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Multiline holding based control for lines merging to a shared transit corridor

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Multiline holding-based control for lines merging to a shared transit corridor

Abstract

In transit corridors, multiple lines share a sequence of consecutive stops to provide higher joint frequency in higher demand areas. A key challenge is to coordinate the transition from single line to joint operation. A holding control strategy aimed at minimizing passenger travel times is introduced for lines merging into a shared corridor, accounting for the coordination of vehicle arrivals from the merging lines as well as the regularity of each line. The criterion is tested using an artificial network and a real-world network to analyze the impact of demand distribution and compare cooperative versus single line control. We illustrate how the real-time strategy yields overall passenger gains, depending on the composition of different user groups. Results are assessed based on operation and passenger performance indicators and show that coordination is achieved. When combined with joint control in the common part, the proposed approach achieves consistent network-wide travel time benefits.

Keywords: Line Coordination, Corridor management, Fork Line Operations, Holding Control

1. Introduction

The majority of an urban network's demand is usually concentrated to areas along key corridors. As a result, multiple public transport lines often share a set of consecutive stops along their route to cater for the high-demand section. This solution yields denser services with shorter headways, hence reducing the need to perform transfers and in turn increasing public transport's attractiveness. Network design subject to passenger cost minimization has been shown to result in such a network topology (Baaj and Mahmassani, 1995). From the operations perspective, networks with a shared transit corridor have mostly been addressed at the tactical planning level by designing timetables for buses that share stops to minimize waiting times (Guihaire and Hao, 2010) and maximize the number of synchronization events (Ceder et al., 2001). In the case of a joint schedule, buses follow a specific sequence of arrivals at the common parts to reduce the congestion of the transit corridor and to provide shorter waiting times for the passengers at these stops (Ibarra-Rojas and Muñoz, 2015).

Aside from planning a timetable that accounts for coordination, transit lines are still subject to travel time and passenger demand variability, which is known to propagate along a route and cause undesirable bunching (Schmöcker et al., 2016). This has negative consequences for service regularity, passenger and vehicle travel time and the overall service quality. Furthermore, due to conflicting interests among various passenger groups, regulating the service of each individual line may not necessarily yield network-wide benefits.

Real-time control dynamically manages disturbances occurring during transit operation. The deployment of control strategies is enabled by the sources of data provided by Advanced Public Transport Systems (APTS). Facilitated by such technologies, various control strategies have been introduced in the literature, mainly focused, apart from a few recent works, on single line operations without considering its interaction with other lines at a network level. By applying control to a single line, a high performance can indeed be maintained. However, single line control ignores the existence of other lines and the benefits that can be obtained for the passengers by coordinating all the additional lines available that serve their destination.

This study introduces a control strategy for multiple lines for a network with merging routes, i.e. routes serving separate branches followed by a set of consecutive common stops. We propose a novel real-time holding control strategy that integrates single line regularity objectives and shared corridor management based on the expected demand distribution over the lines. To the best of our knowledge, research on control beyond a single line has focused mainly on synchronizing transfers at a single transfer location (Abkowitz et al., 1987; Hall et al., 2001). Only a few recent examples have examined real-time holding on a segment of overlapping routes. These studies focused on evaluating different operation schemes and on comparing regularity-based and schedule-based strategies (Hernández et al., 2015; Fabian and Sánchez-Martínez, 2017). The most relevant example in multiline control is the work of (Argote-Cabanero, 2014; Argote-Cabanero et al., 2015) who successfully extended the applicability of an isolated line holding criterion to multiline networks. Furthermore, it is the first work, to our knowledge, that applies control accounting for the transition from individual to joint operation by gradually altering the decision rule for holding from line regularity to line coordination, while at the same time accounting for the different passenger cost components.

Our approach is shown to increase the overall performance of the network compared to single line control by using different simulation environments and assuming various demand levels and distributions, as well as by considering empirical data from a real-world example. The performance is evaluated and compared to single line control strategies to assess potential benefits from both operation and passenger perspectives at a network level.

The remainder of the paper is organized as follows: in Section 2 the literature on single line holding control and multiline operations is reviewed. In Section 3, the holding principles are formulated and the control strategies are derived. Section 4 details the setup of the case studies and the scenarios tested. In Section 5 the analysis of the results is presented and finally conclusions and future research directions are drawn in Section 6.

2. Literature Review

Real-time control was recently thoroughly reviewed by Ibarra-Rojas et al., (2015). Different classifications exist to distinguish control strategies for transit operations. A first classification is based on the level of integration of APTS, and set as milestone the transition from schedule adherence and long-term planning actions towards the availability of real-time information and control (Zolfaghari et al., (2004) and Eberlein et al., (2001)). A second classification is based on the location at which they are applied, and divides strategies into station strategies, interstation strategies and other strategies. In the category of station strategies, holding is an extensively investigated research topic, and it represents a common practice in transit operations.

This literature review is organized as follows: section 2.1 covers single line holding based control, while section 2.2 is devoted to overlapping routes and multiline control.

2.1. Single line holding based control

Considering holding strategies, different approaches have been developed based on line characteristics and availability of information. For lines operated with long headways, it is conventional to use holding strategies aiming at schedule adherence, while for lines with short headways the aim is to maintain service regularity. The criterion for the former category is that a vehicle should not depart earlier than its scheduled time. For the latter category, holding time is calculated by taking into account the headway between consecutive vehicles. Fu and Yang (2002) compared threshold-based holding rules subject to preceding and succeeding vehicles, concluding that the optimal holding time lies between 60% and 80% of the planned headway of the line. Daganzo (2009) proposed a dynamic holding scheme that reduces or increases the speed of a succeeding vehicle depending on the headway with the preceding vehicle. Xuan et al. (2011), based on the work of Daganzo, formulated a family of dynamic holding strategies to maintain schedule reliability and maximize commercial speed.

Cats et al. (2011) compared schedule- and headway-based control with limitation on the maximum allowed headway. They concluded that headway-based control that considers both forward and backward headways outperforms the other strategies and brings substantial benefits for the passengers. Bartholdi and Eisenstein (2012) proposed a self-coordinating control method, which adjusts dynamically headways depending on the actual bus capacity utilizations and a minimum headway to be maintained to avoid bunching. In the same context, Liang et al (2016) formulated a self-adaptive control scheme to regulate headways with fast headway recovery time and as a result they showed substantial benefits in terms of travel times. On the same track, Zhang and Lo (2018) analyzed a framework of equalizing headways subject to preceding and succeeding vehicles accounting for both deterministic and stochastic travel times as well as the number of vehicles in the network.

Holding time can be determined as the decision variable in passenger cost optimization problems. Barnett (1974) formulated a single stop holding model that minimizes the main components of travel cost, namely waiting times and in-vehicle delays. Zhao et al. (2003) treated stops and buses as agents and developed a negotiation algorithm based on marginal costs to determine the optimal conditions for applying holding. Zolfaghari et al. (2004) added waiting times induced by capacity constraints in the objective function. Yu and Yang (2007) determined the optimal holding times by minimizing the total users cost using a Genetic Algorithm. In addition, the authors developed a forecasting model for early departures, based on a support vector machine (SVM) approach. Delgado et al. (2009) combined holding based on minimizing the travel time of individual users with boarding limits and found that the combination should be applied when the preceding vehicle closes in. More recently, Berrebi et al. (2015) used holding in the dispatching policy aiming to reduce passenger waiting time by minimizing the sum of square headways, while Sánchez-Martínez et al. (2016) formulated a holding control optimization accounting for time-dependent changes in passenger demand and running times. Wu et al. (2017) introduced the effects of overtaking and queue swapping behavior to schedule based and headway based holding control strategies.

Holding strategies have also been used for transfer synchronization, starting from the work of Abkowitz et al., (1987), which compared four simple holding-based rules on a single transfer point. Hall et al (2001) examined a set of dispatching policies for transfer stops based on minimizing the expected travel time of all passengers. Nesheli and Ceder (2015) presented a framework to maximize the number of direct transfers and minimize the total passenger travel time. Additionally, Wu et al., (2016) combined holding strategy from operation's perspective with schedule coordination from tactical planning to further assist transfer events, a combination not explored in existing work. Recently, Gavriilidou and Cats (2018) introduced a controller which calculates holding time for regularity and synchronization and the controller decision is taken based on minimization of passenger cost given different levels of passenger information. Based on the state of network, an optimal set of operational tactics was chosen and validated using simulation, showing to achieve a considerable improvement to the network performance.

2.2.Multiline control

User cost minimization in transit network design problems often result with a number of overlapping lines (Baaj and Mahmassani, 1995). However, this design solution does not explicitly take into account service reliability and the related operational challenges. Early work on corridors with overlapping routes focused on modelling waiting time behavior of passengers that can be served by multiple lines (Chriqui and Robillard (1975), Marguier and Ceder (1984). Han and Wilson (1982) investigated the allocation of additional buses on busy networks, which included a shared transit corridor. In the area of tactical design, Ibarra-Rojas and Muñoz, (2016, 2015) introduced a timetable optimization problem for maximizing the synchronization events of different bus lines at common stops on overlapping segments and later they extended their problem to ensure even headways between consecutive vehicles of different lines while limiting diversions from a given timetable .

Only recently has the control of transit corridors gained the attention of the research community. The most relevant work to be mentioned is that of Hernandez et al (2015), who tested holding on a shared transit corridor comparing different operation schemes. However, service performance outside of the corridor was not considered in their study. Argote-Cabanero et al. (2015) extended the single line holding control strategy by Xuan et al (2011) to multiline control, and tested it on the real network of San Sebastian. They proved that the single line control can also be applied to more complex systems with multiple lines with resilient results with line and inter-line metrics with or without the addition of driver guidance, which was also a part of the study. Fabian and Sánchez-Martínez (2017) compared scheduled- and headway-based holding for the trunk and multi-branch light rail network of Boston. The control was applied for each line independently, while satisfying rail infrastructure limitations. Based on their findings, they concluded that headway-based holding based on a joint headway and applied at the shared transit corridor itself can be more beneficial than obeying to the line headway. Schmöcker et al. (2016) formulated a queuing model to describe the effect of shared corridors on bunching and tested several operational scenarios, concluding that cooperation and overtaking between lines can assist in reducing bunching along the shared section.

2.3. Synthesis and motivation

Regularity of transit lines has been analyzed mostly for single lines. The coordination of multiple lines via control has been addressed mainly at the tactical planning phase. A valid research question is how shared transit corridors can be controlled in real-time so that passengers' waiting times – along separate line branches as well as the trunk - are minimized. This question seems not to be properly addressed when looking at the reported literature. Moreover, only few works accounted for coordination between lines with overlapping routes sharing more than one consecutive common stop, and only few quantified the benefits of cooperative schemes on passengers' journey times. To partly fill this gap, in this study we develop a novel rule-based control strategy for real-time corridor management focusing on merging lines. The proposed formulation considers the impact of the holding control measure on all relevant passenger groups and accounts for the demand distribution on the lines at the branches and within the common section. The performance of the cooperative control is compared to the case of independent single-line control and the advantages and disadvantages are quantified from both passenger and operator perspectives, at the line as well as at the network level.

3. Methodology

3.1. Notation

The notation that is used for the formulation of the problem is given below. For the sake of simplicity and without loss of generality, we expect the formulated criterion to be applied every time when a vehicle enters a stop.

Sets

I	set of lines;
J_i	set of stops served by line i;
K_i	set of trips of line i;
N_i	number of stops of line i; and
N_i^w	number of stops of the subset w of line i.

Network related labels

c	index for the shared transit corridor;
b	index for the branches;
cb	index for the shared transit corridor to branch variables.

Time related variables

$t_{ijk}^{arrival}$	Arrival time at stop j of trip k of line i in [time units];
t_{ijk}^{dwell}	Dwell time at stop j of trip k of line i in [time units];
t_{ijk}^{exit}	Exit (departure) time at stop j of trip k of line i in [time units];
$\tau_{j-1,j}^{riding}$	Scheduled riding time between stops j-1 and j in [time units];
$t_{j-1,j}^{riding}$	Actual riding time between stops j-1 and j in [time units];
t_{ijk}^{hold}	Holding time at stop j of trip k of line i in [time units];
$h_{i,j,k,k-1}$	Actual headway at stop j between trips k and k-1 of line i in [time units];
$\hat{h}_i$	Planned headway of line i in [time units];

$\hat{h}^{join}$	Planned joint headway in [time units];
t_{ijk}^{wait}	Waiting time at stop j of trip k of line i in [time units];
t_{ijk}^{inveh}	In vehicle time between stop j and j+1 of trip k of line i in [time units];
t_{ijk}^{travel}	Travel time between stop j and j+1of trip k of line i in [time units];

Passenger related variables

o	origin stop;
d	destination stop;
$\lambda_{o,d}$	arrival rate between origin o and destination d in [passengers per hour];
q_{ijk}	passengers on board on trip k of line i at stop j in [passengers].

3.2. Network configuration

Consider a network that consists of a set of lines $I = \{i_1, i_2 \dots, i_n\}$. Each line i serves a set of stops $J_i = \{j_{i1}, j_{i2}, \dots, j_{in}\}$, which consists of subsets of stops common to multiple lines, such as $J_{c_{i_1 i_2}} = \{J_1 \cap J_2\}$ (the set of stops line i_1 and i_2 share) and a subset of stops served by a single line $\left(J_i^b = J_i \setminus \{J_c\}\right)$ (branch stops). The common stops are considered to be consecutive. At a specific stop (from now on referred to as the merging stop denoted by j^{merg}) lines merge and thereafter operate jointly until the end of their routes. The set of stops of each line, which is served exclusively by a line, is at the beginning of the route, and prior to the subset of common stops. Only one direction is considered, operating from the branches to the shared transit corridor. Given this network configuration, all passengers can reach their destination using all lines that serve the origin stop.

3.3. Problem Formulation

The main objective is to develop a holding criterion for the lines that operate in network configurations similar to the network presented in Figure 1.

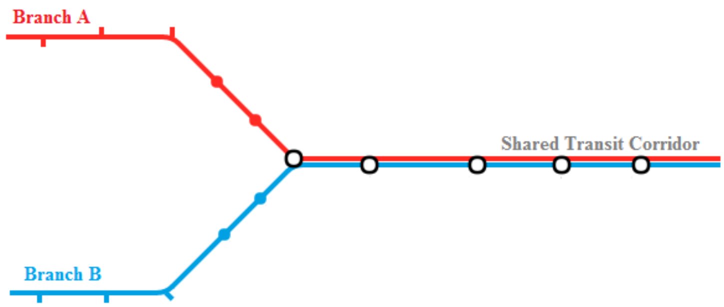


Figure 1: Schematic network configuration

The key decision variable is the holding time, which is optimally computed to attain the minimum total generalized passenger travel time. This consists of two components: the passengers waiting time at stops and the in-vehicle delay. These are formulated in Section 3.5. The holding criterion is then formulated by considering both service regularity on each line branch and the joint headway along the trunk. The inter-arrival of vehicles from the branches to the corridor is therefore a key factor. The control decision rule formulation depends on passenger costs and hence on the number of passengers benefiting from single vs. multi-line regularity, as one may compromise the other in real-time control settings.

3.4. Assumptions

The formulation is based on the following assumptions:

- Passengers do not perform transfers in this network configuration;
- Capacity constraints are not considered;
- Historical data for the demand of the lines are available; and
- AVL data are available in real time.

We consider only networks with lines that after operating independently, they merge and operate jointly on a sequence of common stops (shared transit corridor). Transfers are not taken into account. Transfers and transferring operations are part of a more general problem with more complex networks consisting of lines merging and later diverging after a common segment. This is subject of future studies. The holding criterion is based on the expected values for the number of passengers and the arrival of the succeeding vehicles. The former is based on the historical arrival rates of the passenger demand at each stop, while the latter on scheduled riding times. The variability of these two variables is not taken into consideration. Therefore, historical data for the demand are needed in order to estimate the number of passengers waiting at stops and on board to formulate the passenger cost function. Scheduled riding times are needed in order to estimate the arrival of a succeeding vehicle at a stop. AVL data are needed to know to exact location of all vehicles in the network and based on it to apply control if needed in real time.

3.5. Formulation of the holding criteria

Single line criterion

The holding criterion proposed in this paper is based on the passenger cost minimization control strategy for a single line introduced in Laskaris et al., (2016). The objective of the holding criterion is to minimize the additional cost experienced by the passengers due to the extra holding time.

The optimal holding time is obtained by minimizing travel time subject to holding time t_{ijk}^{hold}, which is expressed by the following formula:

$$t_{ijk}^{hold} = \max\left\{ \frac{(t_{ijk+1}^{exit} - t_{ijk}^{exit}) - (t_{ijk}^{exit} - t_{ijk-1}^{exit})}{2} - \frac{q_{ijk}}{4 \sum_{stop=j+1}^{N_i} \lambda_{stop}}, 0 \right\} \quad (1)$$

This rule is used as a starting point for considering passengers from other lines at the shared transit corridor via the line coordination term. In the following sections, we detail how the passenger cost is extended beyond a single-line level.

Deriving the Branch Stop Holding Criterion

The holding criterion is formulated by including all the terms composing the total passenger travel time for the passengers $t_{ijk}^{travel}(t_{ijk}^{hold})$, which is a function of holding time t_{ijk}^{hold}, and by explicitly considering the influence of common downstream stops. The travel time consists of the additional waiting time t^{wait} passengers experience when a vehicle is instructed to remain at a stop due to a control decision, and the in-vehicle delay t^{inveh} expresses the additional travel time that passengers experience on board while a vehicle is held due to a control decision. Waiting time is perceived as a greater disturbance for passengers, therefore its effects on the total travel time are considered more crucial than the in-vehicle time. This is given by adding a weight   for the waiting time. For this study is set to 2, which is in line with the findings of Wardman (2004). Travel time t_{ijk}^{travel} is thus expressed by the following formula:

$$t_{ijk}^{travel} = \beta^{wait} t_{ijk}^{wait} + t_{ijk}^{inveh} \quad (2)$$

In this study, t^{wait} stands for the waiting time at a branch stop. Waiting time at a branch stop consists of the waiting time between consecutive vehicles at the current stop and the expected waiting time at the first common stop. The second is based on the arrivals of consecutive vehicles regardless the line.

Accounting for passenger arrival rates

At the branches stops are considered to be served only by a line i that operates at this specific part of the network. In this section, each vehicle regulates its departure from a stop depending on its actual headways from both the preceding and the succeeding vehicle. Assuming that passengers arrive uniformly at stops, the expected number of passengers boarding on a vehicle k of line i at stop j, v_j^{board}, is the product of the arrival rates at each stop λ_j and the current headway h_j between the bus arriving at stop j and its preceding vehicle:

$$v_{ijk}^{board} = h_{ijk,k-1}\lambda_j$$

It is assumed that the actual headway between consecutive vehicles, due to either early or delayed departure, is affecting not only the passengers at the current stop, but also the passengers at the remaining downstream stops until the end of the line ($\sum_{stop=j}^{N} \lambda_{stop}$). Considering that multiple lines operate on the common corridor, the sum of the arrival rates for N stops is given by the following formula:

$$\sum_{m=j}^{N}\sum_{n=m+1}^{N} \lambda_{mn} = \sum_{m=j}^{Nj}\sum_{n=m+1}^{N_b} \lambda_{mn}^{b} + \sum_{m=j}^{N_b}\sum_{n=m+1}^{N_c} \lambda_{mn}^{bc} + \sum_{m=j}^{N_c}\sum_{n=m+1}^{N_c} \lambda_{mn}^{c} \tag{3}$$

where

N_b	the number of stops in the subset of branch stops J_b;
N_c	the number of stops in the subset of corridor stops J_c;
$\sum_{m=j}^{N_b} \sum_{n=m+1}^{N_b} \lambda b_{mn}$	the arrival rates of the passengers travelling within the branch;
$\sum_{m=j}^{N_b} \sum_{n=m+1}^{N_c} \lambda bc_{mn}$	the arrival rates travelling from the branch to the corridor; and
$\sum_{m=j_c}^{N_c} \sum_{n=m+1}^{N_c} \lambda c_{mn}$	the arrival rates within the shared transit corridor.

The expected number of passengers that are expressed by the first two components on the right hand side of Equation (3) (the sum of the arrival rates that travel within the branch and the sum of the arrival rates that travel from the branch to the shared transit corridor) depend on the headway of the line at the branch. However, the expected number of passengers that travel within the corridor is overestimated when using the actual headway of the line, since the actual headway will be greater or equal to the joint headway, which will be experienced by the passengers on the shared stops. Therefore, we correct Equation (3) by considering the ratio of the line and the joint headway within the sum of the arrival rates for passengers travelling within the shared transit corridor:

$$\sum_{m=j}^{N}\sum_{n=m+1}^{N} \lambda_{m,n} = \sum_{m=j}^{N_b}\sum_{n=m+1}^{N_b} \lambda_{mn}^{b} + \sum_{m=j}^{N_b}\sum_{n=m+1}^{N_c} \lambda_{mn}^{bc} + \sum_{m=j}^{N_c}\sum_{n=m+1}^{N_c} \frac{\lambda_{mn}^{c}}{\frac{\hat{h}_i}{\hat{h}^{join}}} \tag{4}$$

where $\hat{h}_i$ is the headway of a single line and $\hat{h}^{join}$ is the joint headway at the common segment of the lines. The joint headway is given by the arrival separation time between lines in the tactical planning phase (.i.e. timetable design) is defined by the following formula introduced by Ibarra Rojas and Munoz (2016):

$$\sigma_{sp} = \min\left\{\frac{avrg\_h(L(s),p)}{|L(s)|}, \frac{\min_{l\in L(s)} h_p^l}{2}\right\} \tag{5}$$

Where

h_p^l	The ideal even headway for line l in planning period p

$\text{avrg\_h}(L(s),p)$ The average headway of all lines L(s) in planning period p

To give an example, for a network with two lines A and B with equal headway of 10min and a joint headway at their shared segment of 5min, at the corridor stops of the lines the arrival rates λc will be divided by $\hat{h}_A / \hat{h}^{join} = 5/10 = 1/2$. This penalty factor captures the expected number of passengers affected by control measures applied to a vehicle serving a given line, whereas the demand along the trunk will be distributed over the corresponding lines.

For the sake of simplicity, let:

$$\begin{aligned} &\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}=\Lambda_j \\ &\sum_{m=j}^{Nj}\sum_{n=m+1}^{N_b}\lambda_{mn}^{b}=\Lambda_j^{b} \\ &\sum_{m=j}^{N_b}\sum_{n=m+1}^{N_c}\lambda_{mn}^{bc}=\Lambda_j^{bc} \\ &\sum_{m=j^{merg}}^{N_{J_c}}\sum_{n=m+1}^{N_{J_c}}\frac{\lambda c_{m,n}}{\frac{\hat{h}_i}{\hat{h}^{join}}}=\Lambda_j^{c} \end{aligned} \qquad (6)$$

Where Λ expresses the sum of the arrival rates from a stop j until the end of the line and consists of all subgroups of the demand from the current stop until the end of the line. Given that, Equation 4 can be written as:

$$\Lambda_j=\Lambda_j^{b}+\Lambda_j^{bc}+\Lambda_j^{c} \qquad (7)$$

Regulating headway at the current stop

Assuming uniform arrivals at stops, the waiting time experienced by the passengers is the product of half of the actual headway:

$$t_{jk}^{wait}=\frac{h_j}{2}v_{jk}^{board} \text{ or } t_{jk}^{wait}=h_j^2\frac{\lambda_j}{2} \qquad (8)$$

At a branch stop j, let t^{wait_p0} be the waiting time from the preceding vehicle p, while t^{wait_s0} be the one from the succeeding vehicle s, for a vehicle k of line i when no control action is taken. It should be noted that the departure time of the succeeding vehicle is calculated by adding the scheduled riding times between the last visited stop and the current stop to the departure time from last visited stop. These waiting times can be formulated as the following Equations (9) and (10):

$$t_{ijk}^{wait\_p0}=\frac{\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)^2}{2}\left(\Lambda_j^{b}+\Lambda_j^{bc}\right) \qquad (9)$$

$$t_{ijk}^{wait\_s0}=\frac{\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)^2}{2}\left(\Lambda_j^b+\Lambda_j^{bc}\right) \tag{10}$$

The sum of Equation (9) and Equation (10) yields the total waiting time when no holding is applied:

$$t_{ijk}^{wait\_0}=t_{ijk}^{wait\_p0}+t_{ijk}^{wait\_s0} \tag{11}$$

Similarly, when holding time is assigned to vehicle k of line i at stop j, the waiting times t^{wait_pH} and t^{wait_sH}, subject to the headways from the preceding and the succeeding vehicles, can be respectively formulated as:

$$t_{ijk}^{wait\_pH}=\frac{\left(\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)-t_{ijk-1}^{exit}\right)^2}{2}\left(\Lambda_j^b+\Lambda_j^{bc}\right) \tag{12}$$

$$t_{ijk}^{wait\_sH}=\frac{\left(t_{ijk+1}^{exit}-\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)\right)^2}{2}\left(\Lambda_j^b+\Lambda_j^{bc}\right) \tag{13}$$

The total waiting time in case of holding is then:

$$t_{ijk}^{wait\_H}=t_{ijk}^{wait\_pH}+t_{ijk}^{wait\_sH} \tag{14}$$

Finally, the additional waiting time due to control is the difference between waiting time with and without holding time:

$$t_{ijk}^{wait}=t_{ijk}^{wait\_H}-t_{ijk}^{wait\_0} \tag{15}$$

Equation (16) expresses the waiting time as a function of holding time by using Eq. (11) and (16) into Eq. (15):

$$\begin{aligned} t_{ijk}^{wait}(t_{ijk}^{hold}) &= \left(\Lambda_j^b+\Lambda_j^{bc}\right)\left(t_{ijk}^{hold}\right)^2 + \\ &\left\{\left(\Lambda_j^b+\Lambda_j^{bc}\right)\left[\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)-\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)\right]\right\} t_{ijk}^{hold} \end{aligned} \tag{16}$$

Regulating the transition from the branch to the shared transit corridor

At the branch stops, apart from the regularization of the headways of consecutive vehicles, the transition from the branches to the shared transit corridor needs to be considered to ensure that any potential control decision at branch stops will not propagate as delay to the shared transit corridor. For this reason, a term related to the expected headway at the first common stop is added, accounting for all vehicles that will share the same stops downstream.

Let vehicle k from line i arrive at branch stop j at arrival time $t_{ijk}^{arrival}$. After the completion of dwell time t_{ijk}^{dwell}, the sum of the actual arrival time and dwell time will be the expected departure (exit) time t_{ijk}^{exit}. Using as reference line i, since branches may consist of different number of stops, assume that between the current stop j and the first common stop there are n stops and n-1 links that connect the stops. Between stops, there are n-1 scheduled riding times ($\tau_{j,j-1}^{riding}$) for example estimated from historical data. The projected departure time from the first common stop will be estimated by the sum of the scheduled riding times between the current stop j and the first common stop j^{merg}:

$$\tilde{t}^{exit}_{i,j_i^{merg},k} = t^{arrival}_{ijk} + t^{dwell}_{ijk} + \sum_{s=j}^{j^{merg}} \tau^{riding}_{i,s,s+1,k} \tag{17}$$

In order to estimate the sequence of vehicles at the first common stop, irrespective of the line they serve, we need to project the expected departure time of the preceding vehicle and the succeeding vehicle of the same line as well as the expected and actual departure times from the vehicles of the other line. For each vehicle, the actual departure time from the last visited stop is retrieved and the expected departure time from the first common stop is estimated. The expected departure time of the current vehicle needs to be regulated in the case of uneven headways between consecutive vehicles regardless of the line. Then the expected headway between vehicles at the first common stop is calculated based on the potential waiting time, which is expressed as the difference between the waiting times the passengers at the merging stop will experience with and without holding time:

$$\begin{aligned} &\tilde{t}^{wait}_{i,j^{merg},k} = \tilde{t}^{wait\_H}_{i,j^{merg},k} - \tilde{t}^{wait\_0}_{i,j^{merg},k} = \\ &\Lambda_j^c \left(t^{hold}_{i,j^{merg},k} \right)^2 + \\ &\Lambda_j^c \left[\left(\tilde{t}^{exit}_{i,j^{merg},k} - \tilde{t}^{exit}_{i,j^{merg},k-1} \right) - \left(\tilde{t}^{exit}_{i,j^{merg},k} - \tilde{t}^{exit}_{i,j^{merg},k+1} \right) \right] t^{hold}_{i,j^{merg},k} \end{aligned} \tag{18}$$

Finally, the in-vehicle delay due to holding is the product of the passengers on board q_{ijk} and holding time t_{ijk}^{hold}:

$$t^{inveh}_{ijk} = q_{ijk} t^{hold}_{ijk} \tag{19}$$

Integrated real-time corridor management strategy

The total generalized passenger travel time due to holding can be expressed as a function of holding time by substituting the waiting time terms from Equations (16) and (18) and the in-vehicle delay due to holding from Equation (19) respectively in Equation (2). After solving subject to holding time, the total generalized passenger travel time due to holding is expressed by the following formula:

$$\begin{aligned} &t^{travel}_{ijk}\left(t^{hold}\right) = \beta^{wait} t^{wait}_{ijk}(t^{hold}) + t^{inveh}_{ijk}(t^{hold}) = \\ &\beta^{wait}\left(\Lambda_j^b + \Lambda_j^{bc}\right)\left(t^{hold}_{ijk}\right)^2 + \\ &\beta^{wait}\left\{\left(\Lambda_j^b + \Lambda_j^{bc}\right)\left[\left(t^{exit}_{ijk} - t^{exit}_{ijk-1}\right) - \left(t^{exit}_{ijk+1} - t^{exit}_{ijk}\right)\right]\right\} t^{hold}_{ijk} + \\ &\beta^{wait}\Lambda_j^c\left(t^{hold}_{ijk}\right)^2 + \\ &+\beta^{wait}\Lambda_j^c\left[\left(\tilde{t}^{exit}_{j^{merg},k} - \tilde{t}^{exit}_{j^{merg},k-1}\right) - \left(\tilde{t}^{exit}_{j^{merg},k} - \tilde{t}^{exit}_{j^{merg},k+1}\right)\right] t^{hold}_{ijk} + q_{ijk} t^{hold}_{ijk} \end{aligned} \tag{20}$$

The optimal holding time can then be calculated by differentiating the travel time function subject to holding time, and by setting the derivative equal to zero and solving with respect to holding time t_{ijk}^{hold} with the constraint that $t_{ijk}^{hold} \geq 0$, yielding equation (21):

$$t_{ijk}^{hold}=\max\{\frac{\Lambda_j^b+\Lambda_j^{bc}}{\Lambda_j}\frac{\left[\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)-\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)\right]}{2}+ \\ +\frac{\Lambda_j^c}{\Lambda_j}\frac{\left[\left(\tilde{t}_{i,j^{merg},k}^{exit}-\tilde{t}_{i,j^{merg},k-1}^{exit}\right)-\left(\tilde{t}_{i,j^{merg},k}^{exit}-\tilde{t}_{i,j^{merg},k+1}^{exit}\right)\right]}{2}- \\ -\frac{q_{ijk}}{2\beta^{wait}\Lambda_j},0\} \tag{21}$$

Refining the strategy using a distance decay function

It can be observed that the magnitude of each of the terms that regulates the departure time from the current stop j and expected departure time from the merging stop j^{merg} is affected by the corresponding share of passengers over the total remaining demand that will be experienced due to the control action. Each share of the total demand acts therefore as an endogenous weighing factor to the holding criterion, which influences the effect of each term on the final holding time. To avoid coordinating lines too early in operation at great distances from the common segment, we include a distance-based term in the weighing factors, designed to limit the effect of further away downstream demand:

$$\theta_1=\frac{\Lambda_j^b+\Lambda_j^{bc}}{\Lambda_j}+(1-\frac{1}{j^{merg}-j}) \tag{22}$$

$$\theta_2=\frac{\Lambda_j^c}{\Lambda_j}+(\frac{1}{j^{merg}-j}) \tag{23}$$

The final holding criterion for the branch stops is given in eqn. (24).

$$t_{ijk}^{hold}=\max\left\{\begin{array}{l}\theta_1\frac{\left[\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)-\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)\right]}{2}+ \\ +\theta_2\frac{\left[\left(\tilde{t}_{i,j^{merg},k}^{exit}-\tilde{t}_{i,j^{merg},k-1}^{exit}\right)-\left(\tilde{t}_{i,j^{merg},k}^{exit}-\tilde{t}_{i,j^{merg},k+1}^{exit}\right)\right]}{2}- \\ -\frac{q_{ijk}}{2\beta^{wait}\Lambda_j},0\end{array}\right\} \tag{24}$$

As a vehicle approaches the shared transit corridor, the control gradually shifts from single line to multiline control, based on the passenger groups that are affected by each control action. The holding criterion takes therefore into account (i) the regularization of the consecutive headways at the current stop, (ii) the regularization of the expected headways at the first common stop between lines and (iii) an adjustment that accounts for the demand on board and the remaining demand downstream.

Holding Criterion along the Shared Transit Corridor

For the shared transit corridor, we assume that all traversing lines are treated as a single line. Instead of regulating the headway subject to consecutive vehicles of the same line, all vehicles that interact with one another are taken into account. Waiting time with and without holding applied is calculated subject to the vehicle that departed prior to the current bus and the next one expected to arrive. Passengers at stops of overlapping routes board on the bus that arrives first to the stop, given that it minimizes their travel time (Chriqui and Robillard, 1975; Marguier and Ceder, 1984). When considering networks that have a shared transit corridor and no line that diverts from it, lines bear identical characteristics on the overlapping segment without alternation on their routes that may result to differences in the utility of choosing one line over another. Under such conditions, the holding criterion for the shared transit corridor is shown in eq. (24). The current vehicle from line i is regulating its departure based on the preceding vehicle k-1 and succeeding k+1 without considering the line these vehicles belong to according to the following holding criterion:

$$t_{ijk}^{hold}=\max\left\{\frac{(t_{jk+1}^{exit}-t_{ijk}^{exit})-(t_{ijk}^{exit}-t_{jk-1}^{exit})}{2}-\frac{q_{ijk}}{2\beta^{wait}\Lambda_j},0\right\} \tag{24}$$

Hence, the control along the shared corridor is analogous to the single line passenger cost minimization Eq. (1), except that t^{exit} of the preceding and succeeding vehicles in the common section can be from either line.

4. Experimental and Application Setup

The proposed holding strategy is assessed in two different experimental phases, applying different degrees of freedom in testing parameters. The experimental set-up is summarized in Table 1. First, the holding criterion for the branch stops is tested for an artificial network simulated in Mathworks™ MATLAB®; thereafter we proceed to a full network control of a real case study using empirical demand data, and employing the mesoscopic simulation software BusMezzo (Toledo et al., 2010), an agent-based transit operations and assignment simulation model.

Table 1 Summary of the key properties of the experimental set-up

Experiment/Application	Platform	Network	Control	Demand
Numerical simulation	MATLAB	Artificial	Branches only	Artificial
Agent-based Transit simulation	BusMezzo	Real	Full Network Control	Actual Data

In the following sections, the experimental set-up, the scenarios tested and the selected performance indicators are described.

4.1. Numerical simulation

For the first set of experiments, we consider a transit system including two lines that merge after operating independently, like the one illustrated in Figure 1. The two lines consist of 30 stops each, the first 15 of which are single line (branch) stops and the remaining are shared (trunk) stops. Both lines have the same planned headway and trips are dispatched with an offset equal to half of the planned headway, so that vehicles from the two lines are planned to arrive to the first common stop in an alternate fashion. All branch stops of both lines including the first common stop are simulated. All branch stops are considered time control points, i.e. holding can be applied at any of the stops. In addition, apart from the assumptions stated in Section 3.4, all stops are assumed equidistant (i.e. scheduled riding times are the same between stops) and both lines have the same demand profile.

The network is implemented in Mathworks™ MATLAB®. As the simulation progresses, vehicles are dispatched from the origin terminal, their running times between stops are sampled and, when vehicles arrive at stops, passengers are generated according to the actual headways. Vehicle dispatching times, actual riding times and passenger arrival rates are sampled from the corresponding distributions summarized in Table 2. Dispatching times are sampled by Gamma distribution. By varying the shape a and the scale b of the distribution, perfectly regular to perfectly irregular dispatching times can be replicated. For the current experimental setup, a shape parameter $a=10^6$ and scale parameter $b=10^{-5}$ were chosen, eliminating any disturbance in dispatching times allowing vehicles to depart on schedule. The stochasticity sources are the actual riding time and the passenger demand; trip chaining actions (i.e. the complete daily tours of the buses) are not considered in this experiment.

Riding times are sampled from lognormal distributions with scheduled riding times as the mean and a 20% standard deviation of the mean. The passengers generated are sampled from a Poisson distribution given the average arrival rate λ and the actual headway. The Poisson distribution has been used in the literature to replicate random arrivals of passengers at stops (Fu and Yang, 2002; Toledo et al., 2010). Demand is given in terms of arrival rates per origin-destination pair for each stop. The total number of boarding passengers is the sum of the arrival rates that originate from the given stop given the actual headway between vehicles at the stop. The number of alighting passengers depends on the number of passengers generated at upstream stops with the current stop as their destination. We consider the dwell time function as a linear function of boarding passengers B and alighting passengers A, multiplied by the service time needed per passenger to board and alight as estimated in the study of Dueker et al., (2004).

$$t^{dwell} = 3.48B + 1.7A \tag{24}$$

where B is the number of boarding passengers and A the number of alighting passengers.

Table 2 Summary of distributions specified in the experiment

Dispatching Time	Gamma Distribution	(a, b)
Actual Riding Times	Lognormal Distribution	(μ, σ)
Boarding Passengers	Poisson Distribution	(λ)

After updating vehicle occupancy, depending on the scenario, the assigned controller is triggered. Since overtaking is not allowed, the current vehicle cannot depart if its preceding vehicle is still at the stop, following a strict FIFO priority rule. After serving all branch stops, vehicles are sorted at the first common stop based on their arrival time and passengers are generated according to the actual joint headway between vehicles.

4.2. Agent-based transit simulation BusMezzo

The numerical simulation presented in Section 4.1 lacks in monitoring all different passenger groups and their travel times. Therefore, a more sophisticated simulation environment is adopted.

BusMezzo is a mesoscopic transit simulator built on the mesoscopic traffic simulator Mezzo (Burghout et al., 2005). BusMezzo has been shown to replicate phenomena of transit operation such as the propagation of headway variability and bunching (Toledo et al., 2010). Furthermore, demand can be given in terms of origin-destination pairs, and passengers are simulated as agents and can choose the optimal path that corresponds to the maximal individual utility (Cats et al., 2016). The user can monitor the travel time and the path of each passenger separately within the network and retrieve passenger cost of each passenger group, an important factor for the assessment of the performance of the criterion. Finally, the transit simulator has been used previously to compare and assess the performance of holding strategies, both schedule based and regularity based (Cats et al., 2011, 2012).

For the application using BusMezzo, lines 176 and 177 of the city of Stockholm are chosen (Figure 2). The two lines connect the metro station of Mörby centrum with the Ekerö communities via the densely populated municipality of Solna. As shown in Figure 2, the eastbound direction of lines 176 and 177 serve, before the shared transit corridor, 19 and 12 stops respectively. At the shared transit corridor, the two lines provide a tangential connection between the different radial metro lines and commuter trains as well as buses and the light rail connecting the outskirts of the city with the city center. The timetable of the lines is designed so that vehicles of the two lines depart from their terminals in a fashion that allows them to enter the trunk alternately. Overtaking is allowed in any part of the network. The entire fleet is equipped with real-time vehicle positioning data.

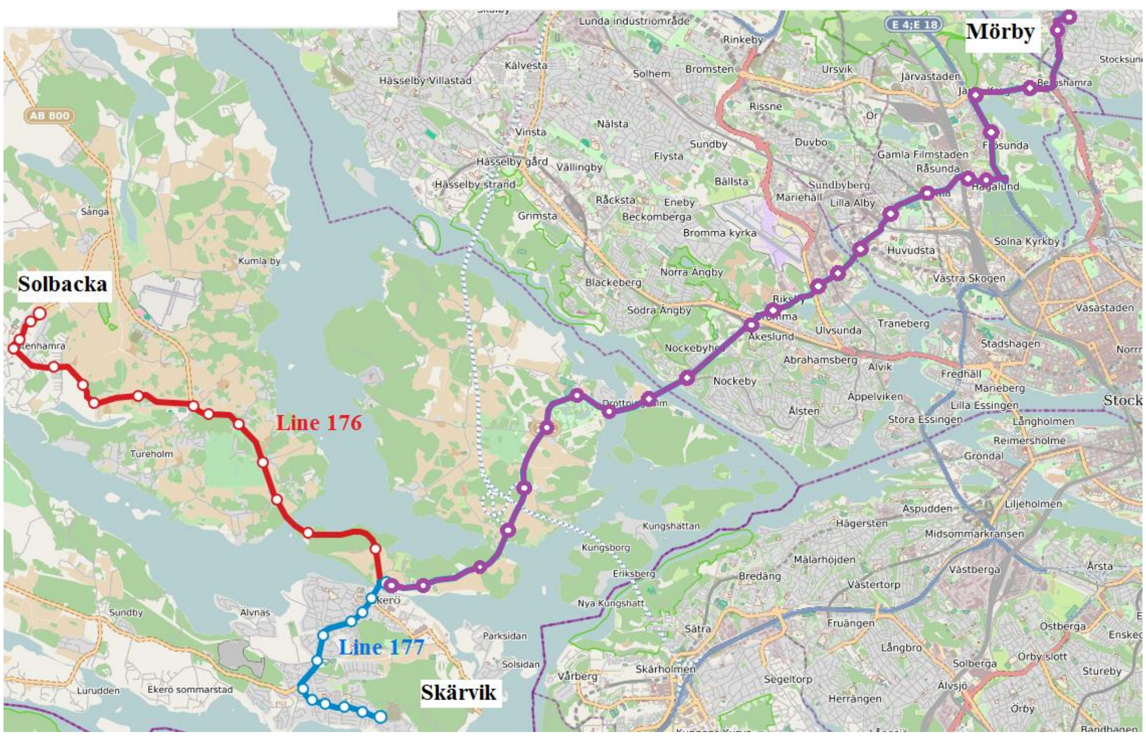
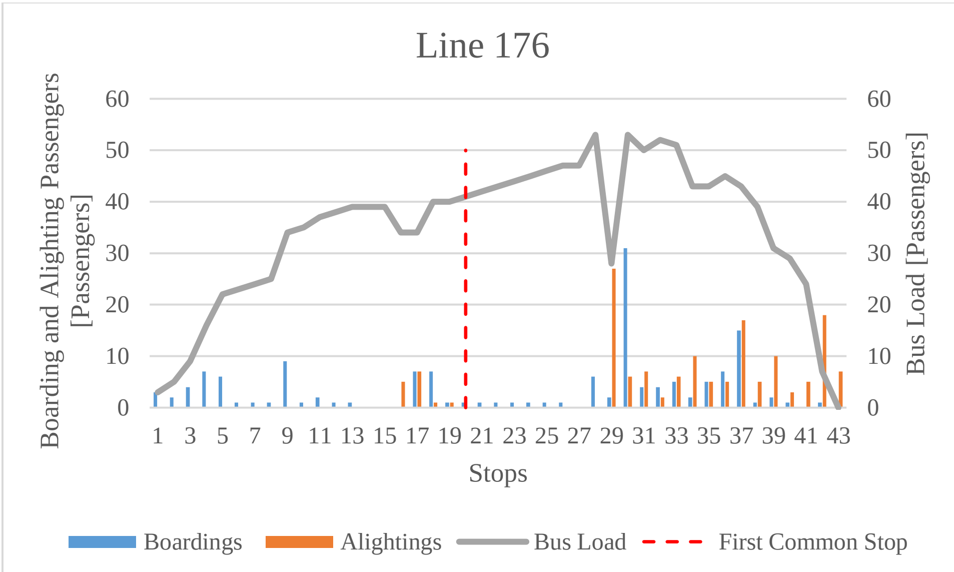
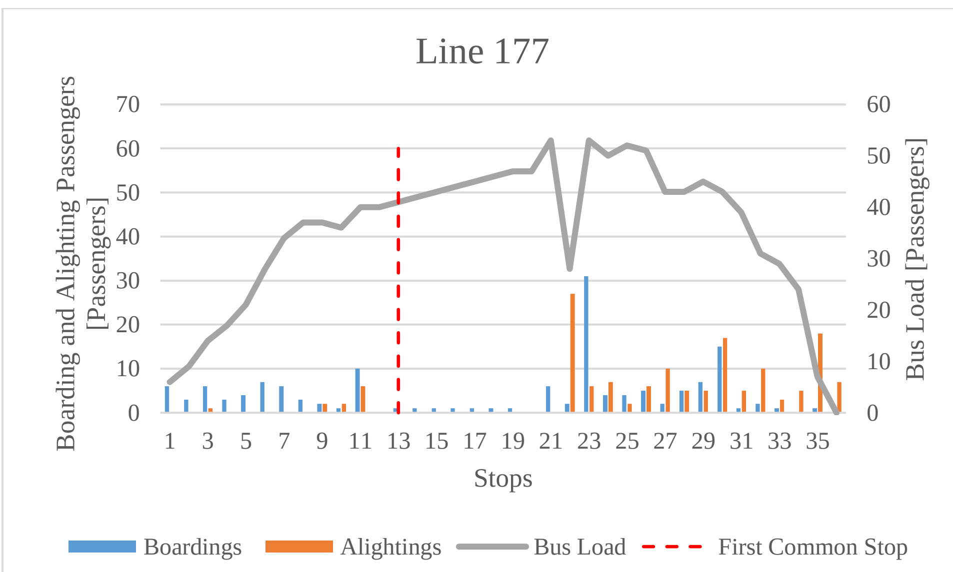


Figure 2: Lines 176 and 177 in Stockholm, Sweden

Empirical data for the demand and travel times of the lines was retrieved and specified as input to the simulation model. As can be observed in Figure 3, the two lines have a similar demand profile, with the majority of the passengers travelling from the branch to the trunk or along the trunk. Only a small share of the passengers has stops along the branch as both its origin and destination. In Table 3, the demand distribution for each of the lines is summarized.



(a)



(b)

Figure 3: Demand profiles of lines 176 (a) and 177 (b) (Westbound)

Table 3: Demand Distribution Breakdown for Lines 176 and 177

	Line 176		Line 177	
	Passengers per vehicle trip	Share of Total Demand	Passengers per vehicle trip	Share of Total Demand
Total Demand	147	100%	144	100%
Demand on Branch	14	9.5%	7	4.9%

Demand on Shared Transit Corridor	133	90.5%	137	95.1%
Corridor Demand generated at branch stops	40	27.2%	44	30.6%
Corridor Demand generated at corridor stops	93	63.3%	93	64.5%

4. 3. Scenarios

Three different schemes are tested: (i) a no control (NC) scheme (vehicles depart immediately after the completion of boarding and alighting operations); (ii) an independent implementation of passenger cost strategy applying the criterion of Eqn. (1) (IPC), and; (iii) the new cooperative scheme formulated in Eqn. (22) (CPC). All scenarios tested using the MATLAB numerical simulation are summarized in Table 4:

Table 4: Scenario design for experiments using numerical simulation

	No Control (NC)	**Independent Passenger Cost (IPC)**	**Cooperative Passenger Cost (CPC)**
Demand Profile 1 (25%-75%)	NC_1	IPC_1	CPC_1
Demand Profile 2 (50%-50%)	NC_2	IPC_2	CPC_2
Demand Profile 3 (75%-25%)	NC_3	IPC_3	CPC_3

For the BusMezzo case study, the first level of comparison concerns differences in tactical planning. Two scenarios with equal headways and unequal headways are tested. For the first scenario, both lines have the same headway of 10 min. For the second scenario, line 177 runs with a headway of 5 min while line 176 has a headway of 10 min. The planned joint headway is calculated as the average headway between the lines. The formulated CPC criteria are compared against a do-nothing scenario (NC) and an Even Headway control strategy (EH), which regulates the departure time based on the headways between consecutive vehicles and, at the same time, limits the maximum allowed holding time to 80% of the planned headway of the line (Cats et al. 2011). The schemes are tested for the actual demand and for a peak demand scenario, which corresponds to a uniform increase of +50% compared to the empirical demand level. The scenario design for the BusMezzo case study are summarized in Table 5.

Table 5: Scenario design for the application using transit simulation

		No Control (NC)	**Even Headway Strategy (EH)**	**Cooperative Passenger Cost CPC)**
Scenario 1: Equal Headways	**Actual Demand**	S1_NC_1	S1_EH_1	S1_CPC_1
	Peak Demand	S1_NC_2	S1_EH_2	S1_CPC_2
Scenario 2: Unequal Headways	**Actual Demand**	S2_NC_1	S2_EH_1	S2_CPC_1
	Peak Demand	S2_NC_2	S2_EH_2	S2_CPC_2

4. 4. Performance Indicators

Regularity performance indicators

The *coefficient of variation of headway*, the ratio between the standard deviation and the average headway, reflects the degree of variability of service headway. The coefficient of variation of the joint headway of both lines is also calculated to examine the impact of line coordination on trunk performance. The coefficient of variation of headway is calculated in line level based on departure-based headways. The coefficient of variation of the joint headway at the merging stop is based on arrivals to demonstrate the arrival with less variability at the common section due to coordinating control at the branch stops prior to the shared transit corridor.

The *level of bunching* is calculated for each line as the share of actual headways that are 50% greater or lower than the planned headway (TCRP, 2003).

Passenger performance indicators

The *generalized travel time* is reported with its components, waiting time and in-vehicle time. In the numerical experiments, the passenger travel times are given per passenger and per route segment for the branch stops. In the real case study, control is applied in BusMezzo also at the shared transit corridor, considering cooperation between lines. Thus, the travel times are reported at the line level and, in the shared transit corridor, for the joint operation. Furthermore, travel times are also given at the network level and compared per passenger group: the passengers travelling on branches, from branches to the shared transit corridor and within the shared transit corridor.

Vehicle performance indicators

Since holding has consequences for vehicle travel times, the *90th percentile of travel time* of vehicle trips, which is the determinant of fleet size requirements, within the branch for both lines is also reported. For the performance of the controller with BusMezzo, the 90th percentile of the total travel time for both lines is used. Moreover, the *average holding time* at each branch stop is investigated. Finally, the *prediction error* of the vehicle arrival projection scheme used by the controller to estimate the expected departure from the first common stop is examined.

Number of Replications

A certain number of replications is needed so that the results are within a certain confidence interval. The sample size needed for reliable and robust results is calculated using the following formula:

$$N' \geq t^2_{\frac{\alpha}{2},N-1} \frac{X_s^2}{X_d^2}$$

Where,

N' sample size;

$t_{\frac{\alpha}{2},N-1}$ student –t value for reliability α and a sample N;

X_d standard deviation of the chosen indicator for the sample N;

X_s accepted standard deviation.

The weighted travel time is used as reference measurement, since it lies at the basis of the formulation of the holding criterion. For the numerical experiment, 200 replications are conducted. Setting as a desired standard deviation a time equal to 1.5% of weighted travel time and for a student –t value of 1.971957 for 5% error and a sample of 200 replications, the maximum number of replications needed is 30, so the chosen number of replications is indeed sufficient. Likewise, for BusMezzo using the same reference measurement, 50 replications are conducted. For a student –t value of 1.677 for 10% error, 20 replications are sufficient.

5. Results and Analysis

5.1. Numerical experiments

Line performance

The performance in terms of regularity and travel time indicators is given in Table 6.

Table 6: Line level performance indicators

Distribution	Scenario	Line A					Line B				
		CV Headway	**Bunching**	**Waiting Time [sec]**	**In vehicle time [sec]**	**Generalized travel time [sec]**	**CV Headway**	**Bunching**	**Waiting Time [sec]**	**In vehicle time [sec]**	**Generalized travel time [sec]**
25-75	NC	0.50	0.10	138.63	308.66	585.92	0.49	0.09	136.79	308.32	581.90
	IPC	0.45	0.07	136.00	307.76	579.77	0.45	0.07	134.53	309.32	578.38
	CPC	0.41	0.05	133.88	307.82	575.58	0.42	0.05	133.73	308.83	576.30
50-50	NC	0.71	0.17	126.95	318.35	572.26	0.72	0.17	125.43	319.32	570.17
	IPC	0.59	0.10	120.94	320.10	561.99	0.59	0.11	119.17	320.33	558.67
	CPC	0.47	0.06	117.48	320.32	555.27	0.48	0.06	116.63	322.17	555.42
75-25	NC	0.66	0.19	163.99	319.28	647.26	0.67	0.18	163.58	321.65	648.81
	IPC	0.54	0.12	154.82	321.18	630.81	0.54	0.12	154.07	325.52	633.65
	CPC	0.41	0.06	148.69	323.22	620.60	0.41	0.06	148.54	322.49	619.57

As expected, applying control reduces service variability, and control strategies are more effective the higher the demand along the branch (case 75-25), which yields higher potential gains for demand-aware control strategies. Equivalent results are also found in terms of bunching. The control schemes reduce headway variability and this is reflected in the results of waiting times at stops. The waiting time gains due to controlling are greater at the third demand scenario for both lines. Since the control scenarios are based on holding, passengers may experience increased on-board time due to the additional time a vehicle remained at a given stop. In-vehicle time with IPC and CPC increases only marginally compared to the do-nothing scenario. This can be explained by the fact that both holding criteria adjust the holding time calculated to the occupancy and the remaining demand downstream, to limit excessive in vehicle time. The cooperative control yields the lowest travel time in all three scenarios.

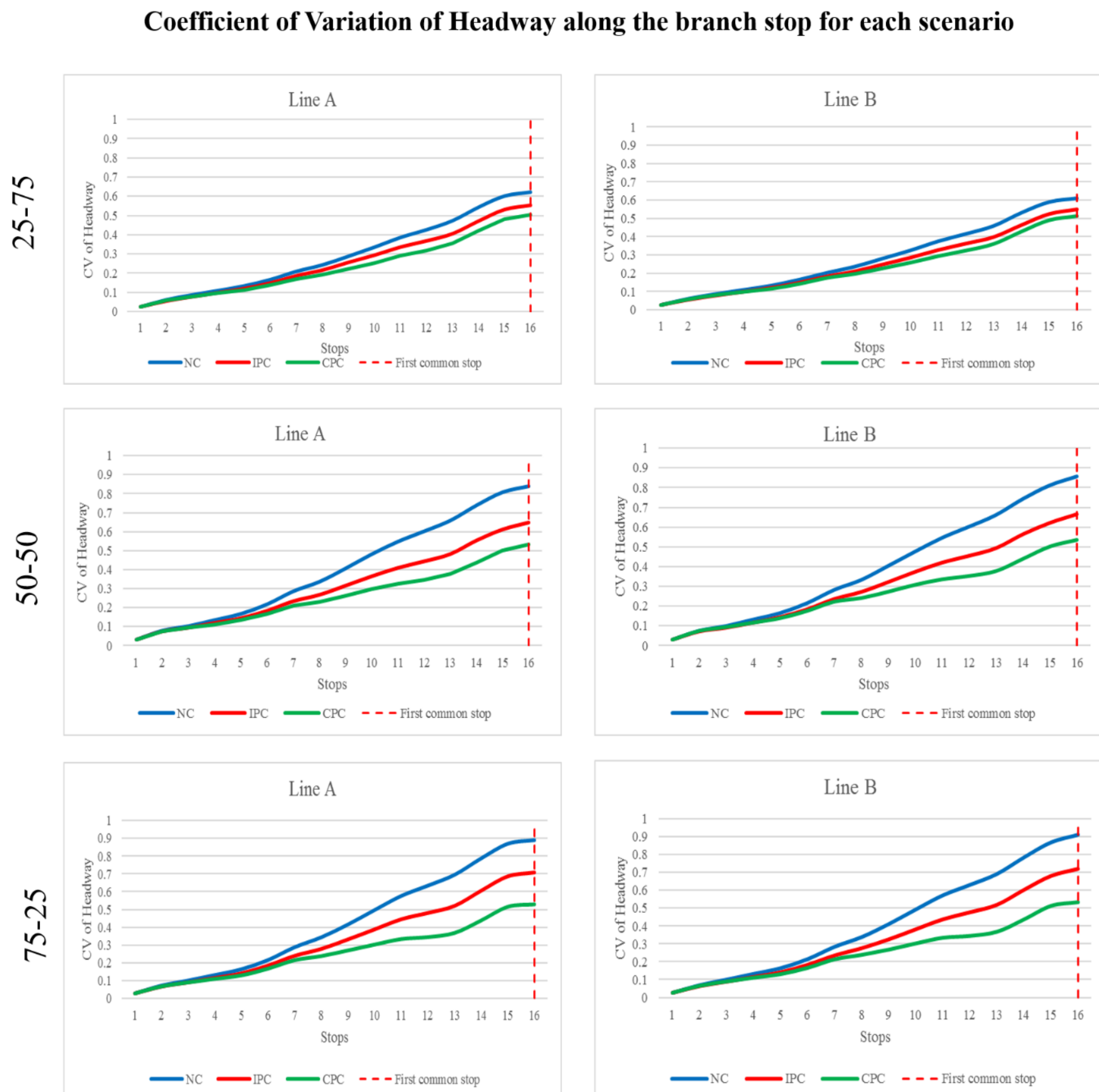


Figure 4: Coefficient of variation of headway along branch stops for each scenario

The effectiveness of the new control scheme is investigated by analyzing the progression of the variability of headways along the branch stops (Figure 4). Both controllers result in improved headway variability compared to the No Control scenario. For the scenario with low demand on the branch, CPC follows the same behavior as the single line controller, which relies only on the consecutive headways at the current stop corrected by the occupancy and the passengers on board. In contrast, in the third demand scenario, CPC is more effective and maintains low headway variability in the part of route where most of the demand is concentrated. Recall that CPC holding criterion is an extension of the IPC holding criterion with the addition of line coordination and a more appropriate adjustment to the remaining demand, considering also the demand that can be served by both lines via the passenger ratio and the weights added to each term. As a result, the main objective of the controller shifts between the importance of line regularity or line coordination based on the demand distribution, resulting in more effective control in this network configuration than single line control.

Arrival at the first common stop

One of the most crucial elements in the current network configuration is the transition from the branches to the trunk. The total holding time before the trunk is estimated with respect to the actual line headway at the current branch stop and the expected headway at the first common stop. It is then adjusted considering the distance from the trunk itself and the number of passengers that will experience the additional time the vehicle remains at the stop. Figure 5 shows the coefficient of variation of headway at the first common stop, based on the arrivals of vehicles from both lines. While IPC yields some beneficial results at the first common stop, CPC outperforms it with a better performance for all three demand scenarios, yielding a greater level of coordination. The most significant reduction is observed for the third demand profile, resulting from control on all branch stops since the majority of the demand is concentrated in that part of the line.

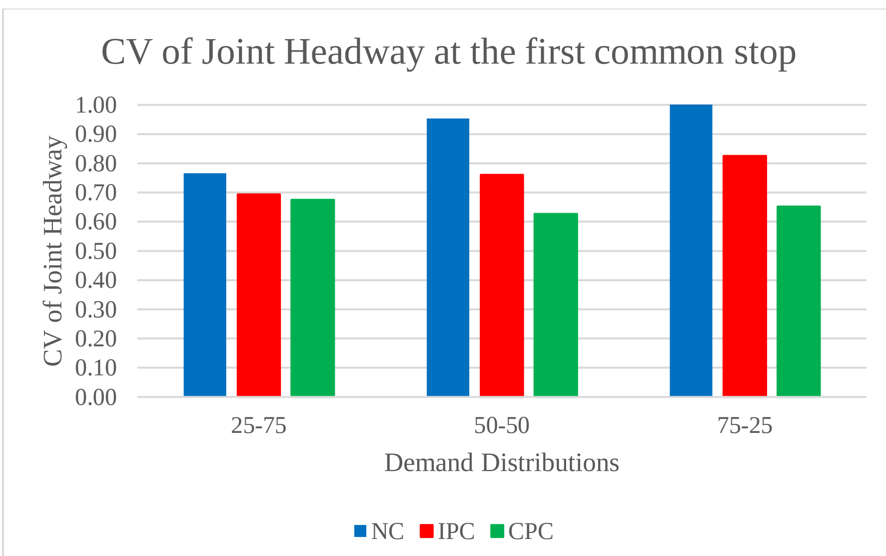


Figure 5: CV of Joint Headway at the first common stop

Travel time distribution

Finally, CPC outperforms all other schemes in travel time variability for most of the scenarios, as can be seen from the travel time distributions in Figure 6. By reducing the variability in travel time until the first common stop, the adherence to the joint headway can be ensured.

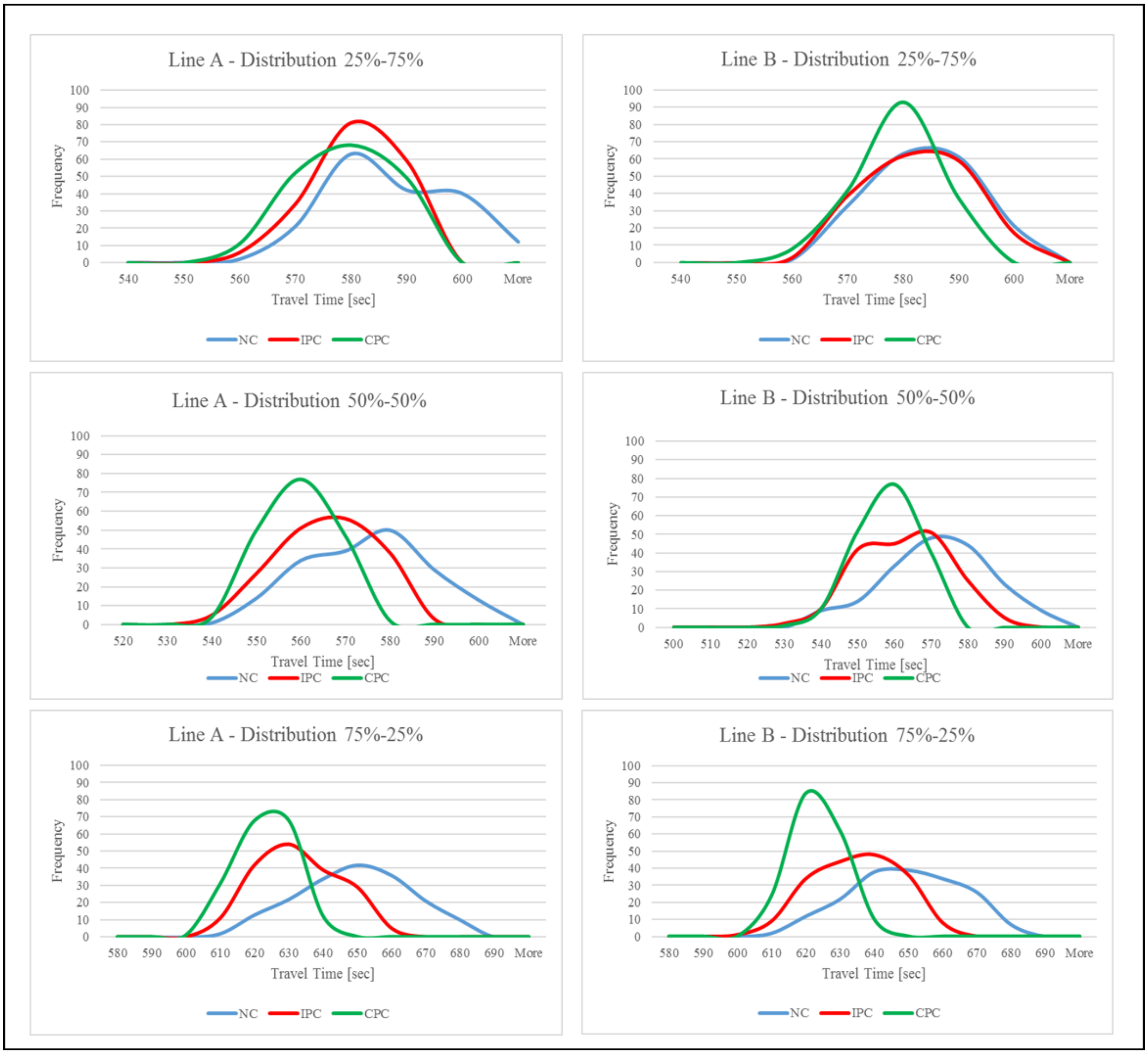


Figure 6: Branch Travel Time Distributions for the different scenarios

Interestingly, at the 25%-75% demand distribution scenario, while line B shows satisfactory results, line A shows an increased travel time variability. It seems that for this demand distribution, one line out of the two is in charge of line coordination, applying additional control, while the other continues to aim for line regularity, which gradually weakens towards the end of the individual segment. For branch demand equal or greater than the corridor demand, both lines perform similarly. In addition to higher variability, the no control scenarios also result in longer vehicle travel times than the controlled scenarios. This can be explained by the experimental setup and the assumption that overtaking is not allowed due to the FIFO departure rule.

5.2. Transit simulation application

Line Results

In terms of regularity measurements at the line level, EH outperforms the other schemes, as it directly relates to its objective in Scenario 1. Line headways vary less with EH for both lines 176 and 177 (Table 7) and almost no bunching occurs under all scenarios. However, travel times at the branches are the lowest with CPC. Compared to EH, CPC estimates the holding time needed at a stop based on the current spacing between vehicles and the expected position current vehicle will have at the first common stop. Therefore, the final holding time with CPC is higher resulting in stronger control compared to EH. However, this comes at the cost of an increased in-vehicle time, especially for the shorter line (Line 177). Overall, CPC is more beneficial in terms of generalized passenger travel time.

For the second scenario, it can be observed that for this specific setup, CPC is contributing less in terms of regularity for the high frequency line (Line 177). The regularity indicators show lower gains in the regularity factors, CV of headway and bunching. Interestingly, CPC performs similarly to EH for line 176 on regularity indicators. With CPC, waiting time and in-vehicle time per passenger are also lower than with single line control for both base and high demand.

Table 7: Line performance indicators for Scenario 1

		Line 176					Line 177				
		CV Headway	**Bunching**	**Waiting Time [sec]**	**In vehicle time [sec]**	**Generalized travel time [sec]**	**CV Headway**	**Bunching**	**Waiting Time [sec]**	**In vehicle time [sec]**	**Generalized travel time [sec]**
Base Demand	NC	0.154	0.015	270.47	1605.17	2146.11	0.151	0.024	269.74	1457.96	1997.44
	EH	0.116	0.00	267.85	1585.34	2121.04	0.114	0.000	265.39	1464.32	1995.09
	CPC	0.190	0.055	225.70	1552.46	2003.85	0.11	0.006	196.84	1518.01	1911.68
Peak Demand	NC	0.179	0.029	362.6	1759.3	2484.5	0.177	0.025	376.9	1694.5	2448.4
	EH	0.145	0.006	348.3	1699.5	2396.0	0.155	0.011	362.2	1685.3	2409.7
	CPC	0.231	0.072	304.2	1748.8	2357.2	0.179	0.024	313.0	1737.1	2363.1

Table 8 Line performance indicators for Scenario 2

		Line 176					Line 177				
		CV Headway	Bunching	Waiting time [sec]	In vehicle time [sec]	Generalized travel time [sec]	CV Headway	Bunching	Waiting time [sec]	In vehicle time [sec]	Generalized travel time [sec]
Base Demand	NC	0.16	0.02	214.45	1615.98	2044.88	0.38	0.22	161.08	1481.26	1803.42
	EH	0.11	0.00	194.65	1667.62	2056.91	0.19	0.02	133.27	1518.39	1784.93
	CPC	0.11	0.01	182.99	1620.80	1986.77	0.26	0.10	131.30	1487.29	1749.88
Peak Demand	NC	0.19	0.15	335.59	1806.73	2477.91	0.34	0.17	253.98	1655.16	2163.13
	EH	0.15	0.02	352.01	1845.77	2549.78	0.20	0.02	225.76	1701.28	2152.80
	CPC	0.14	0.11	280.05	1811.40	2371.50	0.29	0.10	209.01	1695.26	2113.28

When plotting the coefficient of variation of headway along each of the lines, EH is consistent in keeping headway variation low while two patterns for the CPC are observed (Figure 7). Up to the branch stops (prior to the dashed red line), CPC performs similar to EH, maintaining a low coefficient of variation of headway. Close to the first common stop, the control criterion aims mostly for line coordination and vehicles are held to ensure a lower joint headway variability at the first common stop and further downstream. At the shared transit corridor, there is a loss of in-line headway adherence. Line 176 exhibits the highest headway variability with CPC, while for line 177 coefficient of variation of headway increases faster for the peak demand scenario.

The coefficient of variation of headway is plotted against the stops for both lines under scenario 2 in Figure 8. According to the results, CPC manages to maintain lower or equal variability compared to the single line strategy prior to the overlapping segment. It can also be observed that line 177 is severely penalized at the shared transit corridor, where the headways of the line are regulated also subject to vehicles of line 176. This leads to a lower performance on the shared transit corridor and, as shown before, lower overall performance of the line in terms of regularity.

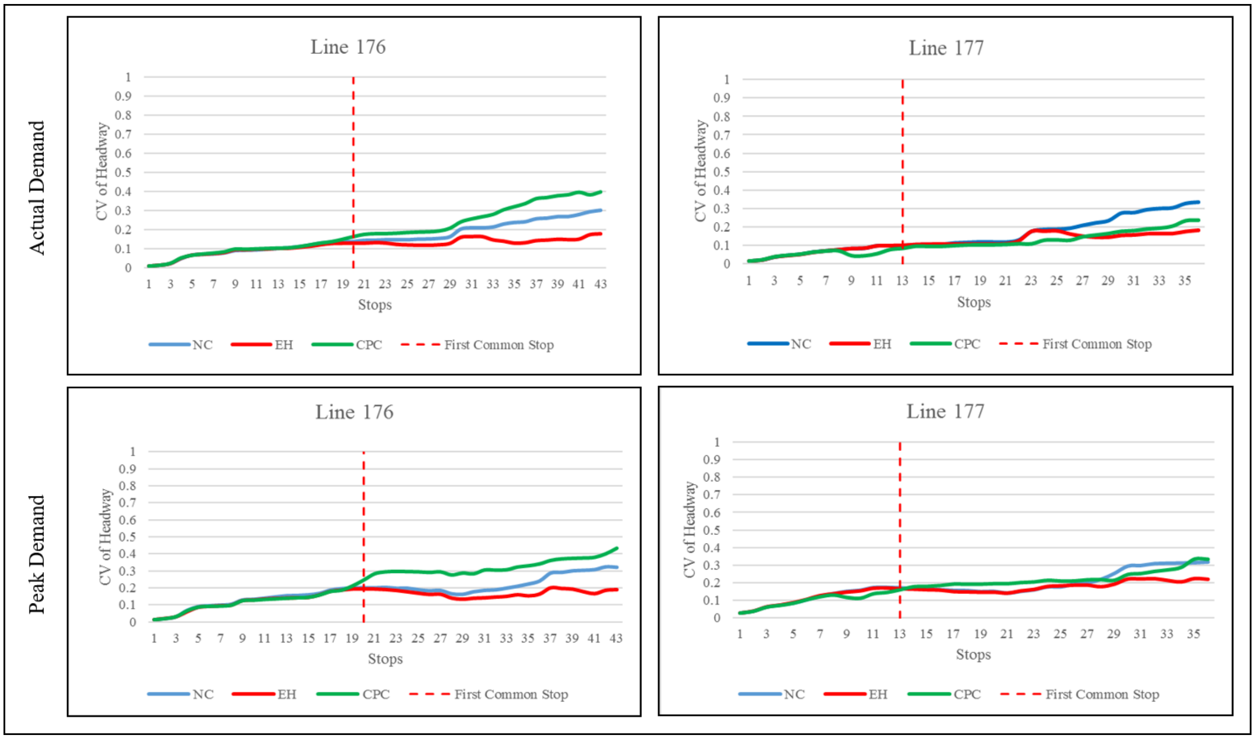


Figure 7: CV of Headway of lines 176 and 177 for Scenario 1

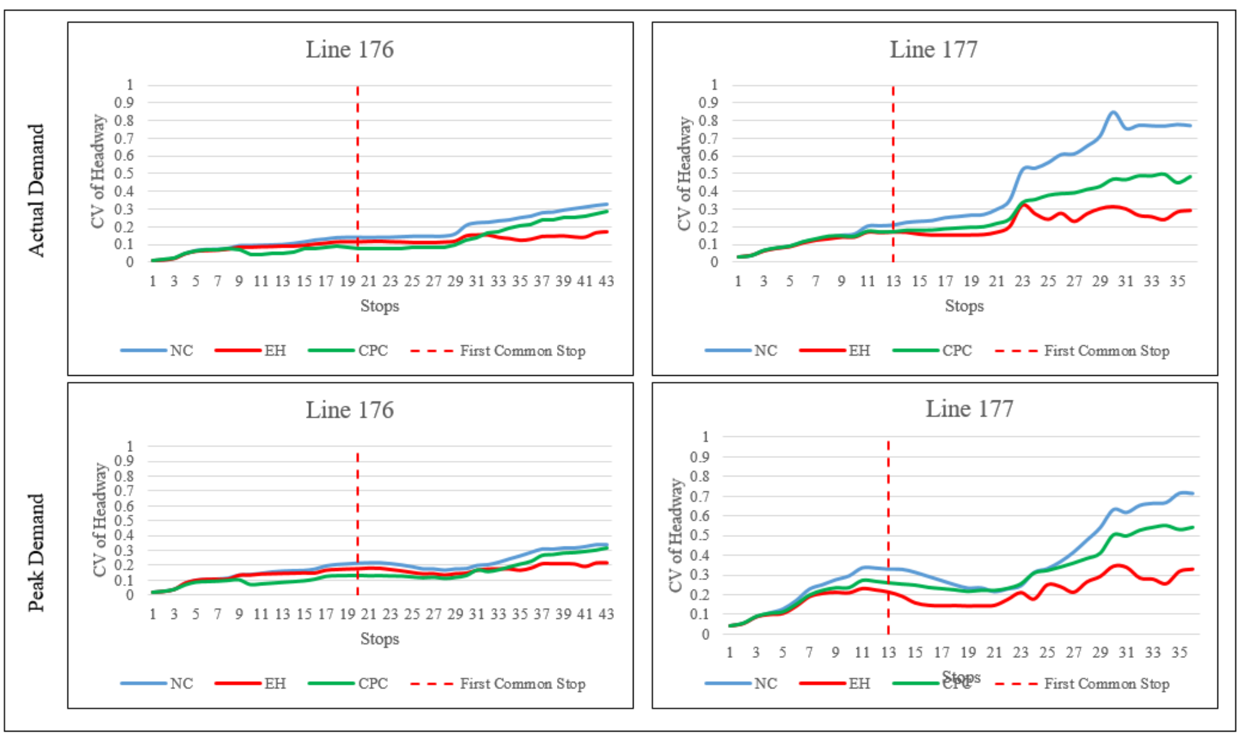


Figure 8 CV of Headway of lines 176 and 177 for Scenario 2

Shared Transit Corridor

In this section, the results of the joint performance of the two lines are discussed. Table 9 summarizes the performance indicators for the joint operation in the shared part for the first scenario. The proposed cooperative control results in a smoother transition to the common part. As can be seen by the arrival pattern at the first common stop, the cooperative scheme outperforms all other schemes. The variability of the joint headway remains low compared to EH. This is also reflected by travel times per passenger, where the greater gains are in terms of waiting time.

Table 9: Performance Indicators for the joint operation in the shared transit corridor for Scenario 1

<table>
<tr><th colspan="2">Shared Transit Corridor</th><th>CV of Headway at the merging stop</th><th>CV of the Joint Headway (Corridor)</th><th>Waiting Time per passenger [sec]</th><th>In vehicle time per passenger [sec]</th><th>Generalized Travel Time per passenger [sec]</th></tr>
<tr><th rowspan="3">Actual Demand</th><th>NC</th><td>0.722</td><td>0.833</td><td>252.99</td><td>1233.06</td><td>1739.03</td></tr>
<tr><th>EH</th><td>0.728</td><td>0.832</td><td>248.04</td><td>1227.03</td><td>1723.10</td></tr>
<tr><th>CPC</th><td>0.488</td><td>0.420</td><td>168.77</td><td>1215.91</td><td>1553.46</td></tr>
<tr><th rowspan="3">Peak Demand</th><th>NC</th><td>0.735</td><td>0.812</td><td>397.2</td><td>1407.9</td><td>2202.2</td></tr>
<tr><th>EH</th><td>0.748</td><td>0.823</td><td>374.9</td><td>1367.1</td><td>2116.9</td></tr>
<tr><th>CPC</th><td>0.616</td><td>0.490</td><td>303.5</td><td>1399.1</td><td>2006.1</td></tr>
</table>

In case of lines with different headways (Scenario 2), with CPC vehicles arrive at the merging stop with significantly lower variability of headway compared to the other schemes. In the case of cooperation between lines, the coefficient of variation of headway is lower too. The benefits of cooperation are also reflected in the travel times per passenger, which is the lowest under CPC. The results are similar for both demand levels. The results for the shared transit corridor in Scenario 2 are shown in Table 10.

Table 10 Performance Indicators for the joint operation in the shared transit corridor for Scenario 2

<table>
<tr><th colspan="2">Shared Transit Corridor</th><th>CV Headway at the merging stop</th><th>CV Headway (Corridor)</th><th>Waiting time per passenger [sec]</th><th>In vehicle time per passenger [sec]</th><th>Generalized travel time per passenger [sec]</th></tr>
<tr><th rowspan="3">Base Demand</th><th>NC</th><td>0.56</td><td>0.84</td><td>165.36</td><td>1252.87</td><td>1583.58</td></tr>
<tr><th>EH</th><td>0.54</td><td>0.65</td><td>125.85</td><td>1300.87</td><td>1552.58</td></tr>
<tr><th>CPC</th><td>0.44</td><td>0.52</td><td>122.30</td><td>1249.84</td><td>1494.44</td></tr>
<tr><th rowspan="3">Peak Demand</th><th>NC</th><td>0.59</td><td>0.70</td><td>303.95</td><td>1401.31</td><td>2009.21</td></tr>
<tr><th>EH</th><td>0.52</td><td>0.59</td><td>262.76</td><td>1462.68</td><td>1988.19</td></tr>
<tr><th>CPC</th><td>0.46</td><td>0.48</td><td>236.54</td><td>1443.22</td><td>1916.29</td></tr>
</table>

Network travel times per passenger group

Table 11 summarizes the relative differences in time per passenger when compared against the No Control scenario. There is a significant reduction in waiting time with CPC, with a marginal increase in in-vehicle time in both Scenario 1 and Scenario 2. Overall, the EH gives a very small improvement in travel time at the network level, whereas passengers receive time saving of more than 10% and even 15% with the CPC.

Table 11: Network performance with control compared to NC

	Network		Waiting Time	In Vehicle Time	Net Network Total Gains
Scenario 1	**Actual Demand**	**EH**	-1.1%	-0.34%	-1.45%
		CPC	-18.9%	2.79%	-16.19%
	Peak Demand	**EH**	-1.16%	0.81%	-0.35%
		CPC	-15.8%	1.83%	-13.9%
Scenario 2	**Actual Demand**	**EH**	-4.6%	2.4%	-2.1%
		CPC	-17.0%	1.6%	-15.4%
	Peak Demand	**EH**	-5.1%	1.9%	-3.2%
		CPC	-18.4%	0.0%	-18.4%

Network users consist of three passenger groups, which have different stakes in the control logic, depending on their travel paths. As illustrated in Figure 11, the two strategies impact the passengers travelling within the branch similarly, with marginal differences compared to the uncontrolled scenario. The passengers traversing the merging point are exposed to line coordination control at the branch, and line or corridor regularity control depending on their final destination. This penalizes their travel time by increasing the in-vehicle time. On the other hand, passengers travelling within the shared transit corridor, which constitute most of the total demand, are favored by the better coordination between lines through reductions in waiting time.

In scenario 2, the results are similar to the equal headway setting. Passengers travelling from the branch to the corridor experience longer in-vehicle times, because of holding time to regulate the transition to the common route segment. On the shared transit corridor, CPC manages to reduce the waiting time per passenger in both cases, with an additional decrease of 20 sec for the peak demand scenarios.

Travel times

Holding strategies trade off an increase the travel time of the vehicles against a reduction of variability. When comparing the 90th percentile of vehicle travel times, it can be observed that CPC leads to different results for the lines under the first scenario setup (Figure 9). Under CPC, Line 176 has a lower average travel time than with EH and NC but greater variability, while the travel time of line 177 is prolonged by almost 5 min with lower variability compared to the other schemes. Hence, there is no conclusive relation between the introduction of CPC and vehicle travel time variability. As can be expected, variability for both lines increases with the demand. When applying CPC, vehicles regulate their departures at the majority of stops of both lines (the shared transit corridor stops) subject to parameters that are exogenous with respect to their own line to achieve coordination in a corridor level. It is therefore expectable to encounter a loss in line performance to achieve higher benefits at the network level.

However, when regulating lines with different headways as in Scenario 2 (Figure 10), CPC shows results that are more robust. More specifically, line 176 has a higher average travel time but lower variability than in Scenario 1. Furthermore, the average travel time of line 177 is shorter with CPC compared to EH with the same level of variability. In Scenario 2, EH again outperforms all other schemes but the results with CPC also allow the operator to better administer the available fleet resources.

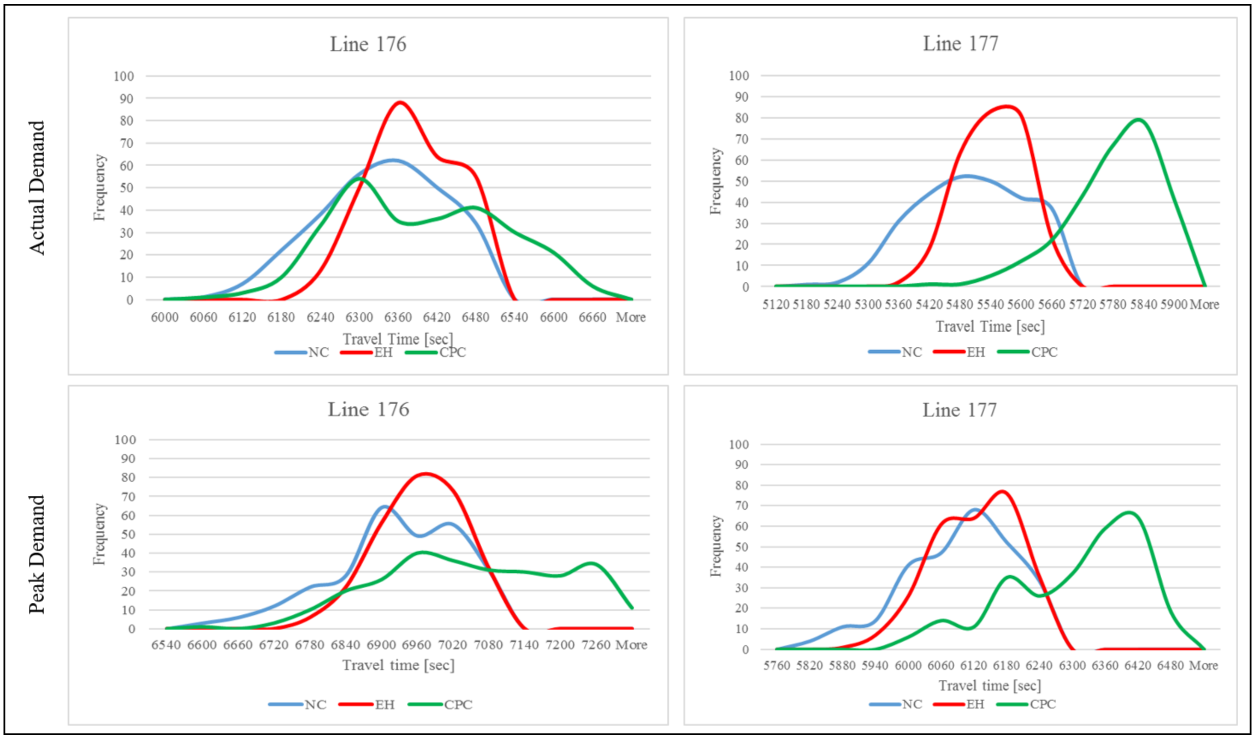


Figure 9: Travel time distribution for lines 176 and 177 for Scenario 1

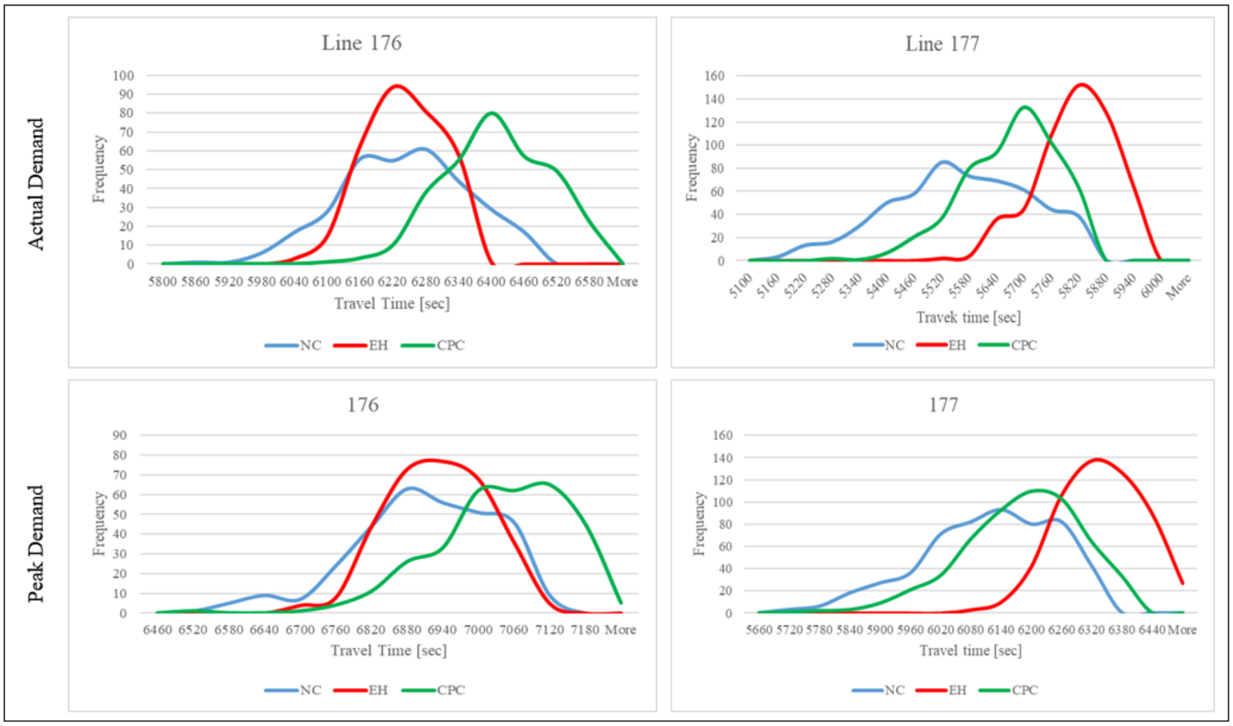


Figure 10 Travel time distribution for lines 176 and 177 for Scenario 2

Holding times and frequency of holding

In this specific case study, the shorter line (line 177) is consistently charged with extra holding time, the greatest share of which is aimed at line coordination.

One feature of the cooperative control scheme is that the control objective on the branches gradually shifts from single line regularity towards line coordination. The transition and the main source of holding time depends on the remaining downstream demand and the distance from the merging stop. Figure 13 shows the average holding time at each branch stop of line 176 and 177, respectively, and the contribution of each of the holding criterion terms. Aiming for line coordination adds significantly more holding time with respect to the average holding times of the line, especially towards the end of the branch. It is worth noting that at the beginning of the route, where branch regularity is more important, control is rarely necessary since variability has not propagated to undesired levels and the demand on the branch is low, and hence it does not lead to service disturbances. When line coordination becomes the most crucial factor for control on the branches, vehicles are held for significantly longer times. Holding time for line coordination is introduced at the last stops of the branch. This additional time penalizes the passengers travelling from the branch to the shared transit corridor. If this passenger group is the majority of the demand, the control can yield to longer travel times due to control, reducing the overall net gains achieved by CPC.

Figure 11: Travel time difference between NC and EH, CPC for the different passenger groups for Scenario 1

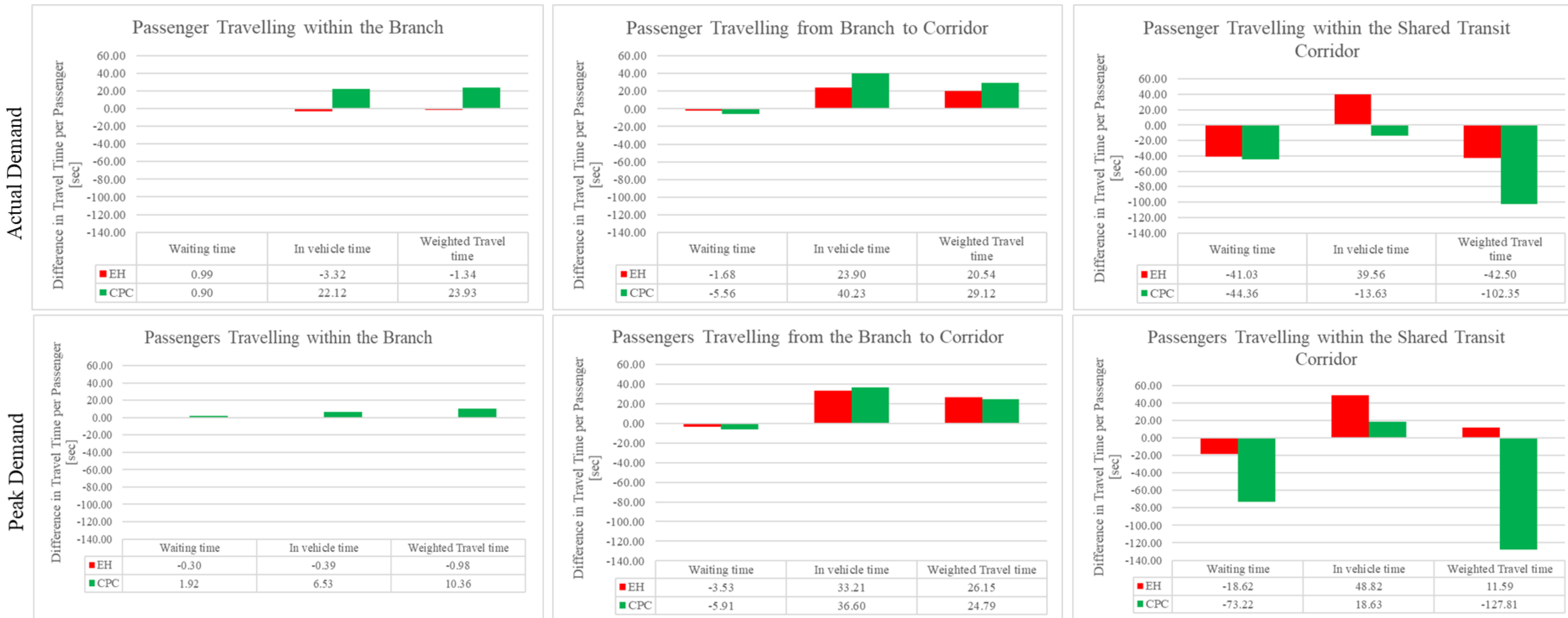
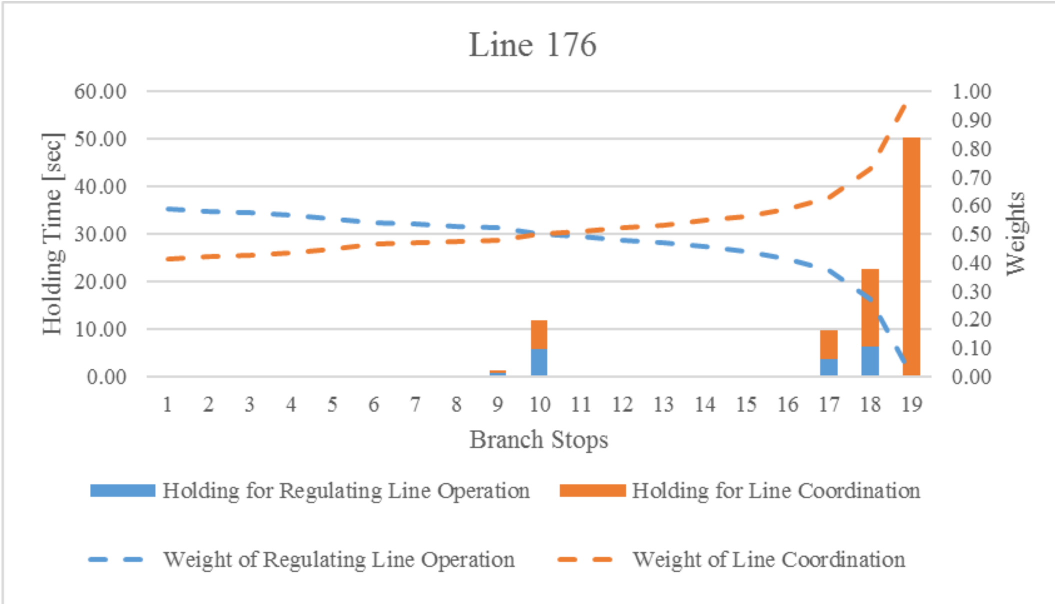


Figure 12 Travel time difference between NC and EH, CPC for the different passenger groups for Scenario 2



(a)

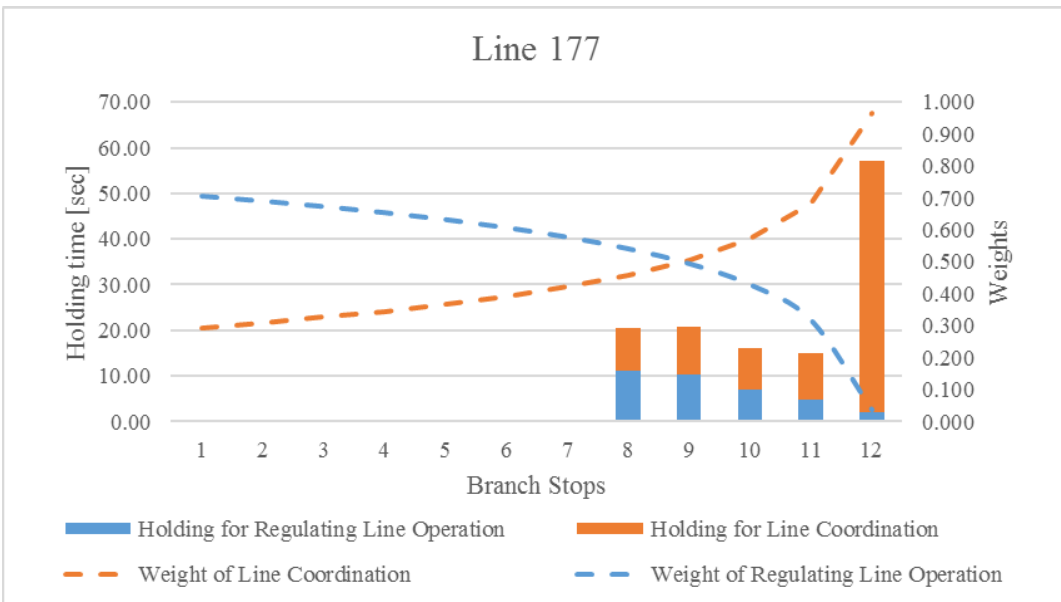
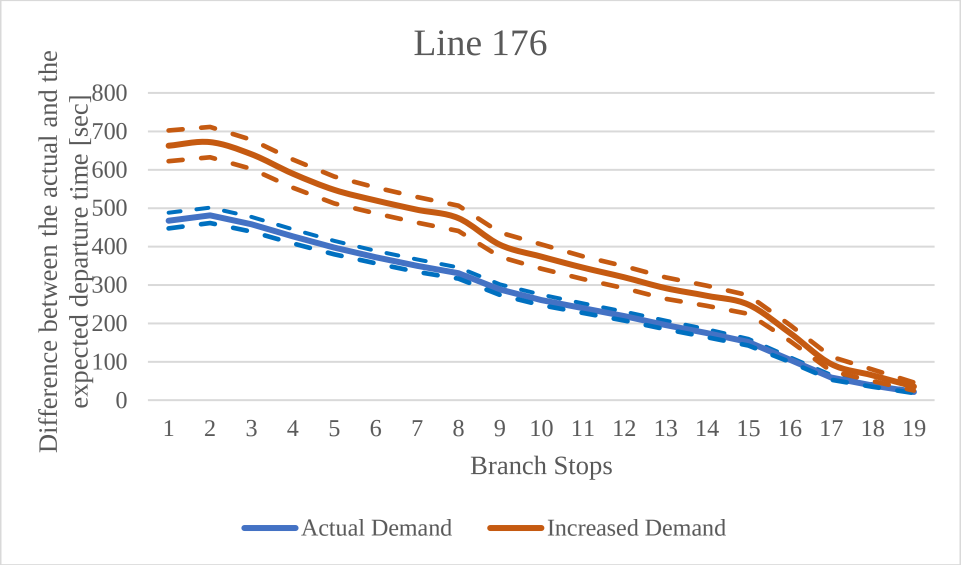


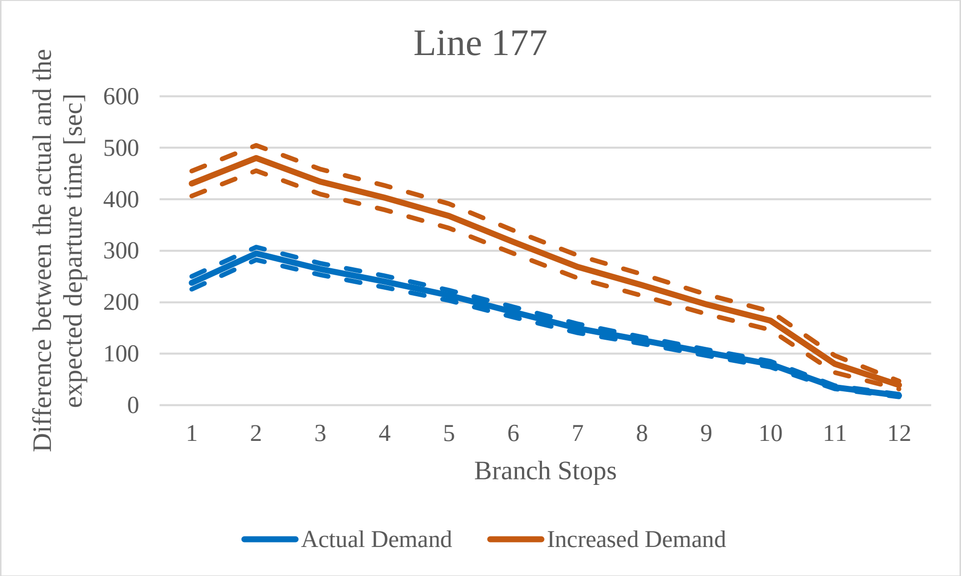
Figure 13: Average holding time at the branch stops of line 176 (a) and 177 (b)

Projection accuracy

The new cooperative control scheme on the branches includes a term for line coordination that incorporates predictions of vehicle departure times for both lines from the first common stop. The expected departure time is calculated by summing the scheduled riding times between the current branch stop and the merging stop. The estimation error increases for increased prediction horizons, i.e. the further upstream the stop is, and hence further away from the first common stop (Cats and Loutos, 2016). Figure 14 depicts the average difference between the projected time to departure as calculated at each stop and the corresponding actual departure time from the merging stop for lines 176 and 177. The average difference and its deviation decrease as vehicles approach the first common stop and, for the longer branch, line coordination is exposed to more inaccurate estimations at the beginning of the route. These results are consistent with the empirical analysis of the same prediction scheme reported in Cats and Loutos (2016). The prediction error also increases for higher demand.



(a)



(b)

Figure 14: Prediction Error on every branch stop of Line 176 (a) and Line 177 (b)

The weights introduced in the holding criterion for the branch stop include a distance correction term that amplifies the impact of the line coordination term as vehicles approach the common segment. However, the holding criterion still includes holding time from the line coordination term, the calculation of which can be based on projected departure times with high prediction error. Ultimately, the quality of controllers can be improved by improving the quality of the prediction schemes deployed in their application.

6. Conclusions

6.1. Key findings

A typical transit network configuration consists of lines that operate initially on individual stops and then merge serving a shared transit corridor. For such network configurations, in this work we develop a real-time holding criterion gradually accounting for coordination, which is extended beyond the single line level. Furthermore, the additional benefits of applying control on the joint operation are tested by regulating the departures from the shared stops based on all vehicles that serve the stop. The developed control criteria are tested using an artificial network and a real-world network through simulation.

The addition of a line coordination term in the control strategy reduces the variability of the joint headway when vehicles enter the shared transit corridor. The extent of this reduction depends both on the demand distribution along the line and on the demand level. From the numerical analysis conducted, the holding criterion is proven to adjust to the demand distribution at the branches, by prioritizing the regularization of line headway or the joint headway. In all scenarios, the arrival to the shared transit corridor is achieved with lower variability compared to single line control.

The real-world network was simulated for two different scenarios, one for lines with equal headways and one with different headways. As we showed in the results section, the proposed control approach enacts a tradeoff between network-wide results and single line, passenger group related indicators. While comparing the regularity performance indicators in line level, it is observed that CPC’s gains are not in the level of single line control. In addition, due to control decisions to coordinate the lines, passengers travelling from branch to shared transit corridor are penalized with additional in vehicle time. On the other hand, the significant reduction of travel time with coordinated control in the shared transit corridor, where the majority of the demand is, without loses for any passenger group in the remaining parts of the network sum up to higher time savings for passengers in network level.

Overall, the multiline holding criterion is sensitive in two main factors:

- Length of the lines;
- Demand profile.

Both factors have been assessed in the current study as of same and critical importance as they were introduced as weighting factors in the holding criterion. Depending on the size of each stop set (branch-corridor) and their corresponding demand, the results may differ, since the criterion will always prioritize regulating the headway that benefit the majority of the passengers. The flexibility of the criterion demonstrated by numerical experiments at the branches, where the criterion behaves differently depending on the segmentation if the demand. Also, control has different effects for lines with different lengths, with one line losing further in performance for the overall benefit of the system. A full sensitivity analysis based on the demand segmentation can validate when it is more beneficial and for which passenger groups to apply single line or multiline control.

We furthermore compared the operation of single line control and coordinated control for the shared transit corridor. The results demonstrate that cooperation between different lines outperforms single line control at the shared transit corridor, drastically reducing the waiting times of the passenger groups that travel within the common segment. These savings yield an overall reduction in total passenger cost. Hence, cooperation can be a viable solution, depending on the distribution of passengers along the network.

6.2. Limitations and recommendations for future research

The proposed control strategy includes a passenger ratio term that is based on historical demand data. This can be potentially substituted by the actual number of boarding passengers if such data is available in real time. Moreover, the criteria presented can be tested for a greater network including applications with more than two lines or different service and demand characteristics, in order to investigate further the transferability and scalability of the proposed approach. The current study is performed in the context that lines either belong to the same operator or (in the case of different operators) there is a cooperation scheme in effect with centralized control. Future studies will also include scenarios with more than one operators and different operation schemes in order to check the applicability of such a strategy in contexts where lines are operated by competing parties.

Managing networks of this type in terms of control, including all different configurations of this network type, is a part of future study. Networks of different size and branch and corridor lengths as well as demand segmentations will be assessed to determine when multiline control can be beneficial. Finally, future research efforts will focus on extending the results and control criteria presented in this work to the more general instances of shared transit corridors where the branching point lies after the common portion (diverging network), as well as multiple branching situations (merging *and* diverging), where transferring passengers must also be explicitly accounted for.

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Appendix

I. Derivation of single line holding criterion

We assume that passenger travel time t^{travel} consists of passenger waiting time at stops and the in vehicle time of passengers on board. Passengers perceive waiting at stops as greater disturbance, therefore waiting time is weighted by a weight β^{wait} :

$$t_{ijk}^{travel}=\beta^{wait}t_{ijk}^{wait}+t_{ijk}^{inveh}$$

The expected waiting time per passenger is half the current headway at a stop. Following the assumption of random passenger arrivals at stops, the number of passengers arriving at the current stop is the product of the sum of the arrival rates $\sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}$ and the current headway. We assume that the headway is preserved and it affects the passengers at the downstream stops too. For a departure time t_{ijk}^{exit} of the current vehicle, the waiting time between the preceding $t_{ijk}^{wait\_p0}$ and the succeeding vehicle $t_{ijk}^{wait\_s0}$ is given by the following equations respectively:

$$t_{ijk}^{wait\_p0}=\frac{\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)}{2}\sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}$$

$$t_{ijk}^{wait\_s0}=\frac{\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)}{2}\sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}$$

The total waiting time is:

$$t_{ijk}^{wait\_0}=\sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}\left[\frac{\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)}{2}+\frac{\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)}{2}\right]=\sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}\left[\frac{\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)+\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)}{2}\right]$$

When a control action is taken, current vehicle remains for additional time (holding time) at the current stops. Therefore the waiting time when holding is applied is given by the following formulas:

$$t_{ijk}^{wait\_pH}=\frac{\left(\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)-t_{ijk-1}^{exit}\right)^{2}}{2}\left(\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}\right)$$

$$t_{ijk}^{wait\_sH}=\frac{\left(t_{ijk+1}^{exit}-\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)\right)^{2}}{2}\left(\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}\right)$$

The total waiting time when holding is applied is:

$$t_{ijk}^{wait\_H} = \sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}\left[\frac{\left(\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)-t_{ijk-1}^{exit}\right)}{2}+\frac{\left(t_{ijk+1}^{exit}-\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)\right)}{2}\right]=\sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}\left[\frac{\left(\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)-t_{ijk-1}^{exit}\right)+\left(t_{ijk+1}^{exit}-\left(t_{ijk}^{exit}+t_{ijk}^{hold}\right)\right)}{2}\right]$$

The additional waiting time due to holding is:

$$t_{ijk}^{wait}=t_{ijk}^{wait\_H}-t_{ijk}^{wait\_0}=\sum_{m=j}^{N}\sum_{n=j+1}^{N}\lambda_{m,n}t_{ijk}^{hold}\left[t_{ijk}^{hold}+\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)-\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)\right]$$

In vehicle time due to holding a stop j is the product

$$t_{ijk}^{inveh}=q_{ijk}t_{ijk}^{hold}$$

The optimal holding time is obtained by minimizing the travel time cost:

$$\min_{t^{hold}} t_{ijk}^{travel}=\beta^{wait}t_{ijk}^{wait}+t_{ijk}^{inveh} \text{s.t.} t^{hold}\geq 0$$

$$\rightarrow \beta^{wait}\frac{\partial t_{ijk}^{wait}}{\partial t^{hold}}+\frac{\partial_{ijk}^{inveh}}{\partial t^{hold}}=0$$

$$\rightarrow \beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}\left(2t^{hold}+\left(t_{ijk}^{exit}-t_{ijk}^{exit}\right)-\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)\right)+q_{ijk}=0$$

$$\rightarrow 2\beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}t^{hold}=2\beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}\left[\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)-\left(t_{ijk}^{exit}-t_{ijk}^{exit}\right)\right]-q_{ijk}$$

$$\rightarrow t^{hold}=\frac{2\beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}\left[\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)-\left(t_{ijk}^{exit}-t_{ijk}^{exit}\right)\right]}{2\beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}}-\frac{q_{ijk}}{2\beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}}$$

$$\rightarrow t^{hold}=\frac{\left[\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)-\left(t_{ijk}^{exit}-t_{ijk}^{exit}\right)\right]}{2}-\frac{q_{ijk}}{2\beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}}$$

And the final holding criterion is:

$$t^{hold}=\max\left\{\frac{\left[\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)-\left(t_{ijk}^{exit}-t_{ijk}^{exit}\right)\right]}{2}-\frac{q_{ijk}}{2\beta^{wait}\sum_{m=j}^{N}\sum_{n=m+1}^{N}\lambda_{m,n}},0\right\}$$