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A perishable food supply chain problem considering demand uncertainty and time deadline constraints: Modeling and application to a high-speed railway catering service

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ABSTRACT

This paper attempts to optimize the flow patterns in a perishable food supply chain network for a high-speed rail catering service. The proposed variational inequality models describe the uncertain demand on trains using the Newsvendor model and impose time deadline constraints on paths considering flow-dependent lead time. The constraints are then reformulated based on the Dirac delta function so that they can be directly dualized. An Euler algorithm with an Augmented Lagrangian Dual algorithm is developed to solve the model. A case study using 246 trains in the Beijing-Shanghai high-speed corridor is applied to demonstrate the applicability of the method.

1. Introduction

Timely delivery is becoming a critical strategy that is as important as lean manufacturing and innovation strategy in modern supply chain management, particularly in perishable food supply chains (PFSCs) (Nagurney et al., 2013; Yu and Nagurney, 2013). Timely delivery imposes a time-critical mode on PFSCs in which each task is executed within a tight time frame (Federgruen et al., 1986; Zhang et al., 2003).

This research is motivated by catering services for high-speed railways (CSHRs). This paper focuses on developing a PFSC for CSHRs (PFSC-CSHRs) in China. It is estimated that China Railway must provide food products for more than 1000 high-speed trains per day across the rail network. With the development of high-speed passenger service, it is important for China Railway to develop profitable catering services for high-quality cold chain meals while guaranteeing time-sensitive quality and food safety (Wu et al., 2015, 2017a,b).

A PFSC-CSHRs network is composed of pathways from food suppliers (FSs), involves distribution centers (DCs) and rail stations (RSs), and ends at high-speed trains (HTs) throughout a given rail network. The food products demanded by HTs within a planning horizon are outsourced from cooperative FSs. DCs operated by rail companies order food products from the FSs and then deliver the meals to RSs. Each train is labeled with a train number (i.e. a trip line) in a train timetable. HTs are served as the end user of the distribution network (see Fig. 1). The PFSC-CSHRs problem is a product flow-assignment problem that aims to assign product flows in each distribution network. On HTs, services provided to different travel classes are quite different. However, either high level or economy class passengers can buy cold chain or ambient food products on the trains. This paper focuses on the on-demand catering retail services for cold chain food products provided in the dinning compartments of HTs for all travel classes.

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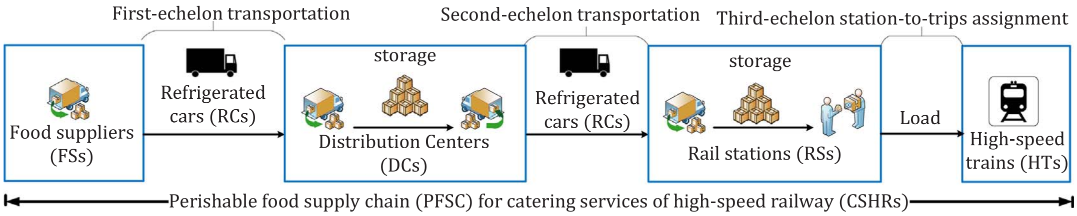


Fig. 1. The catering service process in the CSHR supply chain (Wu et al., 2017a).

1.1. Characteristics of the PFSC-CSHRs problem and literature review

The PFSC-CSHRs problem shares common features with other supply chain problems (Sloof et al., 1996; Miller, 2012; Trienekens and Zuurbier, 2008). However, particular characteristics merit further discussions.

(1) **Influence by rail transport plan**

First, all possible station-to-train assignments in the PFSC-CSHRs network are characterized by the HTs' operating line plan and timetable (Bussieck, 1997; Goossens et al., 2006; Peeters, 2003; Zhang and Nie, 2016).The paper's first task is to incorporate the information from line plans and train timetables into PFSC-CSHRs networks.

(2) **Deterioration of food products**

Second, the quality of cold chain meals deteriorates throughout the distribution process (Zhang et al., 2003; Yu and Nagurney, 2013; van der Vorst, 2000; Akkerman et al., 2010). Zhang et al. (2003) considered the perishability of chilled and frozen meals by limiting the total network degradation within a permitted range. However, that study did not investigate the degradation from the perspective of path-based models. In supply chain analytics for perishable products,path-based supply chain models are widely used to describe the degradation that occurs over the relevant links (Masoumi et al., 2012; Nagurney and Nagurney, 2012; Yu and Nagurney, 2013). The paper's second task is to introduce the framework to describe the deterioration of cold chain rail catering meals.

(3) **Flow-dependent lead time and time deadline constraints**

Third, the delivery process of cold chain meals must be restricted by time deadline constraints that restrict the pathways'lead time within a lifespan. In this paper, a pathway's lead time is different from a pathway's time impedance. Here, we define a pathway's time impedance as the total time impedance of the pathway from a food supplier to a high-speed train (as shown in Fig. 1). Differently, we define a pathway's lead time as the time impedance from a food supplier to its served trains' destination, which is the sum of the pathway's time impedance and the travel time of the train from its catered station to its destination.

Liu and Nagurney (2012) formulated a path-based model to impose that a path whose lead time is longer than a given time deadline would not be assigned any flows. However, the model does not describe the flow-dependent property of the lead time. Congestion effects usually occur in real-world situations because of the limited availability of skilled workers, redundant trimming and equipment turnover. As the amount of shipping on a path increases, the pathways' lead time increases. As a result, it is important to restrict the flow-dependent lead time within a given time deadline. In traffic equilibrium problems, Larsson and Patriksson (1999) proposed a strategy to price generalized side constraints (Lasdon, 1970, chap.8). Patriksson (1994) developed the Augmented Lagrangian Dual (ALD) algorithm to efficiently price capacity constraints. Nagurney and Nagurney (2012) introduced the same idea into a medical nuclear supply chain problem to achieve the dualization of capacity constraints. However, the flow-dependent lead-time property and time deadline constraints have not been addressed simultaneously. The time deadline constraint is described by conditional statements as follows:

If the amount of the flow on a path is greater than zero, then the pathway's flow-dependent lead time should be limited within a given time deadline.A free-flow pathway might have an arbitrarily long lead time, where a free-flow path is defined as a pathway without flows on it.

The third task of this paper is to formulate the time deadline constraints and limit the flow-dependent lead time of any pathway within a given deadline.

(4) **Uncertainty of food demand**

Fourth, the number of meals required by HTs is uncertain. For one thing, train tickets usually do not include meals. For another thing, numerous passengers purchase their tickets on the day of departure or even a few minutes prior to departure. The final number of passengers is unknown until minutes before departure. Thus, it is difficult to estimate demand based on the number of booked passengers. The literature has discussed the uncertainty of airline food products (Ho and Leung, 2010). Goto et al. (2004) indicated that the final number of passengers on a flight varies from the booked number of passengers on a flight by as much as 10% even one hour prior to the departure.

The literature usually exploits the uncertain demands of supply chain networks from the perspective of spatial pricing equilibrium problems (Nagurney and Aronson, 1989, 1999, chap.4). In spatial pricing equilibrium problems, market demand is associated with the price using demand functions (Nagurney, 1999, chap.2,3). In supply chain network equilibrium problems, the spatial pricing equilibrium condition is then used to account for consumer behaviors (Nagurney et al., 2002) in the long term. Nagurney et al. (2002) proposed a supply chain network equilibrium problem consisting of manufacturers, retailers and consumers in which competition occurs in a non-cooperative manner (Nash, 1950, 1951; Dafermos and Nagurney, 1987). The problem corresponds to the analogs of "user-optimized" traffic-network equilibrium problems in previous studies (Dafermos and Nagurney, 1985; Sheffi, 1985; Patriksson, 1994; Nagurney, 1999). However, the problem cannot describe PFSC-CSHRs directly, because the selling and disposal prices in CSHRs are fixed and independent from market demand in the short term (e.g. one day). Dong et al. (2004) developed a supply chain model considering random demand whose probability follows a uniform distribution. Random variables that represent excess supply (inventory) and excess demand (shortage) are proposed. A penalty is imposed on surpluses and shortages. The model is appropriate for describing the supply chains used for expensive necessities, such as a blood bank network system (Nagurney et al., 2013, chap.2). However, this formulation cannot apply to PFSC-CSHRs because cold chain meals are substitutable products. In CSHRs, although there might be a surplus penalty cost, each unit of demand above the inventory level only leads to a loss of potential sales without any penalty. Accordingly, the paper's fourth task is to introduce a proper model to describe the uncertain food demand on each high-speed train.

1.2. Contributions and objectives

In this paper, CSHRs is conducted in a regionalized manner. In each city, a DC is in charge of purchasing from FSs and providing food products for local RSs. The PFSC-CSHRs problem is formulated as two system-optimized (arc flow-based/path flow-based) optimization models to coordinate FSs, DCs and other stakeholders to maximize profit. We reformulate the optimization models as variational inequality(VI) models since they provide a rigorous theoretical framework to explore the related mathematical properties and economic marginal changes.This research can be regarded as an extension of supply chain analytics for perishable products (Nagurney et al., 2013),which is founded on network economics (Nagurney et al., 1994; Nagurney, 1999).

The PFSC-CSHRs model developed in this paper is distinct from other studies on perishable food products in several ways. First, time deadline constraints are considered in the VI models to ensure food security. Time deadline constraints with the flow-dependent lead time are described as conditional statements that cannot be dualized directly. Therefore we propose a reformulation approach based on the Dirac delta function to bring them into the VI models. Second, the on-demand sale mode on trains is formulated using the Newsvendor model (Gallego and Moon, 1993) to describe uncertain food demand with fixed selling, purchase and disposal prices. Third, the VI formulations result in an elegant computational procedure (Nagurney and Nagurney, 2012). In this paper,the ALD algorithm is combined with the Euler algorithm to solve the proposed VI model.We compare the proposed models with other supply chain models in A. To the best of our knowledge, we provide the first food supply chain models to include the deterioration, limited lifespan, time-dependent lead time, and uncertain demand aspects of rail catering food products.

The remainder of this paper is organized as follows. Section 2 describes the network and other basic notations for the PFSC-CSHRs problem. Section 3 introduces the decision variables, constraints and objective function. In Section 4, we develop the novel optimization models for PFSC-CSHRs and derive corresponding VI formulations.We also provide qualitative properties of the VI models in the section. Section 5 proposes an Euler algorithm combined with the ALD algorithm to solve the proposed VI models. In Section 6, a sensitivity analysis is implemented to help rail companies develop reasonable food product pricing strategy. Median scale numerical examples are provided to assess the proposed model and solution algorithm. A large scale numerical case study using 246 trains in the Beijing-Shanghai high-speed corridor is applied to demonstrate the applicability of the proposed approach. Conclusions are drawn in Section 7.

2. Network representation and basic notations

Rail catering operations are conducted in a regionalized manner. In each city that contains service RSs, DCs purchase requested meals from cooperative FSs, pack those meals into refrigerated cars, and deliver them to the local RSs. Here, we consider an existing distribution network, consisting of the set of FSs denoted by $FS_1,FS_2,...,FS_{n_{\mathrm{FS}}}$ and the set of DCs denoted by $DC_1,DC_2,...,DC_{n_{\mathrm{DC}}}$, where n_{FS} is the number of FSs and n_{DC} is the number of DCs. Meals are prepared in food supplier kitchens, and DCs operated by the rail company order the meals from them. Next, we define a directed graph for the distribution network of PFSC-CSHRs. Let a directed graph $\mathscr{G} = \mathscr{G}(\boldsymbol{N},\boldsymbol{A})$ be a collection of basic elements of the distribution network for PFSC-CSHRs, where $\boldsymbol{N} = \boldsymbol{N}_{\mathrm{FS}} \cup \boldsymbol{N}_{\mathrm{DC}} \cup \boldsymbol{N}_{\mathrm{RS}} \cup \boldsymbol{N}_{\mathrm{HT}}$ is the set of nodes. Trains in $\boldsymbol{N}_{\mathrm{HT}}$ are presented by nodes of customers in the graph. $\boldsymbol{A} = \boldsymbol{A}_{\mathrm{PD}} \cup \boldsymbol{A}_{\mathrm{DD}} \cup \boldsymbol{A}_{\mathrm{DR}} \cup \boldsymbol{A}_{\mathrm{RT}}$ is the set of links expressing railway company's activities between the nodes.

2.1. Definition of nodes

First, we let the first and second tiers of nodes as follows:

$$\boldsymbol{N}_{\mathrm{FS}} = \{FS_1,FS_2,...,FS_{n_{\mathrm{FS}}}\},$$

$$\boldsymbol{N}_{\mathrm{DC}} = \{DC_1,DC_2,...,DC_{n_{\mathrm{DC}}}\}.$$

At the DCs, the food products are stored and then packed into refrigerated cars. Meals are typically placed in the cars only a few hours before the scheduled departure time of the catered train. To maintain microbial safety, freshly prepared meals must be kept at a certain temperature and cannot be left unrefrigerated for longer than a certain period. Thus, the third tier of the networks is considered to be the following fictitious nodes of DCs.

$$\boldsymbol{N}'_{\mathrm{DC}} = \{DC'_1,DC'_2,...,DC'_{n_{\mathrm{DC}}}\}.$$

The fourth tier of the network consists of the RSs. The number of RSs is assumed to be n_{RS}, with the component RSs denoted by $RS_1,RS_2,...,RS_{n_{\mathrm{RS}}}$. We define

$$\boldsymbol{N}_{\mathrm{RS}} = \{RS_1,RS_2,...,RS_{n_{\mathrm{RS}}}\}.$$

RSs corresponding to DC_d include $RS_1(d),RS_2(d),...,RS_{n_{\mathrm{RS}(d)}}(d)$ (e.g. $RS_2(d)$ is the second RS served by DC_d). We define the set of regional RSs corresponding to DC_d as follows:

$$\boldsymbol{N}_{\mathrm{RS}}(d) = \{RS_1(d),RS_2(d),...,RS_{n_{\mathrm{RS}(d)}}(d)\}.$$

In railway management, a line is a possible traveling path of trains in a railway network, and a line plan is given by a set of lines corresponding to their frequencies and halting patterns. A timetable is represented by a space-time graph for describing the arrival and departure time of train $HT_t \in \boldsymbol{N}_{\mathrm{HT}}$ at station $RS_s \in \boldsymbol{N}_{\mathrm{RS}}$. In this paper, we define a high-speed train pool $\boldsymbol{N}_{\mathrm{HT}}$ as a set of trains in a timetable, where

$$\boldsymbol{N}_{\mathrm{HT}} = \{HT_1,HT_2,...,HT_{n_{\mathrm{HT}}}\}.$$

2.2. Definition of links

The links shown in graph $\mathscr{G}$ are four activities implemented by a railway company in CSHRs. The costs of the activities constitute the total cost of the rail catering service.

At the first echelon, the meals are shipped from FSs to DCs via refrigerated cars operated by the rail company. We define the set of links connecting the first and second tiers as

$$\boldsymbol{A}_{\mathrm{FD}} = \{(FS_i,DC_d)|\ i = 1,2,...,n_{FS},\ d = 1,2,...,n_{\mathrm{DC}}\}.$$

The next set of links connects node DC_d to corresponding node DC'_d, for $d = 1,2,...,n_{\mathrm{DC}}$, which represents the storage links.

$$\boldsymbol{A}_{\mathrm{DD}} = \{(DC_d,DC'_d)|\ d = 1,2,...,n_{\mathrm{DC}}\}.$$

We then define the set of links connecting the third and fourth tiers as follows. The definition implies that there is at most one DC in a city (thus, "DC" is synonymous with "a city", and each DC can only provide meals to its corresponding RSs in the city (e.g., the DC in Beijing can serve Beijing south, Beijing north, Beijing west and Beijing station).

$$\boldsymbol{A}_{\mathrm{DR}} = \bigcup_{d=1,2,...,n_{\mathrm{DC}}} \{(DC'_d,RS_s(d))|RS_s(d) \in \boldsymbol{N}_{\mathrm{RS}}(d)\}.$$

A station-to-trip possible assignment matrix $\psi(s,t)$ is generated, where $RS_s \in \boldsymbol{N}_{\mathrm{RS}}$ and $HT_t \in \boldsymbol{N}_{\mathrm{HT}}$:

$$\psi(s,t) = \begin{cases} 1 & \text{if } HT_t \text{ can be catered at station } RS_s \\ 0 & \text{otherwise} \end{cases} \quad \forall\ s = 1,2,...,n_{\mathrm{RS}},\ t = 1,2,...,n_{\mathrm{HT}} \tag{1}$$

where $\psi(s,t)$ can be generated based on the operating line plan and timetable (Peeters, 2003; Wu et al., 2017a):

(1) Only the trains whose travel times overlap the given mealtimes must be catered, and at least one-time catering for the trains is provided before the mealtime.
(2) A train can be catered only at the stations at which it stops according to the line plan.
(3) If the trip overlaps with only the lunch mealtime, then the train will be catered before the lunch mealtime.
(4) If the trains trip overlaps with only the dinner mealtime, then the train will be catered before the dinner mealtime.
(5) If the journal time of the trip overlaps both the lunch mealtime and the dinner mealtime, the meal will be catered before the lunch mealtime.
(6) If the train departs from its origin station during the lunch mealtime or the dinner mealtime, the train must be catered at its origin station.

It is noted that, these principles may not be the case for overnight HTs that serve dinner and breakfast to passengers. In this paper, we assume that overnight trains are not considered. The following framework can easily be extended when overnight trains are involved.

Finally, the set of links representing possible station-to-trip assignments is as follows:

$$\boldsymbol{A}_{\mathrm{RT}} = \{(RS_s,HT_t) \in \boldsymbol{N}_{\mathrm{RS}} \times \boldsymbol{N}_{\mathrm{HT}}|\psi(s,t) = 1\}.$$

2.3. Definition of pathways

Finally, we define a set of paths as $p \in \boldsymbol{P}$. For $t = 1,2,n_{\mathrm{HT}}$, a set of associated paths is denoted P_t, where

$$\bigcup_{t=1,2,...,n_{\mathrm{HT}}} \boldsymbol{P}_t = \boldsymbol{P}.$$

For an original/destination pair, i.e., FT_i and HT_t, we define a set of path P_{it} between pair (FT_i,HT_t), where

Table 1
Illustrative example of a train timetable.

	RT_1	RT_2	RT_3	RT_4	RT_5
HT_1	–	**8:20**	**10:00**	11:40	13:20
HT_2	**7:30**	–	**9:10**	**10:50**	12:30
HT_3	–	**13:20**	**15:30**	16:40	17:30
HT_4	**12:20**	–	14:00	15:40	17:40

Note: 11:00–13:00 is the time interval for lunch; 16:00–20:00 is the time interval for supper. Boldtypes indicate possible time and stations of catering service for trains in the timetable.

$$\bigcup_{t=1,2,\ldots,n_{\mathrm{FS}}} \boldsymbol{P}_{it} = \boldsymbol{P}_t \quad \forall\ t = 1,2,\ldots,n_{\mathrm{HT}}.$$

2.4. Illustrative example

Throughout this paper, we assume that the lunch mealtime is 11:00–13:00 and that the dinner mealtime is 16:00–20:00. Given a timetable with four HTs and five RSs in Table 1, we have that (i) HT_1 can be catered at RT_2 and RT_3; (ii) HT_2 can be served at RT_1, RT_3 and RT_4; (iii) HT_3 can be served at RT_2 and RT_3; and (iv) HT_4 can be catered at RT_1.

Fig. 2 displays a simple network consisting of n_{FS} FSs, n_{DC} DCs, n_{RS} RSs and n_{HT} HTs. The relationship among two FSs (FS_1,FS_2), four DCs (DC_1,DC_2,DC_3,DC_4), five RSs (RS_1,RS_2,RS_3,RS_4,RS_5) and the four HTs (HT_1,HT_2,HT_3,HT_4) is shown in Table 1. For example, $\boldsymbol{P}_1$ contains paths $FS_1 \to DC_1 \to DC_1' \to RS_2 \to HT_1$,$FS_2 \to DC_1 \to DC_1' \to RS_2 \to HT_1$,$FS_1 \to DC_2 \to DC_2' \to RS_3 \to HT_1$,$FS_2 \to DC_2 \to DC_2' \to RS_3 \to HT_1$ and so forth. $\boldsymbol{P}_{11}$ contains paths $FS_1 \to DC_1 \to DC_1' \to RS_2 \to HT_1$ and $FS_1 \to DC_2 \to DC_2' \to RS_3 \to HT_1$. The four activities implemented by a rail company are also shown in the figure:

(1) $\boldsymbol{A}_{\mathrm{FD}}$: The railway company transports the food products from food suppliers to operating distribution centers.
(2) $\boldsymbol{A}_{\mathrm{DD}}$: The railway company stores the food products in operating distribution centers.
(3) $\boldsymbol{A}_{\mathrm{DR}}$: The railway company delivers the food products to service rail stations.
(4) $\boldsymbol{A}_{\mathrm{RT}}$: The railway company caters high-speed trains dwelling at the station.

2.5. Basic notations

Table 2 lists all indexes, sets and parameters used in the rest of this article.

3. Problem statement

The PFSC-CSHRs problem aims to provide the optimal levels of procurement, consolidation, transportation and retail under the flow-balance and time deadline constraints, given the demand distribution at the various trains. Decision variables are listed in Section 3.1. Flow balance constraints will be introduced in Section 3.2. The essential time deadline constraints will be addressed in

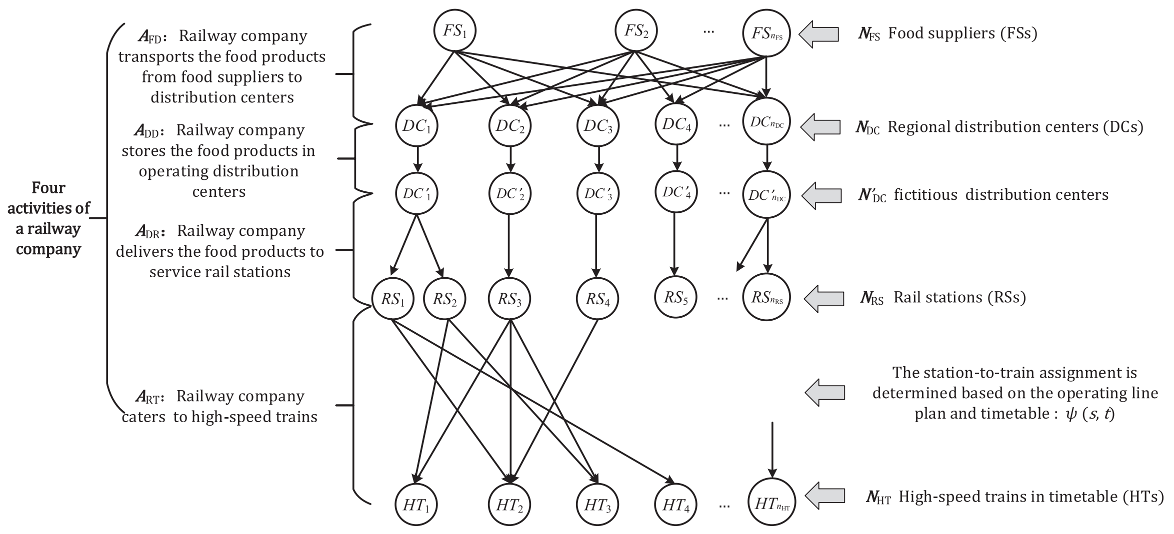


Fig. 2. Illustrative network of PFSC-CSHRs and input data.

Table 2
Sets, indexes and parameters used to express PFSC-CSHRs problem.

Sets	Description
$\boldsymbol{N}_{\mathrm{FS}}$	Set of outsourcing food suppliers
$\boldsymbol{N}_{\mathrm{DC}}$	Set of distribution centers
$\boldsymbol{N}'_{\mathrm{DC}}$	Set of fictitious distribution centers
$\boldsymbol{N}_{\mathrm{RS}}$	Set of rail stations
$\boldsymbol{N}_{\mathrm{HT}}$	Set of high-speed trains in timetable
$\boldsymbol{N}$	Set of nodes on the supply chain network, where $\boldsymbol{N} = \boldsymbol{N}_{\mathrm{FS}} \cup \boldsymbol{N}_{\mathrm{DC}} \cup \boldsymbol{N}_{\mathrm{RS}} \cup \boldsymbol{N}_{\mathrm{HT}}$
$\boldsymbol{A}_{\mathrm{FD}}$	Set of links connecting food suppliers to distribution centers
$\boldsymbol{A}_{\mathrm{DD}}$	Set of links connecting distribution centers to fictitious distribution centers
$\boldsymbol{A}_{\mathrm{DR}}$	Set of links connecting distribution centers to rail stations
$\boldsymbol{A}_{\mathrm{RT}}$	Set of links connecting RSs to high-speed trains
$\boldsymbol{A}$	Set of links on the supply chain network, where $\boldsymbol{A} = \boldsymbol{A}_{\mathrm{PD}} \cup \boldsymbol{A}_{\mathrm{DD}} \cup \boldsymbol{A}_{\mathrm{DR}} \cup \boldsymbol{A}_{\mathrm{RT}}$
$\boldsymbol{P}$	Set of paths of the network
$\boldsymbol{P}_t$	Set of paths associated with train HT_t, where $t = 1,2,...,n_{\mathrm{HT}}$
$\boldsymbol{P}_{it}$	Set of paths between supplier/train pair (FT_i,HT_t), where $i = 1,2,...,n_{\mathrm{FS}},t = 1,2,...,n_{\mathrm{HT}}$
$\hat{\boldsymbol{P}}$	Restricted set of paths on the network
$\hat{\boldsymbol{P}}_t$	Restricted set of paths associated with train HT_t, where $t = 1,2,...,n_{\mathrm{HT}}$
$\hat{\boldsymbol{P}}_{it}$	Restricted set of paths between food supplier/train pair (FT_i,HT_t), where $i = 1,2,...,n_{\mathrm{FS}},t = 1,2,...,n_{\mathrm{HT}}$
Indexes	**Description**
$i = 1,2,...,n_{\mathrm{FS}}$	Indexes of an outsourcing food supplier
$d = 1,2,...,n_{\mathrm{DC}}$	Indexes of a distribution center
$s = 1,2,...,n_{\mathrm{RS}}$	Indexes of a rail station
$t = 1,2,...,n_{\mathrm{HR}}$	Indexes of a high-speed train
(i,t)	Abbr.(FT_i,HT_t). Indexes of a food supplier/train pair
$a \in \boldsymbol{A}$	Indexes of a link/arc
$p \in \boldsymbol{P}$	Indexes of a path, and we use $\delta_{ap} = 1$ to indicate link/arc a on path p
$t(p)$	The index of the train served by path p
$s(p)$	The index of the rail station through which path p passes
Parameters	**Description**
α_a	Probability of each unit not being contaminated through link $a \in \boldsymbol{A}$
μ_p	The multiplier of the throughput on path $p \in \boldsymbol{P}$
δ_{ap}	$\delta_{ap} = 1$ indicates link/arc a is contained in path p;$\delta_{ap} = 0$ otherwise, where $a \in \boldsymbol{A},p \in \boldsymbol{P}$
α_{ap}	An auxiliary multiplier, where $a \in \boldsymbol{A},p \in \boldsymbol{P}$
$t_a^0,t_a(f_a)$	Free-flow and flow-dependent time impendence associated with link $a \in \boldsymbol{A}$
θ_a,β_a	Two parameters to be calibrated in flow-dependent time impendence function $t_a(f_a)$, where $a \in \boldsymbol{A}$
Cap_a	Capacity of vehicles (Refrigerated cars $a \in \boldsymbol{A}_{\mathrm{FD}} \cup \boldsymbol{A}_{\mathrm{DS}}$, forklift $a \in \boldsymbol{A}_{\mathrm{DD}}$, battery truck $a \in \boldsymbol{A}_{\mathrm{RT}}$)
$FCap_a$	Capacity of the distribution center, when $a \in \boldsymbol{A}_{\mathrm{DD}}$; Capacity of the rail station when $a \in \boldsymbol{A}_{\mathrm{RT}}$
$T_p^0,T_p(x)$	Free-flow and flow-dependent time impedance of path p (from a food supplier to a train), where $p \in \boldsymbol{P}$
FC_a	Fixed cost associated with link a, where $a \in \boldsymbol{A}$
VC_a	Variable cost associated with link a, where $a \in \boldsymbol{A}$
τ	Lifespan of food products
$\varkappa^s$	Unit selling price for one food product fixed by the railway administrator
$\varkappa^p$	Unit purchase price for one food product fixed by the railway administrator
$\varkappa^d$	Unit distress price for one food product fixed by the railway administrator
$\mathscr{P}_t(w)$	Probability density function of demand on train t, where $t = 1,2,...,n_{\mathrm{HT}}$
$P_t(w)$	Cumulative distribution function of demand on train t, where $t = 1,2,...,n_{\mathrm{HT}}$
$L(s,t)$	Traveling time of train HT_t from RS_s to its destination, where $s = 1,2,...,n_{\mathrm{RS}}$, $t = 1,2,...,n_{\mathrm{HT}}$
$DR_t(\varkappa^s)$	Dining rate of train HT_t when the sale price of a single food product is $\varkappa^s$, where $t = 1,2,...,n_{\mathrm{HT}}$
D_t	The number of passengers on train HT_t during the lunch or dinner mealtime, where $t = 1,2,...,n_{\mathrm{HT}}$
Ω	The upper bound of the order quantities for each passenger

Section 3.3. The Newsvendor-based objective function to express the cost and revenue under uncertain demand will be proposed in Section 3.4.

3.1. Decision variables

To assign product flows on a given network for PFSC-CSHRs, a rail company's managers must make the decisions shown in Table 3.

Table 3
Decision variables.

Variables	Descriptions
v_t	Projected inventory level for meals for train HT_t, where $t = 1,2,...,n_{\text{HT}}$ and $v = (...,v_t,...)$
x_p	Product flows on path p,$x = (...,x_p,...)$, where $p \in \boldsymbol{P}$
f_a	Product flows on link/arc a,$f = (...,f_a,...)$, where $a \in \boldsymbol{A}$

3.2. Flow balance constraints in supply chain analytics for perishable products

In this section, we recall the general flow balance constraints used in supply chain analytics for perishable products. Food products are a type of perishable product that deteriorates over time. Because of the perishability of the food products, a quality test is usually implemented after each activity. If a meal unit turns out to be contaminated, that unit will be discarded at the corresponding successor node. In a fresh food chain, the expected quantity surviving at the end of an activity is time-dependent, and the temperature and other environmental conditions associated with each link are given and fixed (Yu and Nagurney, 2013) because of the continuous change in the quality of fresh food.

However, PFSC-CSHRs focus on the finished cold chain meals, whose quality safety is guaranteed by their lifespans. Thus, the deterioration of finished food products is not caused by quality degradation but by incidental changes in environmental conditions, equipment failures or staff mishandling. Thus, we assume that each unit on link $a \in \boldsymbol{A}$ has a statistical probability of $\alpha_a(>0)$ of surviving after finishing the corresponding activities. We define f_a as the initial flow of product on link a, and f_a' denotes the final flow on link a, i.e., the flow that reaches the successor node of the link after deterioration has occurred. Therefore, following Yu and Nagurney (2013), we have

$$f_a - f_a' = (1-\alpha_a)f_a \quad \forall\, a \in \boldsymbol{A} \tag{2}$$

Furthermore, let μ_p be the multiplier of the throughput on path p, which is defined as the product of all α_a on the links that comprise that path:

$$\mu_p = \prod_{a\in p} \alpha_a \quad \forall\, p \in \boldsymbol{P}_{it}\,,\ \forall\, i = 1,2,...,n_{\text{FS}},\ t = 1,2,...,n_{\text{HT}} \tag{3}$$

where we assume that $\mu_p > 0$. The demand for train HR_t, $t = 1,2,...,n_{\text{HT}}$, is equal to the sum of all perishable flows on paths in $\boldsymbol{P}_t$. Because of $\cup_{i=1,2,...,n_{\text{FS}}} \boldsymbol{P}_{it} = \boldsymbol{P}_t$, we have

$$\sum_{i=1,2,...,n_{\text{FS}}} \sum_{p\in \boldsymbol{P}_{it}} x_p \mu_p = v_t \quad \forall\, t = 1,2,...,n_{\text{HT}} \tag{4}$$

We define the multiplier α_{ap}, which is the product of the multipliers of the links on path p that precede link a in the path, as follows:

$$\alpha_{ap} = \begin{cases} \delta_{ap} \prod_{\{a'\prec a\}_p} \alpha_a & \text{if } \{a'\prec a\}_p \neq \varnothing \\ \delta_{ap} & \text{if } \{a'\prec a\}_p = \varnothing \end{cases} \quad \forall\, a \in \boldsymbol{A},\, p \in \boldsymbol{P} \tag{5}$$

where $\{a'\prec a\}_p$ indicates the set of the links preceding link a in path p and $\varnothing$ denotes the null set. Moreover, δ_{ap} is defined as equal to 1 if link a is contained in path p, and 0 otherwise. The link flow, f_a, and the path flows, x_p, are connected by the following equations:

$$f_a = \sum_{i=1,2,...,n_{\text{FS}}} \sum_{t=1,2,...,n_{\text{HT}}} \sum_{p\in \boldsymbol{P}_{it}} x_p \alpha_{ap} \quad \forall\, a \in \boldsymbol{A} \tag{6}$$

For additional details about Eqs. (2)–(6), see Nagurney et al. (2013).

3.3. Flow-dependent lead time and time deadline constraints

The PFSC-CSHRs problem is formulated in a one-day cyclic manner, in which the planning horizon (i.e., one day) spans the activities of procurement, storage, distribution and station-to-train assignment.

3.3.1. Free-flow lead time

Associated with each link (activity) $a \in \boldsymbol{A}$, there is a free-flow time impedance t_a^0. The free-flow lead time of a path from an FS to an accessible train's destination can then be calculated by

$$T_p^0 + L(s(p),t(p)) = \sum_{a\in \boldsymbol{A}} t_a^0 \delta_{ap} + L(s(p),t(p)) \quad \forall\, p \in \boldsymbol{P} \tag{7}$$

where T_p^0 the free-flow time impedance of path p. Furthermore, $RS_{s(p)}$ is the unique station associated with path p, and $HT_{t(p)}$ is the unique train associated with path p. Then, $L(s(p),t(p))$ implies the travel time of train $HT_{t(p)}$ from $RS_{s(p)}$ to $HT_{t(p)}$'s destination. It should be noted that the time impedance of path p is different from the lead time of path p in this paper.

Next,we introduce notations to express the restricted set $\hat{\boldsymbol{P}}$ of paths whose free-flow lead time is less than a given time deadline.

Notation: Restricted set $\hat{\boldsymbol{P}}$ is used to define appropriate path definitions. That is,

$$T_p^0 + L(s(p),t(p)) \leqslant \tau \Rightarrow p \in \hat{\boldsymbol{P}}_{it}, \forall p \in \boldsymbol{P}, \forall i = 1,2,...,n_{\mathrm{FS}}, t = 1,2,...,n_{\mathrm{HT}}$$

where $\cup_{i=1,2,...,n_{\mathrm{FS}}} \hat{\boldsymbol{P}}_{it} = \hat{\boldsymbol{P}}_t$ and $\cup_{t=1,2,...,n_{\mathrm{HT}}} \hat{\boldsymbol{P}}_t = \hat{\boldsymbol{P}}$.

The total number of passengers served by train HT_t, $t = 1,2,...,n_{\mathrm{HT}}$, is denoted by D_t. Then, x_p is assumed bounded, because the number of food products required by each passenger is also finite. Then, we can define

$$\begin{cases} 0 \leqslant x_p \leqslant \Omega \times D_t & \forall p \in \hat{\boldsymbol{P}} \\ x_p = 0 & \forall p \notin \hat{\boldsymbol{P}} \end{cases} \tag{8}$$

where Ω is the upper bound of the order quantities of a passenger.

3.3.2. Time-dependent lead time

Furthermore, we allow $t_a(f_a)$ to denote the flow-dependent time impedance function incurred on link a. One should calibrate the functional forms of $t_a(f_a)$ with historical data. The function developed by the US Bureau of Public Roads (BPR) has been employed in many freight transportation studies because it reflects the congestion effect in an oversaturated transportation system by involving the volume/capacity ratio (Federgruen et al., 1986; Yamada et al., 2009; Meng and Wang, 2011). In this paper, we assume that the transportation, storage and handling time function in PFSC-CSHRs has a BPR form

$$t_a(f_a) = t_a^0\left[1 + \theta_a\left(\frac{f_a}{FCap_a}\right)^{\beta_a}\right] \forall a \in \boldsymbol{A} \tag{9}$$

The equations indicate that we should consider the congestion effect of transportation links. If more food products are loaded to a refrigerated truck, the time of loading will be extended, and loading time is a part of the transportation time. The equations also indicate we consider the congestion effect in facilities, i.e., DCs and RSs (storage and assignment links) because of limited availability of skilled workers, redundant trimming and equipment turnover.

Then, we define the flow-dependent lead time for each path

$$T_p(x) + L(s(p),t(p)) = \sum_{a\in\boldsymbol{A}} t_a(f_a)\delta_{ap} + L(s(p),t(p)) \quad \forall p \in \boldsymbol{P} \tag{10}$$

where $T_p(x)$ is the flow-dependent time impedance of path p.

3.3.3. Failure of generalized side constraints to express time deadline constraints

The time deadline constraints considering the time-dependent lead time of pathways cannot be described directly as generalized side constraints discussed in Larsson and Patriksson (1995, 1999), such as

$$T_p(x) + L(s(p),t(p)) \leqslant \tau \quad \forall p \in \hat{\boldsymbol{P}} \tag{11}$$

Fig. 3 illustrates two paths p_1 (A-B-C) and p_2 (A-B-D) on a simple network. We assume the following flow-dependent time impedance functions $t_{a_1}(f_{a_1}) = f_{a_1}, t_{a_2}(f_{a_2}) = 9 + f_{a_2}, t_{a_3}(f_{a_3}) = f_{a_3}$ and $\tau = 10$. We also assume that the market price in node C is 0.5 Yuan per unit and that the market price in node D is 1 Yuan per unit. We do not consider other costs and let $L(s(p_1),t(p_1)) = L(s(p_2),t(p_2)) = 0$. The side constraints (11) leads to $x_{p_1} = 0$ and $x_{p_2} = 1$ because $T_{p_1}(x)$ is bounded. That is,

$$T_{p_1}(x) = f_{a_1} + 9 + f_{a_2} = 1 + 9 + 1 = 10 = \tau$$

The total income is only 1 Yuan, which is unreasonable.

$$T_{p_2}(x) = f_{a_1} + f_{a_3} = 1 + 1 = 2 < \tau$$

However, decision makers can make $x_{p_2} = 5$

$$T_{p_2}(x) = f_{a_1} + f_{a_3} = 5 + 5 = 10 = \tau$$

Now, the lead time of p_1 is calculated as follows

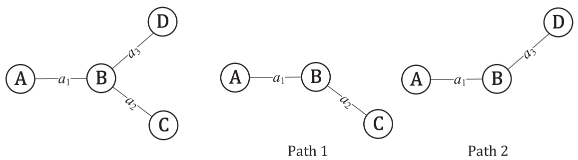


Fig. 3. Illustrative example to indicate the inapplicability of conventional generalized side constraints.

$$T_{p_1}(x) = f_{a_1} + 9 + f_{a_2} = 14 > \tau$$

Because $x_{p_1} = 0$, which implies a free-flow path, it is not restricted by the given time deadline $\tau = 10$.

3.3.4. Reformulation of time deadline constraints based on the Dirac delta function

Time deadline constraints in PFSC-CSHRs should be defined as the following conditional statements.

$$\begin{cases} T_p(x) + L(s(p),t(p)) \leqslant \tau & \text{if } x_p > 0 \\ T_p(x) + L(s(p),t(p)) \leqslant M & \text{if } x_p = 0 \end{cases} \forall\, p \in \boldsymbol{P} \tag{12}$$

where M is a large enough positive number.

The conditional statement requires that if path flow $x_p = 0$, we will not consider the deadline. Otherwise, we should restrict the time-dependent lead time within τ, which is described as a piecewise function. In the field of mathematical optimization, the conditional statements can be reformulated as a constraint with binary 0–1 variables. However, the feasible set of an optimization model with binary variables becomes non-convex. Then the proposed model becomes a non-linear integer programming model which is hard to be solved and cannot be described using VI formulations (Nagurney, 1999).To avoid the discontinuity, this paper approximates the constraints (12) using an expression based on the Dirac delta function.

The Dirac delta function is thought of as an approximation of the "infinitely high/thin" impulse function and is usually constructed by Gaussian distributions centered at the origin with variance tending to zero. Let be a sufficiently small number; then, we define a function as follows:

$$\delta(x_p) = \frac{1}{\sigma\sqrt{\pi}} e^{-\frac{x_p^2}{\sigma^2}} \quad \forall\, p \in \hat{\boldsymbol{P}} \text{ where } \sigma \to 0$$

When the input data of our problem are well defined, we use following approximate time deadline constraints:

$$T_p(x) + L(s(p),t(p)) \leqslant \tau + \delta(x_p) \quad \forall\, p \in \hat{\boldsymbol{P}} \tag{13}$$

The time deadline constraints now become generalized side constraints that can be addressed by Lagrangian dualization. To illustrate the effectiveness of the Dirac delta function,we use MATLAB 2015b to test how $\delta(x_p)$ changes with x_p, see Table 4.

(1) When we assume $\sigma = 1 \times 10^{-3}$, then $\delta(0) = 399$ and $\delta(1) = 0$, which implies the following conditional statement:

$$\begin{cases} T_p(x) + L(s(p),t(p)) \leqslant \tau & \text{if } x_p > 1 \\ T_p(x) + L(s(p),t(p)) \leqslant \tau + 399 & \text{if } x_p = 0 \end{cases} \forall\, p \in \boldsymbol{P}$$

(2) When we assume $\sigma = 1 \times 10^{-4}$, then $\delta(0) = 3989$ and $\delta(1) = 0$, which implies the following conditional statement:

$$\begin{cases} T_p(x) + L(s(p),t(p)) \leqslant \tau & \text{if } x_p \geqslant 1 \\ T_p(x) + L(s(p),t(p)) \leqslant \tau + 3989 & \text{if } x_p = 0 \end{cases} \forall\, p \in \boldsymbol{P}$$

(3) When we assume $\sigma = 1 \times 10^{-5}$, then $\delta(0) = 39894$ and $\delta(1) = 0$, which implies the following conditional statement:

$$\begin{cases} T_p(x) + L(s(p),t(p)) \leqslant \tau & \text{if } x_p \geqslant 0.001 \\ T_p(x) + L(s(p),t(p)) \leqslant \tau + 39894 & \text{if } x_p = 0 \end{cases} \forall\, p \in \boldsymbol{P}$$

The above trials show that if $\sigma = 1 \times 10^{-5}$ and our time deadline is 1440 min (one day), for all paths with $x_p = 0$, the time deadline constraints is $T_p(x) + L(s(p),t(p)) \leqslant 1440 + 39894 = 41334$; for all paths with $x_p \geqslant 1$, the time deadline constraints is $T_p(x) + L(s(p),t(p)) \leq 1440$. The penalty 39894 min (approximately 28 days) is large enough for the relaxation. Empirically, $\sigma = 1 \times 10^{-5}$ is small enough to express the time deadline constraints.

The advantage of the above Dirac delta function-based reformulation is that constraint (12) is continuous and derivative. Furthermore, it even can even simplify our proposed VI formulation. When $\sigma \to 0$, we propose the following assumption.

Table 4
Dirac delta function value varying with path flows and the parameter.

σ	$x_p = 0$	$x_p = 0.001$	$x_p = 0.002$	$x_p = 0.003$	$x_p = 0.004$	$x_p = 0.005$	$x_p \geqslant 0.01$
$\sigma = 1 \times 10^{-3}$	399	242	54	4	0.13	0.0015	0
$\sigma = 1 \times 10^{-4}$	3989	7×10^{-19}	6×10^{-84}	1.5×10^{-192}	0	0	0
$\sigma = 1 \times 10^{-5}$	39894	0	0	0	0	0	0

Assumption 3.1.

$$\frac{d\delta(x_p)}{dx_p} \approx 0 \quad \forall\ p \in \boldsymbol{P} \tag{14}$$

The assumption is empirically reasonable from the perspective of logistics engineering. Assume that $\sigma = 1 \times 10^{-5}$, then we have

$$\begin{cases} \frac{d\delta(x_p)}{dx_p} \equiv 0 & \text{if } x_p \geqslant 1 \times 10^{-3} \\ \frac{d\delta(x_p)}{dx_p} \geqslant 0 & \text{if } x_p \in (0,1 \times 10^{-3})\ \forall\ p \in \boldsymbol{P}. \\ \frac{d\delta(x_p)}{dx_p} = 0 & \text{if } x_p = 0 \end{cases}$$

This implies that if the unit variation of the path flows is greater than 1×10^{-3}, the assumption is never violated.

3.4. Objective function

The objective function of PSFC-CSHRs is to maximize the rail company's profit, which considers sales revenue, purchasing cost, storage cost, handling cost, transportation cost, discarding cost and disposal cost.

3.4.1. Operational cost function

Associated with each link of the proposed network is a unit operational cost function $c_a(f_a)$. Table 5 displays the implications of $c_a(f_a)$ that represent the cost of operation for the four activities in graph $\mathscr{G}$ (see Fig. 2).

In this paper, the strictly increasing function $c_a(f_a) \approx \frac{FC_a}{Cap_a} + V(f_a)$ is used as the approximate unit operational cost on link $a \in \boldsymbol{A}$. Next, the total operational cost on link a is denoted by

$$\hat{c}_a(f_a) = c_a(f_a) \times f_a \approx FC_a \times \frac{f_a}{Cap_a} + V(f_a) \times f_a \quad \forall\ a \in \boldsymbol{A} \tag{15}$$

where $V(f_a) = VC_a \times f_a$ is the flow-dependent variable cost function.

Also associated with each link of the network is a unit discarding cost function $z_a(f_a)$, which represents the cost of products damaged in transit. The total discarding cost on link a is denoted by

$$\hat{z}_a(f_a) = z_a \times f_a \quad \forall\ a \in \boldsymbol{A} \tag{16}$$

where $z_a = (\varkappa^p - \varkappa^d) \times (1-\alpha_a)$ implies the cost of discarding $(1-\alpha_a)$ meals, and $\varkappa^p - \varkappa^d$ is the amount of loss attributable to the low-price treatment.

3.4.2. Sale revenue function

Because meals are sold in a train's dining compartment on trains, we are interested in measuring how many times a selling event occurs in the mealtime interval of lunch/dinner at a counter. In this paper, the demand on each train $t = 1,2,\dots,n_{\mathrm{HT}}$ is assumed to be a random variable following any known continuous probability distribution satisfying:

1. $\mathscr{P}_t(x) > 0$, for all $x \geqslant 0$.
2. The expectation value of the probability distribution is defined as

$$\lambda_t = D_t \times DR_t(\varkappa^s) \quad \forall\ t = 1,2,\dots,n_{\mathrm{HT}} \tag{17}$$

where $DR_t(\varkappa^s) \leqslant \Omega$ is a price-dependent dining rate on each train acquired from our questionnaire investigation. If $DR_t(\varkappa^s) > 1$, it implies that on average each person buys more than one food products on the train. Conversely, if $DR_t(\varkappa^s) < 1$, it implies that on average each person buys less than one food products on the train.

The cumulative distribution function of d_t is denoted by

$$P_t(W) = \int_0^W \mathscr{P}_t(w)dw \quad \forall\ t = 1,2,\dots,n_{\mathrm{HT}} \tag{18}$$

Based on the conventional Newsvendor model (Gallego and Moon, 1993), when the projected inventory level at train HT_t is v_t, the

Table 5
Meaning of $c_a(f_a)$ on links of different echelons.

$c_a(f_a)$	$a \in \boldsymbol{A}_{\mathrm{FD}}$	Transportation cost from food suppliers to distribution centers
	$a \in \boldsymbol{A}_{\mathrm{DD}}$	Inventory cost in distribution centers
	$a \in \boldsymbol{A}_{\mathrm{DR}}$	Transportation cost from distribution centers to rail stations
	$a \in \boldsymbol{A}_{\mathrm{RT}}$	Handling cost to deliver the food products onto trains

expected sales revenues of the shortage ($E^{-}(v_t)$) and the surplus ($E^{+}(v_t)$) (overstock wastes) are given by

$$E^{-}(v_t) = \int_{v_t}^{\infty} (\kappa^s - \kappa^p) v_t \mathscr{P}_t(w) dw \quad \forall\ t = 1,2,...,n_{\mathrm{HT}} \tag{19}$$

$$E^{+}(v_t) = \int_{0}^{v_t} [(\kappa^s - \kappa^p) w + (\kappa^d - \kappa^p)(v_t - w)] \mathscr{P}_t(w) dw \quad \forall\ t = 1,2,...,n_{\mathrm{HT}} \tag{20}$$

4. Model formulation

In this section, we introduce the models for PFSC-CSHRs from the perspectives of the mathematical optimization model as well as VI models. We propose the path flow-based and arc flow-based optimization models and reformulate them as two VI models. The VI formulations provide a framework to enable the exploration of the qualitative properties. The path flow-based VI formulations are important for rail catering management because they express marginal contribution of an additional path flow.

4.1. Optimization models for PFSC-CSHRs

The total profit maximization objective faced by the rail company includes the total cost of operating the various activities (purchasing, storage and distribution), the total discarding cost of waste/loss over the links, and the expected sales revenue considering the overstock waste. This optimization problem can be expressed as.

(**Model 1**)

$$\text{maximize} \sum_{t=1,2,...,n_{\mathrm{HT}}} [E^{+}(v_t) + E^{-}(v_t)] - \sum_{p \in \boldsymbol{P}} [\widehat{Z}_p(x) + \widehat{C}_p(x)]$$

subject to Eqs. (4), (8) and (13).

The operational cost $\widehat{C}_p(x)$ and the discarding cost $\widehat{Z}_p(x)$ are derived from $C_p(x)$ and $Z_p(x)$:

$$\widehat{C}_p(x) = C_p(x) \times x_p, \quad \widehat{Z}_p(x) = Z_p(x) \times x_p \quad \forall\ p \in \boldsymbol{P} \tag{21}$$

$C_p(x)$ and $C_p(x)$ are in turn expressed as follows:

$$C_p(x) \equiv \sum_{a \in \boldsymbol{A}} c_a(f_a) \alpha_{ap}, \quad Z_p(x) \equiv \sum_{a \in \boldsymbol{A}} z_a \alpha_{ap} \quad \forall\ p \in \boldsymbol{P} \tag{22}$$

Model 1 is then equivalent to the following model in terms of arc flows.

(**Model 2**)

$$\text{maximize} \sum_{t=1,2,...,n_{\mathrm{HT}}} [E^{+}(v_t) + E^{-}(v_t)] - \sum_{a \in \boldsymbol{A}} [\hat{z}_a(f_a) + \hat{c}_a(f_a)]$$

subject to Eqs. (4), (6), (8) and (13).

4.2. Preliminary proofs

To reformulate above two optimization models as VI models, we present some preliminaries that enable us to express the partial derivatives of the expected sale revenue and the total costs in terms of path flow variables.

4.2.1. Derivatives of sale revenue function

First, we demonstrate some preliminary proofs that enable us to use the partial derivatives of the expected sale revenue in terms of path flow variables

$$\frac{\partial [E^{-}(v_t) + E^{+}(v_t)]}{\partial x_p} = \left[\frac{\partial E^{-}(v_t)}{\partial v_t} + \frac{\partial E^{+}(v_t)}{\partial v_t}\right] \times \frac{\partial v_t}{\partial x_p} \quad \forall\ p \in \boldsymbol{P}_t,\ \forall\ t = 1,2,...,n_{\mathrm{HT}}$$

Then, for the case of shortage, for all $t = 1,2,...,n_{\mathrm{HT}}$, we have

$$\begin{aligned}\frac{\partial E^{-}(v_t)}{\partial v_t} &= \frac{\partial \int_{v_t}^{\infty} (\kappa^s - \kappa^p) v_t \mathscr{P}_t(w) dw}{\partial v_t} \\ &= \int_{v_t}^{\infty} (\kappa^s - \kappa^p) \mathscr{P}_t(w) dw + v_t \frac{\partial (1 - \int_0^{v_t} (\kappa^s - \kappa^p) \mathscr{P}_t(w) dw)}{\partial v_t} \\ &= (\kappa^s - \kappa^p)(1 - P_t(v_t)) - (\kappa^s - \kappa^p) v_t \mathscr{P}_t(v_t)\end{aligned}$$

Similarly, for the case of surplus, for all $t = 1,2,...,n_{\mathrm{HT}}$, we have

$$\begin{aligned}\frac{\partial E^+(v_t)}{\partial v_t} &= \frac{\partial}{\partial v_t}\int_0^{v_t}[(\kappa^s-\kappa^p)w-(\kappa^p-\kappa^d)(v_t-w)]\mathscr{P}_t(w)dw\\ &= \frac{\partial}{\partial v_t}[\int_0^{v_t}[(\kappa^s-\kappa^p)+(\kappa^p-\kappa^d)]w\mathscr{P}_t(w)dw-v_t\int_0^{v_t}(\kappa^p-\kappa^d)\mathscr{P}_t(w)dw]\\ &= [(\kappa^s-\kappa^p)+(\kappa^p-\kappa^d)]v_t\mathscr{P}_t(v_t)-\int_0^{v_t}(\kappa^p-\kappa^d)\mathscr{P}_t(w)dw-(\kappa^p-\kappa^d)v_t\mathscr{P}_t(v_t)\\ &= (\kappa^s-\kappa^p)v_t\mathscr{P}_t(v_t)-(\kappa^p-\kappa^d)P_t(v_t)\end{aligned}$$

Then, for all $t = 1,2,\ldots,n_{\mathrm{HT}}$ we have

$$\begin{aligned}\frac{\partial E^-(v_t)}{\partial v_t}+\frac{\partial E^+(v_t)}{\partial v_t} &= (\kappa^s-\kappa^p)(1-P_t(v_t))-(\kappa^p-\kappa^d)P_t(v_t)\\ &= (\kappa^s-\kappa^p)-(\kappa^p-\kappa^d+\kappa^s-\kappa^p)P_t(v_t)\\ &= (\kappa^s-\kappa^p)-(\kappa^s-\kappa^d)P_t(v_t)\end{aligned}$$

Furthermore, we have

$$\frac{\partial v_t}{\partial x_p} = \frac{\partial}{\partial x_p}\sum_{i=1,2,\ldots,n_{\mathrm{FS}}}\sum_{q\in \boldsymbol{P}_{it}} x_q\mu_q = \mu_p \quad \forall\ p\in \boldsymbol{P}_t\ ,\ \forall\ t = 1,2,\ldots,n_{\mathrm{HT}}$$

By combining the above two equations, we can easily prove the concavity of the function $E^-(v_t)+E^+(v_t)$.

Lemma 4.1. *If $\kappa^s > \kappa^d$ (realistic assumption), then sales revenue function $E^-(v_t)+E^+(v_t)$ is concave.*

Proof. We already know

$$\frac{\partial[E^-(v_t)+E^+(v_t)]}{\partial x_p} = (\kappa^s-\kappa^p)\mu_p-(\kappa^s-\kappa^d)P_t\left(\sum_{p\in \boldsymbol{P}_t}x_p\mu_p\right)\mu_p$$

If $\kappa^s>\kappa^d$, $\mu_p>0$ and $\mathscr{P}_t(\sum_{p\in \boldsymbol{P}_t}x_p\mu_p)>0$

$$\frac{\partial^2[E^-(v_t)+E^+(v_t)]}{\partial x_p^2} = -(\kappa^s-\kappa^d)\mu_p^2\mathscr{P}_t\left(\sum_{p\in \boldsymbol{P}_t}x_p\mu_p\right)<0$$

Thus, the function $E^-(v_t)+E^+(v_t)$ is concave. □

4.2.2. Derivatives of operational cost function

Second, we introduce a lemma to connect the partial derivatives of the path operational cost, and the path discarding cost to their related path/link flows.

Lemma 4.2. *The partial derivatives of the total operational cost and the total discarding cost with respect to the corresponding path flow are given, respectively, by*

$$\frac{\partial\sum_{q\in \boldsymbol{P}}\widehat{C}_q(x)}{\partial x_p} = \sum_{a\in \boldsymbol{A}}\frac{\widehat{c}_a(f_a)}{f_a}\alpha_{ap} = \sum_{a\in \boldsymbol{A}}\left(c_a(f_a)+\frac{\partial c_a(f_a)}{\partial f_a}f_a\right)\alpha_{ap}\quad \forall\ p\in \boldsymbol{P} \tag{23}$$

$$\frac{\partial\sum_{q\in \boldsymbol{P}}\widehat{Z}_q(x)}{\partial x_p} = \sum_{a\in \boldsymbol{A}}\frac{\widehat{z}_a(f_a)}{f_a}\alpha_{ap} = \sum_{a\in \boldsymbol{A}}z_a\alpha_{ap}\quad \forall\ p\in \boldsymbol{P} \tag{24}$$

Proof. Eqs. (23) and (24) parallels the case of a blood supply chain system; see (Nagurney et al., 2013, chap.2). □

4.2.3. Derivatives of time deadline constraints

Third, we prove a theorem to connect the partial derivatives of the time deadline constraints.

Theorem 4.1. *The partial derivatives of the time deadline constraints with respect to the corresponding path flow are given by*

$$\frac{\partial\{T_q(x)+L(s(q),t(q))-\tau-\delta(x_q)\}}{\partial x_p} = \sum_{a\in \boldsymbol{A}}\frac{\partial t_a(f_a)}{\partial f_a}\alpha_{ap}\delta_{ap}\quad \forall\ p,q\in \boldsymbol{P} \tag{25}$$

Proof. Because τ and $L(s(q),t(q))$ are constant, and Assumption 3.1:

$$\frac{d\delta(x_p)}{dx_p}\approx 0\quad \forall\ p\in \widehat{\boldsymbol{P}}$$

We know that

$$\frac{\partial\{T_q(x)+L(s(q),t(q))-\tau-\delta(x_q)\}}{\partial x_p}=\frac{\partial T_q(x)}{\partial x_p}$$

Then, we have

$$\frac{\partial T_q(x)}{\partial x_p}=\frac{\partial\sum_{a\in \boldsymbol{A}}t_a(f_a)\delta_{aq}}{\partial x_p}=\sum_{a\in \boldsymbol{A}}\frac{\partial t_a(f_a)}{\partial x_p}\delta_{aq}=\sum_{a\in \boldsymbol{A}}\frac{\partial t_a(f_a)}{\partial f_a}\frac{\partial f_a}{\partial x_p}\delta_{aq}\quad \forall\ p,q\in \boldsymbol{P}$$

$$\frac{\partial f_a}{\partial x_p}=\frac{\partial\sum_{p\in \boldsymbol{P}}x_p\alpha_{ap}}{\partial x_p}=\alpha_{ap}\quad \forall\ a\in \boldsymbol{A},p\in \boldsymbol{P}$$

Finally, we have

$$\frac{\partial T_q(x)}{\partial x_p}=\sum_{a\in \boldsymbol{A}}\frac{\partial t_a(f_a)}{\partial f_a}\alpha_{ap}\delta_{aq}\quad \forall\ p,q\in \boldsymbol{P}$$

Eq. (25) is proved. □

4.3. Variational inequality models for PFSC-CSHRs

In this subsection, we attempt to transform the optimization models into VI formulations. A path flow-based VI-Model is used to prove the existence property and an arc flow-based VI model is used to prove uniqueness property of optimal link flows under given Lagrangian multipliers. Here,the Lagrange multiplier γ_q is associated with a time deadline constraint (13) for each link path q. We group these Lagrange multipliers into the vector γ.

Next,we derive the VI models in terms of path flows and link flows. For simplicity, we assume that each passenger buys at most Ω food products during his/her journey. Let $\boldsymbol{K}$ denote the feasible set

$$\boldsymbol{K}=\{x_p|0\leqslant x_p\leqslant \Omega\times D_t\quad \forall\ p\in \hat{\boldsymbol{P}};\gamma\geqslant 0\}\tag{26}$$

Then, we have the following theorem.

Theorem 4.2. *Model 1, subject to its constraints, is equivalent to the VI to determine the vector of optimal path flows and the vector of optimal Lagrange multipliers for the time deadline constraints:* $(x^*,\gamma^*)\in \boldsymbol{K}$.

(VI-Model 1)

$$\sum_{p\in\hat{\boldsymbol{P}}}\left\{\frac{\partial\sum_{q\in \boldsymbol{P}}\hat{C}_q(x^*)}{\partial x_p}+\frac{\partial\sum_{q\in \boldsymbol{P}}\hat{Z}_P(x^*)}{\partial x_p}-(\kappa^s-\kappa^p)\mu_p+(\kappa^s-\kappa^d)P_t\left(\sum_{p\in \boldsymbol{P}_t}x_p^*\mu_p\right)\mu_p+\sum_{q\in\hat{\boldsymbol{P}}}\gamma_q^*\frac{\partial T_q(x^*)}{\partial x_p}\right\}\times[x_p-x_p^*]$$
$$+\sum_{q\in\hat{\boldsymbol{P}}}[\delta(x_q^*)+\tau-T_q(x^*)-L(s(q),t(q))]\times[\gamma_q-\gamma_q^*]\geqslant 0\quad \forall\ (x,\gamma)\in \boldsymbol{K}\tag{27}$$

VI-Model 1 can be rewritten in terms of link flows to determine the vector of optimal link flows, the vector of inventory levels for trains and the vector of the Lagrange multipliers for the time deadline constraints.

(VI-Model 2)

$$\sum_{a\in \boldsymbol{A}}\left[\frac{\partial\hat{c}_a(f_a^*)}{\partial f_a}+\frac{\partial\hat{z}_a(f_a^*)}{\partial f_a}+\sum_{q\in\hat{\boldsymbol{P}}}\gamma_q^*\sum_{a\in \boldsymbol{A}}\frac{\partial t_a(f_a^*)}{\partial f_a}\delta_{aq}\right]\times[f_a-f_a^*]-\sum_{t=1,2,\dots,n_{\mathrm{HT}}}[(\kappa^s-\kappa^p)-(\kappa^s-\kappa^d)P_t(v_t^*)]\times[v_t-v_t^*]$$
$$+\sum_{q\in\hat{\boldsymbol{P}}}\left[\delta(x_q^*)+\tau-\sum_{a\in \boldsymbol{A}}t_a(f_a^*)\delta_{aq}-L(s(q),t(q))\right]\times[\gamma_q-\gamma_q^*]\geqslant 0\quad \forall\ (f,v,\gamma)\in \boldsymbol{K}^1$$
$$\boldsymbol{K}^1\equiv\{f,v|\text{satisfy Eqs. (4),(6) and }\gamma\geqslant 0\}\tag{28}$$

Proof. The convexity of $C_p(x),Z_p(x)$ and $T_q(x)$ holds for all paths since $c_a(f_a),z_a(f_a)$ and $t_a(f_a)$ are assumed to be convex for all link $a\in \boldsymbol{A}$. Based on Lemma 4.1, $-\sum_{t=1,2,\dots,n_{\mathrm{HT}}}[E^+(v_t)+E^-(v_t)]$ is also convex.

Since the objective function of VI-Model 1/Model 1 is convex and the feasible set $\boldsymbol{K}$ is convex and closed (because Lagrangian multipliers are also bounded), then VI-Model 1 follows from the theory of variational inequalities (see Nagurney, 1999). It implies that a solution of VI-Model 1 is the solution of the optimization Model 1.

As for the proof of VI-Model 2, now that VI-Model 1 is established, we can use the equivalence between partial derivatives of the cost/lead time on paths and partial derivatives of cost/time impendence on links from Lemma 4.2 and Theorem 4.1. We also can use the equivalence between partial derivatives of the revenue function on paths and partial derivatives of revenue on order quantities of trains from Lemma 4.1. Based on Eqs. (4) and (6), Lemmas 4.1 and 4.2, as well as Theorem 4.1, we can rewrite the formulation of VI-Model 1 in terms of VI-Model 2. Thus, the second part of Theorem 4.2 are proved. □

4.3.1. Existence under continuous and compact

VI-Model 1 can be put into a standard form of VI (see Nagurney, 1999) as follows: determine $X^* \in \mathscr{K}$, such that

$$\langle \mathbf{F}(X), X - X^* \rangle \geqslant 0 \quad \forall x \in \mathscr{K} \tag{29}$$

where $\langle \cdot,\cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. We define the feasible set as $\mathscr{K} = \boldsymbol{K}$, the vector $X = (x,\gamma)$ and the vector $\mathbf{F}(X) \equiv (F_1(X), F_2(X))$, where

$$F_1(X) = (F_1(X)_p |\ \forall p \in \widehat{\boldsymbol{P}}) \tag{30}$$

$$F_2(X) = (F_2(X)_q |\ \forall q \in \widehat{\boldsymbol{P}}) \tag{31}$$

$$F_1(X)_p = \frac{\partial \sum_{q\in \boldsymbol{P}} \widehat{C}_q(x)}{\partial x_p} + \frac{\partial \sum_{q\in \boldsymbol{P}} \widehat{Z}_p(x)}{\partial x_p} - (\kappa^s - \kappa^p)\mu_p + (\kappa^s - \kappa^d) P_t\left(\sum_{p\in \boldsymbol{P}_t} x_p \mu_p\right)\mu_p + \sum_{q\in \widehat{\boldsymbol{P}}} \gamma_q^* \frac{\partial T_q(x^*)}{\partial x_p} \tag{32}$$

$$F_2(X)_q = \delta(x_q) + \tau - \sum_{a\in \boldsymbol{A}} t_a(f_a)\delta_{aq} - L(s(q), t(q)) \tag{33}$$

Theorem 4.3 (***Existence under continuous and compact***). *There exists at least one solution to VI-Model 1 (also to Model 1).*

Proof. We can assume the shadow prices of time deadline constraints to be bounded (although it might be large). Thus,

$$\mathscr{K} \equiv \boldsymbol{K} = \{x_p | 0 \leqslant x_p \leqslant \Omega \times D_t, 0 \leqslant \gamma_p \leqslant M,\ \forall p \in \widehat{\boldsymbol{P}}_t,\ \forall t = 1,2,\ldots,n_{\mathrm{HT}}\} \tag{34}$$

is a compact set. Further, $F(x)$ is continuous on $\mathscr{K}$. Based on Brouwers fixed-point theorem, VI-Model 1 admits at least one solution (Nagurney, 1999). □

4.3.2. Uniqueness given Lagrangian multipliers

Given Lagrangian multipliers, the problem becomes similar to the general food supply chain problems. Then, it is reasonable that we have uniqueness of link flows rather than path flows (Nagurney et al., 2013, chap.4) as shown in the following theorem.

Theorem 4.4 (***Uniqueness given Lagrangian multipliers***). *Given Lagrangian multipliers γ^*, the link flow pattern f^* and the optimal demand pattern v^* in VI-Model 2 are unique.*

Proof. Given Lagrangian multipliers $\gamma^* \geqslant 0$ for all time deadline constraints, then we have following standard VI model based on VI-Model 2 with unique solution: determine $X^* \in \mathscr{K}^1$, such that

$$\langle \mathscr{F}(X), X - X^* \rangle \geqslant 0 \quad \forall x \in \mathscr{K}^1 \tag{35}$$

We define the feasible set as $\mathscr{K}^1 = \{f, v | \text{satisfy Eqs. (4) and (6)}\}$, the vector $X = (f, v)$ and the vector $\mathscr{F}(X) \equiv (\mathscr{F}_1(X), \mathscr{F}_2(X))$, where

$$\mathscr{F}_1(X) = (\mathscr{F}_1(X)_a |\ \forall a \in \boldsymbol{A}) \tag{36}$$

$$\mathscr{F}_2(X) = (\mathscr{F}_2(X)_t |\ \forall t = 1,2,\ldots,n_{\mathrm{HT}}) \tag{37}$$

$$\mathscr{F}_1(X)_a = \frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \sum_{q\in \widehat{\boldsymbol{P}}} \gamma_q^* \sum_{a\in \boldsymbol{A}} \frac{\partial t_a(f_a^*)}{\partial f_a} \delta_{aq} \tag{38}$$

$$\mathscr{F}_2(X)_t = -(\kappa^s - \kappa^p) + (\kappa^s - \kappa^d) P_t(v_t) \tag{39}$$

If function $\mathscr{F}(X)$ is strictly monotone on $\mathscr{K}^1$, then the solution to variational inequality $\langle \mathscr{F}(X), X - X^* \rangle \geqslant 0 \quad \forall x \in \mathscr{K}^1$ is unique; that is, given Lagrangian multipliers, the link flow pattern and the optimal demand pattern are uniqueness. □

However, just like side constrained traffic equilibrium problems (Larsson and Patriksson, 1999), the values of the Lagrangian multipliers are not necessarily uniquely determined without further assumptions. The values of the Lagrangian multipliers is meaningful to imply the shadow prices for time deadline constraints. These prices can be interpreted as the marginal profit of relaxing the per unit minute of the time deadline.

5. Solution algorithm

In this section, we introduce an iterative scheme to solve the VI models. The VI formulations result in an elegant computational procedure (iterative scheme) based on the Euler algorithm (Nagurney et al., 2013). In this section, we provide an implementable version of the improved Euler algorithm combined with an augmented Lagrangian dual (ALD) algorithm (Larsson and Patriksson, 1995).

5.1. Euler algorithm with normal pricing strategy

The Euler algorithm is generally induced as a general iterative scheme for solving VI models. Dupuis and Nagurney (1993) applied the Euler algorithm to solve a projected dynamic system that is equivalent to the VI model. Based on Everett's Theorem, it is natural to exploit a price-directive solution strategy based on the basic Euler iterative scheme (Larsson and Patriksson, 1999).

5.1.1. Explicit formulae for the Euler algorithm applied to VI-Model 1

In iterations of the Euler algorithm for solving VI-Model 1, we should compute the following closed-form expressions for the food product path flows at iteration n of the Euler algorithm:

(1) For paths whose free-flow lead time satisfies time deadline constraints, we have

$$x_p^{n+1} = \max\{0, x_p^n - a_p^n \times F_1(x^n)_p\} \quad \forall\, p \in \hat{\boldsymbol{P}} \tag{40}$$

where a_p^n is the step size used in the nth iterations.

(2) For other paths, we have

$$x_p^{\tau+1} = x_p^{\tau} = 0 \quad \forall\, p \notin \hat{\boldsymbol{P}} \tag{41}$$

Let us define $|\boldsymbol{N}_{\mathrm{FS}}|$,$|\boldsymbol{N}_{\mathrm{DC}}|$,$|\boldsymbol{N}_{\mathrm{RS}}|$and $|\boldsymbol{N}_{\mathrm{HT}}|$ as the number of FSs, DCs, RSs, and HTs, respectively. Because every DC can only serve its local RSs, given $\boldsymbol{N}_{\mathrm{FS}}$ and $\boldsymbol{N}_{\mathrm{RS}}$, the number of variables x_p grows linearly in terms of the number of HTs involved in the CSHRs. Furthermore, the line plan and train timetable reduce the search set, since a train cannot be catered at every station. The restricted set $\hat{\boldsymbol{P}}$ further reduces the search space of the Euler algorithm, since most of the paths have a free-flow lead time that is larger than the given time deadline τ. Then,

$$|\hat{\boldsymbol{P}}| \leqslant |\boldsymbol{P}| \leqslant |\boldsymbol{N}_{\mathrm{FS}}| \times |\boldsymbol{N}_{\mathrm{RS}}| \times |\boldsymbol{N}_{\mathrm{HT}}| \tag{42}$$

For convergence of the iterative scheme, we require that

1. For all $p \in \hat{\boldsymbol{P}}$ when $n \to \infty, a_p^n \to 0$.
2. For all $p \in \hat{\boldsymbol{P}}, a_p^n > 0$.
3. $\sum_{n=0}^{\infty} a_p^n = \infty$.

5.1.2. Explicit formulae for time deadline constraints pricing

To determine the correct values of the prices (i.e., Lagrange multipliers), one can solve a Lagrangian dual problem that is typically solved using iterative gradient search methods at iteration n.

$$\gamma_p^{n+1} = \max\{0, \gamma_p^n + \eta_n \times (T_p(x^n) - L(s(p), t(p)) - \tau - \delta(x_p))\} \;\forall\, p \in \hat{\boldsymbol{P}} \tag{43}$$

Vector γ indicates the shadow prices for time deadline constraints.

5.2. The Euler algorithm incorporated by the Augmented Lagrangian Dual Algorithm

The ordinary pricing strategy typically takes too much time for convergences. The ALD algorithm combines the exterior penalty method with Lagrangian dual schemes. This method yields faster convergence than ordinary dual schemes and avoids the numerical ill-conditioning inherent to penalty methods (Larsson and Patriksson, 1995). We modify the explicit formulae in iteration n as follows:

$$x_p^{n+1} = \max\left\{0, x_p^n - a_p^n \times F_1(X^n)_p - \eta_n \times c \times \max[0, T_p(x^n) + L(s(p), t(p)) - \tau - \delta(x_p)] \times \frac{\partial T_p(x^n)}{\partial x_p}\right\} \;\forall\, p \in \hat{\boldsymbol{P}} \tag{44}$$

where sequence $\{\eta_n\}$ is positive/non-decreasing penalty parameters (which are also the step lengths used in Eq. (43). Constant c is a penalty value given properly.

Compared with Eq. (40), the additional term is

$$\eta_n \times c \times \max[0, T_p(x^n) + L(s(p), t(p)) - \tau - \delta(x_p)] \times \frac{\partial T_p(x^n)}{\partial x_p}$$

which is the derivative of the penalty function:

$$P(x^n) = 0.5 \times c \times \eta_n \times \max[0, T_p(x^n) + L(s(p), t(p)) - \tau - \delta(x_p)]^2$$

5.3. An implementable version of the algorithm

We propose an implementable version of the proposed Euler scheme incorporated by the ALD algorithm. The algorithm is at first initialized: for all $p \in \widehat{\boldsymbol{P}}$, $x_p^1 = 0$ and $\gamma_p^1 = 0$. Then, some remarks for implementing the proposed Euler algorithm are empirically proposed.

(1) **Avoid violating Assumption 3.1**
It should be noted that Assumption 3.1 should not be violated during the iterations. Assume that $\sigma = 1 \times 10^{-5}$ and we know that

$$\begin{cases} \frac{d\delta(x_p)}{dx_p} \equiv 0 & \text{if } x_p \geqslant 1 \times 10^{-3} \\ \frac{d\delta(x_p)}{dx_p} \geqslant 0 & \text{if } x_p \in (0, 1 \times 10^{-3}) \ \forall\ p \in \boldsymbol{P}. \\ \frac{d\delta(x_p)}{dx_p} = 0 & \text{if } x_p = 0 \end{cases}$$

It implies that, if the marginal change on x_p in the Euler iterative scheme Eq. (44) do not equal to zero, we can impose the marginal changes greater than 1×10^{-3} to avoid violating Assumption 3.1. Then the following equation can be adapted from Eq. (44).

$$\phi_p = x_p^n - a_p^n \times F_1(X^n)_p - \eta_n \times c \times \max[0, T_p(x^n) + L(s(p), t(p)) - \tau - \delta(x_p)] \times \frac{\partial T_p(x^n)}{\partial x_p} \quad \forall\ p \in \widehat{P}$$

$$x_p^{n+1} = \begin{cases} 0 & \text{if } \phi_p \leqslant 0 \\ \max\{1 \times 10^{-3}, \phi_p\} & \text{if } \phi_p > 0 \end{cases} \quad \forall\ p \in \widehat{P} \tag{45}$$

(2) **Set step sizes properly**
Usually, if the step sizes are set too large, the correct values of the prices cannot be found. For simplicity, we assume that each passenger can only order one meal of lunch/dinner on trains. Then, the probability function $\mathscr{P}_t(v_t)$ can be served as a binomial probability function. Let $D_t = 1200$, $DR_t(\kappa^s) = 0.5$, then $\mathscr{P}_t(v_t) = C_{1200}^{v_t}(1-0.5)^{1200-v_t}0.5^{v_t}$ (see Fig. 4). We find that the interval of v_t from $P_t(v_t) = 0.05$ to $P_t(v_t) = 0.95$ is small. This implies that if the change of v_t on train t at each iteration is larger than the interval, the iterative scheme might fail because it is "skipping" the optimal solution.
At the nth iteration, for each train $t = 1,2,...,n_{\text{HT}}$, we should investigate the relationship between the inventory level v_t^n on train HT_t and the cumulative probability function $P_t(v_t^n)$. Empirically, we require that if $P_t(v_t^n) \in [0.05, 0.95]$, for all $t = 1,2,...,n_{\text{HT}}$, then, for all $p \in \widehat{\boldsymbol{P}}_t$, we let $a_p^{n+1} = a_p^0 \times \delta^n$, where $0 < \delta^n < 1$.

(3) **Update parameters related to time deadline constraints**
To monitor the state of time deadline constraints, we propose the following indicators

$$P_n(p) = \begin{cases} T_p(x_p^n) + L(s(p), t(p)) - \tau & \text{if } x_p^n > 0 \\ 0 & \text{if } x_p^n = 0 \end{cases} \quad \forall\ p \in \widehat{P}$$

$$PP_n = (\max\{0, P_n(p)\} |\ \forall\ p \in \widehat{\boldsymbol{P}})$$

Then,the following principles are intuitive. If the path flows violate a time deadline constraint, we increase the penalty

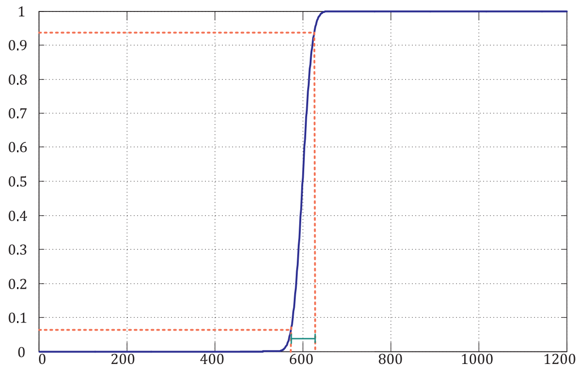


Fig. 4. Cumulative binomial probability function when $D_t = 1200$ and $DR_t(\kappa^s) = 0.5$.

parameter. Otherwise we decrease the corresponding step sizes. At the nth iteration, we have

1. If $|F_1(X)_p| \leqslant \epsilon_1$ and $\|PP_n\| \leqslant \epsilon_3$,$\forall\, p \in \hat{\boldsymbol{P}}$, then we let $a_p^{n+1} = a_p^0 \times \delta^n$, where $0 < \delta^n < 1$.
2. If $\|PP_n\|/\|PP_{n-1}\| \geqslant \epsilon_2$ and $\|PP_n\| \geqslant \epsilon_3$,$\|PP_{n-1}\| \geqslant \epsilon_3$, then we let $\eta_{n+1} = \rho \times \eta_n$, $\rho > 1$.

(4) **Convergence conditions**

If $\|PP_{n+1}\| \leqslant \epsilon_3$ and $\|x^{n+1}-x^n\| \leqslant \epsilon_4$ are satisfied for S_1 or when the iterative number exceeds the maximum number of iteration S_2, the algorithm terminates.

6. Numerical examples

We will perform three experiments to (i) evaluate the effect of selling prices, arc multipliers, and variable costs on a single train market; (ii) identify the function of time deadline constraints and the benefits of combining the ALD algorithm with the Euler algorithm; and (iii) demonstrate the applicability of our method in large-scale networks.

Throughout our experiments, $\theta_a = 0.001$ and $\beta_a = 2$, for all $a \in \boldsymbol{A}_{\mathrm{DD}} \cup \boldsymbol{A}_{\mathrm{RT}}$. Normally, the speed of a refrigerated truck on a transportation link will not be influenced by the number of food products loaded and the loading time is neglected here for simplicity. Hence we assume that $\theta_a = 0$, for all $a \in \boldsymbol{A}_{\mathrm{FD}} \cup \boldsymbol{A}_{\mathrm{DR}}$. $Cap_a = 1000$ units, for all $a \in \boldsymbol{A}_{\mathrm{FD}}$. $Cap_a = 8000$ units, for all $a \in \boldsymbol{A}_{\mathrm{DD}}$. $Cap_a = 1000$ units, for all $a \in \boldsymbol{A}_{\mathrm{DR}}$. Then, $Cap_a = 50$ units, for all $a \in \boldsymbol{A}_{\mathrm{RT}}$. $D_t = 1200$ units for all $t = 1,2,\ldots,n_{\mathrm{HT}}$. $FCap_a$ is the capacity of the facilities; we assume that $FCap_a = 1200$ units, when $a \in \boldsymbol{A}_{\mathrm{RT}}$; $FCap_a = 10{,}000$ units, when $a \in \boldsymbol{A}_{\mathrm{DD}}$. For simplicity, throughout our experiments, for all trains $t = 1,2,\ldots,n_{\mathrm{HT}}$, we restricted that each passenger can only order one meal of lunch/dinner on HTs and $\mathscr{P}_t(v_t)$ follows a binomial distribution system, where $\Omega = 1$.

$$\mathscr{P}_t(v_t) = C_{D_t}^{v_t}\,[1-DR(t)]^{D_t-v_t}\,DR(t)^{v_t}$$

In our experiments, all programs were coded in MATLAB and run under Win7 on a system with an Intel (R) Core (TM) i7-4700 MQ CPU at 2.4 GHz with 8 GB of RAM.

6.1. Sensitivity analysis of prices, costs and arc multipliers in a single train market

We present an example for illustrative purposes. The rail company caters a single high-speed train HT_1, by a single DC, DC_1, from a single FS, FS_1 through a single RS, RS_1. We assume that the train departs from the RS at time 670 min (i.e., 11:10) and arrives at its destination at 870 min (i.e., 14:30) without any other stops. The links are labeled in Fig. 5 as a_1, a_2, a_3, a_4. Fig. 5 also shows the basic input data of this example.

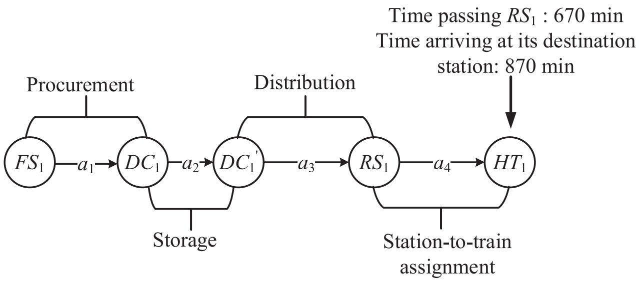
We assume a linear function $DR_t(\kappa^s) = -0.008571 \times \kappa^s + 0.8571$ on train HT_1, which means that if the selling price is 100 Yuan, the dining rate on the train is 0%. However, when the selling price is 30 Yuan, the dining rate on the train is 60%. Furthermore, we set $c = 1$,$\epsilon_1 = 0.005$,$\epsilon_2 = 0.25$,$S_1 = 30$, and $S_2 = 100$. Because only one path is considered in this case, we assume that τ is a sufficiently large time and thus ignore other parameters. We then set the convergence tolerance $\epsilon_4 = 1 \times 10^{-5}$ and the sequence $\{\delta^n\} = (\frac{5}{10},\frac{5}{11},\ldots)$.

6.1.1. Experiment 1

First, we conduct a series of tests by varying the arc multiplier α_{a_2} with fixed prices $\kappa^s = 50$, $\kappa^p = 15$ and $\kappa^d = 5$. Table 6 illustrates the computed optimal path flow, the projected inventory level on train HT_1, the corresponding profit, income and costs. Overall, the value of profit at the optimal solution always continues to increase as α_{a_2} increases. Operational costs increase and then decline as α_{a_2} increases. The reason is that less loss, i.e., $1-\alpha_{a_2}$, leads to less optimal path flows. For instance, when $\alpha_{a_2} = 0.4$, we require 1233 units to satisfy the projected demand of 493 units. When $\alpha_{a_2} = 1$, only 521 units need be transported. This actually decreases operational cost $\widehat{C}_p(x_p)$. Notably, discarding costs also at first increases and then decreases. For example, when α_{a_2} changes from 0.3 to 0.4, the dramatically increasing income (from 7980 Yuan to 17246 Yuan) will cause a decision maker to provide more meals even when discarding costs increase (from 5350.45 to 7396.26 Yuan).

6.1.2. Experiment 2

Second, we conduct a series of tests with varying variable costs VC_{a_2} and prices $\kappa^s = 50$, $\kappa^p = 15$ and $\kappa^d = 5$. Table 7 illustrates the optimal path flow, projected inventory level, and corresponding profit, income and costs. Overall, the value of profit at the optimal



Arc	t_a^0 Min	α_a	FC_a Yuan/vehicle	VC_a Yuan/unit
	Data (Yuan)			
a_1	300	1	50	0.001
a_2	30	0.9	10	0.001
a_3	20	1	50	0.001
a_4	15	1	10	0.001

Fig. 5. PFSC-CSHRs topology and input data for numerical example.

Table 6
Optimal path flows, inventory levels, corresponding profits, incomes and costs under varying α_{a_2}.

α_{a_2}	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	Changing pattern
x_p	0	758	1233	1011	852	735	646	577	521	↗↘
v_t	0	227	493	505	511	515	517	519	521	↗
Profit	0	1322.78	6131.24	9760.56	12091.04	13712.87	14902.56	15800.84	16511.69	↗
$E^{+}(v_t)$	0	0	2101.47	5604.02	7558.42	9207.27	10435.82	11230.21	11993.03	↗
$E^{-}(v_t)$	0	7980.00	15144.05	11949.63	10087.26	8493.69	7295.63	6516.45	5764.89	↗↘
Income	0	7980.00	17245.52	17553.66	17645.68	17700.95	17731.45	17746.66	17757.92	↗
$\widehat{Z}_p(x_p)$	0	5305.45	7396.26	5054.78	3406.40	2206.28	1292.79	577.24	0	↗↘
$\widehat{C}_p(x_p)$	0	1351.77	3718.02	2738.32	2148.24	1781.81	1536.10	1368.58	1246.23	↗↘
Cost	0	6657.22	11114.28	7793.10	5554.64	3988.09	2828.89	1945.82	1246.23	↗↘

Note: ↗↘ indicates that the values increase and then decline as α_{a_2} varies. ↗indicates that the values continue increasing.

Table 7
Optimal path flows, inventory levels, corresponding profits, incomes and costs under varying VC_{a_2}.

VC_{a_2}	0.001	0.005	0.01	0.05	0.1	0.5	1	Changing pattern
x_p	577	571	565	287	147	30	9	↘
v_t	520	514	509	258	133	27	8	↘
Profit	15800.84	14480.40	12855.60	4358.72	2241.71	487.28	219.62	↘
$E^{+}(v_t)$	11230.21	9207.27	6754.63	0.00	0.00	0.00	0.00	↘
$E^{-}(v_t)$	6516.45	8493.69	10857.22	9065.00	4655.00	980.00	315.00	↗↘
Income	17746.66	17700.95	17611.85	9065.00	4655.00	980.00	315.00	↘
$\widehat{Z}_p(x_p)$	577.24	571.45	565.24	287.14	147.24	30.06	9.14	↘
$\widehat{C}_p(x_p)$	1368.58	2649.10	4191.01	4419.14	2266.06	462.66	86.24	↗↘
Cost	1945.82	3220.56	4756.25	4706.28	2413.29	492.72	95.38	↗↘

Note: ↗↘ indicates that the values increase and then decline as VC_{a_2} varies. ↘ indicates that the values continue to decrease.

solution always continues to decrease as VC_{a_2} increases. $E^{+}(v_t)$ declines as v_t decreases. However, $E^{-}(v_t)$ will decrease only within some ranges of VC_{a_2}. The reduction occurs because decreasing v_t will lead to a smaller integrand $(\kappa^s-\kappa^p)v_t\mathscr{P}_t(w)$ over a larger interval $[v_t,\infty)$ in function $E^{-}(v_t)$. Operational costs are directly proportional to VC_{a_2}. Table 7 shows that $\widehat{C}_p(x_p)$ improved when VC_{a_2} changed from 0.001 to 0.01. However, when VC_{a_2} changes from 0.01 to 0.05, the path flow shrinks remarkably (from 565 units to 287 units), and the operational costs also decrease.

6.1.3. Experiment 3

Third, we run a series of tests that have different κ^s, including 30, 40, 50, 60, 70, 80, 90, and 100 Yuan. $\kappa^p = 15$ and $\kappa^d = 5$ are also given. For the same selling prices, we then vary α_{a_2}. The effect of the selling price on objective value (profit) is summarized in Fig. 6(A). Fig. 6(A) shows that profits increase and then decrease, as when selling prices increase in all testing series with different multipliers. In all test series with different selling prices, total profit improves when α_{a_2} increases.

6.1.4. Experiment 4

Fourth, we implement a series of tests that have different κ^s, including 30, 40, 50, 60, 70, 80, 90, and 100 Yuan. $\kappa^p = 15$ and

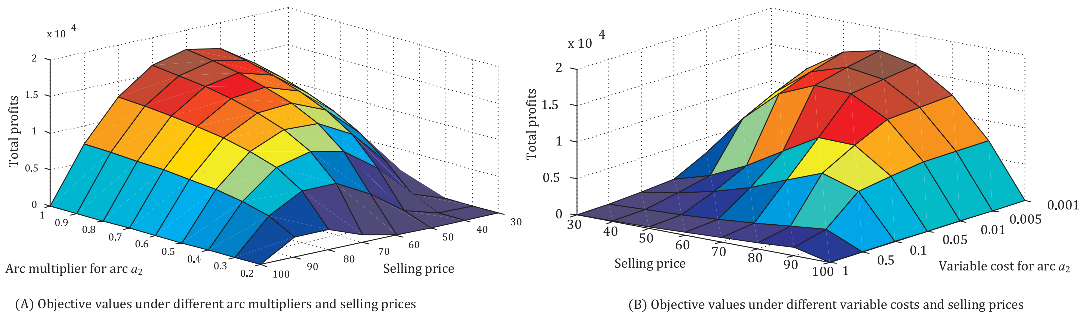
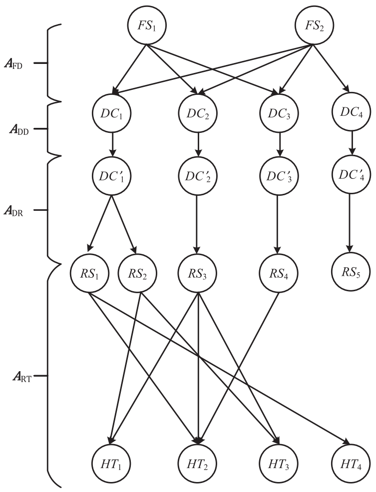


Fig. 6. Objective values under different κ^s and α_{a_2}/Objective values under different κ^s and VC_{a_2}.



Data (Yuan)

Sets	Arc name	t_a^0 Min	α_a	FC_a Yuan/vehicle	VC_a Yuan/unit
A_{FD}	(FS_1, DC_1)	300	0.99	10	0.001
	(FS_1, DC_2)	300	0.97	10	0.001
	(FS_1, DC_3)	300	0.97	10	0.001
	(FS_1, DC_4)	500	0.97	10	0.001
	(FS_2, DC_1)	300	0.99	10	0.001
	(FS_2, DC_2)	400	0.98	10	0.001
	(FS_2, DC_3)	700	1	10	0.001
	(FS_2, DC_4)	100	1	10	0.001
A_{DD}	(DC_1, DC'_1)	30	0.98	50,	0.001
	(DC_2, DC'_2)	30	0.98	30	0.001
	(DC_3, DC'_3)	30	0.98	30	0.001
	(DC_4, DC'_4)	30	0.98	50	0.001
A_{DR}	(DC'_1, RS_1)	20	1	50	0.001
	(DC'_1, RS_2)	20	1	40	0.001
	(DC'_2, RS_3)	15	0.98	30	0.001
	(DC'_3, RS_4)	15	0.98	50	0.001
	(DC'_4, RS_5)	15	0.98	20	0.001
A_{RT}	(RS_1, HT_2)	15	0.98	10	0.001
	(RS_1, HT_4)	15	0.98	10	0.001
	(RS_2, HT_1)	15	0.98	10	0.001
	(RS_2, HT_3)	15	0.98	10	0.001
	(RS_3, HT_1)	3	0.98	10	0.001
	(RS_3, HT_2)	3	0.98	10	0.001
	(RS_3, HT_3)	3	0.98	10	0.001
	(RS_4, HT_2)	3	0.98	10	0.001

Fig. 7. Illustrative network of PFSC-CSHRs and input data.

$\varkappa^d = 5$ are also fixed. For the same selling prices, we then vary VC_{a_2}. Fig. 6(B) shows that profits increase and then decrease, as when the selling prices increase in all testing series with different variable costs. In all series of tests with different selling prices, total profit improves when VC_{a_2} decreases.

6.2. Benefits of integrating the Euler algorithm and the Augmented Lagrangian Dual algorithm

We now apply our method to compute solutions to the network shown in Fig. 7, which is derived from Fig. 2. Free flow time impendence, arc multipliers, fixed costs and variable costs on links are also labeled in Fig. 7. The corresponding timetable for trains HT_1,HT_2,HT_3 and HT_4 is shown in Table 1. $DR_t(\varkappa^s) = -0.008571 \times \varkappa^s + 0.8571$ on trains. We define $\varkappa^s = 50$, $\varkappa^p = 15$ and $\varkappa^d = 5$. $\eta_1 = 1 \times 10^{-4}$, $c = 500$, $\rho = 1.01$, $\epsilon_1 = 5$, $\epsilon_2 = 0.25$, $\epsilon_3 = 1$, $S_1 = 50$ and $S_2 = 300$. Let $\tau = 700$ minutes set the convergence tolerance $\epsilon_4 = 0.05$ and the sequence $\{\delta^n\} = (\frac{9}{10},\frac{9}{11},\ldots)$. For our purposes, we compare three design concepts in Table 8 (i.e., E-RPS, E-NPS and E-ALD).

E-RPS serves as a reference because it represents the general method of addressing lifespan (Liu and Nagurney, 2012). The design concept E-RPS easily removes all the paths from the network whose free-flow lead times exceed a given time deadline and generate a restricted path set $\widehat{\boldsymbol{P}}$ (see, Section 3.3). Table 9 shows all the paths in $\widehat{\boldsymbol{P}}$. We find that path $FS_2 \to DC_3 \to RS_4 \to HT_2$ is removed from the network because of its overlong free-flow lead time ($748\,\text{min} > 700\,\text{min} = \tau$). Then, we can update the flows using Euler algorithm on the paths in the restricted set $\widehat{\boldsymbol{P}}$. However, we find that, when the flow-dependent lead time of the pathways is considered, E-RPS results in two pathways with overlong lead times. For example, path $FS_2 \to DC_1 \to RS_1 \to HT_4$ has an 841 min lead time, which exceeds the given lifespan.

Thus, we continue to consider time deadline constraints in the computational procedure using an iterative scheme of E-NPS or E-ALD. We observe that paths $FS_1 \to DC_1 \to RS_1 \to HT_4$ and $FS_2 \to DC_1 \to RS_1 \to HT_4$ have been bounded within 700 min because of E-NPS or E-ALD. We observe that paths $FS_1 \to DC_1 \to RS_1 \to HT_4$ and $FS_2 \to DC_1 \to RS_1 \to HT_4$ have positive Lagrange multipliers when we use E-ALD. Lagrange multiplier values are the shadow prices for the time deadline constraints. If we relax the time deadline one more minute, the manager might make 0.48 Yuan more money than before.

Further we find that E-RPS earns the largest profit because of the relaxation of time deadline constraints. We also find that E-NPS

Table 8
Three design concepts implemented in this experiment.

Design concept	Abbreviation	Description
Euler algorithm + Restricted Path Set (without flow dependent time deadline constraints)	E-RPS	E-RPS removes the paths whose free-flow lead time exceeds a fixed lifespan from the network (see Section 3.3) and implements the Euler iterative scheme directly without considering time-deadline constraints. The Euler iterative scheme without considering time-deadline constraints can be easily adapted from Eq. (40).
Euler algorithm + Normal Pricing Strategy	E-NPS	E-NPS is the algorithm implementing the Euler iterative scheme Eq. (40) shown in Section 5.1 and using Eq. (43) to determine the Lagrangian multipliers.
Euler algorithm + Augmented Lagrangian Dual algorithm	E-ALD	E-ALD is the algorithm implementing the Euler iterative scheme Eq. (45) shown in Section 5.2 and using Eq. (43) to determine the Lagrangian multipliers.

Table 9
Performance measures of each path for the E-RPS, E-NPS and E-ALD algorithm.

	HTs	RSs	DCs	FSs	Free flow lead time (minutes)	Path flow (unit)			Lead time (minutes)			Lagrangian Multiplier γ_q^* (Yuan)	
						E-NPS	E-ALD	E-RPS	E-NPS	E-ALD	E-RPS	E-NPS	-ALD
Route $\hat{\boldsymbol{P}}$	HT_1	RS_2	DC_1	FS_1	665	74	74	77	679.8	680.3	690.5	0	0
	HT_1	RS_2	DC_1	FS_2	665	74	74	77	679.8	680.3	690.5	0	0
	HT_1	RS_3	DC_2	FS_1	548	299	298	296	577	577	576	0	0
	HT_1	RS_3	DC_2	FS_2	648	106	105	105	677	677	676	0	0
	HT_2	RS_1	DC_1	FS_1	665	36	36	38	672	672	682	0	0
	HT_2	RS_1	DC_1	FS_2	665	36	36	38	672	672	682	0	0
	HT_2	RS_3	DC_2	FS_1	548	149	149	148	565	565	565	0	0
	HT_2	RS_3	DC_2	FS_2	648	53	52	52	665	665	665	0	0
	HT_2	RS_4	DC_3	FS_1	448	280	279	277	457	457	457	0	0
	HT_3	RS_2	DC_1	FS_1	615	95	95	101	637	637	649	0	0
	HT_3	RS_2	DC_1	FS_2	615	95	95	101	637	637	649	0	0
	HT_3	RS_3	DC_2	FS_1	468	225	222	217	494	493	492	0	0
	HT_3	RS_3	DC_2	FS_2	568	137	136	133	594	593	592	0	0
	HT_4	RS_1	DC_1	FS_1	685	62	75	269	697	700	**841**	0	0.48
	HT_4	RS_1	DC_1	FS_2	685	62	74	269	697	700	**841**	0	0.48
Other Routes	HT_2	RS_4	DC_3	FS_2	**748**					N/A			

Boldtypes indicates the lead time is longer than the given time deadline.

and E-ALD implement different pricing (0 Yuan vs. 0.48 Yuan). The reason for the prices is that E-NPS actually does not meet the convergence conditions when the number of iterations exceeds S_2. Table 10 shows that there are differences between the path flows obtained by E-NPS and E-ALD, which results in a gap (approximately 1.4%) between their total profits (The profit obtained by E-ALD is larger than the profit obtained by E-NPS).

Fig. 8 indicates the changes in profits and $\|x^{n+1}-x^n\|$ with respect to the number of generations during the Euler iterative computational procedure of E-NPS, E-ALD and E-RPS. Only the first 120 iterations are displayed in Fig. 8(A) and the first 50 iterations are displayed in Fig. 8(B). We observe that the E-RPS overestimates the profit of CSHRs by neglecting the oversaturated case, see Fig. 8(A). Fig. 8(B) shows the change of $\|x^{n+1}-x^n\|$ with respect to the number of generations. We find that E-ALD yields a faster convergence than E-NPS does under the condition of the above parameter settings. E-NPS uses 300 generations with 282 s of CPU time, but the E-ALD uses only 112 generations to make $\|x^{n+1}-x^n\| \leqslant \epsilon_4 = 0.1$ with 101 s of CPU time. However, the rate of convergence is quite parameter-dependent. The settings of parameters are also case-dependent. Thus, the efficiency of the E-ALD is not always ensured.

6.3. Large-scale application of the Beijing-Shanghai high-speed corridor

To demonstrate the applicability of our proposed framework, this section tests our algorithm on a case based on a timetable for May 2016 with 246 trains on the Beijing-Shanghai high-speed corridor (see Fig. 9). Only HTs that start and end at RSs on the Beijing-Shanghai high-speed lines are considered in this case. The regional trains on the Beijing-Tianjin intercity line and all over-line trains are neglected. The trains on the Nanjing-Shanghai intercity line are considered because of their long average traveling time. In this case, we consider 31 stations on the Beijing-Shanghai corridor, as shown in Fig. 9. The information for 21 DCs with their RSs is also displayed in Table B.13 in Appendix B. The DCs are labeled using the names of the cities. Four FSs - Beijing FS, Jinan FS, Nanjing FS and Shanghai FS - are considered. Because FSs in this case are dispersed far from each other on a large railway network, we assume that a DC orders its required meals only from the nearest FS:

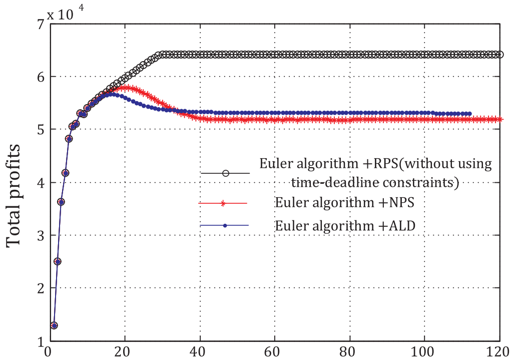
$$\boldsymbol{A}_{\mathrm{FD}} = \{a = (FS_i, DC_d) \in \boldsymbol{N}_{\mathrm{FS}} \times \boldsymbol{N}_{\mathrm{DC}} | t_a^0 = \min_{i \in \boldsymbol{N}_{\mathrm{FS}}} t^0_{FS_i, DC_d}\ \forall\ d \in \boldsymbol{N}_{\mathrm{DC}}\}.$$

Here, we assume $D_t = 1200$, $DR_t(\kappa^s) = -0.008571 \times \kappa^s + 0.8571$ on each train, $\kappa^s = 50$, $\kappa^p = 15$, $\kappa^d = 5$, $\eta_1 = 1 \times 10^{-3}$, $c = 500$, $\rho = 1.1$, $\epsilon_1 = 10$, $\epsilon_2 = 0.25$, $\epsilon_3 = 5$, $S_1 = 35$ and $S_2 = 3000$, $\epsilon_4 = 0.1$ and $\{\delta^n\} = \left(\frac{9}{10}, \frac{9}{11}, \ldots\right)$.

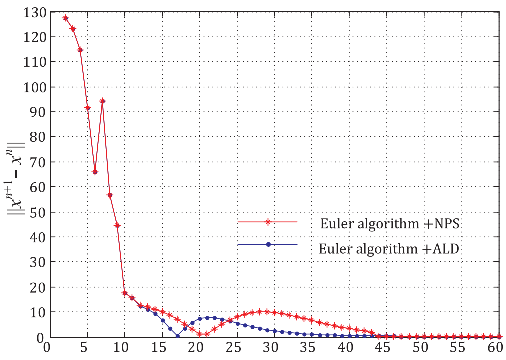
We conduct two tests with different time deadlines, including $\tau = 1000$ min and 1440 min. The rows in Table 11 show the cost contributions, inventory levels, and other basic information about the PFSC-CSHRs' systems. Table 11 illustrates the benefits of time

Table 10
Profit obtained under different design concepts.

Design concept	Profit obtained
E-RPS	64,121 Yuan
E-NPS	52,371 Yuan
E-ALD	53,085 Yuan



(A) Profit in each generation by using Euler algorithm with NPS, ALD algorithm and RPS



(B) $||x^{n+1} - x^n||$ in each generation by using Euler algorithm with NPS and ALD algorithm

Fig. 8. Profit and $\|x^{n+1}-x^n\|$ in each generation using the Euler algorithm with NPS, ALD and RPS.

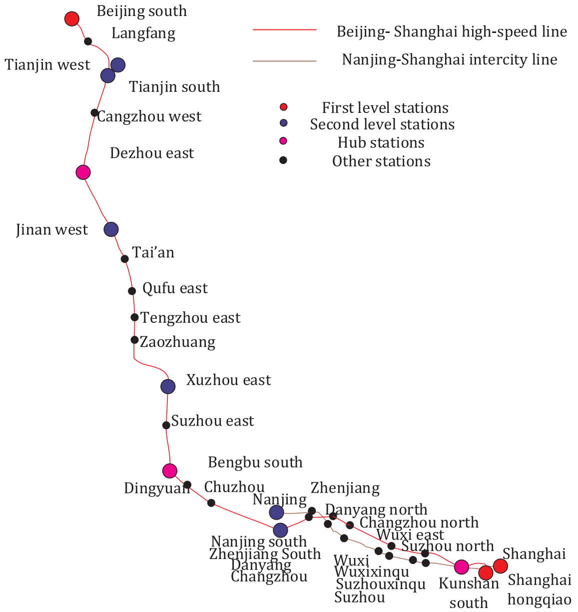


Fig. 9. Map of Beijing-Shanghai high-speed railway corridor.

deadline constraints. If $\tau = 1000$ min is replaced by $\tau = 1440$ min, more money (14,000 Yuan) can be earned (We assume that the longer lifespan do not lead to a decline in food quality). To achieve these earnings, an increase in $E^-(v_t)$ and $E^+(v_t)$ and a decrease in operational and discarding cost are needed. This income increase reflects the increase of the average inventory level on trains. It is noted that although the maximum lead time increases because of the relaxation of the time deadline, the average lead time conversely declines. Finally, we observe that the CPU times used for our proposed method depend upon number of iterations instead of the number of variables.

Fig. 10 displays the change in total profit (from 4×10^5 to 6×10^5) and $\|x^{n+1}-x^n\|$ with respect to the number of generations. The function of time deadline constraints is also illustrated in Fig. 10(A).

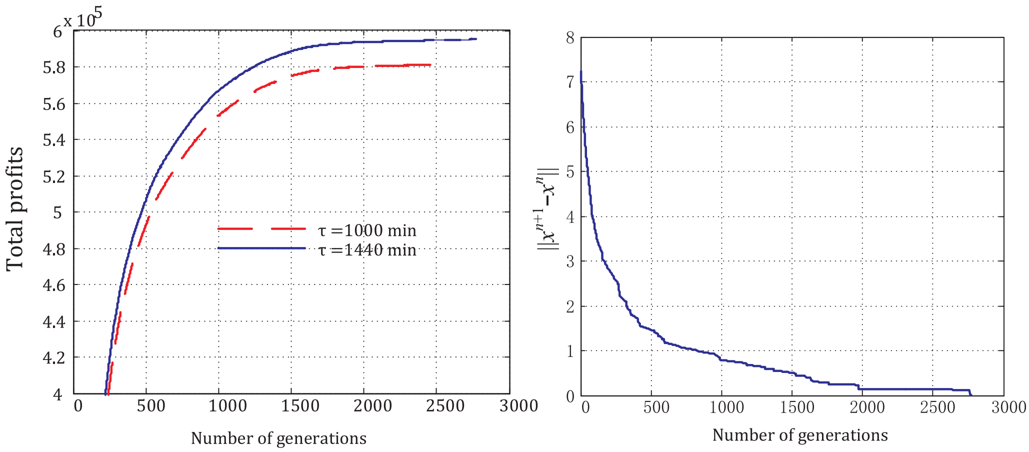
7. Conclusions

In this paper, we propose a novel flow-assignment model for a perishable food supply chain (PFSC) network problem in the context of catering services for high-speed railway (CSHRs). The PFSC-CSHRs is different from traditional food supply chain problems and has some typical challenging features. PFSC-CSHRs problems are affected by decisions concerning the rail transport plan, i.e., a line plan and a timetable. The problems further involve the perishability of the food products, uncertainty of food demand and the flow-dependent lead time which is subject to time deadline constraints.

To formulate the problem, we represented a topological network by introducing the information provided by line plans and timetables. Next, flow balance constraints used in supply chain analytics were introduced to describe the link/path flows in the PFSC-

Table 11
Performance measures for three tests with lifespan 1000 min and 1440 min.

	Test 1	Test 2
τ	1000 min	1440 min
Operational cost	270,240 Yuan	282,940 Yuan
Discarding cost	27,766 Yuan	28,635 Yuan
$E^{-}(v_t)$	861,390 Yuan	887,980 Yuan
$E^{+}(v_t)$	17,732 Yuan	18,708 Yuan
Profit	581,120 Yuan	595,120 Yuan
Average inventory level $\frac{\sum_{t=1,2,\ldots,n_{HT}} v_t}{n_{HT}}$	102 units	105 units
Maximum inventory level $\max_{t=1,2,\ldots,n_{HT}} v_t$	487 units	487 units
Average lead time $\frac{\sum_{p\in\hat{P}} T_p(x)+L(s(p),t(p))}{\lvert\hat{P}\rvert}$	600 min	457 min
Maximum lead time $\max_{p\in\hat{P}} T_p(x) + L(s(p),t(p))$	1004 min	1410 min
Maximum lead time (only containing all paths with positive flows)	1000 min	1410 min
Number of paths violating time deadline constraints	2	0
Number of path variables	569	580
Number of iterations	2458 times	2770 times
CPU times	8.2 h	7.2 h



(A) Profit in each generation by using Euler algorithm when τ =1000 min and 1440 min respectively

(B) $\|x^{n+1}-x^{n}\|$ in each generation by using Euler algorithm when τ = 1440 min

Fig. 10. Profit and $\|x^{(n+1)}-x^{n}\|$ in each generation in different time deadline scenarios.

CSHRs network. A Dirac delta function was introduced to reflect the transition from a free-flow path to a flow-loaded pathway under a time deadline constraint. Operational and discarding costs were then described. A revenue function based on the Newsvendor model was proposed to calculate the income of CSHRs when facing uncertain demand and fixed prices.

This paper contributes to two equivalent VI models (arc flow-based and path flow-based) to determine order quantities for trains and the flow pattern on a PFSC-CSHRs network, considering uncertain demand and constrained time deadlines. Mathematical properties of the VI models are proved. We conduct a sensitivity analysis to evaluate the effect of selling prices, arc multipliers, and variable costs on a single train market.

To solve the VI, this paper provided three design concepts: E-RSP, E-NPS and E-ALD.Using numerical examples, the function of the time deadline constraints is illustrated in our case study because E-RSP overestimated the total profit, unlike E-NPS and E-ALD. E-ALD was also found to perform better than E-NPS in terms of computational time and solution quality. E-ALD was also applied to solve a large-scale PFSC-CSHRs network design problem to assess the applicability of the proposed method. Two large-scale cases with different time deadlines were implemented to illustrate that a higher lifespan without decline in food quality will lead to higher profits.

The presented approach has limitations, which might be the subject of future research. For example, the efficiency of the proposed algorithm is quite parameter-dependent, which occasionally affects the quality of solutions and prolongs computational times. Furthermore, multiple types of food products (including both ambient meals and "cold chain" meals) with different lifespans for trains remain beyond the current scope of our research. Emerging dining reservation services are also not considered in this study.

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Appendix A

See Table A.12.

Table A.12
Comparison of PSFC-CSHRs and other existing supply chain problems in supply chain management.

	Background	Deterioration	Flow-dependent lead time	Side constraints	Demand
Nagurney et al. (2002)	Generalized supply chain	No	No	No	Price-dependent demand
Zhang et al. (2003)	Deep frozen food chain	Yes	No	No	Fixed demand
Dong et al. (2004)	Generalized supply chain	No	No	No	Stochastic demand under unfixed price
Liu and Nagurney (2012)	Multi-period supply chain	No	No	Time deadline	Price-dependent demand
Nagurney and Nagurney (2012)	Medical nuclear supply chain	Yes	No	Link capacity	Fixed demand
Masoumi et al. (2012)	Pharmaceutical supply chain	Yes	No	No	Price-dependent demand
(Nagurney et al., 2013, chap.2)	Blood supply chain	Yes	No	No	Stochastic demand
Yu and Nagurney (2013)	Food supply chain	Yes	No	No	Price-dependent demand
PSFC-CSHRs	High-speed rail catering	Yes	Yes	Time deadline	Stochastic demand under fixed price

Appendix B

See Table B.13.

Table B.13
Stations and information concerning their corresponding DCs.

ID stations	Name of stations	ID DCs	Name of DCs	ID stations	Name of stations	ID DCs	Name of DCs
1	Bengbu South	1	Bengbu	16	Shanghai Hongqiao	12	Shanghai
2	Beijing South	2	Beijing	17	Suzhou	13	Suzhou
3	Cangzhou West	3	Cangzhou	18	Suzhou North	13	Suzhou
4	Changzhou	4	Changzhou	19	Suzhou Yuanqu	13	Suzhou
5	Changzhou North	4	Changzhou	20	Taian	14	Taian
6	Danyang	5	Danyang	21	Tenzhou East	15	Tenzhou
7	Danyang South	5	Danyang	22	Tianjin South	16	Tianjin
8	Dezhou East	6	Dezhou	23	Tianjin West	16	Tianjin
9	Dingyuan	7	Dingyuan	24	Wuxi	17	Wuxi
10	Jinan West	8	Jinan	25	Wuxi East	17	Wuxi
11	Langfang	9	Langfang	26	Wuxi Xinqu	17	Wuxi
12	Nanjing	10	Nanjing	27	Suzhou East	18	Suzhou
13	Nanjing South	10	Nanjing	28	Xuzhou East	19	Xuzhou
14	Qufu East	11	Qufu	29	Zaozhuang	20	Zaozhuang
15	Shanghai	12	Shanghai	30	Zhenjiang	21	Zhenjiang
				31	Zhenjiang South	21	Zhenjiang

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