DESIGN ASPECTS AND PERFORMANCE OF A SETTLING TUBE SYSTEM

R. E. Slot & H. J. Geldof

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Laboratory of Fluid Mechanics

Department of Civil Engineering

Delft University of Technology

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CONTENTS

1. Introduction

The DUST (Delft University Settling Tube) is a settling tube system intended to analyse particle size (settling velocity) of sand ranging from 0.06 mm to 2 mm, with the sample mass varying from 0.5 g to 20 g.

The main parts of the system are (see fig. 1):

a. the sample introduction device (venetian blind with rotating lamellae)

b. the settling tube

c. the measuring device (underwater balance with feedback)

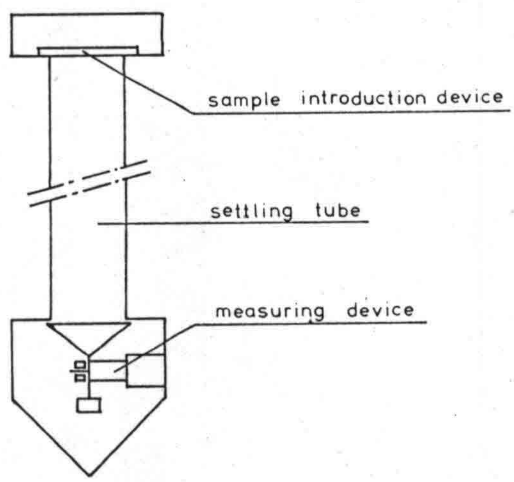

Fig. 1. Settling tube system.

All particles of a sample are released by the sample introduction device at the same time, after which they pass the settling tube. Finally they come at the balance, which measures more or less instantaneously the weight of the particles, thus yielding a cumulative settling velocity distribution.
Dimensions and characteristics of the instrument and the measurement procedures have to be chosen such that the final error in the distribution function attains a minimum. Several of the relevant aspects are treated in the next sections.

2. Design aspects

In order to design an adequate system, the possible sources of errors have to be known and also the physical relation between these errors and the properties of the various parts of the system.

Firstly the errors will be distinguished into systematic and random errors[1), f and σ. Secondly each of them will be seperated into an error in the settling velocity and one in the weight. All errors which contribute to the total error in the settling velocity will be in dicated with the subscript v and in the weight with the subscript w.

2.1. Possible sources of errors

In the following discussion on the possible sources of errors the temperature is assumed to be constant and uniform. Only slight unpredictable variations of the temperature, which may cause random errors, are considered. Furthermore, all the errors mentioned in the following sections are relative errors.

The possible errors in the three parts of the system are:

a. <u>introduction device</u>

- uncertainty in the initial position and velocity of the particles: f_v(intr.dev.) and σ_v(intr.dev.)
- if the initial position of all particles is the same and the initial velocity is zero, a systematic error will be caused by the time the particles need to attain their terminal velocity, depending on the particle diameter: f_v(zero vel
- not all particles are released, as a consequence of adhesion to the lamellae: f_w(adh.) and σ_w(adh.)

b. <u>settling tube</u>

- dependent on the dimensions of the settling tube and the sample volume, there will be a systematic error due to settling convection and hindered settling (concentration effects): f_v(settl.)
- slight variations in the temperature will influence the settling phenomenon and give rise to a random error: σ_v(settl.)

c. <u>weighing system</u>

- a systematic error, due to the delay time of the balance: f_v(delay)
- in addition to the weight, the balance will also measure the impact of the particles (in principle a systematic error in the weight, which may be interpreted as an error in the settling time (see page 5)): f_v(impact)

1) The systematic error is the mean value of the error (accuracy) and the random error is its standard deviation (precision). In principle it is possible to make correctio for the systematic error; for the random error, however, this is impossible.

- environmental changes (especially temperature) will cause an output drift: σ_w(drift)
- noise (electrical and mechanical) in the output signal: σ_w(noise)
- a systematic error due to a non-linear relation between weight and output signal: f_w(lin.)

In addition to the mentioned errors, there will be a random error due to the fact that the sample used for the analysis does not exactly represent the population of origin. It is clear that this error has nothing to do with an error in the measurements. This error will be called the splitting error: σ(split). Other errors, such as the variation in settling tube length due to temperature variations, etc., will not be taken into consideration, as they can be neglected with respect to the errors mentioned before.

In the following sections the physical relation between these errors and the properties of the system is discussed.

2.1.1. Systematic errors

f_w(intr.dev.)

The accuracy of the introduction device, with regard to the initial position of the particles is dependent on the accuracy with which the average initial position of the particles can be determined after opening the lamellae. To release the sample, the lamellae are rotated over about 90^o. If the initial position of the particles is chosen in the (horizontal) plane through the rotation axes of the lamellae, then the maximum error will be D_{lam}/L, in which D_{lam} is the diameter of the rotation circle of the lamellae and L the settling tube length.
By rotating the lamellae, the particles can get an initial velocity, the vertical component of which is the most important. A particle with a certain vertical initial velocity can be considered as a particle with zero initial velocity at a different initial position.
However, visual inspection of the rotating lamellae shows that no particle moves beyond the upper edges of the rotated lamellae. Hence, the contribution to the systematic error in the settling velocity, caused by the introduction device, can be written as

$$f_v(\text{intr.dev.}) = \frac{\Delta D_{lam}}{L} , \qquad (1)$$

where D_{lam} = diameter of rotation circle of lamellae
$|\Delta D_{lam}| \leqslant D_{lam}$
L = length of settling tube.

f_v(zero vel.)

For practical reasons, the initial velocity of the particles is assumed to be equal to the terminal velocity. As in principle, the initial velocity is zero, there will be a gap in distance between a particle with zero initial velocity and a particle with terminal initial velocity, starting from the same position at the same time. The error, due to the mentioned assumption is the ratio between this gap in distance and the length of the settling tube. The contribution of this error to the error in the settling velocity can be written as

$$f_v(\text{zero vel.}) = \frac{1}{L}\int_0^{L/w_\infty} \left[w_\infty - w(t)\right] dt \ , \tag{2}$$

where w_∞ = terminal velocity
$w(t)$ = velocity as function of time.

f_v(settl.)

Measurements made with sand samples of varying weight and average particle diameter show that the median settling velocity deviates from the ideal one. This deviation turns out to be dependent on the sample weight and the average particle diameter as well as on the temperature of the sedimentation fluid (Geldof, 1978). The average particle diameter and the ratio between sample volume and settling tube volume seems to be characteristic for this phenomenon. The scatter in the particle diameter and the ratio between diameter and length of the settling tube are likely to be of influence as well, but this has not yet been verified. Hence, the contribution of this settling phenomenon to the systematic error in the settling velocity can be written as

$$f_v(\text{settl.}) = h(\bar{d}, \sigma_d, D/L)c_\ell \ , \tag{3}$$

where $\bar{d}$ = mean particle diameter
σ_d = standard deviation of particle diameter
D = settling tube diameter
$c_\ell = V_s/\frac{1}{4}\pi D^2 L$ = ratio between sample and settling tube volume

f_v(impact)+f_v(delay)

For the calculation of the error due to the impact of the particles and the delay time of the weighing system (Slot, 1977) an idealized sample is used, so that the weight on the balance will vary linearly with time (see fig. 2).

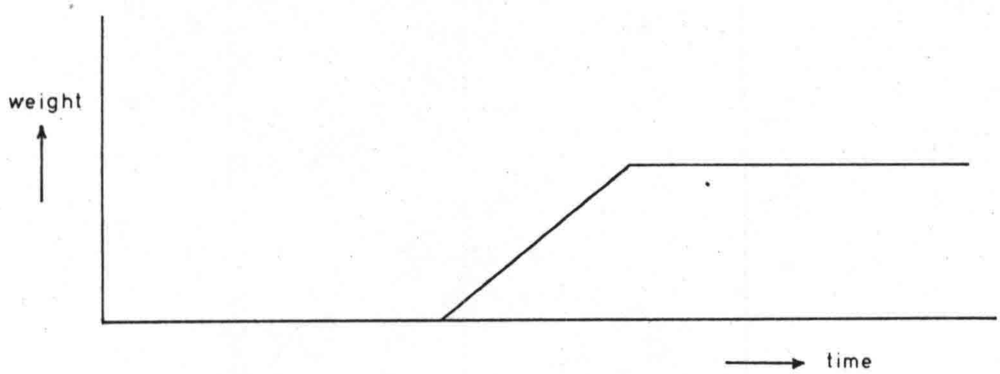

Fig. 2. Weight of idealized sample on balance.

The time of arrival of the particles is assumed to be Poisson-distributed, so that the results of the calculation have to be considered as mean values; they are given in fig. 3.

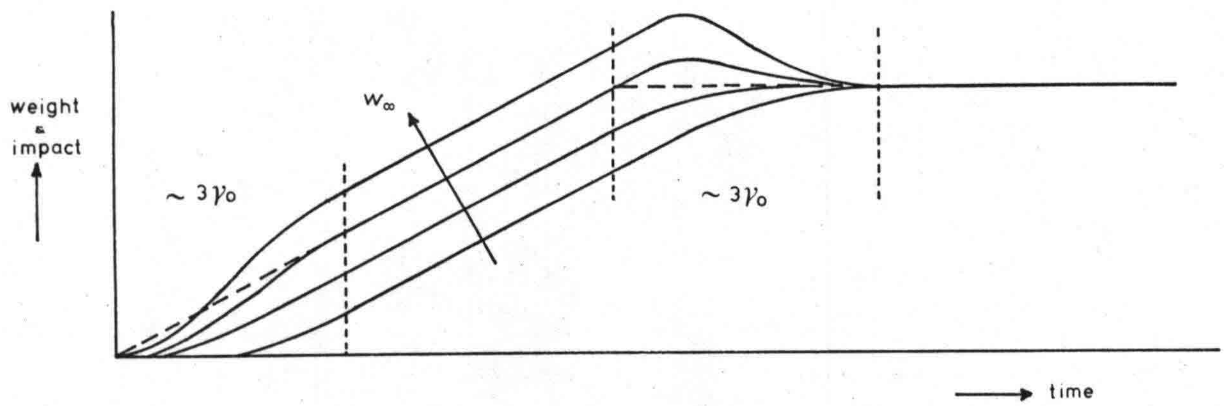

Fig. 3. Response of weighing system to idealized sample.

Actually, there are two independent errors, viz. an error in the weight and an error in the settling time. In the intermediate region of the response curve (fig. 3), the shape of the curve is identical to the ideal (linear) one. Hence, in the intermediate region, the total error due to the impact of the particles and the delay time of the weighing system can be interpreted as an error in the delay time only. At the beginning and at the end of the curve (discontinuities) the shape deviates from the ideal one over a range of about $3\gamma_0$ (γ_0=delay time of weighing system), so that in these regions the above interpretation will not hold.

The total delay time γ_t in the intermediate region can be written as $\gamma_t=\gamma_o-w_\infty/g$, in which w_∞/g is the contribution of the impact of the particles. The systematic error, due to impact and delay can be written as the ratio between γ_t and the settling time L/w_∞ of the particle. Hence, the contribution of impact and delay to the systematic error in the settling velocity can be written as

$$f_v(\text{impact})+f_v(\text{delay}) = \frac{w_\infty(\gamma_o - w_\infty/g)}{L}\,, \tag{4}$$

where γ_o = delay time of weighing system

g = acceleration of gravity.

In general, the curve representing the weight on the balance will not be discontinuous; it will have a more or less smooth shape. If this curve can be approximated by linear segments of a time length of about $3\gamma_o$, however, (4) will still hold.

f_w(adh.)

Especially small particles (<0.1 mm) are likely to adhere to the lamellae, probably as a consequence of the roughness of the lamellae and a certain amount of grease on them. The physical relation is not known quantatively, but in general polishing and degreasing of the lamellae will prevent the adhesion.

f_w(lin.)

The systematic error due to a non-linear relation between weight and output signal can be expressed as the ratio between the maximum deviation from the linear relation and the total sample weight

$$f_w(\text{lin.}) = \frac{\Delta G}{G}\,, \tag{5}$$

where ΔG = maximum deviation between actual and linear relation

G = total sample weight.

2.1.2. Random errors

σ_v(intr.dev.)

The physical relation between the random error and the properties of the introduction device is not known at this moment. It can be stated that the precision depends on the way the sample is spread over the lamellae; hence, it will depend on the skill of the operator.

σ_v(settl.)

The random error due to the settling phenomenon is likely to be caused by slight variation in the temperature (-gradient). The (median) settling velocity turns out to be rather strongly dependent on the temperature of the sedimentation fluid. This effect can not sufficiently be explained by the variation in the settling velocity due to the variation in the kinematic viscosity only. Presumably the viscosity is also of influence on the building up of the particle cluster and determines in this way the rather strong dependence on the temperature.

σ_w(adh.)

The physical relation is not known quantatively, but the random error due to the adhesion is likely to be smaller than the systematic error; see f_w(adh.).

σ_w(drift)

In general, the output signal will change due to environmental variations (especially temperature). The ratio between this drift S (expressed in variation of weight per unit of time) and the quotient of sample weight and measuring time ($G/T=Gw_{min}/L$) determines the error

$$\sigma_w(\text{drift}) = \frac{SL}{Gw_{min}} , \tag{6}$$

where S = output drift (weight per unit of time)

w_{min} = minimum particle velocity in the sample.

σ_w(noise)

In general, the output signal will contain noise due to the electronics and mechanical vibrations. The noise can be characterized by its standard deviation σ_n (σ_n^2=noise power), expressed in terms of weight. Hence, the error will be

$$\sigma_w(\text{noise}) = \frac{\sigma_n}{G} . \tag{7}$$

2.2. Conclusions

As can be seen from section 2.1.1., the length L of the settling tube is an important property; all systematic errors which contribute to the error in the settling velocity will be reduced by increasing the length of the tube. On the other hand the random error in the weight will be increased by increasing L since the drift error is proportional with L: $\sigma_w(\text{drift})=SL/Gw_{min}$.
Other important properties are the diameter D_{lam} of the rotation circle of the lamellae, the diameter D of the settling tube and the delay time γ_o of the weighing system.
The diameter D_{lam} of the rotation circle of the lamellae has to be small compared to the length L of the settling tube. The diameter D of the settling tube has to be chosen as large as possible in order to minimize concentration effects. The delay time γ_o of the weighing system has to be determined in relation to the maximum particle velocity in the sample, such that (4) is minimum over the whole velocity range.
In fig. 4 f_v(impact)+f_v(delay) is plotted as a function of the terminal velocity w_∞. It shows that for small velocities the maximum error is $\frac{1}{4}g\gamma_o^2/L$. In the

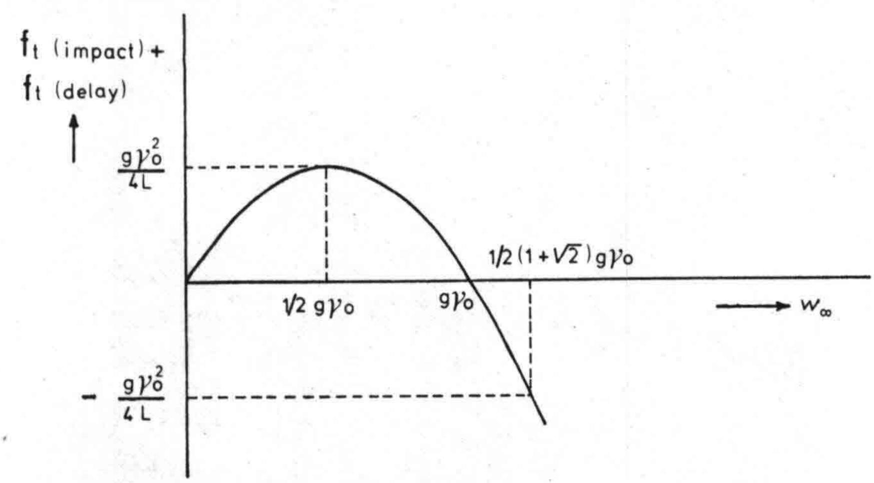

Fig. 4. Systematic error due to impact and delay as function of the terminal velocity.

velocity range from zero to $\frac{1}{2}(1+\sqrt{2})g\gamma_o$ the error will still be smaller than $\frac{1}{4}g\gamma_o^2/L$. Thus the delay time γ_o of the weighing system has to be chosen sufficientl small to reduce the maximum error, but not too small, since the possible range of particle velocity is reduced, as well.

The weighing system has to be linear to such an extent, that the systematic error in the weight is small enough. Furthermore, noise and drift have to be small, since they determine the minimum possible sample weight that can be analysed.

The next section gives a discussion on the possibility of designing a weighing system, the delay time of which is adequate, and the non-linearity, drift and noise of which is minimum.

3. Weighing system

The heart of the DUST is the weighing system, which determines for the greater part the capabilities of the instrument. The weighing system should have a fast, critically-damped response. Drift and noise should be small and the relation between weight and output signal should be linear. In general, the use of feedback will greatly improve the imperfections inherent to the system.

The weighing system of the DUST is composed of (see fig. 5):

a. weighing pan with air chamber, so that the system will float in water
b. special construction of springs, allowing only for axial displacements
c. two inductive transducers to measure displacement
d. coil-magnet system for feedback

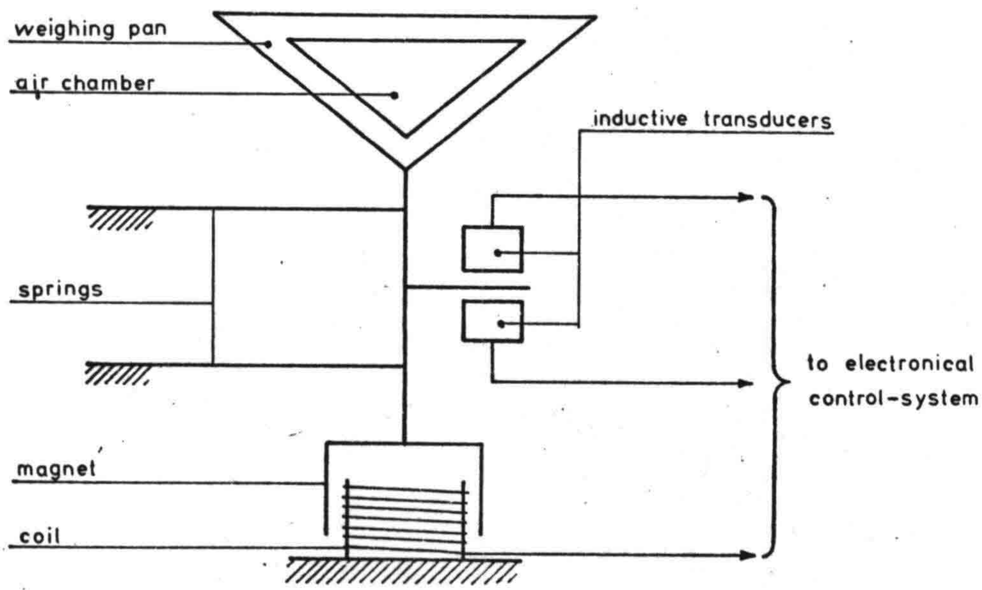

Fig. 5. Weighing system of DUST.

The transfer function $H(\omega)$ of the weighing system without feedback is

$$H(\omega) = A(-M\omega^2 + jk\omega + C)^{-1} , \qquad (8)$$

where	A =	Wheatstone-bridge amplification factor	V/m
	M =	inertial mass of weighing system	kg
	ω =	angular frequency	rad/s
	k =	damping coefficient	Ns/m
	C =	spring constant	N/m

In general, the natural damping coefficient is too small (internal friction in springs and water), so that the weighing system will make an oscillatory motion (see fig. 6). This oscillatory motion will vanish if the damping is critical (see fig. 7).

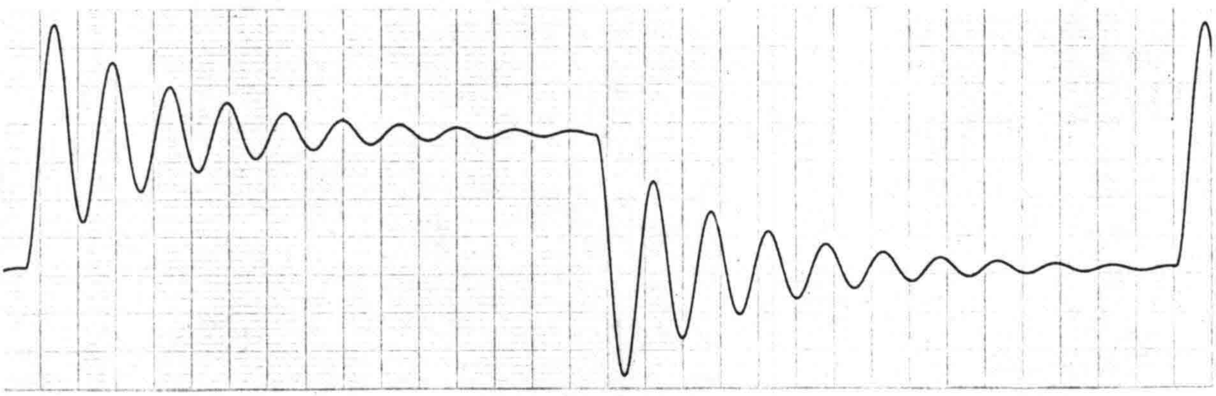

Fig. 6. Oscillatory response of weighing system on square wave function.

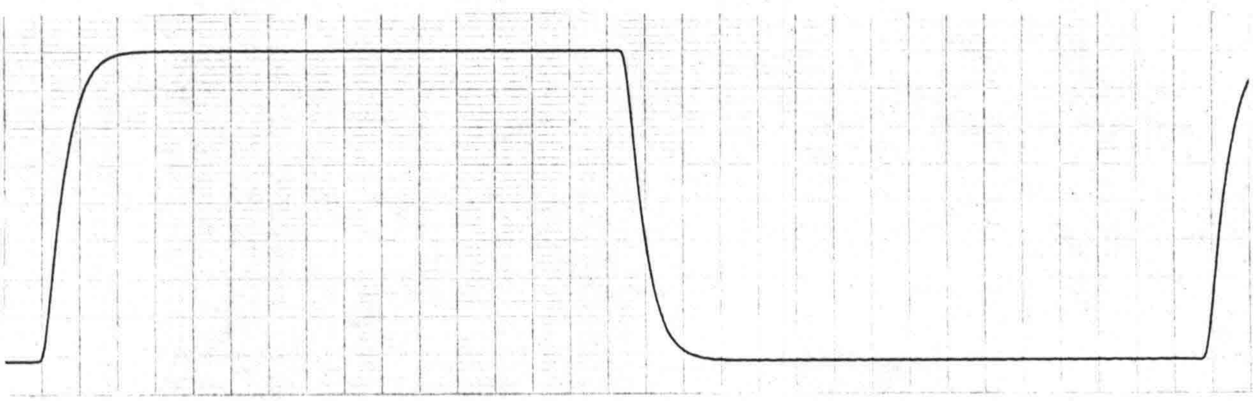

Fig. 7. Critically-damped weighing system.

A measure for the oscillatory motion is the quality factor Q, defined as

$$Q = \frac{1}{k}\sqrt{MC} . \qquad (9)$$

If $Q=\frac{1}{2}$ the system is damped critically, whereas for $Q>\frac{1}{2}$ the system is underdamped and will oscillate.
The natural frequency of the system is

$$\omega_o = \sqrt{\frac{C}{M}} \; . \qquad (10)$$

When a feedback loop is used, the damping and the natural frequency can easily be changed and adjusted. The block-diagram of the system with feedback is given in fig. 8, where the differentiator gives the possibility of adjusting the damping.

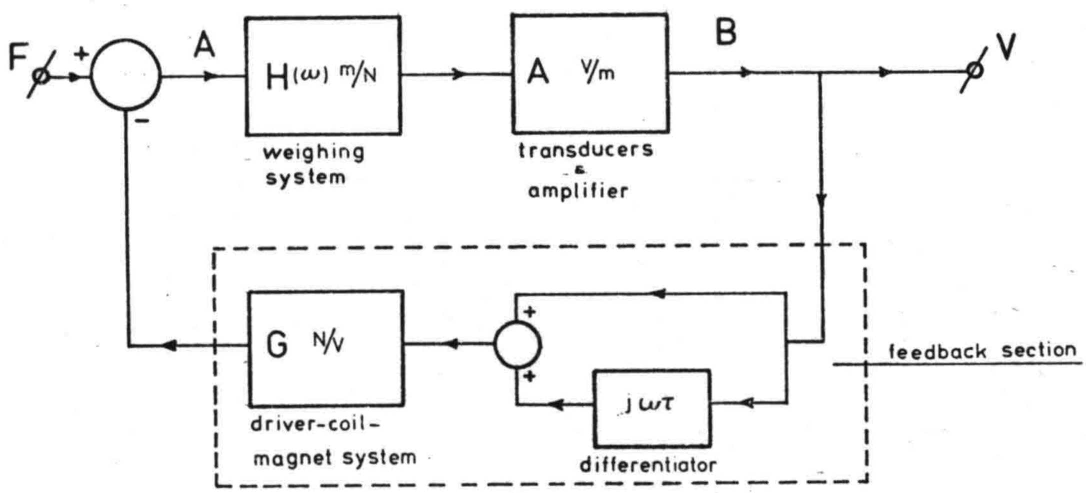

Fig. 8. Block-diagram of weighing system with feedback.

The transfer function of the weighing system with feedback is

$$H_f(\omega) = \left[- \frac{M}{A}\omega^2 + j(\frac{k}{A} + G\tau)\omega + (\frac{C}{A} + G)\right]^{-1} , \qquad (11)$$

where G = amplification factor of the driver-coil-magnet system N/V
τ = RC-time of the weighing system s

The quality factor of the weighing system with feedback is

$$Q_f = \frac{\sqrt{1 + \frac{AG}{C}}}{1 + \frac{AG\tau}{k}} Q \qquad (12)$$

and the natural frequency follows from

$$\omega_{o,f} = \sqrt{1 + \frac{AG}{C}}\, \omega_o \,. \qquad (13)$$

The delay time γ_o of the weighing system, i.e. the time lag between input and output for a linear changing input signal (see fig. 9), is given by

$$\gamma_o = \frac{1}{Q\omega_o} \,. \qquad (14)$$

For a critically-damped system ($Q=\frac{1}{2}$) with natural frequency $\omega_{o,f}$, this becomes

$$\gamma_o = \frac{2}{\omega_{o,f}} \,. \qquad (15)$$

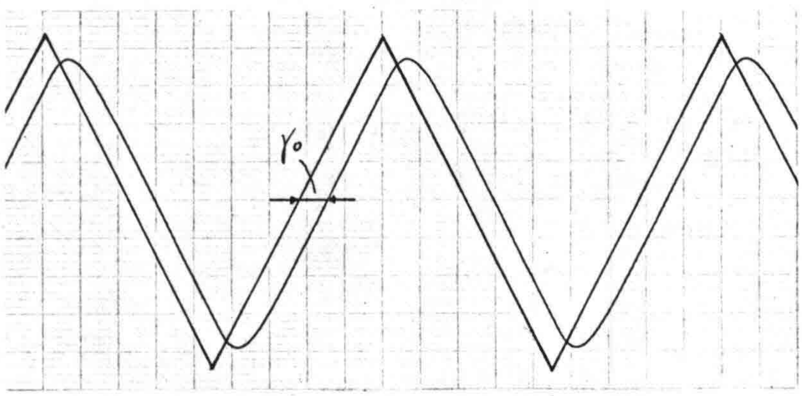

Fig. 9. Delay time γ_o for critically-damped system.

In order to obtain the required delay time γ_o, firstly $\omega_{o,f}$ has to be adjusted by means of G (i.e. the driver of the coil-magnet system). Secondly τ (i.e. the RC-time of the differentiator) has to be adjusted to make the system critically damped.

In principle, the same result can be obtained by adjusting A (i.e. the amplification factor of the Wheatstone-bridge), but, as it will be shown later, it is essential to make A (c.q. AG/C) as large as possible.

For the calculation of the sensitivity of the system to noise, drift and non-linearity, the steady state transfer function ($\omega=0$) has to be used. So for the system without feedback

$$H(0) = \frac{A}{C} \qquad (16)$$

and for the system with feedback

$$H_f(0) = \frac{1}{\frac{C}{A} + G} \,. \qquad (17)$$

For a slight variation ΔC of C (e.g. due to temperature variation) equation (16) leads to

$$\frac{\Delta H(0)}{H(0)} = \frac{\Delta C}{C} , \tag{18}$$

and equation (17) yields

$$\frac{\Delta H_f(0)}{H_f(0)} = \frac{1}{1 + \frac{AG}{C}} \frac{\Delta C}{C} . \tag{19}$$

Similarly, for a slight variation ΔA of A

$$\frac{\Delta H(0)}{H(0)} = \frac{\Delta A}{A} \tag{20}$$

and

$$\frac{\Delta H_f(0)}{H_f(0)} = \frac{1}{1 + \frac{AG}{C}} \frac{\Delta A}{A} . \tag{21}$$

In general, it can be stated that between the points A and B in the block-diagram every variation of system parameters and the influence of noise and drift will be reduced by the factor

$$f = 1 + \frac{AG}{C} . \tag{22}$$

On the other hand, the influence of sources of noise, drift and non-linearity in the feedback section and external sources as well, will not be reduced.
To take full advantage of the feedback, AG/C has to be made as large as possible. The practical limitation is reached, however, if the system becomes unstable as a consequence of phase-shift, attendant system-frequencies, etc.
The main parts in the feedback section are the differentiator and the driver-coil-magnet system. The differentiator can only be a source of noise (its steady state response is zero), whereas the driver-coil-magnet system can also be a source of drift and non-linearity. A proper design of the electronics, however, can make these inherent imperfections small enough.
Temperature variations of the water, in which the weighing pan floats and mechanical vibrations can be considered as external sources of drift and noise, respectively. A constant room (i.e. water) temperature and a quiet place may be essential, although a proper construction of the housing of the balance (spherica housing) and a platform on air springs, damped in glycerine, can reduce the

sensetivity to mechanical vibration for the greater part.

The characteristics of the weighing system of the DUST are:

$M = 8$ kg	$\omega_{o,f} = 58$ rad/s
$C = 170$ N/m	$Q_f = 0.5$
$\omega_o = 4.6$ rad/s	$A = 1.8\ 10^6$ V/m
$k = 5.3$ Ns/m	$G = 15\ 10^{-3}$ N/V
$Q = 7$	$f = 160$
	$\tau = 35\ 10^{-3}$ s

According to (15), the delay time of the weighing system is $\gamma_o = 35\ 10^{-3}$ s. For an additional reduction of noise due to building vibration with a relatively high frequency, a low-pass filter is used, which makes the actual delay time somewhat larger: $\gamma_o = 56\ 10^{-3}$ s.

An additional advantage of the driver-coil-magnet system is the possibility to tare the balance and to use electrical test signals to check and adjust the various parameters of the system.

4. Performance

The dimensions and characteristics of the DUST are:

length of settling tube	$L = 1.70$ m	
diameter of settling tube	$D = 0.17$ m	
diameter of rotation circle of lamellae	$D_{lam} = 8$ mm	
delay time of weighing system	$\gamma_o = 56$ ms	
noise "amplitude"	$\sigma_n = 3\ 10^{-6}$ N	[=0.3 10^{-3} gf]
maximum drift (normal room condition)	$S = 30\ 10^{-9}$ N/s	[=3 10^{-6} gf/s]

accuracy

Using the above numerical values, the systematic errors are found to be

$$f_v(\text{intr.dev.}) = \frac{\Delta D_{lam}}{L} \leq 0.5\% \qquad (23)$$

and

$$f_v(\text{impact}) + f_v(\text{delay}) = \frac{w_\infty(\gamma_o - w_\infty/g)}{L} \leq \frac{g\gamma_o^2}{4L} = 0.5\% \ , \qquad (24)$$

for the velocity ranging from zero to $\frac{1}{2}(1+\sqrt{2})g\gamma_o = 0.66$ m/s.

An approximative calculation of f_v(zero vel.) shows that for spherical particles the error will be 0.6% if the particle diameter is large (2 mm) and less for smaller particle diameter; so, with some safety margin

$$f_v(\text{zero vel.}) = \frac{1}{L}\int_0^{L/w_\infty}\left[w_\infty - w(t)\right]dt < 1\% \; . \qquad (25)$$

Preliminary measurements with sand samples of varoius weight and average particle diameter show that the median settling velocity deviates from the ideal one (found by extrapolation to zero sample weight). For fine sand (0.1 mm) with a sample mass of 0.5 g (c_ℓ=5 10^{-6}) the error will be 5%, whereas for coarse sand (0.8 mm) the same error of 5% will be obtained for a sample mass of 10 g. Thus the error due to the settling phenomenon is

$$f_v(\text{settl.}) = h(\bar{d},\sigma_d,D/L)c_\ell < 5\% \; , \qquad (26)$$

with the restriction that the dependence on σ_d is not taken into account; the used samples being sieve fractions. Furthermore, the dependence on D/L is unknown and also the accuracy of the higher moments of the settling velocity distribution is not known yet.

Measurements made to determine a possible non-linear relation between weight and output signal show that a deviation from a linear relation is not measurable, although a possible deviation of 0.1% would be detectable.

Visual inspection of the adhesion of the particles to the lamellae shows that f_w(adh.) is negligible with respect to the total amount of particles (i.e. total weight).

Hence, with the mentioned restrictions, it can be stated that for the DUST the accuracy of the settling velocity is better than 7%. The accuracy of the weight turns out to be better than 0.1%.

<u>precision</u>

The measurements made for determining the error due to the settling phenomenon also show that the standard deviation is smaller than 2%. This includes not only the error due to the introduction device and the settling phenomenon, but also the error due to the sample splitting method applied here.

For the random error in the weight it is found that

$$\sigma_w(\text{noise}) = \frac{\sigma_n}{G} < 0.06\% \tag{27}$$

and

$$\sigma_w(\text{drift}) = \frac{SL}{Gw_{min}} < 0.4\% \ , \tag{28}$$

for a sample weight of 5 10^{-3} N (0.5 gf) and a minimum velocity of 3 mm/s.

Hence, the precision of the settling velocity turns out to be better than 2%, whereas the precision of the weight is better than 0.5%.

5. Conclusions

a. Using an underwater balance with feedback, it is possible to measure the settling velocity distribution of (sand) particles ranging from 0.1 to 2 mm with an accuracy (systematic part of the error) better than 7%, in which 5% is due to the settling phenomenon itself (concentration effects). The precision (random part of the error) is better than 2%, including the error of sample splitting.
   The accuracy of the (relative] weight-measurement is better than 0.1% with a precision better than 0.5%.
b. The impact of the particles can be interpreted as a contribution to the delay time of the weighing system. As the impact partly compensates the delay time of the weighing system, the demands upon the delay time can be weakened. Given the possible velocity range, there is an optimum value of the delay time, for which the error will be smaller than $\frac{1}{4} g \gamma_0^2 / L$.
c. Further research has to be done on the dependence of the settling phenomenon on the average particle diameter, the scatter in particle diameter, the diameter of the settling tube and the temperature of the sedimentation fluid.

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LIST OF SYMBOLS

A	amplification factor of transducers/Wheatstone-bridge	V/m
C	spring constant	N/m
c_ℓ	ratio between sample volume and settling tube volume	
D	diameter of settling tube	m
D_{lam}	diameter of rotation circle of lamellae	m
$\bar{d}$	average particle diameter in sample	m
F	force on weighing pan	N
f	reduction factor for noise, drift, etc.	
G	total sample weight or	N
	transfer function of driver-coil-magnet system	N/V
g	acceleration of gravity	m/s^2
$H(\omega)$	transfer function of weighing system without feedback	m/N
$H_f(\omega)$	transfer function of weighing system with feedback	m/N
j	$\sqrt{-1}$	
k	damping coefficient	Ns/m
L	length of settling tube	m
M	inertial mass of weighing system	kg
Q	quality factor of weighing system without feedback	
Q_f	quality factor of weighing system with feedback	
S	drift	N/s
V_o	output signal of weighing system	V
$w(t)$	particle velocity	m/s
w_{min}	minimum particle velocity in sample	m/s
w_∞	terminal particle velocity	m/s
γ_o	delay time of weighing system	s
γ_t	total delay time of weighing system, included impact of particles	s
π	3.14....	
ω_o	natural frequency of weighing system without feedback	rad/s
$\omega_{o,f}$	natural frequency of weighing system with feedback	rad/s
σ_n	noise "amplitude"	N
σ_d	standard deviation of particle diameter in sample	m