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Exact Formulation and Comparison Between the User Optimum and System Optimum Solution for Routing Privately Owned Automated Vehicles

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Abstract—The possibility of having driverless cars on the streets seems to be more real than ever. In this paper, we focus on developing exact methods that can determine the effects of privately owned automated vehicles (AVs) and how switching to those vehicles is going to change mobility in urban environments. The considered problem determines the routes of family owned AVs that minimize the transportation costs of that family while considering the possibility of using public transport as an alternative for some trips. We introduce a novel exact linear formulation for this problem which includes a linearized traffic congestion model and which is able to solve the user and system optimum variant of the problem to optimality. The introduced formulation can easily be adapted to consider the current situation with conventional vehicles and a situation where not only the travel time costs of the driver but also costs of the other passengers are taken into account. The main advantage of our novel formulation is that the optimal results can be obtained to explore potential changes of flows with vehicle automation in small networks. We investigated the behavior of the system, given the described scenarios, by applying our formulation to a case study.

Index Terms—Autonomous vehicles, integer linear programming, user and system optimum, routing, value of travel time.

I. INTRODUCTION

VEHICLE automation is becoming one of the main topics of transportation research and also of discussions in the mainstream media and popular science publications. There is a fascination with the possibility of cars being able to drive themselves which was before seen as something out of Science Fiction movies. But the possibility of having driverless cars on the streets seems more real than ever. This is being fostered by the successful experiments and pilots happening all over the world either university based or conducted by car manufacturers. The latter are heavily investing in developing automated and connected vehicles. Vehicle automation has become, along with the shared mobility growth, one of the most disruptive factors of transportation

systems nowadays [1]. In terms of research, there are several main research topics that have been gaining more attention: behavior modeling, traffic flow theory, intelligent vehicles, traffic pattern changes estimation, or shared mobility operations with automated vehicles (AVs).

On the behavior modeling, there has been considerable work on measuring the attitude and perception that travelers have when facing driverless vehicles either as part of public transport networks [2], [3] or as normal private cars [4], [5]. These studies produce, among other relevant results, the attractiveness of automated driving which is connected to generalized costs of traveling and transport mode choice.

Probably the most prolific research stream in automated driving has been on the effects on traffic flow capacity especially in highways [6] where simulation and analytical methods are being employed to measure the effects of both automated and connected vehicles [7]–[10]. While in this work, we do not consider traffic capacity changes, we do recognize that this will be different with AVs but there is still a lot of work to be done particularly regarding changes of capacity in urban networks.

In the field of intelligent vehicles, researchers are defining algorithms that should be used to control the cars and trucks in order to optimize the traffic systems and increase road safety. Many decisions have to be taken in real time by the vehicle's computer including for example braking, lane changing or car following. This is what can be referred to as the low level control of the vehicles and it is essential for there to be any fully automated vehicle on the roads in the future [11]–[15].

Regarding shared mobility, there are several studies that employ agent-based simulation rational to study the fleet of vehicles needed and how to operate a shared automated transport system in order to satisfy part of the current urban mobility demand. This is the case for the model developed in [16] for Texas or the model developed in the International Transport Forum for the city of Lisbon [17] where a significant reduction on the number of vehicles needed has been estimated.

Less attention has been devoted to the effects of privately owned AVs and how switching to those vehicles is going to change mobility, particularly in urban environments. Some recent papers aim at doing so in an aggregated way such as the research of [18] that looks at the trade-offs between capacity, value of time and preference heterogeneity in the population

for a car type. The net effects of automation on emissions are explored in [19] through several illustrative scenarios using an equation of emissions dependent on factors such as modal share, energy intensity and fuel.

Very few papers look at route and parking choice of automated vehicles considering empty trips. One of the few examples is the work of [20] where the 4-steps method was changed to address the option of empty vehicle relocation after a vehicle drops its driver off. However, this is still subject to many of the limitations of the 4-steps method.

The recent paper [21] proposed a mathematical model formulation to determine the routes of family owned AVs that minimize the travel costs of that family, while considering the possibility of parking and of using public transport as an alternative for some trips. The model was first formulated as a system optimum traffic assignment and then converted into the user optimum assignment. The user optimum assignment was solved by an iterative algorithm until similar households have similar costs of traveling.

This paper considers a problem similar to [21], however, we have improved the mathematical problem formulation such that a global optimum solution can be achieved for both the system optimum and user optimum. This entails building a single objective function that results in the referred equilibrium between households as well as a traffic congestion model that can be expressed linearly so that the model can be solved by classic branch-and-bound methods. With this, we lose the possibility of solving the problem optimally for real road networks, due to the NP-hardness of the problems, but we are able to obtain exact global optimum solutions for small networks that can be compared for different scenarios. To this end, we apply our novel formulation to a small toy network and compare the exact system optimum solution to the exact user optimum solution in the case where we have conventional and automated vehicles. Solving the system optimum formulation represents a transportation system in which all the mobility by car would be defined by a central computer, not leaving any freedom for the traveler to make any decisions. The user optimum corresponds to vehicles being able to route themselves empty, but their decisions on routing, departure and arrival times would still be decided by humans who would make choices in a selfish way, implicitly equalizing the generalized cost of traveling between similar families, i.e., families that have the same trips. In addition to providing a novel exact formulation which can solve both the system and user optimum to optimality, we also compare two ways to calculate the travel time costs. While in the user optimum only the travel time costs of the driver are considered, in the system optimum it makes sense to consider the total travel time costs of all the people in the vehicle.

Summarizing, in this paper, we contribute to the state of the art of automated vehicles urban flow patterns estimation with the following:

- Develop a novel mathematical formulation for the routing of privately owned automated vehicles which is able to solve the system optimum and user optimum to optimality. This entails building a single objective function that results in the referred equilibrium between households

TABLE I
USED SETS

Notation	Description
H	set of households
E	set of trips
E_h	subset of E containing trips $e \in E$ corresponding to household $h \in H$
V	set of vehicles
V_h	subset of V containing vehicles $v \in V$ belonging to household $h \in H$
I	set of nodes
R	set of arcs between nodes in set I
T	set of time instants

and includes a linear specification of the traffic delay model inspired by [22],

- Solve the system and user optimum to optimality for a small toy network for different scenarios including the comparison between using or not using the total generalized costs of all travelers in the car and using conventional or automated vehicles.

The paper is structured as follows: we start with the problem formulation in Section II which is divided in the base formulation which is common to all considered scenarios, continues with a section on the objective function, and ends with several modifications that have to be done to get different desired scenarios. The paper continues with Section III on the description of the toy network and synthetic mobility data used for running computational experiments as well as the main scenarios considered. In Section IV, the main results for different scenarios are presented and discussed. The paper ends with the main conclusions that can be taken from this research in Section V.

II. PROBLEM FORMULATION

In this paper, we consider the situation where privately owned automated vehicles are used to fulfill the trips of family members in a household. For each trip, a choice is made whether the trip is done by public transport (PT) or if the trip is fulfilled by one of the family vehicles. This decision is based on the generalized transportation costs described in Section II-C. These generalized transportation costs include public transport costs, travel time costs by AV, fuel costs and penalty costs for early and late arrival. Moreover, we make the following assumptions:

- Automated vehicles are allowed to drive empty in the network without any human supervision.
- Each (automated) vehicle routing adds to the traffic flows of the city.
- Each (automated) vehicle has a certain passenger capacity that must be respected.
- The PT trips do not contribute to the traffic flows in the network.
- No external trips to the city are considered in the network.

We present a model for both the system optimum and the user optimum, which can both be solved to optimality by using a commercial solver. To be able to do this, we model the traffic congestion by using linear constraints only, based on the formulation of [22]. One novelty of this paper is that

Notation	Description
Tr_{ev}	binary variable that equals 1 when trip $e \in E$ is performed by vehicle $v \in V$ and 0 otherwise
P_{evt}	binary variable that equals 1 when trip $e \in E$ is executed by vehicle $v \in V$ and starts at time $t \in T$ and 0 otherwise
A_{evt}	binary variable that equals 1 when trip $e \in E$ is executed by vehicle $v \in V$ and arrives at time $t \in T$ and 0 otherwise
$x^v_{ijt_1t_2}$	binary variable which is 1 when vehicle $v \in V$ drives on arc $(i,j) \in R$ from time instant $t_1 \in T$ to time instant $t_2 \in T$ and 0 otherwise
$F_{ijt_1t_2}$	variable representing the flow resulting from routing variables $x_{ijt_1t_2}$
$\delta_{ijt_1t_2}$	binary variable which is 1 when the travel time from node $i \in I$ to node $j \in I$ starting at time instant $t_1 \in T$ equals $t_2 - t_1$ time steps and 0 otherwise
w^v_{it}	binary variable which is 1 when vehicle $v \in V$ is waiting at node $i \in I$ at time instant $t \in T$
L_{vt}	number of people in vehicle $v \in V$ at time instant $t \in T$
l_{vt}	binary variable which is 1 when vehicle $v \in V$ is occupied at time $t \in T$ and 0 otherwise
ϕ_e	deviation from the preferred arrival time Θ^e_b
$e\phi_e$	amount of time a trip arrives earlier than the preferred arrival time Θ^e_b
$l\phi_e$	amount of the time a trip arrives later than the preferred arrival time Θ^e_b

Notation	Description
μ_h	expansion coefficient which represents the number of households with the same characteristics in the population
C_v	capacity of vehicle $v \in V$
$t^{\min}_{ij}$	minimum travel time of arc $(i,j) \in R$
$t^{\max}_{ij}$	maximum travel time of arc $(i,j) \in R$
t^{PT}_{ij}	travel time by public transport on arc $(i,j) \in R$
Lg_{ij}	length of arc $(i,j) \in R$
Q_{ij}	practical link capacity of arc $(i,j) \in R$
$CF_{ijt_1t_2}$	capacity of spatial link from node $i \in I$ to node $j \in I$ from time instant $t_1 \in T$ to time instant $t_2 \in T$
i_e	origin node of trip $e \in E$ with $i_e \in I$
j_e	destination node of trip $e \in E$ with $j_e \in I$
Θ^e_a	preferred departure time of trip $e \in E$
Θ^e_b	preferred arrival time of trip $e \in E$
a_e	earliest possible departure time of trip $e \in E$
b_e	latest possible arrival time of trip $e \in E$

this enables us to model the problem as an Integer Linear Programming (ILP) problem.

A. Base Formulation

The base formulation is similar to the formulation provided in [21]. In the considered problem, a set of trips E is given, which have to be fulfilled either by (automated) vehicle or public transport. Each trip $e \in E$ is associated with a member who belongs to a certain household $h \in H$. The trips corresponding to household $h \in H$ are given by set $E_h \subseteq E$. For each household $h \in H$, we introduce an expansion coefficient μ_h which represents the number of households with the same characteristics in the population. Using this expansion coefficient μ_h allows us to combine several households with the same characteristics, which reduces the computation time of our developed model. In addition, this expansion coefficient μ_h will enforce congestion in the network. Next to this, each household $h \in H$ has a set of vehicles V_h which they can use to fulfill their trips. The capacity of vehicle $v \in V_h$ of household $h \in H$ is given by C_v. The total set of vehicles is given by $V = \cup_h V_h$.

The network on which the trips take place is described by a set of nodes I and the set of arcs R between these nodes.

Each arc $(i,j) \in R$ has a minimum travel time $t^{\min}_{ij}$ and a maximum travel time $t^{\max}_{ij}$ by car. Parameter t^{PT}_{ij} denotes the travel time in minutes by PT to go from node $i \in I$ to node $j \in I$ which does not depend on congestion. The length of arc $(i,j) \in R$ is given by Lg_{ij} and the practical link capacity of arc $(i,j) \in R$ per time step is given by Q_{ij}.

The origin and destination of trip $e \in E$ is specified by $i_e \in I$ and $j_e \in I$, respectively. For each trip $e \in E$, a preferred departure time Θ^e_a and a preferred arrival time Θ^e_b is specified. Moreover, the earliest possible departure time a_e and latest possible arrival time b_e are given.

For each trip $e \in E$, it has to be decided whether it is executed by an (automated) vehicle or public transport. Binary variable Tr_{ev} equals 1 when trip $e \in E$ is performed by vehicle $v \in V$ and 0 otherwise. This leads to the following constraints which imply that a trip can be executed by at most one vehicle.

$$\sum_{v \in V_h} Tr_{ev} \leq 1, \quad \forall e \in E_h, \ h \in H \quad (1)$$

Note that when $\sum_{v \in V_h} Tr_{ev} = 0$, trip $e \in E$ is performed by public transport. When a trip $e \in E$ is executed by an (automated) vehicle, the start time of the trip is captured by binary variable P_{evt} which is 1 if trip $e \in E$ is executed by vehicle $v \in V$ and starts at time $t \in T$ and 0 otherwise. Here, set T is a set of time instants. The routing of vehicle $v \in V$ is specified by binary variables $x^v_{ijt_1t_2}$ which are one when vehicle $v \in V$ drives on arc $(i,j) \in R$ from time instant $t_1 \in T$ to time instant $t_2 \in T$ and 0 otherwise. The travel time from node $i \in I$ to node $j \in J$ when starting at time instant $t_1 \in T$ depends on the current traffic flow on this arc and is specified by binary variables $\delta_{ijt_1t_2}$ which are 1 when the travel time from node $i \in I$ to node $j \in I$ starting at time instant $t_1 \in T$ equals $t_2 - t_1$ time steps, and 0 otherwise.

The following constraints determine the correct routing of vehicle $v \in V$ defined by variables $x^v_{ijt_1t_2}$ when trip $e \in E$ is executed by vehicle $v \in V$, i.e., when Tr_{ev} is one. Constraints (2) assure that trip $e \in E$ can only be satisfied by vehicle $v \in V$ when this vehicle passes through the origin node $i_e \in I$ after the earliest possible departure time a_e. Constraints (3) assure that trip $e \in E$ can only be satisfied by vehicle $v \in V$ when this vehicle passes through the destination node

$j_e \in I$ before the latest possible arrival time b_e.

$$Tr_{ev} \leq \sum_{j \in I} \sum_{t_1 \geq a_e} \sum_{t_2 \leq b_e} x^v_{i_e j t_1 t_2}, \quad \forall e \in E_h, \; v \in V_h, \; h \in H \quad (2)$$

$$Tr_{ev} \leq \sum_{i \in I} \sum_{t_1 \geq a_e} \sum_{t_2 \leq b_e} x^v_{i j_e t_1 t_2}, \quad \forall e \in E_h, \; v \in V_h, \; h \in H \quad (3)$$

The next constraints ensure that the routing variables $x^v_{ijt_1t_2}$ coincide with the departure time variables P_{evt} and that each trip $e \in E$ has exactly one departure time when it is executed by a vehicle.

$$P_{evt} \leq \sum_{j \in I} \sum_{t_1 \leq b_e} x^v_{i_e j t t_1},$$
$$\forall e \in E_h, \; v \in V_h, h \in H, \; t \geq a_e \in T \quad (4)$$

$$P_{evt} \leq Tr_{ev}, \quad \forall e \in E_h, \; v \in V_h, \; h \in H, \; t \in T \quad (5)$$

$$P_{evt} \geq \sum_{j \in I} \sum_{t_1 \leq b_e} x^v_{i_e j t t_1} + Tr_{ev} - 1,$$
$$\forall e \in E_h, v \in V_h, \; h \in H, \; t \geq a_e \in T \quad (6)$$

$$\sum_{t \in T} P_{evt} \leq 1, \quad \forall e \in E_h, \; v \in V_h, \; h \in H \quad (7)$$

Similar constraints ensure that the binary variables A_{evt}, which are 1 when trip $e \in E$ is executed by vehicle $v \in V$ arriving at time $t \in T$ and 0 otherwise, coincides with the routing variables $x^v_{ijt_1t_2}$.

$$A_{evt} \leq \sum_{i \in I} \sum_{t_1 \geq a_e} x^v_{i j_e t_1 t},$$
$$\forall e \in E_h, v \in V_h, \; h \in H, t \leq b_e \in T \quad (8)$$

$$A_{evt} \leq Tr_{ev}, \quad \forall e \in E_h, \; v \in V_h, \; h \in H \quad (9)$$

$$A_{evt} \geq \sum_{i \in I} \sum_{t_1 \geq a_e} x^v_{i j_e t_1 t} + Tr_{ev} - 1,$$
$$\forall e \in E_h, v \in V_h, \; h \in H, \; t \leq b_e \in T \quad (10)$$

$$\sum_{t \in T} A_{evt} \leq 1, \quad \forall e \in E_h, \; v \in V_h, \; h \in H \quad (11)$$

In addition, it must hold that the departure time of trip $e \in E$ is less than or equal to the arrival time of trip $e \in E$, which is ensured by the following constraint. The departure time can be determined by $\sum_{t \in T} t P_{evt}$ which will return the value of t for which P_{evt} is 1. The same holds for the arrival time.

$$\sum_{t \in T} t P_{evt} \leq \sum_{t \in T} t A_{evt}, \quad \forall e \in E, \; \forall v \in V \quad (12)$$

Given the departure time and arrival time of trip $e \in E$ (when trip $e \in E$ is executed by a vehicle), we can determine the deviation ϕ_e from the preferred arrival time Θ_b^e by means of the following constraint.

$$\phi_e = \sum_{v \in V_h} \left(\Theta_b^e Tr_{ev} - \sum_{t \in T} t A_{evt} \right), \quad \forall e \in E_h, \; h \in H \quad (13)$$

A distinction is made whether a trip arrives earlier or later than preferred. The amount of time a trip arrives earlier than preferred is denoted by $e\phi_e$ and the amount of time a trip arrives later than preferred is denoted by $l\phi_e$. The following

two constraints ensure that $e\phi_e$ and $l\phi_e$ attain the correct value.

$$e\phi_e \geq \phi_e, \quad \forall e \in E \quad (14)$$
$$l\phi_e \geq -\phi_e, \quad \forall e \in E \quad (15)$$
$$e\phi_e \geq 0, \quad \forall e \in E \quad (16)$$
$$l\phi_e \geq 0, \quad \forall e \in E \quad (17)$$

Here, both the amount of time a trip arrives earlier and later have to be bigger than 0, which is ensured by constraints (16) and (17). When trip $e \in E$ arrives earlier than preferred, ϕ_e is positive. This value is then assigned to variable $e\phi_e$. When ϕ_e is negative, trip $e \in E$ arrives later than preferred. This value is then made positive by multiplying it with -1 and this positive value is assigned to variable $l\phi_e$ by constraints (15). Note that constraints (15) are redundant when ϕ_e is positive, because of constraints (17). In a similar way, constraints (14) are redundant when ϕ_e is negative, because of constraints (16).

Next, we have to make sure that the number of people in vehicle $v \in V$ at any time instant $t \in T$, represented by variable L_{vt}, is less than or equal to the capacity C_v of this vehicle. This is ensured by the following two constraints.

$$L_{vt} = \sum_{e \in E} \left(\sum_{t_1 \leq t} (P_{evt_1} - A_{evt_1}) \right), \quad \forall v \in V, \; t \in T \quad (18)$$

$$L_{vt} \leq C_v, \quad \forall v \in V, \; t \in T \quad (19)$$

Constraints (18) only add 1 to variable L_{vt} when a trip $e \in E$ departs before or at time $t \in T$, i.e., when P_{evt_1} is 1 for a $t_1 \leq t$, and arrives after time $t \in T$, i.e., when A_{evt_1} is 1 for a $t_1 \geq t$. Otherwise, $\sum_{t_1 \leq t} (P_{evt_1} - A_{evt_1})$ will equal 0.

At each time instant, a vehicle is either moving on a certain arc or waiting at a certain node. The movement of vehicle $v \in V$ on a certain arc $(i, j) \in R$ from time instant $t_1 \in T$ to time instant $t_2 \in T$ is already specified by binary variable $x^v_{ijt_1t_2}$. In addition to this, a vehicle $v \in V$ waiting at time instant $t_1 \in T$ at node $i \in I$ is represented by binary variable $w^v_{it_1}$. The following two constraints represent flow conservation in the network.

$$\sum_{i, j \in I} \sum_{t_1 > 1} x^v_{ij1t_1} + w^v_{i1} = 1, \quad \forall v \in V \quad (20)$$

$$\sum_{j \in I} \sum_{t_1 > t} x^v_{ijtt_1} + w^v_{it} = \sum_{j \in I} \sum_{t_1 < t} x^v_{jit_1t} + w^v_{i(t-1)},$$
$$\forall i \in I, \; t \in T, \; v \in V \quad (21)$$

In addition, we ensure by the following constraint that a vehicle cannot be waiting at a certain node when there are people in the vehicle.

$$\sum_{i \in I} w^v_{it} \leq \frac{C_v - L_{vt}}{C_v}, \quad \forall t \in T, \; v \in V \quad (22)$$

B. Traffic Congestion

Differently from [21], we use the formulation of [22] to include congestion in our problem formulation. This is one of the contributions of the paper as it allows us to solve the system optimum and (an approximation of the) user optimum to optimality by solving one ILP formulation. In the

TABLE IV
PARAMETERS TRAFFIC CONGESTION

Notation	Description
F	flow in a link
Q	practical link capacity
t_0	free-flow travel time
a	estimation parameter BPR function
b	estimation parameter BPR function

formulation of [22], the Bureau of Public Roads (BPR) function [23] is used to model congestion where the current travel time depends on the flow F in a link and is given by $t_0 \left(1 + a \left(\frac{F}{Q}\right)^b\right)$. Here, t_0 denotes the free-flow travel time, F the current volume, Q the practical link capacity, and a and b are estimation parameters.

Following the terminology of [22], we calculate the spatial link capacity $CF_{ijt_1t_2}$ from node $i \in I$ to node $j \in I$ from time instant $t_1 \in T$ to time instant $t_2 \in T$ beforehand by using the BPR function as follows:

$$CF_{ijt_1t_2} = (t_2 - t_1)Q_{ij}\left(\frac{1}{a}\left(\frac{t_2 - t_1}{t_{ij}^{\min}} - 1\right)\right)^{\frac{1}{b}},$$
$$\forall i, \ j \in I, \ t_1 < t_2 \leq t_1 + t_{ij}^{\max} \in T \quad (23)$$

Note that we use in essence a truncated BPR function, since we limit the travel time to $t_{ij}^{\max}$. This means that flow exceeding the threshold $Q\left(\frac{\left(t_{ij}^{\max}/t_0\right)-1}{a}\right)^{\frac{1}{b}}$ incurs a fictional travel time of infinity.

The flow resulting from routing variables $x_{ijt_1t_2}^v$ is denoted by $F_{ijt_1t_2}$ and provides the flow from node $i \in I$ to node $j \in I$ from time instant $t_1 \in T$ to time instant $t_2 \in T$. Note that the expansion factor μ_h is included in the following constraints to determine the correct value for the flow $F_{ijt_1t_2}$.

$$F_{ijt_1t_2} = \sum_{h \in H} \mu_h \left(\sum_{v \in V_h} x_{ijt_1t_2}^v\right), \quad \forall i, \ j \in I, \ t_1, t_2 \in T \quad (24)$$

To match the flow $F_{ijt_1t_2}$ with the spatial link capacity $CF_{ijt_1t_2}$, [22] introduces binary variables $\delta_{ijt_1t_2}$ which are 1 when the travel time from node $i \in I$ to node $j \in I$ starting at time instant $t_1 \in T$ equals $t_2 - t_1$ time steps, and 0 otherwise. First, for each arc $(i, j) \in R$ and starting time instant $t_1 \in T$, at most one travel time is chosen by the following constraint.

$$\sum_{t_1+t_{ij}^{\max} \geq t_2 \geq t_1+t_{ij}^{\min}} \delta_{ijt_1t_2} \leq 1, \quad \forall i, \ j \in I, \ t_1 \in T \quad (25)$$

Then, the following constraints make sure that the flows fulfill the capacity constraints. Constraints (26) make sure that flow from node $i \in I$ to node $j \in I$ from time instant $t_1 \in T$ to time instant $t_2 \in T$ is only allowed when the travel time from node $i \in I$ to node $j \in I$ at time instant $t_1 \in T$ equals $t_2 - t_1$ time steps. Constraints (26) also make sure that, in this case, flow $F_{ijt_1t_2}$ is limited to the spatial link capacity $CF_{ijt_1t_2}$.

$$F_{ijt_1t_2} \leq CF_{ijt_1t_2}\delta_{ijt_1t_2}, \quad \forall i, \ j \in I, \ t_1, t_2 \in T \quad (26)$$

The following link consistency constraints ensure that vehicles do not pass one another, i.e., the vehicle that enters an arc later can not leave the arc earlier. To ensure this, an inequality should be used. The term $M\left(1 - \sum_{t>t_2} \delta_{ijt_2t}\right)$ makes sure that these constraints also hold when $\sum_{t>t_2} \delta_{ijt_2t} = 0$. This could occur because of the inequality sign in constraints (25).

$$t_1 + \sum_{t>t_1}(t - t_1)\delta_{ijt_1t}$$
$$\leq t_2 + \sum_{t>t_2}(t - t_2)\delta_{ijt_2t}$$
$$+ M\left(1 - \sum_{t>t_2} \delta_{ijt_2t}\right), \quad \forall i, \ j \in I, \ t_1 < t_2 \in T \quad (27)$$

This formulation does not consider any queuing in the network, and more importantly, it does not express the continuity of the relationship between flow and travel time. However, it does allow considering the time-space network changes as a result from the degradation of travel time with the number of vehicles present in the network. The fact that queuing is not considered and that traffic is represented in a simplified way through a time-space network associated with a simplified BPR function, leads necessarily to a reduction of realism on the traffic flows that are estimated in the network, and therefore, also on the mode share between public transport and AVs. We are willing to accept those limitations as we want to isolate potential changes among the scenarios related to the value of travel time and user optimum versus system optimum. A more detailed description of the mode choice and traffic flows requires other types of models and data that virtually make our approach impossible to be applied.

C. Generalized Transportation Costs

As stated before, our aim is to minimize the generalized transportation costs. These costs consist of the following components:
- Public transport cost
- Travel time cost by AV
- Fuel costs
- Early or late arrival

The costs of a trip fulfilled by public transport are given by the value of in-vehicle time, the ticket cost $Tick$ multiplied with a scale factor between fuel costs and ticket costs ς, and a penalty cost ρ for using public transport. The value of in-vehicle time for public transport is given by multiplying the total travel time cost per minute in public transport β by the travel time in public transport in minutes from node $i \in I$ to node $j \in I$ given by t_{ij}^{PT}. Therefore, for a given solution, the public transport costs are given by:

$$\sum_{h \in H} \mu_h \left(\sum_{e \in E_h}\left(1 - \sum_{v \in V_h} Tr_{ev}\right)\left(\beta t_{i_e j_e}^{PT} + Tick \cdot \varsigma + \rho\right)\right). \quad (28)$$

When a trip is fulfilled by a vehicle, travel time costs, fuel costs and costs for early or late arrival are incurred. The travel time costs are given by the total travel time of the driver in

TABLE V
PARAMETERS COST FUNCTIONS

Notation	Description
$Tick$	ticket cost public transport
ς	scale factor between fuel costs and ticket costs
ρ	penalty cost for using public transport
β	travel time cost per minute in public transport
α	travel time cost per time step in a car
ω	fuel costs per kilometer
σ_1	penalty cost for early arrival
σ_2	penalty cost for late arrival

time steps multiplied by the travel time cost per time step in a car α. In order to determine the total travel time of occupied vehicles, we introduce binary variable l_{vt}, which is 1 when vehicle $v \in V$ is occupied at time t and 0 otherwise. To ensure this variable attains the correct value, we introduce the following two constraints:

$$l_{vt} \leq L_{vt}, \quad \forall v \in V, \ t \in T \tag{29}$$

$$l_{vt} \geq \frac{L_{vt}}{C_v}, \quad \forall v \in V, \ t \in T \tag{30}$$

Then, the travel time costs are determined as in equation (31) where only the travel time of the driver is being considered. We argue that, at least under a user optimum perspective, the driver makes his own selfish decisions without considering the travel time of other occupants of the car that he/she is driving.

$$\sum_{h \in H} \mu_h \left(\alpha \sum_{v \in V_h} \sum_{t \in T} l_{vt} \right). \tag{31}$$

The fuel costs can be determined based on the length of the arcs used by the vehicles multiplied by the fuel costs per kilometer ω:

$$\sum_{h \in H} \mu_h \left(\sum_{v \in V_h} \sum_{i,j \in I} \sum_{t_1,t_2 \in T} x^v_{ijt_1t_2} \cdot Lg_{ij} \cdot \omega \right). \tag{32}$$

As specified by constraints (14) - (17), the amount of time a trip arrives earlier or later than preferred when it is executed by car is given by $e\phi_e$ and $l\phi_e$, respectively. The penalty time cost for early and late arrival are given by σ_1 and σ_2, respectively. Therefore, the total penalty cost for early and late arrival is given by:

$$\sum_{h \in H} \mu_h \left(\sum_{e \in E_h} e\phi_e \cdot \sigma_1 + l\phi_e \cdot \sigma_2 \right). \tag{33}$$

D. Formulation for Different Scenarios

In Sections II-A, II-B and II-C, a base formulation for the considered problem is described. One of the contributions of this paper is that this base formulation can be modified to express different scenarios such as 1) conventional vehicles (CVs) instead of AVs, 2) user optimum versus system optimum, and 3) considering the time of all passengers in the vehicle whereas the base formulation does not. These modifications are discussed in more detail in the following sections.

1) Automated Versus Conventional Vehicle: The model introduced in Sections II-A, II-B and II-C can easily be modified to a situation with conventional vehicles by including the following constraint.

$$x^v_{i,j,t,t_1} \leq L_{vt}, \quad \forall i, \ j \in I, t, \ t_1 \in T, \ v \in V \tag{34}$$

These constraints ensure that the routing variables should be zero when there are no people in the vehicle, i.e., when $L_{vt} = 0$. It is assumed that when at least one human is present, he or she can drive the vehicle.

2) User Optimum Versus System Optimum: Minimizing the total costs described in Section II-C leads to the following objective function:

$$\min \sum_{h \in H} \mu_h \left(\sum_{e \in E_h} \left(1 - \sum_{v \in V_h} Tr_{ev} \right) \left(\beta t^{PT}_{i_e j_e} + Tick \cdot \varsigma + \rho \right) \right.$$
$$+ \alpha \sum_{v \in V_h} \sum_{t \in T} l_{vt} + \sum_{v \in V_h} \sum_{i,j \in I} \sum_{t_1,t_2 \in T} x^v_{ijt_1t_2} \cdot Lg_{ij} \cdot \omega$$
$$\left. + \sum_{e \in E_h} e\phi_e \cdot \sigma_1 + l\phi_e \cdot \sigma_2 \right), \tag{35}$$

which yields the system optimum solution for the situation described in Sections II-A and II-B.

In order to attain the user optimum, each household should be able to minimize their own costs. To our knowledge, this is the first attempt to approximate the user optimum in a dynamic traffic assignment setting by including it in an ILP formulation. One way to approximate this user optimum is the following. First, we determine for each household the minimum cost M_h they incur when they are the only one in the network. This can be achieved by solving the problem formulation introduced in Sections II-A and II-B for one household at a time with the objective function given by equation (35). When for each household their minimum cost M_h is known, we can approximate the user optimum by minimizing the maximum relative deviation from this minimum cost. By minimizing this maximum relative deviation, we try to push these relative deviations closer to each other such that the households have similar relative deviations. However, this approach will only give an approximation of the user optimum, because we cannot control the costs of the households that have a lower relative deviation than the maximum relative deviation.

The maximum relative deviation D can be included in the base formulation by adding the following constraints, where K_h denotes the costs of household $h \in H$ for a given solution when also competing with the other households.

$$K_h = \mu_h \left(\sum_{e \in E_h} \left(1 - \sum_{v \in V_h} Tr_{ev} \right) \left(\beta \cdot t^{PT}_{i_e j_e} + Tick \cdot \varsigma + \rho \right) \right.$$
$$+ \alpha \sum_{v \in V_h} \sum_{t \in T} l_{vt} + \sum_{v \in V_h} \sum_{i,j \in I} \sum_{t_1,t_2 \in T} x^v_{ijt_1t_2} \cdot Lg_{ij} \cdot \omega$$
$$\left. + \sum_{e \in E_h} e\phi_e \cdot \sigma_1 + l\phi_e \cdot \sigma_2 \right), \quad \forall h \in H, \tag{36}$$

$$D \geq \frac{K_h - M_h}{M_h}, \quad \forall h \in H. \tag{37}$$

TABLE VI
PARAMETERS AND VARIABLES FOR THE DIFFERENT SCENARIOS

Notation	Description
M_h	parameter representing the minimum cost incurred when a household is the only one in the network
K_h	variable representing the costs of household $h \in H$
D	variable representing the maximum relative deviation from minimum cost M_h
D^*	parameter representing the minimized maximum relative deviation D

The objective function and its attained value are then given by:

$$\min D = D^*. \qquad (38)$$

However, preliminary results showed that using this formulation for the user optimum might result in undesirable behavior. Because we only minimize the maximum relative deviation from the minimum costs over the households, no incentive is given to minimize the relative deviation for the other households. Therefore, preliminary results showed that for households that did not attain this maximum relative deviation, the vehicles would drive around empty without any reason. As these additional costs would not increase the value of objective function (38), this undesirable behavior was allowed by the given base formulation. In order to overcome this limitation, we solve the model again with the system optimum objective function (35), constraints (36)-(37) and the following constraint where D^* is the optimal solution found for the maximum relative deviation D in the previous step:

$$D \le D^*. \qquad (39)$$

For the scenarios considered in Section IV, we could not control the costs for 70% of the households by minimizing the maximum relative deviation from the minimum costs. These costs could be reduced by solving the problem with the system optimum objective function and constraint (39).

3) Unit Load Versus Load of All Passengers: When using automated vehicles, vehicles transporting more people might be given priority in terms of reducing travel time if these decisions are taken by a central computer. Therefore, we can modify the system optimum objective function (35) such that it takes into account the number of people in the vehicle by changing l_{vt} to L_{vt}:

$$\min \sum_{h \in H} \mu_h \left(\sum_{e \in E_h} \left(1 - \sum_{v \in V_h} Tr_{ev} \right) \left(\beta t^{PT}_{i_e j_e} + Tick \cdot \varsigma + \rho \right) \right.$$
$$+ \alpha \sum_{v \in V_h} \sum_{t \in T} L_{vt} + \sum_{v \in V_h} \sum_{i,j \in I} \sum_{t_1, t_2 \in T} x^v_{ijt_1t_2} \cdot Lg_{ij} \cdot \omega$$
$$\left. + \sum_{e \in E_h} e\phi_e \cdot \sigma_1 + l\phi_e \cdot \sigma_2 \right). \qquad (40)$$

III. SETUP EXPERIMENTS

The problem of routing privately owned automated vehicles formulated by the constraints in Sections II-A and II-B and the objective function in Section II-C is an NP-hard problem as it generalizes the well-known multi-commodity integral flow problem [24]. Therefore, we test and compare our various

TABLE VII
CHARACTERISTICS OF HOUSEHOLD TYPES

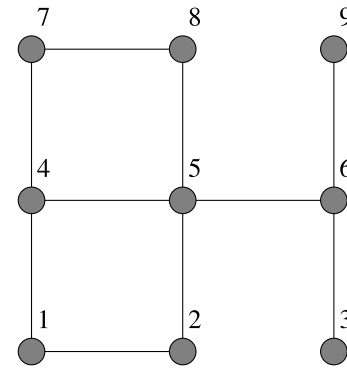| Household type	# members	# trips
1	2	6
2	2	5
3	3	7

Fig. 1. Network for used data set.

models on a small artificial data set that represents three different types of households. Each household type is represented three times in our data set, which gives a total of nine households. For each of these nine households, the expansion factor μ_h is set to 30 to enforce congestion in the network. Each household has its own vehicle with a capacity of 4 people that can be used for their own trips. Table VII shows for each household type how many members the household has and how many trips are done by all members of the household combined.

In addition, we consider a toy network with 9 nodes and 10 edges which is depicted in Figure 1. Each arc has a length Lg_{ij} of 4 kilometers, a practical link capacity Q_{ij} of 120 vehicles, a minimum travel time t^{min}_{ij} of 5 minutes and a maximum travel time of t^{max}_{ij} of 20 minutes.

The trips of the households on the toy network can be described as follows, where the given times are the desired arrival and departure times.

Household type 1

- Two trips from home (node 1) to the workplace (both located at node 5) from 8:00 am until 8:10 am.
- One lunch meeting appointment at node 6 which includes a trip from 12:00 am until 12:10 am to lunch and a trip from 1:00 pm until 1:10 pm back to work.
- Two trips in the afternoon back home (node 1) from the workplace (node 5) from 6:00 pm until 6:10 pm.

TABLE VIII

VALUES FOR REMAINING PARAMETERS

Parameter	Description	Value
a	Parameter BPR function	2
b	Parameter BPR function	4
t_{ij}^{PT}	Travel time in minutes by public transport for all OD-pairs	10
β	Total travel time cost per minute in public transport	0.755
$Tick$	Public transport ticket cost	2
ς	Scale factor fuel and ticket costs	3.11
ρ	Penalty for using public transport	7.622
α	Travel time cost per minute in car	0.806
ω	Fuel costs per kilometer	0.1
σ_1	Penalty time cost for early arrival	0.306
σ_2	Penalty time cost for late arrival	1.309

Household type 2

- Two trips in the morning from home (node 1) to the workplace (node 2 and 5) from 8:00 am until 8:10 am.
- The family member working at node 2 goes for grocery shopping at node 3 at the end of the day with a trip from 5:00 pm until 5:10 pm.
- Both members return back home at the end of the day. One from the workplace (node 5) and the other from the supermarket (node 3) leaving nodes 3 and 5 at 6:30 pm and arriving home at 6:40 pm.

Household type 3

- Three trips in the morning from home (node 1) to university (node 8) and the office (node 5 and 7) from 8:00 am until 8:10 am.
- The student leaves the university (node 8) and goes home (node 1) with a trip from 5:30 pm until 5:40 pm.
- The other two family members leave the office (node 5 and 7) at 6:00 pm. The member working at node 5 returns home (node 1) at 6:10 pm and the member working at node 7 goes to the gym at 6:10 pm at node 4.
- The family member working out at the gym (node 4) returns home (node 1) from 7:00 pm until 7:10 pm.

For each trip $e \in E$, a preferred departure time Θ_a^e and a preferred arrival time Θ_b^e is given. The earliest possible departure time a_e is given by Θ_a^e minus 5 minutes and the latest possible arrival time b_e is given by Θ_b^e plus 5 minutes.

Besides a time step of 5 minutes, we use the values given in Table VIII for the remaining parameters. Note that the PT costs are the same for all origin-destination pairs such that we can isolate the effect between using the UO and SO to route the vehicles.

We compare several scenarios. In the scenarios, we go from the current situation to several future scenarios where AVs are allowed on the public roads.

a) User optimum with conventional vehicles: The first scenario we consider is the current situation on the public roads. Currently, we have conventional vehicles where each household tries to minimize their own costs. A solution to this scenario can be obtained by solving the problem formulation given by the base formulation (constraints (1) - (22) and (24) - (27)), the additional constraints (34) which eliminate empty trips, and constraints (36), (37) and objective function (38) introduced in Section II-D.2 for considering the user optimum. We will refer to this scenario as 'UO CV unit-load.'

b) User optimum with automated vehicles: The first step in the adoption of automated vehicles may be that households will own their own automated vehicle which they control themselves. By this, we mean that the owner is in control of the route that the vehicle takes. Therefore, each household will still try to minimize their own transportation costs in which only the travel time costs of the driver are considered. The main difference between this scenario and the previous scenario is that in this scenario it is allowed for vehicles to drive around without a driver. Therefore, we first solve the formulation given by the base formulation (constraints (1) - (22) and (24) - (27)) and constraints (36), (37), and objective function (38) introduced in Section II-D.2 for considering the user optimum. As solving this formulation might result in unnecessary empty trips as discussed before, we then solve the base formulation in combination with (36), (37), (39) and objective function (35) to eliminate these unnecessary empty trips. We will refer to this scenario as 'UO AV unit-load'.

c) System optimum with automated vehicles: The next step in the adoption of automated vehicles may be that a central operating system decides on which route the vehicles should take in order to minimize the costs of the entire system. Here, only a unit load per vehicle is considered as is the case for both UO scenarios. A solution to this scenario can be obtained by solving the problem formulation given by the base formulation (constraints (1) - (22) and (24) - (27)) and objective function (35). We will refer to this scenario as 'SO AV unit-load'.

d) System optimum with automated vehicles and load: A modification to the previous scenario could be that the travel time costs are not counted per vehicle but per passenger. This means that the travel time costs of a vehicle with 2 people in it will be higher than the travel time costs of a vehicle with 1 person in it. The idea behind this is that preference might be given to a car with more people in it in terms of shorter routes. A solution to this scenario can be obtained by solving the problem formulation given by the base formulation (constraints (1) - (22) and (24) - (27)) and objective function (40). We will refer to this scenario as 'SO AV load-all'.

e) User optimum with automated vehicles and load: For the sake of completeness, we also include the scenario with automated vehicles where households will own their own automated vehicle which they control themselves,

TABLE IX
MINIMUM COST M_h

Household type	UO & SO CV unit-load	UO & SO AV unit-load	UO & SO AV load-all
1	797.40	797.40	1281.00
2	1767.60	1646.70	1778.16
3	2329.77	2052.30	2597.91

TABLE X
RELATIVE DEVIATIONS TO MINIMUM HOUSEHOLD COSTS (NESTED COLUMNS) AND TOTAL COSTS (TWO BOTTOM ROWS)
(IN BOLD WHEN CORRESPONDING TO THE SCENARIO PRESENTED IN A SPECIFIC COLUMN)

Scenario	UO CV unit-load		SO CV unit-load		UO AV unit-load		UO AV load-all		SO AV unit-load		SO AV load-all	
'unit-load' (unit) vs 'load-all' (all)	unit	all	unit	all	unit	all	unit	all	unit	all	unit	all
Min. relative deviation (%)	**12**	7	**0**	0	**21**	22	12	**7**	**2**	4	1	**0**
Avg. relative deviation (%)	**39**	22	**29**	16	**46**	31	36	**21**	**34**	26	37	**22**
Max. relative deviation (%)	**52**	24	**72**	45	**56**	39	63	**30**	**67**	54	70	**41**
Total costs 'unit-load'	**21086.46**		**19014.27**		**19843.17**		19116.60		**18024.00**		18742.89	
Total costs 'load-all'	22779.06		21553.17		22140.27		**20809.20**		21167.40		**20677.29**	

but where the travel time costs are those of all passengers. A solution to this scenario can be obtained by solving first the problem formulation given by the base formulation (constraints (1) - (22) and (24) - (27)), adding the following two constraints

$$
KL_h = \mu_h \left(\sum_{e \in E_h} \left(1 - \sum_{v \in V_h} Tr_{ev} \right) \left(\beta t_{i_e j_e}^{PT} + Tick \cdot \varsigma + \rho \right) \right.
$$

$$
+ \alpha \sum_{v \in V_h} \sum_{t \in T} L_{vt} + \sum_{v \in V_h} \sum_{i,j \in I} \sum_{t_1, t_2 \in T} x_{ijt_1t_2}^v \cdot Lg_{ij} \cdot \omega
$$

$$
\left. + \sum_{e \in E_h} e\phi_e \cdot \sigma_1 + l\phi_e \cdot \sigma_2 \right), \quad \forall h \in H, \tag{41}
$$

$$
DL \geq \frac{KL_h - M_h}{M_h}, \tag{42}
$$

and objective function

$$
\min DL = DL^* \tag{43}
$$

where M_h is now the minimum cost household $h \in H$ incurs when they are the only one in the network and the travel time costs are counted per passenger. Then, the problem formulation given by the base formulation (constraints (1) - (22) and (24) - (27)), constraints (41), (42), and constraint

$$
DL \leq DL^*, \tag{44}
$$

where DL^* is the optimal solution found for DL in the previous step, is solved with objective function (40). We will refer to this scenario as 'UO AV load-all'.

f) System optimum with conventional vehicles: Finally, we also consider the system optimum case with conventional vehicles where only the travel time costs of the driver are considered. A solution to this scenario can be obtained by solving the problem formulation given by the base formulation (constraints (1) - (22) and (24) - (27)), the additional constraints (34) which eliminate empty trips, and objective function (35). We will refer to this scenario as 'SO CV unit-load'.

IV. RESULTS

The introduced problem formulations are implemented in the Mosel language and solved using Xpress 8 on an Intel® Core™ 2 Duo @3.00 GHz with 4.00GB RAM. This mathematical programming tool uses state of the art branch-and-cut methods. For the considered toy network, the different scenarios have approximately 180,000 constraints and 70,000 variables. In this section, we compare all scenarios on several performance indicators. All but the scenario 'UO AV unit-load' are solved to optimality. For scenario 'UO AV unit-load', we present the results after interrupting the solver after 12 hours which resulted in an integrality gap of 14.56%. Running the model for 4 weeks did reduce the integrality gap to 4% since the lower bound increased, but no better solution was found. We compare the scenarios on costs and computation time, and we provide some statistics on trip and arc level.

A. Scenarios Compared on Costs

In this section, we compare the different scenarios based on cost. For each scenario, we present the minimum, average and maximum relative deviation from the household's minimum costs (without competition with the other households) and also the total costs incurred for the entire system. In order to do so, we need for each household the minimum cost M_h they incur when they are the only one in the network. In Table IX, these values are shown for the three household types for all scenarios.

In Table X, the minimum, average, and maximum relative deviation from the minimum costs and also the total costs incurred for the entire system are given. The deviations and total costs are given calculated based on unit load (so travel time costs of a car) and based on the total load (so travel time costs of all the passengers). For each scenario, we present both the 'unit-load' and 'load-all' for the relative deviations (presented in the nested columns) and generalized transportation costs (presented in the two bottom rows) for comparison purposes. The relative deviations and generalized transportation

TABLE XI

STATISTICS PER TRIP

	UO CV unit-load	SO CV unit-load	UO AV unit-load	UO AV load-all	SO AV unit-load	SO AV load-all
# trips by car	990	1110	1260	1170	1320	1200
# trips by public transport	630	510	360	450	300	420
Average penalty costs per trip	1.92	1.98	2.26	1.79	2.31	1.87
Average travel time per trip	8.97	8.78	10.48	8.72	10.11	8.88

TABLE XII

STATISTICS ON ARC LEVEL

	UO CV unit-load	SO CV unit-load	UO AV unit-load	UO AV load-all	SO AV unit-load	SO AV load-all
0 people in car	0	0	420	420	390	330
1 person in car	840	750	1500	1200	1230	1170
2 people in car	420	510	570	420	540	480
3 people in car	0	60	0	0	120	0
Occupancy rate	1.33	1.48	1.06	1.00	1.17	1.08
Occupancy rate ≥ 1	1.33	1.48	1.28	1.26	1.41	1.29
Congestion delay in %	0	0	5.6	0	0	0

costs corresponding to the scenario presented in a specific column are given in bold.

As expected, we see that the total costs decrease when we compare the user optimum case with CVs and AVs. This means that AVs can be used more efficiently compared to CVs.

When we take the adoption of AVs to the next level, i.e., a central operating system being in control of routing the vehicles (scenario 'SO AV unit-load'), we see that the total costs of the network can be reduced even further. However, we see that the costs, and therefore, also the travel time, are spread less evenly over the different households, which denotes no user equilibrium.

The total costs 'load-all' for scenarios 'UO AV load-all' and 'SO AV load-all' are higher than the total costs 'unit-load' for scenarios 'UO AV unit-load' and 'SO AV unit-load', respectively. However, these costs are determined in a different way which makes comparing it unfair. These scenarios are compared on different performance indicators in the coming sections. It is, however, interesting to see that the total costs 'unit-load' are lower for scenario 'UO AV load-all' when compared to scenario 'UO AV unit-load'. This can be explained by the fact that the UO scenarios do not aim at minimizing the total costs but at minimizing the maximum deviation for the minimum costs M_h. When considering these deviations, we see in Table X that the 'UO AV load-all' solution has a higher maximum deviation when considering 'UO AV unit-load' costs when compared to the 'UO AV unit-load' solution.

B. Scenarios Compared per Trip

In this section, we present some statistics on trip level. We compare the number of trips that are fulfilled by car and public transport, the average penalty costs per trip done by car, and the average travel time per trip by car. These results are presented in Table XI.

From Table XI, we see that the number of trips done by car increases when going from scenario 'UO CV unit-load' to 'UO AV unit-load' and to 'SO AV unit load'. This number

decreases when we go from scenario 'UO AV unit-load' to 'UO AV load-all' or from scenario 'SO AV unit-load' to 'SO AV load-all', because the travel time costs for the 'load-all' scenarios are higher as these are counted per passenger instead of per vehicle. When comparing the 'UO' scenarios with the equivalent 'SO' scenarios, we see that the number of trips done by car increases.

In addition, when comparing 'UO CV unit-load' to 'UO AV unit-load', we see that the average penalty costs and average travel time per trip increases, because there is more congestion in the network (shown in Table XII).

When we compare scenario 'UO AV unit-load' to 'UO AV load-all' and scenario 'SO AV unit-load' to 'SO AV load-all', we see that the number of trips done by car decreases, but we also see that the penalty costs and travel time per trip decreases. This is because there is a higher focus on reducing the travel time when there are more people in the car.

C. Scenarios Compared per Arc

In this section, we compare the different scenarios on arc level, where an arc is defined by (i, j, t_1, t_2). For each scenario, we count the number of arcs with empty driving cars, with cars with 1 person, and with cars with 2 or 3 people. We also show the occupancy rate of the vehicles including and excluding empty cars. This way we can measure the effect of using AVs instead of CVs, and the effect the other scenarios have on combined trips in the same vehicle. In addition, we provide the congestion which is given by the travel time above the minimum as a percentage of the total travel time. The results are shown in Table XII.

Table XII shows that using AVs indeed leads to cars driving around empty to pick up family members at different locations. Therefore, the occupancy rate decreases when we include the empty trips.

When comparing scenarios 'UO AV unit-load' and 'UO AV load-all', we see that the number of arcs on which an empty car drives stays the same. Because the number of trips done by AV slightly decreases for 'UO AV load-all' when compared to

TABLE XIII
COMPUTATION TIMES IN MINUTES

	UO CV unit-load	SO CV unit-load	UO AV unit-load	UO AV load-all	SO AV unit-load	SO AV load-all
Computation time (minutes)	14	3	>1440 (>720 + 714)	46 (17 + 29)	15	47

'UO AV unit-load', the occupancy rate including empty trips decreases.

The occupancy rate increases when we compare the UO scenarios with the equivalent SO scenarios. This shows that more trips are combined, which increases the efficient use of the cars in the system optimum.

Comparing scenarios 'SO AV unit-load' and 'SO AV load-all' shows again a decrease in the occupancy rate, because a longer travel time for a car with 3 people in it counts three times as much. Therefore, the number of combined trips reduces in this scenario.

The last part of Table XII compares the scenarios based on congestion. We see that there is only congestion in scenarios 'UO AV unit-load'. When comparing 'UO AV unit-load' with 'SO AV unit-load', we see that the routing is done more efficiently such that no congestion occurs. When comparing 'UO CV unit-load' and 'UO AV unit-load', we see that using AVs instead of CVs increases congestion.

D. Scenarios Compared on Computation Time

In this section, we compare the scenarios on computation time. For scenarios 'UO AV unit-load' and 'UO AV load-all', the total computation time consists of first solving the UO model, and then, the SO model with an additional constraint as described in Section II-D.2. For scenario 'UO CV unit-load', solving the SO model afterwards is not necessary as vehicles are not allowed to drive around empty. The computation times in minutes are given in Table XIII.

Table XIII shows that all scenarios can be solved within one hour, except the 'UO AV unit-load' scenario. This scenario took over 12 hours to solve the UO model, and thus, we decided to interrupt the solver. The integrality gap of the UO model was 14.6%. Solving the SO model for the 'UO AV unit-load' scenario took slightly less than 12 hours. Therefore, the total computation time took over 24 hours. This makes the 'UO AV unit-load' scenario by far the hardest scenario to solve for the considered network. Note that this does not have to be the case in general as, for example, the computation time for scenario 'UO AV load-all' is still reasonable while only the costs for this scenario are calculated differently. Therefore, the long computation time of scenario 'UO AV unit-load' has to do with the NP-hardness of the problem which may cause extremely long computation times for specific instances.

E. Comparision With UO-POAVAP Method [21]

The UO-POAVAP method [21] was also applied to and solved for the small network introduced in this paper. Only the 'UO' scenarios were considered, since the UO-POAVAP method can only be used to solve the user optimum. In order to make a fair comparison, additional

constraints to cope with the fact that travel times change in each iteration had to be added to the UO-POAVAP method:

$$F_{ijt_1t_2} \le CF_{ijt_1t_2}(1-\phi)+CF_{ijt_1(t_2+1)}\phi, \quad \forall i, \ j \in I, \ t_1, t_2 \in T. \tag{45}$$

Here, ϕ is the coefficient for the equilibrium computation that will balance the contribution of the previous and current assignment for the calculation of volumes and other performance indicators in each iteration. Coefficient ϕ was set to $1/n$ for the calculations.

For the 'UO AV' scenarios, the results were similar, although the objective function values were slightly better for the exact method presented in this paper. However, there was a bigger difference in results for the 'UO CV' scenario. An explanation for this is that the UO-POAVAP method is very sensitive to each trip, which is enhanced by the small network considered. The UO-POAVAP method performs better in terms of computation time.

The exact method presented in this paper cannot be applied to the case study discussed in [21] because of the size of the network.

V. CONCLUSIONS

In this paper, we have introduced a novel exact linear formulation, including a linearized traffic congestion model, that can solve both the user optimum and system optimum to optimality for the routing of privately owned automated vehicles. Using this formulation, we compare several scenarios for a small toy network ranging from the current situation on the roads to a future scenario where a central operating system is in charge of routing AVs while taking the time of all people in the car into account. By not using an algorithm to approximate the solution, we make the model harder to be applied in a realistic network given its NP-hard nature. However, by solving our ILP formulation to (near) optimality, we are able to explore the behavior for different model configurations.

The results show that using AVs instead of conventional cars may reduce the costs of the entire system and may enable more trips being done by car. However, because of the latter, also the average travel time increases. In addition, the results show that the total costs of the system should decrease when a central operating system is in charge of routing the vehicles while even executing more trips by AVs. The system optimum solution reduces congestion which in turn reduces the average travel time per trip. However, this slightly increases the average penalty costs for arriving early or late at the destination location of the trip. The occupancy rate of the AVs also increases when we compare the system optimum without considering the number of people in the car with the user optimum. Therefore, the AVs are used more efficiently

in this scenario. When the central operating system is able to also consider the number of people in the car, by counting the travel time of all family members, we see that less trips are done by AV and that the occupancy rate decreases which means less trips are combined. This denotes that the current externalization of all the occupants of car trips may be leading to too many trips being done in this mode. However, we also see that the average travel time reduces significantly and that the average penalty costs also decrease.

For future research, it would be interesting to validate our 'UO CV unit-load' scenario against standard traffic assignments algorithms and to expand these algorithms to be able to deal with the other scenarios considered in this paper. In addition, efficient heuristics should be developed to solve real-life instances under these different scenarios. Determining an exact solution for our formulation already takes a long time for our small data set. In addition, these efficient heuristics should be able to solve the real-time version of the considered problem, i.e., assign trips to AVs or PT and determine the routes for the AVs when new trips are requested. Applying the developed heuristics on the same scenarios for a realistic data set will show if our conclusions also hold up in a realistic setting. The heuristics can also be used to evaluate another scenario even further in the future when AVs are not privately owned anymore, but can be shared between the different households.

ACKNOWLEDGMENT

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