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Distributional Properties of Firms' Growth and the role of the Financial Structure

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"Distributional Properties of Firms' Growth and the role of the Financial Structure"

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Abstract

In this paper, we analyze the distributional properties of firms' growth, using an exhaustive data set of Italian firms between 1987 - 2006. In fact, we want to understand the patterns of growth of firms and the relationship between growth and different financial structures. We use the number of employees and revenues as measure of size. In line with Cabral and Mata (2003) [5], we found that the size distribution in terms of (log) number of employees is pretty stable over time, while the size distribution of the (log) revenue shifts to the right over time. We also noticed that the upper tail of the size distribution follows power law behaviour irrespective of the proxy of size used. On the other hand, our results regarding the power law behaviour in the upper tail of the size distribution shows evidence of higher inequality in firms' revenue compared to that of the number of employees. Furthermore, we noticed that, on average, the firms in our sample displays a positive evolution in terms of revenues and net worth as firms age and over time, while the (log) growth rate (in terms of revenues and net worth) and the profitability measures, ROA (Return on Assets) and ROE (Return on Equity), show a downwards trend over time. In particular, ROI (Return on Investment) appears relatively stable over time. We also observed that young firms experience higher growth rates and profitability levels in the early stages of their business life cycle with respect to older firms. Also, the Laplace benchmark for the growth rate and profitability distribution proved to be extremely robust at both aggregate and disaggregate level.

Keywords: Power-exponential distributions, Paretianity, Laplacianity, Gibrat's law

1. Introduction

In this paper, we explore some fundamental properties of firms' growth using an exhaustive data set of Italian firms. We are mainly interested in understanding the patterns of growth of firms and the relationship between growth and different financial structures. We begin by analyzing the evolution of the size distribution of firms and then the growth rate distribution. We also investigate the distribution of profitability, by considering the ROA (Return on Assets), ROE (Return on Equity) and the ROI (Return on Investment) as measures of profitability. The analysis of the size distribution of firms will be developed using the number of employees and revenues as measure of size, while the analysis for the growth rate distribution is performed in terms of employment, revenues and net worth. It is worth noting that the analysis of the firm size, growth rate and profitability distribution would be developed at firms' age level and over time. In addition, the firms' growth rate and profitability distribution will be examined at firms' revenue level.

Two main aspects of the industrial dynamics appears to have caught the attention of many researchers over the last decades, namely the statistical characterization of firm size distribution and the relation between the growth rates of firms and their size. These two aspects seems to describe the life course of business companies. The distribution of various economic quantities like individual income was first discovered by the Italian economist, Vilfred Pareto, in 1896. He observed that the distribution of individual income follows a Pareto distribution (which is a generic name for power law distribution) and his work made a huge impact on the issue of inequality in economics. Notably, most previous studies have mainly focus on the validity of the well-known "law of proportionate effects", developed by Robert Gibrat (1931), which states that the expected growth rate of firms is independent of their firm size. In this paper, we will also try to investigate whether the "law of proportionate effects" (also known as Gibrat's law) holds for our data.

Previous studies (see for example Refs. [3], [7] and [9]) on industrial data show that two main properties of the size distribution of firms are robust over time and even at firms' age level: the size distribution of firms is positively skewed and also displays heavier long tail, especially in the upper tail. In Cirillo and Hüsler (2009) [7], the power law behaviour in the upper tail of the size distribution has been verified for the Italian firms using the firms' net worth as proxy of size. We will provide similar analysis for the Italian firms using the number of employees and revenues as measure of size and also for different time-period.

In recent papers (see for example Refs. [1] and [3]), the distribution of firms' growth rates and profitability have been shown to be well described by tent-shaped distribution like the Laplace distribution. In fact, the Laplace distribution of growth rates have been shown to be an extremely robust characteristic of the industrial dynamics (see Refs. [3] and [4]). Furthermore, we will verify whether the Laplace distribution is indeed a good fit for the distribution of the firms' growth rates and profitability.

According to Coad, Segarra and Teruel, the authors of Ref. [9], young firms are more vulnerable to selection pressures with respect to older firms. In general, young firms are smaller, less profitable, less productive and generate less revenue compared to older firms. However, they experience higher growth rates and profitability levels in the early stages of their business life cycle. We will also try to understand the effect of the financial fragility on firms' growth patterns (in terms of firms' revenue and net worth).

The structure of this paper is as follow. Section 2 gives a short description of our dataset; Section 3 provides analysis of the size distribution of firms and the power law behaviour in the upper tail of the size distribution; Section 4 gives an empirical analysis of the growth rate and profitability distribution. Finally, concluding remarks are shown in Section 5, while the R-script used for the study and the descriptive statistics of the considered growth variables are reported in the Appendix

2. Data Description

In this paper, we use an unbalanced panel which is part of the larger CEBI data set. CEBI is a comprehensive database first developed by the Bank of Italy and now maintained by Cerved Group S.p.A. This database represents one of the largest Italian industrial data set and it contains firm-level observations and balance sheets of thousands of firms.

It also contains exhaustive data of companies, such as the year of foundation, number of employees, costs, revenues, net worth and many other financial variables. The subpanel contains 6047 companies per year for the observed time-period 1987 - 2006. In our study, we focus mainly on firms that during the observed period haven't undergone any kind of modification of structure, such as merging or acquisition. In other words, we restrict our study to only continuing firms over the observed period. Furthermore, we construct a balanced panel from the raw dataset by restricting our analysis to firms with the following conditions:

- At least one full-time employee.
- Cost of at least 1000 euros per year.
- Revenue of at least 1000 euros per year.
- A total assets of at least 1 each year.
- A net worth of at least 1000 euros per year.

In our study, we use the balanced panel, constructed for the period 1987 - 2006, consisting of 4596 firms each year to analyze the distributional properties of Italian firms. This choice may introduce some bias when studying disaggregation by age. We are aware of this, but we use this for the sake of simplicity. We will perform a more robust analysis in future studies.

3. Firm Size Distribution

In this section, we provide a statistical analysis of the evolution of the size distribution of Italian firms over time and at firms' age level. In most of the literature (see for example Ref. [6]), the size distribution of firms have been fitted with several distributions, such as the generalized beta families. However, our aim in this section is not to provide a plausible fit for the size distribution of Italian firms, but to simply examine the (log) firm size distribution by studying its shape and other relevant properties, while also paying attention to the behaviour of the upper tail of the empirical density. We explore the properties of the size distribution of Italian firms by considering two different proxies of firm size, namely number of employees and revenues. In the first part of this section, we analyze the (log) firm size distribution over time and then later in the section, we take a close look at the firm size distribution at firms' age level. Using Gaussian kernel estimates, we explore the shape of (log) firm size distributions at firms' age level and over time.

Table 1. Descriptive statistics (mean, std, min, max and median) for the number of employees of Italian firms in our balanced panel.

Year	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Std.
1987	1.00	27.00	53.00	150.80	113.00	27960.00	736.0499
1988	1.00	27.00	53.00	148.90	110.00	25240.00	702.8524
1989	1.00	28.00	53.00	147.10	110.00	23780.00	678.6298
1990	1.00	28.00	54.00	146.50	110.00	21600.00	653.2677
1991	1.00	29.00	55.00	145.60	112.00	21880.00	642.5221
1992	2.00	30.00	56.50	146.60	113.00	21060.00	632.9956
1993	1.00	31.00	59.50	147.90	117.20	20440.00	615.5088
1994	1.00	32.00	61.00	151.60	120.00	21410.00	627.5263
1995	1.00	33.00	62.00	155.60	124.00	22970.00	643.2986
1996	1.00	33.00	63.50	156.50	124.00	22650.00	605.6839
1997	2.00	34.00	63.00	156.50	122.00	21570.00	617.4858
1998	1.00	33.00	63.00	154.90	122.00	18890.00	597.8297
1999	1.00	33.00	63.00	155.40	122.00	18540.00	587.8263
2000	1.00	34.00	65.00	159.40	126.00	19840.00	594.0596
2001	1.00	34.00	66.00	160.60	127.00	18320.00	578.8108
2002	1.00	35.00	67.00	162.30	129.00	17880.00	571.7027
2003	1.00	35.00	68.00	175.00	132.00	53130.00	958.2419
2004	1.00	35.00	69.00	177.50	135.00	53660.00	964.0045
2005	2.00	36.00	71.00	178.00	138.00	52960.00	942.6846
2006	1.00	36.00	71.00	180.40	137.00	59460.00	1012.2700

Table 2. Descriptive statistics (mean, std, min, max and median) for the revenues of Italian firms in our balanced panel.

Year	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Std.
1987	6.00	1991.00	3486.00	10030.00	6740.00	2448000.00	56603.19
1988	4.00	2290.00	4017.00	11440.00	7656.00	2683000.00	61404.42
1989	161.00	2749.00	4711.00	13180.00	9164.00	2773000.00	64177.44
1990	30.00	3179.00	5479.00	15080.00	10680.00	3013000.00	70975.89
1991	45.00	3497.00	5958.00	16850.00	11730.00	3263000.00	81625.30
1992	284.00	3913.00	6648.00	17890.00	13080.00	2172000.00	73158.33
1993	299.00	4502.00	7520.00	19980.00	14850.00	2142000.00	75233.06
1994	233.00	5100.00	8483.00	22530.00	16750.00	2464000.00	83477.98
1995	157.00	5476.00	9154.00	24970.00	17860.00	2688000.00	95298.34
1996	5.00	5723.00	9536.00	27230.00	19110.00	2802000.00	105007.61
1997	379.00	5933.00	9888.00	28850.00	20070.00	3099000.00	109953.85
1998	522.00	6052.00	10200.00	29970.00	20740.00	2847000.00	113845.32
1999	670.00	6726.00	11390.00	32510.00	23370.00	3043000.00	116283.03
2000	907.00	7790.00	13300.00	36710.00	27370.00	3700000.00	127849.46
2001	882.00	7848.00	13510.00	39450.00	27990.00	3907000.00	145359.26
2002	436.00	7989.00	14030.00	41830.00	29280.00	4036000.00	151814.29
2003	421.00	8309.00	14840.00	47800.00	30970.00	16060000.00	286190.89
2004	1142.00	8438.00	15040.00	49550.00	32130.00	16150000.00	290511.74
2005	986.00	9124.00	16350.00	53410.00	35000.00	17270000.00	310761.90
2006	670.00	9338.00	16770.00	59010.00	35800.00	22720000.00	397727.34

3.1. **Firm size distribution over time.** First, we analyze the evolution of the (log) firm size distribution using the number of employees and revenues as proxy of size over time and later the behaviour of the upper tail of the empirical density. Table 1 and 2 present the main descriptive statistics of our balanced panel for the different measures of size over the period 1987 - 2006, while Figure 1 shows the kernel density estimates of the considered firm size in four different years obtained using the Gaussian kernel and a bandwidth of 0.2. The size distribution of Italian firms shown in Figure 1 appears to be consistent with the observations made in previous studies regarding the evolution of firm size distribution. In particular, there are at least two properties that comes to light from a simple visual inspection. First, we observe that the (log) size distribution of Italian firms are positively skewed over time. Second, all the distributions for the different size variables provide evidence of heavier tails than the Gaussian distribution, especially in the right tail. Thus, the existence of large firms on the market appears to be remarkably greater than what one would expect with a Gaussian distribution. Moreover, there is evidence that the right tail of the (log) revenue distribution becomes thicker over time. The fat long tail beyond the mode of the size distribution of Italian firms seems to suggest that the right tail of the size distribution obeys a power law. Later in this subsection, we will provide analysis of the behaviour in the upper tail of the size distribution of Italian firms. We observe that the size distribution of Italian firms in all cases appears to have an unimodal structure over time.

In line with Cabral and Mata (2003) [5], the (log) employees distribution of Italian firms presented in Figure 1 seems relatively stable over our window of observation, but this is not quite the case when considering revenues as a measure of size. There is a simple explanation for this behaviour in case of employees, mainly that firms do not dramatically change their numbers of employees from one year to the next. In contrast to the evolution of the (log) employees distribution over time, the plot of the (log) revenue distribution of Italian firms appears to show a shift towards the right over time, indicating that firms in our sample increases their revenues over time. The observation regarding the evolution of the firms' revenue distribution is quite explainable, because irregardless the size of firm, they have to increase their revenues in order to survive. Also our results regarding how the size distribution (in terms of number of employees and revenues) evolves in time is confirmed by simply looking at the evolution of the mode over time. In particular, we notice that the mode increase of the revenue distribution over the 20 years period of observation is quite significant compared to the mode increase of the number of employees.

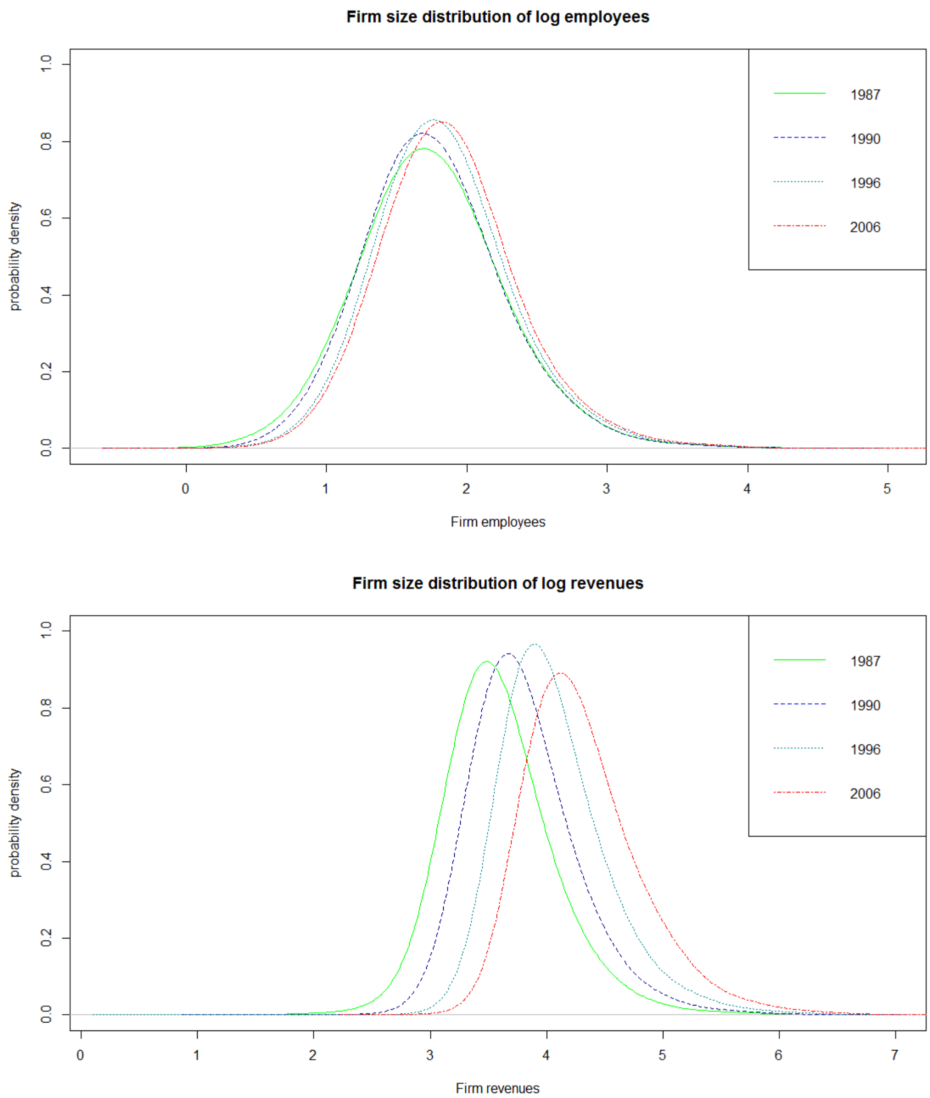


Figure 1. Kernel density estimates of (log) firm size in different years.The kernel density is estimated with a Gaussian kernel bandwidth equal to 0.2.

3.1.1. *Power law and power law exponent estimation.* We now turn our attention to investigating the behaviour of the upper tail of the size distribution of Italian firms. The main aim in this subsection is to explore whether the upper tail of the firm size distribution, which is related to the large firms and their frequency, really obeys a power law. According to Cirillo and Hüsler, the authors of Ref. [7], understanding the behaviour of the upper tail of the firm size distribution is vital to capturing the market structure and the economic behaviour for the considered entities, which in our case are the number of employees and revenues.

In general, a random variable x obeys a power law if it is drawn from the probability distribution

$$f(x) \propto Cx^{-\alpha}, \tag{1}$$

where $C, \alpha > 0$. The parameter α is known as the exponent or scaling parameter of the power law and C is a normalization constant. In most industrial studies, the scaling parameter of the power law falls in the range $2 < \alpha < 3$, although this is not always true. As stated in Aoyama et al. (2010, p. 21) [2], the scaling parameter, α, is used as a measure of inequality in the power law region. In particular, smaller value for α indicates a fatter distribution and thus the existence of larger firms, whereas larger value for α indicate the opposite.

Note that the density, $f(x)$, given in (1) diverges as $x \to 0$. To avoid this problem, there is a lower bound, $x_{min} > 0$, needed in order for the power law behaviour to hold. However, provided that $\alpha > 1$ (condition almost always satisfied in nature), the normalizing constant is easily calculated to obtain the following density function

$$f(x) = \frac{\alpha - 1}{x_{min}} \Big(\frac{x}{x_{min}}\Big)^{-\alpha} \tag{2}$$

The (complementary) cumulative distribution function of a power law is therefore defined as

$$F(x) = \int_x^\infty f(s)ds = \Big(\frac{x}{x_{min}}\Big)^{-\alpha+1} \tag{3}$$

Various techniques have been established in estimating the scaling parameter, α. Note that $\log(1 - F(x)) = C - \alpha \log x$, implying that the power law distribution is linear on a log-log plot of the survival function (also known as Zipf plot). Basically, one concludes that the right tail of the frequency distribution follows a power law with an exponent, α (which is given by the absolute slope of the straight line), if there is presence of linearity in the right tail of the log-log plot of the empirical survival function. The slope of the straight line is obtained by performing a standard OLS regression. As discussed in Cirillo and Hüsler (2009) [7] and Clauset et al. (2007) [8], this method and other variations on this method can be deceptive in claiming power law behaviour that actually do not hold up under closer examination.

In estimating the scaling parameter, α, we use the maximum likelihood method, which has proved to give accurate parameter estimates in the limit of large sample size. Assuming that the data obeys a power law for $x \geq x_{min}$, then the maximum likelihood estimate of the scaling parameter is given by

$$\hat{\alpha} = 1 + n\Big[\sum_{i=1}^{n} \log \frac{x_i}{x_{min}}\Big]^{-1} \tag{4}$$

with standard error

$$\sigma_{\hat{\alpha}} = \frac{\hat{\alpha} - 1}{\sqrt{n}} + O\Big(\frac{1}{n}\Big) \tag{5}$$

Note that x_i, $i = 1, 2, ..., n$ in (4) are the observed values of x, such that $x_i \geq x_{min}$ and n is in this case the number of observation beyond the lower bound, x_{min}.

Nonetheless, to correctly estimate α, we need an accurate method for estimating the lower bound, x_{min}, that represents the value above for which the power law behaviour better hold. The two common ways of estimating x_{min} are either plotting $\hat{\alpha}$ as a function of x_{min} and identifying the point beyond which the value appears relatively stable or by looking for the point beyond which the Zipf plot presents some sort of linearity, as described in Cirillo and Hüsler (2009) [7] and Clauset et al. (2007) [8]. Unfortunately, according to the authors of Ref. [8], these approaches can be sensitive to noise or fluctuations in the tail of a distribution and thus, affect estimating the scaling parameter correctly.

However, the authors of Ref. [8] presented two objective methods to properly estimate the lower bound, x_{min}. One of the method is based on the so-called marginal likelihood which is specific to discrete data and the other is based on minimizing the "distance" between the empirical data and the best-fit power law model above x_{min}, which works either for discrete or continuous data. In our case, we use the latter of the two methodologies proposed in Clauset et al. (2007) [8] to estimate x_{min}, since it's more preferable in most literature on this issue. To quantify the distance between the two distributions, we use the Kolmogorov-Smirnov statistic but other goodness-of-fit statistics such as the Anderson-Darling statistics (see for example Ref. [7]) are also suitable. The Kolmogorov-Smirnov statistic is simply the maximum distance between the empirical cumulative distribution of the actual data, $F_n(x)$, and the cumulative distribution of the fitted power-law model, $F(x)$:

$$D_n = \max_{x \geq x_{min}} |F_n(x) - F(x)|, \tag{6}$$

where $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{x_{min} \leq X_i \leq x}$ is the empirical cumulative distribution function of the data for the observations with value at least x_{min} and $F(x)$ the cumulative distribution function for the best-fit power law model in the region $x \geq x_{min}$.

The best estimate, $\hat{x}_{min}$, is therefore the value of x_{min} that minimizes the Kolmogorov-Smirnov statistic, D_n.

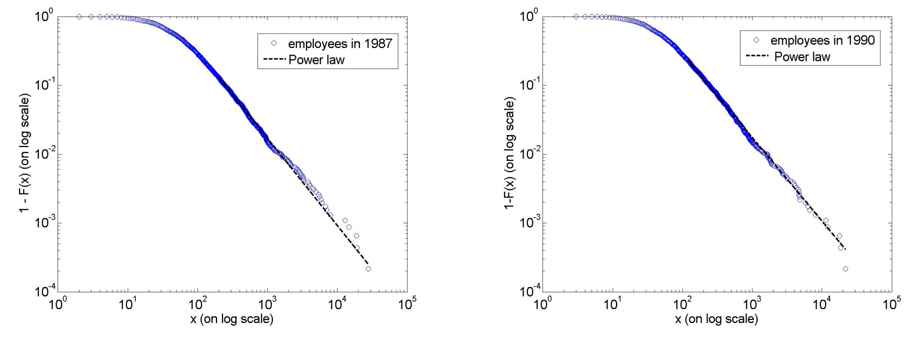


FIGURE 2. Zipf plot of firm size (in terms of number of employees) for the years 1987 and 1990. Dashed lines represent best fit power law model using the method described in the text.

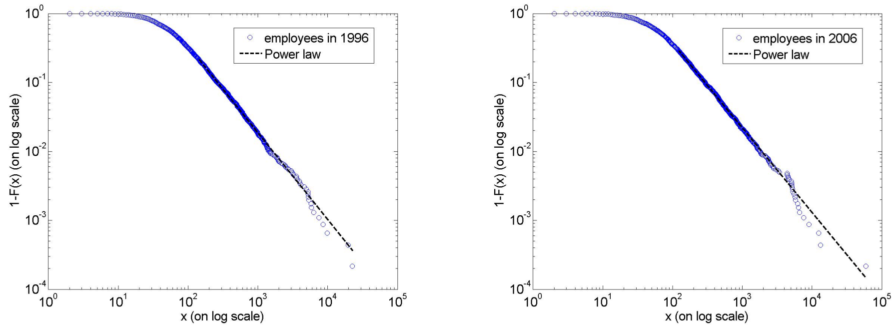


FIGURE 3. Zipf plot of firm size (in terms of number of employees) for the years 1996 and 2006. Dashed lines represent best fit power law model using the method described in the text.

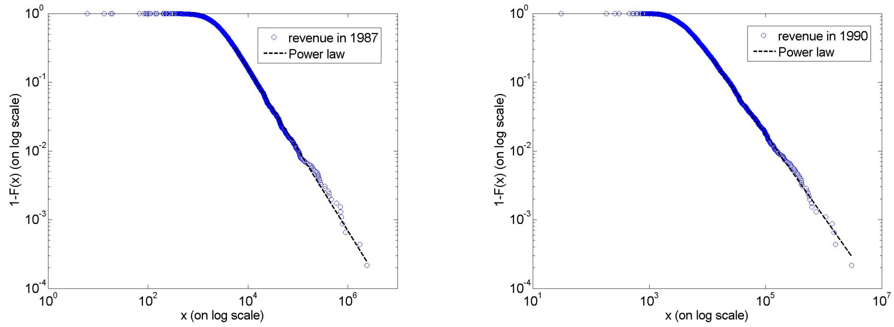


FIGURE 4. Zipf plot of firm size (in terms of revenues) for the years 1987 and 1990. Dashed lines represent best fit power law model using the method described in the text.

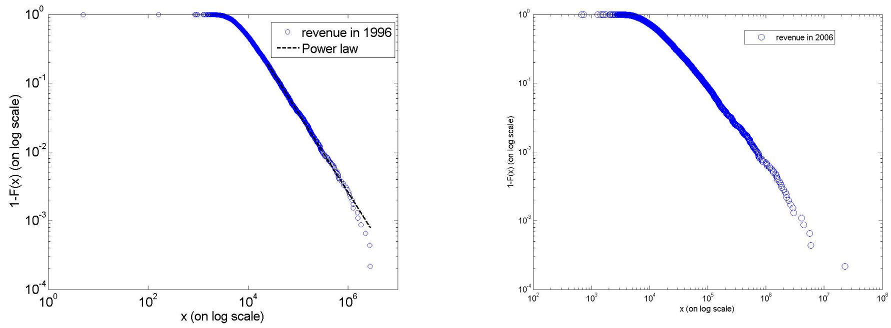


FIGURE 5. Zipf plot of firm size (in terms of revenues) for the years 1996 and 2006. Dashed lines represent best fit power law model using the method described in the text.

We first consider a graphical analysis of the data by plotting the Zipf plot and the mean excess function plot (also known as meplot) of our data to verify the plausibility of power law in the upper tail of the size distribution over time and then employ the objective method described above to estimate α and x_{min} in case of power law behaviour. Figure 2 - 5 display the Zipf plot, while Figure 6 - 7 show the mean excess function plot of the firm size in the four considered years (1987, 1990, 1996 and 2006).

Zipf plot is a simple explanatory tool usually used to investigate whether the tails of a distribution follows a power law behaviour. Furthermore, a straight line on the double logarithmic scale indicates Pareto tail behaviour. In our case, there is a clear evidence in Figure 2 - 5 that the upper tail of the size distribution of Italian firms shows some sort of linearity on the log-log plot of the empirical survival function, indicating at first sight a power law behaviour. In particular, it seems that the linearity on the log-log plot of the empirical survival function is persistent over time.

The empirical mean excess function (MEF) of the sample $X_1, X_2, ..., X_n$ is given by

$$e_n(u) = \frac{\sum_{i=1}^{n}(X_i - u)I_{X_i>u}}{\sum_{i=1}^{n} I_{X_i>u}}, \tag{7}$$

In words: the empirical mean excess function is the sum of the excess over some threshold u divided by the number of observations beyond u. According to Cirillo and Hüsler, the authors of Ref. [7], the mean excess function plot is usually used in extreme value statistics to graphically examine the presence of Generalized Pareto distribution. Notice the upward trend in the mean excess function plots shown in Figure 6 - 7, which is a sign of power law behaviour in the upper tail of the size distribution as described in Cirillo and Hüsler (2009) [7].

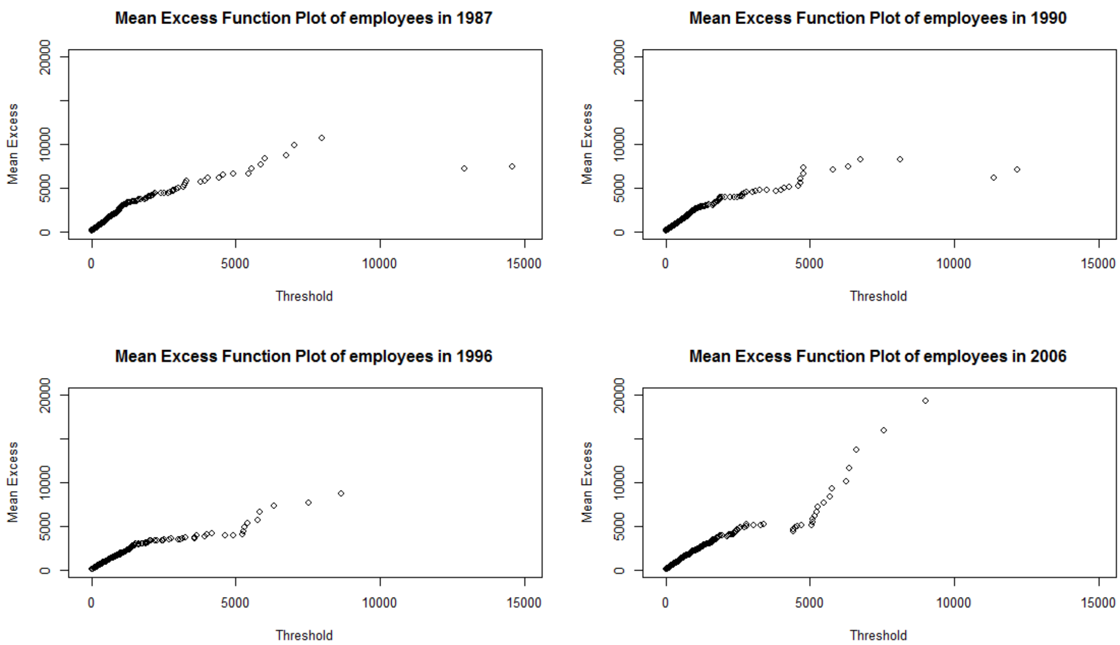


Figure 6. Mean Excess Function plot of firm size (in terms of employees) for four different years.

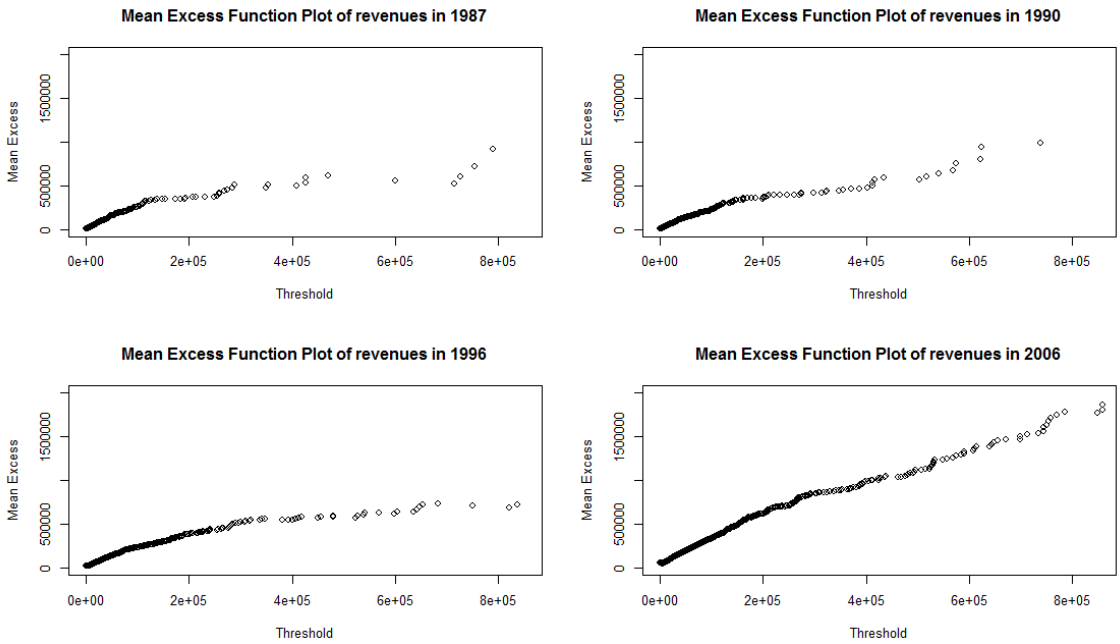


FIGURE 7. Mean Excess Function plot of firm size (in terms of revenues) for four different years.

So from a graphical point of view, one can then conclude that the power law is a plausible fit to our data in the upper tail. The second and third column of Table 3 and 4 show the maximum likelihood estimates of the power law parameters, while the last two columns of Table 3 and 4 present the number of firms for which power law holds for the considered measure of size and the percentage of the firms beyond $\hat{x}_{min}$, respectively.

First of all, note that in Table 3 the scaling parameter of the power law in case of employees yield value close to the range, $\hat{\alpha} \in [2.19, 2.30]$, while in Table 4 the scaling parameter of firms' revenue falls in the range, $\hat{\alpha} \in [2.11, 2.21]$. Figure 8 gives a graphical view of the evolution of the Power law exponent over time for the considered measure of size. Paraphrasing Aoyama et al. (2010, p. 23-24) [2], the inequality in firm size increase with growth in the early stage of economic development, while in advanced state of economic development the firm size inequality reduces. In other words, when the economic structure is changing rapidly, the firm size inequality grows, but reduces again when conditions stabilize. As shown in Figure 8, the evolution of the power law exponent seems relatively stable during the twenty-year period, independently of the proxy of size. Moreover, as observed before in the subsection "Firm size distribution over time" the (log) revenue distribution shows a shift towards the right over time, indicating that the lower bound $\hat{x}_{min}$ is more likely to increase over time as shown in the second column of Table 4. In fact, it seems that the (log) revenue distribution of Italian firms is roughly invariant to translation (as suggested in Cirillo and Hüsler (2009) [7] for the net worth of Italian firms), since firms' revenue is likely to increase over time. It's more interesting to note that, the power law exponent of firms' revenue is relatively smaller than the power law exponent in terms of the number of employees, indicating that the inequality in firm size in terms of revenues is roughly higher for the firms in our sample than the inequality in firm size in terms of employment.

Table 3. Power law fit and Kolmogorov-Smirnov statistic of goodness-of-fit for power law behaviour in the upper tail of employees distribution for every year.

Year	$\hat{x}_{min}$	$\hat{\alpha}$	D_n	P-value	$N_{x \geq \hat{x}_{min}}$	$\%_{N_{x \geq \hat{x}_{min}}}$
1987	193	2.27 ± 0.05	0.02	0.99	626	14 %
1988	228	2.30 ± 0.06	0.02	0.98	515	11 %
1989	243	2.28 ± 0.06	0.02	0.99	464	10 %
1990	115	2.22 ± 0.04	0.02	0.70	1114	24 %
1991	80	2.19 ± 0.03	0.02	0.71	1681	37 %
1992	80	2.19 ± 0.03	0.02	0.66	1711	37 %
1993	160	2.28 ± 0.05	0.02	0.96	799	17 %
1994	82	2.21 ± 0.03	0.02	0.30	1783	39 %
1995	164	2.29 ± 0.04	0.02	0.76	837	18 %
1996	140	2.26 ± 0.04	0.02	0.84	1021	22 %
1997	130	2.23 ± 0.04	0.02	0.95	1081	24 %
1998	137	2.27 ± 0.04	0.02	0.96	1028	22 %
1999	140	2.26 ± 0.04	0.02	0.97	997	22 %
2000	134	2.25 ± 0.04	0.01	0.98	1078	23 %
2001	117	2.24 ± 0.03	0.01	0.97	1285	28 %
2002	131	2.25 ± 0.04	0.02	0.94	1143	25 %
2003	143	2.27 ± 0.04	0.01	0.99	1067	23 %
2004	134	2.25 ± 0.04	0.01	1.00	1166	25 %
2005	129	2.25 ± 0.04	0.01	1.00	1247	27 %
2006	118	2.23 ± 0.03	0.01	0.97	1390	30 %

Table 4. Power law fit and Kolmogorov-Smirnov statistic of goodness-of-fit for power law behaviour in the upper tail of revenue distribution for every year.

Year	$\hat{x}_{min}$	$\hat{\alpha}$	D_n	P-value	$N_{x \geq x_{min}}$	$\%_{N_{x \geq \hat{x}_{min}}}$
1987	5261	2.18 ± 0.03	0.02	0.83	1530	33 %
1988	8835	2.20 ± 0.04	0.01	0.99	1006	22 %
1989	10283	2.21 ± 0.04	0.01	1.00	1040	23 %
1990	11178	2.20 ± 0.04	0.02	0.75	1106	24 %
1991	11261	2.19 ± 0.03	0.01	0.96	1211	26 %
1992	14542	2.21 ± 0.04	0.01	0.99	1022	22 %
1993	18676	2.21 ± 0.04	0.01	0.99	891	19 %
1994	18905	2.20 ± 0.04	0.01	0.98	1005	21 %
1995	20306	2.19 ± 0.04	0.02	0.97	1020	22 %
1996	23131	2.16 ± 0.04	0.01	1.00	937	20 %
1997	18550	2.11 ± 0.03	0.02	0.81	1242	27 %
1998	18620	2.11 ± 0.04	0.02	0.80	1291	28 %
1999	26322	2.16 ± 0.03	0.02	0.96	1018	22 %
2000	26632	2.15 ± 0.03	0.02	0.81	1180	26 %
2001	24510	2.11 ± 0.03	0.02	0.73	1310	29 %
2002	28134	2.12 ± 0.03	0.02	0.80	1208	26 %
2003	43568	2.16 ± 0.04	0.02	0.97	821	18 %
2004	47099	2.14 ± 0.04	0.02	0.91	789	17 %
2005	60990	2.16 ± 0.05	0.02	0.97	657	14 %
2006	56281	2.12 ± 0.04	0.02	0.97	750	16 %

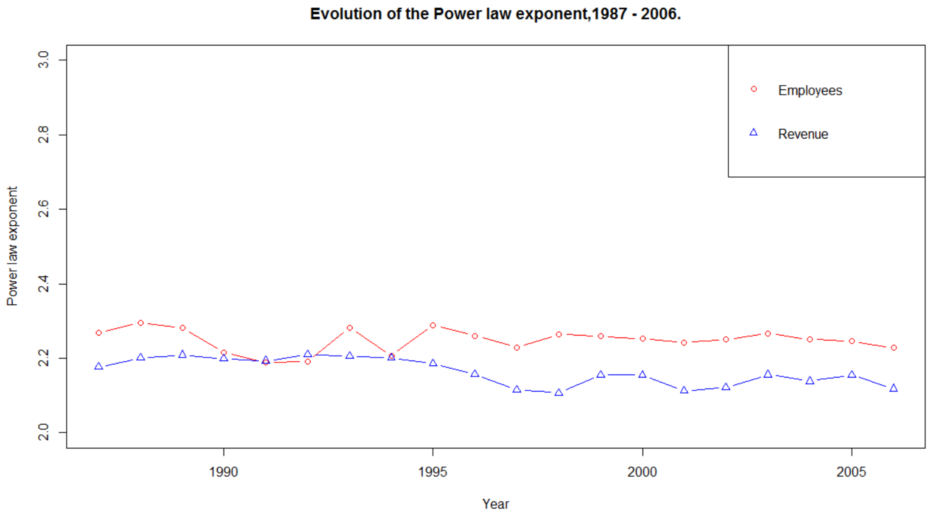


FIGURE 8. Evolution of the Power law exponent,1987 - 2006.

3.1.2. *Testing the power law hypothesis.* In this subsection, we test whether our data set in the upper tail of the size distribution are actually drawn from a hypothesized power law distribution. The basic methods described in the previous subsection allow us to fit a power-law distribution to our data in the upper tail of the size distribution and provides estimates of the parameters α and x_{min}. Even though, this methods are very useful, it's not enough to verify whether the power law is a plausible fit to the data in the upper tail. To explore if the power law is a good model for our dataset in the upper tail of the size distribution, we perform a goodness-of-fit test. For the goodness-of-fit test, we use the Kolmogorov-Smirnov test which we encountered when searching for the best lower bound $\hat{x}_{min}$, but in principle another goodness-of-fit measure can be used instead, such as the Anderson-Darling test. We would like to underline that the obtained results with Kolmogorov-Smirnov are quite reliable (since large sample size are used), although the Kolmogorov-Smirnov statistic seems to be relatively insensitive to difference between distributions at the extreme limits of the range of the quantity x. As already mentioned in the subsection "Power law and power law exponent estimation", the Kolmogorov-Smirnov measures the distance between the empirical cumulative distribution of the actual data and the cumulative distributionof the hypothesized power law model.

The null hypothesis that all the firms beyond the estimated lower bound, $\hat{x}_{min}$ (lower bound is shown in the second column of Table 3 and 4 for each year in our panel), are indeed drawn from the hypothesized power law distribution, is rejected at level α_1[1] for $\sqrt{n}D \geq K_{\alpha_1}$, where K_{α_1} is obtained from $Pr(K \leq K_{\alpha_1}) = 1 - \alpha_1$, according to the Kolmogorov distribution. In our case, since the parameters of theoretical cumulative distribution function $F(x)$ have been estimated from the sample itself, standard Monte Carlo techniques are used to determine the rejection region.

The fourth and fifth column of Table 3 and 4 present, for each year in the panel, the value of the KS-statistic and the corresponding p-value of the goodness-of-fit test, respectively. As one can see, there is a clear evidence that the null hypothesis of a power law in the upper tail is never rejected using 5% significance level, independently

[1]Note that α_1 denotes the critical value and not the scaling parameter of the power law.

of the measure of size. Therefore, one can conclude that the power law is likely to hold for the larger firms in our sample.

3.2. **Size distribution by firms' age.** Finally, we provide an empirical analysis of the size distribution of Italian firms by age. Although our balanced panel doesn't include information about the firms' age, we have knowledge about the year of foundation and the year of observation for every firm, so it's easy to derive the age of the firm. Firm's age is therefore defined as the difference between the year of foundation and the year of observation. Based on the proxies of size used, which are the number of employees and revenues, we classify the firms into the following age groups: 0 - 20 (young firms), 21 - 30 (matured firms), and 31 or more years (older firms). Table 5 presents the number of firms in each age group in five different years, while Table 6 shows the median and standard deviation in 1996 of the main descriptive variables from our balanced panel. We assume that all absolute variables increase with firm age group. Therefore, firms are generally larger when they get older and also their revenues increase and their efficiency and number of employees are higher.

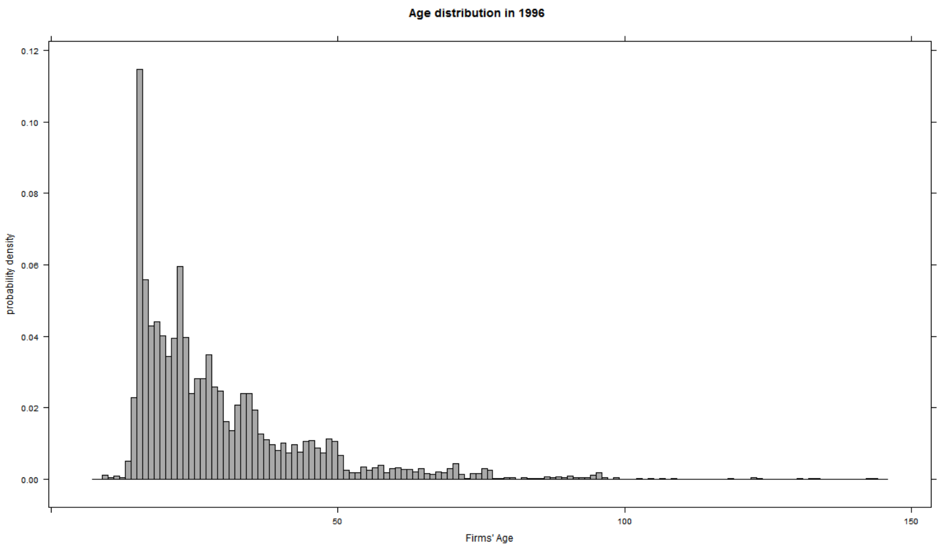
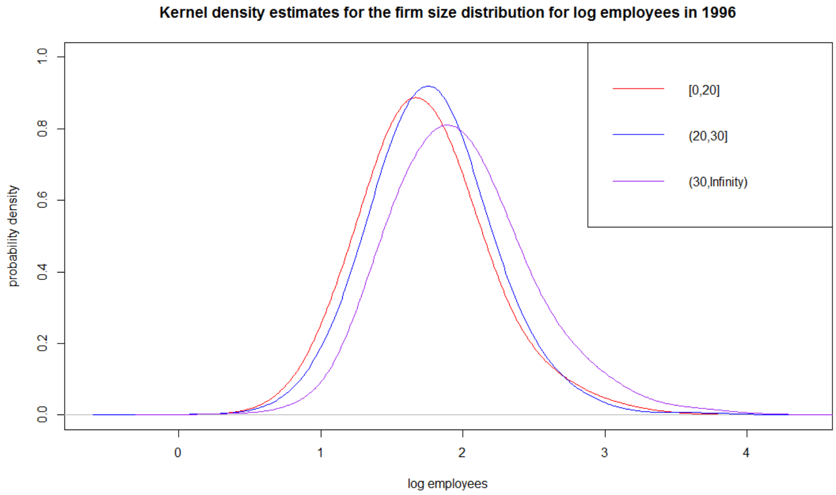


FIGURE 9. Histogram of the firms' age in 1996.

In our balanced panel, the age of firms goes from a minimum of 0 year in 1987 for 3 firms to a maximum of 154 years in 2006 for only one firm. Furthermore, Figure 9 presents the age distribution for the firms in our balanced panel in the year 1996. It appears that young firms (firms with age 0 - 20) are most numerous, indicating probably a sample selection bias due to the balanced panel. Note that the modal age in 1996 is 16 years.

TABLE 5. Number of firms by age group.

Year	Age group	Number of firms
1987	[0,20]	2950
	[21,30]	806
	(30,∞)	840
1993	[0,20]	2121
	[21,30]	1173
	(30,∞)	1302
1996	[0,20]	1508
	[21,30]	1555
	(30,∞)	1533
2001	[0,20]	141
	[21,30]	2272
	(30,∞)	2183
2006	[0,20]	5
	[21,30]	1503
	(30,∞)	3088



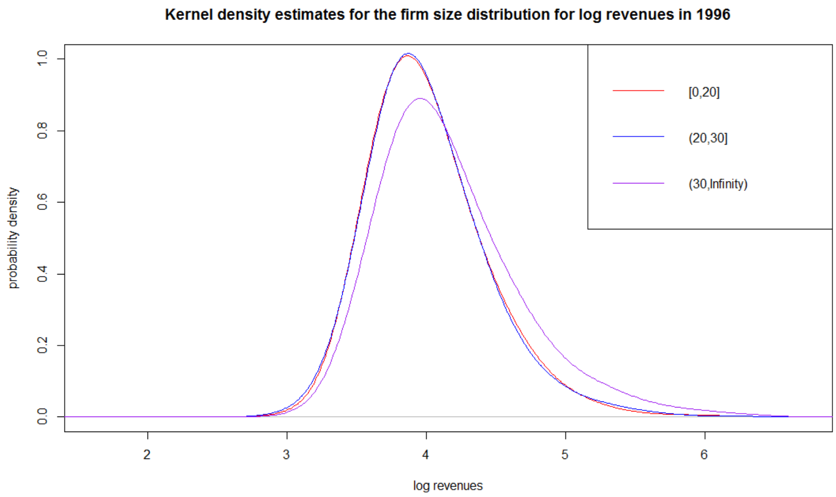
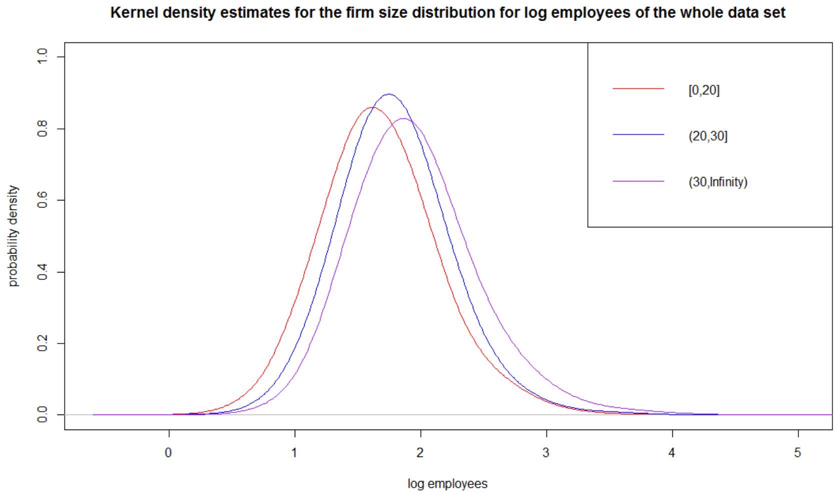


FIGURE 10. Kernel density estimates of firm size by age in 1996. The kernel density is estimated with a Gaussian kernel bandwidth equal to 0.2.

Figure 10 and Figure 11 show the Gaussian kernel density estimate of the size distribution of Italian firms by age in the year 1996 and for the whole dataset, respectively. From a graphical point of view, we observe how the size distribution of Italian firms evolves with age for the considered size variables. It's interesting to see that the size distribution of Italian firms by age is positively skewed and presents heavier tails, independently of the size variable as observed in most literature on this subject. In line with previous studies (see for example Refs. [5], [6] and [9]), it seems that the firm size distribution of the considered size variables slightly shifts towards the right and the right tail becomes thicker as firm's age increases, as shown in Figure 10. However, this behaviour is clearly visible in Figure 11 when using the whole dataset instead of only the observations in the year 1996. Furthermore, the shift in the firm size distribution by age is also verified by the descriptive statistics shown in Table 6, as we see an increase in the median over the age groups regardless of the size variable. A possible explanation regarding the shift and thickness in the right tail of the size distribution as firms' age increases is that older firms are on average larger, so they are more likely to increase their size in terms of revenues and number of employees with respect to younger firms in a sort of natural self-reinforcing process. Figure 10 - 11 also shows that the size distribution of Italian firms by age is unimodal regardless of the size variable, as observed in Cirillo (2010) [6]. However, from a visual inspection, the skewness of the size distribution by age doesn't seems to diminish as firms age, suggested in Cabral and Mata (2003) [5] and Coad, Segarra and Teruel (2012) [9]. This may be due to the quality of our sample.

Table 6. Descriptive statistics (Median and standard deviation in 1996). Standard deviation are given in the parentheses.

	[0,∞)	[0,20]	[21,30]	(30,∞)
Employees	63.5	50.5	60	87
	(605.7)	(256.7)	(661.4)	(763.2)
Revenues	9536	8869.5	8651	11435
	(105007.60)	(59580.71)	(90706.80)	(144789.51)
ROA	0.299	0.317	0.302	0.275
	(10.477)	(17.360)	(5.340)	(1.954)
ROE	0.253	0.276	0.270	0.223
	(0.867)	(1.032)	(0.592)	(0.921)
ROI	0.052	0.053	0.051	0.052
	(0.108)	(0.102)	(0.099)	(0.120)
Growth variables				
Employees	0	0	0	0
	(0.085)	(0.082)	(0.092)	(0.080)
Revenues	0.023	0.022	0.025	0.022
	(0.099)	(0.085)	(0.078)	(0.126)
Net worth	0.043	0.043	0.044	0.042
	(0.113)	(0.116)	(0.108)	(0.114)



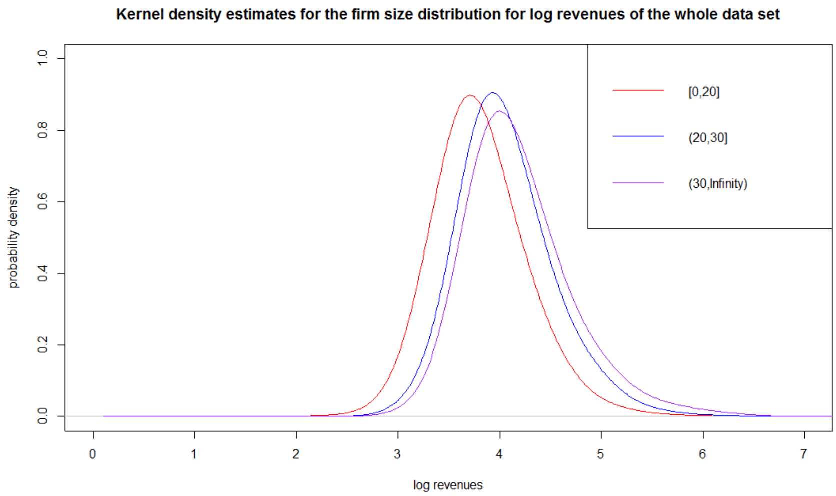


FIGURE 11. Kernel density estimates of firm size by age for the whole dataset. The kernel density is estimated with a Gaussian kernel bandwidth equal to 0.5.

4. EMPIRICAL ANALYSIS OF (LOG) GROWTH RATES AND PROFITABILITY

In this section, we explore the behaviour of the distribution of the firms' (log) growth rate. In particular, we want to investigate the evolution of the firms' (log) growth rate as firms' age, firms' revenue and over time. We will also pay attention to the firms' performance by looking at the distribution of the profitability. We will employ the same analysis as in firms' (log) growth rates to the profitability, since the profitability and the growth rates have the same functional form. Furthermore, we examine whether the empirical, Gibrat's law (i.e. that expected growth rate does not depend on the firm size) holds true for our data.

In most literature on industrial dynamics (see for example Refs. [1] and [3]), the (log) growth rates and profitability distribution seem to be characterized by tent-shaped distribution like the Laplace distribution. We analyze the (log) growth rates distribution by considering the following measure of size: the number of employees, revenues and net worth, while using the following measures of profitability, namely ROA (Return on Assets), ROE (Return on Equity) and ROI (Return on Investment). In contrast to the methodology proposed in most studies, we take a different approach by directly fitting the Laplace distribution to the firms' (log) growth rates and profitability distribution of Italian firms as it seems that the methodology applied in most studies eventually favours the Laplace distribution to other possible distributions.

A random variable x is said to follow a Laplace (μ, b) if its density function is given by

$$f(x|\mu,b) = \frac{1}{2b}\exp\Big(-\frac{|x-\mu|}{b}\Big) \tag{8}$$

where μ is the location parameter and $b > 0$ the scale parameter, which is sometimes referred to as the diversity.

Its cumulative distribution function is expressed as:

$$F(x) = \int_{-\infty}^{x} f(t|\mu,b)dt = \frac{1}{2} + \frac{1}{2}sgn(x-\mu)\Big[1 - \exp\Big(-\frac{|x-\mu|}{b}\Big)\Big] \tag{9}$$

where $sgn(x)$ is the sign function or signum function of a real-valued number x.

The structure of this section is as follow. We first analyze both the firms' (log) growth rates distribution and the profitability distribution over time and then later in this section, investigate the impact of firms' age and firms' revenue on both distributions.

4.1. **Empirical analysis of (log) growth rates and profitability over time.**

4.1.1. *Empirical analysis of (log) growth rates over time.* In this subsection, we present the empirical analysis of (log) growth rates of Italian firms over time. In particular, we investigate whether the Laplace distribution is a reasonable benchmark for the (log) growth rate distribution of Italian firms. We define the logarithmic growth rate as follow:

$$g = \log(S_{i,t}) - \log(S_{i,t-1})$$

where $S_{i,t}$ stands for the firm size (in terms of employment, revenues and net worth) of firm i at time t.

The variable, g, defined above represents the rate at which a firm increases or decreases its size (in terms of employment, revenue and net worth). We will use the maximum likelihood method to estimate the parameters of the Laplace distribution and to verify if indeed the Laplace distribution is plausible fit for both distributions (firms' (log) growth rates and profitability distribution), we use the Kolmogorov-Smirnov test as a goodness-of-fit test. Note that the maximum likelihood method and the Kolmogorov-Smirnov test will be used throughout this section to estimate the parameters of the Laplace distribution and to check whether the Laplace distribution is a good fit for our data, respectively. The maximum likelihood estimate of μ is the median of the sample; thus

$$\hat{\mu} = \begin{cases} x_{((n+1)/2)} & \text{if } n \text{ is odd;} \\ (x_{(n/2)} + x_{((n/2)+1)})/2 & \text{if } n \text{ is even.} \end{cases} \tag{10}$$

and the maximum likelihood estimate of b is given as

$$\hat{b} = \frac{1}{n}\sum_{i=1}^{n} |x_i - \hat{\mu}| \tag{11}$$

where $x_{(1)}, x_{(2)}, ..., x_{(n)}$ are ordered data and n in this case is the number of observations in our whole dataset.

TABLE 7. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution for the employment (log) growth rates.

Year	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
1987/1988	0 ± 0.0002	0.04 ± 0.0007	0.604	0.328
1988/1989	0 ± 0.0002	0.05 ± 0.0007	0.524	0.534
1989/1990	0 ± 0.0002	0.05 ± 0.0007	0.693	0.488
1990/1991	0 ± 0.0002	0.05 ± 0.0007	0.557	0.414
1991/1992	0 ± 0.0003	0.05 ± 0.0007	0.492	0.498
1992/1993	0.01 ± 0.0008	0.05 ± 0.0007	0.547	0.398
1993/1994	0.01 ± 0.0008	0.04 ± 0.0007	0.569	0.360
1994/1995	0 ± 0.0003	0.04 ± 0.0006	0.479	0.550
1995/1996	0 ± 0.0002	0.04 ± 0.0006	0.50	0.456
1996/1997	0 ± 0.0002	0.04 ± 0.0006	0.536	0.380
1997/1998	0 ± 0.0003	0.04 ± 0.0006	0.682	0.322
1998/1999	0 ± 0.0003	0.04 ± 0.0006	0.471	0.568
1999/2000	0.01 ± 0.0008	0.04 ± 0.0006	0.452	0.766
2000/2001	0 ± 0.0002	0.04 ± 0.0005	0.512	0.426
2001/2002	0 ± 0.0003	0.03 ± 0.0005	0.502	0.400
2002/2003	0 ± 0.0006	0.04 ± 0.0005	0.469	0.462
2003/2004	0 ± 0.0002	0.03 ± 0.0005	0.476	0.568
2004/2005	0 ± 0.0003	0.04 ± 0.0006	0.50	0.744
2005/2006	0 ± 0.0002	0.04 ± 0.0006	0.533	0.492

TABLE 8. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution for the (log) growth rate of firms' revenue.

Year	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
1987/1988	0.06 ± 0.001	0.07 ± 0.001	0.602	0.382
1988/1989	0.07 ± 0.001	0.07 ± 0.001	0.515	0.410
1989/1990	0.06 ± 0.001	0.06 ± 0.001	0.507	0.440
1990/1991	0.04 ± 0.001	0.06 ± 0.001	0.460	0.732
1991/1992	0.04 ± 0.001	0.05 ± 0.001	0.581	0.386
1992/1993	0.06 ± 0.001	0.06 ± 0.001	0.427	0.608
1993/1994	0.05 ± 0.001	0.05 ± 0.001	0.495	0.522
1994/1995	0.03 ± 0.001	0.05 ± 0.001	0.506	0.554
1995/1996	0.02 ± 0.001	0.06 ± 0.001	0.793	0.344
1996/1997	0.02 ± 0.001	0.05 ± 0.001	0.674	0.376
1997/1998	0.01 ± 0.001	0.06 ± 0.001	0.622	0.390
1998/1999	0.05 ± 0.001	0.06 ± 0.001	0.419	0.568
1999/2000	0.06 ± 0.001	0.06 ± 0.001	0.611	0.412
2000/2001	0.01 ± 0.001	0.06 ± 0.001	0.364	0.684
2001/2002	0.02 ± 0.001	0.05 ± 0.001	0.568	0.412
2002/2003	0.02 ± 0.001	0.05 ± 0.001	0.448	0.536
2003/2004	0.01 ± 0.001	0.05 ± 0.001	0.422	0.726
2004/2005	0.03 ± 0.001	0.05 ± 0.001	0.412	0.766
2005/2006	0.01 ± 0.001	0.05 ± 0.001	0.527	0.504

TABLE 9. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution for the (log) growth rate of firms' net worth.

Year	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
1987/1988	0.086 ± 0.002	0.11 ± 0.002	0.385	0.564
1988/1989	0.027 ± 0.001	0.07 ± 0.001	0.552	0.434
1989/1990	0.032 ± 0.001	0.06 ± 0.001	0.403	0.710
1990/1991	0.039 ± 0.001	0.06 ± 0.001	0.621	0.308
1991/1992	0.037 ± 0.001	0.06 ± 0.001	0.50	0.492
1992/1993	0.033 ± 0.001	0.06 ± 0.001	0.458	0.540
1993/1994	0.033 ± 0.001	0.06 ± 0.001	0.510	0.454
1994/1995	0.032 ± 0.001	0.05 ± 0.001	0.510	0.384
1995/1996	0.043 ± 0.001	0.06 ± 0.001	0.458	0.332
1996/1997	0.009 ± 0.001	0.06 ± 0.001	0.477	0.676
1997/1998	0.009 ± 0.001	0.06 ± 0.001	0.433	0.718
1998/1999	0.018 ± 0.001	0.06 ± 0.001	0.451	0.642
1999/2000	0.028 ± 0.001	0.07 ± 0.001	0.454	0.554
2000/2001	0.018 ± 0.001	0.06 ± 0.001	0.533	0.418
2001/2002	0.016 ± 0.001	0.05 ± 0.001	0.446	0.754
2002/2003	0.015 ± 0.001	0.06 ± 0.001	0.478	0.414
2003/2004	0.017 ± 0.001	0.06 ± 0.001	0.540	0.502
2004/2005	0.023 ± 0.001	0.07 ± 0.001	0.472	0.652
2005/2006	0.015 ± 0.001	0.06 ± 0.001	0.469	0.688

The first two columns of Table 7 - 9 contain the maximum likelihood estimates of the two parameters of the Laplace distribution for each year of our observed time-period, while the third and fourth column present the value of the Kolgomorov-Smirnov statistic and the corresponding p-value, respectively. Note, however, that all p-values in this section are computed with the non-parametric method, bootstrapping (500 replications) since the parameters of the Laplace distribution are estimated from the data.

Figure12 presents the empirical density of the (log) growth rates for the considered size variables in four different years. In line with previous studies (see for example Refs. [6] and [9]), we observe that the distribution of the (log) growth rates over time irrespective of the used proxy are characterized by a tent-shaped distribution and it also displays heavier tails. From a visual inspection and Table 7 - 9, it seems that the (log) growth rates of the considered size variables are persistent around zero, indicating that most firms in our sample have relatively stable growth rates (in terms of employment, revenues and net worth) over time while a small proportion of the firms experience rapid growth or decline. Interestingly, we observe from the second plot of Figure12 that right tail of the (log) growth rate distribution of the firms' revenue shows negative dependence of growth over time. In particular, we notice that the right tail becomes thinner over time, indicating that the number of firms to experience faster (log) growth rates in terms of revenues decreases over time. Also the heavier tail of (log) growth rate distribution is due to the involvement of relatively frequent extremal growth events as observed in Bottazzi et al. (2007) [3] and Jacoby et al. (2007) [4] for the Italian and French manufacturing firms.

Moreover, from the fourth column of Table 7 - 9, it's clear that the Kolgomorov-Smirnov does not rejects the Laplacian hypothesis at 5% significance level for the considered measure of size.

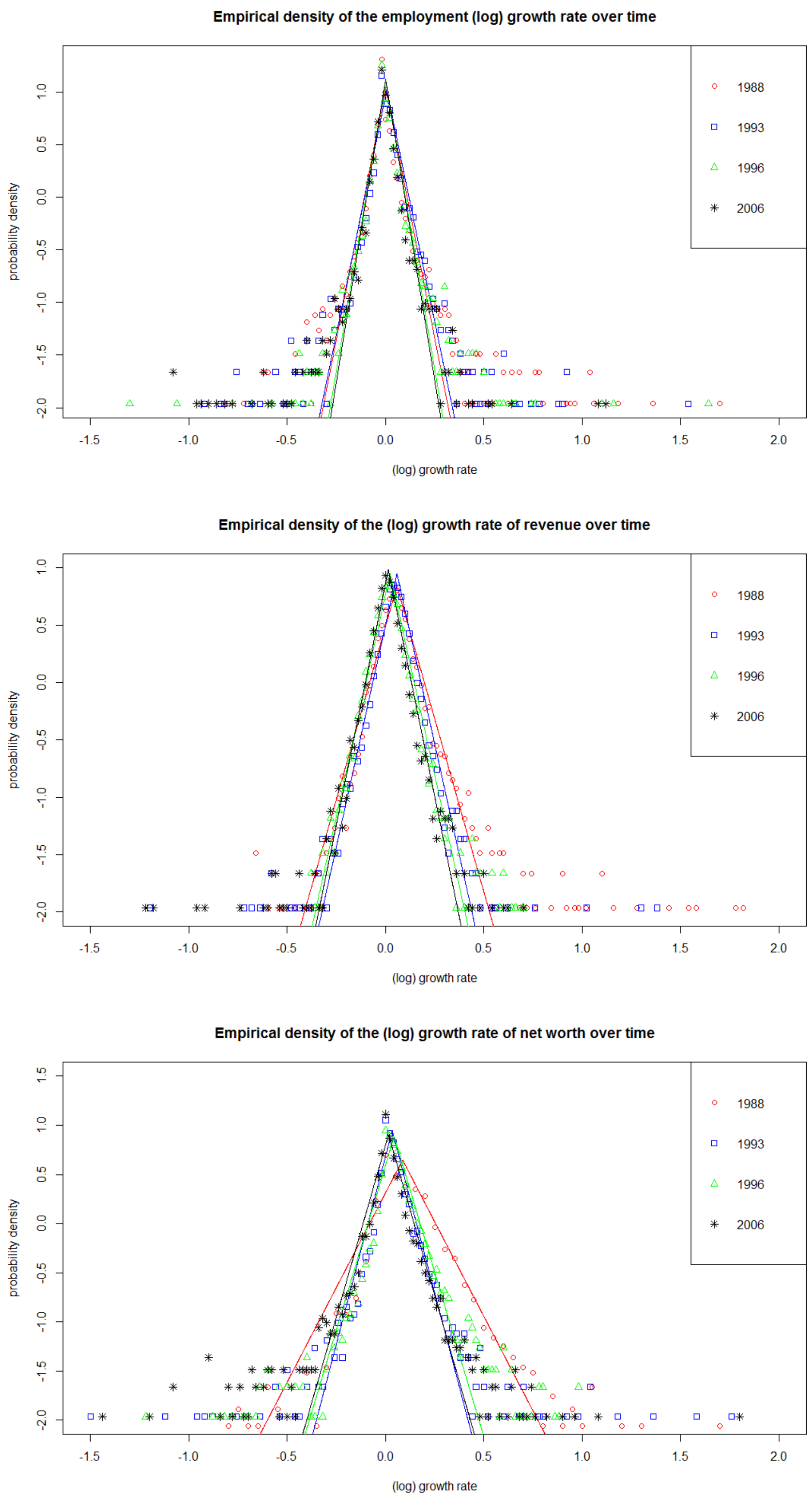


FIGURE 12. Empirical (log) growth rates densities of firm size in 4 different years. (Note the log scale on the y-axis.) Solid lines show the Laplace distribution fit for the corresponding annual (log) growth rates given in the same colour.

Therefore, one can conclude that a tent-shape distribution like the Laplace distribution is a plausible fit for the (log) growth rates distribution of Italian firms as observed in most literature on industrial dynamics.

We now take a closer look into the evolution of the the (log) growth rates over time. The median values will be used to investigate the evolution of the (log) growth rates over time rather than the mean values, since the mean values are heavily influenced by extreme growth events. We will, however, analyze the evolution of the (log) growth rates over time for the revenues and net worth, but not for the employment since most firms do not dramatically change their number of employees from 1 year to the next. Therefore, it does not make sense to investigate the employment growth rate, since the median of the employment growth rate appears to be almost zero over time (see Table 16 - 18 given in the Appendix for the descriptive statistics of the (log) growth rates over time). Figure13 gives a graphical view of the evolution of the (log) firm size and growth rates over time respectively for the firms' revenues and net worth. Based on a graphical analysis, there seems to be, on average, a positive evolution of the firms' revenue and net worth over time. This positive evolution of the median reflects on the pressure exercised by the market on the firms to increase their revenue and also their net worth and thus to be able to survive the competitive environment. From an economic theory point of view, this could also suggest the presence of endogenous growth. Interestingly, we observe from the second plot of Figure13 that the (log) growth rates of both the firms' revenues and net worth display, on average, a downwards trend over time.

4.1.2. *Empirical analysis of the profitability over time.* In this subsection, we provide an empirical analysis of the profitability distribution over time using the following measures, namely ROA (Return on Assets, defined as the ratio of the earnings before interest and taxes (EBIT) to total assets, whereby the total assets is defined as the sum of fixed and financial assets), ROE (Return on Equity, defined as the product of ROA and the ratio of total assets to total equity, where net worth is used as a proxy of equity) and ROI (Return on Investment, defined as the ratio of the earnings before interest and taxes (EBIT) to total costs). We employ the same methodology used in the preceding subsection to analyze the evolution of the profitability over time.

Firms are always looking for ways to improve their performances and thus maximizing their profits. Paraphrasing Alfarano et al. (2012) [1], it seems that firms maximize their profit by seeking increases in market share or revenues through product differentiation, price undercutting, advertising, customer relationship management, etc. At the same time, firms continuously seek to reduce costs, if possible by downsizing operations, by exploiting increasing returns to scale, or by adopting or inventing cost-cutting technologies. In that sense, the profitability seems to play a crucial role in the financial performance of a firm. The ROA (Return On assets) is an indicator of how profitable a company's assets are in generating revenue while the ROI (Return On Investment) is a performance measure used to evaluate the efficiency of an investment. The ROE (Return On Equity), however, measures the company's efficiency at generating profits from every unit of shareholders' equity. Furthermore, Figure 14 displays the evolution of the median values of the considered measure of profitability over time. It seems that both the ROA and ROE relatively decreases over time, especially in the period 1991 - 2006, while on the other hand the ROI appears to be roughly stable over time and varies around the value 0.06, as shown in Figure 14. The stability of the ROI over time seems to suggests that, on average, the revenue generated by firms in relation to the capital invested do not differ much from each other.

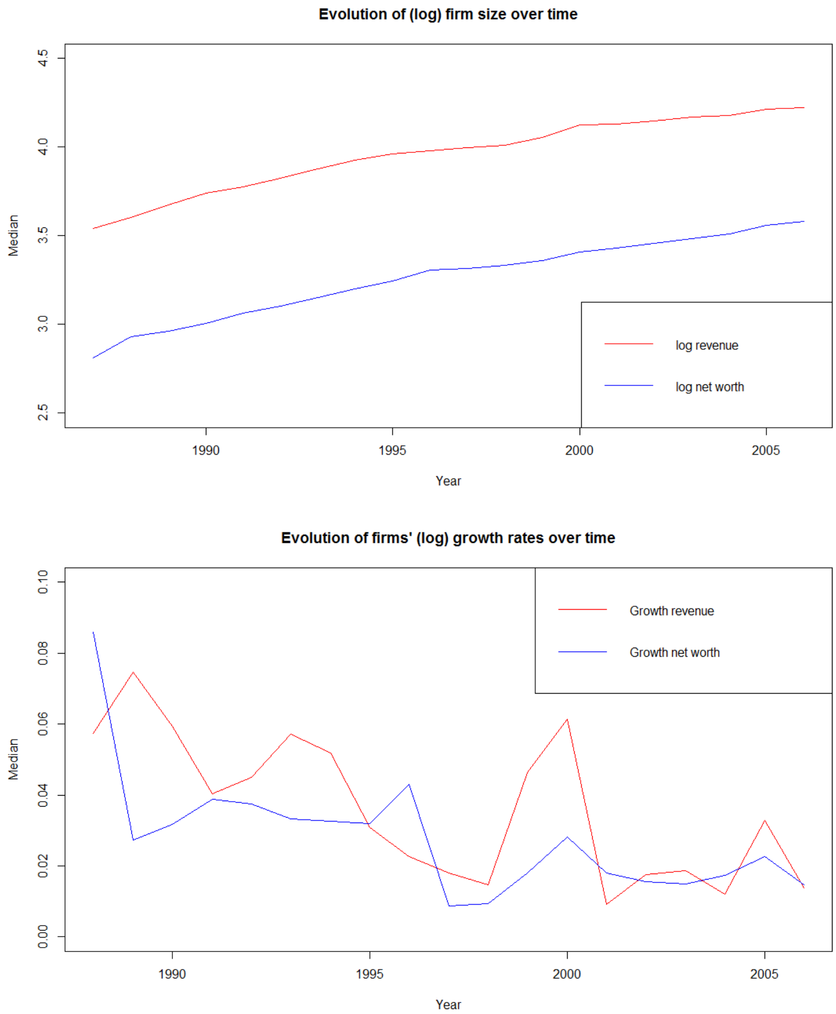


FIGURE 13. Evolution of the median values of the (log) firm size and growth rates over time. Note that the x-axis runs from 1987 to 2006 for the first plot and from 1988 to 2006 for the second plot.

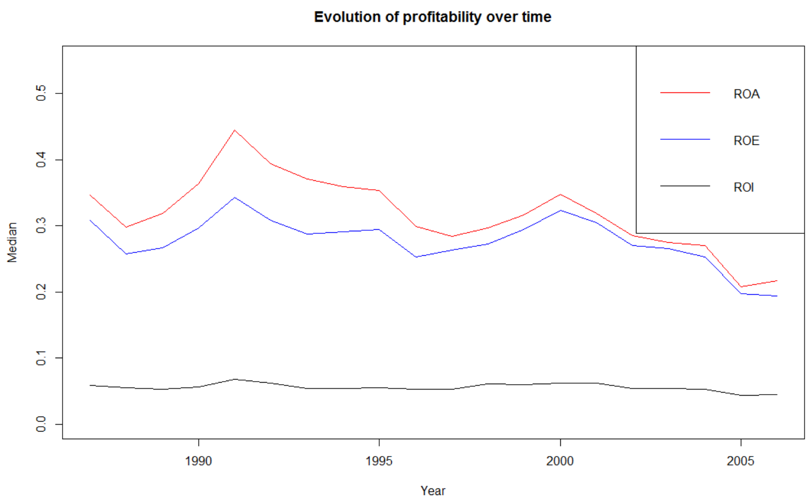


FIGURE 14. Evolution of the median values of the profitability over time. Note that x-axis runs from 1988 - 2006.

TABLE 10. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution for the ROA (Return on Assets).

Year	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
1987	0.35 ± 0.01	1.19 ± 0.02	0.60	0.53
1988	0.30 ± 0.01	0.91 ± 0.01	0.59	0.47
1989	0.32 ± 0.01	1.03 ± 0.02	0.84	0.47
1990	0.36 ± 0.01	1.10 ± 0.02	0.79	0.45
1991	0.45 ± 0.01	1.01 ± 0.01	0.75	0.41
1992	0.39 ± 0.01	0.97 ± 0.01	0.67	0.52
1993	0.37 ± 0.01	0.84 ± 0.01	0.70	0.48
1994	0.36 ± 0.01	0.89 ± 0.01	0.85	0.48
1995	0.35 ± 0.01	0.86 ± 0.01	0.56	0.58
1996	0.30 ± 0.01	0.91 ± 0.01	0.76	0.59
1997	0.28 ± 0.01	0.75 ± 0.01	0.70	0.46
1998	0.30 ± 0.01	0.73 ± 0.01	0.77	0.41
1999	0.32 ± 0.01	0.72 ± 0.01	0.72	0.46
2000	0.35 ± 0.01	0.71 ± 0.01	0.78	0.44
2001	0.32 ± 0.01	0.69 ± 0.01	0.66	0.52
2002	0.29 ± 0.01	0.61 ± 0.01	0.85	0.41
2003	0.27 ± 0.01	0.62 ± 0.01	0.79	0.36
2004	0.27 ± 0.01	0.61 ± 0.01	0.83	0.50
2005	0.21 ± 0.01	0.61 ± 0.01	0.66	0.48
2006	0.22 ± 0.01	0.74 ± 0.01	0.84	0.36

Figure 15 shows the empirical densities of the considered measures of profitability in four different years. Notice that the empirical densities of the profitability display similar properties as the (log) growth rates densities. Visual inspection shows that the distribution of the considered measures of profitability are described by a tent-shaped distribution and possesses heavier tails. It is interesting to note that the ROA distribution displays much heavier right tails than the distribution of the other considered measure of profitability and also the right tail of the ROA distribution becomes thinner over time, suggesting that a noticeable amount of firms in our sample are more likely to generate higher revenue from their assets and that this amount of firms decreases over time.
The dispersion of the profitability (particularly for the ROA and ROE) seems to be much higher than that of the considered (log) growth rates (which is apparent from a simple comparison between the estimated scale parameters of the considered (log) growth rates and the measures of profitability). In line with Erlingsson et al. (2012) [10], the average profitability ratio appears to be less volatile than the average (log) growth rate, especially when the firms' revenue is used as a proxy of size (as shown in Figure 13 and Figure 14), suggesting that the persistency in the profitability of firms are much higher than the way firms manage their assets. As pointed out in Erlingsson et al. (2012) [10], the profitability of firms is presumably the driving force of the firms' dynamics and not the way they grow or decline over time.
Fitting a Laplace distribution to the distribution of profitability, we use the maximum likelihood method to estimate the parameters of the Laplace distribution and then use the Kolmogorov-Smirnov to perform a goodness-of-fit test. Table 10 - 12 presents the results of the maximum likelihood estimation and also the summary results of

Year	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
1987	0.308 ± 0.008	0.55 ± 0.01	0.50	0.70
1988	0.258 ± 0.005	0.43 ± 0.01	0.49	0.70
1989	0.267 ± 0.007	0.48 ± 0.01	0.91	0.28
1990	0.297 ± 0.006	0.53 ± 0.01	0.85	0.44
1991	0.343 ± 0.005	0.41 ± 0.01	0.56	0.65
1992	0.309 ± 0.007	0.41 ± 0.01	0.53	0.70
1993	0.288 ± 0.006	0.38 ± 0.01	0.79	0.36
1994	0.291 ± 0.006	0.35 ± 0.01	0.66	0.35
1995	0.294 ± 0.006	0.35 ± 0.01	0.73	0.33
1996	0.253 ± 0.006	0.34 ± 0.01	0.52	0.50
1997	0.263 ± 0.006	0.37 ± 0.01	0.67	0.36
1998	0.273 ± 0.007	0.43 ± 0.01	0.51	0.55
1999	0.294 ± 0.006	0.39 ± 0.01	0.68	0.33
2000	0.324 ± 0.006	0.47 ± 0.01	0.95	0.36
2001	0.305 ± 0.006	0.44 ± 0.01	0.92	0.33
2002	0.270 ± 0.006	0.43 ± 0.01	0.69	0.48
2003	0.265 ± 0.007	0.41 ± 0.01	0.82	0.31
2004	0.253 ± 0.005	0.37 ± 0.01	0.71	0.50
2005	0.197 ± 0.006	0.36 ± 0.01	0.62	0.65
2006	0.194 ± 0.005	0.37 ± 0.01	0.77	0.34

TABLE 11. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution for the ROE (Return on Equity).

the goodness-of-fit tests that we have performed for each year of our observed time-period. In line with Alfarano et al. (2012) [1], the Laplace distribution seems to be indeed a good fit for the distribution of the considered measures of profitability, since the Kolmogorov-Smirnov test generally does not reject the Laplacian hypothesis at 5% significance level.

4.2. **Empirical analysis of (log) growth rates and profitability by age and revenue.** As shown in most previous studies on the dynamics of firms, financial constraints have quite an impact on firms' investment decision and thus, the growth of firms. In fact, paraphrasing Coad, Segarra and Teruel, the authors of Ref. [9], young firms have less financial resources and therefore presumably suffer from a higher need of external finance. Furthermore, the capital structure of a firm changes as firms' age increases. Especially, younger firms are very limited in obtaining internal equity with respect to older firms. On the other hand, established firms however gain access to resources from their own productive activity and also sources of external finance. Hence, firms tend to increases their internal equity over time and also the equity capital and internal reserves tends to play a more crucial role as firm matures. According to Coad, Segarra and Teruel, the authors of Ref. [9], the internal cash-flow increases over time, specially, among firms with age beyond 50 years. For more information about the effect of firms' age on the firms' performance, see Coad, Segarra and Teruel (2012) [9].
In this section, we analyze the impact of firms' age and revenue on both the (log) growth rate and profitability distribution. Also, we will be analyzing the evolution of the firms' (log) growth rates and profitability as firms' age. We will briefly pay attention to the implications of the financial market on the growth of firms.

Year	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
1987	0.059 ± 0.001	0.079 ± 0.001	0.72	0.38
1988	0.055 ± 0.001	0.071 ± 0.001	0.57	0.47
1989	0.053 ± 0.001	0.070 ± 0.001	0.62	0.53
1990	0.056 ± 0.001	0.069 ± 0.001	0.52	0.42
1991	0.068 ± 0.001	0.071 ± 0.001	0.48	0.42
1992	0.061 ± 0.001	0.071 ± 0.001	0.67	0.38
1993	0.054 ± 0.001	0.069 ± 0.001	0.76	0.38
1994	0.053 ± 0.001	0.065 ± 0.001	0.57	0.37
1995	0.055 ± 0.001	0.065 ± 0.001	0.71	0.30
1996	0.052 ± 0.001	0.064 ± 0.001	0.50	0.51
1997	0.053 ± 0.001	0.065 ± 0.001	0.56	0.39
1998	0.061 ± 0.001	0.070 ± 0.001	0.69	0.38
1999	0.059 ± 0.001	0.067 ± 0.001	0.58	0.42
2000	0.062 ± 0.001	0.068 ± 0.001	0.71	0.42
2001	0.062 ± 0.001	0.068 ± 0.001	0.59	0.41
2002	0.054 ± 0.001	0.066 ± 0.001	0.57	0.37
2003	0.054 ± 0.001	0.067 ± 0.001	0.70	0.32
2004	0.053 ± 0.001	0.066 ± 0.001	0.58	0.44
2005	0.044 ± 0.001	0.063 ± 0.001	0.59	0.44
2006	0.044 ± 0.001	0.064 ± 0.001	0.60	0.36

TABLE 12. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution for the ROI (Return on Investment).

4.2.1. *Empirical analysis of (log) growth rates and profitability ratios by age.* We provide in this subsection, the empirical analysis of the firms' (log) growth rates and profitability distribution of Italian firms by age. In particular, we turn to explore the relationship between (log) growth rates (or profitability) and the firms' age for the considered size variables. Figure 16 and 17 show the empirical densities of the (log) growth rates and profitability of the age groups in the year 1996, respectively. From a graphical point of view, the (log) growth rates and profitability distribution of the age groups seems to be tent-shaped with a strong peak roughly around zero, suggesting that most Italian firms in the classified age groups have relatively stable profitability level and (log) growth rates irrespective of the measure of size. Our results are, in fact, consistent with observation made in previous studies (see Refs. [1], [6] and [9]), concerning the shape of the firms' (log) growth rates and profitability distribution at the firms' age level.
In terms of employment and revenues, we observe a significant age dependency in the upper tail of the (log) growth rates distribution, while the left tail seems to be roughly independent to age in the case of employments. Our results are well in accordance with similar findings on the Spanish manufacturing firms (see Coad, Segarra and Teruel (2012) [9]). This behaviour suggests that younger firms have presumably higher chances of experiencing higher growth rates in terms of employment and revenues, while in the case of employment, age seems to be irrelevant when looking at the patterns of decline. However, when considering revenues as a measure of size, we find an interesting property regarding the left tail of the (log) growth rate distribution. In particular, we find that younger and matured firms are less likely to experience decline with respect to older firms. From a visual inspection, the firms'

age seems to play no role in the (log) growth rates of the firms' net worth, since both younger and older firm experience the same probability of growth and decline. We provide a graphical analysis of the profitability by age. We observe from the first plot of Figure 17 that the ROA distribution shifts towards the right over the age groups, indicating a positive dependence of ROA on firms' age. In other words, the ROA distribution of younger firms are relatively concentrated on lower values of the return on assets with respect to older firms. On the other hand, the second and third plot of Figure 17 seems to suggest that firms' age have negligible effect on the ROE and ROI distribution. We, however, stress that our results may be affected by sample selection bias and thus, should be interpreted with caution.

Table 13. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution for the age groups.

Variable	Age group	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
	[0,20]	0.0003 ± 0.0004	0.0427 ± 0.0011	0.491	0.402
Employment growth rates	[21,30]	0 ± 0.0004	0.0403 ± 0.0010	0.450	0.674
	[31,∞)	- 0.0002 ± 0.0004	0.0353 ± 0.0009	0.672	0.404
	[0,20]	0.0221 ± 0.0017	0.0582 ± 0.0015	0.469	0.380
Growth rates of revenue	[21,30]	0.0249 ± 0.0017	0.0542 ± 0.0014	0.418	0.476
	[31,∞)	0.0215 ± 0.0016	0.0559 ± 0.0014	0.798	0.234
	[0,20]	0.0430 ± 0.0019	0.0652 ± 0.0017	0.380	0.738
Growth rates of net worth	[21,30]	0.0436 ± 0.0015	0.0641 ± 0.0016	0.480	0.384
	[31,∞)	0.0423 ± 0.0017	0.0654 ± 0.0017	0.397	0.416
	[0,20]	0.3169 ± 0.0159	1.2915 ± 0.0333	0.877	0.286
ROA	[21,30]	0.3024 ± 0.0126	0.8057 ± 0.0204	0.916	0.342
	[31,∞)	0.2753 ± 0.0156	0.6397 ± 0.0163	0.839	0.388
	[0,20]	0.2765 ± 0.0087	0.3878 ± 0.0010	0.583	0.416
ROE	[21,30]	0.2703 ± 0.0067	0.3250 ± 0.0082	0.593	0.374
	[31,∞)	0.2229 ± 0.0061	0.3193 ± 0.0082	0.478	0.610
	[0,20]	0.0532 ± 0.0016	0.0617 ± 0.0016	0.553	0.388
ROI	[21,30]	0.0516 ± 0.0017	0.0624 ± 0.0016	0.411	0.442
	[31,∞)	0.0520 ± 0.0015	0.0688 ± 0.0018	0.493	0.418

We examine whether the Laplace distribution is a good fit for the considered firms' (log) growth rates and profitability distribution at firms' age level. In particular, we use the maximum likelihood method to estimate the parameters of the Laplace distribution and then use the Kolmogorov-Smirnov to perform a goodness-of-fit test. Table 13 presents the results of the maximum likelihood estimation and the goodness-of-fit test. It appears that the Laplace distribution is indeed a plausible fit for the conditional firms' (log) growth rates and profitability ratio distribution, because the Kolmogorov-Smirnov test does not reject the Laplacian hypothesis at 5% significance level.

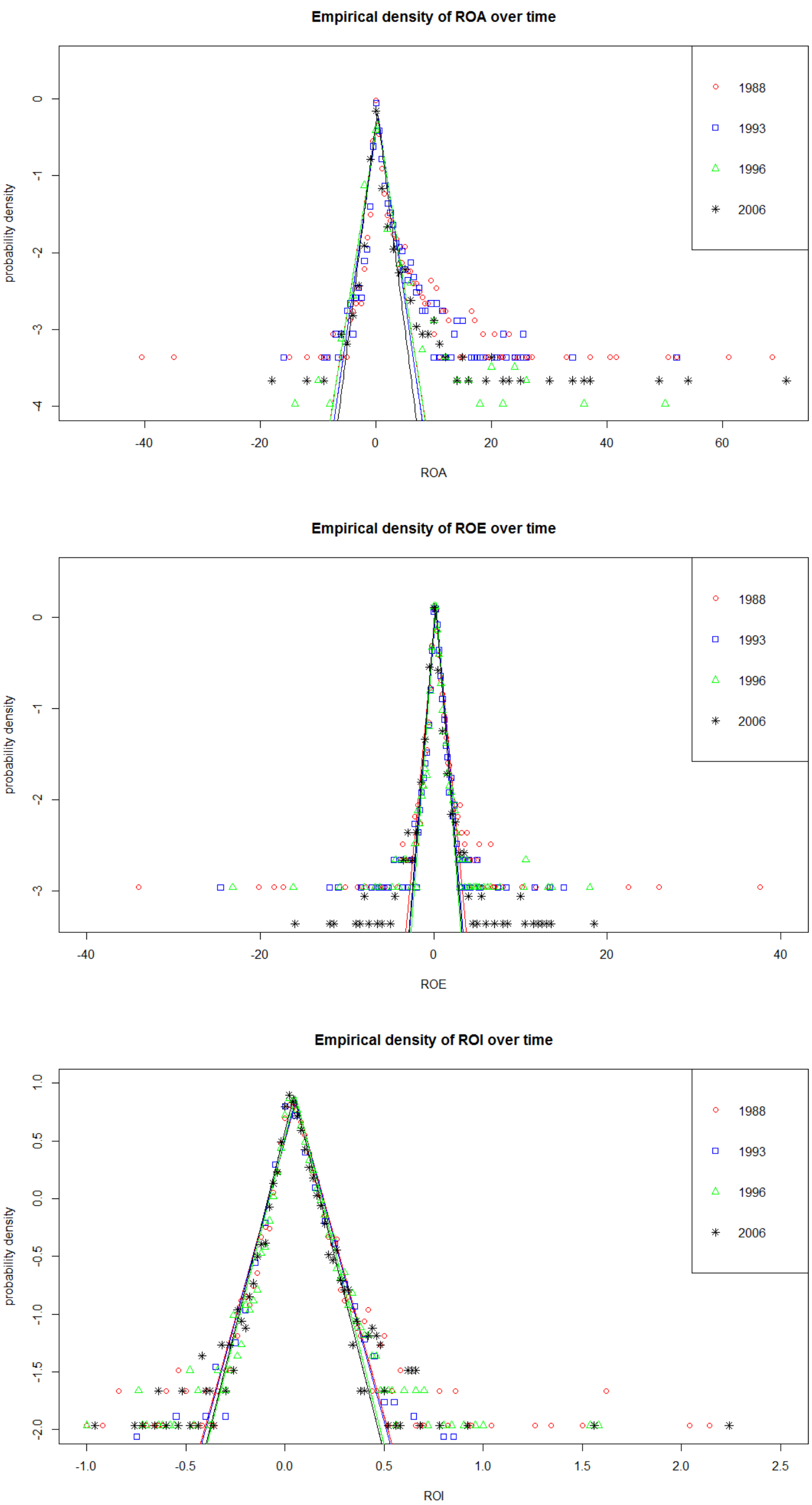


FIGURE 15. Profitability densities in 4 different years. (Note the log scale on the y-axis.) Solid lines show the Laplace distribution fit for the corresponding annual profitability given in the same colour.

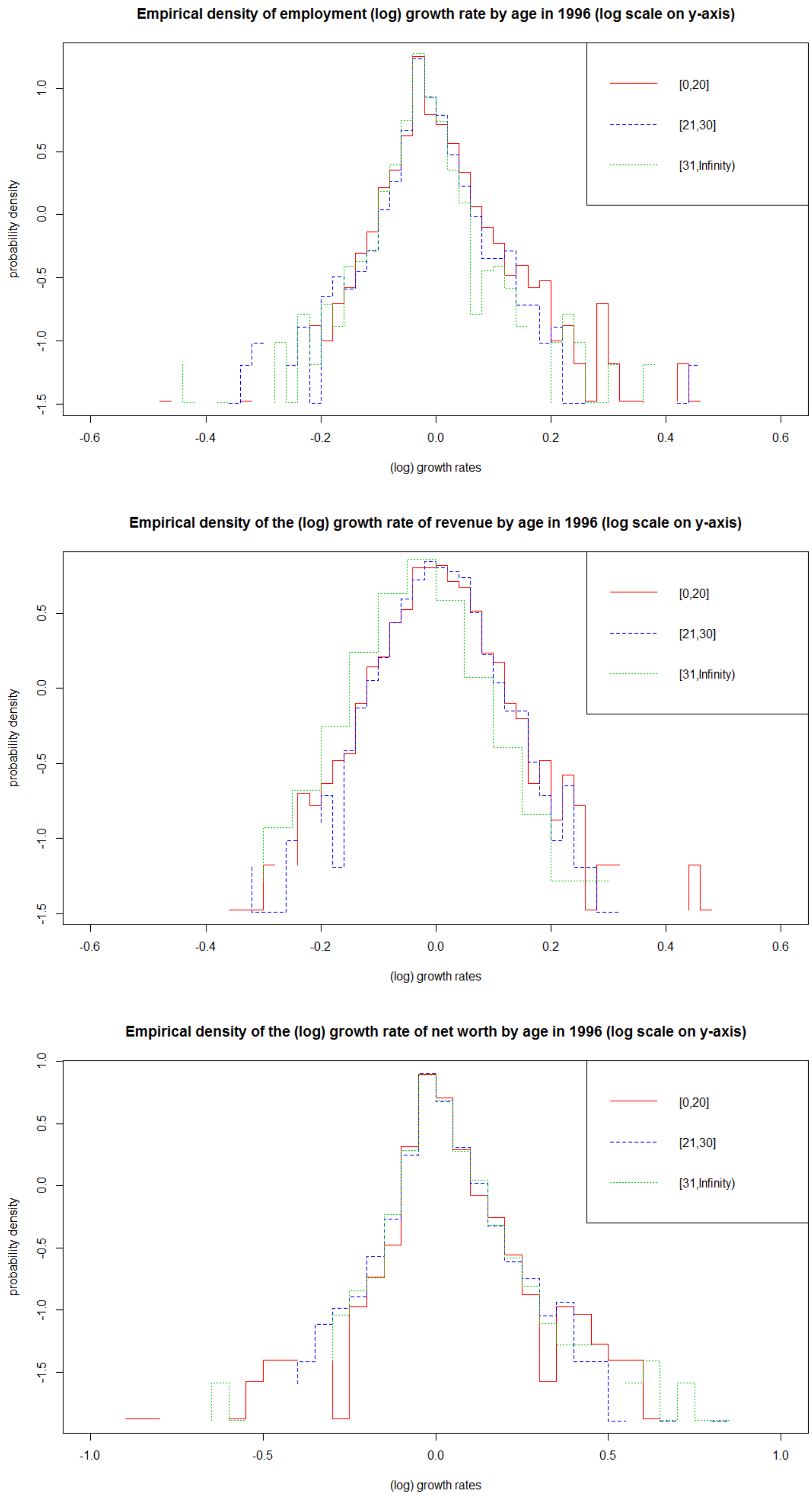


FIGURE 16. Empirical densities of (log) growth rates in 1996 for the three age groups. (Note the log scale on the y-axis.)

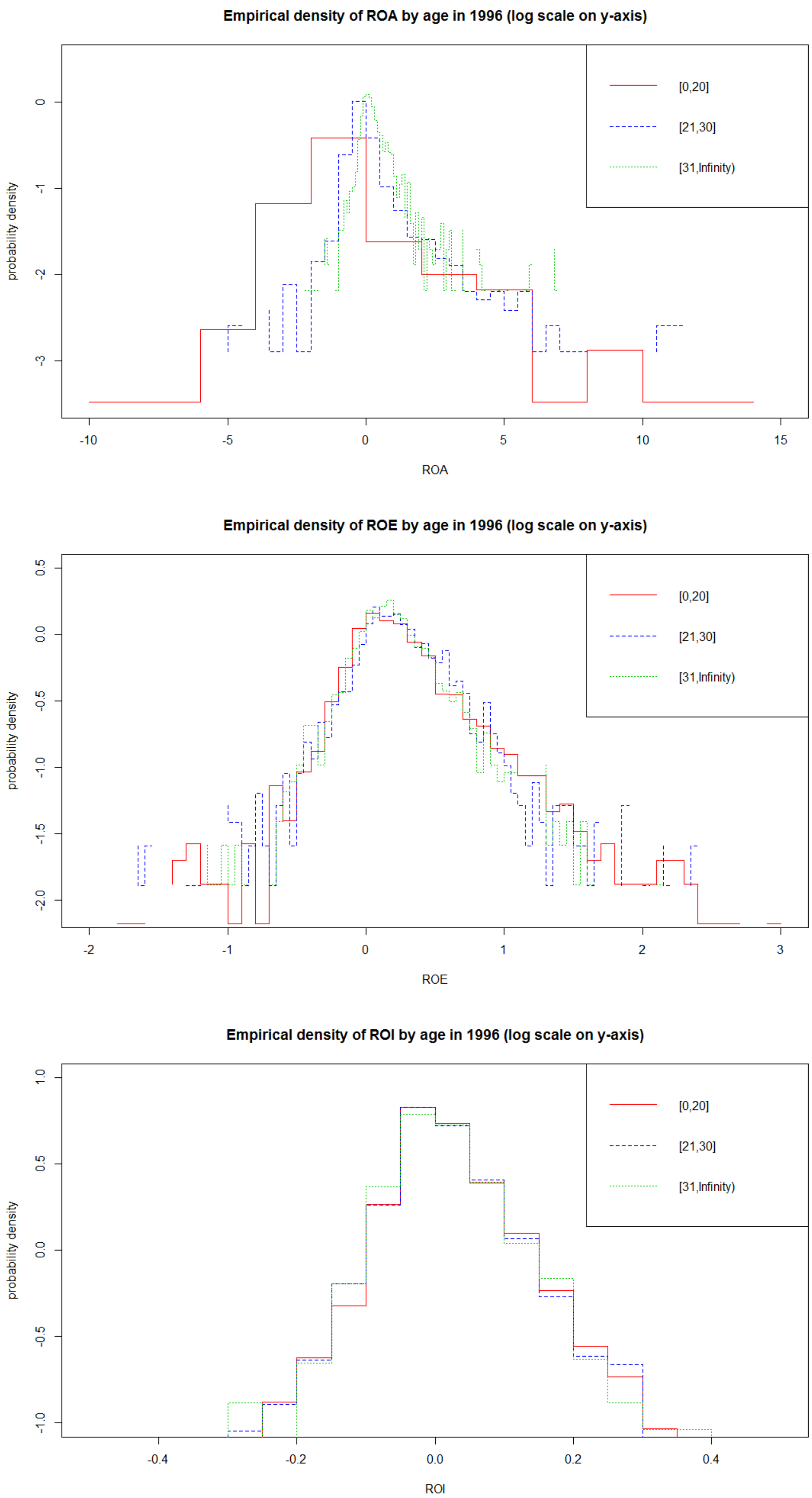


FIGURE 17. Empirical densities of the profitability in 1996 for the three classified age groups. (Note the log scale on the y-axis.)

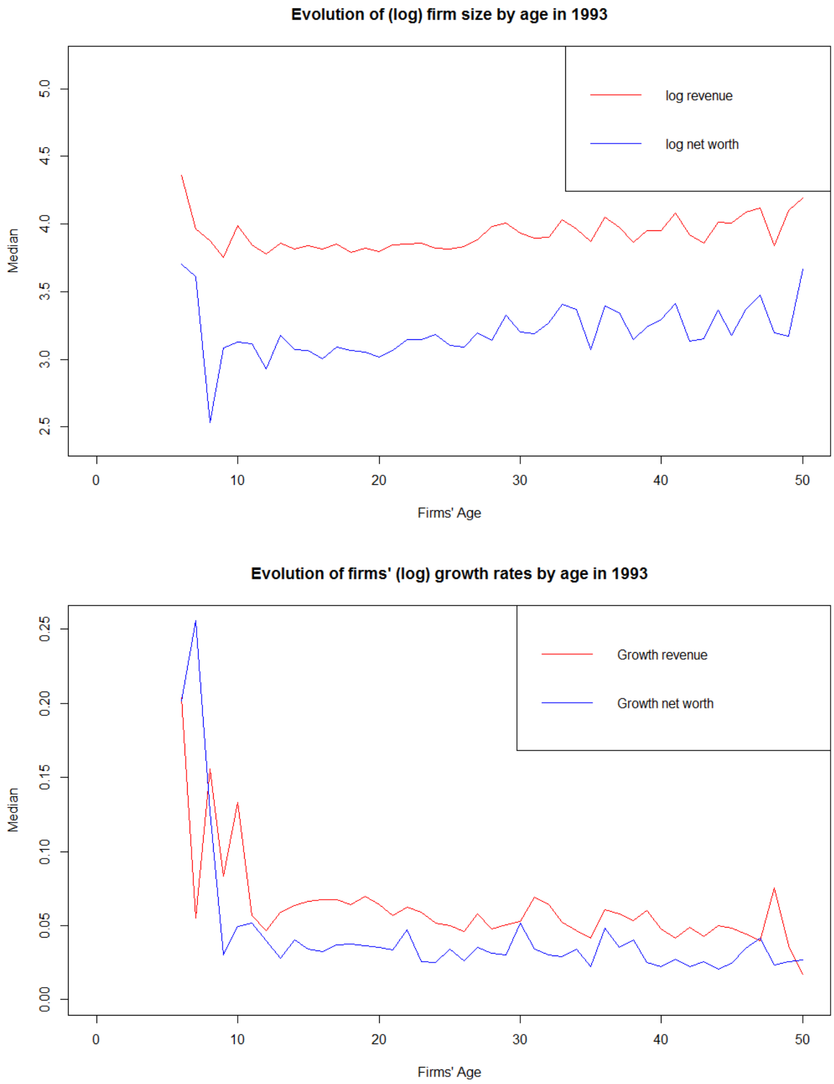


FIGURE 18. Evolution of the median values of the (log) growth rates by age in 1993. Note that the firms' age in 1993 runs from 6 years to 50 years and the rest of the years have been cut.

Next, we explore the evolution of the (log) growth rates (in terms of the revenues and net worth) and profitability (in terms of ROA, ROE and ROI) as firms' age using a graphical analysis. Figure 18 displays the evolution of the median values of the (log) firm size and growth rates as firms' age in 1993 respectively, while Figure 19 shows the evolution of the median values of the considered measures of profitability. From the first plot of Figure 18, we observe a relatively upward trend in the evolution of the (log) firm size, indicating that on the average young firms have less (log) revenues and (log) net worth with respect to older firms that were active in the market in the year 1993. This behaviour also shows that, on average, firms are quite able to cope with the market pressure (so increasing their revenues and net worth in order to survive), and are also capable of growing (in terms of revenues and net worth) over time. Visual inspection shows that firms experience

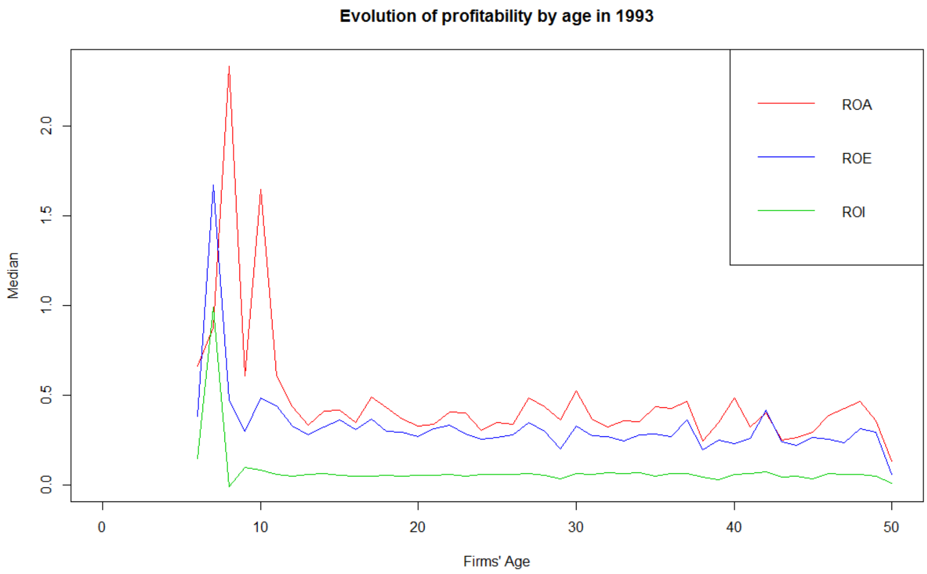


FIGURE 19. Evolution of the median values of the profitability by age in 1993. Note that the firms' age in 1993 runs from 6 years to 50 years and the rest of the years have been cut.

relatively higher profitability levels in their earlier stages, while their profitability roughly stabilizes as age increases, as shown in Figure 19. One possible explanation of this behaviour in the evolution of the firms' profitability is that younger firms are less capital intensive, therefore, generally have higher profitability levels than older firms.

As pointed out in most literature (see for example Coad, Segarra and Teruel (2012) [9]), younger firms have in general higher expected growth rates than older firms. In fact, the second plot of Figure 18 confirms that younger firms indeed experience faster growth rates (in terms of revenues and net worth) than older firms.

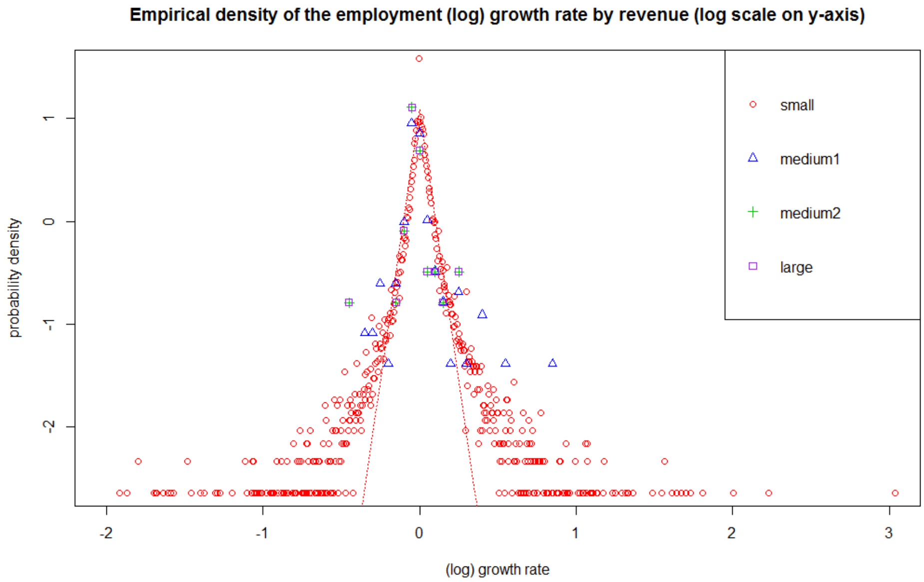
4.2.2. *Empirical analysis of (log) growth rates and profitability by revenue.* In this subsection, we analyze the impact of the firms' revenue on both firms' (log) growth rates and profitability distribution of Italian firms. In fact, we will be investigating whether the distribution of both the (log) growth rates (in terms of employment and net worth) and the profitability (in terms of ROA, ROE and ROI) changes with the firms' revenue. We segregate the firms into categories by dividing the firms' revenue into four groups, namely $[0, 5 * 10^5]$ (small firms), $(5 * 10^5, 1.5 * 10^6]$ (medium firms), $(1.5 * 10^6, 3 * 10^6]$ (medium-large firms) and $(3 * 10^6, \infty)$ (larger firms). Table 14 shows the number of firms in each revenue group. We assume that firms with larger revenues are in general larger. From Table 14, it's clear that our sample consist mostly of small firms, which are firms with revenue in the interval, $[0, 5 * 10^5]$. We therefore stress that our results are more likely affected by sample selection bias and thus, must be interpreted carefully.

Table 14. Number of firms by revenues group.

Year	Revenue group	Number of firms
1987 - 2006	$[0, 5*10^5]$	91280
	$(5*10^5, 1.5*10^6]$	490
	$(1.5*10^6, 3*10^6]$	125
	$(3*10^6, \infty)$	25
1987	$[0, 5*10^5]$	4588
	$(5*10^5, 1.5*10^6]$	6
	$(1.5*10^6, 3*10^6]$	2
	$(3*10^6, \infty)$	0
1993	$[0, 5*10^5]$	4579
	$(5*10^5, 1.5*10^6]$	13
	$(1.5*10^6, 3*10^6]$	4
	$(3*10^6, \infty)$	0
1996	$[0, 5*10^5]$	4567
	$(5*10^5, 1.5*10^6]$	23
	$(1.5*10^6, 3*10^6]$	6
	$(3*10^6, \infty)$	0
2001	$[0, 5*10^5]$	4552
	$(5*10^5, 1.5*10^6]$	33
	$(1.5*10^6, 3*10^6]$	9
	$(3*10^6, \infty)$	2
2006	$[0, 5*10^5]$	4525
	$(5*10^5, 1.5*10^6]$	50
	$(1.5*10^6, 3*10^6]$	16
	$(3*10^6, \infty)$	5

Figure 20 and 21 display the empirical densities of the firms' (log) growth rates and profitability by firms' revenue for the whole sample. Despite the biased sample, it's interesting to notice that the distribution of the firms' (log) growth rates and profitability is described by a tent-shaped distribution with heavier tails and a strong peak around zero (this observation is clearly visible when considering the small firms). Basing our analysis on the distribution of the small firms for both the firms' (log) growth rates and profitability, we observe that the employment (log) growth rates are relatively less spread out compared to (log) growth rates distribution of the firms' net worth (see scaling parameter, $\hat{b}$, given in Table 15), indicating that the distribution of employment (log) growth rates are much more concentrated around zero than the (log) growth rates distribution of the firms' net worth.

Regarding the distribution of the profitability, one observes that the dispersion of the ROA is much higher than that of the ROE and ROA in the case of small firms (compare scaling parameter, $\hat{b}$, given in Table 15). More interestingly, we observe a positively skewed ROA distribution and thus, indicates the emergence of an underlying asymmetric conditional distribution of the ROA. The positively skewed ROA distribution indicates that a noticeable amount of the firms in our sample experience higher return on assets. However, the emergence of an underlying asymmetric distribution of the ROA contradicts with the results shown in Table 15, which suggest that the ROA distribution is symmetric. Moreover, we observe from the first plot of Figure 21 that the right tail of the ROA distribution displays some dependence on firms' revenue over the first three revenue groups, while the distribution of the ROA for the third and fourth revenue group seems to roughly collapse into each other, suggesting that the ROA stabilizes as firms become older. The positive dependence of profitability on firms' revenue over the first three revenue groups indicates that revenue increases the probability of firms experiencing higher profitability levels in terms of ROA.



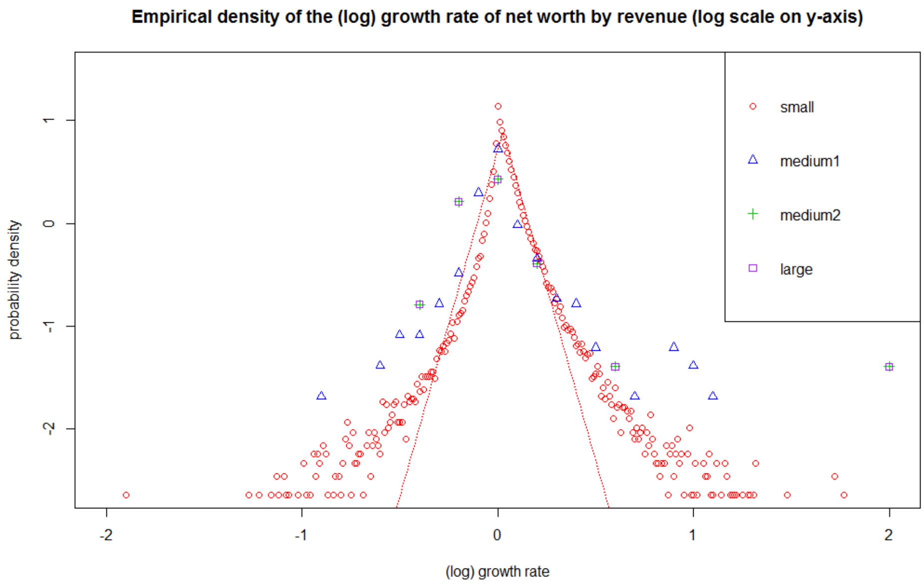


FIGURE 20. Empirical densities of the growth rates for the whole dataset in the four revenue groups. Note the log scale on the y-axis and also that medium 1 stand for medium firms, while medium2 stands for medium-large firms. Red dashed line shows the Laplace fit of the small firms.

We also examine whether the Laplace distribution is a good fit for the considered firms' (log) growth rates and profitability distribution at firms' revenue level. Table

15 presents the results of the maximum likelihood estimation and the goodness-of-fit test for only the small firms (which are firms with revenue in the interval, $[0, 5*10^5]$), since we have enough data for this category and also it seems that the obtained results for the rest of the revenue group are not reliable due to sample selection bias. Nevertheless, it appears that the Laplace distribution is a good fit for the conditional firms' (log) growth rates and profitability distribution, because the Kolmogorov-Smirnov test does not reject the Laplacian hypothesis at 5% significance level for the small firms in terms of revenues.
Due to insufficient data on the rest of revenue group, we couldn't really examine the impact of the firms' revenue on the distribution of firms' (log) growth rates and profitability. Furthermore, it would be interesting to analyze the effect of firms' revenue on the distribution of firms' (log) growth rates (in terms of employment and net worth) and profitability (in terms of ROA, ROE and ROI) using a better selected dataset.

Variable	Revenue group	$\hat{\mu}$	$\hat{b}$	KS-statistic	P-value
Employees	$[0, 5*10^5]$	0.00045 ± 0.00006	0.04121 ± 0.00014	0.572	0.364
Net worth	$[0, 5*10^5]$	0.02569 ± 0.00022	0.06449 ± 0.00022	0.470	0.51
ROA	$[0, 5*10^5]$	0.31398 ± 0.00213	0.82352 ± 0.00273	0.585	0.57
ROE	$[0, 5*10^5]$	0.27601 ± 0.00134	0.40164 ± 0.00133	0.791	0.40
ROI	$[0, 5*10^5]$	0.05580 ± 0.00024	0.06831 ± 0.00023	0.729	0.414

Table 15. Summary results of the maximum likelihood estimation and the Kolmogorov-Smirnov test of the Laplace distribution only for the first revenue group (small firms in terms of firms' revenue). Note that the variables, employees and net worth, are growth variables.

5. Conclusion

In this work, we explored some fundamental properties of the firms' dynamic and also tried to highlight the role of the financial structure in the growth patterns of firms.
We found that the variable used as proxy of size has negligible effect on the two statistical regularities observed in most literature regarding the firms' dynamics, namely that the size distribution of firms is positively skewed and it possesses thick long tails, especially on the right, both at the firms' age level and over time. Therefore, our results also confirms the robustness of the two statistical properties of the firm size distribution. In particular, when the number of employees is used as measure of size, the size distribution seems relatively stable over time, while the size distribution of Italian firms clearly shifts to the right over time when considering revenues as a proxy of size.
Moreover, our results also confirmed that the upper tail of the size distribution of Italian firms obeys a power law behaviour as proposed by Cirillo and Hüsler, the authors of Ref. [7]. Furthermore, the power law exponent, α, appears to be relatively stable over time when we consider the number of employees and revenues as measure of size. In fact, the stability of the power law exponent and shift of the size distribution over time, when we use revenues as a proxy of size, seems to imply that the firms size distribution in terms of revenues is invariant to translation. Generally speaking, it seems that the inequality in firms size in terms of revenues is relatively higher than the inequality in firms size in terms of the number of employees.

Empirical evidence also shows an increase in the thickness of the right tail of the size distribution and a clearly shift of the size distribution towards the right as firms age, indicating an increase in firms' revenue and number of employees as firms' age increases.
Concerning the shape of firms' growth rates and profitability, we found that the tent-shaped distribution like the Laplace distribution seems to be a reasonable benchmark for both the firms' growth rates and profitability distribution, irrespectively of the size proxy used. Our results are well in accordance with most previous studies (see for example Refs. [1], [3], [6] and [9]). Notably, the Laplace benchmark for both the firms' growth rates and profitability distribution seems to be robust over time and also at both firms' age and revenue level, independently of the size variable. Note that the firms' growth rates and profitability distribution shows evidence of heavier tails. This is due to the relatively frequent extremal growth events experienced by a small proportion of the firms in our sample and also note that this regularity emerges at both firms' age and revenue level and also over time. The Gibrat's law, however, appears not to hold true for our data, since we observe a power law behaviour in the size distribution.
Analyzing the evolution of the firms' growth rates and profitability, we found in the case of firms' growth rates that, on average, (log) firms' revenue and (log) net worth increases over time, while, on average, the growth rate relatively decreases over time. Regarding the evolution of the profitability, both the ROA and ROE seems to decrease over time, while the ROI roughly stays stable over time, indicating that, on average, the firms in our sample become more capital intensive over time and thus, generates less revenue from their assets. We also noticed that, on average, the profitability (in terms of ROA, ROE and ROI) are less volatile than the firms' growth rates, especially when considering revenues as a proxy of size. The obtained results concerning the evolution of he firms' growth rates and profitability, suggest that the profitability level of firms plays are very important role in the driving force of the firms' dynamic than the way firms grow or shrink in terms of revenues and net worth over time.
We observe that firms are larger and have, on average, higher revenues and net worth as firms age. We also observe that older firms have lower expected growth rates of revenues and net worth and also lower profitability levels (in terms of ROA, ROA and ROI). In particular, we found in our sample that firms have higher expected growth rates and profitability levels in the early stage of their business life cycle, which is consistent with results obtained in most previous studies on the. In line with Coad, Segarra and Teruel, the authors of Ref. [9], our analysis of the firms' growth rates distribution for different age groups shows that younger firms are more likely to experience higher growth rates (in terms of employment and revenues), while on the other hand, have the same probability as older firms of experiencing decline in terms of employment. However, firms' age seems to have no impact on the firms' net worth distribution and also it appears that older firms are more likely to experience decline with respect to younger and matured firms. We found that the ROA distribution shifts to the right as firms age, while firms' age have negligible effect on the ROE and ROI distribution.
Finally, analyzing the profitability at firms' revenue level, we found that revenues have a positive effect on the return on assets. In fact, we observe that the ROA distribution becomes fairly stable when firms' revenue is more than 3 million euros. It would be interesting to use panel analysis to further study and testing firms' size and growth in order to get a better view of firms' dynamics.

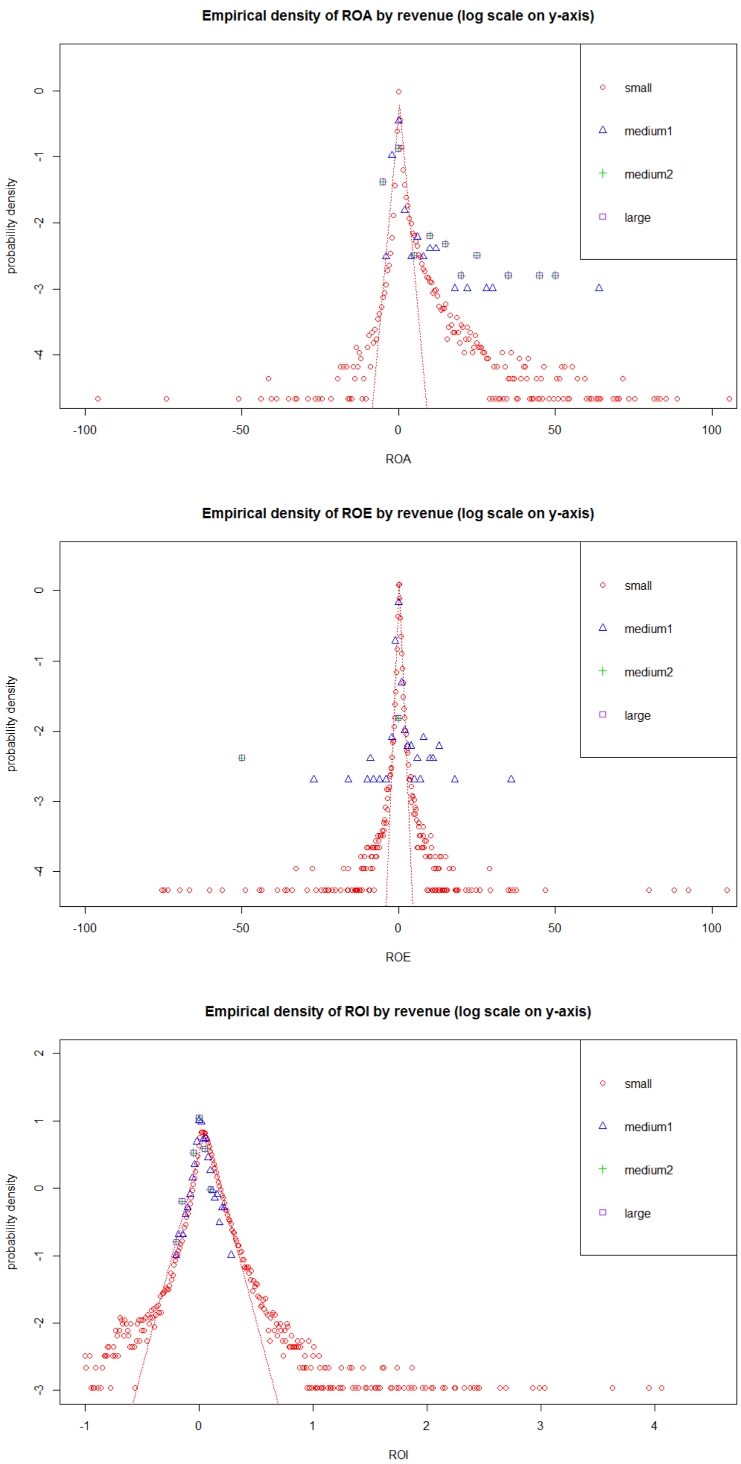


FIGURE 21. Empirical densities of the measures of profitability for the whole dataset in the four revenue groups. Note the log scale on the y-axis and also that medium 1 stand for medium firms, while medium2 stands for medium-large firms. Red dashed line represents the Laplace fit of the small firms.

Appendix

Table 16. Descriptive statistics (mean, std, min, max and median) for the (log) employment growth rates.

Year	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1988	-1.87	-0.02	0.00	0.01	0.02	1.10
1989	-1.11	-0.02	0.00	0.01	0.03	1.09
1990	-1.80	-0.02	0.00	0.01	0.03	1.49
1991	-1.80	-0.01	0.00	0.01	0.03	1.73
1992	-1.48	-0.02	0.00	0.01	0.03	1.81
1993	-1.68	-0.02	0.00	0.01	0.03	2.01
1994	-0.94	-0.02	0.00	0.01	0.03	1.68
1995	-1.37	-0.02	0.00	0.01	0.03	1.62
1996	-1.46	-0.02	0.00	0.01	0.02	1.55
1997	-1.59	-0.02	0.00	0.01	0.03	0.90
1998	-0.94	-0.02	0.00	0.01	0.03	2.23
1999	-1.11	-0.02	0.00	0.01	0.03	1.18
2000	-1.04	-0.02	0.00	0.01	0.03	1.18
2001	-1.91	-0.02	0.00	0.01	0.02	0.95
2002	-1.07	-0.02	0.00	0.01	0.03	1.56
2003	-1.56	-0.02	0.00	0.01	0.03	1.05
2004	-0.90	-0.02	0.00	0.01	0.03	3.04
2005	-1.01	-0.02	0.00	0.01	0.03	1.32
2006	-1.29	-0.02	0.00	0.01	0.02	1.71

Table 17. Descriptive statistics (mean, std, min, max and median) for the (log) growth rate of firms' revenue.

Year	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1988	-1.44	-0.01	0.03	0.04	0.08	1.19
1989	-0.79	-0.00	0.03	0.04	0.08	1.57
1990	-1.17	-0.00	0.04	0.04	0.08	1.38
1991	-2.33	-0.01	0.03	0.04	0.08	1.84
1992	-1.21	-0.00	0.03	0.04	0.08	2.21
1993	-0.86	-0.01	0.03	0.04	0.08	1.45
1994	-1.92	-0.01	0.03	0.04	0.07	1.39
1995	-0.65	-0.01	0.03	0.04	0.08	2.01
1996	-1.11	-0.01	0.03	0.04	0.08	2.27
1997	-1.20	-0.01	0.03	0.04	0.08	1.39
1998	-1.20	-0.01	0.04	0.04	0.08	1.34
1999	-0.58	-0.00	0.03	0.04	0.08	1.00
2000	-1.21	-0.00	0.04	0.04	0.08	0.89
2001	-0.94	-0.01	0.03	0.04	0.08	0.97
2002	-3.85	-0.00	0.04	0.04	0.08	1.58
2003	-0.71	-0.00	0.04	0.04	0.08	2.77
2004	-0.80	-0.01	0.03	0.04	0.08	1.97
2005	-0.63	-0.00	0.04	0.04	0.08	1.05
2006	-0.97	-0.01	0.03	0.04	0.08	1.55

TABLE 18. Descriptive statistics (mean, std, min, max and median) for the (log) growth rate of firms' net worth.

Year	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1988	-1.11	0.00	0.03	0.04	0.07	1.25
1989	-1.05	0.00	0.03	0.04	0.07	1.12
1990	-0.83	0.00	0.02	0.04	0.07	1.23
1991	-1.12	0.00	0.02	0.04	0.07	2.23
1992	-2.09	0.00	0.03	0.04	0.07	1.41
1993	-0.99	0.00	0.03	0.04	0.07	1.81
1994	-0.87	0.00	0.03	0.04	0.07	1.24
1995	-1.89	0.00	0.02	0.04	0.07	1.21
1996	-1.01	0.00	0.03	0.04	0.07	2.55
1997	-1.42	0.00	0.03	0.04	0.07	1.50
1998	-2.34	0.00	0.03	0.04	0.07	2.02
1999	-1.54	0.00	0.03	0.04	0.07	1.37
2000	-1.90	0.00	0.03	0.04	0.07	1.31
2001	-1.07	0.00	0.02	0.04	0.07	1.87
2002	-1.42	0.00	0.03	0.04	0.07	2.19
2003	-1.53	0.00	0.02	0.04	0.07	1.31
2004	-1.67	0.00	0.02	0.04	0.07	2.42
2005	-2.00	0.00	0.03	0.04	0.07	1.72
2006	-0.90	0.00	0.03	0.04	0.07	2.10

R script used for study

```
library(stats4)

# Function 'mlelap' declares the log-likelihood of the Laplace distribution.
# The arguments of 'mlelap' are theta and x, where theta is a vector containing the
# location and scale parameter of the Laplace distribution and x is the dataset.
# logl defines the log-likelihood of the Laplace distribution.

mlelap = function(theta,x) {

  gr <- x;
  n.obs <- length(gr);
  u <- theta[1];
  b <- theta[2];
  logl <- -1*(n.obs*log(2*b)) - (1/b)*sum(abs(gr - u))

  return (- logl)
}


library(VGAM)

# Function 'lapfit' returns the mle of the parameter of Laplace dist.
# and the KS-statistic. Argument of 'lapfit' is x, which is the dataset.
# It also return the plots of the empirical density with the maximum likelihood
# Laplace fit. One with the actual empirical density and the other with
```

```
# log scale on the y-axis. Also compares the plots of the theoretical
# and empirical distribution functions.

lapfit = function(x) {

  gr <- x;
  n.obs <- length(gr);

  # computes mle of Lap. dist.
  opt <- optim(c(0,1),mlelap,x=gr,method="BFGS",hessian=TRUE)
  OI<-solve(opt$hessian)
  se <- sqrt(diag(OI))  # compute standard error for MLE parameters.

  location <- opt$par[1];  # MLE of location parameter
  scale <- opt$par[2];     # MLE of scale parameter

  hist <- hist(gr,breaks="Scott",plot=TRUE);
  xhist<-c(min(hist$breaks),hist$breaks);
  yhist<-c(0,hist$density,0);
  xfit<-seq(min(gr),max(gr),length=n.obs);
  yfit <-dlaplace(xfit,location,scale);

  # pdf with maximum likelihood Laplace fit and log scale on y-axis
  p1 = plot(xhist,log10(yhist),type="p")
  p2 = lines(xfit,log10(yfit),col="red")

  # pdf with maximum likelihood Laplace fit
  p3 <- plot(xhist,yhist,type="p",ylim=c(0,max(yhist,yfit)))
  p4 <- lines(xfit,yfit,col="red")

  # compute empirical cdf
  gr4 <- unique(gr);
  no.obs2 <- length(gr4);
  cq <- c(0:(no.obs2-1))/no.obs2
  cq <- sort(cq)
  xfit2 <-seq(min(gr4),max(gr4),length=no.obs2);

  # Comparing theoretical and empirical distribution functions.
  plot(xfit2,plaplace(xfit2, location = location, scale = scale),type="l",col="red")
  plot(ecdf(gr4),add=TRUE,col="blue")

  # KS- statistic
  cf <- plaplace(xfit2, location = location, scale = scale);
  cf <- sort(cf)
  KS.stat <- max(abs(cq-cf));

  return(list(location = location, scale = scale, location.se = se[1], scale.se = se[2],
  KS.stat= KS.stat));
}


library(VGAM)
```

```
# Function ‘lapfitpval’ computes the p-value of KS test using
# bootstrapping (500 replications).
# Argument of ‘lapfit’ is x, which is the dataset.

lapfitpval = function(x) {

  gr <- x;
  n.obs <- length(gr);

  par <- lapfit(gr)
  location <- par$location;
  scale <- par$scale;
  location.se <- par$location.se;
  scale.se <- par$scale.se;
  Dn <- par$KS.stat;

  Bt <- 500
  bof <- rep(0,Bt)

  for(B in 1:length(bof)){

    # bootstrap resample
    gr2 <- gr[sample(n.obs,n.obs,replace=TRUE)]

    lapsync <- lapfit(gr2);
    D2 <- lapsync$KS.stat;
    bof[B] <- D2;
  }

  #  computes p-value
  pvalue <- sum(bof>=Dn)/length(bof)

  return(list(location = location, scale = scale, location.se = location.se,
  scale.se = scale.se, KS.stat= Dn, KS.pval = pvalue));
}
```

References

[1] Simone Alfarano, Mishael Milaković, Albrecht Irle, and Jonas Kauschke. A statistical equilibrium model of competitive firms. *Journal of Economic Dynamics and Control*, 36:136 – 149, 2012.

[2] Hideaki Aoyama, Yoshi Fujiwara, Yuichi Ikeda, Hiroshi Iyetomi, and Wataru Souma. *Econophysics and Companies*. Cambridge University Press, 2010.

[3] G. Bottazzi, E. Cefis, G. Dosi, and A. Secchi. Invariances and Diversities in the Patterns of Industrial Evolution: Some Evidence from Italian Manufacturing Industries. *Small Business Economics*, 29(1-2):137–159, June 2007.

[4] Giulio Bottazzi, Nadia Jacoby, Angelo Secchi, and Alex Coad. Corporate growth and industrial dynamics: evidence from French manufacturing. *Applied Economics*, November 2007.

[5] Luís M. B. Cabral and José Mata. On the Evolution of the Firm Size Distribution: Facts and Theory. *The American Economic Review*, 93(4):1075–1090, 2003.

[6] Pasquale Cirillo. An analysis of the size distribution of Italian firms by age. *Physica A: Statistical Mechanics and its Applications*, 389:459–466, October 2010.

[7] Pasquale Cirillo and J. Hüsler. On the upper tail of Italian firms’ size distribution. *Physica A: Statistical Mechanics and its Applications*, 388:1546–1554, 2009.

[8] Aaron Clauset, Cosma R. Shalizi, and M. E. J. Newman. Power-law distributions in empirical data. *SIAM Review*, 51(4):661–703, February 2009.

[9] Alex Coad, Agustí Segarra, and Mercedes Teruel. Like milk or wine: Does firm performance improve with age? *Structural Change and Economic Dynamics*, 24:173–189, July 2012.

[10] Einar J. Erlingsson, Simone Alfarano, Marco Raberto, and Hlynur Stefánsson. On the distributional properties of size, profit and growth of Icelandic firms. *Journal of Economic Interaction and Coordination*, pages 1–18, August 2012.