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# A mechanics-based approach for modelling dowel cracking in RC beams

Jiandong Lu[1*][0000-0001-5217-4019], Yuguang Yang[1] and Max A.N. Hendriks[1,2]

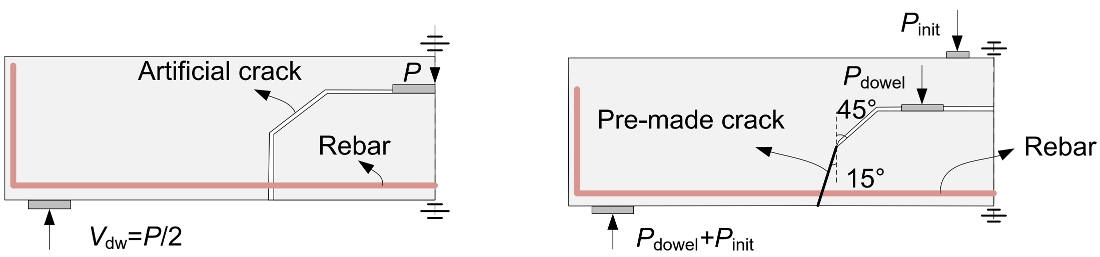
[1] Delft University of Technology, Delft, the Netherlands
[2] Norwegian University of Science and Technology, Trondheim, Norway
J.Lu-1@tudelft.nl

**Abstract.** Dowel action is recognized as one of the major shear resistance mechanisms. Although the dowel action only contributes a relatively small portion of the total shear resistance, the shear failure of reinforced concrete members without shear reinforcement usually occurs accompanied by unstable dowel cracking. This paper presents a new description of the dowel splitting process and a mechanical model based on two theories, namely the Beam on Elastic Foundation (BEF) theory and fracture mechanics. The mechanical model analytically describes the three stages during the dowel splitting of the longitudinal rebar in an RC beam, which are the elastic stage, stable cracking stage and unstable cracking stage. The proposed model can capture the post-peak behaviour of the nature of dowel action. Finally, a simplified equation for engineering practices is proposed. The proposed expression shows promising agreement with the experimental data.

**Keywords:** Dowel action; Unstable dowel splitting; Fracture mechanics; Beam on Elastic Foundation theory; Shear failure.

## 1 Introduction

Dowel action is considered to be one of the elementary shear transfer mechanisms. It stands for the mechanism that reinforcement carries the shear force perpendicular to its axial direction. It was first reported by Friberg [1] in pavement joints. Acharya and Kemp [2] pointed out the importance of taking dowel action into account in shear resistance calculation for reinforced concrete (RC) beams without shear reinforcement. Krefeld and Thurston [3] first developed a block-type beam experiment, as shown in **Fig. 1a**, to investigate the dowel action mechanism. After that, various experimental and analytical studies [4-6] on dowel action were conducted based on the setup shown in **Fig. 1a**. More recently, Autrup et al. [7] further modified the test setup, as shown in **Fig. 1b**, to consider the influence of the axial force in the reinforcement on the dowel action.




(a) (b)

**Fig. 1.** Block-type test setup for testing dowel action: (a) the setup used by Krefeld and Thurston, reproduced from [3]; (b) the setup used by Autrup et al. reproduced from [7].

Although extensive experiments on the dowel action were conducted, the load-displacement responses obtained from literature [3-7] differ. **Fig. 2** summarizes the different responses from literature. A perfect elastoplastic response was observed by Baumann and Rüsch [4], while Taylor [5] and de Resende et al. [6] observed a softening behaviour. With the consideration of axial force, a hardening response was reported by Autrup et al. [7]. Therefore, there is no consistent model to describe the dowel action behaviour. Additionally, most of the existing models in literature can only predict the maximum dowel force and the behaviour after the fracture of the concrete is not considered. The relationship between dowel force and vertical displacement is not well-established. However, this relationship is very crucial to evaluating the shear capacity of RC beams without shear reinforcement using kinematic-based models - Critical Shear Displacement Theory (CSDT) [8], the Critical Shear Crack Theory (CSCT) [9], the Shear Crack Propagation Theory (CSPT) [10], etc.

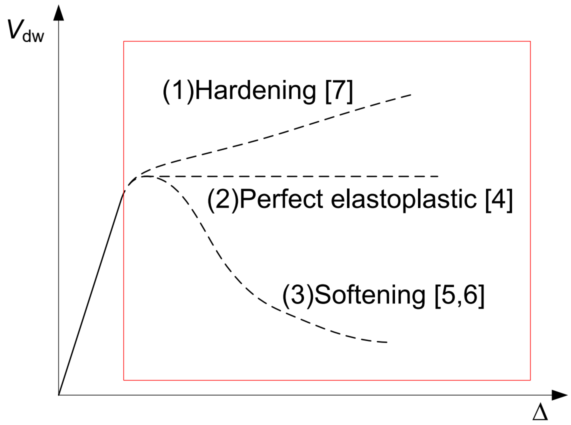



**Fig. 2.** Schematic illustration for different relationships between dowel force and vertical displacement observed in literature [3-7].

In this paper, an analytical model for describing dowel action behaviour is introduced. The model is developed based on the Beam on Elastic Foundation (BEF) theory and fracture mechanics. The proposed model can predict the full force-displacement response of dowel action. Besides, the unstable dowel splitting, which is usually associated with the shear failure in RC beams without shear reinforcement, is theoretically proved in the proposed model. The proposed model is validated by the data collected from literature. Finally, a simplified equation for maximum dowel force is proposed and it can still provide good accuracy. This paper aims to provide a general description of the proposed model, readers are advised to check the detailed derivation procedure in authors' paper [11].

## 2 Proposed mechanical model for dowel action

### 2.1 Derivation of the equilibrium equation

**Fig. 3** illustrates the overview framework of the proposed model for dowel action. For the uncracked part, the reinforcement embedded in concrete is modelled using the BEF theory, in which the rebar is modelled as a beam and the concrete is modelled as the foundation. For the cracked part, the rebar and part of the concrete are considered as a composite beam and it is modelled as a cantilever beam. The concrete softening behaviour is considered for the cracked part. Accordingly, the evolution of dowel action in the proposed model can be characterized into three different stages.. The first stage is the elastic stage where the concrete is not cracked and the relationship between displacement and force remains linear. Then, after concrete cracks, the crack propagation is driven by both force and moment at the crack tip, which is defined as the stable cracking stage in this paper. The crack propagates stably as the dowel force or displacement increases. In the last cracking stage, the crack is fully driven by the moment due to the cantilever effect of the already detached rebar along the dowel crack, which means that the crack can propagate without additional external force until the boundary condition changes. The corresponding derivation for these three stages can be seen as follows.

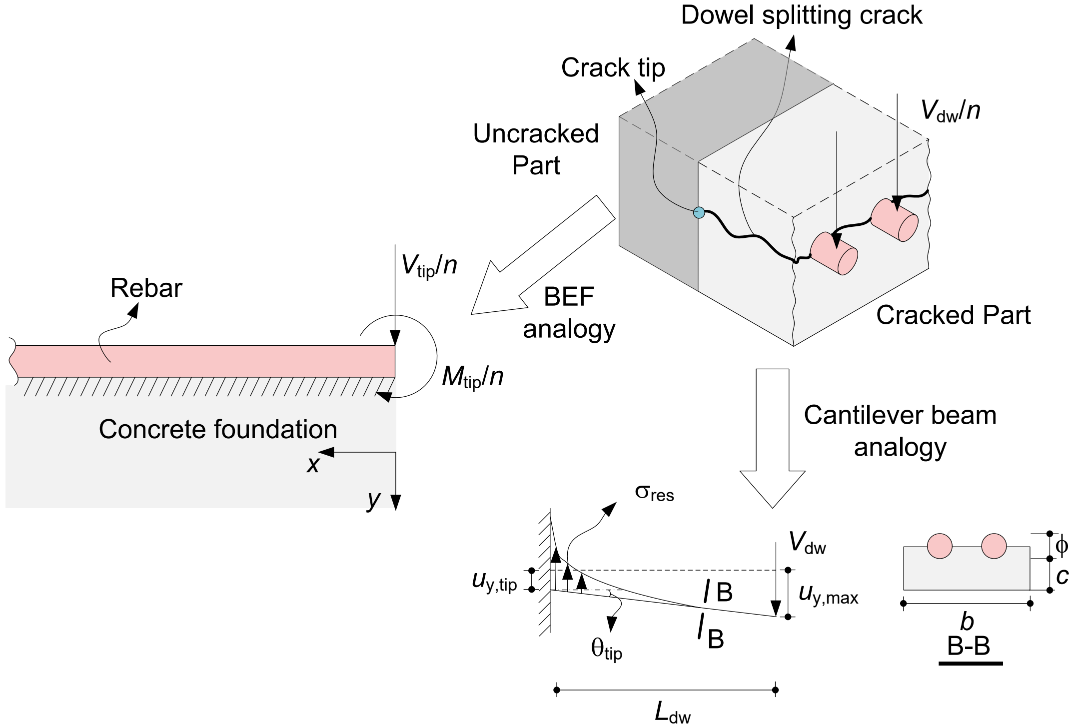



**Fig. 3.** Schematic illustration of the proposed model for dowel action.

**Elastic Stage.** To establish the equilibrium equation, an infinitesimal slice along the x-direction is chosen for analyzing forces, as shown in **Fig. 4a**. Then, an arbitrary cross-section A-A along the y-direction is selected to analyze the stress distribution along the width direction, as shown in **Fig. 4b**. When the cross-section passes through the centre line of the rebar, the critical state can be obtained since the net width is smallest. The stress distribution along the width direction is considered to be uniformly distributed for simplicity. According to the free body diagram in **Fig. 4a** and **b**, the force equilibrium in the vertical direction can be established:

$$b_n \sigma_{ct,max} = (1 - \alpha_{crit}) np(x) \tag{1}$$

where $b_n$ is the net width of the beam, $\sigma_{ct,max}$ is the maximum concrete tensile stress in the width direction obtained at the height through the centre line of the rebar, $n$ is the number of the rebar, $p$ is the reaction force in the concrete foundation, $\alpha_{crit}$ is a critical factor indicating shear force carried by the partial concrete cross-section and $\underline{V}_c$ is the shear force carried by concrete.

According to the solution for BEF theory suggested by Hetényi [12], a relationship between the reaction force $p$ and the dowel force $V_{dw}$ can be obtained and it reaches the maximum value when $x = 0$.

$$p(0) = ku_y(0) = 2\lambda \frac{V_{dw}}{n} \tag{2}$$

In the above Eq. (2), $\lambda$ is the characteristic value of the beam system and it can be calculated using Eq. (3):

$$\lambda = \sqrt[4]{\frac{k}{4E_s I_s}} = \sqrt[4]{\frac{\phi k_f}{4E_s I_s}} \tag{3}$$

where $\phi$ is rebar diameter, $k_f = 127 c_f f_c^{0.5} \phi^{-2/3}$ is the concrete foundation stiffness calculated by the equation proposed by Soroushian et.al [13], $c_f$ is an empirical coefficient ranging from 0.6 to 1.0, $E_s$ is the elastic modulus of steel and $I_s = \pi\phi^4/64$ is the moment of inertia of one rebar. In this paper, the $c_f$ is adopted as 0.6 for multiple rebar situations and 1 for single rebar situations.

When the stress in the width direction $\sigma_{ct,max}$ reaches the concrete tensile strength $f_{ct}$, the cracking occurs. Substituting $\sigma_{ct,max} = f_{ct}$ and Eq. (2) into Eq. (1), the cracking dowel force $V_{dw,cr}$ and the cracking displacement $u_{y,cr}$ can be derived:

$$V_{dw,cr} = \frac{b_n f_{ct}}{2(1-\alpha_{crit})\lambda} \tag{4}$$

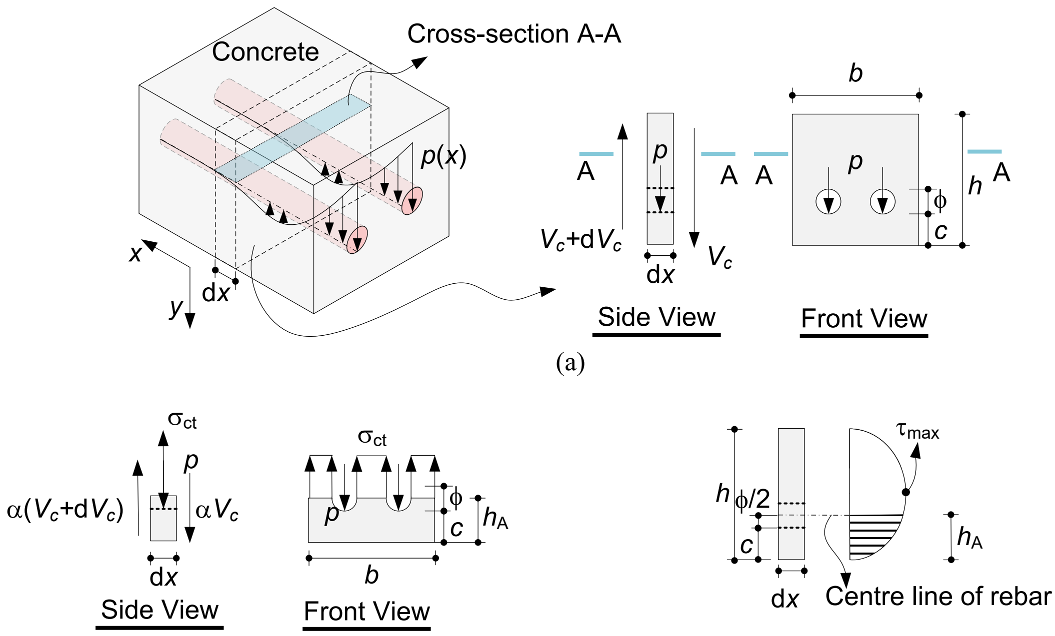
$$u_{y,cr} = \frac{b_n f_{ct}}{(1-\alpha_{crit})nk} \tag{5}$$

The only remaining unknown in Eq. (4) is the critical factor $\alpha_{crit}$ which indicates the shear force carried by the partial concrete. To determine $\alpha_{crit}$ , it is assumed the shear stress distribution in the cross-section follows a parabolic shape and reaches its maximum shear stress at the middle height. Then, $\alpha_{crit}$ can be determined using the following equation.

$$\alpha_{crit} = 3\left(\frac{c+\phi/2}{h}\right)^2 - 2\left(\frac{c+\phi/2}{h}\right)^3 \tag{6}$$

Combining Eqs. (4), (5) and (6), the response of dowel action in the elastic stage can be described using the following equation.

$$V_{dw} = \frac{u_{y,max}}{u_{y,cr}} V_{dw,cr} (u_{y,max} \le u_{y,cr}) \tag{7}$$




(a)

(b) (c)

**Fig. 4.** Illustration for the derivation of equilibrium equation: (a) free body diagram of an infinitesimal element in the longitudinal direction; (b) cross-sectional analysis at an arbitrary section A-A for transversal distribution of concrete stress; (c) assumed shear stress distribution along the height.

**Stable Cracking Stage.** After cracking, the whole system is divided into two parts as shown in **Fig. 3**. The cracked part can be treated as a rigid cantilever beam and the crack tip is considered as the fixed end of the beam. Besides, the cracked part is subjected to the concentrated dowel force $V_{dw}$ and the residual tensile stress $\sigma_{res}$ due to cracking. The simple power law relationship, as shown in Eq. (8), proposed by Reinhardt [14] is adopted as the constitutive model for residual tensile stress $\sigma_{res}$:

$$\sigma_{res} = f_{ct}(1-(\frac{w}{w_c})^{c_1}) \geq 0 \tag{8}$$

where $f_{ct}$ is the concrete tensile strength, $w$ is the crack width, $c_1 = 0.31$ is an empirical coefficient, $w_c = G_f/f_{ct}\cdot(1+c_1)/c_1$ is the characteristic crack width that can transfer the residual strength and $G_f$ is the fracture energy of concrete, which can be calculated using $G_f = 0.073f_c^{0.18}$ according to the *fib* Model Code 2010 [15].

Since the cracked part is assumed to be a rigid cantilever beam, the crack width distribution is linear and it can be calculated according to the rotation at the crack tip:

$$w = \theta_{tip} x \tag{9}$$

If the cracked part is separated to perform force analysis, the following force and moment equilibrium can be established:

$$V_{tip} = V_{dw} - b_n \int_0^{L_{dw}} f_{ct}\left[1-(\frac{\theta_{tip}x}{w_c})^{c_1}\right]dx \tag{10}$$

$$M_{tip} = V_{dw}L_{dw} - b_n \int_0^{L_{dw}} f_{ct}\left[1-(\frac{\theta_{tip}x}{w_c})^{c_1}\right]xdx \tag{11}$$

where $V_{tip}$ and $M_{tip}$ are the reaction force and moment acting at the crack tip, and $L_{dw}$ is the length of the dowel crack.

On the other hand, the BEF theory is still applicable for the uncracked part, but the concentrated moment $M_{tip}$ needs to be considered besides the concentrated load $V_{tip}$. The concrete tensile stress $\sigma_{ct,max}$ at the crack tip along the width direction and the rotation at the crack tip $\theta_{tip}$ is induced by two actions, i.e., concentrated moment and force. The condition for crack propagation is that the concrete tensile stress reaches the concrete tensile strength. Then, the following relationships can be established.

$$\theta_{tip} = \frac{2\lambda^2}{k}\frac{V_{tip}}{n} + \frac{4\lambda^3}{k}\frac{M_{tip}}{n} \tag{12}$$

$$\sigma_{ct,max} = (1-\alpha_{crit})\frac{2\lambda(V_{tip}+\lambda M_{tip})}{b_n} = f_{ct} \tag{13}$$

The moment acting at the crack tip $M_{tip}$ can be obtained by solving Eqs. (8) to (13) using some numerical techniques. With a known $M_{tip}$, the behaviour of dowel action during the stable cracking stage can be obtained by utilizing Eqs. (9) and (10). However, an analytical solution can not be derived due to the complexity of the equations.

**Unstable Cracking Stage.** As indicated by Eq. (13), the concrete tensile stress $\sigma_{ct,max}$ is induced by the moment and force. Since the dowel crack continues to propagate, the length of the dowel crack $L_{dw}$ increases and the moment acting at the crack tip $M_{tip}$ increases accordingly. Then, it can be foreseen that the contribution from $V_{tip}$ may vanish at a certain moment and all the driven force comes from the moment $M_{tip}$. In other words, the crack propagates continuously without additional external force until the boundary condition changes, which is referred to as the unstable cracking stage in this paper.

For the case where $u_{y,max} \geq u_{y,tip} + w_c$, i.e., the maximum crack width is larger than the characteristic crack width $w_c$ that can transfer the residual tensile strength, the analytical solution for the critical displacement $u_{y,crit}$ and critical length for dowel crack $L_{dw,crit}$ can be obtained by setting $V_{tip} = 0$.

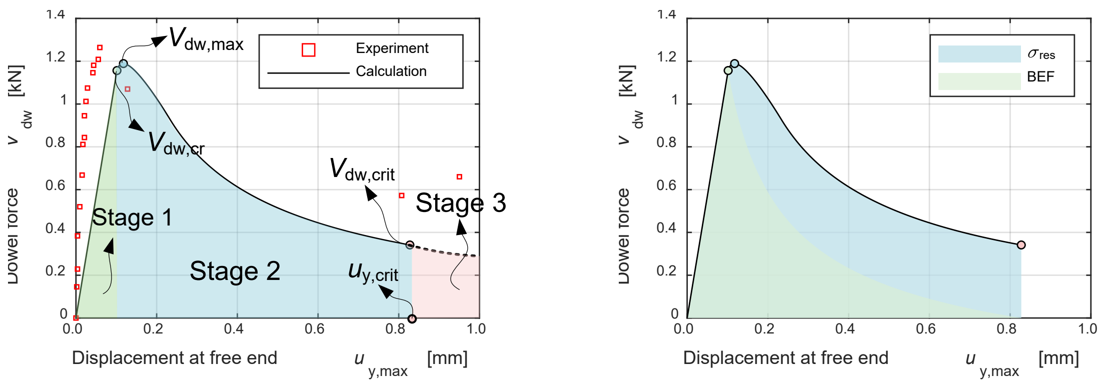
$$L_{dw,crit} = \frac{(1+c_1) f_{ct} b_n}{(1-\alpha_{crit})^2 c_1 w_c \lambda k n} + \frac{(1-\alpha_{crit})(1+c_1) w_c k n}{4(2+c_1)\lambda f_{ct} b_n} \tag{14}$$

$$u_{y,crit} = \frac{2(1+c_1) f_{ct}^2 b_n^2}{(1-\alpha_{crit})^3 c_1 w_c k^2 n^2} + \frac{(1+c_1) w_c}{2(2+c_1)} + \frac{f_{ct} b_n}{(1-\alpha_{crit}) k n} \tag{15}$$

For the case where $u_{y,max} < u_{y,tip} + w_c$, the critical displacement $u_{y,crit}$ can be only obtained by implementing a numerical technique.

## 2.2 Demonstration and simplified equation for maximum dowel force

Taking Beam 2.2 in [8] as an example, the full response is shown in **Fig. 5a** and the experimental data is also plotted as a comparison. The comparison shows that the proposed model can capture the maximum dowel resistance as well as the post-peak softening behaviour. **Fig. 5b** presents the evolution of contribution for dowel resistance from two mechanisms. After cracking, the contribution from residual tensile strength starts to increase and it becomes stabilized as the displacement increases. While the contribution from the BEF starts to diminish and eventually vanish when the unstable cracking occurs.




(a) (b)

**Fig. 5.** The displacement versus force curve of Beam 2.2: (a) comparison against Beam 2.2 from [8] (b) contributions from different mechanisms.

In the proposed model, the maximum dowel resistance $V_{dw,max}$ cannot be derived analytically due to the complexity. According to **Fig. 5a**, the magnitudes of the cracking force $V_{dw,cr}$ and maximum force $V_{dw,max}$ are very close, which agrees with the observation in conventional concrete in [6]. Therefore, if the differences between $V_{dw,cr}$ and $V_{dw,max}$ are ignored, a simplified expression for $V_{dw,max}$ can be derived by substituting Eq. (3) into Eq. (4) and setting Young's modulus of steel $E_s = 210$ GPa and $c_f = 0.6$.

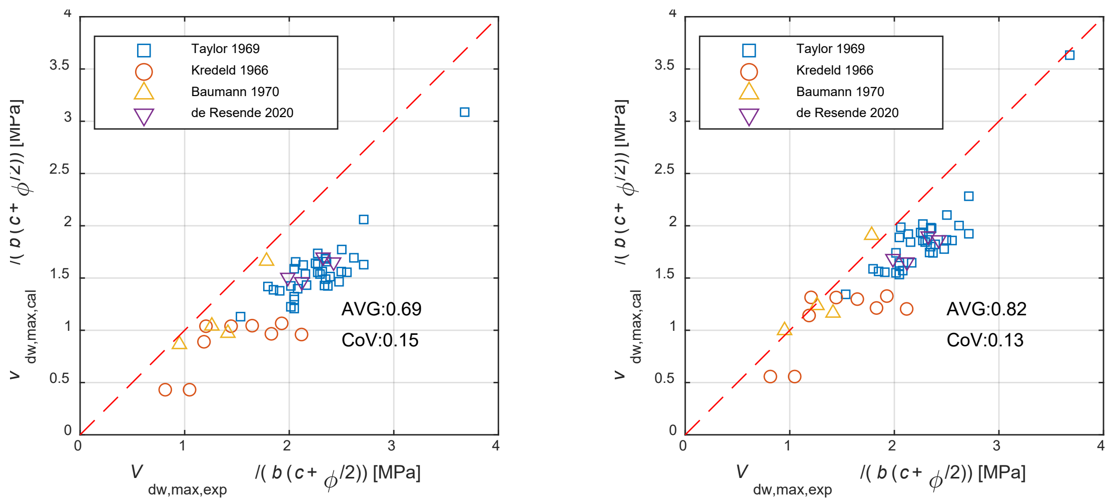
$$V_{dw,max} \approx V_{dw,cr} = \frac{2.4}{1-\alpha_{crit}} b_n \phi^{\frac{11}{12}} f_{ct} f_c^{-\frac{1}{8}} \tag{16}$$

# 3 Validation

To validate the proposed model, some experimental data were collected from the literature [3-6]. It was found that the concrete tensile strength of some data was not reported and the type of concrete tensile strength was not specified. Therefore, the validation is performed using two different tensile strengths, namely direct tensile strength $f_{ct}$ and splitting tensile strength $f_{ct,sp}$. In addition, considering the scatter of concrete tensile strength by nature, it is decided to calculate the concrete tensile strength based on the compressive strength $f_c$ to obtain a more consistent comparison. For direct tensile strength, the relationship proposed in the *fib* Model Code 2010 [15] is adopted, while for splitting tensile strength, the relationship proposed by Bentz et al. [16], as shown in Eq. (17), is used.

$$f_{\mathrm{ct,sp}} = 0.62\sqrt{f_{\mathrm{c}}} \tag{17}$$

**Fig. 6** summarizes the comparisons using different tensile strengths. The results show that using a consistent way to determine the tensile strength can reduce the scatter. Regarding the ratio between calculated results and experimental results, using direct tensile strength results in a relatively conservative estimation while using splitting tensile strength can lead to a closer prediction. Therefore, it is suggested to use splitting tensile strength to evaluate the maximum dowel force.




(a) (b)

**Fig. 6.** Comparison between calculated results and experimental data: (a) using direct tensile strength based on fib Model Code 2010 [15]; (b) using splitting tensile strength based on Eq. (17) proposed in [16].

Based on the results shown in **Fig. 6**, using the splitting tensile strength predicted by Eq. (17) can lead to a very pleasing prediction of the maximum dowel force. A more simplified equation for the maximum dowel force can be further derived by substituting Eq. (17) into Eq. (16).

$$V_{\mathrm{dw,max}} = \frac{2.4}{1-\alpha_{\mathrm{crit}}} b_{\mathrm{n}} \phi^{\frac{11}{12}} f_{ct} f_{\mathrm{c}}^{-\frac{1}{8}} \approx \frac{1.5}{1-\alpha_{\mathrm{crit}}} b_{\mathrm{n}} \phi f_{\mathrm{c}}^{\frac{3}{8}} \tag{18}$$

**Fig. 7** shows the comparison between the results calculated by Eq. (17) and the experimental data. The CoV is higher using the simplified equation because the contribution of rebar is increased during the simplification. However, using the simplified equation can result in pleasing estimations for maximum dowel force with an average ratio of 0.96.

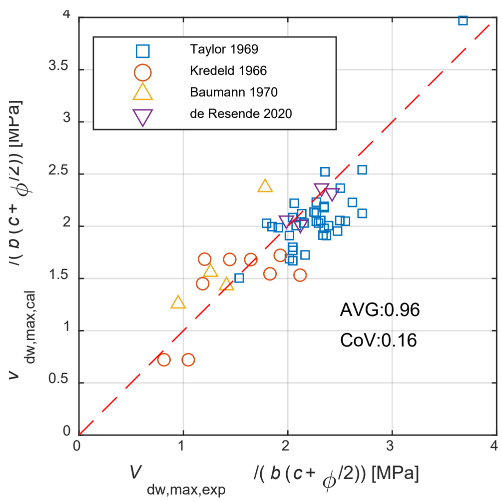



**Fig. 7.** Comparison between the results calculated by simplified equation Eq. (17) and experimental data.

## 4 Conclusions

This paper proposed an analytical model for dowel action based on the Beam on Elastic Foundation theory and fracture mechanics. Unlike the other models in literature, the proposed model considers the residual stressed in the dowel crack, that provides the additional possibility to capture the post-peak behaviour of dowel action and yield the full load-displacement relationship to describe the dowel action. Some conclusions can be summarized as follows:

- The proposed model shows that dowel action has a post-peak softening response and the softening behaviour is mainly determined by the residual tensile strength of concrete and rebar configuration;
- The unstable dowel cracking is theoretically proved in the proposed model, which may be further linked to the shear failure of RC beams without shear reinforcement;
- Splitting tensile strength is more suitable for predicting the maximum dowel force, also considering the stress condition in dowel splitting process;
- A simplified analytical expression for the maximum dowel force is proposed based on splitting tensile strength and it is validated by the experimental data with pleasing accuracy.

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