Design Theory Manual For
ARMORFORM® Erosion Protection Mats

TEXICON
REVETMENT MATS

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1.0   Determination Of Forces Generated By Moving Water

Water flowing in channels or water runup on slopes due to wave action creates forces on the sides and bottom of the channel or on the face of the slope due to wave runup which tends to erode soil. The potential for erosion is a function of the velocity of the water, the steepness of the slope, and the type of soil. Water flowing in channels tends to erode soil from the bottom and sides of the channel due to forces created by the water as it moves past the particles of soil. Wave runup creates forces in a similar way as the waves generate forces as they impinge upon the slope. It is possible, using the laws of energy conservation, to calculate the forces created by moving water. This can be done for channel bottoms and sides as well as slopes subject to wave runup. It is also possible to estimate the forces generated as water impinges on slopes due to turns in the channels of flowing water. A description of the calculation of these forces follows.

1.1   Forces Due To Flowing Water
1.1.1   Active Force On Channel Bottom

When water flows in a channel, a force that acts in the direction of flow is developed on the channel bed. This force, which is simply the pull of water on the wetted area, is called the tractive force, (T_b). The average tractive stress, (τ_b), may be analytically ascertained by the assumption that all frictional losses are caused by frictional forces on the boundary of the channel lining (Ref. 1). From Bernoulli's equation of conservation of energy, the tractive force, T_b, acting on a moving body of water in a direction opposite to that of the flow (Fig. 1) is calculated by:

$$T_b = \gamma_w \, a_a \, h_f \qquad\qquad (1)$$

where:

γ_w = the unit weight of water (pcf)

$a_a = b_a \dfrac{(y_1 + y_2)}{2}$ is the average flow area (sq ft)

b_a = average width of the channel (ft)

y_1 and y_2 = depths of water in two sections at distance L apart (ft)

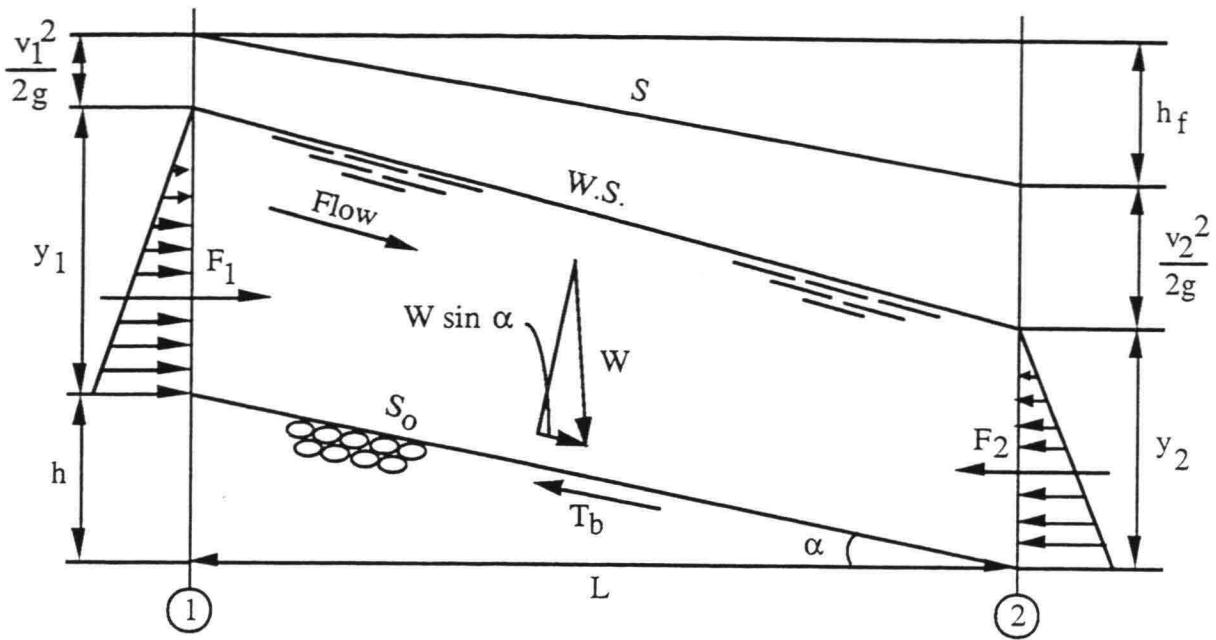h_f = friction head loss (ft-lb/lb)

Figure 1. Forces Acting On A Moving Body Of Water

The average tractive stress, τ_b, in pounds per unit of wetted area, on the boundary of the channel bottom, is equal to:

$$\tau_b \; = \; \frac{T_b}{pL} \; = \; \gamma_w \frac{a}{p} \; x \; \frac{h_f}{L} \; = \; \gamma_w \, RS \qquad (2)$$

where:

T_b = tractive force (lb)

L = horizontal length of a portion of a channel (ft)

a = flow area (sq ft.)

p = wetted perimeter (ft)

γ_w = 62.43 pcf = unit weight of water

R = $\frac{a}{p}$ = hydraulic radius (ft)

S = $S_o \, (Q/Q_{n,y})^2$ = rate of friction loss or slope of energy grade line (ft/ft)

Q = design discharge (cfs)

$Q_{n,y}$ = normal discharge corresponding to depth of flow, y (cfs)

S_o = slope of channel bottom (ft/ft)

This equation is valid for the general case of gradually varied flow. From Figure 1, one can see that S becomes equal to S_O for uniform flow or normal discharge, which is defined by Manning's equation:

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \tag{3}$$

where:

V = Velocity for uniform flow and normal depth (fps)

n = Manning's roughness coefficient

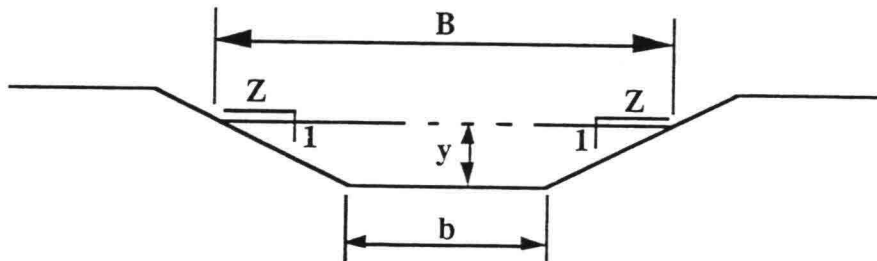The hydraulic radius has the following values for various channel shapes (using the notations in Figure 2):

Figure 2. Cross-Section Of A Channel

$$\text{trapezoidal:} \quad R = \frac{y\,(b + y\,Z)}{b + 2y\sqrt{1 + Z^2}} \tag{4}$$

$$\text{rectangular:} \quad R = \frac{by}{b + 2y} \tag{5}$$

$$\text{parabolic:} \quad R = \frac{2B^2 y}{3B^2 + 8y^2} \tag{6}$$

$$\text{triangular:} \quad R = \frac{yZ}{2\sqrt{1 + Z^2}} \tag{7}$$

where:

b = bottom width (ft)

B = width of the channel at the water surface (ft)

y = depth of water (ft)

Z = side slope of trapezoidal or triangular section expressed as a ratio of horizontal to vertical (ft/ft)

For steady uniform flow, the average tractive shear stress on the channel bottom is given by:

$$\tau_b = \gamma_w\, R\, S_o \qquad (8)$$

For this condition of flow, the tractive shear stress can be expressed as a function of the velocity:

$$\tau_b = \frac{\gamma_w\, n^2}{1.486^2} \times \frac{V^2}{R^{1/3}} \qquad (9)$$

where:

n = Manning's coefficient of roughness

Usually, in a channel with gradually varied flow, the actual flow depth is either larger or smaller than the normal depth and it is conservative to calculate the tractive force based on Equations 8 or 9. Note that V in Equation 9 is the maximum velocity for a steady uniform flow, greater than the velocity corresponding to the gradually varied flow.

1.1.2 Active Force On Channel Side Slopes

The tractive stress in channels, except for wide-open channels, is not uniformly distributed along the wetted perimeter. A typical distribution of tractive stresses in a trapezoidal channel is shown in Figure 3 (Ref. 2). The maximum tractive stress on slopes is related to the tractive stress on bottom by (Figure 4, Reference 3, Appendix C, and Figure 5, Reference 4):

$$\tau_s = 0.94\,\tau_b \text{ for } Z = 4 \qquad (10)$$

$$\tau_s = 0.85\,\tau_b \text{ for } Z = 3 \qquad (11)$$

$$\tau_s = 0.79\,\tau_b \text{ for } Z = 2 \qquad (12)$$

$$\tau_s = 0.76\,\tau_b \text{ for } Z = 1.5 \text{ or smaller} \qquad (13)$$

where:

τ_s = maximum tractive stress on slopes (lb/sq ft)

τ_b = maximum tractive stress on bottom (lb/sq ft)

Z = side slope of trapezoidal section expressed as a ratio of horizontal to vertical (ft/ft)

Equations 10 through 13 give conservative values for side slopes of channels.
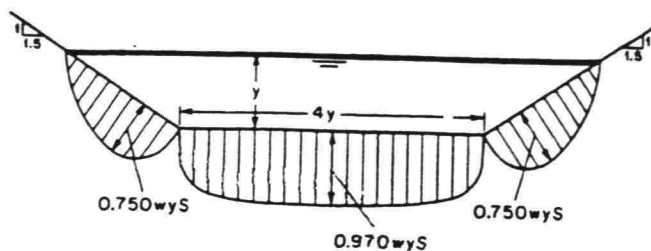
Figure 3. Distribution Of Tractive Shear Stress In A Trapezoidal Channel Section
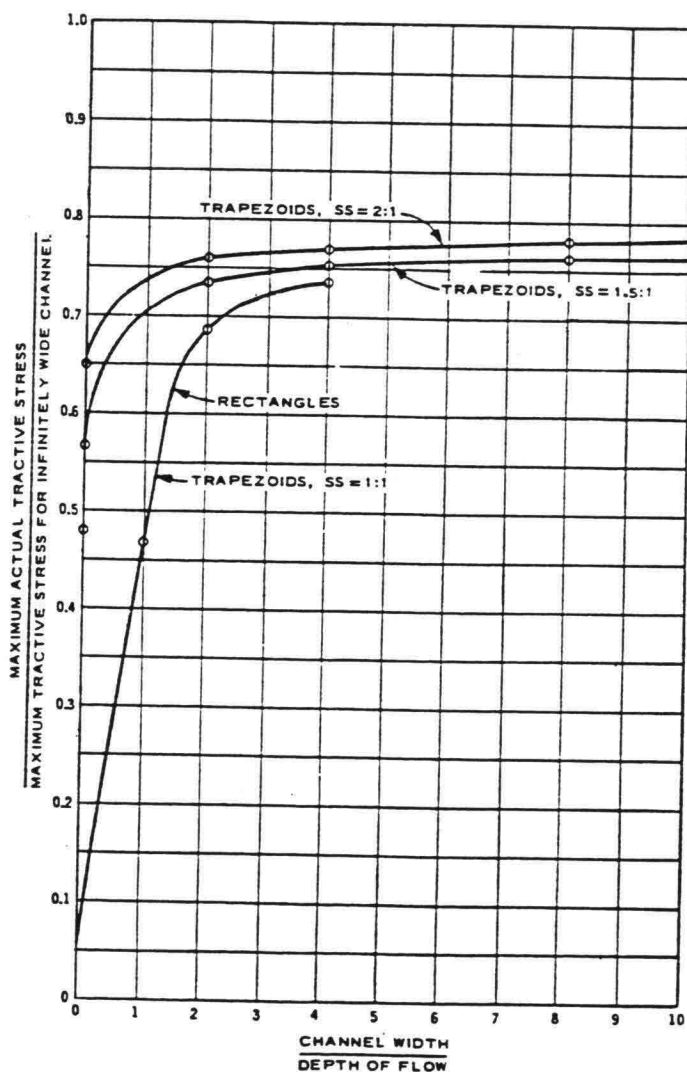
Figure 4. Ratio Of Actual Maximum Tractive Shear Stress On Side Of Channel Of Infinite Width
To Maximum Tractive Shear Stress On Bed Of Infinitely Wide Channel
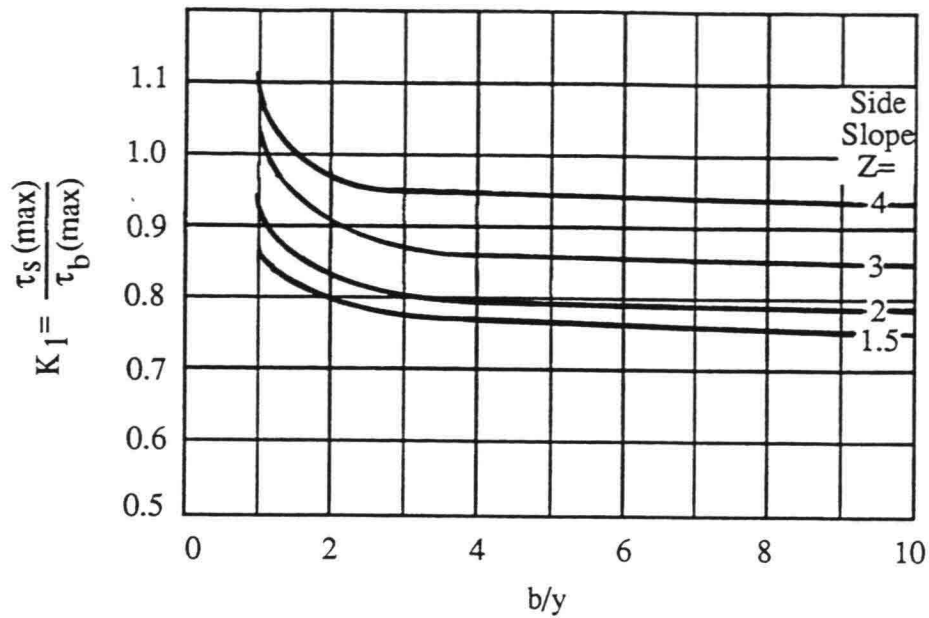
Figure 5. Distribution Of Boundary Shear Around Wetted Perimeter Of Trapezoidal Channels

1.1.3 Tractive Force In Curves

The tractive stress is increased along the channel slopes in a curve. Flow in curves or benches creates a higher velocity of flow on the outside of the bend (concave bank) during normal flow and a higher velocity on the inside of the bend (convex bank) during flood flow. Figure 6 shows a graph recommended by the U.S. Army Corps of Engineers for estimation of the relationship between the forces in straight sections and in curves (Ref. 3, Appendix C).

Figure 6. Ratio Of Tractive Shear Stress On Bend To Tractive Shear Stress On Straight Reach (From SOIL CONSERVATION SERVICE, 1977).

1.2 Forces Due To Wave Action

The following parameters must be known to determine the height of the slope to be protected and to design the type of protection:

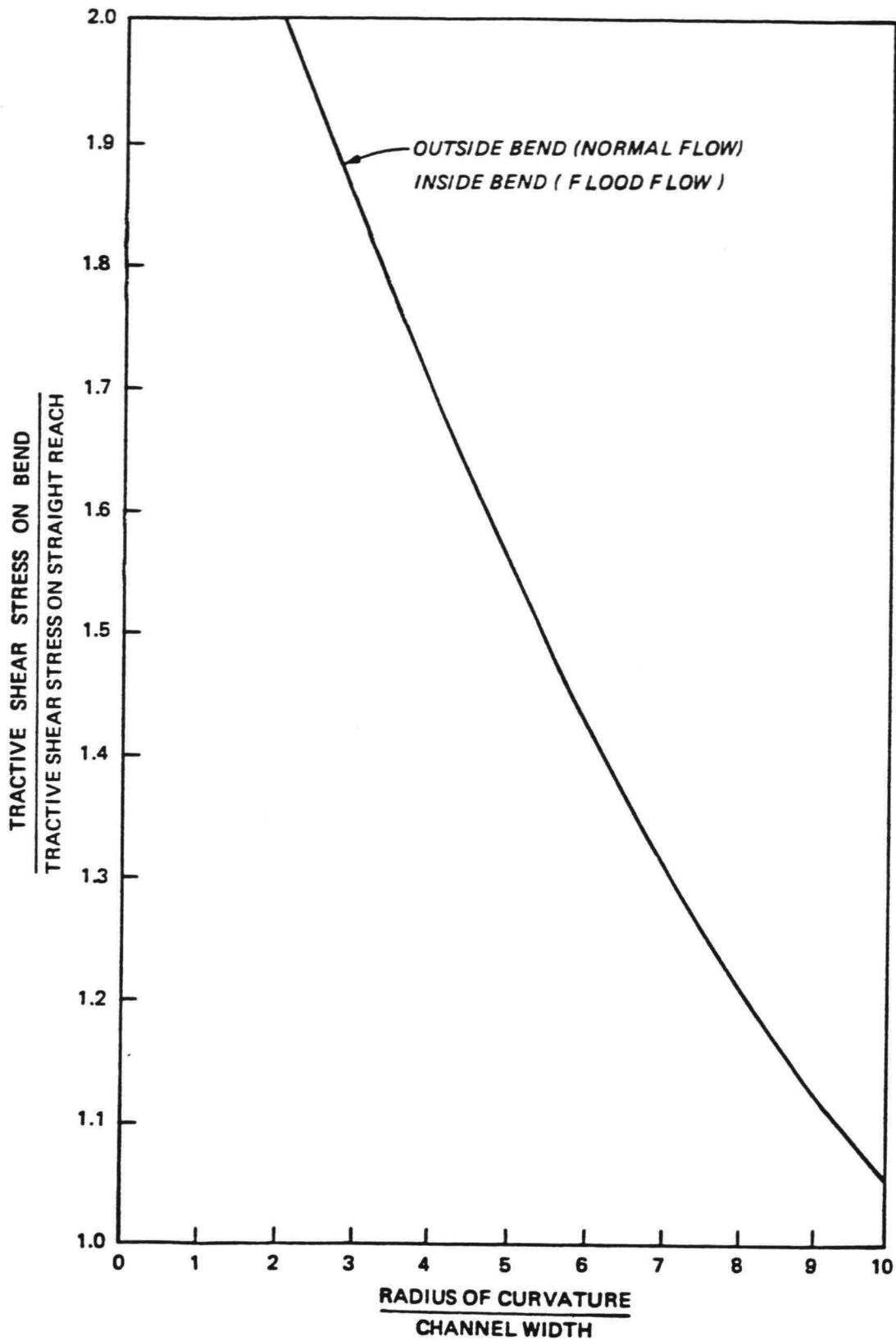- WIND SETUP OR STORM SURGE which is the vertical rise in the normal level caused by wind stresses on the surface of the water.
- WAVE SETUP which is the super-elevation of the water surface over normal elevation due to wave action alone.
- WAVE UPRUSH. The rush of water up onto the beach following the breaking of a wave.
- RUNUP. The rush of water up a structure or beach on the breaking of a wave. The amount of runup is the vertical height above still-water level to which the rush of water reaches.
- WAVE BACKRUSH (LIMIT OF). The point of farthest return of the water following the uprush of the waves.
- WAVE HEIGHT is the vertical distance between a crest and the preceding trough of a wave.

The above parameters are described in References 6 and 7.

Depending on the protection class/importance, the design value may be H_S, the "significant wave height" (average height of one-third of the highest waves) or more. For example, for critical structures at open exposed sites where failure would be disastrous, and in absence of reliable wave records, the design wave height "H" should be H_1, the average height of the highest 1% of all waves expected during an extreme event; for less critical structures, where some risk of exceeding design assumptions is allowable, wave heights between H_{10} (average height of the highest 10% of all waves) and H_1 are acceptable (Ref. 7, Vol. II, p. 7-242).

Approximate relationships are:

$$H_S = \frac{H_{10}}{1.27} = \frac{H_5}{1.37} = \frac{H_1}{1.67} \qquad (14)$$

When in a wave-attack the run-up has reached its maximum value, the water on the slope starts to flow back due to gravity. During this stage water may flow through voids or holes in the protection layer, which may result in an increase of the water level in the underlying layer depending on the permeability of the slope protection (k') and the underlying layer (k).

When the water on the slope flows back, pressures on the slope decrease. When a rough slope is present, this back flow may result in drag forces, intertia forces and lift forces (Failure Mechanism a, see Figure 7-1). Depending on k', k, and the geometry, the water in the underlying layer cannot flow out immediately, which results in uplift pressures against the slope protection. These uplift pressures may cause failure of the slope protection (Figure 7-1, Failure Mechanism b1). In general, wave run-up is larger than wave run-down. Therefore, seepage into the underlying layer takes place over a larger surface than seepage out of the underlying layer, resulting in a higher elevation of the mean phreatic level and the pore pressures within the underlying layer. This is a cumulative effect of a number of waves (Mechanism b2, Figure 7-1).

When the next wave approaches the slope, an increase of pressures on the slope below this wave occurs. These pressures may be transmitted under the slope protection just in front of the wave front, resulting in uplift pressures (Figure 7-1, Mechanism c). These uplift pressures will be present over a very limited area in front of the wave front. At this stage, considerable changes in the velocity field due to the approaching wave also occur (Figure 7-1, Mechanism d). Depending on the slope geometry and wave parameters, wave breaking may occur. A wave breaking on the slope will have an impact on the slope revetment. This causes a strong increase in pressures on the slope with a duration on the order of 0.1 seconds. These pressures on the slope may be transmitted under the slope protection resulting in short duration uplift pressures (Mechanism e, Figure 7-1). After this short duration phenomenon, a mass of water falls on the slope resulting in high pressures on the slope. These high pressures may propagate below the protection just in front of the place where the wave breaks on the slope, thus resulting in uplift pressures on the protection (Mechanism f, Figure 7-1).

After the wave hits the slope, a strong reduction (even "negative" related to atmospheric pressure) of pressures in the slope may occur during a period on the order of 0.1 seconds. This phenomenon has been explained as being a result of oscillations of the air pocket entrapped in the breaking wave. These low pressures on the slope may cause failure (Mechanism g, Figure 7-1).

After wave breaking, wave run-up occurs. During this stage, pressures on the slope protection increase. At this stage, no critical conditions are present, except when the slope is not smooth or when water is beneath the protection.

It should be noted that combinations of failure mechanisms presented above may occur (Ref. 8). The forces to be taken into account are (Ref. 9, 10, See Figure 7-2):

- Drag force, F_D, which acts upward, parallel to the slope.
- Inertia Force, F_I, which acts downward, parallel to the slope.
- Lift Force, F_L, which acts upward, normal to the slope since the flow tends to be parallel to the slope.

a = forces due to down-rush
b = uplift pressures due to water in filter
c = uplift pressures due to approaching
    wave front
d = change in velocity field

(7-1)

e = wave impact
f = uplift pressures due to mass of water
    falling on slope
g = low pressures on slope due to air
    entrainment
h = forces due to up-rush

(7-2)

Simplified model of actions:

F_D = Drag Force

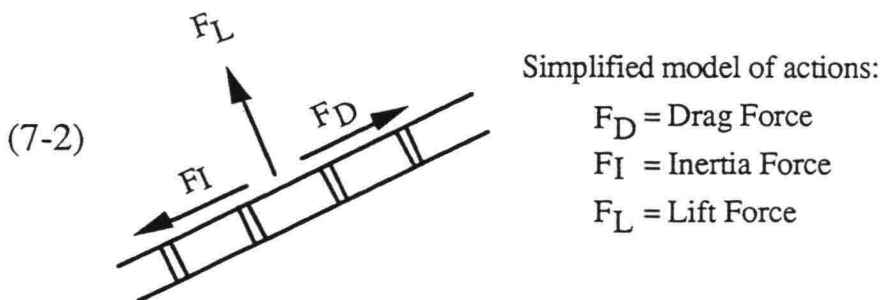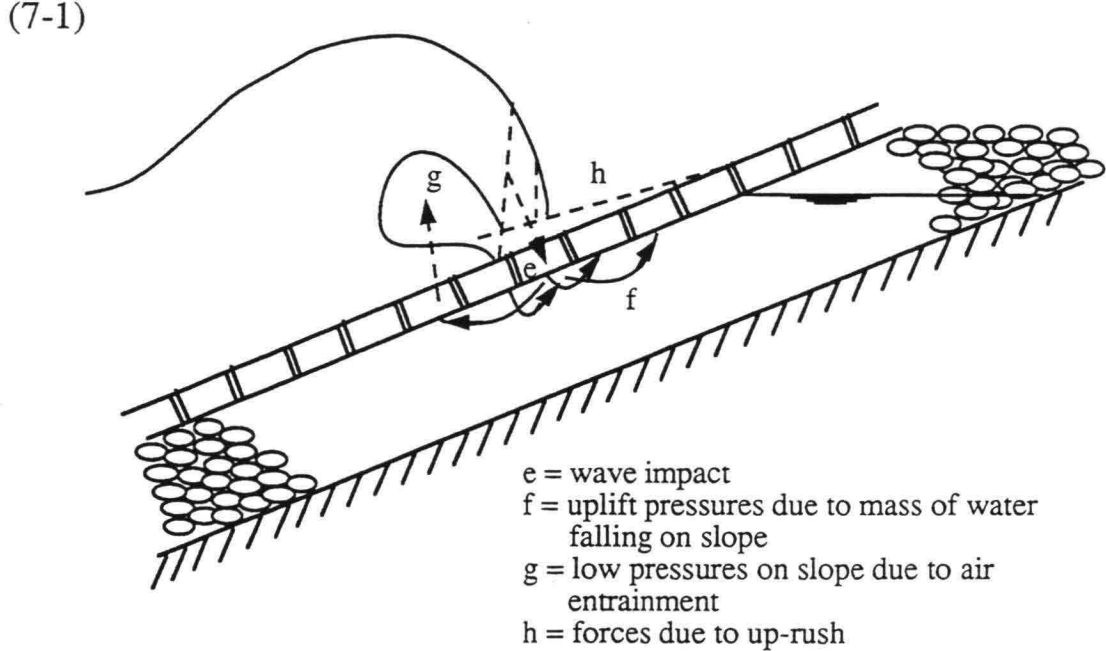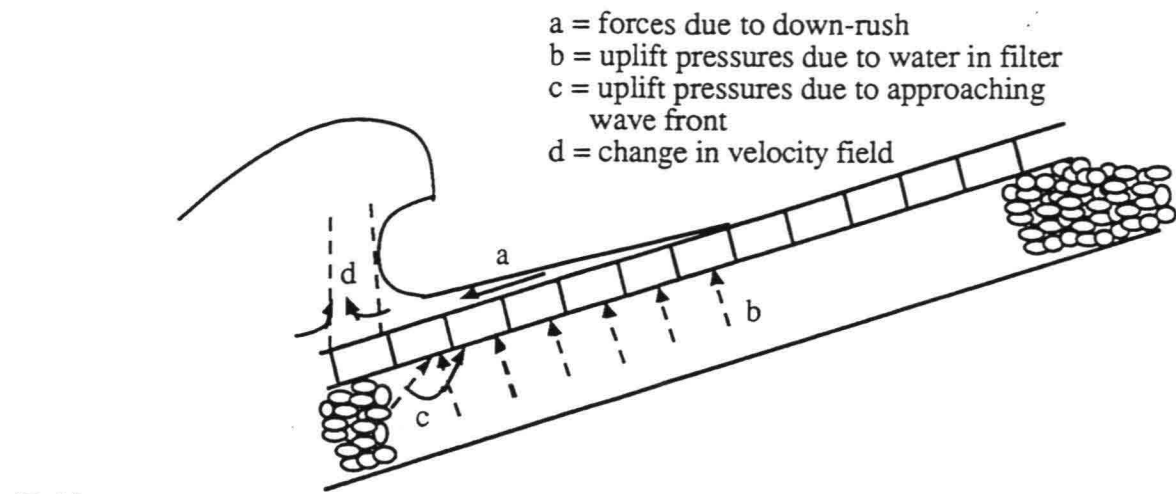F_I = Inertia Force

F_L = Lift Force

Figure 7. Wave Action Mechanism

2.0 Resisting Forces Provided By Erosion Protection

The forces referenced in Section 1.0 provide the potential for creating erosion of stream beds and slopes subject to water motion. The amount of erosion is a function of the amount of tractive force and soil type. Using the tractive forces as calculated in Section 1.0 and the resistance to these forces provided by various types of erosion protection it is possible to calculate the requirements for protection of stream banks, channel banks, and wave runup protection. Utilizing the tractive forces and the resisting forces provided by erosion protection, the design of all types of erosion protection is based on the forces referenced in Section 1.0, whether the protection be conventional riprap, concrete, or some type of erosion protection mat. The following sections describe the design of ARMORFORM protection mats based on the forces as referenced in Section 1.0. A brief description of the various types of ARMORFORM protection is given in Section 2.1

2.1 ARMORFORM Mat Characteristics

Table 1 shows the characteristics of standard ARMORFORM erosion protection mats.

Table 1

ARMORFORM Style	Weight Per Unit Area (lb/sq ft)	Average Thickness t (in)*	Average Manning's Roughness Coefficient, n
Uniform Section Mats:			0.015
3" USM	35	3.0	
4" USM	47	4.0	
6" USM	70	6.0	
8" USM	93	8.0	
Filter Point Mats:			0.025
5" FPM	26	2.2	
8" FPM	47	4.0	
10" FPM	70	6.0	
Articulating Block Mats:			0.045
4" ABM	41	3.5	
6" ABM	64	5.5	
8" ABM	86	7.4	

*The unit weight of the standard grout (fine aggregate concrete) has been conservatively assumed $\gamma_c = 140$ pounds/cubic foot for calculation of the average thickness.

These characteristics have been used in the calculation of resisting forces in the subsequent sections.

2.2 Resisting Forces On Channel Bottom

When an erosion mat is constructed, the resisting force given by friction between the mat and the foundation soil or rock must be high enough to compensate the active forces. A conservative estimation of the resisting force provided by ARMORFORM mats can be made using the following principles:

- The normal stress at the contact between the mat and soil is due to the submerged weight of the mat only:

$$\sigma' = (\gamma_c - \gamma_w)\,t \qquad\qquad (15)$$

    where:

    σ' = the effective normal stress at the contact between revetment and soil (psf)

    γ_c = total unit weight of the structural grout (psf)

    t = average thickness of the mat (ft)

- The submerged unit weight $(\gamma_c - \gamma_w)$ is conservatively considered in the calculations even if weep tubes are not provided and the mat is essentially impervious.

- Although some cement is expected to pass through the fabric and provide additional shear resistance in the soil in the immediate vicinity of the ARMORFORM protection, this effect is not considered and the friction between clean surfaces is used. Accordingly, the angle of friction is considered to be the smallest of the following:

    - δ_f = the angle of friction between the ARMORFORM fabric and the filter fabric underneath (if a separate filter fabric is used),

    - δ_s = the angle of friction between the filter fabric (or the ARMORFORM mat if a separate fabric is not used) and the soil/rock,

    - ϕ' = the angle of internal friction of the soil, if granular; and

    - ϕ_e = an equivalent angle of internal friction of the soil if cohesive, given by the formula:

$$\phi_e = \tan^{-1}\left(\frac{C'}{\sigma'} + \tan\phi'\right) \qquad\qquad (16)$$

    where:

    C' is the effective cohesion intercept (psf)

The angle δ_f between the materials usually used in the ARMORFORM mat and a non-woven filter fabric has been measured in the laboratory. The results of these tests are shown in Figures 8 and 9.
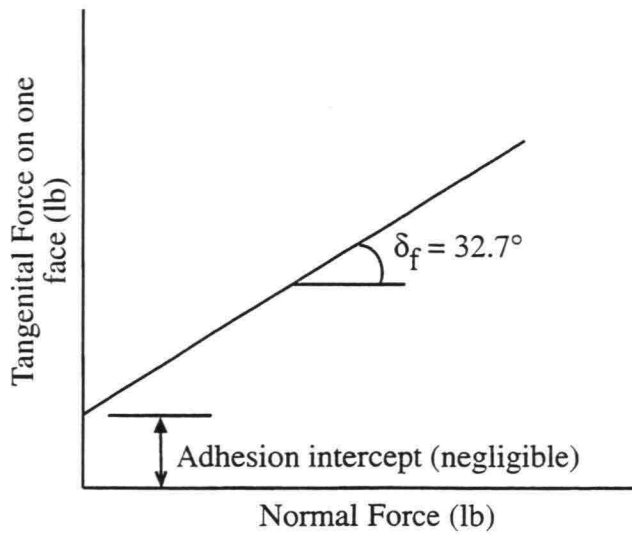
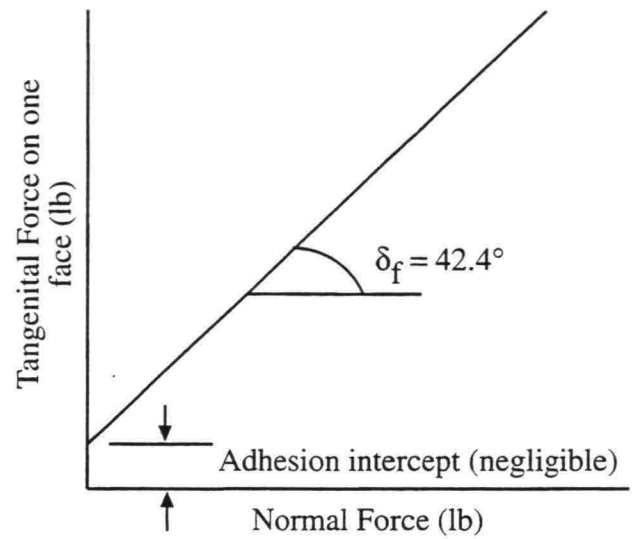Figure 8. Friction Polyester Vs. Filter Fabric
(Use With FPM And ABM)

Figure 9. Friction Polypropylene Vs. Filter Fabric
(Use With USM)

The determination of δ_s based on data in literature is given in Table 2 (based on Ref. 11 and 12).

Table 2

Soil-To-Fabric Friction Angles and Efficiencies[1]
(In Parentheses) In Cohesionless Soil

Geotextile Type	Manufacturer's Designation	Concrete Sand $\emptyset'= 30°$	Rounded Sand $\emptyset'= 28°$	Sandy Silt $\emptyset'= 26°$
Woven, Monofilament	NICOLON N.C. 70/06	25° (80%)	---	---
Nonwoven, Needled	GEOLON N-160	28° (90%)	---	---

The angle of internal friction $\emptyset'$, of a granular soil may be determined using the graph in Figure 10 (Ref. 3, Appendix C).

[1] Efficiencies are calculated as follows:
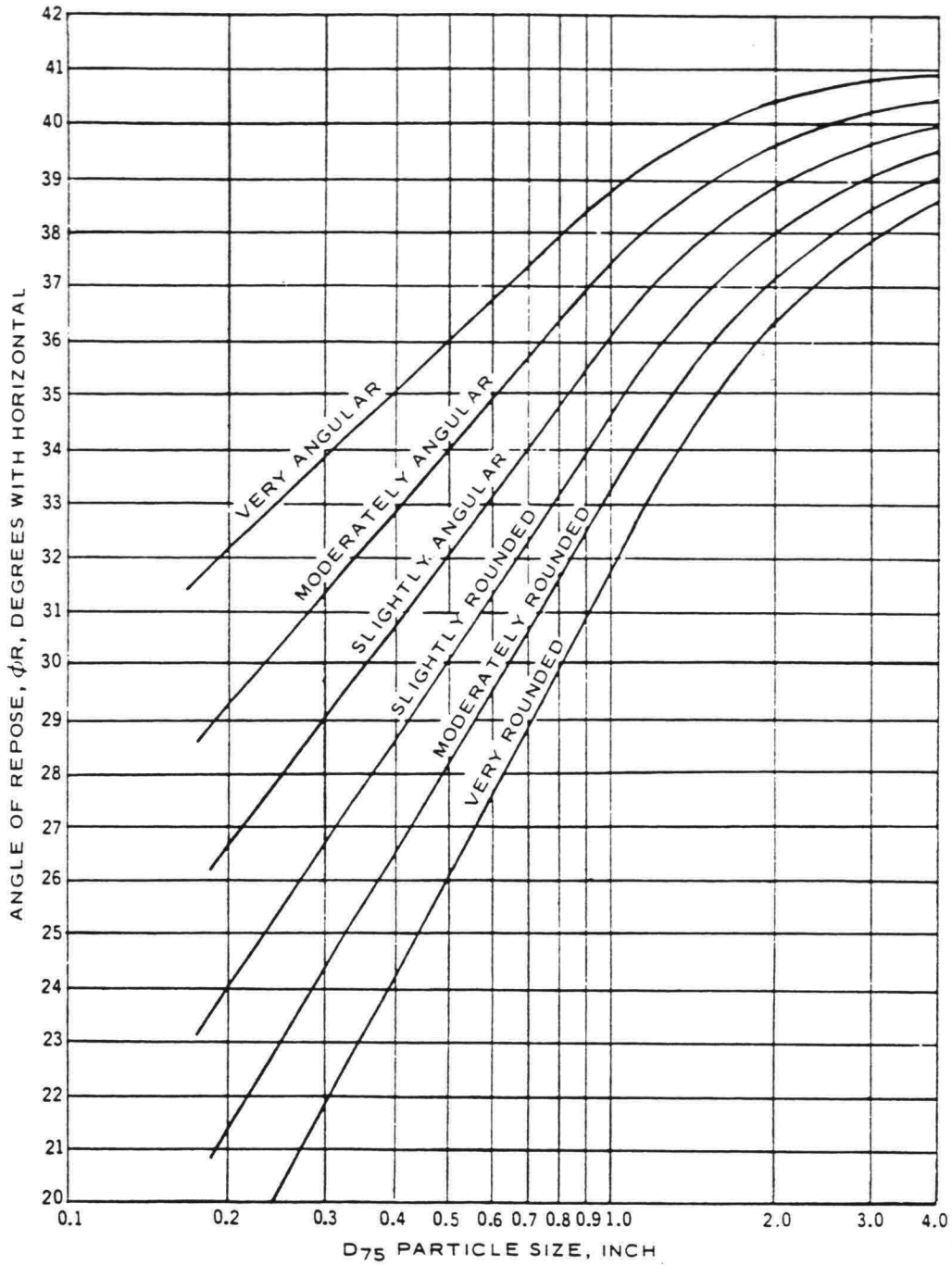$$EFF. = TAN\ \delta / TAN\ \emptyset'$$

Figure 10. Relationship Between Angle Of Repose And D_{75}
Particle Size For Cohesionless Soil

Generally, the equivalent angle of internal friction for cohesive protected soils is not critical. For the range of submerged weight per unit area for standard ARMORFORM styles (between 19.4 psf for 3 inch USM and 51.7 for 8 inch USM), even a low shear resistance cohesive soil (e.g. $\phi' = 10°$ and $C' = 0.3$ psi) ensures an equivalent angle of internal friction in excess of 45°. Therefore, the angle of friction between the filter fabric and soil is critical for cohesive soils. In the absence of a filter fabric underneath the ARMORFORM mat, the friction between the fabric form and the soil determines the minimum angle of friction, δ.

The following values are suggested for use in calculations:

Table 3

Angle Of Friction Between Mat And Soil, δ

Condition	Type of Protected Soil			
	Sand and Gravel, Coarse Grained Materials	Sand, Fine Sand, Fine Grained Cohesionless Materials	Silty Sand, Sandy Silt, Clayey Sand, Low Cohesion Materials	Silt, Clay, Cohesive Materials
ARMORFORM Mat on Filter Fabric Laying on Protected Soil	30°	25°	25°	32.5°
ARMORFORM Mat Laying Directly on Protected Soil	25°	25°	30°	45°

The maximum friction stress which can be mobilized inside the protective mat, at the interface of the mat and the protected soil, or inside the soil is obtained as shown in Figure 11 and has the value:

$$\tau_{f,b} = t \, (\gamma_c - \gamma_w) \cos \alpha \, \tan \delta \qquad (17)$$

Figure 11. Active And Resisting Forces On Channel Bottom
(Longitudinal Section Along Bottom Axis)

To obtain the resistance at the bottom of the channel, $\tau_{r,b}$, the component of the submerged weight of the protective mat in the direction of flow must be subtracted from $\tau_{f,b}$.

$$\tau_{r,b} = \tau_{f,b} - t\,(\gamma_c - \gamma_w)\,\sin\alpha$$

$$= \frac{t\,(\gamma_c - \gamma_w)}{\sqrt{1 + S_o^2}}\,(\tan\delta - S_o) \qquad (18)$$

2.3 Resisting Force On Slopes

The normal force which ensures the mobilization of friction between the mat and the protected slope becomes smaller on slopes than on the bottom. This is shown in Figure 12.

a.

$$\sin \theta = \frac{1}{\sqrt{1 + Z^2}}$$

$$\cos \theta = \frac{1}{\sqrt{1 + Z^2}}$$

b.
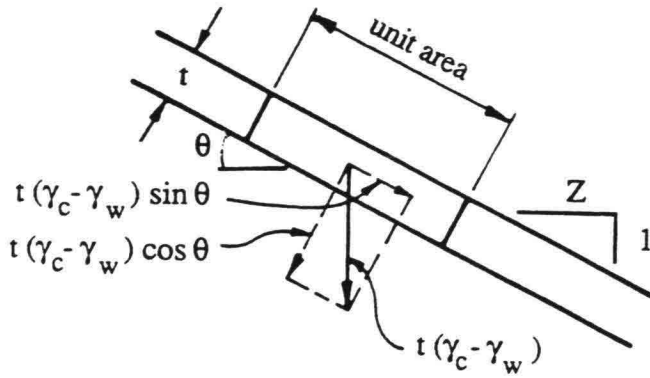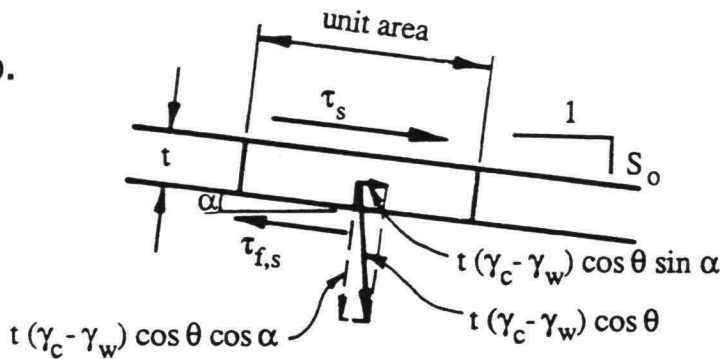
τ_s = Tractive force acting on unit area of side slopes.

$\tau_{f,s}$ = Maximum friction force per unit area.

Figure 12. Active And Resisting Force On Channel Side Slopes:
a. Cross Section; b. Longitudinal Section

The friction stress on slopes becomes:

$$\tau_{f,s} = t\ (\gamma_c - \gamma_w)\ \cos(\text{slope})\ \cos \alpha\ \tan \delta \qquad (19)$$

And, the resistance stress is:

$$\tau_{r,s} = \tau_{f,s} - t\ (\gamma_c - \gamma_w)\ \sin \alpha$$

$$= \frac{t\ (\gamma_c - \gamma_w)}{\sqrt{1 + S_o^2}} \left(\frac{Z \tan \delta}{\sqrt{1 + Z^2}} - S_o \right) \qquad (20)$$

The ratio between the resistance stress on the slope and the resistance stress on the bottom of a channel as a function of the slope is shown in Table 4.

Table 4

Ratio $\dfrac{\tau_{r,s}}{\tau_{r,b}}$ For Usual S_O, δ And Z Values

Z	$S_O = 0.01$		$S_O = 0.1$		$S_O = 0.3$	
	$\delta=25°$	$\delta=45°$	$\delta=25°$	$\delta=45°$	$\delta=25°$	$\delta=45°$
1.5	0.83	0.83	0.79	0.81	0.53	0.76
2	0.89	0.89	0.87	0.88	0.70	0.85
3	0.95	0.95	0.94	0.94	0.86	0.93
5	0.98	0.98	0.98	0.98	0.95	0.97

2.4 Additional Resisting Force Given By Anchors

If necessary, the resisting force may be increased using anchors as shown in Figure 13. The depth of anchorage must be enough for the anchor not to be pulled out when subjected to a force directed perpendicular to the anchor. The maximum depth depends on the grout used and the surrounding protected material, but need not exceed 2 feet in most cases.
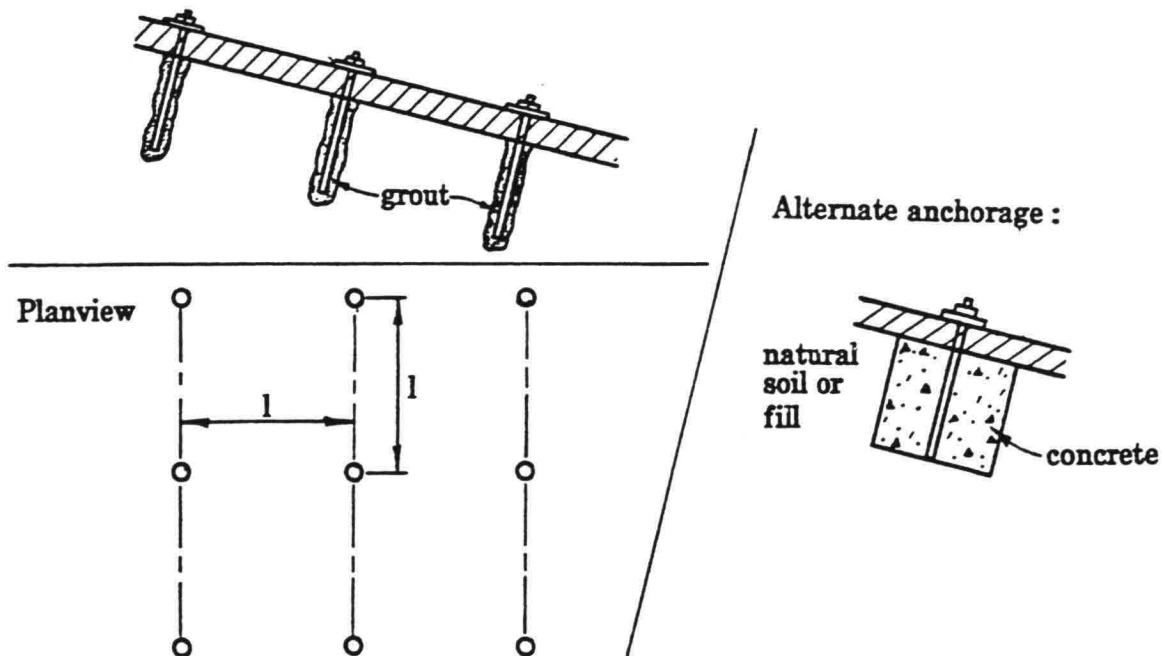
Figure 13. Mat Anchoring By Steel Bolts

The shear strength capacity of an anchor, relative to the unit area of protective mat to be anchored, is:

$$\tau_{r,a} = A_s \, \phi \, \frac{f_s}{l^2} \qquad (21)$$

where:

A_s = the area of the bolt used as anchor (sq in)

ϕ = 0.85 = capacity reduction factor for shear

f_s = 20,000 psi = allowable stress for steel (grades 40, 50)

l = distance between anchors in a square grid (ft)

At distances greater than about 20 feet, the anchors may not ensure the stability of the mat between them. Therefore, the maximum distance in design should be 20 feet.

3.0 Design Of ARMORFORM Mat Erosion Protection

3.1 Mats In Channels With Flowing Water

The factor of safety against failure of an erosion protection mat can be calculated as a ratio of the tractive force to the resisting force as calculated before. With channels it is necessary to calculate the factor of safety for both bottom and side slope protection.

The protective mat is stable if:

$$\tau_b \leq \tau_{r,b} \text{ and} \tag{22}$$

$$\tau_s \leq \tau_{r,s} \tag{23}$$

or, if anchors are used:

$$\tau_b \leq \tau_{r,b} + \tau_{r,a} \text{ and} \tag{24}$$

$$\tau_s \leq \tau_{r,s} + \tau_{r,a} \tag{25}$$

The above equations give an equilibrium situation. Some discussion of factors of safety is deemed appropriate. The factor of safety should be related to the design storm and consequence of a failure. In all cases, if a failure will cause only a maintenance problem, a factor of safety of 1.5 is recommended. Where a failure would result in damage to a dam, a factor of safety of 2.0 is recommended except for design of a 100-year and less frequent storms where 1.5 is recommended. It is conservative to consider the bottom slope (S_o) instead of the slope of energy grade line (S). Therefore, the maximum tractive stress may be defined by Equation 8 instead of Equation 2 in developing a design relationship. The equation for design is then:

$$F_s \gamma_w R S_o = \frac{t (\gamma_c - \gamma_w)}{\sqrt{1 + S_o{}^2}} (\tan \delta - S_o) \tag{26}$$

where:　F_s is the safety factor (1.5 or 2.0, as discussed above)

Equation 26 can be used in design to obtain the necessary thickness, t, of the protective mat on the channel bottom when all the other variables are known.

If anchors are used, the following equation should be used:

$$F_s \gamma_w R S_o = \frac{t (\gamma_c - \gamma_w)}{\sqrt{1 + S_o{}^2}} (\tan \delta - S_o) + A_s \phi \frac{f_s}{l^2} \tag{27}$$

It must be noted that f_s is allowable stress, so that an additional factor of safety of about 2 is applied for anchor design.

Equations 23 and 25 show the necessary equilibrium of forces on side slopes of the channel. However, usually the height of protection is greater than the maximum depth of water since a freeboard is usually included in design. Taking into account the effect of the protection above the maximum water surface, the design equations become:

$$F_s \, \tau_s = \tau_{r,s} \, (d/y) \qquad (28)$$

$$F_s \, \tau_s = \tau_{r,s} \, (d/y) + \tau_{r,a} \qquad (29)$$

where d is the maximum depth of protected channel.

Equation 20 should be used for calculations of $\tau_{r,s}$ and Equation 21 for $\tau_{r,a}$. Determine τ_s using Equations 8 and 10 through 13. Make the necessary correction for bends if necessary, using Figure 6.

Both the active and the resisting forces are smaller on slopes than on the bottom for the general case of a straight channel. From Equations 10 through 13, one can see that the tractive stress on the slope (τ_s) is at least 76% of the force on the bottom (τ_b). Table 4 shows that except in some specific cases (bottom slopes in excess of 30%, low friction angle between protection and soil, and steep side slopes) $\tau_{r,s}$ is more than 76% of $\tau_{r,b}$. Therefore, it can be stated that in most cases the protective mat designed for the channel bottom will be stable on the side slopes as well.

The factors of safety to be checked for various conditions are as follows:

- On channel bottom, without anchors:

$$F_{s,b} = \tau_{r,b}/\tau_b \qquad (30)$$

with $\tau_{r,b}$ obtained from Equation 18 and τ_b from Equations 8 or 9,

- On channel side slope, without anchors:

$$F_{s,s} = \tau_{r,s} \, (d/y)/ \, \tau_s \qquad (31)$$

where $\tau_{r,s}$ is obtained from Equation 20 and τ_s from Equations 10 through 13,

- On channel bottom when anchors are used:

$$F_{s,b} = (\tau_{r,b} + \tau_{r,a})/ \, \tau_b \qquad (32)$$

where $\tau_{r,a}$ is obtained from Equation 21 and the other parameters are as above; and

- On channel side slope when anchors are used:

$$F_{s,s} = [\, \tau_{r,s} \, (d/y) + \tau_{r,a} \,]/ \, \tau_s \qquad (33)$$

with parameters determined as above.

As shown before, the actual factor of safety is greater when anchors are used (Eqs. 32 and 33), as the calculation of resisting stress provided by anchors (Eq. 21) is based on the allowable stress of steel, which already incorporates a factor of safety of about 2.

3.2   Protection Against Wave Action
3.2.1   Active And Reacting Forces

As shown in Section 1.2, the active forces may be summarized as follows:

F_D = drag force, acting upward, parallel to the slope

F_I = inertia force, acting downward, parallel to the slope

F_L = lift force, acting upward, normal to the slope

The reaction of a part of the ARMORFORM mat, considered independent of the surrounding mat, generates the following forces:

- Submerged Weight, W_s, vertically directed
- Frictional Force, $F_R = (W_s \cos \theta - F_L) \tan \delta$, directed along the slope

Where:

θ = slope angle (degrees)

δ = frictional angle of the protection (degrees)

3.2.2   Stability Of ARMORFORM Mat

Projection of forces in the direction of the slope, and on its perpendicular direction gives the conditions for mat stability, based on the stability of an armor unit (Ref. 9).

- Stability against lifting:

$$F_L \leq W_s \cos \theta \qquad (34)$$

(friction between armor units is neglected)

- Stability against upward or downward sliding or rolling:

$$|F_D + F_I - W_s \sin \theta| \leq F_R \qquad (35)$$

U.S. Army Engineer Waterways Experiment Station has developed a formula to determine the stability of armor units on rubble structures. The stability formula, based on the results of extensive small-scale model testing and some verification by large-scale model testing is (Ref. 7):

$$W = \frac{\gamma_r H^3}{K_D (G_r - 1)^3 \cot \theta} \qquad (36)$$

where:

W = mean weight of individual armor unit (lb)

γ_r = unit weight of rock (saturated surface dry) (pcf)

H = design wave height (ft)

$G_r = \gamma_r / \gamma_w$ = specific gravity of rubble or armor stone relative to the water on which the structure is situated

γ_w = unit weight of water (fresh water = 62.43 pcf, seawater = 64.0 pcf)

K_D = stability coefficient that varies primarily with the slope of the armor units, roughness of the armor unit surface, sharpness of edges and degree of interlocking obtained in placement

θ = angle of structure slope measured from horizontal (degrees)

The suggested values of K_D (Ref. 7) vary between 1.1 for smooth-rounded quarrystone randomly placed, and 7.0 for rough angular quarrystone specially placed with the long axis of stone placed perpendicular to structure face. In usual cases, the stability coefficient, K_D, varies from 2 to 4.5 for rough angular quarrystone (K_D = 2 for 2-unit thickness of the armor layer and breaking wave, K_D = 4.5 for greater than 3-unit thickness and non-breaking wave).

Knowing the weight of the protective element, the average dimensions of the stone, d_m, is assumed to be the average value between the diameter of the sphere and the side of a cube of weight, W, and specific gravity, γ_r :

$$d_m = 0.5 \left(\sqrt[3]{\frac{6}{\pi}} + 1 \right) \sqrt[3]{\frac{W}{\gamma_r}} = 1.12 \sqrt[3]{\frac{W}{\gamma_r}} \quad (37)$$

The riprap lining thickness, t, is normally required to be not less than two stone diameters. In addition, at least 8 to 12 inches of gravel filler must be provided underneath. Therefore, the minimum thickness of the riprap is:

$$t = 2 d_m = 2.24 \sqrt[3]{\frac{W}{\gamma_r}} \quad (38)$$

Using Equation 38 together with Equation 36, the following design equation is obtained:

$$t = 2.24 \frac{H}{(G_r - 1) \sqrt[3]{K_D \cot \theta}} \quad (39)$$

By using K_D = 2 and G_r = 2.4 (corresponding to γ_r = 150 pcf), the following relationships result:

$$\left. \begin{array}{l} \text{For 2:1 slope: } t = 1.01\,H \\ \text{For 3:1 slope: } t = 0.88\,H \\ \text{For 4:1 slope: } t = 0.80\,H \end{array} \right\} \quad (40)$$

Extensive studies have been performed on flexible revetments with stones encased in steel wire mesh (Ref. 15). Based on laboratory research at the University of N.S.W. in Manly Vale, Australia, the following equations have been developed to determine the necessary thickness of this type of revetment:

- For slopes steeper than 3.5:1:

$$t = \frac{H}{3(1 - V_o)(G_r - 1)\cot\theta} \qquad (41)$$

- For slopes gentler than 3.5:1:

$$t = \frac{H}{7(1 - V_o)(G_r - 1)\sqrt[3]{\cot\theta}} \qquad (42)$$

where:

$$V_o = \text{the proportion of voids in stone fill.}$$

For common quarrystone $(1 - V_o)(G_r - 1) \cong 1$, so that

$$\left.\begin{array}{l}
\text{For 2:1 slope: } t = 0.17\,H \\
\text{For 3:1 slope: } t = 0.11\,H \\
\text{For 4:1 slope: } t = 0.09\,H
\end{array}\right\} \qquad (43)$$

Equations 43 leads to a substantial decrease of thickness as compared with riprap, due to containment of stones in a woven hexagonal steel wire netting.

Another type of slope protection for which hydraulic model tests have been made is the ARMORFLEX® mat, formed by an interlocking precast concrete grid interconnected with cables. These tests have been performed by Delft Hydraulic Laboratory (Ref. 16). The results of these tests, which involved some other revetment types also, are summarized in Table 5 (based on Ref. 8).

Table 5

Type of Slope Protection	Thickness, t, (in)	Slope	Parameter $H/G_r t$
Square Blocks, 10x10 in.	5.9	3:1	2.7 - 2.8
Hexagonal Prisms, Nonconnected	7.1	3:1	4.8
Hexagonal Prisms, Grouted	7.1	3:1	> 10.0
ARMORFLEX Mat	4.3	3:1	5.8
ARMORFLEX Mat, Grouted	4.3	3:1	≥ 8.0

Using the suggested formula (Ref. 15):

$$H/G_r t = \sqrt[3]{K_D \cot \theta} \qquad (44)$$

and using $G_r = 2.4$, the following experimental values of the ratio t/H may be compared with the corresponding values in Equations 40 and 43:

Table 6

Type of Slope Protection	t/H Ratio for Various Slopes		
	2:1	3:1	4:1
Square Blocks, 10x10 in.	0.173	0.152	0.138
Free Hexagonal Prisms	0.099	0.087	0.079
Grouted Hexagonal Prisms	0.048	0.042	0.038
ARMORFLEX Mat	0.082	0.072	0.065
Grouted ARMORFLEX Mat	0.062	0.052	0.047

In Table 7, suggested values of the t/H ratio to be used in ARMORFORM mat design are presented. They are based on the conservative assumption that ABM behavior is intermediate between free hexagonal prisms and ARMORFLEX mat behavior and USM and FPM behave as well as a grouted mat as long as the fabric form is effective and cannot have a worse behavior than square blocks 10 x 10 inches or mattresses with stones encased in steel wire mesh when the fabric becomes completely destroyed.

Table 7

Type of Slope Protection	t/H Ratio for Various Slopes		
	2:1	3:1	4:1
USM and FPM, Short-Term Application	0.060	0.052	0.047
USM and FPM, Long-Term Application	0.173	0.152	0.138
ABM	0.091	0.080	0.072

The formula to be used in a conservative design of ARMORFORM mats for wave action is:

$$t = \frac{C_w H}{(G_r - 1)\sqrt[3]{\cot \theta}} \qquad (45)$$

where:

t = $12 (W/\gamma_c)$ = average thickness of the mat (in)

W = weight per unit area of the ARMORFORM mat (lb/sq ft)

γ_c = unit weight of the grout (lb/cu ft)

C_w = coefficient with the following values for various mat types:
- for USM or FPM, short-term: $C_w = 1.3$
- for USM or FPM, long-term: $C_w = 3.7$
- for ABM, $C_w = 2.0$

H = design height of waves, see Section 1.2 (ft)

G_r = γ_c/γ_w = relative specific gravity of the grout

γ_w = specific gravity of the water acting on the protection (lb/cu ft)

θ = slope angle of the surface to be protected (degrees)

The average thickness of the ARMORFORM mat should be equal or greater than the value obtained by the above formula.

3.2.3 Use Of Anchors

Anchors may be used to provide additional resistance to uplift. The necessary equivalent weight to be provided by anchors to a unit area of protection mat may be calculated by subtracting the submerged weight per unit area of protection mat from the required weight. The required weight is:

$$W' = t (\gamma_c - \gamma_w) / 12 \qquad (46)$$

where t is given by Equation 45. It results:

$$W' = C_w H \gamma_w / \left(12 \sqrt[3]{Z}\right) \qquad (47)$$

Of this, part is provided by the mat itself:

$$W'_m = t (\gamma_c - \gamma_w) / 12 \qquad (48)$$

where t is the average thickness of the mat, in inches, as given in Table 1 for various types of mats.

The following table gives the submerged unit weight of the standard ARMORFORM mats:

Table 8

Style of Mat	Submerged Weight*, W'_m (lb/sq ft)
3" USM	19.39
4" USM	26.04
6" USM	38.78
8" USM	51.53
5" FPM	14.41
8" FPM	26.04
10" FPM	38.78
4" ABM	22.72
6" ABM	35.46
8" ABM	48.19

*Using the unit weight of the grout of 140 lb/cu ft.

The force to be provided by anchors for each unit area of the mat is the difference between the two submerged weights:

$$F = W' - W'_m \qquad (49)$$

The force to be provided by anchors must be considered by its two components, parallel to the surface to be protected and perpendicular to this surface:

$$F_{\parallel} = F \sin \theta \qquad (50)$$

$$F_{\perp} = F \cos \theta \qquad (51)$$

where θ is the angle of the protective surface with the horizontal.

The resistance to be provided by the anchor must be checked for the forces both perpendicular and parallel to the slope.

In order to determine the necessary resistance of the anchor parallel to the slope, the allowable shear of the anchors must be calculated. The resistance parallel to the slope is equal to the shear strength of the anchors. The resistance per square foot of mat is thus calculated by Equation 52:

$$F_{\parallel} \le A_s \, \emptyset \, f_s / l^2 \qquad (52)$$

where:

A_s = the cross sectional area of the anchor (sq in)

$\emptyset$ = 0.85 is the capacity reduction factor for shear

$$f_s = 20,000 \text{ psi (the allowable stress for steel grade 40, 50)}$$

l = distance between anchors in a square grid (ft). This distance should not be greater than 20 feet and should be a multiple of block dimensions for use with ABM, or of chord spacing for use with USM and FPM.

In like manner, the resistance perpendicular to the slope is provided by the pullout capacity of the anchor.

$$F_\perp \leq (P_s/12)(f_a/F_s)\, d_a/l^2 \qquad (53)$$

where:

P_s = the perimeter of the anchor (in)

f_a = adhesion steel - soil, or pullout capacity per unit area of contact between anchor and soil (lb/sq ft)

F_s = factor of safety - it is recommended that the $F_s = 1.2$ when pullout tests in the field are available and $F_s = 1.5$ when the adhesion is estimated on other bases.

d_a = necessary length of anchors (ft)

l = distance between anchors in square grid (ft)

Alternatively, the pullout capacity may be ensured by the weight of soil adhering to the anchor. A simplified analysis takes into account the component of the weight of a cone in soil with a 60° apex in the anchor direction.

$$F_\perp \leq 0.35\, d^3\, \gamma'\, \cos\theta / F_s \qquad (54)$$

where:

γ' = submerged weight of soil (lb/cu ft)

θ = slope angle (degrees)

Except for particular cases, with very close anchors to each other and/or weak soil, Equation 53 governs the pullout capacity.

3.3 Design Charts

It is possible using the tractive forces and resisting forces to derive design charts for various types of ARMORFORM mats and various slopes and velocities of water flow. The design charts for the various conditions are included in the "ARMORFORM Design Manual".

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