

# Water Balance Modeling for Estimation of Residence Time of Water in a Full-Scale Landfill using a Data-Assimilation Approach

T.J. Heimovaara, A. Bun and A.G. van Turnhout

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Department of Geoscience and Engineering, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands

## 1 Introduction

In order to develop novel approaches for reducing the after-care of Municipal Solid Waste (MSW) sanitary landfills methods are required with which we can quantify the emission potential present in waste bodies. Currently full-scale experiments are being prepared at three Dutch landfills based on enhanced infiltration by irrigation and leachate recirculation in combination with landfill aeration. The aim is to reduce the emission potential as fast as possible by stimulating the biological degradation of organic matter in the waste body. Since the summer of 2012 a base line monitoring program has been implemented at the three landfills which has resulted in a data set which can be used to quantify the water balance using high frequency measurements of meteorological data and pumped volumes of leachate. A simple landfill-scale water balance model has been developed which links rainfall, evapo-transpiration, infiltration and leachate drainage. This approach provides us with a method which allows us to obtain a quantitative estimate of the probability distribution of flow velocities and the pore volume in the waste body associated with this flowing water. The parameters in this model are obtained using a data assimilation approach, where the complete parameter distributions using a Markov Chain Monte-Carlo approach with the objective to obtain the best fit of measured leachate volumes and solute concentrations.

## 2 Theory

The approach for modeling the water balance is based on a similar concept developed by O'Reilly (2004). For a sanitary landfill we have the unique situation that it is possible to have accurate measurements of leachate discharge because of the presence of a bottom liner and all water pumped from the drainage system is measured. At the sites we investigated, cumulative pumped leachate ( $\text{m}^3$ ), levels in the pump pit (m) and the number of times that the pump switched on and off were recorded at a fifteen minute interval. The water produced in the drainage system derives from water infiltrating in to the waste body, which is the difference between the rainfall and the evapotranspiration. Rainfall is easily measured, leaving evapotranspiration to be quantified. The approach presented here is that evapotranspiration is optimized using long term data sets of measured leachate production which are combined with rainfall and potential evapotranspiration data provided by the Royal Dutch Meteorological Institute (KNMI).

The water balance model (shown in figure 1) consists of three layers: a cover layer, the waste body and finally the drainage system. Evapotranspiration is assumed to only take place in the top layer over the rooting depth ( $\Delta z_{root}$  [m]). If the top layer contains sufficient water, a fraction of this water infiltrates in to the waste body by gravity. We model the flow through the waste body using a stochastic transfer function approach. We assume that the flow is based on a bimodal lognormal distribution which allows for a certain fraction of the water in the waste body to flow (much) faster than the rest. Finally the last layer is the drainage layer. Because the pumps at our sites can easily maintain the preset levels in the pump pits we chose to disregard the drainage layer from the model for the analysis presented in this paper. The pumped leachate ( $q_{pump}$ ) is therefore equal to the leachate discharge ( $q_{leach}$ ).

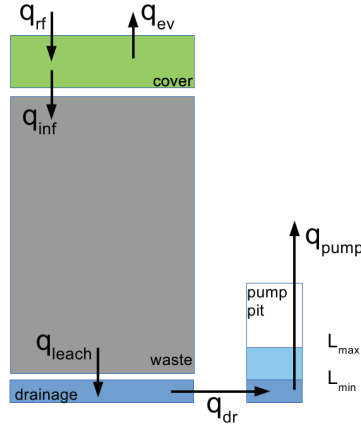


Figure 1: Conceptual water balance model for a landfill consisting of three layers. Arrows are water fluxes which are described in detail in the text.

## 2.1 Soil Cover (rooting zone)

The cover layer is modeled with the mass balance equation

$$\frac{\partial \theta}{\partial t} + \nabla \cdot q + Ev = 0 \quad (1)$$

which is discretised as

$$\frac{\Delta \theta}{\Delta t} + \frac{q_{rf} - q_{inf}}{\Delta z_{root}} + \frac{q_{Ev}}{\Delta z_{root}} = 0 \quad (2)$$

in which  $\theta$  is the volumetric water content;  $\Delta t$  is the time step used for the model which is based on the measurement frequency of the rainfall and potential evaporation data;  $q_{rf}$ ,  $q_{inf}$  and  $q_{Ev}$  are the rainfall, infiltration flux and evaporation flux in m/day;  $\Delta z_{root}$  is the rooting depth which we consider equal to the depth of the cover layer from which evaporation can occur.

The infiltration flux is based on an empirical power law relationship

$$q_{inf} = -K_{sat} \cdot S^m \quad 0 \leq S \leq 1 \quad (3)$$

where  $K_{sat}$  is the maximum saturated hydraulic conductivity in m/day,  $S$  is the effective saturation and  $m$  is an empirical parameter. The effective saturation is calculated with the well known equation

$$S = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad 0 \leq \theta \leq \theta_s \quad (4)$$

in which  $\theta_s$  and  $\theta_r$  are the saturated and residual water contents respectively. Please note that the gravity driven flow ceases when the water content is equal or smaller than the residual water content. In this model the water content can become smaller than the residual water content due to evaporation calculated from the potential evaporation rate ( $Ev_{pot}$  [m/day]) provided by the KNMI multiplied by an empirical crop factor in order to correct for the fact that growth on the landfill cover differs from the reference crop assumed for the calculation of  $Ev_{pot}$

$$q_{Ev} = Ev_{pot} \cdot C_{fact}. \quad (5)$$

We solve the mass balance of the cover layer with a straight forward approach. For every new time ( $t_{n+1}$ ) we calculate the new water content ( $\theta_{n+1}$ ) from the old water content ( $\theta_n$ ) and the known values of rainfall, infiltration and evaporation rates. However we need to ensure that new water content remains in the interval  $0 \leq \theta_{n+1} \leq \theta_s$ . We do this with the algorithm described in algorithm 1.

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**Algorithm 1** Approach to solve the water balance of the cover layer

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1. Estimate infiltration flux based on current water content and calculate the new water content

$$q_{inf_n} = -K_{sat} \cdot S(\theta_n)^m \quad (6)$$

$$\theta_{est_{n+1}} = \theta_n + (q_{rf_{n+1}} - q_{inf_n} + q_{Ev_{n+1}}) \cdot \frac{\Delta t}{\Delta z_{root}} \quad (7)$$

2. if  $\theta_{est_{n+1}} > \theta_s$

this implies that the layer became saturated during time step and excess water needs to be drained to the waste layer:

$$q'_{inf_n} = q_{inf_n} + (\theta_s - \theta_{est_{n+1}}) \cdot \frac{\Delta z_{root}}{\Delta t}, \quad (8)$$

so  $\theta_{n+1}$  becomes equal to  $\theta_s$  with

$$\theta_{est_{n+1}} = \theta_n + (q_{rf_{n+1}} - q'_{inf_n} + q_{Ev_{n+1}}) \cdot \frac{\Delta t}{\Delta z_{root}}. \quad (9)$$

3. else if ( $\theta_{est_{n+1}} < 0$  and  $\theta_n > \theta_r$ ) then too much water has drained or evaporated from the layer. We solve this by limiting the amount of water infiltrating to the waste layer with:

$$q'_{inf_n} = (\theta_r - \theta_n) \cdot \frac{\Delta z_{root}}{\Delta t} \quad (10)$$

and recalculate the estimated updated water content with equation 9.

4. if ( $\theta_{est_{n+1}} < 0$ ), even after correcting the infiltration rate, the evaporation rate is too high and can be corrected with

$$q'_{Ev_{n+1}} = q_{Ev_{n+1}} + \theta_{est_{n+1}} \cdot \frac{\Delta z_{root}}{\Delta t} \quad (11)$$

and the estimated water content can be calculated with

5. Once all conditions are fulfilled ( $0 \leq \theta_{est_{n+1}} \leq \theta_s$ ) then the updated value of the water content is set to the estimate

$$\theta_{n+1} = \theta_{est_{n+1}}. \quad (12)$$

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## 2.2 Waste Body: Bi-modal log normal travel time distribution

We chose to model the flow of water through the waste body with a transfer function approach Jury and Roth (1990); Zacharof and Butler (2004b,a); Rosqvist and Destouni (2000); Rosqvist *et al.* (2005)

$$q_{leach}(t) = \int_0^{\infty} q_{inf}(t - \tau) \cdot f(\tau) d\tau. \quad (13)$$

This approach assumes water to move through the waste body with a fixed distribution of velocities leading to a distribution of residence times. The distribution we chose is a bimodal lognormal distribution allowing us to incorporate two modes of flow. A fraction  $\beta$  of water flows relatively fast, the remainder  $(1 - \beta)$  flows slowly:

$$f(\tau; \mu_{fast}, \sigma_{fast}, \mu_{slow}, \sigma_{slow}) = \frac{\beta}{\tau \sigma_{fast} \sqrt{2\pi}} \exp\left(-\frac{(\ln \tau - \ln \mu_{fast})^2}{2\sigma_{fast}^2}\right) + \frac{1 - \beta}{\tau \sigma_{slow} \sqrt{2\pi}} \exp\left(-\frac{(\ln \tau - \ln \mu_{slow})^2}{2\sigma_{slow}^2}\right) \quad (14)$$

where  $\tau$  is the residence time of the water in the waste body,  $\mu_{fast}$  and  $\mu_{slow}$  are the mean residence times of the fast and slow flow water and  $\sigma_{fast}$  and  $\sigma_{slow}$  are the standard deviations of the log normal distributions. The fast flow fraction is given by  $\beta$ .

## 2.3 Data assimilation approach

In order to run our model we need to find the values for 12 parameters in our model:  $\Delta z_{root}$ ,  $C_{fact}$ ,  $\theta_r$ ,  $\theta_s$ ,  $K_{sat}$ ,  $m$ ,  $\mu_{fast}$ ,  $\sigma_{fast}$ ,  $\mu_{slow}$ ,  $\sigma_{slow}$ ,  $A_{LF}$  and  $\beta$ . Obtaining these parameters using a simple fitting approach is not feasible, as many local minima are present in the objective space. In stead, we chose to use a stochastic optimization approach based on a Bayesian inference scheme. We used the algorithm implemented in DREAM(ZS) Laloy and Vrugt (2012) to find the posterior distribution of these 12 parameters. A major advantage of this approach is that we not only obtain the parameter values that optimally describe the measured data set, but we obtain the complete statistical distribution of these parameters. This provides us with information about model sensitivity, model correctness, parameter identifiability and finally the presence of correlations between different parameters.

## 3 Results

Figure 2 shows the optimal model prediction compared with the measured data, both as cumulative discharged leachate and 5 day averaged leachate fluxes. The model gives a close fit and is able to capture the leachate dynamics as function of time. The optimal parameter values are given in table 1.

## 4 Conclusion and Outlook

The three year water balance for the Braambergen landfill can be described with a very simple model which is driven by daily rain fall and potential evaporation. The model has enough degrees of freedom to closely approximate the measured results. However, interpreting the parameters in a physical sense should be done with care. The model is empirical and although we have given the concepts a physical meaning this does not necessarily have to be true. In principle we may state that the storage volume in the top layer allowing for sufficient evapotranspiration to occur is 4.1 mm/m<sup>2</sup>. The parameters for the gravimetric flow of water from the top layer to the waste are reasonable for natural soils ( $m \approx 3$ ) and the travel times are very reasonable, fast flow occurs in about 8.5 days, whereas slow flow occurs in about 64 days. The estimated surface area for the landfill is underestimated (8.33 ha in comparison with the expected 9.7 ha).

The information on residence time of the water in the waste body will be important input for a solute leaching model in order to predict the decrease with time of the emission potential.

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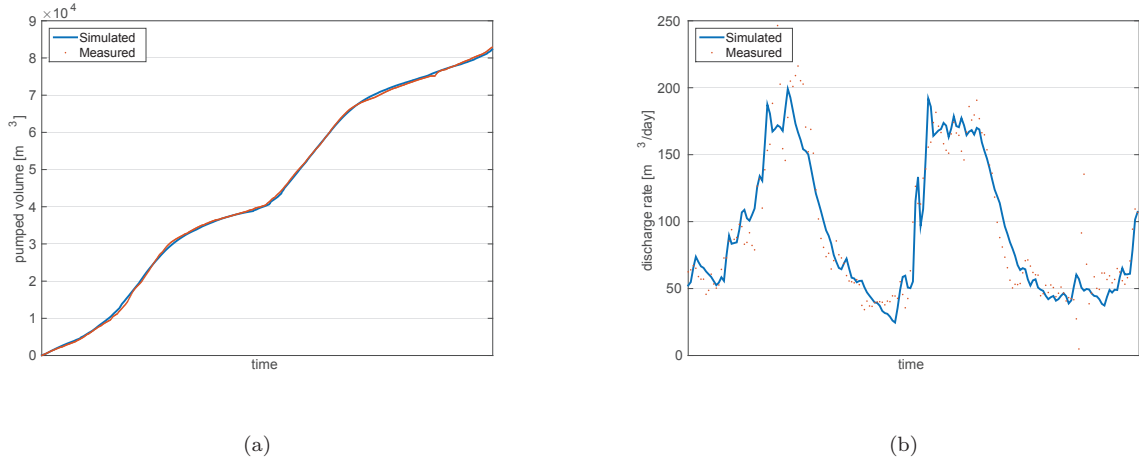


Figure 2: Comparison between simulated and measured cumulative discharge (left) and corresponding 5 day average fluxes (right)

Table 1: Optimal parameters for water balance model.

name	value	name	value	name	value
$\Delta z_{root}$	0.11 [m]	$K_{sat}$	$3.5 \times 10^{-3}$ [m/s]	$\mu_{slow}$	63.56 [days]
$C_{fact}$	0.86 [-]	$m$	3.23 [-]	$\sigma_{slow}$	0.76
$\theta_r$	0.00 [-]	$\mu_{fast}$	8.47 [days]	$A_{LF}$	8.33 [ha]
$\theta_s$	0.37 [-]	$\sigma_{fast}$	$2.74 \times 10^{-6}$ [days]	$\beta$	0.13 [-]

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