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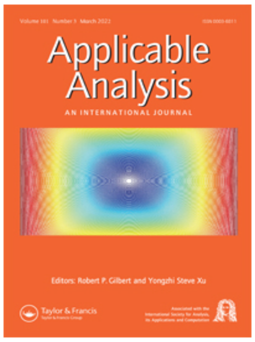

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A hyperbolic-type azimuthal velocity model for equatorial currents

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ABSTRACT
We discuss a mathematical model for the equatorial current across the Pacific Ocean, obtained as a leading-order solution to the Navier-Stokes governing equations for geophysical flows in a rotating frame.

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1. Introduction

Equatorial currents are among the most significant currents in the oceans, being an essential part of the flow. Classical theories ignore their effect, but in recent years their significance was recognized and various theoretical investigations were devoted to this important aspect of geophysical flows.

The main features of the equatorial current are that the current is mainly azimuthal and presents a strong depth-variation in the upper 500 m of the ocean (of about 4 km mean depth), while at depths larger than 500 m there is practically no motion. The most significant features of the depth-variation consist of a westward near-surface current (induced by the trade winds that blow towards the west) and a stronger eastward flow – the Equatorial Undercurrent (EUC) that is a major feature of equatorial circulation. It occurs in Pacific (Cromwell Current), Atlantic (Lomonosov Current) and Indian oceans, but only in the first two of them has a permanent character. In the equatorial Indian Ocean region it is temporary and appears in its western part in spring towards the end of the northeast monsoon season (see [1]).

A lot of attention is paid to study and mathematical modeling of the ocean flows in the equatorial regions and their underlying currents, using the f- and β-plain approximations and dealing with wave solutions for the governing equations in Lagrangian coordinates (see discussions in [2–8], [14–35]). In addition, a recent research (see [9]) proposed an approach to explain how such a current with flow-reversal might be induced by the action of the wind. Moreover the velocity profile derived in [10], also allowed to include the westward Equatorial Intermediate Current (EIC), found under the EUC, and which was not taken into account in [9]. The present paper is a natural continuation of [10], where

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we prove that not only polynomial profiles of the velocity components, but also the hyperbolic ones allow us to model the aforementioned flow behavior in the equatorial regions.

2. Preliminaries

Consider the Navier–Stokes equation and the equation of mass conservation in rotating spherical coordinates (see discussion in [9])

$$\begin{aligned}
&\left(\frac{\partial}{\partial t'}+\frac{u'}{r'\cos\theta}\frac{\partial}{\partial\varphi}+\frac{v'}{r'}\frac{\partial}{\partial\theta}+w'\frac{\partial}{\partial r'}\right)(u',v',w')\\
&\quad+\frac{1}{r'}\left(-u'v'\tan\theta+u'w',u'^2\tan\theta+v'w',-u'^2-v'^2\right)\\
&\quad+2\Omega'(-v'\sin\theta+w'\cos\theta,u'\sin\theta,-u'\cos\theta)\\
&\quad+r'\Omega'^2(0,\sin\theta\cos\theta,-\cos^2\theta)\\
&=-\frac{1}{\rho'}\left(\frac{1}{r'\cos\theta}\frac{\partial p'}{\partial\varphi},\frac{1}{r'}\frac{\partial p'}{\partial\theta},\frac{\partial p'}{\partial r'}\right)+(0,0,-g')\\
&\quad+\nu_1'\left(\frac{\partial^2}{\partial r'^2}+\frac{2}{r'}\frac{\partial}{\partial r'}\right)(u',v',w')\\
&\quad+\frac{\nu_2'}{r'^2}\left(\frac{1}{\cos^2\theta}\frac{\partial^2}{\partial\varphi^2}-\tan\theta\frac{\partial}{\partial\theta}+\frac{\partial^2}{\partial\theta^2}\right)(u',v',w'),
\end{aligned}\tag{1}$$

and

$$\frac{1}{r'\cos\theta}\frac{\partial u'}{\partial\varphi}+\frac{1}{r'\cos\theta}\frac{\partial}{\partial\theta}(v'\cos\theta)+\frac{1}{r'^2}\frac{\partial}{\partial r'}(r'^2w')=0,\tag{2}$$

respectively, where

- (φ,θ,r') are the near the Equator spherical coordinates with r' is the radius;
- $\theta\in(-\pi/2,\pi/2)$ is the angle of latitude and $\varphi\in[0,2\pi)$ is the angle of longitude;
- $(\mathbf{e}_\varphi,\mathbf{e}_\theta,\mathbf{e}_r)$ is the system of unit vectors, with $\mathbf{e}_\varphi$ pointing from West to East, $\mathbf{e}_\theta$ from South to North and $\mathbf{e}_r$ upwards;
- (u',v',w') are the ocean flow velocity components;
- $p'(\varphi,\theta,r',t')$ is the pressure in the fluid;
- ρ' the constant density;
- $\Omega'\approx 7.29\times10^{-5}$ rad s^{-1} is the constant rate of rotation of the Earth;
- $g'=$ constant ≈ 9.81 m s^{-2} is the gravitational acceleration.

Here the notations with primes mean the physical (dimensional) variables. These will be removed after the non-dimensionalization of Equations (1) and (2).

In the Navier–Stokes equation (1) the coefficients of the viscous terms are taken to be constant, where ν_1' represents the vertical and ν_2' the horizontal kinematic eddy viscosities.

At the ocean's surface the dynamic and kinematic boundary constraints and the wind-stress are expressed by

$$p'=P_s'(\varphi,\theta,t')\quad\text{on}\quad r'=R'+h'(\varphi,\theta,t'),\tag{3}$$

$$w'=\frac{\partial h'}{\partial t'}+\frac{u'}{r'\cos\theta}\frac{\partial h'}{\partial\varphi}+\frac{v'}{r'}\frac{\partial h'}{\partial\theta}\quad\text{on}\quad r'=R'+h'(\varphi,\theta,t'),\tag{4}$$

$$\begin{cases} \tau_1'(\varphi,\theta,t') = \rho'\nu_1'\dfrac{\partial u'}{\partial r'}, \\ \tau_2'(\varphi,\theta,t') = \rho'\nu_1'\dfrac{\partial v'}{\partial r'}, \end{cases} \qquad \text{on} \quad r' = R' + h'(\varphi,\theta,t'), \tag{5}$$

respectively, where

- $R' \approx 6378\,\text{km}$ is the radius of the spherical Earth;
- P_s' is the surface pressure;
- (τ_1', τ_2') is the surface wind stress, related to the vertical eddy viscosity by $\nu_1' = \sigma'|(\tau_1', \tau_2')|$ on the surface;
- σ' is a (dimensional) constant;
- $(\tau_1', \tau_2') = c_D\rho_{\text{air}}' U_{\text{wind}}' |U_{\text{wind}}'|$, $\rho_{\text{air}}' \approx 1.2\,\text{kg/m}^3$ is the density of air;
- $c_D \approx 0.0013$ is a (dimensionless) drag coefficient.

The boundary condition at the bottom of the ocean $r' = R' + d'(\varphi,\theta)$ for the viscous flow is given by relation

$$u' = v' = w' = 0 \quad \text{on} \quad r' = R' + d'(\varphi,\theta). \tag{6}$$

By setting $r' = R' + z'$ and

$$p' = \rho'\left(-g'r' + \tfrac{1}{2}r'^2\Omega'^2\cos^2\theta\right) + P'(\varphi,\theta,r',t'), \tag{7}$$

the problem can be non-dimensionalized via following substitutions (see the discussion in [9]):

$$z' = D'z, \quad (u',v',w') = U'(u,v,kw), \quad P' = \rho' U'^2 P, \tag{8}$$

where

- $D' \approx 200\,\text{m}$ is the average depth of the near-surface layer;
- $U' \approx 0.1\,\text{m}\,\text{s}^{-1}$ is the typical speed of mid-latitude ocean currents at the surface;
- $k < 10^{-4}$ is the scaling factor for the vertical velocity.

In terms of the the shallow-water parameter

$$\varepsilon = D'/R',$$

with a typical value of the order 10^{-5}, we rewrite the governing equation (1) and the equation of mass conservation (2) for steady flow in the non-dimensional form

$$\begin{aligned} &\left(\frac{u}{(1+\varepsilon z)\cos\theta}\frac{\partial}{\partial\varphi} + \frac{v}{1+\varepsilon z}\frac{\partial}{\partial\theta} + \frac{k}{\varepsilon}w\frac{\partial}{\partial z}\right)(u,v,kw) \\ &\quad + \frac{1}{1+\varepsilon z}\left(-uv\tan\theta + kuw, u^2\tan\theta + kvw, -u^2 - v^2\right) \\ &\quad + 2\omega(-v\sin\theta + kw\cos\theta, u\sin\theta, -u\cos\theta) \\ &= -\left(\frac{1}{(1+\varepsilon z)\cos\theta}\frac{\partial P}{\partial\varphi}, \frac{1}{1+\varepsilon z}\frac{\partial P}{\partial\theta}, \frac{1}{\varepsilon}\frac{\partial P}{\partial z}\right) \\ &\quad + \frac{1}{R_{e1}}\left(\frac{1}{\varepsilon^2}\frac{\partial^2}{\partial z^2} + \frac{2}{(1+\varepsilon z)}\frac{1}{\varepsilon}\frac{\partial}{\partial z}\right)(u,v,kw) \\ &\quad + \frac{1}{R_{e2}(1+\varepsilon z)^2}\left(\frac{1}{\cos^2\theta}\frac{\partial^2}{\partial\varphi^2} - \tan\theta\frac{\partial}{\partial\theta} + \frac{\partial^2}{\partial\theta^2}\right)(u,v,kw), \end{aligned} \tag{9}$$

and

$$\frac{1}{(1+\varepsilon z)\cos\theta}\left\{\frac{\partial u}{\partial \varphi}+\frac{\partial}{\partial \theta}(v\cos\theta)\right\}+\frac{k/\varepsilon}{(1+\varepsilon z)^2}\frac{\partial}{\partial z}\left\{(1+\varepsilon z)^2 w\right\}=0, \tag{10}$$

respectively, where $\omega = \Omega' R'/U' = \mathrm{O}(1)$ and $R_{ei} = U'R'/\nu_j'$ $(j = 1, 2)$ are the inverse Rossby number and the pair of Reynolds numbers, respectively.

In addition, for $(h', d') = D'(h, d)$, the non-dimensional boundary restrictions are

$$\begin{cases} P = \overline{P}_s(\varphi,\theta), \\ \dfrac{\partial u}{\partial z} = \tau_1(\varphi,\theta), \\ \dfrac{\partial v}{\partial z} = \tau_2(\varphi,\theta), \end{cases} \quad \text{on} \quad z = h(\varphi,\theta), \tag{11}$$

$$\frac{k}{\varepsilon}w = \frac{u}{(1+\varepsilon h)\cos\theta}\frac{\partial h}{\partial \varphi}+\frac{v}{1+\varepsilon h}\frac{\partial h}{\partial \theta} \quad \text{on} \quad z = h(\varphi,\theta), \tag{12}$$

$$(u, v) \quad \text{decays rapidly below } z = h(\varphi,\theta). \tag{13}$$

In [9] Constantin, Johnson show that for the equatorial Pacific flows it is adequate to consider

$$\omega = \mathrm{O}(1), \quad \frac{1}{R_{e2}} = \varepsilon^2\mu \quad \text{with } \mu = \frac{\nu_2'}{\nu_1'}.$$

By multiplication of the third component in the governing equation (9) by ε, we obtain the Navier-Stokes system with Coriolis effects in the non-dimensional form:

$$\begin{aligned} &\left(\frac{u}{(1+\varepsilon z)\cos\theta}\frac{\partial}{\partial \varphi}+\frac{v}{1+\varepsilon z}\frac{\partial}{\partial \theta}+\frac{k}{\varepsilon}w\frac{\partial}{\partial z}\right)(u, v, \varepsilon k w) \\ &\quad + \frac{1}{1+\varepsilon z}\left(-uv\tan\theta + kuw, u^2\tan\theta + kvw, -\varepsilon u^2 - \varepsilon v^2\right) \\ &\quad + 2\omega(-v\sin\theta + kw\cos\theta, u\sin\theta, -\varepsilon u\cos\theta) \\ &= -\left(\frac{1}{(1+\varepsilon z)\cos\theta}\frac{\partial P}{\partial \varphi}, \frac{1}{1+\varepsilon z}\frac{\partial P}{\partial \theta}, \frac{\partial P}{\partial z}\right) \\ &\quad + \left(\frac{\partial^2}{\partial z^2}+\frac{2\varepsilon}{(1+\varepsilon z)}\frac{\partial}{\partial z}\right)(u, v, \varepsilon k w) \\ &\quad + \frac{\varepsilon^2\mu}{(1+\varepsilon z)^2}\left(\frac{1}{\cos^2\theta}\frac{\partial^2}{\partial \varphi^2} - \tan\theta\frac{\partial}{\partial \theta}+\frac{\partial^2}{\partial \theta^2}\right)(u, v, \varepsilon k w). \end{aligned} \tag{14}$$

In the non-dimensional system of Equations (14) and (10), the parameters $(\omega, \mu, k, \varepsilon)$ are held fixed.

In fact the shallow-water model of the equatorial current may be obtained by passing to the limit for $\varepsilon \to 0$ and $k/\varepsilon \to 0$ (see the discussion in [9]). This regime ignores the vertical velocity component and the flow dynamics, governed by the horizontal flow components u and v, subject to the nonlinear system

$$\left(\frac{u}{\cos\theta}\frac{\partial}{\partial \varphi}+v\frac{\partial}{\partial \theta}\right)u - uv\tan\theta - 2\omega v\sin\theta = -\frac{1}{\cos\theta}\frac{\partial P}{\partial \varphi}+\frac{\partial^2 u}{\partial z^2}, \tag{15}$$

$$\left(\frac{u}{\cos\theta}\frac{\partial}{\partial \varphi}+v\frac{\partial}{\partial \theta}\right)v + u^2\tan\theta + 2\omega u\sin\theta = -\frac{\partial P}{\partial \theta}+\frac{\partial^2 v}{\partial z^2}, \tag{16}$$

$$\frac{\partial u}{\partial \varphi}+\frac{\partial}{\partial \theta}(v\cos\theta) = 0, \tag{17}$$

which features Coriolis terms, viscous terms and the horizontal pressure gradients. On the other hand, the boundary conditions (11)–(13) are simplified to

$$\begin{cases} P = \overline{P}_s(\varphi,\theta), \\ \dfrac{\partial u}{\partial z} = \tau_1(\varphi,\theta), \\ \dfrac{\partial v}{\partial z} = \tau_2(\varphi,\theta), \end{cases} \quad \text{on} \quad z=0, \tag{18}$$

$$(u,v) \quad \text{decays rapidly below} \quad z=0. \tag{19}$$

since setting $w = 0$ and $h = 0$ makes (12) irrelevant.

3. Main results

Relation (17) ensures (see the discussion in [11]) the existence of a stream function in spherical coordinates, $\psi(\varphi,\theta,z)$, such that

$$u = -\psi_\theta, \quad v = \frac{1}{\cos\theta}\,\psi_\varphi. \tag{20}$$

Then we can exclude the pressure terms from Equations (15) and (16) to derive (see [9]) the vorticity equation

$$\left(\psi_\varphi \frac{\partial}{\partial\theta} - \psi_\theta \frac{\partial}{\partial\varphi}\right)\left(\frac{1}{\cos^2\theta}\psi_{\varphi\varphi} - \psi_\theta \tan\theta + \psi_{\theta\theta} + 2\omega\sin\theta\right)$$
$$= \cos\theta \left(\frac{1}{\cos^2\theta}\psi_{\varphi\varphi} - \psi_\theta\tan\theta + \psi_{\theta\theta}\right)_{zz}. \tag{21}$$

We restrict our attention to the equatorial zonal band located in the Pacific between between 160°E and 80°W. Its characteristic feature is the symmetry about the Equator and it corresponds to the angle of latitude θ close to 0 and the angle of longitude φ near $\varphi_0 \in (8\pi/9, 14\pi/9)$. For the twice differentiable functions $\alpha(z)$ and $\beta(z)$, we look for longitude-independent meridional velocity components under a linear dependence of the azimuthal velocity component on the longitude. As showed in [9], this leads to the stream function

$$\psi(\varphi,\theta,z) = \{\varphi\alpha(z) + \beta(z)\}\ln\left[\frac{\cos\theta}{1-\sin\theta}\right] - \omega\left\{\sin\theta + \ln\left[\frac{\cos\theta}{1-\sin\theta}\right]\right\}, \tag{22}$$

satisfying the governing equation (21), valid for $|\varphi-\varphi_0| < \hat{\varphi}$ and $|\theta| < \hat{\theta}$ with some fixed (and small) values $\hat{\varphi} > 0$ and $\hat{\theta} > 0$.

Taking into account relations (20), we obtain the horizontal velocity field and the boundary constraints given by

$$u(\varphi,\theta,z) = -\frac{\varphi\alpha(z)+\beta(z)}{\cos\theta} + \omega\frac{\sin^2\theta}{\cos\theta},$$
$$v(\varphi,\theta,z) = \frac{\alpha(z)}{\cos\theta}\ln\left[\frac{\cos\theta}{1-\sin\theta}\right], \tag{23}$$

and

$$\tau_1(\theta,\varphi) = -\frac{\varphi\alpha'(0)+\beta'(0)}{\cos\theta},$$
$$\tau_2(\theta,\varphi) = \frac{\alpha'(0)}{\cos\theta}\ln\left[\frac{\cos\theta}{1-\sin\theta}\right] \tag{24}$$

respectively.

We will interpret the solution (23) as defined in the near-surface ocean region above the thermocline $z=-T$, where we require the no-stress boundary condition

$$u_z = v_z = 0 \quad \text{on} \quad z=-T. \tag{25}$$

The fundamental characteristic of the trade winds in the equatorial Pacific is that, in each hemisphere, they are oriented towards the Equator, blowing westwards with a more stronger westward direction as the Equator ($\theta=0$) is approached. This property holds for the wind stress specified in (24) if

$$\alpha'(0) < 0 < \varphi\alpha'(0)+\beta'(0) \tag{26}$$

for all relevant values of the longitude φ, that is, for $-\hat{\varphi} < \varphi-\varphi_0 < \hat{\varphi}$. Note that this orientation of the wind, in combination with the fact that the vanishing of the meridional component of the Coriolis force, $\omega\sin\theta$, at the Equator prevents it from inducing a deflection from the wind direction (as is typical in Ekman theory at mid-latitudes), leads to a near-surface current that moving westward. This is ensured for the solution (23) if

$$\varphi\alpha(0)+\beta(0) > 0. \tag{27}$$

Moreover, if

$$\alpha(0) > 0, \tag{28}$$

then the meridional flow described by (23) is poleward near the surface. Note that the main features of the near-surface flow in the equatorial Pacific are a westward motion at the surface and a poleward meridional flow (see the discussion in [12]). One can see that a linear or quadratic dependence on the z-variable of the azimuthal velocity profile $u(\varphi,\theta,z)$ of type (23) can not accommodate the constraints (26)–(28) and (25).

In [9] Constantin, Johnson showed that at any fixed longitude $\varphi \in (\varphi_0, \frac{13}{12}\varphi_0)$ the constraints (25)–(28) are accommodated by the cubic azimuthal profile that represents the flow which feathtures the westward drift at the surface and an eastward jet along the termocline.

In addition, a recent result [see discussion in [10]] proved that it is possible to capture not only the presence of the eastward EUC but also the presence of the weaker westward EIC below the EUC, which wasn't taken into account in [9]. In particular a suitable quintic polynomial expressions for the azimuthal velocity profile

$$\alpha(z) = \frac{144}{2125}\frac{(195T^2-2264)z^5}{T^5} + \frac{36}{245}\frac{(195T^2-2264)z^4}{T^4} - \frac{1}{2}z^2 - Tz + 1,$$
$$\beta(z) = \varphi_0\left\{\frac{18}{2125}\frac{(2285T^2-24982)z^4}{T^4} + \frac{1}{4250}\frac{141180T^2-1487411}{T^3}z^3 + \left(\frac{3}{340}\frac{1260T^2-11531}{T^2}+1\right)z^2 + 2Tz\right\}$$

can accommodate this regime.

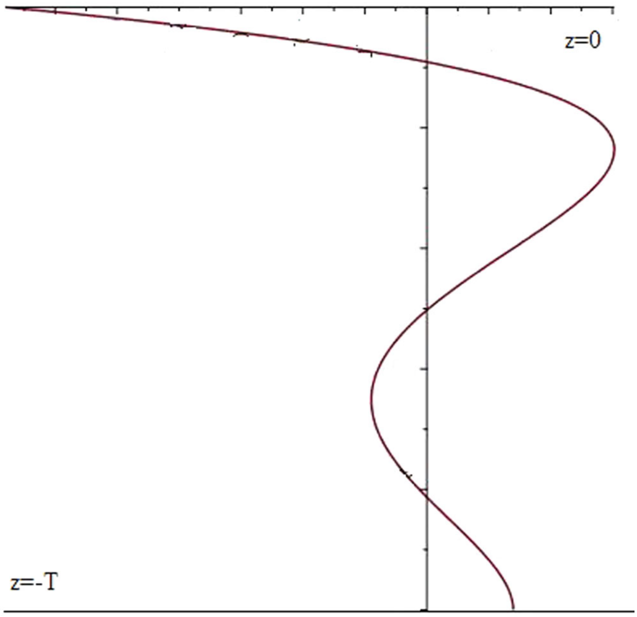

Figure 1. Depiction of the vertical profile of the current in the upper 200–400 m of the Pacific Ocean along the Equator: the westward wind-drift current is near the surface $z = 0$, below it is the eastward EUC that dominates the subsurface flows, with the weaker westward EIC found directly below the EUC (see the data provided in [13]).

Moreover the following theorem proves that also a hyperbolic azimuthal velocity profile coupled with the third-order polynomial is suitable in modeling of the aforementioned currents.

Theorem 3.1. *An equatorial current profile featuring an eastward jet at mid-depth of the near-surface layer above the thermocline $z = -T$, a weak eastward jet just above the thermocline and a strong westward jet near the surface $z = 0$ is accommodated by hyperbolic functions coupled with the third-order polynomial*

$$\alpha(z) = \left(z^3 + \frac{21T}{40}z^2 - \frac{17T^2}{40}z + \frac{T^3}{20}\right)\sinh(z+T),$$
$$\beta(z) = \varphi_0\left(\left(z^2 + Tz\right)T\sinh(z+T) - \frac{1}{100}\right).$$

Proof: It is easy to show that the azimuthal velocity component $u(\varphi, \theta, z)$ defined in the first relation of system (23), vanishes at $z = -T/10$, $z = -T/2$ and $z = -4T/5$, giving the profile depicted in Figure 1.

Simple calculations prove that

$$\alpha(0) = \frac{T^3}{20}\sinh(T) > 0,$$
$$\phi_0\alpha(0) + \beta(0) = \phi_0\left(\frac{T^3}{20}\sinh(T) - \frac{1}{100}\right) > 0$$
$$\alpha'(0) = -\frac{17T^2}{40}\sinh(T) + \frac{T^3}{20}\cosh(T) < 0,$$

$$\phi_0\alpha'(0) + \beta'(0) = \frac{T^2}{40}\phi_0(2T\cosh(T) + 23\sinh(T)) > 0,$$
$$\alpha'(-T) = 0,$$
$$\beta'(-T) = 0.$$

i.e. all the constraints (25)–(28) hold. ■

Disclosure statement

No potential conflict of interest was reported by the authors.

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