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Publication date
2019

Document Version
Final published version

Published in
ISCHP 2019 - 7th International Scientific Conference on Hardwood Processing

Yield optimization and surface image-based strength prediction of beech

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Abstract

Wood is a strongly anisotropic and heterogeneous material with natural defects that are affecting the uniform scatter of the fiber patterns. European beech (Fagus sylvatica) is the species, used for this study. The logs of these species have generally complicated shapes with frequent curvatures, which is in contrast to most of the softwood species with relatively straight log shapes. From structural point of view, these species have fewer knots and natural features, but stronger fiber deviations compared to different softwood species that have complicated knot configurations. This study consists of two parts: 1) log reconstruction and optimization of the cutting pattern, and 2) board reconstruction and strength prediction. Due to the complex structural pattern of hardwoods, the visual grading method is a relatively weak strength predictor for these species. The aim of this study is to develop a numerical method based on the finite element (FE)-analysis to provide a better prediction for the tensile strength of the boards. The analysis covers the scatter of 200 beech boards. By resembling the tensile test setup numerically, the stress concentration factors (SCFs) are calculated, considering the average and maximum stresses around the imperfections. SCFs in combination with the longitudinal stress wave velocity are the numerical parameters, used in the nonlinear regression model for tensile strength prediction. The nonlinear model is checked for different combinations of the numerical parameters to estimate and visualize the potential of the virtual predictions. Performance of the novel criteria is compared to the typical grading criteria (knottiness and the dynamic MoE (MoE_{dyn})), and is shown that the coefficient of determination is higher, when using the virtual methods for tensile strength predictions.

1. Introduction

Due to easier accessibility and sustainability of beech hardwood in central Europe, as well as its higher strength and durability in engineering applications, the interest of the market for this species, and its usage in different studies and applications are being increased. It needs to be remarked that beech logs have generally strong frequent curvatures that affect their yield. Therefore, the aim of this study is to optimize the yield of the straight boards out of shorter logs initially, and to virtually predict the tensile strength of the beech boards later. Therefore, two separate numerical steps are the focus of this study, giving us the opportunity to 1) reconstruct and analyse the complex beech logs and predict their yield based on point clouds, and 2) to reconstruct beech boards with detailed consideration of the knots on the basis of their surface images and to predict the tensile strength.

In case of the log reconstruction (first step of this study), 3D X-ray and CT-scanning are making it possible to analyze the surface, as well as its internal features. This allows extracting the optimized yield of the logs to categorize timber boards for different applications. Internal features in logs, such as knots and cracks and their effects on yield has been studied previously (Steele et.al. 1993, Bhandakar et.al. 2006, Boukadida et.al. 2012). Additionally, an automatic algorithm has been provided for the geometrical recognition of the knots (Longuetaud et.al. 2004, 2012). Therefore, logs with strong geometrical non-uniformities can be reconstructed in order to predict the possible yield and to reduce the waste of the material for engineering applications. In case of the board reconstruction (second step of this study), consideration of the natural features with their comprehensive geometrical representation including the angle of rotation, shape and the coordinate directions in the bulk material helps for better strength predictions. These natural features (knots) influence the fiber pattern in wood, which correspondingly affect the global stiffness direction along

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the boards and finally the strength of the material. As knots are the main strength governing parameters, numerical methods with more accurate geometrical reconstruction of the knots may be an alternative method to improve the strength prediction of the material. Different studies focused on the structural modelling of wood and predicting the strength reduction and failure, resulting from structural non-uniformities (Goodman and Bodig 1980; Phillips et al. 1981; Cramer and Goodman 1986; Foley 2001; Baño et al. 2010; Jenkel and Kaliske 2013; Hackspiel et al. 2014; Lukacevic and Füssl 2014). The basis of most of these models is a 2D flow-grain analogy (Goodman and Bodig 1978; Foley 2001), which is extended for the 3D case here to consider the vector component in the third direction as well (Khaloian et al. 2017).

As higher stresses are developing around the geometrical non-uniformities, stress concentration factors around knots (Khaloian and Van de Kuilen 2019a) as well as the stress wave velocity in the boards (Khaloian and Van de Kuilen 2019b) are multiple parameters that are used in this study for prediction of the strength of beech boards. These are the main parameters that are influenced by the natural features in timber. Small and linear dislocation of the elements due to the small impact of the stress wave, especially over the knot boundaries, is the main reason for reduction of the velocity of the wave, when traveling forth and back though the board.

2. MATERIALS AND METODS

2.1. *GENERAL*

European beech *(Fagus sylvatica)* is the species, used for this study. The temporal experimental plots are located in the low mountain region Spessart in the north of Bavaria. In total, 16 different stands in two different Bavarian forest enterprises are selected in order to find appropriate beech trees. Altogether, 100 sampled beech trees are equally contributed to one of the five following stand types: pure beech stand and mixed stands of beech with Douglas-fir, Norway spruce, oak and pine. A number of 50 samples are randomly selected for the numerical analysis. The age of the sampled stands and trees ranges from 80 to 140 years.

The site conditions are assumed to be constant as the source rocks are sandstone of similar properties and these rocks developed to brown earths. In the same way, climate with temperature and precipitation is assumed to be comparable for all stands as well.

The logs have been sawn to boards when the top diameter was at least about 230mm without bark. Logs have been sawn using a band saw. It was tried to get as many center-boards as possible with the preferred cross-section of 50mm×150mm (discussed later). From the outer parts of the stems, side-boards of smaller cross-section (40mm×80mm) were cut out (Rais et.al. 2018).

The roundwood was scanned with a Riegl LMS-Z420i laser scanning system. Selected and virtually reconstructed 50 logs of 4.1 m in this study cover diameters between 250-660 mm. The shapes of the logs are covering a scatter, including logs with strong frequent curvatures in different coordinate directions (shown in Figure 1), and the logs with relatively straight shapes.

In the second step of this study, for the geometrical reconstruction of the boards and their strength predictions, 200 lower quality beech boards with up to 22 knots are used for the numerical analysis. All beech boards that are numerically reconstructed in this study are tested in tension (EN 408 (2010)), and the strength values (presented as average in Table 1) are used for the validation of the model. Additionally, the visual grading parameters (DEB, DAB) are provided in an average form in Table 1, which represent the knot parameters and are later used for validation of the extracted virtual knot parameters.

Table 1: Material and mechanical properties of the boards used for the simulations

Species		PK	Length (mm)		Thickness (mm)		Width (mm)		Strength (Mpa)		Density (Kg/m^3)		MoE$_{static}$ (MPa)		DEB (-)		DAB (-)	
			avg.	CoV	avg.	CoV	avg	CoV	avg	CoV	avg	CoV	avg	CoV	avg	CoV	avg	CoV
Beech	Beech$_1$	100	3102	0.003	24	0.01	151	0.003	31	0.43	758	0.05	11100	0.18	0.18	0.55	0.20	0.61
	Beech$_2$	100	3102	0.002	24	0.01	100	0.002	34	0.44	773	0.05	11300	0.24	0.21	0.55	0.24	0.59

2.2. *LOG AND BOARD RECONSTRUCTION*

For estimation of the yield, the geometrical model of the logs are reconstructed virtually. This process is done based on the observed point clouds from the laser scanning of the logs. An example of a geometrical configuration of a beech log is shown in Figure 1 and is compared to a spruce log. It is qualitatively shown in this figure that the beech log has strong geometrical non-uniformities with frequent curvatures, whereas the spruce one has a relatively straight shape. Therefore, yield may be strongly influenced by the shape of the logs, especially when the length of the boards is the point of interest.

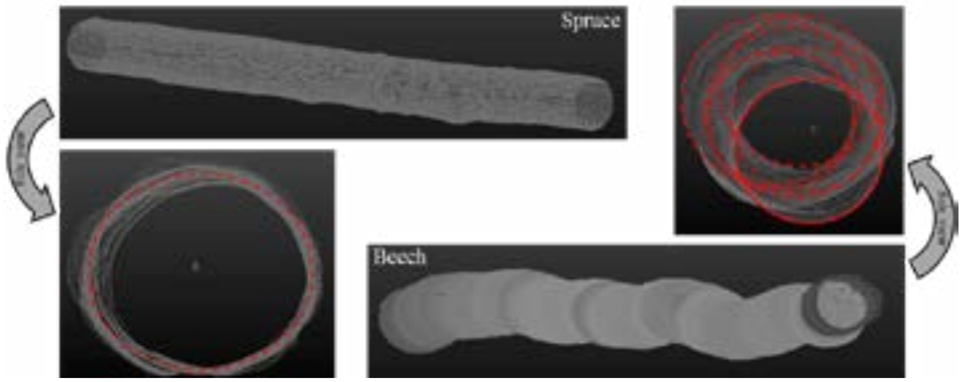


Figure 1: Comparison of log curvature of spruce and beech (the red circles on the side view of the samples help to understand how strong the curvature is)

The side view of the spruce log in Figure 1 has a more uniform geometrical shape, close to a circle that makes it possible to cut longer straight boards. However, in the case of the beech log, multiple circles can be seen in the cross-section of the log, which show the strong curvatures of the structure.

After approximating the surface normals for the reconstruction of the surface of the log and applying the mesh algorithms, the surface is reconstructed with a relatively fine mesh. Dense points are giving the opportunity for the reconstruction of the curved and non-smooth surfaces in this case.

Normals are computed as a weighted product over the nearest neighbors, after finding the tangential planes by computing the centroid of each vertex as the average of all nearest neighbors. Poisson Surface Reconstruction and the Ball Pivoting algorithms are used for the surface reconstruction in this study. Poisson reconstruction is a global solution that considers all the data at once in contrast to many implicit surface fitting methods that segment the data into regions for local fitting. Thus, the data noise is visible in this formulation. However, Poisson reconstruction can create smooth surfaces that robustly approximate noisy data (Kazhdan et.al. 2006).

The second meshing algorithm that is used in this study is the Ball Pivoting algorithm. By combining the points, got from the laser scanning, and after interpolating the given point cloud, this algorithm computes a triangle mesh for the structure. The principle of this method is that a ball of a user specified radius, ρ, pivots between the points and if it touches the three points without containing any other point, these three points form a triangle. Therefore, in this method the edges are being constructed by pivoting the ball from one sample point to the others. This process is continuing until the time when all the points are considered (Bernardini et.al. 1999). When the sampling density of the points are too low, some of the edges are not being created and this causes formation of holes in the structure during the reconstruction process.

The way that the whole log reconstruction process works in this study as well as the required file formats during this process are presented in Figure 2.

Figure 2: Flowchart, showing the process of the log reconstruction

Cutting process in this study is done solely by considering the curvature of the logs, without considering the effect of the internal features on the quality of the boards (invisible), i.e. the outermost points on the log surface are considered as boundaries that are limiting the cutting pattern. Information about the bark (thickness), the defects, etc. is not extractable from the point clouds. Due to the complex geometrical configuration of the beech logs, different cutting patterns need to be taken into account to optimize yield. This includes especially the length and the width effects. Therefore, initially an unstructured mesh is being created by considering all different cutting configurations and then the unnecessary cuts are being deleted to come to the optimized number of the predictions (Figure 3).

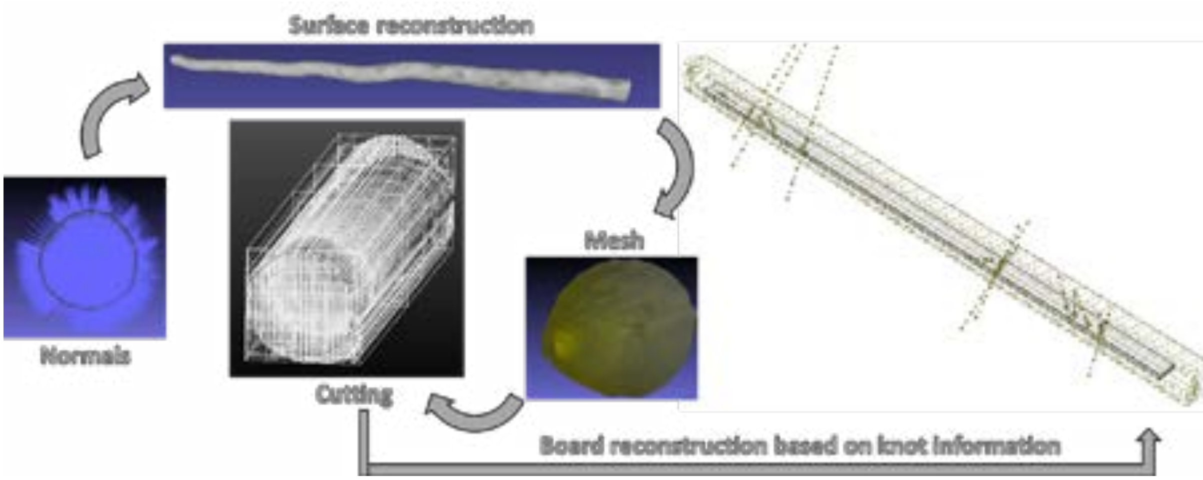


Figure 3: Surface reconstruction, mesh generation, cutting pattern and board reconstruction with knots

In the second step, a comprehensive geometrical model of the boards is reconstructed based on the surface information of the knots from the visual grading. For each knot, separate axis of rotation and plane is defined, to represent the complete geometry of the knots based on the location of the pith. The reconstructed surface of the log, the generated mesh, considering different board configurations with different dimensions and the reconstructed board are presented in Figure 3. It is shown in Figure 3 that if CT-scan is used for the scanning of the logs, the reconstruction from log to boards can be done almost automatically. Laser-scanning in this study provides the surface information of the logs, thus the information about the boards can not be extracted from the logs directly. Therefore, this process is done in two steps here (as explained before).

2.3. Case analysis

In total, 30 beech logs are chosen for the validation of the numerical procedure. The beech logs are chosen randomly from the mixture of beech-pine and beech-Douglas fir stands. As mentioned before, the logs have been cut in the sawmills to the boards of 40*80*4000 mm^3 and 50*150*4000 mm^3. To be able to be comparable to the actual conditions, similar procedure is used for the numerical analyses as well. Saw thickness of 5 mm is considered for the analysis, which is implemented numerically by distancing the boards from each other.

In order to do the sensitivity analysis for the model, and as virtual cutting gives the opportunity to analyze different configurations, four different configurations are considered here for the reference samples, and the results are compared to the real values, obtained from the saw mills. The four configurations are as follows:

- Case1: If only boards of small dimensions (40*80 mm^2 cross-section) are extracted from the logs
- Case2: If only boards of big dimensions (50*150 mm^2 cross-section) are extracted from the logs
- Case3: If as many boards are extracted from the logs, regardless of the dimensional aspects. Therefore, it is possible to extract more boards of smaller dimensions and fewer boards of bigger dimensions.
- Case4: Similar to the real condition, to extract as many center-boards of bigger dimension first as possible and then to extract smaller boards from the rest of the log volume.

Each of the above mentioned cases are analyzed separately and are compared to the actual condition (similar to Case4 configuration).

2.4. Set-up of cutting pattern for current study

In the current study, 20 extra logs have been selected to analyze the dimensional effects.These logs represent the scatter of the curvatures and diameters of the beech logs. By running totally 540 analyses (to analyze the four Cases, explained before), considering 27 board configurations (shown in Table 2) for each of the 20 logs, the dimensional effects are analyzed on the yield. Similarly, a sawing thickness of 5 mm is implemented for this set of the simulations as well.

Table 2: Dimensions of the boards

	Thickness (mm)	Width (mm)	Length (mm)
Log	20	80	2000
	25	120	3000
	30	160	4000

3. Results and discussion

3.1 Case and sensativity analysis

For all above mentioned cases (1-4), the correlation is found (in Table 3) with the number and the volume of the boards that are extracted from the logs in reality (a case similar to Case4). As the focus of this part is to analyze how sensitive the model is, when considering a different cutting pattern than the one applied in the reality, this correlation analysis is performed. Case1 and Case2 are considered the extreme limit conditions of these analyses, by including boards with only small or big cross-sections.

Table 3: Correlation between the actual and virtual number of the boards and their volume

y=ax+b							
Number of boards (actual-virtual)	**a**	**b**	**R^2**	**Volume of boards (actual-virtual)**	**a**	**b**	**R^2**
Case1	0.498	-0.29	0.87	**Case1**	1.071	0.03	0.89
Case2	1.107	0.69	0.83	**Case2**	1.046	-0.01	0.90
Case3	1.012	-0.89	0.84	**Case3**	1.013	0.04	0.94
Case4	1.001	-1.38	0.96	**Case4**	0.955	-0.03	0.94

By knowing the real log volume in each case, the yield (in percent) is calculated (k) using the following equation:

$$k = \left(\frac{V_{boards}}{V_{real\_\log}}\right) * 100 \tag{1}$$

where V_{board} is the total volume of the extracted boards from the log, and V_{real_log} is the total volume of the log.

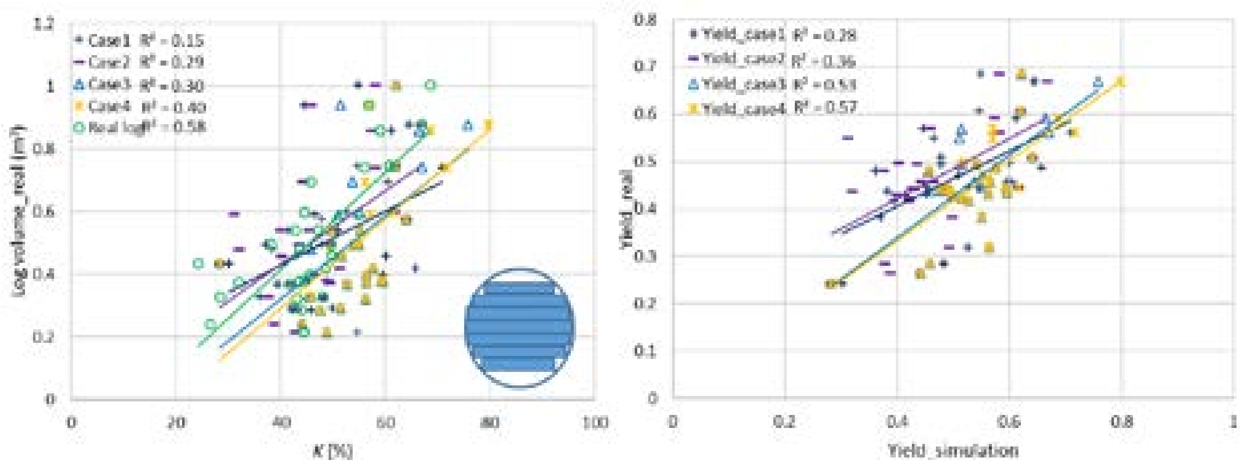
3.2. Validation of the model for the test samples

By correlating the total number of the virtual boards with the real boards, a R^2 value of 0.96 is obtained, showing the high accuracy of the virtual procedure (shown in Table 3).

The k value is calculated for the four mentioned cases as well as the real case of the sawn boards. By considering the average percentage of k, case1 model gives 7.6% increase in volume usage compared to the real case, case2= 0.9%, case3= 17% and case4=19.5% increase respectively. This shows that case2 model with only big boards has the lowest yield. However, by optimizing the cutting pattern, in total 19.5% increase in yield can be expected, neglecting the quality of the boards. As mentioned before, this increase may be due to the lack of information about the quality of the log, the location of the bark, the knots and the defects and correspondingly, consideration of the outermost points of the logs for the analysis. Therefore, as virtual board extraction is done based on the maximum volume of the logs, yield may be slightly overestimated numerically. A comparison between the virtual *k* value of each case and the real (total) log volume is presented in Figure 4.

The correlation between *k* and the V_{real_log} in the real case is R^2=0.58 (shown in Figure 4). This correlation reduces during the sensativity analysis from Case4 to Case1, as the volume of the boards that are being extracted from the logs in each case is reducing respectively (based on equation 1 and as shown in Table 3).

By correlating the yield of each of four cases to the real yield of the log (*Y=k/100*), Case4 is found to be the optimized condition, with higher R^2 value (presented in Figure 4).



y=ax+b							
Log_{volume}-*k*	**a**	**b**	**R^2**	**$Yield_{real}$-$Yield_{simulation}$**	**a**	**b**	**R^2**
Case1	0.009	0.08	0.15	**Case1**	0.583	0.17	0.28
Case2	0.012	-0.04	0.29	**Case2**	0.637	0.17	0.34
Case3	0.013	-0.21	0.30	**Case3**	0.873	-0.01	0.53
Case4	0.014	-0.28	0.40	**Case4**	0.85	-0.01	0.57
Real log	0.016	-0.21	0.58				

Figure 4: Relation between actual and virtual log volumes and yield. Coefficients of the linear correlation and the resulting R^2 values are shown under each figure.

The correlation is much lower in Case1 and Case2 models. This confirms that an optimum mixture of the small- and big-sized boards needs to be considered to be able to increase the yield.

3.3. VIRTUAL CUTTING AND DIMENSIONAL EFFECTS

As the focus in this part of the study is to analyze the length effects, boards with lengths of 2, 3, and 4 m are cut from logs of 4.1 m length. The number of the boards, in case of cutting directly 4 m long boards and deviding them to half , or cutting directly 2 m long boards from the log, has been analyzed. This case was checked for the boards with 20, 25, 30 mm thickness and 80, 120, and 160 mm width, respectively. The results are shown in Figure 5. To show how the length of the boards may affect the volume and the yield, a schematic presentation of the cutting is provided in Figure 5 as well, for straight and curved logs. Similar analyses are performed to figure out the width effects (for the boards of 80 mm or 160 mm width). It is shown that among the coordinate dimensions of the beech boards, length has the main effect on the yield of the logs. This is due to the strong curvaturs and geometrical non-uniformities of beech logs. Therefore, cutting shorter boards (2m) in comparison to the longer ones (4m) results in a 15.5% increase in the yield. The difference between the number of the boards in cases of cutting 3m or 4m boards is negligible, due to the strong curvature of the logs. However, the board thickness and the width do not have considerable effects on the number of the boards, extracted from the log and correspondingly the volume and the yield. By comparing the total number of the boards with width of 80 mm to the ones with width of 160 mm, an increase of 1.5% is obtained, when cutting smaller boards.

As the thickness of 20, 25, and 30mm are considered in this project, it is difficult to make a clear conclusion about the influence of the thickness of the boards on the yield of the material. However, as generally a mixed cutting pattern is used (similar to the logs in this study, where boards with bigger dimensions are cut first from the center and then smaller boards from the rest of the material), an assumption can be made that the thickness of the boards, similar to the width effects, does not have a significant influence on the yield of the material.

Therefore, by considering the economic costs and aspects, the dimensional effects can be neglected in straight logs in contrast to the logs with curvatures.

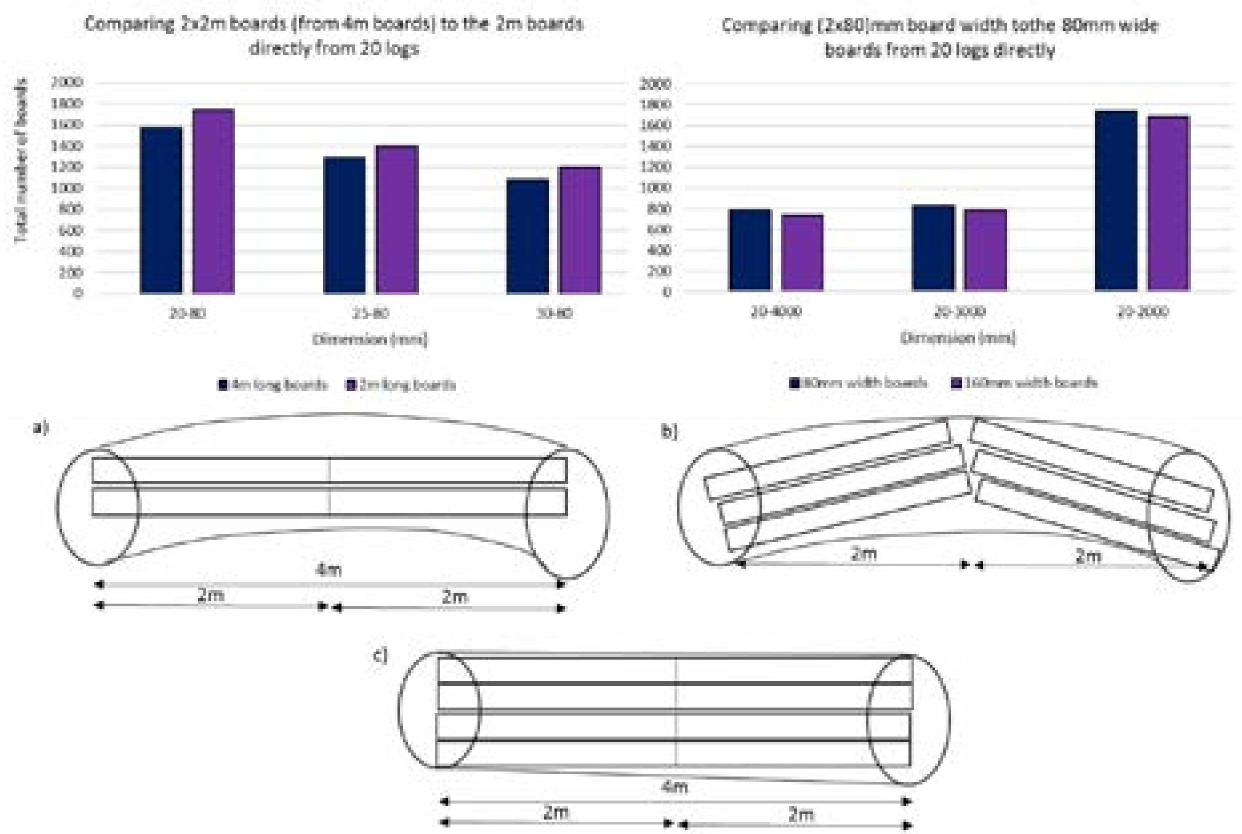


Figure 5: Comparison between 2m and 2x2m boards as well as 80mm and 2x80mm wide boards. a,b) Representation of an exaggerated view of a curved log with the boards, and c) is a schematic view of an example straight log

3.4. *VIRTUAL STRENGTH PREDICTION*

As knots are the main causes of the geometrical non-uniformities in wood, the stress distribution and the stress wave velocity do not remain constant through the board. By considering the single and multiple effects of the maximum and average stresses that are developing around the knots (SCF_i) as well as the velocity of the stress wave, when passing through the knots, four material parameters are introduced to predict the strength of the material virtually (Khaloian and Van de Kuilen 2019a,b).

In order to reduce the dependency of the numerical simulations on the input parameters, the average density of beech samples (ρ=760 kg/m^3) is used as an input parameter for the density.

By running the numerical analysis for about 200 lower quality beech boards with strong fiber deviations, it is shown that due to the strong geometrical complexity of beech boards, MoE_{dyn} is not enough as a single parameter for strength predictions (R^2=0.5 compared to R^2=0.4 for tests and simulations, respectively). Knot factors and their geometrical representations and effects on the development of the stresses are playing an important role as well for the strength prediction of the lower quality boards (Figure 6).

When comparing only the simulated knot parameters based on the stress developments under tensile loading (SCF_1, SCF_2 and SCF_3) to the measured knot parameters from visual grading (DEB, DAB), significant improvements are seen in the quality of the predictions based on the numerical parameters (R^2= 0.54 compared to R^2= 0.15).

By performing a nonlinear multiple regression analysis between the numerical and test parameters with the tensile strength, mathematical equation 2 is given.

$$f = \sum_{i=1}^{n} a_i . e^{b_i . SCF_i} + c.MoE_{dyn} + d \tag{2}$$

Where: *n* is the number of the SCFs required for the strength predictions, SCF_i are the stress concentration factors, presented above. The parameter *f* is the tensile strength, MoE_{dyn} is the dynamic modulus of elasticity, and *a,b,c,d,e* are the constants, provided in Table 4.

Table 4: Coefficients of Equation 2

	a_1	a_2	b_1	b_2	c	d
f	-15.89	-35.35	0.11	0.21	0.0024	73.24

Additionally, a comparison between the numerical and experimental parameters and the tensile strength is presented in Figure 6.

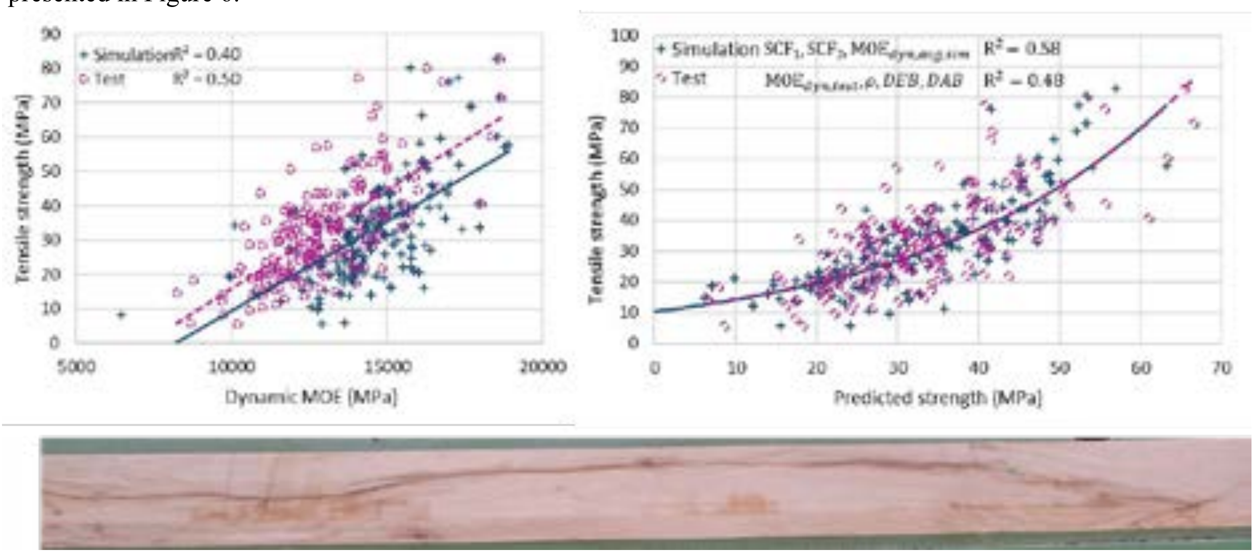


Figure 6: Comparison of the tested and simulated parameters to the measured tensile strength and an example of beech board with extreme fiber deviation

By considering the geometrical effects of the knots on the tensile strength of the boards in the numerical simulations, the predictions are improving considerably. However, for the low quality boards in this study, usage of only two stress concentration factors in combination with the numerical MoE_{dyn} (by application of the average density) in multiple regression analysis are enough for the prediction of the tensile strength (R^2=0.58). Addition of the third SCF is not improving the quality of the predictions to more than 10% (R^2=0.59 compared to R^2=0.58). Therefore, this parameter is not considered as an extra parameter for the tensile strength predictions.

4. CONCLUSION

Due to the strong anisotropy, heterogeneity and geometrical non-uniformity of beech samples, different aspects need to be considered in the numerical simulations to be able to improve the quality of the predictions. First, due to the possible frequent curvatures in beech logs, an optimum configuration needs to be defined to be able to increase the yield. In this study, by considering the outermost points of the beech logs from observed point clouds for the reconstruction of the logs, and without considering the quality of the extracted boards, an increase of about 15.5% in the yield of the logs is expected by cutting shorter boards. It is shown that the width and thickness effects can be neglected, due to their small influence (1.5% increase) on the yield. Additionally, by considering the economic costs, the dimensional effects can be neglected in very straight logs. Therefore, length effects need to be considered in the cases of logs with curvatures.

In the case of modelling low-quality beech boards with strong fiber deviations, it is shown that consideration of the knot geometries has a considerable influence on the prediction of the tensile strength, beside the numerically simulated MoE_{dyn}, when applying the average density of the specimen. It is shown that usage of two numerically extracted stress concentration factors in combination with the calculated MoE_{dyn} in a multiple regression analysis gives a higher correlation with the tensile strength compared to the measured knot parameters (DEB and DAB) in combination with the actual board density and the measured MoE_{dyn}. The R^2=0.58 compared to R^2=0.48 respectively for the simulations and the tests show the strength of the developed virtual method for tensile strength predictions.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support of the Bayerischer Landesanstalt für Wald und Forstwirtschaft for funding the project X042 "Beechconnect" which allowed for the work presented in this paper.

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