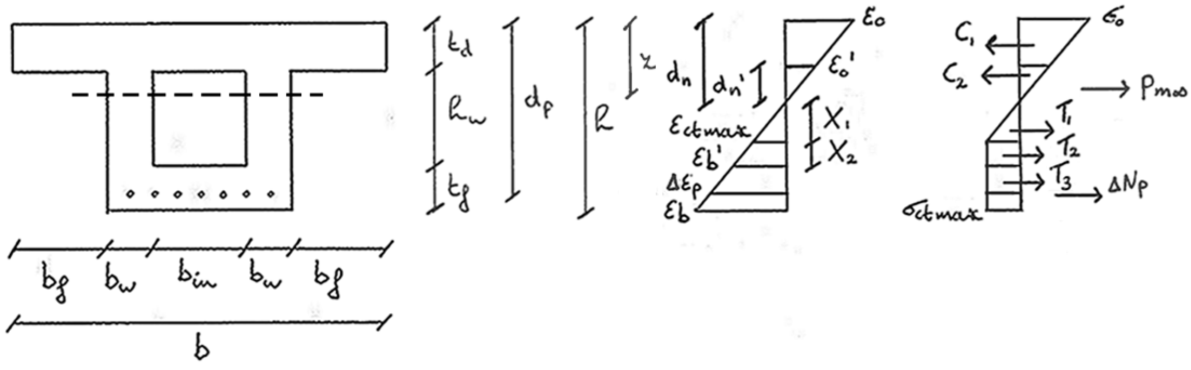


Figure 10.5: deformation and stress diagram when $\varepsilon_b' > \varepsilon_{ctmax}$ & $\varepsilon_b < \varepsilon_{t,p}$.

10.5 When $\boldsymbol{\varepsilon_b' > \varepsilon_{ctmax}}$ & $\boldsymbol{\varepsilon_b < \varepsilon_{t,p}}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.5] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0{}'}{d_n{}'} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b{}' = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow \varepsilon_b{}' = \frac{\varepsilon_0}{d_n}\left(h - d_n - t_f\right)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + (2b_w + b_{in})\sigma_{ctmax} t_f$$
$$A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} t_f$$
$$+2b_{in}\sigma_{ctmax} t_f + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n' + 2b_f E_c \frac{\varepsilon_0}{d_n} d_n' t_d - b_{in} E_c \varepsilon_0 d_n' + b_{in} E_c \frac{\varepsilon_0}{d_n} d_n' t_d = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$+4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n + 2b_{in}\sigma_{ctmax} t_f + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in} E_c \varepsilon_0 d_n + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n + 2b_{in}\sigma_{ctmax} t_f + 2A_p E_p \frac{\varepsilon_0}{d_n} d_p$$
$$-2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0^2 d_n^2 - 2b_f E_c \varepsilon_0^2 d_n^2 + 4b_f E_c \varepsilon_0^2 t_d d_n - 2b_f E_c \varepsilon_0^2 t_d^2 - b_{in} E_c \varepsilon_0^2 d_n^2 + 2b_{in} E_c \varepsilon_0^2 t_d d_n - b_{in} E_c \varepsilon_0^2 t_d^2$$
$$= -2b_w \sigma_{ctmax} \varepsilon_{ctmax} d_n^2 + 4b_w \sigma_{ctmax} \varepsilon_0 h d_n - 4b_w \sigma_{ctmax} \varepsilon_0 d_n^2 + 2b_{in}\sigma_{ctmax} \varepsilon_0 t_f d_n + 2A_p E_p \varepsilon_0^2 d_p$$
$$-2A_p E_p \varepsilon_0^2 d_n \rightarrow$$

$$\left((b - 2b_f - b_{in}) E_c \varepsilon_0^2 + 2b_w \sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)\right) d_n^2$$

$$+2\varepsilon_0 \left((2b_f + b_{in}) E_c \varepsilon_0 t_d - \sigma_{ctmax}\left(2b_w h + b_{in} t_f\right) + A_p E_p \varepsilon_0\right) d_n$$

$$-\left((2b_f + b_{in}) E_c t_d^2 + 2A_p E_p d_p\right) \varepsilon_0^2 = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \left(b - 2b_f - b_{in}\right)E_c\varepsilon_0^2 + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax} + 2\varepsilon_0)$$

$$b = 2\varepsilon_0\left(\left(2b_f + b_{in}\right)E_c\varepsilon_0 t_d - \sigma_{ctmax}\left(2b_w h + b_{in}t_f\right) + A_p E_p \varepsilon_0\right)$$

$$c = -\left(\left(2b_f + b_{in}\right)E_c t_d^2 + 2A_p E_p d_p\right)\varepsilon_0^2$$

The corresponding bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d_n' + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3\left(X_1 + X_2 + \frac{1}{2}t_f\right) + P_{m\infty}(e - d_n)$$
$$+\Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

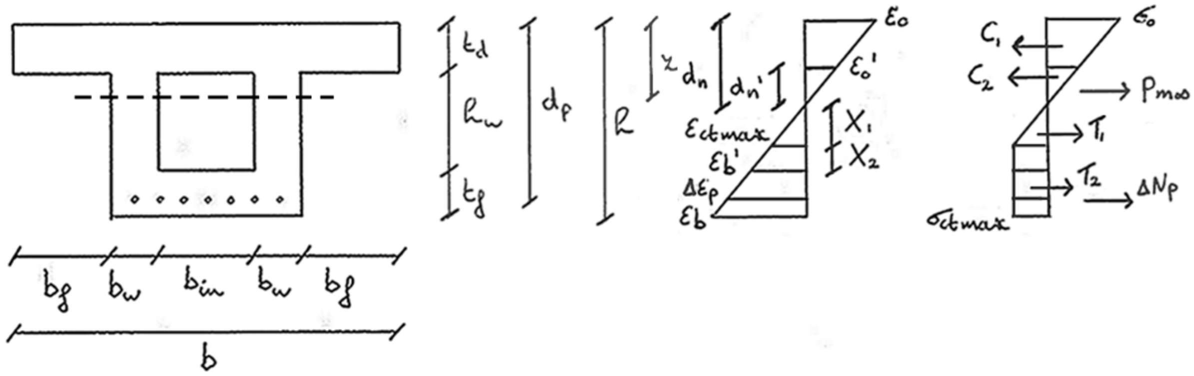


Figure 10.6: deformation and stress diagram when $\varepsilon_b = \varepsilon_{t,p}$.

10.6 When $\varepsilon_b = \varepsilon_{t,p}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.6] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{{\varepsilon_0}'}{{d_n}'} \rightarrow {\varepsilon_0}' = \frac{\varepsilon_0}{d_n}{d_n}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow {\varepsilon_b}' = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow {\varepsilon_b}' = \frac{\varepsilon_0}{d_n}\left(h - d_n - t_f\right)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \frac{\varepsilon_{t,p}}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}(h - d_n) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}h - \frac{\varepsilon_0}{\varepsilon_{t,p}}d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,p}}d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}h$$

$$\rightarrow d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}}h \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{t,p}}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}}h \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,p}}h$$

$${d_n}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + (2b_w + b_{in})\sigma_{ctmax} t_f$$
$$+ A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} t_f$$
$$+ 2b_{in} \sigma_{ctmax} t_f + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n' + 2b_f E_c \frac{\varepsilon_0}{d_n} d_n' t_d - b_{in} E_c \varepsilon_0 d_n' + b_{in} E_c \frac{\varepsilon_0}{d_n} d_n' t_d = 2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$+ 4b_w \sigma_{ctmax}\left(h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n\right) + 4b_w \sigma_{ctmax} t_f + 2b_{in} \sigma_{ctmax} t_f + 2A_p E_p \left(\frac{\varepsilon_0}{d_n} d_p - \varepsilon_0\right) \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in} E_c \varepsilon_0 d_n + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n + 2b_{in} \sigma_{ctmax} t_f + 2A_p E_p \frac{\varepsilon_0}{d_n} d_p$$
$$- 2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{t,p}} h - 2b_f E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{t,p}} h + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0 + \varepsilon_{t,p}}{h} t_d^2 - b_{in} E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{t,p}} h$$
$$+ 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0 + \varepsilon_{t,p}}{h} t_d^2 = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0 + \varepsilon_{t,p}} h + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,p}} h$$
$$+ 2b_{in} \sigma_{ctmax} t_f + 2A_p E_p \frac{\varepsilon_0 + \varepsilon_{t,p}}{h} d_p - 2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0^2 h^2 - 2b_f E_c \varepsilon_0^2 h^2 + 4b_f E_c \varepsilon_0 \left(\varepsilon_0 + \varepsilon_{t,p}\right) t_d h - 2b_f E_c \left(\varepsilon_0 + \varepsilon_{t,p}\right)^2 t_d^2 - b_{in} E_c \varepsilon_0^2 h^2$$
$$+ 2b_{in} E_c \varepsilon_0 \left(\varepsilon_0 + \varepsilon_{t,p}\right) t_d h - b_{in} E_c \left(\varepsilon_0 + \varepsilon_{t,p}\right)^2 t_d^2 = -2b_w \sigma_{ctmax} \varepsilon_{ctmax} h^2$$
$$+ 4b_w \sigma_{ctmax} \left(\varepsilon_0 + \varepsilon_{t,p}\right) h^2 - 4b_w \sigma_{ctmax} \varepsilon_0 h^2 + 2b_{in} \sigma_{ctmax} \left(\varepsilon_0 + \varepsilon_{t,p}\right) t_f h + 2A_p E_p \left(\varepsilon_0 + \varepsilon_{t,p}\right)^2 d_p$$
$$- 2A_p E_p \varepsilon_0 \left(\varepsilon_0 + \varepsilon_{t,p}\right) h \rightarrow$$

$$bE_c\varepsilon_0^2h^2 - 2b_fE_c\varepsilon_0^2h^2 + 4b_fE_c\varepsilon_0^2t_dh + 4b_fE_c\varepsilon_0\varepsilon_{t,p}t_dh - 2b_fE_c\varepsilon_0^2t_d^2 - 4b_fE_c\varepsilon_0\varepsilon_{t,p}t_d^2$$
$$-2b_fE_c\varepsilon_{t,p}^2t_d^2 - b_{in}E_c\varepsilon_0^2h^2 + 2b_{in}E_c\varepsilon_0^2t_dh + 2b_{in}E_c\varepsilon_0\varepsilon_{t,p}t_dh - b_{in}E_c\varepsilon_0^2t_d^2 - 2b_{in}E_c\varepsilon_0\varepsilon_{t,p}t_d^2$$
$$-b_{in}E_c\varepsilon_{t,p}^2t_d^2 + 2b_w\sigma_{ctmax}\varepsilon_{ctmax}h^2 - 4b_w\sigma_{ctmax}\varepsilon_{t,p}h^2 - 2b_{in}\sigma_{ctmax}\varepsilon_0t_fh - 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_fh$$
$$-2A_pE_p\varepsilon_0^2d_p - 4A_pE_p\varepsilon_0\varepsilon_{t,p}d_p - 2A_pE_p\varepsilon_{t,p}^2d_p + 2A_pE_p\varepsilon_0^2h + 2A_pE_p\varepsilon_0\varepsilon_{t,p}h = 0 \rightarrow$$

$$\left(E_c\left(\left(b - 2b_f - b_{in}\right)h^2 + \left(2b_f + b_{in}\right)(2h - t_d)t_d\right) + 2A_pE_p\left(h - d_p\right)\right)\varepsilon_0^2$$

$$+2\left(E_c\varepsilon_{t,p}t_d\left(2b_f + b_{in}\right)(h - t_d) - b_{in}\sigma_{ctmax}t_fh + A_pE_p\varepsilon_{t,p}\left(h - 2d_p\right)\right)\varepsilon_0$$

$$-E_c\varepsilon_{t,p}^2t_d^2\left(2b_f + b_{in}\right) + 2\sigma_{ctmax}h\left(b_wh\left(\varepsilon_{ctmax} - 2\varepsilon_{t,p}\right) - b_{in}\varepsilon_{t,p}t_f\right) - 2A_pE_p\varepsilon_{t,p}^2d_p = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(\left(b - 2b_f - b_{in}\right)h^2 + \left(2b_f + b_{in}\right)(2h - t_d)t_d\right) + 2A_pE_p\left(h - d_p\right)$$

$$b = 2\left(E_c\varepsilon_{t,p}t_d\left(2b_f + b_{in}\right)(h - t_d) - b_{in}\sigma_{ctmax}t_fh + A_pE_p\varepsilon_{t,p}\left(h - 2d_p\right)\right)$$

$$c = -E_c\varepsilon_{t,p}^2t_d^2\left(2b_f + b_{in}\right) + 2\sigma_{ctmax}h\left(b_wh\left(\varepsilon_{ctmax} - 2\varepsilon_{t,p}\right) - b_{in}\varepsilon_{t,p}t_f\right) - 2A_pE_p\varepsilon_{t,p}^2d_p$$

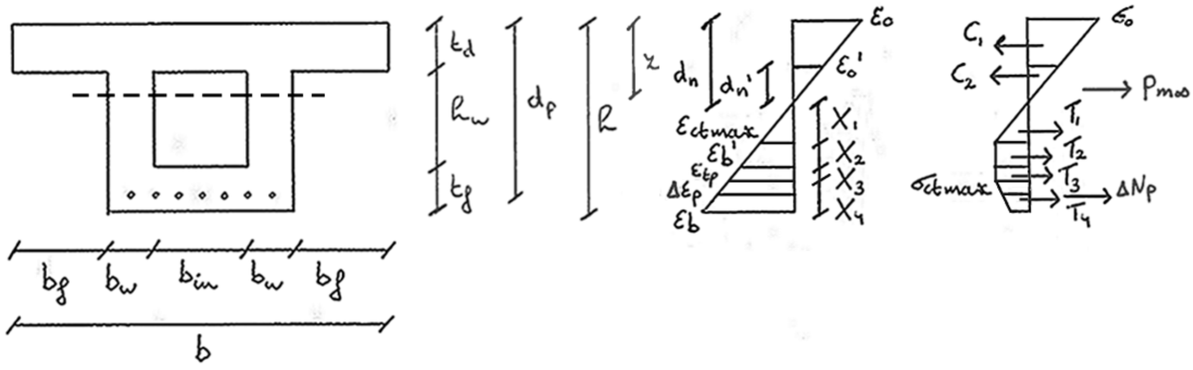


Figure 10.7: deformation and stress diagram when $\varepsilon_b' < \varepsilon_{t,p} < \varepsilon_b$.

10.7 When $\varepsilon_b' < \varepsilon_{t,p} < \varepsilon_b$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.7] the following relations are valid:

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_0{}'}{d_n{}'} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b{}' = \frac{\varepsilon_0}{d_n}(X_1 + X_2) \rightarrow \varepsilon_b{}' = \frac{\varepsilon_0}{d_n}\left(h - d_n - t_f\right)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{t,p}}{X_1 + X_2 + X_3} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - h + d_n + t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_0}{d_n} = \frac{\varepsilon_b}{h - d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$X_4 = h - (X_1 + X_2 + X_3) - d_n = h - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - d_n$$

$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + (2b_w + b_{in})\sigma_{ctmax} X_3 +$$
$$+(2b_w + b_{in})\sigma_{ctmax} X_4 - \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} X_4 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} X_3$$
$$+2b_{in} \sigma_{ctmax} X_3 + 4b_w \sigma_{ctmax} X_4 + 2b_{in} \sigma_{ctmax} X_4 - 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} X_4$$
$$-b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} X_4 + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n' + 2b_f E_c \frac{\varepsilon_0}{d_n} d_n' t_d - b_{in} E_c \varepsilon_0 d_n' + b_{in} E_c \frac{\varepsilon_0}{d_n} d_n' t_d = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$+4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n + 2b_{in} \sigma_{ctmax} t_f - 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} h$$
$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} d_n - b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} h$$
$$+b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} d_n + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in} E_c \varepsilon_0 d_n + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n + 2b_{in} \sigma_{ctmax} t_f - 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} h$$
$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} d_n - b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} h$$
$$+b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}} d_n + 2A_p E_p \frac{\varepsilon_0}{d_n} d_p - 2A_p E_p \varepsilon_0 \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})d_n-2b_fE_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})d_n+4b_fE_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})t_d-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2$$

$$=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{t,u}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})h-4b_w\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})d_n$$

$$+2b_{in}\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})t_f-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2+4b_w\sigma_{ctmax}\varepsilon_0h+4b_w\sigma_{ctmax}\varepsilon_{t,p}h$$

$$-4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n-2b_w\sigma_{ctmax}\varepsilon_0d_n-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2+2b_{in}\sigma_{ctmax}\varepsilon_0h$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}h-2b_{in}\sigma_{ctmax}\varepsilon_{t,p}d_n-b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n-b_{in}\sigma_{ctmax}\varepsilon_0d_n$$

$$+2A_pE_p\frac{\varepsilon_0}{d_n}(\varepsilon_{t,u}-\varepsilon_{t,p})d_p-2A_pE_p\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_n^2+4b_fE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_dd_n$$
$$-2b_fE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2-b_{in}E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_dd_n$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})d_n^2+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})hd_n$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})d_n^2+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})t_fd_n-2b_w\sigma_{ctmax}\varepsilon_0^2h^2$$
$$+4b_w\sigma_{ctmax}\varepsilon_0^2hd_n+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2d_n^2-b_{in}\sigma_{ctmax}\varepsilon_0^2h^2+2b_{in}\sigma_{ctmax}\varepsilon_0^2hd_n+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2-b_{in}\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2-b_{in}\sigma_{ctmax}\varepsilon_0^2d_n^2+2A_pE_p\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_p$$
$$-2A_pE_p\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_n\rightarrow$$

$$\Big((b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})$$
$$+\sigma_{ctmax}\big(2b_w\varepsilon_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})+4b_w\varepsilon_0\varepsilon_{t,u}+(2b_w+b_{in})(\varepsilon_{t,p}^2+\varepsilon_0^2)$$
$$+2b_{in}\varepsilon_0\varepsilon_{t,p}\big)\Big)d_n^2$$

$$+2\varepsilon_0\Big((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})t_d$$
$$-\sigma_{ctmax}\big(b_{in}(\varepsilon_{t,u}-\varepsilon_{t,p})t_f+2b_w(\varepsilon_0+\varepsilon_{t,u})h+b_{in}(\varepsilon_0+\varepsilon_{t,p})h\big)$$
$$+A_pE_p\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})\Big)d_n$$

$$-\Big((2b_f+b_{in})E_c(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2-(2b_w+b_{in})\sigma_{ctmax}h^2+2A_pE_p(\varepsilon_{t,u}-\varepsilon_{t,p})d_p\Big)\varepsilon_0^2=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a = \left(b - 2b_f \; - b_{in}\right)E_c\varepsilon_0^2\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)$$
$$+ \sigma_{ctmax}\left(2b_w\varepsilon_{ctmax}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right) + 4b_w\varepsilon_0\varepsilon_{t,u} + (2b_w + b_{in})\left(\varepsilon_{t,p}^2 + \varepsilon_0^2\right)\right.$$
$$\left. + 2b_{in}\varepsilon_0\varepsilon_{t,p}\right)$$

$$b = 2\varepsilon_0\left(\left(2b_f + b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_d\right.$$
$$- \sigma_{ctmax}\left(b_{in}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_f + 2b_w\left(\varepsilon_0 + \varepsilon_{t,u}\right)h + b_{in}\left(\varepsilon_0 + \varepsilon_{t,p}\right)h\right)$$
$$\left. + A_pE_p\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)\right)$$

$$c = -\left(\left(2b_f + b_{in}\right)E_c\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_d^2 - (2b_w + b_{in})\sigma_{ctmax}h^2 + 2A_pE_p\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_p\right)\varepsilon_0^2$$

In order to determine the bending moment capacity the centre of gravity of part X_4 is required:

$$y = \frac{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_4 \cdot \frac{1}{2}X_4 + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 \cdot \frac{1}{3}X_4}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_4 + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4} \rightarrow$$

$$y = \frac{\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_4^2 + \frac{1}{6}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4^2}{\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}X_4} \rightarrow$$

$$y = \frac{\left(\frac{1}{2}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}X_4^2}{\left(1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}X_4} \rightarrow$$

$$y = \frac{\left(\frac{1}{2} - \frac{1}{3}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_4}{1 - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d'_n + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3\left(X_1 + X_2 + \frac{1}{2}X_3\right)$$
$$+T_4(X_1 + X_2 + X_3 + y) + P_{m\infty}(e - d_n) + \Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

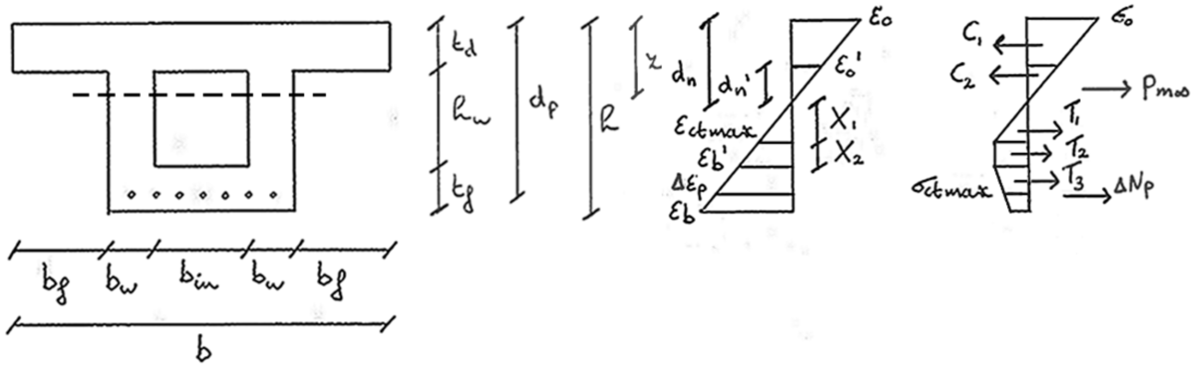


Figure 10.8: deformation and stress diagram when $\varepsilon_b' = \varepsilon_{t,p}$.

10.8 When $\varepsilon_b' = \varepsilon_{t,p}$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.8] the following relations are valid:

$$\frac{{\varepsilon_0}'}{{d_n}'} = \frac{\varepsilon_0}{d_n} \rightarrow {\varepsilon_0}' = \frac{\varepsilon_0}{d_n}{d_n}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_{t,p}}{h - d_n - t_f} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}\left(h - d_n - t_f\right) \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,p}}h - \frac{\varepsilon_0}{\varepsilon_{t,p}}d_n - \frac{\varepsilon_0}{\varepsilon_{t,p}}t_f \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,p}}d_n$$
$$= \frac{\varepsilon_0}{\varepsilon_{t,p}}h - \frac{\varepsilon_0}{\varepsilon_{t,p}}t_f \rightarrow d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}}\left(h - t_f\right) \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{t,p}}{\varepsilon_{t,p}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,p}}\left(h - t_f\right) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,p}}\left(h - t_f\right)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}h - \varepsilon_0$$

$${d_n}' = d_n - t_d$$

$$X_1 + X_2 = h - d_n - t_f \rightarrow X_2 = h - d_n - t_f - X_1 = h - d_n - t_f - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + (2b_w + b_{in})\sigma_{ctmax}t_f$$
$$-\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}t_f + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}t_f$$
$$+2b_{in}\sigma_{ctmax}t_f - 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}t_f + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n' + 2b_f E_c\frac{\varepsilon_0}{d_n}d_n' t_d - b_{in}E_c\varepsilon_0 d_n' + b_{in}E_c\frac{\varepsilon_0}{d_n}d_n' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$+4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n + 2b_{in}\sigma_{ctmax}t_f - 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}t_f$$
$$+2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n + 2b_{in}\sigma_{ctmax}t_f$$
$$-2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{t,u} - \varepsilon_{t,p}}t_f + 2A_pE_p\frac{\varepsilon_0}{d_n}d_p - 2A_pE_p\varepsilon_0 \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_n - 2b_f E_c\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_n + 4b_f E_c\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_d^2$$
$$-b_{in}E_c\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_n + 2b_{in}E_c\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_n + 4b_w\sigma_{ctmax}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)h - 4b_w\sigma_{ctmax}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_n$$
$$+2b_{in}\sigma_{ctmax}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_f - 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f + 2b_w\sigma_{ctmax}\varepsilon_0 t_f + 2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f + b_{in}\sigma_{ctmax}\varepsilon_0 t_f + b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f + 2A_pE_p\frac{\varepsilon_0}{d_n}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_p$$
$$-2A_pE_p\varepsilon_0\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right) \rightarrow$$

$$bE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)-2b_fE_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)+4b_fE_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})t_d$$

$$-2b_fE_c\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2-b_{in}E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)+2b_{in}E_c\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})t_d$$

$$-b_{in}E_c\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)$$

$$+4b_w\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})h-4b_w\sigma_{ctmax}\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,p}}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)+2b_{in}\sigma_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})t_f$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}ht_f+2b_w\sigma_{ctmax}\varepsilon_0t_f+2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f-b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}ht_f$$

$$+b_{in}\sigma_{ctmax}\varepsilon_0t_f+b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f+2A_pE_p\frac{\varepsilon_0+\varepsilon_{t,p}}{h-t_f}(\varepsilon_{t,u}-\varepsilon_{t,p})d_p-2A_pE_p\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2-2b_fE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2$$

$$+4b_fE_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_d-2b_fE_c(\varepsilon_0+\varepsilon_{t,p})^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2$$

$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2+2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_d$$

$$-b_{in}E_c(\varepsilon_0+\varepsilon_{t,p})^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2$$

$$+4b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)h-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2$$

$$+2b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_f-2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})^2ht_f$$

$$+2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(h-t_f)t_f+2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})\varepsilon_{t,p}(h-t_f)t_f$$

$$-b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})^2ht_f+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(h-t_f)t_f$$

$$+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,p})\varepsilon_{t,p}(h-t_f)t_f+2A_pE_p(\varepsilon_0+\varepsilon_{t,p})^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_p$$

$$-2A_pE_p\varepsilon_0(\varepsilon_0+\varepsilon_{t,p})(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2-2b_fE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2$$
$$+4b_fE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_d+4b_fE_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_d$$
$$-2b_fE_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2-2b_fE_c\varepsilon_{t,p}^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_d$$
$$+2b_{in}E_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_d-b_{in}E_c\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2$$
$$-b_{in}E_c\varepsilon_{t,p}^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2$$
$$+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)h+4b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)h$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)^2+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)t_f-2b_w\sigma_{ctmax}\varepsilon_0^2ht_f-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_f$$
$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2ht_f+2b_w\sigma_{ctmax}\varepsilon_0^2(h-t_f)t_f+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(h-t_f)t_f$$
$$+2b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(h-t_f)t_f+2b_w\sigma_{ctmax}\varepsilon_{t,p}^2(h-t_f)t_f-b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f-2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_f$$
$$-b_{in}\sigma_{ctmax}\varepsilon_{t,p}^2ht_f+b_{in}\sigma_{ctmax}\varepsilon_0^2(h-t_f)t_f+b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(h-t_f)t_f$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(h-t_f)t_f+b_{in}\sigma_{ctmax}\varepsilon_{t,p}^2(h-t_f)t_f+2A_pE_p\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_p$$
$$+4A_pE_p\varepsilon_0\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})d_p+2A_pE_p\varepsilon_{t,p}^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_p-2A_pE_p\varepsilon_0^2(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)$$
$$-2A_pE_p\varepsilon_0\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f)\rightarrow$$

$$\left(E_c(\varepsilon_{t,u}-\varepsilon_{t,p})\left((b-2b_f-b_{in})(h-t_f)^2+(2b_f+b_{in})(2(h-t_f)-t_d)t_d\right)\right.$$
$$\left.+\sigma_{ctmax}(2b_w+b_{in})t_f^2+2A_pE_p(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f-d_p)\right)\varepsilon_0^2$$

$$+\left(E_c\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})(2b_f+b_{in})\left((h-t_f)-t_d\right)2t_d\right.$$
$$-\sigma_{ctmax}\left((2(2b_w+b_{in})\varepsilon_{t,u}t_f)(h-t_f)-2(2b_w+b_{in})\varepsilon_{t,p}ht_f\right)$$
$$\left.+2A_pE_p\varepsilon_{t,p}(\varepsilon_{t,u}-\varepsilon_{t,p})(h-t_f-2d_p)\right)\varepsilon_0$$

$$-(2b_f+b_{in})E_c\varepsilon_{t,p}^2(\varepsilon_{t,u}-\varepsilon_{t,p})t_d^2$$
$$+\sigma_{ctmax}\left((2(\varepsilon_{t,u}-\varepsilon_{t,p})(b_w\varepsilon_{ctmax}(h-t_f)-2b_w\varepsilon_{t,p}h-b_{in}\varepsilon_{t,p}t_f)\right.$$
$$\left.-(2b_w+b_{in})\varepsilon_{t,p}^2t_f)(h-t_f)+(2b_w+b_{in})\varepsilon_{t,p}^2ht_f\right)-2A_pE_p\varepsilon_{t,p}^2(\varepsilon_{t,u}-\varepsilon_{t,p})d_p$$
$$=0\rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)\left(\left(b - 2b_f - b_{in}\right)\left(h - t_f\right)^2 + \left(2b_f + b_{in}\right)\left(2\left(h - t_f\right) - t_d\right)t_d\right) + \sigma_{ctmax}(2b_w + b_{in})t_f^2 + 2A_pE_p\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)\left(h - t_f - d_p\right)$$

$$b = E_c\varepsilon_{t,p}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)\left(2b_f + b_{in}\right)\left(\left(h - t_f\right) - t_d\right)2t_d - \sigma_{ctmax}\left(\left(2(2b_w + b_{in})\varepsilon_{t,u}t_f\right)\left(h - t_f\right) - 2(2b_w + b_{in})\varepsilon_{t,p}ht_f\right) + 2A_pE_p\varepsilon_{t,p}\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)\left(h - t_f - 2d_p\right)$$

$$c = -\left(2b_f + b_{in}\right)E_c\varepsilon_{t,p}^2\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)t_d^2 + \sigma_{ctmax}\left(\left(2\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)\left(b_w\varepsilon_{ctmax}\left(h - t_f\right) - 2b_w\varepsilon_{t,p}h - b_{in}\varepsilon_{t,p}t_f\right) - (2b_w + b_{in})\varepsilon_{t,p}^2t_f\right)\left(h - t_f\right) + (2b_w + b_{in})\varepsilon_{t,p}^2ht_f\right) - 2A_pE_p\varepsilon_{t,p}^2\left(\varepsilon_{t,u} - \varepsilon_{t,p}\right)d_p$$

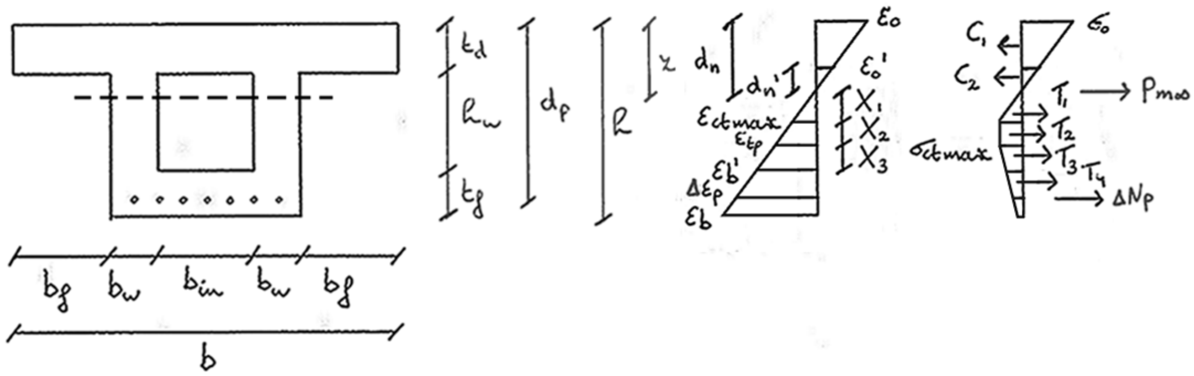


Figure 10.9: deformation and stress diagram when $\varepsilon_{t,p} < \varepsilon_b'$ & $\varepsilon_{t,u} > \varepsilon_b$.

10.9 When $\varepsilon_{t,p} < \varepsilon_b'$ & $\varepsilon_{t,u} > \varepsilon_b$

$d_n > t_d$.

With respect to the deformation and stress diagram of [Figure 10.9] the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3$$
$$-\frac{1}{2}(2b_w)\sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3 + (2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f$$
$$+\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} X_3$$
$$-2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3 + 4b_w \sigma_{ctmax} t_f - 4b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in} \sigma_{ctmax} t_f$$
$$-2b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n' + 2b_f E_c \frac{\varepsilon_0}{d_n} d_n' t_d - b_{in} E_c \varepsilon_0 d_n' + b_{in} E_c \frac{\varepsilon_0}{d_n} d_n' t_d = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$+4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n - 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n$$
$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - 4b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in} \sigma_{ctmax} t_f$$
$$-2b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_p E_p \frac{\varepsilon_0}{d_n} d_p$$
$$-2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n + 4b_f E_c \varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in} E_c \varepsilon_0 d_n + 2b_{in} E_c \varepsilon_0 t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n - 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$
$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$
$$-4b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in} \sigma_{ctmax} t_f - 2b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$
$$+b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_p E_p \frac{\varepsilon_0}{d_n} d_p - 2A_p E_p \varepsilon_0 \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n-2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$

$$=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_b'h+2b_w\sigma_{ctmax}\varepsilon_{t,p}h+2b_w\sigma_{ctmax}\varepsilon_b'd_n-2b_w\sigma_{ctmax}\varepsilon_{t,p}d_n+2b_w\sigma_{ctmax}\varepsilon_b't_f$$

$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_b'd_n-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n-4b_w\sigma_{ctmax}\varepsilon_bt_f$$

$$+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f-2b_{in}\sigma_{ctmax}\varepsilon_bt_f+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f$$

$$+2b_w\sigma_{ctmax}\varepsilon_bt_f-2b_w\sigma_{ctmax}\varepsilon_b't_f+b_{in}\sigma_{ctmax}\varepsilon_bt_f-b_{in}\sigma_{ctmax}\varepsilon_b't_f$$

$$+2A_pE_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})d_p-2A_pE_p(\varepsilon_{tu}-\varepsilon_{t,p})\varepsilon_0\rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n-2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$

$$=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2+4b_w\sigma_{ctmax}\varepsilon_0h+4b_w\sigma_{ctmax}\varepsilon_{t,p}h-2b_w\sigma_{ctmax}\varepsilon_0d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n$$

$$-4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f$$

$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0t_f+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}t_f^2$$

$$+2A_pE_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})d_p-2A_pE_p(\varepsilon_{tu}-\varepsilon_{t,p})\varepsilon_0\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$

$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$

$$=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_n-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2$$

$$-2b_w\sigma_{ctmax}\varepsilon_0^2h^2+4b_w\sigma_{ctmax}\varepsilon_0^2hd_n+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n-2b_w\sigma_{ctmax}\varepsilon_0^2d_n^2$$

$$-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_fd_n-2b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0^2t_fd_n+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_n$$

$$+b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2+2A_pE_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p-2A_pE_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n\rightarrow$$

$$\left((b - 2b_f - b_{in})E_c\varepsilon_0^2(\varepsilon_{tu} - \varepsilon_{t,p}) + 2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p}) + 2\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p}) + (\varepsilon_0 + \varepsilon_{t,p})^2\right)\right)d_n^2$$

$$+2\left((2b_f + b_{in})E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})t_d - \sigma_{ctmax}(2b_wh + b_{in}t_f)(\varepsilon_0 + \varepsilon_{tu}) + A_pE_p\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})\right)d_n$$

$$-\left((2b_f + b_{in})E_c(\varepsilon_{tu} - \varepsilon_{t,p})t_d^2 - \sigma_{ctmax}\left(2b_wh^2 + b_{in}t_f(2h - t_f)\right) + 2A_pE_p(\varepsilon_{tu} - \varepsilon_{t,p})d_p\right)\varepsilon_0^2 = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = (b - 2b_f - b_{in})E_c\varepsilon_0^2(\varepsilon_{tu} - \varepsilon_{t,p}) + 2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p}) + 2\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p}) + (\varepsilon_0 + \varepsilon_{t,p})^2\right)$$

$$b = 2\left((2b_f + b_{in})E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})t_d - \sigma_{ctmax}(2b_wh + b_{in}t_f)(\varepsilon_0 + \varepsilon_{tu}) + A_pE_p\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})\right)$$

$$c = -\left((2b_f + b_{in})E_c(\varepsilon_{tu} - \varepsilon_{t,p})t_d^2 - \sigma_{ctmax}\left(2b_wh^2 + b_{in}t_f(2h - t_f)\right) + 2A_pE_p(\varepsilon_{tu} - \varepsilon_{t,p})d_p\right)\varepsilon_0^2$$

In order to determine the bending moment capacity the centre of gravity of part X_3 is required:

$$y = \frac{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 \cdot \frac{1}{2}X_3 + \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3 \cdot \frac{1}{3}X_3}{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 + \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3} \rightarrow$$

$$y = \frac{b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3^2 + \frac{1}{3}b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3^2}{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 + b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3} \rightarrow$$

$$y = \frac{\left(\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{3}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3^2}{\left(2\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3} \rightarrow$$

$$y = \frac{\left(1 - \frac{2}{3}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_3}{2 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

In order to determine the bending moment capacity the centre of gravity of part X_4 is required:

$$z = \frac{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f \cdot \frac{1}{2} t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f \cdot \frac{1}{3} t_f}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f} \rightarrow$$

$$z = \frac{\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f^2 + \frac{1}{6}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f^2}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax} t_f^2}{\left(\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax} t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2} - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f}{1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3} d_n + C_2 \cdot \frac{2}{3} d_n' + T_1 \cdot \frac{2}{3} X_1 + T_2\left(X_1 + \frac{1}{2} X_2\right) + T_3(X_1 + X_2 + y) + T_4(X_1 + X_2 + X_3 + z)$$
$$+ P_{m\infty}(e - d_n) + \Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

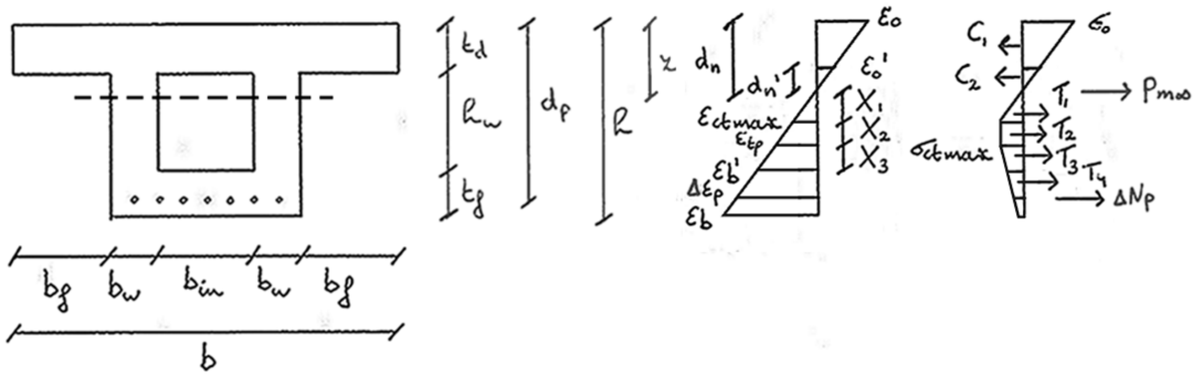


Figure 10.10: deformation and stress diagram when $\varepsilon_p = \varepsilon_{py}$.

10.10 When $\varepsilon_p = \varepsilon_{py}$ $(d_n > t_d)$

With respect to the deformation and stress diagram of [Figure 10.10] the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}(d_p - d_n) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p - \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_n \rightarrow$$

$$d_n + \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_n = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p \rightarrow d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p \rightarrow$$

$$d_n\left(\frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{\varepsilon_{py} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} d_p$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \varepsilon_{py} \rightarrow \Delta\varepsilon_p + \varepsilon_{p\infty} = \varepsilon_{py} \rightarrow \Delta\varepsilon_p = \varepsilon_{py} - \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2} bE_c\varepsilon_0 d_n - \frac{1}{2}(2b_f + b_{in})E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w\sigma_{ctmax} X_2 + 2b_w\sigma_{ctmax} X_3$$
$$-\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3 + (2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f$$
$$+\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax} X_1 + 4b_w\sigma_{ctmax} X_2 + 4b_w\sigma_{ctmax} X_3$$
$$-2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3 + 4b_w\sigma_{ctmax} t_f - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in}\sigma_{ctmax} t_f$$
$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n' + 2b_f E_c\frac{\varepsilon_0}{d_n} d_n' t_d - b_{in}E_c\varepsilon_0 d_n' + b_{in}E_c\frac{\varepsilon_0}{d_n} d_n' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$+4b_w\sigma_{ctmax} h - 4b_w\sigma_{ctmax} d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in}\sigma_{ctmax} t_f$$
$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_pE_p(\varepsilon_{py} - \varepsilon_{p\infty})$$
$$\rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c \frac{\varepsilon_0}{d_n} t_d^2$$

$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_pE_p(\varepsilon_{py} - \varepsilon_{p\infty}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n - 2b_fE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 4b_fE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})t_d - 2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 2b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})t_d^2$$

$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})h - 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_b' h + 2b_w\sigma_{ctmax}\varepsilon_{t,p}h + 2b_w\sigma_{ctmax}\varepsilon_b' d_n - 2b_w\sigma_{ctmax}\varepsilon_{t,p}d_n + 2b_w\sigma_{ctmax}\varepsilon_b' t_f$$

$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_b' d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n - 4b_w\sigma_{ctmax}\varepsilon_b t_f$$

$$+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})t_f - 2b_{in}\sigma_{ctmax}\varepsilon_b t_f + 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f$$

$$+2b_w\sigma_{ctmax}\varepsilon_b t_f - 2b_w\sigma_{ctmax}\varepsilon_b' t_f + b_{in}\sigma_{ctmax}\varepsilon_b t_f - b_{in}\sigma_{ctmax}\varepsilon_b' t_f$$

$$+2A_pE_p(\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n - 2b_fE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 4b_fE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})t_d - 2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 2b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})t_d^2$$

$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})h - 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})d_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2 + 4b_w\sigma_{ctmax}\varepsilon_0 h + 4b_w\sigma_{ctmax}\varepsilon_{t,p}h - 2b_w\sigma_{ctmax}\varepsilon_0 d_n - 4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n$$

$$-4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n + 4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})t_f$$

$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f + 2b_{in}\sigma_{ctmax}\varepsilon_0 t_f + 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}t_f^2$$

$$+2A_pE_p(\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) \rightarrow$$

$$bE_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} (\varepsilon_{tu} - \varepsilon_{t,p}) d_p - 2b_f E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} (\varepsilon_{tu} - \varepsilon_{t,p}) d_p + 4b_f E_c \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d$$

$$-2b_f E_c \frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{d_p} (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2 - b_{in} E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} (\varepsilon_{tu} - \varepsilon_{t,p}) d_p$$

$$+2b_{in} E_c \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d - b_{in} E_c \frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{d_p} (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2$$

$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} (\varepsilon_{tu} - \varepsilon_{t,p}) d_p + 4b_w \sigma_{ctmax} (\varepsilon_{tu} - \varepsilon_{t,p}) h$$

$$-4b_w \sigma_{ctmax} \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} (\varepsilon_{tu} - \varepsilon_{t,p}) d_p - 2b_w \sigma_{ctmax} \frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{d_p} h^2 + 4b_w \sigma_{ctmax} \varepsilon_0 h$$

$$+4b_w \sigma_{ctmax} \varepsilon_{t,p} h - 2b_w \sigma_{ctmax} \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} d_p - 4b_w \sigma_{ctmax} \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} \varepsilon_{t,p} d_p$$

$$-4b_w \sigma_{ctmax} \varepsilon_{t,p} t_f - 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}^2}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} d_p + 4b_w \sigma_{ctmax} \varepsilon_{t,p} t_f + 2b_{in} \sigma_{ctmax} (\varepsilon_{tu} - \varepsilon_{t,p}) t_f$$

$$-2b_{in} \sigma_{ctmax} \frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{d_p} h t_f + 2b_{in} \sigma_{ctmax} \varepsilon_0 t_f + 2b_{in} \sigma_{ctmax} \varepsilon_{t,p} t_f$$

$$+b_{in} \sigma_{ctmax} \frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{d_p} t_f^2 + 2A_p E_p (\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) \rightarrow$$

$$bE_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) d_p^2 - 2b_f E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) d_p^2 + 4b_f E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) t_d d_p$$

$$-2b_f E_c (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})^2 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2 - b_{in} E_c \varepsilon_0^2 (\varepsilon_{tu} - \varepsilon_{t,p}) d_p^2$$

$$+2b_{in} E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) t_d d_p - b_{in} E_c (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})^2 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2$$

$$= -2b_w \sigma_{ctmax} \varepsilon_{ctmax} (\varepsilon_{tu} - \varepsilon_{t,p}) d_p^2 + 4b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) h d_p$$

$$-4b_w \sigma_{ctmax} \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) d_p^2 - 2b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})^2 h^2$$

$$+4b_w \sigma_{ctmax} \varepsilon_0 (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}) h d_p + 4b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}) \varepsilon_{t,p} h d_p - 2b_w \sigma_{ctmax} \varepsilon_0^2 d_p^2$$

$$-4b_w \sigma_{ctmax} \varepsilon_0 \varepsilon_{t,p} d_p^2 - 4b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}) \varepsilon_{t,p} t_f d_p - 2b_w \sigma_{ctmax} \varepsilon_{t,p}^2 d_p^2$$

$$+4b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}) \varepsilon_{t,p} t_f d_p + 2b_{in} \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) t_f d_p$$

$$-2b_{in} \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})^2 h t_f + 2b_{in} \sigma_{ctmax} \varepsilon_0 (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}) t_f d_p$$

$$+2b_{in} \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}) \varepsilon_{t,p} t_f d_p + b_{in} \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})^2 t_f^2$$

$$+2A_p E_p (\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) d_p \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2 - 2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2 + 4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d d_p$$
$$+4b_fE_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_d d_p - 2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-4b_fE_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_fE_c(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2$$
$$+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d d_p + 2b_{in}E_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_d d_p - b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-2b_{in}E_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - b_{in}E_c(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$= -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2 + 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_p$$
$$+4b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})hd_p - 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2 - 2b_w\sigma_{ctmax}\varepsilon_0^2h^2$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})h^2 - 2b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})^2h^2 + 4b_w\sigma_{ctmax}\varepsilon_0^2hd_p$$
$$+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})hd_p + 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_p + 4b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})\varepsilon_{t,p}hd_p$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2d_p^2 - 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_p^2 - 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_p - 4b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})\varepsilon_{t,p}t_fd_p$$
$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_p^2 + 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_p + 4b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})\varepsilon_{t,p}t_fd_p$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_fd_p + 2b_{in}\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_fd_p$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f - 4b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})ht_f - 2b_{in}\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})^2ht_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0^2t_fd_p + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})t_fd_p + 2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_fd_p$$
$$+2b_{in}\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})\varepsilon_{t,p}t_fd_p + b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})t_f^2$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})^2t_f^2 + 2A_pE_p\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})d_p$$
$$+2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p \rightarrow$$

$$\Big(\Big(\big((b-2b_f-b_{in})d_p^2 + (2b_f+b_{in})(2d_p - t_d)t_d\big)E_c(\varepsilon_{tu}-\varepsilon_{t,p}) + \big(2b_w(h-d_p)^2 + b_{in}(2(h-d_p)-t_f)t_f\big)\sigma_{ctmax}\Big)\varepsilon_0^2$$

$$+2\Big(\Big(\big((2b_f+b_{in})(d_p-t_d)t_d\big)E_c(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p}) - \big(2b_w\varepsilon_{tu}d_p(h-d_p) + b_{in}\varepsilon_{tu}t_fd_p - (2b_wh(h-d_p) + b_{in}(2h-d_p-t_f)t_f)(\varepsilon_{py}-\varepsilon_{p\infty})\big)\sigma_{ctmax} - A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})d_p\Big)\varepsilon_0$$

$$
\begin{aligned}
-&\left(2b_f + b_{in}\right)E_c\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2 \\
&+ \left(2b_w\left(\varepsilon_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) + \varepsilon_{t,p}^2\right)d_p^2\right. \\
&- \left(2\left(2b_w h + b_{in}t_f\right)\varepsilon_{tu}d_p - \left(2b_w h^2 + 2b_{in}ht_f - b_{in}t_f^2\right)\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)\right)\left(\varepsilon_{py}\right. \\
&\left.\left. - \varepsilon_{p\infty}\right)\right)\sigma_{ctmax} - 2A_pE_p\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_p = 0 \rightarrow
\end{aligned}
$$

$$
\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

$$
\begin{aligned}
a = &\left(\left(\left(b - 2b_f - b_{in}\right)d_p^2 + \left(2b_f + b_{in}\right)\left(2d_p - t_d\right)t_d\right)E_c\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\right. \\
&\left. + \left(2b_w\left(h - d_p\right)^2 + b_{in}\left(2\left(h - d_p\right) - t_f\right)t_f\right)\sigma_{ctmax}\right)
\end{aligned}
$$

$$
\begin{aligned}
b = 2&\left(\left(\left(2b_f + b_{in}\right)\left(d_p - t_d\right)t_d\right)E_c\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\right. \\
&- \left(2b_w\varepsilon_{tu}d_p\left(h - d_p\right) + b_{in}\varepsilon_{tu}t_f d_p\right. \\
&\left. - \left(2b_w h\left(h - d_p\right) + b_{in}\left(2h - d_p - t_f\right)t_f\right)\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)\right)\sigma_{ctmax} \\
&\left. - A_pE_p\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_p\right)
\end{aligned}
$$

$$
\begin{aligned}
c = -&\left(2b_f + b_{in}\right)E_c\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)t_d^2 \\
&+ \left(2b_w\left(\varepsilon_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) + \varepsilon_{t,p}^2\right)d_p^2\right. \\
&- \left(2\left(2b_w h + b_{in}t_f\right)\varepsilon_{tu}d_p - \left(2b_w h^2 + 2b_{in}ht_f - b_{in}t_f^2\right)\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)\right)\left(\varepsilon_{py}\right. \\
&\left.\left. - \varepsilon_{p\infty}\right)\right)\sigma_{ctmax} - 2A_pE_p\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)d_p
\end{aligned}
$$

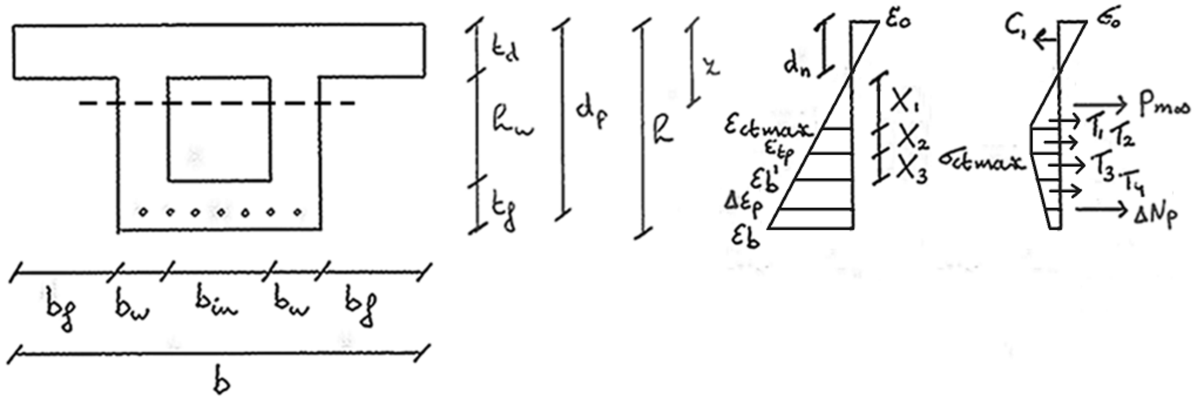


Figure 10.11: deformation and stress diagram when $d_n = t_d$.

10.11 When $d_n = t_d$ $(\varepsilon_p < \varepsilon_{py})$

With respect to the deformation and stress diagram of [Figure 10.11] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 - \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$

$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}X_3 - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$

$$+4b_w\sigma_{ctmax}t_f - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$

$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_pE_p\frac{\varepsilon_0}{d_n}d_p - 2A_pE_p\varepsilon_0 \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}\varepsilon_b'h + 2b_w\sigma_{ctmax}\varepsilon_{t,p}h + 2b_w\sigma_{ctmax}\varepsilon_b'd_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}d_n + 2b_w\sigma_{ctmax}\varepsilon_b't_f - 2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_b'd_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n - 4b_w\sigma_{ctmax}\varepsilon_bt_f + 4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})t_f$$

$$-2b_{in}\sigma_{ctmax}\varepsilon_bt_f + 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\varepsilon_bt_f - 2b_w\sigma_{ctmax}\varepsilon_b't_f + b_{in}\sigma_{ctmax}\varepsilon_bt_f$$

$$-b_{in}\sigma_{ctmax}\varepsilon_b't_f + 2A_pE_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})d_p - 2A_pE_p\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p}) \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n+4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)h$$
$$-4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2+4b_w\sigma_{ctmax}\varepsilon_0 h+4b_w\sigma_{ctmax}\varepsilon_{t,p}h$$
$$-2b_w\sigma_{ctmax}\varepsilon_0 d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n$$
$$+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_{in}\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_f-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0 t_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}t_f^2+2A_pE_p\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_p-2A_pE_p\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\rightarrow$$

$$bE_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d^2+4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)ht_d$$
$$-4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d^2-2b_w\sigma_{ctmax}\varepsilon_0^2h^2+4b_w\sigma_{ctmax}\varepsilon_0^2ht_d+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}ht_d$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2t_d^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_d^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_ft_d-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2t_d^2$$
$$+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_ft_d+2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_ft_d-2b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0^2t_ft_d$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}t_ft_d+b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2+2A_pE_p\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_p-2A_pE_p\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d\rightarrow$$

$$-\Big(bE_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d^2+\left(2b_w(h-t_d)^2+b_{in}\left(2(h-t_d)-t_f\right)t_f\right)\sigma_{ctmax}$$
$$-2A_pE_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(d_p-t_d\right)\Big)\varepsilon_0^2$$

$$+2\sigma_{ctmax}\varepsilon_{tu}\left(2b_w(h-t_d)+b_{in}t_f\right)t_d\varepsilon_0$$

$$-2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+\varepsilon_{t,p}^2\right)t_d^2=0\rightarrow$$

$$\varepsilon_0=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=-\Big(bE_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d^2+\left(2b_w(h-t_d)^2+b_{in}\left(2(h-t_d)-t_f\right)t_f\right)\sigma_{ctmax}$$
$$-2A_pE_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(d_p-t_d\right)\Big)$$

$$b=2\sigma_{ctmax}\varepsilon_{tu}\left(2b_w(h-t_d)+b_{in}t_f\right)t_d$$

$$c=-2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+\varepsilon_{t,p}^2\right)t_d^2$$

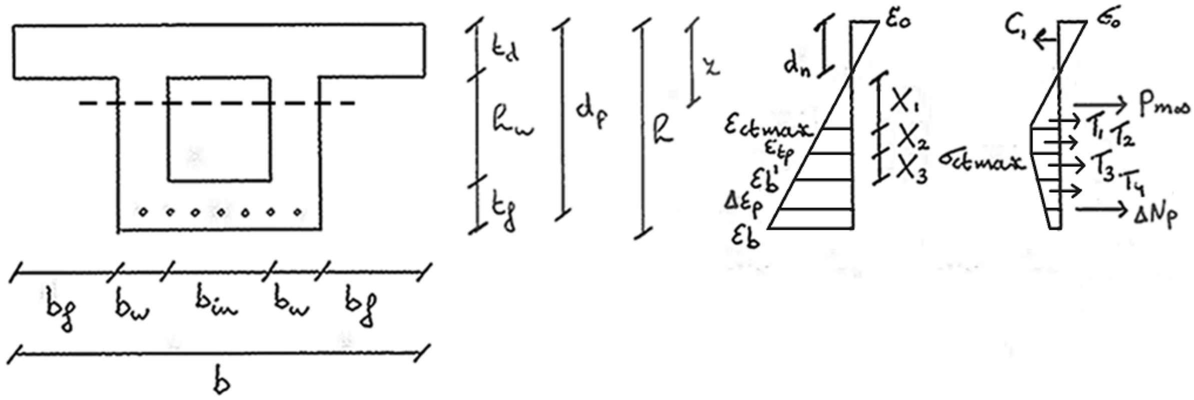


Figure 10.12: deformation and stress diagram when $d_n = t_d$ & $\varepsilon_p = \varepsilon_{py}$.

10.12 When $d_n = t_d$ & $\varepsilon_p = \varepsilon_{py}$

With respect to the deformation and stress diagram of [Figure 10.12] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \varepsilon_0 = \frac{\Delta\varepsilon_p}{d_p - d_n} d_n \rightarrow \varepsilon_0 = \frac{\varepsilon_{py} - \varepsilon_{p\infty}}{d_p - d_n} d_n$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \varepsilon_{py} \rightarrow \Delta\varepsilon_p + \varepsilon_{p\infty} = \varepsilon_{py} \rightarrow \Delta\varepsilon_p = \varepsilon_{py} - \varepsilon_{p\infty}$$

The amount of prestressing steel A_p can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 - \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$

$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}X_3 - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$

$$+4b_w\sigma_{ctmax}t_f - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_pE_p\Delta\varepsilon_p \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$

$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_pE_p(\varepsilon_{py} - \varepsilon_{p\infty}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}\varepsilon_b' h + 2b_w\sigma_{ctmax}\varepsilon_{t,p}h + 2b_w\sigma_{ctmax}\varepsilon_b' d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}d_n + 2b_w\sigma_{ctmax}\varepsilon_b' t_f - 2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_b' d_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n - 4b_w\sigma_{ctmax}\varepsilon_b t_f + 4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})t_f$$

$$-2b_{in}\sigma_{ctmax}\varepsilon_b t_f + 2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_w\sigma_{ctmax}\varepsilon_b t_f - 2b_w\sigma_{ctmax}\varepsilon_b' t_f + b_{in}\sigma_{ctmax}\varepsilon_b t_f$$

$$-b_{in}\sigma_{ctmax}\varepsilon_b' t_f + 2A_pE_p(\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})h$$

$$-4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2 + 4b_w\sigma_{ctmax}\varepsilon_0 h + 4b_w\sigma_{ctmax}\varepsilon_{t,p}h$$

$$-2b_w\sigma_{ctmax}\varepsilon_0 d_n - 4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n - 4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}d_n$$

$$+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f + 2b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f + 2b_{in}\sigma_{ctmax}\varepsilon_0 t_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}t_f^2 + 2A_pE_p(\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p}) \rightarrow$$

$$bE_c(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-d_n}d_n^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})\frac{d_p-d_n}{\varepsilon_{py}-\varepsilon_{p\infty}}$$
$$+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-d_n}h^2$$
$$+4b_w\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-d_n}d_nh+4b_w\sigma_{ctmax}\varepsilon_{t,p}h-2b_w\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-d_n}d_n^2-4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n$$
$$-4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2\frac{d_p-d_n}{\varepsilon_{py}-\varepsilon_{p\infty}}+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f$$
$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-d_n}ht_f+2b_{in}\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-d_n}d_nt_f+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-d_n}t_f^2+2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})\rightarrow$$

$$bE_c(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}t_d^2+2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})\frac{d_p-t_d}{\varepsilon_{py}-\varepsilon_{p\infty}}$$
$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_d+2b_w\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}h^2$$
$$-4b_w\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}t_dh-4b_w\sigma_{ctmax}\varepsilon_{t,p}h+2b_w\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}t_d^2+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_d$$
$$+4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f+2b_w\sigma_{ctmax}\varepsilon_{t,p}^2\frac{d_p-t_d}{\varepsilon_{py}-\varepsilon_{p\infty}}-4b_w\sigma_{ctmax}\varepsilon_{t,p}t_f-2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f$$
$$+2b_{in}\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}ht_f-2b_{in}\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}t_dt_f-2b_{in}\sigma_{ctmax}\varepsilon_{t,p}t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}t_f^2=2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})\rightarrow$$

$$\begin{aligned}2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})&\\ &=bE_c(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}t_d^2\\ &+\sigma_{ctmax}\Bigg(2b_w\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+\varepsilon_{t,p}^2\right)\frac{d_p-t_d}{\varepsilon_{py}-\varepsilon_{p\infty}}+4b_w\varepsilon_{tu}(t_d-h)-2b_{in}\varepsilon_{tu}t_f\\ &+\left(2b_w(h-t_d)^2+b_{in}(2(h-t_d)-t_f)t_f\right)\frac{\varepsilon_{py}-\varepsilon_{p\infty}}{d_p-t_d}\Bigg)\rightarrow\end{aligned}$$

$$A_p = \frac{\alpha}{\gamma}$$

$$\alpha = bE_c\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\frac{\varepsilon_{py} - \varepsilon_{p\infty}}{d_p - t_d}t_d^2$$
$$+ \sigma_{ctmax}\left(2b_w\left(\varepsilon_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) + \varepsilon_{t,p}^2\right)\frac{d_p - t_d}{\varepsilon_{py} - \varepsilon_{p\infty}} + 4b_w\varepsilon_{tu}(t_d - h) - 2b_{in}\varepsilon_{tu}t_f\right.$$
$$\left. + \left(2b_w(h - t_d)^2 + b_{in}\left(2(h - t_d) - t_f\right)t_f\right)\frac{\varepsilon_{py} - \varepsilon_{p\infty}}{d_p - t_d}\right)$$

$$\gamma = 2E_p\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)$$

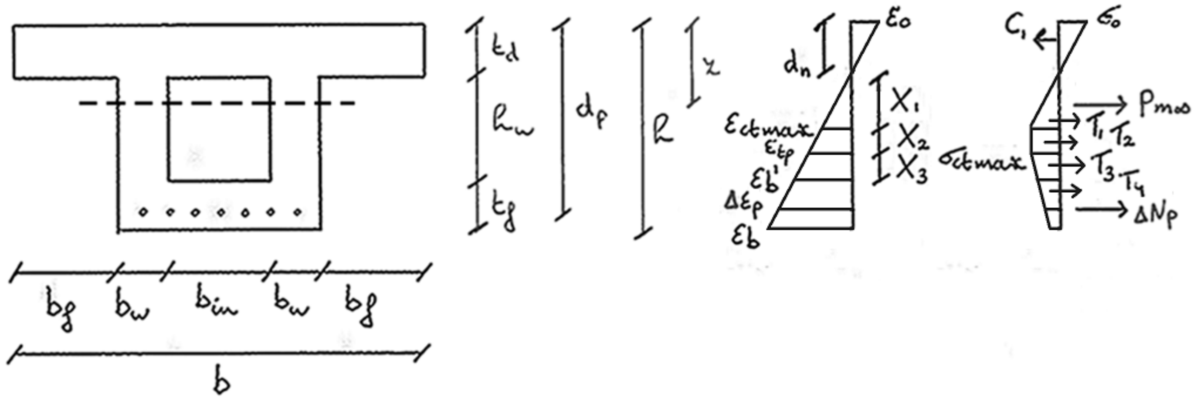


Figure 10.13: deformation and stress diagram when $d_n = t_d$.

10.13 When $d_n = t_d \, (\varepsilon_p > \varepsilon_{py})$

With respect to the deformation and stress diagram of [Figure 10.13] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 - \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$

$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}X_3 - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$

$$+4b_w\sigma_{ctmax}t_f - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2P_{m\infty}$$

$$\rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$

$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_p f_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$+4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})h - 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})h + 2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})h + 2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})t_f - 2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$-4b_w\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f + 4b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+2b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_f - 2b_{in}\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})t_f + 2b_w\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f - 2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+b_{in}\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f - b_{in}\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})t_f + 2A_p f_{pd}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})$$

$$+2A_p(\varepsilon_p - \varepsilon_{py})\,(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h-4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)h^2+4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)h+4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)h$$

$$-2b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2b_{in}\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f h$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2$$

$$+2A_p f_{pd}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+2A_p\frac{\varepsilon_0}{d_n}d_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+2A_p\varepsilon_{p\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_{py}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2P_{m\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$bE_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$+4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)ht_d-4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$-2b_w\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)h^2+4b_w\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)ht_d+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)ht_d$$

$$-2b_w\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f t_d-2b_{in}\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f h$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f t_d+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f t_d+b_{in}\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2$$

$$+2A_p f_{pd}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d+2A_p\varepsilon_0^2 d_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)t_d+2A_p\varepsilon_0\varepsilon_{p\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)t_d$$

$$-2A_p\varepsilon_0\varepsilon_{py}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)t_d-2P_{m\infty}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d\rightarrow$$

$$-\left(bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2+\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\left(2b_w(h-t_d)^2+b_{in}\left(2(h-t_d)-t_f\right)t_f\right)\right.$$

$$\left.-2A_p\,(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(d_p-t_d)\right)\varepsilon_0^2$$

$$+\left(2\left(2b_w(h-t_d)+b_{in}t_f\right)\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_d\right.$$

$$+\left(2A_p\left(f_{pd}(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{p\infty}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)-2P_{m\infty}(\varepsilon_{uk}-\varepsilon_{py})\right)(\varepsilon_{tu}$$

$$\left.-\varepsilon_{t,p})t_d\right)\varepsilon_0$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})t_d^2\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+\varepsilon_{t,p}^2\right)=0\rightarrow$$

$$\varepsilon_0=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a-\left(bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2+\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\left(2b_w(h-t_d)^2+b_{in}\left(2(h-t_d)-t_f\right)t_f\right)\right.$$

$$\left.-2A_p\,(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(d_p-t_d)\right)$$

$$b=2\left(2b_w(h-t_d)+b_{in}t_f\right)\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$+\left(2A_p\left(f_{pd}(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{p\infty}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)-2P_{m\infty}(\varepsilon_{uk}-\varepsilon_{py})\right)(\varepsilon_{tu}$$

$$-\varepsilon_{t,p})t_d$$

$$c=-2b_w\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})t_d^2\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+\varepsilon_{t,p}^2\right)$$

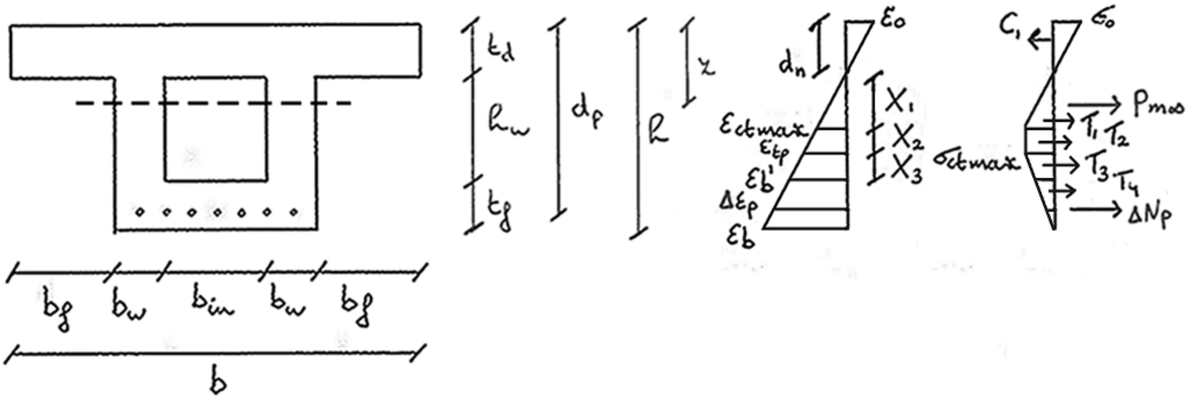


Figure 10.14: deformation and stress diagram when $d_n = t_d$ & $\varepsilon_b = \varepsilon_{t,u}$.

10.14 When $d_n = t_d$ & $\varepsilon_b = \varepsilon_{t,u}$ $\left(\varepsilon_p > \varepsilon_{py}\right)$

With respect to the deformation and stress diagram of [Figure 10.14] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \frac{\varepsilon_{t,u}}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0 = \frac{\varepsilon_{t,u}}{h - d_n} d_n$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The amount of prestressing steel A_p can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}\left(X_3 + t_f\right)$$
$$+\frac{1}{2}b_{in}\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2b_w\sigma_{ctmax}t_f + b_{in}\sigma_{ctmax}t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n$$
$$+b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_pf_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_w\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)h$$
$$-2b_w\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + b_{in}\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_f$$
$$-b_{in}\sigma_{ctmax}\left(\varepsilon_b' - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_f + 2A_pf_{pd}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)$$
$$+2A_p\left(\varepsilon_p - \varepsilon_{py}\right)\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_w\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)h$$
$$-2b_w\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + b_{in}\sigma_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)ht_f + b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_f^2$$
$$+b_{in}\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_f + 2A_pf_{pd}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)$$
$$+2A_p\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p - 2A_p\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$+2A_p\varepsilon_{p\infty}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c\frac{\varepsilon_{t,u}}{h-d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{h-d_n}{\varepsilon_{t,u}}$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{h-d_n}{\varepsilon_{t,u}}+2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{h-d_n}(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{h-d_n}d_n(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{h-d_n}(\varepsilon_{uk}-\varepsilon_{py})t_f^2+b_{in}\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_f+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p\frac{\varepsilon_{t,u}}{h-d_n}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\frac{\varepsilon_{t,u}}{h-d_n}d_n(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})(h-t_d)^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})(h-t_d)^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h(h-t_d)$$

$$-2b_w\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_d)$$

$$+b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f(h-t_d)-b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})ht_f$$

$$+b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_dt_f+b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2$$

$$+b_{in}\sigma_{ctmax}\varepsilon_{tu}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})(h-t_d)t_f+2A_pf_{pd}\varepsilon_{tu}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})(h-t_d)$$

$$+2A_p\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)t_d$$

$$+2A_p\varepsilon_{p\infty}\varepsilon_{tu}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_d)-2A_p\varepsilon_{py}\varepsilon_{tu}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_d)$$

$$-2A_p\sigma_{pm\infty}\varepsilon_{tu}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})(h-t_d)\rightarrow$$

$$
\begin{aligned}
bE_c\varepsilon_{t,u}^2&\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2\\
&+\sigma_{ctmax}\big(\varepsilon_{uk}\\
&-\varepsilon_{py}\big)\left(2b_w\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\left(\varepsilon_{ctmax}-\varepsilon_{t,p}\right)(h-t_d)+\varepsilon_{tu}(t_d-h)\right)(h-t_d)\right.\\
&\left.-b_{in}\varepsilon_{t,u}^2t_f^2\right)\\
&=2\varepsilon_{tu}\big(\varepsilon_{tu}\\
&-\varepsilon_{t,p}\big)\left(\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+\left(\varepsilon_{p\infty}-\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(h-t_d)\right.\\
&\left.+\varepsilon_{tu}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\left(d_p-t_d\right)\right)A_p
\end{aligned}
$$

$$
A_p=\frac{\alpha}{\gamma}
$$

$$
\begin{aligned}
\alpha=bE_c\varepsilon_{t,u}^2&\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2\\
&+\sigma_{ctmax}\big(\varepsilon_{uk}\\
&-\varepsilon_{py}\big)\left(2b_w\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\left(\varepsilon_{ctmax}-\varepsilon_{t,p}\right)(h-t_d)+\varepsilon_{tu}(t_d-h)\right)(h-t_d)\right.\\
&\left.-b_{in}\varepsilon_{t,u}^2t_f^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\gamma=2\varepsilon_{tu}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)&\left(\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+\left(\varepsilon_{p\infty}-\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(h-t_d)\right.\\
&\left.+\varepsilon_{tu}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\left(d_p-t_d\right)\right)
\end{aligned}
$$

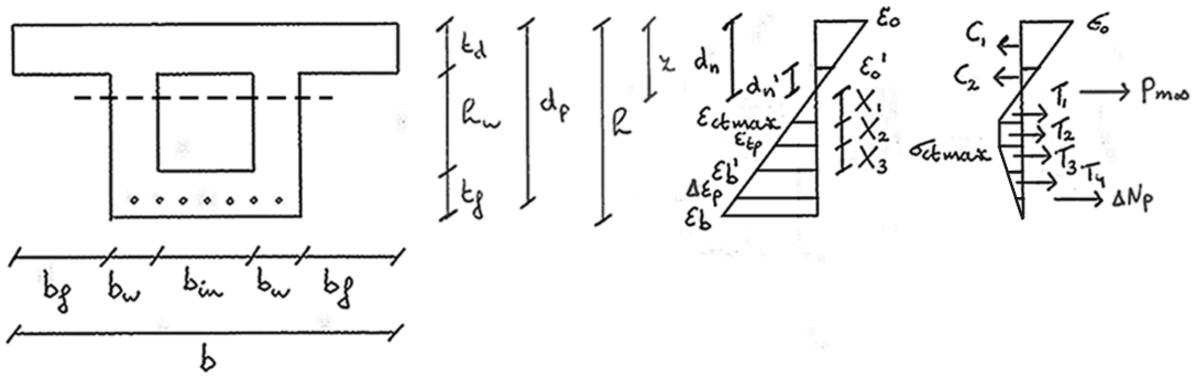


Figure 10.15: deformation and stress diagram when $\varepsilon_b = \varepsilon_{t,u}$.

10.15 When $\boldsymbol{\varepsilon_b = \varepsilon_{t,u}}$ $\boldsymbol{(d_n > t_d \;\&\; \varepsilon_p > \varepsilon_{py})}$

With respect to the deformation and stress diagram of [Figure 10.15] the following relations are valid:

$$\frac{{\varepsilon_0}'}{{d_n}'} = \frac{\varepsilon_0}{d_n} \to {\varepsilon_0}' = \frac{\varepsilon_0}{d_n}{d_n}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \to X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \to X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \to X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \to \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \to \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \to \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_{t,u}}{h - d_n} = \frac{\varepsilon_0}{d_n} \to d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}}(h - d_n) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h - \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n \to d_n + \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}} h \to$$
$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h \to d_n\left(\frac{\varepsilon_0 + \varepsilon_{t,u}}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h \to d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,u}} h$$

$${d_n}' = d_n - t_d$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \to X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}\left(X_3 + t_f\right)$$
$$+\frac{1}{2}b_{in}\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3$$
$$+2b_w\sigma_{ctmax}t_f + b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$
$$-2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n' + 2b_f E_c\frac{\varepsilon_0}{d_n}d_n' t_d - b_{in}E_c\varepsilon_0 d_n' + b_{in}E_c\frac{\varepsilon_0}{d_n}d_n' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n + b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$
$$+2A_p f_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n + b_{in}\sigma_{ctmax}t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_p f_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n-2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$-b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p(\varepsilon_p-\varepsilon_{py})(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2P_{m\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n-2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})t_f^2$$

$$+b_{in}\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_f+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2P_{m\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}h-2b_fE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}h$$
$$+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d-2b_fE_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-b_{in}E_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}h+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$
$$-b_{in}E_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{t,u}}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0+\varepsilon_{t,u}}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h+2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$
$$-2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,u}}h+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{uk}-\varepsilon_{py})t_f^2+b_{in}\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_f+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})$$
$$+2A_p\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$
$$+2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$
$$-2P_{m\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_0^2h^2-2b_fE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_0^2h^2$$
$$+4b_fE_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh-2b_fE_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-b_{in}E_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_0^2h^2+2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh$$
$$-b_{in}E_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2+2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_fh$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})t_fh$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_fh$$
$$+2A_pf_{pd}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h+2A_p(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$
$$-2A_p\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h+2A_p(\varepsilon_0+\varepsilon_{t,u})\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h$$
$$-2A_p(\varepsilon_0+\varepsilon_{t,u})\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h-2P_{m\infty}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h\rightarrow$$

$$
\begin{aligned}
& bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_0^2h^2-2b_fE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_0^2h^2 \\
& +4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh+4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh \\
& -2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 \\
& -2b_fE_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-b_{in}E_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\varepsilon_0^2h^2 \\
& +2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh+2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh \\
& -b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 \\
& -b_{in}E_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2 \\
& +2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2 \\
& +b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_fh+b_{in}\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_fh \\
& -b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})ht_f-b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2 \\
& +2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_f^2+b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2+b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_fh \\
& +b_{in}\sigma_{ctmax}\varepsilon_{t,u}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_fh+2A_pf_{pd}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h \\
& +2A_pf_{pd}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h+2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p \\
& +4A_p\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p+2A_p\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p \\
& -2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h-2A_p\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h \\
& +2A_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h+2A_p\varepsilon_{t,u}\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h \\
& -2A_p\varepsilon_0\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h-2A_p\varepsilon_{t,u}\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h \\
& -2P_{m\infty}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h-2P_{m\infty}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h\rightarrow
\end{aligned}
$$

$$\left(E_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)\left(\left(b-2b_f-b_{in}\right)h^2+\left(2b_f+b_{in}\right)(2h\ -t_d)t_d\right)\right.$$

$$\left.-\,b_{in}\sigma_{ctmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2+2A_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\left(h-d_p\right)\right)\varepsilon_0^2$$

$$+2\left(E_c\varepsilon_{t,u}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)\left(2b_f+b_{in}\right)(h-t_d)t_d-b_{in}\sigma_{ctmax}\varepsilon_{t,u}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2\right.$$

$$-\,A_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h\right.$$

$$\left.\left.+\left(\varepsilon_{t,u}\left(2d_p-h\right)+\left(\varepsilon_{p\infty}-\varepsilon_{py}\right)h\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)\varepsilon_0$$

$$-\left(2b_f+b_{in}\right)E_c\varepsilon_{t,u}^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$
$$+\,2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u}\right)\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h^2$$
$$-\,b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2$$
$$-\,2A_p\varepsilon_{t,u}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h\right.$$
$$\left.+\left(\varepsilon_{t,u}d_p+\left(\varepsilon_{p\infty}-\varepsilon_{py}\right)h\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)=0\rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})\left((b - 2b_f - b_{in})h^2 + (2b_f + b_{in})(2h - t_d)t_d\right) - b_{in}\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})t_f^2 + 2A_p(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)(h - d_p)$$

$$b = 2\left(E_c\varepsilon_{t,u}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})(2b_f + b_{in})(h - t_d)t_d - b_{in}\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})t_f^2 - A_p(\varepsilon_{tu} - \varepsilon_{t,p})\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})h + \left(\varepsilon_{t,u}(2d_p - h) + (\varepsilon_{p\infty} - \varepsilon_{py})h\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)$$

$$c = -(2b_f + b_{in})E_c\varepsilon_{t,u}^2(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d^2 + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u})(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})h^2 - b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk} - \varepsilon_{py})t_f^2 - 2A_p\varepsilon_{t,u}(\varepsilon_{tu} - \varepsilon_{t,p})\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})h + \left(\varepsilon_{t,u}d_p + (\varepsilon_{p\infty} - \varepsilon_{py})h\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

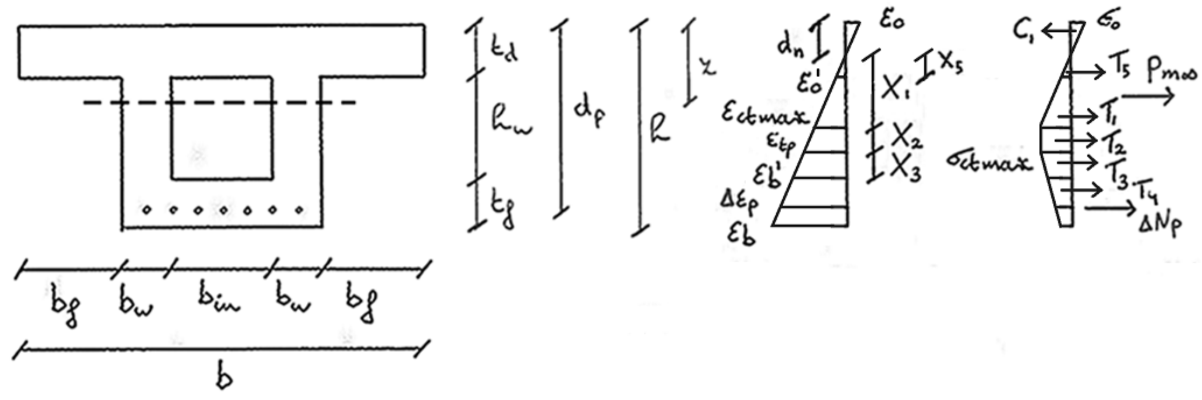


Figure 10.16: deformation and stress diagram when $\varepsilon_p = \varepsilon_{py}$.

10.16 When $\boldsymbol{\varepsilon_p = \varepsilon_{py}}$ $(\boldsymbol{d_n < t_d})$

With respect to the deformation and stress diagram of [Figure 10.16] the following relations are valid:

$$\frac{{\varepsilon_0}'}{X_5} = \frac{\varepsilon_0}{d_n} \to \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \to X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \to X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \to X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \to \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \to d_n = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}(d_p - d_n) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p - \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_n \to$$

$$d_n + \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_n = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p \to d_n \left(1 + \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p \to$$

$$d_n \left(\frac{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}}{\varepsilon_{py} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{py} - \varepsilon_{p\infty}} d_p \to d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{py} - \varepsilon_{p\infty}} d_p$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \to \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \to \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \to X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$X_5 = t_d - d_n$

$\varepsilon_p = \varepsilon_{py} \rightarrow \Delta\varepsilon_p + \varepsilon_{p\infty} = \varepsilon_{py} \rightarrow \Delta\varepsilon_p = \varepsilon_{py} - \varepsilon_{p\infty}$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + T_5 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3 - \frac{1}{2}(2b_w)\sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3$$

$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' X_5$$

$$+A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} X_3 - 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3$$

$$+4b_w \sigma_{ctmax} t_f - 4b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in} \sigma_{ctmax} t_f - 2b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_f E_c \varepsilon_0' X_5 + b_{in} E_c \varepsilon_0' X_5 + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n - 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$-4b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in} \sigma_{ctmax} t_f - 2b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$

$$+b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_f E_c \varepsilon_0' t_d - 2b_f E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + 2A_p E_p\left(\varepsilon_{py} - \varepsilon_{p\infty}\right)$$

$$\rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$
$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n-2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})h+2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})d_n$$
$$+2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})t_f+2b_w\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n-4b_w\sigma_{ctmax}(\varepsilon_b-\varepsilon_{t,p})t_f$$
$$+2b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})t_f-2b_{in}\sigma_{ctmax}(\varepsilon_b-\varepsilon_{t,p})t_f+2b_w\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')t_f$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')t_f+2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})\rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$
$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n-2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)h+2b_w\sigma_{ctmax}\varepsilon_{t,p}h$$
$$+2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)d_n-2b_w\sigma_{ctmax}\varepsilon_{t,p}d_n+2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f$$
$$+2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n-2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0\right)t_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_{tu}t_f-b_{in}\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0\right)t_f-b_{in}\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)t_f$$
$$+2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n$$
$$+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n$$
$$+2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})\rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$
$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2+4b_w\sigma_{ctmax}\varepsilon_0h+4b_w\sigma_{ctmax}\varepsilon_{t,p}h$$
$$-2b_w\sigma_{ctmax}\varepsilon_0d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n+2b_{in}\sigma_{ctmax}\varepsilon_{tu}t_f-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}t_f^2+2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n+2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})\rightarrow$$

$$bE_c(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}d_p=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}(\varepsilon_{tu}-\varepsilon_{t,p})d_p$$
$$+4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}d_p$$
$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}{d_p}h^2+4b_w\sigma_{ctmax}\varepsilon_0h+4b_w\sigma_{ctmax}\varepsilon_{t,p}h$$
$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}d_p-4b_w\sigma_{ctmax}\varepsilon_{t,p}\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}d_p$$
$$-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}d_p+2b_{in}\sigma_{ctmax}\varepsilon_{tu}t_f-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}{d_p}ht_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}{d_p}t_f^2+2b_fE_c\frac{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}{d_p}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d+2b_fE_c(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}d_p$$
$$+b_{in}E_c\frac{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}{d_p}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$+b_{in}E_c(\varepsilon_{tu}-\varepsilon_{t,p})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{py}-\varepsilon_{p\infty}}d_p+2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2$$
$$+4b_w\sigma_{ctmax}\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)(\varepsilon_{tu}-\varepsilon_{t,p})hd_p-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2$$
$$-2b_w\sigma_{ctmax}\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)^2h^2+4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)hd_p$$
$$+4b_w\sigma_{ctmax}\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)\varepsilon_{t,p}hd_p-2b_w\sigma_{ctmax}\varepsilon_0^2d_p^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_p^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_p^2+2b_{in}\sigma_{ctmax}\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)\varepsilon_{tu}t_fd_p$$
$$-2b_{in}\sigma_{ctmax}\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)^2ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)t_fd_p$$
$$+b_{in}\sigma_{ctmax}\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)^2t_f^2+2b_fE_c\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-4b_fE_c\varepsilon_0\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_p+2b_fE_c(\varepsilon_{tu}-\varepsilon_{t,p})\varepsilon_0^2d_p^2$$
$$+b_{in}E_c\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2-2b_{in}E_c\varepsilon_0\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_p$$
$$+b_{in}E_c(\varepsilon_{tu}-\varepsilon_{t,p})\varepsilon_0^2d_p^2+2A_pE_p\left(\varepsilon_0+(\varepsilon_{py}-\varepsilon_{p\infty})\right)(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})d_p\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2 + 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_p$$
$$+4b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})hd_p - 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_p^2 - 2b_w\sigma_{ctmax}\varepsilon_0^2h^2$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})h^2 - 2b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})^2h^2 + 4b_w\sigma_{ctmax}\varepsilon_0^2hd_p$$
$$+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})hd_p + 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_p + 4b_w\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})\varepsilon_{t,p}hd_p$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2d_p^2 - 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_p^2 - 2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_p^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}t_fd_p$$
$$+2b_{in}\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})\varepsilon_{tu}t_fd_p - 2b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f - 4b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})ht_f$$
$$-2b_{in}\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})^2ht_f + 2b_{in}\sigma_{ctmax}\varepsilon_0^2t_fd_p + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})t_fd_p$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})t_f^2 + b_{in}\sigma_{ctmax}(\varepsilon_{py}-\varepsilon_{p\infty})^2t_f^2$$
$$+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + 4b_fE_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + 2b_fE_c(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_p - 4b_fE_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_p + 2b_fE_c(\varepsilon_{tu}-\varepsilon_{t,p})\varepsilon_0^2d_p^2$$
$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + 2b_{in}E_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + b_{in}E_c(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_p - 2b_{in}E_c\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_p + b_{in}E_c(\varepsilon_{tu}-\varepsilon_{t,p})\varepsilon_0^2d_p^2$$
$$+2A_pE_p\varepsilon_0(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})d_p + 2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p \rightarrow$$

$$-\Big(E_c(\varepsilon_{tu}-\varepsilon_{t,p})\big((b-b_{in}-2b_f)d_p^2 + (2b_f+b_{in})(2d_p-t_d)t_d\big) + \sigma_{ctmax}\left(2b_w(h-d_p)^2 + b_{in}(2(h-d_p)-t_f)t_f\right)\Big)\varepsilon_0^2$$

$$+2\Big(\sigma_{ctmax}\Big(\big(2b_w(h-d_p)+b_{in}t_f\big)\varepsilon_{tu}d_p + \big(2b_wh(d_p-h) + b_{in}(d_p-2h+t_f)t_f\big)(\varepsilon_{py}-\varepsilon_{p\infty})\Big) + (2b_f+b_{in})(t_d-d_p)E_c(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})t_d + A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})(\varepsilon_{tu}-\varepsilon_{t,p})d_p\Big)\varepsilon_0$$

$$+\sigma_{ctmax}\Big(\Big(2b_wh\big(2\varepsilon_{tu}d_p-(\varepsilon_{py}-\varepsilon_{p\infty})h\big) + b_{in}t_f\big(2\varepsilon_{tu}d_p - 2(\varepsilon_{py}-\varepsilon_{p\infty})h + (\varepsilon_{py}-\varepsilon_{p\infty})t_f\big)\Big)(\varepsilon_{py}-\varepsilon_{p\infty}) - 2b_wd_p^2\big(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p}) + \varepsilon_{t,p}^2\big)\Big) + (2b_f+b_{in})E_c(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 + 2A_pE_p(\varepsilon_{py}-\varepsilon_{p\infty})^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\left(E_c(\varepsilon_{tu} - \varepsilon_{t,p})\left((b - b_{in} - 2b_f)d_p^2 + (2b_f + b_{in})(2d_p - t_d)t_d\right) + \sigma_{ctmax}\left(2b_w(h - d_p)^2 + b_{in}(2(h - d_p) - t_f)t_f\right)\right)$$

$$b = 2\left(\sigma_{ctmax}\left((2b_w(h - d_p) + b_{in}t_f)\varepsilon_{tu}d_p + (2b_w h(d_p - h) + b_{in}(d_p - 2h + t_f)t_f)(\varepsilon_{py} - \varepsilon_{p\infty})\right) + (2b_f + b_{in})(t_d - d_p)E_c(\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p})t_d + A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty})(\varepsilon_{tu} - \varepsilon_{t,p})d_p\right)$$

$$c = \sigma_{ctmax}\left(\left(2b_w h(2\varepsilon_{tu}d_p - (\varepsilon_{py} - \varepsilon_{p\infty})h) + b_{in}t_f(2\varepsilon_{tu}d_p - 2(\varepsilon_{py} - \varepsilon_{p\infty})h + (\varepsilon_{py} - \varepsilon_{p\infty})t_f)\right)(\varepsilon_{py} - \varepsilon_{p\infty}) - 2b_w d_p^2(\varepsilon_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p}) + \varepsilon_{t,p}^2)\right) + (2b_f + b_{in})E_c(\varepsilon_{py} - \varepsilon_{p\infty})^2(\varepsilon_{tu} - \varepsilon_{t,p})t_d^2 + 2A_p E_p(\varepsilon_{py} - \varepsilon_{p\infty})^2(\varepsilon_{tu} - \varepsilon_{t,p})d_p$$

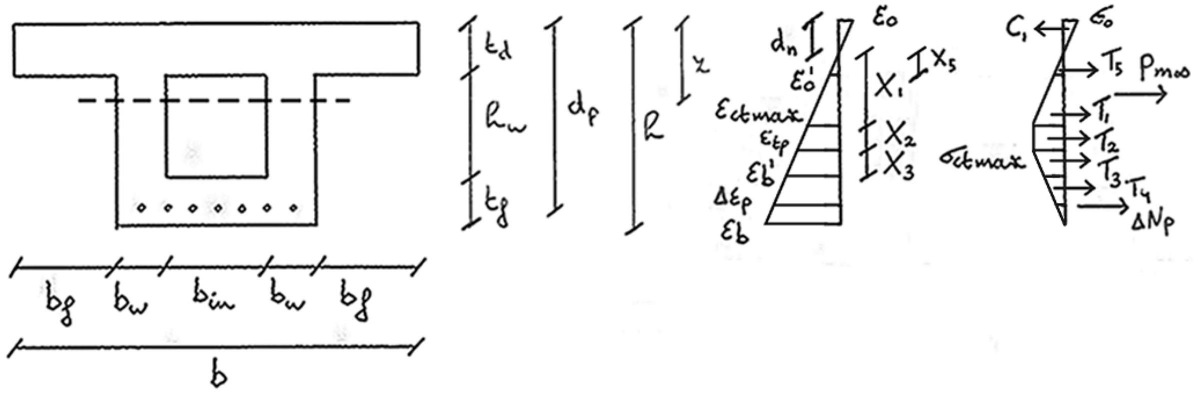


Figure 10.17: deformation and stress diagram when $\varepsilon_b = \varepsilon_{t,u}$.

10.17 When $\varepsilon_b = \varepsilon_{t,u}\left(d_n < t_d \;\&\; \varepsilon_p > \varepsilon_{py}\right)$

With respect to the deformation and stress diagram of [Figure 10.17] the following relations are valid:

$$\frac{\varepsilon_0{}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n}X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n}t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n}h - \varepsilon_0 - \frac{\varepsilon_0}{d_n}t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}d_p - \varepsilon_0$$

$$\frac{\varepsilon_{t,u}}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}}(h - d_n) = \frac{\varepsilon_0}{\varepsilon_{t,u}}h - \frac{\varepsilon_0}{\varepsilon_{t,u}}d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,u}}d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}}h \rightarrow$$
$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}}h \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{t,u}}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}}h \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,u}}h$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$

$$X_5 = t_d - d_n$$

$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + T_5 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}(X_3 + t_f)$$
$$+\frac{1}{2}b_{in}\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_f + b_{in})E_c\varepsilon_0' X_5 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2b_w\sigma_{ctmax}t_f + b_{in}\sigma_{ctmax}t_f$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_fE_c\varepsilon_0' X_5 + b_{in}E_c\varepsilon_0' X_5 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$
$$-2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n$$
$$+b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_fE_c\varepsilon_0' t_d - 2b_fE_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n$$
$$+2A_pf_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})h$$
$$-2b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n + b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_f$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_b' - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_f + 2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d^2$$
$$-4b_fE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d + 2b_fE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$
$$+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d$$
$$+b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})$$
$$+2A_p(\varepsilon_p - \varepsilon_{py})(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + 2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})ht_f + b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})t_f^2$$

$$+2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d + b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) + 2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) + 2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) - 2P_{m\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}h = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{t,u}}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0+\varepsilon_{t,u}}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h + 2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,u}}h + b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{uk}-\varepsilon_{py})ht_f + b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{uk}-\varepsilon_{py})t_f^2 + 2b_fE_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d + 2b_fE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}h$$

$$+b_{in}E_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$+b_{in}E_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}h + 2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p\frac{\varepsilon_0+\varepsilon_{t,u}}{h}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p - 2A_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) - 2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2P_{m\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2 + 2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2 + b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_f h$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{uk}-\varepsilon_{py})ht_f + b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})t_f h$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2 + 2b_f E_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-4b_f E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d h + 2b_f E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$
$$+b_{in}E_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d h$$
$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2 + 2A_p f_{pd}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$
$$+2A_p(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p - 2A_p\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h$$
$$+2A_p\varepsilon_{p\infty}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h - 2A_p\varepsilon_{py}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h$$
$$-2P_{m\infty}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$

$$+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_f^2+b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2$$

$$+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2+4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$+2b_fE_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh$$

$$-4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh+2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$

$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2+2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$+b_{in}E_c\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh$$

$$-2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dh+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h^2$$

$$+2A_pf_{pd}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h+2A_pf_{pd}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$+2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p+4A_p\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$+2A_p\varepsilon_{t,u}^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h$$

$$-2A_p\varepsilon_0\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h+2A_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h$$

$$+2A_p\varepsilon_{t,u}\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h-2A_p\varepsilon_0\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h$$

$$-2A_p\varepsilon_{t,u}\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)h-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2A_p\sigma_{pm\infty}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h\rightarrow$$

$$+\left(b_{in}\sigma_{ctmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2\right.$$

$$-\left(\left(b-2b_f-b_{in}\right)h^2+\left(2b_f+b_{in}\right)(2h-t_d)t_d\right)E_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$\left.+2A_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\left(d_p-h\right)\right)\varepsilon_0^2$$

$$+2\left(b_{in}\sigma_{ctmax}\varepsilon_{t,u}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2+\left(\left(2b_f+b_{in}\right)(t_d-h)\right)E_c\varepsilon_{t,u}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d\right.$$

$$+A_p\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h\right.$$

$$\left.\left.+\left(\varepsilon_{t,u}\left(2d_p-h\right)+\left(\varepsilon_{p\infty}-\varepsilon_{py}\right)h\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\right)\varepsilon_0$$

$$-\sigma_{ctmax}\left(2b_w\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)h^2\left(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u}\right)-b_{in}\varepsilon_{t,u}^2t_f^2\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$+\left(2b_f+b_{in}\right)E_c\varepsilon_{t,u}^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$+2A_p\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h\right.$$

$$\left.+\left(\varepsilon_{t,u}d_p+\left(\varepsilon_{p\infty}-\varepsilon_{py}\right)h\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\varepsilon_{t,u}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)=0\rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} a = {} & b_{in}\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})t_f^2 \\ & - \left((b - 2b_f - b_{in})h^2 + (2b_f + b_{in})(2h - t_d)t_d\right) E_c(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py}) \\ & + 2A_p(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)(d_p - h) \end{aligned}$$

$$\begin{aligned} b = 2\Bigg(& b_{in}\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})t_f^2 + \left((2b_f + b_{in})(t_d - h)\right) E_c\varepsilon_{t,u}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d \\ & + A_p\Bigg((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})h \\ & + \left(\varepsilon_{t,u}(2d_p - h) + (\varepsilon_{p\infty} - \varepsilon_{py})h\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\Bigg)(\varepsilon_{tu} - \varepsilon_{t,p})\Bigg) \end{aligned}$$

$$\begin{aligned} c = {} & -\sigma_{ctmax}\left(2b_w(\varepsilon_{tu} - \varepsilon_{t,p})h^2(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) - b_{in}\varepsilon_{t,u}^2 t_f^2\right)(\varepsilon_{uk} - \varepsilon_{py}) \\ & + (2b_f + b_{in})E_c\varepsilon_{t,u}^2(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d^2 \\ & + 2A_p\Bigg((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})h \\ & + \left(\varepsilon_{t,u}d_p + (\varepsilon_{p\infty} - \varepsilon_{py})h\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\Bigg)\varepsilon_{t,u}(\varepsilon_{tu} - \varepsilon_{t,p}) \end{aligned}$$

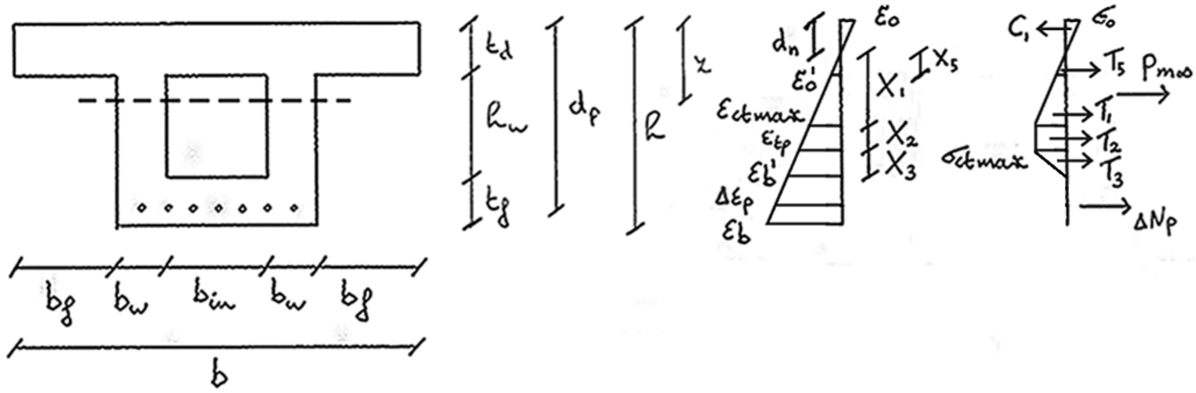


Figure 10.18: deformation and stress diagram when $\varepsilon_b' = \varepsilon_{t,u}$.

10.18 When $\boldsymbol{\varepsilon_b' = \varepsilon_{t,u}}$ $\boldsymbol{(d_n < t_d \ \& \ \varepsilon_p > \varepsilon_{py})}$

With respect to the deformation and stress diagram of [Figure 10.18] the following relations are valid:

$$\frac{{\varepsilon_0}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \frac{\varepsilon_{t,u}}{h - d_n - t_f} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}}(h - d_n - t_f) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h - \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n - \frac{\varepsilon_0}{\varepsilon_{t,u}} t_f$$
$$\rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}} h - \frac{\varepsilon_0}{\varepsilon_{t,u}} t_f \rightarrow d_n \left(1 + \frac{\varepsilon_0}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}}(h - t_f) \rightarrow d_n \left(\frac{\varepsilon_0 + \varepsilon_{t,u}}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}}(h - t_f)$$
$$\rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,u}}(h - t_f)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}(d_p - d_n) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$X_5 = t_d - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_5 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}X_3 + \frac{1}{2}(2b_f + b_{in})E_c\varepsilon_0' X_5 + A_p\sigma_p$$
$$-P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2b_f E_c\varepsilon_0' X_5 + b_{in}E_c\varepsilon_0' X_5$$
$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n$$
$$-2b_w\sigma_{ctmax}t_f + 2b_f E_c\varepsilon_0' t_d - 2b_f E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + 2A_p f_{pd}$$
$$+2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n$$
$$+2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})h - 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})d_n - 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})t_f$$
$$+2b_f E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{uk} - \varepsilon_{py})t_d^2 - 4b_f E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})t_d + 2b_f E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n$$
$$+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{uk} - \varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})t_d + b_{in}E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_p f_{pd}(\varepsilon_{uk} - \varepsilon_{py})$$
$$+2A_p(\varepsilon_p - \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2P_{m\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}(h-t_f)=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{t,u}}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0+\varepsilon_{t,u}}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)+2b_w\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{t,u}}(h-t_f)-2b_w\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$+2b_fE_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h-t_f}(\varepsilon_{uk}-\varepsilon_{py})t_d^2-4b_fE_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d+2b_fE_c(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}(h-t_f)$$

$$+b_{in}E_c\frac{\varepsilon_0+\varepsilon_{t,u}}{h-t_f}(\varepsilon_{uk}-\varepsilon_{py})t_d^2-2b_{in}E_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d+b_{in}E_c(\varepsilon_{uk}-\varepsilon_{py})\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{t,u}}(h-t_f)$$

$$+2A_pf_{pd}(\varepsilon_{uk}-\varepsilon_{py})+2A_p\frac{\varepsilon_0+\varepsilon_{t,u}}{h-t_f}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2P_{m\infty}(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2+2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})h(h-t_f)$$

$$-2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2-2b_w\sigma_{ctmax}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})t_f(h-t_f)$$

$$+2b_fE_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2-4b_fE_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)$$

$$+2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2+b_{in}E_c(\varepsilon_0+\varepsilon_{t,u})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-2b_{in}E_c\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)+b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$

$$+2A_pf_{pd}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)+2A_p(\varepsilon_0+\varepsilon_{t,u})^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0(\varepsilon_0+\varepsilon_{t,u})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)+2A_p(\varepsilon_0+\varepsilon_{t,u})\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)$$

$$-2A_p(\varepsilon_0+\varepsilon_{t,u})\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)-2P_{m\infty}(\varepsilon_0+\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2 + 2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})h(h-t_f)$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})h(h-t_f) - 2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$

$$-2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f(h-t_f) - 2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_f(h-t_f)$$

$$+2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 2b_fE_c\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-4b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f) - 4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)$$

$$+2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2 + b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$+b_{in}E_c\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f) - 2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)$$

$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2 + 2A_pf_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})(h-t_f) + 2A_pf_{pd}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)$$

$$+2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p + 4A_p\varepsilon_0\varepsilon_{t,u}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p + 2A_p\varepsilon_{t,u}^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f) - 2A_p\varepsilon_0\varepsilon_{t,u}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f) + 2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)$$

$$+2A_p\varepsilon_{t,u}\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f) - 2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)$$

$$-2A_p\varepsilon_{t,u}\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f) - 2P_{m\infty}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})(h-t_f) - 2P_{m\infty}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)$$

$\rightarrow$

$$-\left(E_c(\varepsilon_{uk}-\varepsilon_{py})\left((b-2b_f-b_{in})(h-t_f)^2+(2b_f+b_{in})(2(h-t_f)-t_d)t_d\right)\right.$$

$$\left.+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\left((h-t_f)-d_p\right)\right)\varepsilon_0^2$$

$$+2\left(E_c\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})(2b_f+b_{in})\left(t_d-(h-t_f)\right)t_d\right.$$

$$+A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)\right.$$

$$\left.\left.+\left(\varepsilon_{t,u}\left(2d_p-(h-t_f)\right)+(\varepsilon_{p\infty}-\varepsilon_{py})(h-t_f)\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)\varepsilon_0$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\left((\varepsilon_{ctmax}-\varepsilon_{t,p})(h-t_f)+\varepsilon_{t,u}(t_f-h)\right)(h-t_f)$$

$$+(2b_f+b_{in})E_c\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$+2A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)\right.$$

$$\left.+\left(\varepsilon_{t,u}d_p+(\varepsilon_{p\infty}-\varepsilon_{py})(h-t_f)\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\varepsilon_{t,u}\rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\left(E_c(\varepsilon_{uk} - \varepsilon_{py})\left((b - 2b_f - b_{in})(h - t_f)^2 + (2b_f + b_{in})(2(h - t_f) - t_d)t_d\right) + 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\left((h - t_f) - d_p\right)\right)$$

$$b = 2\left(E_c\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})(2b_f + b_{in})\left(t_d - (h - t_f)\right)t_d + A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + \left(\varepsilon_{t,u}\left(2d_p - (h - t_f)\right) + (\varepsilon_{p\infty} - \varepsilon_{py})(h - t_f)\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)$$

$$c = -2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})\left((\varepsilon_{ctmax} - \varepsilon_{t,p})(h - t_f) + \varepsilon_{t,u}(t_f - h)\right)(h - t_f) + (2b_f + b_{in})E_c\varepsilon_{t,u}^2(\varepsilon_{uk} - \varepsilon_{py})t_d^2 + 2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + \left(\varepsilon_{t,u}d_p + (\varepsilon_{p\infty} - \varepsilon_{py})(h - t_f)\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\varepsilon_{t,u}$$

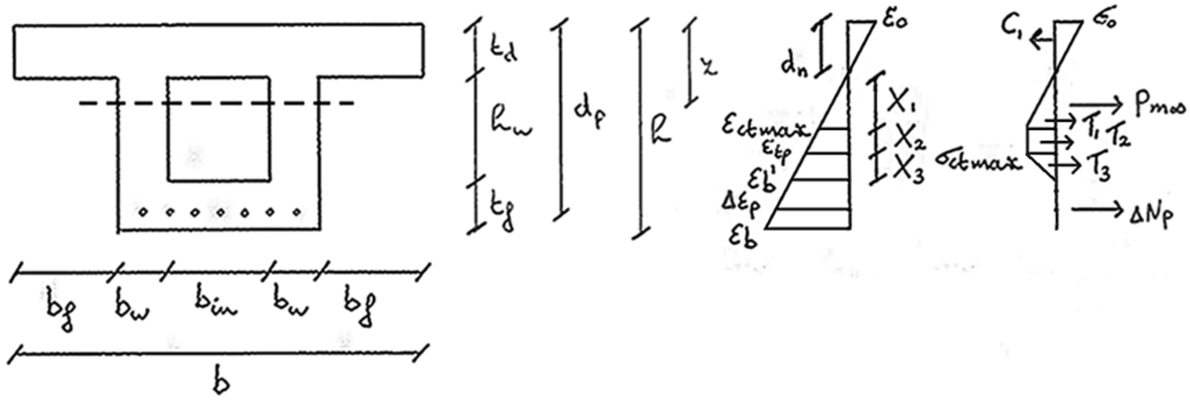


Figure 10.19: deformation and stress diagram when $d_n = t_d$ & $\varepsilon_b' = \varepsilon_{t,u}$.

10.19 When $d_n = t_d$ & $\varepsilon_b' = \varepsilon_{t,u}$ $(\varepsilon_p > \varepsilon_{py})$

With respect to the deformation and stress diagram of [Figure 10.19] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0 = \frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} d_n \rightarrow \varepsilon_0 = \frac{\varepsilon_{t,u}}{h - d_n - t_f} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}(d_p - d_n) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The amount of prestressing steel A_p can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$-2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}h - 2b_w\sigma_{ctmax}d_n$$

$$-2b_w\sigma_{ctmax}t_f + 2A_pf_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})h - 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})d_n - 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p\frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p - 2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$-2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\frac{\varepsilon_{t,u}}{h - t_d - t_f}(\varepsilon_{uk} - \varepsilon_{py})t_d^2 = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_{t,u}}(\varepsilon_{uk} - \varepsilon_{py})(h - t_d - t_f)$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_{t,u}}(\varepsilon_{uk} - \varepsilon_{py})(h - t_d - t_f) + 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})h - 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})t_d$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})t_f + 2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p\frac{\varepsilon_{t,u}}{h - t_d - t_f}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$

$$-2A_p\frac{\varepsilon_{t,u}}{h - t_d - t_f}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)t_d + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\frac{\varepsilon_{t,u}}{h - t_d - t_f}(\varepsilon_{uk} - \varepsilon_{py})t_d^2$$
$$+ 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})\left(\frac{\varepsilon_{ctmax} - \varepsilon_{t,p}}{\varepsilon_{t,u}}(h - t_d - t_f) - h + t_d + t_f\right)$$
$$= A_p2\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py})\right.$$
$$\left. + \left(\frac{\varepsilon_{t,u}}{h - t_d - t_f}(d_p - t_d) + \varepsilon_{p\infty} - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) \rightarrow$$

$$A_p = \frac{\alpha}{\gamma}$$

$$\alpha = bE_c \frac{\varepsilon_{t,u}}{h - t_d - t_f}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 + 2b_w\sigma_{ctmax}\left(\varepsilon_{uk} - \varepsilon_{py}\right)\left(\frac{\varepsilon_{ctmax} - \varepsilon_{t,p}}{\varepsilon_{t,u}}\left(h - t_d - t_f\right) - h + t_d + t_f\right)$$

$$\gamma = 2\left(\left(f_{pd} - \sigma_{pm\infty}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) + \left(\frac{\varepsilon_{t,u}}{h - t_d - t_f}\left(d_p - t_d\right) + \varepsilon_{p\infty} - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

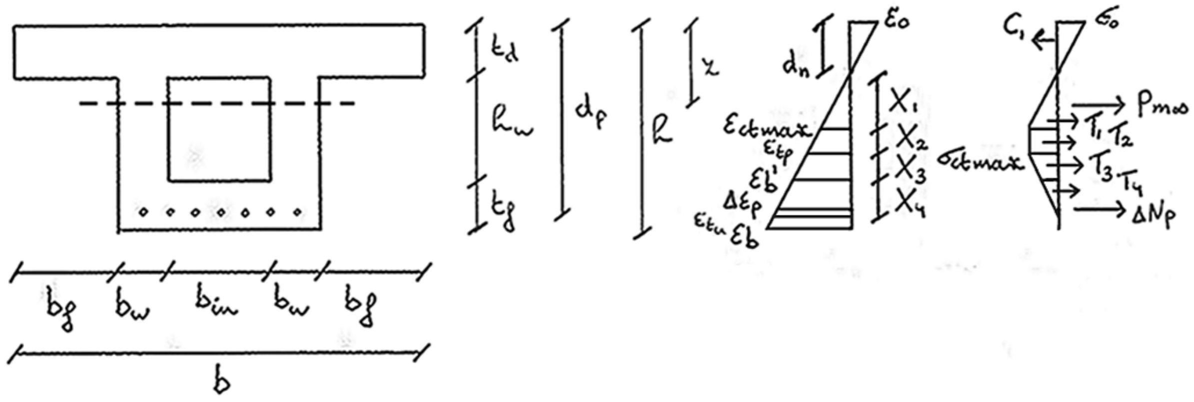


Figure 10.20: deformation and stress diagram when $d_n = t_d$.

10.20 When $d_n = t_d$ $(\varepsilon_p > \varepsilon_{py}$ & $\varepsilon_b' < \varepsilon_{t,u} < \varepsilon_b)$

With respect to the deformation and stress diagram of [Figure 10.20] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3 + X_4} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 + X_4 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_4 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2 + X_3)$$
$$= \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - h + d_n + t_f$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}(X_3 + X_4)$$
$$+\frac{1}{2} b_{in}\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2b_w\sigma_{ctmax}X_4$$
$$+b_{in}\sigma_{ctmax}X_4 - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n$$
$$-b_{in}\sigma_{ctmax}h + b_{in}\sigma_{ctmax}d_n + b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}h$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}d_n - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_pf_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n - b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})h$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n + b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_f$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_b' - \varepsilon_{t,p})\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + b_{in}\sigma_{ctmax}(\varepsilon_b' - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})h$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_b' - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n - b_{in}\sigma_{ctmax}(\varepsilon_b' - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_f$$
$$+2A_pf_{pd}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p(\varepsilon_p - \varepsilon_{py})(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n-2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})h$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_{in}\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})h^2-2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})h-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})ht_f$$

$$+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})t_f^2$$

$$+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})+2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\sigma_{pm\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$+b_{in}\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})ht_d$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_d^2+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_ft_d$$

$$+b_{in}\sigma_{ctmax}\varepsilon_{t,u}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_d^2+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})h^2-2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})ht_d$$

$$-2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2+2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_ft_d$$

$$+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2+2A_pf_{pd}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$+2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)t_d$$

$$+2A_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)t_d-2A_p\varepsilon_0\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)t_d$$

$$-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d\rightarrow$$

$$-\left(bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-b_{in}\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\left((h-t_d)^2-\left(2(h-t_d)-t_f\right)t_f\right)\right.$$

$$\left.-2A_p(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(d_p-t_d)\right)\varepsilon_0^2$$

$$+2\left(b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})(t_d-h+t_f)\right.$$

$$\left.+A_p\left(\left(f_{pd}-\sigma_{pm\infty}\right)(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{p\infty}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(\varepsilon_{tu}-\varepsilon_{t,p})\right)t_d\varepsilon_0$$

$$-\sigma_{ctmax}\left(2b_w(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})-b_{in}\varepsilon_{t,u}^2\right)(\varepsilon_{uk}-\varepsilon_{py})t_d^2=0\rightarrow$$

$$\varepsilon_0=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=-\left(bE_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-b_{in}\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\left((h-t_d)^2-\left(2(h-t_d)-t_f\right)t_f\right)\right.$$

$$\left.-2A_p(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(d_p-t_d)\right)$$

$$b=2\left(b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})(t_d-h+t_f)\right.$$

$$\left.+A_p\left(\left(f_{pd}-\sigma_{pm\infty}\right)(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{p\infty}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(\varepsilon_{tu}-\varepsilon_{t,p})\right)t_d$$

$$c=-\sigma_{ctmax}\left(2b_w(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})-b_{in}\varepsilon_{t,u}^2\right)(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

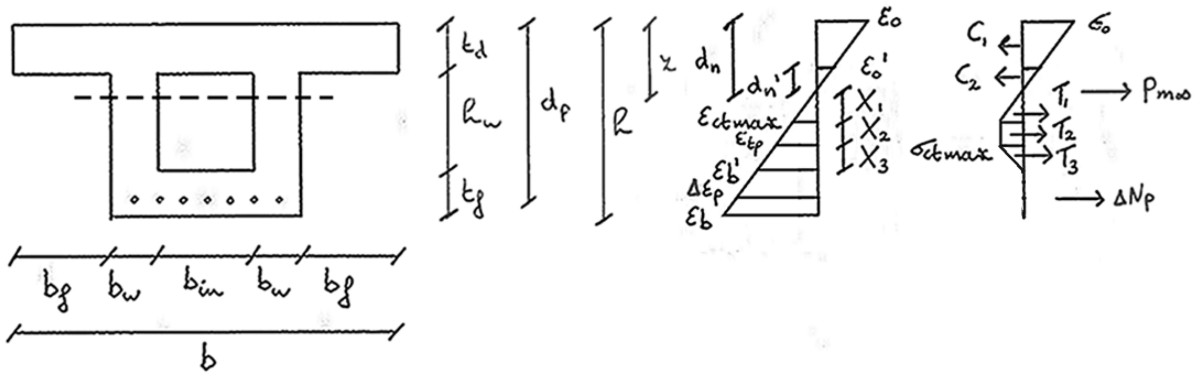


Figure 10.21: deformation and stress diagram when $\varepsilon_b' = \varepsilon_{t,u}$.

10.21 When $\boldsymbol{\varepsilon_b' = \varepsilon_{t,u}\ (d_n > t_d\ \&\ \varepsilon_p > \varepsilon_{py})}$

With respect to the deformation and stress diagram of [Figure 10.21] the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \frac{\varepsilon_{t,u}}{h - d_n - t_f} = \frac{\varepsilon_0}{d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}}\left(h - d_n - t_f\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}} h - \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n - \frac{\varepsilon_0}{\varepsilon_{t,u}} t_f$$

$$\rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{t,u}} d_n = \frac{\varepsilon_0}{\varepsilon_{t,u}} h - \frac{\varepsilon_0}{\varepsilon_{t,u}} t_f \rightarrow d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}}\left(h - t_f\right) \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{t,u}}{\varepsilon_{t,u}}\right) = \frac{\varepsilon_0}{\varepsilon_{t,u}}\left(h - t_f\right)$$

$$\rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,u}}\left(h - t_f\right)$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax} X_3$$
$$+A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3$$
$$+2A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) \right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n' + 2b_f E_c \frac{\varepsilon_0}{d_n} d_n' t_d - b_{in} E_c \varepsilon_0 d_n' + b_{in} E_c \frac{\varepsilon_0}{d_n} d_n' t_d = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$+2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} h - 2b_w \sigma_{ctmax} d_n - 2b_w \sigma_{ctmax} t_f + 2A_p f_{pd}$$
$$+2A_p \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n - 2b_f E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 4b_f E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$
$$-b_{in} E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_{in} E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$
$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_w \sigma_{ctmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) h$$
$$-2b_w \sigma_{ctmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n - 2b_w \sigma_{ctmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_f + 2A_p f_{pd} \left(\varepsilon_{uk} - \varepsilon_{py}\right)$$
$$+2A_p \left(\varepsilon_p - \varepsilon_{py}\right) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2A_p \sigma_{pm\infty} \left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$b E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n - 2b_f E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 4b_f E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$
$$-b_{in} E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_{in} E_c \varepsilon_0 \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d - b_{in} E_c \frac{\varepsilon_0}{d_n} \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$
$$= -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_w \sigma_{ctmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) h$$
$$-2b_w \sigma_{ctmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n - 2b_w \sigma_{ctmax} \left(\varepsilon_{uk} - \varepsilon_{py}\right) t_f + 2A_p f_{pd} \left(\varepsilon_{uk} - \varepsilon_{py}\right)$$
$$+2A_p \frac{\varepsilon_0}{d_n} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p - 2A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) + 2A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$
$$-2A_p \sigma_{pm\infty} \left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{t,u}} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f) - 2b_f E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{t,u}} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + 4b_f E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) t_d$$

$$-2b_f E_c \frac{\varepsilon_0 + \varepsilon_{t,u}}{h - t_f} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 - b_{in} E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{t,u}} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + 2b_{in} E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) t_d$$

$$-b_{in} E_c \frac{\varepsilon_0 + \varepsilon_{t,u}}{h - t_f} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0 + \varepsilon_{t,u}} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f)$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0 + \varepsilon_{t,u}} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + 2b_w \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) h$$

$$-2b_w \sigma_{ctmax} \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{t,u}} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f) - 2b_w \sigma_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) t_f + 2A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py})$$

$$+2A_p \frac{\varepsilon_0 + \varepsilon_{t,u}}{h - t_f} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p - 2A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) + 2A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$

$$-2A_p \sigma_{pm\infty} (\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py})(h - t_f)^2 - 2b_f E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py})(h - t_f)^2$$

$$+4b_f E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py}) t_d (h - t_f) - 2b_f E_c (\varepsilon_0 + \varepsilon_{t,u})^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2$$

$$-b_{in} E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py})(h - t_f)^2 + 2b_{in} E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py}) t_d (h - t_f)$$

$$-b_{in} E_c (\varepsilon_0 + \varepsilon_{t,u})^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 = -2b_w \sigma_{ctmax} \varepsilon_{ctmax} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f)^2$$

$$+2b_w \sigma_{ctmax} \varepsilon_{t,p} (\varepsilon_{uk} - \varepsilon_{py})(h - t_f)^2 + 2b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py}) h (h - t_f)$$

$$-2b_w \sigma_{ctmax} \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py})(h - t_f)^2 - 2b_w \sigma_{ctmax} (\varepsilon_0 + \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py}) t_f (h - t_f)$$

$$+2A_p f_{pd} (\varepsilon_0 + \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + 2A_p (\varepsilon_0 + \varepsilon_{t,u})^2 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p$$

$$-2A_p \varepsilon_0 (\varepsilon_0 + \varepsilon_{t,u}) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) (h - t_f) + 2A_p (\varepsilon_0 + \varepsilon_{t,u}) \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) (h - t_f)$$

$$-2A_p (\varepsilon_0 + \varepsilon_{t,u}) \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) (h - t_f) - 2A_p \sigma_{pm\infty} (\varepsilon_0 + \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py})(h - t_f) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2-2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2+4b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)$$

$$+4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)-2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2-4b_fE_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-2b_fE_c\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2-b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)$$

$$+2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d(h-t_f)-b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2-2b_{in}E_c\varepsilon_0\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-b_{in}E_c\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2+2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})h(h-t_f)$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})h(h-t_f)-2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$

$$-2b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f(h-t_f)-2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})t_f(h-t_f)$$

$$+2A_pf_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)+2A_pf_{pd}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)+2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$+4A_p\varepsilon_0\varepsilon_{t,u}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p+2A_p\varepsilon_{t,u}^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)$$

$$-2A_p\varepsilon_0\varepsilon_{t,u}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)+2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)$$

$$+2A_p\varepsilon_{t,u}\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)-2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)$$

$$-2A_p\varepsilon_{t,u}\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)(h-t_f)-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)$$

$$-2A_p\sigma_{pm\infty}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)\rightarrow$$

$$\left(E_c(\varepsilon_{uk}-\varepsilon_{py})\left((b-2b_f-b_{in})(h-t_f)^2+(2b_f\ +b_{in})(2(h-t_f)-t_d)t_d\right)\right.$$

$$\left.+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\left((h-t_f)-d_p\right)\right)\varepsilon_0^2$$

$$+2\left(E_c\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})\left((2b_f+b_{in})\left((h-t_f)-t_d\right)\right)t_d\right.$$

$$+A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)\right.$$

$$\left.\left.+\left((\varepsilon_{t,u}-\varepsilon_{p\infty}+\varepsilon_{py})(h-t_f)-2\varepsilon_{t,u}d_p\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)\varepsilon_0$$

$$-(2b_f+b_{in})E_c\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$+2b_w\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\left((\varepsilon_{ctmax}-\varepsilon_{t,p})(h-t_f)+\varepsilon_{t,u}(t_f-h)\right)(h-t_f)$$

$$+2A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)\right.$$

$$\left.+\left((\varepsilon_{py}-\varepsilon_{p\infty})(h-t_f)-\varepsilon_{t,u}d_p\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\varepsilon_{t,u}=0\rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = E_c(\varepsilon_{uk} - \varepsilon_{py})\left((b - 2b_f - b_{in})(h - t_f)^2 + (2b_f + b_{in})(2(h - t_f) - t_d)t_d\right) + 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\left((h - t_f) - d_p\right)$$

$$b = 2\left(E_c\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})\left((2b_f + b_{in})\left((h - t_f) - t_d\right)\right)t_d + A_p\left((\sigma_{pm\infty} - f_{pd})(\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + \left((\varepsilon_{t,u} - \varepsilon_{p\infty} + \varepsilon_{py})(h - t_f) - 2\varepsilon_{t,u}d_p\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)$$

$$c = -(2b_f + b_{in})E_c\varepsilon_{t,u}^2(\varepsilon_{uk} - \varepsilon_{py})t_d^2 + 2b_w\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})\left((\varepsilon_{ctmax} - \varepsilon_{t,p})(h - t_f) + \varepsilon_{t,u}(t_f - h)\right)(h - t_f) + 2A_p\left((\sigma_{pm\infty} - f_{pd})(\varepsilon_{uk} - \varepsilon_{py})(h - t_f) + \left((\varepsilon_{py} - \varepsilon_{p\infty})(h - t_f) - \varepsilon_{t,u}d_p\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\varepsilon_{t,u}$$

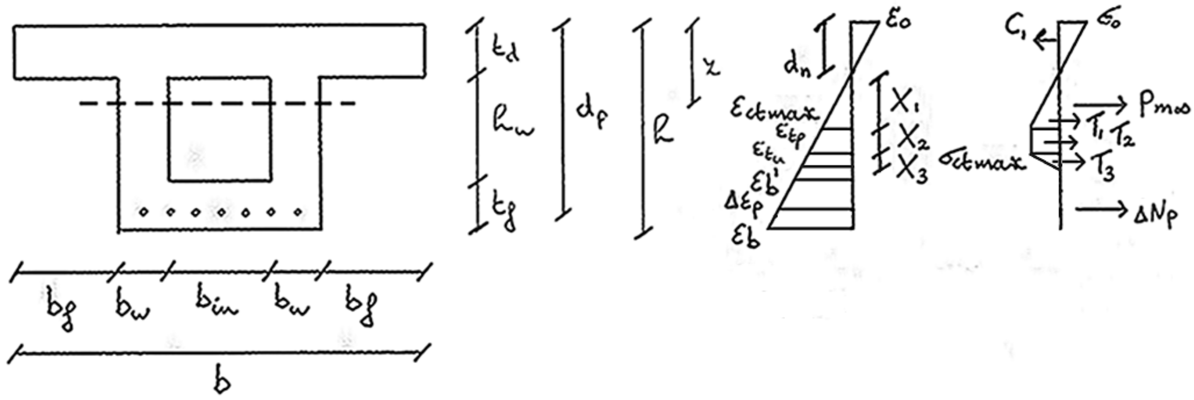


Figure 10.22: deformation and stress diagram when $d_n = t_d$.

10.22 When $d_n = t_d$ $(\varepsilon_p > \varepsilon_{py}$ & $\varepsilon_{t,u} < \varepsilon_b')$

With respect to the deformation and stress diagram of [Figure 10.22] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$-2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + 2A_pf_{pd}$$

$$+2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p\frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk} - \varepsilon_{py})t_d^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})t_d^2 + 2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})t_d^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk} - \varepsilon_{py})t_d^2 + 2A_pf_{pd}\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})t_d + 2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)t_d + 2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)t_d - 2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)t_d$$

$$-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})t_d \rightarrow$$

$$-\left(bE_c(\varepsilon_{uk} - \varepsilon_{py})t_d^2 + 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)(t_d - d_p)\right)\varepsilon_0^2$$

$$+2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) + (\varepsilon_{p\infty} - \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)t_d\varepsilon_0$$

$$-2b_w\sigma_{ctmax}(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u})(\varepsilon_{uk} - \varepsilon_{py})t_d^2 = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\left(bE_c\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 + 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\left(t_d - d_p\right)\right)$$

$$b = 2A_p\left(\left(f_{pd} - \sigma_{pm\infty}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) + \left(\varepsilon_{p\infty} - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)t_d$$

$$c = -2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2$$

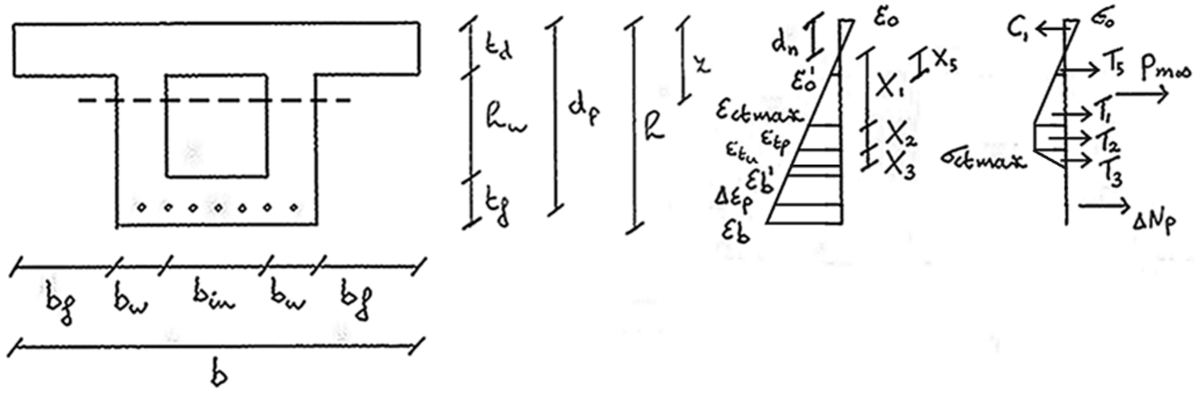


Figure 10.23: deformation and stress diagram when $\varepsilon_p = \varepsilon_{ud}$.

10.23 When $\varepsilon_p = \varepsilon_{ud}$ $(d_n < t_d)$

With respect to the deformation and stress diagram of [Figure 10.23] the following relations are valid:

$$\frac{\varepsilon_0{}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ud} - \varepsilon_{p\infty}}{d_p - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}(d_p - d_n) \rightarrow$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p - \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \rightarrow$$

$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \rightarrow$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$X_5 = t_d - d_n$$

$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \varepsilon_{ud} = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \Delta\varepsilon_p = \varepsilon_{ud} - \varepsilon_{p\infty}$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_5 + \Delta N_p$$

$$\frac{1}{2} bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}X_3 + \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' X_5 + A_p\sigma_p$$
$$-P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2b_f E_c\varepsilon_0' X_5 + b_{in}E_c\varepsilon_0' X_5$$
$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + 2b_f E_c\varepsilon_0' t_d$$
$$-2b_f E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + 2A_p f_{pd} + 2A_p\frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_f E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 - 4b_f E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d$$
$$+2b_f E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 - 2b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d$$
$$+b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2A_p f_{pd}\left(\varepsilon_{uk} - \varepsilon_{py}\right) + 2A_p\left(\varepsilon_{ud} - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty}\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} (\varepsilon_{uk} - \varepsilon_{py}) d_p = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} (\varepsilon_{uk} - \varepsilon_{py}) d_p$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} (\varepsilon_{uk} - \varepsilon_{py}) d_p + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} (\varepsilon_{uk} - \varepsilon_{py}) d_p$$

$$+2b_f E_c \frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{d_p} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 - 4b_f E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) t_d$$

$$+2b_f E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} (\varepsilon_{uk} - \varepsilon_{py}) d_p + b_{in} E_c \frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{d_p} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2$$

$$-2b_{in} E_c \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) t_d + b_{in} E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} (\varepsilon_{uk} - \varepsilon_{py}) d_p + 2A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py})$$

$$+2A_p (\varepsilon_{ud} - \varepsilon_{py}) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2A_p \sigma_{pm\infty} (\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 = -2b_w \sigma_{ctmax} \varepsilon_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 + 2b_w \sigma_{ctmax} \varepsilon_{t,p} (\varepsilon_{uk} - \varepsilon_{py}) d_p^2$$

$$+2b_w \sigma_{ctmax} \varepsilon_{t,u} (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 + 2b_f E_c (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty})^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2$$

$$-4b_f E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) t_d d_p + 2b_f E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) d_p^2$$

$$+b_{in} E_c (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty})^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 - 2b_{in} E_c \varepsilon_0 (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) t_d d_p$$

$$+b_{in} E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 + 2A_p f_{pd} (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) d_p$$

$$+2A_p (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{ud} - \varepsilon_{py}) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p - 2A_p \sigma_{pm\infty} (\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) d_p \rightarrow$$

$$bE_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 = -2b_w \sigma_{ctmax} \varepsilon_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 + 2b_w \sigma_{ctmax} \varepsilon_{t,p} (\varepsilon_{uk} - \varepsilon_{py}) d_p^2$$

$$+2b_w \sigma_{ctmax} \varepsilon_{t,u} (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 + 2b_f E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 + 4b_f E_c \varepsilon_0 (\varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) t_d^2$$

$$+2b_f E_c (\varepsilon_{ud} - \varepsilon_{p\infty})^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 - 4b_f E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d d_p$$

$$-4b_f E_c \varepsilon_0 (\varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) t_d d_p + 2b_f E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) d_p^2 + b_{in} E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2$$

$$+2b_{in} E_c \varepsilon_0 (\varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 + b_{in} E_c (\varepsilon_{ud} - \varepsilon_{p\infty})^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2$$

$$-2b_{in} E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) t_d d_p - 2b_{in} E_c \varepsilon_0 (\varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) t_d d_p + b_{in} E_c \varepsilon_0^2 (\varepsilon_{uk} - \varepsilon_{py}) d_p^2$$

$$+2A_p f_{pd} \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_p + 2A_p f_{pd} (\varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) d_p$$

$$+2A_p \varepsilon_0 (\varepsilon_{ud} - \varepsilon_{py}) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p + 2A_p (\varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{ud} - \varepsilon_{py}) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p$$

$$-2A_p \sigma_{pm\infty} \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) d_p - 2A_p \sigma_{pm\infty} (\varepsilon_{ud} - \varepsilon_{p\infty}) (\varepsilon_{uk} - \varepsilon_{py}) d_p \rightarrow$$

$$-E_c(\varepsilon_{uk}-\varepsilon_{py})\left((b-2b_f-b_{in})d_p^2+(2b_f+b_{in})(2d_p-t_d)t_d\right)\varepsilon_0^2$$

$$+2\left(E_c(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})(2b_f+b_{in})(t_d-d_p)t_d\right.$$
$$\left.+A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)d_p\right)\varepsilon_0$$

$$+(2b_f+b_{in})E_c(\varepsilon_{ud}-\varepsilon_{p\infty})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2-2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$
$$+2A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(\varepsilon_{ud}-\varepsilon_{p\infty})d_p=0\rightarrow$$

$$\varepsilon_0=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=-E_c(\varepsilon_{uk}-\varepsilon_{py})\left((b-2b_f-b_{in})d_p^2+(2b_f+b_{in})(2d_p-t_d)t_d\right)$$

$$b=2\left(E_c(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})(2b_f+b_{in})(t_d-d_p)t_d\right.$$
$$\left.+A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)d_p\right)$$

$$c=(2b_f+b_{in})E_c(\varepsilon_{ud}-\varepsilon_{p\infty})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2-2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$
$$+2A_p\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(\varepsilon_{ud}-\varepsilon_{p\infty})d_p$$

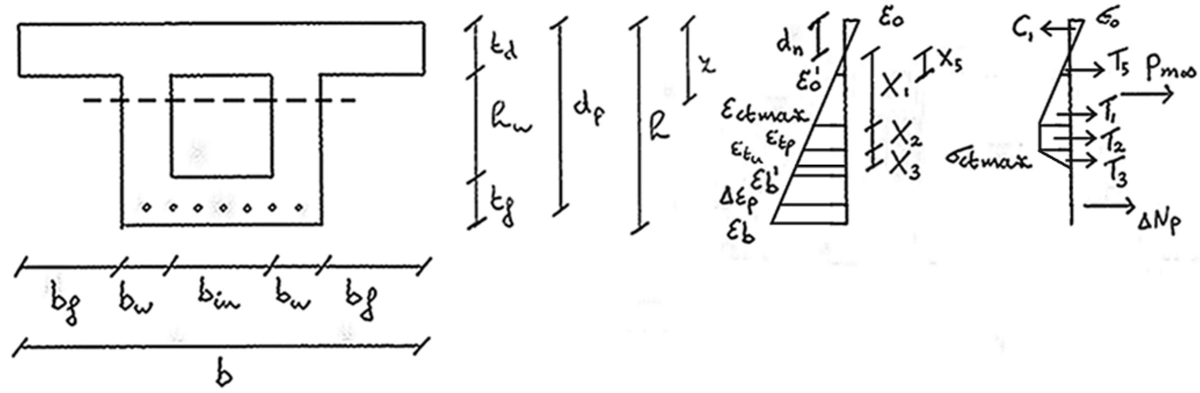


Figure 10.24: deformation and stress diagram when $\varepsilon_p = \varepsilon_{ud}$ & $\varepsilon_0 = \varepsilon_{cmax}$.

10.24 When $\boldsymbol{\varepsilon_p = \varepsilon_{ud}}$ & $\boldsymbol{\varepsilon_0 = \varepsilon_{cmax}}$ $(\boldsymbol{d_n < t_d})$

With respect to the deformation and stress diagram of [Figure 10.24] the following relations are valid:

$$\frac{\varepsilon_0{}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ud} - \varepsilon_{p\infty}}{d_p - d_n} \rightarrow d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}\left(d_p - d_n\right) \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p - \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n \rightarrow d_n + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \rightarrow$$

$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \rightarrow d_n\left(\frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \rightarrow$$
$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$X_5 = t_d - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \varepsilon_{ud} = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \Delta\varepsilon_p = \varepsilon_{ud} - \varepsilon_{p\infty}$$

The amount of prestressing steel A_p can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_5 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax} X_3 + \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' X_5 + A_p \sigma_p$$
$$-P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3 + 2b_f E_c \varepsilon_0' X_5 + b_{in} E_c \varepsilon_0' X_5$$
$$+2A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) \right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n + 2b_f E_c \varepsilon_0' t_d$$
$$-2b_f E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + 2A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - \sigma_{pm\infty} \right) \rightarrow$$

$$b E_c \varepsilon_0 d_n = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n + 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2$$
$$-4b_f E_c \varepsilon_0 t_d + 2b_f E_c \varepsilon_0 d_n + b_{in} E_c \frac{\varepsilon_0}{d_n} t_d^2 - 2b_{in} E_c \varepsilon_0 t_d + b_{in} E_c \varepsilon_0 d_n$$
$$+2A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - \sigma_{pm\infty} \right) \rightarrow$$

$$b E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$
$$+2b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + 2b_f E_c \frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{d_p} t_d^2 - 4b_f E_c \varepsilon_0 t_d$$
$$+2b_f E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + b_{in} E_c \frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{d_p} t_d^2 - 2b_{in} E_c \varepsilon_0 t_d + b_{in} E_c \frac{\varepsilon_0^2}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$
$$+2A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - \sigma_{pm\infty} \right) \rightarrow$$

$$bE_c \frac{\varepsilon_{cmax}^2}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$+2b_f E_c \frac{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}}{d_p} t_d^2 - 4b_f E_c \varepsilon_{cmax} t_d + 2b_f E_c \frac{\varepsilon_{cmax}^2}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$+b_{in} E_c \frac{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}}{d_p} t_d^2 - 2b_{in} E_c \varepsilon_{cmax} t_d + b_{in} E_c \frac{\varepsilon_{cmax}^2}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$+2A_p \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - \sigma_{pm\infty} \right) \rightarrow$$

$$A_p = \frac{\alpha}{\gamma}$$

$$\alpha = \left(b - 2b_f - b_{in}\right) E_c \frac{\varepsilon_{cmax}^2}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p + 2b_w \sigma_{ctmax} \left(\frac{\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}}{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}} \right) d_p$$

$$+\left(2b_f + b_{in}\right) E_c \left(2\varepsilon_{cmax} - \frac{\varepsilon_{cmax} + \varepsilon_{ud} - \varepsilon_{p\infty}}{d_p} t_d \right) t_d$$

$$\gamma = 2 \left(f_{pd} + \frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - \sigma_{pm\infty} \right)$$

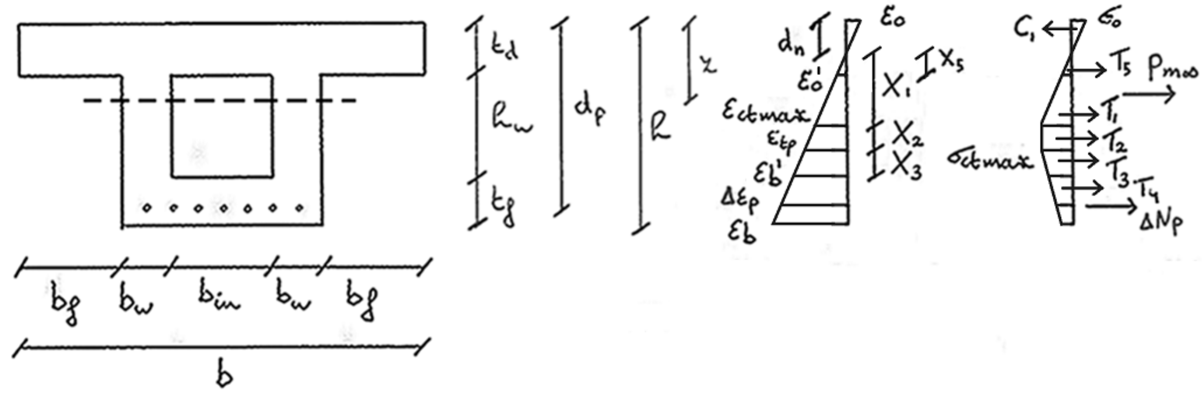


Figure 10.25: deformation and stress diagram when $\varepsilon_b < \varepsilon_{t,u}$.

10.25 When $\boldsymbol{\varepsilon_b < \varepsilon_{t,u}} \left(\boldsymbol{d_n < t_d} \;\&\; \boldsymbol{\varepsilon_p < \varepsilon_{py}}\right)$

With respect to the deformation and stress diagram of [Figure 10.25] the following relations are valid:

$$\frac{{\varepsilon_0}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$X_5 = t_d - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + T_5 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3 - \frac{1}{2}(2b_w)\sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3$$

$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + \frac{1}{2}(2b_f + b_{in}) E_c \varepsilon_0' X_5$$

$$+A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} X_3 - 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3$$

$$+4b_w \sigma_{ctmax} t_f - 4b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in} \sigma_{ctmax} t_f - 2b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_f E_c \varepsilon_0' X_5 + b_{in} E_c \varepsilon_0' X_5 + 2A_p E_p \Delta\varepsilon_p \rightarrow$$

$$b E_c \varepsilon_0 d_n = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n - 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$-4b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in} \sigma_{ctmax} t_f - 2b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$

$$+b_{in} \sigma_{ctmax} \frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_f E_c \varepsilon_0' t_d - 2b_f E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + 2A_p E_p \frac{\varepsilon_0}{d_n} d_p$$

$$-2A_p E_p \varepsilon_0 \rightarrow$$

$$b E_c \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) d_n = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{tu} - \varepsilon_{t,p}) d_n + 4b_w \sigma_{ctmax} (\varepsilon_{tu} - \varepsilon_{t,p}) h$$

$$-4b_w \sigma_{ctmax} (\varepsilon_{tu} - \varepsilon_{t,p}) d_n - 2b_w \sigma_{ctmax} (\varepsilon_b' - \varepsilon_{t,p}) h + 2b_w \sigma_{ctmax} (\varepsilon_b' - \varepsilon_{t,p}) d_n$$

$$+2b_w \sigma_{ctmax} (\varepsilon_b' - \varepsilon_{t,p}) t_f + 2b_w \sigma_{ctmax} (\varepsilon_b' - \varepsilon_{t,p}) \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - 4b_w \sigma_{ctmax} (\varepsilon_b - \varepsilon_{t,p}) t_f$$

$$+2b_{in} \sigma_{ctmax} (\varepsilon_{tu} - \varepsilon_{t,p}) t_f - 2b_{in} \sigma_{ctmax} (\varepsilon_b - \varepsilon_{t,p}) t_f + 2b_w \sigma_{ctmax} (\varepsilon_b - \varepsilon_b') t_f$$

$$+b_{in} \sigma_{ctmax} (\varepsilon_b - \varepsilon_b') t_f + 2b_f E_c \frac{\varepsilon_0}{d_n} (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2 - 4b_f E_c \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d$$

$$+2b_f E_c \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) d_n + b_{in} E_c \frac{\varepsilon_0}{d_n} (\varepsilon_{tu} - \varepsilon_{t,p}) t_d^2 - 2b_{in} E_c \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) t_d$$

$$+b_{in} E_c \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) d_n + 2A_p E_p \frac{\varepsilon_0}{d_n} (\varepsilon_{tu} - \varepsilon_{t,p}) d_p - 2A_p E_p \varepsilon_0 (\varepsilon_{tu} - \varepsilon_{t,p}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$
$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)h + 2b_w\sigma_{ctmax}\varepsilon_{t,p}h$$
$$+2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)d_n - 2b_w\sigma_{ctmax}\varepsilon_{t,p}d_n + 2b_w\sigma_{ctmax}\varepsilon_{t,p}t_f$$
$$+2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n - 2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0\right)t_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_{tu}t_f - b_{in}\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0\right)t_f - b_{in}\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)t_f$$
$$+2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d + 2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n$$
$$+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d + b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n$$
$$+2A_pE_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})d_p - 2A_pE_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})d_n + 4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})h$$
$$-4b_w\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}h^2 + 4b_w\sigma_{ctmax}\varepsilon_0h + 4b_w\sigma_{ctmax}\varepsilon_{t,p}h$$
$$-2b_w\sigma_{ctmax}\varepsilon_0d_n - 4b_w\sigma_{ctmax}\varepsilon_{t,p}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}d_n + 2b_{in}\sigma_{ctmax}\varepsilon_{tu}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}ht_f$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}t_f^2 + 2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})t_d$$
$$+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n + 2A_pE_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})d_p - 2A_pE_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p}) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2 + 4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})hd_n$$
$$-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2 - 2b_w\sigma_{ctmax}\varepsilon_0^2h^2 + 4b_w\sigma_{ctmax}\varepsilon_0^2hd_n + 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}hd_n$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2d_n^2 - 4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}d_n^2 - 2b_w\sigma_{ctmax}\varepsilon_{t,p}^2d_n^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}t_fd_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0^2ht_f + 2b_{in}\sigma_{ctmax}\varepsilon_0^2t_fd_n + b_{in}\sigma_{ctmax}\varepsilon_0^2t_f^2 + 2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n + 2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2 + b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_d^2$$
$$-2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})t_dd_n + b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n^2 + 2A_pE_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_p$$
$$-2A_pE_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})d_n \rightarrow$$

$$-\left(\left(b-2b_f-b_{in}\right)E_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+2\varepsilon_0\varepsilon_{tu}+\varepsilon_0^2+\varepsilon_{t,p}^2\right)\right)d_n^2$$

$$+2\varepsilon_0\left(\sigma_{ctmax}\left(2b_wh+b_{in}t_f\right)\left(\varepsilon_0+\varepsilon_{tu}\right)-\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d-A_pE_p\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\right)d_n$$

$$+\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d^2-\sigma_{ctmax}\left(2b_wh^2+b_{in}t_f\left(2h-t_f\right)\right)+2A_pE_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_p\right)\varepsilon_0^2$$
$$=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=-\left(\left(b-2b_f-b_{in}\right)E_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+2\varepsilon_0\varepsilon_{tu}+\varepsilon_0^2+\varepsilon_{t,p}^2\right)\right)$$

$$b=2\varepsilon_0\left(\sigma_{ctmax}\left(2b_wh+b_{in}t_f\right)\left(\varepsilon_0+\varepsilon_{tu}\right)-\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d-A_pE_p\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\right)$$

$$c=\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)t_d^2-\sigma_{ctmax}\left(2b_wh^2+b_{in}t_f\left(2h-t_f\right)\right)+2A_pE_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)d_p\right)\varepsilon_0^2$$

In order to determine the bending moment capacity the centre of gravity of part X_3 is required:

$$y=\frac{2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3\cdot\frac{1}{2}X_3+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3\cdot\frac{1}{3}X_3}{2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3+\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3}\rightarrow$$

$$y=\frac{b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3^2+\frac{1}{3}b_w\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3^2}{2b_w\sigma_{ctmax}\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3+b_w\sigma_{ctmax}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}X_3}\rightarrow$$

$$y=\frac{\left(\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)+\frac{1}{3}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3^2}{\left(2\left(1-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)+\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3}\rightarrow$$

$$y=\frac{\left(1-\frac{2}{3}\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}\right)X_3}{2-\frac{\varepsilon_b'-\varepsilon_{t,p}}{\varepsilon_{tu}-\varepsilon_{t,p}}}\rightarrow$$

In order to determine the bending moment capacity the centre of gravity of part X_4 is required:

$$z = \frac{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f \cdot \frac{1}{2}t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \cdot \frac{1}{3}t_f}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f} \rightarrow$$

$$z = \frac{\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f^2 + \frac{1}{6}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f^2}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}t_f^2}{\left(\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2} - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f}{1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3(X_1 + X_2 + y) + T_4(X_1 + X_2 + X_3 + z) + T_5 \cdot \frac{2}{3}X_5$$
$$+P_{m\infty}(e - d_n) + \Delta N_p(d_p - d_n)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

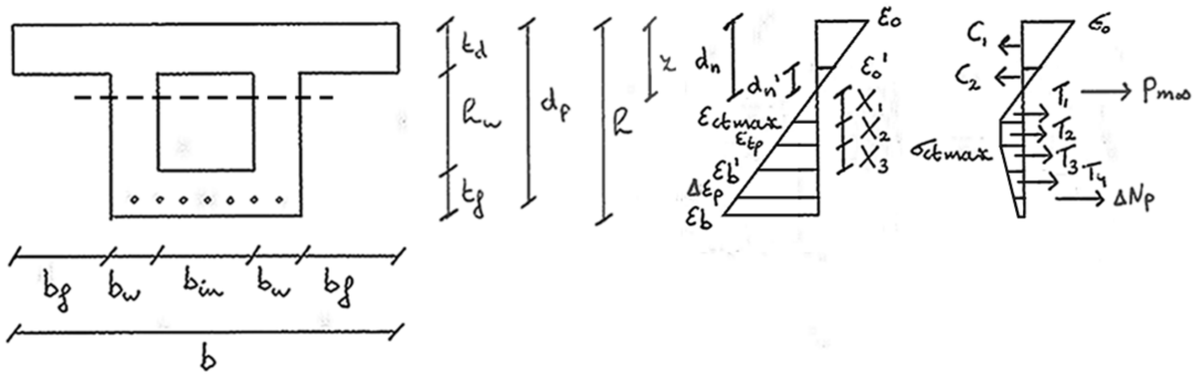


Figure 10.26: deformation and stress diagram when $\varepsilon_{t,p} < \varepsilon_b'$ & $\varepsilon_b < \varepsilon_{t,u}$.

10.26 When $\boldsymbol{\varepsilon_{t,p} < \varepsilon_b'}$ & $\boldsymbol{\varepsilon_b < \varepsilon_{t,u}}$ $\boldsymbol{(d_n > t_d}$ & $\boldsymbol{\varepsilon_p > \varepsilon_{py})}$

With respect to the deformation and stress diagram of [Figure 10.26] the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3$$
$$-\frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3 + (2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) t_f$$
$$+\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0' d_n' - b_{in} E_c \varepsilon_0' d_n' = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 4b_w \sigma_{ctmax} X_3$$
$$-2b_w \sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_3 + 4b_w \sigma_{ctmax} t_f - 4b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in}\sigma_{ctmax} t_f$$
$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$
$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n - 2b_f E_c \varepsilon_0 d_n' + 2b_f E_c \frac{\varepsilon_0}{d_n} d_n' t_d - b_{in} E_c \varepsilon_0 d_n' + b_{in} E_c \frac{\varepsilon_0}{d_n} d_n' t_d = -2b_w \sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$
$$+4b_w \sigma_{ctmax} h - 4b_w \sigma_{ctmax} d_n - 2b_w \sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h + 2b_w \sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n$$
$$+2b_w \sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - 4b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in}\sigma_{ctmax} t_f$$
$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w \sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_p f_{pd}$$
$$+2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n + 4b_f E_c\varepsilon_0 t_d - 2b_f E_c \frac{\varepsilon_0}{d_n} t_d^2 - b_{in}E_c\varepsilon_0 d_n + 2b_{in}E_c\varepsilon_0 t_d - b_{in}E_c \frac{\varepsilon_0}{d_n} t_d^2$$

$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2A_p f_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n - 2b_f E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$+4b_f E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d^2$$

$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_{in}E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d$$

$$-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d^2 = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$+4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})h - 4b_w\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})h + 2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})h + 2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})t_f - 2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$-4b_w\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f + 4b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+2b_{in}\sigma_{ctmax}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_f - 2b_{in}\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk} - \varepsilon_{py})t_f + 2b_w\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f - 2b_w\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})t_f$$

$$+b_{in}\sigma_{ctmax}\varepsilon_b(\varepsilon_{uk} - \varepsilon_{py})t_f - b_{in}\sigma_{ctmax}\varepsilon_b'(\varepsilon_{uk} - \varepsilon_{py})t_f + 2A_p f_{pd}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})$$

$$+2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p - 2A_p\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-2b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+4b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d-2b_fE_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$-b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n+2b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d$$

$$-b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h-4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)h^2+4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)h+4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)h$$

$$-2b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n+4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f$$

$$+2b_{in}\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)ht_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_{in}\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2$$

$$+2A_pf_{pd}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+2A_p\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+2A_p\varepsilon_{p\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\varepsilon_{py}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\sigma_{pm\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dd_n-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dd_n$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})hd_n-4b_w\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})h^2+4b_w\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})hd_n+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})hd_n$$
$$-2b_w\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_fd_n-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_fd_n+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_fd_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_fd_n$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})t_fd_n+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2$$
$$+2A_pf_{pd}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$
$$-2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n+2A_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$-2A_p\varepsilon_0\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n\rightarrow$$

$$\left((b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})+2\varepsilon_0\varepsilon_{tu}+\varepsilon_0^2+\varepsilon_{t,p}^2\right)\right)(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$+2\varepsilon_0\left((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d-\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})(2b_wh+b_{in}t_f)(\varepsilon_0+\varepsilon_{tu})+A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)(\varepsilon_{tu}-\varepsilon_{t,p})\right)d_n$$

$$-\left((2b_f+b_{in})E_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2-\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})\left(2b_wh^2+(2h-t_f)b_{in}t_f\right)+2A_p(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2=0\rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \left(\left(b - 2b_f - b_{in}\right)E_c\varepsilon_0^2\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) + 2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}\left(\varepsilon_{tu} - \varepsilon_{t,p}\right) + 2\varepsilon_0\varepsilon_{tu} + \varepsilon_0^2 + \varepsilon_{t,p}^2\right)\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)$$

$$b = 2\varepsilon_0\left(\left(2b_f + b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d - \sigma_{ctmax}\left(\varepsilon_{uk} - \varepsilon_{py}\right)\left(2b_wh + b_{in}t_f\right)\left(\varepsilon_0 + \varepsilon_{tu}\right) + A_p\left(\left(\sigma_{pm\infty} - f_{pd}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) + \left(\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\right)$$

$$c = -\left(\left(2b_f + b_{in}\right)E_c\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 - \sigma_{ctmax}\left(\varepsilon_{uk} - \varepsilon_{py}\right)\left(2b_wh^2 + \left(2h - t_f\right)b_{in}t_f\right) + 2A_p\left(\varepsilon_{tu} - \varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p\right)\varepsilon_0^2$$

In order to determine the bending moment capacity the centre of gravity of part X_3 is required:

$$y = \frac{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 \cdot \frac{1}{2}X_3 + \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3 \cdot \frac{1}{3}X_3}{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 + \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3} \rightarrow$$

$$y = \frac{b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3^2 + \frac{1}{3}b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3^2}{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 + b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3} \rightarrow$$

$$y = \frac{\left(\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{3}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3^2}{\left(2\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3} \rightarrow$$

$$y = \frac{\left(1 - \frac{2}{3}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3}{2 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

In order to determine the bending moment capacity the centre of gravity of part X_4 is required:

$$z = \frac{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f \cdot \frac{1}{2}t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \cdot \frac{1}{3}t_f}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f} \rightarrow$$

$$z = \frac{\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f^2 + \frac{1}{6}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f^2}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}t_f^2}{\left(\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2} - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f}{1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d_n' + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3(X_1 + X_2 + y) + T_4(X_1 + X_2 + X_3 + z)$$
$$+P_{m\infty}(e - d_n) + \Delta N_p\left(d_p - d_n\right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

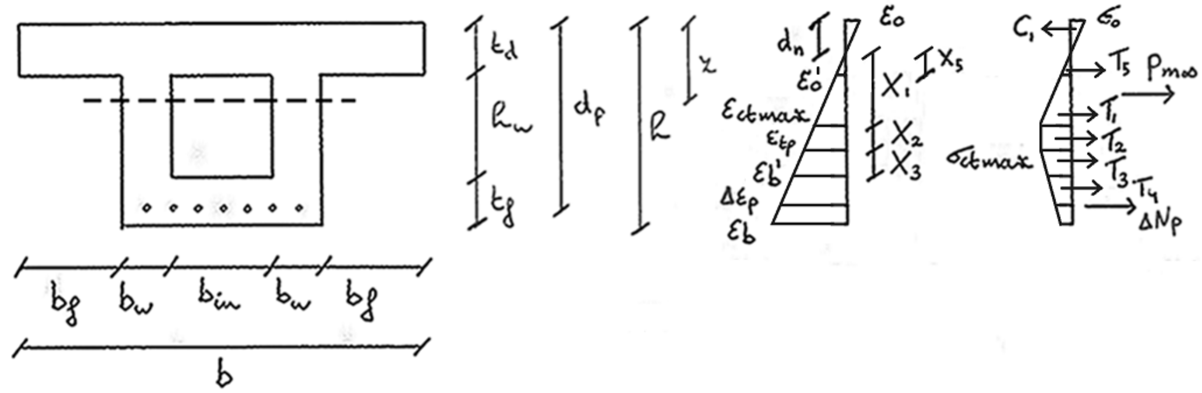


Figure 10.27: deformation and stress diagram when $\varepsilon_{t,p} < \varepsilon_b' \ \& \ \varepsilon_b < \varepsilon_{t,u}$.

10.27 When $\boldsymbol{\varepsilon_{t,p} < \varepsilon_b' \ \& \ \varepsilon_b < \varepsilon_{t,u} \left(d_n < t_d \ \& \ \varepsilon_p > \varepsilon_{py}\right)}$

With respect to the deformation and stress diagram of [Figure 10.27] the following relations are valid:

$$\frac{{\varepsilon_0}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{{\varepsilon_b}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_b}{h - d_n} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n}(h - d_n) \rightarrow \varepsilon_b = \frac{\varepsilon_0}{d_n} h - \varepsilon_0$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$X_5 = t_d - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + T_5 + \Delta N_p$$

$$\frac{1}{2} bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 - \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$
$$+(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' X_5$$
$$+A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 4b_w\sigma_{ctmax}X_3 - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3$$
$$+4b_w\sigma_{ctmax}t_f - 4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_f E_c\varepsilon_0' X_5 + b_{in}E_c\varepsilon_0' X_5$$
$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 4b_w\sigma_{ctmax}h - 4b_w\sigma_{ctmax}d_n - 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}h$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n$$
$$-4b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_{in}\sigma_{ctmax}t_f - 2b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_w\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2b_f E_c\varepsilon_0' t_d - 2b_f E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + 2A_p f_{pd}$$
$$+2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h-4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$-2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h+2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_w\sigma_{ctmax}\left(\varepsilon_b'-\varepsilon_{t,p}\right)\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$-4b_w\sigma_{ctmax}\left(\varepsilon_b-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_{in}\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f$$

$$-2b_{in}\sigma_{ctmax}\left(\varepsilon_b-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_w\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f$$

$$+b_{in}\sigma_{ctmax}(\varepsilon_b-\varepsilon_b')\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_fE_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$-4b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d+2b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-2b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d$$

$$+b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n+2A_pf_{pd}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$+2A_p\left(\varepsilon_p-\varepsilon_{py}\right)\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\sigma_{pm\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h-4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$-2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h+2b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)h$$

$$+2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-2b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-2b_w\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{tu}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f-b_{in}\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f$$

$$-b_{in}\sigma_{ctmax}\left(\frac{\varepsilon_0}{d_n}h-\varepsilon_0-\frac{\varepsilon_0}{d_n}t_f\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f+2b_fE_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$-4b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d+2b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-2b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d$$

$$+b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n+2A_pf_{pd}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$+2A_p\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\varepsilon_{py}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)h-4b_w\sigma_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$-2b_w\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)h^2+4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)h+4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)h$$

$$-2b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-4b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}^2}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{tu}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)ht_f+2b_{in}\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2+2b_fE_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$-4b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d+2b_fE_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-2b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d$$

$$+b_{in}E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n+2A_pf_{pd}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$+2A_p\frac{\varepsilon_0}{d_n}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\varepsilon_{py}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$bE_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)hd_n-4b_w\sigma_{ctmax}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$-2b_w\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)h^2+4b_w\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)hd_n+4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)hd_n$$

$$-2b_w\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2-4b_w\sigma_{ctmax}\varepsilon_0\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2-2b_w\sigma_{ctmax}\varepsilon_{t,p}^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_fd_n-2b_{in}\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)ht_f$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_fd_n+b_{in}\sigma_{ctmax}\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_f^2+2b_fE_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2$$

$$-4b_fE_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_dd_n+2b_fE_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+b_{in}E_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-2b_{in}E_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_dd_n$$

$$+b_{in}E_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2+2A_pf_{pd}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2A_p\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$

$$+2A_p\varepsilon_0\varepsilon_{p\infty}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n-2A_p\varepsilon_0\varepsilon_{py}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$

$$-2A_p\sigma_{pm\infty}\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n\rightarrow$$

$$-\left(\left(b-2b_f-b_{in}\right)E_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+2\varepsilon_0\varepsilon_{tu}+\varepsilon_0^2+\varepsilon_{t,p}^2\right)\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+2\varepsilon_0\left(\sigma_{ctmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)\left(2b_wh+b_{in}t_f\right)\left(\varepsilon_0+\varepsilon_{tu}\right)-\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d\right.$$
$$\left.+A_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)-\left(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)d_n$$

$$+\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-\sigma_{ctmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)\left(2b_wh^2+\left(2h-t_f\right)b_{in}t_f\right)\right.$$
$$\left.+2A_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=-\left(\left(b-2b_f-b_{in}\right)E_c\varepsilon_0^2\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\right.$$
$$\left.+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)+2\varepsilon_0\varepsilon_{tu}+\varepsilon_0^2+\varepsilon_{t,p}^2\right)\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$b=2\varepsilon_0\left(\sigma_{ctmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)\left(2b_wh+b_{in}t_f\right)\left(\varepsilon_0+\varepsilon_{tu}\right)-\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d\right.$$
$$\left.+A_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\left(f_{pd}-\sigma_{pm\infty}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)-\left(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)$$

$$c=\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-\sigma_{ctmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)\left(2b_wh^2+\left(2h-t_f\right)b_{in}t_f\right)\right.$$
$$\left.+2A_p\left(\varepsilon_{tu}-\varepsilon_{t,p}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2$$

In order to determine the bending moment capacity the centre of gravity of part X_3 is required:

$$y = \frac{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 \cdot \frac{1}{2}X_3 + \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3 \cdot \frac{1}{3}X_3}{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 + \frac{1}{2}(2b_w)\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3} \rightarrow$$

$$y = \frac{b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3^2 + \frac{1}{3}b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3^2}{2b_w\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3 + b_w\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_3} \rightarrow$$

$$y = \frac{\left(\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{3}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3^2}{\left(2\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)b_w\sigma_{ctmax}X_3} \rightarrow$$

$$y = \frac{\left(1 - \frac{2}{3}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_3}{2 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

In order to determine the bending moment capacity the centre of gravity of part X_4 is required:

$$z = \frac{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f \cdot \frac{1}{2}t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f \cdot \frac{1}{3}t_f}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f} \rightarrow$$

$$z = \frac{\frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f^2 + \frac{1}{6}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f^2}{(2b_w + b_{in})\sigma_{ctmax}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f + \frac{1}{2}(2b_w + b_{in})\sigma_{ctmax}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2}\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}t_f^2}{\left(\left(1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)(2b_w + b_{in})\sigma_{ctmax}t_f} \rightarrow$$

$$z = \frac{\left(\frac{1}{2} - \frac{1}{2}\frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{6}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)t_f}{1 - \frac{\varepsilon_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} + \frac{1}{2}\frac{\varepsilon_b - \varepsilon_b'}{\varepsilon_{tu} - \varepsilon_{t,p}}} \rightarrow$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right) + T_3 (X_1 + X_2 + y) + T_4 (X_1 + X_2 + X_3 + z) + T_5 \cdot \frac{2}{3} X_5$$
$$+ P_{m\infty} (e - d_n) + \Delta N_p \left(d_p - d_n \right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

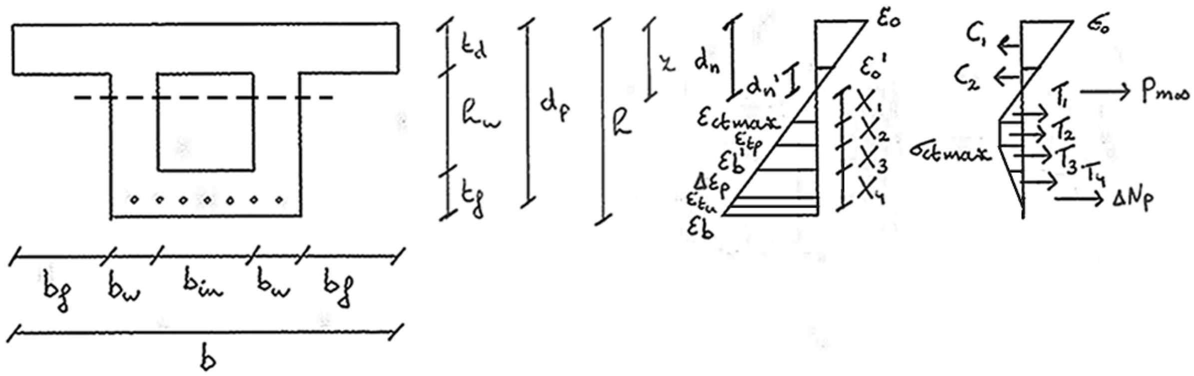


Figure 10.28: deformation and stress diagram when $\varepsilon_b > \varepsilon_{t,u} > \varepsilon_b'$.

10.28 When $\boldsymbol{\varepsilon_b > \varepsilon_{t,u} > \varepsilon_b'}$ $(\boldsymbol{d_n > t_d}$ & $\boldsymbol{\varepsilon_p > \varepsilon_{py}})$

With respect to the deformation and stress diagram of [Figure 10.28] the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3 + X_4} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 + X_4 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_4 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2 + X_3)$$
$$= \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - h + d_n + t_f$$

$$d_n{}' = d_n - t_d$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + T_4 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}(X_3 + X_4)$$
$$+\frac{1}{2}b_{in}\sigma_{ctmax}\left(1 - \frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right)X_4 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3$$
$$+2b_w\sigma_{ctmax}X_4 + b_{in}\sigma_{ctmax}X_4 - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}X_4 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$
$$-2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n' + 2b_f E_c\frac{\varepsilon_0}{d_n}d_n' t_d - b_{in}E_c\varepsilon_0 d_n' + b_{in}E_c\frac{\varepsilon_0}{d_n}d_n' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n - b_{in}\sigma_{ctmax}h + b_{in}\sigma_{ctmax}d_n$$
$$+b_{in}\sigma_{ctmax}t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}h - b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}d_n$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon_b' - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}t_f + 2A_p f_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n-2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$
$$-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n-b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$
$$+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n+b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$
$$-b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n-b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$
$$+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})+2A_p(\varepsilon_p-\varepsilon_{py})(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n-2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d-2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$
$$-b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}^2}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n-2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})h+2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_f+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})h^2-2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})h$$
$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n+2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f$$
$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})t_f^2+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})$$
$$+2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$
$$+2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})\rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+4b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dd_n-2b_fE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_dd_n$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2-2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})hd_n+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_fd_n+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})h^2-2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})hd_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})ht_f+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2+2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_fd_n$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2+2A_pf_{pd}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$+2A_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n-2A_p\varepsilon_0\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n\rightarrow$$

$$\Big((b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})$$
$$+\sigma_{ctmax}\big(2b_w(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})-b_{in}(\varepsilon_0+\varepsilon_{tu})^2\big)\Big)(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$+2\varepsilon_0\Bigg((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d+b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{tu})(h-t_f)(\varepsilon_{uk}-\varepsilon_{py})$$
$$+A_p(\varepsilon_{tu}-\varepsilon_{t,p})\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\Bigg)d_n$$

$$-\Bigg((2b_f+b_{in})E_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2+b_{in}\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$
$$+2A_p(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\Bigg)\varepsilon_0^2=0\rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \Big((b - 2b_f - b_{in})E_c\varepsilon_0^2(\varepsilon_{tu} - \varepsilon_{t,p}) + \sigma_{ctmax}\big(2b_w(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) - b_{in}(\varepsilon_0 + \varepsilon_{tu})^2\big)\Big)(\varepsilon_{uk} - \varepsilon_{py})$$

$$b = 2\varepsilon_0\left((2b_f + b_{in})E_c\varepsilon_0(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d + b_{in}\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{tu})(h - t_f)(\varepsilon_{uk} - \varepsilon_{py}) + A_p(\varepsilon_{tu} - \varepsilon_{t,p})\left((\sigma_{pm\infty} - f_{pd})(\varepsilon_{uk} - \varepsilon_{py}) + (\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)$$

$$c = -\left((2b_f + b_{in})E_c(\varepsilon_{tu} - \varepsilon_{t,p})(\varepsilon_{uk} - \varepsilon_{py})t_d^2 + b_{in}\sigma_{ctmax}(\varepsilon_{uk} - \varepsilon_{py})(h - t_f)^2 + 2A_p(\varepsilon_{tu} - \varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p\right)\varepsilon_0^2$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + C_2 \cdot \frac{2}{3}d_n' + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3\left(X_1 + X_2 + \frac{1}{3}(X_3 + X_4)\right) + T_4\left(X_1 + X_2 + X_3 + \frac{1}{3}X_4\right) + P_{m\infty}(e - d_n) + \Delta N_p(d_p - d_n)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

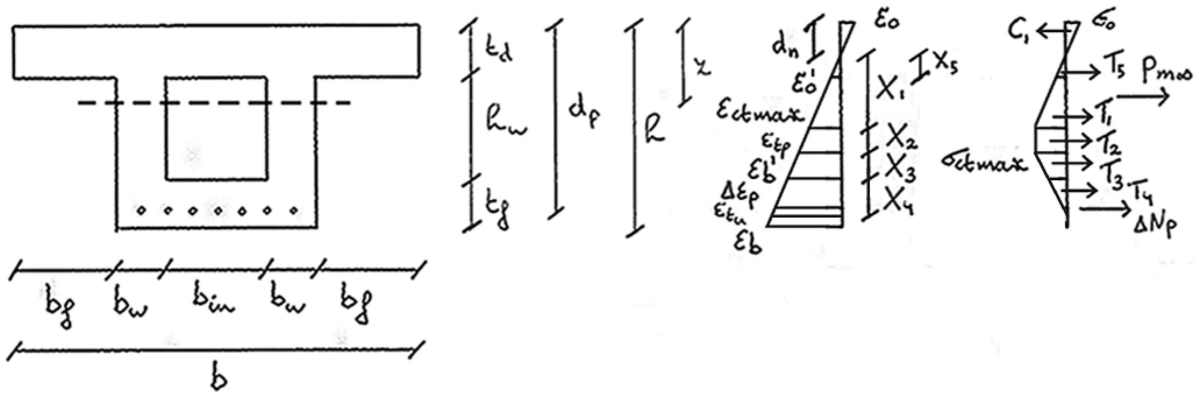


Figure 10.29: deformation and stress diagram when $\varepsilon_b > \varepsilon_{t,u} > \varepsilon_b'$.

10.29 When $\varepsilon_b > \varepsilon_{t,u} > \varepsilon_b'$ $(d_n < t_d$ & $\varepsilon_p > \varepsilon_{py})$

With respect to the deformation and stress diagram of [Figure 10.29] the following relations are valid:

$$\frac{\varepsilon_0{}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_b{}'}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_b' = \frac{\varepsilon_0}{d_n}(X_1 + X_2 + X_3) = \frac{\varepsilon_0}{d_n} h - \varepsilon_0 - \frac{\varepsilon_0}{d_n} t_f$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3 + X_4} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 + X_4 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_4 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2 + X_3)$$
$$= \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - h + d_n + t_f$$

$$X_1 + X_2 + X_3 = h - d_n - t_f \rightarrow X_3 = h - d_n - t_f - (X_1 + X_2) = h - d_n - t_f - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$X_5 = t_d - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_4 + T_5 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}(X_3 + X_4)$$
$$+\frac{1}{2} b_{in}\sigma_{ctmax}\left(1 - \frac{\varepsilon'_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\right) X_4 + \frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon'_0 X_5 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3 + 2b_w \sigma_{ctmax} X_4 + b_{in}\sigma_{ctmax} X_4$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon'_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} X_4 + 2b_f E_c \varepsilon'_0 X_5 + b_{in} E_c \varepsilon'_0 X_5 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$
$$-2A_p\sigma_{pm\infty} \rightarrow$$

$$b E_c \varepsilon_0 d_n = -2b_w \sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n + b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n$$
$$-b_{in}\sigma_{ctmax} h + b_{in}\sigma_{ctmax} d_n + b_{in}\sigma_{ctmax} t_f - b_{in}\sigma_{ctmax}\frac{\varepsilon'_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}}\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n + b_{in}\sigma_{ctmax}\frac{\varepsilon'_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} h$$
$$-b_{in}\sigma_{ctmax}\frac{\varepsilon'_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} d_n - b_{in}\sigma_{ctmax}\frac{\varepsilon'_b - \varepsilon_{t,p}}{\varepsilon_{tu} - \varepsilon_{t,p}} t_f + 2b_f E_c \varepsilon'_0 t_d - 2b_f E_c \varepsilon'_0 d_n + b_{in} E_c \varepsilon'_0 t_d$$
$$-b_{in} E_c \varepsilon'_0 d_n + 2A_p f_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n - b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$+b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + b_{in}\sigma_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$-b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n + b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n - b_{in}\sigma_{ctmax}(\varepsilon_b'-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$+2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$+2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d + b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) + 2A_p(\varepsilon_p-\varepsilon_{py})(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_{t,u}^2}{\varepsilon_0}(\varepsilon_{uk}-\varepsilon_{py})d_n - 2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})h + 2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+2b_{in}\sigma_{ctmax}\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_f + b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})h^2 - 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})h$$

$$-2b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})ht_f + b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n + 2b_{in}\sigma_{ctmax}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_f$$

$$+b_{in}\sigma_{ctmax}\frac{\varepsilon_0}{d_n}(\varepsilon_{uk}-\varepsilon_{py})t_f^2 + 2b_fE_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-4b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d + 2b_fE_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$

$$+b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$+b_{in}E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})$$

$$+2A_p\frac{\varepsilon_0}{d_n}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p - 2A_p\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$+2A_p\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right) - 2A_p\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+b_{in}\sigma_{ctmax}\varepsilon_{t,u}^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 - 2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})hd_n + 2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+2b_{in}\sigma_{ctmax}\varepsilon_0\varepsilon_{tu}(\varepsilon_{uk}-\varepsilon_{py})t_f d_n + b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})h^2 - 2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})hd_n$$
$$-2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})ht_f + b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_f d_n$$
$$+b_{in}\sigma_{ctmax}\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_f^2 + 2b_f E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$-4b_f E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d d_n + 2b_f E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d d_n$$
$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2A_p f_{pd}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n$$
$$+2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p - 2A_p\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$+2A_p\varepsilon_0\varepsilon_{p\infty}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n - 2A_p\varepsilon_0\varepsilon_{py}(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})d_n \rightarrow$$

$$-\Big((b-2b_f-b_{in})E_c\varepsilon_0^2(\varepsilon_{tu}-\varepsilon_{t,p})$$
$$+\sigma_{ctmax}\big(2b_w(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u}) - b_{in}(\varepsilon_0+\varepsilon_{tu})^2\big)\Big)(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$-2\varepsilon_0\Bigg((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d + b_{in}\sigma_{ctmax}(\varepsilon_0+\varepsilon_{tu})(h-t_f)(\varepsilon_{uk}-\varepsilon_{py})$$
$$+A_p(\varepsilon_{tu}-\varepsilon_{t,p})\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py}) + (\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\Bigg)d_n$$

$$+\Bigg((2b_f+b_{in})E_c(\varepsilon_{tu}-\varepsilon_{t,p})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + b_{in}\sigma_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})(h-t_f)^2$$
$$+2A_p(\varepsilon_{tu}-\varepsilon_{t,p})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\Bigg)\varepsilon_0^2 = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -\Big(\big(b - 2b_f - b_{in}\big)E_c\varepsilon_0^2\big(\varepsilon_{tu} - \varepsilon_{t,p}\big) + \sigma_{ctmax}\big(2b_w\big(\varepsilon_{tu} - \varepsilon_{t,p}\big)\big(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}\big) - b_{in}(\varepsilon_0 + \varepsilon_{tu})^2\big)\Big)\big(\varepsilon_{uk} - \varepsilon_{py}\big)$$

$$b = -2\varepsilon_0\Bigg(\big(2b_f + b_{in}\big)E_c\varepsilon_0\big(\varepsilon_{tu} - \varepsilon_{t,p}\big)\big(\varepsilon_{uk} - \varepsilon_{py}\big)t_d + b_{in}\sigma_{ctmax}(\varepsilon_0 + \varepsilon_{tu})\big(h - t_f\big)\big(\varepsilon_{uk} - \varepsilon_{py}\big) + A_p\big(\varepsilon_{tu} - \varepsilon_{t,p}\big)\left(\big(\sigma_{pm\infty} - f_{pd}\big)\big(\varepsilon_{uk} - \varepsilon_{py}\big) + \big(\varepsilon_0 - \varepsilon_{p\infty} + \varepsilon_{py}\big)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\Bigg)$$

$$c = \left(\big(2b_f + b_{in}\big)E_c\big(\varepsilon_{tu} - \varepsilon_{t,p}\big)\big(\varepsilon_{uk} - \varepsilon_{py}\big)t_d^2 + b_{in}\sigma_{ctmax}\big(\varepsilon_{uk} - \varepsilon_{py}\big)\big(h - t_f\big)^2 + 2A_p\big(\varepsilon_{tu} - \varepsilon_{t,p}\big)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p\right)\varepsilon_0^2$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3}d_n + T_1 \cdot \frac{2}{3}X_1 + T_2\left(X_1 + \frac{1}{2}X_2\right) + T_3\left(X_1 + X_2 + \frac{1}{3}(X_3 + X_4)\right) + T_4\left(X_1 + X_2 + X_3 + \frac{1}{3}X_4\right) + T_5 \cdot \frac{2}{3}X_5 + P_{m\infty}(e - d_n) + \Delta N_p\big(d_p - d_n\big)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

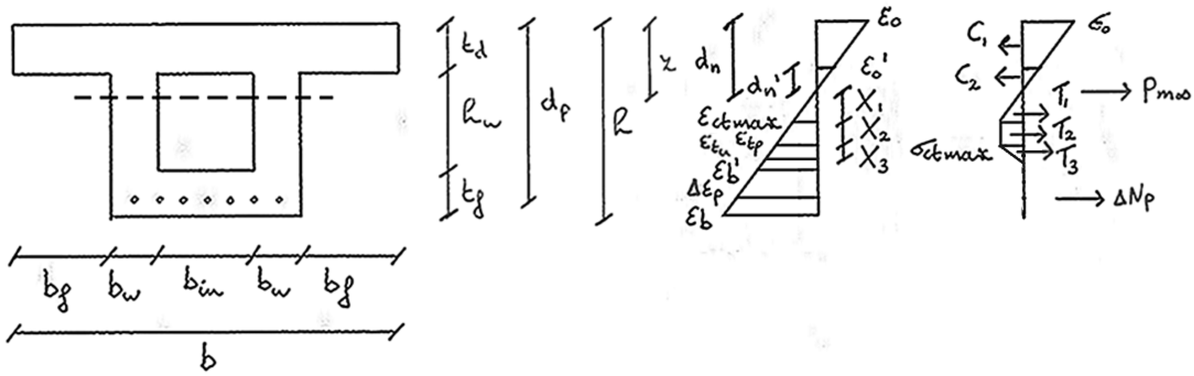


Figure 10.30: deformation and stress diagram when $\varepsilon_b' > \varepsilon_{t,u}$.

10.30 When $\boldsymbol{\varepsilon_b' > \varepsilon_{t,u}}$ $\boldsymbol{(d_n > t_d \ \& \ \varepsilon_p > \varepsilon_{py})}$

With respect to the deformation and stress diagram of [Figure 10.30] the following relations are valid:

$$\frac{{\varepsilon_0}'}{{d_n}'} = \frac{\varepsilon_0}{d_n} \rightarrow {\varepsilon_0}' = \frac{\varepsilon_0}{d_n} {d_n}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$${d_n}' = d_n - t_d$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n - \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' d_n' = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}X_3$$
$$+A_p\sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0' d_n' - b_{in}E_c\varepsilon_0' d_n' = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3$$
$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n - 2b_f E_c\varepsilon_0 d_n' + 2b_f E_c\frac{\varepsilon_0}{d_n}d_n' t_d - b_{in}E_c\varepsilon_0 d_n' + b_{in}E_c\frac{\varepsilon_0}{d_n}d_n' t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + 2A_p f_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n - 2b_f E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 4b_f E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2$$
$$-b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2A_p f_{pd}\left(\varepsilon_{uk} - \varepsilon_{py}\right) + 2A_p\left(\varepsilon_p - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty}\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n - 2b_f E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 4b_f E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d - 2b_f E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2$$
$$-b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d - b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2$$
$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2A_p f_{pd}\left(\varepsilon_{uk} - \varepsilon_{py}\right) + 2A_p\frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$
$$-2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2-2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2+4b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_dd_n$$
$$-2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2-b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2+2b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_dd_n$$
$$-b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_n^2+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})d_n^2+2A_pf_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n+2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$
$$-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n+2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n-2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n\rightarrow$$

$$\left((b-2b_f-b_{in})E_c\varepsilon_0^2+2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$+2\varepsilon_0\Bigg((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d$$
$$+A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\Bigg)d_n$$

$$-\left((2b_f+b_{in})E_c(\varepsilon_{uk}-\varepsilon_{py})t_d^2+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=\left((b-2b_f-b_{in})E_c\varepsilon_0^2+2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})$$

$$b=2\varepsilon_0\Bigg((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d$$
$$+A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\Bigg)$$

$$c=-\left((2b_f+b_{in})E_c(\varepsilon_{uk}-\varepsilon_{py})t_d^2+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3} d_n + C_2 \cdot \frac{2}{3} d_n' + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right) + T_3 \left(X_1 + X_2 + \frac{1}{3} X_3 \right) + P_{m\infty} (e - d_n)$$
$$+ \Delta N_p \left(d_p - d_n \right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

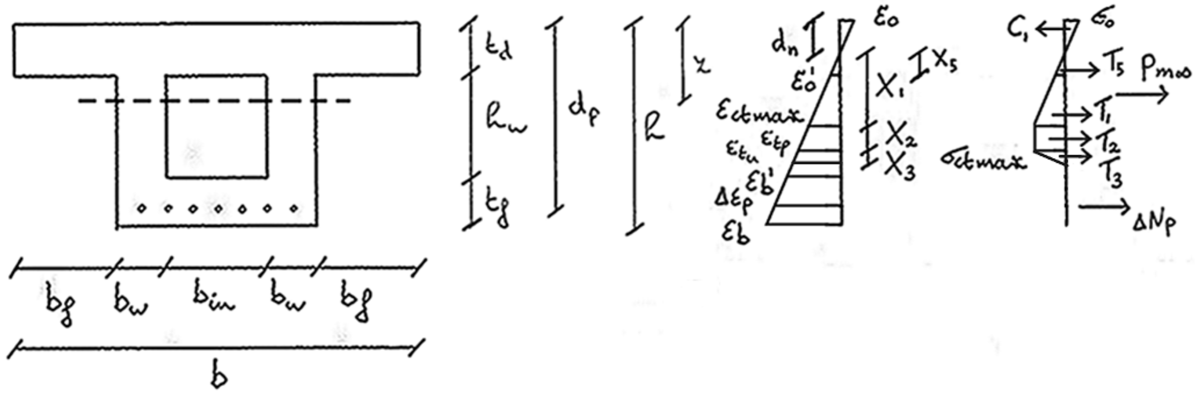


Figure 10.31: deformation and stress diagram when $\varepsilon_b' > \varepsilon_{t,u}$.

10.31 When $\varepsilon_b' > \varepsilon_{t,u}$ $(d_n < t_d$ & $\varepsilon_p > \varepsilon_{py})$

With respect to the deformation and stress diagram of [Figure 10.31] the following relations are valid:

$$\frac{\varepsilon_0{}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$X_5 = t_d - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_5 + \Delta N_p$$

$$\frac{1}{2}bE_c\varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}X_3 + \frac{1}{2}\left(2b_f + b_{in}\right)E_c\varepsilon_0' X_5 + A_p\sigma_p$$
$$-P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2b_fE_c\varepsilon_0' X_5 + b_{in}E_c\varepsilon_0' X_5$$
$$+2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + 2b_fE_c\varepsilon_0' t_d$$
$$-2b_fE_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + 2A_pf_{pd} + 2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_fE_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 - 4b_fE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d$$
$$+2b_fE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 - 2b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d$$
$$+b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2A_pf_{pd}\left(\varepsilon_{uk} - \varepsilon_{py}\right) + 2A_p\left(\varepsilon_p - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$-2A_p\sigma_{pm\infty}\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2b_fE_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 - 4b_fE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d$$
$$+2b_fE_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2 - 2b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d$$
$$+b_{in}E_c\varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right)d_n + 2A_pf_{pd}\left(\varepsilon_{uk} - \varepsilon_{py}\right) + 2A_p\frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p - 2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)$$
$$+2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$bE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$
$$+2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 4b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d d_n$$
$$+2b_fE_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d d_n$$
$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_n^2 + 2A_pf_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n + 2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$
$$-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n + 2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n - 2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$
$$-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_n \rightarrow$$

$$-\left((b-2b_f-b_{in})E_c\varepsilon_0^2 + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})d_n^2$$

$$-2\varepsilon_0\left((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d + A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py}) + (\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)d_n$$

$$+\left((2b_f+b_{in})E_c(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2 = 0 \rightarrow$$

$$d_n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$a = -\left((b-2b_f-b_{in})E_c\varepsilon_0^2 + 2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})$$

$$b = -2\varepsilon_0\left((2b_f+b_{in})E_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d + A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py}) + (\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)$$

$$c = \left((2b_f+b_{in})E_c(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2$$

The bending moment capacity and curvature:

$$M = C_1 \cdot \frac{2}{3} d_n + T_1 \cdot \frac{2}{3} X_1 + T_2 \left(X_1 + \frac{1}{2} X_2 \right) + T_3 \left(X_1 + X_2 + \frac{1}{3} X_3 \right) + T_5 \cdot \frac{2}{3} X_5 + P_{m\infty}(e - d_n)$$
$$+ \Delta N_p \left(d_p - d_n \right)$$

$$\kappa = \frac{\varepsilon_0}{d_n}$$

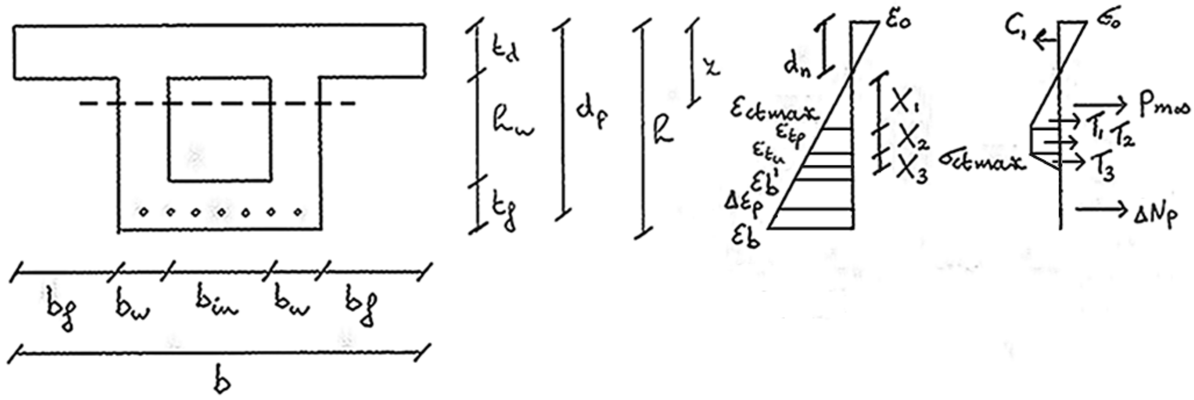


Figure 10.32: deformation and stress diagram when $d_n = t_d$ & $\varepsilon_0 = \varepsilon_{cmax}$.

10.32 When $d_n = t_d$ & $\varepsilon_0 = \varepsilon_{cmax}$ $(\varepsilon_p > \varepsilon_{py}$ & $\varepsilon_{t,u} < \varepsilon_b')$

With respect to the deformation and stress diagram of [Figure 10.32] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The amount of prestressing steel A_p can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + \Delta N_p$$

$$\frac{1}{2} b E_c \varepsilon_0 d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)$$

$$-2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0 d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n + 2A_pf_{pd}$$

$$+2A_p\frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$bE_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p\frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$

$$-2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$bE_c\varepsilon_{cmax}(\varepsilon_{uk} - \varepsilon_{py})t_d = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_{cmax}}(\varepsilon_{uk} - \varepsilon_{py})t_d + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_{cmax}}(\varepsilon_{uk} - \varepsilon_{py})t_d$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_{cmax}}(\varepsilon_{uk} - \varepsilon_{py})t_d + 2A_pf_{pd}(\varepsilon_{uk} - \varepsilon_{py}) + 2A_p\frac{\varepsilon_{cmax}}{t_d}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)d_p$$

$$-2A_p\varepsilon_{cmax}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$\left(bE_c\varepsilon_{cmax} + 2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}}{\varepsilon_{cmax}}\right)(\varepsilon_{uk} - \varepsilon_{py})t_d$$

$$= 2A_p\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) + \left(\frac{\varepsilon_{cmax}}{t_d}d_p - \varepsilon_{cmax} + \varepsilon_{p\infty} - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) \rightarrow$$

$$A_p = \frac{\left(bE_c\varepsilon_{cmax} + 2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}}{\varepsilon_{cmax}}\right)(\varepsilon_{uk} - \varepsilon_{py})t_d}{2\left((f_{pd} - \sigma_{pm\infty})(\varepsilon_{uk} - \varepsilon_{py}) + \left(\frac{\varepsilon_{cmax}}{t_d}d_p - \varepsilon_{cmax} + \varepsilon_{p\infty} - \varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)}$$

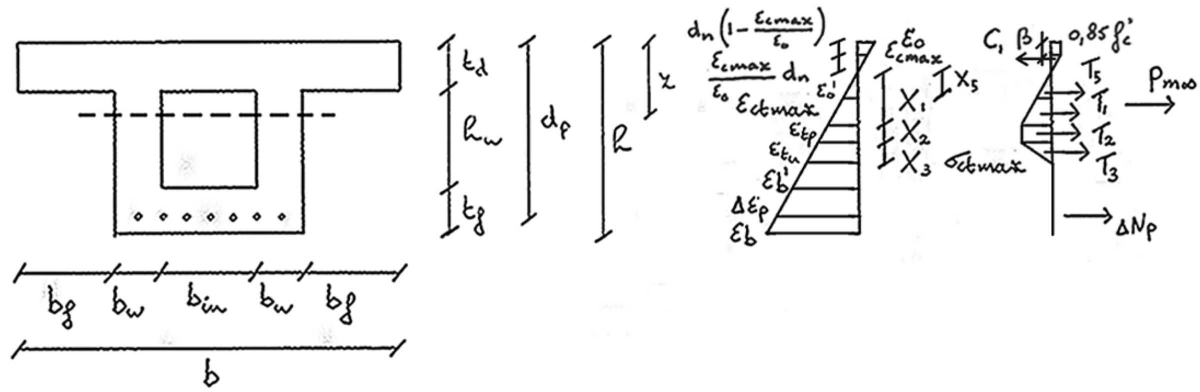


Figure 10.33: deformation and stress diagram when $\varepsilon_p = \varepsilon_{ud}$ & $\varepsilon_0 > \varepsilon_{cmax}$.

10.33 When $\boldsymbol{\varepsilon_p = \varepsilon_{ud}}$ & $\boldsymbol{\varepsilon_0 > \varepsilon_{cmax}}$ $\boldsymbol{(d_n < t_d)}$

With respect to the deformation and stress diagram of [Figure 10.33] the following relations are valid:

$$\frac{{\varepsilon_0}'}{X_5} = \frac{\varepsilon_0}{d_n} \to \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \to X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \to X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \to X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \to X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \to X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \to \frac{\varepsilon_0}{d_n} = \frac{\varepsilon_{ud} - \varepsilon_{p\infty}}{d_p - d_n} \to d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}\left(d_p - d_n\right) \to$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p - \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n \to d_n + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_n = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \to$$

$$d_n\left(1 + \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \to d_n\left(\frac{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}}{\varepsilon_{ud} - \varepsilon_{p\infty}}\right) = \frac{\varepsilon_0}{\varepsilon_{ud} - \varepsilon_{p\infty}} d_p \to$$

$$d_n = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_{ud} - \varepsilon_{p\infty}} d_p$$

$$X_5 = t_d - d_n$$

$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \varepsilon_{ud} = \Delta\varepsilon_p + \varepsilon_{p\infty} \rightarrow \Delta\varepsilon_p = \varepsilon_{ud} - \varepsilon_{p\infty}$

The compressive strain at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_5 + \Delta N_p$$

$$0{,}85 f_c' \left(1 - \frac{\varepsilon_{cmax}}{2\varepsilon_0}\right) b d_n = \frac{1}{2}(2b_w)\sigma_{ctmax}X_1 + 2b_w\sigma_{ctmax}X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax}X_3$$
$$+\frac{1}{2}(2b_f + b_{in})E_c\varepsilon_0' X_5 + A_p\sigma_p - P_{m\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0} d_n = 2b_w\sigma_{ctmax}X_1 + 4b_w\sigma_{ctmax}X_2 + 2b_w\sigma_{ctmax}X_3 + 2b_f E_c \varepsilon_0' X_5$$
$$+b_{in}E_c\varepsilon_0' X_5 + 2A_p\left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0} d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0} d_n + 2b_f E_c\varepsilon_0' t_d - 2b_f E_c\varepsilon_0' d_n + b_{in}E_c\varepsilon_0' t_d - b_{in}E_c\varepsilon_0' d_n + 2A_p f_{pd}$$
$$+2A_p\frac{\varepsilon_{ud} - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b(\varepsilon_{uk} - \varepsilon_{py})d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n = -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n$$
$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}(\varepsilon_{uk} - \varepsilon_{py})d_n + 2b_f E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{uk} - \varepsilon_{py})t_d^2$$
$$-4b_f E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})t_d + 2b_f E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n + b_{in}E_c\frac{\varepsilon_0}{d_n}(\varepsilon_{uk} - \varepsilon_{py})t_d^2$$
$$-2b_{in}E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})t_d + b_{in}E_c\varepsilon_0(\varepsilon_{uk} - \varepsilon_{py})d_n + 2A_p f_{pd}(\varepsilon_{uk} - \varepsilon_{py})$$
$$+2A_p(\varepsilon_{ud} - \varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p\sigma_{pm\infty}(\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$2\cdot 0{,}85 f_c' b\frac{\varepsilon_0}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p - 0{,}85 f_c' b\frac{\varepsilon_{cmax}}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$= -2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p + 2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p + 2b_f E_c\frac{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}{d_p}(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-4b_f E_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d + 2b_f E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+b_{in}E_c\frac{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}{d_p}(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 2b_{in}E_c\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})t_d$$

$$+b_{in}E_c\frac{\varepsilon_0^2}{\varepsilon_0+\varepsilon_{ud}-\varepsilon_{p\infty}}(\varepsilon_{uk}-\varepsilon_{py})d_p + 2A_p f_{pd}(\varepsilon_{uk}-\varepsilon_{py}) + 2A_p(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}(\varepsilon_{uk}-\varepsilon_{py}) \rightarrow$$

$$2\cdot 0{,}85 f_c' b\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_p^2 - 0{,}85 f_c' b\varepsilon_{cmax}(\varepsilon_{uk}-\varepsilon_{py})d_p^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_p^2 + 2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$

$$+2b_f E_c\left(\varepsilon_0+(\varepsilon_{ud}-\varepsilon_{p\infty})\right)^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 - 4b_f E_c\varepsilon_0\left(\varepsilon_0+(\varepsilon_{ud}-\varepsilon_{p\infty})\right)(\varepsilon_{uk}-\varepsilon_{py})t_d d_p$$

$$+2b_f E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_p^2 + b_{in}E_c\left(\varepsilon_0+(\varepsilon_{ud}-\varepsilon_{p\infty})\right)^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-2b_{in}E_c\varepsilon_0\left(\varepsilon_0+(\varepsilon_{ud}-\varepsilon_{p\infty})\right)(\varepsilon_{uk}-\varepsilon_{py})t_d d_p + b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$

$$+2A_p f_{pd}\left(\varepsilon_0+(\varepsilon_{ud}-\varepsilon_{p\infty})\right)(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+2A_p\left(\varepsilon_0+(\varepsilon_{ud}-\varepsilon_{p\infty})\right)(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\sigma_{pm\infty}\left(\varepsilon_0+(\varepsilon_{ud}-\varepsilon_{p\infty})\right)(\varepsilon_{uk}-\varepsilon_{py})d_p \rightarrow$$

$$2\cdot 0{,}85 f_c' b\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_p^2 - 0{,}85 f_c' b\varepsilon_{cmax}(\varepsilon_{uk}-\varepsilon_{py})d_p^2 = -2b_w\sigma_{ctmax}\varepsilon_{ctmax}(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}(\varepsilon_{uk}-\varepsilon_{py})d_p^2 + 2b_w\sigma_{ctmax}\varepsilon_{t,u}(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$

$$+2b_f E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 4b_f E_c\varepsilon_0(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 2b_f E_c(\varepsilon_{ud}-\varepsilon_{p\infty})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-4b_f E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d d_p - 4b_f E_c\varepsilon_0(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})t_d d_p + 2b_f E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$

$$+b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + 2b_{in}E_c\varepsilon_0(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})t_d^2 + b_{in}E_c(\varepsilon_{ud}-\varepsilon_{p\infty})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$

$$-2b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})t_d d_p - 2b_{in}E_c\varepsilon_0(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})t_d d_p + b_{in}E_c\varepsilon_0^2(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$

$$+2A_p f_{pd}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_p + 2A_p f_{pd}(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})d_p$$

$$+2A_p\varepsilon_0(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p + 2A_p(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p$$

$$-2A_p\sigma_{pm\infty}\varepsilon_0(\varepsilon_{uk}-\varepsilon_{py})d_p - 2A_p\sigma_{pm\infty}(\varepsilon_{ud}-\varepsilon_{p\infty})(\varepsilon_{uk}-\varepsilon_{py})d_p \rightarrow$$

$$+E_c(\varepsilon_{uk}-\varepsilon_{py})(2b_f+b_{in})(t_d-d_p)^2\varepsilon_0^2$$

$$-2\left((\varepsilon_{uk}-\varepsilon_{py})(0{,}85f_c'bd_p^2+(2b_f+b_{in})E_c(\varepsilon_{ud}-\varepsilon_{p\infty})(d_p-t_d)t_d)\right.$$

$$\left.+A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})d_p-(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\right)\varepsilon_0$$

$$+\left(0{,}85f_c'b\varepsilon_{cmax}-2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$
$$+(2b_f+b_{in})E_c(\varepsilon_{ud}-\varepsilon_{p\infty})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$+2A_p(\varepsilon_{ud}-\varepsilon_{p\infty})\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)d_p=0\rightarrow$$

$$\varepsilon_0=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=E_c(\varepsilon_{uk}-\varepsilon_{py})(2b_f+b_{in})(t_d-d_p)^2$$

$$b=-2\left((\varepsilon_{uk}-\varepsilon_{py})(0{,}85f_c'bd_p^2+(2b_f+b_{in})E_c(\varepsilon_{ud}-\varepsilon_{p\infty})(d_p-t_d)t_d)\right.$$

$$\left.+A_p\left((\sigma_{pm\infty}-f_{pd})(\varepsilon_{uk}-\varepsilon_{py})d_p-(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\right)$$

$$c=\left(0{,}85f_c'b\varepsilon_{cmax}-2b_w\sigma_{ctmax}(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u})\right)(\varepsilon_{uk}-\varepsilon_{py})d_p^2$$
$$+(2b_f+b_{in})E_c(\varepsilon_{ud}-\varepsilon_{p\infty})^2(\varepsilon_{uk}-\varepsilon_{py})t_d^2$$
$$+2A_p(\varepsilon_{ud}-\varepsilon_{p\infty})\left((f_{pd}-\sigma_{pm\infty})(\varepsilon_{uk}-\varepsilon_{py})+(\varepsilon_{ud}-\varepsilon_{py})\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)d_p$$

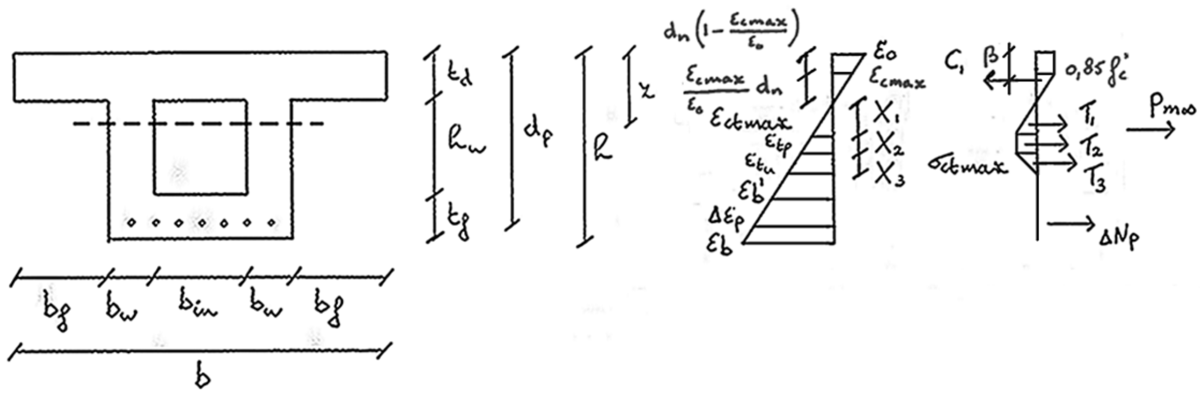


Figure 10.34: deformation and stress diagram when $d_n = t_d$ & $\varepsilon_0 > \varepsilon_{cmax}$.

10.34 When $\boldsymbol{d_n = t_d}$ & $\boldsymbol{\varepsilon_0 > \varepsilon_{cmax}}$ $\left(\boldsymbol{\varepsilon_p > \varepsilon_{py}}\right)$

With respect to the deformation and stress diagram of [Figure 10.34] the following relations are valid:

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The compressive stress at the top of the beam ε_0 can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + \Delta N_p$$

$$0{,}85 f_c' \left(1 - \frac{\varepsilon_{cmax}}{2\varepsilon_0}\right) b d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax} X_3 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' \frac{\varepsilon_{cmax}}{\varepsilon_0} b d_n = 2 b_w \sigma_{ctmax} X_1 + 4 b_w \sigma_{ctmax} X_2 + 2 b_w \sigma_{ctmax} X_3$$

$$+2A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) \right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' \frac{\varepsilon_{cmax}}{\varepsilon_0} b d_n = -2 b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2 b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$+2 b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n + 2A_p f_{pd} + 2A_p \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b (\varepsilon_{uk} - \varepsilon_{py}) d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n = -2 b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n$$

$$+2 b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2 b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} (\varepsilon_{uk} - \varepsilon_{py}) d_n + 2A_p f_{pd} (\varepsilon_{uk} - \varepsilon_{py})$$

$$+2A_p \frac{\varepsilon_0}{d_n} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p - 2A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) + 2A_p \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) - 2A_p \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right)$$

$$-2A_p \sigma_{pm\infty} (\varepsilon_{uk} - \varepsilon_{py}) \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 - 0{,}85 f_c' b \varepsilon_{cmax} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 = -2 b_w \sigma_{ctmax} \varepsilon_{ctmax} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2$$

$$+2 b_w \sigma_{ctmax} \varepsilon_{t,p} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 + 2 b_w \sigma_{ctmax} \varepsilon_{t,u} (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 + 2A_p f_{pd} \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) t_d$$

$$+2A_p \varepsilon_0^2 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) d_p - 2A_p \varepsilon_0^2 \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) t_d + 2A_p \varepsilon_0 \varepsilon_{p\infty} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) t_d$$

$$-2A_p \varepsilon_0 \varepsilon_{py} \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) t_d - 2A_p \sigma_{pm\infty} \varepsilon_0 (\varepsilon_{uk} - \varepsilon_{py}) t_d \rightarrow$$

$$+2A_p \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) (d_p - t_d) \varepsilon_0^2$$

$$-2 \left(0{,}85 f_c' b (\varepsilon_{uk} - \varepsilon_{py}) t_d + A_p \left((\sigma_{pm\infty} - f_{pd})(\varepsilon_{uk} - \varepsilon_{py}) + (\varepsilon_{py} - \varepsilon_{p\infty}) \left(\frac{f_{pk}}{\gamma_s} - f_{pd} \right) \right) \right) t_d \varepsilon_0$$

$$+ \left(0{,}85 f_c' b \varepsilon_{cmax} - 2 b_w \sigma_{ctmax} (\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}) \right) (\varepsilon_{uk} - \varepsilon_{py}) t_d^2 = 0 \rightarrow$$

$$\varepsilon_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2A_p\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\left(d_p - t_d\right)$$

$$b = -2\left(0{,}85f_c'b\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d + A_p\left(\left(\sigma_{pm\infty} - f_{pd}\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right) + \left(\varepsilon_{py} - \varepsilon_{p\infty}\right)\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right)\right)t_d$$

$$c = \left(0{,}85f_c'b\varepsilon_{cmax} - 2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax} - \varepsilon_{t,p} - \varepsilon_{t,u}\right)\right)\left(\varepsilon_{uk} - \varepsilon_{py}\right)t_d^2$$

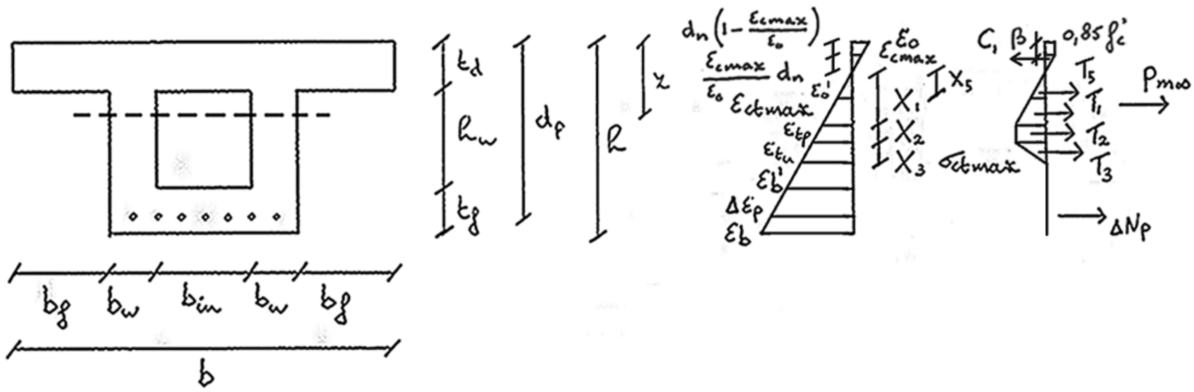


Figure 10.35: deformation and stress diagram when $\varepsilon_0 > \varepsilon_{cmax}$

10.35 When $\boldsymbol{\varepsilon_0 > \varepsilon_{cmax}}$ $\boldsymbol{(\varepsilon_p > \varepsilon_{py}\ \&\ d_n < t_d)}$

With respect to the deformation and stress diagram of [Figure 10.35] the following relations are valid:

$$\frac{\varepsilon_0{}'}{X_5} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0' = \frac{\varepsilon_0}{d_n} X_5 = \frac{\varepsilon_0}{d_n}(t_d - d_n) = \frac{\varepsilon_0}{d_n} t_d - \varepsilon_0$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}(d_p - d_n) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$X_5 = t_d - d_n$$

$$\varepsilon_p = \Delta\varepsilon_p + \varepsilon_{p\infty}$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 = T_1 + T_2 + T_3 + T_5 + \Delta N_p$$

$$0{,}85 f_c' \left(1 - \frac{\varepsilon_{cmax}}{2\varepsilon_0}\right) b d_n = \frac{1}{2}(2b_w)\sigma_{ctmax} X_1 + 2b_w \sigma_{ctmax} X_2 + \frac{1}{2}(2b_w)\sigma_{ctmax} X_3$$

$$+\frac{1}{2}\left(2b_f + b_{in}\right) E_c \varepsilon_0' X_5 + A_p \sigma_p - P_{m\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0} d_n = 2b_w \sigma_{ctmax} X_1 + 4b_w \sigma_{ctmax} X_2 + 2b_w \sigma_{ctmax} X_3 + 2b_f E_c \varepsilon_0' X_5$$

$$+b_{in} E_c \varepsilon_0' X_5 + 2A_p \left(f_{pd} + \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right)\right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0} d_n = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n + 2b_f E_c \varepsilon_0' t_d - 2b_f E_c \varepsilon_0' d_n + b_{in} E_c \varepsilon_0' t_d - b_{in} E_c \varepsilon_0' d_n + 2A_p f_{pd}$$

$$+2A_p \frac{\varepsilon_p - \varepsilon_{py}}{\varepsilon_{uk} - \varepsilon_{py}} \cdot \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p \sigma_{pm\infty} \rightarrow$$

$$2 \cdot 0{,}85 f_c' b\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n - 0{,}85 f_c' b \frac{\varepsilon_{cmax}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n = -2b_w \sigma_{ctmax} \frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n$$

$$+2b_w \sigma_{ctmax} \frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_w \sigma_{ctmax} \frac{\varepsilon_{t,u}}{\varepsilon_0}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2b_f E_c \frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$

$$-4b_f E_c \varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d + 2b_f E_c \varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + b_{in} E_c \frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$

$$-2b_{in} E_c \varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d + b_{in} E_c \varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n + 2A_p f_{pd}\left(\varepsilon_{uk} - \varepsilon_{py}\right) + 2A_p \frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_p$$

$$-2A_p \varepsilon_0 \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) + 2A_p \varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p \varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) - 2A_p \sigma_{pm\infty}\left(\varepsilon_{uk} - \varepsilon_{py}\right) \rightarrow$$

$$2 \cdot 0{,}85 f_c' b \varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 - 0{,}85 f_c' b \varepsilon_{cmax}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 = -2b_w \sigma_{ctmax} \varepsilon_{ctmax}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2$$

$$+2b_w \sigma_{ctmax} \varepsilon_{t,p}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 + 2b_w \sigma_{ctmax} \varepsilon_{t,u}\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 + 2b_f E_c \varepsilon_0^2\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$

$$-4b_f E_c \varepsilon_0^2\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d d_n + 2b_f E_c \varepsilon_0^2\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 + b_{in} E_c \varepsilon_0^2\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d^2$$

$$-2b_{in} E_c \varepsilon_0^2\left(\varepsilon_{uk} - \varepsilon_{py}\right) t_d d_n + b_{in} E_c \varepsilon_0^2\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n^2 + 2A_p f_{pd} \varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n$$

$$+2A_p \varepsilon_0^2 \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_p - 2A_p \varepsilon_0^2 \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_n + 2A_p \varepsilon_0 \varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_n$$

$$-2A_p \varepsilon_0 \varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) d_n - 2A_p \sigma_{pm\infty} \varepsilon_0\left(\varepsilon_{uk} - \varepsilon_{py}\right) d_n \rightarrow$$

$$-\left(0{,}85f_c'b(2\varepsilon_0-\varepsilon_{cmax})+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u}\right)-\left(2b_f+b_{in}\right)E_c\varepsilon_0^2\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$-2\varepsilon_0\left(\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d + A_p\left(\left(\sigma_{pm\infty}-f_{pd}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+\left(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)d_n$$

$$+\varepsilon_0^2\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=-\left(0{,}85f_c'b(2\varepsilon_0-\varepsilon_{cmax})+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u}\right)-\left(2b_f+b_{in}\right)E_c\varepsilon_0^2\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$b=-2\varepsilon_0\left(\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d + A_p\left(\left(\sigma_{pm\infty}-f_{pd}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+\left(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)$$

$$c=\varepsilon_0^2\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta=\frac{bd_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)\cdot\frac{1}{2}d_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{1}{2}b\frac{\varepsilon_{cmax}}{\varepsilon_0}d_n\cdot\left(d_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{1}{3}\frac{\varepsilon_{cmax}}{\varepsilon_0}d_n\right)}{bd_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{1}{2}b\frac{\varepsilon_{cmax}}{\varepsilon_0}d_n}$$

The corresponding bending moment capacity:

$$M=C_1(d_n-\beta)+T_1\cdot\frac{2}{3}X_1+T_2\left(X_1+\frac{1}{2}X_2\right)+T_3\left(X_1+X_2+\frac{1}{3}X_3\right)+T_5\cdot\frac{2}{3}X_5+P_{m\infty}(e-d_n)$$
$$+\Delta N_p\left(d_p-d_n\right)$$

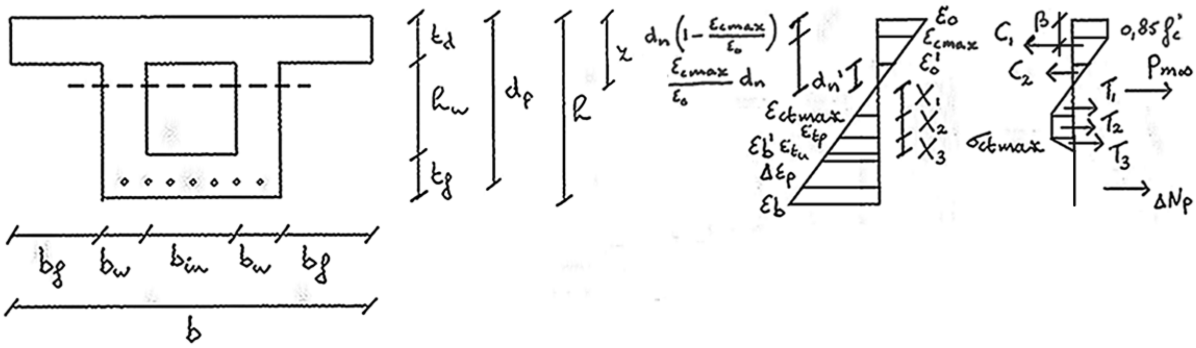


Figure 10.36: deformation and stress diagram when $\varepsilon_0 > \varepsilon_{cmax}$.

10.36 When $\boldsymbol{\varepsilon_0 > \varepsilon_{cmax}} \left(\boldsymbol{\varepsilon_p > \varepsilon_{py}} \,\&\, \boldsymbol{d_n > t_d}\right)$

With respect to the deformation and stress diagram of [Figure 10.36] the following relations are valid:

$$\frac{\varepsilon_0{}'}{d_n{}'} = \frac{\varepsilon_0}{d_n} \rightarrow \varepsilon_0{}' = \frac{\varepsilon_0}{d_n} d_n{}'$$

$$\frac{\varepsilon_{ctmax}}{X_1} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 = \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,p}}{X_1 + X_2} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n \rightarrow X_2 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - X_1 = \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n - \frac{\varepsilon_{ctmax}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_{t,u}}{X_1 + X_2 + X_3} = \frac{\varepsilon_0}{d_n} \rightarrow X_1 + X_2 + X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n \rightarrow X_3 = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - (X_1 + X_2) = \frac{\varepsilon_{t,u}}{\varepsilon_0} d_n - \frac{\varepsilon_{t,p}}{\varepsilon_0} d_n$$

$$\frac{\varepsilon_0}{d_n} = \frac{\Delta\varepsilon_p}{d_p - d_n} \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n}\left(d_p - d_n\right) \rightarrow \Delta\varepsilon_p = \frac{\varepsilon_0}{d_n} d_p - \varepsilon_0$$

$$d_n{}' = d_n - t_d$$

The concrete compressive zone height d_n can be derived from equilibrium of horizontal forces:

$$\sum F_H = 0 \rightarrow C_1 + C_2 = T_1 + T_2 + T_3 + \Delta N_p$$

$$0{,}85f_c'\left(1-\frac{\varepsilon_{cmax}}{2\varepsilon_0}\right)bd_n-\frac{1}{2}\left(2b_f+b_{in}\right)E_c\varepsilon_0'd_n'=\frac{1}{2}(2b_w)\sigma_{ctmax}X_1+2b_w\sigma_{ctmax}X_2$$

$$+\frac{1}{2}(2b_w)\sigma_{ctmax}X_3+A_p\sigma_p-P_{m\infty}\rightarrow$$

$$2\cdot 0{,}85f_c'bd_n-0{,}85f_c'\frac{\varepsilon_{cmax}}{\varepsilon_0}bd_n-2b_fE_c\varepsilon_0'd_n'-b_{in}E_c\varepsilon_0'd_n'=2b_w\sigma_{ctmax}X_1+4b_w\sigma_{ctmax}X_2$$

$$+2b_w\sigma_{ctmax}X_3+2A_p\left(f_{pd}+\frac{\varepsilon_p-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)-2A_p\sigma_{pm\infty}\rightarrow$$

$$2\cdot 0{,}85f_c'bd_n-0{,}85f_c'\frac{\varepsilon_{cmax}}{\varepsilon_0}bd_n-2b_fE_c\varepsilon_0d_n'+2b_fE_c\frac{\varepsilon_0}{d_n}d_n't_d-b_{in}E_c\varepsilon_0d_n'+b_{in}E_c\frac{\varepsilon_0}{d_n}d_n't_d$$

$$=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}d_n+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}d_n+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}d_n+2A_pf_{pd}$$

$$+2A_p\frac{\varepsilon_p-\varepsilon_{py}}{\varepsilon_{uk}-\varepsilon_{py}}\cdot\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\sigma_{pm\infty}\rightarrow$$

$$2\cdot 0{,}85f_c'b\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n-0{,}85f_c'\frac{\varepsilon_{cmax}}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)bd_n-2b_fE_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+4b_fE_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d-2b_fE_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-b_{in}E_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2b_{in}E_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d-b_{in}E_c\frac{\varepsilon_0}{d_n}\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2=-2b_w\sigma_{ctmax}\frac{\varepsilon_{ctmax}}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,p}}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n+2b_w\sigma_{ctmax}\frac{\varepsilon_{t,u}}{\varepsilon_0}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n+2A_pf_{pd}\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$+2A_p\frac{\varepsilon_0}{d_n}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)+2A_p\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)-2A_p\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)$$

$$-2A_p\sigma_{pm\infty}\left(\varepsilon_{uk}-\varepsilon_{py}\right)\rightarrow$$

$$2\cdot 0{,}85f_c'b\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2-0{,}85f_c'\varepsilon_{cmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)bd_n^2-2b_fE_c\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+4b_fE_c\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_dd_n-2b_fE_c\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2-b_{in}E_c\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+2b_{in}E_c\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_dd_n-b_{in}E_c\varepsilon_0^2\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2=-2b_w\sigma_{ctmax}\varepsilon_{ctmax}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+2b_w\sigma_{ctmax}\varepsilon_{t,p}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2+2b_w\sigma_{ctmax}\varepsilon_{t,u}\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2+2A_pf_{pd}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n$$

$$+2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p-2A_p\varepsilon_0^2\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n+2A_p\varepsilon_0\varepsilon_{p\infty}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n$$

$$-2A_p\varepsilon_0\varepsilon_{py}\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_n-2A_p\sigma_{pm\infty}\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n\rightarrow$$

$$\left(0{,}85f_c'b(2\varepsilon_0-\varepsilon_{cmax})-\left(2b_f+b_{in}\right)E_c\varepsilon_0^2+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u}\right)\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)d_n^2$$

$$+2\varepsilon_0\left(\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d\right.$$

$$\left.+A_p\left(\left(\sigma_{pm\infty}-f_{pd}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+\left(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)d_n$$

$$-\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2=0\rightarrow$$

$$d_n=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=\left(0{,}85f_c'b(2\varepsilon_0-\varepsilon_{cmax})-\left(2b_f+b_{in}\right)E_c\varepsilon_0^2+2b_w\sigma_{ctmax}\left(\varepsilon_{ctmax}-\varepsilon_{t,p}-\varepsilon_{t,u}\right)\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)$$

$$b=2\varepsilon_0\left(\left(2b_f+b_{in}\right)E_c\varepsilon_0\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d\right.$$

$$\left.+A_p\left(\left(\sigma_{pm\infty}-f_{pd}\right)\left(\varepsilon_{uk}-\varepsilon_{py}\right)+\left(\varepsilon_0-\varepsilon_{p\infty}+\varepsilon_{py}\right)\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)\right)\right)$$

$$c=-\left(\left(2b_f+b_{in}\right)E_c\left(\varepsilon_{uk}-\varepsilon_{py}\right)t_d^2+2A_p\left(\frac{f_{pk}}{\gamma_s}-f_{pd}\right)d_p\right)\varepsilon_0^2$$

In order to determine the bending moment resistance the distance from the top fibre to the center of gravity of the concrete compressive zone needs to be known:

$$\beta=\frac{bd_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)\cdot\frac{1}{2}d_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{1}{2}b\frac{\varepsilon_{cmax}}{\varepsilon_0}d_n\cdot\left(d_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{1}{3}\frac{\varepsilon_{cmax}}{\varepsilon_0}d_n\right)}{bd_n\left(1-\frac{\varepsilon_{cmax}}{\varepsilon_0}\right)+\frac{1}{2}b\frac{\varepsilon_{cmax}}{\varepsilon_0}d_n}$$

The corresponding bending moment capacity:

$$M=C_1(d_n-\beta)+C_2\cdot\frac{2}{3}d_n'+T_1\cdot\frac{2}{3}X_1+T_2\left(X_1+\frac{1}{2}X_2\right)+T_3\left(X_1+X_2+\frac{1}{3}X_3\right)+P_{m\infty}(e-d_n)$$

$$+\Delta N_p\left(d_p-d_n\right)$$

10.37 Shear

The ultimate shear capacity is given by:

$$V_u = V_{Rb} + V_f + V_s$$

In case of prestressed concrete, the participation of the concrete will be:

$$V_{Rb} = \frac{1}{\gamma_E}\frac{0{,}24}{\gamma_b}\sqrt{f_c'}bz$$

$$b = 2b_w$$

The contribution of the fibres can be expressed by:

$$V_f = \frac{S_{eff}\sigma(w0{,}3)_k}{\gamma_{bf}\tan\beta_u}$$

$$S_{eff} = bz$$

$$z = d_p - \frac{1}{3}d_n$$

The shear reinforcement:

$$V_s = \frac{A_{sw}}{s}zf_{ywd}\cot\beta_u$$

The angle of the compression struts should be limited to 30° as opposed to 21,8° ,which NEN-EN 1992-1-1 prescribes.

10.38 Crack width

Requirement: the box girder remains uncracked in the serviceability limit state.

At $t = 0$, no time-dependent losses are present, so the prestressing force will be at its maximum. Because of the positioning of the tendons the box girder will be slightly cambered and tensile stresses will occur at the top. Bending moments cause compressive stresses at the top and tensile stresses at the bottom of the beam.

$t = 0 \rightarrow$ check top fibre:

$$-\frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{top}} - \frac{M}{W_{top}} \leq f'_{ct} \rightarrow \frac{M}{W_{top}} \geq -\frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{top}} - f'_{ct} \rightarrow$$

$$M \geq -\frac{P_{m0}}{A_c} W_{top} + P_{m0} \cdot e - f'_{ct} W_{top}$$

$t = 0 \rightarrow$ check bottom fibre:

$$-\frac{P_{m0}}{A_c} - \frac{P_{m0} \cdot e}{W_{bot}} + \frac{M}{W_{bot}} \leq f'_{ct} \rightarrow \frac{M}{W_{bot}} \leq \frac{P_{m0}}{A_c} + \frac{P_{m0} \cdot e}{W_{bot}} + f'_{ct}$$

$$\rightarrow M \leq \frac{P_{m0}}{A_c} W_{bot} + P_{m0} \cdot e + f'_{ct} W_{bot}$$

At $t = \infty$, the prestressing force has been reduced by time-dependent losses, which means that the compressive stresses working on the cross-section will be limited. Dead and live loads are present and will cause tensile stresses at the bottom fibre in the span. The bending moment caused by these loads should be limited:

$t = \infty \rightarrow$ check top fibre:

$$-\frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{top}} - \frac{M}{W_{top}} \leq f'_{ct} \rightarrow \frac{M}{W_{top}} \geq -\frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{top}} - f'_{ct} \rightarrow$$

$$M \geq -\frac{P_{m\infty}}{A_c} W_{top} + P_{m\infty} \cdot e - f'_{ct} W_{top}$$

$t = \infty \rightarrow$ check bottom fibre:

$$-\frac{P_{m\infty}}{A_c} - \frac{P_{m\infty} \cdot e}{W_{bot}} + \frac{M}{W_{bot}} \leq f'_{ct} \rightarrow \frac{M}{W_{bot}} \leq \frac{P_{m\infty}}{A_c} + \frac{P_{m\infty} \cdot e}{W_{bot}} + f'_{ct}$$

$$\rightarrow M \leq \frac{P_{m\infty}}{A_c} W_{bot} + P_{m\infty} \cdot e + f'_{ct} W_{bot}$$

11 The cross-sectional capacity

11.1 Rectangular reinforced NSC beam

Rectangular reinforced NSC beam

My < Mc,pl

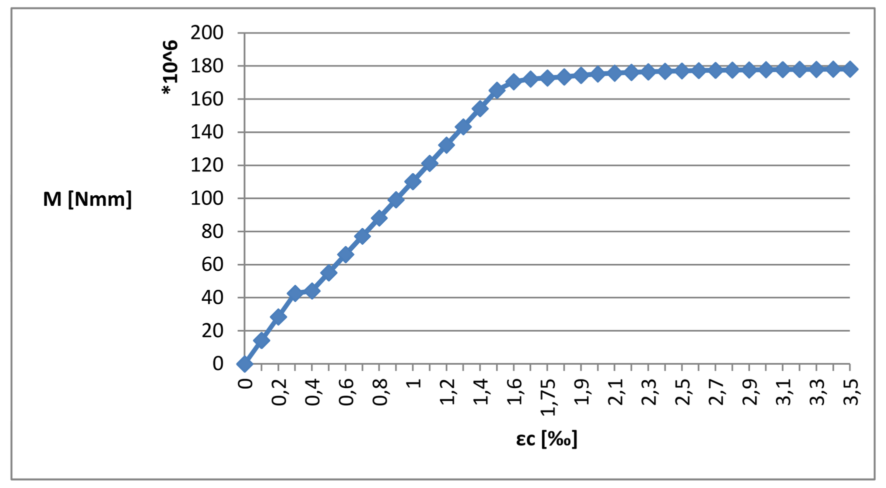
Moment capacity (ULS)

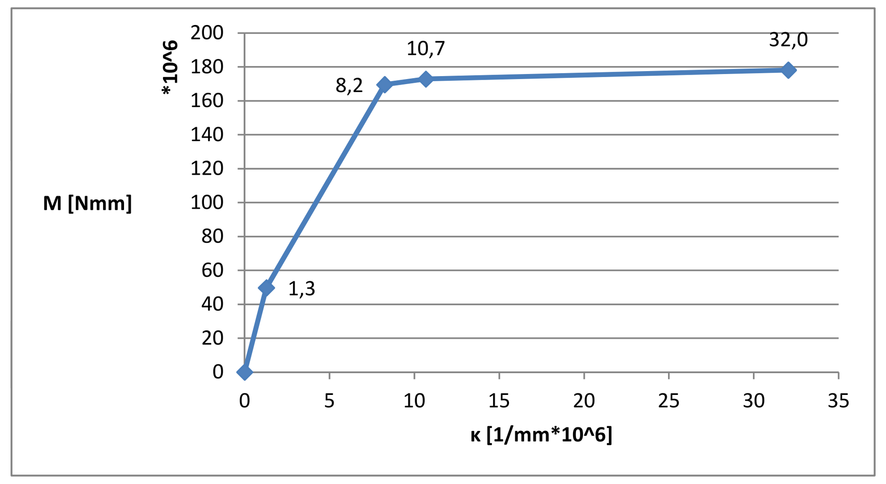
b	400	mm
h	500	mm
c	30	mm
d	450	mm
Ac	2,00E+05	mm^2
Ø main reinforcement	16	mm
number of bars	5	
As	1005	mm^2
Ø stirrups	12	mm
ρ	5,59E-03	
bar spacing	59	mm
fck	20	N/mm^2
fcd	13,3	N/mm^2
fcm	28	N/mm^2
fctm	2,2	N/mm^2
Ec	7619	N/mm^2
εc3	0,00175	
εcu3	0,0035	
fyk	500	N/mm^2
fyd	435	N/mm^2
Es	200000	N/mm^2
εsy	0,00217	
εuk	0,025	
εud	0,0225	
Asmin	207	mm^2
Asmax	8000	mm^2

εct	2,90E-04	
εs	2,26E-04	
εc	0,00035	
x	273	mm
κr	1,28E-06	1/mm
Mcr	**4,97E+07**	**Nmm**
εc	0,0015	
εsy	2,17E-03	
x	186	mm
κy	8,25E-06	1/mm
My	**1,70E+08**	**Nmm**
εc3	0,00175	
εs	3,05E-03	
x	164	mm
κc,pl	1,07E-05	1/mm
Mc,pl	**1,73E+08**	**Nmm**
εcu3	0,0035	
εs	1,09E-02	
x	109	mm
κRd	3,20E-05	1/mm
β	42	mm
MRd	**1,78E+08**	**Nmm**

Check: My < Mc,pl	ok!
Check: εs < εud	ok!

εc [-]	εs [-]	εctensile [-]	x [mm]	κ [1/mm]	Nctop [N]	Ncbot [N]	Ns [N]	ΣH = 0	β [mm]	M [Nmm]	εc [‰]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	6,46E-05	8,29E-05	273	3,66E-07	4,16E+04	2,86E+04	1,30E+04	0	0	1,42E+07	0,1	1,42E+07
0,0002	1,29E-04	1,66E-04	273	7,32E-07	8,33E+04	5,73E+04	2,60E+04	0	0	2,84E+07	0,2	2,84E+07
0,0003	1,94E-04	2,49E-04	273	1,10E-06	1,25E+05	8,59E+04	3,90E+04	0	0	4,26E+07	0,3	4,26E+07
0,0004	5,65E-04	0,00E+00	186	2,15E-06	1,14E+05	0,00E+00	1,14E+05	0	0	4,41E+07	0,4	4,41E+07
0,0005	7,07E-04	0,00E+00	186	2,68E-06	1,42E+05	0,00E+00	1,42E+05	0	0	5,51E+07	0,5	5,51E+07
0,0006	8,48E-04	0,00E+00	186	3,22E-06	1,70E+05	0,00E+00	1,70E+05	0	0	6,61E+07	0,6	6,61E+07
0,0007	9,89E-04	0,00E+00	186	3,75E-06	1,99E+05	0,00E+00	1,99E+05	0	0	7,71E+07	0,7	7,71E+07
0,0008	1,13E-03	0,00E+00	186	4,29E-06	2,27E+05	0,00E+00	2,27E+05	0	0	8,82E+07	0,8	8,82E+07
0,0009	1,27E-03	0,00E+00	186	4,83E-06	2,56E+05	0,00E+00	2,56E+05	0	0	9,92E+07	0,9	9,92E+07
0,0010	1,41E-03	0,00E+00	186	5,36E-06	2,84E+05	0,00E+00	2,84E+05	0	0	1,10E+08	1	1,10E+08
0,0011	1,55E-03	0,00E+00	186	5,90E-06	3,13E+05	0,00E+00	3,13E+05	0	0	1,21E+08	1,1	1,21E+08
0,0012	1,70E-03	0,00E+00	186	6,44E-06	3,41E+05	0,00E+00	3,41E+05	0	0	1,32E+08	1,2	1,32E+08
0,0013	1,84E-03	0,00E+00	186	6,97E-06	3,69E+05	0,00E+00	3,69E+05	0	0	1,43E+08	1,3	1,43E+08
0,0014	1,98E-03	0,00E+00	186	7,51E-06	3,98E+05	0,00E+00	3,98E+05	0	0	1,54E+08	1,4	1,54E+08
0,0015	2,12E-03	0,00E+00	186	8,04E-06	4,26E+05	0,00E+00	4,26E+05	0	0	1,65E+08	1,5	1,65E+08
0,0016	2,42E-03	0,00E+00	179	8,92E-06	4,37E+05	0,00E+00	4,37E+05	0	0	1,71E+08	1,6	1,71E+08
0,0017	2,83E-03	0,00E+00	169	1,01E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,72E+08	1,7	1,72E+08
0,00175	3,05E-03	0,00E+00	164	1,07E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,73E+08	1,75	1,73E+08
0,0018	3,28E-03	0,00E+00	159	1,13E-05	4,37E+05	0,00E+00	4,37E+05	0	53	1,73E+08	1,8	1,73E+08
0,0019	3,73E-03	0,00E+00	152	1,25E-05	4,37E+05	0,00E+00	4,37E+05	0	51	1,74E+08	1,9	1,74E+08
0,0020	4,18E-03	0,00E+00	146	1,37E-05	4,37E+05	0,00E+00	4,37E+05	0	49	1,75E+08	2	1,75E+08
0,0021	4,63E-03	0,00E+00	140	1,49E-05	4,37E+05	0,00E+00	4,37E+05	0	48	1,76E+08	2,1	1,76E+08
0,0022	5,08E-03	0,00E+00	136	1,62E-05	4,37E+05	0,00E+00	4,37E+05	0	47	1,76E+08	2,2	1,76E+08
0,0023	5,52E-03	0,00E+00	132	1,74E-05	4,37E+05	0,00E+00	4,37E+05	0	46	1,77E+08	2,3	1,77E+08
0,0024	5,97E-03	0,00E+00	129	1,86E-05	4,37E+05	0,00E+00	4,37E+05	0	45	1,77E+08	2,4	1,77E+08
0,0025	6,42E-03	0,00E+00	126	1,98E-05	4,37E+05	0,00E+00	4,37E+05	0	45	1,77E+08	2,5	1,77E+08
0,0026	6,87E-03	0,00E+00	124	2,10E-05	4,37E+05	0,00E+00	4,37E+05	0	44	1,77E+08	2,6	1,77E+08
0,0027	7,32E-03	0,00E+00	121	2,23E-05	4,37E+05	0,00E+00	4,37E+05	0	44	1,77E+08	2,7	1,77E+08
0,0028	7,77E-03	0,00E+00	119	2,35E-05	4,37E+05	0,00E+00	4,37E+05	0	44	1,78E+08	2,8	1,78E+08
0,0029	8,22E-03	0,00E+00	117	2,47E-05	4,37E+05	0,00E+00	4,37E+05	0	44	1,78E+08	2,9	1,78E+08
0,0030	8,67E-03	0,00E+00	116	2,59E-05	4,37E+05	0,00E+00	4,37E+05	0	43	1,78E+08	3	1,78E+08
0,0031	9,12E-03	0,00E+00	114	2,71E-05	4,37E+05	0,00E+00	4,37E+05	0	43	1,78E+08	3,1	1,78E+08
0,0032	9,57E-03	0,00E+00	113	2,84E-05	4,37E+05	0,00E+00	4,37E+05	0	43	1,78E+08	3,2	1,78E+08
0,0033	1,00E-02	0,00E+00	112	2,96E-05	4,37E+05	0,00E+00	4,37E+05	0	43	1,78E+08	3,3	1,78E+08
0,0034	1,05E-02	0,00E+00	110	3,08E-05	4,37E+05	0,00E+00	4,37E+05	0	43	1,78E+08	3,4	1,78E+08
0,0035	1,09E-02	0,00E+00	109	3,20E-05	4,37E+05	0,00E+00	4,37E+05	0	42	1,78E+08	3,5	1,78E+08





κ [1/mm*10^6]	M [Nmm]
0	0
1,3	4,97E+07
8,2	1,70E+08
10,7	1,73E+08
32,0	1,78E+08

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
ρl	5,59E-03	
k1	0,15	
σcp	0	N/mm^2
αcw	1	
z	408	mm
v1	0,6	
θ	21,8	°
cot θ	2,5	
tan θ	0,4	
Asw	226	mm^2
s	150	mm

vmin	0,34	N/mm^2
VRd,cmin	6,06E+04	N
VRd,c	**8,05E+04**	**N**
VRd,max	**4,50E+05**	**N**
VRd,s	**6,15E+05**	**N**
VRd	**4,50E+05**	**N**

Crack width (SLS)

exposure class	XD3	
wmax	0,2	mm
wk	0,2	mm
k1	0,8	
k2	0,5	
k3	3,4	
k4	0,425	
kt	0,6	
Ecm	30000	N/mm^2
αe	6,7	
fct,eff	2,2	N/mm^2
x	186	mm
hc,eff	105	mm
Ac,eff	4,18E+04	mm^2
ρp,eff	2,40E-02	
sr,max	215	mm
εsm-εcm	9,30E-04	
σs	250	N/mm^2

εc	0,0009	
εs	1,25E-03	
Nc	2,51E+05	N
Ns	2,51E+05	N
Mqp	**9,75E+07**	**Nmm**

Compressive zone height (ULS)

f	435	N/mm^2
xu	109	mm
xumax	241	mm

Check: xu < xumax	ok!

11.2 Rectangular reinforced HSC beam

Rectangular reinforced HSC beam

My < Mc,pl

Moment capacity (ULS)

b	400	mm
h	500	mm
c	30	mm
d	450	mm
Ac	2,00E+05	mm^2
Ø main reinforcement	16	mm
number of bars	5	
As	1005	mm^2
Ø stirrups	12	mm
ρ	5,59E-03	
bar spacing	59	mm
fck	90	N/mm^2
fcd	60,0	N/mm^2
fcm	98	N/mm^2
fctm	5,0	N/mm^2
Ec	26087	N/mm^2
εc3	0,0023	
εcu3	0,0026	
fyk	500	N/mm^2
fyd	435	N/mm^2
Es	200000	N/mm^2
εsy	0,00217	
εuk	0,025	
εud	0,0225	
Asmin	472	mm^2
Asmax	8000	mm^2

εct	1,93E-04	
εs	1,54E-04	
εc	0,00021	
x	257	mm
κr	7,97E-07	1/mm
Mcr	**9,28E+07**	**Nmm**
εc	0,0007	
εsy	2,17E-03	
x	114	mm
κy	6,47E-06	1/mm
My	**1,80E+08**	**Nmm**
εc3	0,0023	
εs	2,61E-02	
x	36	mm
κc,pl	6,31E-05	1/mm
Mc,pl	**1,91E+08**	**Nmm**
εcu3	0,0026	
εs	3,32E-02	
x	33	mm
κRd	7,96E-05	1/mm
β	11	mm
MRd	**1,92E+08**	**Nmm**

Check: My < Mc,pl	ok!
Check: εs < εud	not ok!

εc [-]	εs [-]	εctensile [-]	x [mm]	κ [1/mm]	Nctop [N]	Ncbot [N]	Ns [N]	ΣH = 0	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0
0,0001	7,48E-05	9,42E-05	257	3,88E-07	1,34E+05	1,19E+05	1,50E+04	0	0	4,52E+07
0,0002	1,50E-04	1,88E-04	257	7,77E-07	2,69E+05	2,39E+05	3,01E+04	0	0	9,05E+07
0,0003	8,86E-04	0,00E+00	114	2,64E-06	1,78E+05	0,00E+00	1,78E+05	0	0	7,34E+07
0,0004	1,18E-03	0,00E+00	114	3,51E-06	2,38E+05	0,00E+00	2,38E+05	0	0	9,79E+07
0,0005	1,48E-03	0,00E+00	114	4,39E-06	2,97E+05	0,00E+00	2,97E+05	0	0	1,22E+08
0,0006	1,77E-03	0,00E+00	114	5,27E-06	3,56E+05	0,00E+00	3,56E+05	0	0	1,47E+08
0,0007	2,07E-03	0,00E+00	114	6,15E-06	4,16E+05	0,00E+00	4,16E+05	0	0	1,71E+08
0,0008	2,64E-03	0,00E+00	105	7,64E-06	4,37E+05	0,00E+00	4,37E+05	0	0	1,81E+08
0,0009	3,45E-03	0,00E+00	93	9,67E-06	4,37E+05	0,00E+00	4,37E+05	0	0	1,83E+08
0,0010	4,37E-03	0,00E+00	84	1,19E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,84E+08
0,0011	5,40E-03	0,00E+00	76	1,44E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,86E+08
0,0012	6,53E-03	0,00E+00	70	1,72E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,87E+08
0,0013	7,78E-03	0,00E+00	64	2,02E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,87E+08
0,0014	9,13E-03	0,00E+00	60	2,34E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,88E+08
0,0015	1,06E-02	0,00E+00	56	2,69E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,89E+08
0,0016	1,22E-02	0,00E+00	52	3,06E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,89E+08
0,0017	1,38E-02	0,00E+00	49	3,45E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,90E+08
0,00175	1,47E-02	0,00E+00	48	3,66E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,90E+08
0,0018	1,56E-02	0,00E+00	47	3,87E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,90E+08
0,0019	1,75E-02	0,00E+00	44	4,31E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,90E+08
0,0020	1,95E-02	0,00E+00	42	4,77E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,91E+08
0,0021	2,16E-02	0,00E+00	40	5,26E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,91E+08
0,0022	2,38E-02	0,00E+00	38	5,78E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,91E+08
0,0023	2,61E-02	0,00E+00	36	6,31E-05	4,37E+05	0,00E+00	4,37E+05	0	0	1,91E+08
0,0024	2,85E-02	0,00E+00	35	6,86E-05	4,37E+05	0,00E+00	4,37E+05	0	12	1,92E+08
0,0025	3,09E-02	0,00E+00	34	7,41E-05	4,37E+05	0,00E+00	4,37E+05	0	11	1,92E+08
0,0026	3,32E-02	0,00E+00	33	7,96E-05	4,37E+05	0,00E+00	4,37E+05	0	11	1,92E+08
0,0027	3,56E-02	0,00E+00	32	8,51E-05	4,37E+05	0,00E+00	4,37E+05	0	11	1,92E+08
0,0028	3,80E-02	0,00E+00	31	9,06E-05	4,37E+05	0,00E+00	4,37E+05	0	11	1,92E+08
0,0029	4,03E-02	0,00E+00	30	9,61E-05	4,37E+05	0,00E+00	4,37E+05	0	10	1,92E+08
0,0030	4,27E-02	0,00E+00	30	1,02E-04	4,37E+05	0,00E+00	4,37E+05	0	10	1,92E+08
0,0031	4,51E-02	0,00E+00	29	1,07E-04	4,37E+05	0,00E+00	4,37E+05	0	10	1,92E+08
0,0032	4,75E-02	0,00E+00	28	1,13E-04	4,37E+05	0,00E+00	4,37E+05	0	10	1,92E+08
0,0033	4,98E-02	0,00E+00	28	1,18E-04	4,37E+05	0,00E+00	4,37E+05	0	10	1,92E+08
0,0034	5,22E-02	0,00E+00	28	1,24E-04	4,37E+05	0,00E+00	4,37E+05	0	10	1,92E+08
0,0035	5,46E-02	0,00E+00	27	1,29E-04	4,37E+05	0,00E+00	4,37E+05	0	10	1,92E+08

εc [‰]	M [Nmm]
0	0
0,1	4,52E+07
0,2	9,05E+07
0,3	7,34E+07
0,4	9,79E+07
0,5	1,22E+08
0,6	1,47E+08
0,7	1,71E+08
0,8	1,81E+08
0,9	1,83E+08
1	1,84E+08
1,1	1,86E+08
1,2	1,87E+08
1,3	1,87E+08
1,4	1,88E+08
1,5	1,89E+08
1,6	1,89E+08
1,7	1,90E+08
1,75	1,90E+08
1,8	1,90E+08
1,9	1,90E+08
2	1,91E+08
2,1	1,91E+08
2,2	1,91E+08
2,3	1,91E+08
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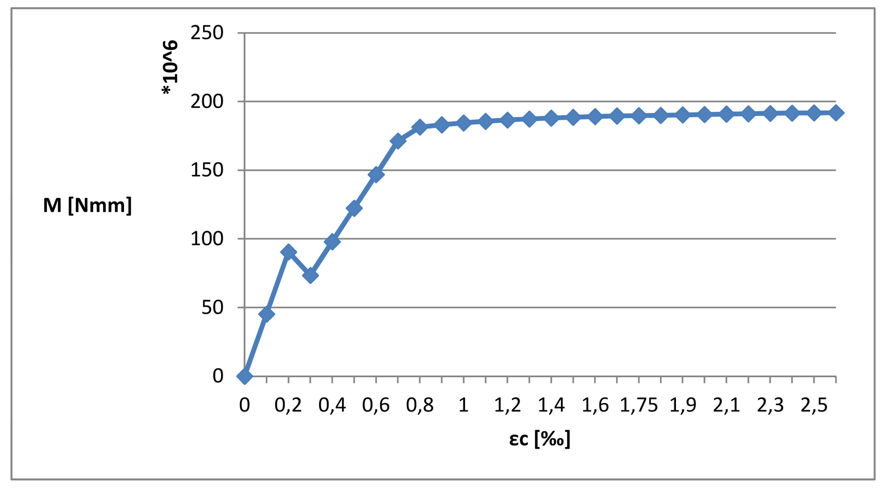
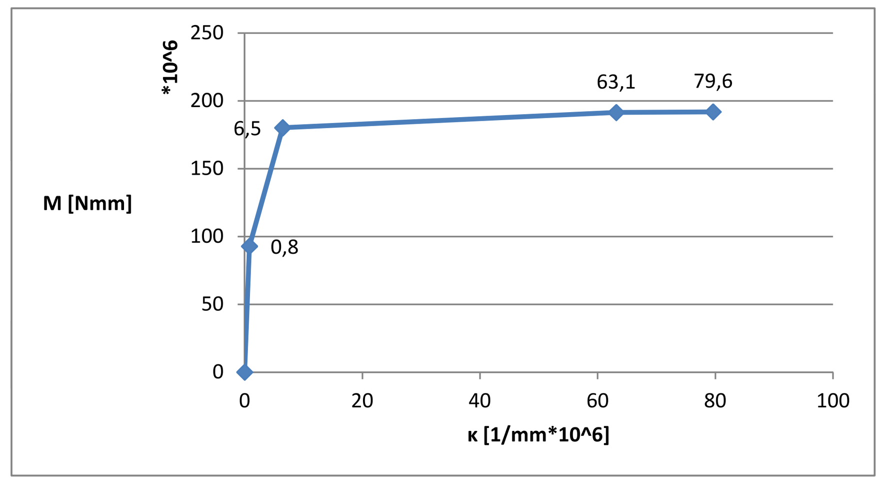
2,5	1,92E+08
2,6	1,92E+08





κ [1/mm*10^6]	M [Nmm]
0	0
0,8	9,28E+07
6,5	1,80E+08
63,1	1,91E+08
79,6	1,92E+08

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
ρl	5,59E-03	
k1	0,15	
σcp	0	N/mm^2
αcw	1	
z	440	mm
v1	0,5	
θ	21,8	°
cot θ	2,5	
tan θ	0,4	
Asw	226	mm^2
s	150	mm

vmin	0,71	N/mm^2
VRd,cmin	1,29E+05	N
VRd,c	**1,33E+05**	**N**
VRd,max	**1,82E+06**	**N**
VRd,s	**6,64E+05**	**N**
VRd	**6,64E+05**	**N**

Crack width (SLS)

exposure class	XD3	
wmax	0,2	mm
wk	0,2	mm
k1	0,8	
k2	0,5	
k3	3,4	
k4	0,425	
kt	0,6	
Ecm	44000	N/mm^2
αe	4,5	
fct,eff	5,0	N/mm^2
x	114	mm
hc,eff	125	mm
Ac,eff	5,00E+04	mm^2
ρp,eff	2,01E-02	
sr,max	237	mm
εsm-εcm	8,43E-04	
σs	333	N/mm^2

εc	0,0006	
εs	1,66E-03	
Nc	3,35E+05	N
Ns	3,35E+05	N
Mqp	**1,38E+08**	**Nmm**

Compressive zone height (ULS)

f	435	N/mm^2
xu	33	mm
xumax	207	mm

Check: xu < xumax	ok!

11.3 Rectangular heavily reinforced HSC beam

Rectangular reinforced HSC beam

My < Mc,pl

Moment capacity (ULS)

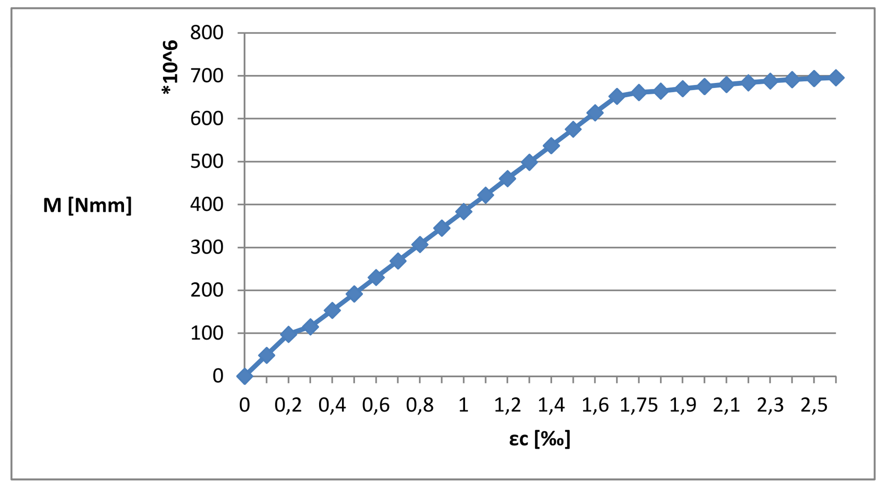
b	400	mm
h	500	mm
c	30	mm
d	442	mm
Ac	2,00E+05	mm^2
Ø main reinforcement	32	mm
number of bars	5	
As	4021	mm^2
Ø stirrups	12	mm
ρ	2,27E-02	
bar spacing	39	mm
fck	90	N/mm^2
fcd	60,0	N/mm^2
fcm	98	N/mm^2
fctm	5,0	N/mm^2
Ec	26087	N/mm^2
εc3	0,0023	
εcu3	0,0026	
fyk	500	N/mm^2
fyd	435	N/mm^2
Es	200000	N/mm^2
εsy	0,00217	
εuk	0,025	
εud	0,0225	
Asmin	464	mm^2
Asmax	8000	mm^2

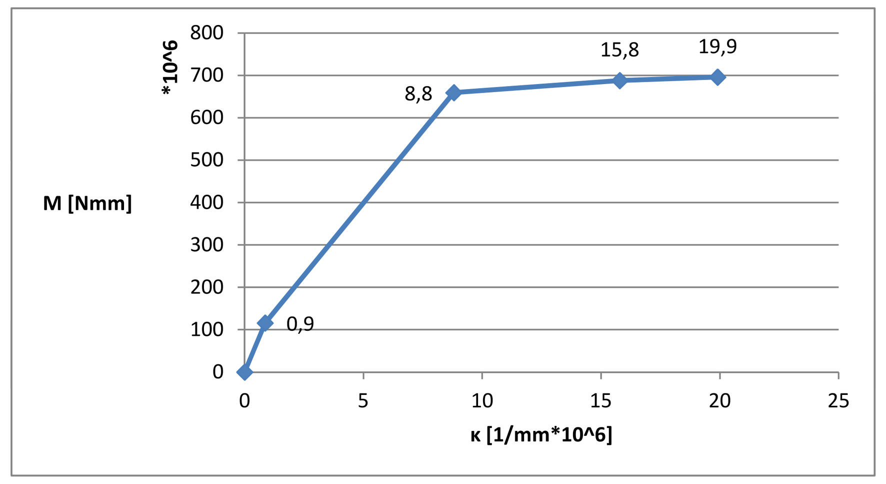
εct	1,93E-04	
εs	1,43E-04	
εc	0,00024	
x	276	mm
κr	8,62E-07	1/mm
Mcr	**1,16E+08**	**Nmm**
εc	0,0017	
εsy	2,17E-03	
x	195	mm
κy	8,80E-06	1/mm
My	**6,59E+08**	**Nmm**
εc3	0,0023	
εs	4,68E-03	
x	146	mm
κc,pl	1,58E-05	1/mm
Mc,pl	**6,88E+08**	**Nmm**
εcu3	0,0026	
εs	6,20E-03	
x	131	mm
κRd	1,99E-05	1/mm
β	44	mm
MRd	**6,96E+08**	**Nmm**

Check: My < Mc,pl	ok!
Check: εs < εud	ok!

εc [-]	εs [-]	εctensile [-]	x [mm]	κ [1/mm]	Nctop [N]	Ncbot [N]	Ns [N]	ΣH = 0	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0
0,0001	6,04E-05	8,14E-05	276	3,63E-07	1,44E+05	9,53E+04	4,85E+04	0	0	4,88E+07
0,0002	1,21E-04	1,63E-04	276	7,26E-07	2,88E+05	1,91E+05	9,71E+04	0	0	9,75E+07
0,0003	3,80E-04	0,00E+00	195	1,54E-06	3,05E+05	0,00E+00	3,05E+05	0	0	1,15E+08
0,0004	5,06E-04	0,00E+00	195	2,05E-06	4,07E+05	0,00E+00	4,07E+05	0	0	1,53E+08
0,0005	6,33E-04	0,00E+00	195	2,56E-06	5,09E+05	0,00E+00	5,09E+05	0	0	1,92E+08
0,0006	7,59E-04	0,00E+00	195	3,08E-06	6,11E+05	0,00E+00	6,11E+05	0	0	2,30E+08
0,0007	8,86E-04	0,00E+00	195	3,59E-06	7,13E+05	0,00E+00	7,13E+05	0	0	2,69E+08
0,0008	1,01E-03	0,00E+00	195	4,10E-06	8,14E+05	0,00E+00	8,14E+05	0	0	3,07E+08
0,0009	1,14E-03	0,00E+00	195	4,61E-06	9,16E+05	0,00E+00	9,16E+05	0	0	3,45E+08
0,0010	1,27E-03	0,00E+00	195	5,13E-06	1,02E+06	0,00E+00	1,02E+06	0	0	3,84E+08
0,0011	1,39E-03	0,00E+00	195	5,64E-06	1,12E+06	0,00E+00	1,12E+06	0	0	4,22E+08
0,0012	1,52E-03	0,00E+00	195	6,15E-06	1,22E+06	0,00E+00	1,22E+06	0	0	4,60E+08
0,0013	1,65E-03	0,00E+00	195	6,66E-06	1,32E+06	0,00E+00	1,32E+06	0	0	4,99E+08
0,0014	1,77E-03	0,00E+00	195	7,18E-06	1,43E+06	0,00E+00	1,43E+06	0	0	5,37E+08
0,0015	1,90E-03	0,00E+00	195	7,69E-06	1,53E+06	0,00E+00	1,53E+06	0	0	5,76E+08
0,0016	2,02E-03	0,00E+00	195	8,20E-06	1,63E+06	0,00E+00	1,63E+06	0	0	6,14E+08
0,0017	2,15E-03	0,00E+00	195	8,71E-06	1,73E+06	0,00E+00	1,73E+06	0	0	6,52E+08
0,00175	2,29E-03	0,00E+00	191	9,14E-06	1,75E+06	0,00E+00	1,75E+06	0	0	6,61E+08
0,0018	2,47E-03	0,00E+00	186	9,67E-06	1,75E+06	0,00E+00	1,75E+06	0	0	6,64E+08
0,0019	2,86E-03	0,00E+00	176	1,08E-05	1,75E+06	0,00E+00	1,75E+06	0	0	6,70E+08
0,0020	3,28E-03	0,00E+00	168	1,19E-05	1,75E+06	0,00E+00	1,75E+06	0	0	6,75E+08
0,0021	3,72E-03	0,00E+00	160	1,32E-05	1,75E+06	0,00E+00	1,75E+06	0	0	6,80E+08
0,0022	4,18E-03	0,00E+00	152	1,44E-05	1,75E+06	0,00E+00	1,75E+06	0	0	6,84E+08
0,0023	4,68E-03	0,00E+00	146	1,58E-05	1,75E+06	0,00E+00	1,75E+06	0	0	6,88E+08
0,0024	5,18E-03	0,00E+00	140	1,72E-05	1,75E+06	0,00E+00	1,75E+06	0	47	6,91E+08
0,0025	5,69E-03	0,00E+00	135	1,85E-05	1,75E+06	0,00E+00	1,75E+06	0	45	6,94E+08
0,0026	6,20E-03	0,00E+00	131	1,99E-05	1,75E+06	0,00E+00	1,75E+06	0	44	6,96E+08
0,0027	6,70E-03	0,00E+00	127	2,13E-05	1,75E+06	0,00E+00	1,75E+06	0	43	6,97E+08
0,0028	7,21E-03	0,00E+00	124	2,26E-05	1,75E+06	0,00E+00	1,75E+06	0	42	6,99E+08
0,0029	7,72E-03	0,00E+00	121	2,40E-05	1,75E+06	0,00E+00	1,75E+06	0	42	7,00E+08
0,0030	8,22E-03	0,00E+00	118	2,54E-05	1,75E+06	0,00E+00	1,75E+06	0	41	7,01E+08
0,0031	8,73E-03	0,00E+00	116	2,68E-05	1,75E+06	0,00E+00	1,75E+06	0	41	7,02E+08
0,0032	9,24E-03	0,00E+00	114	2,81E-05	1,75E+06	0,00E+00	1,75E+06	0	40	7,02E+08
0,0033	9,74E-03	0,00E+00	112	2,95E-05	1,75E+06	0,00E+00	1,75E+06	0	40	7,03E+08
0,0034	1,03E-02	0,00E+00	110	3,09E-05	1,75E+06	0,00E+00	1,75E+06	0	40	7,04E+08
0,0035	1,08E-02	0,00E+00	108	3,23E-05	1,75E+06	0,00E+00	1,75E+06	0	39	7,04E+08

εc [‰]	M [Nmm]
0	0
0,1	4,88E+07
0,2	9,75E+07
0,3	1,15E+08
0,4	1,53E+08
0,5	1,92E+08
0,6	2,30E+08
0,7	2,69E+08
0,8	3,07E+08
0,9	3,45E+08
1	3,84E+08
1,1	4,22E+08
1,2	4,60E+08
1,3	4,99E+08
1,4	5,37E+08
1,5	5,76E+08
1,6	6,14E+08
1,7	6,52E+08
1,75	6,61E+08
1,8	6,64E+08
1,9	6,70E+08
2	6,75E+08
2,1	6,80E+08
2,2	6,84E+08
2,3	6,88E+08
2,4	6,91E+08
2,5	6,94E+08
2,6	6,96E+08





κ [1/mm*10^6]	M [Nmm]
0	0
0,9	1,16E+08
8,8	6,59E+08
15,8	6,88E+08
19,9	6,96E+08

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
ρl	2,00E-02	
k1	0,15	
σcp	0	N/mm^2
αcw	1	
z	403	mm
v1	0,5	
θ	21,8	°
cot θ	2,5	
tan θ	0,4	
Asw	226	mm^2
s	150	mm

vmin	0,72	N/mm^2
VRd,cmin	1,27E+05	N
VRd,c	**2,00E+05**	**N**
VRd,max	**1,67E+06**	**N**
VRd,s	**6,07E+05**	**N**
VRd	**6,07E+05**	**N**

Crack width (SLS)

exposure class	XD3	
wmax	0,2	mm
wk	0,2	mm
k1	0,8	
k2	0,5	
k3	3,4	
k4	0,425	
kt	0,6	
Ecm	44000	N/mm^2
αe	4,5	
fct,eff	5,0	N/mm^2
x	195	mm
hc,eff	102	mm
Ac,eff	4,07E+04	mm^2
ρp,eff	9,89E-02	
sr,max	157	mm
εsm-εcm	1,27E-03	
σs	299	N/mm^2

εc	0,0012	
εs	1,50E-03	
Nc	1,20E+06	N
Ns	1,20E+06	N
Mqp	**4,53E+08**	**Nmm**

Compressive zone height (ULS)

f	435	N/mm^2
xu	131	mm
xumax	204	mm

Check: xu < xumax	ok!

11.4 Rectangular doubly reinforced NSC beam

Rectangular doubly reinforced NSC beam

Mybot < Mc,pl

Moment Capacity (ULS)

b	400	mm
h	500	mm
c	30	mm
dtop	50	mm
dbot	450	mm
Ac	2,00E+05	mm^2
Ø top reinforcement	16	mm
Ø bottom reinforcement	16	mm
number of bars at top	5	
number of bars at bottom	5	
Astop	1005	mm^2
Asbot	1005	mm^2
Ø stirrups	12	mm
ρstop	5,03E-02	
ρsbot	5,59E-03	
bar spacing	59	mm
fck	20	N/mm^2
fcd	13,3	N/mm^2
fcm	28	N/mm^2
fctm	2,2	N/mm^2
Ec	7619	N/mm^2
εc3	0,00175	
εcu3	0,0035	
fyk	500	N/mm^2
fyd	435	N/mm^2
Es	200000	N/mm^2
εsy	0,00217	
εuk	0,025	
εud	0,0225	
Asmin	207	mm^2
Asmax	8000	mm^2

Check: Mybot < Mc,pl	ok!
Check: εs < εud	ok!

εct	2,90E-04	
εstop	2,32E-04	
εsbot	2,32E-04	
εc	0,00029	
x	250	mm
κr	1,16E-06	1/mm
Mcr	**5,55E+07**	**Nmm**
εc	0,0012	
εstop	7,92E-04	
εsbot	2,17E-03	
x	157	mm
κybot	7,41E-06	1/mm
Mybot	**1,74E+08**	**Nmm**
εc3	0,00175	
εstop	8,64E-04	
εsbot	6,22E-03	
x	99	mm
κc,pl	1,77E-05	1/mm
Mc,pl	**1,79E+08**	**Nmm**
εcu3	0,0035	
εstop	8,57E-04	
εsbot	2,03E-02	
x	66	mm
κRd	5,29E-05	1/mm
β	26	mm
MRd	**1,81E+08**	**Nmm**

εc [-]	εstop [-]	εsbot [-]	εctensile [-]	x [mm]	κ [1/mm]	Nctop [N]	Ncbot [N]	Nstop [N]	Nsbot [N]	ΣH = 0	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	8,00E-05	8,00E-05	1,00E-04	250	4,00E-07	3,81E+04	3,81E+04	1,61E+04	1,61E+04	2E-11	0	1,91E+07
0,0002	1,60E-04	1,60E-04	2,00E-04	250	8,00E-07	7,62E+04	7,62E+04	3,22E+04	3,22E+04	4E-11	0	3,83E+07
0,0003	2,04E-04	5,61E-04	0,00E+00	157	1,91E-06	7,17E+04	0,00E+00	4,11E+04	1,13E+05	0	0	4,49E+07
0,0004	2,72E-04	7,48E-04	0,00E+00	157	2,55E-06	9,56E+04	0,00E+00	5,48E+04	1,50E+05	0	0	5,99E+07
0,0005	3,41E-04	9,35E-04	0,00E+00	157	3,19E-06	1,19E+05	0,00E+00	6,85E+04	1,88E+05	0	0	7,49E+07
0,0006	4,09E-04	1,12E-03	0,00E+00	157	3,83E-06	1,43E+05	0,00E+00	8,22E+04	2,26E+05	0	0	8,99E+07
0,0007	4,77E-04	1,31E-03	0,00E+00	157	4,46E-06	1,67E+05	0,00E+00	9,59E+04	2,63E+05	0	0	1,05E+08
0,0008	5,45E-04	1,50E-03	0,00E+00	157	5,10E-06	1,91E+05	0,00E+00	1,10E+05	3,01E+05	0	0	1,20E+08
0,0009	6,13E-04	1,68E-03	0,00E+00	157	5,74E-06	2,15E+05	0,00E+00	1,23E+05	3,38E+05	0	0	1,35E+08
0,001	6,81E-04	1,87E-03	0,00E+00	157	6,38E-06	2,39E+05	0,00E+00	1,37E+05	3,76E+05	0	0	1,50E+08
0,0011	7,49E-04	2,06E-03	0,00E+00	157	7,01E-06	2,63E+05	0,00E+00	1,51E+05	4,14E+05	0	0	1,65E+08
0,0012	8,02E-04	2,38E-03	0,00E+00	151	7,96E-06	2,76E+05	0,00E+00	1,61E+05	4,37E+05	0	0	1,75E+08
0,0013	8,25E-04	2,97E-03	0,00E+00	137	9,50E-06	2,71E+05	0,00E+00	1,66E+05	4,37E+05	0	0	1,76E+08
0,0014	8,42E-04	3,62E-03	0,00E+00	126	1,12E-05	2,68E+05	0,00E+00	1,69E+05	4,37E+05	0	0	1,77E+08
0,0015	8,54E-04	4,31E-03	0,00E+00	116	1,29E-05	2,65E+05	0,00E+00	1,72E+05	4,37E+05	0	0	1,78E+08
0,0016	8,61E-04	5,05E-03	0,00E+00	108	1,48E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	0	1,79E+08
0,0017	8,64E-04	5,82E-03	0,00E+00	102	1,67E-05	2,63E+05	0,00E+00	1,74E+05	4,37E+05	0	0	1,79E+08
0,00175	8,64E-04	6,22E-03	0,00E+00	99	1,77E-05	2,63E+05	0,00E+00	1,74E+05	4,37E+05	0	0	1,79E+08
0,0018	8,64E-04	6,63E-03	0,00E+00	96	1,87E-05	2,63E+05	0,00E+00	1,74E+05	4,37E+05	0	32	1,80E+08
0,0019	8,63E-04	7,43E-03	0,00E+00	92	2,07E-05	2,64E+05	0,00E+00	1,74E+05	4,37E+05	0	31	1,80E+08
0,002	8,62E-04	8,24E-03	0,00E+00	88	2,28E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	30	1,80E+08
0,0021	8,62E-04	9,04E-03	0,00E+00	85	2,48E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	29	1,80E+08
0,0022	8,61E-04	9,85E-03	0,00E+00	82	2,68E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	28	1,81E+08
0,0023	8,61E-04	1,07E-02	0,00E+00	80	2,88E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	28	1,81E+08
0,0024	8,60E-04	1,15E-02	0,00E+00	78	3,08E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	27	1,81E+08
0,0025	8,60E-04	1,23E-02	0,00E+00	76	3,28E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	27	1,81E+08
0,0026	8,59E-04	1,31E-02	0,00E+00	75	3,48E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	27	1,81E+08
0,0027	8,59E-04	1,39E-02	0,00E+00	73	3,68E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	27	1,81E+08
0,0028	8,59E-04	1,47E-02	0,00E+00	72	3,88E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	26	1,81E+08
0,0029	8,58E-04	1,55E-02	0,00E+00	71	4,08E-05	2,64E+05	0,00E+00	1,73E+05	4,37E+05	0	26	1,81E+08
0,003	8,58E-04	1,63E-02	0,00E+00	70	4,28E-05	2,65E+05	0,00E+00	1,73E+05	4,37E+05	0	26	1,81E+08
0,0031	8,58E-04	1,71E-02	0,00E+00	69	4,48E-05	2,65E+05	0,00E+00	1,72E+05	4,37E+05	0	26	1,81E+08
0,0032	8,58E-04	1,79E-02	0,00E+00	68	4,68E-05	2,65E+05	0,00E+00	1,72E+05	4,37E+05	0	26	1,81E+08
0,0033	8,57E-04	1,87E-02	0,00E+00	68	4,89E-05	2,65E+05	0,00E+00	1,72E+05	4,37E+05	0	26	1,81E+08
0,0034	8,57E-04	1,95E-02	0,00E+00	67	5,09E-05	2,65E+05	0,00E+00	1,72E+05	4,37E+05	0	26	1,81E+08
0,0035	8,57E-04	2,03E-02	0,00E+00	66	5,29E-05	2,65E+05	0,00E+00	1,72E+05	4,37E+05	0	26	1,81E+08

εc [‰]	M [Nmm]
0	0
0,1	1,91E+07
0,2	3,83E+07
0,3	4,49E+07
0,4	5,99E+07
0,5	7,49E+07
0,6	8,99E+07
0,7	1,05E+08
0,8	1,20E+08
0,9	1,35E+08
1	1,50E+08
1,1	1,65E+08
1,2	1,75E+08
1,3	1,76E+08
1,4	1,77E+08
1,5	1,78E+08
1,6	1,79E+08
1,7	1,79E+08
1,75	1,79E+08
1,8	1,80E+08
1,9	1,80E+08
2	1,80E+08
2,1	1,80E+08
2,2	1,81E+08
2,3	1,81E+08
2,4	1,81E+08
2,5	1,81E+08
2,6	1,81E+08
2,7	1,81E+08
2,8	1,81E+08
2,9	1,81E+08
3	1,81E+08
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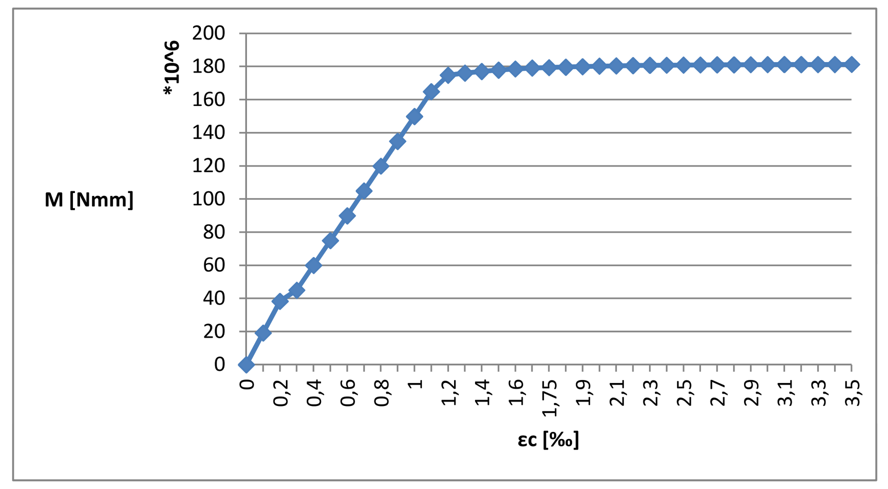
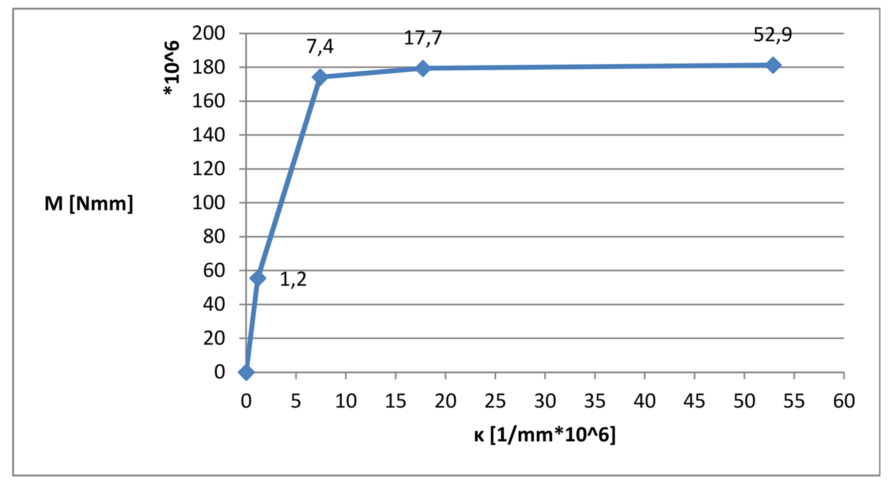
3,5	1,81E+08





κ [1/mm*10^6]	M [Nmm]
0	0
1,2	5,55E+07
7,4	1,74E+08
17,7	1,79E+08
52,9	1,81E+08

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
ρl	5,59E-03	
k1	0,15	
σcp	0	N/mm^2
αcw	1	
z	424	mm
v1	0,6	
θ	21,8	°
cot θ	2,5	
tan θ	0,4	
Asw	226	mm^2
s	150	mm

vmin	0,34	N/mm^2
VRd,cmin	6,06E+04	N
VRd,c	**8,05E+04**	**N**
VRd,max	**4,68E+05**	**N**
VRd,s	**6,40E+05**	**N**
VRd	**4,68E+05**	**N**

Crack width (SLS)

exposure class	XD3	
wmax	0,2	mm
wk	0,2	mm
k1	0,8	
k2	0,5	
k3	3,4	
k4	0,425	
kt	0,6	
Ecm	30000	N/mm^2
αe	6,7	
fct,eff	2,2	N/mm^2
x	157	mm
hc,eff	114	mm
Ac,eff	4,58E+04	mm^2
ρp,eff	2,20E-02	
sr,max	226	mm
εsm-εcm	8,86E-04	
σs	246	N/mm^2

εc	0,0007	
εstop	4,49E-04	
εsbot	1,23E-03	
Nctop	1,57E+05	N
Nstop	9,02E+04	N
Nsbot	2,48E+05	N
Mqp	**9,87E+07**	**Nmm**

Compressive zone height (ULS)

f	435	N/mm^2
xu	66	mm
xumax	241	mm

Check: xu < xumax	ok!

11.5 Rectangular doubly reinforced HSC beam

Rectangular doubly reinforced HSC beam

Mybot < Mc,pl

Moment Capacity (ULS)

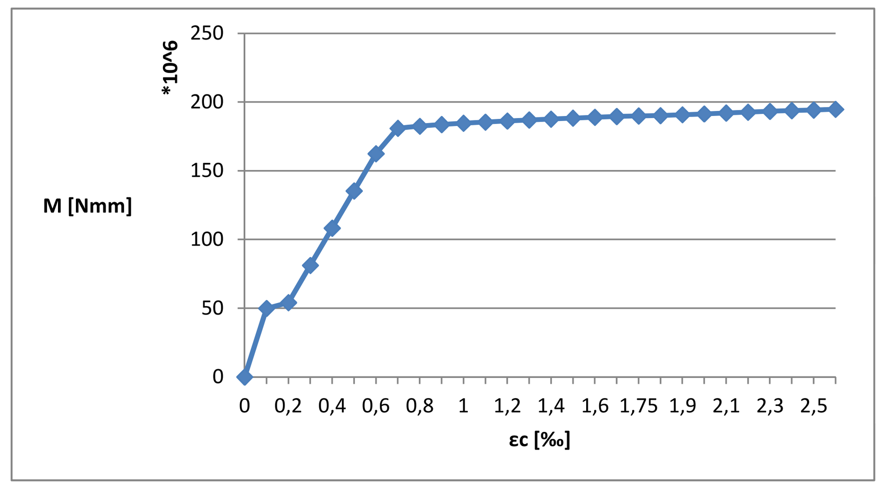
b	400	mm
h	500	mm
c	30	mm
dtop	50	mm
dbot	450	mm
Ac	2,00E+05	mm^2
Ø top reinforcement	16	mm
Ø bottom reinforcement	16	mm
number of bars at top	5	
number of bars at bottom	5	
Astop	1005	mm^2
Asbot	1005	mm^2
Ø stirrups	12	mm
ρstop	5,03E-02	
ρsbot	5,59E-03	
bar spacing	59	mm
fck	90	N/mm^2
fcd	60,0	N/mm^2
fcm	98	N/mm^2
fctm	5,0	N/mm^2
Ec	26087	N/mm^2
εc3	0,0023	
εcu3	0,0026	
fyk	500	N/mm^2
fyd	435	N/mm^2
Es	200000	N/mm^2
εsy	0,00217	
εuk	0,025	
εud	0,0225	
Asmin	472	mm^2
Asmax	8000	mm^2

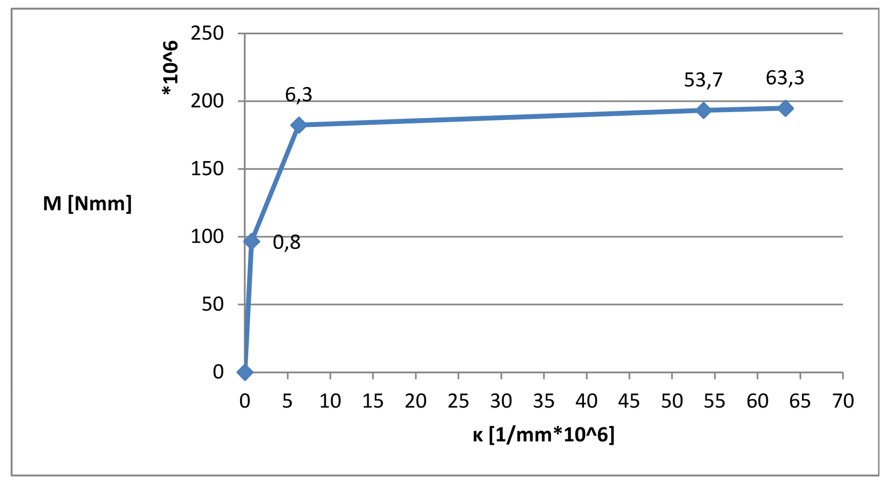
εct	1,93E-04	
εstop	1,55E-04	
εsbot	1,55E-04	
εc	0,00019	
x	250	mm
κr	7,74E-07	1/mm
Mcr	**9,65E+07**	**Nmm**
εc	0,0007	
εstop	3,50E-04	
εsbot	2,17E-03	
x	106	mm
κybot	6,31E-06	1/mm
Mybot	**1,82E+08**	**Nmm**
εc3	0,0023	
εstop	-3,84E-04	
εsbot	2,19E-02	
x	43	mm
κc,pl	5,37E-05	1/mm
Mc,pl	**1,93E+08**	**Nmm**
εcu3	0,0026	
εstop	-5,63E-04	
εsbot	2,59E-02	
x	41	mm
κRd	6,33E-05	1/mm
β	14	mm
MRd	**1,95E+08**	**Nmm**

Check: Mybot < Mc,pl	ok!
Check: εs < εud	not ok!

εc [-]	εstop [-]	εsbot [-]	εctensile [-]	x [mm]	κ [1/mm]	Nctop [N]	Ncbot [N]	Nstop [N]	Nsbot [N]	ΣH = 0	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	8,00E-05	8,00E-05	1,00E-04	250	4,00E-07	1,30E+05	1,30E+05	1,61E+04	1,61E+04	0E+00	0	4,99E+07
0,0002	1,05E-04	6,53E-04	0,00E+00	106	1,90E-06	1,10E+05	0,00E+00	2,12E+04	1,31E+05	0E+00	0	5,41E+07
0,0003	1,58E-04	9,79E-04	0,00E+00	106	2,84E-06	1,65E+05	0,00E+00	3,17E+04	1,97E+05	0	0	8,12E+07
0,0004	2,10E-04	1,31E-03	0,00E+00	106	3,79E-06	2,20E+05	0,00E+00	4,23E+04	2,63E+05	0	0	1,08E+08
0,0005	2,63E-04	1,63E-03	0,00E+00	106	4,74E-06	2,75E+05	0,00E+00	5,29E+04	3,28E+05	0	0	1,35E+08
0,0006	3,16E-04	1,96E-03	0,00E+00	106	5,69E-06	3,30E+05	0,00E+00	6,35E+04	3,94E+05	0	0	1,62E+08
0,0007	3,51E-04	2,44E-03	0,00E+00	100	6,98E-06	3,66E+05	0,00E+00	7,06E+04	4,37E+05	0	0	1,81E+08
0,0008	3,46E-04	3,29E-03	0,00E+00	88	9,08E-06	3,68E+05	0,00E+00	6,95E+04	4,37E+05	0	0	1,82E+08
0,0009	3,30E-04	4,23E-03	0,00E+00	79	1,14E-05	3,71E+05	0,00E+00	6,64E+04	4,37E+05	0	0	1,84E+08
0,001	3,06E-04	5,25E-03	0,00E+00	72	1,39E-05	3,76E+05	0,00E+00	6,14E+04	4,37E+05	0	0	1,85E+08
0,0011	2,74E-04	6,34E-03	0,00E+00	67	1,65E-05	3,82E+05	0,00E+00	5,50E+04	4,37E+05	0	0	1,85E+08
0,0012	2,36E-04	7,48E-03	0,00E+00	62	1,93E-05	3,90E+05	0,00E+00	4,74E+04	4,37E+05	0	0	1,86E+08
0,0013	1,93E-04	8,66E-03	0,00E+00	59	2,21E-05	3,98E+05	0,00E+00	3,88E+04	4,37E+05	0	0	1,87E+08
0,0014	1,46E-04	9,89E-03	0,00E+00	56	2,51E-05	4,08E+05	0,00E+00	2,94E+04	4,37E+05	0	0	1,88E+08
0,0015	9,55E-05	1,11E-02	0,00E+00	53	2,81E-05	4,18E+05	0,00E+00	1,92E+04	4,37E+05	0	0	1,88E+08
0,0016	4,20E-05	1,24E-02	0,00E+00	51	3,12E-05	4,29E+05	0,00E+00	8,45E+03	4,37E+05	0	0	1,89E+08
0,0017	-1,39E-05	1,37E-02	0,00E+00	50	3,43E-05	4,40E+05	0,00E+00	-2,79E+03	4,37E+05	0	0	1,90E+08
0,00175	-4,26E-05	1,44E-02	0,00E+00	49	3,59E-05	4,46E+05	0,00E+00	-8,57E+03	4,37E+05	0	0	1,90E+08
0,0018	-7,19E-05	1,50E-02	0,00E+00	48	3,74E-05	4,52E+05	0,00E+00	-1,44E+04	4,37E+05	0	0	1,90E+08
0,0019	-1,32E-04	1,64E-02	0,00E+00	47	4,06E-05	4,64E+05	0,00E+00	-2,65E+04	4,37E+05	0	0	1,91E+08
0,002	-1,93E-04	1,77E-02	0,00E+00	46	4,39E-05	4,76E+05	0,00E+00	-3,88E+04	4,37E+05	0	0	1,91E+08
0,0021	-2,55E-04	1,91E-02	0,00E+00	45	4,71E-05	4,88E+05	0,00E+00	-5,13E+04	4,37E+05	0	0	1,92E+08
0,0022	-3,19E-04	2,05E-02	0,00E+00	44	5,04E-05	5,01E+05	0,00E+00	-6,41E+04	4,37E+05	0	0	1,93E+08
0,0023	-3,84E-04	2,19E-02	0,00E+00	43	5,37E-05	5,14E+05	0,00E+00	-7,71E+04	4,37E+05	0	0	1,93E+08
0,0024	-4,47E-04	2,32E-02	0,00E+00	42	5,69E-05	5,27E+05	0,00E+00	-8,98E+04	4,37E+05	0	14	1,94E+08
0,0025	-5,06E-04	2,46E-02	0,00E+00	42	6,01E-05	5,39E+05	0,00E+00	-1,02E+05	4,37E+05	0	14	1,94E+08
0,0026	-5,63E-04	2,59E-02	0,00E+00	41	6,33E-05	5,50E+05	0,00E+00	-1,13E+05	4,37E+05	0	14	1,95E+08
0,0027	-6,16E-04	2,71E-02	0,00E+00	41	6,63E-05	5,61E+05	0,00E+00	-1,24E+05	4,37E+05	0	14	1,95E+08
0,0028	-6,67E-04	2,84E-02	0,00E+00	40	6,93E-05	5,71E+05	0,00E+00	-1,34E+05	4,37E+05	0	14	1,95E+08
0,0029	-7,15E-04	2,96E-02	0,00E+00	40	7,23E-05	5,81E+05	0,00E+00	-1,44E+05	4,37E+05	0	14	1,96E+08
0,003	-7,61E-04	3,09E-02	0,00E+00	40	7,52E-05	5,90E+05	0,00E+00	-1,53E+05	4,37E+05	0	14	1,96E+08
0,0031	-8,06E-04	3,21E-02	0,00E+00	40	7,81E-05	5,99E+05	0,00E+00	-1,62E+05	4,37E+05	0	14	1,96E+08
0,0032	-8,48E-04	3,32E-02	0,00E+00	40	8,10E-05	6,08E+05	0,00E+00	-1,71E+05	4,37E+05	0	14	1,97E+08
0,0033	-8,89E-04	3,44E-02	0,00E+00	39	8,38E-05	6,16E+05	0,00E+00	-1,79E+05	4,37E+05	0	14	1,97E+08
0,0034	-9,28E-04	3,56E-02	0,00E+00	39	8,66E-05	6,24E+05	0,00E+00	-1,87E+05	4,37E+05	0	14	1,97E+08
0,0035	-9,66E-04	3,67E-02	0,00E+00	39	8,93E-05	6,31E+05	0,00E+00	-1,94E+05	4,37E+05	0	14	1,97E+08

εc [‰]	M [Nmm]
0	0
0,1	4,99E+07
0,2	5,41E+07
0,3	8,12E+07
0,4	1,08E+08
0,5	1,35E+08
0,6	1,62E+08
0,7	1,81E+08
0,8	1,82E+08
0,9	1,84E+08
1	1,85E+08
1,1	1,85E+08
1,2	1,86E+08
1,3	1,87E+08
1,4	1,88E+08
1,5	1,88E+08
1,6	1,89E+08
1,7	1,90E+08
1,75	1,90E+08
1,8	1,90E+08
1,9	1,91E+08
2	1,91E+08
2,1	1,92E+08
2,2	1,93E+08
2,3	1,93E+08
2,4	1,94E+08
2,5	1,94E+08
2,6	1,95E+08





κ [1/mm*10^6]	M [Nmm]
0	0
0,8	9,65E+07
6,3	1,82E+08
53,7	1,93E+08
63,3	1,95E+08

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
ρl	5,59E-03	
k1	0,15	
σcp	0	N/mm^2
αcw	1	
z	436	mm
v1	0,5	
θ	21,8	°
cot θ	2,5	
tan θ	0,4	
Asw	226	mm^2
s	150	mm

vmin	0,71	N/mm^2
VRd,cmin	1,29E+05	N
VRd,c	**1,33E+05**	**N**
VRd,max	**1,80E+06**	**N**
VRd,s	**6,57E+05**	**N**
VRd	**6,57E+05**	**N**

Crack width (SLS)

exposure class	XD3	
wmax	0,2	mm
wk	0,2	mm
k1	0,8	
k2	0,5	
k3	3,4	
k4	0,425	
Ecm	44000	N/mm^2
kt	0,6	
αe	4,5	
fct,eff	5,0	N/mm^2
x	106	mm
hc,eff	125	mm
Ac,eff	5,00E+04	mm^2
ρp,eff	2,01E-02	
sr,max	237	mm
εsm-εcm	8,43E-04	
σs	333	N/mm^2

εc	0,0005	
εstop	2,68E-04	
εsbot	1,66E-03	
Nctop	2,81E+05	N
Nstop	5,39E+04	N
Nsbot	3,35E+05	N
Mqp	**1,38E+08**	**Nmm**

Compressive zone height (ULS)

f	435	N/mm^2
xu	41	mm
xumax	207	mm

Check: xu < xumax	ok!

11.6 Rectangular doubly reinforced NSC beam + normal force

Rectangular doubly reinforced NSC beam + normal force

Mybot > Mytop > Mc,pl

Moment capacity (ULS)

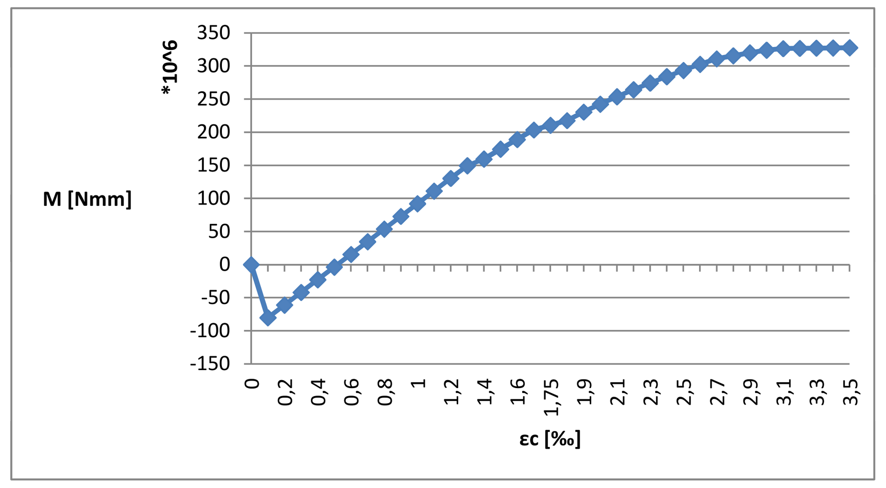
b	400	mm
h	500	mm
c	30	mm
dtop	50	mm
dbot	450	mm
Ac	2,00E+05	mm^2
Ø top reinforcement	16	mm
Ø bottom reinforcement	16	mm
number of bars at top	5	
number of bars at bottom	5	
Astop	1005	mm^2
Asbot	1005	mm^2
Ø stirrups	12	mm
ρstop	5,03E-02	
ρsbot	5,59E-03	
bar spacing	59	mm
fck	20	N/mm^2
fcd	13,3	N/mm^2
fcm	28	N/mm^2
fctm	2,2	N/mm^2
Ec	7619	N/mm^2
εc3	0,00175	
εcu3	0,0035	
fyk	500	N/mm^2
fyd	435	N/mm^2
Es	200000	N/mm^2
εsy	0,00217	
εuk	0,025	
εud	0,0225	
Asmin	207	mm^2
Asmax	8000	mm^2
Ned	1,0E+06	N

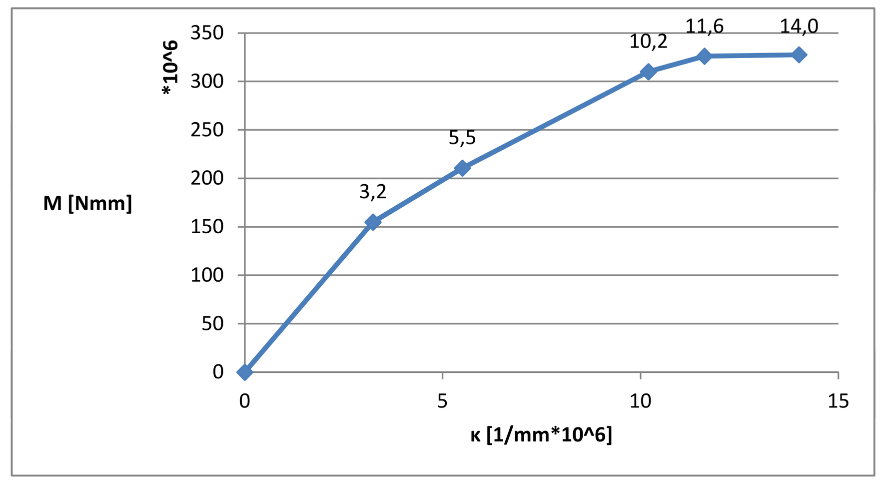
εct	2,90E-04	
εstop	1,17E-03	
εsbot	1,28E-04	
εc	0,00133	
x	410	mm
κr	3,24E-06	1/mm
Mcr	**1,55E+08**	**Nmm**
εc3	0,00175	
εstop	1,48E-03	
εsbot	7,24E-04	
x	318	mm
κc,pl	5,50E-06	1/mm
Mc,pl	**2,11E+08**	**Nmm**
εstop	2,17E-03	
εsbot	1,91E-03	
εc	0,0027	
x	263	mm
κytop	1,02E-05	1/mm
β	96	mm
Mytop	**3,10E+08**	**Nmm**
εstop	2,47E-03	
εsbot	2,17E-03	
εc	0,0031	
x	263	mm
κybot	1,16E-05	1/mm
β	99	mm
Mybot	**3,26E+08**	**Nmm**
εcu3	0,0035	
εstop	2,80E-03	
εsbot	2,80E-03	
x	250	mm
κRd	1,40E-05	1/mm
β	97	mm
MRd	**3,28E+08**	**Nmm**

Check: Mybot > Mytop > Mc,pl	ok!
Check: εc < εcu3	ok!
Check: εs < εud	ok!

εc [-]	εstop [-]	εsbot [-]	εctensile [-]	x [mm]	κ [1/mm]	Nctop [N]	Ncbot [N]	Nstop [N]	Nsbot [N]	ΣH = 0	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	1,84E-04	-8,55E-04	-9,38E-04	-60	-1,68E-06	-9,09E+03	-8,00E+05	3,70E+04	-1,72E+05	0	0	-8,02E+07
0,0002	2,64E-04	-7,75E-04	-8,38E-04	-157	-1,28E-06	-4,77E+04	-8,39E+05	5,30E+04	-1,56E+05	0	0	-6,11E+07
0,0003	3,44E-04	-6,95E-04	-7,38E-04	-342	-8,77E-07	-1,56E+05	-9,48E+05	6,91E+04	-1,40E+05	0	0	-4,19E+07
0,0004	4,24E-04	-6,15E-04	-6,38E-04	-839	-4,77E-07	-5,11E+05	-1,30E+06	8,52E+04	-1,24E+05	0	0	-2,28E+07
0,0005	5,04E-04	-5,35E-04	-5,38E-04	-6501	-7,69E-08	-4,95E+06	-5,74E+06	1,01E+05	-1,07E+05	0	0	-3,68E+06
0,0006	5,84E-04	-4,55E-04	-4,38E-04	1857	3,23E-07	1,70E+06	9,07E+05	1,17E+05	-9,14E+04	0	0	1,55E+07
0,0007	6,64E-04	-3,75E-04	-3,38E-04	968	7,23E-07	1,03E+06	2,41E+05	1,33E+05	-7,53E+04	0	0	3,46E+07
0,0008	7,44E-04	-2,95E-04	-2,38E-04	712	1,12E-06	8,68E+05	7,72E+04	1,50E+05	-5,92E+04	0	0	5,37E+07
0,0009	8,24E-04	-2,15E-04	-1,38E-04	591	1,52E-06	8,10E+05	1,92E+04	1,66E+05	-4,32E+04	0	0	7,29E+07
0,001	9,04E-04	-1,35E-04	-3,85E-05	520	1,92E-06	7,92E+05	1,17E+03	1,82E+05	-2,71E+04	0	0	9,20E+07
0,0011	9,84E-04	-5,46E-05	6,15E-05	474	2,32E-06	7,94E+05	2,48E+03	1,98E+05	-1,10E+04	0	0	1,11E+08
0,0012	1,06E-03	2,54E-05	1,62E-04	441	2,72E-06	8,06E+05	1,46E+04	2,14E+05	5,10E+03	0	0	1,30E+08
0,0013	1,14E-03	1,05E-04	2,62E-04	416	3,12E-06	8,25E+05	3,34E+04	2,30E+05	2,12E+04	0	0	1,49E+08
0,0014	1,22E-03	2,62E-04	0,00E+00	379	3,69E-06	8,08E+05	0,00E+00	2,44E+05	5,28E+04	0	0	1,59E+08
0,0015	1,29E-03	3,86E-04	0,00E+00	358	4,19E-06	8,18E+05	0,00E+00	2,59E+05	7,76E+04	0	0	1,74E+08
0,0016	1,36E-03	5,16E-04	0,00E+00	340	4,70E-06	8,29E+05	0,00E+00	2,74E+05	1,04E+05	0	0	1,89E+08
0,0017	1,44E-03	6,53E-04	0,00E+00	325	5,23E-06	8,42E+05	0,00E+00	2,89E+05	1,31E+05	0	0	2,03E+08
0,00175	1,48E-03	7,24E-04	0,00E+00	318	5,50E-06	8,49E+05	0,00E+00	2,97E+05	1,46E+05	0	0	2,11E+08
0,0018	1,51E-03	7,94E-04	0,00E+00	312	5,77E-06	8,56E+05	0,00E+00	3,04E+05	1,60E+05	0	104	2,18E+08
0,0019	1,59E-03	9,32E-04	0,00E+00	302	6,29E-06	8,69E+05	0,00E+00	3,19E+05	1,87E+05	0	101	2,30E+08
0,002	1,66E-03	1,07E-03	0,00E+00	294	6,81E-06	8,81E+05	0,00E+00	3,34E+05	2,14E+05	0	99	2,42E+08
0,0021	1,73E-03	1,20E-03	0,00E+00	287	7,32E-06	8,92E+05	0,00E+00	3,49E+05	2,41E+05	0	98	2,54E+08
0,0022	1,81E-03	1,32E-03	0,00E+00	281	7,83E-06	9,02E+05	0,00E+00	3,64E+05	2,66E+05	0	97	2,64E+08
0,0023	1,88E-03	1,45E-03	0,00E+00	276	8,33E-06	9,12E+05	0,00E+00	3,79E+05	2,91E+05	0	96	2,74E+08
0,0024	1,96E-03	1,57E-03	0,00E+00	272	8,82E-06	9,22E+05	0,00E+00	3,94E+05	3,16E+05	0	96	2,84E+08
0,0025	2,03E-03	1,69E-03	0,00E+00	268	9,31E-06	9,31E+05	0,00E+00	4,09E+05	3,40E+05	0	96	2,93E+08
0,0026	2,11E-03	1,81E-03	0,00E+00	265	9,80E-06	9,39E+05	0,00E+00	4,24E+05	3,64E+05	0	96	3,03E+08
0,0027	2,19E-03	1,92E-03	0,00E+00	263	1,03E-05	9,49E+05	0,00E+00	4,37E+05	3,86E+05	0	96	3,11E+08
0,0028	2,27E-03	1,99E-03	0,00E+00	263	1,07E-05	9,64E+05	0,00E+00	4,37E+05	4,01E+05	0	97	3,15E+08
0,0029	2,35E-03	2,07E-03	0,00E+00	263	1,10E-05	9,78E+05	0,00E+00	4,37E+05	4,16E+05	0	97	3,20E+08
0,003	2,43E-03	2,14E-03	0,00E+00	263	1,14E-05	9,93E+05	0,00E+00	4,37E+05	4,30E+05	0	98	3,24E+08
0,0031	2,51E-03	2,24E-03	0,00E+00	261	1,19E-05	1,00E+06	0,00E+00	4,37E+05	4,37E+05	0	99	3,26E+08
0,0032	2,58E-03	2,38E-03	0,00E+00	258	1,24E-05	1,00E+06	0,00E+00	4,37E+05	4,37E+05	0	98	3,27E+08
0,0033	2,65E-03	2,52E-03	0,00E+00	255	1,29E-05	1,00E+06	0,00E+00	4,37E+05	4,37E+05	0	98	3,27E+08
0,0034	2,73E-03	2,66E-03	0,00E+00	252	1,35E-05	1,00E+06	0,00E+00	4,37E+05	4,37E+05	0	98	3,27E+08
0,0035	2,80E-03	2,80E-03	0,00E+00	250	1,40E-05	1,00E+06	0,00E+00	4,37E+05	4,37E+05	0	97	3,28E+08

εc [‰]	M [Nmm]
0	0
0,1	-8,02E+07
0,2	-6,11E+07
0,3	-4,19E+07
0,4	-2,28E+07
0,5	-3,68E+06
0,6	1,55E+07
0,7	3,46E+07
0,8	5,37E+07
0,9	7,29E+07
1	9,20E+07
1,1	1,11E+08
1,2	1,30E+08
1,3	1,49E+08
1,4	1,59E+08
1,5	1,74E+08
1,6	1,89E+08
1,7	2,03E+08
1,75	2,11E+08
1,8	2,18E+08
1,9	2,30E+08
2	2,42E+08
2,1	2,54E+08
2,2	2,64E+08
2,3	2,74E+08
2,4	2,84E+08
2,5	2,93E+08
2,6	3,03E+08
2,7	3,11E+08
2,8	3,15E+08
2,9	3,20E+08
3	3,24E+08
3,1	3,26E+08
3,2	3,27E+08
3,3	3,27E+08
3,4	3,27E+08
3,5	3,28E+08





κ [1/mm*10^6]	M [Nmm]
0	0
3,2	1,55E+08
5,5	2,11E+08
10,2	3,10E+08
11,6	3,26E+08
14,0	3,28E+08

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
ρl	5,59E-03	
k1	0,15	
σcp	2,67	N/mm^2
αcw	1,25	
z	353	mm
v1	0,6	
θ	21,8	°
cot θ	2,5	
tan θ	0,4	
Asw	226	mm^2
s	150	mm

vmin	0,34	N/mm^2
VRd,cmin	1,33E+05	N
VRd,c	**1,52E+05**	**N**
VRd,max	**4,87E+05**	**N**
VRd,s	**5,32E+05**	**N**
VRd	**4,87E+05**	**N**

Crack width (SLS)

exposure class	XD3	
wmax	0,2	mm
wk	0,2	mm
k1	0,8	
k2	0,5	
k3	3,4	
k4	0,425	
kt	0,6	
Ecm	30000	N/mm^2
αe	6,7	
fct,eff	2,2	N/mm^2
x	289	mm
hc,eff	70	mm
Ac,eff	2,81E+04	mm^2
ρp,eff	3,57E-02	
sr,max	178	mm
εsm-εcm	1,12E-03	
σs	225	N/mm^2

εc	0,0020	
εstop	1,67E-03	
εsbot	1,13E-03	
Nctop	8,90E+05	N
Nstop	3,36E+05	N
Nsbot	2,26E+05	N
Mqp	**2,49E+08**	**Nmm**

Compressive zone height (ULS)

f	435	N/mm^2
xu	250	mm
xumax	241	mm

Check: xu < xumax	Ned > 0,1fcdAc: no check required!

11.7 Rectangular prestressed NSC beam

Rectangular prestressed NSC beam

Moment capacity (ULS)

b	400	mm
h	500	mm
c	30	mm
d	450	mm
Ac	2,00E+05	mm^2
Ic	4,17E+09	mm^4
z	250	mm
Wbot	1,67E+07	mm^3
Wtop	1,67E+07	mm^3

εcu3	0,0035	
Δεp	0,0204	
εp	0,0257	
xu	66	mm
Ncu	2,64E+05	N
ΔNp	2,64E+05	N
β	26	mm
MRd	**1,99E+08**	**Nmm**

Check: εp < εud	ok!

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	3	
Ap	450	mm^2
strand spacing	134	mm
fck	20	N/mm^2
fcd	13,3	N/mm^2
fcm	28	N/mm^2
fctm	2,2	N/mm^2
Ec	7619	N/mm^2
εc3	0,00175	
εcu3	0,0035	
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	195000	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

(assuming 25% prestress losses due to time dependent behavior)

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
pl	0	
k1	0,15	
σcp	2,35	N/mm^2
αcw	1,18	
z	384	mm
v1	0,6	
α	90	°
sin α	1	
cot α	0	
θ	21,8	°
sin θ	0,37	
cot θ	2,5	
Ø stirrups	12	mm
Asw	226	mm^2
s	100	mm
fyk	500	N/mm^2

vmin	0,34	N/mm^2
VRd,cmin	1,24E+05	N
VRd,c	**6,36E+04**	**N**
VRd,max	**4,98E+05**	**N**
VRd,s	**8,69E+05**	**N**
VRd	**4,98E+05**	**N**

Crack width (SLS)

Requirement: beam remains uncracked in SLS

t = 0: check top fibre	M ≥	3,64E+07	Nmm
t = 0: check bottom fibre	M ≤	2,15E+08	Nmm
t = ∞: check top fibre	M ≥	1,81E+07	Nmm
t = ∞: check bottom fibre	M ≤	1,70E+08	Nmm

Compressive zone height (ULS)

f	645	N/mm^2
xu	66	mm
xumax	197	mm

Check: xu < xumax	ok!

11.8 Rectangular prestressed HSC beam

Rectangular prestressed HSC beam

Moment capacity (ULS)

b	400	mm
h	500	mm
c	30	mm
d	450	mm
Ac	2,00E+05	mm^2
Ic	4,17E+09	mm^4
z	250	mm
Wbot	1,67E+07	mm^3
Wtop	1,67E+07	mm^3

εcu3	0,0026	
Δεp	0,0446	
εp	0,0499	
xu	25	mm
Ncu	3,32E+05	N
ΔNp	3,32E+05	N
β	8	mm
MRd	2,53E+08	Nmm

Check: εp < εud	not ok!

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	3	
Ap	450	mm^2
strand spacing	134	mm
fck	90	N/mm^2
fcd	60,0	N/mm^2
fcm	98	N/mm^2
fctm	5,0	N/mm^2
Ec	26087	N/mm^2
εc3	0,0023	
εcu3	0,0026	
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	195000	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

(assuming 25% prestress losses due to time dependent behavior)

Shear capacity (ULS)

CRd,c	0,12	
k	1,67	
ρl	0	
k1	0,15	
σcp	2,35	N/mm^2
αcw	1,04	
z	425	mm
v1	0,5	
α	90 °	
sin α	1	
cot α	0	
θ	21,8 °	
sin θ	0,37	
cot θ	2,5	
Ø stirrups	12	mm
Asw	226	mm^2
s	100	mm
fyk	500	N/mm^2

vmin	0,71	N/mm^2
VRd,cmin	1,92E+05	N
VRd,c	6,36E+04	N
VRd,max	1,83E+06	N
VRd,s	9,62E+05	N
VRd	9,62E+05	N

Crack width (SLS)

Requirement: beam remains uncracked in SLS

t = 0: check top fibre	M ≥	-1,08E+07	Nmm
t = 0: check bottom fibre	M ≤	2,62E+08	Nmm
t = ∞: check top fibre	M ≥	-2,91E+07	Nmm
t = ∞: check bottom fibre	M ≤	2,17E+08	Nmm

Compressive zone height (ULS)

f	645	N/mm^2
xu	25	mm
xumax	164	mm

Check: xu < xumax	ok!

11.9 Reinforced NSC box girder

Reinforced NSC box girder

My < Mc,pl

Moment Capacity (ULS)

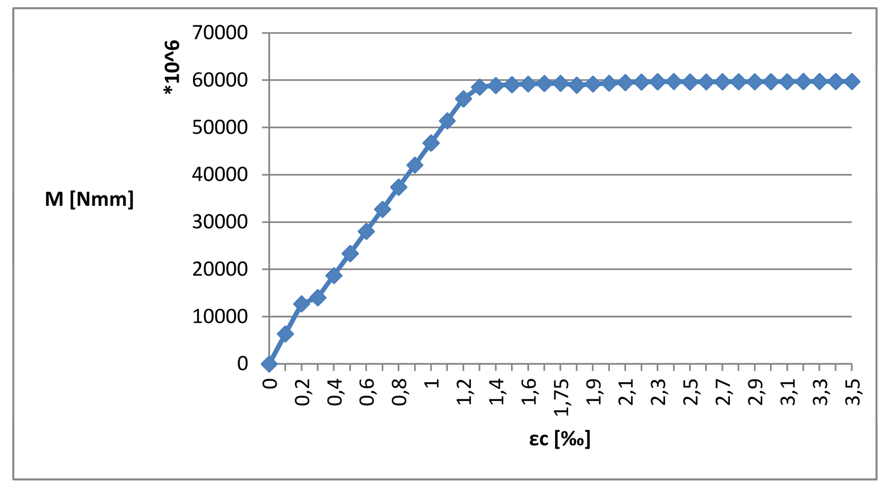
bf	2000	mm
bw	350	mm
bin	4300	mm
b	9000	mm
td	250	mm
tf	150	mm
hw	2600	mm
h	3000	mm
c	30	mm
dbot	2938	mm
Ac	4,82E+06	mm^2
Ø main reinforcement	32	mm
number of bars	60	
As	48255	mm^2
Ø stirrups	16	mm
ρ	1,07E-02	
bar spacing	51	mm
fck	20	N/mm^2
fcd	13,3	N/mm^2
fcm	28	N/mm^2
fctm	2,2	N/mm^2
Ec	7619	N/mm^2
εc3	0,00175	
εcu3	0,0035	
fyk	500	N/mm^2
fyd	435	N/mm^2
Es	200000	N/mm^2
εsy	0,00217	
εuk	0,025	
εud	0,0225	

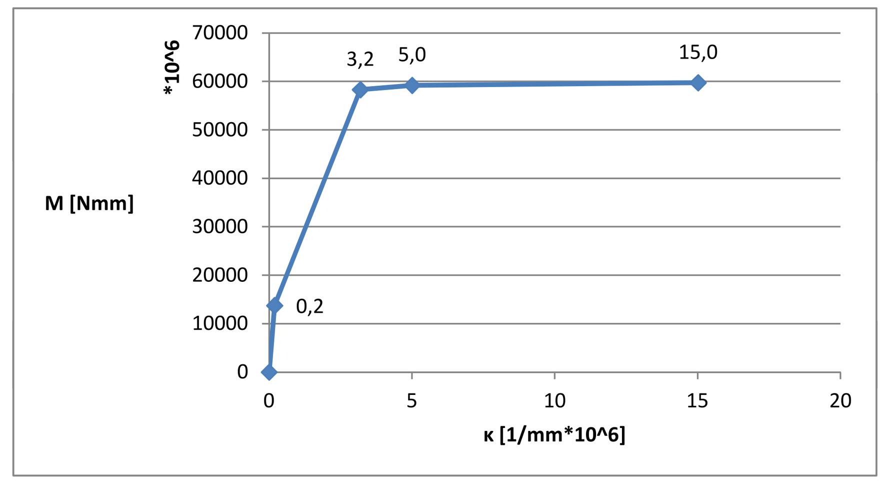
Check: My < Mc,pl	ok!
Check: εs < εud	not ok!

When x = td: εc =	0,0024

εct	2,90E-04	
εs	2,78E-04	
εc	0,00028	
εc'	2,35E-04	
εc"	2,61E-04	
x	1482	mm
x'	1232	mm
x"	1368	mm
κr	1,91E-07	1/mm
Mcr	**1,37E+10**	**Nmm**
εc	0,0012	
εc'	9,56E-04	
εsy	2,17E-03	
x	1071	mm
x'	821	mm
κy	3,19E-06	1/mm
My	**5,83E+10**	**Nmm**
εc3	0,00175	
εs	1,30E-02	
x	350	mm
κc,pl	5,00E-06	1/mm
Mc,pl	**5,92E+10**	**Nmm**
εcu3	0,0035	
εs	4,06E-02	
x	233	mm
κRd	1,50E-05	1/mm
β	91	mm
MRd	**5,97E+10**	**Nmm**

εc [-]	εc' [-]	εc" [-]	εs [-]	εctensile [-]	x [mm]	x' [mm]	x" [mm]	κ [1/mm]	NC1 [N]	NC2 [N]	NC3 [N]	NC4 [N]	NS [N]	ΣH = 0	β [mm]	M [Nmm]	εc [‰]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	8,31E-05	9,24E-05	9,83E-05	1,02E-04	1482	1232	1368	6,75E-08	5,08E+06	-3,24E+06	2,96E+06	-2,07E+06	9,49E+05	0E+00	0	6,35E+09	0,1	6,35E+09
0,0002	1,66E-04	1,85E-04	1,97E-04	2,05E-04	1482	1232	1368	1,35E-07	1,02E+07	-6,47E+06	5,93E+06	-4,14E+06	1,90E+06	0E+00	0	1,27E+10	0,2	1,27E+10
0,0003	2,30E-04	0,00E+00	5,23E-04	0,00E+00	1071	821	0	2,80E-07	1,10E+07	-5,97E+06	0,00E+00	0,00E+00	5,05E+06	0E+00	0	1,40E+10	0,3	1,40E+10
0,0004	3,07E-04	0,00E+00	6,97E-04	0,00E+00	1071	821	0	3,73E-07	1,47E+07	-7,96E+06	0,00E+00	0,00E+00	6,73E+06	0E+00	0	1,87E+10	0,4	1,87E+10
0,0005	3,83E-04	0,00E+00	8,71E-04	0,00E+00	1071	821	0	4,67E-07	1,84E+07	-9,95E+06	0,00E+00	0,00E+00	8,41E+06	0E+00	0	2,34E+10	0,5	2,34E+10
0,0006	4,60E-04	0,00E+00	1,05E-03	0,00E+00	1071	821	0	5,60E-07	2,20E+07	-1,19E+07	0,00E+00	0,00E+00	1,01E+07	0E+00	0	2,80E+10	0,6	2,80E+10
0,0007	5,37E-04	0,00E+00	1,22E-03	0,00E+00	1071	821	0	6,54E-07	2,57E+07	-1,39E+07	0,00E+00	0,00E+00	1,18E+07	0E+00	0	3,27E+10	0,7	3,27E+10
0,0008	6,13E-04	0,00E+00	1,39E-03	0,00E+00	1071	821	0	7,47E-07	2,94E+07	-1,59E+07	0,00E+00	0,00E+00	1,35E+07	0E+00	0	3,74E+10	0,8	3,74E+10
0,0009	6,90E-04	0,00E+00	1,57E-03	0,00E+00	1071	821	0	8,40E-07	3,31E+07	-1,79E+07	0,00E+00	0,00E+00	1,51E+07	0E+00	0	4,21E+10	0,9	4,21E+10
0,0010	7,67E-04	0,00E+00	1,74E-03	0,00E+00	1071	821	0	9,34E-07	3,67E+07	-1,99E+07	0,00E+00	0,00E+00	1,68E+07	0E+00	0	4,67E+10	1	4,67E+10
0,0011	8,43E-04	0,00E+00	1,92E-03	0,00E+00	1071	821	0	1,03E-06	4,04E+07	-2,19E+07	0,00E+00	0,00E+00	1,85E+07	0E+00	0	5,14E+10	1,1	5,14E+10
0,0012	9,20E-04	0,00E+00	2,09E-03	0,00E+00	1071	821	0	1,12E-06	4,41E+07	-2,39E+07	0,00E+00	0,00E+00	2,02E+07	0E+00	0	5,61E+10	1,2	5,61E+10
0,0013	9,49E-04	0,00E+00	2,83E-03	0,00E+00	925	675	0	1,41E-06	4,12E+07	-2,02E+07	0,00E+00	0,00E+00	2,10E+07	0E+00	0	5,85E+10	1,3	5,85E+10
0,0014	9,14E-04	0,00E+00	4,31E-03	0,00E+00	720	470	0	1,94E-06	3,46E+07	-1,36E+07	0,00E+00	0,00E+00	2,10E+07	0E+00	0	5,89E+10	1,4	5,89E+10
0,0015	8,58E-04	0,00E+00	6,04E-03	0,00E+00	584	334	0	2,57E-06	3,01E+07	-9,08E+06	0,00E+00	0,00E+00	2,10E+07	0E+00	0	5,91E+10	1,5	5,91E+10
0,0016	7,88E-04	0,00E+00	7,94E-03	0,00E+00	493	243	0	3,25E-06	2,70E+07	-6,05E+06	0,00E+00	0,00E+00	2,10E+07	0E+00	0	5,92E+10	1,6	5,92E+10
0,0017	7,08E-04	0,00E+00	9,95E-03	0,00E+00	429	179	0	3,97E-06	2,50E+07	-4,00E+06	0,00E+00	0,00E+00	2,10E+07	0E+00	0	5,93E+10	1,7	5,93E+10
0,00175	6,66E-04	0,00E+00	1,10E-02	0,00E+00	404	154	0	4,34E-06	2,42E+07	-3,23E+06	0,00E+00	0,00E+00	2,10E+07	0E+00	0	5,94E+10	1,75	5,94E+10
0,0018	6,22E-04	0,00E+00	1,20E-02	0,00E+00	382	132	0	4,71E-06	2,36E+07	-2,60E+06	0,00E+00	0,00E+00	2,10E+07	-2E+04	127	5,90E+10	1,8	5,90E+10
0,0019	5,32E-04	0,00E+00	1,42E-02	0,00E+00	347	97	0	5,47E-06	2,25E+07	-1,63E+06	0,00E+00	0,00E+00	2,10E+07	-1E+05	116	5,92E+10	1,9	5,92E+10
0,0020	4,38E-04	0,00E+00	1,64E-02	0,00E+00	320	70	0	6,25E-06	2,16E+07	-9,72E+05	0,00E+00	0,00E+00	2,10E+07	-3E+05	108	5,94E+10	2	5,94E+10
0,0021	3,43E-04	0,00E+00	1,86E-02	0,00E+00	299	49	0	7,03E-06	2,09E+07	-5,28E+05	0,00E+00	0,00E+00	2,10E+07	-6E+05	102	5,95E+10	2,1	5,95E+10
0,0022	2,45E-04	0,00E+00	2,08E-02	0,00E+00	281	31	0	7,82E-06	2,03E+07	-2,43E+05	0,00E+00	0,00E+00	2,10E+07	-9E+05	97	5,96E+10	2,2	5,96E+10
0,0023	1,47E-04	0,00E+00	2,30E-02	0,00E+00	267	17	0	8,61E-06	1,99E+07	-7,93E+04	0,00E+00	0,00E+00	2,10E+07	-1E+06	93	5,97E+10	2,3	5,97E+10
0,0024	4,76E-05	0,00E+00	2,52E-02	0,00E+00	255	5	0	9,41E-06	1,94E+07	-7,63E+03	0,00E+00	0,00E+00	2,10E+07	-2E+06	90	5,98E+10	2,4	5,98E+10
0,0025	0,00E+00	0,00E+00	2,48E-02	0,00E+00	269	0	0	9,29E-06	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	96	5,96E+10	2,5	5,96E+10
0,0026	0,00E+00	0,00E+00	2,64E-02	0,00E+00	264	0	0	9,87E-06	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	95	5,96E+10	2,6	5,96E+10
0,0027	0,00E+00	0,00E+00	2,80E-02	0,00E+00	259	0	0	1,04E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	94	5,97E+10	2,7	5,97E+10
0,0028	0,00E+00	0,00E+00	2,95E-02	0,00E+00	254	0	0	1,10E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	93	5,97E+10	2,8	5,97E+10
0,0029	0,00E+00	0,00E+00	3,11E-02	0,00E+00	250	0	0	1,16E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	93	5,97E+10	2,9	5,97E+10
0,0030	0,00E+00	0,00E+00	3,27E-02	0,00E+00	247	0	0	1,22E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	92	5,97E+10	3	5,97E+10
0,0031	0,00E+00	0,00E+00	3,43E-02	0,00E+00	244	0	0	1,27E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	92	5,97E+10	3,1	5,97E+10
0,0032	0,00E+00	0,00E+00	3,59E-02	0,00E+00	241	0	0	1,33E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	92	5,97E+10	3,2	5,97E+10
0,0033	0,00E+00	0,00E+00	3,75E-02	0,00E+00	238	0	0	1,39E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	91	5,97E+10	3,3	5,97E+10
0,0034	0,00E+00	0,00E+00	3,90E-02	0,00E+00	235	0	0	1,44E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	91	5,97E+10	3,4	5,97E+10
0,0035	0,00E+00	0,00E+00	4,06E-02	0,00E+00	233	0	0	1,50E-05	2,10E+07	0,00E+00	0,00E+00	0,00E+00	2,10E+07	0E+00	91	5,97E+10	3,5	5,97E+10





κ [1/mm*10^6]	M [Nmm]
0	0
0,2	1,37E+10
3,2	5,83E+10
5,0	5,92E+10
15,0	5,97E+10

Shear capacity (ULS)

CRd,c	0,12	
k	1,26	
ρl	1,07E-02	
k1	0,15	
σcp	0	N/mm^2
αcw	1	
z	2847	mm
v1	0,6	
θ	21,8	°
cot θ	2,5	
tan θ	0,4	
Asw	402	mm^2
s	100	mm

vmin	0,22	N/mm^2
VRd,cmin	4,56E+05	N
VRd,c	**8,64E+05**	**N**
VRd,max	**5,50E+06**	**N**
VRd,s	**1,14E+07**	**N**
VRd	**5,50E+06**	**N**

Crack width (SLS)

exposure class	XD3	
wmax	0,2	mm
wk	0,2	mm
k1	0,8	
k2	0,5	
k3	3,4	
k4	0,425	
kt	0,6	
Ecm	30000	N/mm^2
αe	6,7	
fct,eff	2,2	N/mm^2
x	1071	mm
x'	821	mm
hc,eff	155	mm
Ac,eff	7,75E+05	mm^2
ρp,eff	6,23E-02	
sr,max	189	mm
εsm-εcm	1,06E-03	
σs	241	N/mm^2

εc	0,0007	
εc'	5,31E-04	
εs	1,21E-03	
Nc1	2,54E+07	N
Nc2	-1,38E+07	N
Ns	1,16E+07	N
Mqp	**3,24E+10**	**Nmm**

Compressive zone height (ULS)

f	435	N/mm^2
xu	233	mm
xumax	1571	mm

Check: xu < xumax	ok!

11.10 Prestressed NSC box girder

Prestressed NSC box girder

Moment Capacity (ULS)

bf	2000	mm
bw	350	mm
bin	4300	mm
b	9000	mm
td	250	mm
tf	150	mm
hw	2600	mm
h	3000	mm
c	30	mm
dbot	2875	mm
Ac	4,82E+06	mm^2
Ic	6,04E+12	mm^4
z	1099	mm
Wtop	5,50E+09	mm^3
Wbot	3,18E+09	mm^3

εcu3	0,0035	
εc'	0,0013	
Δεp	0,0221	
εp	0,0275	
xu	393	mm
Ncu1	3,53E+07	N
Ncu2	-5,73E+06	N
ΔNp	2,96E+07	N
β	153	mm
MRd	1,30E+11	Nmm

Check: εp < εud	ok!
Check: ΣFH = 0	0

Y1860S7 prestressing

Østrand	15,7	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	6	
Ap	49500	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	147	mm
fck	20	N/mm^2
fcd	13,3	N/mm^2
fcm	28	N/mm^2
fctm	2,2	N/mm^2
Ec	7619	N/mm^2
εc3	0,00175	
εcu3	0,0035	
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	195000	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	
When xu = td & εc = εcu3: Ap =	32663	mm^2

Shear capacity (ULS)

CRd,c	0,12	
k	1,26	
ρl	0	
k1	0,15	
σcp	2,67	N/mm^2
αcw	0,49	
z	2722	mm
v1	0,6	
α	90 °	
sin α	1	
cot α	0	
θ	21,8 °	
sin θ	0,37	
cot θ	2,5	
Ø stirrups	12	mm
Asw	226	mm^2
s	100	mm
fyk	500	N/mm^2

vmin	0,22	N/mm^2
VRd,cmin	1,25E+06	N
VRd,c	8,05E+05	N
VRd,max	1,28E+06	N
VRd,s	6,16E+06	N
VRd	1,28E+06	N

Crack width (SLS)

Requirement: beam remains uncracked in SLS

t = 0: check top fibre	M ≥	3,17E+10	Nmm
t = 0: check bottom fibre	M ≤	1,75E+11	Nmm
t = ∞: check top fibre	M ≥	2,07E+10	Nmm
t = ∞: check bottom fibre	M ≤	1,33E+11	Nmm

Compressive zone height (ULS)

f	645	N/mm^2
xu	393	mm
xumax	1256	mm

Check: xu < xumax	ok!

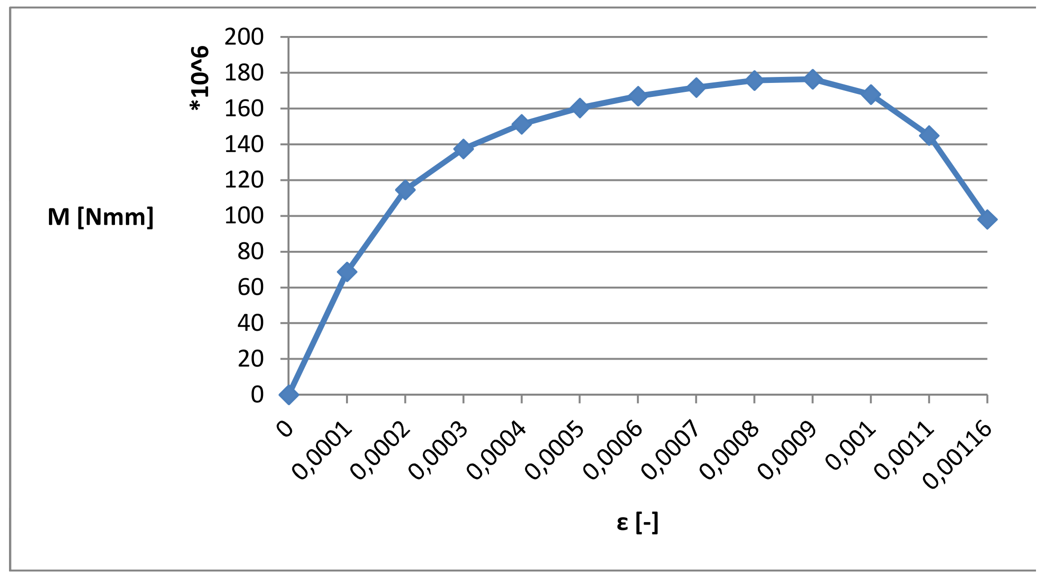
11.11 Rectangular unreinforced UHPFRC beam

Rectangular unreinforced UHPC beam

Moment capacity (ULS)

b	330	mm
D	500	mm
Ac	1,65E+05	mm^2
Ig	3,44E+09	mm^4
Z	1,38E+07	mm^3
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,003	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΣH = 0	y [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	250	250	0	0	1,00E-04	4,00E-07	2,06E+05	2,06E+05	-9,38E-11	0,00E+00	0	0	6,88E+07
0,0002	222	111	167	0	2,50E-04	9,00E-07	3,67E+05	9,17E+04	2,75E+05	0,00E+00	0	0	1,15E+08
0,0003	188	63	250	0	5,00E-04	1,60E-06	4,64E+05	5,16E+04	4,13E+05	0,00E+00	0	0	1,38E+08
0,0004	160	40	300	0	8,50E-04	2,50E-06	5,28E+05	3,30E+04	4,95E+05	0,00E+00	0	0	1,51E+08
0,0005	139	28	333	0	1,30E-03	3,60E-06	5,73E+05	2,29E+04	5,50E+05	0,00E+00	0	0	1,60E+08
0,0006	122	20	357	0	1,85E-03	4,90E-06	6,06E+05	1,68E+04	5,89E+05	0,00E+00	0	0	1,67E+08
0,0007	109	16	375	0	2,50E-03	6,40E-06	6,32E+05	1,29E+04	6,19E+05	0,00E+00	0	0	1,72E+08
0,0008	99	12	389	0	3,25E-03	8,10E-06	6,52E+05	1,02E+04	6,42E+05	0,00E+00	0	0	1,76E+08
0,0009	89	10	334	66	4,13E-03	1,01E-05	6,64E+05	8,19E+03	5,52E+05	1,04E+05	0	33	1,76E+08
0,001	79	8	267	146	5,31E-03	1,26E-05	6,54E+05	6,54E+03	4,40E+05	2,07E+05	0	69	1,68E+08
0,0011	67	6	205	222	7,12E-03	1,64E-05	6,07E+05	5,02E+03	3,38E+05	2,64E+05	0	97	1,45E+08
0,00116	52	4	151	293	1,00E-02	2,23E-05	4,94E+05	3,70E+03	2,49E+05	2,42E+05	0	98	9,81E+07



When εb = εctmax: ε0 =	0,0001
When εb = εt,p: ε0 =	0,00083
When εb = εt,u: ε0 =	0,00116

Mcr	1,10E+08	Nmm
κcr	6,40E-07	1/mm

Mu	1,76E+08	Nmm

Crack width (SLS)

wmax	0,3	mm
εb	5,60E-04	
dn	182	mm
ε0	0,0003	
X1	57	mm
X2	262	mm

C	4,78E+05	N
T1	4,69E+04	N
T2	4,32E+05	N

Mqp	1,41E+08	Nmm

11.12 Rectangular prestressed UHPFRC beam with low prestressing

Rectangular prestressed UHPC beam

Moment Capacity (ULS)

b	200	mm
D	400	mm
Ac	8,00E+04	mm^2
c	30	mm
dp	350	mm
Ig	1,07E+09	mm^4
z	200	mm
Wtop	5,33E+06	mm^3
Wbot	5,33E+06	mm^3

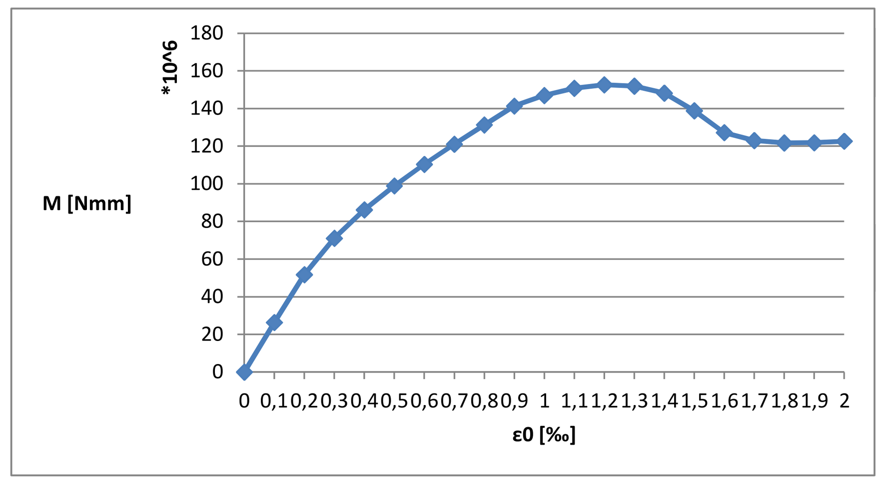
Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	2	
Ap	300	mm^2
strand spacing	84	mm
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,004	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

When εb = εctmax: ε0 =	0,00010	
When Δεp = εctmax: ε0 =	0,00013	
When εp = εp,y: ε0 =	0,00092	
When εb = εt,p: ε0 =	0,00108	
When εb = εt,u: ε0 =	0,00151	
When εp = εud: ε0 =	0,00210	
When εp = εud & ε0 = εcmax: Ap =	501	mm^2
Mu	1,53E+08	Nmm

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	Δεp [-]	εp [-]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΔNP [N]	ΣH = 0	y [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	202	198	0	0	7,31E-05	5,44E-03	9,79E-05	4,95E-07	1,01E+05	9,68E+04	0,00E+00	0,00E+00	4,28E+03	0	0	0	2,63E+07
0,0002	183	91	126	0	1,83E-04	5,55E-03	2,38E-04	1,10E-06	1,83E+05	4,56E+04	1,26E+05	0,00E+00	1,07E+04	0	0	0	5,17E+07
0,0003	158	53	189	0	3,65E-04	5,73E-03	4,60E-04	1,90E-06	2,37E+05	2,63E+04	1,89E+05	0,00E+00	2,13E+04	0	0	0	7,11E+07
0,0004	139	35	226	0	6,05E-04	5,97E-03	7,48E-04	2,87E-06	2,79E+05	1,74E+04	2,26E+05	0,00E+00	3,54E+04	0	0	0	8,62E+07
0,0005	126	25	249	0	8,93E-04	6,26E-03	1,09E-03	3,98E-06	3,14E+05	1,26E+04	2,49E+05	0,00E+00	5,22E+04	0	0	0	9,89E+07
0,0006	115	19	265	0	1,22E-03	6,58E-03	1,48E-03	5,20E-06	3,46E+05	9,62E+03	2,65E+05	0,00E+00	7,13E+04	0	0	0	1,10E+08
0,0007	108	15	277	0	1,58E-03	6,94E-03	1,90E-03	6,50E-06	3,77E+05	7,69E+03	2,77E+05	0,00E+00	9,22E+04	0	0	0	1,21E+08
0,0008	102	13	286	0	1,96E-03	7,32E-03	2,35E-03	7,87E-06	4,06E+05	6,35E+03	2,86E+05	0,00E+00	1,14E+05	0	0	0	1,31E+08
0,0009	97	11	292	0	2,35E-03	7,72E-03	2,82E-03	9,30E-06	4,36E+05	5,38E+03	2,92E+05	0,00E+00	1,38E+05	0	0	0	1,41E+08
0,001	90	9	301	0	2,90E-03	8,26E-03	3,45E-03	1,11E-05	4,49E+05	4,49E+03	3,01E+05	0,00E+00	1,44E+05	0	0	0	1,47E+08
0,0011	83	8	295	14	3,53E-03	8,89E-03	4,19E-03	1,32E-05	4,58E+05	3,78E+03	2,95E+05	1,41E+04	1,45E+05	0	7	0	1,51E+08
0,0012	77	6	249	68	4,28E-03	9,64E-03	5,06E-03	1,56E-05	4,60E+05	3,20E+03	2,49E+05	6,16E+04	1,46E+05	0	33	0	1,53E+08
0,0013	70	5	210	115	5,20E-03	1,06E-02	6,13E-03	1,86E-05	4,55E+05	2,69E+03	2,10E+05	9,44E+04	1,48E+05	0	53	0	1,52E+08
0,0014	63	4	175	158	6,42E-03	1,18E-02	7,54E-03	2,23E-05	4,39E+05	2,24E+03	1,75E+05	1,12E+05	1,50E+05	0	68	0	1,48E+08
0,0015	54	4	139	204	8,31E-03	1,37E-02	9,71E-03	2,80E-05	4,01E+05	1,78E+03	1,39E+05	1,07E+05	1,54E+05	0	71	0	1,39E+08
0,0016	44	3	106	163	1,13E-02	1,66E-02	1,31E-02	3,68E-05	3,48E+05	1,36E+03	1,06E+05	8,16E+04	1,59E+05	0	0	0	1,27E+08
0,0017	37	2	86	132	1,42E-02	1,96E-02	1,65E-02	4,55E-05	3,17E+05	1,10E+03	8,56E+04	6,59E+04	1,65E+05	0	0	0	1,23E+08
0,0018	33	2	72	110	1,72E-02	2,26E-02	1,99E-02	5,43E-05	2,98E+05	9,20E+02	7,18E+04	5,52E+04	1,70E+05	0	0	0	1,22E+08
0,0019	30	2	62	95	2,02E-02	2,56E-02	2,34E-02	6,31E-05	2,86E+05	7,92E+02	6,18E+04	4,75E+04	1,76E+05	0	0	0	1,22E+08
0,002	28	1	54	83	2,32E-02	2,85E-02	2,68E-02	7,20E-05	2,78E+05	6,95E+02	5,42E+04	4,17E+04	1,81E+05	0	0	0	1,23E+08
0,0021	26	1	48	74	2,62E-02	3,15E-02	3,02E-02	8,08E-05	2,73E+05	6,19E+02	4,83E+04	3,71E+04	1,87E+05	0	0	0	1,24E+08
0,0022	25	1	44	67	2,92E-02	3,45E-02	3,36E-02	8,96E-05	2,70E+05	5,58E+02	4,35E+04	3,35E+04	1,93E+05	0	0	0	1,25E+08
0,0023	23	1	40	61	3,22E-02	3,75E-02	3,71E-02	9,84E-05	2,69E+05	5,08E+02	3,96E+04	3,05E+04	1,98E+05	0	0	0	1,27E+08
0,0024	22	1	36	56	3,51E-02	4,05E-02	4,05E-02	1,07E-04	2,68E+05	4,66E+02	3,64E+04	2,80E+04	2,04E+05	0	0	0	1,29E+08
0,0025	22	1	34	52	3,81E-02	4,35E-02	4,39E-02	1,16E-04	2,69E+05	4,31E+02	3,36E+04	2,58E+04	2,09E+05	0	0	0	1,31E+08
0,0026	21	1	31	48	4,11E-02	4,65E-02	4,74E-02	1,25E-04	2,70E+05	4,00E+02	3,12E+04	2,40E+04	2,15E+05	0	0	7	1,32E+08
0,0027	20	1	29	45	4,40E-02	4,94E-02	5,07E-02	1,33E-04	2,72E+05	3,75E+02	2,92E+04	2,25E+04	2,20E+05	0	0	7	1,34E+08
0,0028	20	1	28	42	4,68E-02	5,22E-02	5,39E-02	1,42E-04	2,74E+05	3,53E+02	2,75E+04	2,12E+04	2,25E+05	0	0	7	1,36E+08
0,0029	19	1	26	40	4,95E-02	5,48E-02	5,70E-02	1,50E-04	2,77E+05	3,34E+02	2,61E+04	2,00E+04	2,30E+05	0	0	7	1,38E+08
0,003	19	1	25	38	5,21E-02	5,75E-02	6,00E-02	1,57E-04	2,79E+05	3,18E+02	2,48E+04	1,91E+04	2,35E+05	0	0	6	1,39E+08
0,0031	19	1	24	36	5,46E-02	6,00E-02	6,29E-02	1,65E-04	2,82E+05	3,03E+02	2,36E+04	1,82E+04	2,40E+05	0	0	6	1,41E+08
0,0032	19	1	23	35	5,71E-02	6,25E-02	6,57E-02	1,72E-04	2,85E+05	2,90E+02	2,26E+04	1,74E+04	2,45E+05	0	0	6	1,42E+08
0,0033	18	1	22	33	5,95E-02	6,49E-02	6,85E-02	1,79E-04	2,88E+05	2,79E+02	2,17E+04	1,67E+04	2,49E+05	0	0	6	1,44E+08
0,0034	18	1	21	32	6,18E-02	6,72E-02	7,12E-02	1,86E-04	2,91E+05	2,68E+02	2,09E+04	1,61E+04	2,53E+05	0	0	6	1,45E+08
0,0035	18	1	20	31	6,41E-02	6,95E-02	7,38E-02	1,93E-04	2,94E+05	2,59E+02	2,02E+04	1,55E+04	2,58E+05	0	0	6	1,47E+08
0,0036	18	1	20	30	6,63E-02	7,17E-02	7,63E-02	2,00E-04	2,97E+05	2,50E+02	1,95E+04	1,50E+04	2,62E+05	0	0	6	1,48E+08
0,0037	18	0	19	29	6,85E-02	7,39E-02	7,89E-02	2,06E-04	3,00E+05	2,42E+02	1,89E+04	1,45E+04	2,66E+05	0	0	6	1,50E+08
0,0038	18	0	18	28	7,07E-02	7,60E-02	8,13E-02	2,13E-04	3,03E+05	2,35E+02	1,83E+04	1,41E+04	2,70E+05	0	0	6	1,51E+08
0,0039	18	0	18	27	7,28E-02	7,81E-02	8,37E-02	2,19E-04	3,06E+05	2,28E+02	1,78E+04	1,37E+04	2,74E+05	0	0	6	1,52E+08
0,0040	18	0	17	27	7,48E-02	8,02E-02	8,61E-02	2,25E-04	3,09E+05	2,22E+02	1,73E+04	1,33E+04	2,78E+05	0	0	6	1,53E+08

ε0 [‰]	M [Nmm]
0	0
0,1	2,63E+07
0,2	5,17E+07
0,3	7,11E+07
0,4	8,62E+07
0,5	9,89E+07
0,6	1,10E+08
0,7	1,21E+08
0,8	1,31E+08
0,9	1,41E+08
1	1,47E+08
1,1	1,51E+08
1,2	1,53E+08
1,3	1,52E+08
1,4	1,48E+08
1,5	1,39E+08
1,6	1,27E+08
1,7	1,23E+08
1,8	1,22E+08
1,9	1,22E+08
2	1,23E+08



Shear Capacity (ULS)

γE*γb	1,5	
Seff	6,83E+04	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	26	mm
z	341	mm
cot βu	1,00	
Ø stirrups	12	mm
fyk	500	N/mm^2
Asw	226	mm^2
s	100	mm

VRb	1,34E+05	N
Vf	4,20E+05	N
Vs	3,09E+05	N
Vu	8,63E+05	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: beam remains uncracked in SLS

Span

t = 0: check top fibre	M ≥	-7,79E+06	Nmm
t = 0: check bottom fibre	M ≤	1,33E+08	Nmm
t = ∞: check top fibre	M ≥	-1,65E+07	Nmm
t = ∞: check bottom fibre	M ≤	1,11E+08	Nmm

Support

t = 0: check top fibre	M ≤	1,33E+08	Nmm
t = 0: check bottom fibre	M ≥	-7,79E+06	Nmm
t = ∞: check top fibre	M ≤	1,11E+08	Nmm
t = ∞: check bottom fibre	M ≥	-1,65E+07	Nmm

11.13 Rectangular prestressed UHPFRC beam with high prestressing

Rectangular prestressed UHPC beam

Moment Capacity (ULS)

b	200	mm
D	400	mm
Ac	8,00E+04	mm^2
c	30	mm
dp	350	mm
Ig	1,07E+09	mm^4
z	200	mm
Wtop	5,33E+06	mm^3
Wbot	5,33E+06	mm^3

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	8	
Ap	1200	mm^2
strand spacing	-2	mm

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,004	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

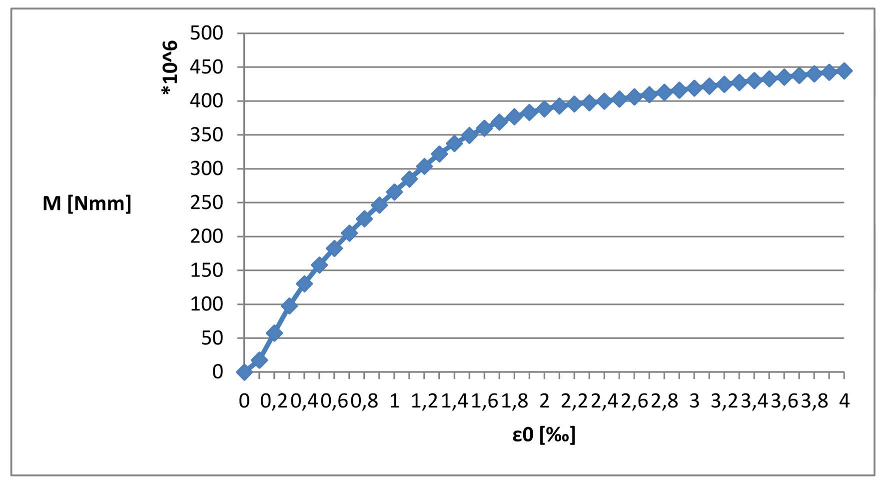
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

When εb = εctmax: ε0 =	0,00011
When Δεp = εctmax: ε0 =	0,00014
When εp = εp,y: ε0 =	0,00135
When εb = εt,p: ε0 =	0,00155
When εb = εt,u: ε0 =	0,00227
When εp = εud: ε0 =	0,00408

When εp = εud & ε0 = εcmax: Ap =	501	mm^2

Mu	4,45E+08	Nmm

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	Δεp [-]	εp [-]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΔNP [N]	ΣH = 0	γ [mm]	β [mm]	M [Nmm]	ε0 [‰]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	208	192	0	0	6,80E-05	5,43E-03	9,20E-05	4,80E-07	1,04E+05	8,82E+04	0,00E+00	0,00E+00	1,59E+04	0	0	0	1,76E+07	0,1	1,76E+07
0,0002	194	97	108	0	1,60E-04	5,53E-03	2,11E-04	1,03E-06	1,94E+05	4,86E+04	1,08E+05	0,00E+00	3,74E+04	0	0	0	5,76E+07	0,2	5,76E+07
0,0003	176	59	165	0	2,97E-04	5,66E-03	3,82E-04	1,70E-06	2,64E+05	2,93E+04	1,65E+05	0,00E+00	6,94E+04	0	0	0	9,77E+07	0,3	9,77E+07
0,0004	163	41	197	0	4,61E-04	5,83E-03	5,84E-04	2,46E-06	3,25E+05	2,03E+04	1,97E+05	0,00E+00	1,08E+05	0	0	0	1,30E+08	0,4	1,30E+08
0,0005	153	31	216	0	6,44E-04	6,01E-03	8,07E-04	3,27E-06	3,82E+05	1,53E+04	2,16E+05	0,00E+00	1,51E+05	0	0	0	1,58E+08	0,5	1,58E+08
0,0006	146	24	230	0	8,38E-04	6,20E-03	1,04E-03	4,11E-06	4,38E+05	1,22E+04	2,30E+05	0,00E+00	1,96E+05	0	0	0	1,83E+08	0,6	1,83E+08
0,0007	141	20	239	0	1,04E-03	6,41E-03	1,29E-03	4,97E-06	4,93E+05	1,01E+04	2,39E+05	0,00E+00	2,43E+05	0	0	0	2,05E+08	0,7	2,05E+08
0,0008	137	17	246	0	1,25E-03	6,61E-03	1,54E-03	5,85E-06	5,47E+05	8,54E+03	2,46E+05	0,00E+00	2,92E+05	0	0	0	2,26E+08	0,8	2,26E+08
0,0009	133	15	252	0	1,46E-03	6,83E-03	1,80E-03	6,74E-06	6,01E+05	7,42E+03	2,52E+05	0,00E+00	3,42E+05	0	0	0	2,46E+08	0,9	2,46E+08
0,001	131	13	256	0	1,67E-03	7,04E-03	2,06E-03	7,64E-06	6,54E+05	6,54E+03	2,56E+05	0,00E+00	3,92E+05	0	0	0	2,66E+08	1	2,66E+08
0,0011	129	12	260	0	1,89E-03	7,26E-03	2,32E-03	8,55E-06	7,08E+05	5,85E+03	2,60E+05	0,00E+00	4,43E+05	0	0	0	2,85E+08	1,1	2,85E+08
0,0012	127	11	263	0	2,11E-03	7,47E-03	2,58E-03	9,46E-06	7,61E+05	5,29E+03	2,63E+05	0,00E+00	4,94E+05	0	0	0	3,04E+08	1,2	3,04E+08
0,0013	125	10	265	0	2,33E-03	7,69E-03	2,85E-03	1,04E-05	8,15E+05	4,82E+03	2,65E+05	0,00E+00	5,45E+05	0	0	0	3,22E+08	1,3	3,22E+08
0,0014	121	9	270	0	2,65E-03	8,02E-03	3,23E-03	1,16E-05	8,47E+05	4,32E+03	2,70E+05	0,00E+00	5,72E+05	0	0	0	3,37E+08	1,4	3,37E+08
0,0015	114	8	278	0	3,09E-03	8,46E-03	3,75E-03	1,31E-05	8,57E+05	3,81E+03	2,78E+05	0,00E+00	5,76E+05	0	0	0	3,49E+08	1,5	3,49E+08
0,0016	108	7	264	21	3,57E-03	8,93E-03	4,31E-03	1,48E-05	8,67E+05	3,39E+03	2,64E+05	2,02E+04	5,79E+05	0	10	0	3,60E+08	1,6	3,60E+08
0,0017	103	6	236	56	4,09E-03	9,46E-03	4,92E-03	1,66E-05	8,73E+05	3,02E+03	2,36E+05	5,14E+04	5,83E+05	0	27	0	3,69E+08	1,7	3,69E+08
0,0018	97	5	211	86	4,67E-03	1,00E-02	5,60E-03	1,85E-05	8,76E+05	2,70E+03	2,11E+05	7,49E+04	5,87E+05	0	41	0	3,77E+08	1,8	3,77E+08
0,0019	92	5	189	114	5,32E-03	1,07E-02	6,35E-03	2,06E-05	8,75E+05	2,42E+03	1,89E+05	9,16E+04	5,92E+05	0	52	0	3,83E+08	1,9	3,83E+08
0,002	87	4	170	139	6,03E-03	1,14E-02	7,18E-03	2,30E-05	8,71E+05	2,18E+03	1,70E+05	1,02E+05	5,97E+05	0	61	0	3,89E+08	2	3,89E+08
0,0021	82	4	153	161	6,83E-03	1,22E-02	8,11E-03	2,55E-05	8,64E+05	1,96E+03	1,53E+05	1,06E+05	6,03E+05	0	67	0	3,93E+08	2,1	3,93E+08
0,0022	78	4	137	181	7,73E-03	1,31E-02	9,15E-03	2,84E-05	8,53E+05	1,76E+03	1,37E+05	1,04E+05	6,10E+05	0	68	0	3,96E+08	2,2	3,96E+08
0,0023	73	3	124	190	8,75E-03	1,41E-02	1,03E-02	3,16E-05	8,38E+05	1,58E+03	1,24E+05	9,50E+04	6,18E+05	0	0	0	3,98E+08	2,3	3,98E+08
0,0024	69	3	112	172	9,82E-03	1,52E-02	1,16E-02	3,49E-05	8,25E+05	1,43E+03	1,12E+05	8,59E+04	6,26E+05	0	0	0	4,00E+08	2,4	4,00E+08
0,0025	65	3	102	157	1,09E-02	1,63E-02	1,28E-02	3,83E-05	8,15E+05	1,30E+03	1,02E+05	7,83E+04	6,34E+05	0	0	0	4,03E+08	2,5	4,03E+08
0,0026	62	2	93	144	1,20E-02	1,74E-02	1,41E-02	4,18E-05	8,09E+05	1,20E+03	9,33E+04	7,18E+04	6,42E+05	0	0	21	4,06E+08	2,6	4,06E+08
0,0027	60	2	86	133	1,31E-02	1,85E-02	1,54E-02	4,52E-05	8,04E+05	1,11E+03	8,63E+04	6,64E+04	6,50E+05	0	0	20	4,10E+08	2,7	4,10E+08
0,0028	58	2	80	124	1,42E-02	1,95E-02	1,66E-02	4,85E-05	8,02E+05	1,03E+03	8,04E+04	6,18E+04	6,58E+05	0	0	19	4,13E+08	2,8	4,13E+08
0,0029	56	2	75	116	1,52E-02	2,06E-02	1,78E-02	5,18E-05	8,00E+05	9,66E+02	7,53E+04	5,79E+04	6,66E+05	0	0	19	4,16E+08	2,9	4,16E+08
0,003	55	2	71	109	1,62E-02	2,16E-02	1,90E-02	5,50E-05	8,00E+05	9,09E+02	7,09E+04	5,46E+04	6,74E+05	0	0	19	4,19E+08	3	4,19E+08
0,0031	53	2	67	103	1,72E-02	2,26E-02	2,01E-02	5,81E-05	8,01E+05	8,60E+02	6,71E+04	5,16E+04	6,81E+05	0	0	18	4,22E+08	3,1	4,22E+08
0,0032	52	2	64	98	1,82E-02	2,36E-02	2,13E-02	6,12E-05	8,02E+05	8,17E+02	6,37E+04	4,90E+04	6,88E+05	0	0	18	4,25E+08	3,2	4,25E+08
0,0033	51	2	61	93	1,92E-02	2,45E-02	2,24E-02	6,42E-05	8,04E+05	7,78E+02	6,07E+04	4,67E+04	6,96E+05	0	0	18	4,28E+08	3,3	4,28E+08
0,0034	51	1	58	89	2,01E-02	2,55E-02	2,35E-02	6,72E-05	8,06E+05	7,44E+02	5,80E+04	4,46E+04	7,03E+05	0	0	18	4,30E+08	3,4	4,30E+08
0,0035	50	1	56	86	2,11E-02	2,64E-02	2,46E-02	7,02E-05	8,09E+05	7,13E+02	5,56E+04	4,28E+04	7,10E+05	0	0	18	4,33E+08	3,5	4,33E+08
0,0036	49	1	53	82	2,20E-02	2,73E-02	2,56E-02	7,31E-05	8,12E+05	6,84E+02	5,34E+04	4,11E+04	7,16E+05	0	0	18	4,35E+08	3,6	4,35E+08
0,0037	49	1	51	79	2,29E-02	2,82E-02	2,67E-02	7,59E-05	8,15E+05	6,59E+02	5,14E+04	3,95E+04	7,23E+05	0	0	17	4,38E+08	3,7	4,38E+08
0,0038	48	1	50	76	2,38E-02	2,91E-02	2,77E-02	7,87E-05	8,18E+05	6,35E+02	4,95E+04	3,81E+04	7,30E+05	0	0	17	4,40E+08	3,8	4,40E+08
0,0039	48	1	48	74	2,46E-02	3,00E-02	2,87E-02	8,15E-05	8,21E+05	6,14E+02	4,79E+04	3,68E+04	7,36E+05	0	0	17	4,42E+08	3,9	4,42E+08
0,0040	47	1	46	71	2,55E-02	3,08E-02	2,97E-02	8,42E-05	8,25E+05	5,94E+02	4,63E+04	3,56E+04	7,43E+05	0	0	17	4,45E+08	4	4,45E+08



Shear Capacity (ULS)

γE*γb	1,5	
Seff	6,69E+04	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	47	mm
z	334	mm
cot βu	1,00	
Ø stirrups	12	mm
fyk	500	N/mm^2
Asw	226	mm^2
s	100	mm

VRb	1,31E+05	N
Vf	4,11E+05	N
Vs	3,02E+05	N
Vu	8,45E+05	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: beam remains uncracked in SLS

Span

t = 0: check top fibre	M ≥	9,68E+07	Nmm
t = 0: check bottom fibre	M ≤	4,05E+08	Nmm
t = ∞: check top fibre	M ≥	6,20E+07	Nmm
t = ∞: check bottom fibre	M ≤	3,15E+08	Nmm

Support

t = 0: check top fibre	M ≤	4,05E+08	Nmm
t = 0: check bottom fibre	M ≥	9,68E+07	Nmm
t = ∞: check top fibre	M ≤	3,15E+08	Nmm
t = ∞: check bottom fibre	M ≥	6,20E+07	Nmm

11.14 Unreinforced UHPFRC box girder

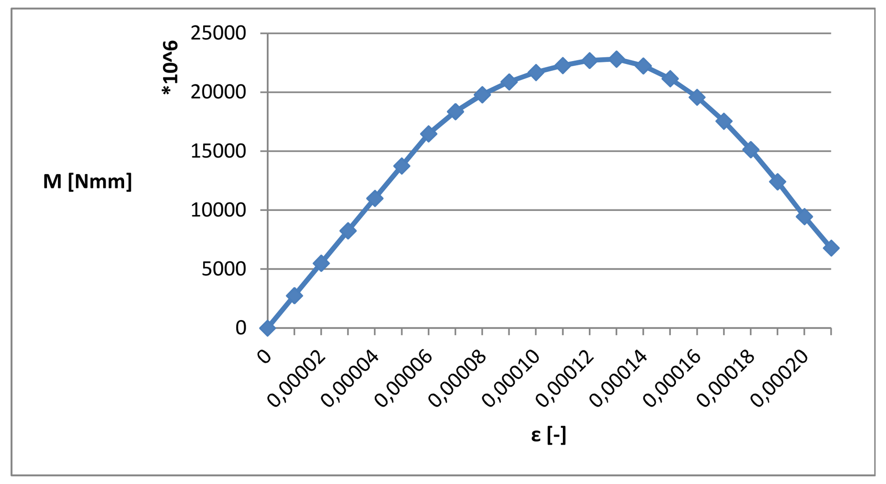
Unreinforced UHPC box girder

Moment Capacity (ULS)

bf	2000	mm
bw	350	mm
bin	4300	mm
b	9000	mm
td	250	mm
tf	150	mm
hw	2600	mm
h	3000	mm
Ac	4,82E+06	mm^2
e	1099	mm
Ic	6,04E+12	mm^4
W0	5,50E+09	mm^3
Wb	3,18E+09	mm^3
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,0036	
εt,p	0,00058	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

ε0 [-]	ε0' [-]	εb' [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0
0,00001	7,72E-06	1,59E-05	1,73E-05	1099	849	1751	0	0	0	0	9,10E-09
0,00002	1,54E-05	3,19E-05	3,46E-05	1099	849	1751	0	0	0	0	1,82E-08
0,00003	2,32E-05	4,78E-05	5,19E-05	1099	849	1751	0	0	0	0	2,73E-08
0,00004	3,09E-05	6,38E-05	6,92E-05	1099	849	1751	0	0	0	0	3,64E-08
0,00005	3,86E-05	7,97E-05	8,65E-05	1099	849	1751	0	0	0	0	4,55E-08
0,00006	4,63E-05	9,61E-05	1,04E-04	1096	846	1754	71	79	0	0	5,48E-08
0,00007	5,28E-05	1,26E-04	1,36E-04	1018	768	1454	378	0	0	0	6,88E-08
0,00008	5,84E-05	1,67E-04	1,80E-04	924	674	1155	771	0	0	0	8,66E-08
0,00009	6,28E-05	2,20E-04	2,37E-04	826	576	918	1106	0	0	0	1,09E-07
0,00010	6,58E-05	2,89E-04	3,10E-04	732	482	732	1386	0	0	0	1,37E-07
0,00011	6,74E-05	3,76E-04	4,02E-04	645	395	586	1619	0	0	0	1,71E-07
0,00012	6,72E-05	4,82E-04	5,13E-04	568	318	474	1808	0	0	0	2,11E-07
0,00013	6,48E-05	6,13E-04	6,53E-04	498	248	383	1832	136	150	0	2,61E-07
0,00014	5,80E-05	7,95E-04	8,44E-04	427	177	305	1456	662	150	0	3,28E-07
0,00015	4,66E-05	1,03E-03	1,09E-03	363	113	242	1156	1090	150	0	4,13E-07
0,00016	3,03E-05	1,32E-03	1,40E-03	308	58	193	921	1428	150	0	5,19E-07
0,00017	8,84E-06	1,67E-03	1,76E-03	264	14	155	741	1690	150	0	6,45E-07
0,00018	1,72E-05	2,07E-03	2,19E-03	228	0	127	606	1889	150	22	7,89E-07
0,00019	4,70E-05	2,51E-03	2,65E-03	200	0	105	504	2040	150	50	9,48E-07
0,00020	7,99E-05	2,99E-03	3,16E-03	179	0	89	427	2155	150	71	1,12E-06
0,00021	1,10E-04	3,42E-03	3,61E-03	164	0	79	375	2232	150	86	1,27E-06

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΣH = 0	y [mm]	z [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0
0,00001	2,47E+06	-1,36E+06	4,11E+06	-3,00E+06	0,00E+00	0,00E+00	0,00E+00	0	0	0	2,75E+09
0,00002	4,94E+06	-2,72E+06	8,22E+06	-6,00E+06	0,00E+00	0,00E+00	0,00E+00	0	0	0	5,50E+09
0,00003	7,42E+06	-4,08E+06	1,23E+07	-9,00E+06	0,00E+00	0,00E+00	0,00E+00	0	0	0	8,25E+09
0,00004	9,89E+06	-5,44E+06	1,64E+07	-1,20E+07	0,00E+00	0,00E+00	0,00E+00	0	0	0	1,10E+10
0,00005	1,24E+07	-6,80E+06	2,06E+07	-1,50E+07	0,00E+00	0,00E+00	0,00E+00	0	0	0	1,38E+10
0,00006	1,48E+07	-8,12E+06	2,28E+07	-1,81E+07	1,96E+06	0,00E+00	0,00E+00	0	0	0	1,65E+10
0,00007	1,60E+07	-8,41E+06	2,54E+06	1,32E+06	3,75E+06	0,00E+00	0,00E+00	0	0	0	1,84E+10
0,00008	1,66E+07	-8,16E+06	2,02E+06	2,70E+06	3,75E+06	0,00E+00	0,00E+00	0	0	0	1,98E+10
0,00009	1,67E+07	-7,51E+06	1,61E+06	3,87E+06	3,75E+06	0,00E+00	0,00E+00	0	0	0	2,09E+10
0,00010	1,65E+07	-6,58E+06	1,28E+06	4,85E+06	3,75E+06	0,00E+00	0,00E+00	0	0	0	2,17E+10
0,00011	1,60E+07	-5,52E+06	1,03E+06	5,67E+06	3,75E+06	0,00E+00	0,00E+00	0	0	0	2,23E+10
0,00012	1,53E+07	-4,44E+06	8,29E+05	6,33E+06	3,75E+06	0,00E+00	0,00E+00	0	0	0	2,27E+10
0,00013	1,46E+07	-3,34E+06	6,71E+05	6,41E+06	4,75E+05	3,68E+06	0,00E+00	0	68	75	2,28E+10
0,00014	1,34E+07	-2,13E+06	5,33E+05	5,10E+06	2,23E+06	3,45E+06	0,00E+00	0	327	75	2,22E+10
0,00015	1,22E+07	-1,09E+06	4,23E+05	4,04E+06	3,53E+06	3,15E+06	0,00E+00	0	530	75	2,12E+10
0,00016	1,11E+07	-3,66E+05	3,37E+05	3,22E+06	4,39E+06	2,79E+06	0,00E+00	0	681	75	1,96E+10
0,00017	1,01E+07	-2,51E+04	2,71E+05	2,59E+06	4,85E+06	2,34E+06	0,00E+00	0	783	74	1,76E+10
0,00018	9,24E+06	0,00E+00	2,22E+05	2,12E+06	4,99E+06	1,83E+06	7,77E+04	0	842	74	1,51E+10
0,00019	8,57E+06	0,00E+00	1,85E+05	1,76E+06	4,86E+06	1,27E+06	4,84E+05	0	861	73	1,24E+10
0,00020	8,04E+06	0,00E+00	1,56E+05	1,49E+06	4,54E+06	6,63E+05	1,18E+06	0	840	71	9,46E+09
0,00021	7,69E+06	0,00E+00	1,37E+05	1,31E+06	4,15E+06	1,18E+05	1,96E+06	0	788	50	6,79E+09



When εb = εctmax: ε0 =	0,000058
When εb' = εctmax: ε0 =	0,000061
When εb = εt,p: ε0 =	0,000125
When εb' = εt,p: ε0 =	0,000128
When dn = td: ε0 =	0,000174
When εb = εt,u: ε0 =	0,000209

Mu	**2,28E+10**	**Nmm**

Shear Capacity (ULS)

Crack Width (SLS)

wmax	0,3	mm
εb	2,27E-04	
dn	842	mm
dn'	592	mm
X1	952	mm
X2	1056	mm

ε0	0,00009	
ε0'	6,22E-05	
εb'	2,11E-04	
C1	1,67E+07	N
C2	-7,63E+06	N
T1	1,67E+06	N
T2	3,70E+06	N
T3	3,75E+06	N

Mqp	**2,07E+10**	**Nmm**

11.15 Prestressed UHPFRC box girder with low prestressing

Prestressed box UHPC box girder

Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td	250	mm
tf	150	mm
hw	2800	mm
h	3200	mm
Ac	7,04E+06	mm^2
e	1062	mm
Ic	1,05E+13	mm^4
W0	9,88E+09	mm^3
Wb	4,91E+09	mm^3
c	30	mm
dp	3109	mm

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	15	
number of tendons	8	
Ap	18000	mm^2
Øduct	90	mm
Øanchor	310	mm
anchor spacing	675	mm

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1046	N/mm^2
εp∞	0,0054	

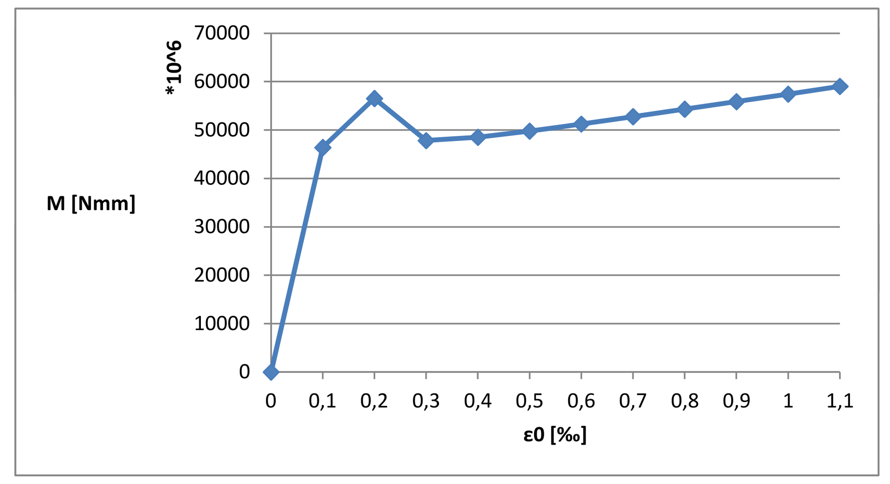
When εb = εctmax: ε0 =	0,000051
When εb' = εctmax: ε0 =	0,000054
When εb = εt,p: ε0 =	0,000106
When εb' = εt,p: ε0 =	0,000108
When εp = εp,y: ε0 =	0,000197
When dn = td: ε0 =	0,000183
When εb = εt,u: ε0 =	0,000217
When εb' = εt,u: ε0 =	0,000221
When εp = εud: ε0 =	0,001147

When dn = td & εp = εp,y: Ap =	24301	mm^2
When dn = td & εb = εt,u: Ap =	45323	mm^2
When dn = td & εb' = εt,u: Ap =	49270	mm^2
When εp = εud & ε0 = εcmax: Ap =	433457	mm^2
When dn = td & ε0 = εcmax: Ap =	383929	mm^2

Mu	5,90E+10	Nmm

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	5,53E-05	4,46E-04	4,56E-04	5,82E-03	4,72E-04	559	309	559	1932	0	0	0	1,79E-07	2,17E+07	-6,32E+06	9,78E+05	6,76E+06	6,00E+06	0,00E+00	0,00E+00	1,60E+06	0,00E+00	0	0	0	4,64E+10
0,0002	2,13E-05	2,50E-03	2,55E-03	7,92E-03	2,63E-03	226	0	113	499	2212	0	24	8,85E-07	1,75E+07	0,00E+00	1,98E+05	1,75E+06	5,08E+06	1,73E+06	1,89E+05	8,57E+06	0,00E+00	912	87	0	5,65E+10
0,0003	1,74E-04	0,00E+00	5,60E-03	1,10E-02	0,00E+00	158	0	53	233	1498	0	92	1,90E-06	1,84E+07	0,00E+00	9,22E+04	8,15E+05	2,62E+06	0,00E+00	5,93E+06	8,91E+06	0,00E+00	0	0	0	4,78E+10
0,0004	2,85E-04	0,00E+00	8,12E-03	1,35E-02	0,00E+00	146	0	36	161	1038	0	104	2,74E-06	2,26E+07	0,00E+00	6,38E+04	5,64E+05	1,82E+06	0,00E+00	1,10E+07	9,20E+06	-9,31E-08	0	0	0	4,85E+10
0,0005	3,91E-04	0,00E+00	1,06E-02	1,59E-02	0,00E+00	140	0	28	124	798	0	110	3,56E-06	2,72E+07	0,00E+00	4,91E+04	4,34E+05	1,40E+06	0,00E+00	1,58E+07	9,47E+06	7,08E-08	0	0	0	4,98E+10
0,0006	4,94E-04	0,00E+00	1,30E-02	1,84E-02	0,00E+00	137	0	23	101	650	0	113	4,37E-06	3,19E+07	0,00E+00	4,00E+04	3,53E+05	1,14E+06	0,00E+00	2,06E+07	9,74E+06	-4,10E-07	0	0	0	5,12E+10
0,0007	5,96E-04	0,00E+00	1,54E-02	2,08E-02	0,00E+00	135	0	19	85	549	0	115	5,18E-06	3,66E+07	0,00E+00	3,38E+04	2,98E+05	9,60E+05	0,00E+00	2,53E+07	1,00E+07	-7,08E-07	0	0	0	5,28E+10
0,0008	6,97E-04	0,00E+00	1,78E-02	2,32E-02	0,00E+00	134	0	17	74	475	0	116	5,99E-06	4,14E+07	0,00E+00	2,92E+04	2,58E+05	8,31E+05	0,00E+00	3,00E+07	1,03E+07	-4,32E-07	0	0	0	5,43E+10
0,0009	7,98E-04	0,00E+00	2,02E-02	2,56E-02	0,00E+00	133	0	15	65	419	0	117	6,79E-06	4,62E+07	0,00E+00	2,58E+04	2,28E+05	7,33E+05	0,00E+00	3,47E+07	1,05E+07	6,26E-07	0	0	0	5,59E+10
0,001	8,98E-04	0,00E+00	2,26E-02	2,80E-02	0,00E+00	132	0	13	58	374	0	118	7,59E-06	5,10E+07	0,00E+00	2,30E+04	2,04E+05	6,55E+05	0,00E+00	3,93E+07	1,08E+07	9,09E-07	0	0	0	5,74E+10
0,0011	9,99E-04	0,00E+00	2,50E-02	3,04E-02	0,00E+00	131	0	12	53	339	0	119	8,40E-06	5,59E+07	0,00E+00	2,08E+04	1,84E+05	5,93E+05	0,00E+00	4,40E+07	1,11E+07	0,00E+00	0	0	0	5,90E+10
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,1	4,64E+10
0,2	5,65E+10
0,3	4,78E+10
0,4	4,85E+10
0,5	4,98E+10
0,6	5,12E+10
0,7	5,28E+10
0,8	5,43E+10
0,9	5,59E+10
1	5,74E+10
1,1	5,90E+10



Shear Capacity (ULS)

γE*γb	1,5	
Seff	2,15E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	30	
tan βu	0,58	
dn	131	mm
z	3065	mm
cot βu	1,73	
Ø stirrups	16	mm
fyk	500	N/mm^2
Asw	402	mm^2
s	100	mm

VRb	4,20E+06	N
Vf	2,29E+07	N
Vs	8,54E+06	N
Vu	3,56E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: box girder remains uncracked in SLS

t = 0: check top fibre	M ≥	-8,77E+10	Nmm
t = 0: check bottom fibre	M ≤	1,08E+11	Nmm
t = ∞: check top fibre	M ≥	-8,55E+10	Nmm
t = ∞: check bottom fibre	M ≤	9,09E+10	Nmm

11.16 Prestressed UHPFRC box girder with high prestressing

Prestressed UHPC box girder

Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td	250	mm
tf	150	mm
hw	2800	mm
h	3200	mm
Ac	7,04E+06	mm^2
e	1062	mm
Ic	1,05E+13	mm^4
W0	9,88E+09	mm^3
Wb	4,91E+09	mm^3
c	30	mm
dp	3071	mm

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	16	
Ap	132000	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	-138	mm

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1046	N/mm^2
εp∞	0,0054	

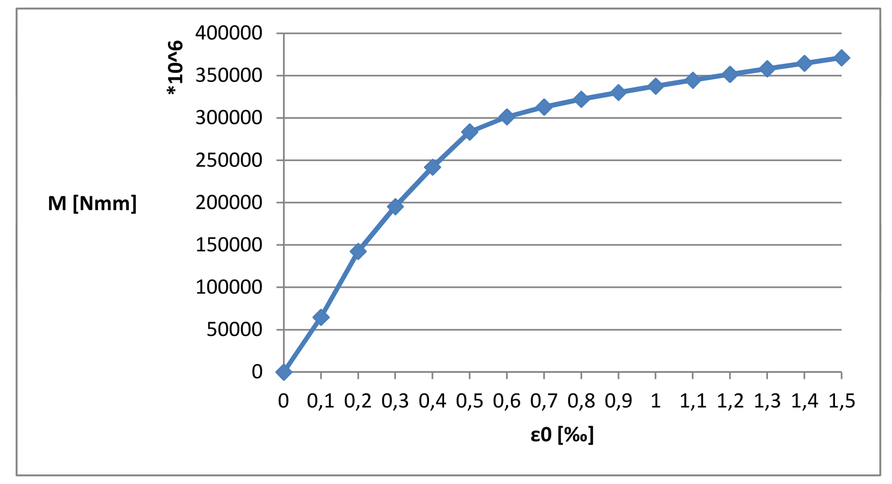
When εb = εctmax: ε0 =	0,000060
When εb' = εctmax: ε0 =	0,000064
When εb = εt,p: ε0 =	0,000160
When εb' = εt,p: ε0 =	0,000166
When εp = εp,y: ε0 =	0,000480
When dn = td: ε0 =	0,000720
When εb = εt,u: ε0 =	0,000506
When εb' = εt,u: ε0 =	0,000512
When εp = εud: ε0 =	0,001550

When dn = td & εp = εp,y: Ap =	25148	mm^2
When dn = td & εb = εt,u: Ap =	45349	mm^2
When dn = td & εb' = εt,u: Ap =	49300	mm^2
When εp = εud & ε0 = εcmax: Ap =	428931	mm^2
When dn = td & ε0 = εcmax: Ap =	385395	mm^2

Mu	3,71E+11	Nmm

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	7,32E-05	2,27E-04	2,29E-04	5,59E-03	2,43E-04	934	684	934	1182	0	0	0	1,07E-07	3,62E+07	-1,85E+07	1,63E+06	4,14E+06	6,00E+06	0,00E+00	0,00E+00	5,89E+06	0,00E+00	0	0	0	6,48E+10
0,0002	1,24E-04	7,30E-04	7,36E-04	6,10E-03	7,76E-04	656	406	328	1448	618	0	0	3,05E-07	5,08E+07	-1,86E+07	5,74E+05	5,07E+06	2,09E+06	5,55E+06	0,00E+00	1,90E+07	0,00E+00	305	76	0	1,43E+11
0,0003	1,67E-04	1,32E-03	1,33E-03	6,70E-03	1,40E-03	565	315	188	832	1465	0	0	5,31E-07	6,57E+07	-1,95E+07	3,30E+05	2,91E+06	4,43E+06	4,27E+06	0,00E+00	3,42E+07	0,00E+00	694	78	0	1,95E+11
0,0004	2,09E-04	1,93E-03	1,94E-03	7,31E-03	2,04E-03	524	274	131	579	1816	0	0	7,63E-07	8,13E+07	-2,12E+07	2,29E+05	2,03E+06	4,81E+06	2,96E+06	0,00E+00	5,00E+07	0,00E+00	810	81	0	2,42E+11
0,0005	2,11E-04	3,02E-03	3,05E-03	8,41E-03	3,19E-03	433	183	87	383	2148	0	0	1,15E-06	8,39E+07	-1,43E+07	1,52E+05	1,34E+06	4,24E+06	5,85E+05	0,00E+00	6,33E+07	0,00E+00	797	179	0	2,84E+11
0,0006	1,08E-04	0,00E+00	5,45E-03	1,08E-02	0,00E+00	305	55	51	224	1444	0	0	1,97E-06	7,08E+07	-2,17E+06	8,88E+04	7,85E+05	2,53E+06	0,00E+00	0,00E+00	6,52E+07	1,79E-07	0	0	0	3,01E+11
0,0007	1,77E-05	0,00E+00	7,68E-03	1,30E-02	0,00E+00	256	6	37	162	1042	0	0	2,73E-06	6,96E+07	-4,25E+04	6,41E+04	5,66E+05	1,82E+06	0,00E+00	0,00E+00	6,71E+07	3,28E-07	0	0	0	3,13E+11
0,0008	6,94E-05	0,00E+00	9,88E-03	1,52E-02	0,00E+00	230	0	29	127	818	0	20	3,48E-06	7,13E+07	0,00E+00	5,03E+04	4,45E+05	1,43E+06	0,00E+00	5,12E+05	6,89E+07	0,00E+00	0	0	0	3,22E+11
0,0009	1,55E-04	0,00E+00	1,21E-02	1,74E-02	0,00E+00	213	0	24	105	674	0	37	4,22E-06	7,44E+07	0,00E+00	4,15E+04	3,66E+05	1,18E+06	0,00E+00	2,11E+06	7,07E+07	-1,94E-07	0	0	0	3,30E+11
0,001	2,40E-04	0,00E+00	1,42E-02	1,96E-02	0,00E+00	202	0	20	89	573	0	48	4,96E-06	7,81E+07	0,00E+00	3,53E+04	3,12E+05	1,00E+06	0,00E+00	4,30E+06	7,25E+07	0,00E+00	0	0	0	3,38E+11
0,0011	3,25E-04	0,00E+00	1,64E-02	2,18E-02	0,00E+00	193	0	18	77	499	0	57	5,70E-06	8,23E+07	0,00E+00	3,07E+04	2,71E+05	8,73E+05	0,00E+00	6,85E+06	7,42E+07	3,13E-07	0	0	0	3,45E+11
0,0012	4,09E-04	0,00E+00	1,86E-02	2,39E-02	0,00E+00	186	0	16	69	442	0	64	6,44E-06	8,67E+07	0,00E+00	2,72E+04	2,40E+05	7,73E+05	0,00E+00	9,63E+06	7,60E+07	2,53E-07	0	0	0	3,51E+11
0,0013	4,94E-04	0,00E+00	2,07E-02	2,61E-02	0,00E+00	181	0	14	62	396	0	69	7,17E-06	9,13E+07	0,00E+00	2,44E+04	2,15E+05	6,94E+05	0,00E+00	1,26E+07	7,78E+07	5,22E-07	0	0	0	3,58E+11
0,0014	5,78E-04	0,00E+00	2,29E-02	2,83E-02	0,00E+00	177	0	13	56	359	0	73	7,91E-06	9,60E+07	0,00E+00	2,21E+04	1,95E+05	6,29E+05	0,00E+00	1,56E+07	7,96E+07	3,13E-07	0	0	0	3,65E+11
0,0015	6,62E-04	0,00E+00	2,50E-02	3,04E-02	0,00E+00	173	0	12	51	329	0	77	8,65E-06	1,01E+08	0,00E+00	2,02E+04	1,79E+05	5,76E+05	0,00E+00	1,87E+07	8,13E+07	-7,30E-07	0	0	0	3,71E+11
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,1	6,48E+10
0,2	1,43E+11
0,3	1,95E+11
0,4	2,42E+11
0,5	2,84E+11
0,6	3,01E+11
0,7	3,13E+11
0,8	3,22E+11
0,9	3,30E+11
1	3,38E+11
1,1	3,45E+11
1,2	3,51E+11
1,3	3,58E+11
1,4	3,65E+11
1,5	3,71E+11



Shear Capacity (ULS)

γE*γb	1,5	
Seff	2,11E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	30	
tan βu	0,58	
dn	172	mm
z	3013	mm
cot βu	1,73	
Ø stirrups	16	mm
fyk	500	N/mm^2
Asw	402	mm^2
s	100	mm

VRb	4,13E+06	N
Vf	2,25E+07	N
Vs	8,39E+06	N
Vu	3,50E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: box girder remains uncracked in SLS

t = 0: check top fibre	M ≥	-1,42E+11	Nmm
t = 0: check bottom fibre	M ≤	5,38E+11	Nmm
t = ∞: check top fibre	M ≥	-1,26E+11	Nmm
t = ∞: check bottom fibre	M ≤	4,13E+11	Nmm

11.17 Prestressed UHPC box girder with no fibers

Prestressed UHPC box girder without fibers

Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td	250	mm
tf	150	mm
hw	2800	mm
h	3200	mm
c	30	mm
dbot	3109	mm
Ac	7,04E+06	mm^2
Ic	1,05E+13	mm^4
z	1062	mm
Wtop	9,88E+09	mm^3
Wbot	4,91E+09	mm^3

εc	4,98E-04	
Δεp	0,0261	
εp	0,0315	
x	58	mm
Nc	1,12E+07	N
ΔNp	1,12E+07	N
Check: ΣFH = 0	-1,61E-04	N
MRd	5,33E+10	Nmm

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	15	
number of tendons	8	
Ap	18000	mm^2
Øduct	90	mm
Øanchor	310	mm
anchor spacing	675	mm
fck	150	N/mm^2
fcd	128	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
εc3	0,00255	
εcu3	0,004	
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	195000	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	
When xu = td & εc = εcu3: Ap =	451895	mm^2

Shear capacity (ULS)

CRd,c	0,12	
k	1,25	
ρl	0	
k1	0,15	
σcp	2,68	N/mm^2
αcw	1,02	
z	3051	mm
v1	0,6	
α	90 °	
sin α	1	
cot α	0	
θ	21,8 °	
sin θ	0,37	
cot θ	2,5	
Ø stirrups	16	mm
Asw	402	mm^2
s	100	mm
fyk	500	N/mm^2

vmin	0,60	N/mm^2
VRd,cmin	2,18E+06	N
VRd,c	8,74E+05	N
VRd,max	2,88E+07	N
VRd,s	1,23E+07	N
VRd	1,23E+07	N

Crack width (SLS)

Requirement: beam remains uncracked in SLS

t = 0: check top fibre	M ≥	-3,33E+10	Nmm
t = 0: check bottom fibre	M ≤	9,34E+10	Nmm
t = ∞: check top fibre	M ≥	-3,73E+10	Nmm
t = ∞: check bottom fibre	M ≤	7,62E+10	Nmm

Compressive zone height (ULS)

f	645	N/mm^2
xu	58	mm
xumax	309	mm

Check: xu < xumax	ok!

11.18 Prestressed VHPFRC box girder

Prestressed VHPC box girder

Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td	250	mm
tf	150	mm
hw	2800	mm
h	3200	mm
Ac	7,04E+06	mm^2
e	1062	mm
Ic	1,05E+13	mm^4
W0	9,88E+09	mm^3
Wb	4,91E+09	mm^3
c	30	mm
dp	3109	mm

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	15	
number of tendons	8	
Ap	18000	mm^2
Øduct	90	mm
Øanchor	310	mm
anchor spacing	675	mm

f'c	120	N/mm^2
f'ct	6,4	N/mm^2
σctmax	4	N/mm^2
Ec	40000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1046	N/mm^2
εp∞	0,0054	

When εb = εctmax: ε0 =	0,000051
When εb' = εctmax: ε0 =	0,000054
When εb = εt,p: ε0 =	0,000108
When εb' = εt,p: ε0 =	0,000110
When εp = εp,y: ε0 =	0,000209
When dn = td: ε0 =	0,000204
When εb = εt,u: ε0 =	0,000228
When εb' = εt,u: ε0 =	0,000233
When εp = εud: ε0 =	0,001163

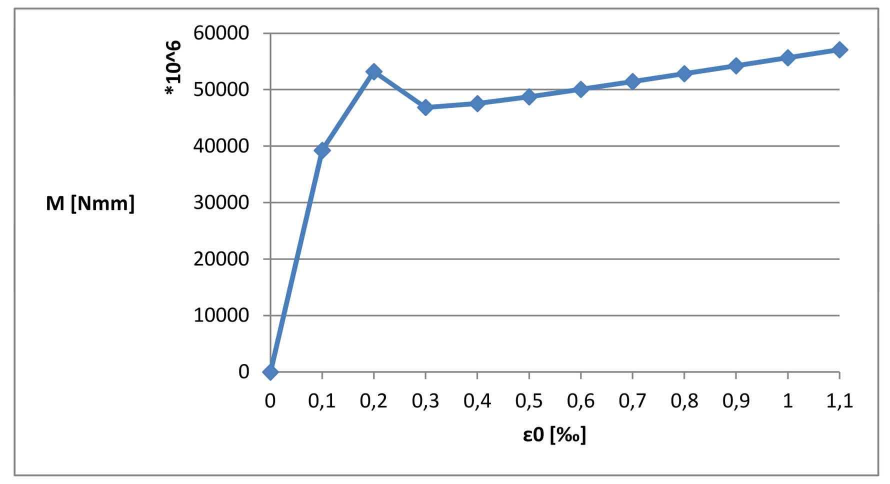
When dn = td & εp = εp,y: Ap =	19440	mm^2
When dn = td & εb = εt,u: Ap =	36259	mm^2
When dn = td & εb' = εt,u: Ap =	39416	mm^2
When εp = εud & ε0 = εcmax: Ap =	346765	mm^2
When dn = td & ε0 = εcmax: Ap =	307143	mm^2

Mu	5,71E+10	Nmm

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	5,70E-05	4,25E-04	4,35E-04	5,80E-03	4,51E-04	581	331	581	1888	0	0	0	1,72E-07
0,0002	2,84E-06	2,21E-03	2,25E-03	7,62E-03	2,32E-03	254	4	127	560	2110	0	0	7,89E-07
0,0003	1,49E-04	0,00E+00	5,29E-03	1,07E-02	0,00E+00	167	0	56	246	1583	0	83	1,80E-06
0,0004	2,60E-04	0,00E+00	7,81E-03	1,32E-02	0,00E+00	151	0	38	167	1077	0	99	2,64E-06
0,0005	3,65E-04	0,00E+00	1,03E-02	1,56E-02	0,00E+00	144	0	29	128	822	0	106	3,46E-06
0,0006	4,68E-04	0,00E+00	1,27E-02	1,80E-02	0,00E+00	140	0	23	103	666	0	110	4,27E-06
0,0007	5,69E-04	0,00E+00	1,51E-02	2,04E-02	0,00E+00	138	0	20	87	560	0	112	5,08E-06
0,0008	6,70E-04	0,00E+00	1,75E-02	2,28E-02	0,00E+00	136	0	17	75	484	0	114	5,88E-06
0,0009	7,70E-04	0,00E+00	1,99E-02	2,52E-02	0,00E+00	135	0	15	66	426	0	115	6,68E-06
0,001	8,70E-04	0,00E+00	2,23E-02	2,76E-02	0,00E+00	134	0	13	59	380	0	116	7,48E-06
0,0011	9,70E-04	0,00E+00	2,46E-02	3,00E-02	0,00E+00	133	0	12	53	344	0	117	8,28E-06
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	1,80E+07	-5,58E+06	8,13E+05	5,29E+06	4,80E+06	0,00E+00	0,00E+00	1,53E+06	0,00E+00	0	0	0	3,92E+10
0,0002	1,57E+07	-3,04E+03	1,78E+05	1,57E+06	4,18E+06	1,89E+06	0,00E+00	7,90E+06	0,00E+00	909	82	0	5,32E+10
0,0003	1,55E+07	0,00E+00	7,79E+04	6,88E+05	2,22E+06	0,00E+00	3,67E+06	8,88E+06	2,61E-08	0	0	0	4,69E+10
0,0004	1,88E+07	0,00E+00	5,30E+04	4,68E+05	1,51E+06	0,00E+00	7,59E+06	9,16E+06	4,47E-08	0	0	0	4,75E+10
0,0005	2,24E+07	0,00E+00	4,05E+04	3,57E+05	1,15E+06	0,00E+00	1,14E+07	9,44E+06	-2,98E-08	0	0	0	4,87E+10
0,0006	2,61E+07	0,00E+00	3,28E+04	2,90E+05	9,32E+05	0,00E+00	1,52E+07	9,71E+06	1,22E-06	0	0	0	5,01E+10
0,0007	2,99E+07	0,00E+00	2,76E+04	2,44E+05	7,84E+05	0,00E+00	1,89E+07	9,98E+06	-1,72E-06	0	0	0	5,14E+10
0,0008	3,37E+07	0,00E+00	2,38E+04	2,10E+05	6,77E+05	0,00E+00	2,26E+07	1,02E+07	-9,46E-07	0	0	0	5,28E+10
0,0009	3,76E+07	0,00E+00	2,10E+04	1,85E+05	5,96E+05	0,00E+00	2,63E+07	1,05E+07	1,13E-06	0	0	0	5,43E+10
0,001	4,14E+07	0,00E+00	1,87E+04	1,65E+05	5,32E+05	0,00E+00	3,00E+07	1,08E+07	-6,11E-07	0	0	0	5,57E+10
0,0011	4,53E+07	0,00E+00	1,69E+04	1,49E+05	4,81E+05	0,00E+00	3,36E+07	1,10E+07	0,00E+00	0	0	0	5,71E+10
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,1	3,92E+10
0,2	5,32E+10
0,3	4,69E+10
0,4	4,75E+10
0,5	4,87E+10
0,6	5,01E+10
0,7	5,14E+10
0,8	5,28E+10
0,9	5,43E+10
1	5,57E+10
1,1	5,71E+10



Shear Capacity (ULS)

γE*γb	1,5	
Seff	2,15E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	30	
tan βu	0,58	
dn	132	mm
z	3065	mm
cot βu	1,73	
Ø stirrups	16	mm
fyk	500	N/mm^2
Asw	402	mm^2
s	100	mm

VRb	3,76E+06	N
Vf	2,29E+07	N
Vs	8,54E+06	N
Vu	3,52E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: beam remains uncracked in SLS

t = 0: check top fibre	M ≥	-7,19E+10	Nmm
t = 0: check bottom fibre	M ≤	1,00E+11	Nmm
t = ∞: check top fibre	M ≥	-6,97E+10	Nmm
t = ∞: check bottom fibre	M ≤	8,31E+10	Nmm

12 Transverse direction and mobile loads

1. Contents

2. Node

Name	Coord X [m]	Coord Y [m]	Coord Z [m]
K1	-3,925	0,000	3,000
K2	0,000	0,000	3,000
K3	0,000	0,000	0,000
K4	7,650	0,000	0,000
K5	7,650	0,000	3,000
K6	11,575	0,000	3,000

3. Member 1D

Name	CrossSection	Length [m]	Shape	Beg. node	End node	Type	FEM type	Layer
S1	CS1 - Rechthoek (250; 1000)	3,925	Line	K1	K2	general (0)	standard	Laag1
S2	CS2 - Rechthoek (350; 1000)	3,000	Line	K2	K3	general (0)	standard	Laag1
S3	CS3 - Rechthoek (150; 1000)	7,650	Line	K3	K4	general (0)	standard	Laag1
S4	CS2 - Rechthoek (350; 1000)	3,000	Line	K4	K5	general (0)	standard	Laag1
S5	CS1 - Rechthoek (250; 1000)	7,650	Line	K2	K5	general (0)	standard	Laag1
S6	CS1 - Rechthoek (250; 1000)	3,925	Line	K5	K6	general (0)	standard	Laag1

4. Supports in node

Name	Node	System	Type	X	Y	Z	Rx	Ry	Rz
Sn1	K3	GCS	Standard	Free	Rigid	Rigid	Free	Free	Free
Sn2	K4	GCS	Standard	Rigid	Rigid	Rigid	Free	Free	Free
Sn3	K2	GCS	Standard	Rigid	Rigid	Free	Free	Free	Free

5. Point forces on beam

Name	Member Load case	System Dir	F [kN] Type	x [m]	Coor Orig	Rep (n)
F1	S1 BG1 - Load case 1	GCS Z	-59,00 Force	1,860	Abso From start	1
F2	S1 BG2 - Load case 2	GCS Z	-59,00 Force	1,860	Abso From start	1
F3	S6 BG2 - Load case 2	GCS Z	-40,00 Force	2,065	Abso From start	1
F4	S5 BG3 - Load case 3	GCS Z	-16,00 Force	0,985	Abso From start	1

Name	Member Load case	System Dir	F [kN] Type	x [m]	Coor Orig	Rep (n)
F5	S5 BG3 - Load case 3	GCS Z	-24,00 Force	2,985	Abso From start	1
F6	S5 BG3 - Load case 3	GCS Z	-51,00 Force	3,985	Abso From start	1
F7	S5 BG3 - Load case 3	GCS Z	-17,00 Force	5,985	Abso From start	1
F8	S5 BG4 - Load case 4	GCS Z	-104,00 Force	0,985	Abso From start	1
F9	S5 BG4 - Load case 4	GCS Z	-84,00 Force	2,985	Abso From start	1
F10	S5 BG4 - Load case 4	GCS Z	-46,00 Force	3,985	Abso From start	1
F11	S5 BG4 - Load case 4	GCS Z	-28,00 Force	5,985	Abso From start	1
F12	S1 BG4 - Load case 4	GCS Z	-22,00 Force	1,910	Abso From start	1

6. Line forces on beam

Name	Member Load case	Type System	Dir Distribution	P1 [kN/m]	x1 [m] x2 [m]	Coor Loc	Orig	Ecc ey [m] Ecc ez [m]
Lijnlast1	S1 BG1 - Load case 1	Force LCS	Z Uniform	-10,35	1,410 3,925	Abso Length	From start	0,000 0,000
Lijnlast2	S1 BG2 - Load case 2	Force LCS	Z Uniform	-10,35	1,410 3,925	Abso Length	From start	0,000 0,000
Lijnlast3	S6 BG2 - Load case 2	Force LCS	Z Uniform	-3,50	0,000 2,515	Abso Length	From start	0,000 0,000
Lijnlast4	S5 BG3 - Load case 3	Force LCS	Z Uniform	-3,50	0,485 3,485	Abso Length	From start	0,000 0,000
Lijnlast5	S5 BG3 - Load case 3	Force LCS	Z Uniform	-10,35	3,485 6,485	Abso Length	From start	0,000 0,000
Lijnlast6	S5 BG4 - Load case 4	Force LCS	Z Uniform	-10,35	0,485 3,485	Abso Length	From start	0,000 0,000
Lijnlast7	S5 BG4 - Load case 4	Force LCS	Z Uniform	-3,50	3,485 6,485	Abso Length	From start	0,000 0,000
Lijnlast8	S1 BG4 - Load case 4	Force LCS	Z Uniform	-3,50	1,410 3,925	Abso Length	From start	0,000 0,000
Lijnlast9	S5 BG4 - Load case 4	Force LCS	Z Uniform	-3,50	0,000 0,485	Abso Length	From start	0,000 0,000

7. Load cases

7.1. Load cases - BG1

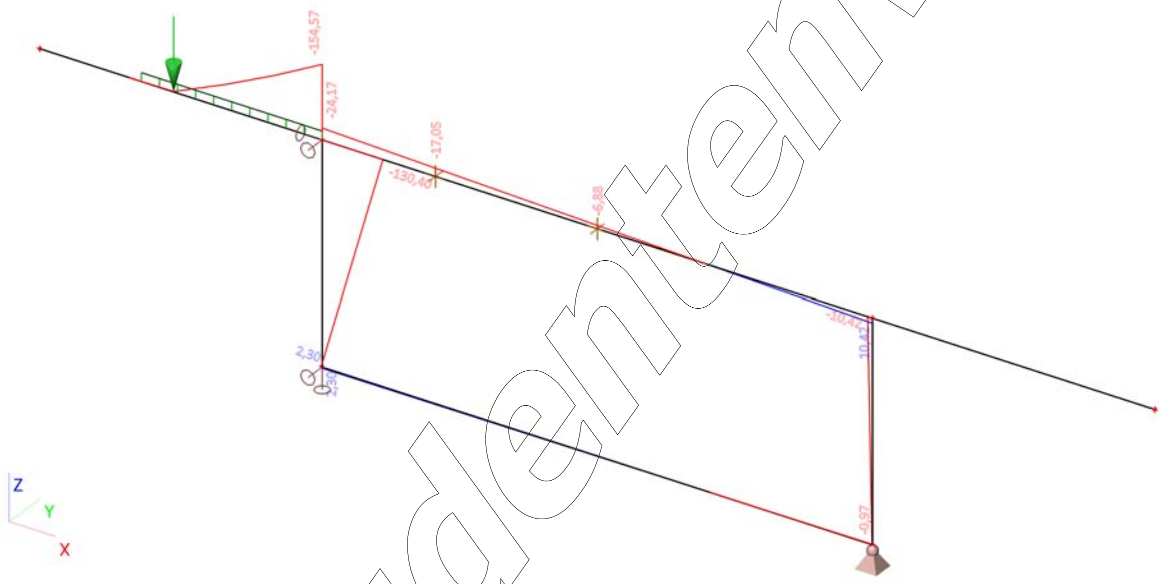
Name	Description	Action type	LoadGroup	Load type	Spec	Duration	Master load case
BG1	Load case 1	Variable	LG2	Static	Standard	Short	None

7.1.1. Internal forces on member

Linear calculation, Extreme : Global, System : Principal
Selection : All
Load cases : BG1

Member	Case	dx [m]	N [kN]	Vy [kN]	Vz [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
S2	BG1	0,000	**-89,55**	0,00	**44,23**	0,00	-130,40	0,00
S4	BG1	0,000	**4,52**	0,00	-3,15	0,00	-0,97	0,00
S1	BG1	0,000	0,00	**0,00**	0,00	**0,00**	0,00	**0,00**
S1	BG1	3,925	0,00	0,00	**-85,03**	0,00	**-154,57**	0,00
S5	BG1	7,650	3,15	0,00	4,52	0,00	**10,42**	0,00

7.1.2. My



7.2. Load cases - BG2

Name	Description	Action type	LoadGroup	Load type	Spec	Duration	Master load case
BG2	Load case 2	Variable	LG2	Static	Standard	Short	None

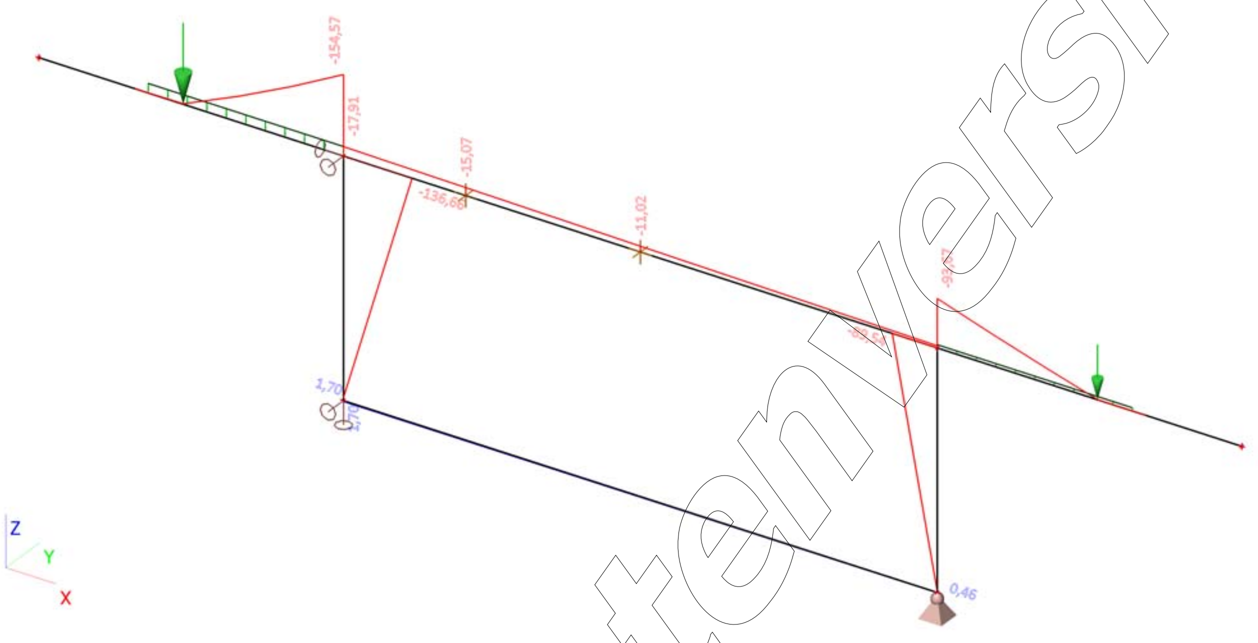
7.2.1. Internal forces on member

Linear calculation, Extreme : Global, System : Principal
Selection : All
Load cases : BG2

Member	Case	dx [m]	N [kN]	Vy [kN]	Vz [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
S2	BG2	0,000	**-86,83**	0,00	46,12	0,00	-136,66	0,00
S5	BG2	0,000	**30,00**	0,00	1,80	0,00	-17,91	0,00

Member	Case	dx [m]	N [kN]	Vy [kN]	Vz [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
S1	BG2	0,000	0,00	**0,00**	0,00	**0,00**	0,00	**0,00**
S1	BG2	3,925	0,00	0,00	**-85,03**	0,00	**-154,57**	0,00
S6	BG2	0,000	0,00	0,00	**48,80**	0,00	-93,67	0,00
S2	BG2	3,000	-86,83	0,00	46,12	0,00	**1,70**	0,00

7.2.2. My



7.3. Load cases - BG3

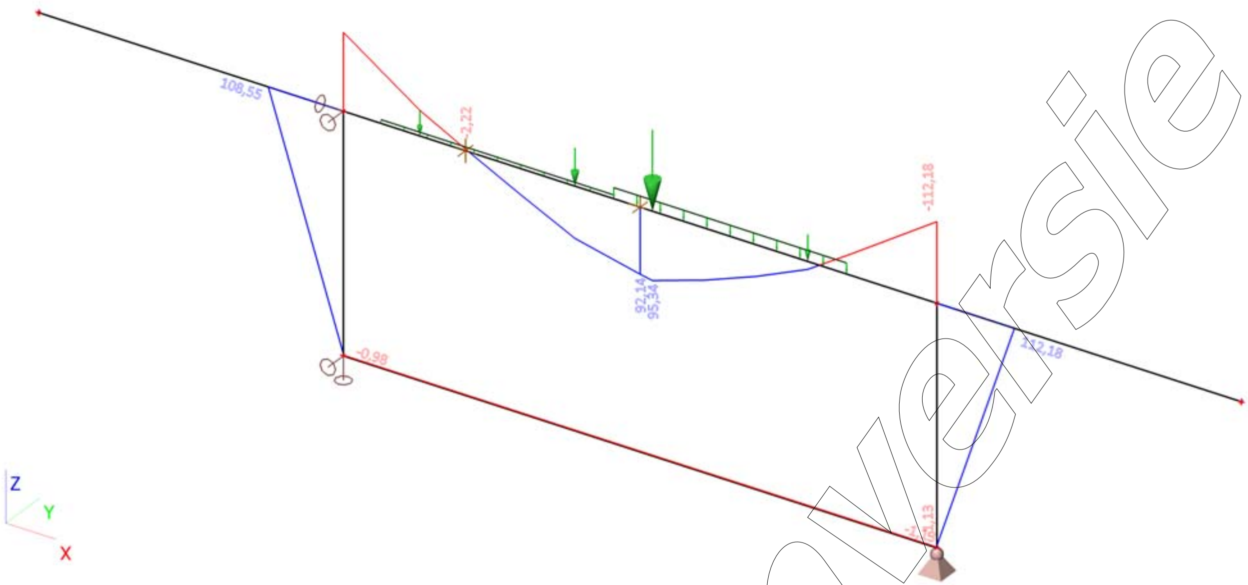
Name	Description	Action type	LoadGroup	Load type	Spec	Duration	Master load case
BG3	Load case 3	Variable	LG2	Static	Standard	Short	None

7.3.1. Internal forces on member

Linear calculation, Extreme : Global, System : Principal
Selection : All
Load cases : BG3

Member	Case	dx [m]	N [kN]	Vy [kN]	Vz [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
S2	BG3	0,000	**-74,83**	0,00	-36,51	0,00	108,55	0,00
S3	BG3	0,000	**36,51**	0,00	-0,02	0,00	-0,98	0,00
S1	BG3	0,000	0,00	**0,00**	0,00	**0,00**	0,00	**0,00**
S5	BG3	6,540	-37,77	0,00	**-74,72**	0,00	-29,24	0,00
S5	BG3	0,000	-37,77	0,00	**74,83**	0,00	-108,55	0,00
S5	BG3	7,650	-37,77	0,00	-74,72	0,00	**-112,18**	0,00
S4	BG3	3,000	-74,72	0,00	37,77	0,00	**112,18**	0,00

7.3.2. My



7.4. Load cases - BG4

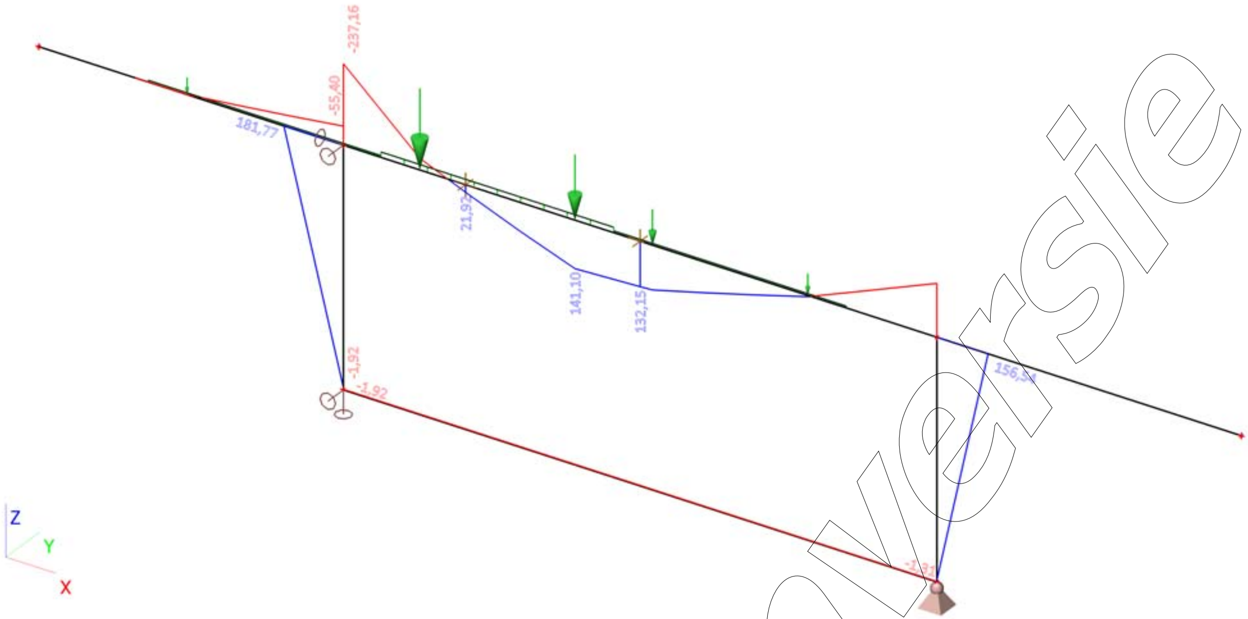
Name	Description	Action type	LoadGroup	Load type	Spec	Duration	Master load case
BG4	Load case 4	Variable	LG2	Static	Standard	Short	None

7.4.1. Internal forces on member

Linear calculation, Extreme : Global, System : Principal
Selection : All
Load cases : BG4

Member	Case	dx [m]	N [kN]	Vy [kN]	Vz [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
S2	BG4	0,000	**-239,60**	0,00	-61,23	0,00	**181,77**	0,00
S3	BG4	0,000	**61,23**	0,00	0,08	0,00	-1,92	0,00
S1	BG4	0,000	0,00	**0,00**	0,00	**0,00**	0,00	**0,00**
S5	BG4	6,540	-52,62	0,00	**-96,45**	0,00	-49,48	0,00
S5	BG4	0,000	-52,62	0,00	**208,80**	0,00	**-237,16**	0,00

7.4.2. My



13 Contribution of axle load 1 to the bending moment at mid span

restart;
4th order differential equation:
$DV1 := EI \cdot diff(w1(x), x\$4) = 0; DV2 := EI \cdot diff(w2(x), x\$4) = 0;$

$$EI \left(\frac{d^4}{dx^4} w1(x) \right) = 0$$

$$EI \left(\frac{d^4}{dx^4} w2(x) \right) = 0 \quad (1)$$

The general solution:
$solution := dsolve(\{DV1, DV2\}, \{w1(x), w2(x)\});$

$$\left\{ w1(x) = \frac{1}{6}\_C5\, x^3 + \frac{1}{2}\_C6\, x^2 + \_C7\, x + \_C8,\ w2(x) = \frac{1}{6}\_C1\, x^3 + \frac{1}{2}\_C2\, x^2 + \_C3\, x + \_C4 \right\} \quad (2)$$

$assign(solution);$
$w1 := w1(x) : w2 := w2(x) :$

Mechanical relations:
$phi1 := diff(w1, x) : kappa1 := diff(phi1, x) : M1 := EI \cdot kappa1 : V1 := diff(M1, x) :$
$phi2 := diff(w2, x) : kappa2 := diff(phi2, x) : M2 := EI \cdot kappa2 : V2 := diff(M2, x) :$

Boundary conditions at x=0:
$x := 0 : eq1 := w1 = 0 : eq2 := phi1 = 0 :$

Interface conditions at x=a:
$\mathrm{x} := \mathrm{a} : \mathrm{eq3} := \mathrm{w1} = \mathrm{w2} : \mathrm{eq4} := \mathrm{phi1} = \mathrm{phi2} : \mathrm{eq5} := \mathrm{M1} = \mathrm{M2} : \mathrm{eq6} := \mathrm{V1} = \mathrm{V2} + \mathrm{F} :$

Boundary conditions at x=l:
$\mathrm{x} := \mathrm{l} : \mathrm{eq7} := \mathrm{w2} = 0 : \mathrm{eq8} := \mathrm{phi2} = 0 :$

Solving the unknowns:
$\mathrm{solution} := \mathrm{solve}(\{\mathrm{eq1}, \mathrm{eq2}, \mathrm{eq3}, \mathrm{eq4}, \mathrm{eq5}, \mathrm{eq6}, \mathrm{eq7}, \mathrm{eq8}\}, \{\_C1, \_C2, \_C3, \_C4, \_C5, \_C6, \_C7, \_C8\}) :$
$assign(solution);$
$x :='x'$

$$x \quad (3)$$

Parameters:
$\mathrm{a} := 0.985 : \mathrm{l} := 7.65 : \mathrm{x} := \frac{1}{2}\mathrm{l} :$

The bending moment:
$M2;$

$$0.0634133988\, F \quad (4)$$

14 Contribution of axle load 1 to the bending moment at the support

restart;
4th order differential equation:
$DV1 := EI \cdot diff(w1(x), x\$4) = 0; DV2 := EI \cdot diff(w2(x), x\$4) = 0;$

$$EI \left(\frac{d^4}{dx^4} w1(x) \right) = 0$$

$$EI \left(\frac{d^4}{dx^4} w2(x) \right) = 0 \qquad (1)$$

The general solution:
$solution := dsolve(\{DV1, DV2\}, \{w1(x), w2(x)\});$

$$\left\{ w1(x) = \frac{1}{6} \_C5\, x^3 + \frac{1}{2} \_C6\, x^2 + \_C7\, x + \_C8, w2(x) = \frac{1}{6} \_C1\, x^3 + \frac{1}{2} \_C2\, x^2 + \_C3\, x + \_C4 \right\} \qquad (2)$$

$assign(solution);$
$w1 := w1(x) : w2 := w2(x) :$

Mechanical relations:
$phi1 := diff(w1, x) : kappa1 := diff(phi1, x) : M1 := EI \cdot kappa1 : V1 := diff(M1, x) :$
$phi2 := diff(w2, x) : kappa2 := diff(phi2, x) : M2 := EI \cdot kappa2 : V2 := diff(M2, x) :$

Boundary conditions at x=0:
$x := 0 : eq1 := w1 = 0 : eq2 := phi1 = 0 :$

Interface conditions at x=a:
x := a : eq3 := w1 = w2 : eq4 := phi1 = phi2 : eq5 := M1 = M2 : eq6 := V1 = V2 + F :

Boundary conditions at x=l:
x := l : eq7 := w2 = 0 : eq8 := phi2 = 0 :

Solving the unknowns:
solution := solve({eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8}, {_C1, _C2, _C3, _C4, _C5, _C6, _C7, _C8}) :
$assign(solution);$
$x := 'x'$

$$x \qquad (3)$$

Parameters:
a := 0.985 : l := 7.65 : x := 0 :

The bending moment:
$M1;$

$$-0.7476763915\, F \qquad (4)$$

15 Cross-sectional capacity of the flange at the support

Flange support

Moment Capacity (ULS)

b	1000	mm
D	300	mm
Ac	3,00E+05	mm^2
c	30	mm
dp	262	mm
Ig	2,25E+09	mm^4
z	150	mm
Wtop	1,50E+07	mm^3
Wbot	1,50E+07	mm^3

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	5	
Ap	750	mm^2
strand spacing	215	mm

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,004	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

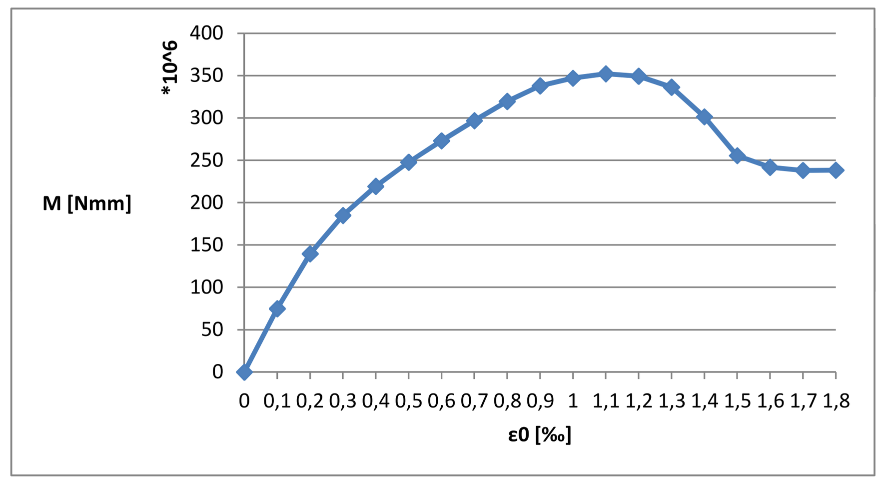
When εb = εctmax: ε0 =	0,00010
When Δεp = εctmax: ε0 =	0,00013
When εp = εp,y: ε0 =	0,00087
When εb = εt,p: ε0 =	0,00102
When εb = εt,u: ε0 =	0,00141
When εp = εud: ε0 =	0,00184

When εp = εud & ε0 = εcmax: Ap =	1874	mm^2

Mu	3,52E+08	Nmm

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	Δεp [-]	εp [-]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΔNP [N]	ΣH = 0	y [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	151	149	0	0	7,34E-05	5,44E-03	9,86E-05	6,62E-07	3,78E+05	3,67E+05	0,00E+00	0,00E+00	1,07E+04	0	0	0	7,48E+07
0,0002	136	68	96	0	1,86E-04	5,55E-03	2,42E-04	1,47E-06	6,79E+05	1,70E+05	4,82E+05	0,00E+00	2,72E+04	0	0	0	1,40E+08
0,0003	117	39	145	0	3,74E-04	5,74E-03	4,72E-04	2,57E-06	8,75E+05	9,72E+04	7,23E+05	0,00E+00	5,47E+04	0	0	0	1,85E+08
0,0004	102	25	173	0	6,29E-04	5,99E-03	7,78E-04	3,93E-06	1,02E+06	6,37E+04	8,63E+05	0,00E+00	9,19E+04	0	0	0	2,19E+08
0,0005	91	18	191	0	9,40E-04	6,31E-03	1,15E-03	5,50E-06	1,14E+06	4,55E+04	9,54E+05	0,00E+00	1,37E+05	0	0	0	2,48E+08
0,0006	83	14	203	0	1,30E-03	6,66E-03	1,57E-03	7,25E-06	1,24E+06	3,45E+04	1,02E+06	0,00E+00	1,90E+05	0	0	0	2,73E+08
0,0007	76	11	213	0	1,70E-03	7,06E-03	2,05E-03	9,15E-06	1,34E+06	2,73E+04	1,06E+06	0,00E+00	2,48E+05	0	0	0	2,97E+08
0,0008	72	9	219	0	2,13E-03	7,49E-03	2,55E-03	1,12E-05	1,43E+06	2,24E+04	1,10E+06	0,00E+00	3,11E+05	0	0	0	3,20E+08
0,0009	67	7	226	0	2,63E-03	7,99E-03	3,14E-03	1,35E-05	1,50E+06	1,86E+04	1,13E+06	0,00E+00	3,58E+05	0	0	0	3,38E+08
0,001	62	6	232	0	3,26E-03	8,63E-03	3,88E-03	1,63E-05	1,54E+06	1,54E+04	1,16E+06	0,00E+00	3,61E+05	0	0	0	3,47E+08
0,0011	57	5	201	38	3,99E-03	9,36E-03	4,73E-03	1,94E-05	1,56E+06	1,29E+04	1,00E+06	1,76E+05	3,64E+05	0	18	0	3,52E+08
0,0012	51	4	167	77	4,91E-03	1,03E-02	5,80E-03	2,33E-05	1,54E+06	1,07E+04	8,36E+05	3,28E+05	3,68E+05	0	36	0	3,49E+08
0,0013	46	4	137	114	6,16E-03	1,15E-02	7,25E-03	2,85E-05	1,48E+06	8,77E+03	6,84E+05	4,16E+05	3,74E+05	0	50	0	3,36E+08
0,0014	38	3	105	155	8,34E-03	1,37E-02	9,75E-03	3,72E-05	1,32E+06	6,73E+03	5,25E+05	4,03E+05	3,84E+05	0	54	0	3,01E+08
0,0015	28	2	73	113	1,25E-02	1,78E-02	1,45E-02	5,33E-05	1,06E+06	4,69E+03	3,66E+05	2,81E+05	4,03E+05	0	0	0	2,56E+08
0,0016	23	1	56	87	1,65E-02	2,19E-02	1,92E-02	6,92E-05	9,24E+05	3,61E+03	2,82E+05	2,17E+05	4,22E+05	0	0	0	2,42E+08
0,0017	20	1	46	71	2,06E-02	2,59E-02	2,38E-02	8,50E-05	8,50E+05	2,94E+03	2,29E+05	1,76E+05	4,41E+05	0	0	0	2,38E+08
0,0018	18	1	39	60	2,46E-02	2,99E-02	2,84E-02	1,01E-04	8,05E+05	2,49E+03	1,94E+05	1,49E+05	4,60E+05	0	0	0	2,38E+08
0,0019	16	1	34	52	2,85E-02	3,39E-02	3,29E-02	1,16E-04	7,78E+05	2,15E+03	1,68E+05	1,29E+05	4,78E+05	0	0	0	2,40E+08
0,002	15	1	30	46	3,24E-02	3,78E-02	3,74E-02	1,31E-04	7,61E+05	1,90E+03	1,48E+05	1,14E+05	4,97E+05	0	0	0	2,44E+08
0,0021	14	1	27	41	3,63E-02	4,17E-02	4,19E-02	1,47E-04	7,52E+05	1,70E+03	1,33E+05	1,02E+05	5,15E+05	0	0	0	2,47E+08
0,0022	14	1	24	37	4,02E-02	4,56E-02	4,64E-02	1,62E-04	7,47E+05	1,54E+03	1,20E+05	9,27E+04	5,33E+05	0	0	0	2,51E+08
0,0023	13	1	22	34	4,41E-02	4,94E-02	5,08E-02	1,77E-04	7,47E+05	1,41E+03	1,10E+05	8,47E+04	5,51E+05	0	0	0	2,55E+08
0,0024	12	1	20	31	4,79E-02	5,33E-02	5,52E-02	1,92E-04	7,50E+05	1,30E+03	1,02E+05	7,81E+04	5,69E+05	0	0	0	2,60E+08
0,0025	12	0	19	29	5,18E-02	5,71E-02	5,96E-02	2,07E-04	7,54E+05	1,21E+03	9,42E+04	7,24E+04	5,87E+05	0	0	0	2,64E+08
0,0026	12	0	18	27	5,56E-02	6,09E-02	6,40E-02	2,22E-04	7,61E+05	1,13E+03	8,78E+04	6,76E+04	6,04E+05	0	0	4	2,68E+08
0,0027	11	0	16	25	5,92E-02	6,46E-02	6,82E-02	2,36E-04	7,69E+05	1,06E+03	8,25E+04	6,34E+04	6,22E+05	0	0	4	2,73E+08
0,0028	11	0	16	24	6,28E-02	6,81E-02	7,23E-02	2,50E-04	7,77E+05	9,99E+02	7,79E+04	5,99E+04	6,38E+05	0	0	4	2,77E+08
0,0029	11	0	15	23	6,62E-02	7,16E-02	7,62E-02	2,64E-04	7,86E+05	9,48E+02	7,40E+04	5,69E+04	6,54E+05	0	0	4	2,81E+08
0,003	11	0	14	22	6,95E-02	7,49E-02	8,00E-02	2,77E-04	7,95E+05	9,04E+02	7,05E+04	5,42E+04	6,69E+05	0	0	4	2,85E+08
0,0031	11	0	13	21	7,27E-02	7,81E-02	8,37E-02	2,89E-04	8,04E+05	8,64E+02	6,74E+04	5,19E+04	6,84E+05	0	0	4	2,88E+08
0,0032	11	0	13	20	7,58E-02	8,12E-02	8,73E-02	3,02E-04	8,14E+05	8,29E+02	6,47E+04	4,97E+04	6,99E+05	0	0	4	2,92E+08
0,0033	11	0	12	19	7,88E-02	8,42E-02	9,07E-02	3,13E-04	8,24E+05	7,98E+02	6,22E+04	4,79E+04	7,13E+05	0	0	4	2,96E+08
0,0034	10	0	12	18	8,18E-02	8,71E-02	9,41E-02	3,25E-04	8,33E+05	7,69E+02	6,00E+04	4,61E+04	7,27E+05	0	0	4	2,99E+08
0,0035	10	0	12	18	8,46E-02	9,00E-02	9,74E-02	3,36E-04	8,43E+05	7,43E+02	5,80E+04	4,46E+04	7,40E+05	0	0	4	3,03E+08
0,0036	10	0	11	17	8,74E-02	9,28E-02	1,01E-01	3,47E-04	8,53E+05	7,19E+02	5,61E+04	4,32E+04	7,53E+05	0	0	4	3,06E+08
0,0037	10	0	11	17	9,02E-02	9,56E-02	1,04E-01	3,58E-04	8,63E+05	6,98E+02	5,44E+04	4,19E+04	7,66E+05	0	0	4	3,09E+08
0,0038	10	0	11	16	9,29E-02	9,82E-02	1,07E-01	3,69E-04	8,73E+05	6,78E+02	5,29E+04	4,07E+04	7,78E+05	0	0	4	3,12E+08
0,0039	10	0	10	16	9,55E-02	1,01E-01	1,10E-01	3,79E-04	8,82E+05	6,59E+02	5,14E+04	3,95E+04	7,91E+05	0	0	4	3,15E+08
0,0040	10	0	10	15	9,81E-02	1,03E-01	1,13E-01	3,90E-04	8,92E+05	6,42E+02	5,01E+04	3,85E+04	8,03E+05	0	0	4	3,18E+08

ε0 [‰]	M [Nmm]
0	0
0,1	7,48E+07
0,2	1,40E+08
0,3	1,85E+08
0,4	2,19E+08
0,5	2,48E+08
0,6	2,73E+08
0,7	2,97E+08
0,8	3,20E+08
0,9	3,38E+08
1	3,47E+08
1,1	3,52E+08
1,2	3,49E+08
1,3	3,36E+08
1,4	3,01E+08
1,5	2,56E+08
1,6	2,42E+08
1,7	2,38E+08
1,8	2,38E+08



Shear Capacity (ULS)

γE*γb	1,5	
Seff	2,56E+05	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	17	mm
z	256	mm
cot βu	1,00	
Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

VRb	5,02E+05	N
Vf	1,58E+06	N
Vs	0,00E+00	N
Vu	2,08E+06	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: beam remains uncracked in SLS

Span

t = 0: check top fibre	M ≥	-5,51E+07	Nmm
t = 0: check bottom fibre	M ≤	2,89E+08	Nmm
t = ∞: check top fibre	M ≥	-7,13E+07	Nmm
t = ∞: check bottom fibre	M ≤	2,47E+08	Nmm

Support

t = 0: check top fibre	M ≤	2,89E+08	Nmm
t = 0: check bottom fibre	M ≥	-5,51E+07	Nmm
t = ∞: check top fibre	M ≤	2,47E+08	Nmm
t = ∞: check bottom fibre	M ≥	-7,13E+07	Nmm

16 Cross-sectional capacity of the deck at the support

Deck support

Moment Capacity (ULS)

b	1000	mm
D	400	mm
Ac	4,00E+05	mm^2
c	30	mm
dp	362	mm
Ig	5,33E+09	mm^4
z	200	mm
Wtop	2,67E+07	mm^3
Wbot	2,67E+07	mm^3

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	5	
Ap	750	mm^2
strand spacing	215	mm

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,004	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

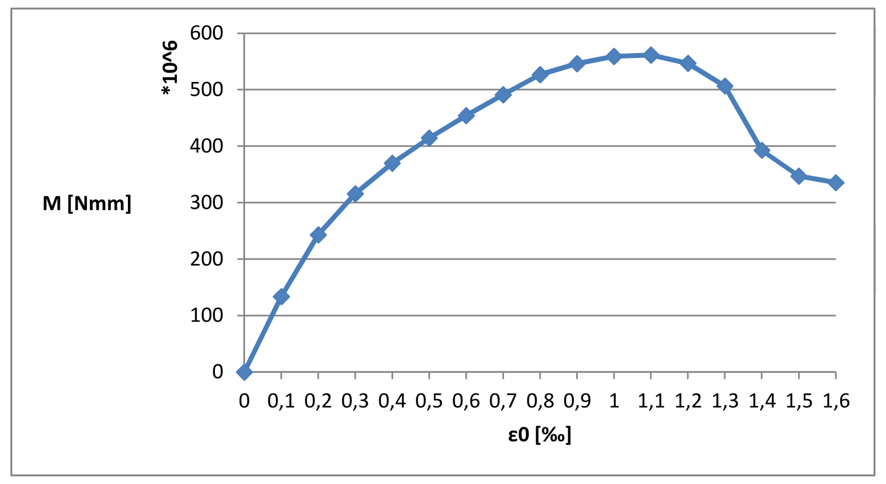
When εb = εctmax: ε0 =	0,00010
When Δεp = εctmax: ε0 =	0,00012
When εp = εp,y: ε0 =	0,00082
When εb = εt,p: ε0 =	0,00099
When εb = εt,u: ε0 =	0,00135
When εp = εud: ε0 =	0,00168

When εp = εud & ε0 = εcmax: Ap =	2590	mm^2

Mu	5,61E+08	Nmm

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	Δεp [-]	εp [-]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΔNP [N]	ΣH = 0	γ [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	201	199	0	0	7,99E-05	5,45E-03	9,88E-05	4,97E-07	5,03E+05	4,91E+05	0,00E+00	0,00E+00	1,17E+04	0	0	0	1,34E+08
0,0002	180	90	129	0	2,01E-04	5,57E-03	2,43E-04	1,11E-06	9,02E+05	2,25E+05	6,47E+05	0,00E+00	2,94E+04	0	0	0	2,43E+08
0,0003	154	51	194	0	4,03E-04	5,77E-03	4,77E-04	1,94E-06	1,16E+06	1,29E+05	9,71E+05	0,00E+00	5,90E+04	0	0	0	3,16E+08
0,0004	134	34	232	0	6,78E-04	6,04E-03	7,91E-04	2,98E-06	1,34E+06	8,40E+04	1,16E+06	0,00E+00	9,91E+04	0	0	0	3,70E+08
0,0005	119	24	257	0	1,02E-03	6,38E-03	1,18E-03	4,19E-06	1,49E+06	5,97E+04	1,28E+06	0,00E+00	1,49E+05	0	0	0	4,14E+08
0,0006	108	18	274	0	1,41E-03	6,78E-03	1,62E-03	5,55E-06	1,62E+06	4,50E+04	1,37E+06	0,00E+00	2,06E+05	0	0	0	4,54E+08
0,0007	99	14	286	0	1,85E-03	7,22E-03	2,12E-03	7,05E-06	1,74E+06	3,55E+04	1,43E+06	0,00E+00	2,71E+05	0	0	0	4,91E+08
0,0008	92	12	296	0	2,33E-03	7,70E-03	2,66E-03	8,65E-06	1,85E+06	2,89E+04	1,48E+06	0,00E+00	3,41E+05	0	0	0	5,27E+08
0,0009	85	9	306	0	2,94E-03	8,30E-03	3,34E-03	1,06E-05	1,91E+06	2,36E+04	1,53E+06	0,00E+00	3,59E+05	0	0	0	5,46E+08
0,001	78	8	304	10	3,64E-03	9,00E-03	4,12E-03	1,28E-05	1,95E+06	1,95E+04	1,52E+06	4,76E+04	3,62E+05	0	5	0	5,59E+08
0,0011	71	6	253	69	4,48E-03	9,85E-03	5,07E-03	1,54E-05	1,96E+06	1,62E+04	1,27E+06	3,15E+05	3,66E+05	0	33	0	5,61E+08
0,0012	64	5	208	123	5,59E-03	1,10E-02	6,30E-03	1,87E-05	1,92E+06	1,33E+04	1,04E+06	4,96E+05	3,71E+05	0	56	0	5,47E+08
0,0013	55	4	165	175	7,24E-03	1,26E-02	8,14E-03	2,36E-05	1,79E+06	1,06E+04	8,26E+05	5,75E+05	3,79E+05	0	72	0	5,07E+08
0,0014	39	3	109	168	1,15E-02	1,69E-02	1,29E-02	3,57E-05	1,37E+06	7,00E+03	5,46E+05	4,20E+05	3,99E+05	0	0	0	3,93E+08
0,0015	30	2	77	118	1,69E-02	2,22E-02	1,88E-02	5,07E-05	1,11E+06	4,93E+03	3,84E+05	2,96E+05	4,24E+05	0	0	0	3,47E+08
0,0016	25	2	60	92	2,20E-02	2,74E-02	2,45E-02	6,53E-05	9,80E+05	3,83E+03	2,99E+05	2,30E+05	4,48E+05	0	0	0	3,35E+08
0,0017	21	1	49	75	2,71E-02	3,25E-02	3,01E-02	7,95E-05	9,09E+05	3,14E+03	2,45E+05	1,89E+05	4,72E+05	0	0	0	3,34E+08
0,0018	19	1	42	64	3,20E-02	3,74E-02	3,56E-02	9,35E-05	8,66E+05	2,67E+03	2,09E+05	1,60E+05	4,95E+05	0	0	0	3,37E+08
0,0019	18	1	36	56	3,69E-02	4,23E-02	4,10E-02	1,07E-04	8,41E+05	2,33E+03	1,82E+05	1,40E+05	5,17E+05	0	0	0	3,42E+08
0,002	17	1	32	50	4,17E-02	4,71E-02	4,63E-02	1,21E-04	8,28E+05	2,07E+03	1,61E+05	1,24E+05	5,40E+05	0	0	0	3,49E+08
0,0021	16	1	29	45	4,65E-02	5,19E-02	5,16E-02	1,34E-04	8,21E+05	1,86E+03	1,45E+05	1,12E+05	5,62E+05	0	0	0	3,55E+08
0,0022	15	1	26	41	5,12E-02	5,66E-02	5,69E-02	1,48E-04	8,20E+05	1,69E+03	1,32E+05	1,02E+05	5,84E+05	0	0	0	3,62E+08
0,0023	14	1	24	37	5,59E-02	6,13E-02	6,20E-02	1,61E-04	8,22E+05	1,55E+03	1,21E+05	9,32E+04	6,06E+05	0	0	0	3,69E+08
0,0024	14	1	22	34	6,06E-02	6,60E-02	6,72E-02	1,74E-04	8,28E+05	1,44E+03	1,12E+05	8,62E+04	6,28E+05	0	0	0	3,77E+08
0,0025	13	1	21	32	6,52E-02	7,06E-02	7,23E-02	1,87E-04	8,35E+05	1,34E+03	1,04E+05	8,02E+04	6,49E+05	0	0	0	3,84E+08
0,0026	13	0	19	30	6,98E-02	7,52E-02	7,74E-02	2,00E-04	8,45E+05	1,25E+03	9,75E+04	7,50E+04	6,71E+05	0	0	4	3,91E+08
0,0027	13	0	18	28	7,42E-02	7,96E-02	8,23E-02	2,13E-04	8,55E+05	1,18E+03	9,18E+04	7,06E+04	6,91E+05	0	0	4	3,99E+08
0,0028	12	0	17	27	7,85E-02	8,38E-02	8,70E-02	2,25E-04	8,66E+05	1,11E+03	8,69E+04	6,68E+04	7,11E+05	0	0	4	4,06E+08
0,0029	12	0	17	25	8,26E-02	8,79E-02	9,15E-02	2,36E-04	8,78E+05	1,06E+03	8,26E+04	6,35E+04	7,30E+05	0	0	4	4,12E+08
0,003	12	0	16	24	8,65E-02	9,19E-02	9,59E-02	2,47E-04	8,89E+05	1,01E+03	7,88E+04	6,07E+04	7,49E+05	0	0	4	4,19E+08
0,0031	12	0	15	23	9,04E-02	9,57E-02	1,00E-01	2,58E-04	9,01E+05	9,68E+02	7,55E+04	5,81E+04	7,67E+05	0	0	4	4,25E+08
0,0032	12	0	15	22	9,41E-02	9,94E-02	1,04E-01	2,69E-04	9,13E+05	9,30E+02	7,26E+04	5,58E+04	7,84E+05	0	0	4	4,31E+08
0,0033	12	0	14	22	9,77E-02	1,03E-01	1,08E-01	2,79E-04	9,25E+05	8,96E+02	6,99E+04	5,38E+04	8,01E+05	0	0	4	4,37E+08
0,0034	12	0	13	21	1,01E-01	1,07E-01	1,12E-01	2,89E-04	9,38E+05	8,65E+02	6,75E+04	5,19E+04	8,17E+05	0	0	4	4,43E+08
0,0035	12	0	13	20	1,05E-01	1,10E-01	1,16E-01	2,99E-04	9,50E+05	8,37E+02	6,53E+04	5,02E+04	8,33E+05	0	0	4	4,48E+08
0,0036	12	0	13	19	1,08E-01	1,13E-01	1,20E-01	3,08E-04	9,62E+05	8,11E+02	6,33E+04	4,87E+04	8,49E+05	0	0	4	4,54E+08
0,0037	12	0	12	19	1,11E-01	1,17E-01	1,23E-01	3,18E-04	9,74E+05	7,87E+02	6,14E+04	4,72E+04	8,64E+05	0	0	4	4,59E+08
0,0038	12	0	12	18	1,14E-01	1,20E-01	1,27E-01	3,27E-04	9,85E+05	7,65E+02	5,97E+04	4,59E+04	8,79E+05	0	0	4	4,64E+08
0,0039	12	0	12	18	1,18E-01	1,23E-01	1,30E-01	3,36E-04	9,97E+05	7,45E+02	5,81E+04	4,47E+04	8,94E+05	0	0	4	4,69E+08
0,0040	12	0	11	17	1,21E-01	1,26E-01	1,34E-01	3,44E-04	1,01E+06	7,26E+02	5,66E+04	4,36E+04	9,08E+05	0	0	4	4,74E+08

ε0 [‰]	M [Nmm]
0	0
0,1	1,34E+08
0,2	2,43E+08
0,3	3,16E+08
0,4	3,70E+08
0,5	4,14E+08
0,6	4,54E+08
0,7	4,91E+08
0,8	5,27E+08
0,9	5,46E+08
1	5,59E+08
1,1	5,61E+08
1,2	5,47E+08
1,3	5,07E+08
1,4	3,93E+08
1,5	3,47E+08
1,6	3,35E+08



Shear Capacity (ULS)

γE*γb	1,5	
Seff	3,55E+05	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	22	mm
z	355	mm
cot βu	1,00	
Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

VRb	6,95E+05	N
Vf	2,18E+06	N
Vs	0,00E+00	N
Vu	2,88E+06	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: beam remains uncracked in SLS

Span

t = 0: check top fibre	M ≥	-1,14E+08	Nmm
t = 0: check bottom fibre	M ≤	4,53E+08	Nmm
t = ∞: check top fibre	M ≥	-1,39E+08	Nmm
t = ∞: check bottom fibre	M ≤	3,93E+08	Nmm

Support

t = 0: check top fibre	M ≤	4,53E+08	Nmm
t = 0: check bottom fibre	M ≥	-1,14E+08	Nmm
t = ∞: check top fibre	M ≤	3,93E+08	Nmm
t = ∞: check bottom fibre	M ≥	-1,39E+08	Nmm

17 Cross-sectional capacity of the deck at the haunch’s end

Deck end haunch

Moment Capacity (ULS)

b	1000	mm
D	200	mm
Ac	2,00E+05	mm^2
c	30	mm
dp	80	mm
Ig	6,67E+08	mm^4
z	100	mm
Wtop	6,67E+06	mm^3
Wbot	6,67E+06	mm^3

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	5	
Ap	750	mm^2
strand spacing	215	mm

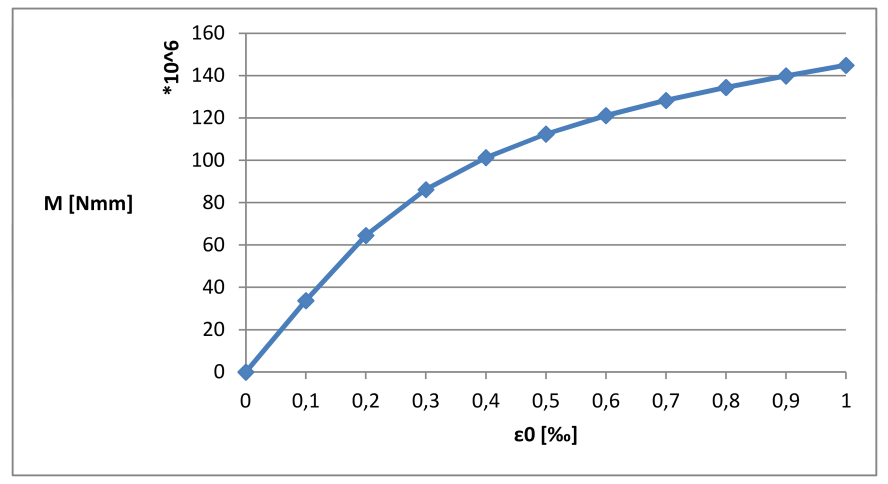
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,004	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

When εb = εctmax: ε0 =	0,00010	
When Δεp = εctmax: ε0 =	0,00041	
When εp = εp,y: ε0 =	0,00154	
When εb = εt,p: ε0 =	0,00107	
When εb = εt,u: ε0 =	0,00149	
When εp = εud: ε0 =	0,00288	
When εp = εud & ε0 = εcmax: Ap =	572	mm^2
Mu	1,45E+08	Nmm

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	Δεp [-]	εp [-]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΔNP [N]	ΣH = 0	y [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	100	100	1	0	-1,98E-05	5,35E-03	1,01E-04	1,00E-06	2,49E+05	2,49E+05	2,89E+03	0,00E+00	-2,89E+03	0	0	0	3,37E+07
0,0002	89	44	67	0	-1,95E-05	5,35E-03	2,51E-04	2,26E-06	4,43E+05	1,11E+05	3,35E+05	0,00E+00	-2,85E+03	0	0	0	6,45E+07
0,0003	75	25	100	0	1,91E-05	5,38E-03	4,98E-04	3,99E-06	5,64E+05	6,27E+04	4,99E+05	0,00E+00	2,79E+03	0	0	0	8,62E+07
0,0004	65	16	119	0	9,33E-05	5,46E-03	8,33E-04	6,17E-06	6,49E+05	4,05E+04	5,95E+05	0,00E+00	1,36E+04	0	0	0	1,01E+08
0,0005	57	11	131	0	2,00E-04	5,56E-03	1,25E-03	8,74E-06	7,15E+05	2,86E+04	6,57E+05	0,00E+00	2,92E+04	0	0	0	1,12E+08
0,0006	51	9	140	0	3,34E-04	5,70E-03	1,74E-03	1,17E-05	7,71E+05	2,14E+04	7,00E+05	0,00E+00	4,89E+04	0	0	0	1,21E+08
0,0007	47	7	146	0	4,94E-04	5,86E-03	2,28E-03	1,49E-05	8,21E+05	1,68E+04	7,32E+05	0,00E+00	7,22E+04	0	0	0	1,28E+08
0,0008	43	5	151	0	6,75E-04	6,04E-03	2,89E-03	1,84E-05	8,68E+05	1,36E+04	7,56E+05	0,00E+00	9,87E+04	0	0	0	1,34E+08
0,0009	41	5	155	0	8,73E-04	6,24E-03	3,53E-03	2,22E-05	9,13E+05	1,13E+04	7,74E+05	0,00E+00	1,28E+05	0	0	0	1,40E+08
0,001	38	4	158	0	1,09E-03	6,45E-03	4,22E-03	2,61E-05	9,58E+05	9,58E+03	7,89E+05	0,00E+00	1,59E+05	0	0	0	1,45E+08
0,0011	36	3	129	31	1,32E-03	6,68E-03	4,94E-03	3,02E-05	1,00E+06	8,28E+03	6,46E+05	1,43E+05	1,92E+05	12152	15	0	1,48E+08
0,0012	35	3	113	49	1,55E-03	6,92E-03	5,69E-03	3,44E-05	1,05E+06	7,26E+03	5,66E+05	2,10E+05	2,27E+05	34370	23	0	1,49E+08
0,0013	34	3	101	63	1,80E-03	7,17E-03	6,46E-03	3,88E-05	1,09E+06	6,45E+03	5,03E+05	2,52E+05	2,64E+05	64797	29	0	1,49E+08
0,0014	32	2	90	75	2,06E-03	7,42E-03	7,25E-03	4,32E-05	1,13E+06	5,78E+03	4,51E+05	2,74E+05	3,01E+05	101589	33	0	1,48E+08
0,0015	31	2	82	126	2,32E-03	7,69E-03	8,06E-03	4,78E-05	1,18E+06	5,23E+03	4,08E+05	3,14E+05	3,40E+05	110423	0	0	1,52E+08
0,0016	20	1	49	75	4,77E-03	1,01E-02	1,43E-02	7,96E-05	8,04E+05	3,14E+03	2,45E+05	1,88E+05	3,68E+05	0	0	0	1,16E+08
0,0017	17	1	39	60	6,31E-03	1,17E-02	1,83E-02	1,00E-04	7,22E+05	2,50E+03	1,95E+05	1,50E+05	3,75E+05	0	0	0	1,10E+08
0,0018	15	1	32	50	7,88E-03	1,32E-02	2,24E-02	1,21E-04	6,69E+05	2,07E+03	1,61E+05	1,24E+05	3,82E+05	0	0	0	1,07E+08
0,0019	13	1	27	42	9,50E-03	1,49E-02	2,66E-02	1,42E-04	6,34E+05	1,75E+03	1,37E+05	1,05E+05	3,90E+05	0	0	0	1,06E+08
0,002	12	1	24	37	1,11E-02	1,65E-02	3,09E-02	1,64E-04	6,09E+05	1,52E+03	1,19E+05	9,13E+04	3,97E+05	0	0	0	1,06E+08
0,0021	11	1	21	32	1,28E-02	1,82E-02	3,52E-02	1,86E-04	5,91E+05	1,34E+03	1,05E+05	8,05E+04	4,05E+05	0	0	0	1,06E+08
0,0022	11	0	19	29	1,45E-02	1,99E-02	3,96E-02	2,09E-04	5,79E+05	1,20E+03	9,34E+04	7,18E+04	4,13E+05	0	0	0	1,06E+08
0,0023	10	0	17	26	1,62E-02	2,16E-02	4,40E-02	2,32E-04	5,71E+05	1,08E+03	8,42E+04	6,48E+04	4,21E+05	0	0	0	1,06E+08
0,0024	9	0	15	24	1,80E-02	2,33E-02	4,85E-02	2,55E-04	5,66E+05	9,82E+02	7,66E+04	5,89E+04	4,29E+05	0	0	0	1,07E+08
0,0025	9	0	14	22	1,97E-02	2,51E-02	5,31E-02	2,78E-04	5,62E+05	9,00E+02	7,02E+04	5,40E+04	4,37E+05	0	0	0	1,08E+08
0,0026	9	0	13	20	2,15E-02	2,69E-02	5,76E-02	3,01E-04	5,61E+05	8,30E+02	6,47E+04	4,98E+04	4,46E+05	0	0	3	1,08E+08
0,0027	8	0	12	19	2,32E-02	2,86E-02	6,21E-02	3,24E-04	5,61E+05	7,72E+02	6,02E+04	4,63E+04	4,54E+05	0	0	3	1,09E+08
0,0028	8	0	11	17	2,49E-02	3,03E-02	6,64E-02	3,46E-04	5,62E+05	7,22E+02	5,63E+04	4,33E+04	4,61E+05	0	0	3	1,09E+08
0,0029	8	0	11	16	2,65E-02	3,19E-02	7,06E-02	3,68E-04	5,63E+05	6,80E+02	5,30E+04	4,08E+04	4,69E+05	0	0	3	1,10E+08
0,003	8	0	10	15	2,81E-02	3,35E-02	7,48E-02	3,89E-04	5,66E+05	6,43E+02	5,02E+04	3,86E+04	4,76E+05	0	0	3	1,11E+08
0,0031	8	0	10	15	2,96E-02	3,50E-02	7,88E-02	4,09E-04	5,68E+05	6,11E+02	4,76E+04	3,66E+04	4,84E+05	0	0	3	1,11E+08
0,0032	7	0	9	14	3,12E-02	3,65E-02	8,27E-02	4,29E-04	5,72E+05	5,82E+02	4,54E+04	3,49E+04	4,91E+05	0	0	3	1,12E+08
0,0033	7	0	9	13	3,26E-02	3,80E-02	8,65E-02	4,49E-04	5,75E+05	5,57E+02	4,34E+04	3,34E+04	4,97E+05	0	0	3	1,12E+08
0,0034	7	0	8	13	3,41E-02	3,94E-02	9,03E-02	4,68E-04	5,78E+05	5,34E+02	4,16E+04	3,20E+04	5,04E+05	0	0	3	1,13E+08
0,0035	7	0	8	12	3,55E-02	4,09E-02	9,40E-02	4,87E-04	5,82E+05	5,13E+02	4,00E+04	3,08E+04	5,11E+05	0	0	3	1,13E+08
0,0036	7	0	8	12	3,69E-02	4,22E-02	9,76E-02	5,06E-04	5,86E+05	4,94E+02	3,85E+04	2,96E+04	5,17E+05	0	0	3	1,14E+08
0,0037	7	0	7	11	3,82E-02	4,36E-02	1,01E-01	5,24E-04	5,90E+05	4,77E+02	3,72E+04	2,86E+04	5,24E+05	0	0	3	1,14E+08
0,0038	7	0	7	11	3,96E-02	4,49E-02	1,05E-01	5,42E-04	5,94E+05	4,61E+02	3,60E+04	2,77E+04	5,30E+05	0	0	3	1,15E+08
0,0039	7	0	7	11	4,09E-02	4,62E-02	1,08E-01	5,60E-04	5,98E+05	4,47E+02	3,48E+04	2,68E+04	5,36E+05	0	0	3	1,15E+08
0,0040	7	0	7	10	4,22E-02	4,75E-02	1,11E-01	5,77E-04	6,02E+05	4,33E+02	3,38E+04	2,60E+04	5,42E+05	0	0	3	1,16E+08

ε0 [‰]	M [Nmm]
0	0
0,1	3,37E+07
0,2	6,45E+07
0,3	8,62E+07
0,4	1,01E+08
0,5	1,12E+08
0,6	1,21E+08
0,7	1,28E+08
0,8	1,34E+08
0,9	1,40E+08
1	1,45E+08
1,1	1,48E+08
1,2	1,49E+08
1,3	1,49E+08
1,4	1,48E+08
1,5	1,52E+08
1,6	1,16E+08
1,7	1,10E+08
1,8	1,07E+08
1,9	1,06E+08
2	1,06E+08
2,1	1,06E+08
2,2	1,06E+08
2,3	1,06E+08
2,4	1,07E+08
2,5	1,08E+08
2,6	1,08E+08
2,7	1,09E+08
2,8	1,09E+08



Shear Capacity (ULS)

γE*γb	1,5	
Seff	6,72E+04	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	38	mm
z	67	mm
cot βu	1,00	
Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

VRb	1,32E+05	N
Vf	4,14E+05	N
Vs	0,00E+00	N
Vu	5,45E+05	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: beam remains uncracked in SLS

Span

t = 0: check top fibre	M ≥	-1,09E+08	Nmm
t = 0: check bottom fibre	M ≤	6,73E+07	Nmm
t = ∞: check top fibre	M ≥	-9,52E+07	Nmm
t = ∞: check bottom fibre	M ≤	6,38E+07	Nmm

Support

t = 0: check top fibre	M ≤	6,73E+07	Nmm
t = 0: check bottom fibre	M ≥	-1,09E+08	Nmm
t = ∞: check top fibre	M ≤	6,38E+07	Nmm
t = ∞: check bottom fibre	M ≥	-9,52E+07	Nmm

18 Cross-sectional capacity of the deck at mid span

Deck mid span

Moment Capacity (ULS)

b	1000	mm
D	200	mm
Ac	2,00E+05	mm^2
c	30	mm
dp	162	mm
Ig	6,67E+08	mm^4
z	100	mm
Wtop	6,67E+06	mm^3
Wbot	6,67E+06	mm^3

Y1860S7 prestressing

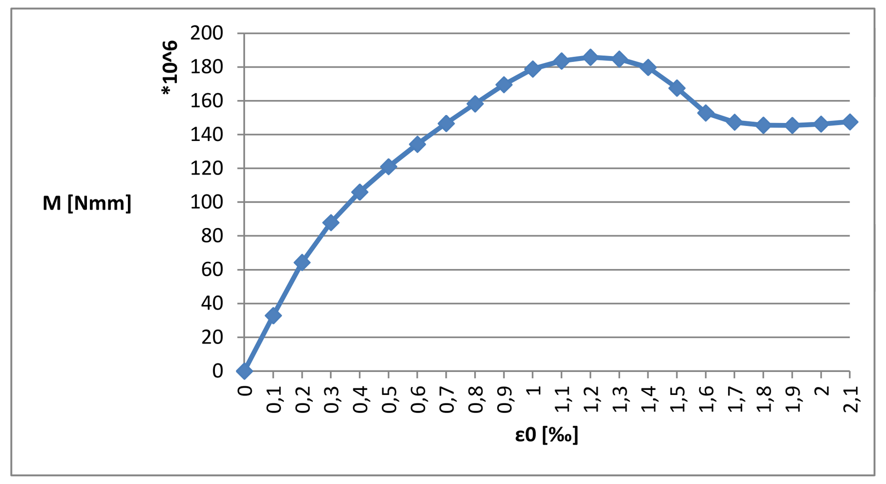
Østrand	16	mm
Astrand	150	mm^2
number of strands	5	
Ap	750	mm^2
strand spacing	215	mm
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,004	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

When εb = εctmax: ε0 =	0,00010	
When Δεp = εctmax: ε0 =	0,00015	
When εp = εp,y: ε0 =	0,00097	
When εb = εt,p: ε0 =	0,00107	
When εb = εt,u: ε0 =	0,00151	
When εp = εud: ε0 =	0,00216	
When εp = εud & ε0 = εcmax: Ap =	1159	mm^2
Mu	1,86E+08	Nmm

Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	Δεp [-]	εp [-]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΔNP [N]	ΣH = 0	y [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	101	99	0	0	6,06E-05	5,43E-03	9,82E-05	9,91E-07	2,52E+05	2,43E+05	0,00E+00	0,00E+00	8,86E+03	0	0	0	3,29E+07
0,0002	91	45	64	0	1,56E-04	5,52E-03	2,40E-04	2,20E-06	4,55E+05	1,14E+05	3,18E+05	0,00E+00	2,29E+04	0	0	0	6,43E+07
0,0003	79	26	95	0	3,19E-04	5,68E-03	4,64E-04	3,82E-06	5,89E+05	6,54E+04	4,77E+05	0,00E+00	4,67E+04	0	0	0	8,79E+07
0,0004	69	17	114	0	5,39E-04	5,90E-03	7,59E-04	5,79E-06	6,90E+05	4,32E+04	5,68E+05	0,00E+00	7,88E+04	0	0	0	1,06E+08
0,0005	62	12	125	0	8,05E-04	6,17E-03	1,11E-03	8,05E-06	7,76E+05	3,10E+04	6,27E+05	0,00E+00	1,18E+05	0	0	0	1,21E+08
0,0006	57	9	134	0	1,11E-03	6,47E-03	1,51E-03	1,05E-05	8,54E+05	2,37E+04	6,68E+05	0,00E+00	1,62E+05	0	0	0	1,34E+08
0,0007	53	8	139	0	1,44E-03	6,81E-03	1,94E-03	1,32E-05	9,27E+05	1,89E+04	6,97E+05	0,00E+00	2,11E+05	0	0	0	1,47E+08
0,0008	50	6	144	0	1,80E-03	7,16E-03	2,41E-03	1,60E-05	9,98E+05	1,56E+04	7,19E+05	0,00E+00	2,63E+05	0	0	0	1,58E+08
0,0009	47	5	147	0	2,17E-03	7,54E-03	2,89E-03	1,90E-05	1,07E+06	1,32E+04	7,36E+05	0,00E+00	3,18E+05	0	0	0	1,70E+08
0,001	45	4	151	0	2,61E-03	7,98E-03	3,46E-03	2,23E-05	1,12E+06	1,12E+04	7,53E+05	0,00E+00	3,57E+05	0	0	0	1,79E+08
0,0011	42	4	147	7	3,19E-03	8,55E-03	4,20E-03	2,65E-05	1,14E+06	9,44E+03	7,36E+05	3,64E+04	3,60E+05	0	4	0	1,84E+08
0,0012	38	3	124	34	3,88E-03	9,24E-03	5,07E-03	3,13E-05	1,15E+06	7,98E+03	6,22E+05	1,55E+05	3,63E+05	0	16	0	1,86E+08
0,0013	35	3	105	58	4,73E-03	1,01E-02	6,15E-03	3,72E-05	1,13E+06	6,71E+03	5,24E+05	2,37E+05	3,67E+05	0	27	0	1,85E+08
0,0014	31	2	87	80	5,87E-03	1,12E-02	7,57E-03	4,48E-05	1,09E+06	5,57E+03	4,35E+05	2,80E+05	3,73E+05	0	34	0	1,80E+08
0,0015	27	2	69	103	7,65E-03	1,30E-02	9,80E-03	5,65E-05	9,96E+05	4,43E+03	3,45E+05	2,65E+05	3,81E+05	0	35	0	1,68E+08
0,0016	22	1	53	81	1,04E-02	1,58E-02	1,33E-02	7,43E-05	8,62E+05	3,37E+03	2,63E+05	2,02E+05	3,94E+05	0	0	0	1,53E+08
0,0017	18	1	42	65	1,32E-02	1,86E-02	1,67E-02	9,21E-05	7,84E+05	2,71E+03	2,12E+05	1,63E+05	4,07E+05	0	0	0	1,47E+08
0,0018	16	1	35	54	1,60E-02	2,14E-02	2,02E-02	1,10E-04	7,36E+05	2,27E+03	1,77E+05	1,36E+05	4,20E+05	0	0	0	1,46E+08
0,0019	15	1	30	47	1,89E-02	2,42E-02	2,37E-02	1,28E-04	7,04E+05	1,95E+03	1,52E+05	1,17E+05	4,33E+05	0	0	0	1,45E+08
0,002	14	1	27	41	2,17E-02	2,70E-02	2,72E-02	1,46E-04	6,84E+05	1,71E+03	1,33E+05	1,03E+05	4,46E+05	0	0	0	1,46E+08
0,0021	13	1	24	37	2,45E-02	2,99E-02	3,08E-02	1,64E-04	6,71E+05	1,52E+03	1,19E+05	9,13E+04	4,60E+05	0	0	0	1,48E+08
0,0022	12	1	21	33	2,74E-02	3,27E-02	3,43E-02	1,82E-04	6,63E+05	1,37E+03	1,07E+05	8,22E+04	4,73E+05	0	0	0	1,49E+08
0,0023	11	0	19	30	3,02E-02	3,56E-02	3,78E-02	2,01E-04	6,59E+05	1,25E+03	9,72E+04	7,48E+04	4,86E+05	0	0	0	1,51E+08
0,0024	11	0	18	27	3,30E-02	3,84E-02	4,14E-02	2,19E-04	6,58E+05	1,14E+03	8,91E+04	6,86E+04	4,99E+05	0	0	0	1,53E+08
0,0025	11	0	16	25	3,59E-02	4,13E-02	4,49E-02	2,37E-04	6,59E+05	1,05E+03	8,23E+04	6,33E+04	5,13E+05	0	0	0	1,55E+08
0,0026	10	0	15	24	3,87E-02	4,41E-02	4,84E-02	2,55E-04	6,62E+05	9,80E+02	7,64E+04	5,88E+04	5,26E+05	0	0	3	1,57E+08
0,0027	10	0	14	22	4,15E-02	4,68E-02	5,18E-02	2,73E-04	6,66E+05	9,17E+02	7,15E+04	5,50E+04	5,39E+05	0	0	3	1,59E+08
0,0028	10	0	13	21	4,41E-02	4,95E-02	5,51E-02	2,90E-04	6,71E+05	8,63E+02	6,73E+04	5,18E+04	5,51E+05	0	0	3	1,61E+08
0,0029	9	0	13	20	4,67E-02	5,21E-02	5,83E-02	3,06E-04	6,77E+05	8,16E+02	6,37E+04	4,90E+04	5,63E+05	0	0	3	1,63E+08
0,003	9	0	12	19	4,92E-02	5,46E-02	6,14E-02	3,22E-04	6,83E+05	7,76E+02	6,05E+04	4,66E+04	5,75E+05	0	0	3	1,64E+08
0,0031	9	0	12	18	5,16E-02	5,70E-02	6,45E-02	3,38E-04	6,89E+05	7,40E+02	5,77E+04	4,44E+04	5,86E+05	0	0	3	1,66E+08
0,0032	9	0	11	17	5,40E-02	5,93E-02	6,74E-02	3,53E-04	6,95E+05	7,08E+02	5,53E+04	4,25E+04	5,97E+05	0	0	3	1,68E+08
0,0033	9	0	11	16	5,63E-02	6,16E-02	7,02E-02	3,68E-04	7,02E+05	6,80E+02	5,30E+04	4,08E+04	6,08E+05	0	0	3	1,69E+08
0,0034	9	0	10	16	5,85E-02	6,39E-02	7,30E-02	3,82E-04	7,09E+05	6,54E+02	5,10E+04	3,93E+04	6,18E+05	0	0	3	1,71E+08
0,0035	9	0	10	15	6,07E-02	6,61E-02	7,57E-02	3,96E-04	7,16E+05	6,31E+02	4,92E+04	3,79E+04	6,28E+05	0	0	3	1,73E+08
0,0036	9	0	10	15	6,28E-02	6,82E-02	7,84E-02	4,10E-04	7,23E+05	6,10E+02	4,76E+04	3,66E+04	6,38E+05	0	0	3	1,74E+08
0,0037	9	0	9	14	6,49E-02	7,03E-02	8,10E-02	4,24E-04	7,30E+05	5,90E+02	4,60E+04	3,54E+04	6,48E+05	0	0	3	1,76E+08
0,0038	9	0	9	14	6,70E-02	7,23E-02	8,36E-02	4,37E-04	7,37E+05	5,72E+02	4,46E+04	3,43E+04	6,58E+05	0	0	3	1,77E+08
0,0039	9	0	9	13	6,90E-02	7,43E-02	8,61E-02	4,50E-04	7,44E+05	5,56E+02	4,34E+04	3,34E+04	6,67E+05	0	0	3	1,79E+08
0,0040	9	0	8	13	7,09E-02	7,63E-02	8,85E-02	4,63E-04	7,51E+05	5,41E+02	4,22E+04	3,24E+04	6,76E+05	0	0	3	1,80E+08

ε0 [‰]	M [Nmm]
0	0
0,1	3,29E+07
0,2	6,43E+07
0,3	8,79E+07
0,4	1,06E+08
0,5	1,21E+08
0,6	1,34E+08
0,7	1,47E+08
0,8	1,58E+08
0,9	1,70E+08
1	1,79E+08
1,1	1,84E+08
1,2	1,86E+08
1,3	1,85E+08
1,4	1,80E+08
1,5	1,68E+08
1,6	1,53E+08
1,7	1,47E+08
1,8	1,46E+08
1,9	1,45E+08
2	1,46E+08
2,1	1,48E+08



Crack Width (SLS)

Requirement: beam remains uncracked in SLS

Span

t = 0: check top fibre	M ≥	-2,33E+07	Nmm
t = 0: check bottom fibre	M ≤	1,53E+08	Nmm
t = ∞: check top fibre	M ≥	-3,08E+07	Nmm
t = ∞: check bottom fibre	M ≤	1,28E+08	Nmm

Support

t = 0: check top fibre	M ≤	1,53E+08	Nmm
t = 0: check bottom fibre	M ≥	-2,33E+07	Nmm
t = ∞: check top fibre	M ≤	1,28E+08	Nmm
t = ∞: check bottom fibre	M ≥	-3,08E+07	Nmm

19 Bending moment and shear force at the support due to the UDL

$restart;$

4th order differential equation:

$DV1 := EI\cdot diff(w1(x), x\$4) = \mathrm{q1} : \mathrm{DV2} := EI\cdot diff(w2(x), x\$4) = q2 :$

The general solution:

$solution := dsolve(\{DV1, DV2\}, \{w1(x), w2(x)\}) :$

$assign(solution);$

$w1 := w1(x) : w2 := w2(x) :$

Mechanical relations:

$phi1 := diff(w1, x) : kappa1 := diff(phi1, x) : M1 := EI\cdot kappa1 : V1 := diff(M1, x) :$

$phi2 := diff(w2, x) : kappa2 := diff(phi2, x) : M2 := EI\cdot kappa2 : V2 := diff(M2, x) :$

Boundary conditions at x=0:

$x := 0 : eq1 := w1 = 0 : eq2 := phi1 = 0 :$

Interface conditions at x=a:

$\mathrm{x} := \mathrm{a} : \mathrm{eq3} := \mathrm{w1} = \mathrm{w2} : \mathrm{eq4} := \mathrm{phi1} = \mathrm{phi2} : \mathrm{eq5} := \mathrm{M1} = \mathrm{M2} : \mathrm{eq6} := \mathrm{V1} = \mathrm{V2} :$

Boundary conditions at x=l:

$\mathrm{x} := \mathrm{l} : \mathrm{eq7} := \mathrm{w2} = 0 : \mathrm{eq8} := \mathrm{phi2} = 0 :$

Solving the unknowns:

$\mathrm{solution} := \mathrm{solve}(\{\mathrm{eq1}, \mathrm{eq2}, \mathrm{eq3}, \mathrm{eq4}, \mathrm{eq5}, \mathrm{eq6}, \mathrm{eq7}, \mathrm{eq8}\}, \{\_C1, \_C2, \_C3, \_C4, \_C5, \_C6, \_C7, \_C8\}) :$

$assign(solution);$

$x :='x':$

UDL of slow lane applied over full deck length:

$\mathrm{a} := 0.41 : \mathrm{l} := 7.5 : \mathrm{x} := 0 : \mathrm{q1} := 10.35 : \mathrm{q2} := 10.35 :$

The bending moment:

$M1;$

$$48.5156250100 \quad (1)$$

The shear force:

$V1;$

$$-38.8125000000 \quad (2)$$

Subtract the part where the slow lane is not present:

$\mathrm{a} := 0.41 : \mathrm{l} := 7.5 : \mathrm{x} := 0 : \mathrm{q1} := 6.85 : \mathrm{q2} := 0 :$

The bending moment:

$M1;$

$$0.5346375562 \quad (3)$$

The shear force:

$V1;$

$$-2.8003363650 \quad (4)$$

Subtract the part where the slow lane is not present:

$\mathrm{a} := 3.41 : \mathrm{l} := 7.5 : \mathrm{x} := 0 : \mathrm{q1} := 0 : \mathrm{q2} := 6.85 :$

The bending moment:

$M1;$

$$12.3102098900 \quad (5)$$

The shear force:

$V1;$

$$-6.0599837500 \quad (6)$$

20 Bending moment and shear force at the haunch’s end due to the UDL

restart;
4th order differential equation:
$DV1 := EI \cdot diff(w1(x), x\$4) = q1 : DV2 := EI \cdot diff(w2(x), x\$4) = q2 :$
The general solution:
$solution := dsolve(\{DV1, DV2\}, \{w1(x), w2(x)\}) :$
$assign(solution);$
$w1 := w1(x) : w2 := w2(x) :$

Mechanical relations:
$phi1 := diff(w1, x) : kappa1 := diff(phi1, x) : M1 := EI \cdot kappa1 : V1 := diff(M1, x) :$
$phi2 := diff(w2, x) : kappa2 := diff(phi2, x) : M2 := EI \cdot kappa2 : V2 := diff(M2, x) :$

Boundary conditions at x=0:
$x := 0 : eq1 := w1 = 0 : eq2 := phi1 = 0 :$

Interface conditions at x=a:
$x := a : eq3 := w1 = w2 : eq4 := phi1 = phi2 : eq5 := M1 = M2 : eq6 := V1 = V2 :$

Boundary conditions at x=l:
$x := l : eq7 := w2 = 0 : eq8 := phi2 = 0 :$

Solving the unknowns:
$solution := solve(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8\}, \{\_C1, \_C2, \_C3, \_C4, \_C5, \_C6, \_C7, \_C8\}) :$
$assign(solution);$
$x :='x':$

UDL of slow lane applied over full deck length:
$a := 0.91 : l := 8.5 : x := 0.2 \cdot l : q1 := 10.35 : q2 := 10.35 :$

The bending moment:
M2;

$$2.4926250100 \quad (1)$$

The shear force:
V2;

$$-26.3925000000 \quad (2)$$

Subtract the part where the slow lane is not present:
$a := 0.91 : l := 8.5 : x := 0.2 \cdot l : q1 := 6.85 : q2 := 0 :$

The bending moment:
M2;

$$-0.2736494436 \quad (3)$$

The shear force:
V2;

$$0.0676213785 \quad (4)$$

Subtract the part where the slow lane is not present:
$a := 3.91 : l := 8.5 : x := 0.2 \cdot l : q1 := 0 : q2 := 6.85 :$

The bending moment:
M1;

$$4.0783838600 \quad (5)$$

The shear force:
V1;

$$-6.6928892150 \quad (6)$$

21 Bending moment and shear force at mid span due to the UDL

$restart;$

4th order differential equation:

$DV1 := EI \cdot diff(w1(x), x\$4) = q1 : DV2 := EI \cdot diff(w2(x), x\$4) = q2 :$

The general solution:

$solution := dsolve(\{DV1, DV2\}, \{w1(x), w2(x)\}) :$

$assign(solution);$

$w1 := w1(x) : w2 := w2(x) :$

Mechanical relations:

$phi1 := diff(w1, x) : kappa1 := diff(phi1, x) : M1 := EI \cdot kappa1 : V1 := diff(M1, x) :$

$phi2 := diff(w2, x) : kappa2 := diff(phi2, x) : M2 := EI \cdot kappa2 : V2 := diff(M2, x) :$

Boundary conditions at x=0:

$x := 0 : eq1 := w1 = 0 : eq2 := phi1 = 0 :$

Interface conditions at x=a:

$x := a : eq3 := w1 = w2 : eq4 := phi1 = phi2 : eq5 := M1 = M2 : eq6 := V1 = V2 :$

Boundary conditions at x=l:

$x := l : eq7 := w2 = 0 : eq8 := phi2 = 0 :$

Solving the unknowns:

$solution := solve(\{eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8\}, \{\_C1, \_C2, \_C3, \_C4, \_C5, \_C6, \_C7, \_C8\}) :$

$assign(solution);$

$x := 'x':$

UDL of slow lane applied over full deck length:

$a := 4.41 : l := 9.5 : x := 0.5 \cdot l : q1 := 10.35 : q2 := 10.35 :$

The bending moment:

$M2;$

$$-38.9203125000 \quad (1)$$

The shear force:

$V2;$

$$0.0000000000 \quad (2)$$

Subtract the part where the slow lane is not present:

$a := 4.41 : l := 9.5 : x := 0.5 \cdot l : q1 := 6.85 : q2 := 0 :$

The bending moment:

$M2;$

$$-10.3069812100 \quad (3)$$

The shear force:

$V2;$

$$4.9987431310 \quad (4)$$

Subtract the part where the slow lane is not present:

$a := 7.41 : l := 9.5 : x := 0.5 \cdot l : q1 := 0 : q2 := 6.85 :$

The bending moment:

$M1;$

$$-1.0971211080 \quad (5)$$

The shear force:

$V1;$

$$-0.6166975420 \quad (6)$$

22 Cross-sectional capacity of the floor at the support

Floor support

Moment Capacity (ULS)

b	1000	mm
D	150	mm
Ac	1,50E+05	mm^2
c	30	mm
dp	112	mm
Ig	2,81E+08	mm^4
z	75	mm
Wtop	3,75E+06	mm^3
Wbot	3,75E+06	mm^3

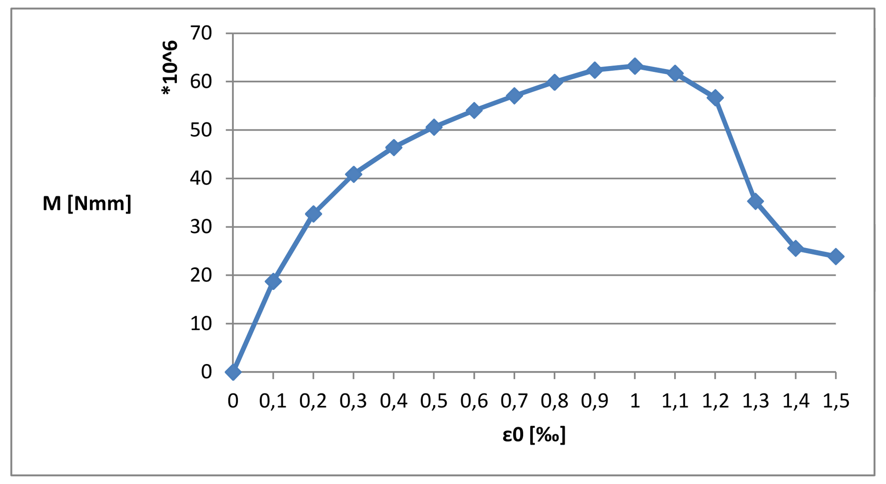
Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	1	
Ap	150	mm^2
strand spacing	#DEEL/0!	mm
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,01	
εt,p	0,004	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	0,0078	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σpm∞	1046	N/mm^2
εp∞	0,0054	

When εb = εctmax: ε0 =	0,00010	
When Δεp = εctmax: ε0 =	0,00017	
When εp = εp,y: ε0 =	0,00089	
When εb = εt,p: ε0 =	0,00094	
When εb = εt,u: ε0 =	0,00127	
When εp = εud: ε0 =	0,00152	
When εp = εud & ε0 = εcmax: Ap =	801	mm^2
Mu	6,32E+07	Nmm

ε0 [-]	dn [mm]	X1 [mm]	X2 [mm]	X3 [mm]	Δεp [-]	εp [-]	εb [-]	κ [1/mm]	C [N]	T1 [N]	T2 [N]	T3 [N]	ΔNP [N]	ΣH = 0	γ [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	75	75	0	0	4,90E-05	5,41E-03	9,96E-05	1,33E-06	1,88E+05	1,86E+05	0,00E+00	0,00E+00	1,43E+03	0	0	0	1,87E+07
0,0002	67	34	49	0	1,34E-04	5,50E-03	2,48E-04	2,98E-06	3,35E+05	8,38E+04	2,47E+05	0,00E+00	3,93E+03	0	0	0	3,27E+07
0,0003	57	19	74	0	2,91E-04	5,66E-03	4,91E-04	5,27E-06	4,27E+05	4,74E+04	3,71E+05	0,00E+00	8,50E+03	0	0	0	4,09E+07
0,0004	49	12	89	0	5,15E-04	5,88E-03	8,25E-04	8,17E-06	4,90E+05	3,06E+04	4,44E+05	0,00E+00	1,51E+04	0	0	0	4,64E+07
0,0005	43	9	98	0	8,03E-04	6,17E-03	1,25E-03	1,16E-05	5,37E+05	2,15E+04	4,92E+05	0,00E+00	2,35E+04	0	0	0	5,06E+07
0,0006	38	6	105	0	1,15E-03	6,52E-03	1,74E-03	1,56E-05	5,76E+05	1,60E+04	5,26E+05	0,00E+00	3,37E+04	0	0	0	5,41E+07
0,0007	35	5	110	0	1,55E-03	6,92E-03	2,32E-03	2,01E-05	6,09E+05	1,24E+04	5,51E+05	0,00E+00	4,54E+04	0	0	0	5,71E+07
0,0008	32	4	114	0	2,00E-03	7,37E-03	2,96E-03	2,50E-05	6,39E+05	9,98E+03	5,70E+05	0,00E+00	5,86E+04	0	0	0	5,99E+07
0,0009	30	3	117	0	2,51E-03	7,87E-03	3,67E-03	3,04E-05	6,65E+05	8,21E+03	5,86E+05	0,00E+00	7,14E+04	0	0	0	6,24E+07
0,001	27	3	106	15	3,14E-03	8,50E-03	4,54E-03	3,70E-05	6,77E+05	6,77E+03	5,28E+05	7,01E+04	7,20E+04	0	7	0	6,32E+07
0,0011	24	2	86	37	3,96E-03	9,33E-03	5,68E-03	4,52E-05	6,70E+05	5,53E+03	4,32E+05	1,60E+05	7,28E+04	0	18	0	6,17E+07
0,0012	21	2	69	59	5,17E-03	1,05E-02	7,34E-03	5,69E-05	6,33E+05	4,39E+03	3,43E+05	2,12E+05	7,39E+04	0	26	0	5,67E+07
0,0013	14	1	41	62	9,48E-03	1,48E-02	1,31E-02	9,63E-05	4,39E+05	2,60E+03	2,03E+05	1,56E+05	7,79E+04	0	0	0	3,53E+07
0,0014	8	1	23	36	1,73E-02	2,27E-02	2,37E-02	1,67E-04	2,93E+05	1,50E+03	1,17E+05	8,97E+04	8,52E+04	0	0	0	2,56E+07
0,0015	6	0	17	26	2,47E-02	3,00E-02	3,35E-02	2,34E-04	2,41E+05	1,07E+03	8,35E+04	6,42E+04	9,21E+04	0	0	0	2,39E+07
0,0016	5	0	13	20	3,16E-02	3,70E-02	4,29E-02	2,97E-04	2,16E+05	8,42E+02	6,57E+04	5,05E+04	9,86E+04	0	0	0	2,37E+07
0,0017	5	0	11	17	3,84E-02	4,37E-02	5,20E-02	3,58E-04	2,02E+05	6,99E+02	5,45E+04	4,19E+04	1,05E+05	0	0	0	2,39E+07
0,0018	4	0	9	14	4,49E-02	5,03E-02	6,07E-02	4,17E-04	1,94E+05	6,00E+02	4,68E+04	3,60E+04	1,11E+05	0	0	0	2,43E+07
0,0019	4	0	8	13	5,13E-02	5,66E-02	6,93E-02	4,75E-04	1,90E+05	5,27E+02	4,11E+04	3,16E+04	1,17E+05	0	0	0	2,48E+07
0,002	4	0	7	11	5,75E-02	6,29E-02	7,77E-02	5,32E-04	1,88E+05	4,70E+02	3,67E+04	2,82E+04	1,23E+05	0	0	0	2,54E+07
0,0021	4	0	7	10	6,37E-02	6,91E-02	8,60E-02	5,88E-04	1,88E+05	4,25E+02	3,32E+04	2,55E+04	1,28E+05	0	0	0	2,60E+07
0,0022	3	0	6	9	6,98E-02	7,52E-02	9,42E-02	6,43E-04	1,88E+05	3,89E+02	3,03E+04	2,33E+04	1,34E+05	0	0	0	2,65E+07
0,0023	3	0	6	9	7,58E-02	8,12E-02	1,02E-01	6,98E-04	1,90E+05	3,58E+02	2,80E+04	2,15E+04	1,40E+05	0	0	0	2,71E+07
0,0024	3	0	5	8	8,18E-02	8,72E-02	1,10E-01	7,52E-04	1,92E+05	3,33E+02	2,59E+04	2,00E+04	1,45E+05	0	0	0	2,77E+07
0,0025	3	0	5	7	8,77E-02	9,31E-02	1,18E-01	8,05E-04	1,94E+05	3,10E+02	2,42E+04	1,86E+04	1,51E+05	0	0	0	2,83E+07
0,0026	3	0	5	7	9,35E-02	9,89E-02	1,26E-01	8,58E-04	1,97E+05	2,91E+02	2,27E+04	1,75E+04	1,56E+05	0	0	1	2,89E+07
0,0027	3	0	4	7	9,92E-02	1,05E-01	1,34E-01	9,10E-04	2,00E+05	2,75E+02	2,14E+04	1,65E+04	1,62E+05	0	0	1	2,95E+07
0,0028	3	0	4	6	1,05E-01	1,10E-01	1,41E-01	9,59E-04	2,03E+05	2,61E+02	2,03E+04	1,56E+04	1,67E+05	0	0	1	3,00E+07
0,0029	3	0	4	6	1,10E-01	1,15E-01	1,48E-01	1,01E-03	2,06E+05	2,49E+02	1,94E+04	1,49E+04	1,71E+05	0	0	1	3,05E+07
0,003	3	0	4	6	1,15E-01	1,20E-01	1,55E-01	1,05E-03	2,09E+05	2,38E+02	1,85E+04	1,43E+04	1,76E+05	0	0	1	3,11E+07
0,0031	3	0	4	5	1,20E-01	1,25E-01	1,61E-01	1,10E-03	2,12E+05	2,28E+02	1,78E+04	1,37E+04	1,81E+05	0	0	1	3,15E+07
0,0032	3	0	3	5	1,24E-01	1,30E-01	1,68E-01	1,14E-03	2,16E+05	2,20E+02	1,71E+04	1,32E+04	1,85E+05	0	0	1	3,20E+07
0,0033	3	0	3	5	1,29E-01	1,34E-01	1,74E-01	1,18E-03	2,19E+05	2,12E+02	1,65E+04	1,27E+04	1,89E+05	0	0	1	3,25E+07
0,0034	3	0	3	5	1,33E-01	1,39E-01	1,80E-01	1,22E-03	2,22E+05	2,05E+02	1,60E+04	1,23E+04	1,93E+05	0	0	1	3,29E+07
0,0035	3	0	3	5	1,38E-01	1,43E-01	1,86E-01	1,26E-03	2,25E+05	1,98E+02	1,55E+04	1,19E+04	1,97E+05	0	0	1	3,34E+07
0,0036	3	0	3	5	1,42E-01	1,47E-01	1,91E-01	1,30E-03	2,28E+05	1,92E+02	1,50E+04	1,15E+04	2,01E+05	0	0	1	3,38E+07
0,0037	3	0	3	4	1,46E-01	1,51E-01	1,97E-01	1,34E-03	2,31E+05	1,87E+02	1,46E+04	1,12E+04	2,05E+05	0	0	1	3,42E+07
0,0038	3	0	3	4	1,50E-01	1,55E-01	2,02E-01	1,37E-03	2,34E+05	1,82E+02	1,42E+04	1,09E+04	2,09E+05	0	0	1	3,47E+07
0,0039	3	0	3	4	1,54E-01	1,59E-01	2,08E-01	1,41E-03	2,37E+05	1,77E+02	1,38E+04	1,06E+04	2,13E+05	0	0	1	3,51E+07
0,0040	3	0	3	4	1,58E-01	1,63E-01	2,13E-01	1,45E-03	2,40E+05	1,73E+02	1,35E+04	1,04E+04	2,16E+05	0	0	1	3,54E+07

ε0 [‰]	M [Nmm]
0	0
0,1	1,87E+07
0,2	3,27E+07
0,3	4,09E+07
0,4	4,64E+07
0,5	5,06E+07
0,6	5,41E+07
0,7	5,71E+07
0,8	5,99E+07
0,9	6,24E+07
1	6,32E+07
1,1	6,17E+07
1,2	5,67E+07
1,3	3,53E+07
1,4	2,56E+07
1,5	2,39E+07



Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,10E+05	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	6	mm
z	110	mm
cot βu	1,00	
Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

VRb	2,15E+05	N
Vf	6,77E+05	N
Vs	0,00E+00	N
Vu	8,92E+05	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

Crack Width (SLS)

Requirement: beam remains uncracked in SLS

Span

t = 0: check top fibre	M ≥	-2,75E+07	Nmm
t = 0: check bottom fibre	M ≤	4,30E+07	Nmm
t = ∞: check top fibre	M ≥	-2,81E+07	Nmm
t = ∞: check bottom fibre	M ≤	3,97E+07	Nmm

Support

t = 0: check top fibre	M ≤	4,30E+07	Nmm
t = 0: check bottom fibre	M ≥	-2,75E+07	Nmm
t = ∞: check top fibre	M ≤	3,97E+07	Nmm
t = ∞: check bottom fibre	M ≥	-2,81E+07	Nmm

23 Launch phase

23.1 Launch phase front spans loads

Launch phase loads

Dimensions

ltot	550000	mm
number of spans	10	
lmid	57895	mm
lend	43421	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
e0	630	mm
eb	630	mm
dp	1734	mm
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	16	
Ap	117600	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	193	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Steel nose

l	15000	mm
h	3000	mm
b	1200	mm
tw	60	mm
tf	120	mm
γ	7,85E-05	N/mm^3
A	4,54E+05	mm^2

Partial factors

Launch phase: CC1

6.10a

γG	1,2
γQ	1,35

6.10b

γG	1,1
γQ	1,35

Bending moments in launch phase

Self-weight

MGk,sw,sup	-4,87E+10	Nmm
MGk,sw,span	2,43E+10	Nmm
MGk,sw,cant	-1,87E+11	Nmm

Nose

MGk,nose,cant	4,01E+09	Nmm

SLS

MEd,sls,sup	-4,87E+10	Nmm
MEd,sls,span	2,43E+10	Nmm
MEd,sls,cant	-1,87E+11	Nmm
σc,sls,sup	5,56	N/mm^2
σc,sls,span	5,27	N/mm^2
σc,sls,cant	21,38	N/mm^2

ULS

MEd,uls,sup	-5,84E+10	Nmm
MEd,uls,span	2,92E+10	Nmm
MEd,uls,cant	-2,25E+11	Nmm

Time-dependent losses

Creep

σcc	-24,48	N/mm^2
εcc	9,79E-05	
Δσpc	19	N/mm^2

Relaxation

Assume: the launch phase lasts half a year

t	4380	hours
Δσpr	28	N/mm^2
Δσp,c+r	47	N/mm^2
σp∞	1348	N/mm^2

Central prestressing

σc	-2,28	N/mm^2
Check: σc ≤ 0	ok!	
n0	8	
nb	8	
Check: n0*e0 = nb*eb	ok!	

Maximum compressive stress

σc	-64	N/mm^2
Check: σc ≤ 0.6fck	ok!	

compressive stress after tensioning

σc	24	N/mm^2

compressive strength after day

1	30	N/mm^2
8	60	N/mm^2

Shear forces in the launch phase

Self-weight

VGk,sw,sup	5,04E+06	N
VGk,sw,cant	8,01E+06	N

Nose

VGk,nose,cant	5,34E+05	N

SLS

VEd,sls,sup	5,04E+06	N
VEd,sls,cant	8,01E+06	N

ULS

VEd,uls,sup	6,05E+06	N
VEd,uls,cant	9,61E+06	N

Nose optimization

lnose 0 mm

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	0	0
5	4,38E+10	22418	22418
10,17	8,90E+10	31967	31967
15	1,31E+11	38829	38829
20	1,75E+11	44836	44836

lnose 10000 mm

Anose	2,02E+05	mm
Mnose	7,91E+08	Nmm
Vnose	1,58E+05	N

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	#GETAL!	#GETAL!
5	4,38E+10	21325	31325
10,17	8,90E+10	30929	40929
15	1,31E+11	37815	47815
20	1,75E+11	43836	53836

lnose 15000 mm

Anose	4,54E+05	mm
Mnose	4,01E+09	Nmm
Vnose	5,34E+05	N

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	#GETAL!	#GETAL!
5	4,38E+10	18521	33521
10,17	8,90E+10	28324	43324
15	1,31E+11	35290	50290
20	1,75E+11	41361	56361

lnose 20000 mm

Anose	8,06E+05	mm
Mnose	1,27E+10	Nmm
Vnose	1,27E+06	N

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	#GETAL!	#GETAL!
5	4,38E+10	12983	32983
10,17	8,90E+10	23218	43218
15	1,31E+11	30353	50353
20	1,75E+11	36526	56526

lnose [mm]	Anose [mm^2]	Vnose [N]	Mnose [Nmm]	Qnose [kg]	Cost nose [€]	lcant [mm]	Mcant [Nmm]	Ap,req [mm^2]	ΔAp [mm^2]	ΔQp [kg/m]	ΔQp [kg/two spans]	Cost additional central prestressing [€]	Total cost [€]
10000	2,02E+05	1,58E+05	7,91E+08	1,61E+04	€ 48.396	47895	2,08E+11	118208	13868	1,09E+03	1,26E+05	€ 315.132	€ 363.529
11000	2,44E+05	2,11E+05	1,16E+09	2,15E+04	€ 64.416	46895	2,03E+11	115033	10693	8,39E+02	9,72E+04	€ 242.986	€ 307.401
12000	2,90E+05	2,73E+05	1,64E+09	2,79E+04	€ 83.629	45895	1,98E+11	112235	7895	6,20E+02	7,18E+04	€ 179.404	€ 263.033
13000	3,41E+05	3,48E+05	2,26E+09	3,54E+04	€ 106.327	44895	1,93E+11	109832	5493	4,31E+02	4,99E+04	€ 124.815	€ 231.142
14000	3,95E+05	4,34E+05	3,04E+09	4,43E+04	€ 132.800	43895	1,90E+11	107844	3504	2,75E+02	3,18E+04	€ 79.621	€ 212.421
15000	4,54E+05	5,34E+05	4,01E+09	5,44E+04	€ 163.338	42895	1,87E+11	106285	1945	1,53E+02	1,77E+04	€ 44.201	€ 207.538
16000	5,16E+05	6,48E+05	5,19E+09	6,61E+04	€ 198.231	41895	1,85E+11	105172	832	6,53E+01	7,56E+03	€ 18.908	€ 217.140
17000	5,83E+05	7,78E+05	6,61E+09	7,93E+04	€ 237.771	40895	1,84E+11	104519	179	1,41E+01	1,63E+03	€ 4.073	€ 241.844
18000	6,53E+05	9,23E+05	8,31E+09	9,41E+04	€ 282.247	39895	1,84E+11	104340	0	0,00E+00	0,00E+00	€ 0	€ 282.247
19000	7,28E+05	1,09E+06	1,03E+10	1,11E+05	€ 331.950	38895	1,84E+11	104646	307	2,41E+01	2,79E+03	€ 6.970	€ 338.920
20000	8,06E+05	1,27E+06	1,27E+10	1,29E+05	€ 387.171	37895	1,86E+11	105450	1111	8,72E+01	1,01E+04	€ 25.238	€ 412.408

23.2 Launch phase rear spans loads

Launch phase rear spans

Dimensions

ltot	550000	mm
number of spans	10	
lmid	57895	mm
lend	43421	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
e0	630	mm
eb	630	mm
dp	1734	mm
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	4	
Ap	29400	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	2067	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
µ	0,75	

Partial factors

Launch phase: CC1

6.10a

γG	1,2
γQ	1,35

6.10b

γG	1,1
γQ	1,35

Bending moments in launch phase

Self-weight

MGk,sw,sup	-4,87E+10	Nmm
MGk,sw,span	2,43E+10	Nmm

SLS

MEd,sls,sup	-4,87E+10	Nmm
MEd,sls,span	2,43E+10	Nmm
σc,sls,sup	5,56	N/mm^2
σc,sls,span	5,27	N/mm^2

ULS

MEd,uls,sup	-5,84E+10	Nmm
MEd,uls,span	2,92E+10	Nmm

Time-dependent losses

Creep

σcc	-6,12	N/mm^2
εcc	2,45E-05	
Δσpc	5	N/mm^2

Relaxation

Assume: the launch phase lasts half a year

t	4380	hours
Δσpr	28	N/mm^2
Δσp,c+r	33	N/mm^2
σp∞	1362	N/mm^2

Central prestressing

σc	-0,42	N/mm^2
Check: σc ≤ 0	ok!	
n0	2	
nb	2	
Check: n0*e0 = nb*eb	ok!	

Shear forces in the launch phase

Self-weight

VGk,sw,sup	5,04E+06	N

SLS

VEd,sls,sup	5,04E+06	N

ULS

VEd,uls,sup	6,05E+06	N

23.3 Launch phase mid span capacity

Launch phase mid span capacity

Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td = d*	235	mm
tf	150	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	1105	mm
Ic	9,67E+12	mm^4
e0	630	mm
eb	630	mm
dp	1734	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	2	
Ap	14700	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	6200	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1348	N/mm^2
εp∞	6,91E-03	

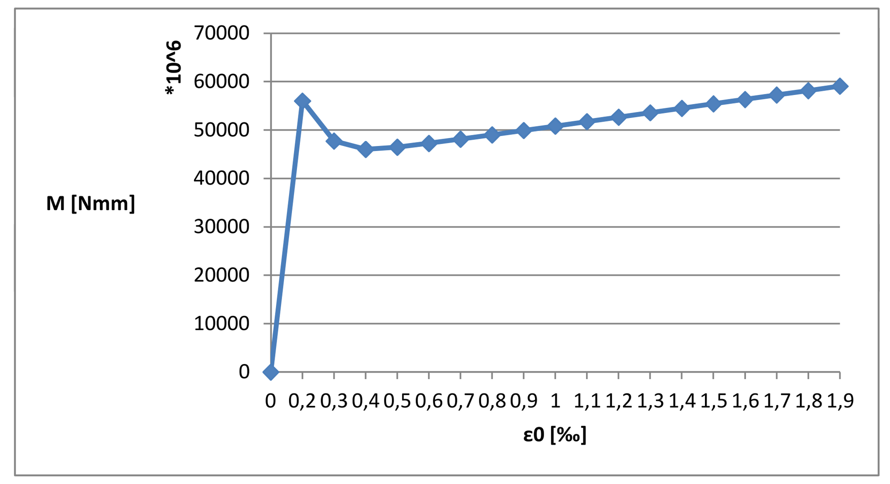
When εb = εctmax: ε0 =	0,000139
When εb' = εctmax: ε0 =	0,000144
When εb = εt,p: ε0 =	0,000201
When εb' = εt,p: ε0 =	0,000204
When εp = εp,y: ε0 =	0,000259
When dn = td: ε0 =	0,000303
When εb = εt,u: ε0 =	0,000291
When εb' = εt,u: ε0 =	0,000296
When εp = εud: ε0 =	0,001917

When dn = td & εp = εp,y: Ap =	663	mm^2
When dn = td & εb = εt,u: Ap =	12087	mm^2
When dn = td & εb' = εt,u: Ap =	13239	mm^2
When εp = εud & ε0 = εcmax: Ap =	78090	mm^2
When dn = td & ε0 = εcmax: Ap =	143344	mm^2

Mu	5,91E+10	Nmm
MEd	2,92E+10	Nmm
Unity check: MEd/Mu ≤ 1	0,49	ok!

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0002	1,46E-04	4,96E-04	1,96E-04	7,11E-03	5,30E-04	877	642	438	1735	0	0	0	2,28E-07	6,80E+07	-3,47E+07	7,67E+05	6,07E+06	6,00E+06	0,00E+00	0,00E+00	5,61E+05	0,00E+00	0	0	0	5,60E+10
0,0003	4,98E-06	0,00E+00	1,87E-03	8,79E-03	0,00E+00	239	4	80	352	2268	0	0	1,25E-06	2,78E+07	-7,31E+03	1,40E+05	1,23E+06	3,97E+06	0,00E+00	0,00E+00	2,65E+06	0,00E+00	0	0	0	4,77E+10
0,0004	1,20E-04	0,00E+00	3,43E-03	1,03E-02	0,00E+00	181	0	45	200	1286	0	54	2,21E-06	2,80E+07	0,00E+00	7,92E+04	6,99E+05	2,25E+06	0,00E+00	2,41E+06	2,79E+06	0,00E+00	0	0	0	4,60E+10
0,0005	2,30E-04	0,00E+00	4,88E-03	1,18E-02	0,00E+00	161	0	32	142	916	0	74	3,10E-06	3,12E+07	0,00E+00	5,64E+04	4,98E+05	1,60E+06	0,00E+00	6,32E+06	2,92E+06	-1,04E-07	0	0	0	4,65E+10
0,0006	3,36E-04	0,00E+00	6,30E-03	1,32E-02	0,00E+00	151	0	25	111	715	0	84	3,98E-06	3,51E+07	0,00E+00	4,40E+04	3,88E+05	1,25E+06	0,00E+00	1,05E+07	3,05E+06	2,02E-06	0	0	0	4,72E+10
0,0007	4,40E-04	0,00E+00	7,71E-03	1,46E-02	0,00E+00	144	0	21	91	587	0	91	4,85E-06	3,92E+07	0,00E+00	3,61E+04	3,19E+05	1,03E+06	0,00E+00	1,48E+07	3,18E+06	-1,56E-07	0	0	0	4,81E+10
0,0008	5,43E-04	0,00E+00	9,10E-03	1,60E-02	0,00E+00	140	0	18	77	498	0	95	5,71E-06	4,34E+07	0,00E+00	3,06E+04	2,71E+05	8,71E+05	0,00E+00	1,91E+07	3,31E+06	3,84E-07	0	0	0	4,90E+10
0,0009	6,46E-04	0,00E+00	1,05E-02	1,74E-02	0,00E+00	137	0	15	67	433	0	98	6,57E-06	4,78E+07	0,00E+00	2,66E+04	2,35E+05	7,57E+05	0,00E+00	2,35E+07	3,43E+06	-4,84E-08	0	0	0	4,99E+10
0,001	7,48E-04	0,00E+00	1,19E-02	1,88E-02	0,00E+00	135	0	13	59	383	0	101	7,43E-06	5,21E+07	0,00E+00	2,35E+04	2,08E+05	6,70E+05	0,00E+00	2,79E+07	3,56E+06	-5,48E-07	0	0	0	5,08E+10
0,0011	8,50E-04	0,00E+00	1,33E-02	2,02E-02	0,00E+00	133	0	12	53	343	0	103	8,29E-06	5,66E+07	0,00E+00	2,11E+04	1,86E+05	6,00E+05	0,00E+00	3,23E+07	3,69E+06	-3,20E-07	0	0	0	5,17E+10
0,0012	9,52E-04	0,00E+00	1,47E-02	2,16E-02	0,00E+00	131	0	11	48	311	0	104	9,15E-06	6,10E+07	0,00E+00	1,91E+04	1,69E+05	5,44E+05	0,00E+00	3,66E+07	3,81E+06	-8,12E-07	0	0	0	5,27E+10
0,0013	1,05E-03	0,00E+00	1,60E-02	2,30E-02	0,00E+00	130	0	10	44	284	0	105	1,00E-05	6,55E+07	0,00E+00	1,75E+04	1,55E+05	4,97E+05	0,00E+00	4,10E+07	3,94E+06	3,28E-07	0	0	0	5,36E+10
0,0014	1,15E-03	0,00E+00	1,74E-02	2,43E-02	0,00E+00	129	0	9	41	262	0	106	1,09E-05	6,99E+07	0,00E+00	1,61E+04	1,42E+05	4,58E+05	0,00E+00	4,54E+07	4,07E+06	6,71E-07	0	0	0	5,45E+10
0,0015	1,26E-03	0,00E+00	1,88E-02	2,57E-02	0,00E+00	128	0	9	38	243	0	107	1,17E-05	7,44E+07	0,00E+00	1,49E+04	1,32E+05	4,25E+05	0,00E+00	4,98E+07	4,19E+06	1,12E-06	0	0	0	5,54E+10
0,0016	1,36E-03	0,00E+00	2,02E-02	2,71E-02	0,00E+00	127	0	8	35	226	0	108	1,26E-05	7,89E+07	0,00E+00	1,39E+04	1,23E+05	3,96E+05	0,00E+00	5,42E+07	4,32E+06	-8,57E-07	0	0	0	5,63E+10
0,0017	1,46E-03	0,00E+00	2,16E-02	2,85E-02	0,00E+00	127	0	7	33	212	0	109	1,34E-05	8,34E+07	0,00E+00	1,30E+04	1,15E+05	3,71E+05	0,00E+00	5,86E+07	4,45E+06	-1,94E-07	0	0	0	5,72E+10
0,0018	1,56E-03	0,00E+00	2,30E-02	2,99E-02	0,00E+00	126	0	7	31	199	0	109	1,43E-05	8,79E+07	0,00E+00	1,23E+04	1,08E+05	3,48E+05	0,00E+00	6,30E+07	4,57E+06	2,16E-07	0	0	0	5,82E+10
0,0019	1,66E-03	0,00E+00	2,44E-02	3,13E-02	0,00E+00	126	0	7	29	188	0	110	1,51E-05	9,24E+07	0,00E+00	1,16E+04	1,02E+05	3,29E+05	0,00E+00	6,74E+07	4,70E+06	-5,74E-07	0	0	0	5,91E+10
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,2	5,60E+10
0,3	4,77E+10
0,4	4,60E+10
0,5	4,65E+10
0,6	4,72E+10
0,7	4,81E+10
0,8	4,90E+10
0,9	4,99E+10
1	5,08E+10
1,1	5,17E+10
1,2	5,27E+10
1,3	5,36E+10
1,4	5,45E+10
1,5	5,54E+10
1,6	5,63E+10
1,7	5,72E+10
1,8	5,82E+10
1,9	5,91E+10



23.4 Launch phase support capacity

Launch phase support capacity

Moment Capacity (ULS)

bf	0	mm
bw	350	mm
bin	7300	mm
b	8000	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	2095	mm
Ic	9,67E+12	mm^4
e0	630	mm
eb	630	mm
dp	2725	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	2	
Ap	14700	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	6200	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1348	N/mm^2
εp∞	6,91E-03	

Shear reinforcement

Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

When εb = εctmax: ε0 =	0,000301
When εb' = εctmax: ε0 =	0,000330
When εb = εt,p: ε0 =	0,000453
When εb' = εt,p: ε0 =	0,000468
When εp = εp,y: ε0 =	0,000529
When dn = td: ε0 =	0,000827
When εb = εt,u: ε0 =	0,000536
When εb' = εt,u: ε0 =	0,000535
When εp = εud: ε0 =	0,001126

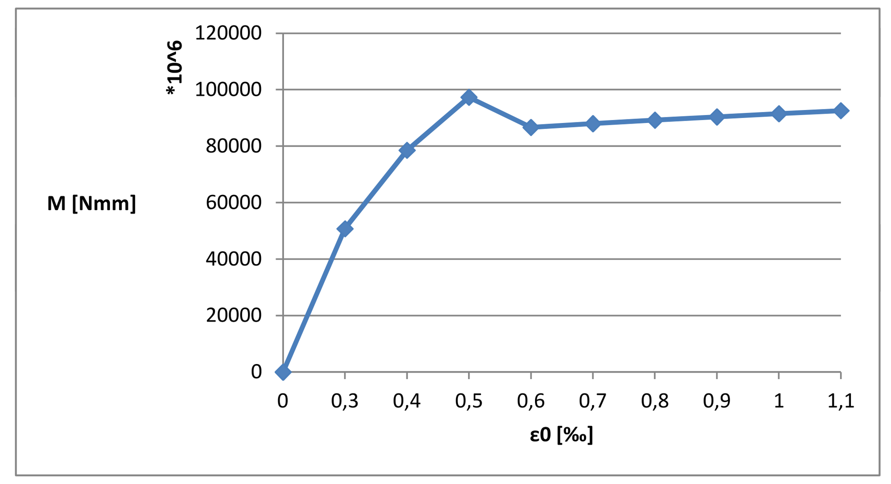
When dn = td & εp = εp,y: Ap =	-10052	mm^2
When dn = td & εb = εt,u: Ap =	-934	mm^2
When dn = td & εb' = εt,u: Ap =	-102	mm^2
When εp = εud & ε0 = εcmax: Ap =	65587	mm^2
When dn = td & ε0 = εcmax: Ap =	42554	mm^2

Mu	9,73E+10	Nmm	MEd	5,84E+10	Nmm	Unity check: MEd/Mu ≤ 1	0,60 ok!

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0003	2,81E-04	6,97E-05	3,98E-05	6,95E-03	9,90E-05	2406	2256	559	0	0	0	0	1,25E-07
0,0004	3,66E-04	2,73E-04	2,19E-04	7,13E-03	3,27E-04	1761	1611	440	763	0	0	0	2,27E-07
0,0005	4,37E-04	7,53E-04	6,52E-04	7,56E-03	8,53E-04	1183	1033	237	1045	501	0	0	4,23E-07
0,0006	2,40E-04	0,00E+00	5,94E-03	1,29E-02	0,00E+00	250	100	42	184	1185	0	0	2,40E-06
0,0007	1,29E-04	0,00E+00	9,68E-03	1,66E-02	0,00E+00	184	34	26	116	746	0	0	3,81E-06
0,0008	2,66E-05	0,00E+00	1,33E-02	2,02E-02	0,00E+00	155	5	19	86	552	0	0	5,16E-06
0,0009	7,20E-05	0,00E+00	1,68E-02	2,37E-02	0,00E+00	139	0	15	68	439	0	11	6,48E-06
0,001	1,69E-04	0,00E+00	2,02E-02	2,71E-02	0,00E+00	128	0	13	57	365	0	22	7,79E-06
0,0011	2,65E-04	0,00E+00	2,37E-02	3,06E-02	0,00E+00	121	0	11	49	313	0	29	9,10E-06
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0003	1,44E+08	-1,16E+08	1,57E+07	-7,10E+06	0,00E+00	0,00E+00	0,00E+00	1,14E+05	0,00E+00	0	0	0	5,08E+10
0,0004	1,41E+08	-1,08E+08	7,70E+05	2,67E+06	9,41E+06	0,00E+00	0,00E+00	6,28E+05	0,00E+00	0	0	0	7,86E+10
0,0005	1,18E+08	-8,23E+07	4,14E+05	3,66E+06	1,69E+06	8,54E+06	0,00E+00	1,87E+06	0,00E+00	247	122	0	9,73E+10
0,0006	3,00E+07	-4,38E+06	7,29E+04	6,44E+05	2,07E+06	0,00E+00	0,00E+00	3,02E+06	3,73E-08	0	0	0	8,67E+10
0,0007	2,57E+07	-7,91E+05	4,59E+04	4,06E+05	1,31E+06	0,00E+00	0,00E+00	3,36E+06	0,00E+00	0	0	0	8,80E+10
0,0008	2,48E+07	-2,50E+04	3,39E+04	3,00E+05	9,65E+05	0,00E+00	0,00E+00	3,69E+06	0,00E+00	0	0	0	8,92E+10
0,0009	2,50E+07	0,00E+00	2,70E+04	2,39E+05	7,68E+05	0,00E+00	1,46E+05	4,01E+06	0,00E+00	0	0	0	9,04E+10
0,001	2,57E+07	0,00E+00	2,25E+04	1,98E+05	6,39E+05	0,00E+00	6,68E+05	4,32E+06	0,00E+00	0	0	0	9,15E+10
0,0011	2,66E+07	0,00E+00	1,92E+04	1,70E+05	5,47E+05	0,00E+00	1,41E+06	4,64E+06	0,00E+00	0	0	0	9,26E+10
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-1,98E+07	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,3	5,08E+10
0,4	7,86E+10
0,5	9,73E+10
0,6	8,67E+10
0,7	8,80E+10
0,8	8,92E+10
0,9	9,04E+10
1	9,15E+10
1,1	9,26E+10



Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,88E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	119	mm
z	2685	mm
cot βu	1,00	

VRb	3,68E+06	N
Vf	1,16E+07	N
Vs	0,00E+00	N
Vu	1,16E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	6,05E+06	N	Unity check: VEd/Vu ≤ 1	0,52 ok!

23.5 Launch phase cantilever capacity

Launch phase cantilever capacity

Moment Capacity (ULS)

bf	0	mm
bw	350	mm
bin	7300	mm
b	8000	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	2095	mm
Ic	9,67E+12	mm^4
e0	630	mm
eb	630	mm
dp	2725	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	6	
Ap	44100	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	800	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1348	N/mm^2
εp∞	6,91E-03	

Shear reinforcement

Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

When εb = εctmax: ε0 =	0,000651
When εb' = εctmax: ε0 =	0,000708
When εb = εt,p: ε0 =	0,000860
When εb' = εt,p: ε0 =	0,000894
When εp = εp,y: ε0 =	0,001024
When dn = td: ε0 =	0,002664
When εb = εt,u: ε0 =	0,001142
When εb' = εt,u: ε0 =	0,001153
When εp = εud: ε0 =	0,001961

When dn = td & εp = εp,y: Ap =	-10052	mm^2
When dn = td & εb = εt,u: Ap =	-934	mm^2
When dn = td & εb' = εt,u: Ap =	-102	mm^2
When εp = εud & ε0 = εcmax: Ap =	65587	mm^2
When dn = td & ε0 = εcmax: Ap =	42554	mm^2

Mu	2,70E+11	Nmm	MEd	2,25E+11	Nmm	Unity check: MEd/Mu ≤ 1	0,83	ok!

Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,86E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	201	mm
z	2658	mm
cot βu	1,00	

VRb	3,65E+06	N
Vf	1,14E+07	N
Vs	0,00E+00	N
Vu	1,14E+07	N

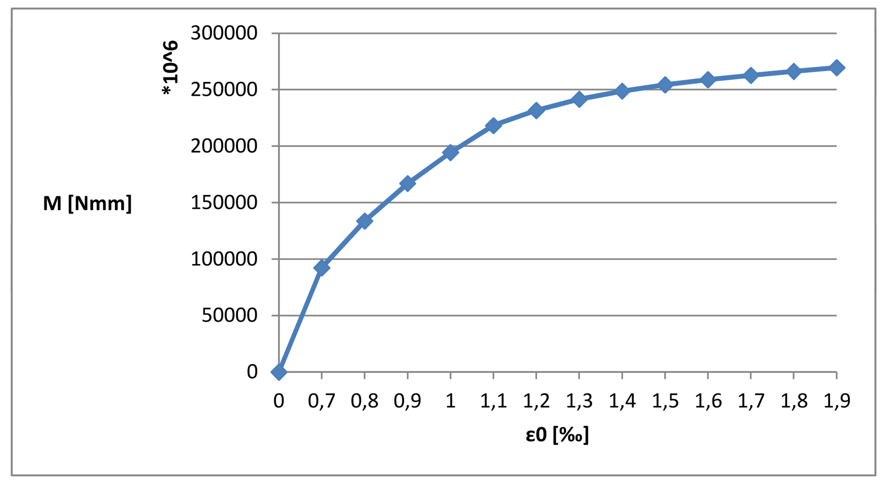
if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	9,61E+06	N	Unity check: VEd/Vu ≤ 1	0,84	ok!

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0007	6,60E-04	8,84E-05	2,46E-05	6,94E-03	1,51E-04	2632	2482	332	44	192	0	0	2,66E-07
0,0008	7,45E-04	2,87E-04	1,99E-04	7,11E-03	3,74E-04	2181	2031	273	511	0	0	0	3,67E-07
0,0009	8,26E-04	5,60E-04	4,42E-04	7,35E-03	6,76E-04	1827	1677	203	897	38	0	0	4,93E-07
0,001	9,01E-04	9,48E-04	7,91E-04	7,70E-03	1,10E-03	1522	1372	152	672	619	0	0	6,57E-07
0,0011	9,49E-04	1,88E-03	1,64E-03	8,55E-03	2,11E-03	1096	946	100	440	1330	0	0	1,00E-06
0,0012	9,20E-04	0,00E+00	3,89E-03	1,08E-02	0,00E+00	642	492	54	236	1522	0	0	1,87E-06
0,0013	8,94E-04	0,00E+00	6,07E-03	1,30E-02	0,00E+00	481	331	37	163	1052	0	0	2,70E-06
0,0014	8,52E-04	0,00E+00	8,55E-03	1,55E-02	0,00E+00	383	233	27	121	779	0	0	3,65E-06
0,0015	7,99E-04	0,00E+00	1,12E-02	1,81E-02	0,00E+00	321	171	21	95	609	0	0	4,67E-06
0,0016	7,40E-04	0,00E+00	1,40E-02	2,09E-02	0,00E+00	279	129	17	77	496	0	0	5,73E-06
0,0017	6,77E-04	0,00E+00	1,69E-02	2,38E-02	0,00E+00	249	99	15	65	417	0	0	6,82E-06
0,0018	6,11E-04	0,00E+00	1,98E-02	2,67E-02	0,00E+00	227	77	13	56	359	0	0	7,93E-06
0,0019	5,43E-04	0,00E+00	2,28E-02	2,97E-02	0,00E+00	210	60	11	49	314	0	0	9,05E-06
0,002	4,73E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	196	46	10	43	279	0	0	0,00E+00
0,0021	4,03E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	186	36	9	39	251	0	0	0,00E+00
0,0022	3,32E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	177	27	8	35	228	0	0	0,00E+00
0,0023	2,60E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	169	19	7	32	209	0	0	0,00E+00
0,0024	1,88E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	163	13	7	30	193	0	0	0,00E+00
0,0025	1,16E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	157	7	6	28	179	0	0	0,00E+00
0,0026	4,36E-05	0,00E+00	0,00E+00	0,00E+00	0,00E+00	153	3	6	26	167	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0043	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0044	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0045	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0046	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0007	3,69E+08	-2,99E+08	7,52E+06	-5,36E+06	7,66E+06	0,00E+00	0,00E+00	2,12E+05	1,49E-07	0	0	0	9,23E+10
0,0008	3,49E+08	-2,76E+08	4,77E+05	1,79E+06	9,41E+06	0,00E+00	0,00E+00	1,71E+06	0,00E+00	0	0	0	1,34E+11
0,0009	3,29E+08	-2,53E+08	3,55E+05	3,14E+06	1,32E+05	9,16E+06	0,00E+00	3,80E+06	0,00E+00	19	122	0	1,67E+11
0,001	3,04E+08	-2,26E+08	2,66E+05	2,35E+06	2,01E+06	7,81E+06	0,00E+00	6,80E+06	0,00E+00	301	124	0	1,94E+11
0,0011	2,41E+08	-1,64E+08	1,74E+05	1,54E+06	3,56E+06	4,60E+06	0,00E+00	7,87E+06	0,00E+00	597	138	0	2,18E+11
0,0012	1,54E+08	-8,27E+07	9,37E+04	8,28E+05	2,66E+06	0,00E+00	0,00E+00	8,49E+06	0,00E+00	0	0	0	2,32E+11
0,0013	1,25E+08	-5,40E+07	6,47E+04	5,72E+05	1,84E+06	0,00E+00	0,00E+00	9,09E+06	0,00E+00	0	0	0	2,42E+11
0,0014	1,07E+08	-3,63E+07	4,79E+04	4,23E+05	1,36E+06	0,00E+00	0,00E+00	9,77E+06	0,00E+00	0	0	0	2,49E+11
0,0015	9,64E+07	-2,50E+07	3,75E+04	3,31E+05	1,07E+06	0,00E+00	0,00E+00	1,05E+07	0,00E+00	0	0	0	2,54E+11
0,0016	8,93E+07	-1,74E+07	3,05E+04	2,70E+05	8,68E+05	0,00E+00	0,00E+00	1,13E+07	0,00E+00	0	0	0	2,59E+11
0,0017	8,47E+07	-1,23E+07	2,57E+04	2,27E+05	7,30E+05	0,00E+00	0,00E+00	1,21E+07	0,00E+00	0	0	0	2,63E+11
0,0018	8,17E+07	-8,58E+06	2,21E+04	1,95E+05	6,28E+05	0,00E+00	0,00E+00	1,29E+07	0,00E+00	0	0	0	2,66E+11
0,0019	7,98E+07	-5,94E+06	1,93E+04	1,71E+05	5,50E+05	0,00E+00	0,00E+00	1,37E+07	0,00E+00	0	0	0	2,70E+11
0,002	7,86E+07	-4,02E+06	1,72E+04	1,52E+05	4,89E+05	0,00E+00	0,00E+00	0,00E+00	1,45E+07	0	0	0	2,36E+11
0,0021	7,80E+07	-2,62E+06	1,55E+04	1,37E+05	4,40E+05	0,00E+00	0,00E+00	0,00E+00	1,53E+07	0	0	0	2,37E+11
0,0022	7,77E+07	-1,61E+06	1,41E+04	1,24E+05	4,00E+05	0,00E+00	0,00E+00	0,00E+00	1,61E+07	0	0	0	2,37E+11
0,0023	7,78E+07	-9,08E+05	1,29E+04	1,14E+05	3,66E+05	0,00E+00	0,00E+00	0,00E+00	1,70E+07	0	0	0	2,38E+11
0,0024	7,81E+07	-4,37E+05	1,19E+04	1,05E+05	3,37E+05	0,00E+00	0,00E+00	0,00E+00	1,78E+07	0	0	0	2,38E+11
0,0025	7,86E+07	-1,53E+05	1,10E+04	9,72E+04	3,13E+05	0,00E+00	0,00E+00	0,00E+00	1,86E+07	0	0	0	2,39E+11
0,0026	7,93E+07	-2,04E+04	1,03E+04	9,07E+04	2,92E+05	0,00E+00	0,00E+00	0,00E+00	1,94E+07	0	0	0	2,43E+11
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0043	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0044	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0045	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0046	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,7	9,23E+10
0,8	1,34E+11
0,9	1,67E+11
1	1,94E+11
1,1	2,18E+11
1,2	2,32E+11
1,3	2,42E+11
1,4	2,49E+11
1,5	2,54E+11
1,6	2,59E+11
1,7	2,63E+11
1,8	2,66E+11
1,9	2,70E+11



24 Use phase

24.1 Use phase loads

Use phase loads

Dimensions

ltot	550000	mm
number of spans	10	
lmid	57895	mm
lend	43421	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	4	
Ap	29400	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	1700	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Partial factors

Use phase: CC3

6.10a	
γG	1,4
γQ traffic	1,5
γQ other	1,65
ψ traffic	0,8
6.10b	
γG	1,25
γQ	1,5
6.15b	
ψ traffic	0,8

Loads in the use phase

Self-weight

qGk,sw	174	N/mm

SIDL

asphalt	44	N/mm
footpath	3,9	N/mm
edge element	3,1	N/mm
parapet	0,75	N/mm
safety barrier	1	N/mm
qGk,sidl	62	N/mm

Traffic

αq1*q1k	10,35	kN/m^2
αqo*qok	3,5	kN/m^2
qQk,udl	65	N/mm
Q1k	150000	N
Q2k	100000	N
Q3k	50000	N

Time-dependent losses

Assume:	6	%
σp∞	1311	N/mm^2

Creep

σcc	-17	N/mm^2
εcc	6,77E-05	
Δσpc	13	N/mm^2

Relaxation

t	876000	hours
Δσpr	75	N/mm^2
Δσp,c+r	89	N/mm^2
σp∞	1306	N/mm^2

Continuity prestressing

e0	630	mm
eb	1670	mm
a	28947	mm
tan θ	0,08	
θ	4,54	°
sin θ	0,08	
Fv	3,05E+06	N

Bending moments by prestressing

Msup	4,42E+10	Nmm
Mspan	4,42E+10	Nmm
σc,sup	-3,48	N/mm^2
σc,span	-3,45	N/mm^2
Check: σc ≤ 0	ok!	

Bending moments in the use phase (SLS)

Intermediate beam support

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	8,43E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	21432	mm
θQk,ts,a	4,23E-04	rad
θQk,ts,b	3,61E-04	rad
θa,int	5,24E-03	rad
θb,int	5,18E-03	rad
Ma	-8,90E+10	Nmm
Mb	-7,64E+10	Nmm
σc,sup	10,2	N/mm^2

Intermediate beam span

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	6,59E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	28947	mm
θQk,ts,a	4,13E-04	rad
θQk,ts,b	4,18E-04	rad
θa,int	5,23E-03	rad
θb,int	5,23E-03	rad
Ma	-7,94E+10	Nmm
Mb	-7,97E+10	Nmm
Mint	1,34E+11	Nmm
Mspan	5,47E+10	Nmm
σc,span	11,9	N/mm^2

End beam span

k0	6,68E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Mb,ext	6,60E+10	Nmm
θGk	1,67E-03	rad
θQk,udl	3,66E-04	rad
a	16168	mm
θQk,ts,b	2,05E-04	rad
θb,int	2,24E-03	rad
Mb	-7,00E+10	Nmm
Mint	7,48E+10	Nmm
Mspan	4,68E+10	Nmm

Bending moments in the use phase (ULS 6.10a)

Intermediate beam support

Ma,ext	1,20E+11	Nmm
Mb,ext	9,26E+10	Nmm
θGk	5,53E-03	rad
θQk,udl	1,30E-03	rad
θQk,ts,a	6,35E-04	rad
θQk,ts,b	5,41E-04	rad
θa,int	7,46E-03	rad
θb,int	7,37E-03	rad
Ma	-1,27E+11	Nmm
Mb	-1,08E+11	Nmm
MEd,sup	-1,27E+11	Nmm

Intermediate beam span

Ma,ext	9,23E+10	Nmm
Mb,ext	9,26E+10	Nmm
θGk	5,53E-03	rad
θQk,udl	1,30E-03	rad
θQk,ts,a	6,19E-04	rad
θQk,ts,b	6,28E-04	rad
θa,int	7,45E-03	rad
θb,int	7,46E-03	rad
Ma	-1,13E+11	Nmm
Mb	-1,13E+11	Nmm
Mint	1,92E+11	Nmm
MEd,span	7,88E+10	Nmm

End beam span

Mb,ext	9,23E+10	Nmm
θGk	2,33E-03	rad
θQk,udl	5,50E-04	rad
θQk,ts,b	3,07E-04	rad
θb,int	3,19E-03	rad
Mb	-9,89E+10	Nmm
Mint	1,07E+11	Nmm
MEd,span	6,73E+10	Nmm

Bending moments in the use phase (ULS 6.10b)

Intermediate beam support

Ma,ext	1,17E+11	Nmm
Mb,ext	8,27E+10	Nmm
θGk	4,93E-03	rad
θQk,udl	1,63E-03	rad
θQk,ts,a	7,93E-04	rad
θQk,ts,b	6,76E-04	rad
θa,int	7,36E-03	rad
θb,int	7,24E-03	rad
Ma	-1,26E+11	Nmm
Mb	-1,02E+11	Nmm
MEd,sup	-1,26E+11	Nmm

Intermediate beam span

Ma,ext	8,24E+10	Nmm
Mb,ext	8,27E+10	Nmm
θGk	4,93E-03	rad
θQk,udl	1,63E-03	rad
θQk,ts,a	7,74E-04	rad
θQk,ts,b	7,84E-04	rad
θa,int	7,34E-03	rad
θb,int	7,35E-03	rad
Ma	-1,08E+11	Nmm
Mb	-1,08E+11	Nmm
Mint	1,90E+11	Nmm
MEd,span	8,20E+10	Nmm

End beam span

Mb,ext	8,25E+10	Nmm
θGk	2,08E-03	rad
θQk,udl	6,87E-04	rad
θQk,ts,b	3,84E-04	rad
θb,int	3,15E-03	rad
Mb	-9,31E+10	Nmm
Mint	1,07E+11	Nmm
MEd,span	6,97E+10	Nmm

Shear forces in the use phase (ULS 6.10a)

Intermediate beam support

Ma,ext	1,20E+11	Nmm
Mb,ext	9,26E+10	Nmm
θGk	5,53E-03	rad
θQk,udl	1,30E-03	rad
a	0	mm
θQk,ts,a	3,34E-05	rad
θQk,ts,b	1,72E-05	rad
θa,int	6,86E-03	rad
θb,int	6,85E-03	rad
Ma	-1,20E+11	Nmm
Mb	-1,03E+11	Nmm
VEd	1,36E+07	N

Shear forces in the use phase (ULS 6.10b)

Intermediate beam support

Ma,ext	1,17E+11	Nmm
Mb,ext	8,27E+10	Nmm
θGk	4,93E-03	rad
θQk,udl	1,63E-03	rad
a	0	mm
θQk,ts,a	4,18E-05	rad
θQk,ts,b	2,15E-05	rad
θa,int	6,60E-03	rad
θb,int	6,58E-03	rad
Ma	-1,17E+11	Nmm
Mb	-9,53E+10	Nmm
VEd	1,35E+07	N

24.2 Use phase mid span capacity

Use phase mid span capacity

Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td = d*	235	mm
tf	150	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	1105	mm
Ic	9,67E+12	mm^4
eb	1670	mm
dp	2775	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	6	
Ap	44100	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	800	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1306	N/mm^2
εp∞	6,70E-03	

When εb = εctmax: ε0 =	0,000055
When εb' = εctmax: ε0 =	0,000058
When εb = εt,p: ε0 =	0,000119
When εb' = εt,p: ε0 =	0,000122
When εp = εp,y: ε0 =	0,000176
When dn = td: ε0 =	0,000205
When εb = εt,u: ε0 =	0,000221
When εb' = εt,u: ε0 =	0,000225
When εp = εud: ε0 =	0,001182

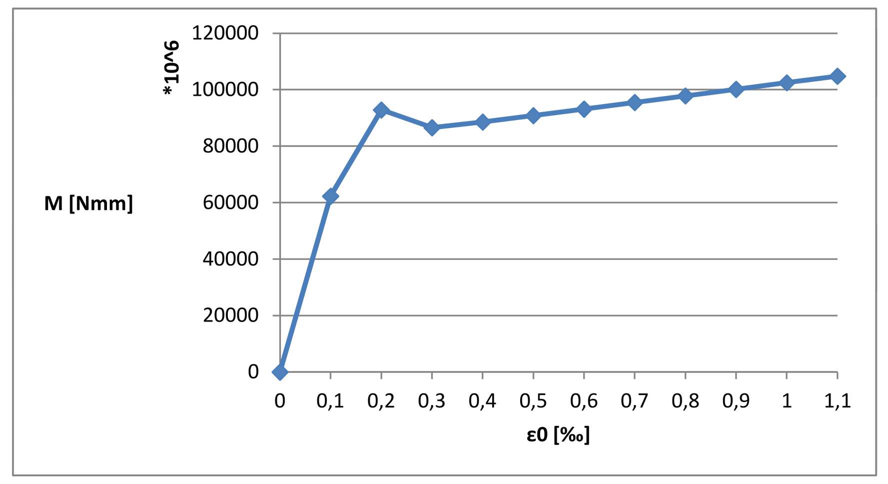
When dn = td & εp = εp,y: Ap =	-18323	mm^2
When dn = td & εb = εt,u: Ap =	81428	mm^2
When dn = td & εb' = εt,u: Ap =	88845	mm^2
When εp = εud & ε0 = εcmax: Ap =	697187	mm^2
When dn = td & ε0 = εcmax: Ap =	610402	mm^2

Mu	1,05E+11	Nmm
MEd	8,20E+10	Nmm
Unity check: MEd/Mu ≤ 1	0,78	ok!

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	6,59E-05	3,42E-04	3,02E-04	7,00E-03	3,64E-04	690	455	690	1670	0	0	0	1,45E-07
0,0002	1,37E-05	2,22E-03	2,00E-03	8,70E-03	2,33E-03	253	17	126	558	2113	0	0	7,92E-07
0,0003	1,48E-04	0,00E+00	4,98E-03	1,17E-02	0,00E+00	158	0	53	232	1494	0	78	1,90E-06
0,0004	2,55E-04	0,00E+00	7,33E-03	1,40E-02	0,00E+00	144	0	36	159	1021	0	92	2,78E-06
0,0005	3,57E-04	0,00E+00	9,61E-03	1,63E-02	0,00E+00	137	0	27	121	781	0	98	3,64E-06
0,0006	4,56E-04	0,00E+00	1,19E-02	1,86E-02	0,00E+00	134	0	22	98	634	0	102	4,49E-06
0,0007	5,54E-04	0,00E+00	1,41E-02	2,08E-02	0,00E+00	131	0	19	83	533	0	104	5,33E-06
0,0008	6,51E-04	0,00E+00	1,63E-02	2,30E-02	0,00E+00	130	0	16	72	461	0	106	6,17E-06
0,0009	7,48E-04	0,00E+00	1,85E-02	2,52E-02	0,00E+00	128	0	14	63	406	0	107	7,01E-06
0,001	8,45E-04	0,00E+00	2,08E-02	2,75E-02	0,00E+00	128	0	13	56	363	0	108	7,84E-06
0,0011	9,41E-04	0,00E+00	2,30E-02	2,97E-02	0,00E+00	127	0	12	51	328	0	108	8,68E-06
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	2,67E+07	-1,11E+07	1,21E+06	5,84E+06	6,00E+06	0,00E+00	0,00E+00	2,60E+06	0,00E+00	0	0	0	6,23E+10
0,0002	1,96E+07	-8,78E+04	2,21E+05	1,95E+06	5,22E+06	2,34E+06	0,00E+00	9,75E+06	0,00E+00	910	82	0	9,28E+10
0,0003	1,83E+07	0,00E+00	9,20E+04	8,12E+05	2,62E+06	0,00E+00	4,24E+06	1,06E+07	0,00E+00	0	0	0	8,65E+10
0,0004	2,23E+07	0,00E+00	6,28E+04	5,55E+05	1,79E+06	0,00E+00	8,65E+06	1,12E+07	0,00E+00	0	0	0	8,85E+10
0,0005	2,66E+07	0,00E+00	4,81E+04	4,24E+05	1,37E+06	0,00E+00	1,29E+07	1,18E+07	1,60E-07	0	0	0	9,08E+10
0,0006	3,11E+07	0,00E+00	3,90E+04	3,44E+05	1,11E+06	0,00E+00	1,71E+07	1,25E+07	4,21E-06	0	0	0	9,31E+10
0,0007	3,56E+07	0,00E+00	3,28E+04	2,90E+05	9,34E+05	0,00E+00	2,13E+07	1,31E+07	1,35E-06	0	0	0	9,55E+10
0,0008	4,02E+07	0,00E+00	2,84E+04	2,51E+05	8,07E+05	0,00E+00	2,54E+07	1,37E+07	0,00E+00	0	0	0	9,78E+10
0,0009	4,48E+07	0,00E+00	2,50E+04	2,21E+05	7,10E+05	0,00E+00	2,96E+07	1,43E+07	-5,22E-07	0	0	0	1,00E+11
0,001	4,94E+07	0,00E+00	2,23E+04	1,97E+05	6,35E+05	0,00E+00	3,37E+07	1,49E+07	-1,42E-07	0	0	0	1,02E+11
0,0011	5,40E+07	0,00E+00	2,02E+04	1,78E+05	5,74E+05	0,00E+00	3,78E+07	1,55E+07	4,10E-07	0	0	0	1,05E+11
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,1	6,23E+10
0,2	9,28E+10
0,3	8,65E+10
0,4	8,85E+10
0,5	9,08E+10
0,6	9,31E+10
0,7	9,55E+10
0,8	9,78E+10
0,9	1,00E+11
1	1,02E+11
1,1	1,05E+11



24.3 Use phase support capacity

Use phase support capacity

Moment Capacity (ULS)

bf	0	mm
bw	350	mm
bin	7300	mm
b	8000	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	2095	mm
Ic	9,67E+12	mm^4
eb	630	mm
dp	2725	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	9	
Ap	66150	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	294	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1306	N/mm^2
εp∞	6,70E-03	

Shear reinforcement

Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

When εb = εctmax: ε0 =	0,000132
When εb' = εctmax: ε0 =	0,000150
When εb = εt,p: ε0 =	0,000291
When εb' = εt,p: ε0 =	0,000307
When εp = εp,y: ε0 =	0,000443
When dn = td: ε0 =	0,000667
When εb = εt,u: ε0 =	0,000425
When εb' = εt,u: ε0 =	0,000422
When εp = εud: ε0 =	0,001122

When dn = td & εp = εp,y: Ap =	-64565	mm^2
When dn = td & εb = εt,u: Ap =	-6330	mm^2
When dn = td & εb' = εt,u: Ap =	-685	mm^2
When εp = εud & ε0 = εcmax: Ap =	300606	mm^2
When dn = td & ε0 = εcmax: Ap =	158291	mm^2

Mu	1,83E+11	Nmm
MEd	1,27E+11	Nmm

Unity check: MEd/Mu ≤ 1	0,69	ok!

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]	ε0 [‰]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,00002	1,84E-05	1,26E-05	9,95E-06	6,71E-03	1,52E-05	1820	1670	1145	0	0	0	0	1,10E-08	7,28E+06	-5,59E+06	4,19E+06	-2,63E+06	0,00E+00	0,00E+00	0,00E+00	1,28E+05	0,00E+00	0	0	0	2,84E+10	0,02	2,84E+10
0,00004	3,67E-05	2,52E-05	1,99E-05	6,72E-03	3,03E-05	1820	1670	1145	0	0	0	0	2,20E-08	1,46E+07	-1,12E+07	8,38E+06	-5,26E+06	0,00E+00	0,00E+00	0,00E+00	2,57E+05	0,00E+00	0	0	0	3,30E+10	0,04	3,30E+10
0,00006	5,51E-05	3,78E-05	2,99E-05	6,73E-03	4,55E-05	1820	1670	1145	0	0	0	0	3,30E-08	2,18E+07	-1,68E+07	1,26E+07	-7,89E+06	0,00E+00	0,00E+00	0,00E+00	3,85E+05	0,00E+00	0	0	0	3,75E+10	0,06	3,75E+10
0,00008	7,34E-05	5,03E-05	3,98E-05	6,74E-03	6,07E-05	1820	1670	1145	0	0	0	0	4,40E-08	2,91E+07	-2,24E+07	1,68E+07	-1,05E+07	0,00E+00	0,00E+00	0,00E+00	5,13E+05	0,00E+00	0	0	0	4,21E+10	0,08	4,21E+10
0,0001	9,18E-05	6,29E-05	4,98E-05	6,75E-03	7,59E-05	1820	1670	1145	0	0	0	0	5,50E-08	3,64E+07	-2,80E+07	2,09E+07	-1,32E+07	0,00E+00	0,00E+00	0,00E+00	6,42E+05	0,00E+00	0	0	0	4,67E+10	0,1	4,67E+10
0,00012	1,10E-04	7,55E-05	5,97E-05	6,76E-03	9,10E-05	1820	1670	1145	0	0	0	0	6,59E-08	4,37E+07	-3,35E+07	2,51E+07	-1,58E+07	0,00E+00	0,00E+00	0,00E+00	7,70E+05	0,00E+00	0	0	0	5,12E+10	0,12	5,12E+10
0,00014	1,28E-04	8,89E-05	7,04E-05	6,77E-03	1,07E-04	1813	1663	1151	144	92	0	0	7,72E-08	5,08E+07	-3,90E+07	2,59E+07	-1,87E+07	3,66E+06	0,00E+00	0,00E+00	9,08E+05	1,68E-08	0	0	0	5,63E+10	0,14	5,63E+10
0,00016	1,46E-04	1,18E-04	9,51E-05	6,79E-03	1,40E-04	1709	1559	1068	187	0	0	0	9,36E-08	5,47E+07	-4,15E+07	1,87E+06	6,56E+05	9,41E+06	0,00E+00	0,00E+00	1,23E+06	0,00E+00	0	0	0	6,88E+10	0,16	6,88E+10
0,00018	1,63E-04	1,57E-04	1,30E-04	6,83E-03	1,84E-04	1584	1434	880	501	0	0	0	1,14E-07	5,70E+07	-4,26E+07	1,54E+06	1,75E+06	9,41E+06	0,00E+00	0,00E+00	1,67E+06	0,00E+00	0	0	0	8,26E+10	0,18	8,26E+10
0,0002	1,80E-04	2,03E-04	1,70E-04	6,87E-03	2,35E-04	1472	1322	736	757	0	0	0	1,36E-07	5,89E+07	-4,33E+07	1,29E+06	2,65E+06	9,41E+06	0,00E+00	0,00E+00	2,20E+06	0,00E+00	0	0	0	9,50E+10	0,2	9,50E+10
0,00022	1,96E-04	2,55E-04	2,16E-04	6,92E-03	2,92E-04	1374	1224	624	967	0	0	0	1,60E-07	6,04E+07	-4,38E+07	1,09E+06	3,38E+06	9,41E+06	0,00E+00	0,00E+00	2,79E+06	0,00E+00	0	0	0	1,06E+11	0,22	1,06E+11
0,00024	2,12E-04	3,12E-04	2,68E-04	6,97E-03	3,56E-04	1289	1139	537	1139	0	0	0	1,86E-07	6,19E+07	-4,41E+07	9,40E+05	3,99E+06	9,41E+06	0,00E+00	0,00E+00	3,45E+06	0,00E+00	0	0	0	1,16E+11	0,24	1,16E+11
0,00026	2,28E-04	3,74E-04	3,23E-04	7,02E-03	4,25E-04	1215	1065	467	1282	0	0	0	2,14E-07	6,32E+07	-4,43E+07	8,18E+05	4,49E+06	9,41E+06	0,00E+00	0,00E+00	4,17E+06	0,00E+00	0	0	0	1,25E+11	0,26	1,25E+11
0,00028	2,44E-04	4,41E-04	3,83E-04	7,08E-03	4,98E-04	1151	1001	411	1402	0	0	0	2,43E-07	6,45E+07	-4,45E+07	7,20E+05	4,91E+06	9,41E+06	0,00E+00	0,00E+00	4,94E+06	0,00E+00	0	0	0	1,33E+11	0,28	1,33E+11
0,0003	2,59E-04	5,12E-04	4,47E-04	7,15E-03	5,77E-04	1095	945	365	1505	107	129	0	2,74E-07	6,57E+07	-4,46E+07	6,39E+05	5,27E+06	4,26E+06	5,11E+06	0,00E+00	5,76E+06	0,00E+00	64	0	0	1,40E+11	0,3	1,40E+11
0,00032	2,74E-04	5,97E-04	5,23E-04	7,22E-03	6,70E-04	1034	884	323	1427	180	235	0	3,09E-07	6,62E+07	-4,41E+07	5,66E+05	5,00E+06	6,25E+05	9,10E+06	0,00E+00	6,75E+06	0,00E+00	90	120	0	1,48E+11	0,32	1,48E+11
0,00034	2,88E-04	6,90E-04	6,06E-04	7,31E-03	7,71E-04	979	829	288	1272	426	235	0	3,47E-07	6,66E+07	-4,36E+07	5,04E+05	4,45E+06	1,45E+06	8,78E+06	0,00E+00	7,82E+06	0,00E+00	211	121	0	1,55E+11	0,34	1,55E+11
0,00036	3,02E-04	7,88E-04	6,95E-04	7,39E-03	8,79E-04	930	780	258	1141	635	235	0	3,87E-07	6,70E+07	-4,30E+07	4,52E+05	3,99E+06	2,13E+06	8,45E+06	0,00E+00	8,96E+06	0,00E+00	313	121	0	1,61E+11	0,36	1,61E+11
0,00038	3,16E-04	8,91E-04	7,88E-04	7,49E-03	9,91E-04	887	737	233	1031	814	235	0	4,29E-07	6,74E+07	-4,24E+07	4,08E+05	3,61E+06	2,67E+06	8,09E+06	0,00E+00	1,02E+07	0,00E+00	398	122	0	1,67E+11	0,38	1,67E+11
0,0004	3,29E-04	9,98E-04	8,85E-04	7,58E-03	1,11E-03	848	698	212	936	968	235	0	4,72E-07	6,78E+07	-4,19E+07	3,71E+05	3,28E+06	3,12E+06	7,71E+06	0,00E+00	1,14E+07	0,00E+00	470	123	0	1,73E+11	0,4	1,73E+11
0,00042	3,43E-04	1,11E-03	9,86E-04	7,69E-03	1,23E-03	814	664	194	856	1102	235	0	5,16E-07	6,84E+07	-4,15E+07	3,39E+05	2,99E+06	3,47E+06	7,33E+06	0,00E+00	1,27E+07	0,00E+00	530	123	0	1,78E+11	0,42	1,78E+11
0,00044	3,56E-04	1,23E-03	1,09E-03	7,79E-03	1,36E-03	783	633	178	786	1218	235	0	5,62E-07	6,89E+07	-4,11E+07	3,11E+05	2,75E+06	3,75E+06	6,93E+06	0,00E+00	1,41E+07	0,00E+00	581	124	0	1,83E+11	0,44	1,83E+11
0,00046	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00048	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0005	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00052	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00054	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00056	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00058	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0006	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00062	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00064	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00066	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00068	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0007	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00072	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00074	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00076	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00078	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
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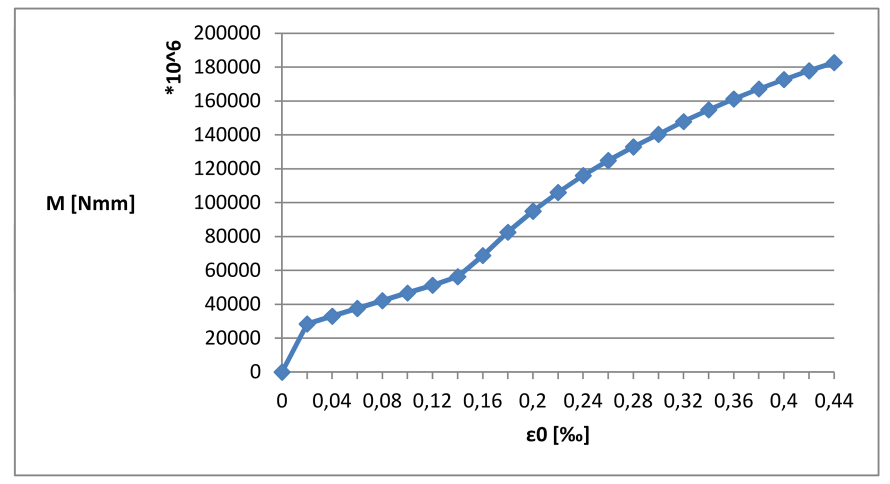


Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,72E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	30	°
tan βu	0,58	
dn	783	mm
z	2464	mm
cot βu	1,73	

VRb	3,38E+06	N
Vf	1,84E+07	N
Vs	0,00E+00	N
Vu	1,84E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	1,36E+07	N

Unity check: VEd/Vu ≤ 1	0,74	ok!

24.4 Derivation to find maximum value of M_A

$$\varphi_{A,TS} = \frac{Fa(a^2 - 3al + 2l^2) + F(a + 1.2)(a^2 + 2 \cdot 1.2a + 1.2^2 - 3al - 3 \cdot 1.2l + 2l^2)}{6EIl}$$

$$\frac{d\varphi_{A,TS}}{da} =$$

$$\frac{F}{6EIl}(2a^3 - 6a^2l + 4al^2 + 3(1.2)a^2 + 3(1.2)^2a - 6(1.2)al + (1.2)^3 - 3(1.2)^2l + 2(1.2)l^2) \rightarrow$$

$$6a^2 + 6(1.2)a - 12al + 4l^2 - 6(1.2)l + 3(1.2)^2$$

Insert into formula for M_A:

$$-\left(2 + \frac{k_0}{k_B}\right)k_0\{6a^2 + 6(1.2)a - 12al + 4l^2 - 6(1.2)l + 3(1.2)^2\}$$

$$\varphi_{B,TS} = \frac{Fa}{6EI}\left(3l - \frac{(a^2 + 2l^2)}{l}\right) + \frac{F(a + 1.2)}{6EI}\left(3l - \frac{((a + 1.2)^2 + 2l^2)}{l}\right)$$

$$\frac{d\varphi_{B,TS}}{da} = \frac{F}{6EIl}(-2a^3 - 3(1.2)a^2 - 3(1.2)^2a + 2al^2 + (1.2)l^2 - (1.2)^3) =$$

$$-6a^2 - 6(1.2)a - 3(1.2)^2 + 2l^2$$

Insert into formula for M_A:

$$k_0\{-6a^2 - 6(1.2)a - 3(1.2)^2 + 2l^2\}$$

$$\frac{dM_A}{da} = 0 \rightarrow$$

$$\left(2 + \frac{k_0}{k_B}\right)\{6a^2 + 6(1.2)a - 12al + 4l^2 - 6(1.2)l + 3(1.2)^2\} + 6a^2 + 6(1.2)a + 3(1.2)^2 - 2l^2 = 0$$

$$6\left(3 + \frac{k_0}{k_B}\right)a^2 + 6\left((1.2)\left(3 + \frac{k_0}{k_B}\right) - \left(2 + \frac{k_0}{k_B}\right)2l\right)a + 2\left(3 + 2\frac{k_0}{k_B}\right)l^2 - 6(1.2)\left(2 + \frac{k_0}{k_B}\right)l$$

$$+3(1.2)^2\left(3 + \frac{k_0}{k_B}\right) = 0$$

24.5 Contribution of external spans to the bending moment at mid span

> $restart;$

> $eq1 := \frac{M_c \cdot \frac{3}{4} l}{3\,EI} - \frac{q \cdot \left(\frac{3}{4} l\right)^3}{24\,EI} = 0;$

$$eq1 := \frac{1}{4} \frac{M_c l}{EI} - \frac{9}{512} \frac{q l^3}{EI} = 0 \quad \textbf{(1)}$$

> $solution := solve(\{eq1\}, \{M_c\});$

$$solution := \left\{M_c = \frac{9}{128} q l^2\right\} \quad \textbf{(2)}$$

> $assign(solution);$

> $M_c;$

$$\frac{9}{128} q l^2 \quad \textbf{(3)}$$

>

> $restart;$

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \qquad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot \frac{3}{4}\,l}{3\,EI} - \frac{q \cdot \left(\frac{3}{4}\,l\right)^3}{24\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{4}\,\frac{M_d\,l}{EI} - \frac{9}{512}\,\frac{q\,l^3}{EI} \qquad (2)$$

> $solution := solve(\{eq1, eq2\}, \{M_c, M_d\});$

$$solution := \left\{M_c = \frac{133}{1536}\,q\,l^2, M_d = \frac{59}{768}\,q\,l^2\right\} \qquad (3)$$

> $assign(solution);$

> $M_c;$

$$\frac{133}{1536}\,q\,l^2 \qquad (4)$$

>

> restart;

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \quad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_e \cdot l}{6\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{3}\,\frac{M_d\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_e\,l}{EI} \quad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_e \cdot l}{3\,EI} = \frac{M_e \cdot \frac{3}{4}\,l}{3\,EI} - \frac{q \cdot \left(\frac{3}{4}\,l\right)^3}{24\,EI};$

$$eq3 := -\frac{1}{6}\,\frac{M_d\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_e\,l}{EI} = \frac{1}{4}\,\frac{M_e\,l}{EI} - \frac{9}{512}\,\frac{q\,l^3}{EI} \quad (3)$$

>

> $solution := solve(\{eq1, eq2, eq3\}, \{M_c, M_d, M_e\});$

$$solution := \left\{M_c = \frac{95}{1152}\,q\,l^2, M_d = \frac{49}{576}\,q\,l^2, M_e = \frac{89}{1152}\,q\,l^2\right\} \quad (4)$$

> assign(solution);

> $M_c;$

$$\frac{95}{1152}\,q\,l^2 \quad (5)$$

>

> restart;

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \qquad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_e \cdot l}{6\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{3}\,\frac{M_d\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_e\,l}{EI} \qquad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_e \cdot l}{3\,EI} = \frac{M_e \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_f \cdot l}{6\,EI};$

$$eq3 := -\frac{1}{6}\,\frac{M_d\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_e\,l}{EI} = \frac{1}{3}\,\frac{M_e\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_f\,l}{EI} \qquad (3)$$

> $eq4 := -\frac{M_e \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_f \cdot l}{3\,EI} = \frac{M_f \cdot \frac{3}{4}\,l}{3\,EI} - \frac{q \cdot \left(\frac{3}{4}\,l\right)^3}{24\,EI};$

$$eq4 := -\frac{1}{6}\,\frac{M_e\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_f\,l}{EI} = \frac{1}{4}\,\frac{M_f\,l}{EI} - \frac{9}{512}\,\frac{q\,l^3}{EI} \qquad (4)$$

> $solution := solve(\{eq1, eq2, eq3, eq4\}, \{M_c, M_d, M_e, M_f\});$

$$solution := \left\{M_c = \frac{599}{7168}\,q\,l^2, M_d = \frac{297}{3584}\,q\,l^2, M_e = \frac{87}{1024}\,q\,l^2, M_f = \frac{277}{3584}\,q\,l^2\right\} \qquad (5)$$

> assign(solution);

> $M_c;$

$$\frac{599}{7168}\,q\,l^2 \qquad (6)$$

>

> restart;

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \qquad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_e \cdot l}{6\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{3}\,\frac{M_d\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_e\,l}{EI} \qquad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_e \cdot l}{3\,EI} = \frac{M_e \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_f \cdot l}{6\,EI};$

$$eq3 := -\frac{1}{6}\,\frac{M_d\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_e\,l}{EI} = \frac{1}{3}\,\frac{M_e\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_f\,l}{EI} \qquad (3)$$

> $eq4 := -\frac{M_e \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_f \cdot l}{3\,EI} = \frac{M_f \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_g \cdot l}{6\,EI};$

$$eq4 := -\frac{1}{6}\,\frac{M_e\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_f\,l}{EI} = \frac{1}{3}\,\frac{M_f\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_g\,l}{EI} \qquad (4)$$

> $eq5 := -\frac{M_f \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_g \cdot l}{3\,EI} = \frac{M_g \cdot \frac{3}{4}\,l}{3\,EI} - \frac{q \cdot \left(\frac{3}{4}\,l\right)^3}{24\,EI};$

$$eq5 := -\frac{1}{6}\,\frac{M_f\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_g\,l}{EI} = \frac{1}{4}\,\frac{M_g\,l}{EI} - \frac{9}{512}\,\frac{q\,l^3}{EI} \qquad (5)$$

> $solution := solve(\{eq1, eq2, eq3, eq4, eq5\}, \{M_c, M_d, M_e, M_f, M_g\});$

$$solution := \left\{M_c = \frac{6683}{80256}\,q\,l^2, M_d = \frac{3349}{40128}\,q\,l^2, M_e = \frac{6653}{80256}\,q\,l^2, M_f = \frac{3409}{40128}\,q\,l^2, M_g = \frac{6203}{80256}\,q\,l^2\right\} \qquad (6)$$

> $assign(solution);$

> $M_c;$

$$\frac{6683}{80256}\,q\,l^2 \qquad (7)$$

>

> restart;

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \tag{1}$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_e \cdot l}{6\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{3}\,\frac{M_d\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_e\,l}{EI} \tag{2}$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_e \cdot l}{3\,EI} = \frac{M_e \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_f \cdot l}{6\,EI};$

$$eq3 := -\frac{1}{6}\,\frac{M_d\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_e\,l}{EI} = \frac{1}{3}\,\frac{M_e\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_f\,l}{EI} \tag{3}$$

> $eq4 := -\frac{M_e \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_f \cdot l}{3\,EI} = \frac{M_f \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_g \cdot l}{6\,EI};$

$$eq4 := -\frac{1}{6}\,\frac{M_e\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_f\,l}{EI} = \frac{1}{3}\,\frac{M_f\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_g\,l}{EI} \tag{4}$$

> $eq5 := -\frac{M_f \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_g \cdot l}{3\,EI} = \frac{M_g \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_h \cdot l}{6\,EI};$

$$eq5 := -\frac{1}{6}\,\frac{M_f\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_g\,l}{EI} = \frac{1}{3}\,\frac{M_g\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_h\,l}{EI} \tag{5}$$

> $eq6 := -\frac{M_g \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_h \cdot l}{3\,EI} = \frac{M_h \cdot \frac{3}{4}\,l}{3\,EI} - \frac{q \cdot \left(\frac{3}{4}\,l\right)^3}{24\,EI};$

$$eq6 := -\frac{1}{6}\,\frac{M_g\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_h\,l}{EI} = \frac{1}{4}\,\frac{M_h\,l}{EI} - \frac{9}{512}\,\frac{q\,l^3}{EI} \tag{6}$$

> $solution := solve(\{eq1, eq2, eq3, eq4, eq5, eq6\}, \{M_c, M_d, M_e, M_f, M_g, M_h\});$

$$solution := \left\{M_c = \frac{4993}{59904}\,q\,l^2,\ M_d = \frac{2495}{29952}\,q\,l^2,\ M_e = \frac{4999}{59904}\,q\,l^2,\ M_f = \frac{191}{2304}\,q\,l^2,\ M_g = \frac{5089}{59904}\,q\,l^2,\ M_h = \frac{2315}{29952}\,q\,l^2\right\} \tag{7}$$

> $assign(solution);$

> $M_c;$

$$\frac{4993}{59904}\,q\,l^2 \tag{8}$$

>

24.6 Contribution of external span to the bending moment at the support

> $restart;$

> $eq1 := \frac{M_c \cdot \frac{3}{4} l}{3\ EI} - \frac{q \cdot \left(\frac{3}{4} l\right)^3}{24\ EI} = 0;$

$$eq1 := \frac{1}{4} \frac{M_c\, l}{EI} - \frac{9}{512} \frac{q\, l^3}{EI} = 0 \qquad \textbf{(1)}$$

> $solution := solve(\{eq1\}, \{M_c\});$

$$solution := \left\{M_c = \frac{9}{128}\, q\, l^2\right\} \qquad \textbf{(2)}$$

> $assign(solution);$

> $M_c;$

$$\frac{9}{128}\, q\, l^2 \qquad \textbf{(3)}$$

>

> $restart;$

> $eq1 := \frac{M_c \cdot l}{3\ EI} - \frac{q \cdot l^3}{24\ EI} + \frac{M_d \cdot l}{6\ EI} = 0;$

$$eq1 := \frac{1}{3}\ \frac{M_c\ l}{EI} - \frac{1}{24}\ \frac{q\ l^3}{EI} + \frac{1}{6}\ \frac{M_d\ l}{EI} = 0 \quad \textbf{(1)}$$

> $eq2 := -\frac{M_c \cdot l}{6\ EI} + \frac{q \cdot l^3}{24\ EI} - \frac{M_d \cdot l}{3\ EI} = \frac{M_d \cdot \frac{3}{4}\ l}{3\ EI};$

$$eq2 := -\frac{1}{6}\ \frac{M_c\ l}{EI} + \frac{1}{24}\ \frac{q\ l^3}{EI} - \frac{1}{3}\ \frac{M_d\ l}{EI} = \frac{1}{4}\ \frac{M_d\ l}{EI} \quad \textbf{(2)}$$

> $solution := solve(\{eq1, eq2\}, \{M_c, M_d\});$

$$solution := \left\{M_c = \frac{5}{48}\ q\ l^2, M_d = \frac{1}{24}\ q\ l^2\right\} \quad \textbf{(3)}$$

> $assign(solution);$

> $M_c;$

$$\frac{5}{48}\ q\ l^2 \quad \textbf{(4)}$$

>

> restart;

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \quad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} + \frac{M_e \cdot l}{6\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{3}\,\frac{M_d\,l}{EI} + \frac{1}{6}\,\frac{M_e\,l}{EI} \quad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} - \frac{M_e \cdot l}{3\,EI} = \frac{M_e \cdot \frac{3}{4}\,l}{3\,EI};$

$$eq3 := -\frac{1}{6}\,\frac{M_d\,l}{EI} - \frac{1}{3}\,\frac{M_e\,l}{EI} = \frac{1}{4}\,\frac{M_e\,l}{EI} \quad (3)$$

> $solution := solve(\{eq1, eq2, eq3\}, \{M_c, M_d, M_e\});$

$$solution := \left\{M_c = \frac{19}{180}\,q\,l^2,\ M_d = \frac{7}{180}\,q\,l^2,\ M_e = -\frac{1}{90}\,q\,l^2\right\} \quad (4)$$

> $assign(solution);$

> $M_c;$

$$\frac{19}{180}\,q\,l^2 \quad (5)$$

>

> $restart;$

> $eq1 := \frac{M_c \cdot l}{3\ EI} - \frac{q \cdot l^3}{24\ EI} + \frac{M_d \cdot l}{6\ EI} = 0;$

$$eq1 := \frac{1}{3}\frac{M_c\, l}{EI} - \frac{1}{24}\frac{q\, l^3}{EI} + \frac{1}{6}\frac{M_d\, l}{EI} = 0 \quad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\ EI} + \frac{q \cdot l^3}{24\ EI} - \frac{M_d \cdot l}{3\ EI} = \frac{M_d \cdot l}{3\ EI} + \frac{M_e \cdot l}{6\ EI};$

$$eq2 := -\frac{1}{6}\frac{M_c\, l}{EI} + \frac{1}{24}\frac{q\, l^3}{EI} - \frac{1}{3}\frac{M_d\, l}{EI} = \frac{1}{3}\frac{M_d\, l}{EI} + \frac{1}{6}\frac{M_e\, l}{EI} \quad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\ EI} - \frac{M_e \cdot l}{3\ EI} = \frac{M_e \cdot l}{3\ EI} + \frac{M_f \cdot l}{6\ EI};$

$$eq3 := -\frac{1}{6}\frac{M_d\, l}{EI} - \frac{1}{3}\frac{M_e\, l}{EI} = \frac{1}{3}\frac{M_e\, l}{EI} + \frac{1}{6}\frac{M_f\, l}{EI} \quad (3)$$

> $eq4 := -\frac{M_e \cdot l}{6\ EI} - \frac{M_f \cdot l}{3\ EI} = \frac{M_f \cdot \frac{3}{4}\, l}{3\ EI};$

$$eq4 := -\frac{1}{6}\frac{M_e\, l}{EI} - \frac{1}{3}\frac{M_f\, l}{EI} = \frac{1}{4}\frac{M_f\, l}{EI} \quad (4)$$

> $solution := solve(\{eq1, eq2, eq3, eq4\}, \{M_c, M_d, M_e, M_f\});$

$$solution := \left\{M_c = \frac{71}{672}\, q\, l^2,\ M_d = \frac{13}{336}\, q\, l^2,\ M_e = -\frac{1}{96}\, q\, l^2,\ M_f = \frac{1}{336}\, q\, l^2\right\} \quad (5)$$

> $assign(solution);$

> $M_c;$

$$\frac{71}{672}\, q\, l^2 \quad (6)$$

>

> restart;

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \qquad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} + \frac{M_e \cdot l}{6\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{3}\,\frac{M_d\,l}{EI} + \frac{1}{6}\,\frac{M_e\,l}{EI} \qquad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} - \frac{M_e \cdot l}{3\,EI} = \frac{M_e \cdot l}{3\,EI} + \frac{M_f \cdot l}{6\,EI};$

$$eq3 := -\frac{1}{6}\,\frac{M_d\,l}{EI} - \frac{1}{3}\,\frac{M_e\,l}{EI} = \frac{1}{3}\,\frac{M_e\,l}{EI} + \frac{1}{6}\,\frac{M_f\,l}{EI} \qquad (3)$$

> $eq4 := -\frac{M_e \cdot l}{6\,EI} - \frac{M_f \cdot l}{3\,EI} = \frac{M_f \cdot l}{3\,EI} + \frac{M_g \cdot l}{6\,EI};$

$$eq4 := -\frac{1}{6}\,\frac{M_e\,l}{EI} - \frac{1}{3}\,\frac{M_f\,l}{EI} = \frac{1}{3}\,\frac{M_f\,l}{EI} + \frac{1}{6}\,\frac{M_g\,l}{EI} \qquad (4)$$

> $eq5 := -\frac{M_f \cdot l}{6\,EI} - \frac{M_g \cdot l}{3\,EI} = \frac{M_g \cdot \frac{3}{4}\,l}{3\,EI};$

$$eq5 := -\frac{1}{6}\,\frac{M_f\,l}{EI} - \frac{1}{3}\,\frac{M_g\,l}{EI} = \frac{1}{4}\,\frac{M_g\,l}{EI} \qquad (5)$$

> $solution := solve(\{eq1, eq2, eq3, eq4, eq5\}, \{M_c, M_d, M_e, M_f, M_g\});$

$$solution := \left\{M_c = \frac{265}{2508}\,q\,l^2, M_d = \frac{97}{2508}\,q\,l^2, M_e = -\frac{13}{1254}\,q\,l^2, M_f = \frac{7}{2508}\,q\,l^2, M_g = -\frac{1}{1254}\,q\,l^2\right\} \qquad (6)$$

> assign(solution);

> $M_c;$

$$\frac{265}{2508}\,q\,l^2 \qquad (7)$$

>

> restart;

> $eq1 := \frac{M_c \cdot l}{3\,EI} - \frac{q \cdot l^3}{24\,EI} + \frac{M_d \cdot l}{6\,EI} = 0;$

$$eq1 := \frac{1}{3}\,\frac{M_c\,l}{EI} - \frac{1}{24}\,\frac{q\,l^3}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} = 0 \quad (1)$$

> $eq2 := -\frac{M_c \cdot l}{6\,EI} + \frac{q \cdot l^3}{24\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} + \frac{M_e \cdot l}{6\,EI};$

$$eq2 := -\frac{1}{6}\,\frac{M_c\,l}{EI} + \frac{1}{24}\,\frac{q\,l^3}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{3}\,\frac{M_d\,l}{EI} + \frac{1}{6}\,\frac{M_e\,l}{EI} \quad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} - \frac{M_e \cdot l}{3\,EI} = \frac{M_e \cdot l}{3\,EI} + \frac{M_f \cdot l}{6\,EI};$

$$eq3 := -\frac{1}{6}\,\frac{M_d\,l}{EI} - \frac{1}{3}\,\frac{M_e\,l}{EI} = \frac{1}{3}\,\frac{M_e\,l}{EI} + \frac{1}{6}\,\frac{M_f\,l}{EI} \quad (3)$$

> $eq4 := -\frac{M_e \cdot l}{6\,EI} - \frac{M_f \cdot l}{3\,EI} = \frac{M_f \cdot l}{3\,EI} + \frac{M_g \cdot l}{6\,EI};$

$$eq4 := -\frac{1}{6}\,\frac{M_e\,l}{EI} - \frac{1}{3}\,\frac{M_f\,l}{EI} = \frac{1}{3}\,\frac{M_f\,l}{EI} + \frac{1}{6}\,\frac{M_g\,l}{EI} \quad (4)$$

> $eq5 := -\frac{M_f \cdot l}{6\,EI} - \frac{M_g \cdot l}{3\,EI} = \frac{M_g \cdot l}{3\,EI} + \frac{M_h \cdot l}{6\,EI};$

$$eq5 := -\frac{1}{6}\,\frac{M_f\,l}{EI} - \frac{1}{3}\,\frac{M_g\,l}{EI} = \frac{1}{3}\,\frac{M_g\,l}{EI} + \frac{1}{6}\,\frac{M_h\,l}{EI} \quad (5)$$

> $eq6 := -\frac{M_g \cdot l}{6\,EI} - \frac{M_h \cdot l}{3\,EI} = \frac{M_h \cdot \frac{3}{4} l}{3\,EI};$

$$eq6 := -\frac{1}{6}\,\frac{M_g\,l}{EI} - \frac{1}{3}\,\frac{M_h\,l}{EI} = \frac{1}{4}\,\frac{M_h\,l}{EI} \quad (6)$$

> $solution := solve(\{eq1, eq2, eq3, eq4, eq5, eq6\}, \{M_c, M_d, M_e, M_f, M_g, M_h\});$

$$solution := \left\{M_c = \frac{989}{9360}\,q\,l^2,\ M_d = \frac{181}{4680}\,q\,l^2,\ M_e = -\frac{97}{9360}\,q\,l^2,\ M_f = \frac{1}{360}\,q\,l^2,\ M_g = -\frac{7}{9360}\,q\,l^2,\ M_h = \frac{1}{4680}\,q\,l^2\right\} \quad (7)$$

> assign(solution);

> $M_c;$

$$\frac{989}{9360}\,q\,l^2 \quad (8)$$

>

24.7 Modeling adjacent spans as a rotational spring

> $restart;$

> $\theta := \frac{T \cdot \frac{3}{4}\, l}{3\, EI};$

$$\theta := \frac{1}{4}\, \frac{T\, l}{EI} \tag{1}$$

> $k_r := \frac{T}{\theta};$

$$k_r := \frac{4\, EI}{l} \tag{2}$$

>

> $restart;$

> $eq1 := -\frac{T \cdot l}{6\,EI} + \frac{M_c \cdot l}{3\,EI} = -\frac{M_c \cdot \frac{3}{4}\,l}{3\,EI};$

$$eq1 := -\frac{1}{6}\,\frac{T\,l}{EI} + \frac{1}{3}\,\frac{M_c\,l}{EI} = -\frac{1}{4}\,\frac{M_c\,l}{EI} \quad \textbf{(1)}$$

> $solution := solve(\{eq1\}, \{M_c\});$

$$solution := \left\{M_c = \frac{2}{7}\,T\right\} \quad \textbf{(2)}$$

> $assign(solution);$

> $\theta := \frac{T \cdot l}{3\,EI} - \frac{M_c \cdot l}{6\,EI};$

$$\theta := \frac{2}{7}\,\frac{T\,l}{EI} \quad \textbf{(3)}$$

> $k_r := \frac{T}{\theta};$

$$k_r := \frac{7}{2}\,\frac{EI}{l} \quad \textbf{(4)}$$

>

> restart;

> $eq1 := -\frac{T \cdot l}{6\,EI} + \frac{M_c \cdot l}{3\,EI} = -\frac{M_c \cdot l}{3\,EI} + \frac{M_d \cdot l}{6\,EI};$

$$eq1 := -\frac{1}{6}\,\frac{T\,l}{EI} + \frac{1}{3}\,\frac{M_c\,l}{EI} = -\frac{1}{3}\,\frac{M_c\,l}{EI} + \frac{1}{6}\,\frac{M_d\,l}{EI} \qquad (1)$$

> $eq2 := \frac{M_c \cdot l}{6\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot \frac{3}{4}\,l}{3\,EI};$

$$eq2 := \frac{1}{6}\,\frac{M_c\,l}{EI} - \frac{1}{3}\,\frac{M_d\,l}{EI} = \frac{1}{4}\,\frac{M_d\,l}{EI} \qquad (2)$$

> $solution := solve(\{eq1, eq2\}, \{M_c, M_d\});$

$$solution := \left\{M_c = \frac{7}{26}\,T, M_d = \frac{1}{13}\,T\right\} \qquad (3)$$

> assign(solution);

> $\theta := \frac{T \cdot l}{3\,EI} - \frac{M_c \cdot l}{6\,EI};$

$$\theta := \frac{15}{52}\,\frac{T\,l}{EI} \qquad (4)$$

> $k_r := \frac{T}{\theta};$

$$k_r := \frac{52}{15}\,\frac{EI}{l} \qquad (5)$$

>

> restart;

> $eq1 := -\frac{T\cdot l}{6\,EI} + \frac{M_c\cdot l}{3\,EI} = -\frac{M_c\cdot l}{3\,EI} + \frac{M_d\cdot l}{6\,EI};$

$$eq1 := -\frac{1}{6}\frac{T\,l}{EI} + \frac{1}{3}\frac{M_c\,l}{EI} = -\frac{1}{3}\frac{M_c\,l}{EI} + \frac{1}{6}\frac{M_d\,l}{EI} \quad \mathbf{(1)}$$

> $eq2 := \frac{M_c\cdot l}{6\,EI} - \frac{M_d\cdot l}{3\,EI} = \frac{M_d\cdot l}{3\,EI} - \frac{M_e\cdot l}{6\,EI};$

$$eq2 := \frac{1}{6}\frac{M_c\,l}{EI} - \frac{1}{3}\frac{M_d\,l}{EI} = \frac{1}{3}\frac{M_d\,l}{EI} - \frac{1}{6}\frac{M_e\,l}{EI} \quad \mathbf{(2)}$$

> $eq3 := -\frac{M_d\cdot l}{6\,EI} + \frac{M_e\cdot l}{3\,EI} = -\frac{M_e\cdot\frac{3}{4}\,l}{3\,EI};$

$$eq3 := -\frac{1}{6}\frac{M_d\,l}{EI} + \frac{1}{3}\frac{M_e\,l}{EI} = -\frac{1}{4}\frac{M_e\,l}{EI} \quad \mathbf{(3)}$$

> $solution := solve(\{eq1, eq2, eq3\}, \{M_c, M_d, M_e\});$

$$solution := \left\{M_c = \frac{26}{97}\,T,\ M_d = \frac{7}{97}\,T,\ M_e = \frac{2}{97}\,T\right\} \quad \mathbf{(4)}$$

> $assign(solution);$

> $\theta := \frac{T\cdot l}{3\,EI} - \frac{M_c\cdot l}{6\,EI};$

$$\theta := \frac{28}{97}\frac{T\,l}{EI} \quad \mathbf{(5)}$$

> $k_r := \frac{T}{\theta};$

$$k_r := \frac{97}{28}\frac{EI}{l} \quad \mathbf{(6)}$$

>

> restart;

> $eq1 := -\frac{T \cdot l}{6\,EI} + \frac{M_c \cdot l}{3\,EI} = -\frac{M_c \cdot l}{3\,EI} + \frac{M_d \cdot l}{6\,EI};$

$$eq1 := -\frac{1}{6}\frac{T\,l}{EI} + \frac{1}{3}\frac{M_c\,l}{EI} = -\frac{1}{3}\frac{M_c\,l}{EI} + \frac{1}{6}\frac{M_d\,l}{EI} \quad (1)$$

> $eq2 := \frac{M_c \cdot l}{6\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} - \frac{M_e \cdot l}{6\,EI};$

$$eq2 := \frac{1}{6}\frac{M_c\,l}{EI} - \frac{1}{3}\frac{M_d\,l}{EI} = \frac{1}{3}\frac{M_d\,l}{EI} - \frac{1}{6}\frac{M_e\,l}{EI} \quad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} + \frac{M_e \cdot l}{3\,EI} = -\frac{M_e \cdot l}{3\,EI} + \frac{M_f \cdot l}{6\,EI};$

$$eq3 := -\frac{1}{6}\frac{M_d\,l}{EI} + \frac{1}{3}\frac{M_e\,l}{EI} = -\frac{1}{3}\frac{M_e\,l}{EI} + \frac{1}{6}\frac{M_f\,l}{EI} \quad (3)$$

> $eq4 := \frac{M_e \cdot l}{6\,EI} - \frac{M_f \cdot l}{3\,EI} = \frac{M_f \cdot \frac{3}{4}\,l}{3\,EI};$

$$eq4 := \frac{1}{6}\frac{M_e\,l}{EI} - \frac{1}{3}\frac{M_f\,l}{EI} = \frac{1}{4}\frac{M_f\,l}{EI} \quad (4)$$

> $solution := solve(\{eq1, eq2, eq3, eq4\}, \{M_c, M_d, M_e, M_f\});$

$$solution := \left\{M_c = \frac{97}{362}\,T,\; M_d = \frac{13}{181}\,T,\; M_e = \frac{7}{362}\,T,\; M_f = \frac{1}{181}\,T\right\} \quad (5)$$

> $assign(solution);$

> $\theta := \frac{T \cdot l}{3\,EI} - \frac{M_c \cdot l}{6\,EI};$

$$\theta := \frac{209}{724}\frac{T\,l}{EI} \quad (6)$$

> $k_r := \frac{T}{\theta};$

$$k_r := \frac{724}{209}\frac{EI}{l} \quad (7)$$

>

> $restart;$

> $eq1 := -\frac{T \cdot l}{6\,EI} + \frac{M_c \cdot l}{3\,EI} = -\frac{M_c \cdot l}{3\,EI} + \frac{M_d \cdot l}{6\,EI};$

$$eq1 := -\frac{1}{6}\frac{T\,l}{EI} + \frac{1}{3}\frac{M_c\,l}{EI} = -\frac{1}{3}\frac{M_c\,l}{EI} + \frac{1}{6}\frac{M_d\,l}{EI} \quad (1)$$

> $eq2 := \frac{M_c \cdot l}{6\,EI} - \frac{M_d \cdot l}{3\,EI} = \frac{M_d \cdot l}{3\,EI} - \frac{M_e \cdot l}{6\,EI};$

$$eq2 := \frac{1}{6}\frac{M_c\,l}{EI} - \frac{1}{3}\frac{M_d\,l}{EI} = \frac{1}{3}\frac{M_d\,l}{EI} - \frac{1}{6}\frac{M_e\,l}{EI} \quad (2)$$

> $eq3 := -\frac{M_d \cdot l}{6\,EI} + \frac{M_e \cdot l}{3\,EI} = -\frac{M_e \cdot l}{3\,EI} + \frac{M_f \cdot l}{6\,EI};$

$$eq3 := -\frac{1}{6}\frac{M_d\,l}{EI} + \frac{1}{3}\frac{M_e\,l}{EI} = -\frac{1}{3}\frac{M_e\,l}{EI} + \frac{1}{6}\frac{M_f\,l}{EI} \quad (3)$$

> $eq4 := \frac{M_e \cdot l}{6\,EI} - \frac{M_f \cdot l}{3\,EI} = \frac{M_f \cdot l}{3\,EI} - \frac{M_g \cdot l}{6\,EI};$

$$eq4 := \frac{1}{6}\frac{M_e\,l}{EI} - \frac{1}{3}\frac{M_f\,l}{EI} = \frac{1}{3}\frac{M_f\,l}{EI} - \frac{1}{6}\frac{M_g\,l}{EI} \quad (4)$$

> $eq5 := -\frac{M_f \cdot l}{6\,EI} + \frac{M_g \cdot l}{3\,EI} = -\frac{M_g \cdot \frac{3}{4} l}{3\,EI};$

$$eq5 := -\frac{1}{6}\frac{M_f\,l}{EI} + \frac{1}{3}\frac{M_g\,l}{EI} = -\frac{1}{4}\frac{M_g\,l}{EI} \quad (5)$$

> $solution := solve(\{eq1, eq2, eq3, eq4, eq5\}, \{M_c, M_d, M_e, M_f, M_g\});$

$$solution := \left\{M_c = \frac{362}{1351}\,T, M_d = \frac{97}{1351}\,T, M_e = \frac{26}{1351}\,T, M_f = \frac{1}{193}\,T, M_g = \frac{2}{1351}\,T\right\} \quad (6)$$

> $assign(solution);$

> $\theta := \frac{T \cdot l}{3\,EI} - \frac{M_c \cdot l}{6\,EI};$

$$\theta := \frac{390}{1351}\frac{T\,l}{EI} \quad (7)$$

> $k_r := \frac{T}{\theta};$

$$k_r := \frac{1351}{390}\frac{EI}{l} \quad (8)$$

>

24.8 Tendon arrangement for the use phase

Use phase loads

Dimensions

ltot	550000	mm
number of spans	10	
lmid	57895	mm
lend	43421	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	4	
Ap	29400	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	1700	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Loads in the use phase

Self-weight

qGk,sw	174	N/mm

SIDL

asphalt	44	N/mm
footpath	3,9	N/mm
edge element	3,1	N/mm
parapet	0,75	N/mm
safety barrier	1	N/mm
qGk,sidl	62	N/mm

Traffic

αq1*q1k	0	kN/m^2
αqo*qok	0	kN/m^2
qQk,udl	0	N/mm
Q1k	0	N
Q2k	0	N
Q3k	0	N

Time-dependent losses

Assume:	6	%
σp∞	1311	N/mm^2

Creep

σcc	-10	N/mm^2
εcc	4,17E-05	
Δσpc	8	N/mm^2

Relaxation

t	876000	hours
Δσpr	75	N/mm^2
Δσp,c+r	84	N/mm^2
σp∞	1311	N/mm^2

Continuity prestressing

e0	630	mm
eb	1670	mm
a	19298	mm
tan θ	0,12	
θ	6,80	°
sin θ	0,12	
Fv	4,56E+06	N

Bending moments by prestressing

Msup	5,87E+10	Nmm
Mspan	2,93E+10	Nmm
σc,sup	-4,91	N/mm^2
σc,span	-4,97	N/mm^2

Check: σc ≤ 0	ok!

Bending moments in the use phase (SLS)

Intermediate beam support

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	6,59E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	0,00E+00	rad
a	21432	mm
θQk,ts,a	0,00E+00	rad
θQk,ts,b	0,00E+00	rad
θa,int	3,95E-03	rad
θb,int	3,95E-03	rad
Ma	-6,59E+10	Nmm
Mb	-6,60E+10	Nmm
σc,sup	7,5	N/mm^2

Intermediate beam span

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	6,59E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	0,00E+00	rad
a	28947	mm
θQk,ts,a	0,00E+00	rad
θQk,ts,b	0,00E+00	rad
θa,int	3,95E-03	rad
θb,int	3,95E-03	rad
Ma	-6,59E+10	Nmm
Mb	-6,60E+10	Nmm
Mint	9,89E+10	Nmm
Mspan	3,30E+10	Nmm
σc,span	7,1	N/mm^2

End beam span

k0	6,68E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Mb,ext	6,60E+10	Nmm
θGk	1,67E-03	rad
θQk,udl	0,00E+00	rad
a	16168	mm
θQk,ts,b	0,00E+00	rad
θb,int	1,67E-03	rad
Mb	-6,12E+10	Nmm
Mint	5,34E+10	Nmm
Mspan	2,89E+10	Nmm

Use phase loads

Dimensions

ltot	550000	mm
number of spans	10	
lmid	57895	mm
lend	43421	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	4	
Ap	29400	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	1700	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Loads in the use phase

Self-weight

qGk,sw	174	N/mm

SIDL

asphalt	44	N/mm
footpath	3,9	N/mm
edge element	3,1	N/mm
parapet	0,75	N/mm
safety barrier	1	N/mm
qGk,sidl	62	N/mm

Traffic

αq1*q1k	10,35	kN/m^2
αqo*qok	3,5	kN/m^2
qQk,udl	65	N/mm
Q1k	150000	N
Q2k	100000	N
Q3k	50000	N

Time-dependent losses

Assume:	6	%
σp∞	1311	N/mm^2

Creep

σcc	-14	N/mm^2
εcc	5,77E-05	
Δσpc	11	N/mm^2

Relaxation

t	876000	hours
Δσpr	75	N/mm^2
Δσp,c+r	87	N/mm^2
σp∞	1308	N/mm^2

Continuity prestressing

e0	630	mm
eb	1670	mm
a	19298	mm
tan θ	0,12	
θ	6,80	°
sin θ	0,12	
Fv	4,56E+06	N

Bending moments by prestressing

Msup	5,87E+10	Nmm
Mspan	2,93E+10	Nmm
σc,sup	-2,28	N/mm^2
σc,span	-0,24	N/mm^2
Check: σc ≤ 0	ok!	

Bending moments in the use phase (SLS)

Intermediate beam support

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	8,43E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	21432	mm
θQk,ts,a	4,23E-04	rad
θQk,ts,b	3,61E-04	rad
θa,int	5,24E-03	rad
θb,int	5,18E-03	rad
Ma	**-8,90E+10**	**Nmm**
Mb	**-7,64E+10**	**Nmm**
σc,sup	**10,2**	**N/mm^2**

Intermediate beam span

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	6,59E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	28947	mm
θQk,ts,a	4,13E-04	rad
θQk,ts,b	4,18E-04	rad
θa,int	5,23E-03	rad
θb,int	5,23E-03	rad
Ma	-7,94E+10	Nmm
Mb	-7,97E+10	Nmm
Mint	1,34E+11	Nmm
Mspan	**5,47E+10**	**Nmm**
σc,span	**11,9**	**N/mm^2**

End beam span

k0	6,68E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Mb,ext	6,60E+10	Nmm
θGk	1,67E-03	rad
θQk,udl	3,66E-04	rad
a	16168	mm
θQk,ts,b	2,05E-04	rad
θb,int	2,24E-03	rad
Mb	-7,00E+10	Nmm
Mint	7,48E+10	Nmm
Mspan	**4,68E+10**	**Nmm**

Use phase loads

Dimensions

ltot	550000	mm
number of spans	10	
lmid	57895	mm
lend	43421	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	4	
Ap	29400	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	1700	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Loads in the use phase

Self-weight

qGk,sw	174	N/mm

SIDL

asphalt	44	N/mm
footpath	3,9	N/mm
edge element	3,1	N/mm
parapet	0,75	N/mm
safety barrier	1	N/mm
qGk,sidl	62	N/mm

Traffic

αq1*q1k	10,35	kN/m^2
αqo*qok	3,5	kN/m^2
qQk,udl	65	N/mm
Q1k	150000	N
Q2k	100000	N
Q3k	50000	N

Time-dependent losses

Assume:	6	%
σp∞	1311	N/mm^2

Creep

σcc	-17	N/mm^2
εcc	6,77E-05	
Δσpc	13	N/mm^2

Relaxation

t	876000	hours
Δσpr	75	N/mm^2
Δσp,c+r	89	N/mm^2
σp∞	1306	N/mm^2

Continuity prestressing

e0	630	mm
eb	1670	mm
a	28947	mm
tan θ	0,08	
θ	4,54	°
sin θ	0,08	
Fv	3,05E+06	N

Bending moments by prestressing

Msup	4,42E+10	Nmm
Mspan	4,42E+10	Nmm
σc,sup	-0,61	N/mm^2
σc,span	-3,45	N/mm^2

Check: σc ≤ 0	ok!

Bending moments in the use phase (SLS)

Intermediate beam support

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	8,43E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	21432	mm
θQk,ts,a	4,23E-04	rad
θQk,ts,b	3,61E-04	rad
θa,int	5,24E-03	rad
θb,int	5,18E-03	rad
Ma	-8,90E+10	Nmm
Mb	-7,64E+10	Nmm
σc,sup	10,2	N/mm^2

Intermediate beam span

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	6,59E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	28947	mm
θQk,ts,a	4,13E-04	rad
θQk,ts,b	4,18E-04	rad
θa,int	5,23E-03	rad
θb,int	5,23E-03	rad
Ma	-7,94E+10	Nmm
Mb	-7,97E+10	Nmm
Mint	1,34E+11	Nmm
Mspan	5,47E+10	Nmm
σc,span	11,9	N/mm^2

End beam span

k0	6,68E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Mb,ext	6,60E+10	Nmm
θGk	1,67E-03	rad
θQk,udl	3,66E-04	rad
a	16168	mm
θQk,ts,b	2,05E-04	rad
θb,int	2,24E-03	rad
Mb	-7,00E+10	Nmm
Mint	7,48E+10	Nmm
Mspan	4,68E+10	Nmm

Use phase loads

Dimensions

ltot	550000	mm
number of spans	10	
lmid	57895	mm
lend	43421	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	4	
Ap	29400	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	1700	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Loads in the use phase

Self-weight

qGk,sw	174	N/mm

SIDL

asphalt	44	N/mm
footpath	3,9	N/mm
edge element	3,1	N/mm
parapet	0,75	N/mm
safety barrier	1	N/mm
qGk,sidl	62	N/mm

Traffic

αq1*q1k	10,35	kN/m^2
αqo*qok	3,5	kN/m^2
qQk,udl	65	N/mm
Q1k	150000	N
Q2k	100000	N
Q3k	50000	N

Time-dependent losses

Assume:	6	%
σp∞	1311	N/mm^2

Creep

σcc	-17	N/mm^2
εcc	6,77E-05	
Δσpc	13	N/mm^2

Relaxation

t	876000	hours
Δσpr	75	N/mm^2
Δσp,c+r	89	N/mm^2
σp∞	1306	N/mm^2

Continuity prestressing

e0	630	mm
eb	1670	mm
a	28947	mm
tan θ	0,08	
θ	4,54	°
sin θ	0,08	
Fv	3,05E+06	N

Bending moments by prestressing

Msup	4,42E+10	Nmm
Mspan	4,42E+10	Nmm
σc,sup	-3,48	N/mm^2
σc,span	-3,45	N/mm^2
Check: σc ≤ 0	ok!	

Bending moments in the use phase (SLS)

Intermediate beam support

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	8,43E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	21432	mm
θQk,ts,a	4,23E-04	rad
θQk,ts,b	3,61E-04	rad
θa,int	5,24E-03	rad
θb,int	5,18E-03	rad
Ma	**-8,90E+10**	**Nmm**
Mb	**-7,64E+10**	**Nmm**
σc,sup	**10,2**	**N/mm^2**

Intermediate beam span

k0	5,01E+13	Nmm/rad
ka	2,89E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Ma,ext	6,59E+10	Nmm
Mb,ext	6,61E+10	Nmm
θGk	3,95E-03	rad
θQk,udl	8,68E-04	rad
a	28947	mm
θQk,ts,a	4,13E-04	rad
θQk,ts,b	4,18E-04	rad
θa,int	5,23E-03	rad
θb,int	5,23E-03	rad
Ma	-7,94E+10	Nmm
Mb	-7,97E+10	Nmm
Mint	1,34E+11	Nmm
Mspan	**5,47E+10**	**Nmm**
σc,span	**11,9**	**N/mm^2**

End beam span

k0	6,68E+13	Nmm/rad
kb	2,89E+13	Nmm/rad
Mb,ext	6,60E+10	Nmm
θGk	1,67E-03	rad
θQk,udl	3,66E-04	rad
a	16168	mm
θQk,ts,b	2,05E-04	rad
θb,int	2,24E-03	rad
Mb	-7,00E+10	Nmm
Mint	7,48E+10	Nmm
Mspan	**4,68E+10**	**Nmm**

25 Reduction of web thickness

Launch phase cantilever capacity

Moment Capacity (ULS)

bf	0	mm
bw	300	mm
bin	7350	mm
b	7950	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,42E+06	mm^2
z	2122	mm
Ic	9,41E+12	mm^4
e0	603	mm
eb	603	mm
dp	2725	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	6	
Ap	44100	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	810	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1347	N/mm^2
εp∞	6,91E-03	

Shear reinforcement

Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

When εb = εctmax: ε0 =	0,000690
When εb' = εctmax: ε0 =	0,000749
When εb = εt,p: ε0 =	0,000894
When εb' = εt,p: ε0 =	0,000928
When εp = εp,y: ε0 =	0,001055
When dn = td: ε0 =	0,002684
When εb = εt,u: ε0 =	0,001158
When εb' = εt,u: ε0 =	0,001169
When εp = εud: ε0 =	0,001969

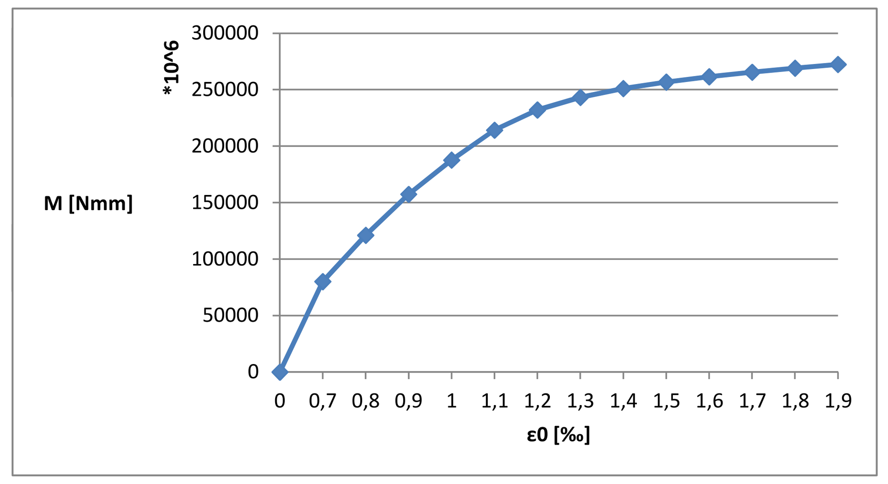
When dn = td & εp = εp,y: Ap =	-9150	mm^2
When dn = td & εb = εt,u: Ap =	-393	mm^2
When dn = td & εb' = εt,u: Ap =	393	mm^2
When εp = εud & ε0 = εcmax: Ap =	65066	mm^2
When dn = td & ε0 = εcmax: Ap =	42319	mm^2

Mu	2,72E+11	Nmm	MEd	2,17E+11	Nmm	Unity check: MEd/Mu ≤ 1	0,80	ok!

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0007	6,62E-04	4,84E-05	-1,23E-05	6,90E-03	1,08E-04	2774	2624	192	205	30	0	0	2,52E-07
0,0008	7,49E-04	2,02E-04	1,21E-04	7,03E-03	2,81E-04	2368	2218	296	301	0	0	0	3,38E-07
0,0009	8,32E-04	4,54E-04	3,44E-04	7,25E-03	5,61E-04	1971	1821	219	775	193	42	0	4,57E-07
0,001	9,08E-04	8,18E-04	6,71E-04	7,58E-03	9,62E-04	1631	1481	163	720	451	0	0	6,13E-07
0,0011	9,67E-04	1,54E-03	1,32E-03	8,23E-03	1,74E-03	1237	1087	112	497	1119	0	0	8,89E-07
0,0012	9,36E-04	0,00E+00	3,60E-03	1,05E-02	0,00E+00	681	531	57	250	1613	0	0	1,76E-06
0,0013	9,10E-04	0,00E+00	5,79E-03	1,27E-02	0,00E+00	500	350	38	170	1094	0	0	2,60E-06
0,0014	8,67E-04	0,00E+00	8,28E-03	1,52E-02	0,00E+00	394	244	28	124	800	0	0	3,55E-06
0,0015	8,13E-04	0,00E+00	1,10E-02	1,79E-02	0,00E+00	328	178	22	96	621	0	0	4,58E-06
0,0016	7,53E-04	0,00E+00	1,38E-02	2,07E-02	0,00E+00	283	133	18	78	504	0	0	5,65E-06
0,0017	6,89E-04	0,00E+00	1,67E-02	2,36E-02	0,00E+00	252	102	15	66	422	0	0	6,74E-06
0,0018	6,23E-04	0,00E+00	1,96E-02	2,65E-02	0,00E+00	229	79	13	56	362	0	0	7,85E-06
0,0019	5,54E-04	0,00E+00	2,25E-02	2,95E-02	0,00E+00	212	62	11	49	317	0	0	8,97E-06
0,002	4,85E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	198	48	10	44	282	0	0	0,00E+00
0,0021	4,15E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	187	37	9	39	253	0	0	0,00E+00
0,0022	3,44E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	178	28	8	36	230	0	0	0,00E+00
0,0023	2,72E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	170	20	7	33	210	0	0	0,00E+00
0,0024	2,00E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	164	14	7	30	194	0	0	0,00E+00
0,0025	1,28E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	158	8	6	28	180	0	0	0,00E+00
0,0026	5,63E-05	0,00E+00	0,00E+00	0,00E+00	0,00E+00	153	3	6	26	168	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0043	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0044	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0045	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00
0,0046	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0007	3,86E+08	-3,19E+08	7,88E+06	-1,70E+06	1,19E+06	0,00E+00	0,00E+00	-1,06E+05	1,79E-07	0	0	0	8,01E+10
0,0008	3,77E+08	-3,05E+08	4,44E+05	9,04E+05	9,33E+06	0,00E+00	0,00E+00	1,04E+06	0,00E+00	0	0	0	1,21E+11
0,0009	3,53E+08	-2,78E+08	3,29E+05	2,32E+06	7,65E+06	1,67E+06	0,00E+00	2,96E+06	0,00E+00	21	0	0	1,57E+11
0,001	3,24E+08	-2,47E+08	2,45E+05	2,16E+06	1,29E+06	8,18E+06	0,00E+00	5,77E+06	0,00E+00	222	123	0	1,88E+11
0,0011	2,71E+08	-1,93E+08	1,69E+05	1,49E+06	2,77E+06	5,72E+06	0,00E+00	7,82E+06	0,00E+00	520	131	0	2,14E+11
0,0012	1,62E+08	-9,12E+07	8,51E+04	7,51E+05	2,42E+06	0,00E+00	0,00E+00	8,45E+06	0,00E+00	0	0	0	2,32E+11
0,0013	1,29E+08	-5,85E+07	5,77E+04	5,10E+05	1,64E+06	0,00E+00	0,00E+00	9,05E+06	0,00E+00	0	0	0	2,43E+11
0,0014	1,10E+08	-3,89E+07	4,22E+04	3,73E+05	1,20E+06	0,00E+00	0,00E+00	9,73E+06	0,00E+00	0	0	0	2,51E+11
0,0015	9,77E+07	-2,65E+07	3,28E+04	2,89E+05	9,32E+05	0,00E+00	0,00E+00	1,05E+07	0,00E+00	0	0	0	2,57E+11
0,0016	9,01E+07	-1,85E+07	2,66E+04	2,35E+05	7,56E+05	0,00E+00	0,00E+00	1,12E+07	0,00E+00	0	0	0	2,61E+11
0,0017	8,52E+07	-1,30E+07	2,23E+04	1,97E+05	6,33E+05	0,00E+00	0,00E+00	1,20E+07	0,00E+00	0	0	0	2,66E+11
0,0018	8,20E+07	-9,08E+06	1,91E+04	1,69E+05	5,43E+05	0,00E+00	0,00E+00	1,28E+07	0,00E+00	0	0	0	2,69E+11
0,0019	8,00E+07	-6,30E+06	1,67E+04	1,48E+05	4,76E+05	0,00E+00	0,00E+00	1,36E+07	0,00E+00	0	0	0	2,72E+11
0,002	7,87E+07	-4,28E+06	1,49E+04	1,31E+05	4,22E+05	0,00E+00	0,00E+00	0,00E+00	1,45E+07	0	0	0	2,39E+11
0,0021	7,80E+07	-2,81E+06	1,34E+04	1,18E+05	3,80E+05	0,00E+00	0,00E+00	0,00E+00	1,53E+07	0	0	0	2,40E+11
0,0022	7,77E+07	-1,75E+06	1,21E+04	1,07E+05	3,45E+05	0,00E+00	0,00E+00	0,00E+00	1,61E+07	0	0	0	2,40E+11
0,0023	7,78E+07	-1,01E+06	1,11E+04	9,80E+04	3,16E+05	0,00E+00	0,00E+00	0,00E+00	1,69E+07	0	0	0	2,41E+11
0,0024	7,81E+07	-5,02E+05	1,02E+04	9,03E+04	2,91E+05	0,00E+00	0,00E+00	0,00E+00	1,78E+07	0	0	0	2,41E+11
0,0025	7,85E+07	-1,90E+05	9,49E+03	8,38E+04	2,70E+05	0,00E+00	0,00E+00	0,00E+00	1,86E+07	0	0	0	2,42E+11
0,0026	7,92E+07	-3,43E+04	8,85E+03	7,81E+04	2,52E+05	0,00E+00	0,00E+00	0,00E+00	1,94E+07	0	0	0	2,46E+11
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0043	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0044	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0045	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00
0,0046	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-5,94E+07	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,7	8,01E+10
0,8	1,21E+11
0,9	1,57E+11
1	1,88E+11
1,1	2,14E+11
1,2	2,32E+11
1,3	2,43E+11
1,4	2,51E+11
1,5	2,57E+11
1,6	2,61E+11
1,7	2,66E+11
1,8	2,69E+11
1,9	2,72E+11



Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,59E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	202	mm
z	2658	mm
cot βu	1,00	

VRb	3,12E+06	N
Vf	9,81E+06	N
Vs	0,00E+00	N
Vu	9,81E+06	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	9,23E+06	N	Unity check: VEd/Vu ≤ 1	0,94	ok!

Use phase support capacity

Moment Capacity (ULS)

bf	0	mm
bw	300	mm
bin	7350	mm
b	7950	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,42E+06	mm^2
z	2122	mm
Ic	9,41E+12	mm^4
eb	603	mm
dp	2725	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	49	
number of tendons	9	
Ap	66150	mm^2
Øduct	157	mm
Øanchor	550	mm
anchor spacing	300	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1306	N/mm^2
εp∞	6,70E-03	

Shear reinforcement

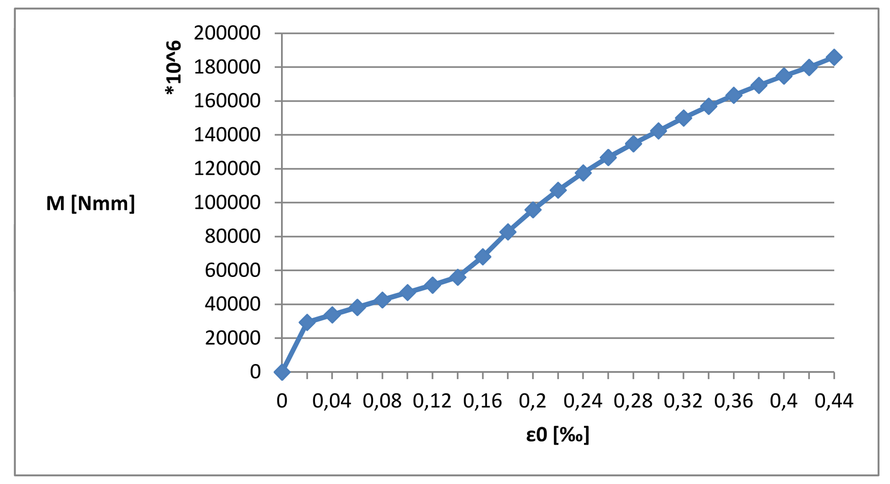
Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

When εb = εctmax: ε0 =	0,000134
When εb' = εctmax: ε0 =	0,000152
When εb = εt,p: ε0 =	0,000289
When εb' = εt,p: ε0 =	0,000305
When εp = εp,y: ε0 =	0,000439
When dn = td: ε0 =	0,000664
When εb = εt,u: ε0 =	0,000416
When εb' = εt,u: ε0 =	0,000415
When εp = εud: ε0 =	0,001124

When dn = td & εp = εp,y: Ap =	-58369	mm^2
When dn = td & εb = εt,u: Ap =	-2660	mm^2
When dn = td & εb' = εt,u: Ap =	2644	mm^2
When εp = εud & ε0 = εcmax: Ap =	298012	mm^2
When dn = td & ε0 = εcmax: Ap =	157320	mm^2

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,00002	1,84E-05	1,24E-05	9,73E-06	6,71E-03	1,49E-05	1833	1683	1133	0	0	0	0	1,09E-08
0,00004	3,67E-05	2,47E-05	1,95E-05	6,72E-03	2,98E-05	1833	1683	1133	0	0	0	0	2,18E-08
0,00006	5,51E-05	3,71E-05	2,92E-05	6,73E-03	4,48E-05	1833	1683	1133	0	0	0	0	3,27E-08
0,00008	7,35E-05	4,94E-05	3,89E-05	6,74E-03	5,97E-05	1833	1683	1133	0	0	0	0	4,36E-08
0,0001	9,18E-05	6,18E-05	4,87E-05	6,75E-03	7,46E-05	1833	1683	1133	0	0	0	0	5,46E-08
0,00012	1,10E-04	7,41E-05	5,84E-05	6,76E-03	8,95E-05	1833	1683	1133	0	0	0	0	6,55E-08
0,00014	1,29E-04	8,69E-05	6,85E-05	6,77E-03	1,05E-04	1830	1680	1136	171	63	0	0	7,65E-08
0,00016	1,46E-04	1,14E-04	9,19E-05	6,79E-03	1,36E-04	1731	1581	1082	153	0	0	0	9,24E-08
0,00018	1,63E-04	1,55E-04	1,28E-04	6,83E-03	1,81E-04	1594	1444	886	486	0	0	0	1,13E-07
0,0002	1,80E-04	2,02E-04	1,70E-04	6,87E-03	2,34E-04	1474	1324	737	755	0	0	0	1,36E-07
0,00022	1,96E-04	2,56E-04	2,18E-04	6,92E-03	2,94E-04	1370	1220	623	973	0	0	0	1,61E-07
0,00024	2,12E-04	3,16E-04	2,71E-04	6,97E-03	3,60E-04	1281	1131	534	1151	0	0	0	1,87E-07
0,00026	2,28E-04	3,80E-04	3,28E-04	7,03E-03	4,30E-04	1205	1055	464	1297	0	0	0	2,16E-07
0,00028	2,43E-04	4,48E-04	3,89E-04	7,09E-03	5,06E-04	1140	990	407	1418	0	0	0	2,46E-07
0,0003	2,58E-04	5,22E-04	4,56E-04	7,15E-03	5,87E-04	1082	932	361	1523	70	165	0	2,77E-07
0,00032	2,73E-04	6,11E-04	5,35E-04	7,23E-03	6,84E-04	1019	869	319	1407	220	235	0	3,14E-07
0,00034	2,87E-04	7,06E-04	6,21E-04	7,32E-03	7,89E-04	964	814	284	1252	466	235	0	3,53E-07
0,00036	3,01E-04	8,06E-04	7,12E-04	7,41E-03	8,99E-04	915	765	254	1123	673	235	0	3,93E-07
0,00038	3,15E-04	9,11E-04	8,07E-04	7,50E-03	1,01E-03	873	723	230	1014	849	235	0	4,36E-07
0,0004	3,28E-04	1,02E-03	9,06E-04	7,60E-03	1,13E-03	835	685	209	922	1000	235	0	4,79E-07
0,00042	3,41E-04	1,13E-03	1,01E-03	7,71E-03	1,26E-03	802	652	191	843	1130	235	0	5,24E-07
0,00044	3,52E-04	1,30E-03	1,16E-03	7,85E-03	1,43E-03	751	601	171	754	1289	235	0	5,86E-07
0,00046	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00048	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0005	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00052	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00054	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00056	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00058	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0006	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00062	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00064	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00066	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00068	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0007	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00072	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00074	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00076	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00078	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0008	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]	ε0 [‰]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,00002	7,29E+06	-5,68E+06	4,05E+06	-2,57E+06	0,00E+00	0,00E+00	0,00E+00	1,26E+05	0,00E+00	0	0	0	2,94E+10	0,02	2,94E+10
0,00004	1,46E+07	-1,14E+07	8,11E+06	-5,14E+06	0,00E+00	0,00E+00	0,00E+00	2,51E+05	0,00E+00	0	0	0	3,38E+10	0,04	3,38E+10
0,00006	2,19E+07	-1,70E+07	1,22E+07	-7,72E+06	0,00E+00	0,00E+00	0,00E+00	3,77E+05	0,00E+00	0	0	0	3,82E+10	0,06	3,82E+10
0,00008	2,91E+07	-2,27E+07	1,62E+07	-1,03E+07	0,00E+00	0,00E+00	0,00E+00	5,02E+05	0,00E+00	0	0	0	4,26E+10	0,08	4,26E+10
0,0001	3,64E+07	-2,84E+07	2,03E+07	-1,29E+07	0,00E+00	0,00E+00	0,00E+00	6,28E+05	0,00E+00	0	0	0	4,70E+10	0,1	4,70E+10
0,00012	4,37E+07	-3,41E+07	2,43E+07	-1,54E+07	0,00E+00	0,00E+00	0,00E+00	7,53E+05	0,00E+00	0	0	0	5,14E+10	0,12	5,14E+10
0,00014	5,09E+07	-3,97E+07	2,60E+07	-1,81E+07	2,52E+06	0,00E+00	0,00E+00	8,84E+05	3,73E-08	0	0	0	5,60E+10	0,14	5,60E+10
0,00016	5,50E+07	-4,24E+07	1,62E+06	4,58E+05	9,33E+06	0,00E+00	0,00E+00	1,19E+06	0,00E+00	0	0	0	6,81E+10	0,16	6,81E+10
0,00018	5,70E+07	-4,33E+07	1,33E+06	1,46E+06	9,33E+06	0,00E+00	0,00E+00	1,65E+06	0,00E+00	0	0	0	8,28E+10	0,18	8,28E+10
0,0002	5,86E+07	-4,37E+07	1,11E+06	2,26E+06	9,33E+06	0,00E+00	0,00E+00	2,19E+06	0,00E+00	0	0	0	9,58E+10	0,2	9,58E+10
0,00022	5,99E+07	-4,39E+07	9,34E+05	2,92E+06	9,33E+06	0,00E+00	0,00E+00	2,81E+06	0,00E+00	0	0	0	1,07E+11	0,22	1,07E+11
0,00024	6,11E+07	-4,40E+07	8,01E+05	3,45E+06	9,33E+06	0,00E+00	0,00E+00	3,49E+06	0,00E+00	0	0	0	1,18E+11	0,24	1,18E+11
0,00026	6,23E+07	-4,41E+07	6,95E+05	3,89E+06	9,33E+06	0,00E+00	0,00E+00	4,23E+06	0,00E+00	0	0	0	1,27E+11	0,26	1,27E+11
0,00028	6,35E+07	-4,42E+07	6,11E+05	4,25E+06	9,33E+06	0,00E+00	0,00E+00	5,02E+06	0,00E+00	0	0	0	1,35E+11	0,28	1,35E+11
0,0003	6,45E+07	-4,42E+07	5,41E+05	4,57E+06	2,78E+06	6,50E+06	0,00E+00	5,88E+06	0,00E+00	82	0	0	1,42E+11	0,3	1,42E+11
0,00032	6,48E+07	-4,36E+07	4,78E+05	4,22E+06	6,53E+05	8,98E+06	0,00E+00	6,91E+06	0,00E+00	110	120	0	1,50E+11	0,32	1,50E+11
0,00034	6,51E+07	-4,29E+07	4,25E+05	3,76E+06	1,36E+06	8,65E+06	0,00E+00	8,01E+06	0,00E+00	230	120	0	1,57E+11	0,34	1,57E+11
0,00036	6,55E+07	-4,23E+07	3,81E+05	3,37E+06	1,92E+06	8,31E+06	0,00E+00	9,18E+06	0,00E+00	331	121	0	1,63E+11	0,36	1,63E+11
0,00038	6,59E+07	-4,18E+07	3,44E+05	3,04E+06	2,38E+06	7,95E+06	0,00E+00	1,04E+07	0,00E+00	415	122	0	1,69E+11	0,38	1,69E+11
0,0004	6,64E+07	-4,13E+07	3,13E+05	2,77E+06	2,75E+06	7,57E+06	0,00E+00	1,17E+07	0,00E+00	485	122	0	1,75E+11	0,4	1,75E+11
0,00042	6,69E+07	-4,09E+07	2,86E+05	2,53E+06	3,04E+06	7,18E+06	0,00E+00	1,30E+07	0,00E+00	543	123	0	1,80E+11	0,42	1,80E+11
0,00044	6,57E+07	-3,89E+07	2,56E+05	2,26E+06	3,35E+06	6,62E+06	0,00E+00	1,43E+07	0,00E+00	612	124	0	1,86E+11	0,44	1,86E+11
0,00046	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00048	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0005	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00052	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00054	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00056	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00058	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0006	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00062	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00064	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00066	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00068	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0007	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00072	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00074	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00076	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00078	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0008	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!



Mu	1,86E+11	Nmm

MEd	1,24E+11	Nmm

Unity check: MEd/Mu ≤ 1 — 0,67 ok!

Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,48E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	30	°
tan βu	0,58	
dn	751	mm
z	2475	mm
cot βu	1,73	

VRb	2,91E+06	N
Vf	1,58E+07	N
Vs	0,00E+00	N
Vu	1,58E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	1,33E+07	N

Unity check: VEd/Vu ≤ 1 — 0,84 ok!

26 Increasing the span length

Launch phase loads

Dimensions

ltot	550000	mm
number of spans	9	
lmid	64706	mm
lend	48529	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
e0	615	mm
eb	615	mm
dp	1719	mm
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	18	
Ap	148500	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	122	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Steel nose

l	15000	mm
h	3000	mm
b	1200	mm
tw	60	mm
tf	120	mm
γ	7,85E-05	N/mm^3
A	4,54E+05	mm^2

Partial factors

Launch phase: CC1

6.10a

γG	1,2
γQ	1,35

6.10b

γG	1,1
γQ	1,35

Bending moments in launch phase

Self-weight

MGk,sw,sup	-6,08E+10	Nmm
MGk,sw,span	3,04E+10	Nmm
MGk,sw,cant	-2,46E+11	Nmm

Nose

MGk,nose,cant	4,01E+09	Nmm

SLS

MEd,sls,sup	-6,08E+10	Nmm
MEd,sls,span	3,04E+10	Nmm
MEd,sls,cant	-2,46E+11	Nmm
σc,sls,sup	6,94	N/mm^2
σc,sls,span	6,58	N/mm^2
σc,sls,cant	28,07	N/mm^2

ULS

MEd,uls,sup	-7,29E+10	Nmm
MEd,uls,span	3,65E+10	Nmm
MEd,uls,cant	-2,95E+11	Nmm

Time-dependent losses

Creep

σcc	-30,91	N/mm^2
εcc	1,24E-04	
Δσpc	24	N/mm^2

Relaxation

Assume: the launch phase lasts half a year

t	4380	hours
Δσpr	28	N/mm^2
Δσp,c+r	52	N/mm^2
σp∞	1343	N/mm^2

Central prestressing

σc	-1,69	N/mm^2
Check: σc ≤ 0		ok!
n0	9	
nb	9	
Check: n0*e0 = nb*eb		ok!

Maximum compressive stress

σc	-83	N/mm^2
Check: σc ≤ 0.6fck		ok!

Shear forces in the launch phase

Self-weight

VGk,sw,sup	5,64E+06	N
VGk,sw,cant	9,19E+06	N

Nose

VGk,nose,cant	5,34E+05	N

SLS

VEd,sls,sup	5,64E+06	N
VEd,sls,cant	9,19E+06	N

ULS

VEd,uls,sup	6,76E+06	N
VEd,uls,cant	1,10E+07	N

Nose optimization

lnose 0 mm

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	0	0
5	4,38E+10	22418	22418
12,63	1,11E+11	35627	35627
15	1,31E+11	38829	38829
20	1,75E+11	44836	44836

lnose 10000 mm

Anose	2,02E+05	mm
Mnose	7,91E+08	Nmm
Vnose	1,58E+05	N

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	#GETAL!	#GETAL!
5	4,38E+10	21325	31325
12,63	1,11E+11	34603	44603
15	1,31E+11	37815	47815
20	1,75E+11	43836	53836

lnose 15000 mm

Anose	4,54E+05	mm
Mnose	4,01E+09	Nmm
Vnose	5,34E+05	N

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	#GETAL!	#GETAL!
5	4,38E+10	18521	33521
12,63	1,11E+11	32045	47045
15	1,31E+11	35290	50290
20	1,75E+11	41361	56361

lnose 20000 mm

Anose	8,06E+05	mm
Mnose	1,27E+10	Nmm
Vnose	1,27E+06	N

P [N/mm^2]	M [Nmm]	lcant [mm]	ltot [mm]
0	0,00E+00	#GETAL!	#GETAL!
5	4,38E+10	12983	32983
12,63	1,11E+11	27038	47038
15	1,31E+11	30353	50353
20	1,75E+11	36526	56526

lnose [mm]	Anose [mm^2]	Vnose [N]	Mnose [Nmm]	Qnose [kg]	Cost nose [€]	lcant [mm]	Mcant [Nmm]	Ap,req [mm^2]	ΔAp [mm^2]	ΔQp [kg/m]	ΔQp [kg/two spans]	Cost additional central prestressing [€]	Total cost [€]
10000	2,02E+05	1,58E+05	7,91E+08	1,61E+04	€ 48.396	54706	2,70E+11	153956	16362	1,28E+03	1,66E+05	€ 415.544	€ 463.940
11000	2,44E+05	2,11E+05	1,16E+09	2,15E+04	€ 64.416	53706	2,64E+11	150297	12702	9,97E+02	1,29E+05	€ 322.596	€ 387.012
12000	2,90E+05	2,73E+05	1,64E+09	2,79E+04	€ 83.629	52706	2,58E+11	147056	9461	7,43E+02	9,61E+04	€ 240.287	€ 323.916
13000	3,41E+05	3,48E+05	2,26E+09	3,54E+04	€ 106.327	51706	2,53E+11	144256	6662	5,23E+02	6,77E+04	€ 169.189	€ 275.516
14000	3,95E+05	4,34E+05	3,04E+09	4,43E+04	€ 132.800	50706	2,49E+11	141920	4325	3,40E+02	4,39E+04	€ 109.848	€ 242.647
15000	4,54E+05	5,34E+05	4,01E+09	5,44E+04	€ 163.338	49706	2,46E+11	140066	2472	1,94E+02	2,51E+04	€ 62.781	€ 226.119
16000	5,16E+05	6,48E+05	5,19E+09	6,61E+04	€ 198.231	48706	2,43E+11	138716	1121	8,80E+01	1,14E+04	€ 28.480	€ 226.711
17000	5,83E+05	7,78E+05	6,61E+09	7,93E+04	€ 237.771	47706	2,42E+11	137886	292	2,29E+01	2,96E+03	€ 7.408	€ 245.179
18000	6,53E+05	9,23E+05	8,31E+09	9,41E+04	€ 282.247	46706	2,41E+11	137594	0	0,00E+00	0,00E+00	€ 0	€ 282.247
19000	7,28E+05	1,09E+06	1,03E+10	1,11E+05	€ 331.950	45706	2,42E+11	137857	262	2,06E+01	2,67E+03	€ 6.665	€ 338.615
20000	8,06E+05	1,27E+06	1,27E+10	1,29E+05	€ 387.171	44706	2,43E+11	138688	1094	8,59E+01	1,11E+04	€ 27.783	€ 414.954

Launch phase rear spans

Dimensions

ltot	550000	mm
number of spans	9	
lmid	64706	mm
lend	48529	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
e0	615	mm
eb	615	mm
dp	1719	mm
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	6	
Ap	49500	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	1112	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Partial factors

Launch phase: CC1

6.10a

γG	1,2
γQ	1,35

6.10b

γG	1,1
γQ	1,35

Bending moments in launch phase

Self-weight

MGk,sw,sup	-6,08E+10	Nmm
MGk,sw,span	3,04E+10	Nmm

SLS

MEd,sls,sup	-6,08E+10	Nmm
MEd,sls,span	3,04E+10	Nmm
σc,sls,sup	6,94	N/mm^2
σc,sls,span	6,58	N/mm^2

ULS

MEd,uls,sup	-7,29E+10	Nmm
MEd,uls,span	3,65E+10	Nmm

Time-dependent losses

Creep

σcc	-10,30	N/mm^2
εcc	4,12E-05	
Δσpc	8	N/mm^2

Relaxation

Assume: the launch phase lasts half a year

t	4380	hours
Δσpr	28	N/mm^2
Δσp,c+r	36	N/mm^2
σp∞	1359	N/mm^2

Central prestressing

σc	-3,10	N/mm^2
Check: σc ≤ 0	ok!	
n0	3	
nb	3	
Check: n0*e0 = nb*eb	ok!	

Shear forces in the launch phase

Self-weight

VGk,sw,sup	5,64E+06	N

SLS

VEd,sls,sup	5,64E+06	N

ULS

VEd,uls,sup	6,76E+06	N

Launch phase mid span capacity

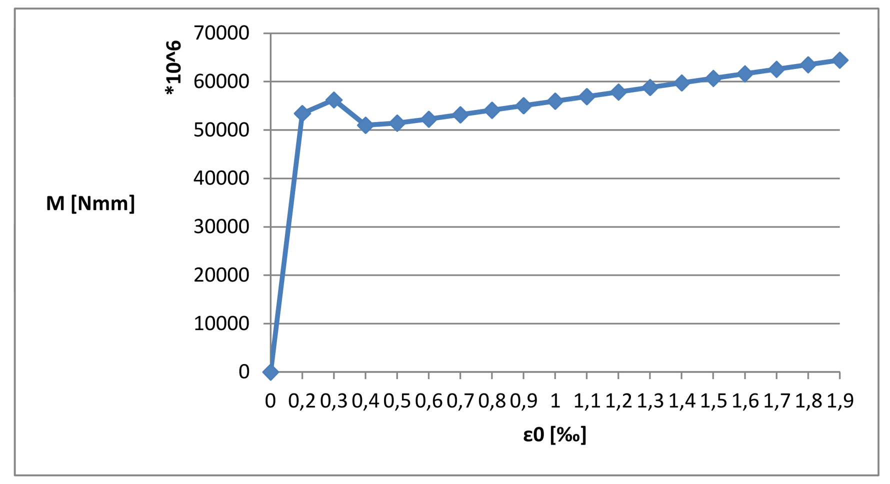
Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td = d*	235	mm
tf	150	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	1105	mm
Ic	9,67E+12	mm^4
e0	615	mm
eb	615	mm
dp	1719	mm
UHPFRC		
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
Y1860S7 prestressing		
Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	2	
Ap	16500	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	6140	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1343	N/mm^2
εp∞	6,89E-03	

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]
0	0	0	0	0	0	0	0	0	0	0	0	0
0,0002	1,55E-04	3,78E-04	1,26E-04	7,01E-03	4,07E-04	1055	819	527	1468	0	0	0
0,0003	5,38E-05	2,89E-03	1,50E-03	8,39E-03	3,05E-03	287	51	96	422	2246	0	0
0,0004	8,72E-05	0,00E+00	3,16E-03	1,00E-02	0,00E+00	193	0	48	213	1373	0	42
0,0005	1,98E-04	0,00E+00	4,60E-03	1,15E-02	0,00E+00	168	0	34	149	958	0	67
0,0006	3,04E-04	0,00E+00	6,01E-03	1,29E-02	0,00E+00	156	0	26	115	740	0	79
0,0007	4,08E-04	0,00E+00	7,40E-03	1,43E-02	0,00E+00	149	0	21	94	604	0	87
0,0008	5,12E-04	0,00E+00	8,78E-03	1,57E-02	0,00E+00	143	0	18	79	510	0	92
0,0009	6,14E-04	0,00E+00	1,02E-02	1,71E-02	0,00E+00	140	0	16	69	442	0	95
0,001	7,16E-04	0,00E+00	1,15E-02	1,84E-02	0,00E+00	137	0	14	61	390	0	98
0,0011	8,18E-04	0,00E+00	1,29E-02	1,98E-02	0,00E+00	135	0	12	54	349	0	100
0,0012	9,19E-04	0,00E+00	1,43E-02	2,12E-02	0,00E+00	133	0	11	49	316	0	102
0,0013	1,02E-03	0,00E+00	1,57E-02	2,25E-02	0,00E+00	132	0	10	45	288	0	103
0,0014	1,12E-03	0,00E+00	1,70E-02	2,39E-02	0,00E+00	131	0	9	41	265	0	105
0,0015	1,22E-03	0,00E+00	1,84E-02	2,53E-02	0,00E+00	130	0	9	38	246	0	106
0,0016	1,32E-03	0,00E+00	1,98E-02	2,67E-02	0,00E+00	129	0	8	36	229	0	107
0,0017	1,43E-03	0,00E+00	2,11E-02	2,80E-02	0,00E+00	128	0	8	33	214	0	107
0,0018	1,53E-03	0,00E+00	2,25E-02	2,94E-02	0,00E+00	127	0	7	31	201	0	108
0,0019	1,63E-03	0,00E+00	2,39E-02	3,08E-02	0,00E+00	127	0	7	29	190	0	109
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0

ε0 [-]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0002	1,90E-07	8,17E+07	-4,71E+07	9,23E+05	5,14E+06	6,00E+06	0,00E+00	0,00E+00	4,05E+05	0,00E+00	0	0	0	5,34E+10
0,0003	1,05E-06	3,33E+07	-1,02E+06	1,67E+05	1,48E+06	4,61E+06	8,76E+05	0,00E+00	3,01E+06	0,00E+00	859	113	0	5,62E+10
0,0004	2,07E-06	2,99E+07	0,00E+00	8,45E+04	7,46E+05	2,40E+06	0,00E+00	1,36E+06	3,18E+06	0,00E+00	0	0	0	5,10E+10
0,0005	2,97E-06	3,26E+07	0,00E+00	5,90E+04	5,21E+05	1,68E+06	0,00E+00	4,90E+06	3,33E+06	1,83E-07	0	0	0	5,14E+10
0,0006	3,84E-06	3,63E+07	0,00E+00	4,55E+04	4,02E+05	1,29E+06	0,00E+00	8,91E+06	3,47E+06	-3,62E-06	0	0	0	5,22E+10
0,0007	4,71E-06	4,03E+07	0,00E+00	3,71E+04	3,28E+05	1,06E+06	0,00E+00	1,31E+07	3,62E+06	-6,26E-07	0	0	0	5,32E+10
0,0008	5,58E-06	4,45E+07	0,00E+00	3,14E+04	2,77E+05	8,93E+05	0,00E+00	1,74E+07	3,76E+06	1,00E-06	0	0	0	5,41E+10
0,0009	6,44E-06	4,88E+07	0,00E+00	2,72E+04	2,40E+05	7,73E+05	0,00E+00	2,17E+07	3,90E+06	2,64E-07	0	0	0	5,50E+10
0,001	7,29E-06	5,31E+07	0,00E+00	2,40E+04	2,12E+05	6,82E+05	0,00E+00	2,60E+07	4,04E+06	2,91E-07	0	0	0	5,60E+10
0,0011	8,15E-06	5,75E+07	0,00E+00	2,15E+04	1,90E+05	6,10E+05	0,00E+00	3,04E+07	4,18E+06	2,64E-07	0	0	0	5,69E+10
0,0012	9,01E-06	6,19E+07	0,00E+00	1,94E+04	1,72E+05	5,52E+05	0,00E+00	3,47E+07	4,32E+06	-7,93E-07	0	0	0	5,79E+10
0,0013	9,87E-06	6,64E+07	0,00E+00	1,77E+04	1,57E+05	5,04E+05	0,00E+00	3,91E+07	4,46E+06	3,54E-07	0	0	0	5,88E+10
0,0014	1,07E-05	7,08E+07	0,00E+00	1,63E+04	1,44E+05	4,64E+05	0,00E+00	4,35E+07	4,60E+06	8,98E-07	0	0	0	5,97E+10
0,0015	1,16E-05	7,53E+07	0,00E+00	1,51E+04	1,34E+05	4,30E+05	0,00E+00	4,78E+07	4,75E+06	-6,67E-07	0	0	0	6,07E+10
0,0016	1,24E-05	7,98E+07	0,00E+00	1,41E+04	1,24E+05	4,00E+05	0,00E+00	5,22E+07	4,89E+06	3,69E-07	0	0	0	6,16E+10
0,0017	1,33E-05	8,43E+07	0,00E+00	1,32E+04	1,16E+05	3,75E+05	0,00E+00	5,66E+07	5,03E+06	-1,16E-06	0	0	0	6,26E+10
0,0018	1,41E-05	8,88E+07	0,00E+00	1,24E+04	1,09E+05	3,52E+05	0,00E+00	6,10E+07	5,17E+06	2,27E-07	0	0	0	6,35E+10
0,0019	1,50E-05	9,33E+07	0,00E+00	1,17E+04	1,03E+05	3,32E+05	0,00E+00	6,54E+07	5,31E+06	2,50E-07	0	0	0	6,44E+10
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,2	5,34E+10
0,3	5,62E+10
0,4	5,10E+10
0,5	5,14E+10
0,6	5,22E+10
0,7	5,32E+10
0,8	5,41E+10
0,9	5,50E+10
1	5,60E+10
1,1	5,69E+10
1,2	5,79E+10
1,3	5,88E+10
1,4	5,97E+10
1,5	6,07E+10
1,6	6,16E+10
1,7	6,26E+10
1,8	6,35E+10
1,9	6,44E+10



When εb = εctmax: ε0 =	0,000150
When εb' = εctmax: ε0 =	0,000155
When εb = εt,p: ε0 =	0,000213
When εb' = εt,p: ε0 =	0,000216
When εp = εp,y: ε0 =	0,000276
When dn = td: ε0 =	0,000329
When εb = εt,u: ε0 =	0,000307
When εb' = εt,u: ε0 =	0,000311
When εp = εud: ε0 =	0,001953

When dn = td & εp = εp,y: Ap =	1145	mm^2
When dn = td & εb = εt,u: Ap =	12089	mm^2
When dn = td & εb' = εt,u: Ap =	13242	mm^2
When εp = εud & ε0 = εcmax: Ap =	76192	mm^2
When dn = td & ε0 = εcmax: Ap =	143448	mm^2

Mu	6,44E+10	Nmm
MEd	3,65E+10	Nmm
Unity check: MEd/Mu ≤ 1	0,57	ok!

Launch phase support capacity

Moment Capacity (ULS)

bf	0	mm
bw	350	mm
bin	7300	mm
b	8000	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	2095	mm
Ic	9,67E+12	mm^4
e0	615	mm
eb	615	mm
dp	2710	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	2	
Ap	16500	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	6140	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1343	N/mm^2
εp∞	6,89E-03	

Shear reinforcement

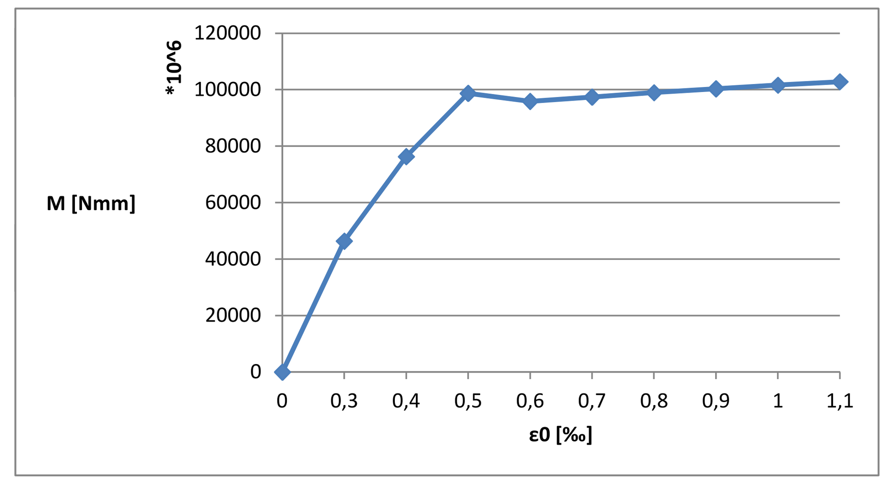
Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

When εb = εctmax: ε0 =	0,000322
When εb' = εctmax: ε0 =	0,000352
When εb = εt,p: ε0 =	0,000478
When εb' = εt,p: ε0 =	0,000494
When εp = εp,y: ε0 =	0,000563
When dn = td: ε0 =	0,000924
When εb = εt,u: ε0 =	0,000576
When εb' = εt,u: ε0 =	0,000575
When εp = εud: ε0 =	0,001183

When dn = td & εp = εp,y: Ap =	-9923	mm^2
When dn = td & εb = εt,u: Ap =	-934	mm^2
When dn = td & εb' = εt,u: Ap =	-102	mm^2
When εp = εud & ε0 = εcmax: Ap =	65386	mm^2
When dn = td & ε0 = εcmax: Ap =	42596	mm^2

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0003	2,82E-04	5,44E-05	2,39E-05	6,91E-03	8,25E-05	2510	2360	455	0	0	0	0	1,20E-07	1,51E+08	-1,21E+08	1,14E+07	-4,52E+06	0,00E+00	0,00E+00	0,00E+00	7,70E+04	0,00E+00	0	0	0	4,64E+10
0,0004	3,69E-04	2,07E-04	1,54E-04	7,04E-03	2,55E-04	1955	1805	489	521	0	0	0	2,05E-07	1,56E+08	-1,22E+08	8,55E+05	1,82E+06	9,41E+06	0,00E+00	0,00E+00	4,97E+05	0,00E+00	0	0	0	7,63E+10
0,0005	4,46E-04	5,76E-04	4,84E-04	7,37E-03	6,61E-04	1378	1228	276	1217	95	0	0	3,63E-07	1,38E+08	-9,98E+07	4,82E+05	4,26E+06	3,29E+05	9,15E+06	0,00E+00	1,56E+06	0,00E+00	47	121	0	9,87E+10
0,0006	3,38E-04	0,00E+00	4,14E-03	1,10E-02	0,00E+00	343	193	57	253	1626	0	0	1,75E-06	4,12E+07	-1,19E+07	1,00E+05	8,84E+05	2,85E+06	0,00E+00	0,00E+00	3,28E+06	0,00E+00	0	0	0	9,59E+10
0,0007	2,22E-04	0,00E+00	7,94E-03	1,48E-02	0,00E+00	220	70	31	139	892	0	0	3,19E-06	3,08E+07	-2,82E+06	5,49E+04	4,85E+05	1,56E+06	0,00E+00	0,00E+00	3,67E+06	0,00E+00	0	0	0	9,74E+10
0,0008	1,21E-04	0,00E+00	1,15E-02	1,84E-02	0,00E+00	177	27	22	98	628	0	0	4,53E-06	2,83E+07	-5,85E+05	3,86E+04	3,41E+05	1,10E+06	0,00E+00	0,00E+00	4,03E+06	0,00E+00	0	0	0	9,90E+10
0,0009	2,31E-05	0,00E+00	1,49E-02	2,18E-02	0,00E+00	154	4	17	76	486	0	0	5,85E-06	2,77E+07	-1,67E+04	2,99E+04	2,64E+05	8,51E+05	0,00E+00	0,00E+00	4,39E+06	0,00E+00	0	0	0	1,00E+11
0,001	7,25E-05	0,00E+00	1,84E-02	2,53E-02	0,00E+00	140	0	14	62	398	0	10	7,15E-06	2,80E+07	0,00E+00	2,45E+04	2,16E+05	6,96E+05	0,00E+00	1,34E+05	4,74E+06	0,00E+00	0	0	0	1,02E+11
0,0011	1,67E-04	0,00E+00	2,18E-02	2,87E-02	0,00E+00	130	0	12	52	337	0	20	8,45E-06	2,86E+07	0,00E+00	2,07E+04	1,83E+05	5,89E+05	0,00E+00	6,03E+05	5,09E+06	-4,47E-08	0	0	0	1,03E+11
0,0012	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-2,22E+07	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,3	4,64E+10
0,4	7,63E+10
0,5	9,87E+10
0,6	9,59E+10
0,7	9,74E+10
0,8	9,90E+10
0,9	1,00E+11
1	1,02E+11
1,1	1,03E+11



Mu	1,03E+11	Nmm
MEd	7,29E+10	Nmm
Unity check: MEd/Mu ≤ 1	0,71	ok!

Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,87E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	124	mm
z	2669	mm
cot βu	1,00	

VRb	3,66E+06	N
Vf	1,15E+07	N
Vs	0,00E+00	N
Vu	1,15E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	6,76E+06	N
Unity check: VEd/Vu ≤ 1	0,59	ok!

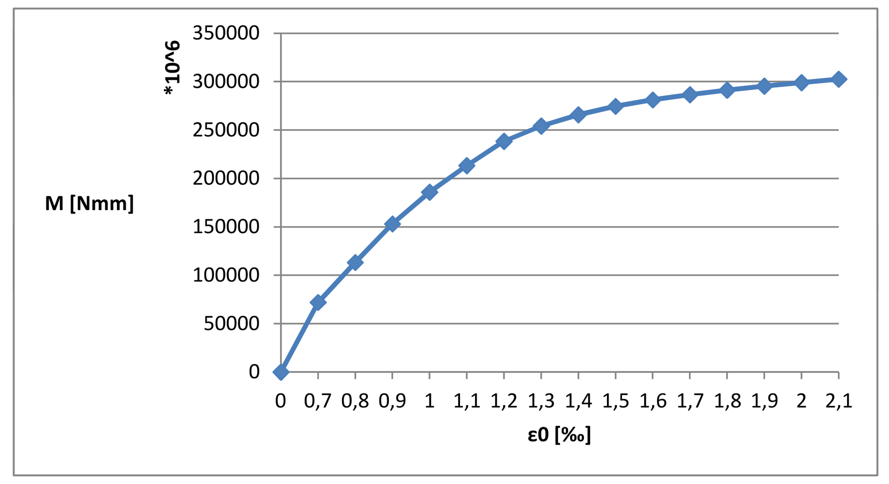
Launch phase cantilever capacity

Moment Capacity (ULS)

bf	0	mm
bw	350	mm
bin	7300	mm
b	8000	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	2095	mm
Ic	9,67E+12	mm^4
e0	615	mm
eb	615	mm
dp	2710	mm
UHPFRC		
f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
Y1860S7 prestressing		
Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	6	
Ap	49500	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	764	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1343	N/mm^2
εp∞	6,89E-03	
Shear reinforcement		
Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0007	6,63E-04	3,23E-05	-3,06E-05	6,86E-03	9,04E-05	2834	2684	131	0	0	0	0	2,47E-07	3,97E+08	-3,25E+08	6,62E+06	-7,71E+05	0,00E+00	0,00E+00	0,00E+00	-2,96E+05	0,00E+00	0	0	0	7,19E+10
0,0008	7,52E-04	1,46E-04	6,52E-05	6,95E-03	2,22E-04	2506	2356	313	146	0	0	0	3,19E-07	4,01E+08	-3,23E+08	5,48E+05	5,10E+05	9,41E+06	0,00E+00	0,00E+00	6,29E+05	0,00E+00	0	0	0	1,13E+11
0,0009	8,36E-04	3,60E-04	2,52E-04	7,14E-03	4,60E-04	2117	1967	235	612	0	0	0	4,25E-07	3,81E+08	-3,00E+08	4,12E+05	2,14E+06	9,41E+06	0,00E+00	0,00E+00	2,43E+06	0,00E+00	0	0	0	1,53E+11
0,001	9,17E-04	6,48E-04	5,06E-04	7,39E-03	7,78E-04	1799	1649	180	795	191	0	0	5,56E-07	3,60E+08	-2,76E+08	3,15E+05	2,78E+06	6,56E+05	8,84E+06	0,00E+00	4,89E+06	0,00E+00	95	123	0	1,86E+11
0,0011	9,92E-04	1,03E-03	8,48E-04	7,73E-03	1,20E-03	1531	1381	139	615	680	0	0	7,19E-07	3,37E+08	-2,50E+08	2,44E+05	2,15E+06	2,18E+06	7,51E+06	0,00E+00	8,18E+06	0,00E+00	330	126	0	2,13E+11
0,0012	1,04E-03	1,96E-03	1,69E-03	8,57E-03	2,21E-03	1127	977	94	415	1330	0	0	1,07E-06	2,70E+08	-1,85E+08	1,64E+05	1,45E+06	3,49E+06	4,31E+06	0,00E+00	9,09E+06	0,00E+00	591	141	0	2,38E+11
0,0013	1,03E-03	0,00E+00	3,66E-03	1,05E-02	0,00E+00	711	561	55	241	1555	0	0	1,83E-06	1,85E+08	-1,05E+08	9,57E+04	8,45E+05	2,72E+06	0,00E+00	0,00E+00	9,70E+06	0,00E+00	0	0	0	2,54E+11
0,0014	1,01E-03	0,00E+00	5,59E-03	1,25E-02	0,00E+00	543	393	39	171	1103	0	0	2,58E-06	1,52E+08	-7,27E+07	6,79E+04	6,00E+05	1,93E+06	0,00E+00	0,00E+00	1,03E+07	0,00E+00	0	0	0	2,66E+11
0,0015	9,83E-04	0,00E+00	7,84E-03	1,47E-02	0,00E+00	435	285	29	128	825	0	0	3,45E-06	1,31E+08	-5,12E+07	5,08E+04	4,49E+05	1,44E+06	0,00E+00	0,00E+00	1,10E+07	0,00E+00	0	0	0	2,75E+11
0,0016	9,41E-04	0,00E+00	1,03E-02	1,72E-02	0,00E+00	364	214	23	100	647	0	0	4,40E-06	1,16E+08	-3,67E+07	3,98E+04	3,52E+05	1,13E+06	0,00E+00	0,00E+00	1,17E+07	0,00E+00	0	0	0	2,81E+11
0,0017	8,90E-04	0,00E+00	1,29E-02	1,98E-02	0,00E+00	315	165	19	82	527	0	0	5,40E-06	1,07E+08	-2,68E+07	3,24E+04	2,86E+05	9,22E+05	0,00E+00	0,00E+00	1,26E+07	0,00E+00	0	0	0	2,87E+11
0,0018	8,34E-04	0,00E+00	1,57E-02	2,25E-02	0,00E+00	279	129	16	69	442	0	0	6,44E-06	1,01E+08	-1,97E+07	2,72E+04	2,40E+05	7,73E+05	0,00E+00	0,00E+00	1,34E+07	0,00E+00	0	0	0	2,91E+11
0,0019	7,74E-04	0,00E+00	1,84E-02	2,53E-02	0,00E+00	253	103	13	59	379	0	0	7,51E-06	9,62E+07	-1,46E+07	2,33E+04	2,06E+05	6,63E+05	0,00E+00	0,00E+00	1,42E+07	0,00E+00	0	0	0	2,95E+11
0,002	7,11E-04	0,00E+00	2,13E-02	2,82E-02	0,00E+00	233	83	12	51	331	0	0	8,59E-06	9,31E+07	-1,08E+07	2,04E+04	1,80E+05	5,79E+05	0,00E+00	0,00E+00	1,51E+07	0,00E+00	0	0	0	2,99E+11
0,0021	6,47E-04	0,00E+00	2,41E-02	3,10E-02	0,00E+00	217	67	10	46	294	0	0	9,69E-06	9,11E+07	-7,89E+06	1,81E+04	1,60E+05	5,14E+05	0,00E+00	0,00E+00	1,60E+07	0,00E+00	0	0	0	3,03E+11
0,0022	5,81E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	204	54	9	41	264	0	0	0,00E+00	8,97E+07	-5,72E+06	1,62E+04	1,43E+05	4,61E+05	0,00E+00	0,00E+00	0,00E+00	1,69E+07	0	0	0	2,64E+11
0,0023	5,15E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	193	43	8	37	239	0	0	0,00E+00	8,89E+07	-4,06E+06	1,47E+04	1,30E+05	4,18E+05	0,00E+00	0,00E+00	0,00E+00	1,78E+07	0	0	0	2,64E+11
0,0024	4,47E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	184	34	8	34	218	0	0	0,00E+00	8,85E+07	-2,80E+06	1,34E+04	1,19E+05	3,82E+05	0,00E+00	0,00E+00	0,00E+00	1,87E+07	0	0	0	2,65E+11
0,0025	3,79E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	177	27	7	31	201	0	0	0,00E+00	8,84E+07	-1,85E+06	1,24E+04	1,09E+05	3,52E+05	0,00E+00	0,00E+00	0,00E+00	1,96E+07	0	0	0	2,66E+11
0,0026	3,11E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	170	20	7	29	186	0	0	0,00E+00	8,86E+07	-1,16E+06	1,15E+04	1,01E+05	3,26E+05	0,00E+00	0,00E+00	0,00E+00	2,05E+07	0	0	0	2,71E+11
0,0027	2,49E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	165	15	6	27	174	0	0	0,00E+00	8,89E+07	-6,92E+05	1,07E+04	9,46E+04	3,05E+05	0,00E+00	0,00E+00	0,00E+00	2,14E+07	0	0	0	2,71E+11
0,0028	1,92E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	161	11	6	25	164	0	0	0,00E+00	8,95E+07	-3,86E+05	1,01E+04	8,89E+04	2,86E+05	0,00E+00	0,00E+00	0,00E+00	2,22E+07	0	0	0	2,72E+11
0,0029	1,39E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	158	8	5	24	155	0	0	0,00E+00	9,01E+07	-1,93E+05	9,51E+03	8,40E+04	2,70E+05	0,00E+00	0,00E+00	0,00E+00	2,30E+07	0	0	0	2,72E+11
0,003	9,09E-05	0,00E+00	0,00E+00	0,00E+00	0,00E+00	155	5	5	23	147	0	0	0,00E+00	9,07E+07	-7,77E+04	9,02E+03	7,97E+04	2,57E+05	0,00E+00	0,00E+00	0,00E+00	2,38E+07	0	0	0	2,72E+11
0,0031	4,58E-05	0,00E+00	0,00E+00	0,00E+00	0,00E+00	152	2	5	22	140	0	0	0,00E+00	9,14E+07	-1,88E+04	8,59E+03	7,59E+04	2,44E+05	0,00E+00	0,00E+00	0,00E+00	2,46E+07	0	0	0	2,72E+11
0,0032	3,82E-06	0,00E+00	0,00E+00	0,00E+00	0,00E+00	150	0	5	21	133	0	0	0,00E+00	9,21E+07	-1,25E+02	8,21E+03	7,25E+04	2,34E+05	0,00E+00	0,00E+00	0,00E+00	2,54E+07	0	0	0	2,72E+11
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0041	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0042	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0043	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0044	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0045	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00
0,0046	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	-6,65E+07	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,7	7,19E+10
0,8	1,13E+11
0,9	1,53E+11
1	1,86E+11
1,1	2,13E+11
1,2	2,38E+11
1,3	2,54E+11
1,4	2,66E+11
1,5	2,75E+11
1,6	2,81E+11
1,7	2,87E+11
1,8	2,91E+11
1,9	2,95E+11
2	2,99E+11
2,1	3,03E+11



When εb = εctmax: ε0 =	0,000712
When εb' = εctmax: ε0 =	0,000774
When εb = εt,p: ε0 =	0,000929
When εb' = εt,p: ε0 =	0,000967
When εp = εp,y: ε0 =	0,001118
When dn = td: ε0 =	0,003209
When εb = εt,u: ε0 =	0,001246
When εb' = εt,u: ε0 =	0,001259
When εp = εud: ε0 =	0,002116

When dn = td & εp = εp,y: Ap =	-9923	mm^2
When dn = td & εb = εt,u: Ap =	-934	mm^2
When dn = td & εb' = εt,u: Ap =	-102	mm^2
When εp = εud & ε0 = εcmax: Ap =	65386	mm^2
When dn = td & ε0 = εcmax: Ap =	42596	mm^2

Mu	3,03E+11	Nmm	MEd	2,95E+11	Nmm	Unity check: MEd/Mu ≤ 1	0,97	ok!

Shear Capacity (ULS)

γE*γb	1,5	
Seff	1,85E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	45	°
tan βu	1,00	
dn	215	mm
z	2638	mm
cot βu	1,00	

VRb	3,62E+06	N
Vf	1,14E+07	N
Vs	0,00E+00	N
Vu	1,14E+07	N

if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	1,10E+07	N	Unity check: VEd/Vu ≤ 1	0,97	ok!

Use phase loads

Dimensions

ltot	550000	mm
number of spans	9	
lmid	64706	mm
lend	48529	mm
lb	15500	mm
lp	7300	mm
l1	3750	mm
lv	1400	mm
d1	150	mm
d2	300	mm
d3	200	mm
d4	350	mm
d5	150	mm
h	3200	mm
hv	200	mm
hw	2650	mm
Ac	6,70E+06	mm^2
S	7,40E+09	mm^3
z0	1105	mm
zb	2095	mm
Ic	9,67E+12	mm^4
W0	8,76E+09	mm^3
Wb	4,62E+09	mm^3
d*	235	mm

Material properties

UHPFRC

fck	150	N/mm^2
fcd	128	N/mm^2
fctk	8	N/mm^2
fctd	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	
γ	2,6E-05	N/mm^3
ϕ	0,2	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	4	
Ap	33000	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	1660	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
ρ1000	2,5	%
μ	0,75	

Partial factors

Use phase: CC3

6.10a	
γG	1,4
γQ traffic	1,5
γQ other	1,65
ψ traffic	0,8
6.10b	
γG	1,25
γQ	1,5
6.15b	
ψ traffic	0,8

Loads in the use phase

Self-weight

qGk,sw	174	N/mm

SIDL

asphalt	44	N/mm
footpath	3,9	N/mm
edge element	3,1	N/mm
parapet	0,75	N/mm
safety barrier	1	N/mm
qGk,sidl	62	N/mm

Traffic

αq1*q1k	10,35	kN/m^2
αqo*qok	3,5	kN/m^2
qQk,udl	65	N/mm
Q1k	150000	N
Q2k	100000	N
Q3k	50000	N

Time-dependent losses

Assume:	6	%
σp∞	1311	N/mm^2

Creep

σcc	-21	N/mm^2
εcc	8,34E-05	
Δσpc	16	N/mm^2

Relaxation

t	876000	hours
Δσpr	75	N/mm^2
Δσp,c+r	92	N/mm^2
σp∞	1303	N/mm^2

Continuity prestressing

e0	615	mm
eb	1655	mm
a	32353	mm
tan θ	0,07	
θ	4,01	°
sin θ	0,07	
Fv	3,03E+06	N

Bending moments by prestressing

Msup	4,90E+10	Nmm
Mspan	4,90E+10	Nmm
σc,sup	-2,59	N/mm^2
σc,span	-2,48	N/mm^2

Check: σc ≤ 0	ok!

Bending moments in the use phase (SLS)

Intermediate beam support

k0	4,48E+13	Nmm/rad
ka	2,59E+13	Nmm/rad
kb	2,59E+13	Nmm/rad
Ma,ext	1,05E+11	Nmm
Mb,ext	8,26E+10	Nmm
θGk	5,51E-03	rad
θQk,udl	1,21E-03	rad
a	24023	mm
θQk,ts,a	5,29E-04	rad
θQk,ts,b	4,51E-04	rad
θa,int	7,25E-03	rad
θb,int	7,17E-03	rad
Ma	-1,11E+11	Nmm
Mb	-9,49E+10	Nmm
σc,sup	12,6	N/mm^2

Intermediate beam span

k0	4,48E+13	Nmm/rad
ka	2,59E+13	Nmm/rad
kb	2,59E+13	Nmm/rad
Ma,ext	8,23E+10	Nmm
Mb,ext	8,26E+10	Nmm
θGk	5,51E-03	rad
θQk,udl	1,21E-03	rad
a	32353	mm
θQk,ts,a	5,16E-04	rad
θQk,ts,b	5,22E-04	rad
θa,int	7,24E-03	rad
θb,int	7,25E-03	rad
Ma	-9,87E+10	Nmm
Mb	-9,90E+10	Nmm
Mint	1,66E+11	Nmm
Mspan	6,72E+10	Nmm
σc,span	14,6	N/mm^2

End beam span

k0	5,98E+13	Nmm/rad
kb	2,59E+13	Nmm/rad
Mb,ext	8,24E+10	Nmm
θGk	2,32E-03	rad
θQk,udl	5,11E-04	rad
a	18212	mm
θQk,ts,b	2,57E-04	rad
θb,int	3,09E-03	rad
Mb	-8,71E+10	Nmm
Mint	9,22E+10	Nmm
Mspan	5,74E+10	Nmm

Bending moments in the use phase (ULS 6.10a)

Intermediate beam support

Ma,ext	1,50E+11	Nmm
Mb,ext	1,16E+11	Nmm
θGk	7,72E-03	rad
θQk,udl	1,82E-03	rad
θQk,ts,a	7,93E-04	rad
θQk,ts,b	6,76E-04	rad
θa,int	1,03E-02	rad
θb,int	1,02E-02	rad
Ma	-1,58E+11	Nmm
Mb	-1,34E+11	Nmm
MEd,sup	-1,58E+11	Nmm

Intermediate beam span

Ma,ext	1,15E+11	Nmm
Mb,ext	1,16E+11	Nmm
θGk	7,72E-03	rad
θQk,udl	1,82E-03	rad
θQk,ts,a	7,74E-04	rad
θQk,ts,b	7,83E-04	rad
θa,int	1,03E-02	rad
θb,int	1,03E-02	rad
Ma	-1,40E+11	Nmm
Mb	-1,40E+11	Nmm
Mint	2,37E+11	Nmm
MEd,span	9,66E+10	Nmm

End beam span

Mb,ext	1,15E+11	Nmm
θGk	3,25E-03	rad
θQk,udl	7,67E-04	rad
θQk,ts,b	3,85E-04	rad
θb,int	4,41E-03	rad
Mb	-1,23E+11	Nmm
Mint	1,32E+11	Nmm
MEd,span	8,25E+10	Nmm

Bending moments in the use phase (ULS 6.10b)

Intermediate beam support

Ma,ext	1,46E+11	Nmm
Mb,ext	1,03E+11	Nmm
θGk	6,89E-03	rad
θQk,udl	2,27E-03	rad
θQk,ts,a	9,91E-04	rad
θQk,ts,b	8,45E-04	rad
θa,int	1,02E-02	rad
θb,int	1,00E-02	rad
Ma	-1,56E+11	Nmm
Mb	-1,26E+11	Nmm
MEd,sup	-1,56E+11	Nmm

Intermediate beam span

Ma,ext	1,03E+11	Nmm
Mb,ext	1,03E+11	Nmm
θGk	6,89E-03	rad
θQk,udl	2,27E-03	rad
θQk,ts,a	9,67E-04	rad
θQk,ts,b	9,79E-04	rad
θa,int	1,01E-02	rad
θb,int	1,01E-02	rad
Ma	-1,34E+11	Nmm
Mb	-1,34E+11	Nmm
Mint	2,34E+11	Nmm
MEd,span	1,00E+11	Nmm

End beam span

Mb,ext	1,03E+11	Nmm
θGk	2,91E-03	rad
θQk,udl	9,59E-04	rad
θQk,ts,b	4,81E-04	rad
θb,int	4,35E-03	rad
Mb	-1,15E+11	Nmm
Mint	1,31E+11	Nmm
MEd,span	8,51E+10	Nmm

Shear forces in the use phase (ULS 6.10a)

Intermediate beam support

Ma,ext	1,50E+11	Nmm
Mb,ext	1,16E+11	Nmm
θGk	7,72E-03	rad
θQk,udl	1,82E-03	rad
a	0	mm
θQk,ts,a	3,75E-05	rad
θQk,ts,b	1,93E-05	rad
θa,int	9,57E-03	rad
θb,int	9,55E-03	rad
Ma	-1,50E+11	Nmm
Mb	-1,28E+11	Nmm
VEd	1,50E+07	N

Shear forces in the use phase (ULS 6.10b)

Intermediate beam support

Ma,ext	1,46E+11	Nmm
Mb,ext	1,03E+11	Nmm
θGk	6,89E-03	rad
θQk,udl	2,27E-03	rad
a	0	mm
θQk,ts,a	4,68E-05	rad
θQk,ts,b	2,41E-05	rad
θa,int	9,21E-03	rad
θb,int	9,19E-03	rad
Ma	-1,47E+11	Nmm
Mb	-1,19E+11	Nmm
VEd	1,49E+07	N

Use phase mid span capacity

Moment Capacity (ULS)

bf	3750	mm
bw	350	mm
bin	7300	mm
b	15500	mm
td = d*	235	mm
tf	150	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	1105	mm
Ic	9,67E+12	mm^4
eb	1655	mm
dp	2760	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	0,003	
εt,p	0,00054	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	6	
Ap	49500	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	764	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1303	N/mm^2
εp∞	6,68E-03	

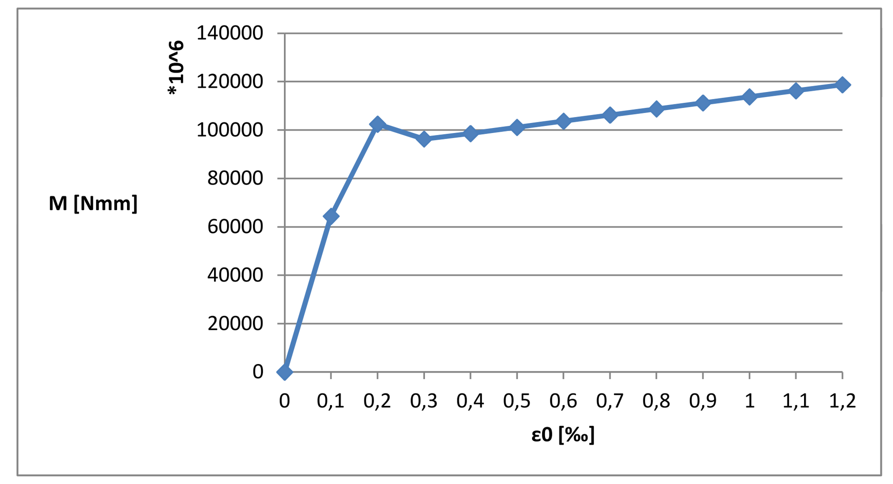
When εb = εctmax: ε0 =	0,000055
When εb' = εctmax: ε0 =	0,000058
When εb = εt,p: ε0 =	0,000121
When εb' = εt,p: ε0 =	0,000125
When εp = εp,y: ε0 =	0,000183
When dn = td: ε0 =	0,000215
When εb = εt,u: ε0 =	0,000228
When εb' = εt,u: ε0 =	0,000233
When εp = εud: ε0 =	0,001202

When dn = td & εp = εp,y: Ap =	-16836	mm^2
When dn = td & εb = εt,u: Ap =	80415	mm^2
When dn = td & εb' = εt,u: Ap =	87746	mm^2
When εp = εud & ε0 = εcmax: Ap =	687864	mm^2
When dn = td & ε0 = εcmax: Ap =	607282	mm^2

Mu	1,19E+11	Nmm
MEd	1,00E+11	Nmm
Unity check: MEd/Mu ≤ 1	0,84	ok!

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,0001	6,68E-05	3,31E-04	2,90E-04	6,97E-03	3,52E-04	708	473	708	1634	0	0	0	1,41E-07	2,74E+07	-1,17E+07	1,24E+06	5,72E+06	6,00E+06	0,00E+00	0,00E+00	2,80E+06	0,00E+00	0	0	0	6,44E+10
0,0002	3,88E-05	1,89E-03	1,69E-03	8,37E-03	1,99E-03	292	57	146	645	1968	0	0	6,85E-07	2,26E+07	-8,13E+05	2,55E+05	2,26E+06	5,25E+06	3,05E+06	0,00E+00	1,10E+07	0,00E+00	882	80	0	1,02E+11
0,0003	1,31E-04	0,00E+00	4,75E-03	1,14E-02	0,00E+00	164	0	55	241	1553	0	71	1,83E-06	1,90E+07	0,00E+00	9,56E+04	8,44E+05	2,72E+06	0,00E+00	3,45E+06	1,19E+07	0,00E+00	0	0	0	9,63E+10
0,0004	2,38E-04	0,00E+00	7,09E-03	1,38E-02	0,00E+00	147	0	37	163	1048	0	88	2,71E-06	2,29E+07	0,00E+00	6,45E+04	5,70E+05	1,83E+06	0,00E+00	7,73E+06	1,27E+07	1,53E-07	0	0	0	9,86E+10
0,0005	3,39E-04	0,00E+00	9,35E-03	1,60E-02	0,00E+00	140	0	28	124	797	0	95	3,57E-06	2,72E+07	0,00E+00	4,91E+04	4,33E+05	1,40E+06	0,00E+00	1,19E+07	1,33E+07	3,87E-07	0	0	0	1,01E+11
0,0006	4,38E-04	0,00E+00	1,16E-02	1,83E-02	0,00E+00	136	0	23	100	645	0	99	4,41E-06	3,16E+07	0,00E+00	3,97E+04	3,50E+05	1,13E+06	0,00E+00	1,61E+07	1,40E+07	3,13E-06	0	0	0	1,04E+11
0,0007	5,35E-04	0,00E+00	1,38E-02	2,05E-02	0,00E+00	133	0	19	84	542	0	102	5,25E-06	3,62E+07	0,00E+00	3,33E+04	2,94E+05	9,48E+05	0,00E+00	2,02E+07	1,47E+07	-7,30E-07	0	0	0	1,06E+11
0,0008	6,32E-04	0,00E+00	1,60E-02	2,27E-02	0,00E+00	131	0	16	73	467	0	104	6,09E-06	4,08E+07	0,00E+00	2,88E+04	2,54E+05	8,18E+05	0,00E+00	2,43E+07	1,54E+07	-4,02E-07	0	0	0	1,09E+11
0,0009	7,28E-04	0,00E+00	1,82E-02	2,49E-02	0,00E+00	130	0	14	64	411	0	105	6,92E-06	4,54E+07	0,00E+00	2,53E+04	2,23E+05	7,19E+05	0,00E+00	2,83E+07	1,61E+07	0,00E+00	0	0	0	1,11E+11
0,001	8,24E-04	0,00E+00	2,04E-02	2,71E-02	0,00E+00	129	0	13	57	367	0	106	7,75E-06	5,00E+07	0,00E+00	2,26E+04	1,99E+05	6,42E+05	0,00E+00	3,24E+07	1,68E+07	-1,12E-07	0	0	0	1,14E+11
0,0011	9,19E-04	0,00E+00	2,26E-02	2,93E-02	0,00E+00	128	0	12	51	331	0	107	8,58E-06	5,46E+07	0,00E+00	2,04E+04	1,80E+05	5,80E+05	0,00E+00	3,64E+07	1,74E+07	-2,46E-07	0	0	0	1,16E+11
0,0012	1,01E-03	0,00E+00	2,48E-02	3,15E-02	0,00E+00	127	0	11	47	302	0	108	9,41E-06	5,93E+07	0,00E+00	1,86E+04	1,64E+05	5,29E+05	0,00E+00	4,05E+07	1,81E+07	-6,63E-07	0	0	0	1,19E+11
0,0013	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0014	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0015	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0016	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0017	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0018	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0019	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,002	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0021	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0022	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0023	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0024	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0025	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0026	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0027	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0028	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0029	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,003	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0031	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0032	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0033	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0034	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0035	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0036	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0037	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0038	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,0039	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00
0,004	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0	0	0	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0	0	0	0,00E+00

ε0 [‰]	M [Nmm]
0	0
0,1	6,44E+10
0,2	1,02E+11
0,3	9,63E+10
0,4	9,86E+10
0,5	1,01E+11
0,6	1,04E+11
0,7	1,06E+11
0,8	1,09E+11
0,9	1,11E+11
1	1,14E+11
1,1	1,16E+11
1,2	1,19E+11



Use phase support capacity

Moment Capacity (ULS)

bf	0	mm
bw	350	mm
bin	7300	mm
b	8000	mm
td = tf	150	mm
tf = d*	235	mm
hw	2815	mm
h	3200	mm
Ac	6,70E+06	mm^2
z	2095	mm
Ic	9,67E+12	mm^4
eb	615	mm
dp	2710	mm

UHPFRC

f'c	150	N/mm^2
f'ct	8	N/mm^2
σctmax	5	N/mm^2
Ec	50000	N/mm^2
Lf	13	mm
εt,u	3,39E-03	
εt,p	5,42E-04	
εctmax	0,0001	
εc,u	0,007	
εc,p	0,004	
εcmax	0,00255	

Y1860S7 prestressing

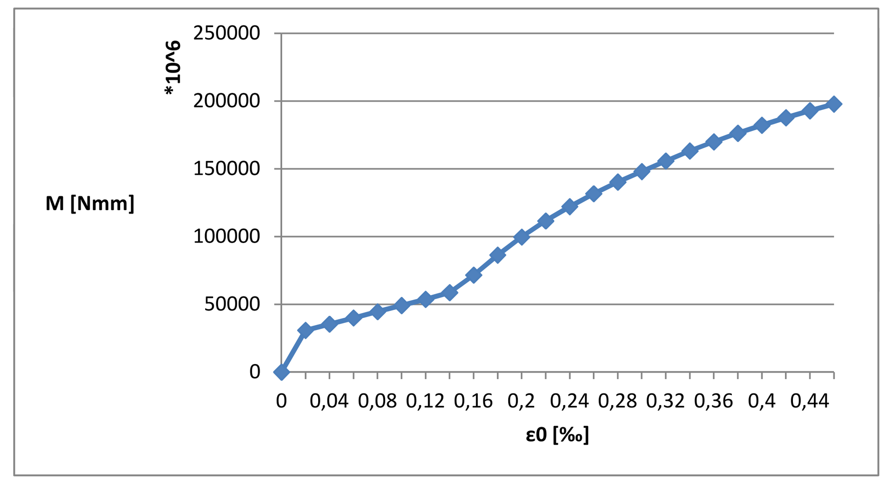
Østrand	16	mm
Astrand	150	mm^2
number of strands	55	
number of tendons	9	
Ap	74250	mm^2
Øduct	167	mm
Øanchor	580	mm
anchor spacing	260	mm
fpk	1860	N/mm^2
fpk/γs	1691	N/mm^2
fp0,1k	1674	N/mm^2
fpd	1522	N/mm^2
Ep	1,95E+05	N/mm^2
εpy	7,80E-03	
εuk	0,035	
εud	0,0315	
σpm0	1395	N/mm^2
σp∞	1303	N/mm^2
εp∞	6,68E-03	

Shear reinforcement

Ø stirrups	0	mm
fyk	500	N/mm^2
Asw	0	mm^2
s	300	mm

ε0 [-]	ε0' [-]	εb' [-]	Δεp [-]	εp [-]	εb [-]	dn [mm]	dn' [mm]	X1 [mm]	X2 [mm]	X3 [mm]	X4 [mm]	X5 [mm]	κ [1/mm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,00002	1,84E-05	1,25E-05	9,71E-06	6,69E-03	1,51E-05	1824	1674	1141	0	0	0	0	1,10E-08
0,00004	3,67E-05	2,50E-05	1,94E-05	6,70E-03	3,02E-05	1824	1674	1141	0	0	0	0	2,19E-08
0,00006	5,51E-05	3,75E-05	2,91E-05	6,71E-03	4,53E-05	1824	1674	1141	0	0	0	0	3,29E-08
0,00008	7,34E-05	5,00E-05	3,89E-05	6,72E-03	6,03E-05	1824	1674	1141	0	0	0	0	4,39E-08
0,0001	9,18E-05	6,25E-05	4,86E-05	6,73E-03	7,54E-05	1824	1674	1141	0	0	0	0	5,48E-08
0,00012	1,10E-04	7,50E-05	5,83E-05	6,74E-03	9,05E-05	1824	1674	1141	0	0	0	0	6,58E-08
0,00014	1,28E-04	8,82E-05	6,86E-05	6,75E-03	1,06E-04	1819	1669	1146	154	81	0	0	7,70E-08
0,00016	1,46E-04	1,15E-04	9,17E-05	6,78E-03	1,37E-04	1723	1573	1077	166	0	0	0	9,29E-08
0,00018	1,63E-04	1,53E-04	1,25E-04	6,81E-03	1,80E-04	1601	1451	889	475	0	0	0	1,12E-07
0,0002	1,80E-04	1,97E-04	1,63E-04	6,85E-03	2,29E-04	1492	1342	746	727	0	0	0	1,34E-07
0,00022	1,96E-04	2,47E-04	2,07E-04	6,89E-03	2,84E-04	1397	1247	635	932	0	0	0	1,57E-07
0,00024	2,13E-04	3,01E-04	2,55E-04	6,94E-03	3,44E-04	1315	1165	548	1102	0	0	0	1,83E-07
0,00026	2,29E-04	3,60E-04	3,07E-04	6,99E-03	4,09E-04	1244	1094	478	1243	0	0	0	2,09E-07
0,00028	2,44E-04	4,22E-04	3,62E-04	7,05E-03	4,78E-04	1182	1032	422	1360	0	0	0	2,37E-07
0,0003	2,60E-04	4,88E-04	4,20E-04	7,10E-03	5,50E-04	1129	979	376	1460	202	33	0	2,66E-07
0,00032	2,75E-04	5,63E-04	4,88E-04	7,17E-03	6,34E-04	1074	924	336	1482	73	235	0	2,98E-07
0,00034	2,90E-04	6,47E-04	5,62E-04	7,25E-03	7,25E-04	1021	871	300	1327	316	235	0	3,33E-07
0,00036	3,05E-04	7,35E-04	6,41E-04	7,32E-03	8,22E-04	975	825	271	1196	523	235	0	3,69E-07
0,00038	3,19E-04	8,27E-04	7,23E-04	7,41E-03	9,23E-04	933	783	246	1085	701	235	0	4,07E-07
0,0004	3,33E-04	9,23E-04	8,09E-04	7,49E-03	1,03E-03	896	746	224	990	855	235	0	4,46E-07
0,00042	3,47E-04	1,02E-03	8,99E-04	7,58E-03	1,14E-03	863	713	206	908	988	235	0	4,87E-07
0,00044	3,61E-04	1,12E-03	9,90E-04	7,67E-03	1,25E-03	834	684	189	837	1105	235	0	5,28E-07
0,00046	3,75E-04	1,23E-03	1,08E-03	7,77E-03	1,36E-03	807	657	175	775	1207	235	0	5,70E-07
0,00048	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0005	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00052	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00054	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00056	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00058	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0006	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00062	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00064	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00066	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00068	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0007	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00072	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00074	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00076	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,00078	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!
0,0008	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	235	#GETAL!	#GETAL!

ε0 [-]	C1 [N]	C2 [N]	T1 [N]	T2 [N]	T3 [N]	T4 [N]	T5 [N]	ΔNp [N]	ΣH = 0	y [mm]	z [mm]	β [mm]	M [Nmm]	ε0 [‰]	M [Nmm]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,00002	7,30E+06	-5,61E+06	4,15E+06	-2,60E+06	0,00E+00	0,00E+00	0,00E+00	1,41E+05	0,00E+00	0	0	0	3,08E+10	0,02	3,08E+10
0,00004	1,46E+07	-1,12E+07	8,30E+06	-5,21E+06	0,00E+00	0,00E+00	0,00E+00	2,81E+05	0,00E+00	0	0	0	3,54E+10	0,04	3,54E+10
0,00006	2,19E+07	-1,68E+07	1,25E+07	-7,81E+06	0,00E+00	0,00E+00	0,00E+00	4,22E+05	0,00E+00	0	0	0	4,00E+10	0,06	4,00E+10
0,00008	2,92E+07	-2,24E+07	1,66E+07	-1,04E+07	0,00E+00	0,00E+00	0,00E+00	5,63E+05	0,00E+00	0	0	0	4,45E+10	0,08	4,45E+10
0,0001	3,65E+07	-2,80E+07	2,08E+07	-1,30E+07	0,00E+00	0,00E+00	0,00E+00	7,03E+05	0,00E+00	0	0	0	4,91E+10	0,1	4,91E+10
0,00012	4,38E+07	-3,36E+07	2,49E+07	-1,56E+07	0,00E+00	0,00E+00	0,00E+00	8,44E+05	0,00E+00	0	0	0	5,37E+10	0,12	5,37E+10
0,00014	5,09E+07	-3,91E+07	2,60E+07	-1,84E+07	3,26E+06	0,00E+00	0,00E+00	9,93E+05	2,24E-08	0	0	0	5,86E+10	0,14	5,86E+10
0,00016	5,51E+07	-4,19E+07	1,88E+06	5,80E+05	9,41E+06	0,00E+00	0,00E+00	1,33E+06	0,00E+00	0	0	0	7,16E+10	0,16	7,16E+10
0,00018	5,76E+07	-4,32E+07	1,56E+06	1,66E+06	9,41E+06	0,00E+00	0,00E+00	1,81E+06	0,00E+00	0	0	0	8,63E+10	0,18	8,63E+10
0,0002	5,97E+07	-4,41E+07	1,31E+06	2,54E+06	9,41E+06	0,00E+00	0,00E+00	2,36E+06	0,00E+00	0	0	0	9,96E+10	0,2	9,96E+10
0,00022	6,15E+07	-4,47E+07	1,11E+06	3,26E+06	9,41E+06	0,00E+00	0,00E+00	2,99E+06	0,00E+00	0	0	0	1,12E+11	0,22	1,12E+11
0,00024	6,31E+07	-4,52E+07	9,59E+05	3,86E+06	9,41E+06	0,00E+00	0,00E+00	3,69E+06	0,00E+00	0	0	0	1,22E+11	0,24	1,22E+11
0,00026	6,47E+07	-4,56E+07	8,37E+05	4,35E+06	9,41E+06	0,00E+00	0,00E+00	4,44E+06	0,00E+00	0	0	0	1,32E+11	0,26	1,32E+11
0,00028	6,62E+07	-4,60E+07	7,39E+05	4,76E+06	9,41E+06	0,00E+00	0,00E+00	5,24E+06	0,00E+00	0	0	0	1,40E+11	0,28	1,40E+11
0,0003	6,77E+07	-4,65E+07	6,58E+05	5,11E+06	8,08E+06	1,32E+06	0,00E+00	6,08E+06	0,00E+00	17	0	0	1,48E+11	0,3	1,48E+11
0,00032	6,87E+07	-4,64E+07	5,87E+05	5,19E+06	2,55E+05	9,22E+06	0,00E+00	7,06E+06	0,00E+00	36	120	0	1,56E+11	0,32	1,56E+11
0,00034	6,95E+07	-4,61E+07	5,26E+05	4,64E+06	1,09E+06	8,93E+06	0,00E+00	8,14E+06	0,00E+00	157	121	0	1,63E+11	0,34	1,63E+11
0,00036	7,02E+07	-4,58E+07	4,74E+05	4,19E+06	1,77E+06	8,63E+06	0,00E+00	9,28E+06	0,00E+00	259	121	0	1,70E+11	0,36	1,70E+11
0,00038	7,09E+07	-4,56E+07	4,30E+05	3,80E+06	2,33E+06	8,31E+06	0,00E+00	1,05E+07	0,00E+00	344	122	0	1,76E+11	0,38	1,76E+11
0,0004	7,17E+07	-4,54E+07	3,92E+05	3,46E+06	2,79E+06	7,97E+06	0,00E+00	1,17E+07	0,00E+00	417	122	0	1,82E+11	0,4	1,82E+11
0,00042	7,25E+07	-4,52E+07	3,60E+05	3,18E+06	3,17E+06	7,63E+06	0,00E+00	1,30E+07	0,00E+00	479	123	0	1,88E+11	0,42	1,88E+11
0,00044	7,34E+07	-4,50E+07	3,32E+05	2,93E+06	3,47E+06	7,28E+06	0,00E+00	1,43E+07	0,00E+00	531	123	0	1,93E+11	0,44	1,93E+11
0,00046	7,43E+07	-4,49E+07	3,07E+05	2,71E+06	3,71E+06	6,91E+06	0,00E+00	1,57E+07	0,00E+00	576	124	0	1,98E+11	0,46	1,98E+11
0,00048	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0005	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00052	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00054	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00056	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00058	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0006	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00062	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00064	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00066	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00068	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0007	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00072	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00074	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00076	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	0,00E+00	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,00078	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!
0,0008	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	#GETAL!	######	######	0	#GETAL!	#GETAL!	#GETAL!



When εb = εctmax: ε0 =	0,000133
When εb' = εctmax: ε0 =	0,000151
When εb = εt,p: ε0 =	0,000298
When εb' = εt,p: ε0 =	0,000315
When εp = εp,y: ε0 =	0,000468
When dn = td: ε0 =	0,000769
When εb = εt,u: ε0 =	0,000455
When εb' = εt,u: ε0 =	0,000454
When εp = εud: ε0 =	0,001182

When dn = td & εp = εp,y: Ap =	-62982	mm^2
When dn = td & εb = εt,u: Ap =	-6251	mm^2
When dn = td & εb' = εt,u: Ap =	-677	mm^2
When εp = εud & ε0 = εcmax: Ap =	297226	mm^2
When dn = td & ε0 = εcmax: Ap =	157839	mm^2

Mu	1,98E+11	Nmm
MEd	1,58E+11	Nmm
Unity check: MEd/Mu ≤ 1	0,80	ok!

Shear Capacity (ULS)

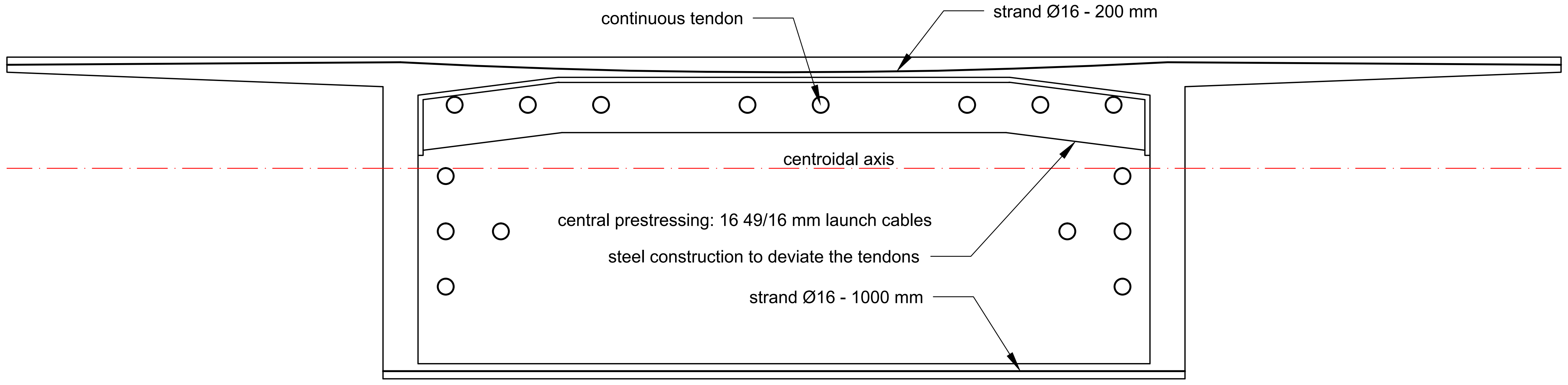
γE*γb	1,5	
Seff	1,71E+06	mm^2
σ(w0,3)k	8	N/mm^2
γbf	1,3	
βu	30	°
tan βu	0,58	
dn	807	mm
z	2441	mm
cot βu	1,73	

VRb	3,35E+06	N
Vf	1,82E+07	N
Vs	0,00E+00	N
Vu	1,82E+07	N

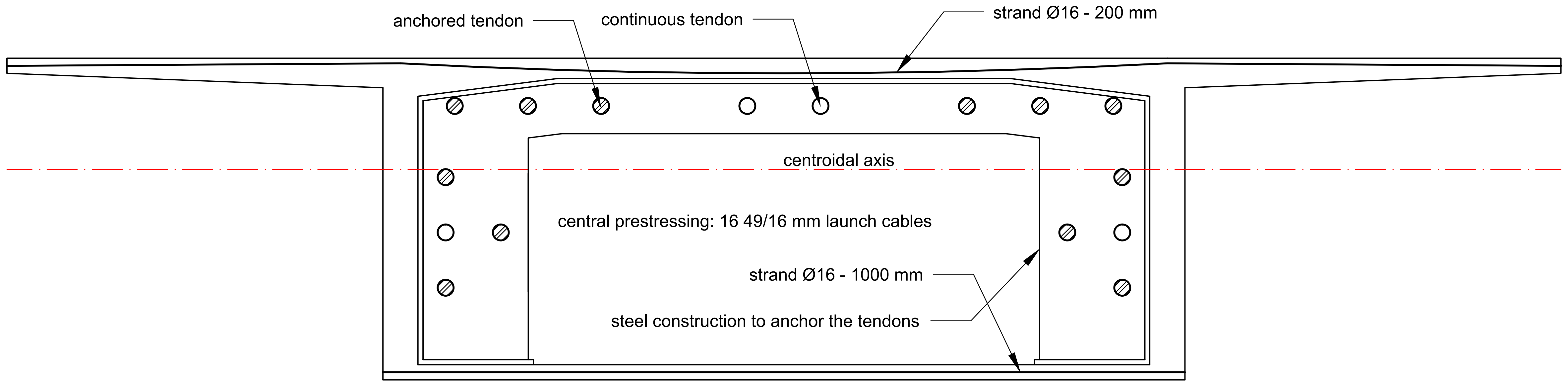
if VEd < VRb → Vu = VRb + Vf + Vs, if VEd > VRb → Vu = Vf+ Vs

VEd	1,50E+07	N
Unity check: VEd/Vu ≤ 1	0,82	ok!

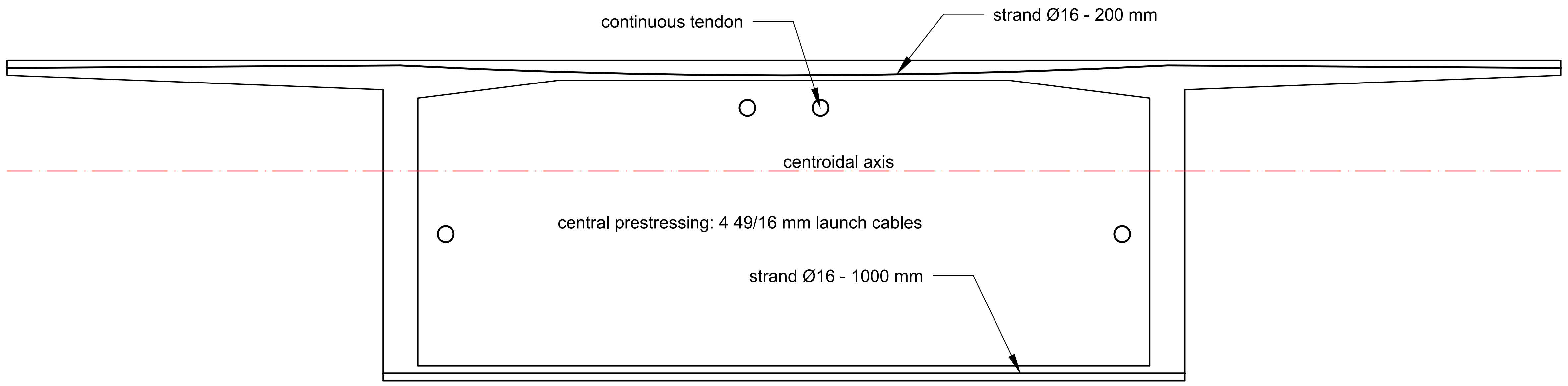
27 Overview of central prestressing arrangement



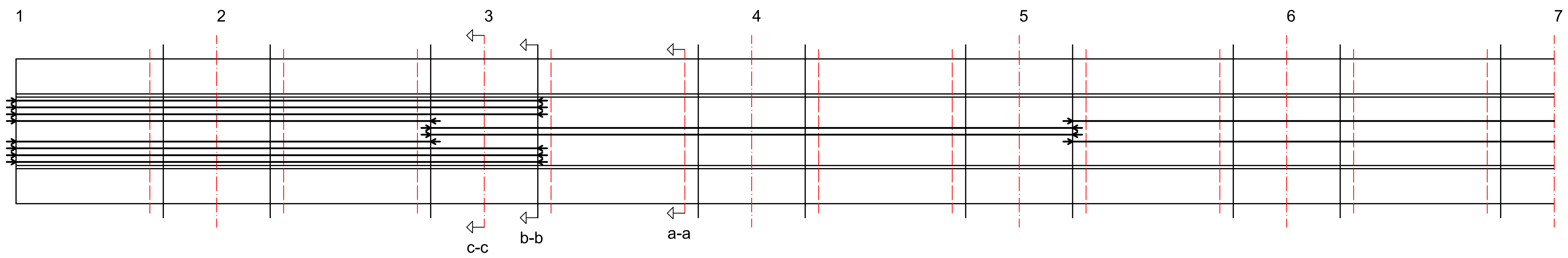
cross-section c-c at the support

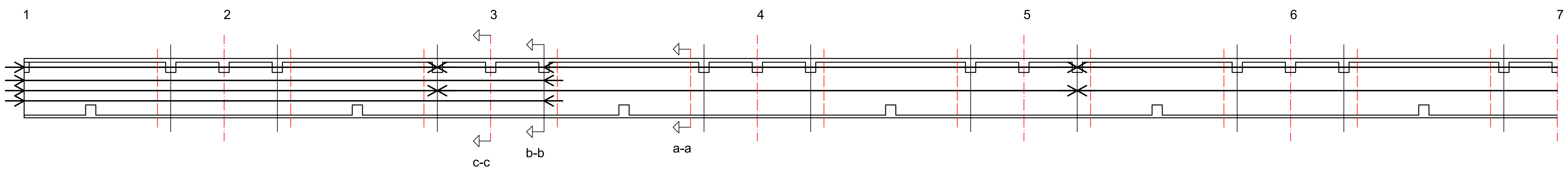


cross-section b-b at the anchor zone

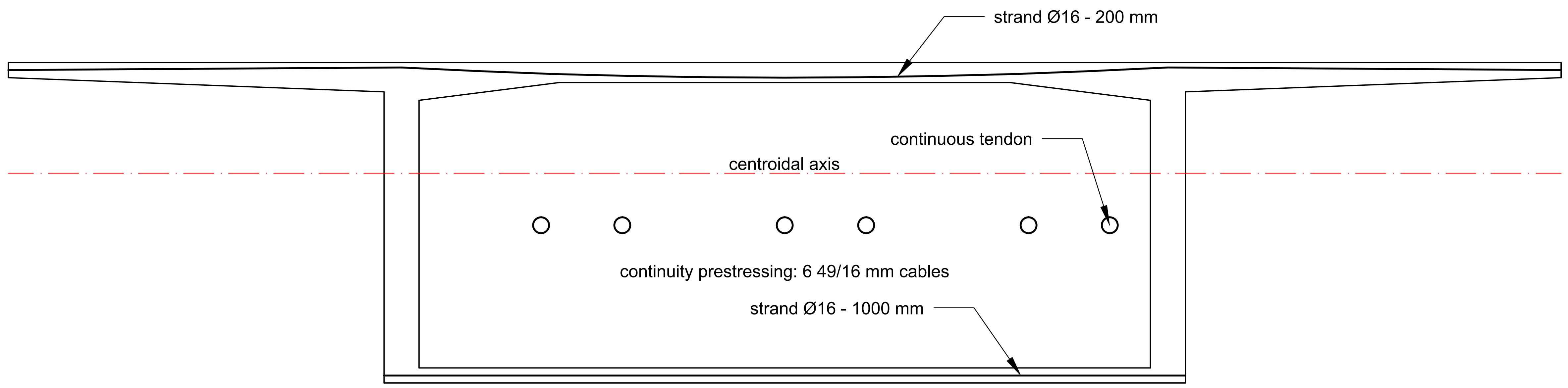


cross-section a-a between segments

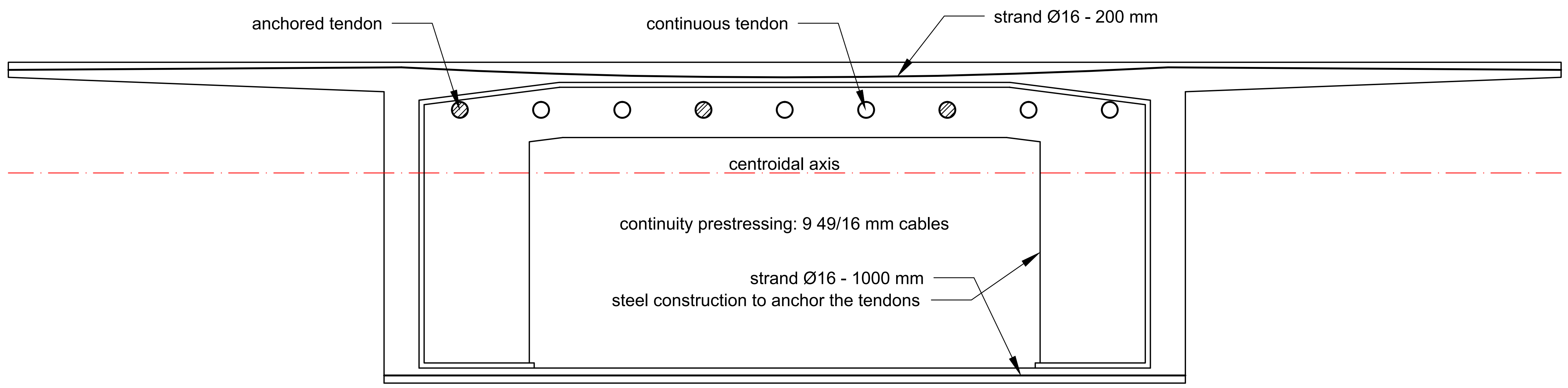




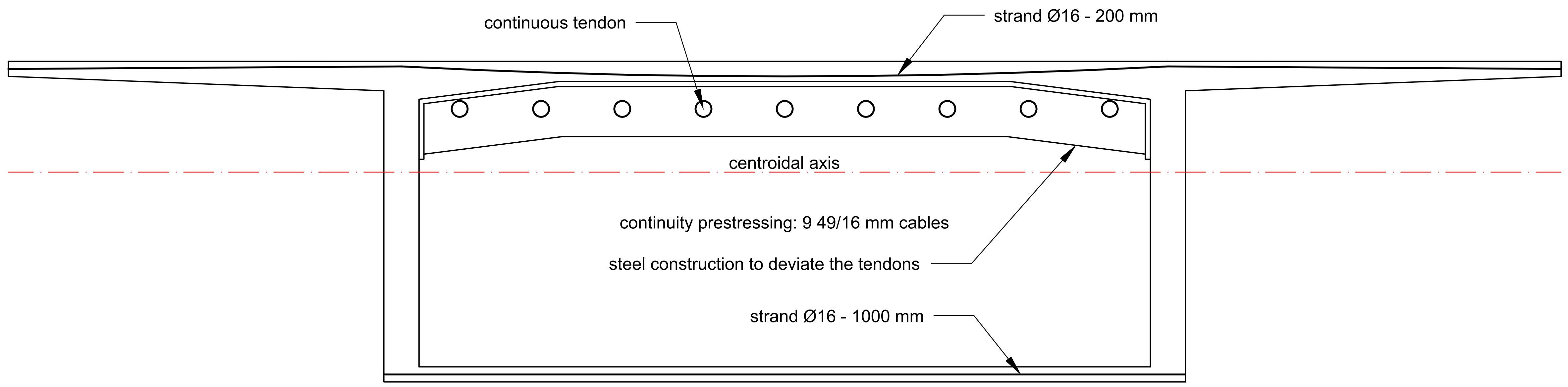
28 Overview of continuity tendon arrangement



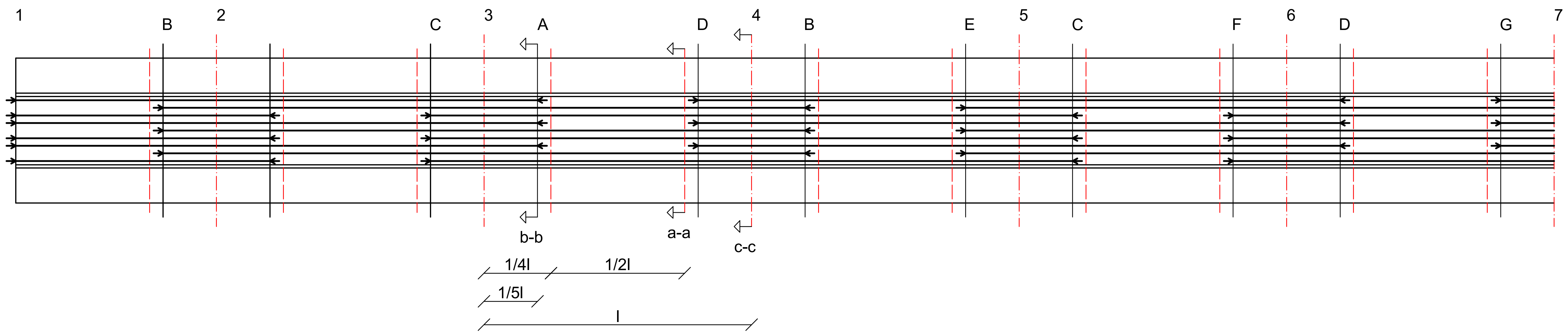
cross-section a-a: between segments

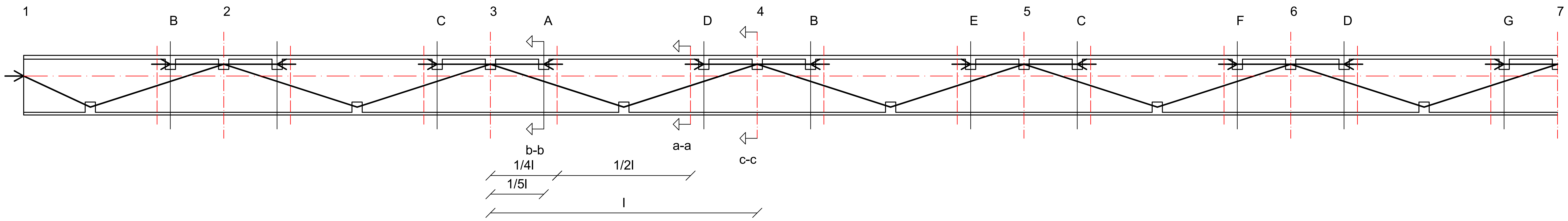


cross-section b-b: at the anchor zone



cross-section c-c: at the support





29 Cost

Cost

UHPFRC B150

material	quantity		ppu [€]	total [€]
cement	705	kg	0,120	84,60
sand/gravel	1010	kg	0,018	18,18
crushed quartz	210	kg	0,030	6,30
silica fume	230	kg	0,300	69,00
superplasticizer	17	kg	0,500	8,50
reinforcing steel	0	kg	1,000	0,00
steel fibers	190	kg	1,500	285,00
prestressing steel	0	kg	2,500	0,00
manufacture	1		30,00	30,00
transport	0		-	-
assembly	0		-	-

€ 502	cost per m^3 concrete

NSC B45

material	quantity		ppu [€]	total [€]
cement	350	kg	0,12	42,00
gravel	1200	kg	0,018	21,60
sand	600	kg	0,012	7,20
filler	25	kg	0,15	3,75
plasticizer	1	kg	0,5	0,50
reinforcing steel	0	kg	1	0,00
prestressing steel	0	kg	2,5	0,00
manufacture	1		15,00	15,00
transport	0		-	-
assembly	0		-	-

€ 90	cost per m^3 concrete

Incrementally launched UHPFRC box girder

material	quantity		ppu [€]	total [€]
cement	705	kg	0,120	84,60
sand/gravel	1010	kg	0,018	18,18
crushed quartz	210	kg	0,030	6,30
silica fume	230	kg	0,300	69,00
superplasticizer	17	kg	0,500	8,50
reinforcing steel	0	kg	1,000	0,00
steel fibers	190	kg	1,500	285,00
prestressing steel	124	kg	2,500	309,10
manufacture	1		30,00	30,00
formwork	1			203,49
launching	1			103,10
nose	1			44,32

€ 1.162	cost per m^3 concrete
€ 502	cost per m^2 bridge deck

Design Zeeburgerbrug

material	quantity		ppu [€]	total [€]
cement	350	kg	0,12	42,00
gravel	1200	kg	0,018	21,60
sand	600	kg	0,012	7,20
filler	25	kg	0,15	3,75
plasticizer	1	kg	0,5	0,50
reinforcing steel	110	kg	1	110,00
prestressing steel	37	kg	2,5	92,09
manufacture	1		15,00	15,00
formwork	1			100,49
launching	1			50,91
nose	1			38
auxiliary bridge piers	1			101

€ 582	cost per m^3 concrete
€ 394	cost per m^2 bridge deck

Incrementally launched UHPFRC box girder

concrete		
Ac	6,70	m^2
lb	15,5	m
formwork		
l	30	m
Kb	25000	€/m
total	750000	€
cost per m^3 concrete	203	€
launching: single unit per week + twelve workmen		
number of launch operations	19	
cost per launch operation	20000	€
total	380000	€
cost per m^3 concrete	103	€
transverse prestressing: Dywidag Y1860S7 strands Ø16-200 mm in the deck, Ø16-1000 mm in the floor		
Ap/m bridge length	800	mm^2
Ap/m^3 concrete	0,0019	m^2
weight trans. pres./m^3 concrete	15	kg
central prestressing: 16 Dywidag Y1860S7 49/16 tendons for first two spans, 4 49/16 tendons for rear spans		
Ap,front	117600	mm^2
Ap/m^3 concrete	0,0175	m^2
weight cent. pres./m^3 concrete	140	kg
Ap,rear	29400	mm^2
Ap/m^3 concrete	0,0044	m^2
weight cent. pres./m^3 concrete	35	kg
continuity prestressing: 6 Dywidag Y1860S7 49/16 tendons		
Ap,cable	44100	mm^2
Ap/m^3 concrete	0,0066	m^2
weight cont. Pres./m^3 concrete	53	kg
nose		
l	15	m
A	0,45	m^2
weight	5,44E+04	kg
total	163338	€
cost per m^3 concrete	44	€

Design Zeeburgerbrug

concrete		
Ac	13,57	m^2
lb	20,03	m
formwork		
l	30	m
Kb	25000	€/m
total	750000	€
cost per m^3 concrete	100	€
launching: single unit per week + twelve workmen		
number of launch operations	19	
cost per launch operation	20000	€
total	380000	€
cost per m^3 concrete	51	€
central prestressing: 26 Dywidag FeP1030 Ø36 bars		
Ap,bar	1018	mm^2
Ap/m^3 concrete	0,0020	m^2
weight cent. pres./m^3 concrete	15,6	kg
continuity prestressing: 20 Dywidag FeP1860 12/15,7 cables		
Ap,cable	1800	mm^2
Ap/m^3 concrete	0,0027	m^2
weight cont. Pres./m^3 concrete	21,2	kg
reinforcing steel		
weight rein. steel/m^3 concrete	110	kg
auxiliary bridge piers		
cost for two bridges (1985)	2650000	fl.
cost per m^3 concrete	113	fl.
price index (2015)	1,96	
cost per m^3 concrete	101	€
nose		
l	18	m
A	0,65	m^2
weight	9,41E+04	kg
total	282247	€
cost per m^3 concrete	38	€

Index 1995-2015 (GWW 1995)

cost component	estimated part [%]	t = 0 (nov 95)	t = 1 (jan 15)	cost t = 0 [€]	cost t = 1 [€]
personnel	40	100	171,5	40	68,60
fuel	1	100	223,1	1	2,23
steel	18	100	176,8	18	31,82
concrete	26	100	126,6	26	32,92
equipment	15	100	105,4	15	15,81
total	100			100	151,38

Index 1985-1991 (van der Meulen)

construction cost	1,16

Index 1985-1995 (estimation)

construction cost	1,29

Index 1985-2015 (estimation)

construction cost	1,96