

MIMO system identification for ship motion prediction

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Master of Science Thesis



MIMO system identification for ship motion prediction

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Abstract

Accurate ship motion prediction is crucial for safe and efficient operations at sea. Previously studied Single Input Single Output (SISO) system-based method often falls short in providing robust and precise predictions. This thesis explores the potential of using a Multiple Inputs Multiple Outputs (MIMO) system identification approach to provide more accurate and robust ship motion prediction in offshore operations.

We investigate two MIMO systems, the force-to-motion system and wave-to-motion system, employing Spectral Analysis method and Subspace method, respectively. Both synthetic and real-field data are utilized to study the methods. While Spectral Analysis method demonstrates better accuracy, it relies on the assumption of perfect accuracy of the pre-computed force Response Amplitude Operators (RAOs), which is a limitation of this approach. Subspace method is less precise, it also requires real-time re-evaluation of the system order due to its sensitivity to wave conditions, making this method less versatile.

Comparing these methods with SISO system method and conventional RAO-based approach, Spectral Analysis method emerged as the most accurate, followed by the Subspace method. These findings strongly support the potential of MIMO system identification to significantly improve ship motion prediction accuracy.

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Chapter 1

Introduction

Since the late 20th century, radar systems have emerged as a valuable tool for measuring ocean surface waves. Analysis techniques have been developed to extract directional wave spectrum from the radar images [2], providing information including wave propagation, the wavelengths and the periods. These information is crucial in accurately predicting wave behavior [25]. Accurate wave prediction allows precise motion prediction, which is essential for the safety and efficiency of offshore operations, illustrated in Figure 1-1.

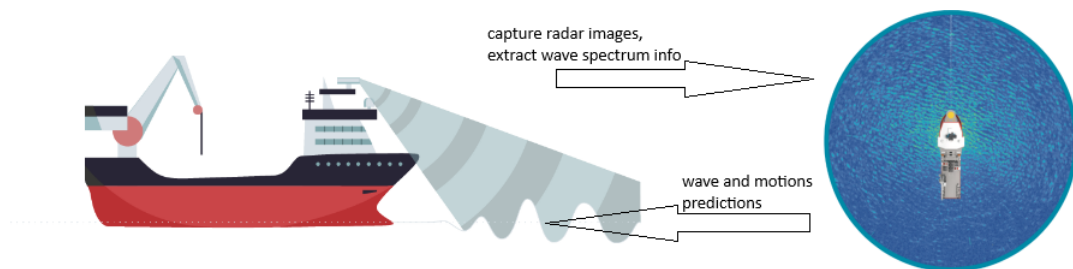


Figure 1-1: Wave and motion prediction from on-board radar

The growing complexity of offshore operations increasingly demand accurate vessel motion prediction. For instance, transferring personnel, equipment, or cargo between vessels or platforms requires precise prediction of ship motion to minimize swing and ensure safety. More accurate predictions of ship motion allow the helicopter pilot to anticipate the movement of the landing platform, minimizing the risk of a collision between the helicopter and the ship during landing [7]. Many more operations could be placed in this list, these operation necessitates more accurate predictions of wave-induced motions to ensure safety and efficiency.

To address the increasing demands of complex offshore operations, this thesis investigates methods to improve the accuracy of motion prediction. In collaboration with Next Ocean B.V., a spin-off from Delft University of Technology, this research aims to enhance the motion prediction algorithm within the WaveAnalyzer and WavePredictor software. These tools

leverage radar technology to observe surrounding ocean waves and convert radar images into wave and motion predictions.

1-1 Motion prediction methods and previous work

Traditionally Response Amplitude Operators (RAOs) have been used to map predicted wave components directly to future ship motions [15]. However, these RAOs are often provided by shipyards and may not account for real-world uncertainties, such as uneven mass distribution or nonlinearities in ship behavior [1]. These limitations can lead to inaccurate motion predictions.

To address this challenge, recent research has explored methods leveraging system identification for motion prediction. System identification is a general technique for building mathematical model of a system from input and output data. The predicted motion is then computed using this model, instead of using the given inaccurate motion RAOs [5].

In this thesis two systems are of interest. One of them is wave-to-motion system, which has the wave components acting on ship body as the system input and induced ship motions as the system output. Another system being considered is the force-to-motion system which maps the wave excitation forces to the ship motions. The systems were first estimated using historical data of waves or forces and ship motions. Subsequently, the estimated system model and predicted future wave or forces are employed to compute the future ship motions.

Motion prediction predict the Six Degrees-of-Freedom (6DoF) displacement of the ship a few minutes into the future. The motion prediction timeline diagram is presented in Figure 1-2. At time t_0 , the history data of waves or forces predictions from previous time window t_{-1} to t_0 and the actual measured ship motion data from t_{-1} to t_0 are used as training data for system identification. At the same time, the predictions for waves or forces are made for a future time window from t_0 to t_1 . The estimated system model, along with the newly made wave/force predictions for time window t_0 to t_1 are used to predict the ship's motion for the same time window. The exact length of the time intervals will be given in later chapters where the system identification techniques are discussed.

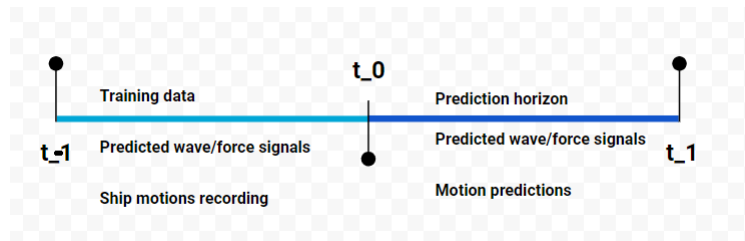


Figure 1-2: Motion prediction timeline

Prior research [5] primarily focused on estimating a Single Input Single Output (SISO) force-to-motion system for each of the 6DoF forces and motions. This approach assumes that motion in one direction is solely influenced by the force acting in the same direction. However in reality the ship motions exhibits coupling, where forces from one direction can influence

motions in other directions. To capture this coupling effect and enhance motion prediction accuracy, our research proposes a Multiple Inputs Multiple Outputs (MIMO) system identification approach.

However the force-to-motion approach relies on the assumption that the input forces are perfectly computed from wave components and force RAOs. While the force RAOs generally exhibit less uncertainty compared to motion RAOs, their inherent limitations can still introduce inaccuracies into the prediction process.

To study the severity of this limitation, our research also proposes the wave-to-motion system. This system bypasses the uncertainties associated with RAOs by directly utilizing actual wave data as input. By leveraging the fundamental characteristics of the wave field, the wave-to-motion system has the potential to achieve more accurate and robust motion predictions, especially in scenarios where RAO-based force computations might be less reliable.

1-2 Research objectives

This thesis explores two MIMO systems for ship motion prediction: the force-to-motion system, denoted G_1 , and the wave-to-motion system, denoted as G_2 (see Figure 1-3).

For system G_1 , the input signals are the wave excitation forces for each Degrees-of-Freedom (DoF), the outputs are motions from 6DOF. For wave-to-motion system G_2 the input signals are the wave components from various directions, where each direction is treated as a separate input.

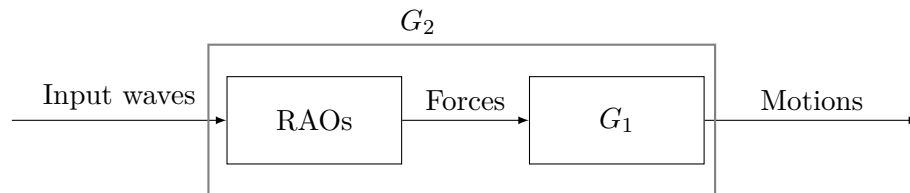


Figure 1-3: System models

We propose estimating the wave excitation force-to-motion system as a MIMO system, incorporating motion coupling effects into the system model, using Spectral Analysis techniques. In this context, we can define our first thesis objective:

1. *To investigate whether motion prediction can be improved by estimating MIMO wave excitation force-to-motion system employing MIMO Spectral Analysis system identification technique.*

To achieve this objective, we will investigate the following sub-objectives:

- To investigate whether the force-to-motion system can be more accurately modeled as MIMO systems than SISO systems.

- Implement Spectral Analysis method identifying the MIMO system.
- Compare the motion prediction results from the MIMO system to that from the SISO system.

The wave-to-motion system approach is an alternative approach to address the limitation from the force-to-motion system approach. We can define our second objective of the thesis:

2. To investigate whether motion prediction can be improved by estimating the wave-to-motion system employing MIMO Subspace identification technique.

To achieve this objective, we will investigate the following sub-objectives:

- To investigate whether wave-to-motion presentation could achieve more accurate motion prediction than the force-to-motion system.
- Implement subspace system identification technique to identify the wave-to-motion system.
- Compare the motion prediction results with the force-to-motion system approach results.

Chapter 2

Related Work

This chapter lays the foundation for our research by providing a comprehensive review of the background theories and previous studies on motion prediction. This review equips readers with understanding of the relevant theoretical concepts and the current challenges faced in this field. We will summarize the past research for SISO system identification, highlighting their key findings and limitations. This overview serves as a starting point for understanding the motivation behind our proposed MIMO system approach.

2-1 System identification and motion prediction

Just as for any other rigid body, the motions of a ship can be classified as three mutually perpendicular translational motions of the Center of Gravity (CoG) and three rotational motions around CoG [12].

These motions are described in the Steadily Translating coordinate system $O_b(x_b, y_b, z_b)$, as in Figure 2-1. The (x_b, y_b) -plane lies in the still water surface with the origin O_b at the time-averaged position of the CoG of the ship. The direction of the positive x_b -axis is in the forward direction, the positive y_b -axis points in the lateral port (left hand) side and z_b points upwards [12]. The six degree of motions are motions along or around x_b , y_b and z_b axes. The 6DoF motions are surge, sway, heave, roll, pitch and yaw, as illustrated in Figure 2-1, the corresponding index of each DoF is given in table 2-1.

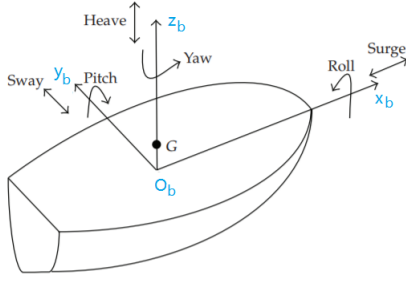


Figure 2-1: Ship schematic diagram showing the six degrees of freedom [12]

DoF	Index
Surge	1
Sway	2
Heave	3
Roll	4
Pitch	5
Yaw	6

Table 2-1: DoF index

2-1-1 System identification

The true system transfer function at each frequency ω is denoted as $G(\omega)$, which can be represented in the time domain as an impulse response or state space model. For simplicity, we will only discuss the frequency domain representation in this section. The aim of system identification is to systematically extract a mathematical model $\hat{G}$ from the sets of input and output data. The model $\hat{G}$ has a certain structure, which can be parameterized by a set of parameter vector θ . The parameter θ can be in the form of system matrices or transfer functions [24].

The estimated model and the true system hold the following relationship [24]

$$\hat{G}(\theta, \omega) = G(\omega) + \epsilon(\omega), \quad (2-1)$$

in which ϵ is the system error. The goal of system identification is to select the best model $\hat{G}$ that ensures the error ϵ between the estimated system $\hat{G}$ and the true system G is minimized.

2-2 Force-to-Motion System

This section is based on the book Offshore Hydromechanics[12], the citation within the section is omitted until stated otherwise.

2-2-1 SISO force-to-motion system model

Prior study [5] considered all the motions are decoupled from each other, which means a motion from i_{th} direction only has influence from the force from i_{th} ($i = 1, 2, \dots, 6$) direction. The equation of motion of i_{th} direction is written as

$$m_i \ddot{x}_i = f_i, \quad (2-2)$$

where m is the mass or inertia of the ship, x_i is the displacement on i_{th} degree of freedom, f_i is total force from i_{th} direction acting on the vessel.

The total force can be split into the wave excitation forces $f_{i_{wave}}$, the hydrostatic forces $f_{i_{hydrostatic}}$ and reaction forces $f_{i_{reaction}}$ from the water:

$$f_i = f_{i_{wave}} + f_{i_{hydrostatic}} + f_{i_{reaction}}. \quad (2-3)$$

The hydrostatic force is a function of displacement of the vessel, expressed as $f_{i_{hydrostatic}} = c_i x_i$, where c_i is the restoring spring coefficient. The hydrodynamic reaction forces $f_{i_{reaction}}$ is the forces experienced on the moving body in the absence of the waves and is a function of its velocity and acceleration, $f_{i_{reaction}} = a_i \ddot{x}_i + b_i \dot{x}_i$, in which a_i is the added mass coefficient and b_i is the radiation damping coefficient. Substitute these expressions into equation 2-2 and equation 2-3 the motion equation can be rewrite as

$$(m_i - a_i) \ddot{x}_i - b_i \dot{x}_i - c_i x_i = f_{i_{wave}}. \quad (2-4)$$

For each direction i of 6DoF, the transfer function of the SISO force-to-motion system is expressed as

$$G_i(e^{j\omega}) = \frac{X_i(e^{j\omega})}{F_{i_{wave}}(e^{j\omega})} \quad (2-5)$$

In previous study the force-to-motion transfer function for i_{th} direction was estimated using Spectral Analysis method:

$$\hat{G}_i(e^{j\omega}) = \frac{\hat{S}_{y_i u_i}(e^{j\omega})}{\hat{S}_{u_i u_i}(e^{j\omega})}, \quad (2-6)$$

where $\hat{S}_{y_i u_i}(e^{j\omega})$ and $\hat{S}_{u_i u_i}(e^{j\omega})$ are the cross and auto Power Spectral Density (PSD) of the input and output signals on i^{th} direction. The PSDs were estimated using equations Welch's method.

2-2-2 MIMO system equation of motion

To include the motion coupling into system model, we will write out the equation of motion for 6DoF,

$$(\mathbf{M} + \mathbf{A})\ddot{\vec{x}} + \mathbf{B}\dot{\vec{x}} + \mathbf{C}\vec{x} = \vec{f}_{wave}, \quad (2-7)$$

where $\vec{x}$ is the vector of displacement in the six degree of freedom, $\vec{f}_{wave}$ is the vector of the wave excitation forces and moments acting on the ship body. Matrices $\mathbf{M}$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C} \in \mathbb{C}^{6 \times 6}$, are the mass/inertia matrix, the added mass coefficient matrix, the radiation damping coefficient matrix and restoring spring coefficient matrix, respectively (bold and capital letter to emphasis that they are matrices).

2-2-3 Motion coupling

In the most general case of a floating object, all entries in the system matrices have nonzero values. The off-diagonal terms in the matrices are referred to as coupling terms, they couple

motion in one direction to forces in any other directions. In a randomly shaped floating object, a coupling between a motion is coupled to forces in all other directions. When axes of symmetry are present in the vessel's underwater hull shape, couplings between certain DOF's are lost.

Ships tend to be symmetrical in the x-z plane. This removes half of the couplings and creates two sets of coupled motions, namely surge-heave-pitch (index 1-3-5) and sway-roll-yaw (index 2-4-6). No coupling between these two sets is present. In this application we consider ships exhibit such a symmetrical shape, which means the matrices $\mathbf{M}$, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ have the structures as

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} a_{11} & 0 & a_{13} & 0 & a_{15} & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 & a_{26} \\ a_{31} & 0 & a_{33} & 0 & a_{35} & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & a_{46} \\ a_{51} & 0 & a_{53} & 0 & a_{55} & 0 \\ 0 & a_{62} & 0 & a_{64} & 0 & a_{66} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & 0 & b_{13} & 0 & b_{15} & 0 \\ 0 & b_{22} & 0 & b_{24} & 0 & b_{26} \\ b_{31} & 0 & b_{33} & 0 & b_{35} & 0 \\ 0 & b_{42} & 0 & b_{44} & 0 & b_{46} \\ b_{51} & 0 & b_{53} & 0 & b_{55} & 0 \\ 0 & b_{62} & 0 & b_{64} & 0 & b_{66} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & c_{35} & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & c_{53} & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

These two sets of motions are called symmetric (index 1, 3, and 5) and anti-symmetric motions (index 2, 4 and 6), respectively. We can re-write the system matrices

$$\text{for the symmetrical system, } \mathbf{M}_{\text{sym}} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_{yy} \end{bmatrix},$$

$$\mathbf{A}_{\text{sym}} = \begin{bmatrix} a_{11} & a_{13} & a_{15} \\ a_{31} & a_{33} & a_{35} \\ a_{51} & a_{53} & a_{55} \end{bmatrix}, \quad \mathbf{B}_{\text{sym}} = \begin{bmatrix} b_{11} & b_{13} & b_{15} \\ b_{31} & b_{33} & b_{35} \\ b_{51} & b_{53} & b_{55} \end{bmatrix}, \quad \mathbf{C}_{\text{sym}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_{33} & c_{35} \\ 0 & c_{53} & c_{55} \end{bmatrix}.$$

$$\text{for asymmetrical motions, } \mathbf{M}_{\text{asym}} = \begin{bmatrix} m & 0 & 0 \\ 0 & I_{xx} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix},$$

$$\mathbf{A}_{\text{asym}} = \begin{bmatrix} a_{22} & a_{24} & a_{26} \\ a_{42} & a_{44} & a_{46} \\ a_{62} & a_{64} & a_{66} \end{bmatrix}, \quad \mathbf{B}_{\text{asym}} = \begin{bmatrix} b_{22} & b_{24} & b_{26} \\ b_{42} & b_{44} & b_{46} \\ b_{62} & b_{64} & b_{66} \end{bmatrix}, \quad \mathbf{C}_{\text{asym}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_{44} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Previous studies often simplified the system by considering only the diagonal elements of the system matrices, effectively reducing the MIMO system to a set of uncoupled SISO systems. This thesis aims to investigate the impact of incorporating off-diagonal elements on motion prediction accuracy by comparing MIMO and SISO system models.

Spectral Analysis Method Identifying Force-to-Motion System

In this chapter we will present details of the Spectral Analysis method for identifying the MIMO force-to-motion system. We will implement and test the Spectral Analysis MIMO algorithm using synthetic data.

We have introduced the symmetrical system and asymmetrical system in section 2-2-3, the transfer functions will be denoted as G_{sym} and G_{asym} , respectively. Both systems can be identified using the same technique, therefore to simplify the notation, we will not make differentiation between the two system notations and refer to both as system G in the rest of this chapter.

3-1 Force-to-motion system

3-1-1 Input and Output signals

To have flexibility in testing our method under various sea conditions, we generated synthetic input forces and output motions. This approach allows us to simulate a wide range of scenarios without relying on limited field measurement data. The synthetic forces and motions are generated by multiplying synthetic wave components with corresponding force and motion RAOs, respectively.

These synthetic wave components are computed from the well-known JONSWAP wave spectrum, which is widely used for its realistic representation of waves on sea surface [23]. Figure 3-1 is a realization of the Jonswap wave spectrum. The magnitude of the Fourier coefficients of a Jonswap wave spectrum indicates the relative contribution of each frequency to the overall wave field.

In practice we rarely encounter cases where the signals are free of noise. Therefore, assessing the robustness of the identification method against noise is a crucial step. This robustness property reveals the method's sensitivity to noisy data.

In this application, noise mainly enters the system as measurement errors in the force signals. We have introduced earlier that the excitation forces are calculated from the wave components and force RAOs. Any inaccuracies or errors in the wave spectrum or for RAOs will be reflected in the force signals. On the other hand, motion signals are acquired from on-board motion sensors and are filtered through a low-pass filter before being used, allowing them to be considered noise-free in this application (information provided by NextOcean B.V.).

When generating synthetic waves, we modeled the measurement error as a sequence of complex white Gaussian noise added to the Fourier coefficients of the wave components. Unfortunately, any statistical properties of the measurement errors that occur in reality are unknown, so we chose the signal-to-noise ratio to be $20dB$ as the worst case. Signal to Noise Ratio (SNR) is calculated using formula $SNR = 20 \log\left(\frac{\text{power signal}}{\text{power noise}}\right)$.

To simulate specific wave propagating directions, we assigned direction to the wave component on the spectrum. Real world seas rarely have all waves traveling in the same direction, there's typically a dominant propagating direction with some spreading of wave components around it. To capture this wave spreading effect, we assigned a random propagation direction to each wave component. This randomness was modeled by a normal distribution, with the main propagating direction as the mean (center) and a chosen standard deviation representing the degree of spreading. A higher standard deviation signifies a wider spread of wave directions. Figure 3-1 shows the histogram of wave spreading we generated, which is representative to the spreading exist on the sea surface.

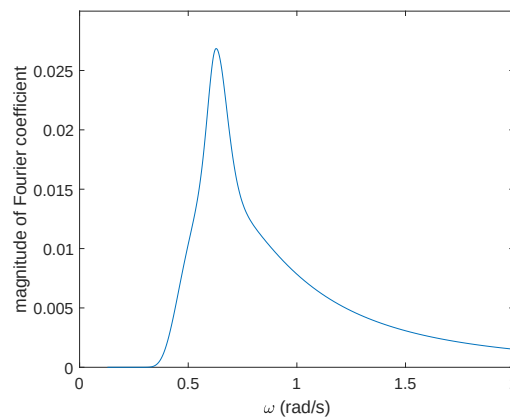


Figure 3-1: Jonswap wave spectrum

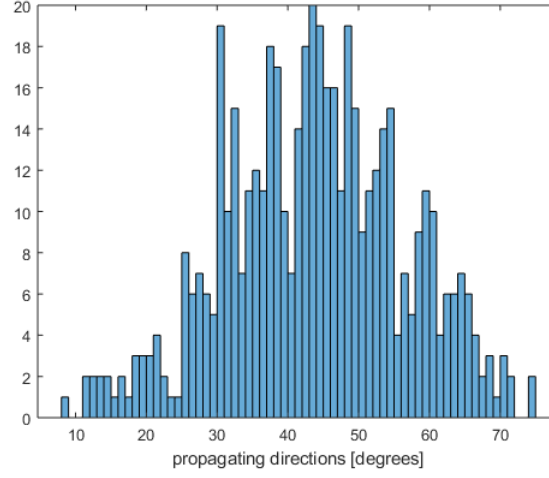


Figure 3-2: Histogram of the wave components propagating directions

The time series of the forces and motions can be computed from the force and motion spectrum. To study the system identification method, we will generate a training set of forces and motions, denoted as $(\vec{u}_i(k), \vec{y}_i(k))$, to identify the system, and a validation set of waves and motions, denoted as $(\vec{u}_v(k), \vec{y}_v(k))$, to validate the estimated system model.

3-2 Spectral analysis method

To evaluate the accuracy of the estimated system model $\hat{G}$ and the quality of motion prediction, the variance accounted for (VAF) is used as a quality measure. The VAF metric calculates the accumulated difference between the validation output $\vec{y}_v(k)$ and the predicted output $\hat{\vec{y}}_v(k)$. The predicted output $\hat{\vec{y}}_v(k)$ is computed from estimated $\hat{G}$ and validation input signals $\vec{u}_v(k)$. To calculate the VAF for each DoF, the following formula is used [24]:

$$\text{VAF}_i = \max(0, 1 - \frac{\frac{1}{N} \sum_{k=1}^N \|\vec{y}_{vi}(k) - \hat{\vec{y}}_i(k)\|_2^2}{\frac{1}{N} \sum_{k=1}^N \|\vec{y}_{vi}(k)\|_2^2}), \quad (3-1)$$

where i denotes the index of the DoF. A VAF value of 1 means that the estimated output matches the true output signals, indicating a perfect estimated system model. Since typical motions are predicted a few minutes into the future, in this thesis we use 3 minutes of data to validate the system model.

3-2-1 Basic principle

The Spectral Analysis Method is a frequency-domain approach in which the Power Spectral Density (PSD) and Cross Spectral Density (CSD) of signals are used to identify the system model. PSD describes the power distribution of a signal across the frequency spectrum and CSD shows the distribution of power between two signals over the frequency spectrum [16].

At each frequency ω , the system transfer function $G(e^{j\omega}) \in \mathbb{C}^{3 \times 3}$ can be derived by the

PSD and CSD of the input and output signals using [16]

$$G(e^{j\omega})S_{uu}(e^{j\omega}) = S_{yu}(e^{j\omega}), \quad (3-2)$$

where $S_{uu}(e^{j\omega}) \in \mathbb{C}^{3 \times 3}$ is the PSD of the input forces and $S_{yu}(e^{j\omega}) \in \mathbb{C}^{3 \times 3}$ is the cross spectral density matrix of the input and output signals. By rearranging the above equation, the transfer function $G(e^{j\omega})$ can be expressed as

$$G(e^{j\omega}) = S_{yu}(e^{j\omega})S_{uu}^{-1}(e^{j\omega}), \quad (3-3)$$

assuming $S_{uu}(e^{j\omega})$ is invertible (the invertibility of the matrix will be discussed in Section 3-2-3).

In practice, however, the matrices $S_{yu}(e^{j\omega})$ and $S_{uu}(e^{j\omega})$ need to be estimated. These estimates are denoted as $\hat{S}_{yu}(e^{j\omega})$ and $\hat{S}_{uu}(e^{j\omega})$ respectively. The derivation of the estimations will be covered in detail in Section 3-2-2. By substituting the estimated PSD and CSD into Equation 3-3, we obtain the estimated system transfer function [16],

$$\hat{G}(e^{j\omega}) = \hat{S}_{yu}(e^{j\omega})\hat{S}_{uu}^{-1}(e^{j\omega}). \quad (3-4)$$

This equation is the fundamental equation of the spectral analysis method. In the following sections, we will delve into the specifics of the operations involved in this equation.

3-2-2 PSD estimation

We use Welch's method to estimate the power spectral densities, which asymptotically yields an unbiased and consistent estimator of the PSD and CSD [14]. The method involves dividing the total sample of size N into M segments, each with a length of n and overlapping by L . A window function is applied to each overlapping segment, and each windowed segment is then transformed into the frequency domain using the discrete Fourier transform (DFT) operation [16],

$$\vec{U}_w(e^{j\omega}) = \frac{1}{W} \sum_{k=0}^{n-1} \vec{u}(k)w(k)e^{j\omega k}, \quad (3-5)$$

$$\vec{Y}_w(e^{j\omega}) = \frac{1}{W} \sum_{k=0}^{n-1} \vec{y}(k)w(k)e^{j\omega k}, \quad (3-6)$$

$$W = \sum_{k=0}^{n-1} w(k), \quad (3-7)$$

where k is the sample index ($k = 0, 1, \dots, n-1$), $\vec{U}_w(e^{j\omega})$ and $\vec{Y}_w(e^{j\omega})$ are the Fourier transform of the windowed input and output signals respectively, $w(k)$ is the window function [21]. The purpose and selection of the window function will be illustrated later in this section.

The estimation of the PSD and CSD are given as [14]:

$$\hat{S}_{uu}(e^{j\omega}) = \frac{1}{M} \sum_{m=1}^M \vec{U}_w^{[m]}(e^{j\omega}) (\vec{U}_w^{[m]}(e^{j\omega}))^H, \quad (3-8)$$

$$\hat{S}_{yu}(e^{j\omega}) = \frac{1}{M} \sum_{m=1}^M \vec{Y}_w^{[m]}(e^{j\omega}) (\vec{U}_w^{[m]}(e^{j\omega}))^H, \quad (3-9)$$

in which the subscript w indicates that the time series are *windowed* before transforming to frequency domain.

Implementing the method requires careful consideration of several factors, such as the total number of data for identification N , the segment length n , window type, and overlapping length. The process of selecting these variables will be covered in detail below.

Segment length

Since we have limited amount total data for system identification, deciding the segment length n will always be a trade off. It is taking trade off between reduced variance of the spectrum and the frequency resolution. With shorter segment length, the total number of data can be splitted into more segments. With more segments we are essentially taking the average of a larger number of smaller segments. This helps to reduce the variance of the overall average. With longer segment, the resulting frequency resolution is higher.

Let's first look at the minimum desired frequency resolution Δf_0 . The frequency resolution is inversely proportional to the segment length n , as expressed by the equation [17]

$$n = \frac{n_w \times f_s}{\Delta f_0}, \quad (3-10)$$

where n is the segment length, $f_s = 10\text{Hz}$ represents the sampling frequency and n_w is the parameter that represents the main lobe width of the window function (which we will explain in later subsection). Selecting a suitable frequency resolution ensures that the analysis captures all essential dynamics in the data.

Wave spectrum and RAOs allow for the calculation of force and motion signals. They provide valuable insights into the frequency content of both force and motion data. They provide valuable insights into the frequency content of both force and motion data. Our objective is to select a suitable frequency resolution that adequately captures the essential features of the wave spectrum and RAOs. A resolution sufficient for describing these key elements will also be appropriate for the force and motion signals.

In this study we use the RAOs of ship model SPA. Figure 3-3 and 3-4 shows the 6DoF force RAOs and motion RAOs, respectively, generated with main wave propagation of 45° .

As observed from examining multiple RAO curves, a frequency resolution of $\Delta f_0 = 0.25\text{rad/s}$

is sufficient for capturing the necessary details of these functions. The samples highlighted in red in the plots represent the samples separated with the chosen frequency resolution.

Returning to Equation 3-10, in order to determine the segment length n , we still need to know the main lobe width, n_w , of the chosen window function. We will explore selection of this parameter in the following section.

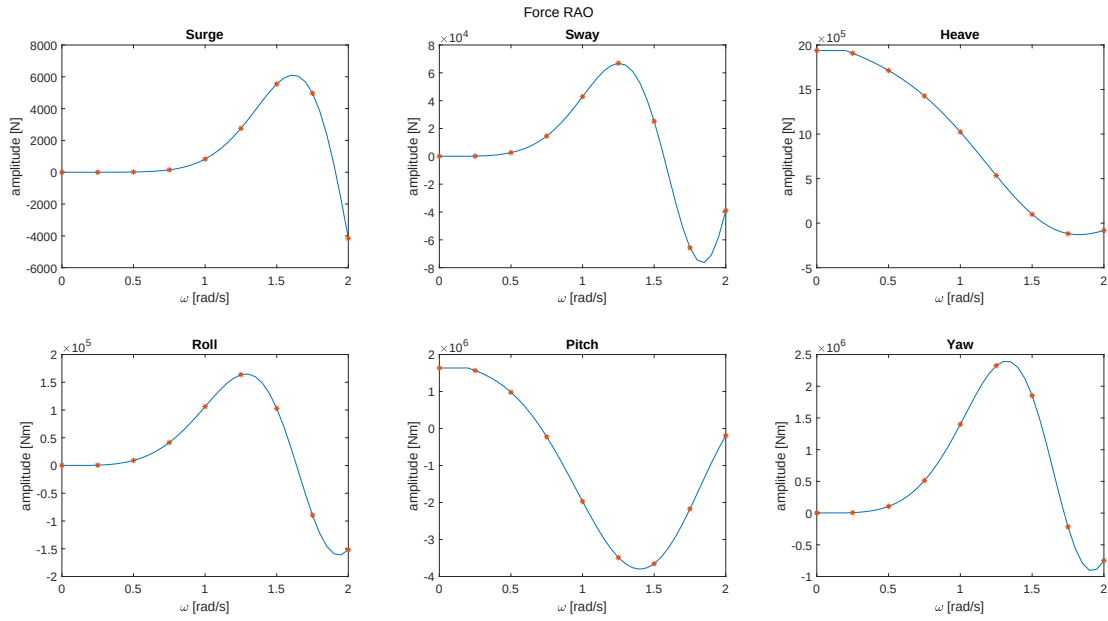


Figure 3-3: Force RAOs

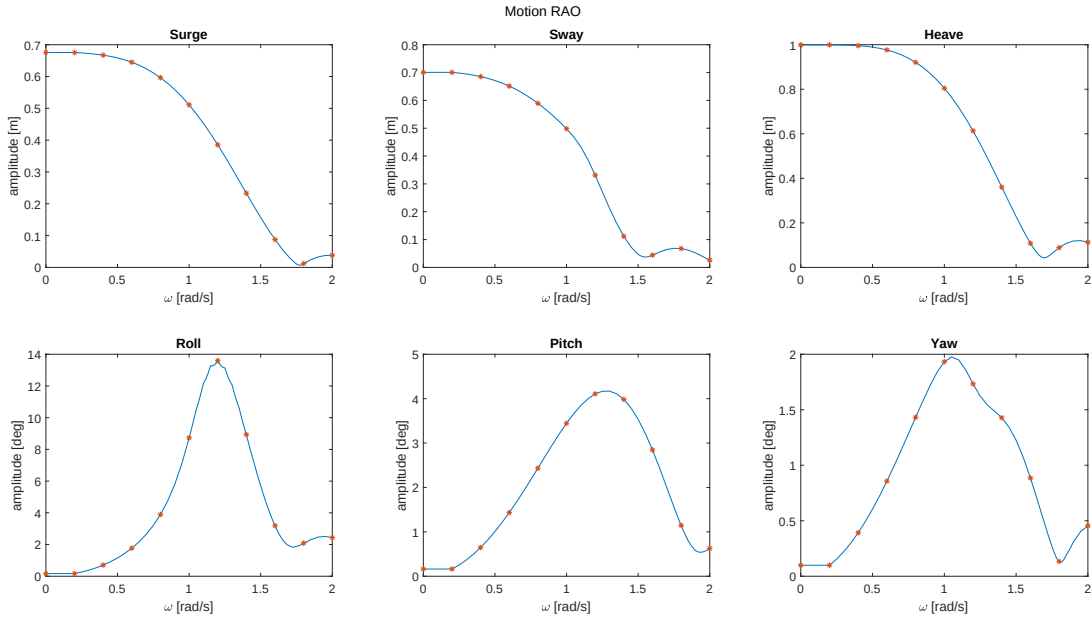


Figure 3-4: Motion RAOs

Window type

As we know, wave excitation forces and motions are non-periodic signals. Transforming non-periodic signals from the time domain to the frequency domain induces spectral leakage in the computed frequency response, because the signal segment has a finite length and is non-periodic. The spectral leakage can be effectively reduced by properly applying a window function [14].

Each window function has its own characteristics and suitability for different applications. Figure 3-5 shows the frequency response of an example of a window function. The main lobe and the side lobe of the frequency response of the window function determine the characteristic of the function.

The width of the main lobe defined by the number of frequency bins affects the frequency resolution of the FFT result. The high resolution provides accurate estimates of the frequency of a sinusoid and results in the separation of two sinusoids that are closely spaced in frequency.

The side lobe level is the highest side lobe peak measured in decibels relative to the peak of the main lobe. The side lobe level of the window directly affects the extent to which a frequency component leaks into adjacent frequency bins. Low side lobe level means low spectral leakage, which improves the detectability of a weak sinusoid in the presence of a strong sinusoid that is not bin-centred.

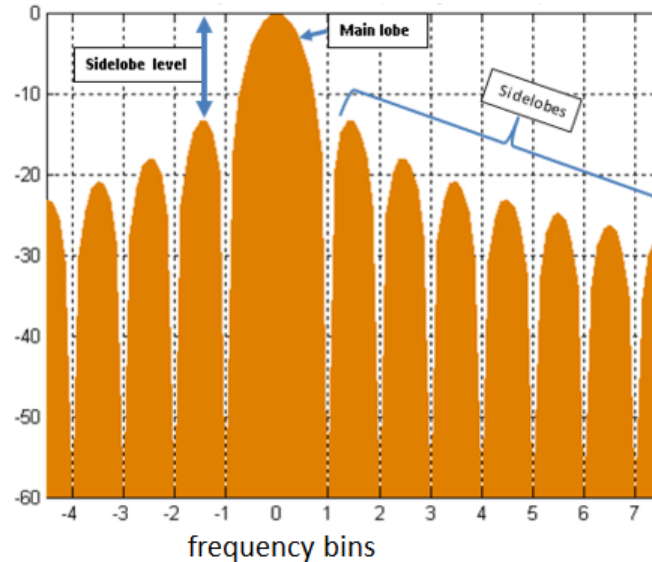


Figure 3-5: Fourier transform of a window function

Generally when reducing the side lobes of a spectral window, the main lobe width must be increased, and vice versa. Therefore, choosing an appropriate window function is to make a trade-off between a narrow main lobe, which provides high resolution, and low side lobes, which reduce spectral leakage [17].

When selecting a window function, knowing the frequency content of the input and out-

put signals is important. Based on the typical wave spectrum showing in Figure 3-1 and typical shape of force and motion RAOs that we focused on in this thesis, we can say that both wave excitation forces and motions are broadband signals.

Generally broadband signals are less sensitive to spectrum leakage compare with signals that have sharp frequency content. Minimizing spectrum leakage is not our priority. Instead we will choose a window function which has narrow main lobe. This approach prioritizes shorten the segment length to achieve the desired frequency resolution.

Comparing among the window functions that have narrow main lobes, Hanning window offers a balance between leakage and main lobe width, making it a good option for this application. The window function has a main lobe width of 2, i.e $n_w = 2$. Using this value in Equation 3-10 and knowing $\Delta f = 0.25\text{rad/s}$, we can calculate the selected segment length to be $n = 512$.

Overlap

When a window is applied to a signal, the data sequence gets tapered to zero at the edges, which will obviously lead to loss of information. This can be avoided by processing the sequence with overlapping windows. In general, 50% to 75% overlap is necessary to retain most of the information in the signal [17]. We will choose 75% as the overlapping parameter in this thesis.

The total length of data N

We have selected the length of each segment previously by looking at the frequency content of the signals. Knowing each segment length, choosing the total length of data to use is essentially choosing the number of segments to use for averaging.

The Welch's method yields the unbiased and consistent PSD asymptotically (equation 3-8 and equation 3-9), which means that, with unlimited number of segments for averaging, the estimated PSD converges to the true PSD. In practical we are not able to use unlimited number of segments, as that would require unlimited number of data.

Furthermore for identification algorithms operating onboard a ship, the total data length used for PSD estimation becomes particularly critical. Longer data sequences provide a more accurate PSD estimate but also increase the computational load on the software.

To find a balance between accuracy and computational efficiency, the system identification algorithm was evaluated with varying training data lengths and compare the achieved VAF. The tests were conducted with main wave direction of 45° and under noisy data conditions (details of noise data will be given in Section 4-1). Here we present the VAF results for 6DoF to illustrate the influence of data length.

Table 3-1 summarizes the VAF values obtained for each data length (denoted by N). We tested lengths ranging from 1 to 15 minutes. For each DoF, the VAF is the average of 50 test runs. The table also includes the average VAF across six directions for each data length.

The results indicate that 5 minutes provides the most effective compromise, achieving similar VAF compared to longer sequences but with significantly less data.

length N (minutes)	surge	sway	heave	roll	pitch	yaw	6DoF average
1	96.0	77.6	93.8	65.7	94.5	79.2	84.5
3	96.8	95.0	97.1	87.9	94.2	93.4	94.1
5	97.1	96.3	97.1	90.2	94.3	94.8	95.0
10	97.2	96.2	97.2	90.7	94.0	94.7	95.0
15	97.2	96.4	97.2	91.4	94.5	94.9	95.3

Table 3-1: VAF with different parameter N - wave 45°

3-2-3 Matrix inversion

Another detail that is worth our attention is the matrix inversion operation in Equation 3-4. This equation is only valid if matrix $\hat{S}_{uu}^{-1}(e^{j\omega})$ is invertable. The invertability of matrix can be evaluated using the condition number of the matrix.

The condition number γ of a matrix is calculated from the singular values [19],

$$\gamma(e^{j\omega}) = \sigma_1(e^{j\omega})/\sigma_k(e^{j\omega}), \quad (3-11)$$

where $\sigma_1(e^{j\omega})$ and $\sigma_k(e^{j\omega})$ are the largest and smallest nonzero singular values, respectively. The condition number is often considered an indicator of the number of precision digits that may be lost when we invert a matrix. When the condition number is too large, the matrix is said to be ill-conditioned, meaning that at a certain point, it cannot be inverted anymore.

Figure 3-6 shows the condition number of the matrix $\hat{S}_{uu}(e^{j\omega})$ along the entire frequency band, the largest value goes up to a scale of 10^{10} . However, when classifying a condition number as large or small it is not enough to simply look at the number, we must look at the application.

Let Z denote the result of $\text{inv}(\hat{S}_{uu}(e^{j\omega}))$ operation in matlab, and X denote the exact inverted matrix of $\hat{S}_{uu}(e^{j\omega})$. we can estimate the relative round-off error in the inversion operation using the following formula [8]:

$$\|E\| = \frac{\|Z - X\|}{\|X\|} = 3 * \text{eps} * \text{cond}(\hat{S}_{uu}(e^{j\omega})), \quad (3-12)$$

where the factor 3 is used because the matrix $\hat{S}_{uu}(e^{j\omega})$ is a 3×3 matrix, eps returns the floating-point relative accuracy and $\text{cond}(\hat{S}_{uu}(e^{j\omega}))$ returns the condition number of the matrix.

For the symmetric system $G_{sym}(e^{j\omega})$, along the frequency band in the worst case we get $\|E\| = 2.4752 * 10^{-11}$ and for the asymmetric system $G_{asym}(e^{j\omega})$ we get $\|E\| = 7.3062 * 10^{-13}$. These results mean that we can expect 11 and 13 (out of 16) significant digits in the results

of the inversion operation, respectively. The smallest entries in the matrix $\hat{S}_{uu}^{-1}(e^{j\omega})$ are on the scale of 10^{-7} , which means that by losing 11 or 13 significant digits we can still derive quite accurate results for the matrix inversion. Therefore we conclude that the matrices are not ill-conditioned for inversion in this case.

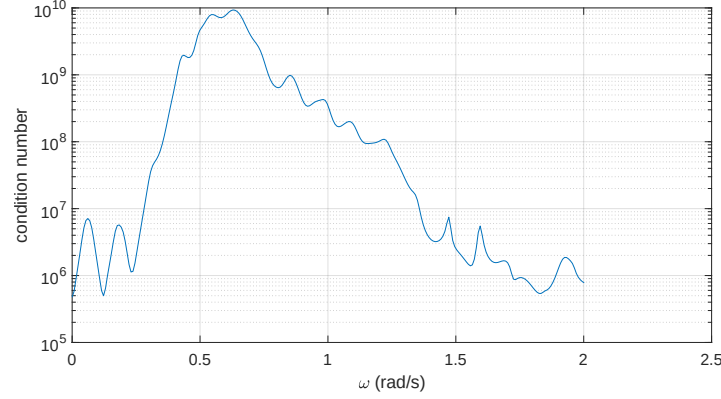


Figure 3-6: $\gamma(S_{uu})$, condition number of matrix S_{uu}

In cases where the matrix is singular or ill-conditioned, pseudo inversion is often used as a solution to compute the best approximation of the matrix inversion.

Chapter 4

Testing results with Spectral Analysis method

An aspect of system identification is assessing the quality of the input-output relationship, which significantly influences the accuracy of the identified model. This chapter focuses on evaluating the coherence between input and output signals as a metric for assessing data quality. The results of the coherence analysis will be correlated with the performance of the Spectral Analysis method for system identification. To validate the performance of the method under various conditions, tests were conducted using both synthetic and real field data.

4-1 Synthetic data testing results

Synthetic data were generated by adding noise to wave components with Signal to Noise Ratio (SNR)s ranging from 20dB to 60 dB (as detailed in section 3-1-1). Figure 4-1 displays the Fourier coefficient magnitudes of the noise-free and noisy Johnswap wave spectrum with a SNR of 20dB. To ensure data independence, two separate noise sequences were injected into the wave components to create training and validation forces, denoted as $\vec{u}_i(k)$ and $\vec{u}_v(k)$, respectively.

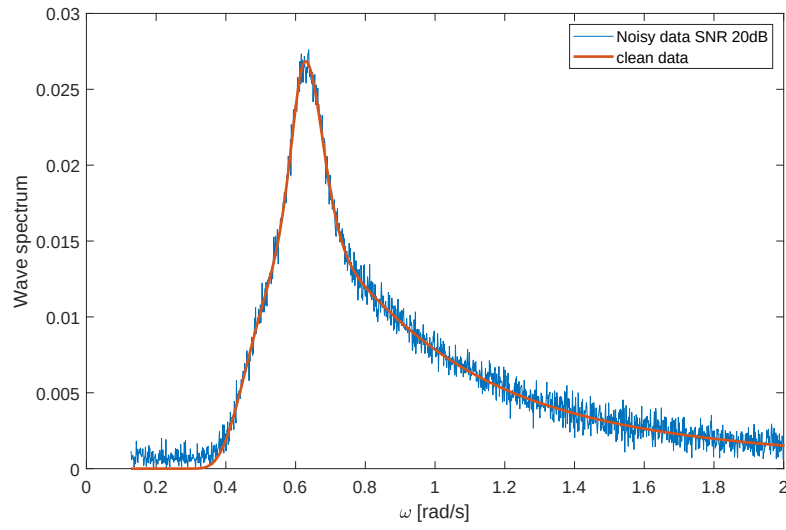


Figure 4-1: Wave components Fourier coefficients magnitude

The test simulated a wave condition where the main wave propagating direction is 45° which excites both symmetrical and asymmetrical forces and motions. The force and motion signals are given in Figure 4-2 and Figure 4-3.

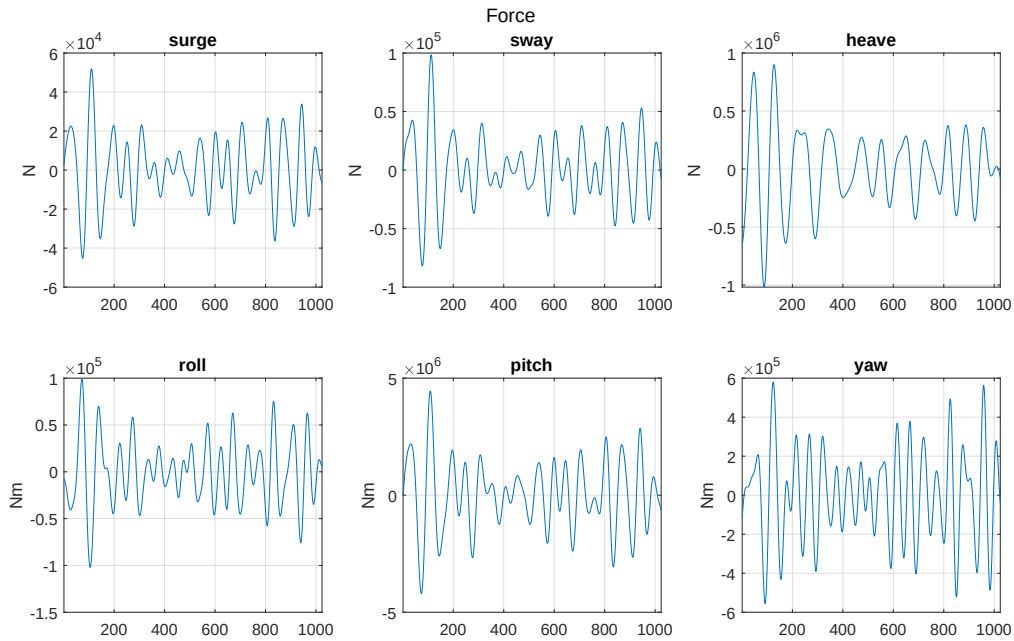
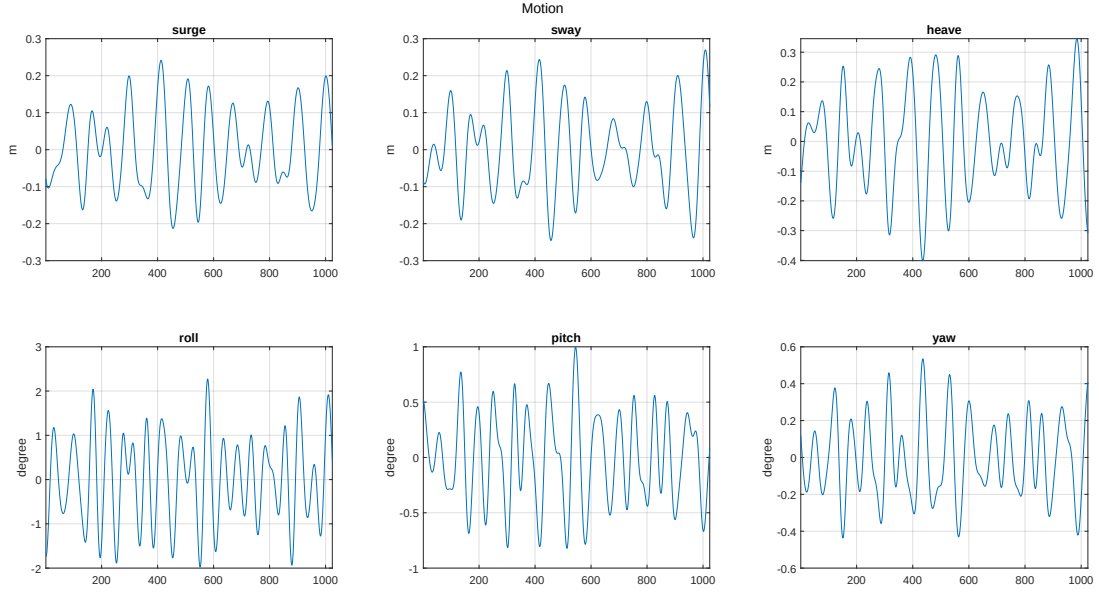


Figure 4-2: Force signals - synthetic data

**Figure 4-3:** Motion signals - synthetic data

The parameters employed in the Spectral Analysis algorithm, as determined in Section 3-2-2 are summarized in the Table 4-1.

segment length	512	remarks
window type	Hann	remarks
total number of data	5 min	remarks
overlap size	75%	remarks

Table 4-1: Parameters in the Spectral Analysis method

4-1-1 Coherence analysis

MIMO force-to-motion system coherence

Coherence is frequently used to measure the linear relationship between the input and output signals. It ranges from 0 and 1, where 1 signifies a perfect linear relationship between the input and output, and a value less than 1 indicates the presence of additional input signals, measurement noise, or nonlinear distortions.

On the i^{th} direction, the MIMO coherence computes the coherence of between the output on the i^{th} direction and all the inputs [16]

$$\gamma_{i,mimo}^2(e^{j\omega}) = S_{y_i u}^H(e^{j\omega}) S_{uu}^{-1}(e^{j\omega}) S_{y_i u}(e^{j\omega}) S_{y_i y_i}^{-1}(e^{j\omega}), \quad (4-1)$$

in which $S_{uu} \in \mathbb{C}^{3 \times 3}$ is the input power spectrum density matrix, $S_{y_i u} \in \mathbb{C}^{3 \times 1}$ is the cross spectrum density of the i_{th} output motion and the inputs and $S_{y_i y_i}$ is the power spectrum

density of the output motion.

Coherence MIMO VS SISO

The SISO coherence is the measure of the coherence between the motion and force from the same direction [16]

$$\gamma_{i,\text{siso}}^2(e^{j\omega}) = \frac{|S_{y_i u_i}^H(e^{j\omega})|^2}{S_{u_i u_i}(e^{j\omega}) S_{y_i y_i}(e^{j\omega})}. \quad (4-2)$$

The power spectrum density matrices employed in equation 4-1 and equation 4-2 were estimated using equation 3-8 and equation 3-9 with parameters summarized in Table 4-1. It's important to note that these PSD matrices, and consequently the coherence estimates, are subject to estimation uncertainty due to the finite data length used in the estimation process. However, for the purpose of this analysis, the relative differences in coherence between the MIMO and SISO systems are of primary interest.

Figure 4-4 presents a comparison of coherence values for MIMO and SISO force-to-motion systems. The MIMO system exhibits consistently high coherence, approaching value of 1 across the entire frequency range, indicating a strong linear relationship between inputs and outputs. In contrast, the SISO system displays slight fluctuations in coherence, suggesting a less robust linear relationship. Notably, coherence for the heave direction in the SISO system closely following that of the MIMO system.

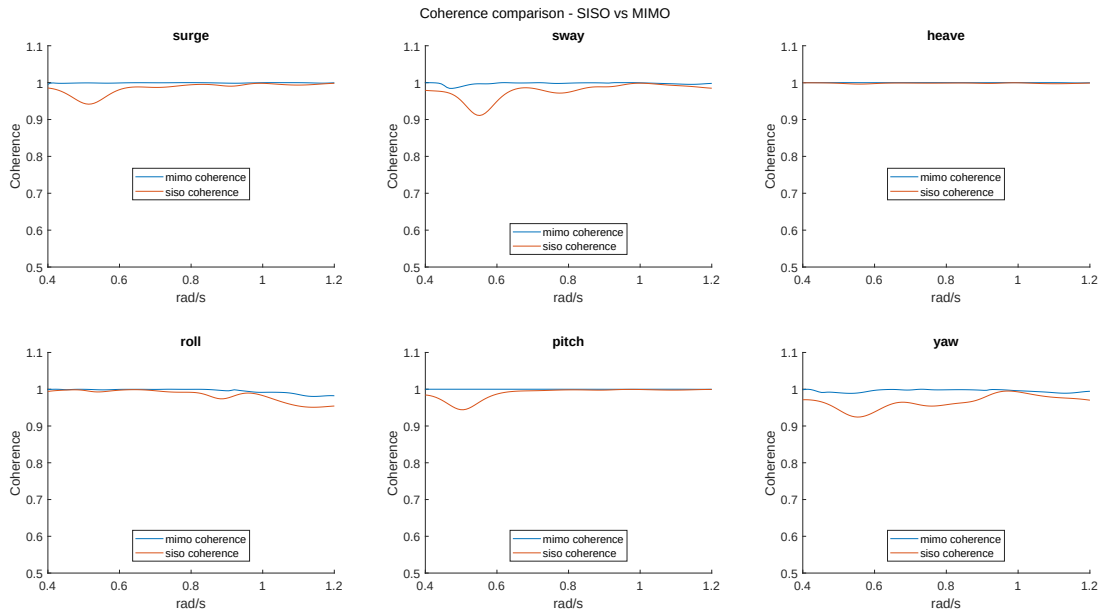


Figure 4-4: Coherence comparison - SISO vs MIMO

4-1-2 Motion prediction performance

MIMO Spectral Analysis results

The VAF values achieved for different noise levels are presented in Figure 4-5. The Spectral Analysis method demonstrates robust performance across all noise levels, with VAF values consistently exceeding 90%. For each direction, as the SNR increased, VAF value approached saturation. The saturation values for all six directions exceed 99%, as summarized in Table 4-2. While these results highlight the method's robustness, it's important to acknowledge that the method's performance might deteriorate with extremely high noise levels, and further investigation might be required to determine its limitations in such scenarios.

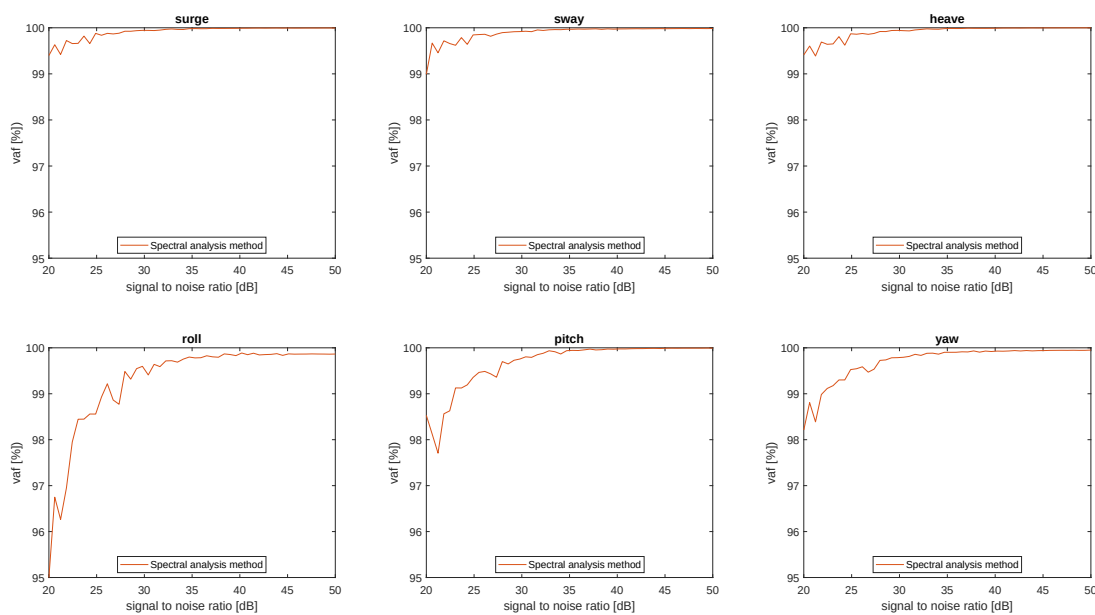


Figure 4-5: Computed VAF of noisy systems

surge	sway	heave	roll	pitch	yaw
99.6%	99.2%	99.4%	99.2%	99.8%	99.2%

Table 4-2: VAF - synthetic data without noise

Comparing with SISO method

MIMO system's improved coherence, as detailed in Section 4-1-1, suggested potential performance advantages over the SISO approach. To validate this hypothesis, the VAF values achieved by MIMO and SISO methods were compared across a range of SNR levels from 20dB to 60dB.

Figure 4-6 presents the VAF values for both methods across all six directions. The results consistently demonstrate superior performance of the MIMO system across all directions and

noise levels. As the SNR increased, VAF values for both methods approached saturation, with final values summarized in Table 4-3. While the MIMO method maintains its advantage across all directions, the performance difference is less pronounced for heave and pitch, aligning with the coherence analysis where minimal differences between the two methods for these directions were indicated (figure 4-4).

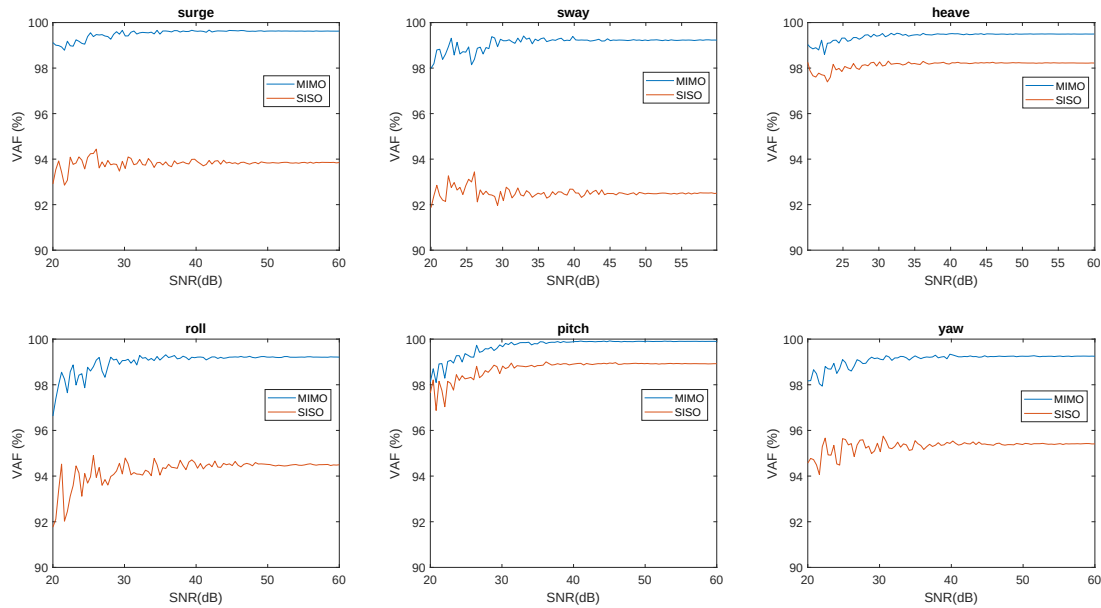


Figure 4-6: VAF of MIMO (blue) VS SISO (red)

DoF	Surge	Sway	Heave	Roll	Pitch	Yaw
MIMO VAF at SNR 60dB	99.6%	99.2%	99.4%	99.2%	99.8%	99.2%
SISO VAF at SNR 60dB	93.8%	92.5%	98.2%	94.4%	98.9%	95.4%
advantage MIMO vs SISO	5.8%	6.7%	1.2%	4.8%	0.9%	3.8%

Table 4-3: VAF comparison of MIMO and SISO method at SNR 60dB

Testing with synthetic data has confirmed the theoretical expectation that a MIMO force-to-motion system model provides a more accurate representation of system dynamics, compared to a SISO model. The VAF values clearly demonstrate the better performance of the MIMO spectral analysis method in predicting motion.

Synthetic data test with different wave conditions were conducted and same findings were concluded. To further validate these findings and assess the performance of MIMO Spectral Analysis method in real-world conditions, tests were conducted using field data presented in following sections.

4-2 Experimental results

In this section testing with real field data are conducted. Each dataset represents 1 hour of data. The wave condition can be described by a two dimensional wave spectrum (details given in Appendix), the wave directions were determined from these spectra. Within each hour of data set the wave spectrum was updated every 15 minutes, from which the variation of wave directions were determined.

It was assumed that the heading of the ship remain constant during the prediction horizon, as significant changes in heading could invalidate the motion predictions.

4-2-1 Experiment 1 - relative wave direction centered around -185°

Wave condition

Figure 4-7 presents a snapshot of the wave spectrum for the first hour of data. The dominant wave direction is 36° counter-clockwise from east, with fluctuations between 33° and 43° over the hour. The ship's heading, as shown in Figure 4-8, maintained a relatively stable course at approximately 232° degrees counter-clockwise from east, with variations within $\pm 2.5^\circ$. Given the limited heading variation, its impact on motion prediction for this dataset is considered negligible.

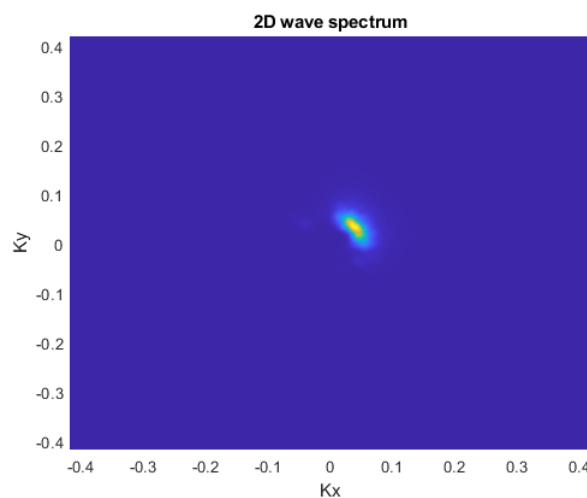


Figure 4-7: 2D wave spectrum - indicating a group of wave components with main direction of 36°

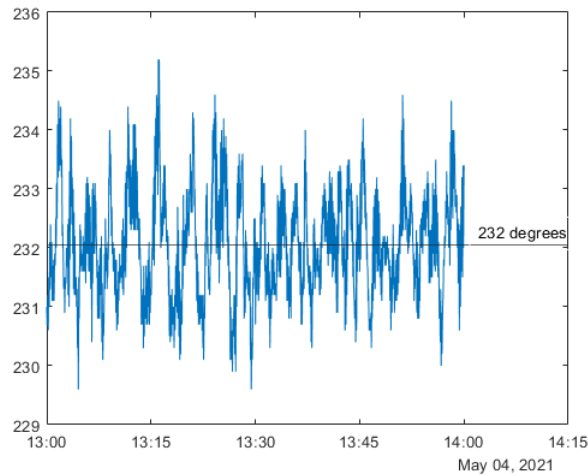


Figure 4-8: Ship heading during the hour

To assess the wave's influence on ship motions, the spectrum of the 1500 most significant wave components was determined (Figure 4-9). These components exhibit relative directions (relative to ship) ranging from -220° to -140° , centered around -185° . This wave direction distribution suggests that both symmetrical and asymmetrical ship motions were excited.

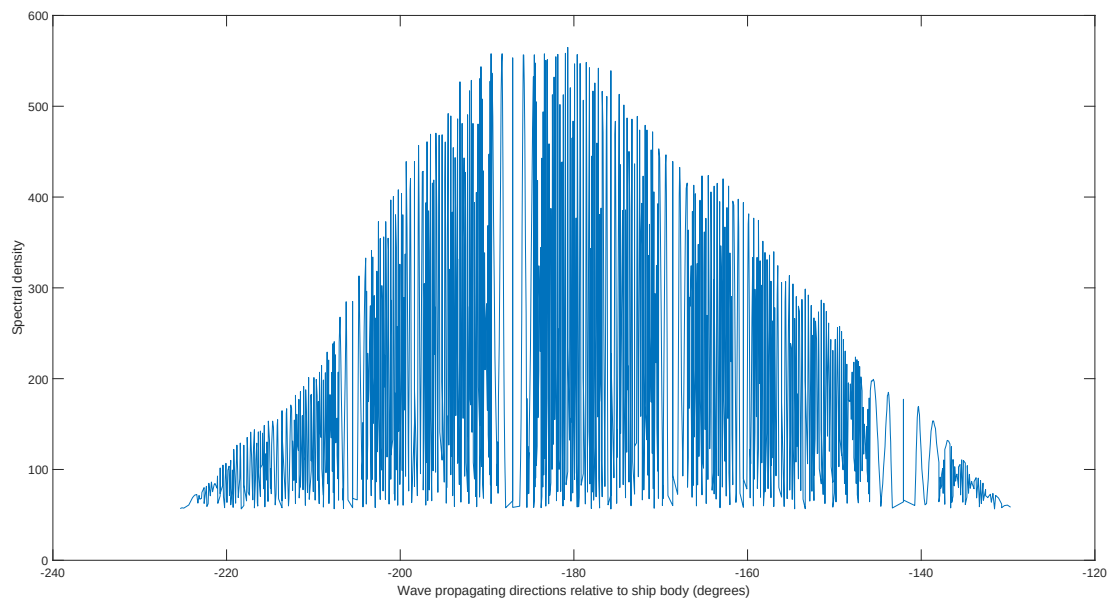


Figure 4-9: Relative wave direction of 1500 most significant wave components

Coherence and VAF analysis

Figure 4-10 shows the coherence between the motions and the forces from both SISO and MIMO systems. Table 4-4 shows the VAF achieved with MIMO Spectral Analysis method and SISO Spectral Analysis method for 6DoF.

Due to insufficient data below 0.5 rad/s and above 1 rad/s, the coherence for both models approaches zero in these frequency ranges. Therefore our coherence analysis will focus on the frequency band between 0.5 and 1 rad/s.

For symmetrical motions (surge, heave, and pitch), both MIMO and SISO models exhibited relatively high coherence values, generally ranging from 0.8 to 0.9. While coherence levels for surge and pitch were comparable between the two models, the MIMO system demonstrated slightly higher coherence for heave. This suggested potential performance advantages for the MIMO method in predicting heave motion, with similar performance expected for surge and pitch.

Table 4-4 presents the corresponding VAF values. As anticipated, the MIMO method outperformed the SISO method for heave motion, aligning with the coherence analysis. However, contrary to expectations, the MIMO method did not surpass the SISO method for pitch, despite the higher coherence value.

For asymmetrical motions (sway, roll, and yaw), both MIMO and SISO models exhibited low coherence values (below 0.5), indicating potential challenges in accurately capturing the underlying system dynamics. While the MIMO model demonstrated slightly higher coherence overall, the VAF results for sway and roll aligned with this expectation, with the MIMO method outperforming the SISO approach. However, unexpectedly, the MIMO model underperformed for yaw motion.

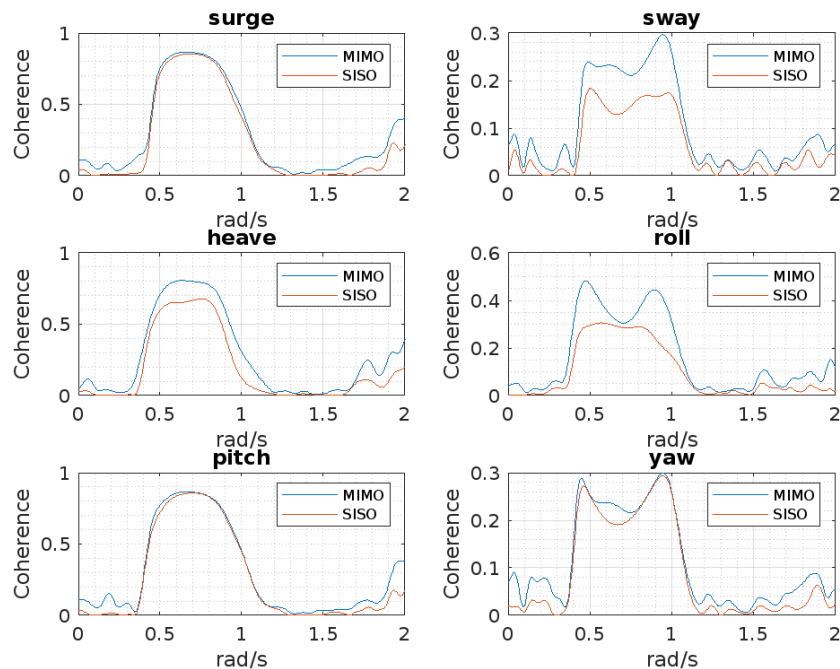


Figure 4-10: Coherence for the 1st hour of data - MIMO vs SISO

DoF	Surge	Sway	Heave	Roll	Pitch	Yaw
MIMO	88.0%	62.7%	82.8%	74.6%	87.6%	81.8%
SISO	87.7%	57.5%	80.1%	73.3%	88.9%	82.6%
MIMO>SISO	0.3%	8.9%	3.7%	1.8%	-1.5%	-2.1%

Table 4-4: Spectral Analysis method achieved Vaf - MIMO vs SISO

Coherence analysis indicated a strong potential for the MIMO approach to outperform the SISO approach, with consistently higher coherence values across all six degrees of freedom. However, the relationship between coherence and VAF performance was not entirely consistent. Despite higher coherence values for the MIMO model in pitch and yaw, the VAF results did not consistently reflect this advantage. To further investigate this inconsistency and confirm the generalization of the findings, additional datasets were analyzed.

4-2-2 Other datasets

Dataset 2 - relative wave direction centered around -216°

For this hour of data, the dominant wave direction is 36° counter-clockwise from east, with fluctuations between 33° and 43° over the hour. The ship heading has a mean of 254° counter-clockwise from east. The wave components relative propagating directions are calculated to be centered around -216° .

The same as with previous batch of data, we focus on frequency range 0.5rad/s to 1.0 rad/s. In overall, both MIMO and SISO models obtain reasonable coherence. For each motion, the MIMO model obtains higher coherence than SISO model across the frequency band, suggesting better motion prediction results. Table 4-5 summarized the VAF achieved with MIMO and SISO systems. MIMO system obtained better performance in all directions except for yaw motion.

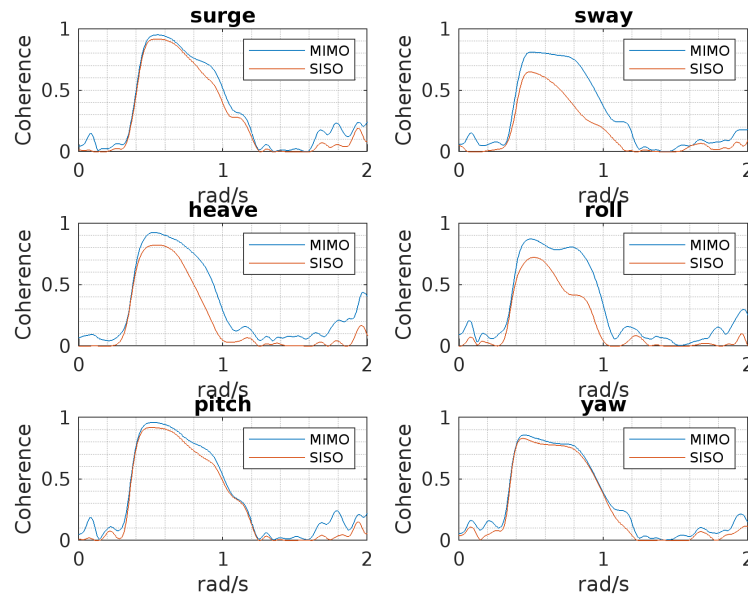


Figure 4-11: Coherence dataset 2 - MIMO vs SISO

DoF	Surge	Sway	Heave	Roll	Pitch	Yaw
MIMO	84.5%	82.8%	84.5%	89.5%	86.6%	87.6%
SISO	84.3%	78.1%	76.8%	86.6%	85.6%	89.9%
MIMO>SISO	0.2%	6.0%	9.9%	3.4%	1.0%	-2.6%

Table 4-5: Spectral Analysis Vaf - dataset 2

Dataset 3 - relative wave direction centered around -220°

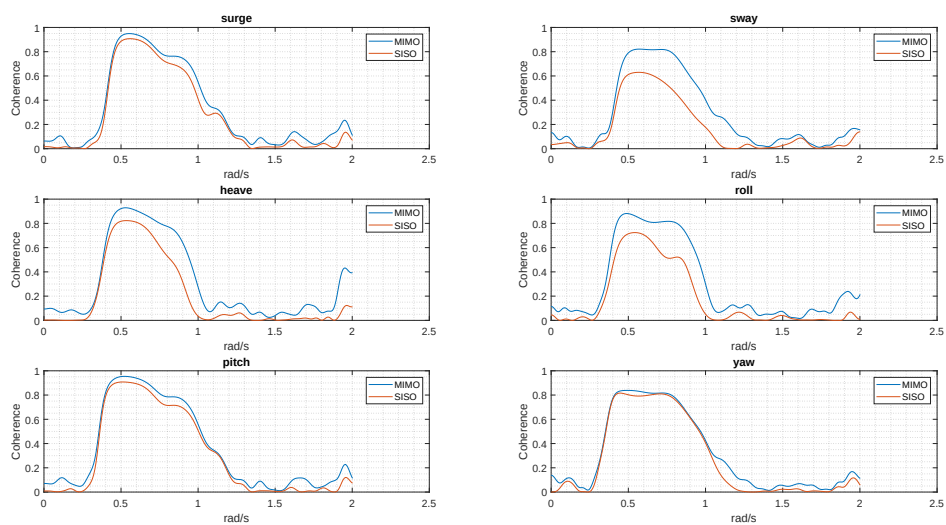


Figure 4-12: Coherence batch 3 - MIMO vs SISO

DoF	Surge	Sway	Heave	Roll	Pitch	Yaw
MIMO	87.7%	85.7%	86.5%	91.8%	89.1%	91.3%
SISO	87.5%	81.7%	78.4%	89.5%	89.0%	91.7%
MIMO>SISO	0.2%	6.0%	9.9%	3.4%	1.0%	-2.6%

Table 4-6: Spectral Analysis Vaf - dataset 3

Coherence analysis of MIMO and SISO models indicates that the MIMO force-to-motion system generally provides a more accurate representation of system dynamics. This finding suggested a potential advantage in MIMO Spectral analysis in motion prediction. However, the relationship between coherence and VAF performance is not entirely consistent.

Looking at the coherence analysis, it seems that yaw motion (also pitch in synthetic data test) MIMO coherence only has marginal advantage over SISO coherence. Which means on this direction, SISO relationship was probably sufficient to represent the force-to-motion system. Using MIMO system model could lead to over fitting or other issues come with higher order system. For future work, we could test the method with various sea conditions and confirm whether this hypothesis holds.

4-3 Summary

This chapter investigated the Spectral Analysis method identifying the force-to-motion systems, which are modeled as MIMO systems. The performance and robustness of the method were evaluated using synthetic data and real field data.

The Spectral Analysis method demonstrated robust motion prediction capabilities, achieving VAF values exceeding 90% for noise levels up to 20 dB. Additionally, the MIMO system approach was compared with SISO approach. Overall MIMO approach proved to obtain more accurate motion predictions. However there are some exceptional cases where further investigation is needed.

State Space Method Identifying Waves-to-motions System

We have studied force-to-motion MIMO system in previous chapter. However the force-to-motion approach relies on the assumption that the input force are perfectly computed from wave components and force RAOs. While the force RAOs generally exhibit less uncertainty compared to motion RAOs, their inherent limitations can still introduce inaccuracies into the prediction process. In this chapter we will study wave-to-motion system and evaluate whether more accurate motion prediction can be achieved. The algorithm we will apply to the system is Past Output Multivariable Output-Error State-space (PO-MOESP) algorithm, a subspace identification method.

5-1 Wave-to-motion system

Similar to the force-to-motion system we studied in previous chapter, we will split the wave-to-motion system into separate symmetrical and asymmetrical systems. For simplicity, we will denote both systems as G throughout this chapter due to the identical identification technique employed.

Figure 5-1 shows a simplified system block diagram of the wave-to-motion system. The system inputs, denoted as vector $\vec{u}(k) \in \mathbb{C}^{m \times 1}$, are the wave components propagating from m distinctive directions. The system outputs $\vec{y}(k) \in \mathbb{C}^{3 \times 1}$ are the motions experienced by the ship (surge, heave and pitch for the symmetrical system, sway, roll and yaw for asymmetrical system), k is the sample index.

The system order, number of input and output signals directly impact the total number

of data we need for the identification [24]:

$$N > 3 * s(m + l), \quad (5-1)$$

where N is the total number of the data to use, s is the dimensional parameter which is often taken to be 3 times of the estimated system order, m is the number of the input and l is the number of output (equal to 3 here). Considering the values of these parameters, we choose 10min length data to be the N .



Figure 5-1: Wave-to-motion MIMO system block diagram

In practice the input wave components are computed from the radar images collected by the onboard radar system. This information is used to compute the time series of the wave components (details on this process can be found in Appendix A). The output motions are collected from the onboard motion sensors.

5-1-1 System identification with Wave Spreading

The synthetic wave components were generated in the same way as described in Section 3-1-1. The waves are generated with a resolution of 1° directional resolution. However, ship behavior may not be significantly affected by wave direction that varies of 1° . To exploit this and improve system identification efficiency, we grouped nearby waves with similar impact on the ship together as one input.

This grouping is achieved by defining a wave direction grid with a chosen resolution (with unit in degrees). Each grid bin represents a range of incoming wave directions. Waves falling within the same bin are indistinguishable of their influence on the ship and are combined into one input to the system. These input waves grouping effectively reducing the input dimensions and improving system identification efficiency.

Table 5-1 summarizes the wave bin sizes from 1° to 15° , corresponding required system orders, the number of input to the system and the achieved VAFs for various bin sizes. Smaller wave bin sizes, result in significantly higher number of inputs to the system and require higher system orders (detailed in section 5-2-1) to achieve the same prediction accuracy compared to larger wave bin sizes. Considering the input complexity, the order of the estimation and the resulting VAF, bin sizes of 15 degrees appear most effective.

5-2 Basic principle of the subspace model

The foundation of the PO-MOESP method is to construct the state space model of the wave-to-motion system. A state space model represents a system's dynamics by relating its inputs, outputs, and internal states. As we previously discussed, the noise in the system mostly

wave bin size(°)	system order (sym, asym)	number of inputs	6DoF average Vaf (%)
1	45, 42	71	59
5	40, 38	16	96.2
10	12, 10	9	96.0
15	12, 10	6	96.6

Table 5-1: performance comparison with different wave bin sizes

coming from, wave components, the input of the system. We can introduce the noise term and rewrite the above equation as:

$$\vec{x}(k+1) = A\vec{x}(k) + B(\vec{u}(k) + \vec{e}(k)), \quad (5-2)$$

$$\vec{y}(k) = C\vec{x}(k) + D(\vec{u}(k) + \vec{e}(k)), \quad (5-3)$$

where $\vec{u}(k) \in \mathbb{R}^m$, $\vec{x}(k) \in \mathbb{R}^n$, $\vec{e}(k) \in \mathbb{R}^m$ and $\vec{y}(k) \in \mathbb{R}^3$. $\vec{u}(k)$ represents the wave components propagating towards the ship from m distinguish directions, $\vec{y}(k)$ represents the motion signals of the ship induced by the waves, $\vec{x}(k)$ is the state of the system, n is the order of the state space and $\vec{e}(k)$ is the error between the true and measured input wave components. In this thesis we will treat the measurement error $e(k)$ as a zero-mean white-noise sequence, assuming it is uncorrelated with $x(k)$ and $u(k)$ for all $k \in \mathbb{Z}$.

Given a finite number of samples of input signals $\vec{u}(k)$ and the output signals $\vec{y}(k)$ of the minimal (reachable and observable) system 5-2 and 5-3, the goal of the system identification is to determine the system matrices (A, B, C, D) up to a similarity transformation T , denoted as (A_T, B_T, C_T, D_T) .

The section repeats the material from Chapter 9 of the book *Filtering and System Identification*. The details in the derivation and proof of the subspace method are outside the scope of this thesis.

Given samples of the input and output signals to the system, the constructed block Hankel matrices are defined as

$$U_{i,s,N} = \begin{bmatrix} u(i) & u(i+1) & \dots & u(i+N-1) \\ u(i+1) & u(i+2) & \dots & u(i+N) \\ \vdots & \dots & \dots & \vdots \\ u(i+s-1) & u(i+s) & \dots & u(i+N+s-2) \end{bmatrix} \in \mathbb{R}^{sm \times N}, \quad (5-4)$$

$$Y_{i,s,N} = \begin{bmatrix} y(i) & y(i+1) & \dots & y(i+N-1) \\ y(i+1) & y(i+2) & \dots & y(i+N) \\ \vdots & \dots & \dots & \vdots \\ y(i+s-1) & y(i+s) & \dots & y(i+N+s-2) \end{bmatrix} \in \mathbb{R}^{sl \times N}, \quad (5-5)$$

where m is the number of inputs and l is the number of the outputs of the system, s is a dimensional parameter and N is the total number of samples used for system identification.

The first entry, i , of the subscripts of $Y_{i,s,N}$ refers to the sample index of its first entry, the second entry s is a dimensional parameter representing the number of block rows, the last entry N refers to the number of columns. The conditions for choosing these parameters are $s > n$ and $N > s$, where n is the order of the system. Generally, we can take the value of s as 2 or 3 times the dimension n of the state space.

The crucial step of the subspace method is to construct the data equation to relate these Hankel matrices. Using the system representation given above we formulate the data equation

$$Y_{i,s,N} = \mathcal{O}_s X_{i,N} + \mathcal{T}_s U_{i,s,N} + \mathcal{T}_s E_{i,s,N}, \quad (5-6)$$

where $E_{i,s,N}$ is the block Hankel matrix constructed from the noise sequence $e(k)$ and $\mathcal{O}_s$ is the extended observability matrix defined as

$$\mathcal{O}_s = \begin{bmatrix} C_T \\ C_T A_T \\ \vdots \\ C_T A_T^{s-1} \end{bmatrix}. \quad (5-7)$$

The data equation allows us to estimate the column space of the matrix $\mathcal{O}_s$. From this subspace, we can then estimate the system matrices A_T and C_T .

5-2-1 PO-MOESP algorithm

The subspace method we will implement in this chapter is PO-MOESP, which stands for Past Output Multivariable Output-Error State-sPace. As the name suggests, the algorithm uses the past input and output Hankel matrices $U_{0,s,N}$ and $Y_{0,s,N}$ as the instrumental variables when estimating the column space,

$$Z_N = \begin{bmatrix} U_{0,s,N} \\ Y_{0,s,N} \end{bmatrix}.$$

In our case the the noise signal $e(k)$ is assumed to be ergodic stochastic process which is uncorrelated with $u(k)$ and $x(k)$. This assumption governs the Equations 9.56 and 9.58 in the book *Filtering and System Identification* are satisfied,

$$\lim_{N \rightarrow \infty} \frac{1}{N} X_{s,N} \Pi_{U_{s,s,N}}^\perp Z_N^T = 0, \quad (5-8)$$

$$\text{rank} \left(\lim_{N \rightarrow \infty} \frac{1}{N} E_{s,s,N} \Pi_{U_{s,s,N}}^\perp \begin{bmatrix} U_{0,s,N} & Y_{0,s,N} \end{bmatrix} \right) = n. \quad (5-9)$$

These two equations ensure that PO-MOESP will asymptotically lead to an unbiased consistent estimation of the column space of the extended observability matrix $\mathcal{O}_s$.

In this algorithm we utilize the RQ factorization. The RQ factorization of a matrix $M \in \mathbb{R}^{m \times n}$ is given by

$$M = RQ, \quad (5-10)$$

where $Q \in \mathbb{R}^{n \times n}$ is an orthogocal matrix and $R \in \mathbb{R}^{m \times n}$ is lower triangular. Applying RQ

factorization on the data matrix $\begin{bmatrix} U_{s,s,N} \\ U_{0,s,N} \\ Y_{0,s,N} \\ Y_{s,s,N} \end{bmatrix}$,

$$\begin{bmatrix} U_{s,s,N} \\ U_{0,s,N} \\ Y_{0,s,N} \\ Y_{s,s,N} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q1 \\ Q2 \\ Q3 \end{bmatrix}, \quad (5-11)$$

we derive the matrix $R_{32} \in \mathbb{R}^{sl \times s(l+m)}$. It is stated by the Theorem 9.4 in the book that asymptotically the column space of the matrix R_{32} is equivalent to the column space of the observability matrix $\mathcal{O}_s$, i.e.

$$\text{range}\left(\lim_{N \rightarrow \infty} \frac{1}{\sqrt{N}} R_{32}\right) = \text{range}(\mathcal{O}_s). \quad (5-12)$$

Hence the matrix R_{32} can be used to asymptotically obtain an unbiased estimation of the system matrix A_T and C_T .

System order estimation

The column space of the matrix R_{32} is computed by the SVD,

$$R_{32} = U\Sigma V^H. \quad (5-13)$$

The matrix Σ contains the singular values of the matrix, which are crucial for estimating the order of the system n . The larger the singular value, the more significant the mode is. On the other hand, the smallest singular values contain the least significant modes and are often attributed to noise in the system. Therefore we assign the number of dominant singular values, n , as the order of the system.

We construct data matrices using a parameter value of $s = 100$, assuming a system order no higher than 100. Subsequently we apply SVD to the resulting matrix R_{32} (defined in equation 5-11). Figure 5-2 presents the obtained singular values in descending order on a logarithmic scale for the symmetrical system (containing surge, heave and pitch motions). The plot reveals a distinct drop-off in magnitude after approximately 52 singular values, suggesting a system order of 52^{th} for the system.

However, a closer inspection of the singular values at the corner of the drop-off indicates that they attenuate to a scale of $\times 10^{-13}$ while the largest singular values remain significantly higher value of 1.6. Compared to the dominant singular values, these smaller values generally could represent minimal information or noise within the system. Therefore we discarded them and estimate the system order based solely on the largest singular values. To assess this possibility of system order reduction, we will analyze the energy ratio of the singular values.

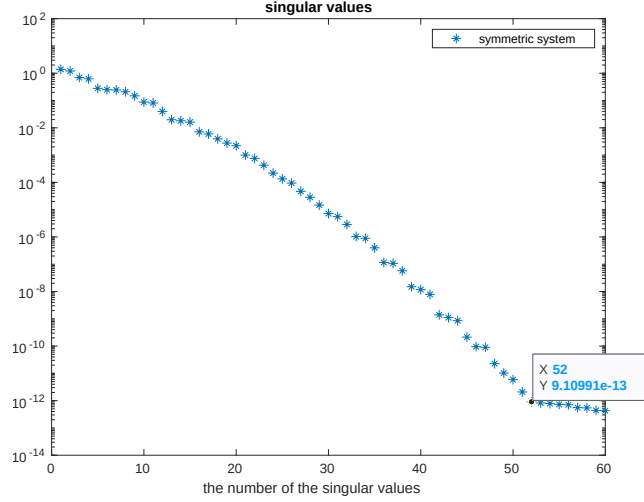


Figure 5-2: Singular values of the symmetrical system

The total energy captured by all the singular values are calculated by summing the square of all the singular values. For each order j , the accumulative energy ratio is calculated by dividing the sum of squared singular values up to that order by the total energy [22]:

$$\text{Energy ratio}(j) = \frac{\sum_{i=1}^{i=j} \sigma_i^2}{\sum_{i=1}^{i=s} \sigma_i^2}, \quad (5-14)$$

in which s represents the total number of the singular values, i is the index of a singular value and σ_i represents the i^{th} singular value of the system.

Figure 5-3 shows the energy ratio plot of the symmetrical system. We can identify a 'knee' point on the curve, where the cumulative energy ratio begins to plateau. This indicates that most of the energy is captured by the components up to that order.

When estimating the system order we should follow the principle of parsimony, that is to use the simplest model that adequately captures the system's behaviour. The goal is to find a balance between the goodness-of-fit and the system complexity [9].

Considering the parsimony principle in mind, the energy ratio plot and the VAF computed with system models of different orders, we estimate a 12th order system for the symmetrical system and a 10th order system for the asymmetrical system.

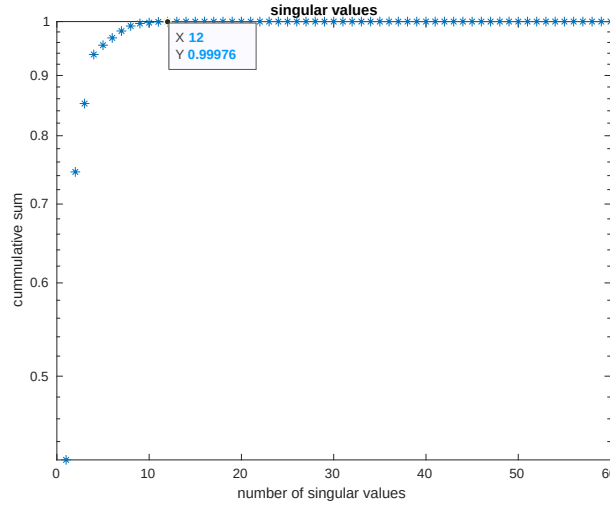


Figure 5-3: Energy ratio of the singular values - symmetrical system

Solving system matrices

Once we have estimated the system order n using the method described earlier, we can approximate matrix R_{32} by considering only the first n singular values. This approximation can be obtained using the following formula

$$R_{32} = U_n \Sigma_n V_n^H, \quad (5-15)$$

where $U_n \in \mathbb{R}^{sl \times n}$, $\Sigma_n \in \mathbb{R}^{n \times n}$ and $V_n \in \mathbb{R}^{s(l+m) \times n}$. The matrix U_n satisfies

$$U_n = \begin{bmatrix} C_T \\ C_T A_T \\ \vdots \\ C_T A_T^{s-1} \end{bmatrix}. \quad (5-16)$$

The system matrix C_T equals the first l rows of U , $C_T = U(1 : l, :)$. The matrix A_T is computed by solving the following overdetermined equation, which due to the condition $s > n$ has a unique solution,

$$U(1 : (s-1)l, :) A_T = U(l+1 : sl, :). \quad (5-17)$$

The system matrix B_T , D_T and the initial state $x_T(0)$ can be computed by solving a least squares problem. Given the estimated matrices A_T and C_T , the output signals $y(k)$ can be expressed by the equation

$$y(k)^T = C_T A_T^k x_T(0) + \left(\sum_{\tau=0}^{k-1} (u(\tau) + e(\tau))^T \otimes C_T A_T^{k-\tau-1} \right) \text{vec}(B_T) + ((u(k) + e(k))^T \otimes I_l) \text{vec}(D_T). \quad (5-18)$$

Taking

$$\theta(k^T) = \begin{bmatrix} C_T A_T^k & (\sum_{\tau=1}^k (u(\tau) + e(\tau))^T \otimes C_T A_T^{k-\tau-1}) & ((u(k) + e(k))^T \otimes I_l) \end{bmatrix}, \quad (5-19)$$

and

$$\theta = \begin{bmatrix} x_T(0) \\ \text{vec}(B_T) \\ \text{vec}(D_T) \end{bmatrix}, \quad (5-20)$$

an unbiased estimate of θ can be obtained by solving the least squares problem

$$\min_{\theta} \frac{1}{N} \sum_{k=0}^{N-1} \|y(k) - \theta(k)\|_2^2. \quad (5-21)$$

The solved system matrices A for both symmetrical and asymmetrical systems exhibit a pronounced diagonal dominance, with significantly larger values along the main diagonal compared to off-diagonal elements. This characteristic suggests a relatively decoupled system behavior, where the influence of each state on its own future evolution is more pronounced than its interaction with other states.

Chapter 6

Testing results with Subspace method

To evaluate the performance of Subspace method identifying wave-to-motion system, coherence analysis is employed to evaluate the quality of the input-output relationship for both systems. Subsequently, the performance of each method is assessed in terms of Variance Accounted For (VAF) values. In this chapter Both synthetic and real field data are utilized for a comprehensive evaluation.

6-1 Synthetic data testing result

The synthetic waves were generated the same way as in Section 3-1-1. One objective of the thesis is to compare motion prediction performances of force-to-motion system and wave-to-motion system using Spectral Analysis method and Subspace method, respectively. Intuitively, a wave-to-motion system, directly mapping waves to motions, was expected to outperform a force-to-motion system, which introduces an additional step of estimating wave forces.

6-1-1 Coherence Check

To quantify the potential advantage of the wave-to-motion system in capturing the underlying system dynamics, coherence analysis was conducted. Figure 6-1 compares the coherence between input and output signals for both systems, focusing on the frequency range of 0.4 to 1.4 rad/s where most signal energy resides.

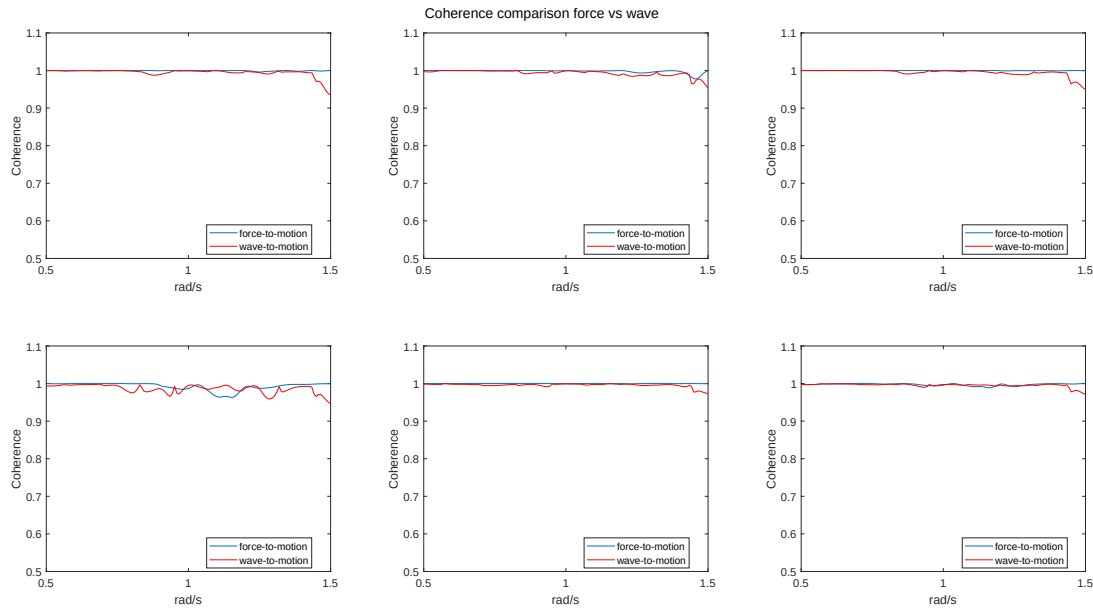


Figure 6-1: Coherence comparison force-to-motion VS wave-to-motion

The wave-to-motion system exhibits a high level of coherence, approaching 1 across the entire frequency band, indicating a strong linear relationship between wave components and motions. However, minor fluctuations in the coherence are observed, potentially due to grouping of waves (section 5-1-1). The waves that grouped together have slight different impact on the ship. On the other hand, the force-to-motion system demonstrates a more consistent coherence close to 1, suggesting a robust linear relationship between input forces and output motions.

It is important to note that the tests we conducted in this section were using synthetic data. Consequently, the potential advantage of wave-to-motion system due to possible error in computed forces in force-to-motion system were minimized. We will repeat the analysis with real field data to draw definitive conclusions on the input-output relationship.

6-1-2 VAF Comparison

The parameters we selected for Subspace algorithm are summarized in Table 6-1.

Total length	10min
wave bin size	15°
order of the system	12 (symmetric) and 10 (asymmetric)

Table 6-1: Parameters for subspace method

To assess the algorithm's robustness, its performance is evaluated with various noise levels, ranging from a SNR 20dB to 50dB. As described in Section 4-1, the noise contained in this system is mostly from the measurement noise in the wave components. Figure 4-1 illustrates

a noisy wave spectrum with an SNR of 20 dB, representative of the noise conditions we have considered in this thesis.

The VAF values obtained for different noise levels are presented in Figure 6-2. Unexpectedly, the wave-to-motion system identified using the Subspace method consistently underperformed the force-to-motion system identified using the Spectral Analysis method at all six directions and different noise levels.

Coherence analysis, as presented in Section 6-1-1, indicated comparable coherence levels for both systems, suggesting that the discrepancy in performance is not primarily due to differences in the input-output relationship. To gain deeper insights into the underlying reasons for this unexpected outcome, future further investigation is required. This includes exploring the impact of model order and the specific characteristics of the subspace and spectral analysis methods employed.

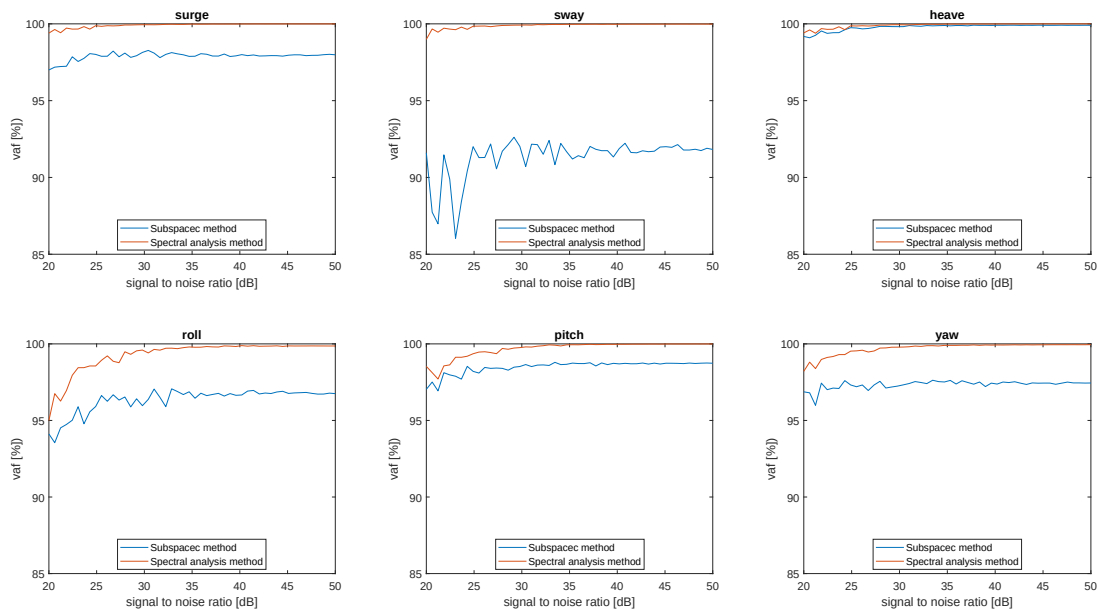


Figure 6-2: VAF robustness test of subspace method, comparing with the spectral analysis method

6-2 Experiment testing result

Since the wave conditions can significantly alter the wave-to motion system dynamics, re-evaluating the system order for different wave scenarios is recommended. As with synthetic data, we will determine the system order by analyzing the singular values and their energy ratio.

6-2-1 Subspace method comparing with Spectral analysis method

Field data 1 - relative wave direction centered around -185°

For this hour of data, the wave condition and the ship heading has been introduced in Section 4-2-1. The singular values computed for this hour of data are plotted in Figure 6-3. Clear drop-offs in magnitude are observed after 26 singular values for both symmetrical system and asymmetrical system. Similar to Section 5-1-1, the largest singular value is 1000 times larger than the smallest dominant singular value. Therefore we will use the energy ratio to implement the order reduction following the principle of parsimony.

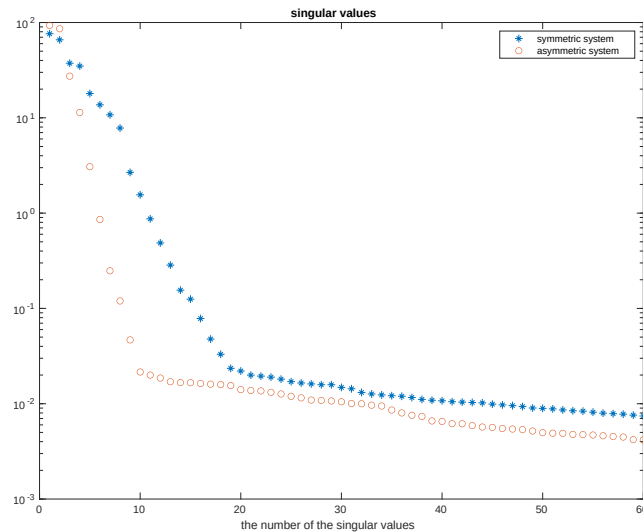


Figure 6-3: Singular values

With 6 singular values for the both systems more than 99.5% of the energy are captured, see the energy ratio shown in Figure 6-4. Therefore we can estimate the system as two 6th order systems. This system orders estimation are confirmed to be valid for the entire hour data.

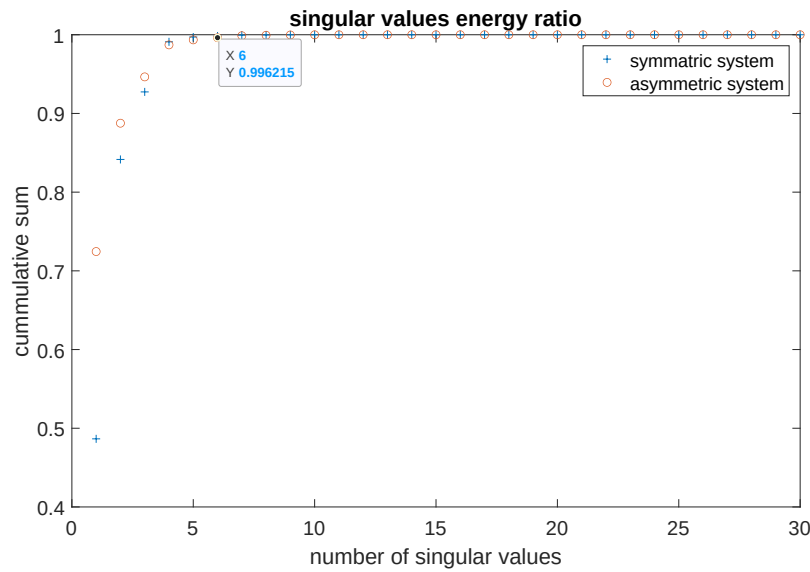


Figure 6-4: Singular values energy ratio

Figure 6-5 presents the coherence values obtained from wave-to-motion system, comparing with force-to-motion system. The wave-to-motion system consistently achieves higher coherence across all 6DoF. This suggests a stronger linear relationship between the wave inputs and the predicted motions, potentially indicating a better ability to capture the system dynamics. Conversely, the force-to-motion system shows significantly lower coherence, particularly for asymmetrical motions (sway, roll, yaw).

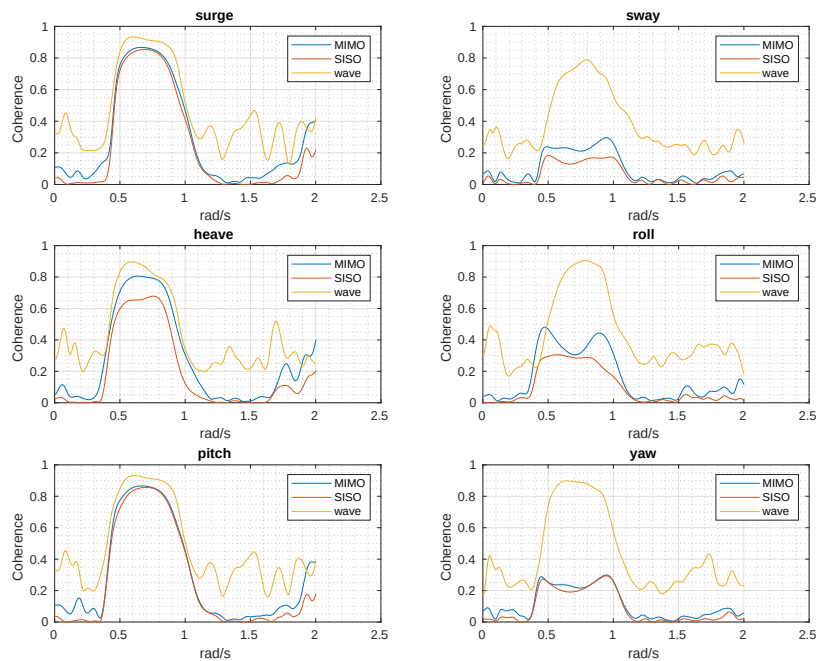


Figure 6-5: Coherence dataset 1 - Wave-to-motion VS MIMO & SISO force-to-motion

Table 6-2 summarizes the VAF obtained from Subspace method, compared to the Spectral Analysis method. Aligned with the coherence analysis, the Subspace method (wave-to-motion) outperforms the Spectral Analysis method (force-to-motion) on asymmetrical (sway, roll and yaw) motions, with VAF improvements of 2.6%, 5.5% and 2.1% for sway, roll, and yaw, respectively. However, on symmetrical motions (surge, heave, and pitch), the Subspace method unexpectedly underperforms, exhibiting VAF reductions of 9.6% and 9.5% for surge and pitch, respectively.

These results highlight an interesting disconnect. A strong correlation exists between coherence and VAF performance on asymmetrical motions. Nonetheless, this relationship is less evident on symmetrical motions, indicating that other factors might be influencing the performance discrepancy. To further investigate this inconsistency, we will analyze coherence and VAF results from a few other data sets in the following section.

DoF	Surge	Sway	Heave	Roll	Pitch	Yaw
Subspace	88.0%	62.7%	82.8%	74.6%	87.6%	81.8%
MIMO Spectral Analysis	81.3%	64.4%	83.4%	78.9%	80.8%	83.6%
Subspace>Spectral	-9.6%	2.6%	0.7%	5.5 %	-9.5%	2.1%

Table 6-2: Subspace vs Spectral Analysis Vaf - dataset 1

Field data 2 - relative wave directions centered around -216°

Figure 6-7 presents a comparison of the coherence values obtained from both wave-to-motion system and force-to-moiont system from another hour of data.

While for this hour of data the wave-to-motion system exhibits slightly higher overall coherence, the corresponding VAF results in Table 6-3 show the Subspace method still underperforming across all directions. This finding reinforces the notion that factors beyond coherence, such as model order, noise sensitivity, or system dynamics, are likely contributing to the observed performance differences.

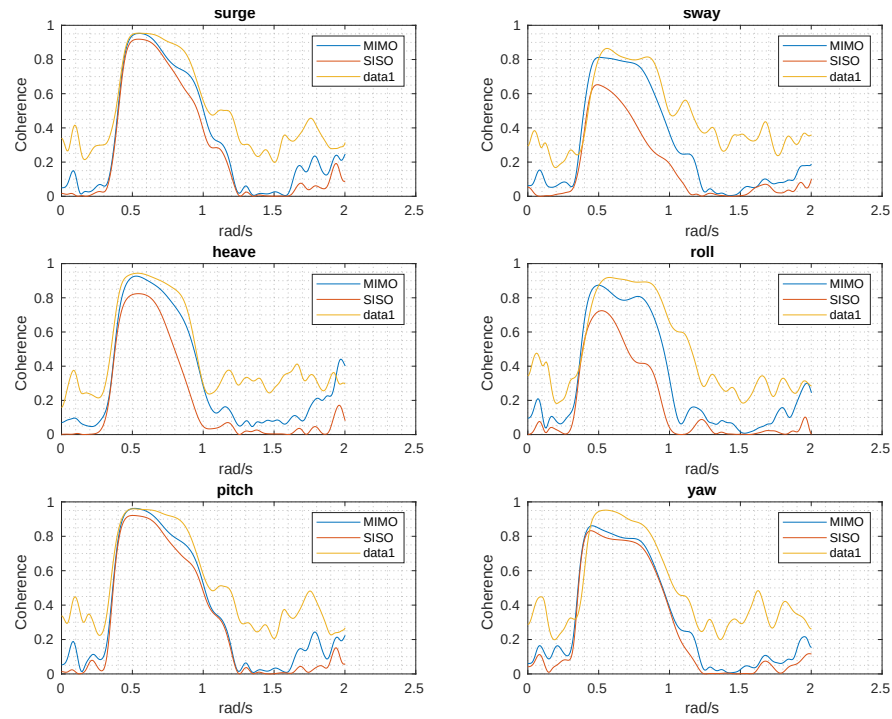


Figure 6-6: Coherence dataset 2 - Wave-to-motion VS MIMO & SISO force-to-motion

DoF	Surge	Sway	Heave	Roll	Pitch	Yaw
Subspace	81.8%	57.8%	74.7%	70.9%	81.6%	72.0%
MIMO Spectral Analysis	87.4%	76.4%	86.4%	82.5%	87.4%	85.4%
Subspace>Spectral	-7.9%	-4.6%	-3.5%	-13.0%	-9.2%	-8.9%
Conventional	57.8%	68.9%	22.7%	55.3%	45.2%	59.9%

Table 6-3: Subspace vs Spectral Analysis vaf - dataset 2

Dataset3 - Field data 2 - relative wave directions centered around -220°

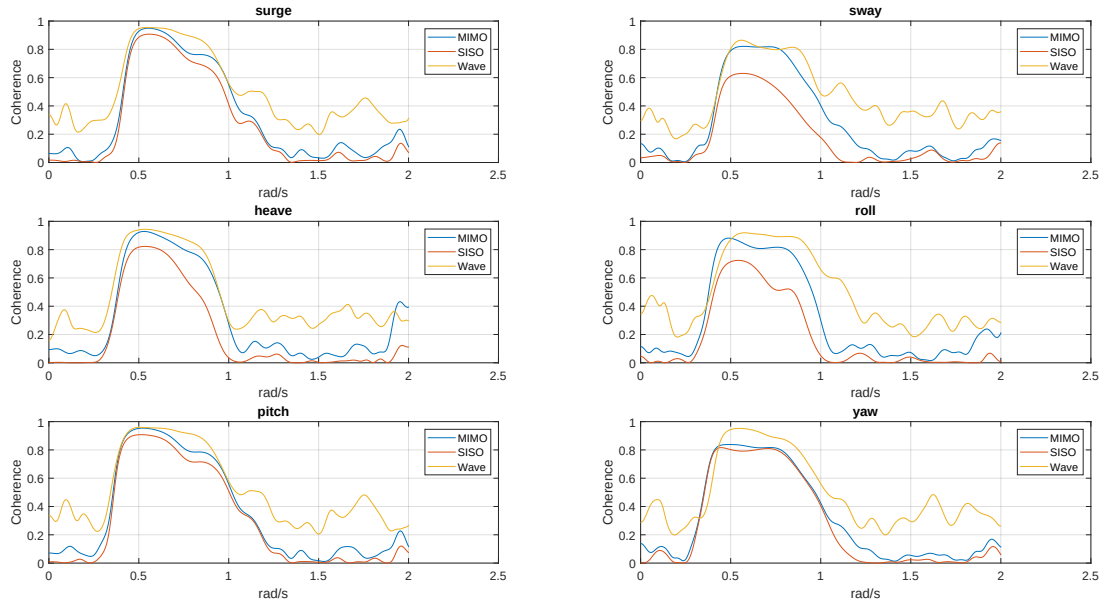


Figure 6-7: Coherence dataset 3 - Wave-to-motion VS MIMO & SISO force-to-motion

DoF	Surge	Sway	Heave	Roll	Pitch	Yaw
Subspace	83.9%	76.7%	85.12%	84.6%	86.6%	85.1%
MIMO Spectral Analysis	87.7%	85.7%	86.5%	91.8%	89.1%	91.3%
Subspace>Spectral	0.2%	6.0%	9.9%	3.4%	1.0%	-2.6%

Table 6-4: Spectral Analysis Vaf - dataset 3

When the wave-to-motion coherence is significant higher than the force-to-motion, as for the asymmetrical motions in field dataset 1, Subspace method on the wave-to-motion system might performance better than Spectral Analysis method on force-to-motion system. However when the wave-to-motion exhibit slightly higher coherence, Subspace method on the wave-to-motion system showed less desired performance. This finding suggests that other factors beyond input-output relationships, such as model complexity, noise sensitivity, or underlying system dynamics, are influencing the performance of system identification method.

To gain a deeper understanding of these factors and improve the predictive capabilities of both methods, further investigations are necessary which may including considering alternative system identification techniques, particularly for the wave-to-motion system.

6-3 Summary

This chapter compared the performance of wave-to-motion and force-to-motion systems in predicting vessel motions using Subspace and Spectral Analysis methods, respectively. While

the wave-to-motion system exhibited stronger input-output relationships, as indicated by higher coherence values, the expected better performance in terms of motion prediction was not consistently observed.

The Subspace method, applied to the wave-to-motion system, encountered challenges due to the high order nature of the system, necessitating careful model order selection. The use of singular values and energy ratios aided in determining appropriate model dimensions. However, the dynamic nature of wave conditions implies that the optimal system order may vary.

Chapter 7

Conclusion

This thesis investigated the potential for improving motion prediction through the application of MIMO system identification techniques. By comparing force-to-motion and wave-to-motion systems using Spectral Analysis and Subspace methods, respectively, the study aimed to assess the advantages of each approach. In this conclusion chapter the main objectives which were defined in Chapter 1 will be answered based on literature review, synthetic data testing and field data testing result. Furthermore, in this chapter also recommendations will be given on how the accuracy of ship motions predictions can be further improved.

7-1 Conclusions

Force-to-motion system with Spectral Analysis method

We constructed two MIMO force-to-motion systems based on the hydrodynamic theory, namely the symmetrical system and asymmetrical system. The coherence analysis on the synthetic data and field data confirmed that MIMO force-to-motion system captured the dynamics between wave excitation forces and motions more accurately than the SISO force-to-motion system.

VAF results generally support the findings from the coherence analysis, with the MIMO method outperforming the SISO approach for most motion directions. However, discrepancies were observed, particularly for yaw motion, where the MIMO system underperformed. This inconsistency suggests that factors beyond the strength of the input-output relationship, such as model complexity or noise sensitivity, influence the predictive performance.

Wave-to-motion system with Subspace method

The wave-to-motion system demonstrated a stronger linear relationship with wave inputs compared to the force-to-motion system, as evidenced by higher coherence values. This sug-

gested potential advantages for motion prediction using the wave-to-motion approach. However, the VAF results revealed that the Subspace method (wave-to-motion) underperforming the Spectral Analysis method (force-to-motion) in many cases.

7-2 Recommendations

This thesis has explored the potential of MIMO system identification for improving motion prediction. While the findings offer valuable insights, several recommendations for future research can further advance this field.

More Investigation on the performance of the MIMO Spectral Analysis method on a wider range of real-world sea states and operational scenarios could be done. This will provide a more comprehensive understanding the reason why certain instances SISO system performs better than the MIMO system for certain direction, specifically on yaw direction.

The wave-to-motion system is expected to achieve better motion prediction according to coherence analysis. As we have already seen, wave-to-motion systems are high order system, which prone to over-fitting the data. For future work, conduct a more in-depth analysis of the impact of model order, one could also investigate in tools to handle over fitting in the model. We could also investigate in other system identification methods which identifies models with simpler structures, such as Spectral Analysis method.

Appendix A

From Waves to Wave Spectra

A-1 Sea Surface and Waves

Scientists and engineers have been studying ocean waves for a wide range of applications, from design of the ship to safety operations on offshore structures for instance. They want to quantify what they see, record the moving sea surface and eventually predict waves.

To have a good understanding of sea waves, the definitions of *surface elevation* and *wave* should be distinguished. The surface elevation is the instantaneous elevation of the sea surface, while a wave is a profile between two down zero-crossings of the surface elevation [6].

The sea surface elevation can be modeled as a summation of cosine waves with different amplitudes, directions, frequencies and phases. The superposition is visualized in the classical drawing as in Figure A-1, it is the popular *random phase/amplitude model* as given in Equation A-1, described in the earth-fixed body-bound reference frame S_e [13]. (Please reference to the description of the Earth-fix body-bound reference frame S_e given in Chapter B.)

$$\eta(\vec{x}_e, t) = \sum_{n=1}^N a_n \cos(\vec{k}_n \cdot \vec{x}_e - \omega_n t + \varphi_n), \quad (\text{A-1})$$

where

$\eta(\vec{x}_e, t)$ represents the surface elevation at certain location (x_e, y_e) at time instant t ,

$\vec{x}_e = (x_e, y_e)$ is an arbitrary point on the still water plane,

a_n is the amplitude of the n -th cosine wave,

N is the total number of cosine waves,

$\vec{k}_n = (k_{n,x}, k_{n,y})$ is the wavenumber of the n -th wave component,

$|\vec{k}_n| = 2\pi/\lambda_n$, λ_n is the wave length,

ω_n is the angular frequency,

and φ_n is the phase of the wave.

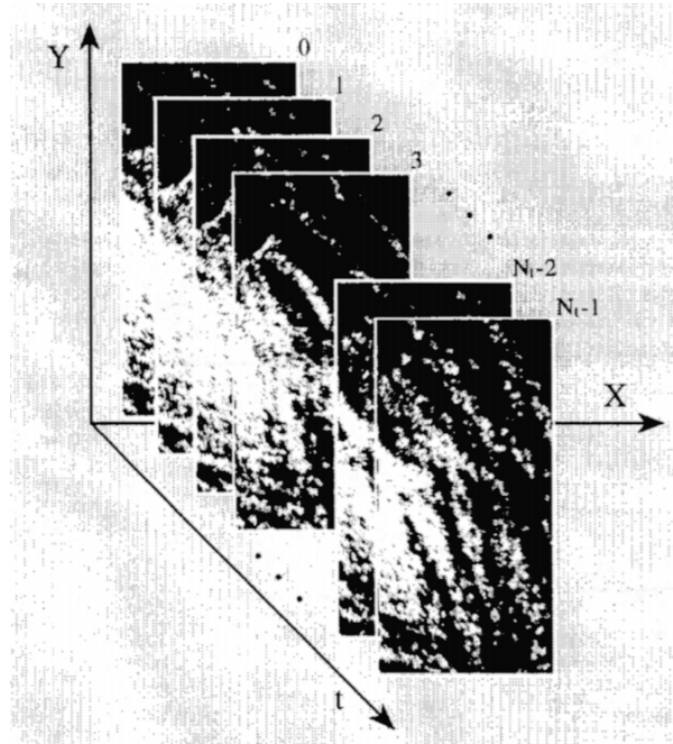


Figure A-1: Sea surface as a superposition of cosine waves [13]

A-2 Radar Image to Wave Field

Various techniques with measurement instruments to observe sea surface have made significant advances during last decades. The instrumental techniques can be divided into *in situ* techniques and *remote-sensing* techniques. For an *in situ* technique the instrument is deployed in the water, while for a remote sensing technique the instrument is deployed at some distance above the sea surface [11].

The application covered in this literature review employs a remote sensing technique, utilizing a marine X-band radar system. The X-band radar is commonly installed on the ship or floating platform, used to monitor moving structures and to derive statistical sea state parameters. With the recent development of X-band radar techniques and computer technologies, it is feasible to extract sea wave components in the sea surface model (Equation A-1), which can be used to predict the sea field at the position of the floating structure in real time [3].

A-2-1 Radar Image

A marine radar sends out electromagnetic waves, the waves interact with the sea surface ripples caused by the local wind. This interaction produces back-scattered electromagnetic energy, the energy is then received by the radar antenna [4].

The back-scattered electromagnetic energy can be modulated by the large structures on the

sea surface, such as swell and wind sea waves. Other radar imaging mechanisms, such as shadowing modulation and tilt modulation, will modulate the energy as well. Altogether they are responsible for the pattern of the collected radar image [4].

Figure A-2 shows an example of a radar image collected by Next Ocean B.V., given in polar coordinates. The horizontal axis displays the azimuth angles with respect to the earth eastern direction, the vertical axis displays the range measured from the radar location.

The brightness intensity in the image indicates the intensity of the reflected electromagnetic energy, it indirectly gives us information about the structure or object that reflects the energy [18].

Looking at the example image, based on the shape and intensity of the bright pattern,

- in the orange highlighted region, electromagnetic energy is reflected by waves,
- the green highlighted bright spot indicates strong electromagnetic energy modulation due to a large offshore structure,
- in the red highlighted area the strong reflections are formed by another radar imaging mechanism called *Range dependency* [20]. Details for *Range dependency* will be spared in this literature review, since it has no relevance to the research.

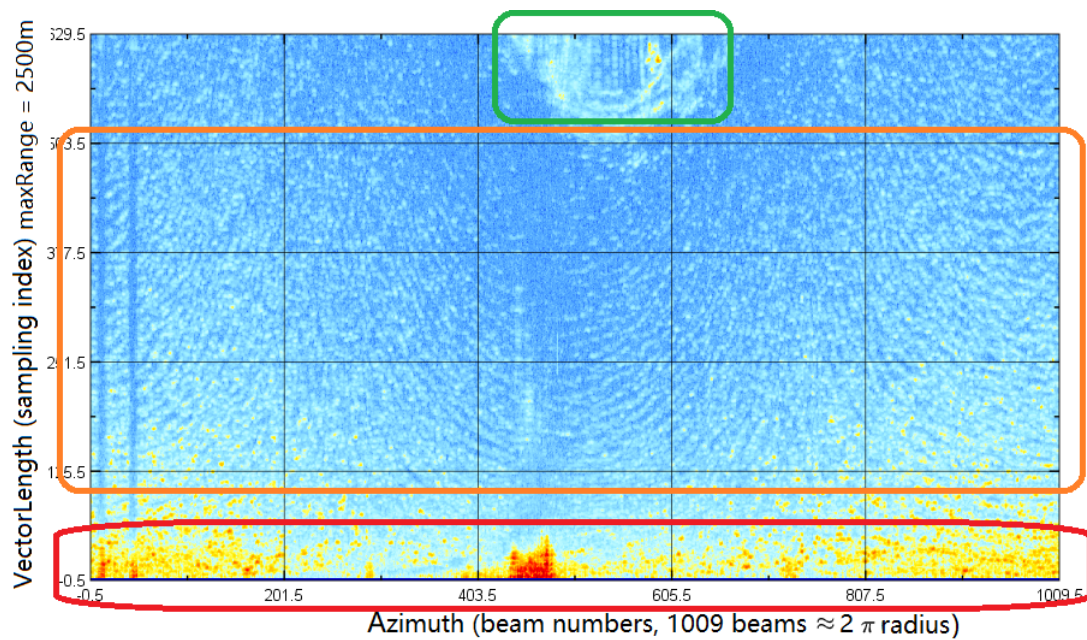


Figure A-2: Example of a radar image, source: Next Ocean

A-2-2 Radar Images to Wave Spectrum

The wave spectrum can be retrieved from the radar images through following steps [4]:

- Collect radar image time series
A collection of N_{im} radar image series is collected consecutively, the images are trans-

formed from polar coordinate to Cartesian coordinate, as shown in Figure A-3. The sampling time is the antenna rotation time.

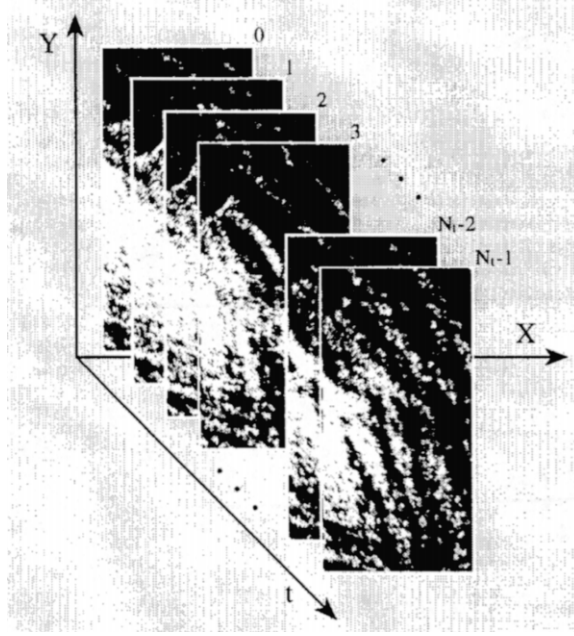


Figure A-3: Radar images time series [4]

- Image normalization

This step is performed by subtracting the mean intensity of the radar image time series to avoid the main contribution of the static pattern present in the data set, e.g. due to the range dependency (the red highlighted area in Figure A-2).

- Three Dimensional Fast Fourier Transform (3DFFT)

As the name suggested, 3DFFT operation applies Fast Fourier Transform (FFT) operations along three dimensions, namely along x-axis, y-axis and time axis.

The resulting spectrum is the so-called image spectrum, denoted as $I_o(k_x, k_y, \omega)$, in which k_x and k_y are the wavenumber projections on x-axis and y-axis respectively. The image spectrum is a consequence of many radar back-scattering mechanisms, rather than a spectrum image of the wave field. To obtain the desired wave field, further procedure steps are required.

- Applying high-pass filter

The purpose of this step is to eliminate the non-stationary and non-homogeneous trends in the images. A good empirical threshold for the frequency of the high-pass filter is $\omega = 2\pi \times 0.03[\text{rad/s}]$, as lower frequencies may not be considered as swell or wind sea waves. The image spectrum after the high pass filter is denoted as $I_f(k_x, k_y, \omega)$.

- Surface current estimation

The wave field is solved based on the assumption that the ocean waves are dispersive,

which means that $\vec{k}$ and ω satisfy the dispersion relationship, given in Equation A-2.

$$\omega = \sqrt{g|\vec{k}| \tanh |\vec{k}|d} + \vec{k} \cdot \vec{U}, \quad (\text{A-2})$$

in which

ω is the wave angular frequency,

$\vec{k} = (k_x, k_y)$ is the two-dimensional wavenumber,

g is the gravitational constant.

$\vec{U}$ is the surface current speed.

The *surface current* is the water on the sea surface moves in a certain pattern. The current speed can be deduced from the image spectrum $I_f(k_x, k_y, \omega)$ and Equation A-2 by the *least square fitting*.

- Filtering the image spectrum

Once the component of the current has been fitted, the next step is to remove all the energy that does not belong to the wave field. For this purpose, the dispersion relationship Equation A-2 is used. All the wave components that away from the dispersion relationship will be filtered out, the three-dimensional spectrum $S(k_x, k_y, \omega)_3$ of the wave field is obtained.

A two-dimensional spectrum $S(k_x, k_y)_2$ can be generated by integrating the 3D spectrum along the ω axis. An example of Next Ocean B.V. generated two-dimensional wave spectrum $S(k_x, k_y)_2$ is shown in Figure A-4.

In this spectrum image the horizontal and vertical axes displays the wavenumber components k_x, k_y respectively. There is one wave system visible in the image, which has the energy concentrated at approximately $(k_x, k_y) = (300, 300)[\text{rad/s}]$.

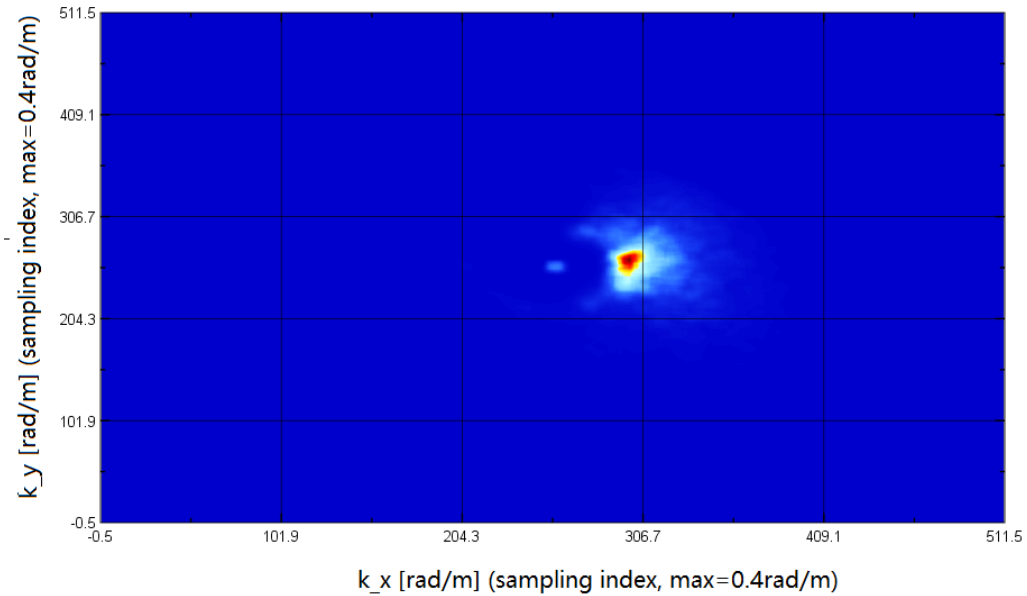


Figure A-4: Example of a directional wave spectrum, source: Next Ocean

A-2-3 Wave Spectrum to Individual Wave Component

A number N of wave components that contain the most wave energy are resolved from the wave spectrum $S(k_x, k_y)_2$, where N is typically 1500.

Given the surface current, the information of the wave frequency ω for each point on the $k_x - k_y$ grid can be retrieved from the Equation A-2. Instead of the wavenumber, an equivalent directional parameter can be used here is the wave propagation angle μ_e . The relationship between the propagation angle and the wavenumber is expressed as:

$$k_{n,x} = |k_n| \cos(\mu_{b_n}), k_{n,y} = |k_n| \sin(\mu_{b_n}). \quad (\text{A-3})$$

A $\mu_e - \omega$ grid thus can be created equivalent to the $k_x - k_y$ grid. The 2D wave spectrum $S(k_x, k_y)_2$ can then be transformed to $S(\omega, \mu_e)$. Correspondingly the resolved N number of wave components are now expressed with the parameter pair (ω, μ_e) .

The wave elevation of the n -th wave component in the frequency domain can be derived from the spectrum using the formula

$$\eta(\omega, \mu_e)_n = \sqrt{2S(\omega, \mu_e)_n \cdot \Delta\omega}. \quad (\text{A-4})$$

These wave components, carrying the information of elevations, wave frequencies and propagation directions, will be used for further procedure to predict the ship motions (will be explained in Chapter B).

Appendix B

System Model

The research topic of this thesis is to investigate if a MIMO system will improve the motion prediction. This chapter draws a complete picture of the motion prediction procedure, starts from the collected radar images until the predicted motions. Section B-1 gives a system model description to the procedure, in Section B-2 the implementation steps for computing the predicted forces and moments are explained, which will be used as the inputs to the MIMO system.

In this chapter the predicted forces and moments are referring to the wave excitation forces and moments. The information contained in this chapter are supplied by Next Ocean B.V..

When describing the wave components on the sea surface and the motions of ships, three different types of reference frames are commonly used, as shown in Figure B-1:

- Geographic coordinate system $G(x,y)$
A geographic coordinate system is a system that uses a three-dimensional spherical surface to determine locations on the Earth. Any location on Earth can be referenced by a point with longitude (x) and latitude (y) coordinates. For the application of Next Ocean B.V. the GPS collected absolute locations of the ship are defined in this reference frame.
- Earth-fixed body-bound coordinate system $S_e(x_e, y_e, z_e)$
The origin of the reference frame is located at the time-averaged Center of Gravity (CoG) of the ship. The positive x_e -axis points at the earth eastern direction, the positive y_e -axis points at the earth north direction.
- Steadily translating coordinate system $O_b(x_b, y_b, z_b)$
The (x_b, y_b) -plane lies in the still water surface with the origin O at the time-averaged position of the CoG of the ship. The direction of the positive x_b -axis is in the forward direction, the positive y_b -axis points in the lateral port (left hand) side and z_b points upwards.

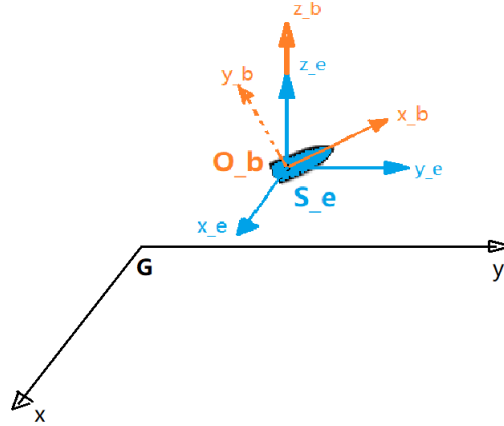


Figure B-1: The reference frames G , S_e , O_b

B-1 System model

The motion prediction sequence contains four parts: the ship path prediction, the wave components determination, the forces, moments prediction and the motion prediction. Every 6 seconds the radar images are updated, a new set of wave components are solved based on these images. At the same time an update of the ship path prediction is computed. The wave excitation forces and moments experienced by the ship can be predicted with the information of the updated wave components and path. We can estimate a MIMO system using the history of the predicted forces, moments and the history of the motion measurements. The new predicted motions can be computed with the estimated MIMO system and the latest generated forces and moments.

The wave components determination has been explained in chapter A, in this section we will discuss the other three parts in detail. In the rest of the report, the word 'loads' is interchangeable with the 'forces and moments'.

B-1-1 The Ship Path Prediction

At time instance t_1 , the beginning of a prediction, the origin of the reference frame S_e and O_b are reset according to the current location of the CoG.

The predicted position of the ship (x_b, y_b) at time instance $t_1 + \tau$ can be estimated by extrapolating the history of the motion measurements. The position is expressed in the reference frame O_b , which is rotated with an angle of heading α_e with respect to the reference frame S_e . The variable $\tau \in [0, 300][s]$ is the prediction time, where $300[s]$ is the prediction horizon.

At time instance t_2 , $t_2 = t_1 + 6[s]$, the next motion prediction sequence begins, the origin for both of the reference frames move with the position of the ship. An illustration of the

position prediction at time instances t_1 and t_2 is given in Figure B-2, the axes z_e and z_b are omitted in the figure for a simple visualization.

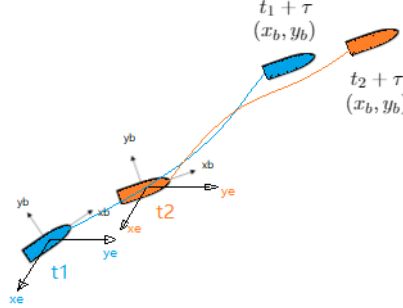


Figure B-2: Ship position prediction at t_1 and t_2

Since the lapse of time between two prediction sequences is shorter than the prediction horizon, there will be overlapped prediction time window $[t_2, t_1 + \tau]$. For computing the predicted forces and moments, only the latest update of the position prediction for this time window will be used.

B-1-2 The loads and motion predictions

A sea surface can be modeled as a sum of cosine waves of various amplitudes, phases, frequencies and propagate directions, as given in Equation A-1. The wave components describing the sea surface are solved and defined in the reference frame S_e , denoted as $\eta_e(\omega, \mu_e)$. In the notation, η_e represents the complex amplitude of the wave, ω is the wave angular frequency and μ_e is the propagation direction with respect to the x_e axis, which points in the earth eastern direction.

The response of a ship to these waves can be described by its Response Amplitude Operators (RAOs). There are two types of RAO, one maps wave components to wave excitation loads, another maps wave components to ship motions. In this report they are denoted as RAO_1 and RAO_2 respectively.

The usage of RAO_2 in the classical applications is to compute the predicted Six Degrees-of-Freedom (6DoF) motions from the determined wave components. Because the RAO_2 depends on e.g. loading condition of the ship, in practise using of the RAO_2 should be avoided. Instead a transfer function G_b will be identified, which maps the wave excitation loads to the motions, such that together with RAO_1 the RAO_2 can be replaced.

The RAO describes the response of the ship in terms of the relative propagation angle of the wave. In order to be able to use RAO_1 for computing the wave excitation loads, the wave components need to be expressed in term of their propagation angles relative to the ship: μ_b . Knowing the ship heading α_e , the angle μ_b of a wave can be calculated with the formula $\mu_b = \mu_e - \alpha_e$. The wave components can now be described in the steadily translating coordinate O_b , as $\eta_b(\omega, \mu_b)$.

We can map the n -th wave component to the 6DoF excitation loads $\vec{F}_b(\mu_b)_n$ that acting at the ship's CoG with the corresponding RAO_1 . Given the predicted position of the ship (x_b, y_b) at time instance t , the predicted total 6DoF force and moments $\mathbf{F}_{tot}$ acting on ship can be computed by summing the loads obtained by all the wave components.

In the previous paragraph the 6DoF excitation loads $\vec{F}_b(\mu_b)_n$ are direction dependant. The directionality is reflected on the ratio of the forces and moments in different modes. The total force and moments $\mathbf{F}_{tot}$ will loss the direction information carried by $\vec{F}_b(\mu_b)$, therefore it will not be denoted in term of μ_b .

With the history of the measured motions and history of the predicted wave excitation loads, a Multiple Inputs Multiple Outputs (MIMO) system G_b can be identified. The motions can be predicted with the system G_b and the latest predicted total forces $\mathbf{F}_{tot}$. The methods that applied to identify the MIMO system will be introduced in Chapter ??.

The above described system model is given in Figure B-3.

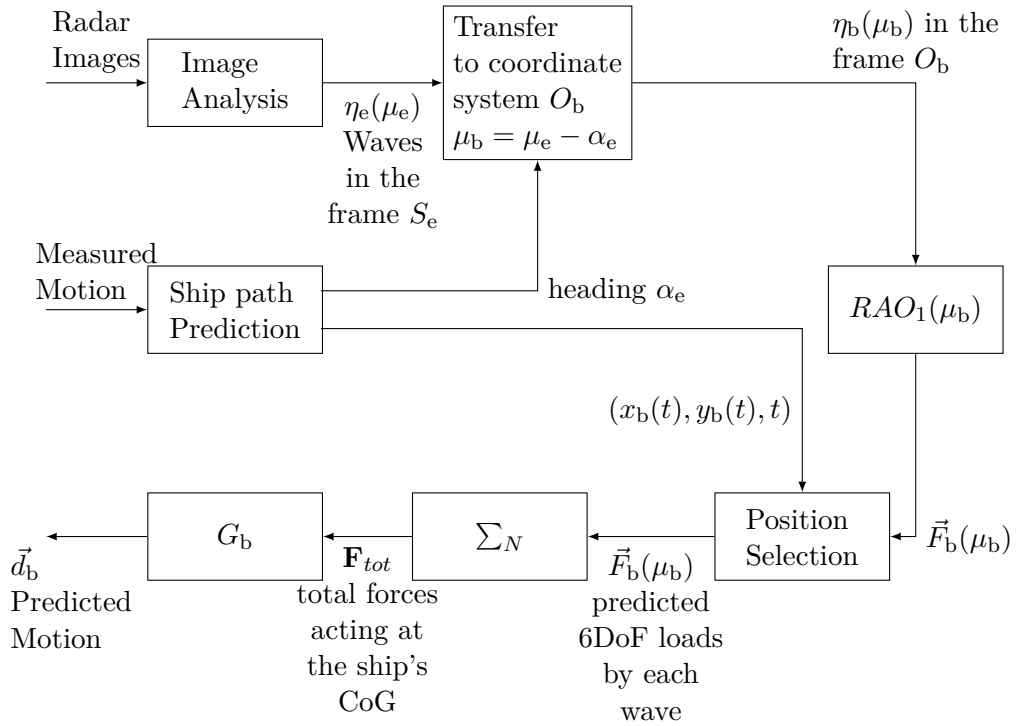


Figure B-3: System model description of the motion prediction procedure

B-2 The Implementation

The subsystem that determines the wave components from the radar images only resolves for a number N of wave components that contain the most wave energy, where N is typically 1500. They are a subset of the points on the μ_b - ω grid.

We can regard this as a sparse discrete Fourier domain representation (where most of the values are zero) or equivalently, as a sum of N phasor domain representations. This sparsity allows us to save computational effort by processing for the motion prediction only the nonzero wave components.

The N number of wave components are stored in a vector $\vec{\eta}_b(\omega, \mu_b)$, in the steadily translating coordinate. Each wave component is uniquely characterized by a pair of parameters, namely the wave frequency ω and relative propagation direction μ_b ,

$$\vec{\eta}_b(\omega, \mu_b) = \begin{bmatrix} \eta_b(\omega, \mu_b)_1 \\ \eta_b(\omega, \mu_b)_2 \\ \vdots \\ \eta_b(\omega, \mu_b)_N \end{bmatrix} \in \mathbb{C}^{N \times 1}. \quad (\text{B-1})$$

B-2-1 The Computation of the Forces in the Phasor Domain

RAO_1 is given in the format of a look-up table $\in \mathbb{C}^{m \times n \times 6}$, the 1st dimension displays the relative directions μ_b , the 2nd dimension displays the wave frequency $\vec{\omega}$ and the 3rd dimension is the DoF.

If the wave angular frequency or the relative direction of certain wave component does not match with any of the values on the grid, the value will be interpolated to the nearest value on the grid. With each pair of parameters (ω, μ_b) , an unique $\text{RAO}_1(\omega, \mu_b)$ values for 6DoF can be found on the table.

When we multiply one complex wave component with the corresponding RAO_1 , the 6DoF forces and moments in the phasor domain for that wave component are obtained,

$$\mathbf{F}_b(\omega, \mu_b) \in \mathbb{C}^{N \times 6} = \begin{bmatrix} \vec{F}_b(\omega, \mu_b)_1 \\ \vec{F}_b(\omega, \mu_b)_2 \\ \vdots \\ \vec{F}_b(\omega, \mu_b)_N \end{bmatrix} = \begin{bmatrix} \eta_b(\omega, \mu_b)_1 \text{RAO}_1(\omega, \mu_b)_1 \\ \eta_b(\omega, \mu_b)_2 \text{RAO}_1(\omega, \mu_b)_2 \\ \vdots \\ \eta_b(\omega, \mu_b)_N \text{RAO}_1(\omega, \mu_b)_N \end{bmatrix}. \quad (\text{B-2})$$

B-2-2 The transformation to the time-spatial domain

A wave on the sea surface propagates in both time and space, therefore we need both time and space information for the inverse phasor operation to transform the wave component from the phasor domain to the spatial-time domain.

Given the predicted location of the ship, the equation for the predicted wave elevation ζ_b , at location $(x_b(t), y_b(t))$ at time instance t , contributed by the n -th wave components is

$$\zeta_b(x_b(t), y_b(t), t)_n = \text{Re}(\eta_b(\omega, \mu_b)_n e^{j(|\vec{k}_n| \cos(\mu_{bn})x_b(t) + |\vec{k}_n| \sin(\mu_{bn})y_b(t) - \omega_n t)}). \quad (\text{B-3})$$

The corresponding 6DoF forces and moment in time-spatial domain are computed with the equation

$$\vec{f}_b(x_b(t), y_b(t), t)_n = \underbrace{\text{Re}(\eta_b(\omega, \mu_b)_n \text{RAO}_1(\omega, \mu_b)_n)}_{\vec{F}_b(\omega, \mu_b)_n} e^{j(|\vec{k}_n| \cos(\mu_{bn})x_b(t) + |\vec{k}_n| \sin(\mu_{bn})y_b(t) - \omega_n t)}. \quad (\text{B-4})$$

The total predicted forces and moments are computed by adding up the predicted loads excited by all the wave components

$$\mathbf{f}_{tot}(x_b(t), y_b(t), t) \in \mathbb{R}^{N_t \times 6} = \sum_{n=1}^N \vec{f}_b(x_b(t), y_b(t), t)_n, \quad (\text{B-5})$$

in which N_t is the length of the prediction horizon.

The above described implementation from the wave components in the phasor domain to the loads in the time domain is summarized in Figure B-4.

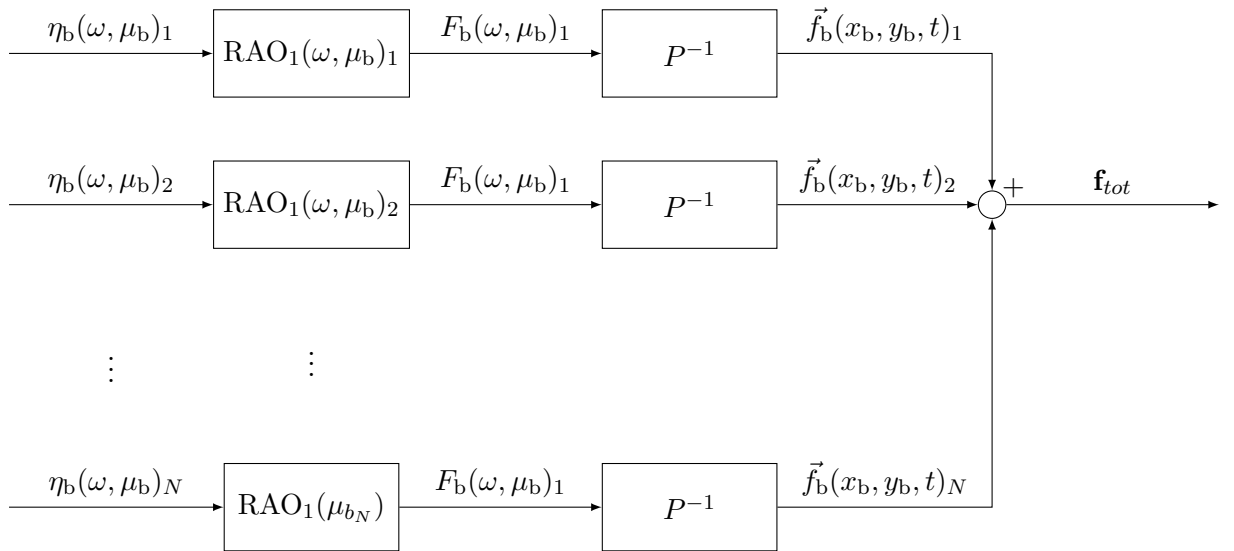


Figure B-4: Diagram from the wave components to the total forces and moments in time domain

Appendix C

Ship dynamics

In this chapter we investigate in the Spectral Analysis identification method. The method will be studied and applied on the force-to-motion MIMO system. We will start the chapter by introducing the ship dynamics, based on which the MIMO system will be constructed. In the part of the chapter we will give detailed explanation on the Spectral Analysis implementation and evaluation.

C-1 Ship dynamics

This section is based on the book *Offshore Hydromechanics* [12], the citation within the section is omitted until stated otherwise.

Just as for any other rigid body, the motions of a ship can be classified as three mutually perpendicular translational motions of the Center of Gravity (CoG) and three rotational motions around CoG. These motions will be described in the steadily translating coordinate system $O_b(x_b, y_b, z_b)$. The (x_b, y_b) -plane lies in the still water surface with the origin O_b at the time-averaged position of the CoG of the ship. The direction of the positive x_b -axis is in the forward direction of the ship, the positive y_b -axis points in the lateral port (left hand) side and z_b points upwards. The six oscillating motions (namely surge, sway, heave, roll, pitch and yaw) of a ship in steadily translating coordinate system are illustrated in Figure C-1.

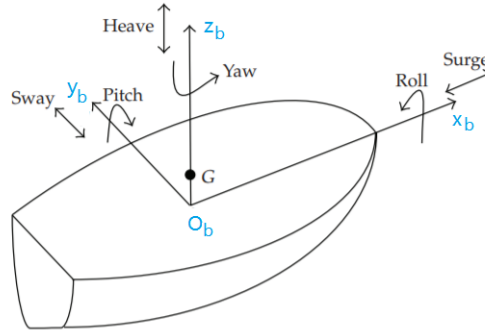


Figure C-1: Ship schematic diagram showing the six degrees of freedom

C-1-1 Motion Equations

Symbolically the equation of motion of a floating body on the sea surface can be written as

$$\mathbf{M}\ddot{\vec{x}}(t) = \vec{F}(t), \quad (\text{C-1})$$

where $\mathbf{M}$ is the mass inertia matrix, $\vec{x} = (x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6)^\top$ is the vector of displacement in the six degree of freedom,

$\vec{F} = (F_1 \ F_2 \ F_3 \ F_4 \ F_5 \ F_6)^\top$ is the vector of the forces and moments acting on the body.

The total force vector can be split into the wave excitation forces $\vec{F}_w$ and reaction forces $\vec{F}_r$ from the water:

$$\vec{F}(t) = \vec{F}_w(t) + \vec{F}_r(t). \quad (\text{C-2})$$

The wave excitation forces are the input of the force-to-motion system that we investigate. The reaction forces $\vec{F}_r$ are the hydrodynamic reaction forces from the water on the moving body in the absence of the waves.

The hydrodynamic force can be approximately characterised by a linear system. Let $\mathbf{A}$ be the added mass coefficient, $\mathbf{B}$ be the radiation damping coefficient and $\mathbf{C}$ be the restoring spring coefficient (more detail on the potential coefficients will be given in section C-1-2). The linear system then can be written as:

$$\vec{F}_r(t) = -\mathbf{A}\ddot{\vec{x}}(t) - \mathbf{B}\dot{\vec{x}}(t) - \mathbf{C}\vec{x}(t) \quad (\text{C-3})$$

Substitute Eq. C-3 and Eq. C-2 into Eq. C-1 the following motion equation yields:

$$(\mathbf{M} + \mathbf{A})\ddot{\vec{x}}(t) + \mathbf{B}\dot{\vec{x}}(t) + \mathbf{C}\vec{x}(t) = \vec{F}_w(t), \quad (\text{C-4})$$

where the matrices $\mathbf{M}$, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are all 6×6 matrices. We will first carefully look at the motion equation in one single direction. The scope will be extended to the situations where coupled directions are considered in later sections.

Motion in single direction

A floating object oscillating in one degree of freedom can be seen as an object connected to a mass-damper-spring system, as shown in Figure C-2.

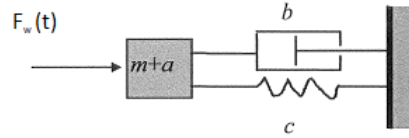


Figure C-2: Motion in one degree of freedom

The motion equation for this system can be written as

$$F_w(t) = (m + a)\ddot{x}(t) + b\dot{x}(t) + cx(t). \quad (\text{C-5})$$

In the equation x is the displacement of the object, a , b and c are the potential coefficients of the object, F_w is the wave excitation force which is the input of the system under investigation in this chapter. The potential coefficients reflect how a object responds in the waves. In the following section we will give a more intuitive introduction on these coefficients.

C-1-2 Potential coefficients

We have all once observed the phenomenon that, when a ship oscillates on the water surface, a certain amount water adjacent will move along with the body. Accelerating this additional water gives reaction forces and moments to the body caused by the pressure field of the water particles. The body mass inertia is increased by an 'added mass a '.

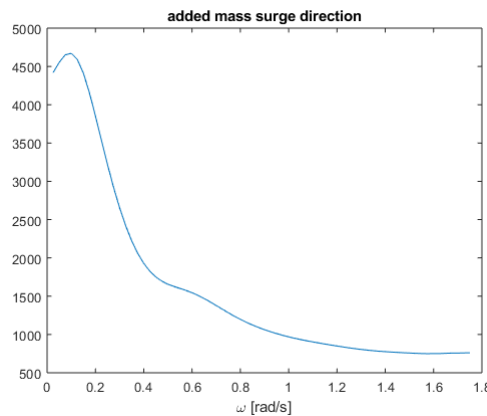


Figure C-3: Added mass along the frequency band for surge

The added mass is strongly influenced by the oscillation frequency. It reflects at different frequencies, the body has variant characteristic on accelerating water particles. As shown in Figure C-3 the added mass significantly declines from low frequencies to high frequencies. The ship accelerates more mass at low frequencies than at high frequencies.

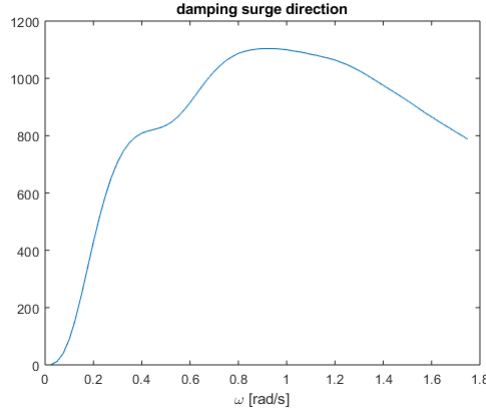


Figure C-4: Damping coefficient along the frequency band for surge

The damping coefficient is influenced by the oscillation frequency as well. See Figure C-4 for an example of radiation damping coefficient on the surge direction. When the ship is oscillating with 1 rad/s, the body has the maximal ability to radiate energy. As ω close to 0, no waves can be formed [10].

The third potential coefficient describes the variation of the buoyancy force due to the motions. This force is proportional to the vertical displacement of the body, the corresponding coefficient is defined as restoring spring coefficient. The restoring spring coefficient is present only for the heave, roll and pitch motions, where the floating body undergoes vertical displacement variations. Unlike previous two potential coefficients, the spring coefficient does not vary with the motion frequency.

C-1-3 Coupled motions

A floating body can exhibit hydrodynamic reactions in any degree of freedom i caused by motions from any direction j . We say that direction i and j are coupled. For a ship with right/left side symmetry, approximate fore/aft symmetry, the matrices $\mathbf{M}$, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ have following structures:

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix}, \quad (\text{C-6})$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 & 0 & a_{15} & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & a_{42} & 0 & a_{44} & 0 & 0 \\ a_{51} & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix} \quad (\text{C-7})$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & b_{15} & 0 \\ 0 & b_{22} & 0 & b_{24} & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 & 0 \\ 0 & b_{42} & 0 & b_{44} & 0 & 0 \\ b_{51} & 0 & 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66} \end{bmatrix} \quad (\text{C-8})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & c_{35} & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & c_{53} & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{C-9})$$

However the potential coefficients matrices in practice will be deviated from this structure, due to e.g. changing mass distribution, asymmetry shape or the speed of the ship. For this reason we will develop an algorithm to compute the force-to-motion transfer function, instead of computing it from the given potential coefficient matrices.

Up to now we developed and discussed the continuous time system model Eq. C-4. From next section we will implement the Spectral Analysis identification method on this model. Since we will deal with sampled signals, from now on the system and signals will be described in discrete time.

Bibliography

- [1] LK Alford, RF Beck, JT Johnson, D Lyzenga, O Nwogu, and A Zundel. Design, implementation, and evaluation of a system for environmental and ship motion forecasting. In *30th Symposium on Naval Hydrodynamics. Hobart, Tasmania, Australia*, 2014.
- [2] Stephen F. Barstow, Jean-Raymon bidlot, Sofia Caries, Mark A. Donelan, and Willian M Drennan. *Measuring and Analysing the Directional Spectrum of ocean waves*. Elsevier Science, 2005.
- [3] Elise Blondel-Couprie and Peter Naaijen. Reconstruction and prediction of short-crested seas based on the application of a 3d-fft on synthetic waves: Part 2 - prediction. volume 5, 07 2012.
- [4] Jose Carlos Nieto Borge. Analysis of wave fields from temporal sequence of x-band marine radar images. 2006.
- [5] Jerome Bresser. improving real-time ship motion predictions by fusing measurements and hydro-dynamical modeling.
- [6] W.H. Buckley, R.D. Pierce, J.B. Peters, and M.J. Davis. Use of the half-cycle analysis method to compare measured wave height and simulated gaussian data having the same variance spectrum. *Ocean Engineering*, 11(4):423 – 445, 1984.
- [7] Zhanyang Chen, Mengchao Du, Xiao Ji, Xingyun Liu, and Weidong Zhao. Prediction of ship future motions at different predictable time scales for shipboard helicopter landing in regular and irregular waves. *Ships and Offshore Structures*, 0(0):1–9, 2024.
- [8] Alan K Cline, Cleve B Moler, George W Stewart, and James H Wilkinson. *An estimate for the condition number of a matrix*.
- [9] M. Deistler. System identification. söderström and p. stoica prentice hall international, 1989. *Econometric Theory*, 10(3–4):813–815, 1994.
- [10] O. Faltinsen. Sea loads on ships and offshore structures. 1 1990.

- [11] L. H Holthuijsen. *Waves in Oceanic and Coastal Waters*. Cambridge Univ.Press, 2007.
- [12] J. M. J Journee and W. W. Massie. *Offshore Hydromechanics*. Delft University of Technology, 2001.
- [13] Willard J. Pierson Jr., Richard W. James, and Gerhard Neumann. Practical methods for observing and forecasting ocean waves by means of wave spectra and statistics. *U. S. Dept. Commerce. Hydrographic Office Publ.*, 603:300, 01 1955.
- [14] Dimitris G. Manolakis and Vinay K. Ingle. *Contents*, pages vii–xii. Cambridge University Press, 2011.
- [15] Peter Naaijen, D.K. Roozen, and Rene Huijsmans. Reducing operational risks by on-board phase resolved prediction of wave induced ship motions. In *Proceedings of the 35th International Conference on Ocean, Offshore and Arctic Engineering*, volume 7 of *ASME Conference Proceedings*, United States, 2016. ASME. Paper No. OMAE2016-54591; ASME 2016 35th International Conference on Ocean, Offshore and Arctic Engineering, OMAE 2016, OMAE 2016 ; Conference date: 19-06-2016 Through 24-06-2016.
- [16] Rik Pintelon and Johan Schoukens. *System Identification : A Frequency Domain Approach*. 2012.
- [17] K. M. M Prabhu. *window function and their application in signal processing*. 2014.
- [18] M A Richards, Jim Scheer, and William A Holm. *Principles of modern radar. Volume I, Basic principles*.
- [19] Ian postlethwaite Sigurd Skogestad. *Multivariable Feedback Control*. 2012.
- [20] Merrill I Skolnik. *Radar Handbook*. 1970.
- [21] O M Solomon, Jr. Psd computations using welch’s method. 1991.
- [22] J. Nathan Kutz Steven L. Brunton. *Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control*.
- [23] M. J. Tucker. *Waves in ocean engineering*. 1991.
- [24] Michel Verhaegen and Vincent Verdult. *Filtering and System Identification*. 2012.
- [25] I.R Young and W. Rosenthal. A three-dimensional analysis of marine radar images for the determination of ocean wave directionality and surface currents. 1985.

Glossary

List of Acronyms

RAOs	Response Amplitude Operators
MIMO	Multiple Inputs Multiple Outputs
3DFFT	Three Dimensional Fast Fourier Transform
CoG	Center of Gravity
DoF	Degrees-of-Freedom
6DoF	Six Degrees-of-Freedom
FFT	Fast Fourier Transform
PSD	Power Spectral Density
CSD	Cross Spectral Density
SISO	Single Input Single Output
DFT	discrete Fourier transform
VAF	variance accounted for
PO-MOESP	Past Output Multivariable Output-Error State-sPace
SNR	Signal to Noise Ratio

