Effect of Minimum Up and Down Time Constraints with Fully Flexible Temporal Resolutions

Integrating Clustered Unit Commitment Constraints in the Tulipa Energy Model

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Abstract

In recent literature fully flexible temporal resolutions have been proposed as a new form of temporal clustering in generation expansion planning models, showing promising benefits in terms of the tradeoff between solution accuracy and computation time. However, unit commitment constraints such as minimum up and down times have not yet been considered in combination with these resolutions. This paper introduces minimum up and down time (MU/MD) constraints to fully flexible time resolutions and shows the effects of including them when doing generation expansion planning. This is done by constructing a case study based on the European energy grid and comparing the effects of adding MU/MD constraints to a model with a fully flexible time resolution in the form of the geographically decreasing resolution. The paper shows that the addition of minimum up and down time constraints maintains the benefits of fully flexible time resolutions, but comes with additional computation time and also has little effect on the optimal solution cost for the case study examined.

1 Introduction

The transition from using fossil fuels to renewable energy sources is a challenge which has become increasingly important in recent years. To support this transition, long-term planning models are needed to determine decisions regarding which technologies to invest into and when to do so. Generation expansion planning (GEP) models are commonly used to determine optimal investment strategies in order to provide energy that meets the future demand, while also minimizing costs [1].

Due to the large size of these models, certain constraints related to the technical limitations of energy generators are not always considered. Such constraints are more common in the field of the Unit Commitment (UC) problem, which concerns the generation of a schedule for the operation of individual energy generators. Unit commitment constraints model the fact that a certain type of generator may be physically limited as to when it can be turned on or off, how long it takes to reach the desired power output, etc. Including UC constraints in GEP is essential when considering renewable energy sources. This is because the fluctuations in the power generation of these sources create a need for flexibility, which causes significant changes to the generation mix [2]. Recent research thus recommends to include short-term operating conditions, such as unit commitment constraints, into GEP models to ensure the viability of the investment decisions [1].

However, a major downside of modeling this is the increased number of variables and constraints it adds to the model, which significantly increases compute time. This is why the recommended approach when considering GEP models with UC constraints is the use of Clustered Unit Commitment (CUC) [2]. This approach involves grouping together assets of the same type or which have similar characteristics, and using an integer variable to represent how many units are turned on (committed) at a given time, instead of using binary variables for each instance of an asset. This has been shown to significantly decrease compute time, but it also introduces some errors in the final cost [3] [4]. However, using CUC can have negative effects on the viability of the generated schedule, such as overestimating system flexibility due to the aggregation [4], although methods to overcome this limitation have also been found [5].

A common constraint used in the unit commitment problem is the minimum up and down time. This constraint aims to model the fact that due to technical limitations, some types of generators cannot be started up or shut down in consecutive hours. This constraint is very commonly explored in literature, as was shown by [6], wherein out of the 142 UC models explored, 129 modeled minimum up and down time. This type of constraint is especially relevant when renewable energy sources are considered alongside thermal generators. This is because the need for flexibility caused by intermittent renewable energy sources can lead to more cycling operations for thermal generators [7], which is sometimes not possible to achieve in the real world due to the technical limitations of these generators. Furthermore, the inclusion of minimum up and down constraints into UC models has been found to provide a more accurate model solution [8]. The most common formulation used for these constraints is the turn on/off inequalities proposed by [9], and while these inequalities are defined for binary variables, adaptations which use clustered unit commitment have also been found [4].

When unit commitment is considered, the most common temporal resolution used is one hour. The literature survey conducted by Montero et al. [6] shows that out of the 142 models studied, only 12 used resolutions different than one hour, with some using more fine grained resolutions (30 min, 15 min), or less fine grained by having a larger time period described by fewer time steps. However, because the use of one hour resolution is computationally infeasible for long-term planning models, aggregation methods such as the selection of representative days or weeks from the planning horizon are used [10]. Newer research also suggests the aggregation method of time period clustering, which clusters the whole planning horizon into blocks, and was found to be more accurate than representative periods while retaining the same performance benefit [11]. Another type of temporal clustering is the use of fully flexible time resolutions, which allows removing variables and constraints which are not needed, and also allows modeling different variables at different resolutions. This has been shown to reduce computation time while affecting the solution accuracy of specific parts of the model less when compared to other time resolution clustering methods [12].

Recently, researchers working on the Tulipa Energy Model [13] have developed a generation expansion planning model which supports fully flexible clustered temporal resolutions [14]. This allows for the clustering of parameters together, resulting in a model with fewer variables and constraints. The inclusion of UC constraints into a model such as Tulipa can provide insights into the effect of fully flexible temporal resolutions on the GEP problem when UC constraints are also included. This represents a knowledge gap in current research, as most models use one hour uniform resolutions for modeling such constraints.

Thus, the aim of this paper is to bridge this knowledge gap by investigating the effect of adding unit commitment constraints, more specifically minimum up and down time constraints, to a model with fully flexible clustered temporal resolution, both in terms of the optimal solution cost and investment mix, as well as computation time. The paper shows that when the constraints are added the computation time increases, the optimal solution cost and investment plan have only minor changes, and the benefit of fully flexible temporal resolutions is still present.

The rest of the paper is structured as follows. First, Section 2 introduces the mathematical formulation used for the added constraints, after which the experimental setup and results are presented in Section 3. In Section 4 the results of the experiments are discussed, and finally the conclusion of the paper is presented in Section 5.

2 Mathematical Formulation

This section describes the mathematical formulation of the model. First, an overview of the Tulipa energy model is presented, after which the unit commitment variables and constraints are introduced. Finally, the formulation of the minimum up and down time constraints which will be tested is presented and verified using a small case study.

2.1 Overview of the Tulipa Energy Model

The Tulipa energy model uses Mixed Integer Linear Programming (MILP) to formally define and solve the GEP problem [14]. This means that the input of the problem is a set of parameters, such as generator properties and demand profiles. The model then generates a set of variables, as well as an objective function to optimize using these variables, namely the cost of satisfying the electricity demand. The variables are subject to a set of constraints, which are based on the parameters given as input, and model limitations such as the maximum energy output of a generator or ensure that the demand is met during every time period. Since it is a MILP model, Tulipa supports both integer and continuous variables, and requires that the constraints added to the model are linear.

The Tulipa energy model uses the Julia programming language [15] and the JuMP [16] modeling language.

In order to consider future investments and demand while keeping the size of the model computationally tractable, instead of modeling every year, a set of milestone years $\mathcal{Y}$ is chosen, these being the years which will have operational constraints and investments. Furthermore, only a set of representative days $\mathcal{K}_y$ is considered for ensuring the feasibility of the investment plan instead of the whole year. This is done again with the aim of reducing model size, and for each representative day constraints regarding the operation of the assets and satisfying the demand are generated.

2.2 Unit Commitment Variables and Constraints

For the assets which have unit commitment constraints ($a \in \mathcal{A}_y^{\text{uc}}$), the variable $v_{a,k_y,b_{k_y}}^{\text{units on}}$ defines how many units of a given asset are on at a given time. This variable is indexed over asset, representative period and also time block $b_{k_y} \in \mathcal{B}^{\text{uc}}$. Because temporal resolutions can be flexible, different variables have different temporal resolutions, so the 24 hours of a day can be split into different time blocks. In the case of the $v^{\text{units on}}$ variable, the ordered set $\mathcal{B}^{\text{uc}}$ gives the set of time blocks defined for this variable. For example, if the temporal resolution is set to be uniform in clusters of 8 hours, $\mathcal{B}^{\text{uc}} = [[1:8],[8:16],[16:24]]$.

The variables $v_{a,k_y,b_{k_y}}^{\text{start up}}$ and $v_{a,k_y,b_{k_y}}^{\text{shut down}}$ represent how many units of a given asset are started up or shut down in a given time block, where b_{k_y} is an element of the temporal resolution sets of the two variables, namely $\mathcal{B}^{\text{su}}$ and $\mathcal{B}^{\text{sd}}$. These sets are defined based on the highest common temporal resolution of variables relating to an asset. This resolution, named $\mathcal{B}^{\text{highest}}$, is defined by considering both the unit commitment variable $v^{\text{units on}}$, as well as all of the flow variables v^{flow} of an asset [12]. The sets $\mathcal{B}^{\text{su}}$ and $\mathcal{B}^{\text{sd}}$ are then defined in the temporal resolution of $\mathcal{B}^{\text{highest}}$, but only the time blocks which start at the same time as a time block in $\mathcal{B}^{\text{uc}}$ are considered. This is done because an asset can only start up or shut down when there is a change in the unit commitment variable. Figure 1 gives an example of how $\mathcal{B}^{\text{highest}}$ is defined, using one flow variable with a uniform 2 hour resolution, and the unit commitment variable modeled at 3 hour uniform resolution. The figure also shows how the sets $\mathcal{B}^{\text{su}}$ and $\mathcal{B}^{\text{sd}}$ are defined for this example. Because they are defined in the same way, the sets $\mathcal{B}^{\text{su}}$ and $\mathcal{B}^{\text{sd}}$ are equal.

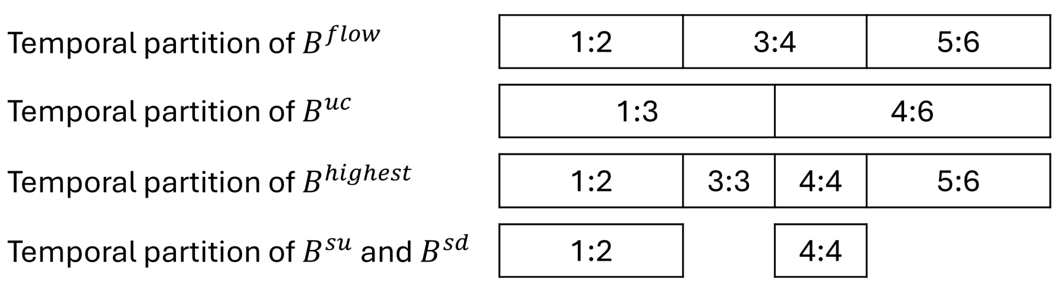


Figure 1: Example of the temporal partition of the $\mathcal{B}^{\text{highest}}$, $\mathcal{B}^{\text{su}}$ and $\mathcal{B}^{\text{sd}}$ sets based on the temporal partition of the flow and UC variables

The start up and shut down variables can be defined in terms of the $v_{a,k_y,b_{k_y}}^{\text{units on}}$ variable using constraints (1a) and (1b). The function $B_{a,y,k_y}^{\text{uc}}(b_{k_y})$ where $b_{k_y} \in \mathcal{B}_{a,y,k_y}^{\text{su}}$ gives the time block in $\mathcal{B}_{a,y,k_y}^{\text{uc}}$ which starts at the same time as b_{k_y}. Because constraint (1a) also refers to the previous time block $(b_{k_y} - 1)$, the constraint is only defined starting from the second time block in $\mathcal{B}_{a,y,k_y}^{\text{su}}$.

$$v^{\text{units on}}_{a,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y})} - v^{\text{units on}}_{a,k_y,B^{\text{uc}}_{a,y,k_y}(b_{k_y}-1)} = v^{\text{start up}}_{a,k_y,b_{k_y}} - v^{\text{shut down}}_{a,k_y,b_{k_y}} \quad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{\text{uc}}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{su}}_{a,y,k_y} \tag{1a}$$

$$v^{\text{start up}}_{a,k_y,b_{k_y}},\ v^{\text{shut down}}_{a,k_y,b_{k_y}} \in \mathbb{Z}_{\geq 0} \quad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{\text{uc}}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{su}}_{a,y,k_y} \tag{1b}$$

2.3 Minimum Up and Down Time Constraints

The minimum up and down time constraints can be seen as a restriction on the number of units which can be turned on or off respectively based on the number of units we have previously started up or shut down. They can therefore be modeled as a pair of linear constraints on the start up, shut down and units on variables of an asset at a given time block (2). The expression start(b) denotes the start time of a given time block, and the variable $v^{\text{available units}}_{a,y}$ gives the number of units of a given asset which are operational in a given year, and therefore includes both initial units as well as those invested into. The parameters $\underline{T}^{\text{up}}_a$ and $\underline{T}^{\text{down}}_a$ define the minimum up and down time respectively for the given asset, in hours.

$$\sum_{\substack{i \in \mathcal{B}^{\text{su}}_{a,y,k_y}:\ \text{start}(b_{k_y}) - \underline{T}^{\text{up}}_a + 1 \\ \leq \text{start}(i) \leq \text{start}(b_{k_y})}} v^{\text{start up}}_{a,k_y,i} \leq v^{\text{units on}}_{a,k_y,b'} \quad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{\text{uc}}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{su}}_{a,y,k_y}, b' = B^{\text{uc}}_{a,y,k_y}(b_{k_y}) \tag{2a}$$

$$\sum_{\substack{i \in \mathcal{B}^{\text{sd}}_{a,y,k_y}:\ \text{start}(b_{k_y}) - \underline{T}^{\text{down}}_a + 1 \\ \leq \text{start}(i) \leq \text{start}(b_{k_y})}} v^{\text{shut down}}_{a,k_y,i} \leq v^{\text{available units}}_{a,y} - v^{\text{units on}}_{a,k_y,b'} \quad \forall y \in \mathcal{Y}, a \in \mathcal{A}^{\text{uc}}_y, k_y \in \mathcal{K}_y, b_{k_y} \in \mathcal{B}^{\text{sd}}_{a,y,k_y}, b' = B^{\text{uc}}_{a,y,k_y}(b_{k_y}) \tag{2b}$$

The formulation of these constraints is based on the turn on/off inequalities introduced in [9], but have been adapted from using binary variables to integer ones with clustered unit commitment, as can be seen in [4]. Furthermore, the sum of start up and shut down variables accounts for the flexible temporal resolutions by only summing over time blocks which start between the previous time block still affected by the minimum up or down time restrictions, and the beginning of the current time block. The final mathematical formulation of the constraints was developed by researchers at TNO [17].

2.4 Verification of Minimum Up and Down Time Constraints

To ensure the correct implementation of the minimum up and down time constraints, a small case study was ran to verify the expected changes to the optimal solution. This case study was not intended to be realistic, and only served as a way to test the constraints. The data files of the case study can be found at [18]. In this case, the optimal solution invests into only one OCGT generator, which has a minimum up time of 3 hours, and a minimum down time of 5 hours. Figure 2 shows that without the added constraints, the optimal solution keeps one OCGT generator off for a single hour between 4 and 5, before turning it back on for one hour between 5 and 6. As can be seen in Figure 3, with the added constraints the generator is never on for less than 3 hours, being kept on for 9 at the start of the period, and is also off for 12 hours before being turned back on.

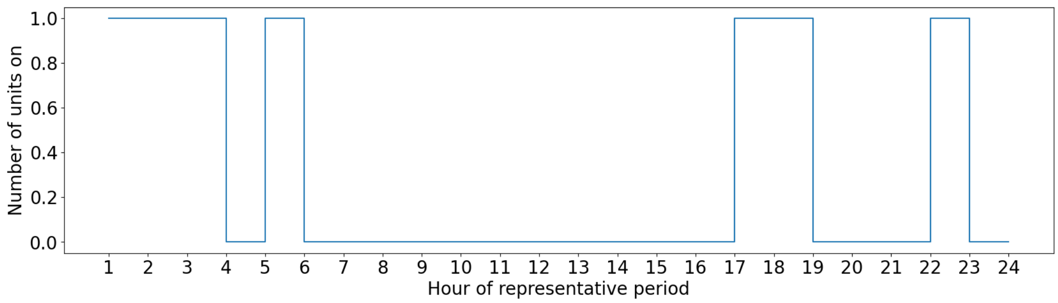


Figure 2: OCGT units on without minimum up and down time constraints

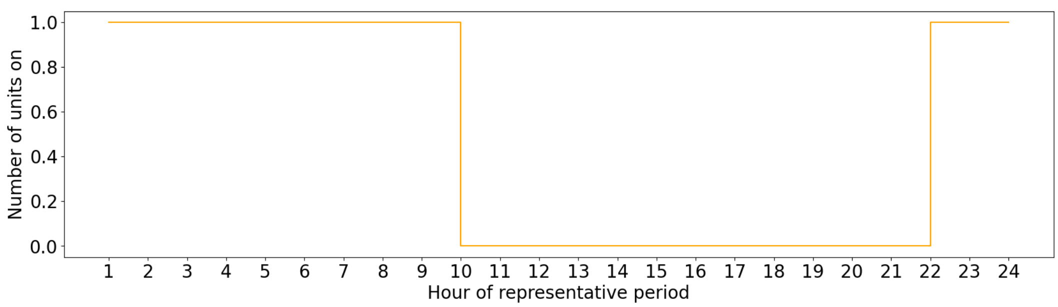


Figure 3: OCGT units on with added minimum up and down time constraints

3 Experimental Setup and Results

This section presents the experimental setup used for evaluating the new constraints, as well as the results that were found. First, the case study used for the experiments is explained, after which the structure of the experiments is described. Finally, the numerical results from the experiments are shown.

3.1 Case Study

The main case study used for the experiments models the energy system of the European countries, with the addition of the UK, Norway and Switzerland. While trade of electricity between countries is not modeled, the objective of this case study is to have a size and complexity which is comparable to real world applications of GEP and UC. The model includes electricity demand profiles, as well as the estimated availability of intermittent renewable energy sources for all countries in 10 distinct representative days of a single year. The parameters for these were aggregated in collaboration with TNO [19].

In terms of assets, the model includes both renewable energy sources in the form solar generation and onshore and offshore wind generation, as well as thermal generators such as Nuclear, Coal, CCGT and OCGT. The model also includes storage in the form of batteries, which can be used both to store excess energy in the case of overproduction, and to deploy stored energy back onto the grid in the case of underproduction.

In order to form a realistic investment plan, real world parameters for both investments as well as unit commitment have been aggregated. The estimated investment costs in the year 2030 from [20] were used for all types of assets. Because clustered unit commitment is used for the generators, for each asset type the capacity in MW was found by taking the mean of real world power plant capacities, using data from the World Resources Institute [21]. Unit commitment parameters for thermal generators such as the minimum operating point and the minimum up and down times were sourced from [22]. The ramping capabilities of thermal generators were sourced from [23], with the exception of nuclear generators for which the ramp up limit specified by [22] was used. Additionally, fuel costs for coal generators [24], CCGT and OCGT generators [25], as well as nuclear generators [26] were included in the model. The values for all of these parameters can be found in Table 1.

Table 1: Parameters used for the generator types in the case study

Asset Type	Investment Cost (kEUR/MW)	Unit Capacity (MW)	Minimum Op. Point	Max Ramp Up	Max Ramp Down	Min Up Time (h)	Min Down Time (h)	Fuel Costs (kEUR/MW)
Solar	485	18	-	-	-	-	-	-
Wind Onshore	1000	49	-	-	-	-	-	-
Wind Offshore	2580	49	-	-	-	-	-	-
Coal	1500	824	0.325	0.5	0.5	5	6	0.0177
Nuclear	5658	2095	0.5	0.3	0.3	6	10	0.0084
CCGT	775	357	0.45	1	1	3	2	0.0330
OCGT	475	357	0.2	1	1	3	2	0.0510

3.2 Experimental Setup

In order to assess the impact of minimum up and down time constraints on the optimal solution, two types of models were ran. The first model included only simple unit commitment constraints for thermal generators, namely the minimum operating point and ramping limits. The second model had these constraints, but also included minimum up and down time constraints. In both cases, the models began with no initial units, meaning that they have to find the optimal mix of generators and storage to support the electricity demand.

For the experiments, different methods of time period clustering were used for the representative periods. These resolutions affect the size of the model by changing the amount of parameters and variables generated for the constraints. The fully flexible temporal resolution test case was implemented by a geographically decreasing (GD) temporal resolution based on the energy connections of the target country, in the same way as [12]. This means that countries further away from the target are modeled at increasingly lower resolutions. In this case study, the target country is the Netherlands, so it is modeled at one hour resolution, whereas other countries are modeled at resolutions between 1 and 5 hours. This is based on the number of connections needed to reach the Netherlands, as can be seen in Figure 4.

This case was evaluated against other non-flexible types of temporal clustering, namely by using uniform clusters of 1, 2, 3, 4 and 6 hours. The uniform one hour resolution is the most commonly used approach, as well as the one with the highest detail and was thus considered as the baseline. The other uniform clusters were chosen in order to compare the benefit of fully flexible temporal resolutions as opposed to regular clustering methods. Thus, two types of models were ran, one with minimum up and down time constraints and one without, each of them having six different types of temporal resolutions, totaling 12 different experiments.

The results to measure in each experiment consisted of the computation time, the objective function cost, as well as the investment plan. The computation time was measured by considering both the time taken to generate the model variables and constraints and the time to solve the model, using a MIP optimality gap of 0.01% and taking 30 samples for each run, using randomized seeds for the solver. The other results, namely the objective function cost and investment plan, were determined by solving each model with the default seed (zero), to ensure replicable results across multiple runs. The cost of the final solution was measured using four different values. First, the total cost of the model, including both investment costs and operational costs such as fuel costs was measured, both for the whole case study as well as just for the target country. Then, investment costs were excluded and only operational costs were measured, both for the entire study and the

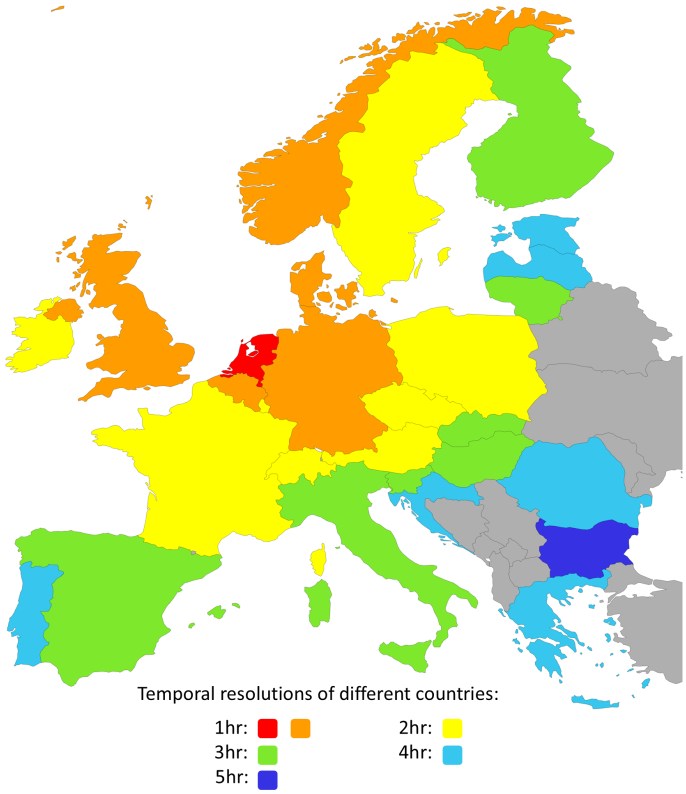


Figure 4: Temporal resolutions used for different countries in the geographically decreasing case

target country. The investment plan was investigated by looking at the capacity invested into by the model of each type of generation technology.

The models which add minimum up and down time constraints had the possibility of changing the investment plan. It was therefore also needed to assessed whether changes to the investment plan were required to make the model feasible, in the sense that without the changes it was impossible to meet the electricity demand at all times, or if they were only there to reduce the overall operational costs. This was checked by running a solution feasibility study, in which investments are turned off and the number of initial units of every asset is set to be equal to the investment plan of the models without the extra constraints. For these separate runs, the optimal solution cost, which now only consisted of operational costs, as well as the amount of demand not served were measured.

The case study and experiments were implemented using the Tulipa Energy Model version 0.15.1 [13] and solved using Gurobi version 11.0.3 [27]. All experiments were ran on a personal computer equipped with an AMD Ryzen 7 5800H 3.2GHz CPU, and 16GB of RAM. The input files for the experiments, as well as the files with the results can be found at [18].

3.3 Numerical Results

Computation Time

Table 2 shows the time taken to create the model for all of the experiments, whereas Table 3 shows the time taken to reach the optimal solution, within the 0.01% MIP gap. These tables include the mean runtime across the 30 runs and the standard deviation. Furthermore, for the models which add the minimum up and down time constraints, the percentage increase in model creation or solving time, when compared to the same model without constraints, is also shown.

Table 2: Time taken for model creation for different models

	Model Resolution					
Mean (SD) Runtime (s)	1hr	2hr	3hr	4hr	6hr	GD
Without MU/MD	4.68 (0.37)	2.57 (0.23)	2.06 (0.21)	1.68 (0.17)	1.34 (0.15)	2.63 (0.18)
With MU/MD	16.74 (1.34)	10.39 (0.87)	9.36 (1.28)	8.83 (0.59)	8.62 (1.52)	10.28 (1.15)
Percentage Increase	257.51%	303.59%	354.30%	424.68%	543.96%	290.63%

Table 3: Time taken for solving the model for different models

	Model Resolution					
Mean (SD) Runtime (s)	1hr	2hr	3hr	4hr	6hr	GD
Without MU/MD	160.9 (7.13)	47.5 (1.77)	25.4 (1.17)	15.6 (0.77)	8.5 (0.61)	38.2 (2.15)
With MU/MD	245.0 (11.9)	79.5 (4.76)	35.9 (2.01)	23.1 (1.00)	11.6 (0.79)	57.9 (3.13)
Percentage Increase	52.27%	67.11%	41.69%	48.11%	35.92%	51.35%

Model creation is shown to be more affected by the addition of minimum up and down time constraints than model solving in terms of percentage difference, with increases up to 543% in model creation, whereas model solving time only increases by at most 68%. However, when wall time is considered, the increases in model solving time are the more significant ones, for example in the one hour uniform resolution with MU/MD constraints, the model creation time increases by around 12 seconds, but model solving time increases by over 84 seconds.

Optimal Solution

The optimal solution cost for different temporal resolutions, without minimum up or down time constraints, is presented in Table 4. The total cost refers to the cost of both investments into assets and the cost of fuel, whereas the operation cost only includes the latter. The costs when only considering the Netherlands are also presented separately.

Table 4: Optimal solution costs for different models, in terms of total costs and operation costs, for the whole model or just the Netherlands.

	Model resolution					
Costs (MEur)	1hr	2hr	3hr	4hr	6hr	GD
Total	63029.93	62822.36	62517.05	62277.18	61525.78	62708.14
Total (NL)	2222.64	2211.72	2207.71	2204.59	2181.32	2222.75
Operation	12633.52	12483.97	12338.02	12495.73	11219.71	12352.62
Operation (NL)	396.47	396.95	408.39	400.58	337.01	395.39

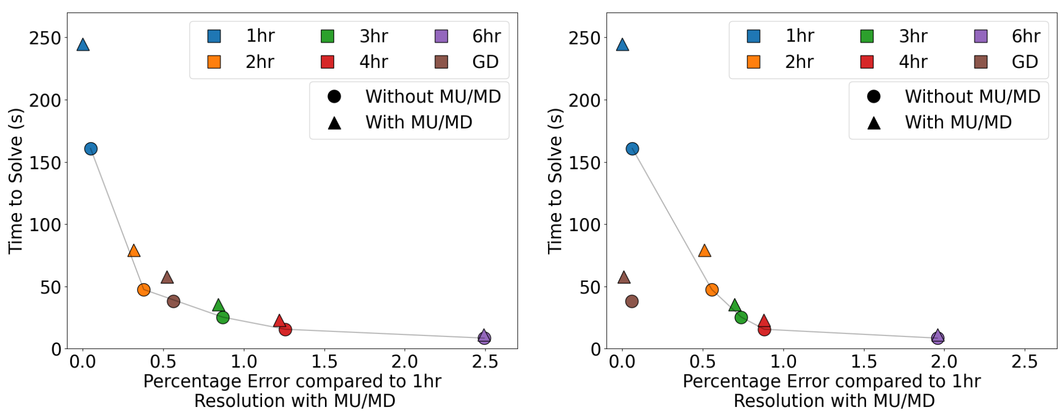
When minimum up and down time constraints are added, the total costs increase. The percentage increase, for both the entire case study and for the Netherlands only, can be seen in Table 5.

Table 5: Percentage increase of the cost for the model when constraints are added

Model Resolution	1hr	2hr	3hr	4hr	6hr	GD
Percentage Increase (total)	0.048%	0.062%	0.028%	0.037%	0.001%	0.041%
Percentage Increase (NL only)	0.062%	0.047%	0.042%	0.004%	0.000%	0.065%

The minimum up and down time constraints provide a small increase in optimal solution cost, for example in the geographically decreasing resolution case only a 0.041% increase compared to the same model without the constraints. Such a difference is however still above the MIP gap of 0.01%, meaning that it originates from a genuine difference in the investment or operational plan for the planned year. It can also be seen that for the 6 hour uniform resolution, the solution found by the model with the added constraints is very similar to the one without the constraints, since the cost increase of 0.001% is now well within the MIP gap. The possible reasons for this will be expanded upon in the discussion section.

Furthermore, the difference in objective function cost can be weighed against the increase in computation time, as can be seen in Figure 5. The percentage error is measured against the uniform one hour resolution, with the minimum up and down time constraints included, as this is considered to be the most accurate solution. This is once again done for both the entire case study and when only considering the target country, namely the Netherlands.



(a) Cost for the whole case study

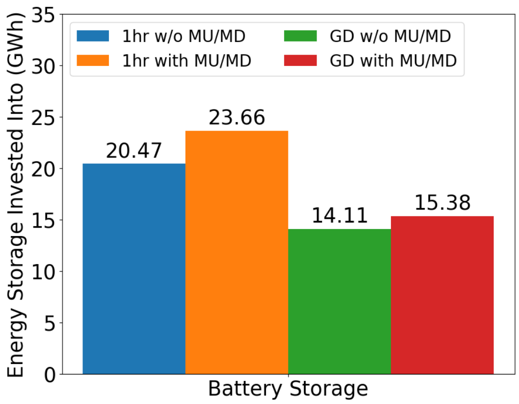
(b) Cost for NL only

Figure 5: Time to solve the model against percentage error compared to 1hr resolution with minimum up and down time constraints

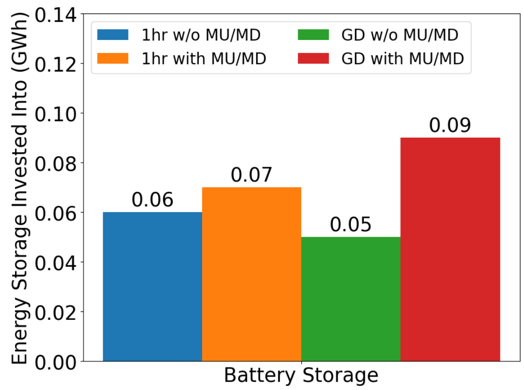
What can be seen is that the addition of minimum up and down time constraints make the solution more accurate, with the exception of the 6 hour resolution where no difference is observed. However, this increase in accuracy also comes with a higher computation time, meaning that the models with the MU/MD constraints do not exceed the Pareto front formed by the models without MU/MD. The sole exception to this is when only the cost for the target country is considered, because in this case both of the GD resolution models exceed the Pareto front.

Investment Plan

The investment plan for generators had insignificant differences when minimum up and down time constraints were added to the model, but some differences were found in the amount of maximum energy storage that was invested into. Figure 6a shows the energy storage investments, in GWh, for the entire case study, and Figure 6b restricts this to only include the Netherlands. Both of these figures consider four of the models, namely the one hour uniform and GD resolutions, with and without minimum up and down time constraints.



(a) Entire case study



(b) NL only

Figure 6: Investments into battery storage for the 1hr and GD resolution models, with and without minimum up and down time constraints

It can be seen that the amount of energy storage has a significant increase when minimum up and down time constraints are added, and this is consistent both for the entire case study and for the target country. It should be noted that in the investment plan the amount of battery deployment assets, which represent how much of the already stored energy can be deployed onto the grid during one hour, did not change significantly between the models with and without MU/MD.

Solution Feasibility

When the solution feasibility study was ran, none of the models had any demand which was not served. The new operational costs for each resolution can be found in Table 6, which also shows the percentage increase for models with the added minimum up and down time constraints. Note that the operational costs differ slightly from those found in Table 4, which is caused by the solver finding a better operational plan, now that the MIP gap of 0.01% only includes operations and not also investments.

Table 6: Feasibility study costs for models without MU/MD for different resolutions, and percentage increase when MU/MD constraints are added

Model Resolution	1hr	2hr	3hr	4hr	6hr	GD
Operational Cost (MEur)	12633.14	12483.83	12338.02	12495.73	11219.71	12352.41
Percentage Increase with MU/MD	0.315%	0.375%	0.178%	0.238%	0.000%	0.250%

What was found is that the percentage increase when MU/MD constraints are added is more significant if the investment plan cannot be changed. For example, in the one hour uniform case the percentage increase when investments were allowed was 0.048% (see Table 5), but when the investment plan was not able to be changed, the cost difference was 0.315%.

4 Discussion

The objective of this paper was to investigate the effect of adding minimum up and down time constraints to a generation expansion planning model with fully flexible temporal resolutions, in terms of computation time, optimal solution cost and investment plan.

In terms of computation time, the models that include the minimum up and down time constraints, using the formulation adapted for fully flexible temporal resolutions (2), were found to be both slower to create and to solve than the models without the constraints. The extra time taken for model creation is an expected result, as for each time block of each representative period new constraints need to be computed and added to the model creation. However, the increase in solving time when the new constraints are added is unexpected when considering findings by [8], which indicated that minimum up and down time constraints on their own had only a small impact on model solve time. A possible explanation for why in this case the constraints had a more significant impact is that [8] only considered the unit commitment problem, without investments into assets. This paper therefore suggests that minimum up and down time constraints have a bigger impact on computation time when generation expansion planning is performed alongside unit commitment.

In this case study, the addition of minimum up and down time constraints had a small effect on the total cost of the model. This is in line with the findings of [28], which showed that unit commitment constraints in generation expansion planning don't influence the total cost when the case study has sources of flexibility other than thermal generators. In our case study the important source of flexibility is energy storage in the form of batteries. This therefore suggests that the high degree of flexibility in the model is the reason why the cost differences are small when MU/MD constraints are added. An interesting finding is that the cost difference for the 6 hour uniform model was within the MIP gap, which shows the MU/MD constraints had no effect on the model. This is indeed the case, and can be explained by the fact that the model did not invest into any nuclear power. None of the minimum up and down time parameters of the other technologies exceed 6 hours (see Table 1), and since the use of a 6 hour uniform resolution means that generators can only be turned on or off every 6 hours, the minimum up and down time constraints were already satisfied. However, the extra constraints still had an effect on the model creation and solving time, which indicates that they should not be part of the model in such cases. Since the minimum up and down time parameters, as well as the temporal resolution used for the unit commitment variable are known for each asset when the model is created, the constraints can be removed as part of a preprocessing step in order to speed up model creation and solving.

The Pareto front in Figure 5b shows that the geographically decreasing fully flexible temporal resolution is useful when we are interested in the cost for the target country more than the cost for the whole model. This is line with findings by [12], but it can be seen that when minimum up and down time constraints are added, the GD resolution model still exceeds the Pareto front, and ends up being slightly more accurate than GD without MU/MD. This indicates that the benefit of fully flexible temporal resolutions is achievable even when unit commitment constraints such as minimum up and down times are added to the model. An unexpected result is the fact that the geographically decreasing resolution is still on the Pareto front when we consider the entire case study (see Figure 5a), which is different from the findings in [12]. More specifically, in both cases the GD resolution accuracy for the whole model lies between those of the 2 hour and 3 hour uniform resolutions, but in our case study the GD model runs faster than the 2 hour, which is where the difference occurs. This difference in computation time could be caused by the fact that different assets were used in this case study, not modeling assets such as pumped hydro and electrolyzers, instead opting to model more types of thermal generators. It could also be caused by the lack of electricity trade between countries in the current case study.

In terms of the investment plan, small changes were found in the investments for generators, with the only significant differences originating from the investments into battery storage. Both of these findings are also supported by [28], which showed that investments into thermal generators and renewable energy sources are not significantly affected by the inclusion of unit commitment constraints, but investments into flexibility providers such as storage are affected. The findings in this paper therefore support the idea that unit commitment constraints, in this case minimum up and down time constraints, are important if generation expansion planning models want to investigate investments into energy storage or other flexibility providers, but they are not needed if the main goal is only finding the total cost or the investments into generators. The feasibility study shows that the changes to the investment plan when the minimum up and down time constraints are added are made to reduce operational costs, since all demand can be served even when the extra storage capacity is not invested into. Furthermore, the fact that the optimal solution cost when MU/MD constraints are added had a more significant increase with investments turned off when compared to running the full model strengthens this belief.

Responsible Research

During the process of this research, steps were taken to ensure the reproducibility and replicability of the presented results. In the interest of reproducibility, the input files for each experiment were published in the form of a GitHub repository, adhering to the principles of F.A.I.R. data. Furthermore, the software used for the experiments, the versions used, as well as the hardware used to run the experiments was also stated. In addition, the output files containing the measured results for each experiment have also been published. Replicability was ensured by using open source software for the creation of the models, namely the Tulipa Energy Model [13]. This allows for the creation of different models using new data, and existing documentation on how input data is structured in the Tulipa Energy Model ensures interoperability. An issue in terms of reproducibility is the use of closed source software for model solving, namely using the Gurobi solver [27]. While open source solvers such as HiGHS [29] were tried, for large models such as the ones ran for this paper computation times between 10 and 100 times longer were observed, which made it infeasible to use such solvers given the limited time frame of this project. Gurobi was chosen since it is free to use for research purposes, but the paper acknowledges that this can change in the future and that the need to switch to a different solver may affect the reproducibility of the results.

5 Conclusions and Future Work

This paper contributes to the existing literature by showing the effects of minimum up and down time constraints on the generation expansion planning problem with fully flexible temporal resolutions. This was done by first creating a realistic case study which models the energy grid of the European countries, with the addition of the UK, Norway and Switzerland. Then, the changes when minimum up and down time constraints are added were measured across different temporal resolutions, with the main focus being the geographically decreasing fully flexible time resolution.

A first conclusion is that the computation time of the generation expansion planning model increases when minimum up and down time constraints are added, both for the creation of the model as well as solving it. A second conclusion is that for the case study used in this paper, the addition of minimum up and down time constraints had a relatively small effect on the optimal solution cost and the investments into generators, but had a more significant effect on the amount of energy storage invested into. A third and final conclusion is that the benefit provided by fully flexible time resolutions in terms of the tradeoff between computation time and solution accuracy is still present when minimum up and down time constraints are added to the model.

The findings presented by this paper should be interpreted while also considering the limitations of this study. Firstly, the case study in this paper used a greenfield mode, meaning that no initial units were present and only one year was considered for the model. While this type of approach is common in literature [28], a case study which includes initial units and also models multiple years would be more realistic. Secondly, the investments into generators had no investment limits, which caused the model to opt for a large capacity of onshore wind generation. Such decisions may or may not be possible when factors such sufficient land area are taken into account. The use of proper investment limits would be able to prevent the model from generating solutions which are infeasible in the real world. Thirdly, some types of energy assets such as pumped hydro and electrolyzers were not considered in this study. Finally, while this paper aimed to model economic parameters as accurately as possible, further improvements can still be made, such as considering maintenance costs for generators and having different fuel costs for each country. This also includes modeling electricity trade between countries, which is a common occurrence in the real world and also represents a different source of model flexibility.

The conclusions presented above indicate that the use of fully flexible temporal resolutions in generation expansion planning can be expanded with the addition of unit commitment constraints such as minimum up and down times. Future research could investigate the impact of minimum up and down time constraints in different case studies. This can be in the form of addressing the limitations of the current case study which were presented above, or by considering the effect of removing other flexibility providers such as batteries. While some experiments with the inclusion of trade were ran as part of this study, as can be seen in Appendix A, further research is needed to investigate the impact of minimum up and down time constraints in case studies which also model trade of electricity. In order to improve the model creation times when minimum up and down time constraints are included, more efficient implementations for creating the constraints can also be researched. One possible optimization could be removing MU/MD constraints which are already satisfied due to the use of a lower temporal resolution in the unit commitment variable, as was shown for the 6 hour uniform case. Future studies are also encouraged to investigate the addition of other types of unit commitment constraints, such as start up and shut down capabilities, in models with fully flexible time resolutions. Finally, another avenue for future research is identifying other use cases for fully flexible temporal resolutions and seeing their effect on the generation expansion planning and unit commitment problems.

6 Acknowledgments

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A Europe Case Study with Trade Between Countries

This appendix will first explain the changes made to the main case study of the paper in order to include trade of electricity between countries, as well as changes to the experimental setup. Then, the numerical results will be presented and shortly discussed in relation to the findings of the paper.

Case Study and Experiment Setup Differences

The main difference between the case study presented in the paper is the inclusion of trade of electricity between counties. This trade is modeled without costs, but is limited to 1 GW per hour for any connection. Moreover, the pairs of countries with connections were chosen to be the same as those found in [12].

In terms of the experimental setup, the uniform hourly resolutions were kept the same, so modeling 1, 2, 3, 4 and 6 hour uniform resolutions respectively. It should be noted that the trade between countries was also modeled at the same uniform resolution for each case. More major changes were made to the geographically decreasing (GD) resolution. In order to test the limits of fully flexible time resolutions, a distinction was made between the temporal resolutions of the unit commitment and flow variables. The mathematical formulation of fully flexible temporal resolutions allows for these two resolutions to not be multiples of each other [12]. To test the effect this has on minimum up and down time constraints, for the GD resolution case some countries used a flow variable modeled at 2 hour uniform resolution, whereas the UC variable was modeled at 3 hour uniform resolution, as can be seen in Figure 7.

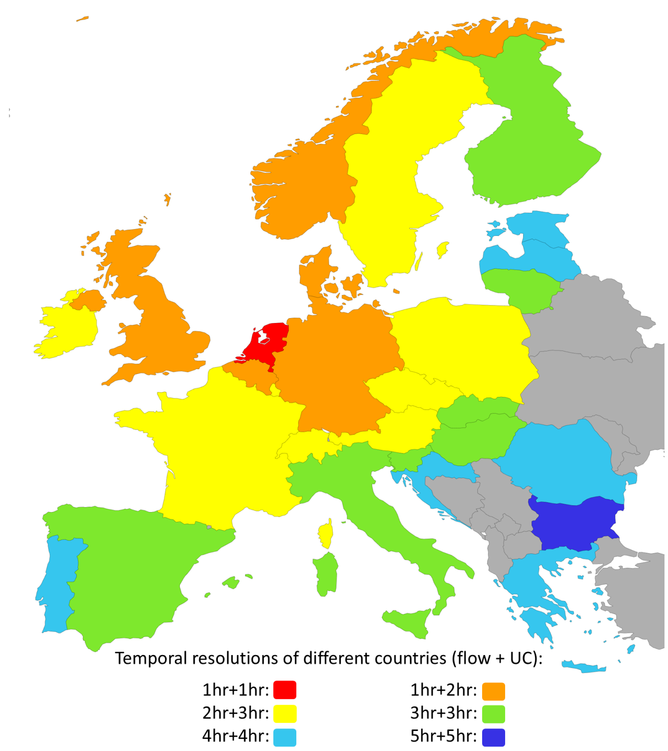

Figure 7: Flow and Unit Commitment temporal resolutions used for different countries in the geographically decreasing case

Finally, because the addition of trade increased solving times considerably, the MIP gap for the optimal solutions was increased from 0.01% to 0.1%. This was done in order to still be able to use 30 samples to evaluate mean computation times and standard deviations across all 12 cases.

Numerical Results

Computation Time

Tables 7 shows the model creation times, standard deviations and percentage increases when minimum up and down time constraints are added for all cases. Table 8 shows the same metrics, but for model solving times. These values were computed using 30 samples, and for solving a MIP gap of 0.1% and randomized seeds were used.

Table 7: Time taken for model creation for different models

	Model Resolution					
Mean (SD) Runtime (s)	1hr	2hr	3hr	4hr	6hr	GD
Without MU/MD	4.94 (0.38)	2.66 (0.19)	2.09 (0.15)	1.72 (0.18)	1.40 (0.14)	2.80 (0.17)
With MU/MD	16.19 (1.17)	10.34 (1.12)	8.82 (0.93)	8.38 (1.21)	8.16 (0.96)	11.46 (1.09)
Percentage Increase	228.01%	289.12%	322.41%	385.80%	482.61%	308.44%

Table 8: Time taken for model solving for different models, within 0.1% MIP Gap

	Model Resolution					
Mean (SD) Runtime (s)	1hr	2hr	3hr	4hr	6hr	GD
Without MU/MD	104.98 (10.17)	37.45 (3.73)	22.80 (6.47)	9.92 (3.04)	3.44 (1.33)	230.00 (71.21)
With MU/MD	232.22 (83.12)	51.55 (10.06)	32.78 (8.72)	12.98 (4.20)	6.98 (2.23)	358.99 (77.98)
Percentage Increase	121.20%	37.65%	43.79%	30.77%	102.78%	56.08%

It can be seen that the percentage increases on the model creation time when minimum up and down time constraints are added are very similar to those found in the case study without trade. However, in terms of model solve time more drastic differences can be seen, for example 1hr uniform resolution used to have a 52.27% increase, whereas now the increase is 121.20%. There are also large differences in the standard deviations for the models, for example GD without MU/MD used to have a standard deviation of 2.15s across the 30 samples for the case study without trade, whereas with trade and the same number of samples, the standard deviation is 71.21s. Finally, the GD temporal resolution was found to be significantly slower than the others, both with and without minimum up and down time constraints.

Optimal Solution Cost

Table 9 shows the optimal solution cost for the different models, as well as the percentage increase when the MU/MD constraints are added.

Table 9: Optimal solution costs for models with and without minimum up and down time constraints, and the percentage increase when the constraints are added.

Model Resolution	1hr	2hr	3hr	4hr	6hr	GD
Total Cost w/o MU/MD (MEur)	59039.25	58868.29	58553.14	58325.94	57611.30	58862.09
Total Cost with MU/MD (MEur)	59044.00	58876.33	58574.25	58343.80	57596.07	58872.42
Percentage Increase with MU/MD	0.008%	0.014%	0.036%	0.031%	-0.026%	0.018%

The cost was found to generally increase when the minimum up and down time constraints are added, but this was below the 0.1% MIP gap for the optimal solution, and therefore these increases in cost are not significant enough. It can also be seen that for the 6 hour uniform resolution, the extra constraints appear to have decreased the cost of the model, but this is still below the MIP gap and can be attributed to the solver finding a better solution for the model with the constraints.

Investment Plan

The investment plan for the whole model, in terms of the capacity invested into for each type of generator (rounded to the nearest integer) can be seen in Figure 8. It should be noted that offshore wind and nuclear generators are missing from the figure, as these were never invested into. The invested storage capacity for batteries, in GWh, can be seen in Figure 9.

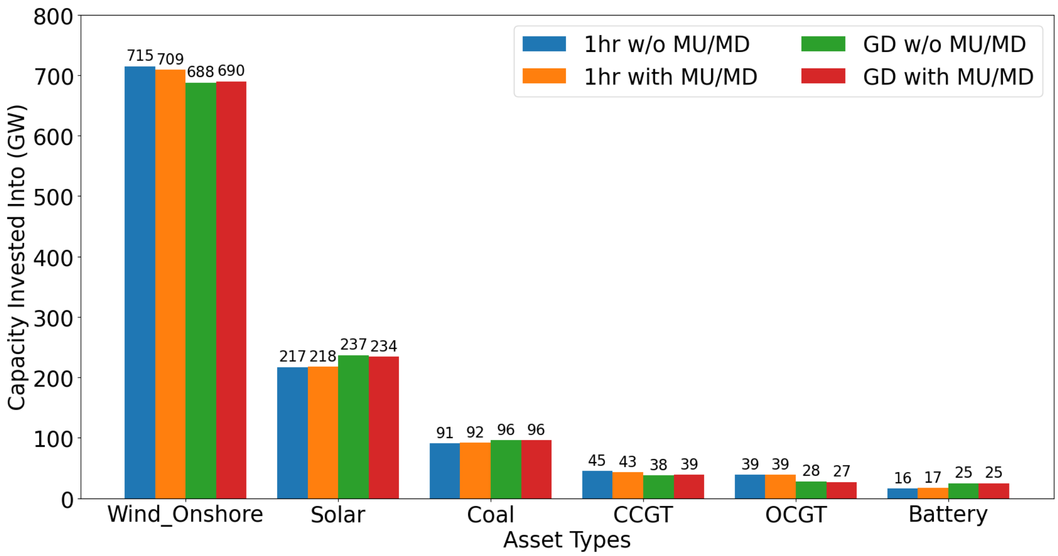

Figure 8: Capacity invested into for each asset type, for 1hr and GD resolutions with and without MU/MD constraints

The effect of minimum up and down time constraints on the investment plan for generators is not significant. More important changes can be seen when comparing the 1hr and GD resolution cases. For geographically decreasing, solar technology replaces some of the wind generators, and more battery assets are invested into. In terms of energy storage, Figure 9 shows that the addition of minimum up and down time constraints leads to slightly higher investments into battery storage.

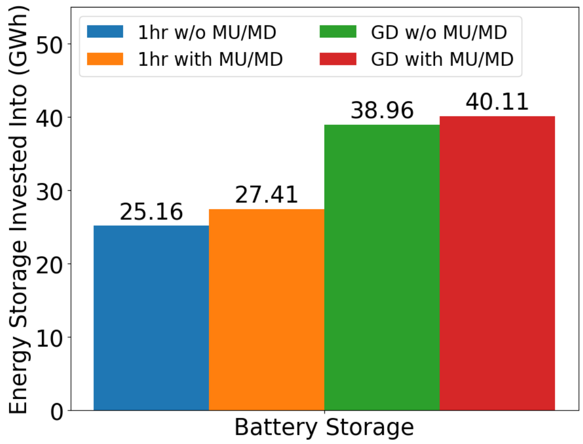

Figure 9: Storage capacity invested into for 1hr and GD resolutions with and without MU/MD constraints

Discussion

The first important difference when comparing the case study with trade to the one without is the increase in standard deviation values for model solving time. This difference is most noticeable for models which already have a high computation time, namely 1hr uniform with minimum up and down time constraints and both cases of GD resolution. This increase might be caused by the fact that the experiments were ran using a 0.1% MIP gap, which is ten times higher than the one used for the experiments without trade of electricity. This higher gap means that there are more solutions which are accurate enough to be below the MIP gap and accepted than previously. It is therefore believed that the high variance in solving times is caused by different seeds being able to find such a solution more quickly than others.

The second difference is the larger percentage increase in model solving time for the one hour uniform resolution. At first, this indicates that for some case studies the impact of minimum up and down times on solving time might be higher. However, it can also be seen that for the one hour uniform resolution with MU/MD, there is also a very high standard deviation, over 8 times larger than the deviation of one hour uniform without MU/MD. This indicates that the number of samples used (30) might not be enough to properly evaluate the impact of MU/MD constraints on solving time with higher MIP gaps.

Another important finding is that for the geographically decreasing resolution case, the time taken to solve the model is considerably longer than any of the other cases, being over two times longer than solving the one hour uniform resolution case. This was an unexpected result, and could be caused by either of the two changes made to this case, namely the inclusion of trade between countries and the use of different resolutions for the flow and UC variables. It should also be noted that for this resolution, the standard deviations in solving time of both the model with and without the minimum up and down time constraints were higher than before. This suggests that running more samples or using a lower MIP gap is needed in order to ensure reliable results.

Results such as the optimal solution cost and investment plan for generators were found to not have significant changes when minimum up and down time constraints were added, which is in line with the findings from the case study without trade. The only difference in investments when MU/MD constraints were added was the invested energy storage capacity, as the addition of the minimum up and down constraints increased the amount of energy storage invested into. This is in line with findings by [28], which state that the addition of unit commitment constraints in GEP models have an impact on investments into dedicated flexibility providers. However, this change is smaller than in the case study without trade. The smaller difference when trade is included can be attributed to the fact that the trade of electricity between countries offers another type of flexibility. Since trade comes at no additional cost, it can also be used to compensate for the need for flexibility introduced by the MU/MD constraints.

Conclusions and Future Work

With all of these findings considered, only two significant differences were found between the case study with trade and the one without trade. The first is the increased runtime for the geographically decreasing resolution, which could have been caused either by the addition of trade or by the use of different temporal resolutions for the flow and unit commitment variables within the same country. The second difference is the much greater variance in the run times for model solving, especially for the experiments with longer run times, which could be explained by the use of a higher MIP gap. Based on this findings, more research is needed when investigating the impact of minimum up and down time constraints with the addition of trade between countries. Firstly, the geographically decreasing case should be tested with the temporal resolutions for the flow and UC variables being identical, in order to have the same temporal configuration as in the case study without trade. Secondly, the experiments should be ran using a lower MIP gap, such as 0.01%, as was done for the experiments presented in the main paper. Finally, the experiments should also be ran with more than 30 samples, so as to have a more accurate estimate of the computation times both with and without minimum up and down time constraints.

References

[1] Nikolaos E. Koltsaklis and Athanasios S. Dagoumas. “State-of-the-art generation expansion planning: A review”. en. In: *Applied Energy* 230 (Nov. 2018), pp. 563–589. ISSN: 03062619. DOI: 10.1016/j.apenergy.2018.08.087.

[2] Bryan Palmintier and Mort Webster. “Impact of unit commitment constraints on generation expansion planning with renewables”. In: *2011 IEEE Power and Energy Society General Meeting*. 2011, pp. 1–7. DOI: 10.1109/PES.2011.6038963.

[3] Bryan S. Palmintier and Mort D. Webster. “Heterogeneous Unit Clustering for Efficient Operational Flexibility Modeling”. en. In: *IEEE Transactions on Power Systems* 29.3 (May 2014), pp. 1089–1098. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2013.2293127.

[4] Jelle Meus, Kris Poncelet, and Erik Delarue. “Applicability of a Clustered Unit Commitment Model in Power System Modeling”. en. In: *IEEE Transactions on Power Systems* 33.2 (Mar. 2018), pp. 2195–2204. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2017.2736441.

[5] Germán Morales-España and Diego A. Tejada-Arango. “Modeling the Hidden Flexibility of Clustered Unit Commitment”. In: *IEEE Transactions on Power Systems* 34.4 (July 2019), pp. 3294–3296. ISSN: 1558-0679. DOI: 10.1109/TPWRS.2019.2908051.

[6] Luis Montero, Antonio Bello, and Javier Reneses. “A Review on the Unit Commitment Problem: Approaches, Techniques, and Resolution Methods”. en. In: *Energies* 15.4 (Feb. 2022), p. 1296. ISSN: 1996-1073. DOI: 10.3390/en15041296.

[7] F. Aliprandi, A. Stoppato, and A. Mirandola. “Estimating CO2 emissions reduction from renewable energy use in Italy”. en. In: *Renewable Energy* 96 (Oct. 2016), pp. 220–232. ISSN: 09601481. DOI: 10.1016/j.renene.2016.04.022.

[8] Rogier Hans Wuijts, Marjan Van Den Akker, and Machteld Van Den Broek. “Effect of modelling choices in the unit commitment problem”. en. In: *Energy Systems* 15.1 (Feb. 2024), pp. 1–63. ISSN: 1868-3967, 1868-3975. DOI: 10.1007/s12667-023-00564-5.

[9] Deepak Rajan and Samer Takriti. *Minimum Up/Down Polytopes of the Unit Commitment Problem with Start-Up Costs*. en. Tech. rep. RC23628. IBM, June 2005.

[10] James H. Merrick. “On representation of temporal variability in electricity capacity planning models”. en. In: *Energy Economics* 59 (Sept. 2016). Publisher: Elsevier BV, pp. 261–274. ISSN: 0140-9883. DOI: 10.1016/j.eneco.2016.08.001.

[11] Salvador Pineda and Juan M. Morales. “Chronological Time-Period Clustering for Optimal Capacity Expansion Planning With Storage”. en. In: *IEEE Transactions on Power Systems* 33.6 (Nov. 2018), pp. 7162–7170. ISSN: 0885-8950, 1558-0679. DOI: 10.1109/TPWRS.2018.2842093.

[12] Zhi Gao et al. *Fully Flexible Temporal Resolution for Energy System Optimization*. en. Apr. 2025. DOI: 10.2139/ssrn.5214263.

[13] Abel Soares Siqueira et al. *Tulipa Energy Model*. Apr. 2025. DOI: 10.5281/ZENODO.8363262.

[14] Diego A. Tejada-Arango et al. *Tulipa Energy Model: Mathematical Formulation*. en. Sept. 2023. DOI: 10.48550/arXiv.2309.07711.

[15] Jeff Bezanson et al. “Julia: A Fresh Approach to Numerical Computing”. en. In: *SIAM Review* 59.1 (Jan. 2017), pp. 65–98. ISSN: 0036-1445, 1095-7200. DOI: 10.1137/141000671.

[16] Miles Lubin et al. “JuMP 1.0: Recent improvements to a modeling language for mathematical optimization”. In: *Mathematical Programming Computation* (2023). DOI: 10.1007/s12532-023-00239-3.

[17] Maaike Elgersma. personal communication. Apr. 24, 2025.

[18] Gabriel Tertelici. *MUMD-data*. June 2025. URL: https://github.com/GabrielTertelici/MUMD-data.

[19] Diego Tejada-Arango. personal communication. May 15, 2025.

[20] lisazeyen. *PyPSA/technology-data: v0.12.0*. May 2025. DOI: 10.5281/zenodo.15462938.

[21] *Global Energy Observatory, Google, KTH Royal Institute of Technology in Stockholm, Enipedia, World Resources Institute. 2021. Global Power Plant Database version 1.3.0.* en. URL: https://datasets.wri.org//datasets/global-power-plant-database (visited on 05/29/2025).

[22] Tom Brown et al. *PyPSA-Eur: An open sector-coupled optimisation model of the European energy system*. Apr. 2025. DOI: 10.5281/zenodo.15163251.

[23] Matthias Silbernagl, Matthias Huber, and René Brandenberg. *Improving Accuracy and Efficiency of Start-up Cost Formulations in MIP Unit Commitment by Modeling Power Plant Temperatures*. June 2015. DOI: 10.48550/arXiv.1408.2644. URL: https://arxiv.org/src/1408.2644v4/anc/ModelD-2025.xls (visited on 05/29/2025).

[24] *Coal-Fired Power*. Tech. rep. E01. IEA ETSAP, Apr. 2010. URL: https://iea-etsap.org/E-TechDS/PDF/E01-coal-fired-power-GS-AD-gct_FINAL.pdf (visited on 05/29/2025).

[25] *Gas-Fired Power*. Tech. rep. E02. IEA ETSAP, Apr. 2010. URL: https://iea-etsap.org/E-TechDS/PDF/E02-gas_fired_power-GS-AD-gct_FINAL.pdf (visited on 05/29/2025).

[26] *Nuclear Power*. Tech. rep. E03. IEA ETSAP, Apr. 2010. URL: https://iea-etsap.org/E-TechDS/PDF/E03-Nuclear-Power-GS-AD-gct_FINAL.pdf (visited on 05/29/2025).

[27] Gurobi Optimization, LLC. *Gurobi Optimizer Reference Manual*. 2024. URL: https://www.gurobi.com.

[28] Kris Poncelet, Erik Delarue, and William D’haeseleer. “Unit commitment constraints in long-term planning models: Relevance, pitfalls and the role of assumptions on flexibility”. en. In: *Applied Energy* 258 (Jan. 2020), p. 113843. ISSN: 03062619. DOI: 10.1016/j.apenergy.2019.113843.

[29] Q. Huangfu and J. A. J. Hall. *Parallelizing the dual revised simplex method*. 2018. DOI: 10.1007/s12532-017-0130-5. URL: https://github.com/ERGO-Code/HiGHS.