Systematic Approach for Tolerance Analysis of Photonic Systems

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ABSTRACT

Passive alignment of photonic components is an assembly method compatible with a high production volume. Its precision performance relies completely on the dimensional accuracies of geometrical alignment features. A tolerance analysis plays a key role in designing and optimizing these passive alignment features. The objective of this paper is to develop a systematic approach for conducting such tolerance analysis, starting with a conceptual package design, setting up the tolerance chain, describing it mathematically and converting the misalignment to a coupling loss probability distribution expressed in dB. The method has successfully been applied to a case study where an indium phosphide (InP) chip is aligned with a TriPleX¹ (SiO₂ cladding with Si₃N₄ core) interposer via a silicon optical bench (SiOB).

Tolerance Analysis, Tolerance Chain, Photonics, Passive Alignment, Optical Coupling, Micro Assembly

1. Introduction

Passive alignment of photonic components is an assembly method compatible with a high production volume². Its precision performance relies completely on the dimensional accuracies of geometrical alignment features. Waveguide to waveguide coupling often requires sub-micron alignment accuracy. In our case, even 0.1 μ m is needed. The coupling and alignment performance of the waveguiding chips in the package depend on the accuracy and precision of a multitude of process steps. That creates a stack-up of tolerances, i.e. a tolerance chain. Each manufacturing step of the alignment features and their assembly has a certain dimensional tolerance. The combined tolerance stack-up could easily exceed the required accuracy. Therefore, a tolerance analysis plays a key role in the design and optimization of such passive alignment features. Tolerance analysis and error budgeting is a common aspect in many engineering fields. Also the mathematical techniques are readily available. However, to the authors' best knowledge, these techniques and concepts

have not yet been combined into one integral approach for tolerance analysis for passive alignment of photonic systems. Therefore, we report in this paper about a systematic approach for conducting a tolerance analysis; starting with a conceptual chip design, setting up the tolerance chain, solving the math and converting the resulting misalignment in a coupling loss probability distribution. The proposed method is applied on a case study, where alignment in the in-plane (IP) and out-of-plane (OOP) direction (figure 1) are evaluated. Application on the propagation direction of the wave (PROP) is not demonstrated in this paper, but could follow the same path.





The steps of the tolerance analysis method are schematically presented in figure 2. The analysis starts with the conceptual design of the package. On one hand, this design has to be translated into a detailed description containing data on the manufacturing processes, masks and dimensions. On the other hand, assembly related effects like friction, contact surface roughness and wafer warpage are to be extracted from the conceptual package design. Although surface roughness and wafer warpage are properties of the individual chips, their contribution to the tolerance chain only becomes visible during assembly. Based on processing and assembly details, the tolerance chain between two waveguides can be constructed. Every element in the tolerance chain contains one (or multiple combined) physical effects with a known type and magnitude of deviation, described by a probability distribution function (PDF), e.g. a Gaussian distribution. The same method can be



applied to an array of waveguides, by evaluating each pair individually, figure 2 Tolerance Analysis Method since the statistical parameters of some tolerance elements may vary with the absolute position of the waveguide.

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The PDFs of the individual elements are combined into the full tolerance chain by computing the convolution integral. This computation is conducted separately for OOP, IP and PROP direction. Angular misalignment can be caught by projecting it on the OOP, IP and PROP axes respectively and accounted for as additional corresponding linear errors. Additional analyses can be implemented to represent assembly related effects, like friction or wafer warpage. Afterwards, the PDFs for the OOP, IP and PROP chains can be combined numerically into the loss distribution, expressed in dB.

3. Case study

The developed method has been applied on a case study (figure 3), which consists of two photonic chips, mounted on a common substrate. The indium phosphide chip forms the photonic heart of the assembly. InP as a photonic platform



offers wide а variety of possibilities for integration of active optical components³. The major drawback is however the mismatch in spotsize between the InP waveguides and a single mode fiber. Therefore, an interposer chip named "TriPleX"¹ is used to overcome both the spotsize and pitch mismatch. The waveguides on the TriPleX chip consist of silicon nitride boxes filled with silicon oxide. The cladding is also formed by SiO₂. The bottom, the filling, the sidewalls, and the top of the nitride box are fabricated using one single mask and three

figure 3 Case Study

layer depositions. Both photonic chips are mounted with their active side down (flip-chip) on a silicon optical bench (SiOB) that functions as a common substrate. The reason for the upside-down mounting of the chips is that it allows to define alignment features in the same layers and using the same processing steps as for the waveguides. This is an essential aspect for minimizing the tolerance chain. The case study focuses on the passive alignment of the InP chip with respect to the TriPleX interposer. For that purpose, both the photonic chips and the silicon substrate have been equipped with passive alignment features. Figure 4 gives a schematic of the alignment features for the out-of-plane (OOP) and in-plane (IP) direction.



figure 4 OOP passive Alignment Features (side-view, not to scale) IP passive Alignment Features (top-view, not to scale)

4. Results

The methodology as described in section 2 has been applied to the case study presented in section 3. In this section, the several steps of the analysis will be explained in more detail and applied to the case.

4.1 From design to tolerance chains

Converting the conceptual design of passive alignment features starts with a detailed description of their processing. For the presented case, the relevant tolerance chain is the one linking the centers of two opposite waveguides. The first step is to determine how the center positions of the waveguides are defined with respect to each other by the different process

and assembly steps. First, a reference node N0 is placed at any arbitrary location. In our case, N0 is defined at the contact area between the TriPleX interposer and the silicon substrate. From there on, nodes are placed on the interface between subsequent process steps forming the tolerance chain towards the two waveguide centers (figure 5).



figure 5 OOP Features with Tolerance Chain

IP Features with Tolerance Chain

On the TriPleX side of the OOP tolerance chain (figure 5 left), the node are labeled with an A, followed by a number. The tolerance chain from one side of the box-shaped waveguide to the other is formed by three layer depositions. Then, the center of the waveguide is assumed to be exactly in the middle of the three layers. On the InP side of the tolerance chain, the nodes are labeled with a B followed by a number. The first step in this tolerance chain takes into account a possible unevenness of the contact pad on the substrate feature. Then, the tolerance chain towards the top and bottom sides of the InP waveguide is defined by various depositions and an etch step. Finally, the waveguides center is nominally halfway between node B3 and B4. The IP tolerance chain is labeled with C and D (figure 5 right). Again, a start node N0 is chosen and from there on the route towards the assumed waveguide centers is formed.

The above described method is used to form the tolerance chain for both OOP and IP direction (figure 6). It can be noticed that the OOP chain is dominated by layer depositions and etching, while the IP chain mainly contains lithography and mating accuracies. The mating is modeled separately as an assembly effect.



figure 6 OOP and IP Tolerance Chains

4.2 From tolerance chain to convolution integral

Based on process data and experience, every element in the tolerance chain can be described by a representative deviation from its nominal position together with the corresponding statistical parameters. Examples are a normal distribution with a mean value μ and a standard deviation σ or a uniform distribution with a minimum and a maximum

value. In the OOP chain, all deposition and etches have been chosen as uniform distributions, all within ± 10 % of their nominal value. In reality, depositions and etches will have a normal distribution with a 6 σ specification for ISO certified laboratories. A uniform distribution is a conservative alternative if no accurate data on the standard deviation is available. The IP chain uses both normal and uniform distributions. For the addition of multiple normal distributions, simple mathematical rules exist. However, the summation of various types of distribution requires the computation of the convolution integral (equation 1), in which **f** and **g** represent two functions of which the convolution is computed. For this we had developed a Matlab routine.

$$f^*g = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau \tag{1}$$

The discrete version of the convolution operator as it is implemented by Matlab, is shown in equation 2. When $\mathbf{w} = \mathbf{u} * \mathbf{v}$ and \mathbf{u} is a vector with dimension \mathbf{n} and \mathbf{v} is a vector with dimension \mathbf{m} , it results in \mathbf{w} having dimension $\mathbf{d} = \mathbf{n} + \mathbf{m} - 1$. The \mathbf{k}^{th} element of \mathbf{w} is computed with equation 2. \mathbf{j} is the counter that runs from 1 to \mathbf{n} .

$$w(k) = \sum_{j} u(j) \cdot v(k-j+1)$$
⁽²⁾

To evaluate the tolerance chain from figure 6, three custom Matlab functions were developed: *NORMdistr*, *UNIFdistr* and *convolution*. NORMdistr and UNIFdistr create row vectors that contain both position data \mathbf{x} and probability distribution data $\mathbf{p}(\mathbf{x})$.

NORMdistr(mu, sigma, stepsize) requires mean value μ , standard deviation σ and stepsize Δx . The output is a 2 × d row vector with the following structure.

$$\begin{bmatrix} p(x) \\ x \end{bmatrix} = \begin{bmatrix} p(\mu - 6\sigma) & \cdots & p(\mu - \Delta x) & p(\mu) & p(\mu + \Delta x) & \cdots & p(\mu + 6\sigma) \\ \mu - 6\sigma & \cdots & \mu - \Delta x & \mu & \mu + \Delta x & \cdots & \mu + 6\sigma \end{bmatrix}$$

UNIF distr(val1, val2, stepsize, type) creates a uniform distribution between val1 and val2 and stepsize Δx . The output is again a 2 × d row vector with the following structure.

$$\begin{bmatrix} p(x) \\ x \end{bmatrix} = \begin{bmatrix} \frac{1}{n \cdot \Delta x} & \cdots & \frac{1}{n \cdot \Delta x} & \frac{1}{n \cdot \Delta x} & \frac{1}{n \cdot \Delta x} & \cdots & \frac{1}{n \cdot \Delta x} \\ low & \cdots & \mu - \Delta x & \mu & \mu + \Delta x & \cdots & high \end{bmatrix}$$

convolution(distr1, distr2, stepsize) combines distribution 1 given by a $2 \times n$ structure and distribution 2 with a $2 \times m$ structure into a $2 \times (n + m - 1)$ distribution. Inside the *convolution* function Matlab runs the standard *conv* function that uses equation 2. After computing the convolution, the output distribution is again normalized.

The convolution integral is defined for two functions only. Therefore, computing the convolution integral for a typical tolerance chain will look as follows (figure 7).



figure 7 Convolution Sequence

The Matlab routines *NORMdistr*, *UNIF distr* and *convolution* have been used to evaluate both tolerance chains. The misalignment distribution for OOP and IP direction are shown in figure 8. Since both curves are a summation (concolution) of multiple symmetric distributions, they can be approximated as a Gaussian distribution, as explained in the Central Limit Theory. Gaussian curve fitting yields a σ_{OOP} of 35 nm and a σ_{IP} of 39 nm.



figure 8 OOP and IP Misalignment Distribution

4.3 Combining OOP and IP distributions

Both the OOP and IP misalignment contribute to the misalignment in the plane perpendicular (PERP) to the propagation direction (PROP). See figure 9. The 2D perpendicular distribution can be computed with the matrix operation in equation 3. Since the IP and OOP direction are orthogonal, the probability functions are independent. Therefore, the distributions can be combined by plain multiplication in stead of convolution. Furthermore, since the OOP and IP distribution were already normalized, the combined distribution will also be normalized.



(3)

$$\begin{bmatrix} OOP_1 \\ \vdots \\ OOP_n \end{bmatrix} \cdot \begin{bmatrix} IP_1 & \cdots & IP_m \end{bmatrix} = \begin{bmatrix} PERP_{1,1} & \cdots & PERP_{1,m} \\ \vdots & \ddots & \vdots \\ PERP_{n,1} & \cdots & PERP_{n,m} \end{bmatrix}$$

Application of equation 3 yields the result indicated in figure 10 that shows the PERP misalignment distribution being elliptic equiprobability lines, with aspect ratios defined by σ_{OOP} and σ_{IP} .



figure 10 Radial Distribution

Now, the probability of the misalignment being within a certain area **A** is given by the integral shown in equation 4. Applying the polar coordinate transformation $d\mathbf{A} = \mathbf{r} \cdot d\mathbf{r} \cdot d\theta$ to equation 4 yields the radial distribution, i.e. the probability distribution that the two waveguides are misaligned over a radial distance **r**. The evaluation of the combined distribution can be restricted to one quarter plane due to the symmetry around the OOP and IP axis. The result is then

$$P_A = \iint_A p(x, y) \, dy \, dx \tag{4}$$

$$P_{RAD}(r) = 4 \cdot \int_{0}^{\pi/2} p(r,\theta) \cdot r \cdot dr \cdot d\theta$$
(5)

multiplied by 4, as shown in equation 5 and visualized in figure 11 (left). The ellipses indicate the combined misalignment distribution. The area bound by the black line is the integration area.

By using equation 5, the combined distribution from figure 10 is converted into the radial misalignment distribution from figure 11 (right). It can be seen that the radial offset with the highest probability is located around 35 nm. Furthermore, the probability distribution goes to zero for \mathbf{r} going to infinity. Next to that, the cumulative distribution, i.e. the integral



converges to one, as was expected. It can also be seen that the probability of being more than 100 nm offset is very small.

However, one should be careful with interpreting the curve. Based on just the curve, one would say the expectancy value, i.e. the expected misalignment, is around 50 nm. Nonetheless, the proper expectancy value is 0, because misalignment on the left and on the right can compensate. During the polar coordinate transformation, all direction information is "stored" in the angle θ . By integrating over θ , this information is lost. Therefore, the significance of this curve is somewhat

limited, because of the difference in influence of OOP and IP misalignment on the coupling loss. The next section will address the coupling loss distribution.

4.4 Coupling losses

The radial distribution does not yet take mode shapes into account, i.e. the fact that a misalignment in IP direction does not yield the same coupling loss as an equal misalignment in OOP direction. Equation 6 and 7⁴ express the coupling losses as a function of the mode sizes $(1/e^2) w_1$ and w_2 , with $w_1 < w_2$. Similar equations are valid for the OOP direction.

$$\eta_{\text{mode,IP}} = \frac{2 \cdot w_{1,IP} \cdot w_{2,IP}}{w_{1,IP}^2 + w_{2,IP}^2} \tag{6}$$

$$\eta_{\text{align},\text{IP}} = e^{-\Delta x^2 / w_{1,\text{IP}}^2} \tag{7}$$

Equation 6 and 7 result in elliptical lines of equal dB coupling loss. The aspect ratio of the ellipses is defined by the respective mode sizes. Then, integrating the misalignment PDF over the equal coupling loss lines will give the probability of achieving a certain coupling loss. The mode size of the two waveguides are shown in table 1. table 1 InP and TriPleX Waveguide Data

	InP	TriPleX
W _{0,IP}	1.13 μm	0.714 µm
W _{0,OOP}	0.43 µm	0.777 μm
n	3.07	1.542

Based on this data, the coupling loss contour plot (figure 12) is computed with equations 6 and 7. The plot does not take into account the back reflections due to different refractive indices.



The difficulty of integrating over an elliptical path is the fact that the polar Jacobian \mathbf{r} varies with angle $\boldsymbol{\theta}$. Therefore, also **dr** varies. \mathbf{r} as a function of $\boldsymbol{\theta}$ is expressed by equation 8. \mathbf{a} is the "radius" of the ellipse in IP direction and \mathbf{b} in OOP direction. The ratio between \mathbf{a} and \mathbf{b} is constant and is given by the waveguide dimensions. Therefore, the cumulative probability of the elliptical path is given by equation 9. Here again, symmetry along the IP and OOP axis is assumed.

$$r(\theta) = \frac{a \cdot b}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$
(8)

$$P_{ellipse}(a) = 4 \cdot \int_0^{\pi/2} p(r(a,\theta),\theta) \cdot r(a,\theta) \cdot dr(a,\theta) \cdot d\theta$$
(9)

The elliptic integration from equation 9 is schematically depicted in figure 13. It has been used to obtain figure 14 (left). Since every value of **a** corresponds to certain coupling loss expressed in dB, it is easy to convert the **a** axis into the **dB** axis (figure 14 middle), yielding the misalignment PDF in dB space. Finally, the dB-PDF can be integrated to obtain the cumulative distribution function (CDF) from figure 14 (right). From the dB-CDF graph, it can be concluded that the best possible alignment gives 1.16 dB loss. Furthermore, almost 60 % will be coupling within 1.2 dB loss and about 99% will be better than 1.4 dB.



figure 14 Towards Coupling Losses in dB

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5. Discussion and conclusion

In this section, the achievements and limitations of the work will be discussed. Also, a general conclusion will be drawn.

5.1 Discussion

In this paper, only the alignment in the IP and OOP direction is taken into account, together with their cumulative contribution in the PERP plane. An analysis for the wave propagation direction (PROP) was not demonstrated in this paper. Nonetheless, the method is applicable to that direction as well.

A second remark that has to be made is the fact that angular misalignment is only taken into account as an additional error projected on the IP, OOP and PROP axis. For our case study, that is sufficient. However, other photonic systems might require a separate evaluation of the angular misalignment, e.g. for fiber to Bragg-grating coupling. Therefore, independent analysis of the angular directions is planned as future work.

Furthermore, some elements from the tolerance chains have been represented by a uniform distribution. In reality, a Gaussian distribution is far more representative. However, the corresponding statistical data was not available, yielding to the choice for the uniform distribution. This gives an overestimation of the dimensional deviations. A good aspect of using the convolution integral is that any preferred distribution can be used and combined with others.

Finally, the conversion from design to the loss distribution expressed in dB offers a very practical tool for comparing several alignment options. However, the conversion method is up to now limited to elliptical mode shapes. For asymmetrical waveguides, like indium phosphide slab waveguides, the modeshape will not be elliptical. The numerical implementation of integrating over random shaped paths could be conducted in the future.

5.2 Conclusion

It can be concluded that the developed method has been successfully applied on a case study where an InP chip is aligned with respect to a TriPleX interposer via a silicon optical bench. A systematic approach is introduced for converting the conceptual package design into various tolerance chains. The tolerance chains have been solved and combined by using standard (numerical) mathematical techniques. The cumulative coupling loss distribution is a convenient criterion for optimization.

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