

Optimal long-haul Trajectories in a Wind Field

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Optimal long-haul Trajectories in a Wind Field

by

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Preface

This report is the result of a graduation project to complete the Master Science at the faculty of Aerospace Engineering (Control and Operations: Air Transport and Operations), Delft University of Technology.

The report discusses the research done to develop an optimization tool which can identify the near optimal trajectory for long-haul flights with respect to the fuel consumption and the flight time with the considerations of en-route wind fields and a set of constraints based on ATC regulations.

I would like to thank the members of my exam committee, which consisted of:

- Prof. Dr. R. Curran; professor of Air Transport and Operations at Delft University of Technology.
- Ir. S. Hartjes; assistant professor of Air Transport and Operations at Delft University of Technology.
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Summary

The long-haul flights of airlines are impacted by the en-route wind fields. Flights on the same route with different directions may have significantly different flight performances. It may benefit to airlines in terms of fuel cost and on-time performance if the optimal long-haul routes are able to plan. However, the wind direction and strength are varying in different regions, at different altitudes and different times. It is difficult to identify the most suitable trajectory in a complex wind field. Consequently, the trajectory optimization problem is proposed. The main goal of this study is to develop an optimization model to identify the near optimal trajectory with minimum consumption for long-haul flights taking into account en-route winds and a set of constraints based on ATC regulations.

One of the methods to solve the problem is the genetic algorithm since it is a global search method. For this purpose the software tool NSGA is applied in this study. However, the computational process of NSGA is not very efficient due to the computational loss for infeasible solutions. In order to overcome the computational inefficiency of NSGA, a 2-phase approach is proposed. Phase 1 is to reduce the search scope for Phase 2, while the outcomes of Phase 2 are detailed solutions with the accurate fuel consumption and flight time of the trajectories. Moreover, approaches based on parameterization are introduced to minimize the number of infeasible solutions and the control parameters.

The control variables of Phase 1 are the coordinates of waypoints, true airspeeds of segments, distances of vertical segments, and the altitude changing locations. Infeasible solutions may be generated with excess flight distance and/or random terminated locations. In order to maintain the feasibility of evaluated solutions, an algorithm is introduced to locate the latitude feasible range of each waypoint. Additionally, approaches based on parameterization are introduced to ensure that the evaluated trajectories terminate at the city pair. By applying these approaches, the number of infeasible solutions due to the excess flight distance and/or random terminated locations is significantly reduced.

Although the approaches proposed are able to reduce the number of the control parameters, long computational time is still required to evaluate each solution with a small time or distance step. In order to increase the computational speed, the equivalent wind speed and equivalent weight concepts are proposed in Phase 1. By applying this approach, the distance step is able to increase to a segment distance. The computational efficiency is increased.

The outcomes of Phase 2 are detailed solutions. With the involved climbing and

descent phases, the number of parameters in Phase 2 is more than the number of parameters in Phase 1. The search scope of Phase 2 is based on the outcomes of Phase 1 to obtain detailed solution within an acceptable computational time.

In this study, the forward simulation is applied to ensure the feasibility of the flight altitude. However, the constant landing weight is hardly achieved by applying this algorithm. The transport losses may occur if there is fuel left when the aircraft arrives at the destination. In order to solve this problem, an iterative algorithm is proposed to adjust the initial take-off weight if the final landing weight is not within the expected range.

The verification in this study shows that the designed tool is quality and credible. The tool is able to detect the tailwind and headwind in a real wind field. The outcomes of the tool are a little deviated to the optimal solution, but the deviation is acceptable after analysing.

In this study, three routes are researched, which are Amsterdam and New York, Amsterdam and Johannesburg, Amsterdam and Singapore. The average saving in terms of flight time and fuel consumption of the optimized solutions are 3.16% and 3.1% respectively compared to the solutions with the great circle path and optimized vertical profile.

The results achieved in this study are promising, but enhancements can be made in the some areas.

The optimization processes discussed in this study were only performed for one type of aircraft. More aircraft models can be analysed to further prove the performance of the designed tool.

The current program assumes a standard atmosphere. As the pressure and temperature are of great influence on both the air density which further impacts the performance of the engine. A more elaborate atmospheric model could be introduced to obtain more accurate solutions.

The tool designed in this study is based on Matlab. With more efficient software, the computational time can be dramatically decreased. Additionally, the use of a better equipped computer with multi processors will significantly increase the computational efficiency.

In addition, well-designed termination criterions are recommended when there are multi closed local optimal solutions in the search space, which will increase the computational efficiency during the optimization process.

List of notations

Abbreviations

AMS	Amsterdam Airport Schiphol (Amsterdam)
ATC	Air Transport Control
CAS	Calibrated Airspeed
DAG-TM	Distributed Air/Ground Traffic Management
FL	Flight Level
GA	Genetic Algorithm
GDPS	Global Deterministic Prediction System
GS	Ground Speed
HKG	Hong Kong International Airport (Hong Kong)
ICAO	International Civil Aviation Organization
IDPGA	Improved Dual-Population GA
IFR	Instrument Flight Rules
I.S.A.	International Standard Atmosphere
JFK	John F. Kennedy International Airport (New York)
JNB	OR Tambo International Airport (Johannesburg)
MMO	Maximum Operating Mach Number
MTOW	Maximum Take-off Weight
NAO	North Atlantic Ocean
NOC	Neighboring Optimal Control
NSGA	Non-dominated Sorting Genetic Algorithm
OEW	Operation Empty Weight
RVSM	Reduced Vertical Separation Minima
SUA	Special Use Airspace
SIN	Singapore Changi Airport (Singapore)
TAS	True Airspeed
VMO	Maximum Operating Speed
VSM	Vertical Separation Minima

Symbol	Description	Unit
c_T	Specific Fuel Consumption	[N/N s]
C_D	Drag Coefficient	[—]
C_L	Lift Coefficient	[—]
D	Drag Force	[N]
$D_{Great\ Circle}$	Great Circle Path Distance	[km]
D_{max}	Maximum Flight Distance	[km]
D_{travel}	Travelled Distance	[km]
D_r	Remaining Available Flight Distance	[km]
$D_{v,b}$	Basic Distance of a Vertical Segment	[°]

$D_{v,n}$	Distance of a Vertical Segment	[°]
ff	Fuel Flow	[kg/s]
h	Flight Altitude	[m]
h_0	Initial Flight Altitude of Phase 2	[m]
L	Processed Distance	[°]
$l_{min,h}$	Minimum Segment Distance	[°]
M	Mach Number	[—]
M_l	Number of Longitude Selection Range	[—]
$MTOW$	Maximum Take-off Weight	[N]
P_c	Cabin Pressure	N/m ²
P_p	Produced Power	J/s
P_r	Required Power	J/s
R	Specific Range	[m]
r	Radius of Earth	[km]
S	Wing Surface Area	[m ²]
SD	Scalar Distance	[km/°]
T_{max}	Maximum Available Thrust	[N]
t	Climb/Descent Time	[s]
V_{CAS}	Calibrated Airspeed	[m/s]
V_{gs}	Ground Speed	[m/s]
V_{TAS}	True Airspeed	[m/s]
V_w	Wind Speed	[m/s]
W	Aircraft Weight	[N]
u	Wind Speed Component	[m/s]
v	Wind Speed Component	[m/s]
Z_0	Initial Flight Altitude of Phase 1	[m]
γ	Flight Path Angle	[°]
η	Control parameter	[—]
ϕ_D	Drift Angle	[°]
ϕ_w	Wind Vector Angle	[°]
ψ	Heading Angle	[°]
λ	Longitude	[°]
ρ	Air Density	[kg/m ³]
φ	Latitude	[°]

Constants	Description	Value	
P_0	Pressure at 0 m ISA	101,325	N/m ²
h_0	Initial Flight Altitude	10,000	ft

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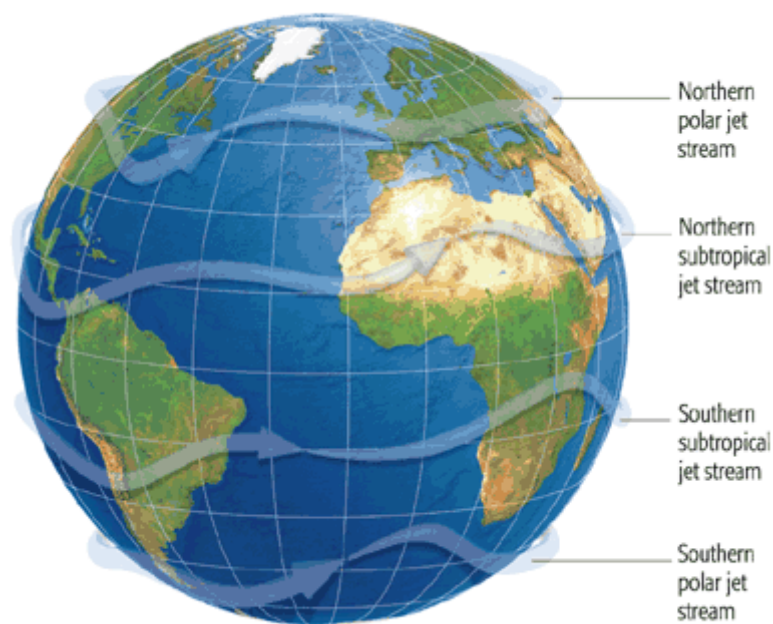
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1. Introduction

With the development of aviation industry and the improvement of the environmental awareness, more and more airlines have paid attention to reduce fuel consumption during daily flight operations. More importantly, with the increasing oil price, the amount spent on fuel accounts for a large fraction of the total operational expenditure of airlines. In 2007, fuel represented 25.4 per cent of the total cost of airlines, which was only 11 per cent in 1994 (Doganis, R., 2010). This is further exacerbated by the strong competition of the airline market, and resulting need to reduce costs. On the other hand, the main attractive factor of air travel service is the efficiency to transport passengers and/or cargo over large distances. Therefore, on-time performance of airlines is critical to fulfil the satisfaction of current customers and attract new ones. Airlines may choose to increase the airspeed to compensate the time loss caused by other operations, such as the ground handling and boarding process, to improve on-time performance and reduce the amount of missed connections. However, this behaviour may lead to increased fuel consumption. Beside the operations of airlines or airports, the en-route wind effect is an essential factor to the flight on-time performance. Airlines pursue to minimize the adverse effects of headwinds, or maximize the beneficial effect of tailwinds when planning flight trajectories. Figure 1 indicates the path of the jet streams in the world.

SUBTROPICAL ZONE EXPANSION



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Figure 1.1. The path of the jet streams (Mann, M. 2014)

According to the jet stream profile of the world shown in figure 1.1, it is easy to notice that the jet streams in the sky are in the east-west direction instead of the north-south direction. Therefore, the flights of east-west routes are affected more significantly by en-route winds than the flights of north-south routes. For example, for flights between Amsterdam (AMS) and New York (JFK) in the summer season, the flight time from AMS to JFK is around 8 hours and 30 minutes, while the flight time from JFK to AMS is around 7 hours and 30 minutes (KLM, 2014) which is 11.8% shorter than the flights from AMS to JFK. This time difference between two directions is caused by winds above North Atlantic Ocean (NAO). Aircraft are impeded by the headwind when flying from the east to the west, while aircraft gain the benefit from the tailwind when flying from the west to the east. Apart from flights across NAO, flights on Europe-Asia routes are also affected by high altitude wind fields. According to KLM, the flight time of flights in summer from AMS to Hong Kong (HKG) is as short as 11 hours and 15 minutes, which is 8.2% shorter compared to 12 hours 15 minutes of flights from HKG to AMS (KLM, 2014). Routes between AMS and Johannesburg (JNB) – in the north-south direction – have a flight time to JNB in summer of about 10 hours and 50 minutes, while the return flight in the same season takes 11 hours and 5 minutes (KLM, 2014). The time difference between two directions of the north-south flights is not as significant as the time difference between two directions of the east-west flights. For short-haul flights, the cruise phase occupies a relatively small proportion within the whole flight. The impacts of other factors such as the departure and arrival procedures to the short-haul flights are more significant than the impacts of the same factors to the long-haul flights. Such that, the en-route wind influence on the flight time is decreased.

Apart from the flight time, wind fields also have significant impacts on the fuel consumption during flight operations. The specific range indicates the efficiency of the fuel consumption with a certain flight condition, which is the ratio of the ground airspeed (GS) and the fuel flow (ff). The ground speed is the sum of the true airspeed (TAS) and the wind speed, which increases when there is a tailwind effect while decreases when the aircraft is against a headwind. Therefore, the specific range decreases as the wind speed decrease from a tailwind to a headwind with the same TAS and fuel flow. That is, for a given flight distance, aircraft with a tailwind consumes less fuel than aircraft with a headwind because the specific range of the aircraft with a tailwind is larger than the specific range of the aircraft with a headwind.

In order to achieve the best flight performance in terms of the flight time and the fuel consumption, airlines may adjust the flight trajectories based on en-route wind profiles. Figure 2 shows the flight trajectories of airlines over the Pacific Ocean between Los Angeles and Tokyo.

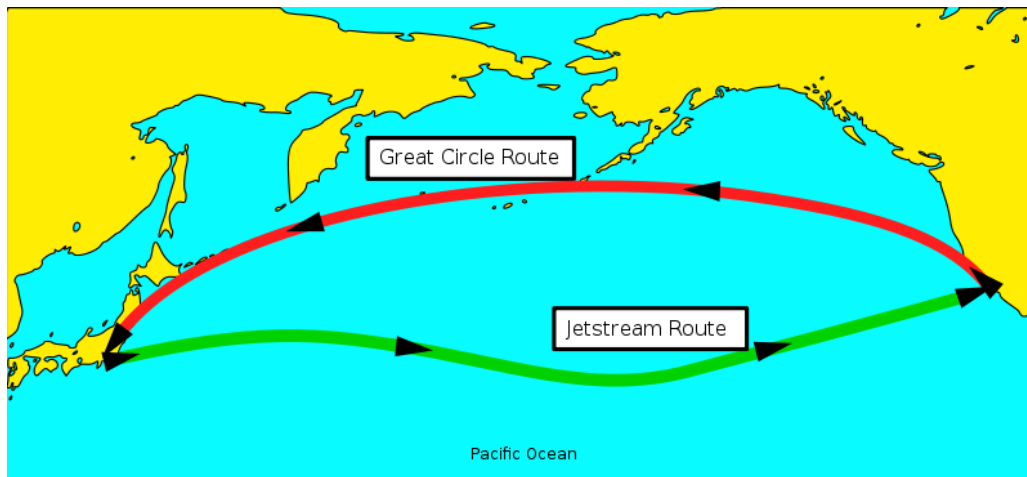


Figure 1.2 The flight trajectories of airlines between Los Angeles and Tokyo. (ChaosNil, 2007)

As shown in figure 1.2, the flight route from Los Angeles to Tokyo follows the direct great circle path (red line) for the shortest distance, while the flight route follows the jet stream (green line) route when heading eastwards. By flying on the detour, aircraft benefit from the strong tailwind which can be as large as 122.9 m/s (239 kts) (National Weather Service, 2011). In 1952, Pan Am cut the flight time from Tokyo to Honolulu from 18 to 11.5 hours with the benefit of the jet stream at the altitude of 24,900 ft (7600 m) (Taylor, Frank J. ,1958). Based on the analysis above, the wind effects on the flight performance are significant. Consequently, it is necessary to consider en-route wind effects when planning flight trajectories. However, it is difficult to identify the most suitable trajectory in a complex wind field. The wind directions and strength are varying in different regions, at different altitudes and different times. Even though the problem is complex to solve, it may benefit to airlines in terms of fuel cost and on-time performance if the optimal long-haul routes are able to plan.

In last decades, the trajectory optimization problem has been researched by many researchers with several computational methods. Among them, optimal control and genetic algorithm (GA) are widely used. Jardin and Bryson (2001) and Jardin (2008) applied the technique of neighboring optimal control (NOC) to optimize horizontal aircraft trajectories in general wind fields. These two researches only considered the horizontal profile of trajectories, while the airspeed and the flight altitude were prescribed. In 2009, Bijlsma (2009) developed an improved NOC approach to identify the so called 'absolute minimums' with moderate computational effort and memory space. Even though the optimal solution obtained through this algorithm was better than the solution obtained by NOC, improvements were still required because the linear interpolation implemented in the research may cause large error during the optimization process. Hok K.Ng et.al (2012) proposed a trajectory optimization algorithm by taking the vertical profile into account. Nevertheless, the airspeed was still not a control variable. The horizontal profile in this research was generated by

interpolation techniques or collocation methods which may cause convergence problems or the inaccuracy solutions. In terms of GA, Oussedik et al. (2000) developed a GA model to identify alternative routes for aircraft in the airspace of France. This research provided an idea for this project that flight routes can be optimized based on beacons instead of a time sequence. The investigated scope can be mapped based on coordinates. However, the potential beacons in the research of Oussedik et al. (2000) are fixed, which may limit the freedom of the optimization process. Yokoyama & Suzuki (2003) introduced a GA model with an improved selection process to solve convergence problems of GA. Although the improved selection process is able to identify the optimal solutions, infeasible solutions are still evaluated by the optimizer, which may waste computational time and memory space. Ji et.al (2013) applied an improved dual-population GA (IDPGA). An additional population was used to maintain the population diversity. The drawback of IDPGA is that infeasible solutions were allowed to be generated during the optimization. Except optimal control and GA, dynamic programming is another potential method for the trajectory optimization problem. Hok K.Ng et.al (2009) researched the trajectory optimization problem by dynamic programming. Dynamic programming is able to provide a provably global optimal solution when feasible solutions exist. Similarly to GA, the computational time of dynamic programming is long because the optimizer computes at each grid for the global optimal solution (Hok K.Ng et.al, 2009). In addition, dynamic programming may only be efficient for problems with low state dimension and coarse grid discretization (Rippel, E. et.al, 2005).

Based on the discussion above, previous research for the trajectory optimization problem considered the airspeed as a constant instead of a control variable. However, the airspeed is essential to flight time and fuel consumption under most circumstances because flight time and fuel consumption of flights are directly dependent on the airspeed. In addition, the outcomes of previous research using optimal control were not guaranteed to be global optimal solutions. On the other hand, the outcomes of research using GA and dynamic programming could theoretically provide global optimal solutions. However, the problem of the evaluation of infeasible solutions was not solved in previous research. Additionally, the computational time of GA and dynamic programming in previous research may rise significantly when the number of evaluation points increases. This study aims to develop an optimization tool to optimize long-haul aircraft trajectories in a wind field by considering the ground path, vertical profile and airspeed simultaneously and to obtain a near-optimal solution within an acceptable computational time. The outcomes of the model are the optimal en-route trajectories on long-haul routes by considering the wind fields and a set of constraints based on ATC regulations. The corresponding research question of this project can be defined as:

How to design an optimization model to identify the minimum consumption trajectory on long-haul routes taking into account en-route winds and a set of constraints based on ATC regulations by a computational method?

According to the analysis above, optimal control has a fast convergent speed but the investigated space is limited which may result in a local optimum. On the other hand, GA is relatively slower compared to optimal control. This is mainly caused by the required number of problem evaluations which is significantly affected by the number of problem parameters and the presence of constraints in the problem formulation. Infeasible solutions of the problem may be evaluated by the optimizer of GA. Therefore, the computational time and memory space are wasted for infeasible solutions. However, GA offers the benefit of being a global search algorithm. This means that methods based on GA consider a wider search space, and theoretically would allow identifying a global optimal solution. In order to minimize the computational time while maintain the global searching standard, a 2-phase approach is proposed in this study. Phase 1 is to reduce the search space from a larger one with less optimal parameters compared to Phase 2. Phase 2, based on the results of Phase 1, searches for detailed solutions in a reduced search space.

The outline of this report is as follows. Chapter 2 describes details of the flight planning for this project, which is followed by the introduction of the tool design in Chapter 3. In Chapter 4, the verification of the designed tool is implemented. After that, several case studies of the designed tool are provided in Chapter 5. Finally, a conclusion is stated in Chapter 6.

2. Flight Planning

In this chapter, the background of the flight planning will be discussed. Firstly, the flight mechanics is discussed. After that, the wind data base applied in this study is introduced, which is followed by the introduction of a set of constraints based on the ATC regulations.

2.1 Flight Mechanics

Theoretically, the optimal ground track between an origin and a destination without any wind effects is the great circle path because it is the shortest route to connect the city pair. Aircraft consume the least fuel and flight time with a given TAS to perform flights along the great circle path. In terms of the vertical profile without wind effects, airlines pursue to fly as fast as possible when flight time is the priority to minimize. However, the airspeed of aircraft is not unlimited. TAS is restricted by the maximum operating speed (VMO) at low altitudes and the maximum operating Mach number (MMO) at high altitudes. Moreover, VMO increases with the increasing of flight altitudes, while MMO decreases with the increasing of flight altitudes. Therefore, the intersection altitude of VMO and MMO, named the crossover altitude, is the altitude where the aircraft is able to access the maximum feasible TAS. The figure below is the diagram of VMO, MMO and the crossover altitude formed by a Boeing 747-400 aircraft model.

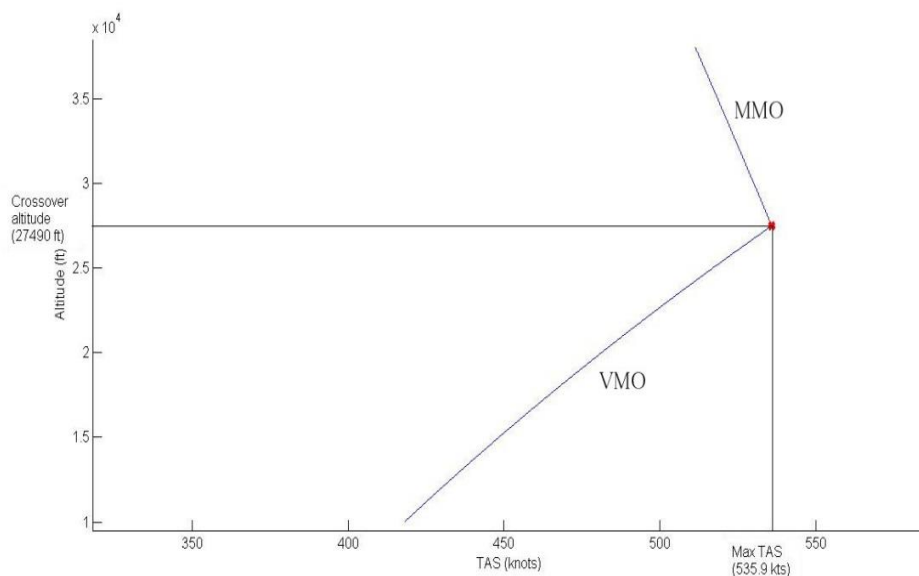


Figure 2.1. The TAS boundary formed by VMO and MMO from 10,000ft to 38,000ft.

As shown in the figure above, the maximum feasible TAS is reached at the crossover altitude. Therefore, aircraft is supposed to fly at the crossover altitude for the maximum feasible TAS to minimize the flight time.

Regarding to the fuel preference profile, aircraft are supposed to fly at the altitude with the highest fuel efficiency. In order to achieve the highest fuel efficiency, the specific range (range per unit weight of fuel) is supposed to be maximized. According to Ruijgrok, G.J.J (2004), with the assumptions that flying with a constant airspeed at a fixed altitude, and at a constant angle of attack as well as constant specific fuel consumption for the duration of flight, the specific can be expressed as:

$$R = \frac{2}{c_T} \sqrt{\frac{2}{S} \frac{C_L}{\rho C_D^2}} [\sqrt{W_1} - \sqrt{W_2}] \quad 2.1$$

Where:

- R is the specific range,
- c_T is the specific fuel consumption,
- S is the aircraft wing surface area,
- ρ is the air density at the flight altitude,
- C_L is the lift coefficient,
- C_D is the drag coefficient,
- W_1 is the initial weight of the flight distance,
- W_2 is the final weight of the flight distance.

As shown in equation 2.1, ρ is present in the denominator. That is, less air density will result in a higher specific range. Within the standard atmosphere, the air density decreases with the increasing of the altitude. Therefore, the maximum fuel efficiency will be achieved when the aircraft flying at high altitudes.

When the aircraft flies at constant airspeed and angle of attack, the specific range is expressed as (Ruijgrok G.J.J,2004):

$$R = \frac{V}{c_T} \frac{C_L}{C_D} \ln \frac{W_1}{W_2} \quad 2.2$$

Where V is the TAS. For a given initial weight and fuel load, the aircraft is supposed to fly at that altitude and airspeed at which the product $V \frac{C_L}{C_D}$ is a maximum. When

V is specified, the maximum $\frac{C_L}{C_D}$ will lead to the maximum specific range.

The table below indicates the specific range of a Boeing 747-400 model with 250,000 kg against to Mach numbers.

Table 2.1. The specific range (m) of a Boeing 747-400 aircraft model applied in this study.

	0.72	0.74	0.76	0.78	0.8	0.82	0.84	0.86
FL 380	112.2	114.1	115.5	116.5	117.2	117.6	117.7	117.5
FL 360	107.3	108.5	109.4	109.8	110.0	109.9	109.5	109.0
FL 340	100.5	101.2	101.5	101.6	101.3	100.8	100.2	99.4
FL 320	92.6	92.8	92.7	92.3	91.8	91.0	90.2	89.3
FL 300	83.9	83.8	83.4	82.8	82.1	81.2	80.3	79.3
FL 280	75.3	74.9	74.3	73.6	72.7	71.8	70.9	69.9

Table 2.1 states that the specific range increases with the increasing of the altitude. The figure above is the ratio of equation 2.2 and 2.1.

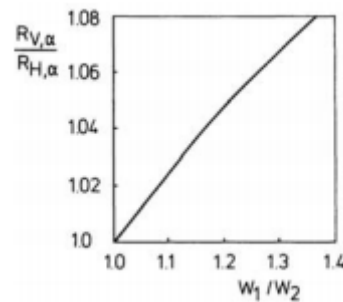


Figure 2.2. Relative range of equation 2.2 and 2.1 (Ruijgrok G.J.J, 2004)

The graph shown in figure 2.2 indicates the slope of the ratio of $\frac{R_{V,\alpha}}{R_{H,\alpha}}$ and $\frac{W_1}{W_2}$ is

larger than 1. That is, flying with a cruise-climb profile is more economical than level flight, especially when the aircraft executes a long-distance flight. (Ruijgrok G.J.J, 2004) Therefore, the theoretical optimal vertical profile with the maximum fuel efficiency is supposed to be a cruise-climb profile.

On the other hand, the temperature of air also affects the performance of engines. The figure below shows the temperature of International Standard Atmosphere (I.S.A) against to altitudes. For altitudes below the service ceiling of most aircraft, the temperature decreases with the increasing of altitudes. Since cool air expands more when heated than warm air and the expansion of the air is the power to drive the engines. Therefore, cool air will produce more power than warm air.

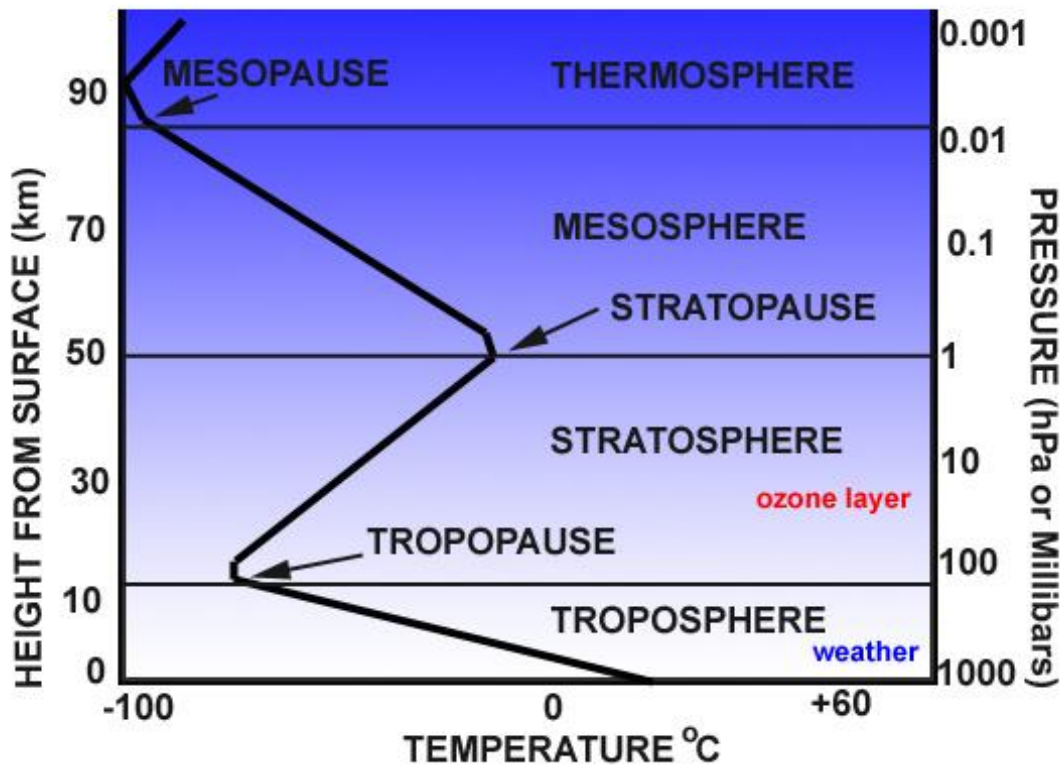


Figure 2.3. The temperatures of I.S.A at different altitudes. (Mason J., 2013)

However, the situation is significantly different when taking wind effects into account. The theoretical optimal trajectories in a wind field may not be the optimal trajectories because GS is not equal to TAS. GS in a wind field is calculated based on TAS, aircraft heading angle, wind vector angle and drift angle. The formula is as follow (H. Hirabayashi and Y. Fukuda, 2014):

$$V_{gs} = V_{TAS} \cos \phi_D + V_w \cos(\phi_w - \psi) \quad 2.3$$

Where

- V_{gs} is the GS,
- V_{TAS} is the TAS,
- V_w is the wind speed,
- ϕ_w is the wind vector angle,
- ψ is the heading angle,
- ϕ_D is the drift angle.

Drift angle is calculated by:

$$\phi_D = \sin^{-1}\left(\frac{V_w}{V_{TAS}} \sin(\phi_w - \psi)\right) \quad 2.4$$

The vector relationship between TAS and GS is shown in figure 2.4.

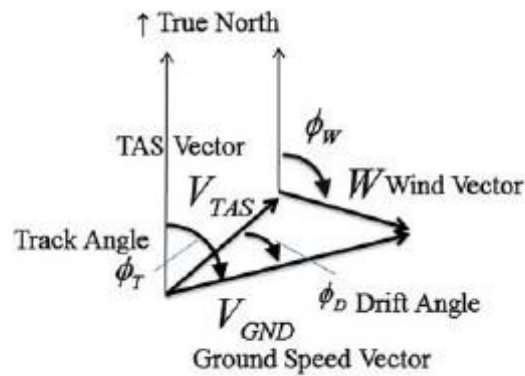


Figure 2.4. Relation between heading direction and wind direction. (H. Hirabayashi and Y. Fukuda, 2014)

In the real world, flight trajectories of airlines are varying to the theoretical trajectories due to wind effects and influences of other factors. The figures below show the ground tracks of the flight from AMS to JFK, and the returned flight from JFK to AMS on 1st October 2014 by a Boeing 747-400.



Figure 2.5. The ground track of the flight from AMS to JFK on 1st October 2014. (Flight Aware, 2014)



Figure 2.6. The ground track of the flight from JFK to AMS on 1st October 2014. (Flight Aware, 2014)

Due to the eastward jet stream over NAO shown in figure 1.1, the ground track from AMS to JFK is more northerly than the ground track from JFK to AMS because the airline tried to avoid the strong headwind when flying from the east to the west, while pursued the tailwind when flying from the west to the east.

2.2 Wind profile

As discussed previously, the wind effect is an important factor for identifying the optimal trajectory. In most cases, wind effects on aircraft during the cruise phase are caused by the jet streams flowing from the west to the east on the earth. Jet streams are fast flowing, narrow air currents found in the atmospheres of some planets, including Earth (National Geographic, n.d.). Jet streams on the earth are caused by the rotation of the earth and the atmospheric heating such as the solar radiation. The typical speed of the jet streams is from 35.8 m/s to 62.5 m/s (69.5 kts to 121.5 kts). Normally, the jet streams are faster in winter when the temperature difference between tropical air currents and polar air currents are greater (National Geographic, n.d.). Jet streams normally form in the area between two air masses with a large temperature differences as shown in the figure below.



Figure 2.7. The jet stream between two air masses. (WW2010, 2010)

The strongest jet streams are the polar jets at the altitude around 7000 m to 12000 m (23000 ft to 39000 ft) near to the poles, while the weaker jet streams are the subtropical jets at round 10000 m to 16000 m (33000 ft to 52000 ft) (Princeton University, n.d.). The polar jets and the subtropical jets both locate on the Northern Hemisphere and the Southern Hemisphere. Hence, there are four major jet streams on the earth.

There are other jet streams formed at different seasons on the earth. During the Northern Hemisphere summer, jet streams can be formed in tropical regions where dry air encounters more humid air at high altitudes. In the winter months of the Northern Hemisphere, the arctic and tropical air masses encounter, which create strong jet streams above the America as shown figure 2.7.

In this project, the wind data is obtained from the official weather website of the government of Canada (Weather, 2014). The data provided in the website is 25 km resolution numerical data of the global deterministic prediction system (GDPS) model (GRIB2 format). There are 27 isobaric wind profiles at 27 pressure altitudes provided in the website from 50 hpa to 1015 hpa. As shown in the wind profiles around AMS on August 31 2014 at different pressure altitudes below, wind profiles in the same region at different altitudes are similar to each other. The error caused by the pressure variation at the same physical altitude is limited. On the other hand, it is difficult to identify the application range of an isobaric wind profile since the pressure in the same region is various at different times. Therefore, the isobaric wind profiles obtained from the official weather website of the government of Canada are applied to different geographic altitudes in this study. By applying NC toolbox, this type of file can be read by Matlab.



Figure 2.8. Wind profiles at various pressure altitudes around AMS.

2.3 Constraints

Currently, aircraft of airlines follow pre-set waypoints to perform flights. Aircraft fly from one waypoint to the next one until arriving at the destination. The figure below is an example of the flight routes between Monterrey and Leon-Guanajuato.

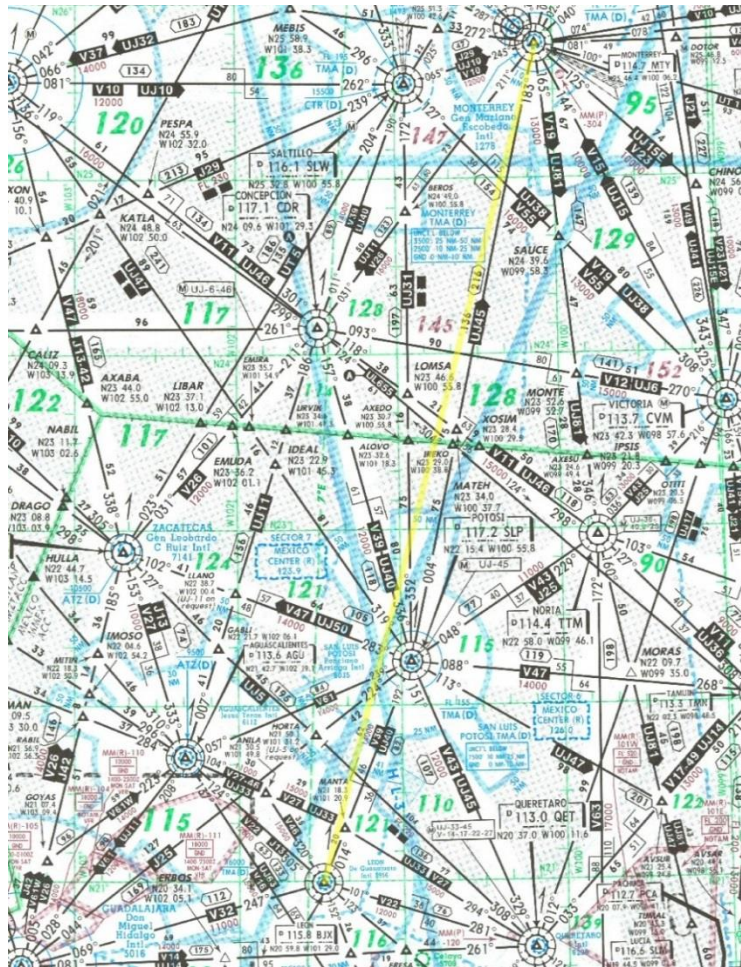


Figure 2.9. The flight routes between Monterrey and Leon-Guanajuato. (Penick .T, 2010)

However, the instrument to restrict the ground tracks by waypoints results in an inefficient utilization of airspace because aircraft have to follow certain trajectories while keeping safe separations between each other. In 1981, a theory of optimal flight routing was investigated during a six month study called Operation Free Flight (J. A. McDonald and Y. Zhao, n.d.). The definition of the free flight was defined in *RTCA Task Force 3 Free Flight Report* and later quoted in *Concept Definition for Distributed Air/Ground Traffic Management (DAG-TM) and National Airspace System Architecture Version 4.0* as the following (Jimmy Krozel, 2000):

“... a safe and efficient flight operating capability under instrument flight rules (IFR) in which the operators have the freedom to select their path and speed in real time. Air traffic restrictions are only imposed to ensure separation, to preclude exceeding airport capacity, to prevent unauthorized flight through Special Use Airspace (SUA), and to ensure safety of flight. Restrictions are limited in extent and duration to correct the identified problem. Any activity which removes restrictions represents a move toward free flight.”

Since airlines are able to select paths and TASs, the wasted flight time and fuel due to the constraints of waypoints are minimized. Consequently, the utilization of airspace under the free flight concept is more efficient than the current air transport control system.

In terms of the vertical profile, ICAO proposed the Reduced Vertical Separation Minima (RVSM) in 1982 to reduce the Vertical Separation Minima (VSM) above 29000 ft from 2000 feet to 1000 feet. However, RVSM is only available for aircraft that meet the equipment requirements of RVSM within specific regions, otherwise 2000 feet vertical separations are required (Eurocontrol, 2014). In addition, available flight levels are different with different flight directions. For eastbound tracks (0° to 179°), odd flight levels, such as FL70, FL90, are applied, while even flight levels (e.g. 80, 100) are applied to the westbound tracks (180° to 359°). The settings of the horizontal and vertical profiles in this study will be discussed in the next chapter.

3. Optimization Tool Description

In this chapter, the designed tool for identifying the optimal trajectory is introduced. The optimization methodology applied in this study is discussed in the section 3.1. After that, detailed introductions about optimization algorithms are provided.

3.1 Optimization method

Based on the literature study, optimal control is a local search algorithm with an efficient computational performance. GA and dynamic programming are global search algorithms but the computational power is not as strong as optimal control due to computational losses for infeasible solutions. In this project, the objective is to identify the global optimal solution. Therefore, optimal control, as a local search algorithm, is not applied in this study. Since this study is a prospective research, the results of this research may apply as a reference for further research. The free flight concept, which is introduced at selected facilities by the close of 2002 (NASA,n.d) because it can maintain the freedom during the optimization process. By applying this concept, the optimization process for the ground track is fully free. No fixed waypoints are provided. Hence, dynamic programming is not suitable in this study because dynamic programming is only efficient for problems with coarse grid discretization. Since GA, a global search algorithm, does not require any discretization for the optimization process, GA is applied as the optimization method in this study. In this project, the Non-dominated Sorting Genetic Algorithm (NSGA) is selected. The NSGA is an approach that evaluates a population of solutions in each generation. For the final outcomes of NSGA, a batch of solutions is generated. Among them, trade-off solutions are identified. Another reason to choose NSGA instead of other GA algorithm is that NSGA describes individuals by a numeric string instead of a binary string. It is able to discretize parameters by dividing parameters into corresponding number of parts.

In order to overcome the computational inefficiency of NSGA, a 2-phase approach is proposed in this study. Phase 1 is to reduce the search scope for Phase 2, while the outcomes of Phase 2 are detailed solutions. Since the objective of Phase 1 is to narrow the search scope for Phase 2, the outcomes of Phase 1 can be rough solutions. The number of parameters is smaller than the number of parameters in Phase 2. The accuracies of flight time and fuel consumption are not the priorities in Phase 1 as long as the optimizer narrows the search space to the optimal solution area. Based on the outcomes of Phase 1, the search space in Phase 2 is reduced and

the outcomes of Phase 2 are the detailed solutions with more parameters and evaluation points. The purpose of this approach is to avoid the ineffective optimization process in the irrelevant search area while obtaining detailed solutions. Beside the 2-Phase approach, approaches based on the parameterization are introduced in this study to reduce the number of parameters and minimize number of infeasible solutions. More details about these approaches will be discussed in the following sections. By applying these approaches, the computational performance is more efficient.

3.2 Phase 1

Phase 1 is supposed to locate a reduced search space for Phase 2. The accuracy is not prior as long as the optimizer is able to distinguish the influence of parameters on the objectives. In order to describe trajectories, a larger number of control variables are required including the longitude and latitude of each waypoint, the TAS, the distances and the altitude change direction for each vertical segment. On the other hand, infeasible solutions may occur with some conditions such as an exceeded flight distance. In order to avoid the ineffective computational process, algorithms based on the parameterization are implemented.

In the rest of this section, the introduction for east-west routes, which apply the longitude axis as the base line, is provided. The difference of the processes for east-west routes and north-south routes is that east-west routes use longitude as the base line, while north-south routes apply the latitude as the base line.

3.2.1 Horizontal Profile

For the horizontal profile, the tool optimizes locations of waypoints from the west to the east for all east-west routes and from the south to the north all north-south routes. Once the coordinate of last waypoint is determined, the last segment is automatically linked from the last waypoint to the east ending city of the route. By applying this algorithm, generated trajectories are guaranteed to terminate at these two cities.

For the optimization process of locations of waypoints, the tool first determines the longitude of each waypoint. Then the latitude is chosen within a certain bound. In order to ensure the waypoints are sequenced from the origin to the destination, the

optimizer divides the total processed distance L into several ranges for waypoints by equation 3.1.

$$M = \frac{L}{l_{min,h}} \quad 3.1$$

Where M is the number of the longitude selection range, if the outcome is not an integer, then M is chosen as the closest lower integer. $l_{min,h}$ is the minimum segment distance between two successive waypoints. As shown in the figures below, the processed distance from the west to the east (from the left to the right) is divided into 3 ranges. These two blanks near the two terminations of the route are both fixed distances for the climbing and descent phases in Phase 2.

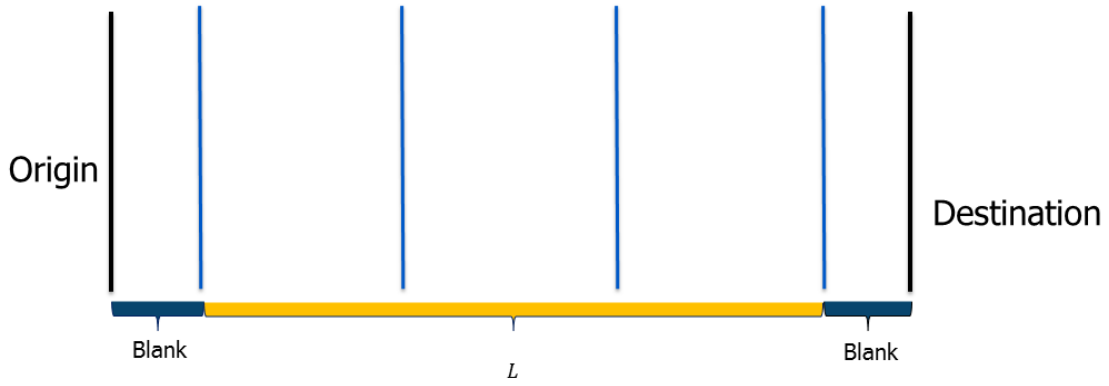


Figure 3.1. Step 1 of the optimization process for the horizontal profile.

In this study, fixed bounds of parameters are required by NSGA. However, the boundaries of parameters such as the longitude of each waypoint are varying with numerous factors. A conservative estimation of these bounds may limit the freedom for the optimizer, while a too wide estimation will result in the evaluations of infeasible solutions. Additional constraints may solve this problem but will lead to longer computational times and slow convergences. Hence, the normalization is implemented to solve the problem. Instead of directly optimizing the value of the control variables, the optimizer optimizes the normalized control variable. Equation 3.2 is applied to determine the longitudes (dash lines) of waypoints as shown in figure 3.2.

$$\lambda_n = (n - 1) \times l_{min,h} + \eta_{n,\lambda} \times l_{min,h} + \lambda_0 \quad 3.2$$

Where n is the integer number between 1 and M , λ_n is the longitude of the n^{th} waypoint, λ_0 is the longitude of the initial point of L , $\eta_{n,\lambda}$ is the control variable between 0 and 1.

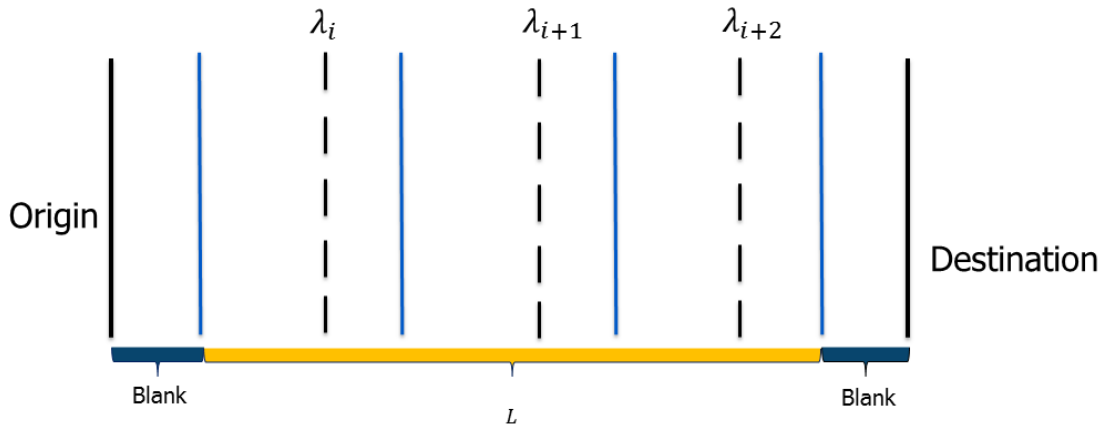


Figure 3.2. Step 2 of the optimization process for the horizontal profile.

In this study, $l_{min,h}$ is set as 10.8° , which is chosen based on the optimization performance of the optimizer. The table below shows computational time of the optimization process and the flight time and the fuel consumptions of the corresponding optimal trajectories with different preferences of Phase 1 between AMS and JFK with 6° vertical basic segment distance.

Table 3.1. The performances of the tool with different $l_{min,h}$.

$l_{min,h}$	Run time (gen)	No. Segment	Pop	No. Variables	Fuel Preference	Time Preference
8	490.0 s (200)	9	88	33	76,871.7 kg 26,910.5 s	104,167 kg 24,194.9 s
10.8	433.0 s (200)	7	88	29	75,965.8 kg 25,375.7 s	98,038.5 kg 23,066.3 s
13.6	436.0 s (200)	6	88	27	76,866.6 kg 27,429 s	101,340 kg 23,904.9 s

As shown in table above, the fuel and time preference trajectories of the optimization process with 10.8° $l_{min,h}$ show better performance in terms of fuel consumption and flight time compared to optimization with other two settings. The computational time of the optimization process with 10.8° $l_{min,h}$ is 11.6 % and 0.7% shorter than the optimization process with 9.6° and 16.8° $l_{min,h}$, respectively. Therefore, 10.8° is set as the minimum segment distance between two successive waypoints.

Once the longitude of each waypoint is determined, the tool turns to determine the latitude of each waypoint. During the optimization process, infeasible solutions will occur if the total flight distance is larger than the designed flight range. That is, the aircraft is out of fuel to perform the whole flight. In order to solve this problem, the Pythagorean Theorem on the flat earth model is applied to determine the feasible

latitude range of each waypoint based on the remaining available flight distance (D_r), the coordinates of the previous waypoint (or the origin) (λ_o, φ_o), the coordinates of the destination (λ_f, φ_f), the longitude of the current waypoint (λ) and the scale distance (SD):

$$D_r = \sqrt{[(\varphi - \varphi_o) \times SD]^2 + [(\lambda - \lambda_o) \times SD]^2} + \sqrt{[(\varphi - \varphi_f) \times SD]^2 + [(\lambda - \lambda_f) \times SD]^2} \quad 3.3$$

The remaining available flight distance is the difference between the designed flight range $D_{designed}$ and the travelled distance $D_{travelled}$ as:

$$D = D_{designed} - D_{travelled} \quad 3.4$$

The scale distance is the average scale distance of the great circle path between the city pair, which is calculated as:

$$SD = \frac{D_{Great\ Circle}}{\sqrt{(\lambda_o - \lambda_f)^2 + (\varphi_o - \varphi_f)^2}} \quad 3.5$$

By solving equation 3.3, the feasible range of the latitude of each waypoint can be obtained such that the total flight distance of each solution is within the limitation of designed flight distance. As shown in figure 3.3, the green lines and grey lines are the ground tracks with the maximum remaining flight distance. The red line indicates the feasible range of the latitude for the i^{th} waypoint. The latitude of the i^{th} waypoint is calculated as:

$$\varphi_i = (\varphi_{i,max} - \varphi_{i,min}) \cdot \eta_{i,\varphi} + \varphi_{i,min} \quad 3.6$$

Where $\varphi_{i,max}$ and $\varphi_{i,min}$ are the latitude bounds of each waypoint, $\eta_{i,\varphi}$ is the parameter between 0 and 1 optimized by the model.

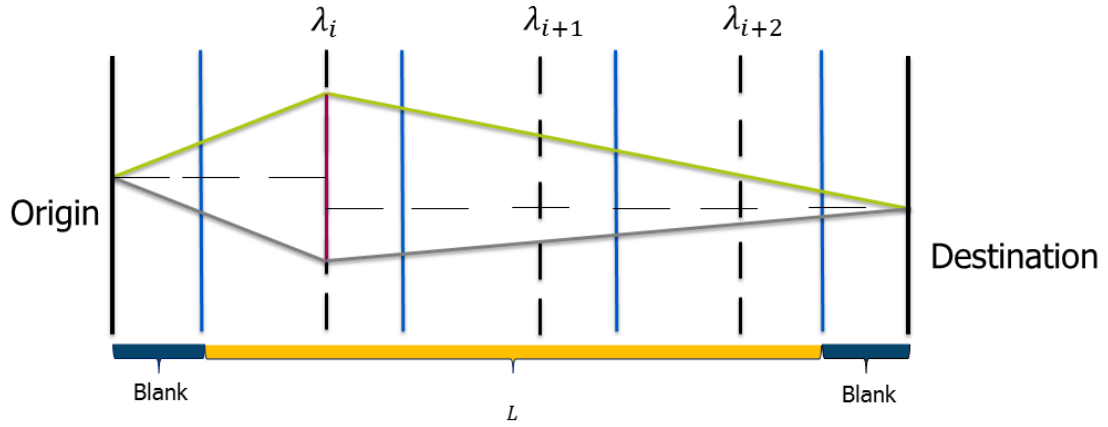


Figure 3.3. Step 3 of the optimization process for the horizontal profile.

The reason to apply the flat earth model instead of the spherical earth model is the difficulty to solve the trigonometric functions in the equations of the spherical earth model. According to Movable Type Scripts (n.d.), the equations for the distance between two points on the earth are stated as equation 3.7, 3.8 and 3.9.

$$a = \left(\sin \frac{\Delta\varphi}{2}\right)^2 + \cos \varphi_1 \cdot \cos \varphi_2 \cdot \left(\sin \frac{\Delta\lambda}{2}\right)^2 \quad 3.7$$

$$c = 2 \cdot \text{atan2}(\sqrt{a}, \sqrt{(1-a)}) \quad 3.8$$

$$d = r \cdot c \quad 3.9$$

Where φ_1 and φ_2 are the longitude of two waypoints, and r is the radius of earth. In order to obtain the available range of the latitude, the optimizer is required to inversely solve the equations above. The explicit solution is difficult to obtain due to the trigonometric functions.

As discussed above, the flat earth model is simpler and more efficient than the spherical earth model. However, there are errors produced during the computation process of the flat earth model, even though the average scale of the great circle path between the city pair is applied. These errors are caused by differences between longitude scales at different latitudes. When the optimizer calculates the largest virtual trajectories, the errors of the longitude distance (the thin dash lines in figure 3.3) may cause errors of the largest virtual trajectories. A larger longitude distance between the processed waypoint and the east city of the trajectory will cause a larger computational error. For example, for the route between AMS and Singapore (SIN) (10516 km), the largest error occurs when processing the closest waypoint to the AMS. The error of the upper latitude bound (φ_{max}) is 3.1%, while the error of the lower latitude (φ_{min}) is 11.2%.

Although there are errors caused by the flat earth model, the algorithm still can eliminate most infeasible solutions, which would significantly enhance the computational performance. In some cases especially relatively short routes, there is no infeasible solution evaluated during optimization if the maximum search range is set smartly. For instance, for the route between AMS and JFK, if the maximum search range set by users and the designed flight range of the aircraft model are 4,859.6 nm (9,000 km) and 6,184.65 nm (11,454 km), respectively. There is 2,454 km ($11,454 - 9,000 = 2,454$) distance buffer before the flight distance reaches the real limitation (11,454 km) of the aircraft model. In this case, the buffer distance between the maximum search range and the designed flight range is able to compensate the error caused by the flat earth model. On the other hand, for north-south routes, the error is not significant because the latitude scalars are the same at different longitudes. In this project, the maximum search distance of east-west routes is set as the direct distance with a 540 nm (1,000 km) additional distance because of the error consideration or the designed flight distance (6,185 nm (11,454 km)) of the aircraft model, while the test distance of north-south routes is set as the direct distance with a 1,350 nm (2,500 km) additional distance or the designed flight distance.

In the optimization process for horizontal profiles in Phase 1, only the location of each waypoint is optimized in order to ensure the evaluated ground track terminates at the origin and destination. The algorithm for the longitude of each waypoint is to guarantee the processed waypoints are sequenced. By smartly parameterizing the problem, infeasible solutions are hardly generated due to the exceeded flight distance, especially relatively short routes.

3.2.2 Vertical Profile

As discussed in Chapter 2, RVSM is only available for aircraft that fulfil the equipment requirements of RVSM within specific regions. For the convenience, all flight levels in this study are discretized in steps of 2,000 ft. Since there is only one aircraft optimized in the optimization process, there is no difference between flying at the even flight levels or the odd flight levels. The flight levels of the tracks from 180° to 359° (even flight levels) are applied in this report for both directions. For the vertical search scope, the flight levels between 10,000 ft and 38,000 ft are considered in this study. The reason to set 10,000 ft as the lower bound is that approach and departure procedures normally control up to 10,000 ft (Karwowski W. 2006). On the other hand, since the main research factor of this study is wind effects, the main jet streams on the earth are located between 23,000 ft and 39,000 ft as introduced in Chapter 2, the up bound of the search scope is set as 38,000 ft in this study. Consequently, 15 wind profiles from the weather website of the government of Canada (Weather, 2014)

between 175 hpa (41,000 ft in I.S.A. conditions) and 700 hpa (8,100 ft in I.S.A. conditions) pressure altitudes are applied to 15 physical altitudes.

When optimizing the vertical profile, TAS, altitude changing directions, and vertical segment distances are control variables. During the optimization process, infeasible solutions may occur when TAS or the flight altitude is beyond the feasible range. Therefore, it is necessary to determine the feasible range of TAS and the flight altitudes in order to minimize the number of infeasible solutions. Moreover, the normalization is implemented to ensure that all value selections are within the feasible ranges.

When locating the feasible ranges of TASs, the impact of the horizontal force balance limitation is significant. Table 3.2 and 3.3 show the maximum excess power, which is the difference between the maximum feasible thrust and the drag force, of a Boeing 747-400 aircraft model with 362,874 kg and 250,000 kg weight at different flight levels with different TASs, respectively. The cells with non-zero numbers indicate that the TASs at corresponding flight levels are within the flight envelope. Specifically, the cells with negative numbers represent that the maximum available thrust is less than the drag force. On the other hand, cells with positive values state the feasible TASs at corresponding flight levels.

Table 3.2. The maximum excess power at different flight levels with various TASs and 362,874 kg.

	280 kts	320 kts	360 kts	400 kts	440 kts	480 kts	520 kts	560 kts
FL 300	0	0	-11,410	-3,045	-4,348	-12,903	-26,203	0
FL 280	0	0	13,309	18,301	14,366	3,814	-10,891	0
FL 260	0	0	37,660	39,296	32,666	19,980	3,657	0

Table 3.3. The maximum excess power at different flight levels with various TASs and 250,000 kg.

	520 kts	522 kts	524 kts	526 kts	528 kts	530 kts	532 kts	535 kts
FL 340	8,394	0	0	0	0	0	0	0
FL 320	19,489	18,833	18,181	0	0	0	0	0
FL 300	30,221	29,500	28,785	28,075	27,372	26,675	0	0
FL 280	40,661	39,867	39,079	38,297	37,522	36,755	35,995	0
Crossover Altitude	43,283	42,469	41,661	40,860	40,065	39,278	38,499	37,347
FL 260	50,865	49,989	49,119	0	0	0	0	0

As shown in the figures above, different aircraft weight will result in different feasible TAS ranges. Specifically, 520 kts at FL280 is a feasible TAS with 25,000 kg aircraft weight as shown in table 3.3, while it is not a feasible TAS table 3.2. The aircraft with 362,874 kg and 520 kts at FL280 is too heavy to maintain the force balance in the horizontal direction even though the TAS is within the flight envelope. Therefore, it is

necessary to consider the impact of the force balance in the horizontal direction when optimizing the TAS.

In this study, the aircraft is assumed to fly at a constant TAS within a single segment because acceleration and deceleration processes are completed in a short time, which means the thrust is set as the drag force. That is,

$$T_{max} = D \quad 3.10$$

The maximal cruise thrust is a function of Mach number and flight altitudes,

$$T_{max} = f(M, h) \quad 3.11$$

While the drag force can be calculated by the drag coefficient, the air density on the flight level, the wing surface area and Mach number.

$$D = f(C_D, \rho, S, M) \quad 3.12$$

By substituting equation 3.11 and 3.12 into equation 3.10, an equation with Mach number as the variable at a given altitude is obtained. Therefore, the feasible range of TAS on a flight level is obtained by multiplying Mach number with the speed of sound at that altitude. In order to maintain the feasibility of selected TASs, the normalization process is implemented. TAS of each segment is obtained as equation 3.13. Where η_{TAS} is the control variable between 0 and 1.

$$V_{TAS} = (V_{TAS,max} - V_{TAS,min}) \cdot \eta_{TAS} + V_{TAS,min} \quad 3.13$$

The control parameters of the altitude change direction are discretized into three commands. Values between 0 and 1/3 mean a descent to the optimizer, while values from 1/3 to 2/3 command that the altitude stays at the same altitude in the next flight segment. Finally, values between 2/3 and 1 present a climb to the next higher flight level. Before the optimizer simulating next vertical segment, the tool searches for the feasible range of TAS at the altitude of the next vertical segment by equation 3.10. If the feasible range is identified, the range will be applied to the next vertical segment. If not, the aircraft will stay at the current altitude until arriving at next altitude changing location. Once the aircraft reaches the ceiling of the search space, any climbing commands will be automatically convert into levelling commands. The same procedure is also applied when the aircraft flies at the bottom of the search space.

Regarding to the initial altitude in Phase 1, for time preference profile, the aircraft is supposed to fly at the crossover altitude in order to access the maximum TAS, while the aircraft is supposed to fly at high altitudes to save fuel for minimum fuel consumption. In this study, the aircraft model applied is the aircraft model developed by Teengs, M. (2006). The VMO in this model is 365 kts, while the MMO is 0.9. The crossover altitude is 27,490 ft. The maximum Take-off Weight (MTOW) is 362,874 kg. Table 3.2 and 3.3 is the data deduced from the aircraft model applied in this study. Consequently, the best choice for both time and fuel preference profiles with MTOW is 28,000 ft, because 28,000 ft is the closest flight altitude to the crossover altitude and the highest flight altitude that aircraft with MTOW can reach base on table 3.2. During the optimization, the initial altitude Z_0 of Phase 1 is set as 26,000 ft. The optimizer is allowed to climb to 28,000 ft, or stay at 26,000 ft, or even descend to 24,000 ft to start trajectories. The purpose to allow the optimizer to choose the starting altitude is to avoid a possible strong headwind in the beginning of trajectories.

In terms of the distance of the vertical segment in the cruise phase, the processed distance (L) is separated into several vertical segments at the altitude changing locations. In order to make sure the vertical profile is terminated at the ending point of the cruise phase, the distance of the last vertical segment is from the ending point of the penultimate segment to the ending point of the cruise phase. The segment distance except the last segment is calculated by:

$$D_{v,n} = D_{v,b} + \eta_{v,n} \cdot D_{v,b} \quad 3.14$$

Where $D_{v,n}$ is the horizontal distance of the n^{th} vertical segment, $D_{v,b}$ is the basic distance of a vertical segment set by users, and $\eta_{v,n}$ is a control parameter between 0 and 1. By applying this algorithm, the distance of each vertical segment is between $D_{v,b}$ and $2D_{v,b}$. Any points from the point $D_{v,b}$ away from the west ending point of the processed distance to the east ending point of the processed distance are available to be chosen, which ensures the freedom for the optimizer in this study.

In this study, the basic distance of a vertical segment $D_{v,n}$ is set as 6° . Similar to $l_{min,h}$, $D_{v,n}$ is set based on the performance of the optimization process. The table below shows run times of the optimization processes and performances of the corresponding optimized trajectories of Phase 1 from AMS to JFK with 10.8° horizontal segment distance.

Table 3.4. The performances with different $D_{v,n}$.

$D_{v,n}$	Run time (gen)	No. Segment	Pop	No. Variables	Fuel Preference	Time Preference
3.6	437.4 s (200)	10	88	41	73158.2 kg 26326.8 s	95954.4 kg 23303.8 s
6	433.0 s (200)	6	88	29	75965.8 kg 25375.7 s	98038.5 kg 23066.3 s
9	423.7 s (200)	4	88	23	79697.3 kg 26203.5 s	94038.1 kg 22911 s

As shown in the table above, the optimization performance in terms of fuel consumption increases with the increasing of $D_{v,n}$, while the optimization performance regarding to the travel time is negative relation to $D_{v,n}$. Choosing either extreme setting will cause a large loss in either the fuel consumption or travel time. Moreover, the variations between the computational times of these three settings are small. Therefore, the compromised solution (6°) is applied in this study.

3.2.3 Simulation Method

Although the approaches proposed above are able to reduce the number of control parameters and the number of infeasible solutions, the optimization process still requires long computational time for the evaluation of each solution with small time/distance step. In order to increase the computational speed, concepts of the equivalent wind speed and the equivalent weight for each segment are proposed in Phase 1. The equivalent wind speed and weight concepts introduce constant wind speed and constant aircraft weight in a vertical segment. By doing this algorithm, the distance step is able to expand to a whole vertical segment. In order to verify these concepts, some tests are implemented. Since the purpose of Phase 1 is to reduce the search space while keeping the optimal solution within the reduced search space, detecting the behaviour of parameters to the objectives is the priority in Phase 1. As long as the optimizer is able to distinguish the impacts of parameters to the objectives, the algorithm is acceptable.

The tested distance for the equivalent concepts in this study is 1,000 km. One of the reasons is that the average distance of the segments tested in this study is lower than 1,000 km. The average distance of AMS-JFK, AMS-JNB, and AMS-SIN are 451 km ($\frac{5,863}{13}$), 691 km ($\frac{8,986}{13}$) and 618.6 km ($\frac{10,516}{17}$). Since segments with distance larger than the average distance is possible to be generated, 1,000 km is chosen as the test

distance. In terms of the tested altitudes, the crossover altitude (27,490 ft) and the typical cruise altitude (35,000 ft) of the aircraft model applied in this study are chosen. Two random TASs (180 m/s and 220m/s) are also chosen to as the tested TASs. The initial aircraft weight of the tests is 260,000 kg.

In terms of the equivalent wind speed, the average wind speed of a segment is calculated before the optimization process. In a segment, the wind speed data is extracted every 0.24° (resolution of wind data) longitude/latitude distance for the average wind speed. Figure 3.4 and 3.5 show the fuel consumptions and the travel times of the equivalent wind speed model and the detailed wind speed model.

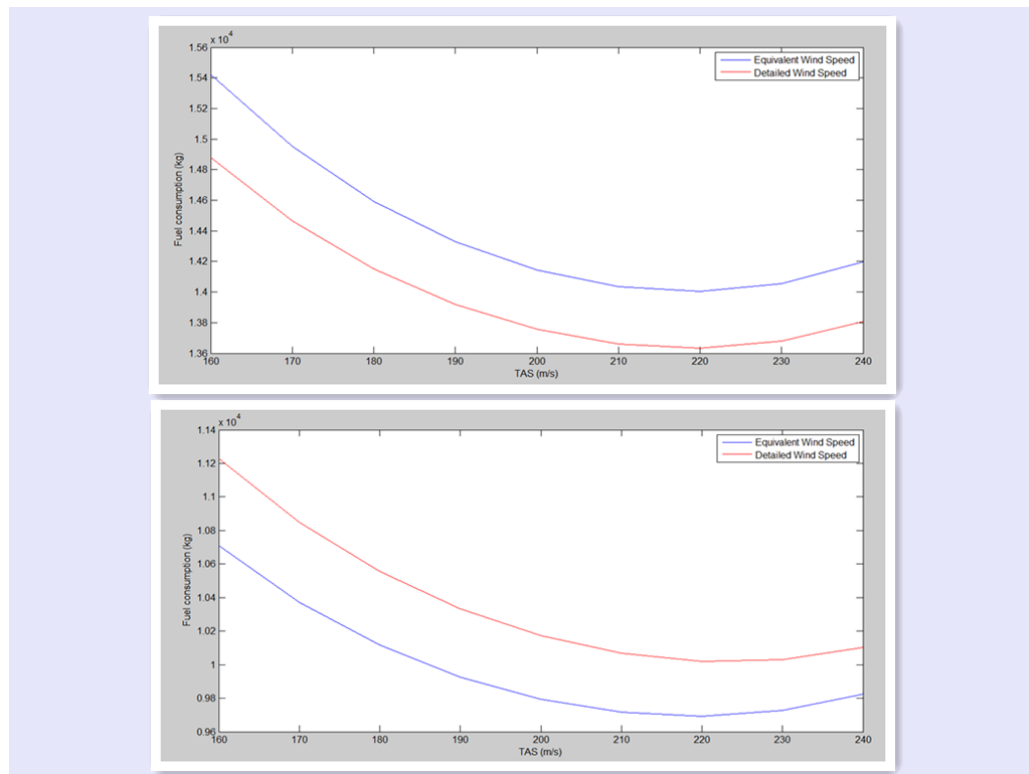


Figure 3.4. Fuel consumption with different TASs at the Crossover altitude (above) and the Typical cruise altitude (below).

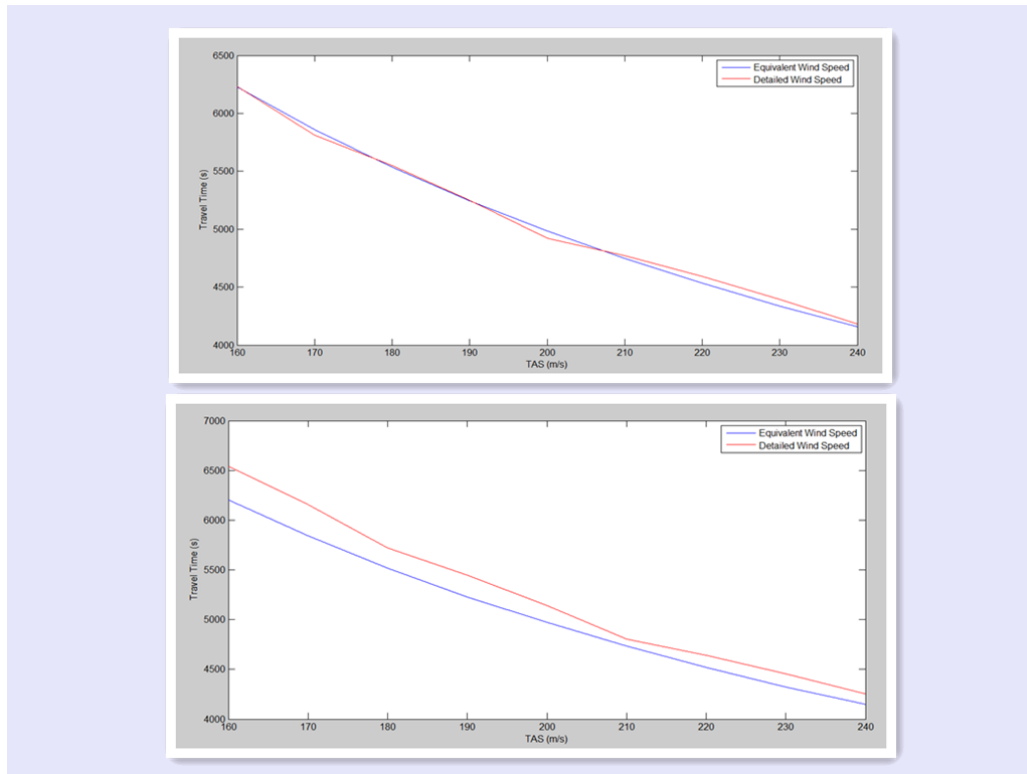


Figure 3.5. Travel time with different TASs (above: Crossover altitude, below: Typical cruise altitude).

As shown in figure 3.4, the differences of the fuel consumptions between these two models at both altitudes are 3.3% and 4.7%. In figure 3.5, the flight times of two models are almost the same, the largest difference is that aircraft fly at the typical cruise altitude with 160 m/s TAS, the error is around 4.5%. Although there are errors between these two models, the shapes of the equivalent wind speed (1,000km) and the detailed wind speed (25km) are nearly the same to each other in terms of the fuel consumption and the flight time. Figure 3.6 indicates the fuel consumptions of the equivalent wind speed model and the detailed wind speed model at different altitudes with 180 m/s and 220 m/s. The shapes of the two models with both TASs are the same to each other. Since the TAS and the wind profiles are the same at different altitudes, the travel time does not change at different altitudes. So the figures of flight times at different altitudes are not shown in the report.

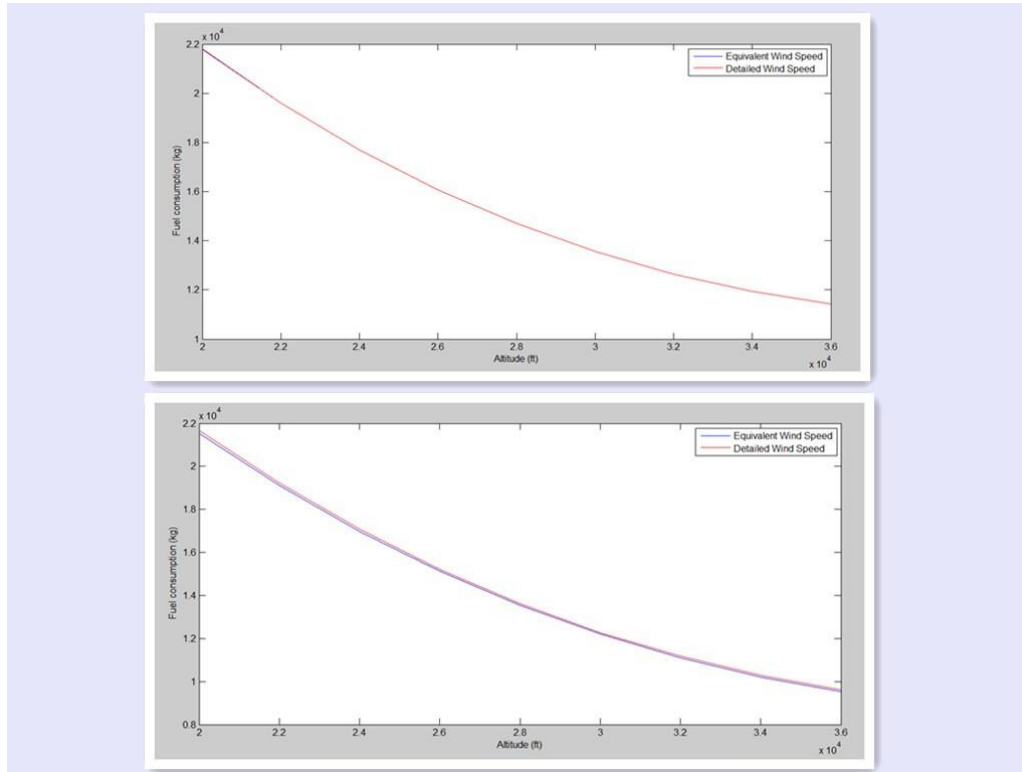


Figure 3.6. The fuel consumptions at different altitudes with a constant TAS (above: 180 m/s, below: 220 m/s).

Since the objective of Phase 1 is to narrow the search scope for Phase 2, so the accurate values of fuel consumption and flight time are not important as long as the search scope is properly reduced to the area of the optimal solution. Therefore, the equivalent wind speed concept is acceptable in Phase 1.

Regarding to the equivalent weight, the weight of an aircraft is assumed to be a constant in a single segment. The fuel consumption rate is calculated based on the weight at the starting point of a segment. Firstly, the estimated final weight of a segment is calculated by the fuel consumption rate with the initial weight of the segment. After that, the average of the initial weight and the estimated final weight is calculated and set as the constant weight of the segment. For example, the initial weight of an aircraft is 260,000 kg, the corresponding fuel consumption rate at 9,000 m (29,527.5 ft) with 250 m/s (485.96 kts) TAS is 2.85 kg/s. If the distance of a segment is 200 km, then the estimated final weight is calculated as $260,000 - 2.85 \times \left(\frac{200,000}{250}\right) = 257,720 \text{ (kg)}$. Hence, the average weight is $\frac{(260,000+257,720)}{2} = 258,860 \text{ (kg)}$. After that, the average weight is applied as the constant weight to calculate the fuel consumption of the segment.

The figure below indicates the fuel consumption of the equivalent weight model and the detailed weight model with different TASs for 1,000 km distance without wind

effects. Since there is no wind effect and the TASs of the equivalent weight model and the detailed weight model are the same. It is not difficult to imagine that the travel time of two models are the same. So the figures about the travel times of the two models are not shown in this report. Figure 3.7 is the fuel consumptions of these two models with different TASs at two altitudes.

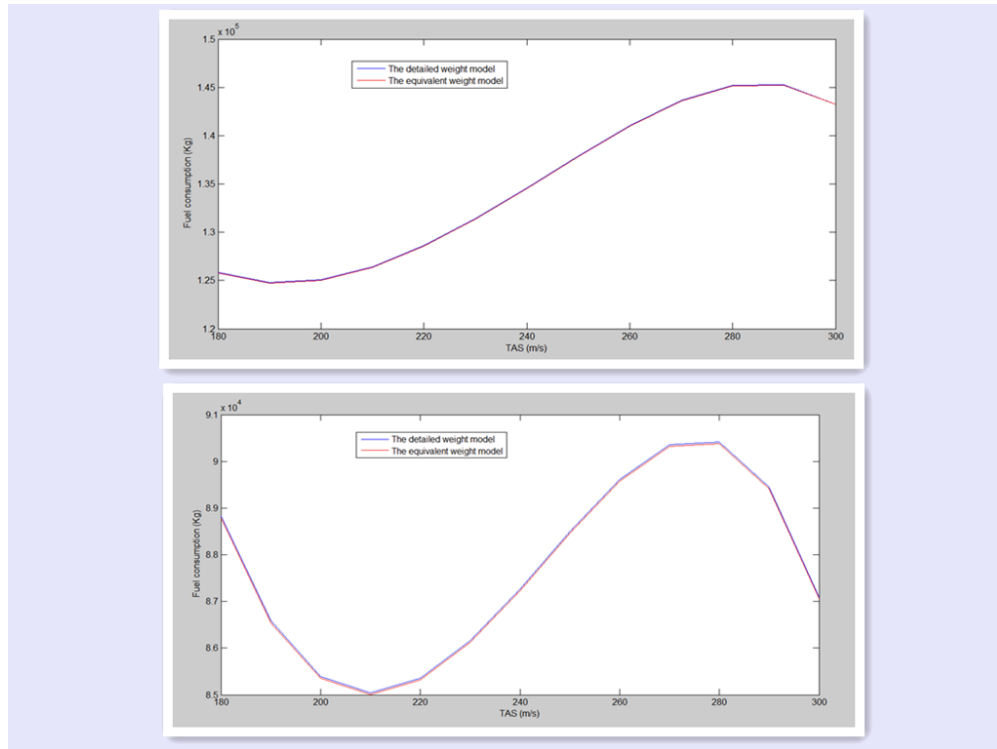


Figure 3.7. Fuel consumptions of two models with different TASs at the crossover altitude (above) and the cruise altitude (below).

As shown in figure 3.7, not only the shape of two models are the same, the fuel consumptions for 1,000 km are the nearly the same to each other. Therefore, the equivalent weight model is also effective in Phase 1.

Figure 3.8 and 3.9 show fuel consumptions and flight times of the detailed model and the equivalent model at a constant altitude with different TASs.

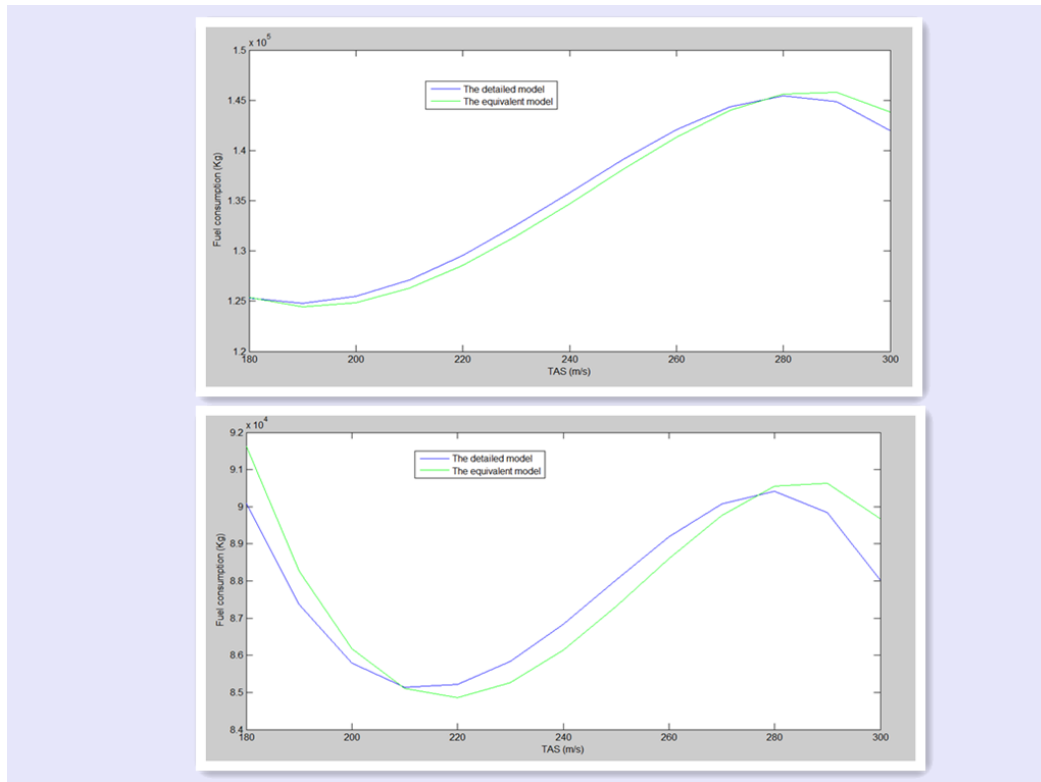


Figure 3.8. Fuel consumptions of two models with different TASs at the crossover altitude (above) and the cruise altitude (below).

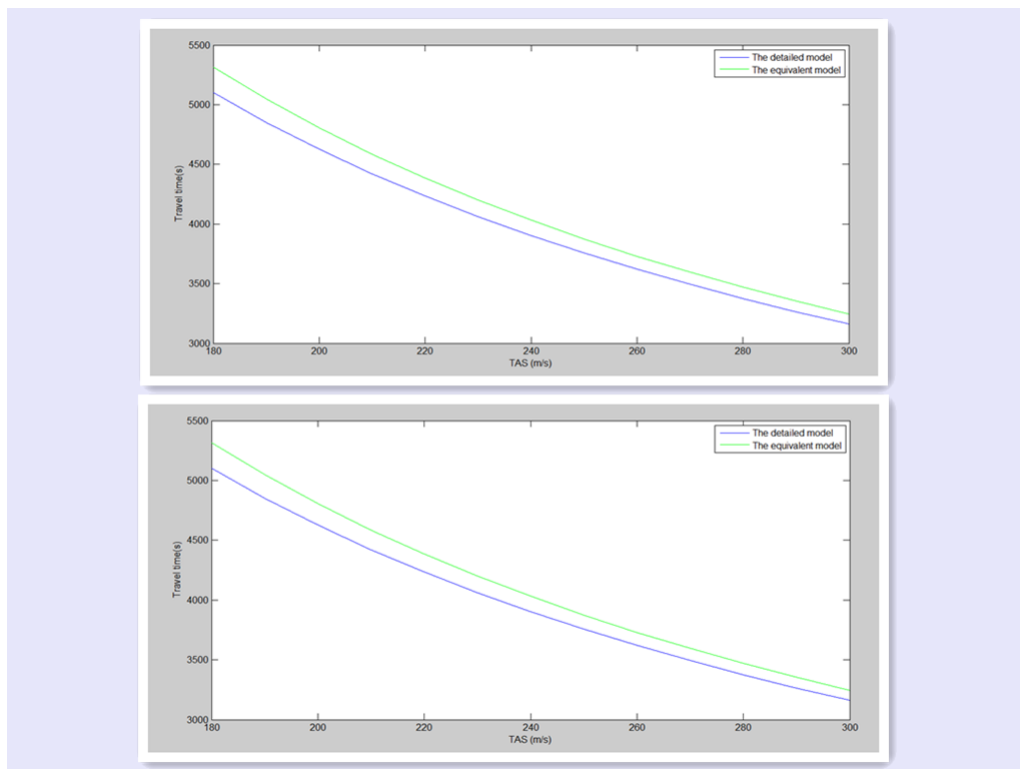


Figure 3.9. Travel times of two models with different TASs at the crossover altitude (above) and the cruise altitude (below).

According to figure 3.8 and 3.9, the equivalent model and the detailed model have similar performance with different TASs at a constant altitude because the shapes of two models are the same to each other. Figure 3.10 indicates the performance of two models in terms of the fuel consumptions at different altitudes with a constant TAS. Since there is no wind effect and the TASs of both models are the same, the travel times of two models are the same. The travel times of two models is not shown in this report.

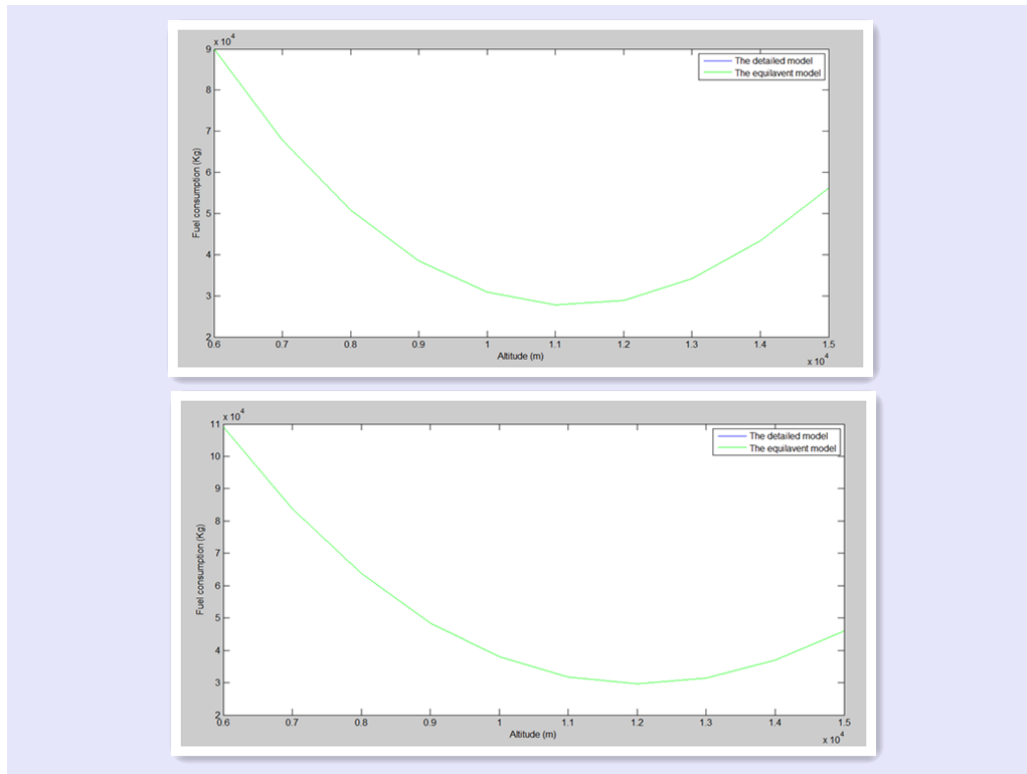


Figure 3.10. Fuel consumptions of two models at different altitudes with two TASs (above: 180 m/s, below: 220 m/s).

As shown in figure 3.10, the fuel consumptions of the two models are the same to each at different altitudes. Therefore, the equivalent model is proven and qualified in Phase 1.

3.3 Phase 2

For Phase 2, the detailed solutions are generated. The outcomes' accuracies of flight time and fuel consumption are important in Phase 2. The search space of Phase 2 is bounded based on the outcomes of Phase 1. That is, the search space of Phase 2 is smaller than the search space of Phase 1. The reason to apply a reduced search scope in Phase 2 is to achieve a high computational efficiency while obtaining

detailed solutions. With the reduced search scope, the computational time of Phase 2 is able to be reduced even though the number of control variables increases due to the consideration of climbing and descent phases. The following sections introduce the details of the model for Phase 2.

3.3.1 Horizontal Profile

In phase 2, the optimization variables of the horizontal profile are the coordinates of the waypoints. The number of waypoints in Phase 2 is the same as the number of waypoints in Phase 1. The search space of Phase 2 is based on the outcomes of Phase 1, the optimal location of each waypoint in Phase 2 is searched in an area made by the outcomes of Phase 1. The longitude and latitude search range of the i^{th} waypoint are bounded by the maximum and minimum longitude and latitude of the i^{th} waypoint in Phase 1 with an amount of margin, respectively. The purpose to add a margin in Phase 2 is to avoid the optimal solution slipping out or on the edges of the reduced search space. As shown in figure 3.11, the red, green and blue lines are the outcome solutions of Phase 1. The search scope of each waypoint in Phase 2 is represented as the boxes with the black dash lines.

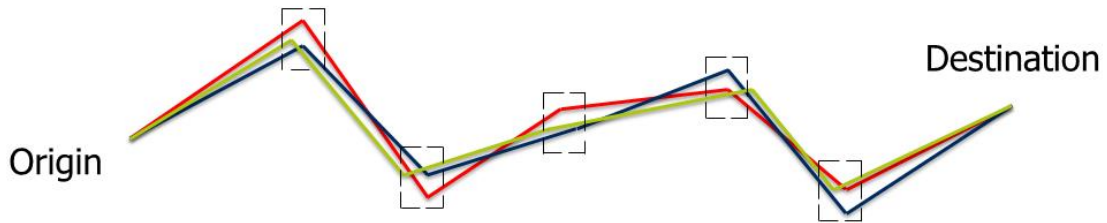


Figure 3.11. The search scope of waypoints in Phase 2.

In order to maintain the feasibility and minimize the number of constraints, the normalization is implemented in the optimization process for the coordination of waypoints in Phase 2. The longitude and latitude of waypoints in Phase 2 are obtained as:

$$\begin{aligned} \lambda_{i,p2} = & \left(\lambda_{i,p1,max} + \lambda_{margin} - (\lambda_{i,p1,min} - \lambda_{margin}) \right) \cdot \eta_{i,\lambda,p2} \\ & + (\lambda_{i,p1,min} - \lambda_{margin}) \end{aligned} \quad 3.15$$

$$\begin{aligned} \varphi_{i,p2} = & \left(\varphi_{i,p1,max} + \varphi_{margin} - (\varphi_{i,p1,min} - \varphi_{margin}) \right) \cdot \eta_{i,\varphi,p2} \\ & + (\varphi_{i,p1,min} - \varphi_{margin}) \end{aligned} \quad 3.16$$

3.3.2 Vertical Profile

Similar to Phase 1, the lengths of vertical segments, TAS and altitude change directions are the control variables of the vertical profile in Phase 2. The optimal altitude changing locations are searched along the ground track while within the range formed by the outcomes of Phase 1. Since the climbing phase is involved in Phase 2, the initial altitude of the cruise phase in Phase 2 may not be the same as the initial altitude of Phase 1 (26,000 ft), the flight altitude of each segment in Phase 2 is bounded by the flight altitude range of Phase 1. The maximum and minimum flight levels in Phase 1 are the bounds of the search space for the flight altitude in Phase 2. In terms of TAS, the search space is determined by the Mach number range of each segment in Phase 1 and the horizontal force balance limitation introduced in the previous section to ensure the feasibility during the optimization process. Similarly, the normalization is also implemented to the lengths of vertical segments and TAS in the optimization process of Phase 2 to ensure the feasibility of each evaluated solution. The parameter of the altitude change direction for each segment is discretized into three commands as the same as Phase 1.

Beside the cruise phase, climbing and descent phases are involved to fix the initial and final altitudes of the evaluated trajectories in Phase 2. This is because unique initial and final altitudes are convenient for users to compare different trajectories. The initial altitude of the climb phase and the final altitude of the descent phase are both set as 10,000 ft (3,048 m), because the approach and departure controls are up to 10,000 ft as discussed in section 3.2.2.

In terms of the climbing phase, the feasible range of the initial cruise altitude is determined based on the horizontal force balance discussed in section 3.2.2. After that, the control parameter of the initial altitude of the cruise phase is discretized according to the number of feasible flight altitudes. By doing this approach, the feasibility and the freedom of the initial flight altitude is guaranteed. For the descent phase, the initial altitude is the altitude of the last cruise segment. After determining the cruise altitudes of the first and the last segments, the calibrated airspeeds (CASs) of the climbing phase and the descent phase are optimized with the normalization process. The maximum and minimum CASs of the climbing and the descent phases are determined by the maximum and minimum CASs at the altitude of the first cruise

segment and the initial climbing altitude or the altitude of the last cruise segment and the final descent altitude with the limitation of the horizontal force balance. The CAS of the climbing or the descent phase is calculated by normalization as:

$$V_{CAS} = (V_{CAS,max} - V_{CAS,min}) \cdot \eta_{CAS} + V_{CAS,min} \quad 3.17$$

Then, the model optimizes the thrust of the climbing phase or the descent phase. In order to locate the feasible range of the thrust, the Maximum Climb/Continuous Corrected Thrust and the Minimum Idle Thrust of the model of Teengs, M. (2006) are included as the maximum available thrust and the minimum available thrust for the climbing phase and the descent phase, respectively. Consequently, the maximum climbing rate and the minimum descent rate are dependent on the maximum and minimum available thrust produced and the feasible CAS ranges of the climbing phase and the descent phase.

In order to ensure the climbing phase and the descent phase do not occupy large portions of the total distance, limitations for the minimum climbing and descent rates are involved. According to Ruijgrok G.J.J (2004), the climbing or descent time is expressed as:

$$t = \frac{P_0(1 - P_c/P_0)}{dP/dt} \quad 3.18$$

Where P_0 is the pressure at sea level, P_c is the cabin pressure, dP/dt is the time rate of change of pressure. Since the aircraft currently operated by airlines are pressurized, the cabin pressure at the normal cruising altitude is equivalent to a geopotential pressure altitude of approximately 1800 m (~6,000 ft), i.e., $P_c/P_0 = 0.8$. By substituting the value into equation 3.18, the climbing or descent duration in the standard atmosphere is calculated as:

$$t = \frac{101325(1 - 0.8)}{dP/dt} \quad 3.19$$

The average climb and descent rates can be obtained as:

$$\frac{dh}{dt} = \frac{\pm(h_{cruise} - h_0)}{t} \quad 3.20$$

Where h_{cruise} is the cruise altitude of the first or the last cruise segment, h_0 is the initial or final altitude, which are both 10,000 ft. For the climbing phase, the average climb rate is a positive number, while it is a negative number for the descent phase.

Table below shows the horizontal distance and the average climbing rate in fpm with different climbing rates in pa/s from 10,000 ft to different cruise altitudes with 300,000 kg take-off weight and minimum feasible CAS.

Table 3.5. The horizontal distance with various climbing rates from 10,000 ft.

Climbing rate (Pa/s)	Cruise altitude (ft)	Horizontal distance (Km)	Average Climbing Rate (fpm)
5	28,000	460	266.5
5	30,000	484	296.1
5	32,000	508	325.7
5	34,000	573	355.3
10	28,000	262	532.9
10	30,000	267	592.2
10	32,000	282	651.4
10	34,000	321	710.6
15	28,000	176	799.4
15	30,000	190	888.2
15	32,000	206	977.1
15	34,000	241	1,065.9
20	28,000	138	1,065.9
20	30,000	151	1,184.3
20	32,000	171	1,302.7
20	34,000	203	1,421.2

The table below indicates the horizontal distance and the average descent rate in fpm with different descent rates in pa/s from different cruise altitudes to 10,000 ft with 225,000 kg landing weight and minimum feasible CAS.

Table 3.6. The horizontal distance with different descent rate to 10,000 ft.

Descent Rate (Pa/s)	Cruise altitude (ft)	Horizontal distance (Km)	Average Descent Rate (fpm)
5	28,000	208	-266.5
5	30,000	225	-296.1
5	36,000	331	-384.9
5	38,000	389	-414.5
10	28,000	104	-532.9
10	30,000	112	-592.2
10	36,000	166	-769.8
10	38,000	195	-829.0
15	28,000	69.0	-799.4
15	30,000	74.5	-888.2
15	36,000	110	-1,154.7
15	38,000	130	-1,243.5

As shown in the tables above, the horizontal distance of the climbing phase and the descent phase are varying with the altitude changing rate and the cruise altitude. Among the possible pairs shown in table 3.5 and 3.6, the total distance of the climbing rate and descent rate with 15 Pa/s (693 fpm at FL320, 600 fpm at FL280 and 333 fpm at FL100) and 5 Pa/s (231 fpm at FL320, 200 fpm at FL280 and 111 fpm at FL100) is around 500 km, which is around 8.5% of the total distance of the relatively short route such as AMS-JFK (5,863 km). In order to ensure the distance of climbing phase and the descent phase do not occupy a large portion of the total distance while maintaining the optimization freedom, 15 Pa/s and 5 Pa/s, as the minimum climb and descent rates, are applied in this study.

In terms of force balance, the climb or descent rate is produced by the difference between the produced power and required power:

$$\frac{dh}{dt} = \frac{P_p - P_r}{W} \quad 3.21$$

Where W is the weight of the aircraft, P_p is the produced power, P_r is the required power:

$$P_p = TV_{TAS} \quad 3.22$$

$$P_r = W \sqrt{\frac{W}{S} \frac{2 C_D^2}{\rho C_L^3}} \quad 3.23$$

Where T is the thrust, V_{TAS} is the TAS, S is the wing surface area, C_D and C_L are the drag and lift coefficient respectively. By substituting equation 3.22 and 3.23 into equation 3.21, the minimum thrusts of the climbing phase $T_{min,c}$ and the maximum descent phases $T_{max,d}$ due to the limitation of the altitude change rate are obtained as:

$$T_{min,c} = \frac{W}{V_{TAS}} \left(\frac{dh}{dt}_{min,c} + \sqrt{\frac{W}{S} \frac{2 C_D^2}{\rho C_L^3}} \right) \quad 3.24$$

$$T_{min,d} = \frac{W}{V_{TAS}} \left(\frac{dh}{dt}_{max,d} + \sqrt{\frac{W}{S} \frac{2 C_D^2}{\rho C_L^3}} \right) \quad 3.45$$

Since the control variables of the climbing and descent thrusts are dependent on the flight altitude, and the weight of the aircraft, normalization process is implemented

to maintain the feasibility of the evaluated solutions. The thrust of the climbing and descent phase are calculated as:

$$T_c = (T_{max,c} - T_{min,c}) \cdot \eta_{T,c} + T_{min,c} \quad 3.26$$

$$T_d = (T_{max,d} - T_{min,d}) \cdot \eta_{T,d} + T_{min,d} \quad 3.27$$

Since all flights in this study departure from or arrive at AMS, the longitude scale at AMS is applied to calculate the blanks for the climbing and descent phase. Based on the spherical earth model introduced in the section 3.1.1, the longitude scale at AMS ($52.3^\circ N$) is 68 km/degree. Since the model processes the route from the east to the west or from the south to the north disregarding the direction of the flight in the beginning, the equalled blank for the climbing phase and the descent phase are required to provide feasible spaces for both phases. Therefore, 5° blank (340 km) is applied as the blank for both phases.

3.3.3 Simulation Method

The objective of Phase 2 is to identify the detailed solutions. Consequently, a large distance step is not suitable for the optimization process of Phase 2. Therefore, the equations of motion with small time step are applied:

$$\dot{\lambda} = \frac{V_{TAS} \cdot \cos \gamma \cdot \cos \psi \cdot \sec \varphi + u}{r + h} \quad 3.28$$

$$\dot{\varphi} = \frac{V_{TAS} \cdot \cos \gamma \cdot \sin \psi + v}{r + h} \quad 3.29$$

$$\dot{m} = -ff \quad 3.30$$

Where λ , φ and h represent the aircraft center of gravity longitude, latitude and altitude, respectively. V_{TAS} is the TAS, γ is the flight path angle, which is set to 0 in this project since the aircraft in this study is assumed as a point-mass. ψ is the Yaw angle (heading angle). ff is the fuel consumption rate in this report, u and v are the wind speed components. In the climb and descent phases, the equation of motion in vertical direction is included as:

$$\dot{h} = \frac{dh}{dt} \quad 3.31$$

Where $\frac{dh}{dt}$ is the climb or descent rate.

Table 3.7 indicates the computation time, fuel consumption and travel time with different time step for the same trajectory.

Table 3.7. Performance differences with different time steps.

Time step (s)	Computation time (s)	Difference to 1s time step	Fuel consumption (kg)	Difference to 1s time step (%)	Travel time (s)	Difference to 1s time step (%)
1	6.68	0%	74795.0	0%	23794.7	0%
30	0.8	-88.02%	74904.5	0.15%	23799.1	0.02%
60	0.55	-91.77%	75017.9	0.30%	23803.2	0.04%
120	0.66	-90.12%	75265.8	0.63%	23818.2	0.10%
300	0.59	-91.17%	75957.8	1.55%	23848.9	0.23%
600	0.58	-91.32%	77405.3	3.50%	23940.0	0.61%

According to table 3.7, the simulation with 1 minute time step has the best comprehensive performance with the shortest computation time while the fuel consumption and flight time deviations compared to the simulation with 1 second time step are limited. Therefore, 60 seconds is set as the time step in Phase 2.

3.4 Simulation direction

For the simulation direction, there are two directions, forward simulation and backward simulation. The forward simulation is to simulate the trajectory from the origin to the destination with a predetermined take-off weight. One of the advantages of the forward simulation is that the process is a subtractive process. The weight of the aircraft is subtracted at every segment. At the starting point of each segment, the weight of the aircraft is maximal for the segment. Therefore, the optimizer is able to evaluate the feasible range of the TAS and verify the feasibility to fly at a given altitude for the segment. However, the constant final landing weight is difficult to maintain by applying this approach. Without a constant landing weight, the redundant fuel may lead to a transport loss during the simulation. The redundant fuel will cause more fuel consumption when performing the flight because the aircraft is heavier. The aircraft has to produce more lift to maintain the force balance in the vertical direction. If the transport loss is involved, users are difficult to determine the optimal solution within a batch of solutions based on the fuel consumption and the flight time.

Regarding to the backward simulation, the optimizer simulates the trajectory from the destination to the origin. The final landing weight is set before the simulation. That is, the simulation is an additive process. By using this approach, the constant final landing weight is guaranteed, which is convenient for the users to compare the fuel consumptions of different trajectories. However, this approach has a fatal flaw. The feasible TAS range is impossible to determine, because the weight at the starting point of a segment, which is the maximal weight in the segment, is unknown before the simulation of the segment. Therefore, the feasibility of the segment altitude is difficult to judge.

As discussed above, the backward simulation is not suitable for this project because the maximum weight of a segment is difficult to evaluate. Therefore, the forward simulation is used in this study. An iterative loop to determine required initial weight roughly to ensure a (more or less) fixed landing weight is included in Phase 1. The weight difference between the expected landing weight and the actual landing weight of each individual in a generation is analysed. The original take-off weight is subtracted by the minimum positive weight difference of the generation in the next generation. With this algorithm, the maximal landing weight difference to the expected landing weight is controlled within 10,000 kg in Phase 1.

In Phase 2, the iterative self-learn program is executed into the individual level. The trajectory is forwards simulated from the origin to the starting point of the last cruise segment to access the initial weight of the last cruise segment. For the descent phase, the expected landing weight is set and the backward simulation is implemented from the destination to the starting point of the descent phase. After that, the location of the starting point of the descent phase, which is also the ending point of the last cruise segment, is obtained. Therefore, the fuel consumption of the whole trajectory is obtained by summing the fuel consumption from the origin to the end of the cruise phase and the fuel consumption of the descent phase. The final landing weight is obtained by subtracting the total fuel consumption from the initial take-off weight. If the difference between the expected landing weight and the actual landing weight is not within a designed range, the optimizer will analyse the difference, adjust the take-off weight, and simulate the trajectory again with the adjusted take-off weight until the final landing weight is within the designed range. By using this program, the final landing weight error can be controlled within 3,000 Kg. The flow chart of the iterative algorithm is shown in Appendix A.

4. Verification

In this chapter, the verification is implemented. The model is tested within a no-wind effect scenario in the beginning. In the next step, simple wind fields are installed to test the ability of the model for avoiding headwinds and pursue tailwinds. Last but not the least, the model is implemented into a real wind field to test the comprehensive ability of the model. In this chapter, all tests are implemented from the point $52.3^{\circ}N, 4.9^{\circ}E(AMS)$ to $52.3^{\circ}N, 74^{\circ}W$. One of the reasons to choose this route is the large distance between these two points. The direct distance of this route is 5085 km. Another reason to choose the same latitude is that it is ease to identify the difference of the trajectories due to the change of the wind profile during the test.

The aircraft model applied in this study is the model developed by Teengs, M. (2006) for Boeing 747-400, which is also the applied aircraft model of the examples in previous chapters. Some analysis in the rest of the report will be based on the results shown in previous chapters. The Maximum Take-off Weight (MTOW) of the aircraft model is 362,874 kg, the Operation Empty Weight (OEW) is 178,756 kg, the maximum payload is 63,917 kg, the design range is 6,185 nm (11,454 km), and the maximum seating capacity is 400 (42 up, 24 first, 32 business, 302 economy). The VMO and MMO of this aircraft model are 365 kts and 0.9, respectively. Consequently, the crossover altitude is 27,490 ft, the crossover flight level in this study is FL280. The fuel consumption model is developed from the Boeing 747 performance manual. The aircraft is assumed as a point-mass in this study when simulating the movement of the aircraft.

4.1 No-wind scenario

In the no-wind scenario, the model is supposed to identify the great circle path between two locations because this is the minimum distance path between two locations on the earth. The great circle path generated by Movable Type Scripts (n.d.) and the optimized horizontal path are shown in figure 4.1 and 4.2.



Figure 4.1. The great circle path between to test points. (Movable Type Scripts, n.d.)

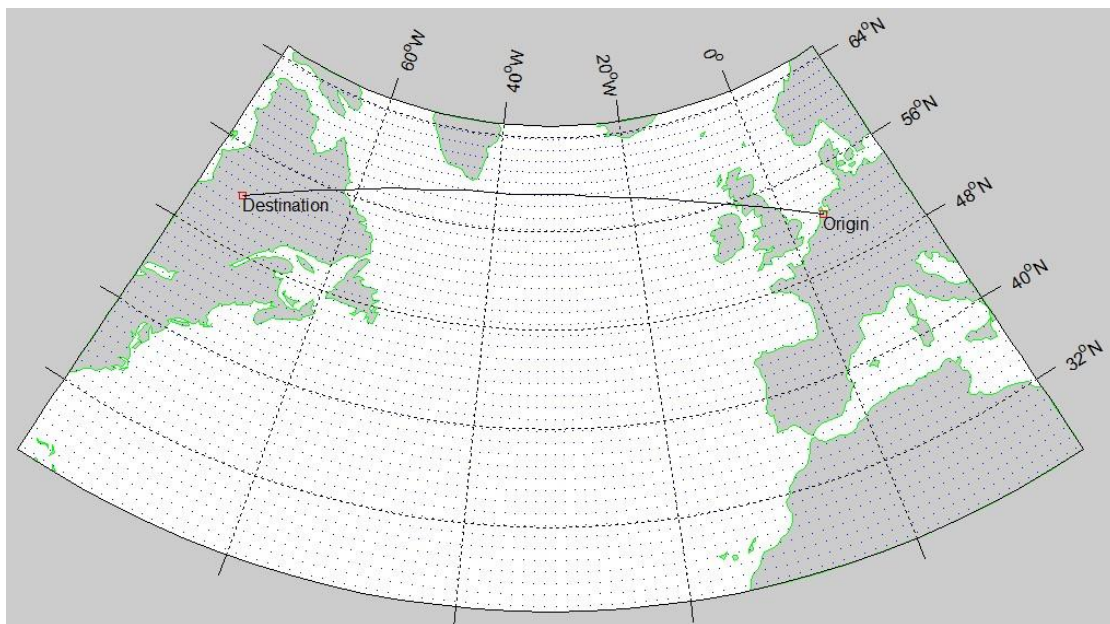


Figure 4.2. Horizontal path between two points with no wind effects.

In figure 4.1, the ground track is just above Ireland, and below Greenland, while ground the track in figure 4.2 is similar. The route is above Ireland and below Greenland. The path in figure 4.1 is more curved than the path in figure 4.2 because figure 4.1 is based on the flat earth model, while figure 4.2 is based on the spherical earth model. This test indicates that the model has the quality to identify the shortest horizontal trajectory when there is no wind effect.

For vertical profile, figure 4.3 shows the near optimal vertical profile of the time preference solution. The cruise altitude for the majority of the vertical path is FL280

which is the crossover flight level in this study. One may notice that the trajectory has a segment at FL300 from around 2,100 nm to 2,300 nm away from the origin. The reason for this phenomenon is that NSGA is a random search method. It is difficult to optimize the solutions in the final generations of the optimization process. As shown in table 3.3, the difference of the maximum feasible TAS at FL280 and FL300 is only 2 kts. The flight time of this segment by flying at FL300 is around 2.5 minutes (0.8% of total flight time) longer than the flight time by flying at FL280. The small difference may cause difficulties for the optimizer to distinguish the performances at these two altitudes. That is, the outcomes of the optimizer may have little variation from the absolutely optimal solutions for the problem. Hence, the result shown in figure 4.2 is acceptable. Moreover, GA is a random search algorithm. With the optimization process progressing, it is more and more difficult to improve the outcomes because the space for improvement becomes smaller.

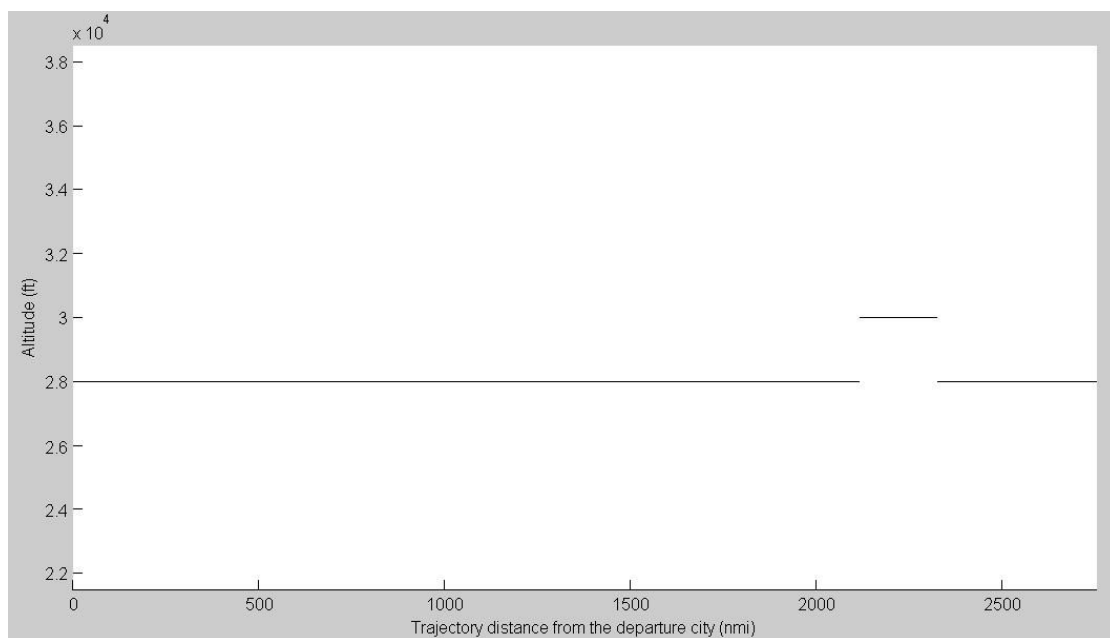


Figure 4.3. The vertical trajectory for the time preference near optimal solution.

Regarding the fuel preference profile, the aircraft is supposed to fly at high altitudes in order to gain a large specific range as shown in table 2.1. Figure 4.3 shows the vertical flight trajectory for the fuel preference profile. The trajectory climbs to the ceiling of the feasible flight levels. There are two reasons that the trajectory is not quick to climb to FL380, the first reason is that the aircraft is too heavy to climb to the next flight level and the second is that the minimum distance ($D_{v,b}$) of each vertical segment is fixed by the model setting discussed in Chapter 3. The aircraft is required to fly for an amount of distance before the next climbing.

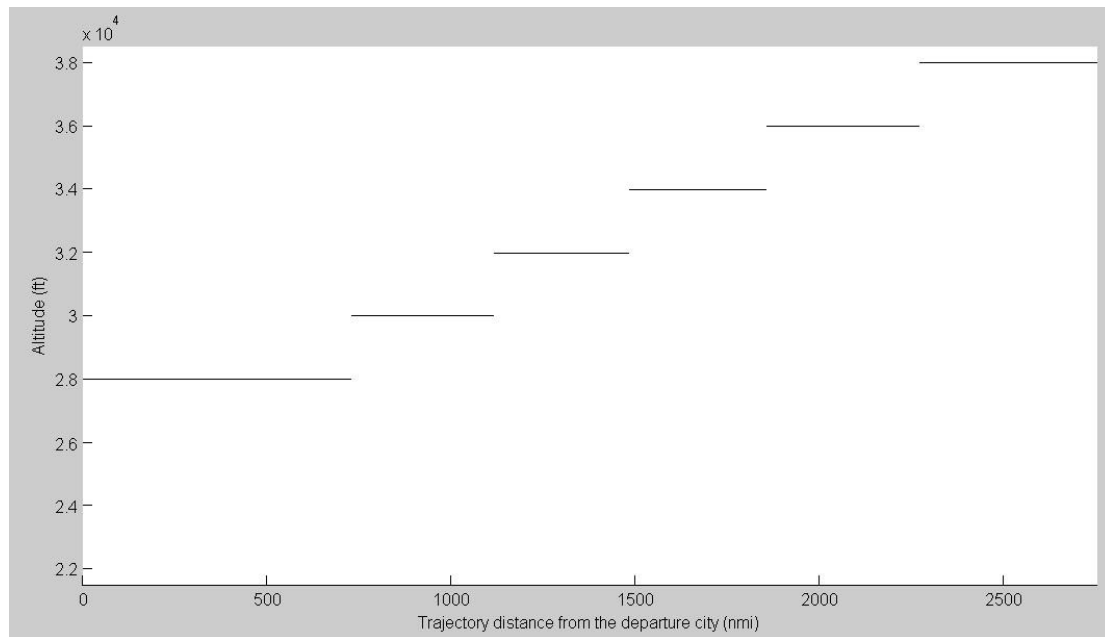


Figure 4.4. The vertical flight trajectory for the fuel preference profile.

Based on the test results shown above, the model has the ability to identify the near optimal trajectories either for the time preference profile or for the fuel preference profile in a no wind scenario.

4.2 Simple wind fields

In this section, simple wind fields are implemented to test the ability of the tool for detecting wind effects and making corresponding reaction. Firstly, a wind field with the east-to-west wind from $26^{\circ}N$ up to $56^{\circ}N$ and the west-to-east wind from $56^{\circ}N$ to $64^{\circ}N$ is pre-set. Since the test route is from the east to the west. That means aircraft will benefit from the tailwind from $26^{\circ}N$ to $56^{\circ}N$ and encounter the headwind between $56^{\circ}N$ and $64^{\circ}N$. In addition, the wind field is applied to all available flight levels. The horizontal trajectory of the solution generated by the model is shown in the figure below:

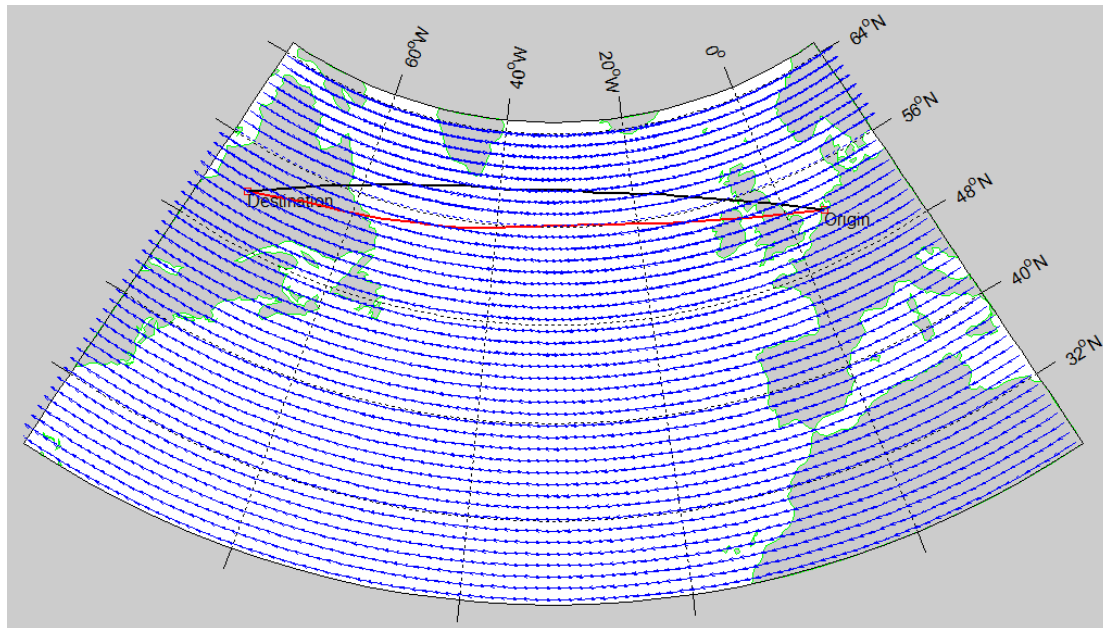


Figure 4.5. The horizontal trajectory of the solution generated by the model.

As shown in figure 4.5, the outcome solution (red line) is more bent to south compared to the great circle path (black line) to avoid the strong headwinds. Additionally, the solution is close to the $56^{\circ}N$ latitude line so that the trajectory can benefit from the tailwinds with a relatively short flight distance.

Not only the unique wind profiles for different flight levels, the model also shows a quality performance when different wind profiles are implemented at different altitudes. Figure 4.6 expresses the vertical trajectory of the time preference solution in a wind field with headwinds between 26,000 ft and 32,000 ft, and tailwinds at other flight levels. The background is the vertical wind profile along the trajectory, the value of the wind speed in meter per second is explained by the colour bar on the right hand side of the figure. The positive values indicate tailwinds, while the negative values of wind speed meaning headwinds.

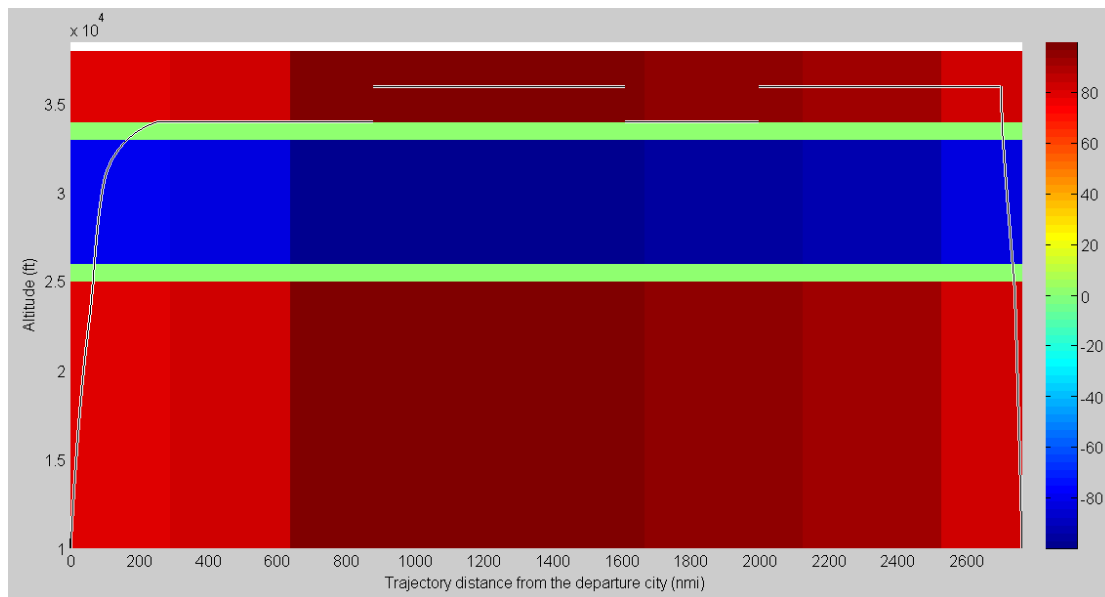


Figure 4.6. The vertical trajectory optimized by the model.

As shown in figure 4.6, the cruise altitude stays at FL340 and FL360 to avoid the headwind around the crossover altitude. As shown in Table 3.3, the decreased slope of MMO is smaller than the decreased slope of VMO. That is, with the same altitude difference to the crossover flight level (FL280), the maximum available TAS at a higher altitude is larger than the maximum available TAS at a lower altitude. This is the reason that the optimizer choose to fly at the higher altitudes (FL340 and FL360) rather than the lower altitude (FL240).

In terms of the fuel preference profile, the aircraft is supposed to fly at high altitudes for high fuel efficiency. A wind field with strong headwinds from 34,000 ft to 38,000 ft and tailwinds from 10,000 ft up to 32,000 ft is installed to test the performance of the tool. Figure 4.7 represents the vertical trajectory of the fuel preference solution.

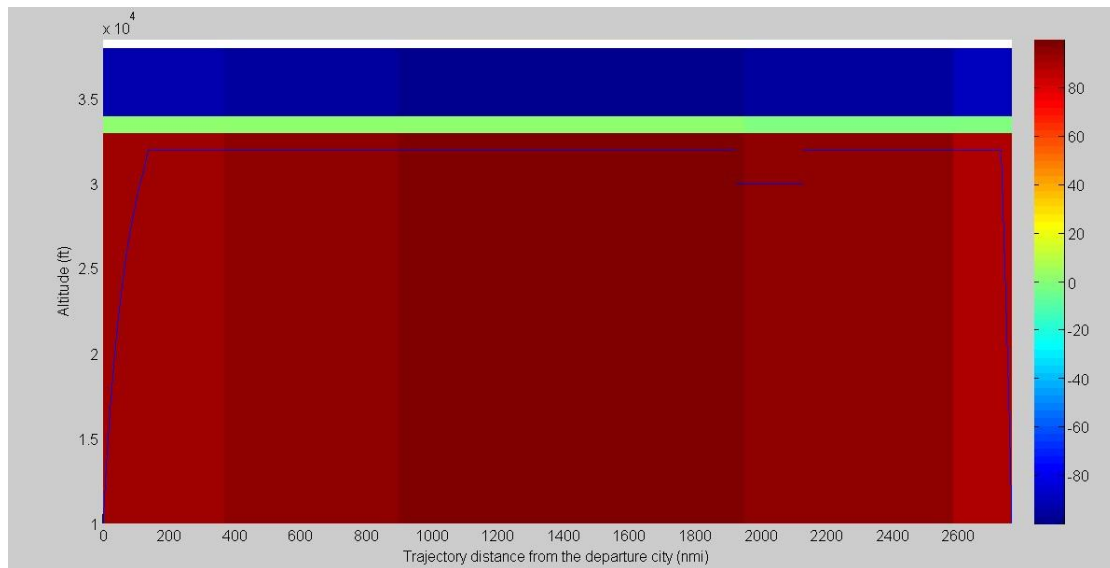


Figure 4.7. The vertical trajectory of the solution obtained by the model.

Figure 4.7 shows that the altitude of the solution stays at FL320 for majority of the trajectory in order to avoid the strong headwinds at the higher altitudes. These two tests show that the model is able to distinguish the vertical wind differences for the optimal time and fuel preference vertical trajectories.

According to the results shown above, the model is qualified to distinguish headwinds and tailwinds, and make corresponding decisions to avoid headwinds and pursue the tailwinds. In the next step, a real wind field on 13 July 2014 is implemented to test the comprehensive ability of the model under a more realistic wind condition.

4.3 Complex wind fields

In this section, the wind condition on 13 July 2014 has been implemented. Figure 4.8, 4.9 and 4.10 indicate the horizontal trajectory, the vertical trajectory and the airspeeds diagram of the time preference profile.

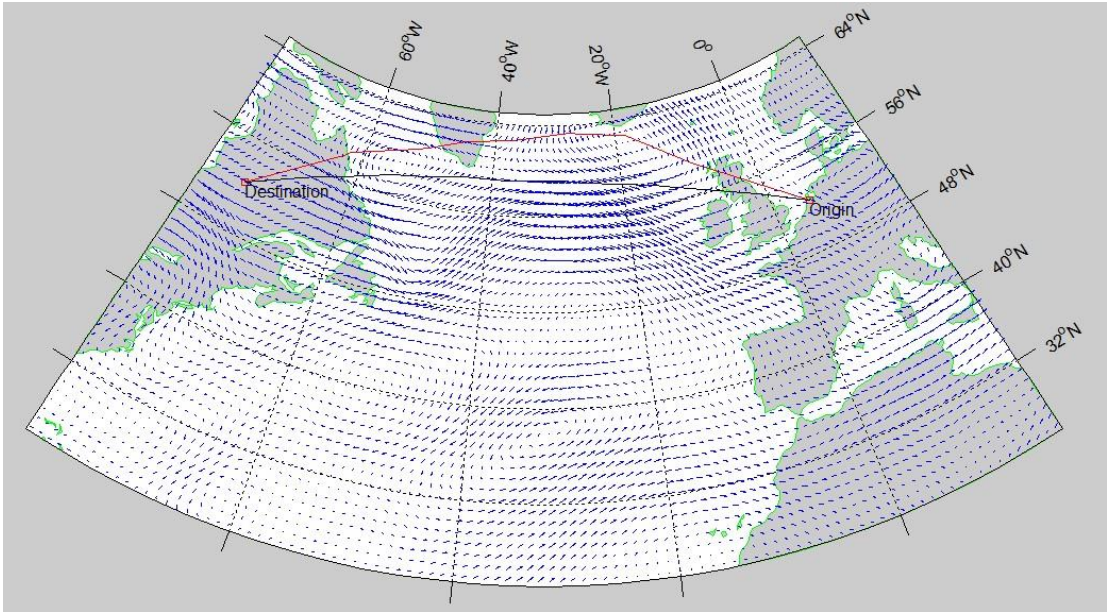


Figure 4.8. The optimal horizontal trajectory of the time preference profile.

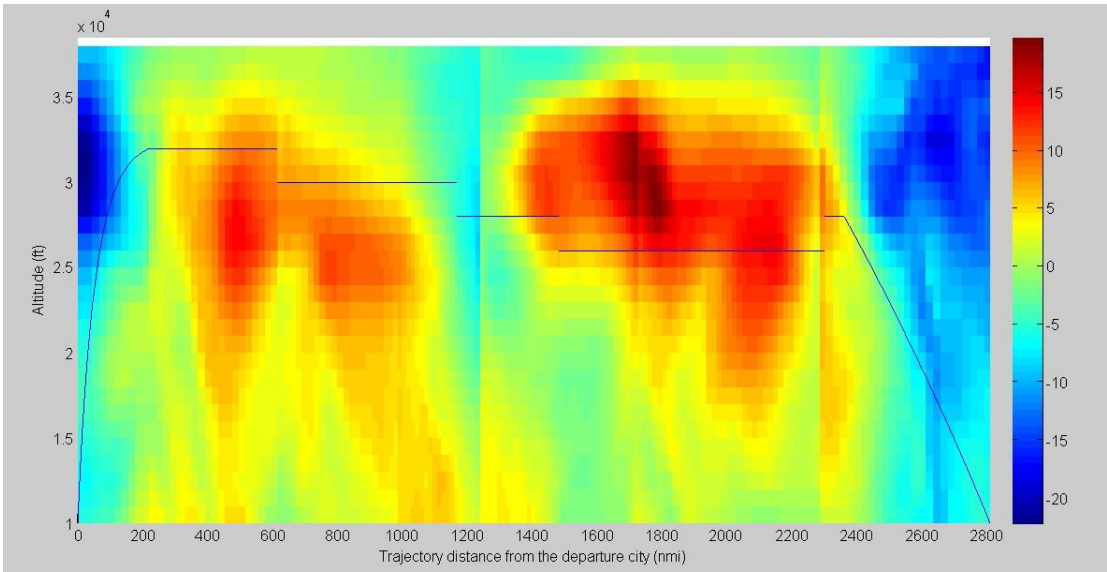


Figure 4.9. The optimal vertical trajectory of the time preference profile.

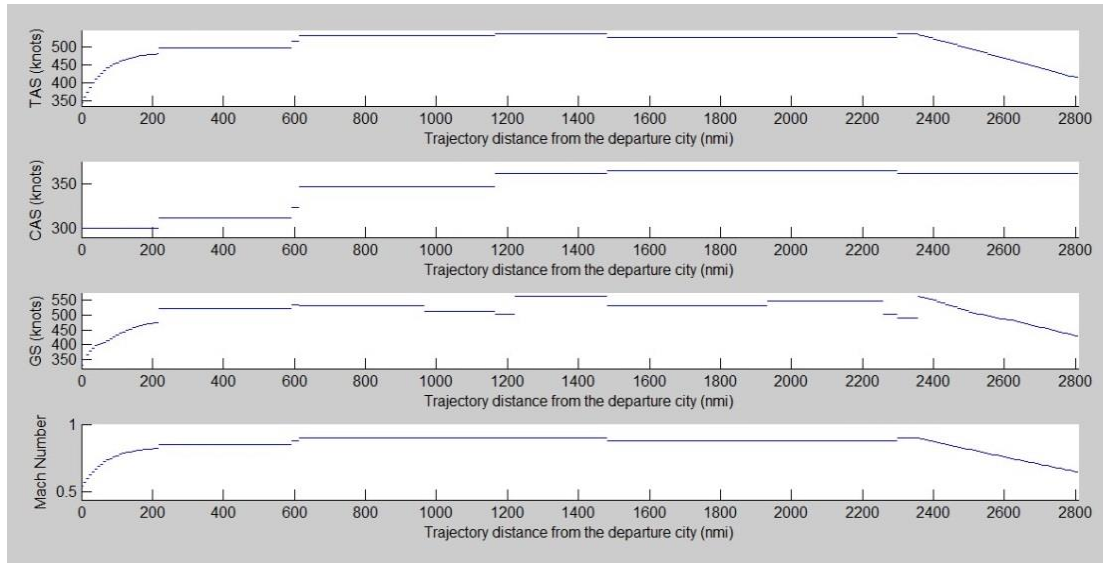


Figure 4.10. The airspeeds diagram of the time preference profile.

As shown in figure 4.8, the horizontal trajectory of the solution enjoys the tailwinds from the south to the north between $0^{\circ}W$ and $20^{\circ}W$. After that, the aircraft flies towards the northwest to avoid the strong headwind along the great circle path. In the final phases of the trajectory, the aircraft benefits slightly from the tailwind until arriving at the destination. Figure 4.9 indicates the vertical trajectory of the outcome of the test, the aircraft climbs to FL320 and then descends to the flight levels around FL280. As discussed in the previous section, the outcomes of the optimizer may be varying to the optimal solution because of the characteristic of NSGA. Therefore, the flight level in figure 4.9 does not constantly stay at FL280. The other reason is that decreased slope of MMO and VMO to altitude change is small. As discussed in Chapter 2, the altitude change will not make a large impact on the maximum feasible TAS, especially when the aircraft climbs. Therefore, it is acceptable that the flight altitude is little various to the crossover altitude.

Even though the flight levels in figure 4.9 are oscillating, the flight levels stay around FL280 in order to reach a large TAS. Regarding the airspeeds in figure 4.10, the Mach number before the aircraft reaches 1450 nm is around 0.9, which is the MMO in this study. After the aircraft reaches 1450 nm , the CAS is around 365 knots (VMO). The airspeeds results indicate that the optimizer tries to increase TAS as large as possible to minimize the time duration of the flight. In other words, the model is capable of identifying the minimum time flight trajectory in a real wind field.

For the fuel preference profile, the horizontal trajectory is shown as figure 4.11. As shown in figure 4.11, the fuel preference profile is more northerly compared to the ground track of minimum time preference profile. The ground track in figure 4.11 is able to benefit more tailwind advantages between $0^{\circ}W$ and $20^{\circ}W$.

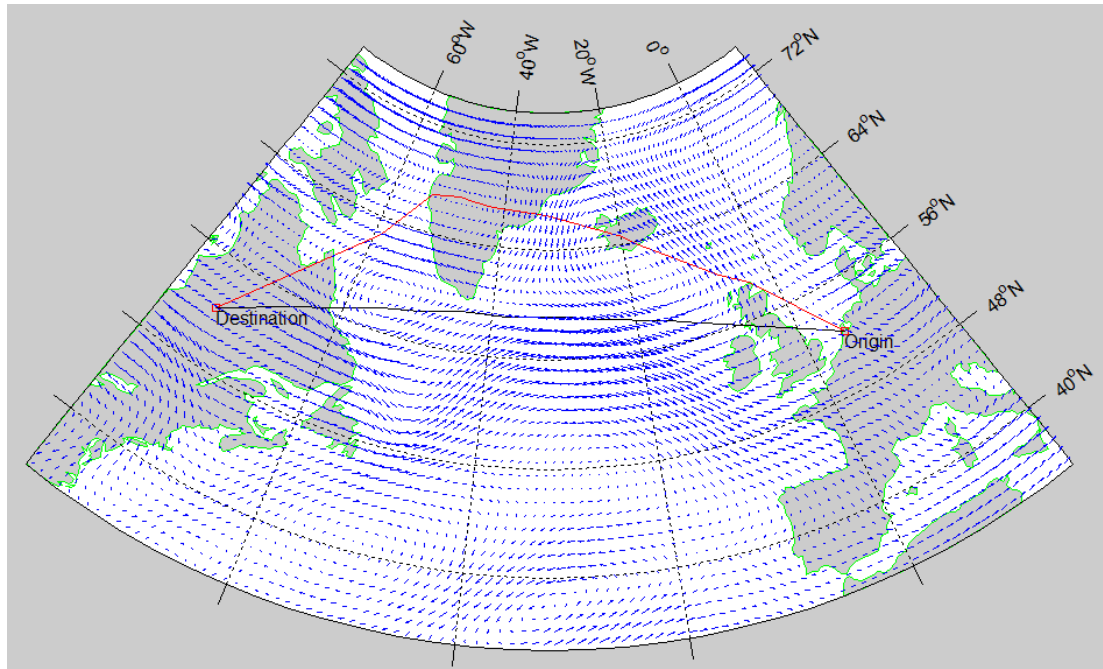


Figure 4.11. The ground track of the minimum fuel consumption solution.

Figure 4.12 and 4.13 present the vertical trajectory and the corresponding airspeeds diagram of the fuel preference profile.

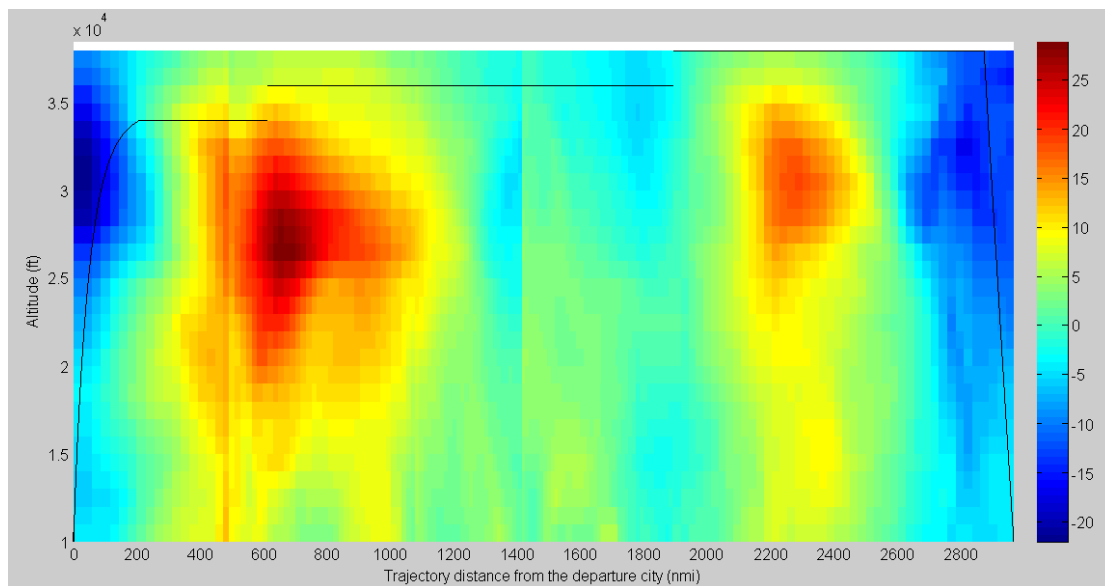


Figure 4.12. The vertical trajectory of the solution of the fuel preference profile.

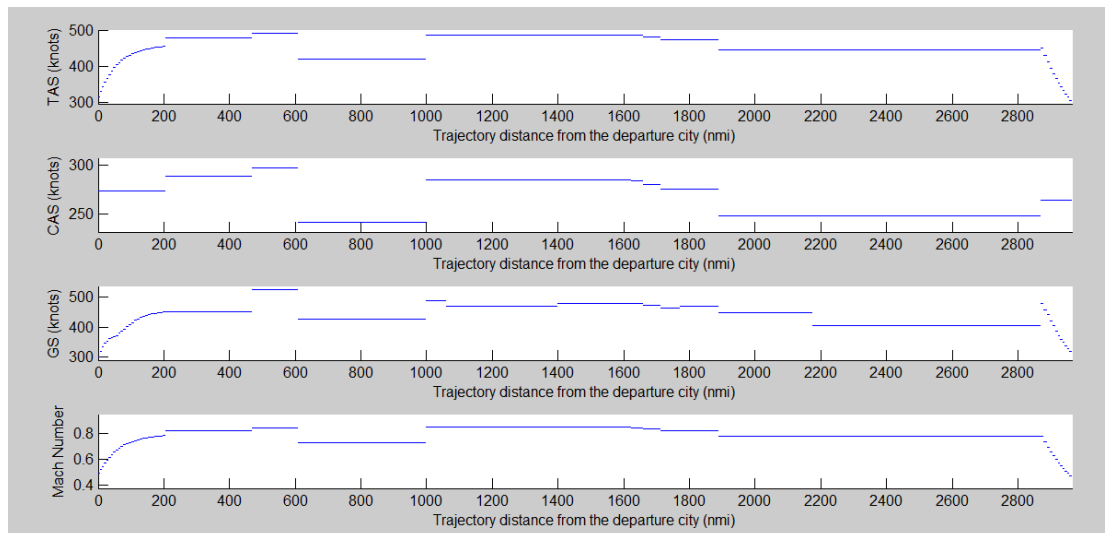


Figure 4.13. Airspeeds diagram of the fuel preference profile.

As shown in figure 4.12, the aircraft climbs to FL340 after the climbing phase because of the high fuel efficiency and the heavy aircraft weight. After the aircraft is light enough to climb to higher flight levels, the aircraft climbs and stays at the highest feasible flight level until the descent phase. Table 4.1 shows the maximum excess thrust of the aircraft with different Mach number and different weight (kg) at FL360. Cells with -1 indicated that the maximum excess thrust is less than 0.

Table 4.1. The maximum excess thrust with different Mach number and weights (kg) at FL360.

	0.60	0.65	0.70	0.75	0.80	0.85	0.90
255,000	13,150.9	17,533.4	18,021.0	15,343.0	10,139.7	3,107.6	-1
260,000	8,749.0	13,706.4	14,622.6	12,250.6	7,245.4	312.7	-1
265,000	4,261.7	9,805.1	11,158.1	9,098.1	4,295.0	-1	-1
270,000	-1	5,829.4	7,627.7	5,885.5	1,288.3	-1	-1
275,000	-1	1,779.5	4,031.2	2,613.0	-1	-1	-1
280,000	-1	-1	368.8	-1	-1	-1	-1

As shown in table 4.1, the aircraft can only fly at 0.7 Mach number with 280,000 kg weight. With the decreasing of the weight, the aircraft is able to accelerate to 0.85 with 260,000 kg and 255,000 kg weight. For the case of figure 4.12 and 4.13, the initial and final weight of the segment at FL360 is around 280,000 kg and 255,000 kg, respectively. Therefore, the aircraft can only fly with 0.7 Mach number at FL360 with the initial weight of the segment at FL360 in figure 4.12. This is the reason that the starting Mach number at FL360 is around 0.7 in figure 4.13. Figure 4.14 below shows the specific range curves with different aircraft weights at FL360 disregarding the feasibility of the aircraft. As shown in figure 4.14, the maximum specific range occurs at 0.8 Mach number for all aircraft weights.

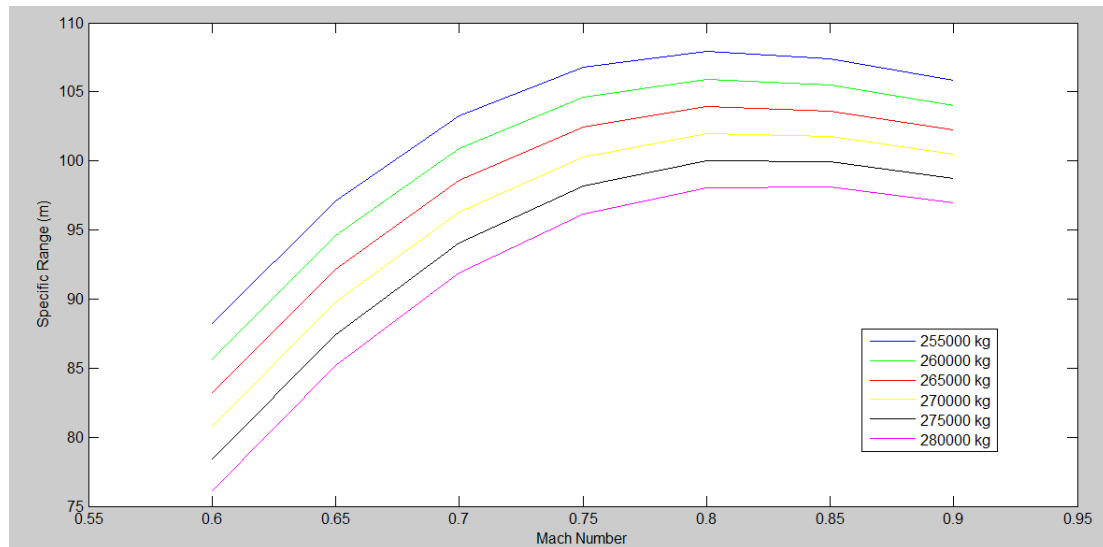


Figure 4.14. The specific range with different weights and Mach numbers at FL360.

Therefore, the aircraft is supposed to fly at 0.8 Mach number at FL360 in order to reach the maximum specific range in this case. However, due to the heavy weight, the aircraft can only fly with 0.7 Mach number in the beginning when it climbs to FL360. As the decreasing of the weight, the aircraft is able to increase the Mach number to around 0.8 in figure 4.13. The reason of the CAS decreasing when the aircraft climbs to FL380 is the same as the reason of the CAS decreasing when the aircraft climbs to FL360 analysed above.

In conclusion, the trajectories generated with different wind fields are almost the same as expected trajectories. Some variations may occur in the outcomes of the process, but the oscillations are acceptable based on the analysis above. Therefore, the designed model is capable of identifying the near optimal trajectory in real wind fields.

5. Case Studies & Results

In this chapter, several realistic cases are studied. The route between AMS and JFK is tested for two directions with the wind profiles of two different days in the beginning since the strong polar jet significantly affects this route. Apart from the routes between Europe and America, the route from Amsterdam to JNB (Johannesburg) is studied, because the wind conditions on this route are different from east-west routes. After that, the route between AMS and SIN (Singapore) is researched to identify the near optimal trajectories for the extremely long route. The wind profile applied in this chapter is the wind profile on 31 August 2014 except otherwise stated. The wind profiles shown in the ground track figures are the wind profiles at 28,000 ft except otherwise stated. This is because wind profiles at different altitudes are not dramatically different to each other as shown in Chapter 2. The aircraft model applied for these case studies is the same as the aircraft model of the test cases in Chapter 4. The studies in this chapter are implemented on a laptop with dual i5 processors and 4 GB RAM.

5.1 AMS-JFK

As discuss in Chapter 2, the strongest jet stream, Polar jets, are near to the poles. The route between AMS and JFK is significantly affected by the north polar jet. Therefore, the route between AMS and JFK is researched in this study. For the AMS-JFK route, the aircraft is assumed to fly with a full load factor. The weight of each passenger with the hand luggage is 100 kg. With the assumption of 6244 kg cargo, the total payload is 46244 kg, and the expected final landing weight is 225,000 kg. There are 7 horizontal segments and 6 vertical segments in this case. In total, 29 and 33 parameters are optimized in Phase 1 and Phase 2, respectively.

5.1.1 AMS to JFK (8.31)

Figure 5.1 shows the outcomes of Phase 1 optimized by the model for the flights from AMS to JFK.

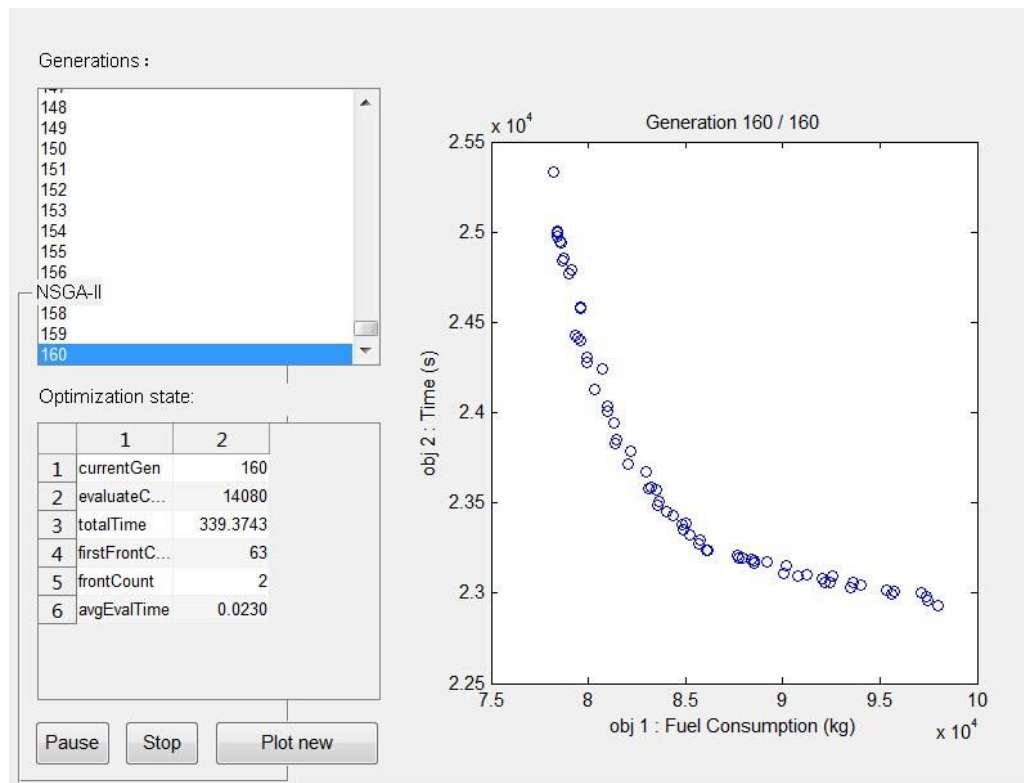


Figure 5.1. Outcomes of Phase 1 for the optimizer for the flights from AMS to JFK.

As shown in the figure above, the computational time is 340 seconds, which is less than 6 minutes. The flight time and the fuel consumption of the minimum time solution is around 23,000 seconds (6.4 hours) with 98,000 kg, while the flight time and fuel consumption of the minimum fuel consumption solution is around 25,200 seconds (7 hours) with 78,000 kg of fuel consumption. The figures below indicate the ground track, vertical profile and the airspeed diagram of the minimum time solution in Phase 1.

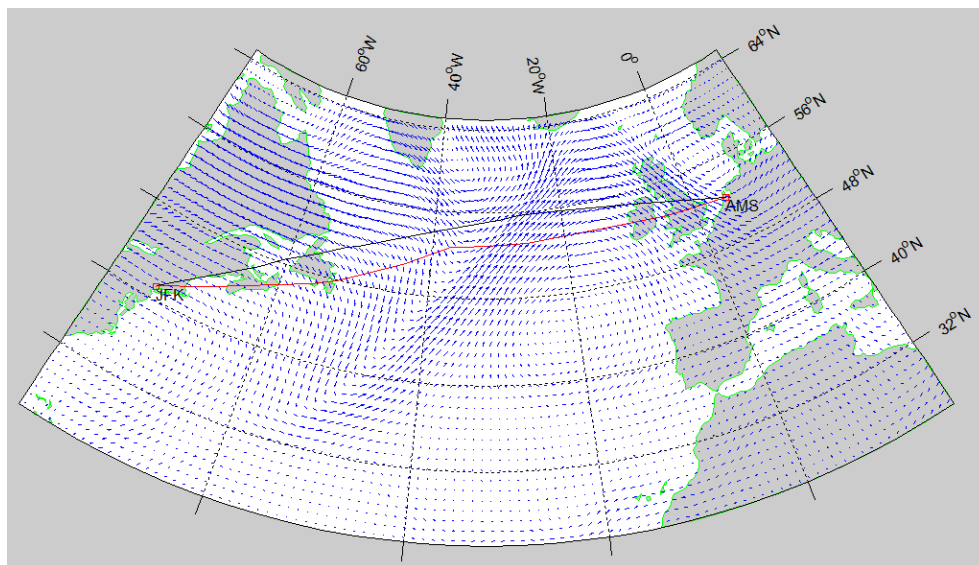


Figure 5.2. The ground track of the minimum time solution from AMS to JFK in Phase 1.

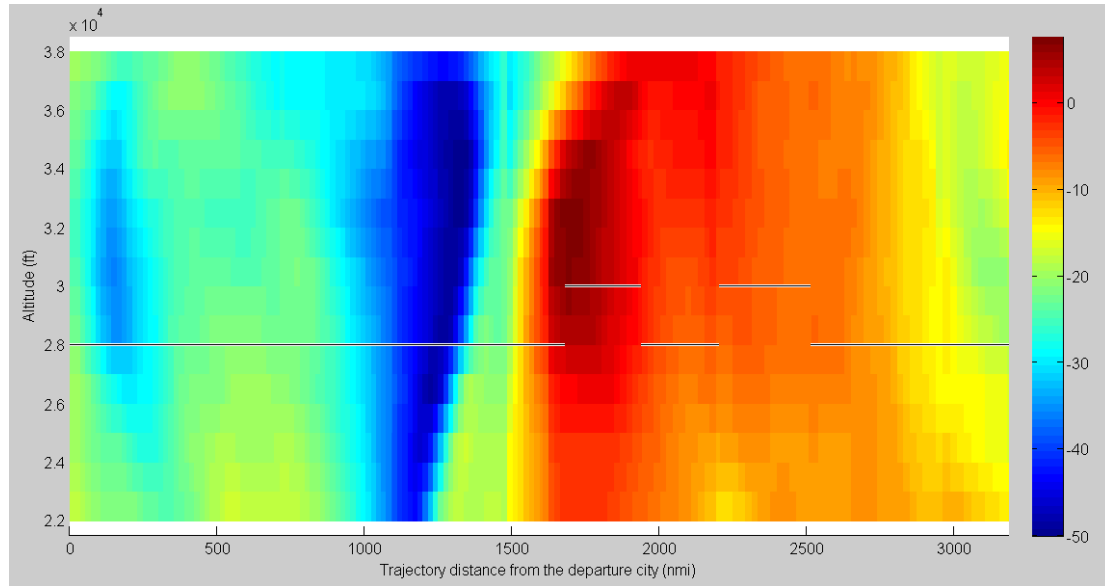


Figure 5.3. The vertical trajectory of the minimum time solution in Phase 1 from AMS to JFK.

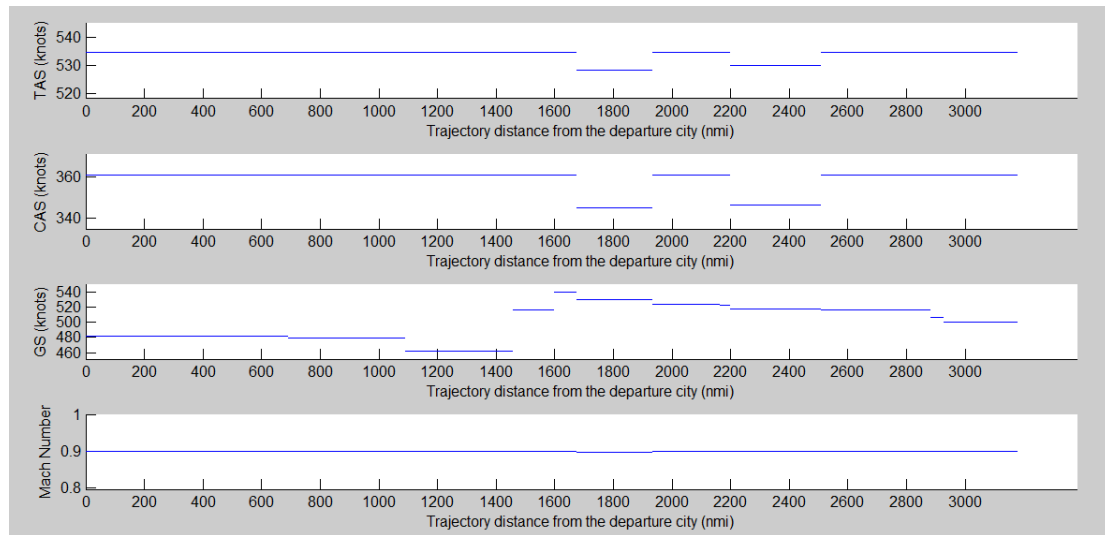


Figure 5.4. The airspeeds diagram of the minimum time solution in Phase 1 from AMS to JFK.

As shown in figure 5.2, the aircraft flies more southerly to avoid the strong headwind in the region from $54^{\circ}N$ to $64^{\circ}N$ and from $0^{\circ}W$ to $26^{\circ}W$. After that, the aircraft flies towards west to minimize the influence of the wind from the south between $20^{\circ}W$ to $35^{\circ}W$. In the second half of this trajectory, there is less wind effect compared to the first half. The aircraft flies towards JFK with tiny turns. In figure 5.3 and 5.4, the aircraft flies at the crossover flight level (FL280) or the adjacent flight level (FL300). The reason of the variation is the same as the test case in Chapter 4. It is difficult to identify the absolutely optimal solution by GA with this aircraft model. Since the flight altitude of the whole trajectory is above the crossover altitude, the maximum feasible TAS is limited by MMO. The Mach number shown in figure 4 indicates that the Mach number of the whole trajectory is constantly equal to the

maximum feasible airspeed (0.9) to minimize the flight time. The figure below shows the outcomes of Phase 2 in this case.

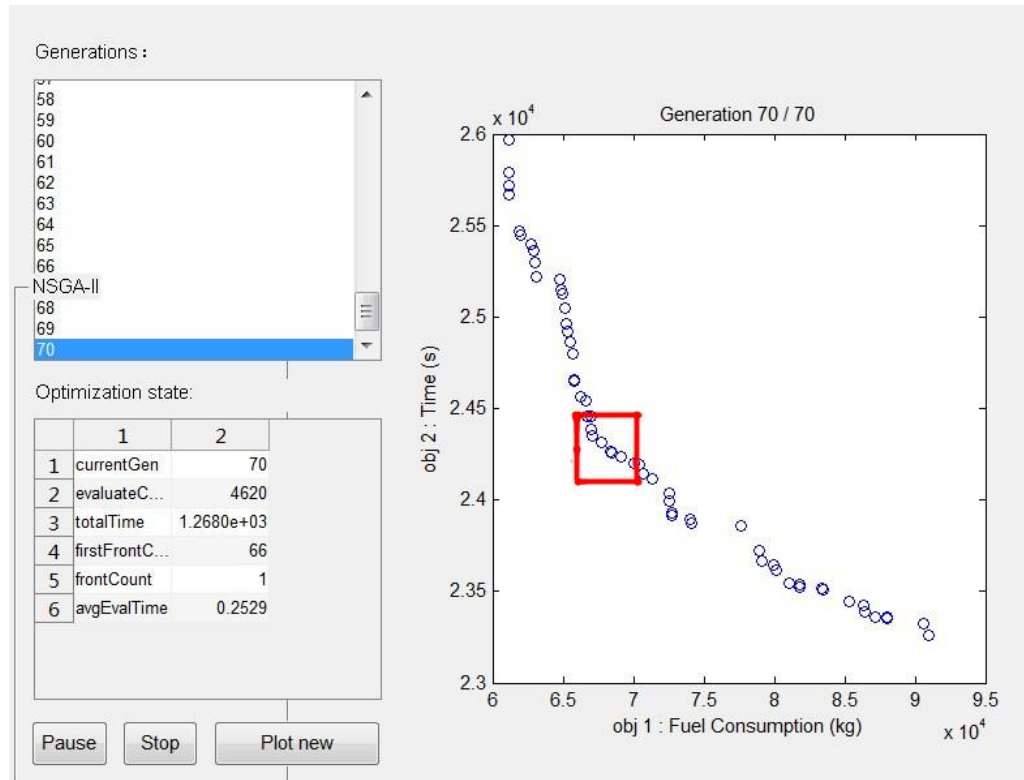


Figure 5.5. Outcomes of the optimizer for the flights from AMS to JFK.

As shown in figure 5.5, the computational time for Phase 2 is around 20 min. Hence, the model is able to generate the outcomes for the route from AMS to JFK in around 26 min with 6 min computational time for Phase 1. Regarding to the outcome solutions, the minimum time solution is around 23,300 seconds (6.47 hours), while the fuel consumption of the minimum fuel solution is 60,500 kg. Additionally, it is interesting to notice the trade-off solutions in the Pareto-front. Compared to the minimum fuel solution, the trade-off solutions in the red box of figure 5.5 with fuel consumption between 65,000 kg and 70,000 kg used are around 7,000 kg more fuel to save around 30 minutes flight time. On the other hand, the trade-off solutions in the red box of figure 5.5 fly around 1,000 seconds (16.6 minutes) longer than the minimum time solution, but save around 25,000 kg fuel consumption. By providing the trade-off solutions, airlines are able to choose the most suitable flight trajectory under different circumstances. If there are operation delays caused by other operations such as ground handling process. Airlines may choose the solutions with less flight time to recover the time delays on the schedule. If the operations are ahead of the schedule and the aircraft takes off on time, airlines are suggested to fly on the trajectories with less fuel consumptions to minimize the operation cost. Not only benefiting to the practical work, the trade-off solutions are also beneficial to the trajectory planning phase. The trade-off solutions may be chosen smartly to balance

the fuel consumption and the flight time in the planning phase. For example, the trade-off solutions discussed above may be suggested when airlines make the flight schedules. The trade-off solutions may save significant amount of the fuel consumption or the flight time with little cost on the other objective. Additionally, the trade-off solutions provide a buffer consumption of fuel and the flight time for the unexpected operational delays. The analysis for individual solutions is provided below.

Figure 5.6 presents the horizontal trajectories of the minimum flight time solution in Phase 2 for the route from AMS to JFK.

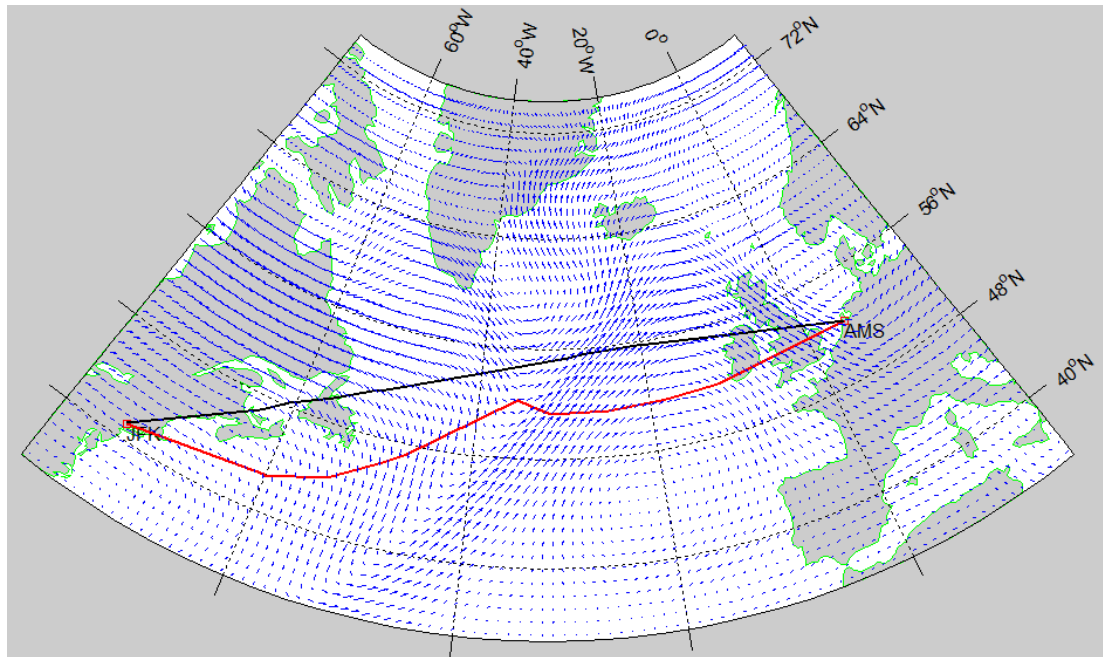


Figure 5.6. The horizontal trajectory of the minimum time profile of the route from AMS to JFK.

As shown in figure 5.6, the horizontal trajectory of the solution (red line) is more southerly compared to the great circle path (black line). The aircraft flies towards the south after taking off at the Amsterdam Schiphol airport for the benefit of the wind flowing from the north. If the aircraft followed the great circle path, then the strong north wind will be a stronger headwind to the aircraft. The aircraft may have to consume more fuel or require more time to fly through the headwind field at around $0^{\circ}W$. After the aircraft flies through the wind field, the aircraft encounters a headwind from the east. Compared to the great circle path, the generated trajectory is less affected by the headwind field in the area at around $18^{\circ}W$. From $20^{\circ}W$ to $36^{\circ}W$ along the generated trajectory, there is a wind field which blows from the south. Therefore, the generated trajectory flies slightly towards the north to minimize the negative impact flowing from the wind instead of heading to the southwest as the great circle path. After that, the aircraft continuously flies down to

the south until the latitude is almost the same as the latitude of JFK. In this segment, the generated trajectory enjoys the wind benefit from the north between $36^{\circ}W$ and $60^{\circ}W$. Compared to the generated trajectory, the great circle path may be against by the wind from the east to the west from $40^{\circ}W$ to $60^{\circ}W$. After the aircraft passes the $62^{\circ}W$ latitude, the impacts from the wind to the great circle path and the generated trajectory are similar to each other. Both of them are against by the wind from the east to the west. However, the distance of the last segment is short. The total flight time of the great circle path is 24,143.7 seconds (6.7 hours) with 94,036.7 kg fuel consumption, while the flight time of the generated trajectory is 23,624.7 seconds (6.56 hours) with 91,160.8 kg fuel consumption. That is, the generated trajectory spends 2.1% less flight time and saves 3.1% fuel compared to the great circle path with the optimized time preference profile. The distance difference between the minimum time solution (5,999 km (3,239.2 nm)) and the great circle (5,830 km (3,147.9 nm)) is 169 km (91.3 nm). Except the horizontal profiles, the vertical profiles of the generated trajectory and the great circle path are also analysed.

The vertical profiles of the minimum time solution and the great circle solution are shown in figure 5.7 and 5.8, respectively. The disconnection of the wind profile at around 1500 nm is due to the heading change of the aircraft. The background wind profile is the wind profile along the aircraft flight direction. Hence, a heading change of the aircraft may result in a wind speed change along the flight direction. The wind profiles shown in figure 5.7 and 5.8 are further evidences of the analysis for the horizontal profile. For the outcome solution, the aircraft gains the benefit from the tailwind between 1600 nm (2963.2 km) and 3000 nm (5556 km) from AMS. With a worse wind condition, the headwind is along the whole trajectory of the great circle path. The worst wind condition occurs at around 1200 nm (2222.4 km) from AMS. The headwind at that position is as large as 60 m/s (134.2 knots).

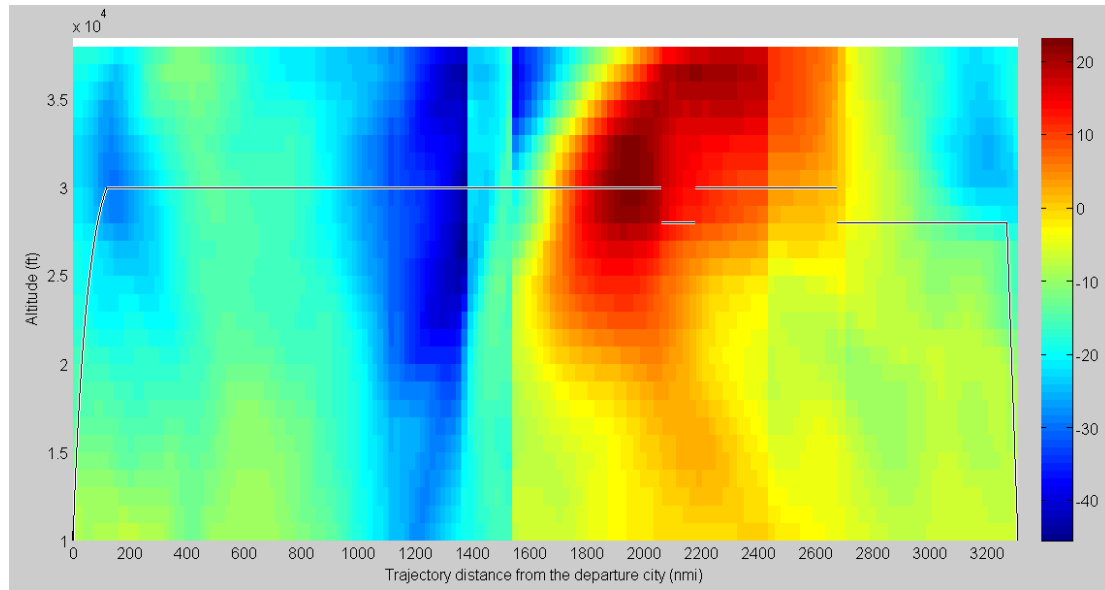


Figure 5.7. The vertical trajectory of the minimum time profile of the route from AMS to JFK.

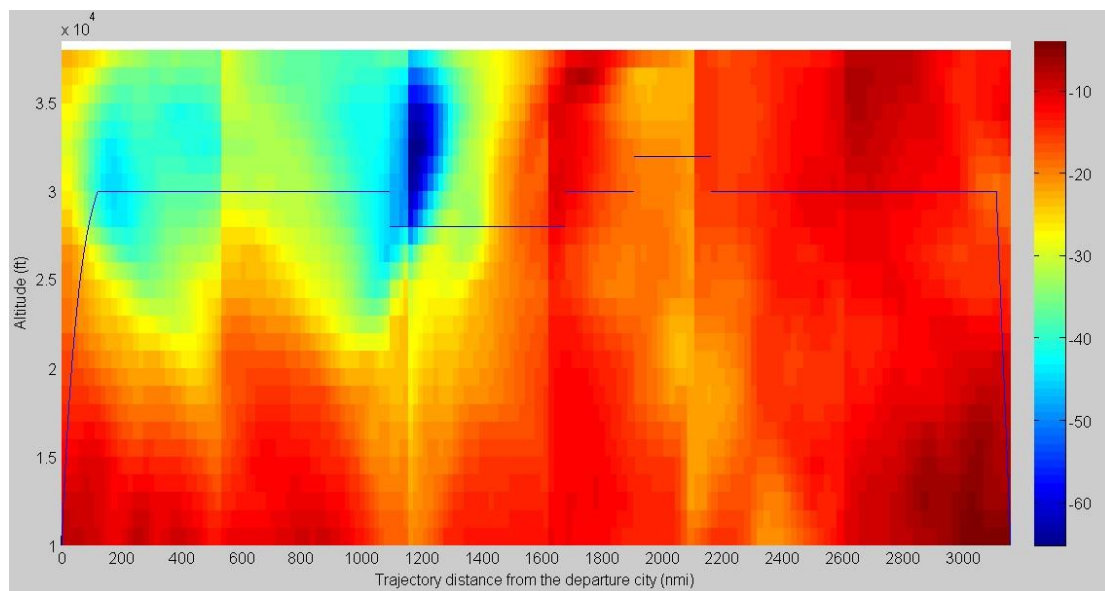


Figure 5.8. The vertical trajectory of the great circle path of the route from AMS to JFK.

Regarding to the flight level, the minimum time solution stays at FL300 until 2,000 nm (3,218.6 km) from AMS. Since changing the altitude will not enable the aircraft to avoid the headwind in the segment from 1,000 nm (1,609.3 km) to 1,500 nm (2,414 km), the aircraft does not descend or climb to the adjacent flight level. From 1,600 nm (2,574.95 km) to 2,000 nm (3,218.6 km), the aircraft can gain the most benefit from the tailwind by staying at 30,000 ft (9,144 m) compared to flying at the adjacent flight level. Since the wind speeds of FL300 and FL280 between 2,100 nm and 2,300 nm are barely different and the maximum feasible TASs at FL 300 and FL280 are nearly the same, the optimizer is not able to distinguish the difference between these two flight levels. Therefore, the aircraft descends at around 2050 nm and

climbs back to FL300 at 2,150 nm. After reaching 2,600 nm from AMS, the aircraft descends from 30,000 ft to 28,000 ft with the reason that there is no performance difference between 30,000 ft and 28,000 ft in the segment between 2,600 nm and 2,800 nm. Also, the aircraft can decrease the impact of the headwind from 3,000 nm to the starting point of the descent phase.

Figure 5.9 indicates the airspeeds of the minimum time solution in this case. Since the flight altitude of the cruise phase is above the crossover altitude, TAS is limited by MMO. According to figure 5.9, the Mach number is a constant at 0.9 during the cruise phase. That means the aircraft flies at the maximum feasible TAS during the cruise phase. For the climbing and descent phase, CASs are constants, which are limited by the weight of the aircraft and the altitude of the first or last segment of the cruise phase. The constantly changing of TAS, GAS and Mach number in the climbing and descent are due to the change of the altitude.

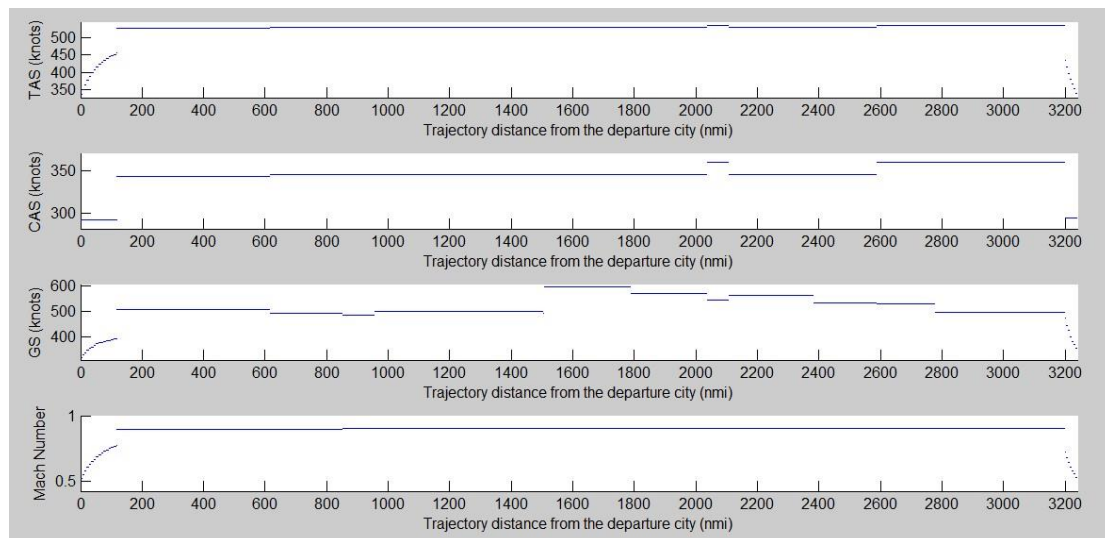


Figure 5.9. The airspeeds diagram of the minimum time profile of the route from AMS to JFK.

Regarding the minimum fuel consumption solution, the horizontal flight route is shown as figure 5.10.

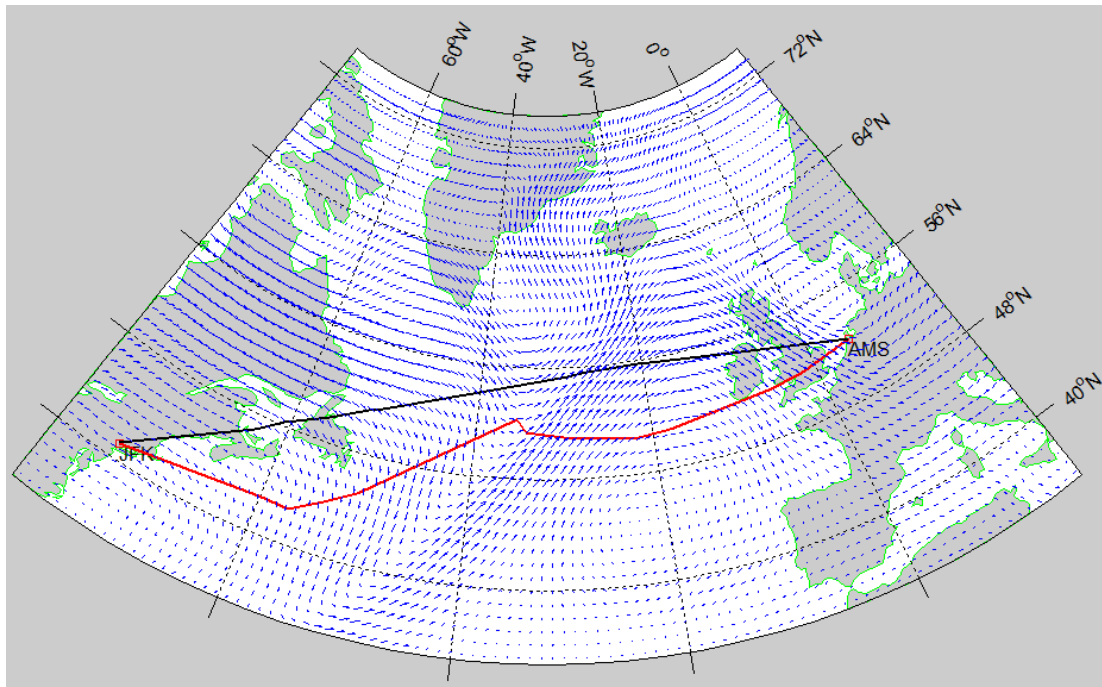


Figure 5.10. The horizontal trajectory of the minimum fuel trajectory from AMS to JFK.

Since the trajectory presented in figure 5.10 is similar to the trajectory of the minimum time profile, detailed analysis of this trajectory in figure 5.10 is not provided in this report. Compared to the great circle path with the optimized vertical trajectory (64,036.2 kg, 26,726.9 seconds), the minimum fuel solution (61,147.4 kg, 25,967 seconds) saves 2,888.8 kg (4.5%) fuel and 759.9 seconds (12.67 minutes). The reason for this is that the minimum fuel solution gains more benefit from the wind along the trajectory even though the distance of the minimum fuel solution (6,065 km, 3,274.8 nm) is 235 km (126.9 nm) larger than the great circle path. Compared to the minimum time solution, the minimum fuel solution spends 45.2 min (11.6%) more than the minimum time solution while saves 29,809.3 kg (32.8%) fuel consumption.

For the vertical trajectory, the vertical profile of the minimum fuel consumption solution is different from the vertical profile of the minimum flight time solution. The vertical trajectory of the minimum fuel consumption solution is shown in figure 5.11.

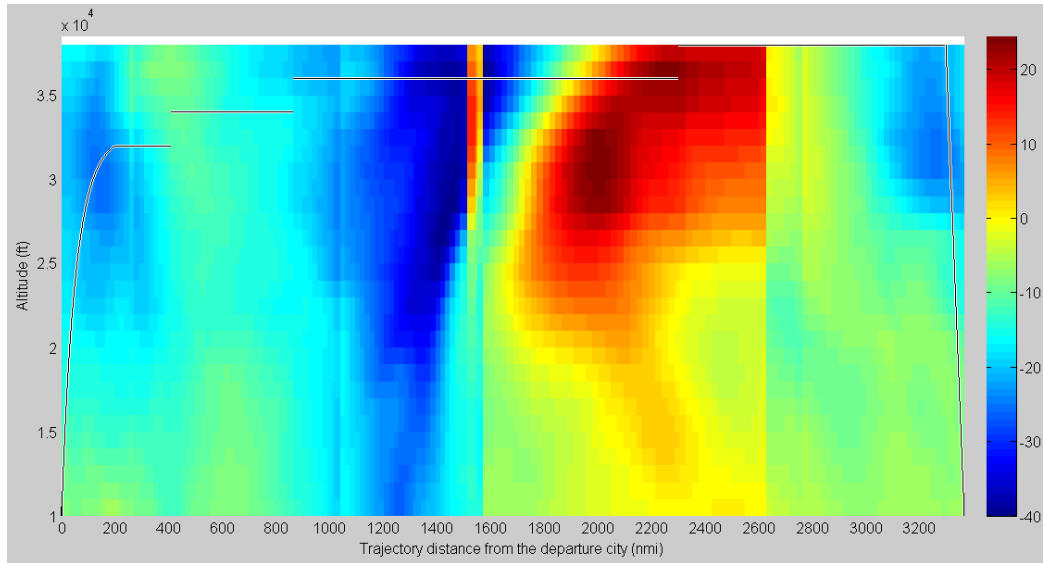


Figure 5.11. The vertical trajectory of the minimum fuel profile of the route from AMS to JFK.

As discussed in chapter 2, the fuel efficiency of the fuel consumption model increases as the flight altitude increases. Therefore, there is no doubt that the flight levels of the vertical flight trajectory of the minimum fuel solution increases constantly to the ceiling of the flight levels. Due to the heavy weight, the aircraft is not able to climb to the highest flight level in the beginning of the trajectory. Hence, the trajectory in figure 5.11 starts at 34,000 ft (10,363.2 m) and climbs to 38,000 ft (11,582.4 m) at approximately 1,300 nmi (2,407.6 km) from AMS.

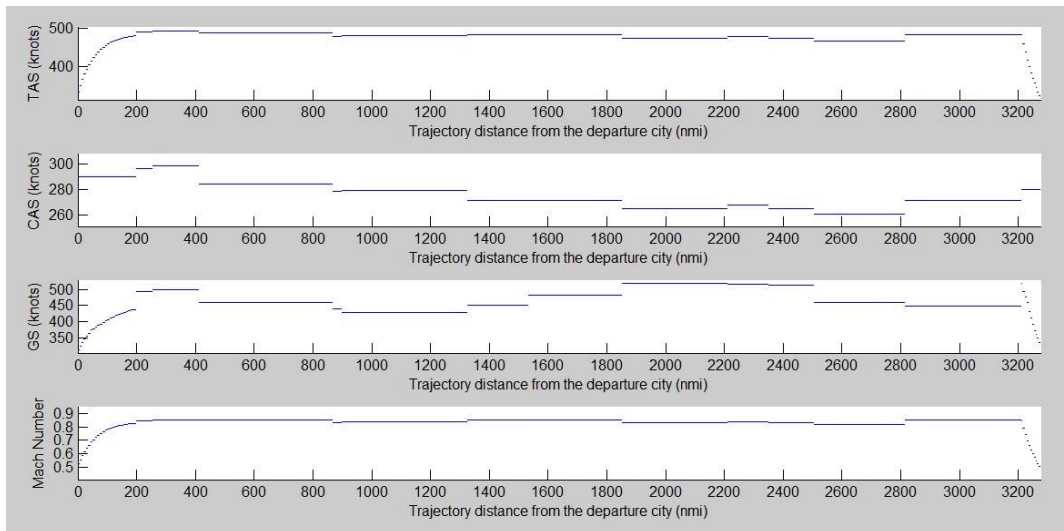


Figure 5.12. The airspeeds diagram of the minimum fuel consumption profile from AMS to JFK.

Figure 5.12 is the airspeeds diagram of the minimum fuel consumption profile from AMS to JFK. Since the most economical Mach numbers at FL 340, FL360 and FL380 are 0.78, 0.8 and 0.84 as show in table 2.1 respectively, the cruise Mach numbers shown in figure 5.12 are between 0.8 and 0.9.

Except two extreme solutions, the trade-off solutions are also important for airlines. Since the ground tracks of the trade-off solutions are similar to the two extreme solutions above, the ground tracks of trade-off solutions will not be provided in this report. The figures below show the vertical profile and the airspeed diagram of one of the trade-off solutions with 70,077.3 kg fuel consumption and 24,202.9 seconds (6.72 h) flight time.

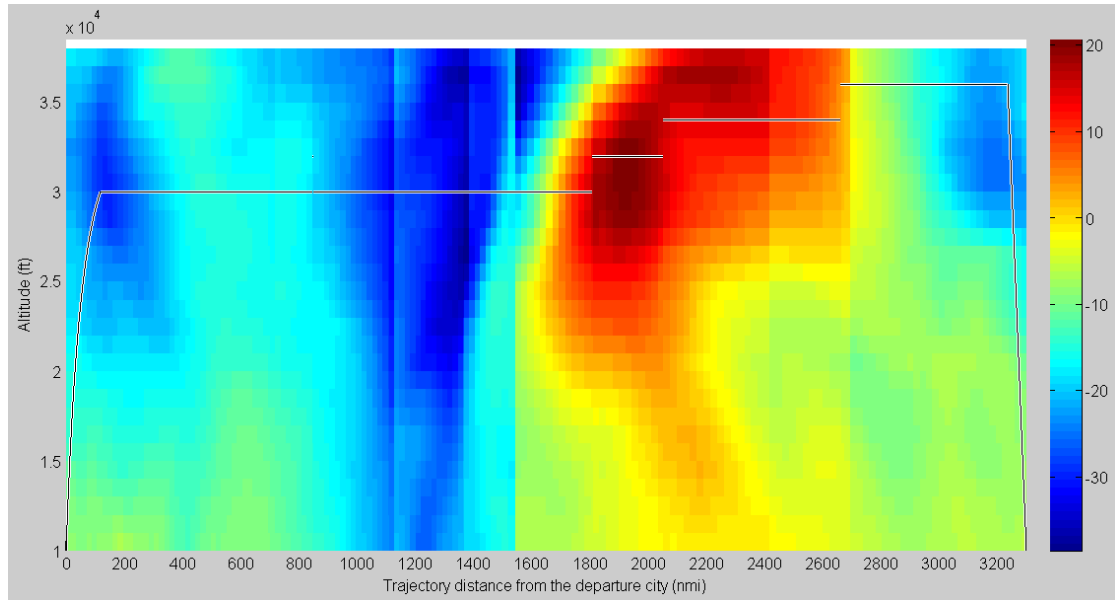


Figure 5.13. The vertical profile of a trade-off solution of route from AMS to JFK.

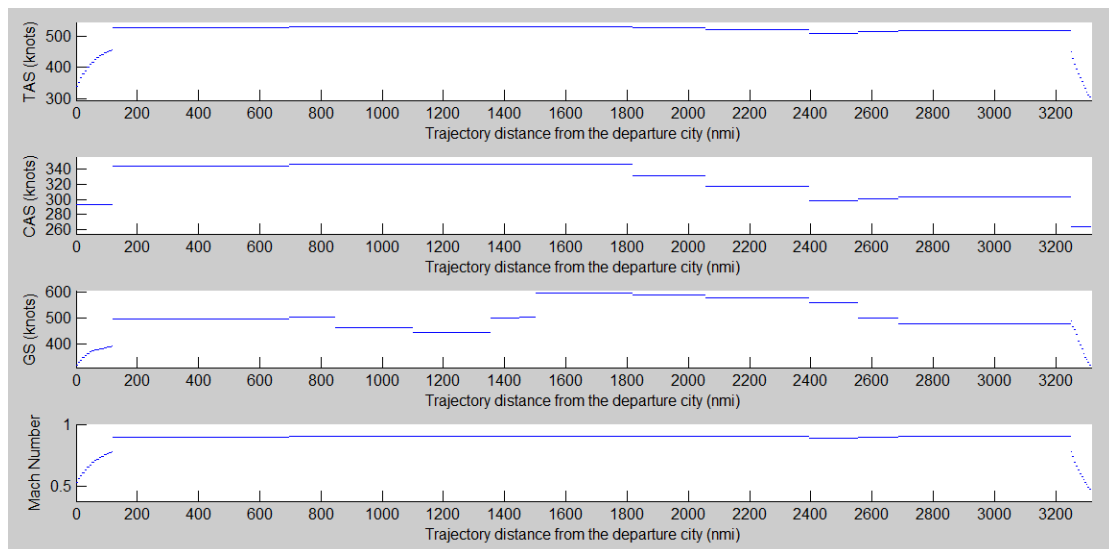


Figure 5.14. The airspeed diagram of a trade-off solution of route from AMS to JFK.

Compared to these two extreme solutions, the vertical profile trade-off solution is more wind-oriented. As shown in figure 5.13, the flight level keeps a constant until the aircraft flies to 1,800 nm. After that, the aircraft climbs because the tailwind effects are stronger at higher altitudes. Regarding to the airspeed, the aircraft keeps

a constant Mach number at 0.9 for the whole trajectory. In the respect of saving time, altitude changes will not significantly affect the maximum feasible TAS, while the fuel efficiency will significantly increase when the flight altitude increasing as discussed in Chapter 2 and 3. Therefore, the aircraft following the trade-off solution climbs to higher altitudes and flies with the maximum feasible TAS to balance the fuel consumption and flight time. The results of several trade-off solutions are shown in the table below:

Table 5.1. The results of trade-off solutions compared to the time and fuel preference solutions.

Fuel consumption (kg)	Compare to the minimum time solution (%)	Compare to the minimum fuel solution (%)	Flight time (hours)	Compare to the minimum time solution (%)	Compare to the minimum fuel solution (%)
70663.6	77.7	115.6	6.7	103.8	92.9
70077.3	77.0	114.6	6.7	104.1	93.2
68399.6	75.2	104.3	6.7	111.9	93.4
66213.0	72.8	108.3	6.8	105.6	94.6

5.1.2 JFK to AMS (8.31)

For the opposite direction, the situation is different. The route has more tailwind effects since the polar jet flows from the west to the east. As shown in figure 5.15, the general wind direction is toward to the east above the NAO. It is not surprised that the performances of the flights from JFK to AMS are better than the flights in the opposite direction. The horizontal trajectory of the minimum time solution from JFK to AMS with the wind profile on 31 August 2014 is shown in figure 5.15.

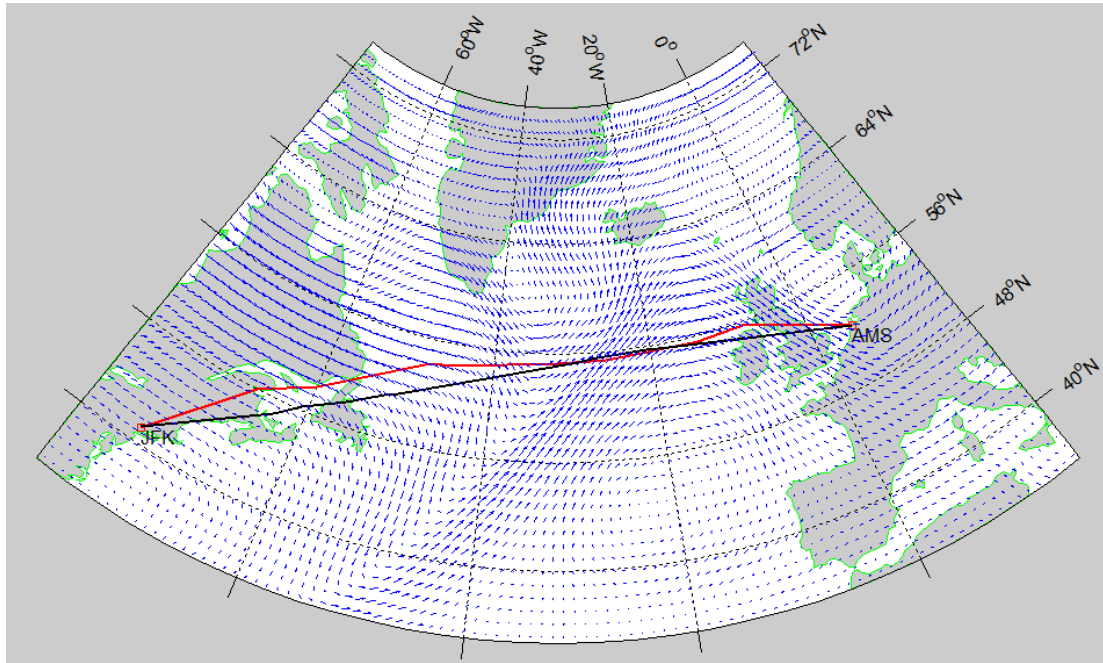


Figure 5.15. The horizontal trajectory of the minimum time solution from JFK to AMS.

Different from the minimum time trajectory from AMS to JFK, the minimum time trajectory in figure 5.15 is closer to the great circle path to minimize the flight distance. As shown in figure 5.15, the aircraft flies more northerly in order to gain the benefit from the west wind from $48^{\circ}N$ earlier compared to the great circle path in the beginning of the trajectory. By flying this trajectory, the flight enables to speed up by the west tailwind from $48^{\circ}N$. After that, the trajectory coincide with the great circle path before reaching $30^{\circ}W$. After the aircraft reaches $15^{\circ}W$, the trajectory breaks away from the great circle path again in order to adjust the heading closely to the wind direction from $8^{\circ}W$ till the end of the trajectory. The total flight time of the solution is 20,913.7 seconds (5.8 hours) with 88,219 kg fuel consumption, while the flight time of the great circle with the optimized vertical trajectory is 21,147.5 seconds (5.87 hours), the fuel consumption of the same trajectory is 89,104.4 kg. In other words, the solution from the model is 1.1% and 1% less than the great circle path in the respect of the flight time and the fuel consumption. The smaller savings compared to the previous case is because the whole trajectory is close to the great circle path. The flight distance of this trajectory (5,896 km) is 66 km (35.6 nm) longer than the distance of the great circle path (5,830 km).

Figure 5.16 and 5.17 present the vertical trajectory and the airspeeds of the minimum time solution from the model. As shown in figure 5.16, the trajectory enjoys the tailwind along the whole trajectory. The flight levels are around the crossover flight level. It is interesting to notice that the trajectory does not climb to higher flight levels for the benefit of the strong tailwind, and even descend to a lower flight level. The reason for this unusual chose is due to the characteristics of NSGA.

Once the optimization process reaches to the final generations, locating the precise solution is difficult for NSGA since the improved space is small. When the trajectory descends to FL260, TAS is limited by VMO in this segment because the altitude is lower than the crossover altitude. According to figure 5.17, the CAS in this segment is at the top of the feasible range as presented in the second diagram of figure 5.17. As shown in the first diagram of figure 5.17, the difference between the resulting TAS and the maximum TAS at the crossover flight level is small. The optimizer is not able to distinguish the difference from the final results.

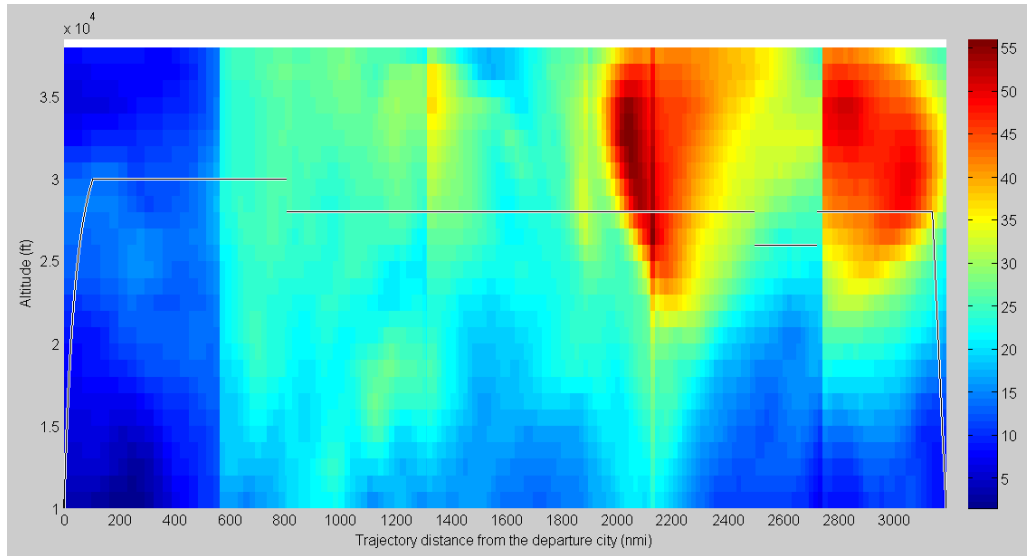


Figure 5.16. The vertical trajectory of the minimum time solution from JFK to AMS.

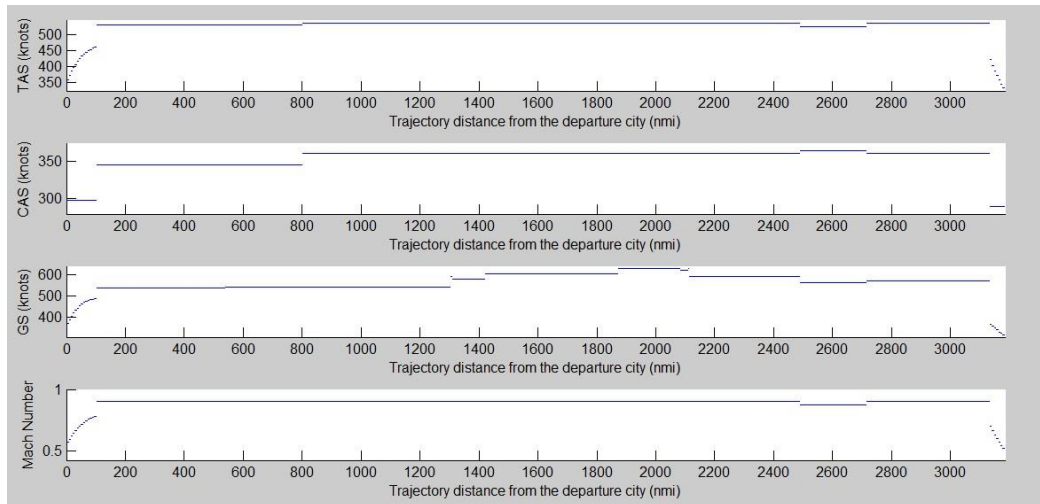


Figure 5.17. The airspeed diagram of the minimum time solution from JFK to AMS.

Since the horizontal trajectory of the fuel preference profile is similar to the horizontal trajectory of the time preference profile, the ground track of the minimum fuel solution will not be presented in this report. The flight time and the fuel consumption of the outcome solution are 23,577.8 seconds (6.55 h) and 53,666.8 kg.

respectively, which are 7.83 minutes (2%) and 599.2 kg (1.1%) less than the great circle path with the optimized vertical trajectory. The flights along the minimum fuel consumption solution fly 44.4 minutes (12.7%) longer in contrast to the minimum time solution with 34,552.2 kg (39.1%) less fuel consumption. Similar to the minimum fuel consumption of the route from AMS to JFK, the flight altitude constantly increases to the ceiling of the feasible altitude and the Mach number keeps as around 0.8 in the case from JFK to AMS.

5.1.3 AMS to JFK (7.13)

The next case study is the route from AMS to JFK with the wind profile on 13 July 2014. Figure 5.18 shows the horizontal trajectory of the minimum time solution.

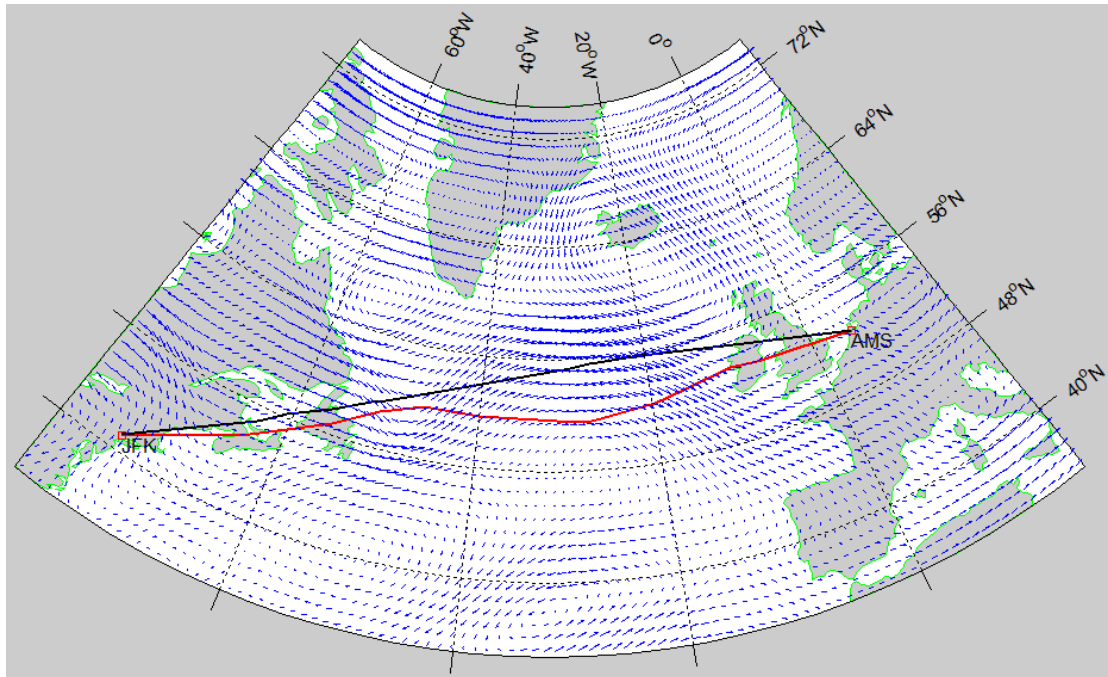


Figure 5.18. The horizontal profile of the minimum time solution for AMS to JFK route.

The time minimum trajectory in figure 5.18 is more southerly compared to the great circle path. The trajectory flies towards the south after taking off at AMS, the reason for this decision is due to the strong wind from the northwest at around $20^{\circ}W$. The great circle path from AMS to $20^{\circ}W$ is northwards, which will encounter a strong headwind in this segment. The other reason of the southward flight in this segment for the minimum time solution is that the trajectory is less resisted by the wind from $25^{\circ}W$ to $40^{\circ}W$ compared to the great circle path. Because the main headwind from $20^{\circ}W$ to $40^{\circ}W$ is between $52^{\circ}N$ and $60^{\circ}N$, the minimum time trajectory is just below the headwind, which minimizes the impact from the headwind. After

passing the westward headwind, the trajectory heads towards the west rather than the southwest of the great circle path to avoid the wind from the south in the area between $36^{\circ}W$ and $44^{\circ}W$. Once the aircraft flies through the wind from the south, the aircraft flies closely to the great circle path in order to minimize the flight distance with the reason that there is not much space for the optimizer to gain the wind benefit in the last segments of the trajectory. The total flight time of this trajectory is 23,624.7seconds (6.56 hours) and the fuel consumption is 91,160.8 kg respectively, which is 252.1seconds (4.2 minutes) and 7,205.9 kg lower than the great circle path (23,876.8 seconds, 98,366.7kg). The total flight distance of this trajectory is 5,928 km (3200.8 nm), which is 98 km (52.9 nm) larger than the flight distance of the great circle path. Similar to the previous cases, the flight levels are around FL280 and the Mach number is 0.9 in this case.

In terms of the minimum fuel consumption solution, the horizontal trajectory is different from the horizontal trajectory of the minimum flight time solution. The optimized horizontal trajectory is shown in figure 5.19. In this case, the wind profile shown in figure 5.19 is the wind profile at 38,000 ft (11582.4 m).

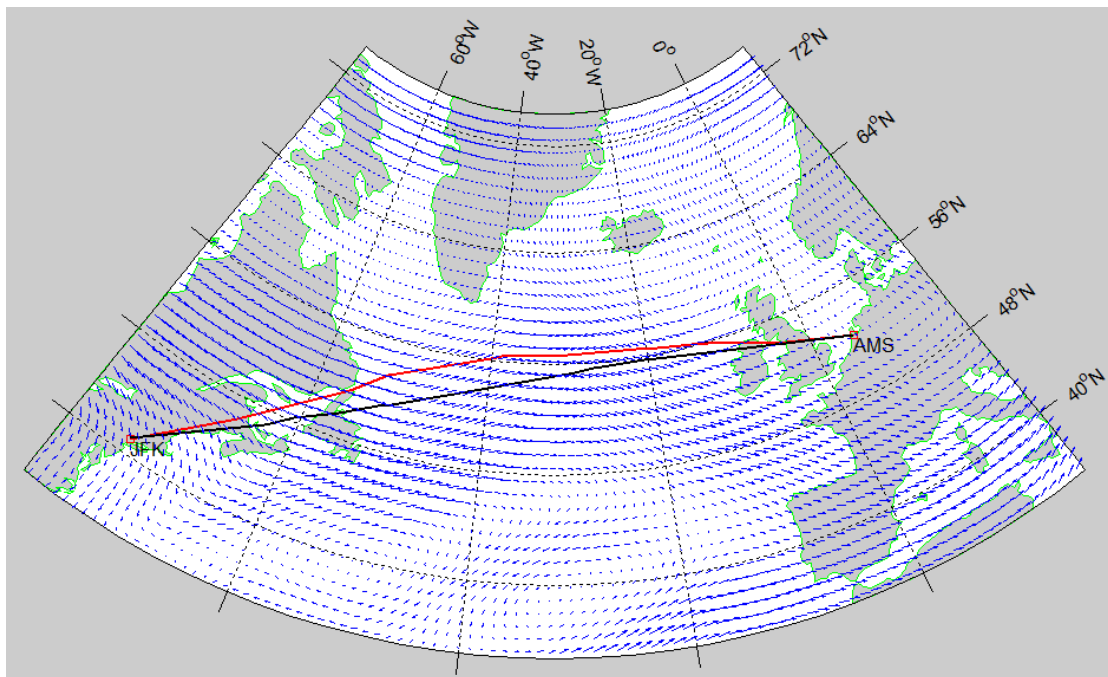


Figure 5.19. The optimized horizontal trajectory of the route from AMS to JFK with the wind profile at 38000 ft.

According to figure 5.19, it can be noted that the eastward headwind in that area from $56^{\circ}N$ to $60^{\circ}N$, and from $20^{\circ}W$ to $40^{\circ}W$ at 38,000 ft (11,582.4 m) is weaker than the headwind in the same region at 28,000 ft (8,534.4 m) shown in figure 5.18. Consequently, the aircraft chooses the direction of north after taking off at AMS. By doing this, the flight distance of the minimum fuel trajectory is 58 km (31.3 nm).

shorter the distance of the minimum time route. Figure 5.20 and 5.21 show the vertical profile of the minimum time trajectory and the minimum fuel consumption trajectory, respectively.

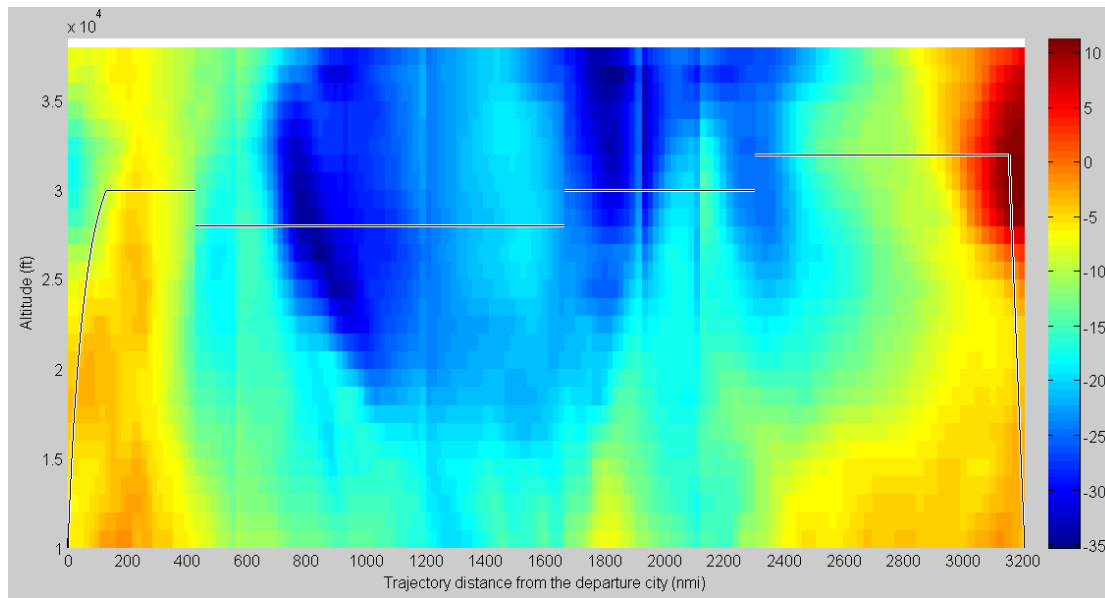


Figure 5.20. The vertical profile of the minimum time trajectory from AMS to JFK.

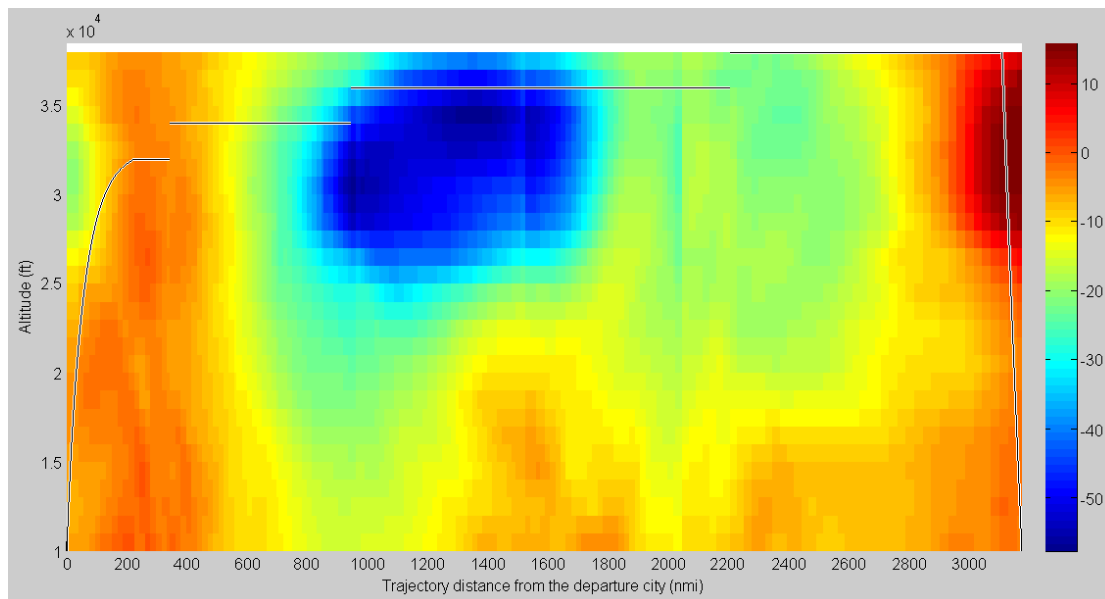


Figure 5.21. The vertical profile of the minimum fuel consumption trajectory from AMS to JFK.

Figure 5.20 and 5.21 provide further evidence for the analysis shown above, the minimum fuel consumption trajectory encounters the strong headwinds (> 30 m/s) at around 800 nm (1,287.5 km). If the aircraft flies along the minimum time ground track with the same vertical profile as the minimum fuel solution, the distance affected by the strong headwind is as large as 2,000 nm (from 700 nm to 2,600 nm). Therefore, the ground track of the minimum time is not a good solution for the

minimum fuel consumption trajectory. However, this explanation leads to another question: why the ground track of the minimum time solution is not the same as the ground track of the minimum fuel trajectory? The answer to this question is also the headwind. As shown in figure 5.20, the wind speed of the headwind along the trajectory is below 35 m/s (68 kts), while the headwind speed along the minimum fuel consumption solution can be more than 50 m/s (97.2 kts) at the same altitude as the minimum time trajectory. That is, the significant difference of the ground tracks between the minimum time solution and the minimum fuel consumption solution is due to the difference of the wind profiles at different altitudes. For the comparison between the minimum time solution and the minimum fuel solution, the minimum fuel solution saves 29,792.1 kg (32%) fuel and costs 3,032.9 s (50.5 min, 12.8%) more time than the minimum time solution.

5.1.4 JFK to AMS (7.13)

For the flights from JFK to AMS, the trajectory is along the great circle path. As shown in figure 5.22, the ground track of the minimum time solution intends to follow the tailwind for the maximum the wind benefit. Since the aircraft enjoys the tailwind from JFK to AMS and the wind profiles at different altitudes are similar, the ground track of the minimum fuel solution is similar to the ground track of the minimum time solution. The table below shows the comparison of the solutions to the optimized solutions with the great circle ground track.

Table 5.2. The comparison of the solutions to the optimized solutions with the great circle ground track.

Solution	Fuel consumption (kg)	Difference to the great circle ground track	Flight time (hours)	Difference to the great circle ground track
Time preference	84,002.1	1,914 kg (2.2%) (save)	5.7	893.5 s (4.2%) (save)
Fuel preference	51,582.4	3,303.4 kg (6%) (save)	6.5	1,538.7 s (6.6%) (save)

In terms for the vertical profile and airspeeds, the track and the airspeeds are the same as the results shown for pervious cases. Therefore, the detailed analysis will not be provided in this report.

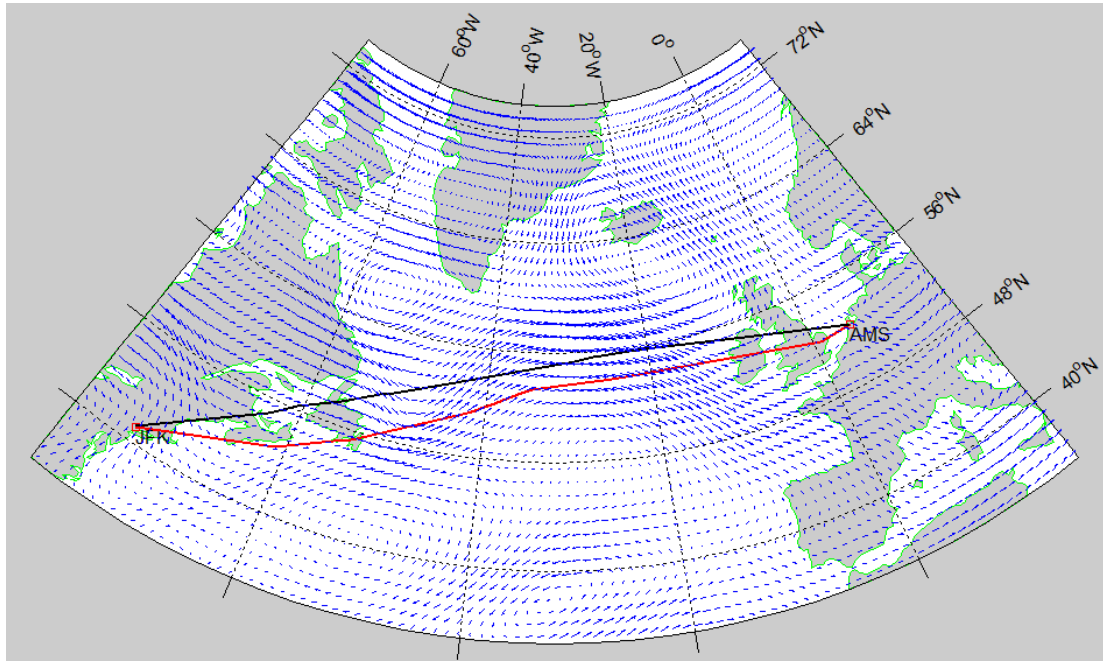


Figure 5.22. The ground track of the minimum time solution from JFK to AMS.

5.2 AMS-JNB

For the route between AMS and JNB, the influence of the wind on the route are less than the influence of the wind on the route between AMS and JFK, because the jet streams on the earth are normally east-west as discussed in Chapter 2. For comparison reason, the research of the route between AMS and JNB is also included in this study. Since the distance of the routes between AMS and JNB is larger than the flight distance between AMS and JFK, the fuel consumptions for the route from AMS to JNB route is more than the fuel consumption for AMS-JFK route. The payload for this route is set as 32,000 kg. That is, the expected final weight is 210,000 kg. In this case, 7 horizontal segments and 6 vertical segments are involved. 29 and 33 parameters are evaluated in Phase 1 and Phase 2.

5.2.1 AMS to JNB (8.31)

Figure 5.23 shows the final generation of Phase 2 for flights from AMS to JNB. The computational time for Phase 2 is 46.4 minutes. With 10 minutes computational time of Phase 1, the total computational time for AMS to JNB route is within 1 hour. As shown in figure 5.23, the flight time of the minimum time solution is just longer than

32,500 seconds (9.03 hours), while the fuel consumption for fuel preference solution is approximately 80,000 kg. It can be noticed that the solutions with approximately 100,000 kg fuel consumption significantly decrease the fuel consumption by more than 20,000kg compared to the minimum time solution with only 2 or 3 minutes longer flight time. In addition, with less than 10,000 kg extra fuel consumption compared to the minimum fuel consumption solution, the solutions with approximately 88,000 kg fuel consumption can decrease the flight time by 40 minutes from around 35,800 seconds (9.9 hours) to 33,200 s (9.2 hours).

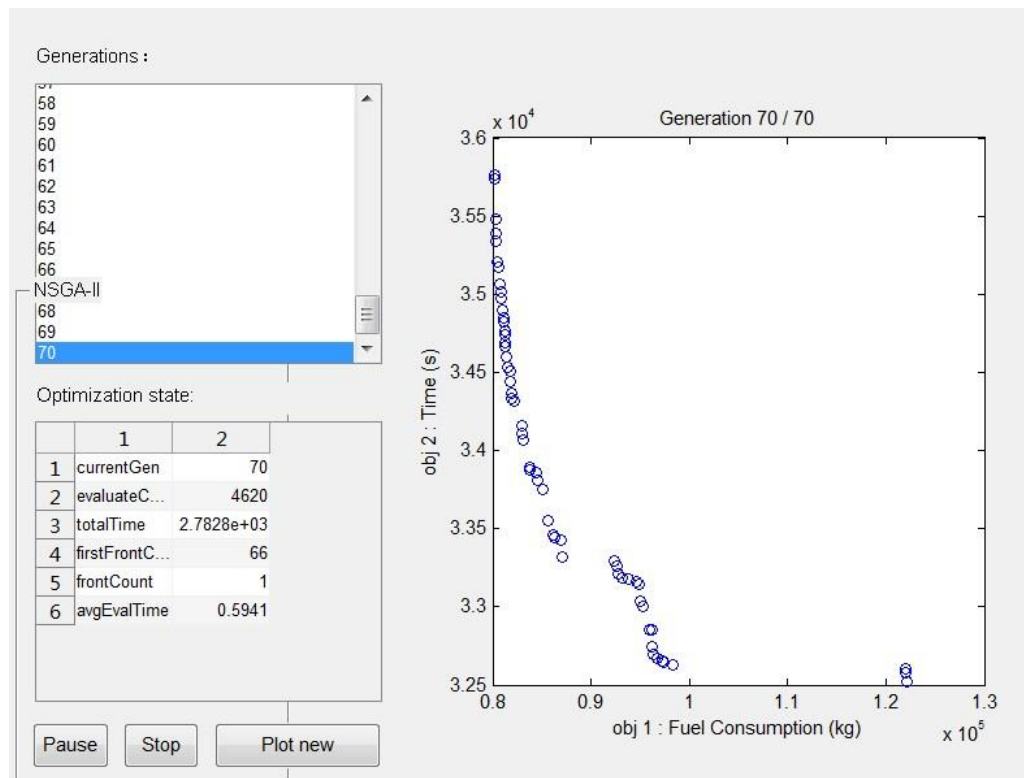


Figure 5.23. The solutions of the final generation for flights from AMS to JNB.

Regarding the individual solution, figure 5.24 and 5.25 present the ground track and the vertical trajectory of the minimum time solution from AMS to JNB.

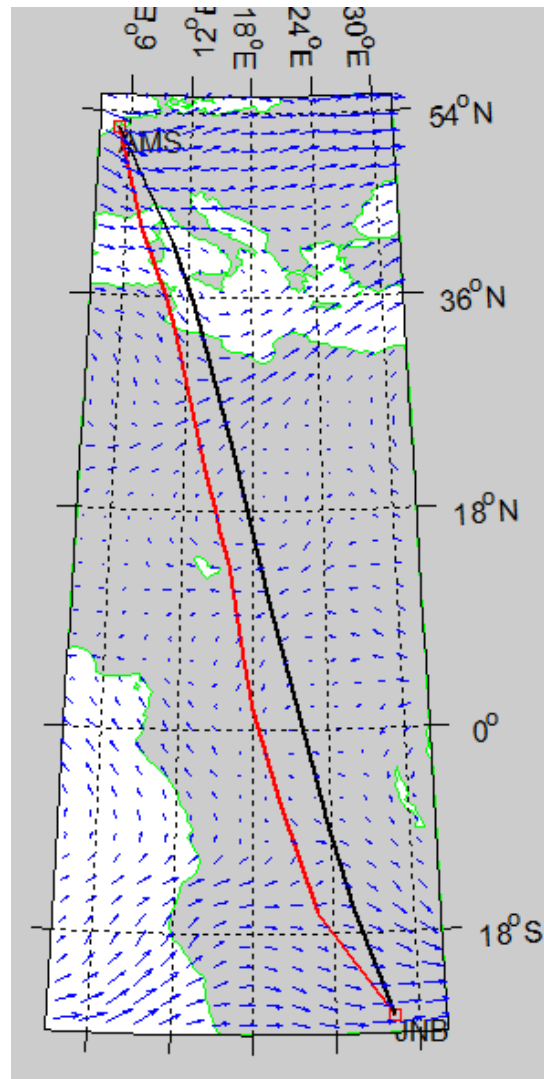


Figure 5.24. The horizontal profile of the minimum time solution for route from AMS to JNB.

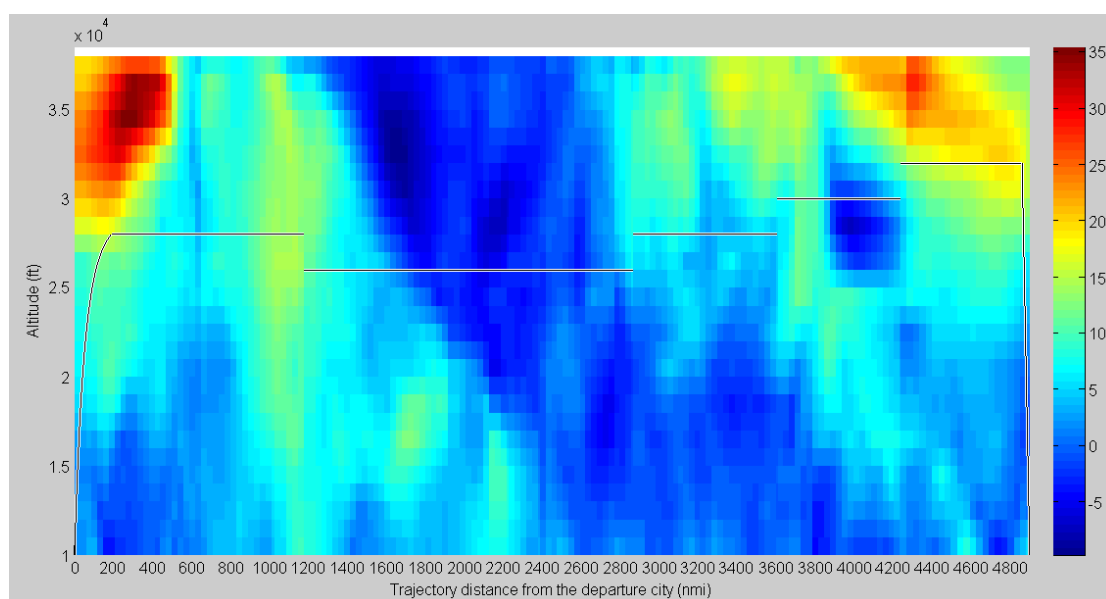


Figure 5.25. The vertical trajectory of the minimum time solution from AMS to JNB.

In figure 5.24, the optimized trajectory is nearly parallel to the great circle path in order to minimize the flight distance since the wind effects along the trajectory is not strong. However, the trajectory separates itself from the great circle path in the beginning of the route and flies back towards the great circle path to JNB at the end of the trajectory. The reason for this phenomenon is that the aircraft has tailwinds from the polar jets in the beginning and at the end of the flight trajectory based on the wind profile shown in figure 5.24. Another unusual phenomenon is that the aircraft climbs to 34,000 ft (10,363.2 m) in the final cruise phase of the trajectory. As analysed in chapter 3, the influence of the altitude to the feasible airspeed range is limited. In fact, the airspeed at 34,000 ft (10,972.8 m) in this case is 268.1 m/s (521.1kts) , while the airspeed at 28,000 ft (8,534.4 m) is 275.2 m/s (534.9 kts) with a Mach number of 0.9 as shown in figure 5.26.

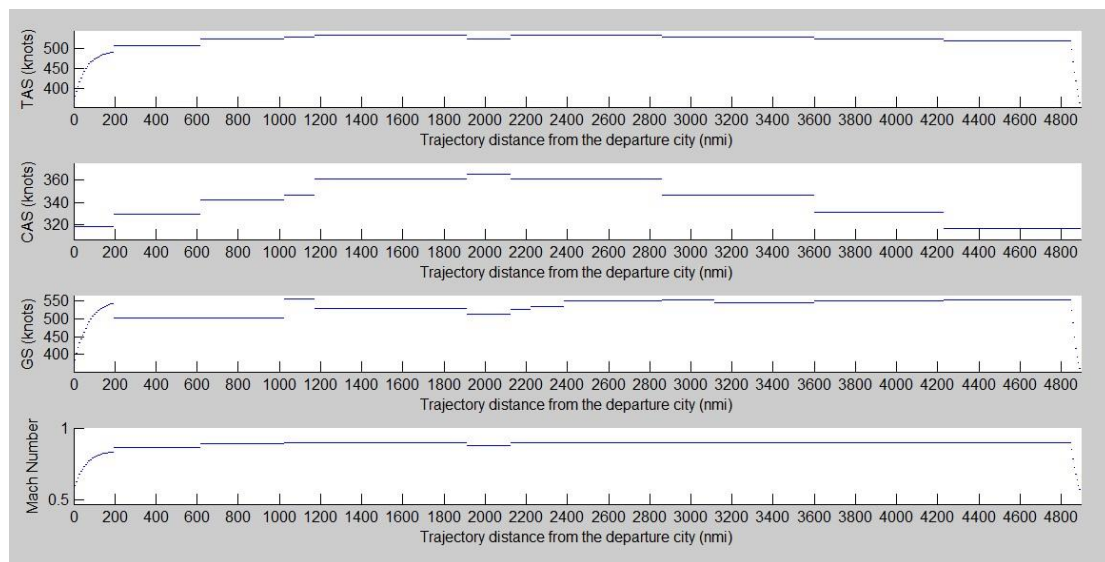


Figure 5.26. The airspeed diagram of the minimum time solution from AMS to JNB.

Compared to the great circle path, the minimum time solution (32,519.5 seconds (9.03 h) 122,065kg) is 3 minutes less than the flight time of the great circle path. The flight distance of minimum trajectory is 78 km larger than the great circle path. The fuel consumption of the minimum time solution is 354 kg larger than the great circle path.

In terms of the minimum fuel solution, the ground track is similar to the ground track of the minimum time solution. The flight altitude constantly increases to 38,000 ft (911,582.4 m) and stay at that altitude for the majority of the trajectory. As expected, the Mach number of the minimum fuel solution is between 0.8 and 0.9 for the whole trajectory. The fuel consumption of this trajectory is 80,212.2 kg with a 35,766.1 s (9.9 hours) flight time, which are 1,465.8 kg (1.8%) and 16.56 minutes (2.7%) lower than the great circle path.

The figures below show the vertical profile and the airspeeds of a trade-off solution with 98,349.5 kg fuel consumption and 32,631 seconds (9.06 hours) flight time.

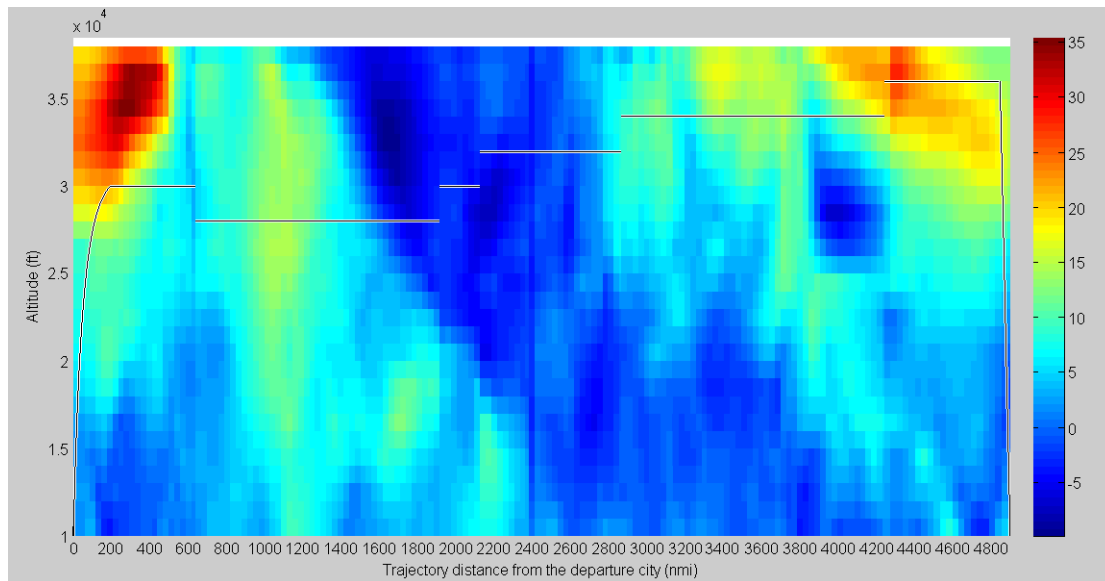


Figure 5.27. The vertical profile of a trade-off solution from AMS to JNB.

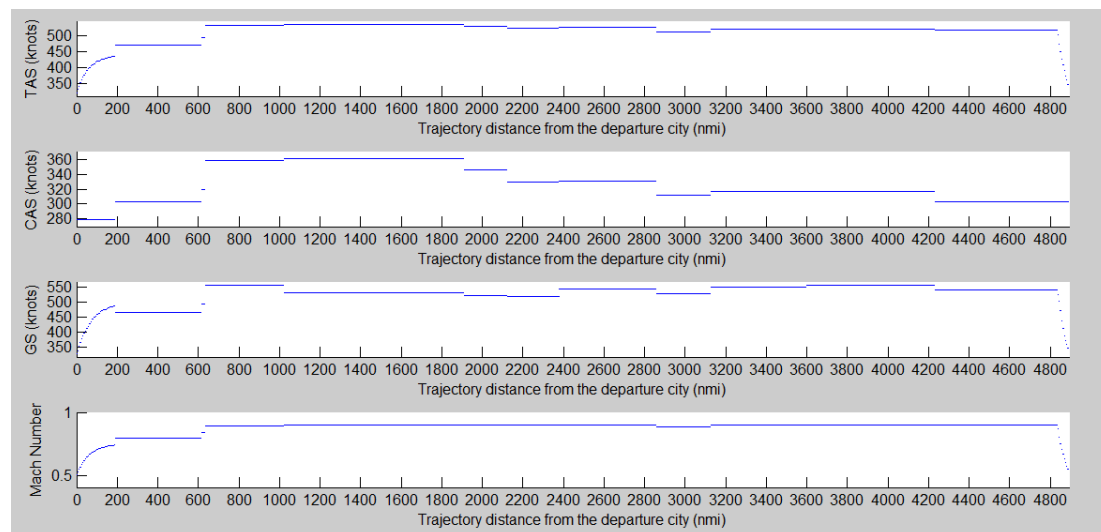


Figure 5.28. The airspeed of a trade-off solution from AMS to JNB.

The results of figure 5.27 and 5.28 are similar to the results of the trade-off solution for the route from AMS to JFK. The aircraft climbs to a high flight level for the strong tailwind at FL360 and high fuel efficiency at high flight levels. Moreover, the climbs do not affect much to the maximum feasible TAS. As shown in figure 5.28, the TAS is almost the same from 600 nm to the end of the trajectory. That is, the trade-off solution well balances the objectives of the flight time and the fuel consumption. The results of several trade-off solutions from AMS to JNB are shown in table 5.3.

Table 5.3. The results of several trade-off solution of route from AMS to JNB.

Fuel consumption (kg)	Compare to the minimum time solution (%)	Compare to the minimum fuel solution (%)	Flight time (hours)	Compare to the minimum time solution (%)	Compare to the minimum fuel solution (%)
95229.0	78.0	118.7	9.2	101.5	92.3
96763.7	79.3	120.6	9.1	100.5	91.4
97311.9	79.7	121.3	9.1	100.4	91.3
98349.5	80.6	122.6	9.1	100.3	91.2

5.2.2 JNB to AMS (8.31)

For flights with the opposite direction, the ground track nearly overlapped the great circle path. Figure 5.29 indicates the ground track of the minimum time solution from JNB to AMS.

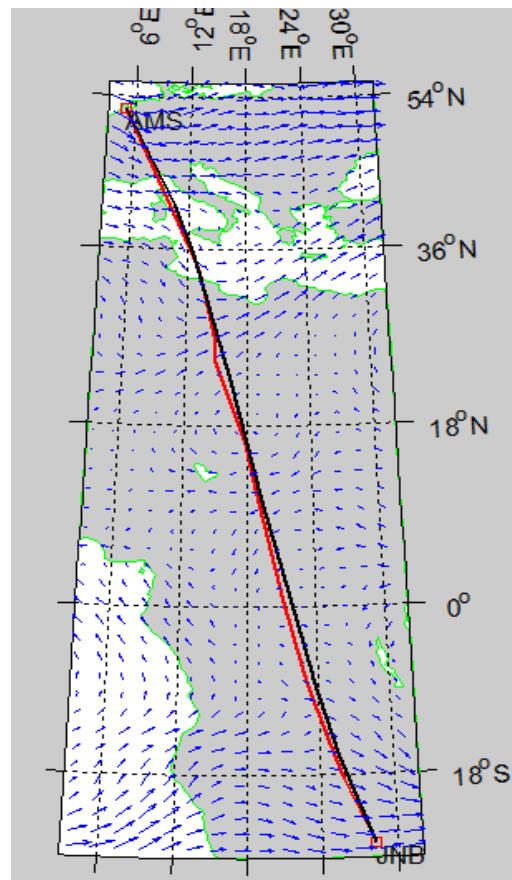


Figure 5.29. The ground track of the minimum time solution of the route from JNB to AMS.

Different from the solution of the route from AMS to JNB, the route from JNB to AMS is with headwinds in the areas near both two cities. The strong wind fields in these are the polar jet streams discussed in Chapter 2. Moreover, there are not many options for the optimizer to adjust the heading of the aircraft. Therefore, the optimizer chooses the shortest route to minimize the flight distance. Since the trajectory is almost the same as the great circle path, the flight time and the fuel consumption are also similar to each other. The flight time and the fuel consumption of this trajectory are 34,195.3 seconds (9.5 hours) and 146,704kg, respectively. The flight distance of this trajectory is 9,022 km. For the minimum fuel solution, the detailed analysis has not been provided in this report because the ground track is similar to the minimum time solution from JNB to AMS, while the vertical profile and airspeeds are similar to the minimum fuel solution from AMS to JNB.

5.3 AMS-SIN

For the routes between AMS and SIN, the expected final weight is set as 210,000 kg because larger payload may cause many infeasible solutions. The direct flight distance is 10,516 km (5,678 nm), which is near to the maximum flight range of the B747-400 aircraft model applied in this study. Therefore, it is also interesting to identify the near optimal solution for this extremely long route. Since the direct distance is already closed to the maximum feasible flight range, some infeasible solutions may be evaluated in this case. The number of the evaluated horizontal segments and the vertical segments are 9 and 8, respectively. The number of the evaluated control parameters is 15 more than previous cases in Phase 2 and equals as 48. Consequently, the computational time of this case is larger than the previous cases.

5.3.1 AMS to SIN (8.31)

Figure 5.30 shows the final generation of the route from AMS to SIN. The minimum flight time of the flights is approximately 36,100 seconds (10 hours) and the minimum fuel consumption is about 91,000 kg. Compared to the minimum time solution, the trade-off solutions can spend around 1,000 seconds (16.6 minutes) to save 2,000 kg fuel consumption. On the other hand, the trade-off solutions can save 2,500 seconds (42 minutes) with about 7,000 kg fuel as the extra cost by setting the minimum fuel solution as the reference.

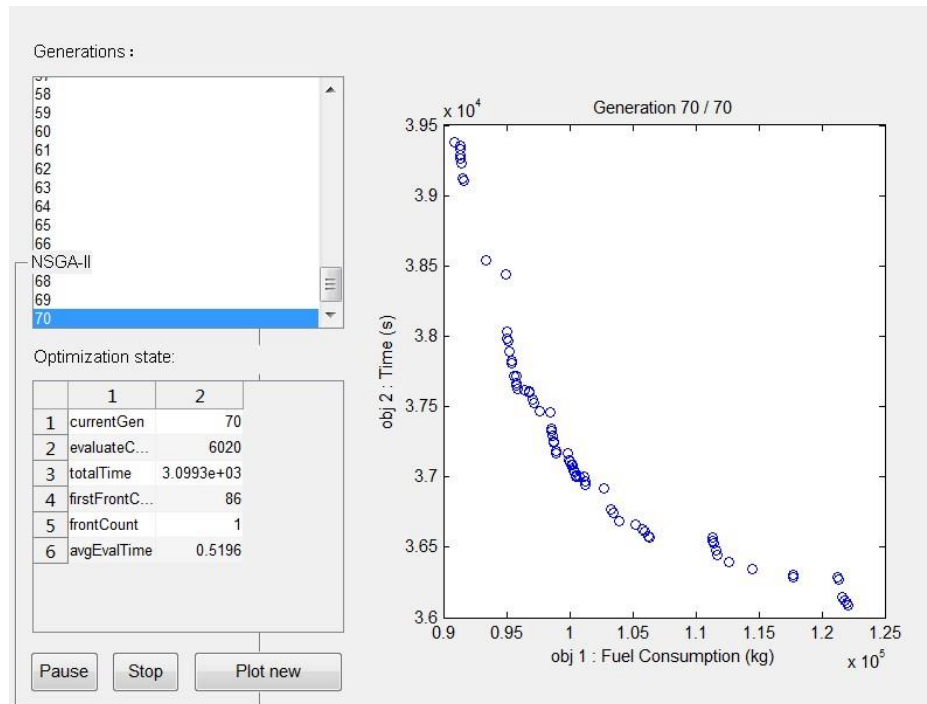


Figure 5.30. The final generation of the route from AMS to SIN.

The figure below shows the horizontal profile of the minimum fuel consumption solution from AMS to SIN.

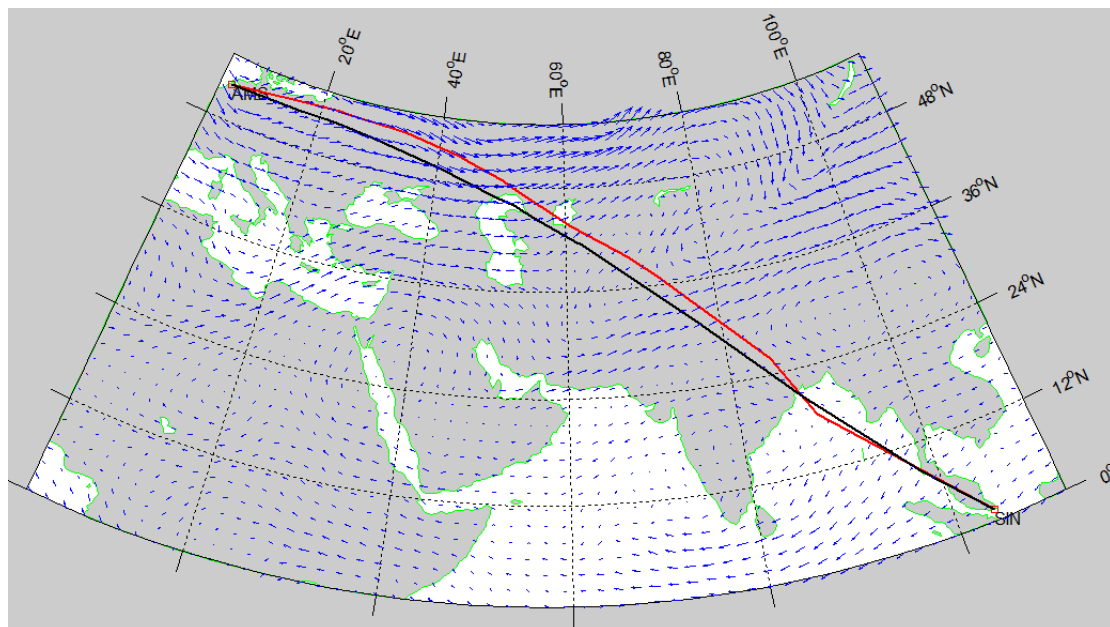


Figure 5.31. The ground track of the minimum time solution from AMS to SIN.

According to the ground track in figure 5.31, the aircraft flies more easterly than the great circle path to gain the benefit from the eastwards wind after taking off at AMS. After that, the trajectory is parallel and gradually returns to the great circle path until

coinciding with it at the end of the trajectory at SIN. Figure 5.32 shows the vertical profile of this trajectory. It can be clearly seen that the aircraft has a strong tailwind at FL320 and FL340. In the second half of the trajectory, there is a headwind against the aircraft, but the strength of this headwind is relatively weak.

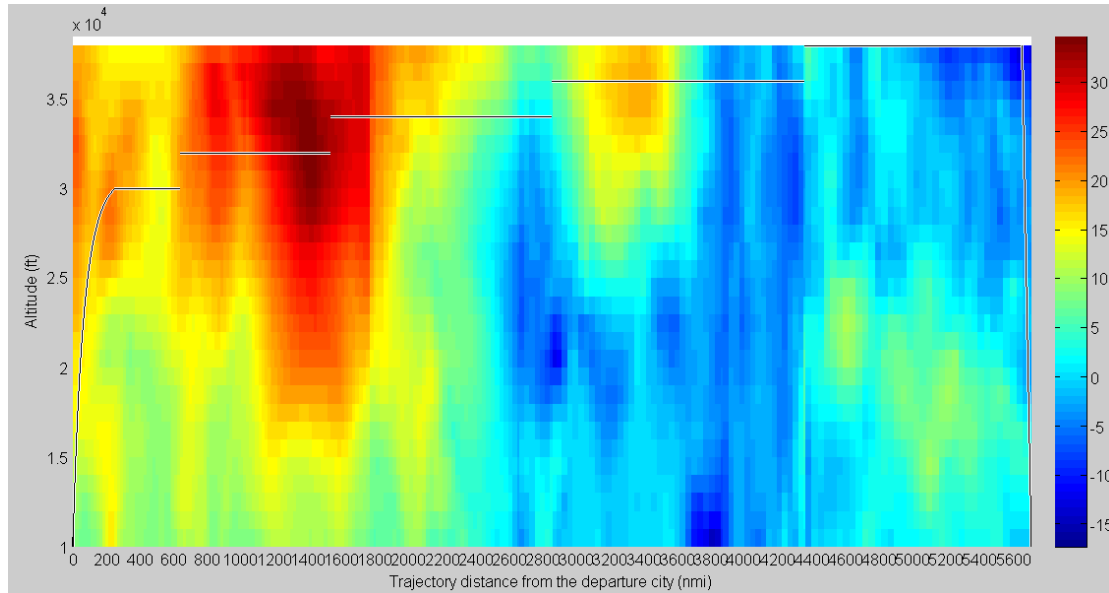


Figure 5.32. The vertical profile of the minimum fuel solution from AMS to SIN.

The fuel consumption of the minimum fuel solution in this case is 90,912.1 kg with 10.9 hours, and the flight distance is 10,553 km (5,698.1 nm), which are 4,248.9 kg (4.5%) and 30 minutes (4.4%) lower than the minimum fuel consumption trajectory along the great circle path with a 37 km larger flight distance.

5.3.2 SIN to AMS (8.31)

As shown in figure 5.31, there is a strong west wind along the trajectory from $48^{\circ}N$ to $54^{\circ}N$. For the route from SIN to AMS, the impact from this west wind is significant. In other words, more infeasible solutions may be evaluated since the flight from SIN to AMS cost more fuel due to the resistance of this eastwards wind on the aircraft. As shown in figure 5.33, the fuel consumption of the solutions in the final generation can be as large as 190000 kg, which will result in an infeasible take-off weight with 36,244 kg payload because the MTOW of the aircraft model is 362,874 kg (Teengs, M., 2006). Moreover, the computational time of Phase 2 is 71 minutes in this case compared to 51.6 minutes for flights from AMS to SIN. In this report, the feasible minimum fuel consumption solution instead of the optimized minimum fuel consumption solution has been analysed.

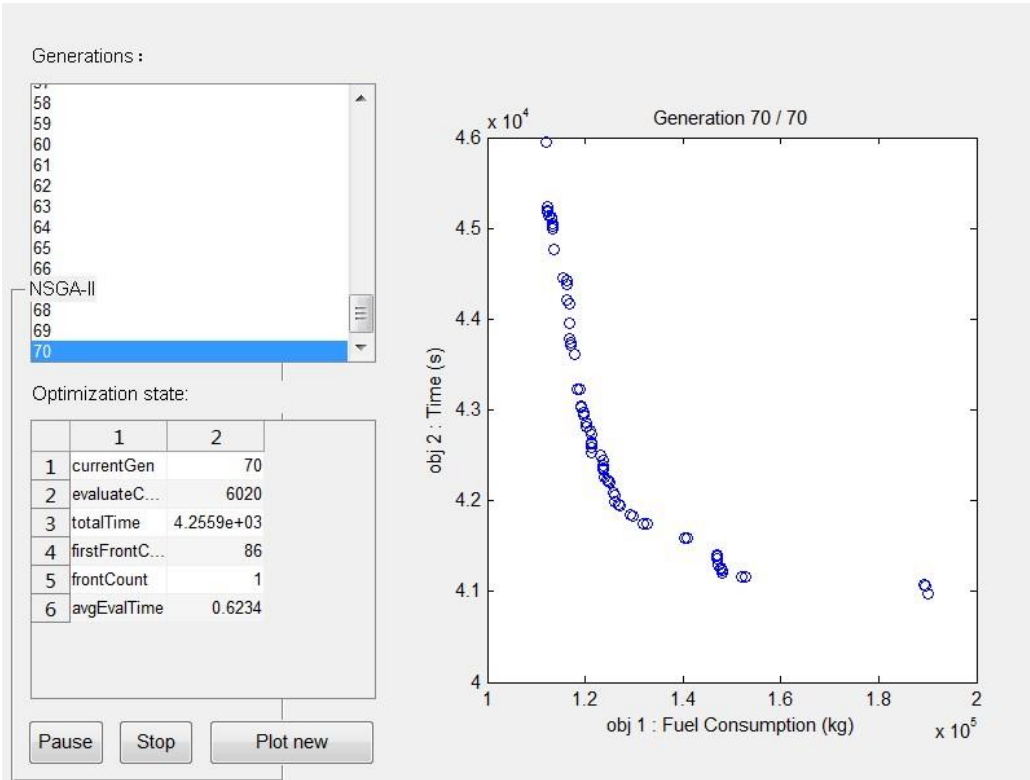


Figure 5.33. The last generation of the route from SIN to AMS.

Since the fuel preference solution is a feasible solution with similar profile as the minimum fuel consumption solution in the previous cases, only the feasible minimum time solution will be discussed in this report. The ground track of the feasible minimum time solution is almost the same as the ground track from AMS to SIN. Hence the horizontal profile is not shown in this report. Figure 5.34 shows the vertical profile of the feasible minimum time solution.

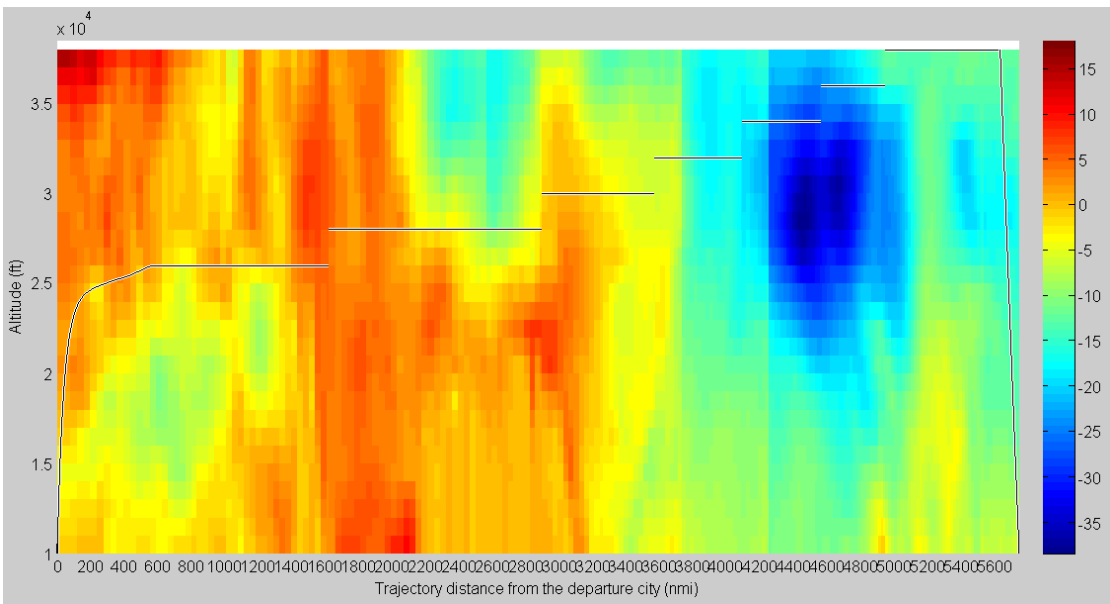


Figure 5.34. The vertical profile of the feasible minimum time solution from SIN to AMS.

Similar to the trade-off solutions of the route between AMS and JFK, AMS and JNB, the altitude of the trajectory in figure 5.34 constantly increases to the ceiling of the vertical limitation. The purpose of this climbing is to increase the fuel efficiency. As discussed in chapter 3, the influence of the altitude to the fuel efficiency is significant. The increased altitude will lead to lower fuel consumption for the trajectory. That is, a feasible take-off weight is obtained. Figure 5.35 indicates the airspeeds of this solution.

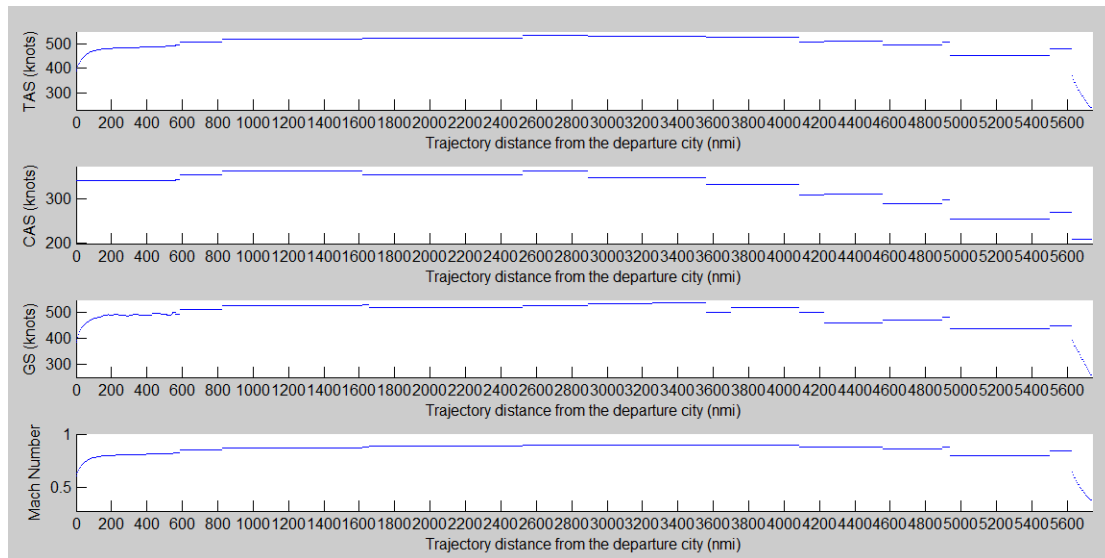


Figure 5.35. The airspeeds diagram of the feasible minimum time solution from SIN to AMS.

Based on figure 5.35, the airspeeds of the trajectory are the maximum feasible airspeeds. As shown in table 2.1, the differences found in the specific range for different Mach numbers at the same altitude are smaller compare to the changing of the altitude. That is, the fuel efficiency is barely affected by the airspeed changes.

6. Conclusion & Recommendations

In this report, research is done on identifying the near optimal solutions within a wind fields taking into account a set of ATC constraints. The developed tool based on NSGA shows a good performance to solve the trajectory optimization problem. In order to overcome the disadvantages of NSGA, a 2-phase approach and some algorithms based on the parameterization are implemented. The outcomes of the tool are better trajectories in wind fields compare to the trajectories with the great circle path (the direct path) and the optimized vertical profile. The trade-off solutions provide extra options for the users to choose the most suitable trajectory based on the priority of users' objectives. By applying the discretized process, the optimizer of NSGA is able to optimize the discretized flight levels efficiently. In fact, not only for the trajectory optimization problem in this study, the discretized process of NSGA can also be applied to other problems.

A parameterization process has been developed and implemented to establish the parameter bounds such that less solutions outside the feasible range are evaluated, and hence that the computational efficiency is greatly improved. For relatively short distance flights, there is no infeasible solution evaluated during the optimization process. For long distance flights, infeasible solutions are generated because the direct distances of such routes are near to the feasible flight ranges of the used aircraft model. The evaluation of the infeasible solutions is caused by the computational error of the flat earth model applied in this study. If a spherical earth model which is able to be solved inversely in a short time is developed. The infeasible solutions are eliminated during the optimization process. The other algorithm applied in this study to minimize the computational time is the equivalent weight and the equivalent wind concepts in Phase 1. The distance step of this algorithm is able to expand to a whole segment distance. By applying the parameterization and the equivalent concept, the total computational time for the optimization process is relatively shortened. The computational time for the route between AMS and JFK becomes approximately 40 minutes. For the large distance routes (AMS-SIN), the computational time is longer around 1 hour. In most cases, the longer evaluation time for large distance routes is caused by the larger number of evaluated segments and parameters. Moreover, the larger distance results in a large span of fuel consumption. The larger fuel consumption span leads to a larger number of evaluations during the iterative process for each individual.

For the climbing phase and the descent phase, the time of the climbing phase is longer than the time of the descent phase because the weight at the end of the cruise phase is much lower than the weight in the beginning of the trajectory.

The outcomes of the optimizer have some variations compared to the expected solutions. The altitude of the minimum time solution does not stay at the crossover flight level because the maximum airspeeds are insensitive to the altitude changes. The decreased slopes of VMO and MMO against to the altitude are small. In terms of the fuel efficiency, the fuel consumption model developed in this study is more sensitive to the altitude changes than the airspeeds.

The verification results in this report prove that the developed tool is efficient and qualify. The tool is able to identify the great circle paths without wind effects. In the test with simple wind fields, the optimizer can distinguish the tailwind and headwind and takes right decisions.

In terms of investigated case studies, the optimized solutions have a better performance in terms of both flight time and fuel consumption. The average saving in terms of flight time and fuel consumption of the optimized solutions are 3.16% and 3.1% compared to the optimized solutions with the great circle ground track. The flight levels of the minimum time solutions vary around the crossover flight level and the airspeeds are at the top of the feasible range. The flight altitudes of the minimum fuel solutions also increase from the departure city to the arrival city. The airspeeds of the minimum fuel solutions are maintained in the range between 0.75 and 0.85 Mach number.

The following recommendations have been proposed.

The optimization processes discussed in this study were only performed for one type of aircraft. More aircraft models can be analysed to further prove the performance of the designed tool. Moreover, the results of the aircraft model are not realistic. An elaborate aircraft model is recommended to further prove the performance of the designed tool.

The current program assumes a standard atmosphere. As the pressure and temperature are of great influence on both the air density which further impacts the performance of the engine. A more elaborate atmospheric model could be introduced to obtain more accurate solutions.

The tool designed in this study is based on Matlab. With more efficient software such as C++, the computational time can be dramatically decreased. Additionally, the use of a better equipped computer with multi processors will significantly increase the computational efficiency.

In addition, well-designed termination criterions are recommended, which will increase the computational efficiency the accuracy of the outcomes of the

optimization process.

Last but not the least, a simple algorithm to calculate the distance between two points on the earth based on the spherical earth model is suggested to be developed in order to fully eliminate infeasible solutions in the optimization process.

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Appendix A. Flow chart of the iterative algorithm

