# Appendix 2: Mathematical Algorithm for Cam Profile Generation

This appendix describes the mathematical relationship between the shape of the cam profile and the encoded ankle torque-angle characteristics. All the symbols used in the mathematical expressions are described in the Table I. The detailed mathematical derivation for generating the cam profile geometry from ankle torque-angle curves is provided in Shepherd and Rouse (2017) [1]. The cam follower is modelled as a point, with the final curve offset by a value equal to the cam follower radius. The leaf spring is treated as a rotary spring due to two factors: (1) the follower does not move vertically when the spring is deflected, and (2) non-vertical forces acting on the follower induce a moment on the spring, as the point of force application is positioned above the spring's neutral plane. As a result, the spring rotates around a "virtual pivot," which is approximated at the location of the simple support shown in Figure 1. While the actual position of the virtual pivot may be closer to the cam and polycentric in nature, small errors in this parameter were deemed mathematically insignificant. Initial experiments with the preliminary cam profile revealed series compliance due to the flexure of the aluminium prosthesis frame, leading to undesirable effects on the ankle's torque-angle curve. To incorporate this series compliance into the mathematical model, the unwanted deflection ($\delta$) is assumed to be proportional to the ankle moment, with an experimentally determined stiffness of approximately 1200 Nm/rad, and is modelled as acting between the cam and the user.

The solution for the cam profile was formulated using the principle of virtual work. This approach assumes no energy loss within the transmission or spring, whereby the energy stored in the ankle (minus the energy stored in the series compliance) equals the energy stored in the leaf spring.

$$\int_0^{\gamma} M_S \, d\gamma = \int_0^{\theta} M_A \, d\theta - \int_0^{\delta} M_A \, d\delta .......(1)$$

Here, $\gamma$ represents the angular deflection of the rotary spring, $M_S$ is the spring moment, $\theta$ is the ankle angle, $\delta$ represents series compliance in the frame, and $M_A$ is the ankle torque. The right-hand side of the equation is defined as a function of $\theta$, as the desired torque-angle curve at the ankle is pre-determined. Numerical integration is performed for both the desired torque-angle curve and the known series compliance. To simplify the process, dorsiflexion and plantarflexion are solved individually, with the lower limit of integration set at 0. At the equilibrium position ($\theta$ = 0°), the spring is preloaded by a small angle $\gamma_0$. Based on this Equation 1 can be written as:

$$\int_0^{\gamma} k(\gamma + \gamma_0) \, d\gamma = \int_0^{\theta} M_A \, d\theta - \int_0^{\delta} M_A \, d\delta .......(2)$$

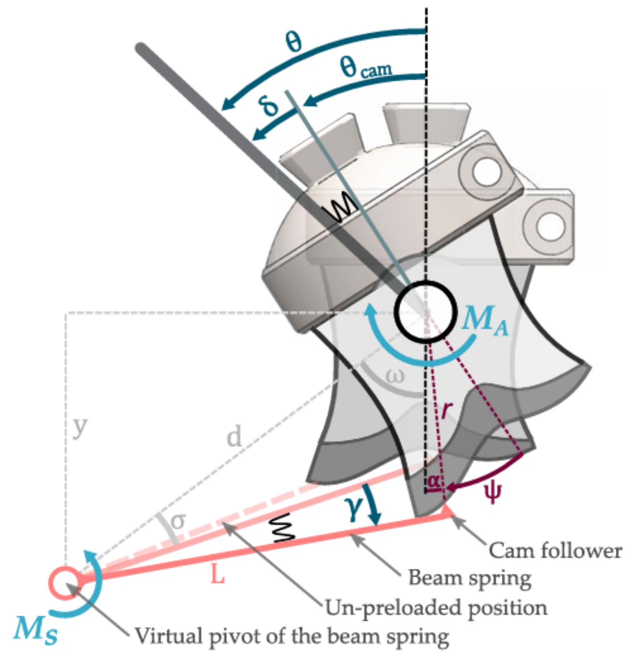


Fig. 1: The energy stored at the ankle joint is equivalent to the energy stored in the leaf spring, which is modelled as a rotational spring with a virtual pivot. The cam follower is simplified as a point (represented by the pink triangle). A downward deflection of the cam follower, denoted as $\gamma$, generates a restoring moment in the leaf spring ($M_S$). The deflection of the ankle, $\theta$, is a combination of the cam deflection ($\theta_{\text{cam}}$) and the series compliance ($\delta$). This deflection results in a moment at the ankle ($M_A$), caused by the interaction force between the cam follower and the cam profile. The leaf spring is preloaded to a small angle. The geometry, illustrated in grey, determines the cam radius ($r$) from $\gamma$, and the angle $\psi$ from $\theta$. The final cam profile is computed in polar coordinates ($r$, $\psi$). It is important to note that the profile requires an offset for the final curve to function with the actual cam follower. For clarity, the curve shown does not include this offset and is presented for illustrative purposes. *Adapted from [1].*

Here k is the rotational stiffness of the cantilever leaf spring. Integrating the left side of Equation 2 with the constant of integration c:

$$\frac{1}{2}k\gamma^2 + k\gamma_0\gamma + c = \int_0^{\theta} M_A \, d\theta - \int_0^{\delta} M_A \, d\delta .......(3)$$

With the initial conditions $\theta$ = 0°and $\gamma$ = 0°, the constant of integration c can be calculated to be zero. Using the quadratic

TABLE I: List of symbols used in the Appendix.

| Symbol | Description |
|---|---|
| $M_S$ | Torque at the interface of the spring and simple support |
| $M_A$ | Torque at the ankle joint |
| $d$ | Absolute distance between the ankle joint and the interface of the spring and simple support |
| $L$ | Absolute distance between the cam follower and the interface of the spring and simple support |
| $r$ | Cam radius |
| $\theta_{\text{cam}}$ | Rotation of the cam profile |
| $\sigma$ | The angle between $d$ and $L$ at the neutral ankle angle $\theta_{\text{cam}}$ |
| $\gamma_0$ | Angular deflection of the rotary spring, due to a preload |
| $\gamma$ | Rotation of the rotary spring |
| $k$ | The rotational stiffness of the cantilever beam spring, around the simple support |
| $\omega$ | Angle between $d$ and the vertical axis |
| $a$ | Angle between $r$ and the vertical axis, caused by the horizontal displacement of the cam follower due to the rotary motion of the spring |
| $\psi$ | Effective cam profile angular deflection, accounting for $a$ |
| $\delta$ | Change in ankle angle due to elastic deflection of the frame |

equation to solve for $\gamma$ in terms of $\theta$:

$$\gamma(\theta) = -\gamma_0 + \sqrt{\gamma_0^2 + \frac{2}{k}(\int_0^\theta M_A \, d\theta - \int_0^\delta M_A \, d\delta)}.......(4)$$

Now from the function of $\gamma$, the leaf spring angle in terms of $\theta$, the ankle angle and the defined geometry; the cam profile radius r can be derived as:

$$r(\theta) = \sqrt{L^2 + d^2 - 2Ldcos(\gamma + \theta)}]......(5)$$

To express the cam profile in polar coordinates (r,$\psi$); $\psi$ can be expressed as a function of $\theta$:

$$\psi(\theta) = \theta_{cam} - \alpha = \theta - \delta - \alpha........(6)$$

$\alpha$ can be determined using the law of sines:

$$\alpha(\theta) = sin^{-1}(Lsin(\sigma + \gamma(\theta))) + \omega........(7)$$

Now the cam profile geometry is expressed with the polar coordinates (r, $\psi$). The polar coordinates can be converted to cartesian coordiantes (x,y), offsetting the evaluated curve by the perpendicular distance equal to the cam follower radius 9.5mm; creating the final cam profile curve.

## References

[1] Max K. Shepherd and Elliott J. Rouse. "The VSPA Foot: A Quasi-Passive Ankle-Foot Prosthesis With Continuously Variable Stiffness". In: *IEEE Transactions on Neural Systems and Rehabilitation Engineering* 25.12 (Dec. 2017), pp. 2375–2386. ISSN: 1534-4320, 1558-0210. DOI: 10.1109/TNSRE.2017.2750113.