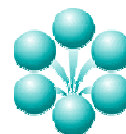
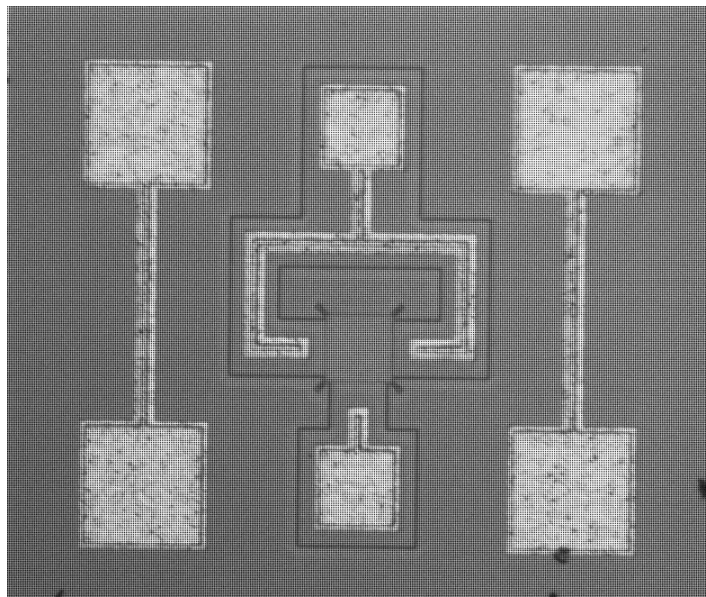


Design, Modeling and Simulation of a 52MHz MEMS Gyroscope Device in 1.5um SOI

By

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**Electronic
Instrumentation
Laboratory**

Design, Modeling and Simulation of a 52MHz MEMS Gyroscope Device in 1.5um SOI

THESIS

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE

in

ELECTRICAL ENGINEERING

By

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Acknowledgements

I wish to thank my university supervisor Prof. Paddy French for his exciting and enlightening lectures on sensors which led me to choose this topic for my Masters thesis. Thank you for stimulating me to perform my research with great independence and providing unlimited amounts of wisdom and reflection as well as guidance in technical and organizational problems.

My sincere thanks go to my daily supervisor at NXP Jan-Jaap Koning. I thank him for not giving up on me. I'm grateful for his continuous guidance, encouragement and support. I will not forget the words of wisdom he shared from his experiences in life and as a student.

I would like to thank Huug van der Vlist, department head of the Device Engineering and Characterization group, for giving me the opportunity to do my work in his group. I'm a better engineer today partly because of the time I've spent working in your group.

I can't thank Joep Bontemps enough for bearing with me, and the countless times I've used the phrase, "*Joep, can I ask you a question*". I enjoyed the discussions we had ranging from the underachieving PSV Eindhoven, to the beautiful football by Arsenal FC. *Hakuna Matata* Joep.

Finally, I am grateful to my fellow interns Di Wu, Pauline Mechet, Samuel, Aftab Qureshi, Krishnan Seetharaman, Andrei Minero and Jindong Yang for the friendly working environment in the office. I wish you guys nothing but the best.

Chapter 1

Introduction

This chapter gives a brief history of NXP Semiconductors, of which this project was done. Furthermore, the activities of the Device Engineering and Characterization group are mentioned. Lastly, the goal and outline of this thesis is presented.

1.1 NXP Semiconductors

NXP is a top ten semiconductors company founded by Philips 54 years ago. Until October 1st, 2006, it was known as Philips Semiconductors, and it was part of Royal Philips Electronics. As a consequence of strategic management decisions of the Board of Directors of Philips, the semiconductor business has been given an independent identity by creating the spin-off company called NXP (which stands for Next eXperience).

The name change announcement follows an agreement between Royal Philips and Kohlberg Kravis Roberts & Co. (KKR), Bain Capital, Silver Lake Partners, Apax and AlpInvest Partners NV that will see the consortium take an 80.1% stake in the semiconductor operation with Philips retaining a 19.9% interest. NXP is Europe second largest semiconductor company, with approximately 37,000 people operating in more than 20 countries world-wide.

NXP Semiconductors has headquarters in Eindhoven, The Netherlands, and the main business units, which have been responsible for net sales of 4.96 billion euros in 2006, are as follows:

- Mobile and Personal
- Home
- Automotive & Identification
- Multimarket Semiconductors
- NXP Software

The company has more than 25 Research & Development and Innovation centers worldwide: the project described in this report has been funded by the innovation center of the Business Line RF products in Nijmegen.

1.1.1 NXP Semiconductors, Nijmegen

The NXP semiconductors site of Nijmegen was founded in 1953 – only six years after the invention of the transistor from John Bardeen and Walter Brattain at the Bell Labs – and it is one of the largest semiconductor locations of Europe, the eldest plant created by Royal Philips Electronics.

The site employs around 4400 people from 60 different nationalities. 2700 Employees work in production (wafer foundries). The other 1700 people are involved in innovation, marketing, logistics, management and support activities. Manufacturing in Nijmegen is done in four wafer foundries that are named after the used wafer size in inches (1 inch = 25.4 mm): ICN4, ICN5, ICN6, and ICN8. The current production of the Nijmegen site is 1.3 million 6" or 725 thousand 8" equivalent wafers per year.

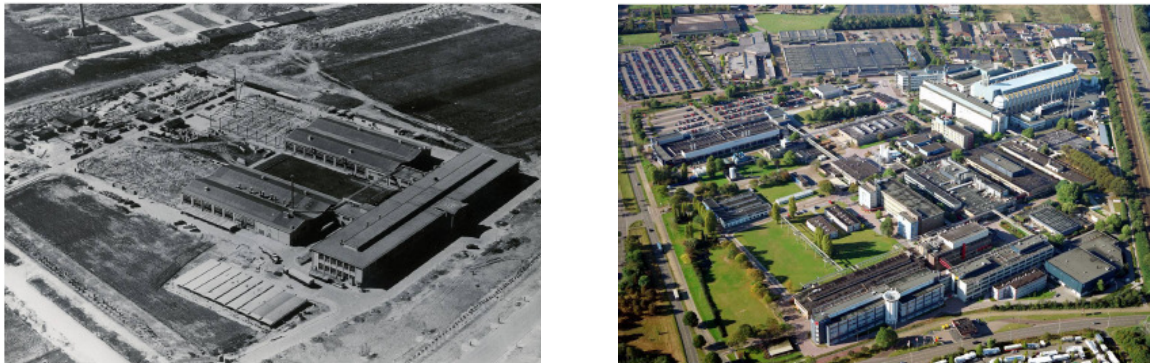


Figure 1.1: On the left side on picture took during the construction of the Nijmegen plant and on the right side an image of how the plant looks like today. The various wafer fabs are highlighted there

1.1.2 Device Engineering and Characterization group

This project was done with the support of the Device Engineering and Characterization group which is part of the Site I&T Organization Nijmegen. The department provides total solutions in the area between technology and the IC development. Its primary scope is the ICN technologies, but it is open for services to all Business lines and fabs. The group specializes in mixed signal, smart power and high voltage applications

The group provides several components such as:

- Device Design; smart power and HV devices for IC processes and discrete. Device Simulation.
- Modeling, parameter extraction and process blocks for circuit simulation (general).
- On Chip ESD protection (devices and structures). Smart power and HV.
- Consultancy for IC Design, circuit simulation, process development and reliability issues.

DE&C has its roots in the consumer business Nijmegen and foundry ICN5. On November, 2004, the modeling and characterization activities at all ICN fabs were integrated in the department.

The group has several missions including following ones:

- Providing Business Lines and IC innovations centers with tools and design support related to Device Design and modeling.
- Doing circuit simulations and on-chip ESD protection to enable state of the art and zero-defect compliant IC Design.
- Being a leader within Semiconductors in the field of High voltage and Analog/Mixed signal.

1.2 Problem Statement

The aim of this work is to design a gyroscope with high mechanical resonance and high qualify factor. This will keep the signal to noise ratio high over the desired bandwidth without increasing the drive amplitude and mass of the device hence save costs.

The equation of mechanical Brownian noise of vibrating gyro is given by [1]:

$$\Omega_{z(Brownian)} \propto \frac{1}{q_{drive}} \sqrt{\frac{4k_B T}{\omega_o M Q_{Effect-Sense}}} \quad (1.1)$$

Where,

$\Omega_{z(Brownian)}$	=	Noise floor
q_{drive}	=	Drive Amplitude
ω_o	=	Natural frequency
M	=	Mass of the body
$Q_{Effect-Sense}$	=	Effective Quality Factor
T	=	Absolute Temperature
k_B	=	Boltzmann Constant

In the case of the design of our gyroscope with $q_{drive} = 3 \times 10^{-11} \text{m}$, $\omega_o = 52 \text{MHz}$ ($3.2 \times 10^8 \text{ rad/s}$), $M = 6 \times 10^{-10} \text{Kg}$, $T = 300 \text{K}$, $Q_{Effect-Sense} = 4 \times 10^4$ and $k_B = 1.3806503 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$, we have a noise floor of $\Omega_z = 0.00322^0 / \text{s} / \sqrt{\text{Hz}}$.

In comparison to the state of the art gyroscopes [2], the proposed gyroscope has a higher frequency and a comparable noise floor. In addition, the device is small in size, hence requires small chip area and saves costs. Furthermore the high operating frequency improves SNR and enables the device to operate at moderate pressure of 1mbar, whilst maintaining a high quality factor [4].

1.3 Published results

Some results of this work have been published in the proceedings of Euroensors XXIII conference [5]. The title of the publication is “Design, modelling and simulation of a high frequency MEMS Gyroscope in 1.5 μm SOI.

In the paper, a simulation model of a MEMS gyroscope is presented. The mode matching and analysis of the dynamic behaviour of a gyroscope using the model is explained. Furthermore, the drive mode s-parameter measurements are presented at different bias voltages.

1.4 Thesis Outline

In this chapter, a brief history of NXP Semiconductors is presented. The major business units and their activities are stated. In addition, the motivation for this work is discussed. The chapter ends with the outline of this thesis.

Chapter 2 focuses on the fundamentals of a vibrating gyroscope. The working principle of a generic gyroscope is explained. Furthermore, the equations governing the dynamics of a gyroscope are derived from equations of motion.

In Chapter 3, the design concepts of a MEMS Gyroscope are covered. The operating principle of a lamé mode resonator is introduced. The factors crucial for matching of the drive and sense frequency are dealt with at the end of the chapter.

The FEM simulations are presented in Chapter 4. The chapter explains how the model was developed using the package COMSOL Multiphysics. Furthermore, the simulation model optimises the gyroscope design for perfect matching of the drive and sense frequency.

In Chapter 5 the electrical design aspects of the gyroscope are covered. The electrostatic actuation method is presented. In addition, piezoresistive and capacitive readout topologies are proposed and compared.

Chapter 6 focused on the fabrication process of the gyroscope prototype. The advantages of using SOI are stated. Furthermore, a brief step by step explanation of the VibrantiN2-ABCD2 process flow is presented.

Chapter 7 reports the measurement results of the drive mode of the gyroscope. The measurement process and techniques are presented. An electrical model was derived and fitted with the measured data.

Finally, Chapter 8 presents a summary of what has been achieved in this thesis and give recommendations for future work.

Chapter 2

Fundamentals of Micromachined Gyroscopes

In this chapter, the general working principle of a gyroscope is presented. Analysis of the dynamics and Coriolis response is followed by discussion of the limitations of the conventional approach, which defines the motivation of this thesis.

2.1 Micro-Electro-Mechanical-Systems Gyroscope (MEMS Gyroscope)

MEMS technology has revolutionized inertial sensors. Since the first demonstrated of a micromachined gyroscope by the Draper Laboratory in 1991, various micromachined gyroscopes designs fabricated in the surface micromachining, bulk micromachining or alternative fabrication techniques have been reported.

Inspired by the promising success of micromachined accelerometers in the same area, extensive research efforts towards commercial micromachined led to several innovative gyroscope topologies, fabrication and integration approaches, and detection technique.

Consequently, vibrating micromachined gyroscopes that utilize vibrating elements to induce and detect Coriolis force have been effectively implemented and demonstrated in various micromachining-based batch fabrication processes.

However, achieving robustness against fabrication variations and environmental fluctuations still remains as one of the greatest challenges in commercialization and high volume production of micromachined vibratory rate gyroscope.

The limitations of the lithography-based micromachining technologies define the upper-bound on the performance and robustness of micromachined gyroscopes. Conventional gyroscopes designs based on matching or near-matching the drive and sense mode resonant frequencies are quite sensitive to variations in oscillatory system parameters.

Thus, providing stable and reliable vibratory micromachined gyroscopes have proven to be extremely challenging, primarily due to the high sensitivity of the dynamical system response to fabrication and environmental variations.

2.2 Working Principle of a Gyroscope

A Gyroscope is a sensor that measures the rate of rotation of an object. Vibrating gyroscopes must be driven at resonance in order to function as angular rate sensors. This direction will be referred as the drive direction (figure 2.1).

When the device is rotated along the rotation axis, a coriolis force is induced in the sense direction. The force will excite the device in sense direction into resonance mode. The sense direction is orthogonal to both the drive direction and the rotation axis.

The sense and drive direction, can each be viewed as a mass-spring damper system (figure 2.2). Hence, a gyroscope can be viewed as a two degrees-of-freedom (2-DOF) mass-spring damper system whereby, one degree of freedom is the sense direction, and the second degree of freedom orthogonal to the first is the sense direction.

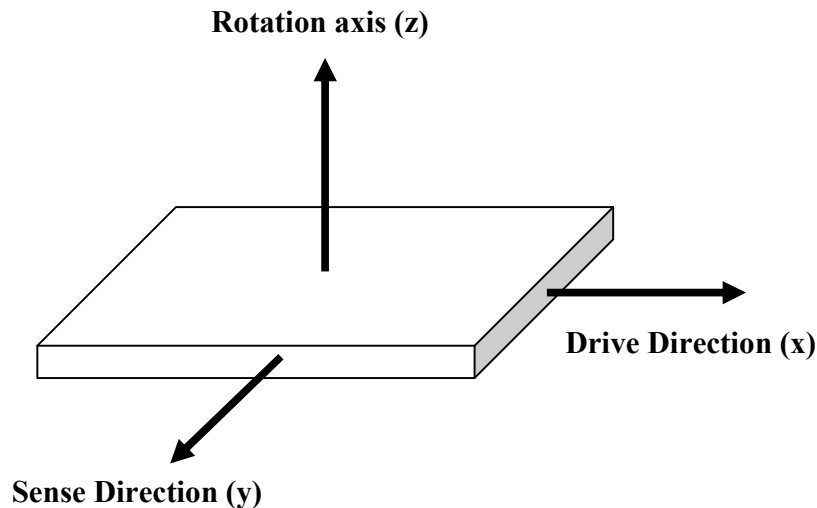


Figure 2.1: Generic model of a gyroscope.

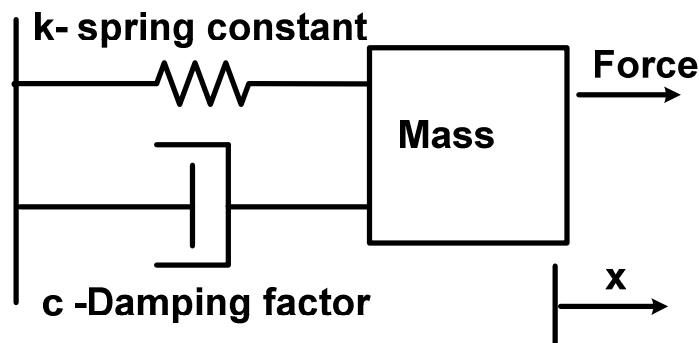


Figure 2.2: One-dimensional mass-spring damper system

The Coriolis force couples the sense and drive directions of the gyroscope (figure 2.3). To understand the dynamics and principle of operation of a gyroscope, the equations of motions of a simple mass-spring damper system (figure 2.2) will be derived.

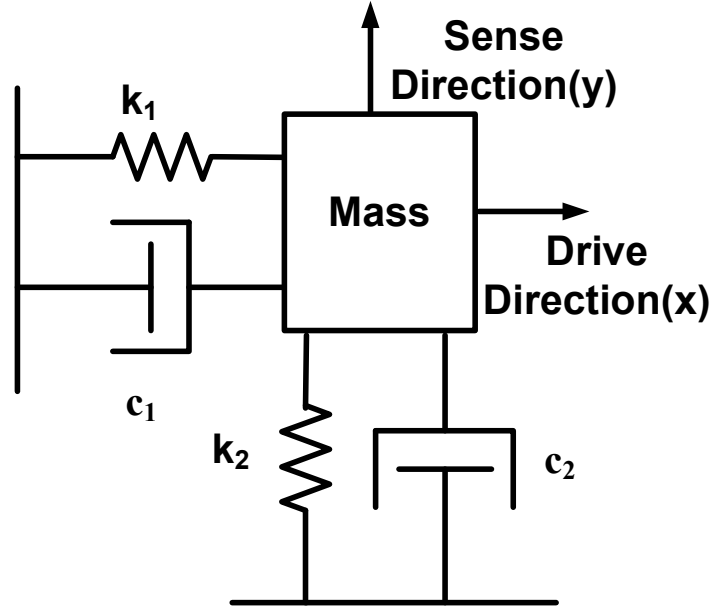


Figure 2.3: Two degrees-of-freedom (2-DOF) mass-spring damper system

From Newton's Second Law of motion, we know that:

$$\sum F = ma \quad (2.1)$$

$$\sum F = m \frac{\partial^2 x}{\partial t^2} \quad (2.2)$$

The forces acting on the mechanical system are the spring forces F_s , damping force F_d and the actuation force F_{el} .

$$F_s = -kx \quad (2.3)$$

$$F_d = -c \frac{\partial x}{\partial t} \quad (2.4)$$

Hence, equation 2.2 becomes: -

$$-kx - c \frac{\partial x}{\partial t} + F_{el} = m \frac{\partial^2 x}{\partial t^2} \quad (2.5)$$

$$F_{el} = m \frac{\partial^2 x}{\partial t^2} + c \frac{\partial x}{\partial t} + kx \quad (2.6)$$

Where,

- m = Mass of the body
- F_{el} = Actuation force on the Drive direction
- c = Damping Coefficient
- k = Spring Constant

Equation 2.6 holds in the inertial frame, where $\Omega = 0$. For an observer in the rotating frame, the rotation induced Coriolis force must be taken into consideration. The acceleration experienced by a moving body in a rotating frame can be derived starting with the following definitions of figure 2.4:

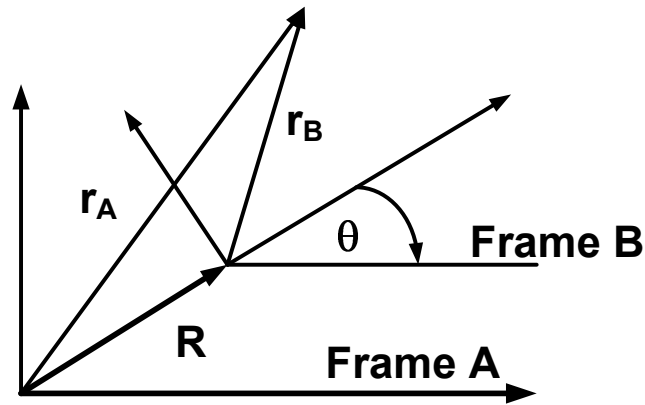


Figure 2.4: Time derivative of a vector in a rotating frame.

Where,

- Frame A = Inertial (stationary) frame
- Frame B = Non-inertial (rotating) reference frame
- r_A = Position vector relative to inertial frame A
- r_B = Position vector relative to inertial frame B
- Ω = Angular velocity of rotating frame B
- R = Position of rotating frame relative to inertial frame.
- θ = Orientation of rotating frame B

The velocity $\dot{r}_A$ as viewed in the inertial frame is related to the velocity in the rotating frame $\dot{r}_B$ as [6]:

$$\dot{r}_A(t) = \dot{r}_B(t) + \dot{\theta} x r_B(t) \quad (2.7)$$

However, the operator equivalent in the rotating frame to taking the time derivative in the inertial frame is: -

$$\left. \frac{\partial}{\partial t} \right|_A (\dot{r}_A(t)) = \left. \frac{\partial}{\partial t} \right|_B (\dot{r}_A(t)) + \Omega x (\dot{r}_A(t)) \quad (2.8)$$

$$a_A = a_B + \dot{\Omega} x \dot{r}_B + \Omega x \dot{r}_B + \Omega x \dot{r}_B + \Omega x (\Omega x r_B) \quad (2.9)$$

$$a_A = a_B + 2\Omega x \dot{r}_B + \Omega x (\Omega x r_B) + \dot{\Omega} x r_B \quad (2.10)$$

By Multiplication of equation 2.10 with mass (m), we get:

$$m a_A = m a_B + 2m\Omega x \dot{r}_B + m\Omega x (\Omega x r_B) + m\dot{\Omega} x r_B \quad (2.11)$$

Where:

$m a_A$	=	F_i (The applied force to the proof mass)
$m a_B$	=	F_r (The force experienced in the rotating frame)
$2m\Omega x \dot{r}_B$	=	Coriolis force
$m\Omega x \dot{r}_B$	=	Euler Force
$m\Omega x (\Omega x r_B)$	=	Centrifugal force

$$F_r = F_i - 2m\Omega x \dot{r}_B - m\Omega x (\Omega x r_B) - m\dot{\Omega} x r_B \quad (2.12)$$

If we apply the result to a gyroscope with 2-DOF as in figure 2.3, we get:

$$F_{r,x} = F_{i,x} + 2m\Omega \dot{y} + m\Omega^2 x + m\dot{\Omega} y \quad (2.13)$$

$$F_{r,y} = F_{i,y} - 2m\Omega \dot{x} + m\Omega^2 y - m\dot{\Omega} x \quad (2.14)$$

In the square plate we assume the angular rate is constant, hence $\dot{\Omega}=0$, and if we take into account the damping effects, our modified equations are:

$$m\ddot{x} + c\dot{x} + kx = F_{i,x} + 2m\Omega\dot{y} + m\Omega^2 x \quad (2.15)$$

$$m\ddot{y} + c\dot{y} + ky = F_{i,y} - 2m\Omega\dot{x} + m\Omega^2 y \quad (2.16)$$

Where,

c = Damping factor

k = $m\omega^2$ - Spring Constant

$$m\ddot{x} + c\dot{x} + m(\omega^2 - \Omega^2)x = F_{i,x} + 2m\Omega\dot{y} \quad (2.17)$$

$$m\ddot{y} + c\dot{y} + m(\omega^2 - \Omega^2)y = F_{i,y} - 2m\Omega\dot{x} \quad (2.18)$$

The two terms $2m\Omega\dot{y}$ and $2m\Omega\dot{x}$ are rotation induced coriolis forces, which show the coupling between the drive direction and the sense direction.

In the case of our device, the square plate is excited in the x-direction. The y-direction is the sense direction of which is excited by the Coriolis force. With the assumption $\omega^2 \gg \Omega^2$ equations 2.17 becomes:

$$m\ddot{x} + c\dot{x} + m\omega^2 x = F_{electrostatic} + 2m\Omega\dot{y} \quad (2.19)$$

But since $F_{electrostatic} \geq 2m\Omega\dot{y}$, equation 1.19 becomes:

$$m\ddot{x} + c\dot{x} + m\omega^2 x = F_{electrostatic} \quad (2.20)$$

$$m\ddot{y} + c\dot{y} + m\omega^2 y = -2m\Omega\dot{x} \quad (2.21)$$

Hence, the Coriolis induced force $2m\Omega\dot{x}$ is used for angular rate measurement.

Chapter 3

Mechanical Design of a MEMS Gyroscope

In this chapter, the design concept of a MEMS Gyroscope will be presented. Most conventional micro mechanical gyroscopes developed are resonant sensors, they are basically resonators. They have high bias stability, resolution and immune to environmental interference. In this work the gyroscope will be made from a lamé-mode resonator.

3.1 Lamé mode resonator

The basic architecture of a vibrating gyroscope is comprised of a drive-mode oscillator that generates and maintains a constant linear or angular momentum, coupled to a sense-mode coriolis accelerometer that measures the sinusoidal Coriolis force induced due to the combination of the drive vibration and an angular rate input.

In this project, the structure of the lamé-mode resonator is a square plate (figure 3.1). The plate is supported by 4 anchors. Two electrodes are used for exciting the resonator, one to sense the resonator motion. The primary mode of vibration is a Lamé-mode, in which the edges of the square plate bend in anti-phase, so preserving the volume of the plate.

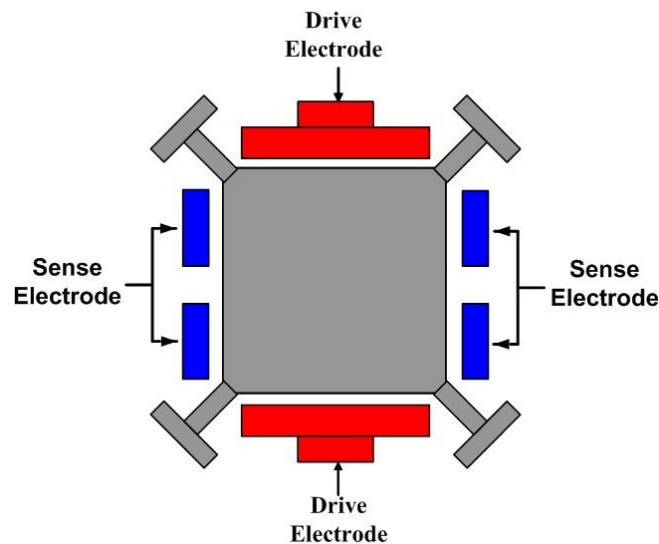


Figure 3.1: Lamé mode resonator layout

3.2 Resonator theory

Resonating systems are governed by equations of motion with one degree of freedom. The resonator can be viewed as a one dimensional mechanical damper system (figure 3.2). We can derive the equations governing the motion of the one dimensional damper system, as in figure 3.1 from Newton's Second Law of motion:

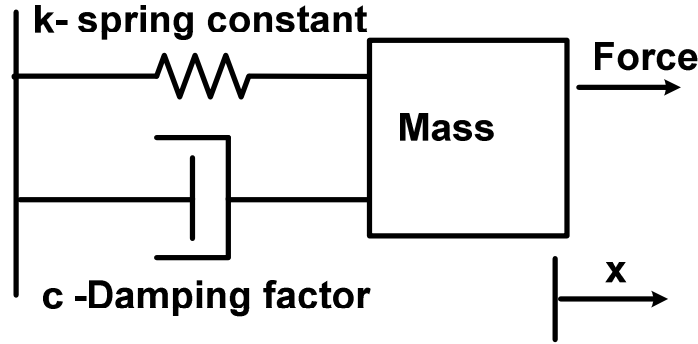


Figure 3.2: One dimensional mechanical damper system

$$\sum F = ma \quad (3.1)$$

$$\sum F = m \frac{\partial^2 x(t)}{\partial t^2} \quad (3.2)$$

The forces acting on the mechanical system are the spring forces F_s , damping force F_d and the actuation force F_{el}

$$F_s = -kx(t) \quad (3.3)$$

$$F_d = -c \frac{\partial x}{\partial t} \quad (3.4)$$

Substituting equations 3.3 and 3.4 into equation 3.2:

$$-kx - c \frac{\partial x}{\partial t} + F_{el} = m \frac{\partial^2 x}{\partial t^2} \quad (3.5)$$

$$F_{el} = m \frac{\partial^2 x}{\partial t^2} + c \frac{\partial x}{\partial t} + kx \quad (3.6)$$

Where,

m	=	Mass of the body
F _{el}	=	Sinusoidal actuation force on the Drive direction
c	=	Damping Coefficient
k	=	Spring Constant

F_{el} is a harmonic force which actuates the device to vibrate at a given resonance frequency as follows: -

$$F_{el}(t) = F_o e^{j\omega t} \quad (3.7)$$

With the definition of the un-damped natural frequency ω_n and the damping factor ξ which represents the ratio of the damping to critical damping ($2\sqrt{km}$), the equation of motion becomes:

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \frac{F_{el}(t)}{m} \quad (3.8)$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad (3.9)$$

$$\xi = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{c}{2m\omega_n} \quad (3.10)$$

We assume the equation is linear, and has a solution of the form: -

$$x(t) = x_o e^{j\omega t + \varphi} \quad (3.11)$$

Furthermore, the derivatives of (3.8) are as follows: -

$$\dot{x}(t) = j\omega x_o e^{j\omega t + \varphi} = j\omega x(t) \quad (3.12)$$

$$\ddot{x}(t) = -\omega^2 x_o e^{j\omega t + \varphi} = -\omega^2 x(t) \quad (3.13)$$

After substitution of equation 3.7 to 3.10 into (3.6) we get:

$$x_o = \frac{\frac{F_o}{k}}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{2\xi \frac{\omega}{\omega_n}\right\}^2}} \quad (3.14)$$

$$\varphi = -\tan^{-1} \left(\frac{2\xi \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right) \quad (3.15)$$

In the presence of a considerable damping, the amplitude expression is maximized at the frequency

$$\omega_r = \omega_n \sqrt{1 - 2\xi^2} \quad (3.16)$$

For lightly damped systems i.e. $\xi \leq 1$, the amplitude is maximized at the natural frequency ω_n , and the amplitude at the resonance becomes

$$|x_o|_{res} = \frac{F_o}{2k\xi} = \frac{F_o}{c\omega_n} \quad (3.17)$$

The Quality factor of the system is defined as the maximum ratio of the amplitude to the static deflection, which is F_o / k . Taking the ratio of the amplitude at resonance to the static deflection, the Q factor of a lightly damped system reduces to

$$Q = \frac{1}{2\xi} \quad (3.18)$$

It should be noticed that the quality factor is one of the most important parameters of a resonator, since it directly scales the amplitude at resonance. For example, for a resonator with a known Q factor, the oscillation amplitude at resonance can be found as

$$|x_o|_{res} = Q \frac{F_o}{k} \quad (3.19)$$

At resonance frequency, the phase is -90° shifted from the forcing function phase. At frequencies lower than the resonant frequency, the phase approaches 0° , meaning that the position follows the forcing function closely.

At frequencies higher than the resonant frequency, the phase approaches -180° . The transition from 0° to -180° around the resonant frequency becomes more abrupt for higher Q values.

The bandwidth or the half-power bandwidth of the system is defined as the difference between the frequencies where the power is half of the resonance power. Since the power is proportional to the square of the oscillation amplitude, the half power frequencies are solved by equating the amplitude expression to $1/\sqrt{2}$ times the resonance amplitude.

For small values of damping, the bandwidth is approximated as:

$$BW \approx \frac{\omega_n}{Q} \quad (3.20)$$

This analysis forms the background for the following discussions on the dynamics and response of the drive and sense oscillators in vibratory gyroscope.

3.3 DRIVE-MODE OPERATION

Most vibratory gyroscopes are based on conservation of momentum between the drive and sense modes. In vibrating gyroscopes, the drive mode oscillator, is the source of momentum. The drive mode oscillator can be modeled as a mass-spring damper system figure 3.2. The derivation of the equations of motion follows the same steps as from equation 3.1 to equation 3.20.

With the definition of the drive mode frequency as ω_d , the drive proof mass m_d , drive spring stiffness k_d , drive damping c_d , drive mode Quality factor Q_d , drive amplitude $x = x_o e^{j\omega_d t + \phi}$ and harmonic drive force $F_d e^{j\omega_d t}$, the drive equations of motion along the drive axis becomes

$$m_d \ddot{x} + c_d \dot{x} + k_d x = F_d e^{j\omega_d t} \quad (3.21)$$

$$x_o = \frac{F_d}{k_d \sqrt{\left\{ 1 - \left(\frac{\omega}{\omega_d} \right) \right\}^2 + \left\{ \frac{1}{Q_d} \frac{\omega}{\omega_d} \right\}^2}} \quad (3.22)$$

$$\varphi_d = -\tan^{-1} \left(\frac{\frac{1}{Q_d} \frac{\omega}{\omega_d}}{1 - \left(\frac{\omega}{\omega_d} \right)^2} \right) \quad (3.23)$$

$$\omega_d = \sqrt{\frac{k_d}{m_d}} \quad (3.24)$$

$$Q_d = \frac{m_d \omega_d}{c_d} \quad (3.25)$$

The scale factor of the gyroscope is directly proportional to the drive-mode oscillation amplitude. The phase and the frequency of the drive oscillation directly determine the phase and frequency of the Coriolis force, and subsequently the sense mode response. Thus, it is important to maintain a drive mode oscillation with stable amplitude, phase and frequency.

For that reason, almost all the gyroscope operates at drive mode resonant frequency in practical implementations. At resonance, the drive-mode phase becomes -90° , and the amplitude simply reduces to;-

$$|x_o|_{res} = Q_d \frac{F_d}{m_d \omega_d^2} \quad (3.26)$$

3.4 SENSE-MODE OPERATION

When the device is rotated along the rotation axis, while vibrating at the drive mode resonant frequency, a coriolis force is induced in the sense direction. The Coriolis force is of the form:

$$F_C = 2m_c \Omega_z \dot{x} \quad (3.27)$$

We know that $\dot{x}(t) = j\omega x_o e^{j\omega t + \varphi} = j\omega x(t)$ from equation 3.12.

$$F_C = 2m_c \Omega_z jx_o \omega_d e^{j\omega_d t} \quad (3.28)$$

Where m_c is the portion of the driven proof mass that contributes to the Coriolis force. In a single mass design, it is assumed $m_c = m_d$. The Coriolis force amplitude, which sets the scale factor of the gyroscope, is directly proportional to the drive-mode oscillation amplitude.

Hence, it's critical that the drive amplitude regulated to achieve a stable scale factor. The one degree of freedom sense mode oscillator is

$$m_s \ddot{y} + c_s \dot{y} + k_y y = 2m_c x_o \dot{x}(t) \quad (3.29)$$

Where m_s is the portion of the proof mass that responds to the Coriolis force. Again, in a simple single mass design, m_c, m_d and m_s are equal.

The amplitude and phase of the steady-state sense mode Coriolis response in a linear system, defining the sense-mode resonant frequency ω_s and the sense-mode Quality factor Q_s becomes

$$y_o = \Omega_z \frac{m_c \omega_d}{m_s \omega_s^2} \frac{2x_o}{\sqrt{\left\{1 - \left(\frac{\omega_d}{\omega_s}\right)^2\right\}^2 + \left\{\frac{1}{Q_s} \frac{\omega_d}{\omega_s}\right\}^2}} \quad (3.30)$$

$$\varphi_s = -\tan^{-1} \left(\frac{\frac{1}{Q_s} \frac{\omega_d}{\omega_s}}{1 - \left(\frac{\omega_d}{\omega_s}\right)^2} \right) + \varphi_d \quad (3.31)$$

$$\omega_s = \sqrt{\frac{k_s}{m_s}} \quad (3.32)$$

$$Q_s = \frac{m_s \omega_s}{c_c} \quad (3.33)$$

To achieve the maximum possible gain in the sense-mode, it is generally desirable to operate at or near peak of the sense mode response curve. This is typically achieved by matching drive and sense resonant frequencies. When operating at sense-mode resonance, i.e. $\omega_d = \omega_s$, the sense-mode phase becomes -90° from the drive velocity, and the amplitude reduces to

$$|y_o|_{res} = \Omega_z \frac{2Q_s x_o m_C}{m_s \omega_s} \quad (3.34)$$

3.5 MODE MATCHING

The matching of the sense and drive mode resonance frequency amplifies the coriolis response of the gyroscope. However, operating close to the resonant peak also makes the system very sensitive to variations in system parameters that cause a shift in the resonant frequencies or damping.

Consider a sense-mode system with a resonant frequency of $\omega_s=10$ kHz and a Q factor $Q_s=1000$ as seen in figure 3.3. When the operating frequency matches the sense-mode frequency ω_s , the amplification factor is 1000, equal to the Q factor. If there is only 5Hz relative shift between the operating frequency and the sense-mode resonant frequency, the gain drops by 29.3%. For a 10Hz relative shift, the gain drop is 55%.

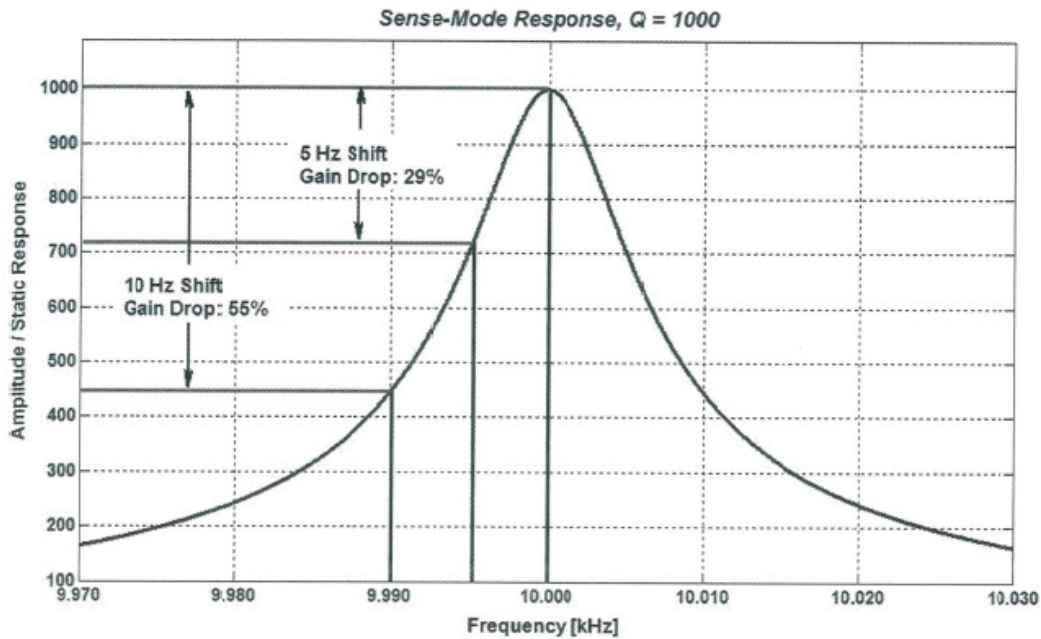


Figure 3.3: The sense-mode amplification of a sense-mode system with a resonant frequency of $\omega_s=10$ kHz and a Q factor of $Q_s=1000$. For a 5Hz relative shift between the operating frequency and the sense-mode resonant frequency, the gain drop is 29% [7].

Under higher quality factor conditions the gain is higher. However, the bandwidth becomes even narrower. For example, let us take the same sense-mode system with a resonant frequency of $\omega_s=10$ kHz, and increase the Q-factor to $Q_s=10,000$ as seen in Fig 3.4. The amplification factor at resonance directly increases to 10,000.

However the half-power bandwidth becomes:

$$BW = \frac{\omega_s}{Q_s} = \frac{10,000\text{Hz}}{10,000} = 1\text{Hz}$$

Hence, the relative position of the sense-mode frequency with respect to the operating frequency has to be controlled with extreme position.

Furthermore, fabrication imperfections are inevitable due to the process spread. They affect the geometry of the device and material properties. This results into variations of the resonant frequency from device to device.

Structural and environmental effects that result in quite large variations in the resonant frequencies, it is extremely difficult to control the drive and sense frequencies with high precision. It is common practice to operate away from the resonant frequency of the sense-mode, where frequency variations have reduced effect on the output gain and phase.

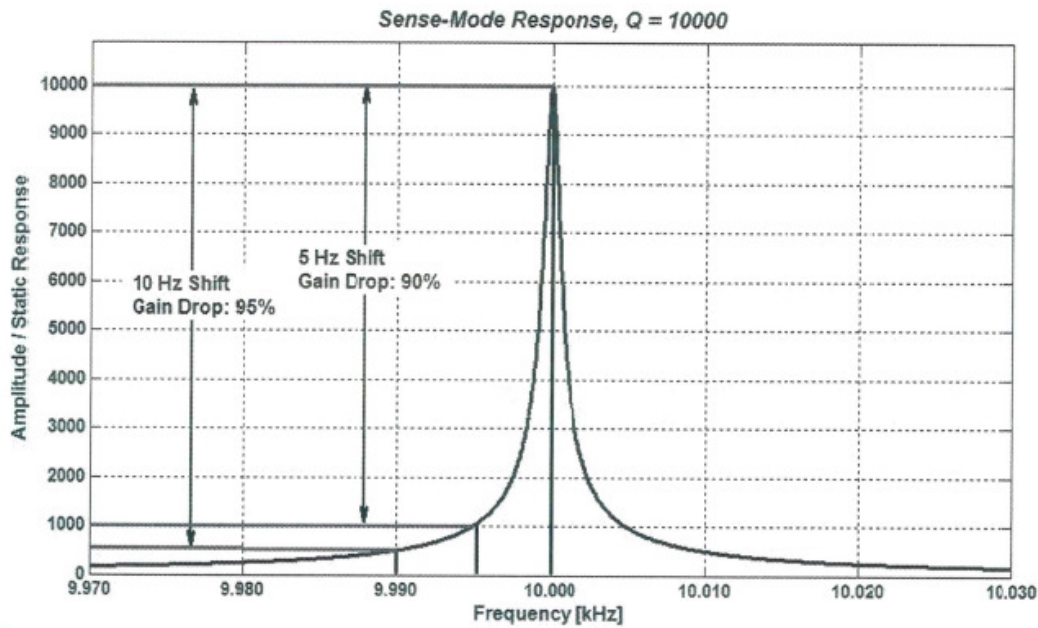


Figure 3.4: High Quality factor devices provide higher gains, though more sensitive to frequency variations. The sense-mode amplification of a system with $\omega_s=10$ kHz and $Q_s=10,000$ drops by over 90% for a 5Hz relative shift between the operating frequency and the sense-mode resonant frequency [7].

Fluctuations in damping cause significant frequency variations. The result is the variations of the Q-factor of sense-mode. To reduce the effect of damping, devices are packaged to maintain a near vacuum pressure. However, damping shift with temperature changes.

Damping stability is obtained by operating devices away from the resonance peak to reduce fluctuations.

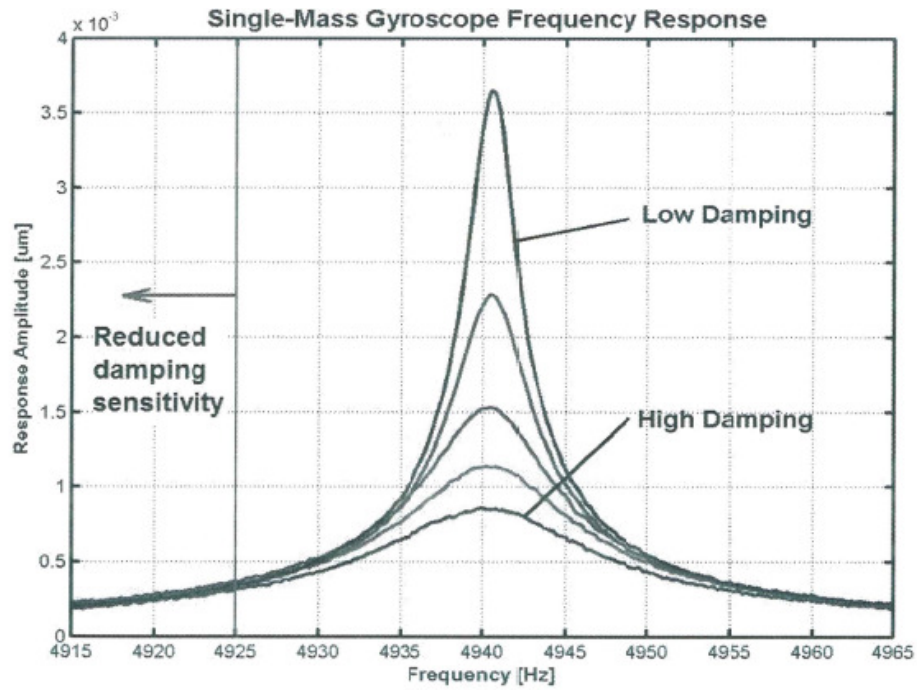


Figure 3.5: The sense-mode gain is sensitive to damping, for devices operated near the resonance peak [7].

Chapter 4

MEMS Gyroscope Simulation

This chapter describes the Finite Element simulations carried out with COMSOL Multiphysics to analyze the performance and optimize the design of the MEMS Gyroscope. The Eigen frequency analysis of drive and sense mode are described in the first part, followed by the mode matching in the second part.

4.1 Simulation model of the gyroscope

4.1.1 Geometry modeling

The geometry of the square plate is shown in Figure 3.1. Refer to Appendix A for a detailed procedure on geometry modeling. The geometry has etching holes in order to speed up the oxide etching below the structure. Furthermore, the square plate is anchored and fixed on the four corners.

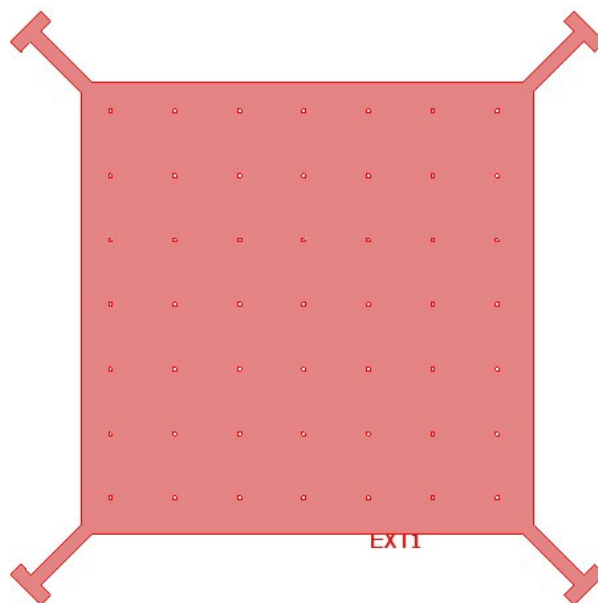


Figure 4.1: Top view of the square plate

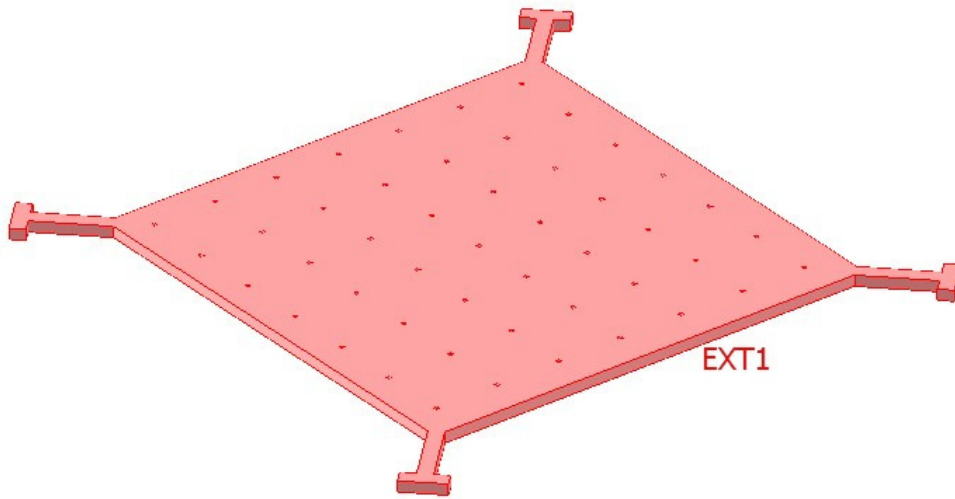


Figure 4.2: Side view of the square plate

4.1.2 Eigen Frequency analysis

The eigenvalue solver from COMSOL Multiphysics uses the relation between complex eigenvalue λ and frequency f as seen in the equation below to solve for Eigen frequencies of geometry.

$$\lambda = i\omega = i2\pi f \quad (4.1)$$

After drawing, the geometries and setting up the model, an eigenfrequency analysis followed. This would allow us to know the different eigenfrequencies of the square plate. For more information on modeling and eigenfrequency analysis in COMSOL refer to Appendix A.

The frequencies of interest are the Lamé-mode as seen in figure 3.3, and the shear mode as seen in figure 3.5. Furthermore, a frequency sweep with high resolution close to the resonant frequencies was performed to gain more insight on the Q-factor of the device. Plot 3.3 and 3.4 were obtained from the simulation.

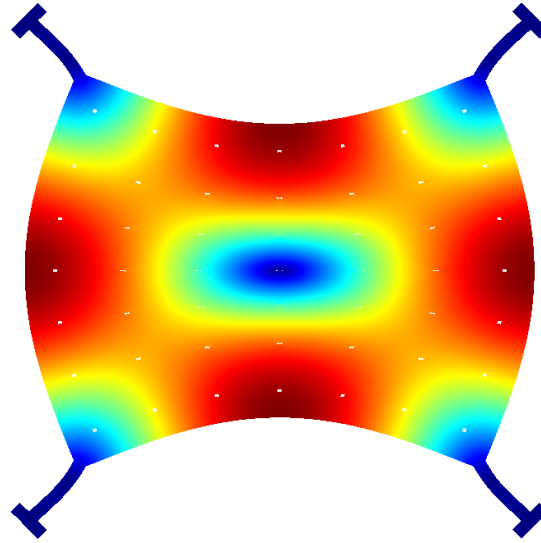


Figure 4.3: The drive mode of the geometry at 5.240142e7Hz; known as Lamé- mode

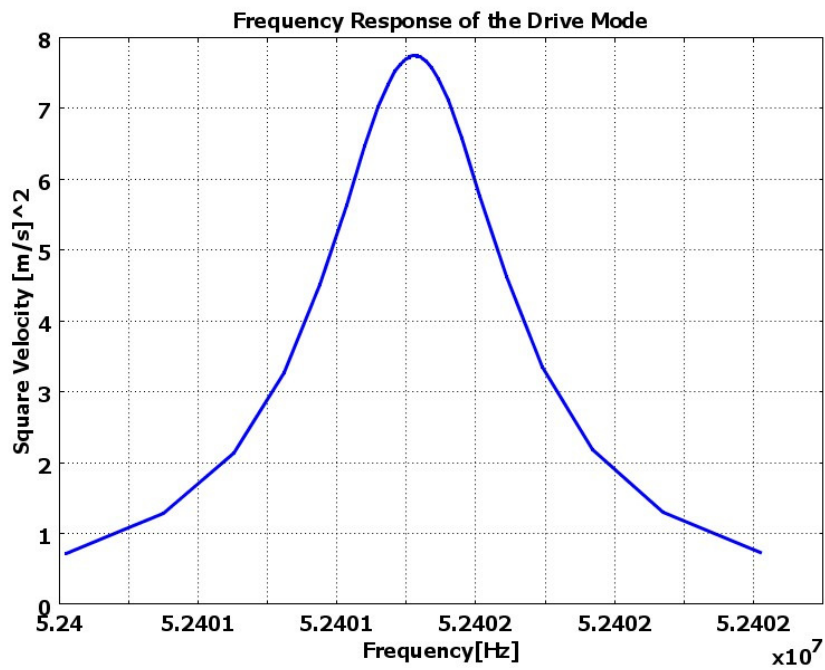


Figure 4.4: The frequency response plot of drive mode of the geometry at 5.240142e7Hz, known as Lamé- mode

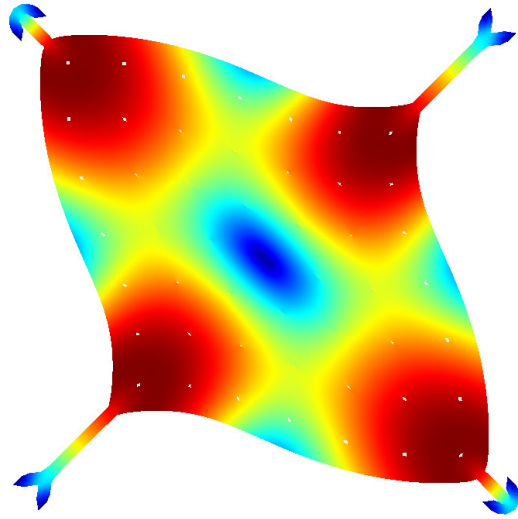


Figure 4.5: The sense mode of the geometry at 59.638MHz, known as shear mode.

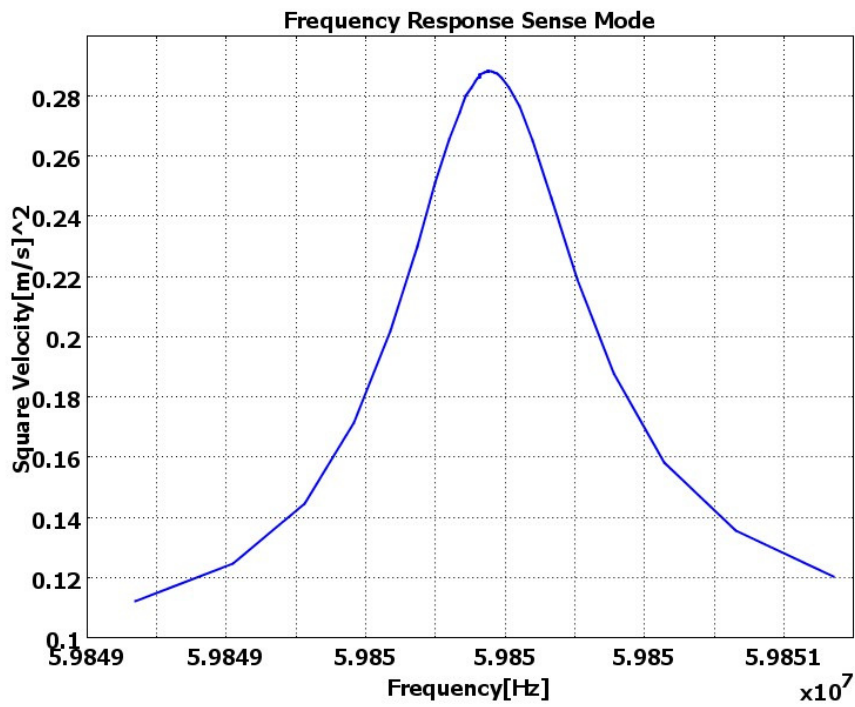


Figure 4.6: The frequency response plot of sense mode of the geometry at 5.984994e7Hz, known as Shear- mode

The lamé-mode at 52.40142MHz is the drive mode, and the shear-mode at 59.8499MHz is the sense-mode. The two modes must be matched closely in order to improve the performance of the gyroscope. The matching of shear-mode and lamé-mode resonance frequencies amplifies the Coriolis force and reduces the response time of the gyroscope.

4.2 Drive and sense mode matching

The matching of drive and sense mode resonance frequencies greatly enhances the sense-mode mechanical response to angular rate input, hence it amplifies the coriolis force and reduce the response time of the gyroscope.

The devices have a Quality factor, $Q = 30,000$ and since $f_o = 52\text{MHz}$ hence,

$$BW \approx \frac{f_o}{Q} = \frac{52\text{MHz}}{30,000} \approx 1730\text{Hz} \quad (4.2)$$

Under high Q factor conditions the gain is high, however the bandwidth is narrow. This makes mode-matching very sensitive to fabrication imperfections, damping factor and structural effects among others. The variations in system parameters cause a shift in resonance frequency, hence frequency mismatch.

In order to match the lamé-mode and shear mode of the gyroscope, the width of the anchors (s1 and s2) of the square plate were varied by using a COMSOL script to match the two resonance frequencies. The table below shows square plate dimensions used.

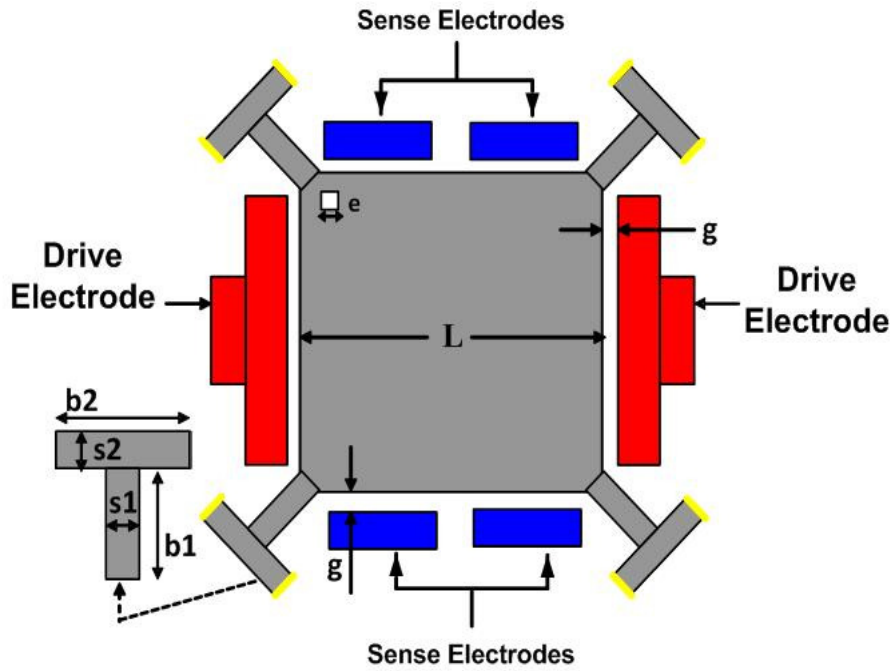


Figure 4.7: The square plate resonator

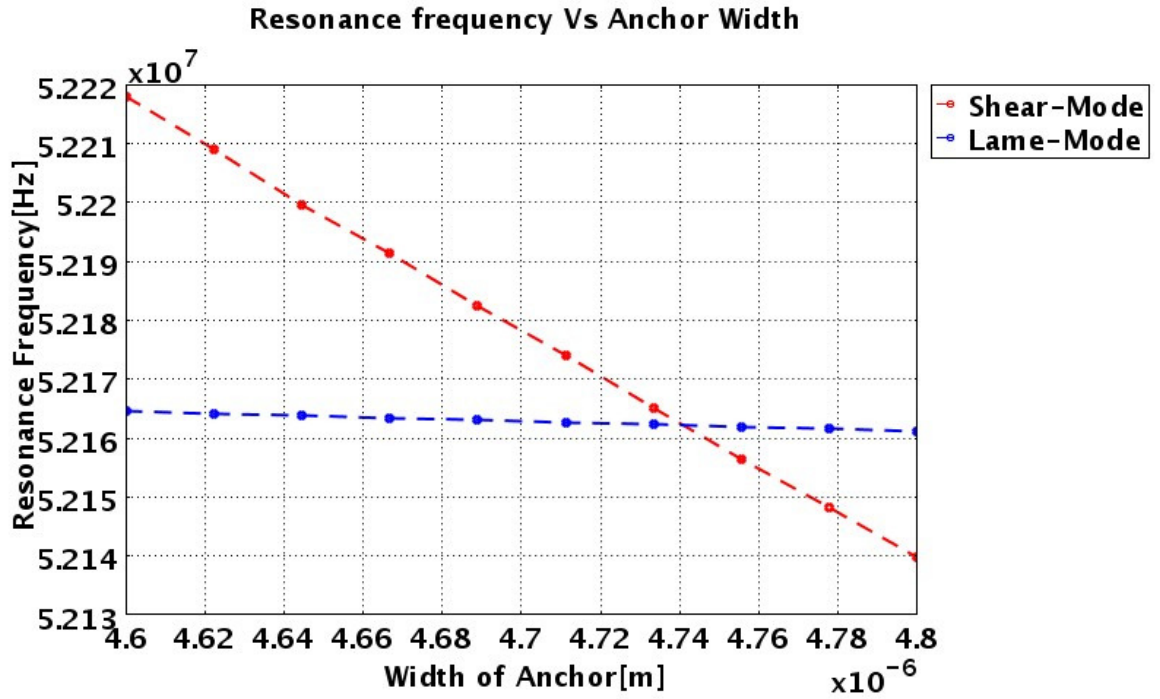


Figure 4.8: A plot of Resonance frequency Versus Thickness of the anchors

Name	L[um]	h[um]	Nh	s1[um]	b1[um]	s2[um]	b2[um]	pillar	Gap[nm]
Sqplate	63	0.8	9	4.74	10	2	25	No	200

Figure 4.8 was obtained from a script from COMSOL Multiphysics. From the plot, it can be deduced that, a thickness of 4.74um of s1, would make the drive (lamé-mode) and sense (shear mode) matched.

4.3 Dynamic behavior of the gyroscope

The dynamic behavior of the gyroscope, under matched-mode conditions was simulated using a COMSOL Multiphysics script. The gyroscope was excited at a drive frequency of 52MHz. The Coriolis force was incorporated on the gyroscope.

The matched sense and drive frequency will start splitting further apart. This splitting is proportional to the Coriolis force, hence proportional to the angular rotation of the device. The response of the gyroscope under the influence of angular rate input is shown in figure 3.9.

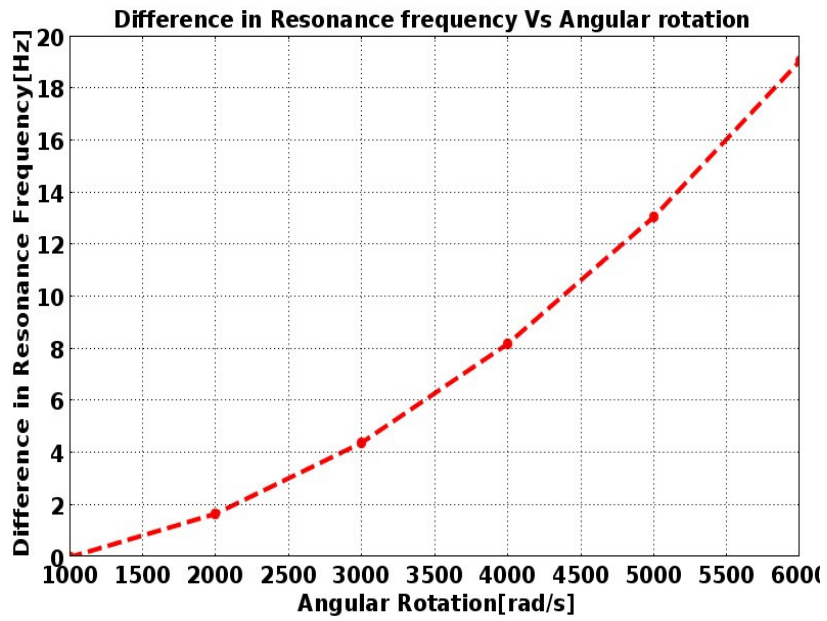


Figure 4.9: Plot of the Sensitivity of the MEMS Gyroscope

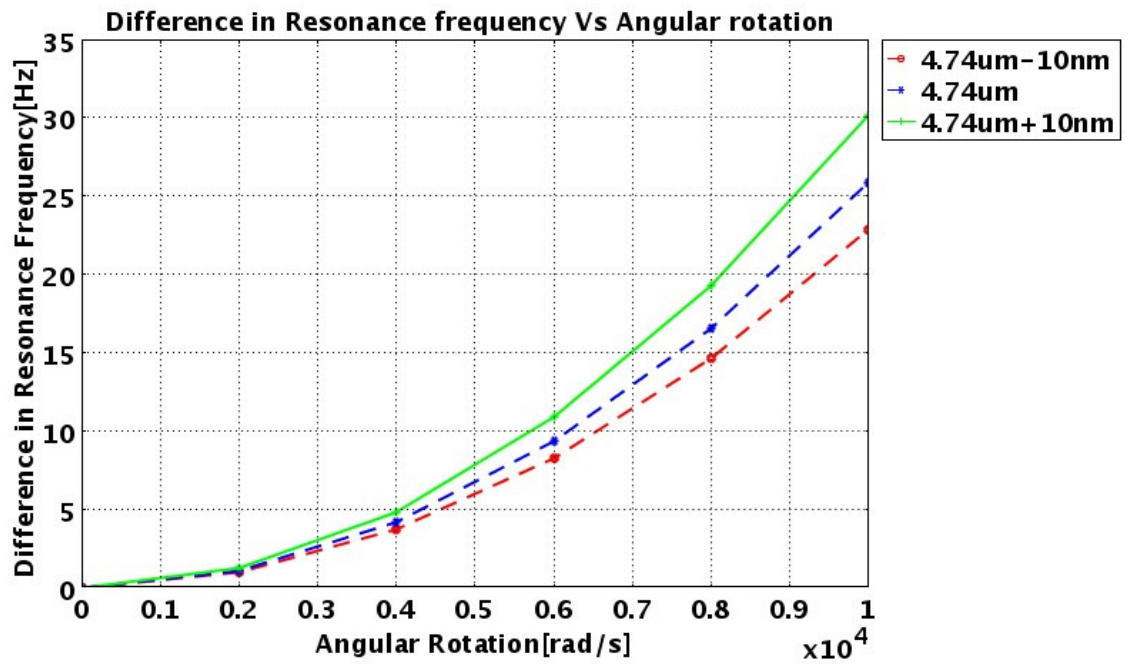


Figure 4.10: Plot of the Sensitivity of the MEMS Gyroscope with mismatch

4.4 Model validation

To validate the simulation model, the analytical amplitude will be compared to the simulated value. The drive mode DC voltage was 30V and power of -25dBm , hence an AC voltage of 40mV

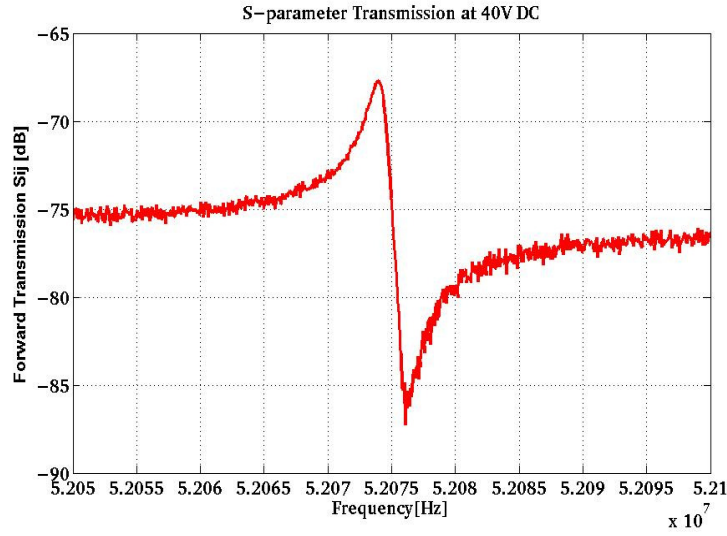


Fig 4.11: Transmission measurements (S_{ij}) of the device at 52MHz. The drive voltage is 40V DC.

The known parameters of the device are:

$$mass = 6 \times 10^{-10} \text{ Kg}$$

$$width = 63 \times 10^{-6} \text{ m}$$

$$height = 1.5 \times 10^{-6} \text{ m}$$

$$gap = 200 \times 10^{-9} \text{ m}$$

$$f_o = 52 \text{ MHz}$$

Furthermore, we know from equation 2.9 that

$$\omega_n = \sqrt{\frac{k}{m}} \quad (4.3)$$

Hence, the analytical value of the spring constant $k = 6.4 \times 10^7$. The Q-factor extracted from the measurements of the device $Q = 30,000$. The force F_o can be calculated using the formula below:

$$F_o = \frac{\varepsilon_o * width * height}{gap^2} * V_{dc} V_{ac} \quad (4.4)$$

$$F_o = 2.5 \times 10^{-8} \text{ N} \quad (4.5)$$

The analytical amplitude of the device at resonance is given by:

$$|x_o|_{res} = Q \frac{F_o}{k} \quad (4.6)$$

Hence, the analytical amplitude of the device:

$$x_{analytical} = 1.5 \times 10^{-11} \text{ m}.$$

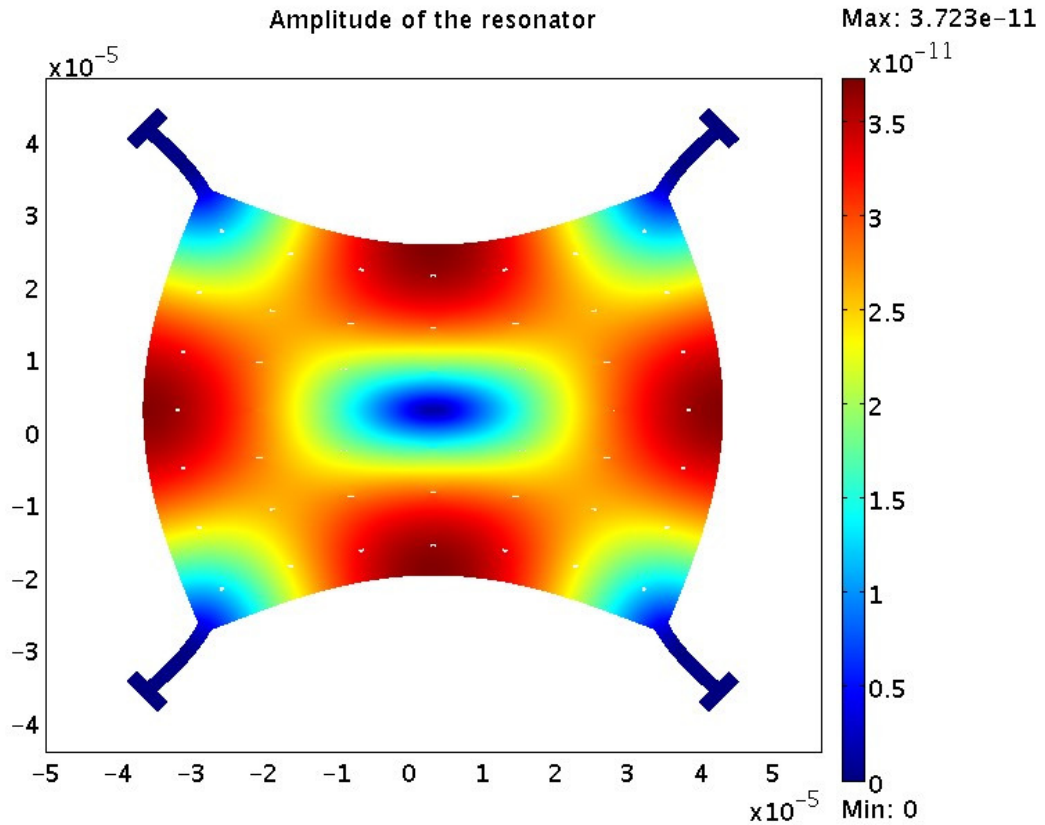


Figure 4.12: The result of the frequency response analysis of the simulation model.

The figure 4.11 shows the amplitude of the device from the simulation model. The amplitude of the resonator is around:

$$x_{simulated} = 3 \times 10^{-11} m .$$

The COMSOL model of the device is a good approximation of the device since it has been demonstrated that $x_{simulated} \approx x_{analytical}$.

The damping and Q-factor of the device were also modeled in COMSOL through Rayleigh method. For more information on Rayleigh Damping refer to COMSOL user's guide.

Chapter 5

Readout Topology

5.1 Electrostatic actuation

An electrostatic force on electrodes on the sides of the resonator, forces the structure to resonate at a desired frequency, causing sinusoidal displacement of the gap across the electrodes and the square plate.

The sinusoidal electrostatic force is applied across the gap is given as follows: -

$$F_{el} = \frac{\partial E}{\partial x} \quad (5.1)$$

Where E is the energy stored in the capacitor and is given by

$$E = \frac{1}{2} C(x) V(t)^2 \quad (5.2)$$

Substituting equation 5.2 into equation 5.1 gives the following

$$F_{el} = \frac{1}{2} \frac{\partial C(x)}{\partial x} V^2(t) \quad (5.3)$$

However, the applied Voltage (V) is a sum of an AC and DC voltages, given by:

$$V(t) = V_{dc} + V_{ac} e^{j\omega t} \quad (5.4)$$

By substituting equation 5.4 into equation 5.3 gives the following

$$F_{el} = \frac{d \frac{\epsilon_o w h}{(g-x)}}{\partial x} (V_{dc}^2 + 2V_{dc} V_{ac} e^{j\omega t} + V_{ac}^2 e^{2j\omega t}) \quad (5.5)$$

The only term of interest is $2V_{dc} V_{ac} e^{j\omega t}$. We can neglect the term V_{dc}^2 since it doesn't contribute to the actuation. Furthermore $V_{ac}^2 e^{2j\omega t}$ is very small, hence negligible.

$$F_{el} = -\frac{d \frac{\epsilon_o w h}{(g-x)}}{\partial x} V_{dc} V_{ac} e^{j\omega t} \quad (5.6)$$

Where,

w = Width of the gap
h = Thickness of the SOI layer.

Hence, if we introduce the electromechanical coupling coefficient η :

$$\eta = V_{dc} \frac{\partial C}{\partial t} \quad (5.7)$$

Equation 5.6 becomes:

$$F_{el} = \eta V_{ac} e^{j\omega t} \quad (5.8)$$

5.2 Response of the resonator

The variation of the actuating voltage causes a variation of charge accumulated over the gap over time. This induces a current through the resonator as seen below:

$$I = \frac{\partial q}{\partial t} = \frac{\partial (C(x)V(t))}{\partial t} \quad (5.9)$$

$$I = (V_{dc} + V_{ac}) \frac{\partial C}{\partial x} \frac{\partial x}{\partial t} \quad (5.10)$$

However, $V_{dc} \geq V_{ac}$ hence we can ignore AC part:

$$I = V_{dc} \frac{\partial C}{\partial x} \frac{\partial x}{\partial t} = \eta \frac{\partial x}{\partial t} \quad (5.11)$$

Where η = electromechanical coupling coefficient:

$$\eta = \frac{d \frac{\epsilon_o w h}{(g-x)}}{\partial x} V_{dc} \approx \frac{\epsilon_o w h}{g^2} V_{dc} \quad (5.12)$$

Hence, the current is proportional to the velocity of the mechanical resonator. Furthermore, the electromechanical coupling - η - transforms a voltage into a force (equation 5.11) and a motion into a current (equation 5.12).

This chapter describes the detection method for MEMS Gyroscope. First, the mechanical signal is transduced an electrical signal. Afterwards, the frequency of the electrical signal is compared to a reference drive frequency by using a phase detector. Finally, a frequency output is obtained as an output signal.

5.3 CAPACITIVE DETECTION

The sense electrode of the gyroscope design is parallel to the square plate resonator. The parallel capacitance between the electrode and the body mass is:

$$C = \frac{\epsilon_o A_{overlap}}{d} \quad (5.13)$$

Where ϵ_o = permittivity of free space, $A_{overlap}$ = the overlap area between the sense electrode and the square plate and d = gap between the electrode and the square plate.

When the device is in the sense mode – shear mode – the capacitance between the sense electrode and the device varies with the deflection caused by the Coriolis force. The change in capacitance ΔC can be presented as shown as:

$$\Delta C = \frac{\epsilon_o A_{overlap}}{d - \Delta d} - \frac{\epsilon_o A_{overlap}}{d} \quad (5.14)$$

The change in capacitance is a nonlinear function of displacement in variable-gap capacitors. However, in the case that $\Delta d \leq d$, the change in capacitance ΔC varies linearly to the displacement.

$$\Delta C \approx \frac{\epsilon_o A_{overlap}}{d^2} \Delta d \quad (5.15)$$

The proposed capacitive readout topology for the gyroscope is shown in figure 4.10. In the presence of angular rotation, the device vibrates in shear mode. The asymmetrical shape of the shear mode results into the gap capacitances on the side sense electrodes to be:

$$C_{s+} = \frac{\epsilon_o A_{overlap}}{d - \Delta d}, \text{ and } C_{s-} = \frac{\epsilon_o A_{overlap}}{d + \Delta d} \quad (5.16)$$

With the help of transimpedance amplifiers the gap capacitances C_{s+} and C_{s-} would be converted to V1 and V2. The differential voltage $V1 - V2 = \Delta V$ from the instrumentation amplifier can be demodulated synchronously.

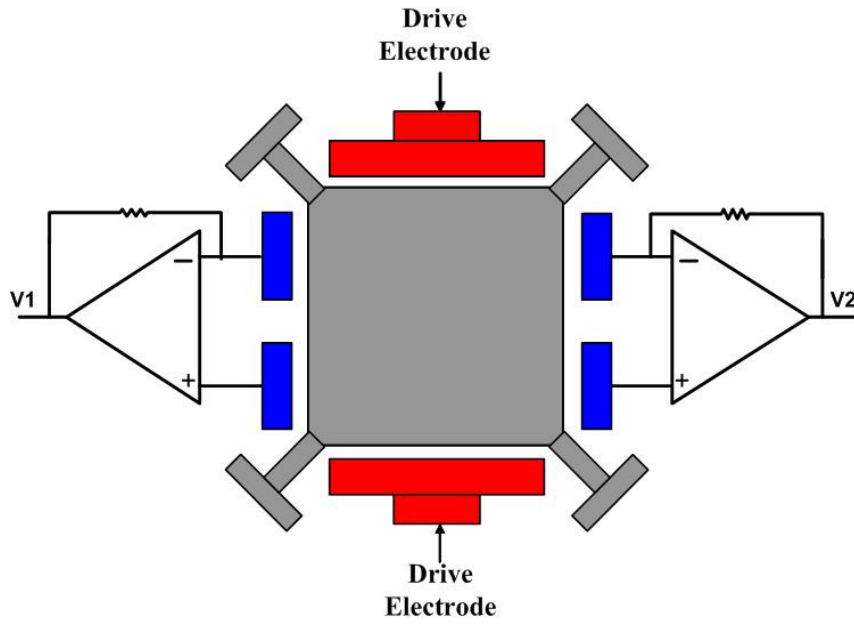


Figure 5.1: A capacitive readout topology

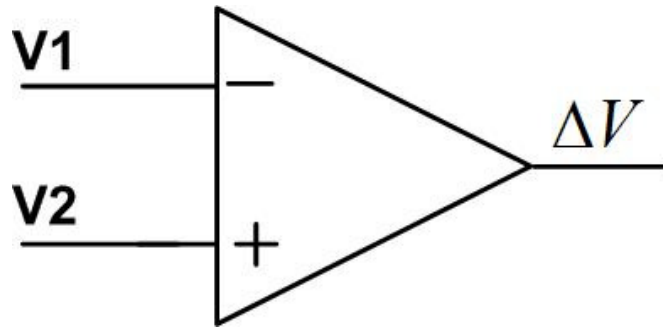


Figure 5.2: Differential Amplifier

In the case of our design with parameters:

$$\begin{aligned}
 A_{\text{overlap}} &= \text{width} * \text{height} = 63 \times 10^{-6} * 1.5 \times 10^{-6} \text{m}^2 \\
 \epsilon_0 &= 8.85 \times 10^{-12} \text{F/m} \\
 d &= 200 \text{nm} \\
 \Delta d &= 3 \times 10^{-11} \text{m}
 \end{aligned}$$

The variable capacitor from equation 5.20

$$\Delta C \approx \frac{2 * 8.85 \times 10^{-12} * 1.5 \times 10^{-6} * 63 \times 10^{-6}}{(2 \times 10^{-7})^2} * 3 \times 10^{-11} \quad (5.17)$$

$$\Delta C = 1.254 \text{aF} \quad (5.18)$$

Hence, the expected sensitivity of the capacitive readout is 1.254aF.

Furthermore, the most basic detection approach is to directly amplify the motional current due to the sense-mode oscillation. By imposing a constant DC bias voltage V_{DC} over a sense electrode with the capacitance $C_s = C_{sn} + \Delta C_s e^{j\omega_s t}$, the motional current becomes

$$i_s = \frac{\partial}{\partial t} [V_{DC} C_s(t)] \quad (5.19)$$

$$i_s = V_{DC} \omega_s \Delta C_s e^{j\omega_s t} \quad (5.20)$$

Hence, the expected current of readout topology with $\omega_s=52\text{MHz}$, $\Delta C = 1.254\text{aF}$ and $V_{DC} = 40\text{V}$ is

$$i_s = 40 * 2 * \pi * 52 \times 10^6 * 1.254\text{aF} \quad (5.21)$$

$$i_s = 1.638 \times 10^{-8} \text{A} \quad (5.22)$$

The typical value of the feedback resistors used in transimpedance amplifiers is $1\text{M}\Omega$. The output voltage from the transimpedance amplifiers in the readout circuit in Figure 5.10 becomes:

$$\Delta V = K * i_s \quad (5.23)$$

$$\Delta V = 1\text{M}\Omega * 1.638 \times 10^{-8} \text{A} \quad (5.24)$$

$$\Delta V = 16.38\text{mV} \quad (5.25)$$

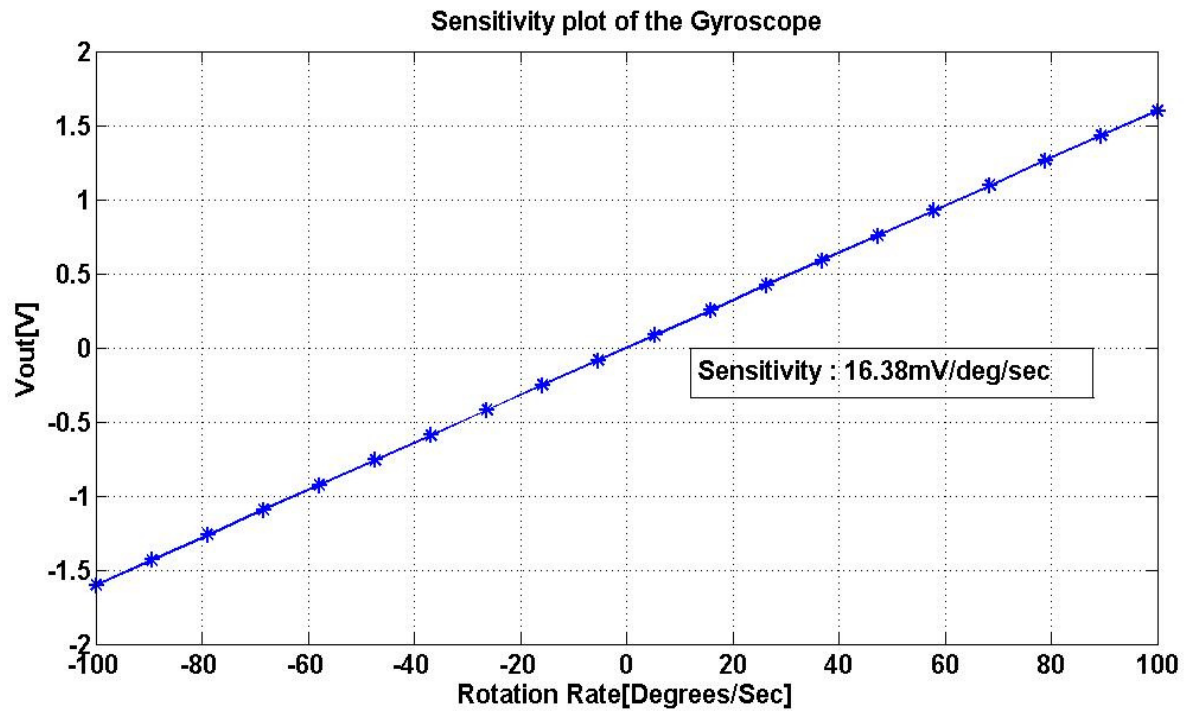


Figure 5.3: The simulated sensitivity plot of the lamé-mode gyroscope.

Furthermore, increasing the overall sensing area provides improved sensitivity. However, the sensing electrode gap d is the foremost factor that defines the capacitance sensitivity. This is because ΔC varies inversely to the squarer of the gap and only linear to the overlap area.

The advantage of this readout method is that the smaller gaps results into higher sensitivities for variable gap capacitors.

Chapter 6

Gyroscope Fabrication

In this chapter, the fabrication process of the Gyroscope is presented. The Gyroscope is fabricated on SOI (Silicon on Insulator) wafers. At Philips Semiconductors in Nijmegen the process that runs SOI wafers is **ABCD**. ABCD is an abbreviation for **A**dvanced-**B**ipolar **C**MOS **D**MOS. In this project the MEMS Gyroscope were developed in ABCD2, using VibrantiN3a.

6.1 ABCD process flow

The MEMS resonators are fabricated on silicon-on-insulator (SOI) wafers. SOI wafers consist of four layers.

1. The top-layer is the active single-crystal silicon layer in which devices are made, the SOI layer.
2. The second layer is an insulating layer of silicon dioxide, the buried oxide (box) layer.
3. These two layers are on top of a thick single-crystal silicon layer, the handle wafer.
4. On the bottom of the handle wafer is a back surface oxide layer.

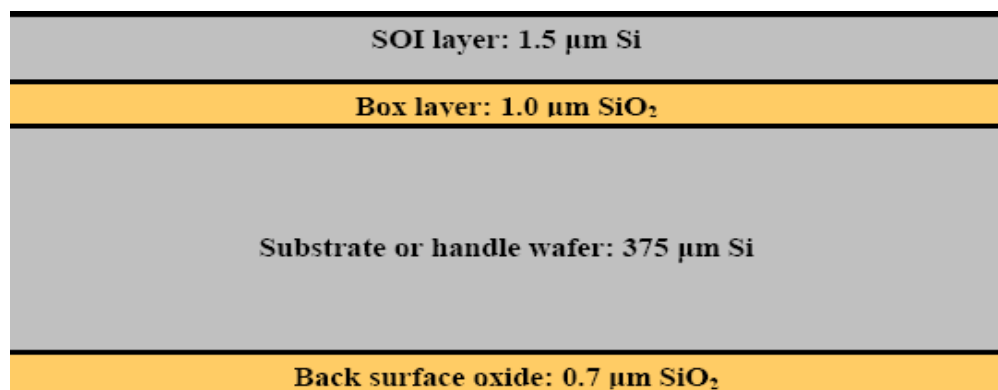


Figure 6.1: SOI wafers for ABCB process flow.

The SOI wafers are preferred for the ABCD process flow because:

1. ABCD2 is a cheap process. That means that the final product can be offered at a very competitive price.
2. The SOI wafers are very well suited for MEMS development. The buried oxide layer can be selectively etched to release silicon structures.
3. In ABCD it is possible to generate large voltages up to 120V. This is a great advantage, since the resonators are electrostatically actuated.

Below are the process steps for the ABCD process:

(a) Well dope

The SOI is doped to improve conductivity of the resonator and the electrodes. Aluminum bond pads and electric wires will be added to minimize the parasitic resistance from and to the resonator.



Figure 6.2: Patterned well drive

(b) Trench etch

The well drive is done after trench etch. In this way the dope cannot diffuse to the other side of the trench and this minimizes the parasitic capacitances. The well drive is essential to minimize stress in the resonator

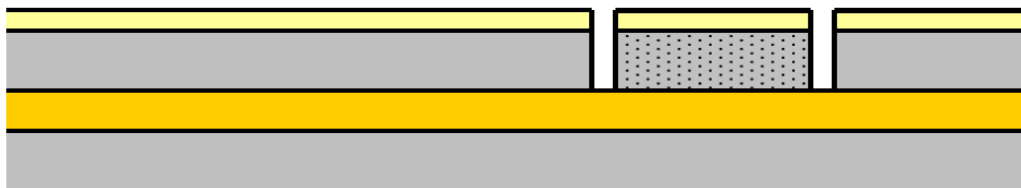


Figure 6.3: Patterned trench etch + well drive

(c) Contact implantation

In order to reduce the contact resistance between bond pads, metal and active silicon surface, contact holes are doped for good conductivity. These contact holes are added where the metal makes contact with the active silicon layer.

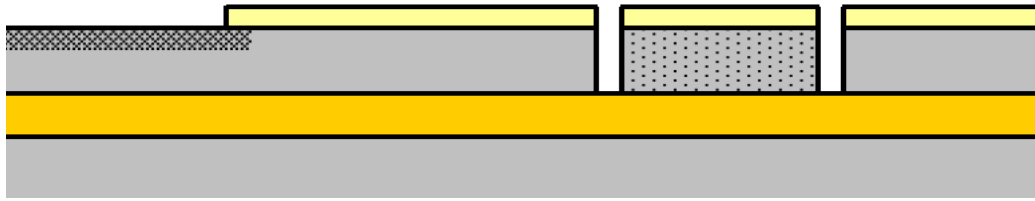


Figure 6.4: Patterned contact implantation + anneal

(d) Interconnect metal deposition

After the contact implantation is done, aluminum is deposited to create the bond pads and the wires. The aluminum is deposited on a thin layer of oxide and only makes contact to the active silicon at the contact point close to the electrodes.

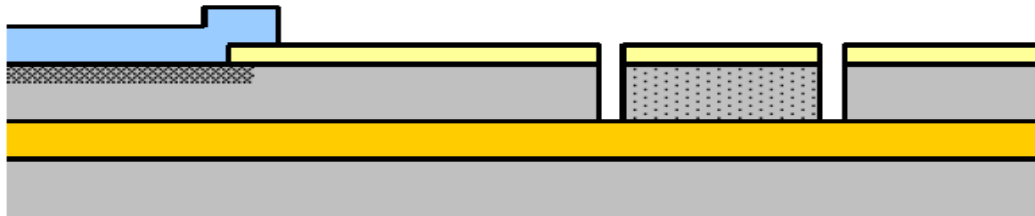


Figure 6.5 Interconnect metal deposition + patterned etch

(e) Box etch

The final step for the non-capped flow is the box etch. After the structures have been released the resonators can be measured and characterized.

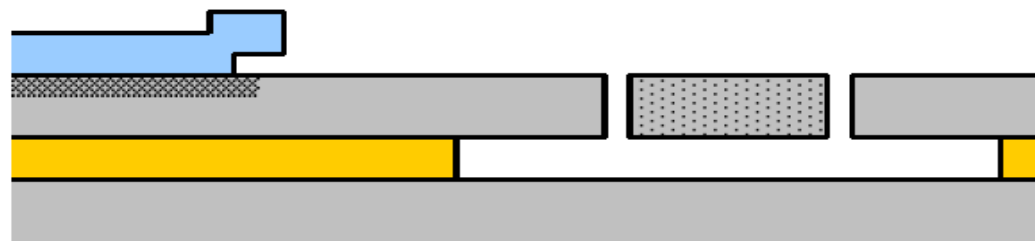


Figure 6.6: Box etch + dry

6.2 DEVICE LAYOUT

Figure 5.7 was obtained from a script from COMSOL Multiphysics. From the plot, it can be deduced that, a thickness of 4.74 μm of s1, would make the drive (lamé-mode) and sense (shear mode) matched. Refer to Appendix D for more information on COMSOL scripting.

However, ABCD process flow has a spread of 10nm. Hence lamé-mode resonators of 4.7 μ m and 4.81 μ m suspension thickness, were designed to accommodate the spread as seen in Figure 6.8 and Figure 6.9

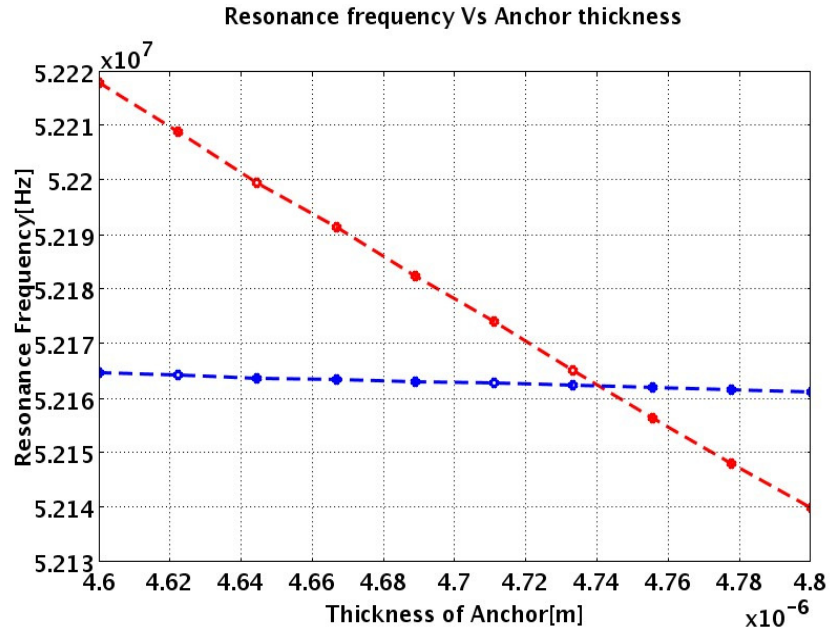


Figure 6.7: A plot of Resonance frequency Versus Thickness of the anchors

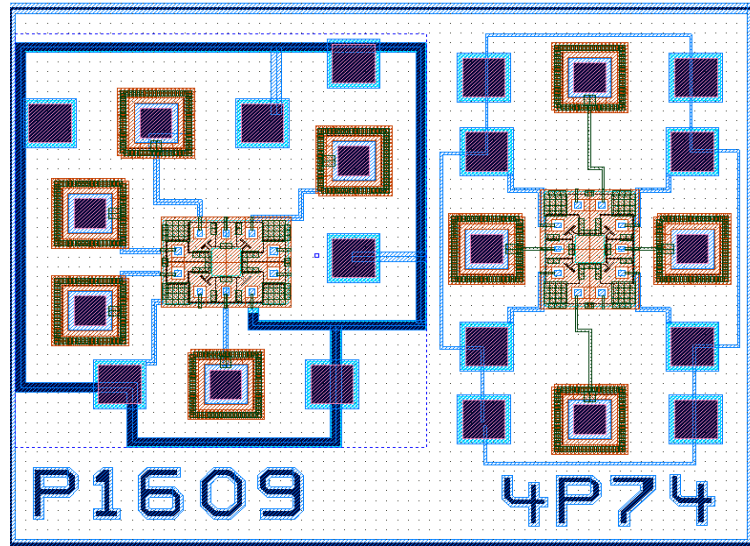


Figure 6.8: 52MHz Lamé mode resonator with suspension width of 4.74 μ m.

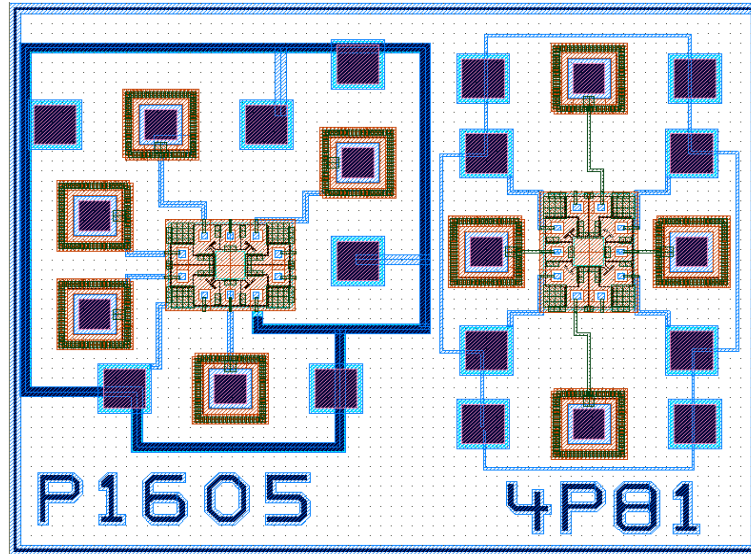


Figure 6.9: 52MHz Lamé resonator with suspension width 4.81um

Both figure 6.8 and 6.9 have two designs on the chip. The resonator on the right is used for sense mode characterization. It has two drive and sense electrode on each side of the square plate. The bond pads can also be used for S-parameters measurements.

The square plate on the right is used for drive mode characterization. It has a pair of bond pads which can be used for differential capacitive detection. The other bond pads can be used for the proposed piezoresistive detection method.

Chapter 7

Measurements of Drive Mode – Lamé Mode

In this chapter, measurements results from the sample devices are presented and discussed. The lamé mode of the square plate was characterized. S-parameter measurements were taken at several fixed sense voltage at varying drive voltages. Furthermore the frequency shift was analyzed at different temperatures.

7.1 Introduction

In figure 7.1 the layout of the lamé-mode resonator is depicted. Two electrodes are used for actuation and only one for readout. The top electrode could not be connected, since this would lead to crossing of metal wires.

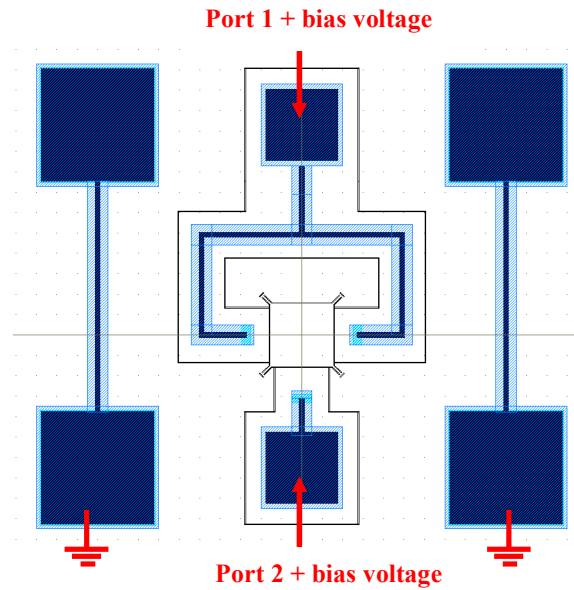


Figure 7.1: Layout of the lamé mode resonator.

In figure 6.1 the layout of the Lamé mode resonator is depicted. There are six bond pads in total in two ground-signal-ground configurations on either side of the resonator. This configuration is suitable for the high frequency probes used to measure the resonator. The resonator is a two-port device with the left and right electrode used for actuation and the bottom electrode for readout (or vice-versa).

For measurements with a network analyzer the 2 ports are connected on the input and output side of the resonator as can be seen in figure 1. Since we do not want to bias the resonator with the high DC voltage, the bias voltage is applied on both ports using bias tees. The resonator itself is grounded by the four outer bond pads to eliminate parasitic cross talk from input to output.

7.2 Temperature measurements of the Lamé mode resonator

Devices samples were diced from wafer ‘VibrantiN2 D13’. The samples were wire bonded as seen in the figure 7.2. The device of interest is sample number 1520. Afterwards the samples were packaged and inserted on the flench as see in figure 7.2.

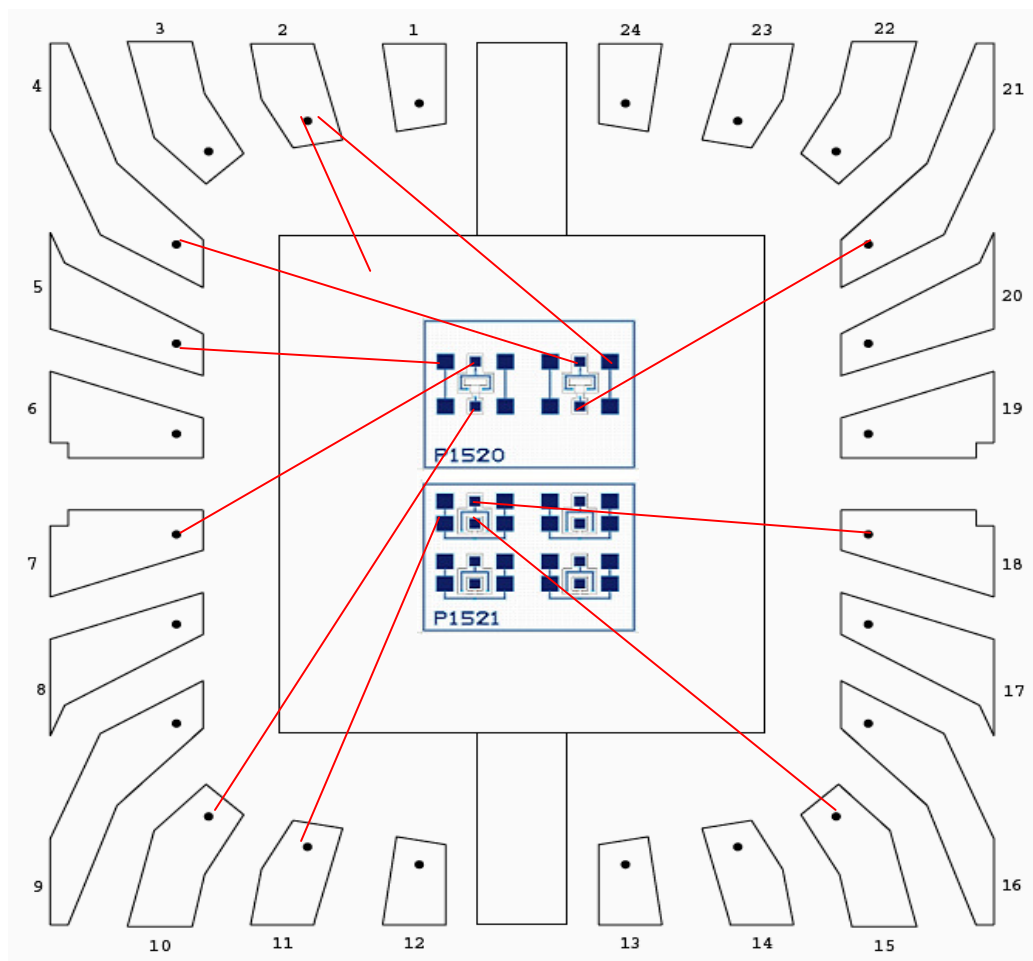


Figure 7.2: Bonding diagram of the dices sample.

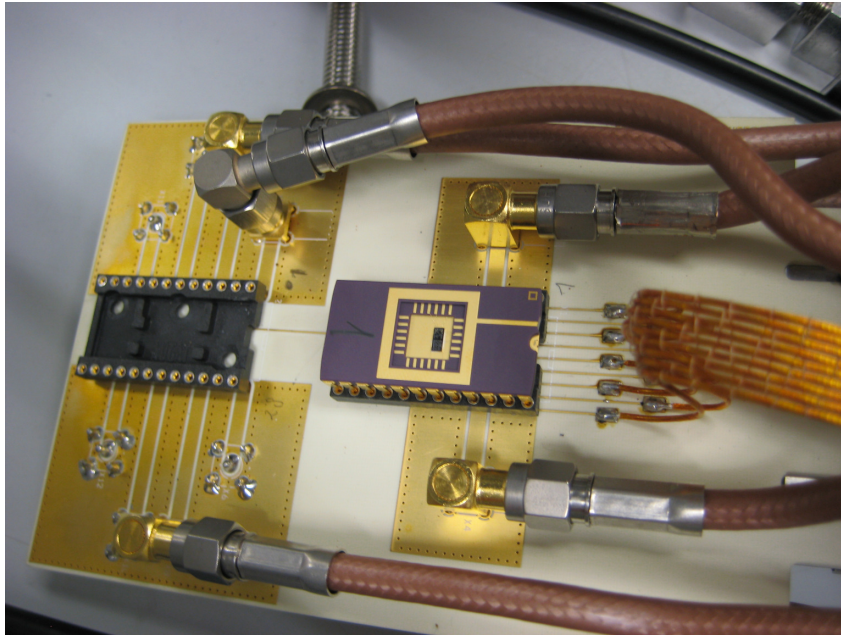


Fig 7.3: A flange with wire connectors

Afterwards the flange was inserted in temperature chamber as seen in figure 7.2.2. The temperature controlled chamber is cover with a jacket which heats up the chamber from outside. In order to determine the temperature within the chamber, a platinum resistor is inserted inside the chamber. The platinum resistor was connected to Pin 7 and 8 on 'MEMS_DIP_16'. The output cables were connected to a resistor box on connectors 4 and 6.

The resistance of the platinum varies linearly with temperature of the jacket given as $R = 3.85 \cdot T (\text{chamber}) + 1000$. Hence, the temperature inside the chamber was obtained from the measurements of the resistance of the platinum resistor. The resistance of the platinum was measured in intervals of 10°C .

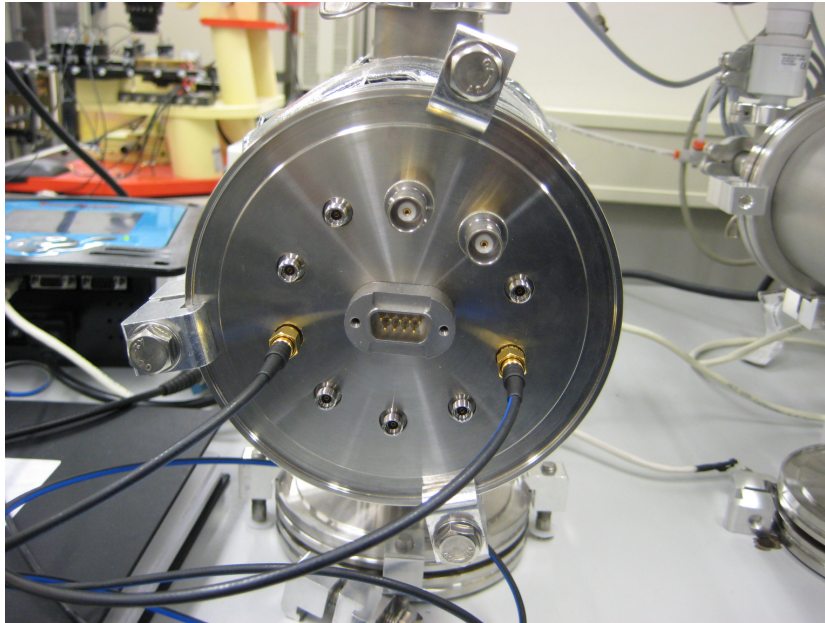


Fig 7.4: Chamber used for temperature measurements.

TABLE 7.1 – Temperature measurements for device P1520A at $V_{dc}=30V$ on wafer D13.

Vdc=30V				
Ambient Temperature [°C]	Chamber Temperature [°C]	Frequency[Hz]	Shift in ppm	Resistance [Ohm]
21,9	20,77922078	52068420,57	0	1080
30	21,2987013	52068086,12	6,42325994	1082
40	22,5974026	52067417,22	19,26974141	1087
50	24,41558442	52065577,76	54,59758466	1094
60	27,27272727	52062567,73	112,4067705	1105
70	31,16883117	52058888,8	183,062457	1120
80	35,58441558	52054206,52	272,9878657	1137
90	40,25974026	52049357,02	366,1248947	1155
100	45,45454545	52043504,18	478,5316652	1175
110	50,90909091	52037818,56	587,7268153	1196
120	56,88311688	52031296,82	712,9800673	1219
130	63,63636364	52023604,52	860,714681	1245
140	70,12987013	52016246,66	1002,026035	1270
150	77,14285714	52007383,78	1172,241991	1297
160	84,41558442	51998353,68	1345,669587	1325
170	92,98701299	51986982,44	1564,059868	1358

TABLE 7.2 – Temperature measurements for device P1520A at Vdc=40V on wafer D13.

Vdc=40V	Chamber Temperature [°C]	Frequency[Hz]	Shift in ppm	Resistance [Ohm]
Ambient Temperature [°C]				
40	24,15584416	52061250	0	1093
50	25,66233766	52059750	28,81221638	1098,8
60	28,05194805	52057250	76,83257701	1108
70	31,42857143	52053750	144,0610819	1121
80	36,36363636	52048750	240,1018032	1140
90	40,51948052	52044250	326,5384523	1156
100	45,71428571	52038250	441,7873178	1176
110	51,94805195	52031250	576,2443276	1200
120	58,18181818	52024250	710,7013374	1224
130	65,19480519	52016250	864,3664914	1251
140	71,68831169	52008250	1018,031645	1276
150	78,7012987	51999250	1190,904944	1303
160	85,97402597	51989750	1373,382314	1331
170	93,76623377	51978750	1584,671901	1361

TABLE 7.3 – Temperature measurements for device P1520A at Vdc=50V on wafer D13.

Vdc=50V	Chamber Temperature [°C]	Frequency[Hz]	Shift in ppm	Resistance [Ohm]
Ambient Temperature[°C]				
30	23,8961039	52109188,54	0	1092
40	24,67532468	52060586,63	932,6937026	1095
50	25,97402597	52059589,95	951,8204253	1100
60	28,57142857	52056599,92	1009,200574	1110
70	32,20779221	52052613,21	1085,707446	1124
80	36,36363636	52047795,93	1178,153253	1140
90	41,55844156	52041815,86	1292,913551	1160
100	46,75324675	52036500,25	1394,92272	1180
110	51,94805195	52030354,07	1512,870805	1200
120	57,66233766	52023709,55	1640,38227	1222
130	64,15584416	52015902,24	1790,208226	1247
140	70,38961039	52008261,05	1936,846396	1271
150	77,4025974	51999457,06	2105,799074	1298
160	84,41558442	51990154,74	2284,314923	1325
170	92,20779221	51980187,96	2475,582284	1355

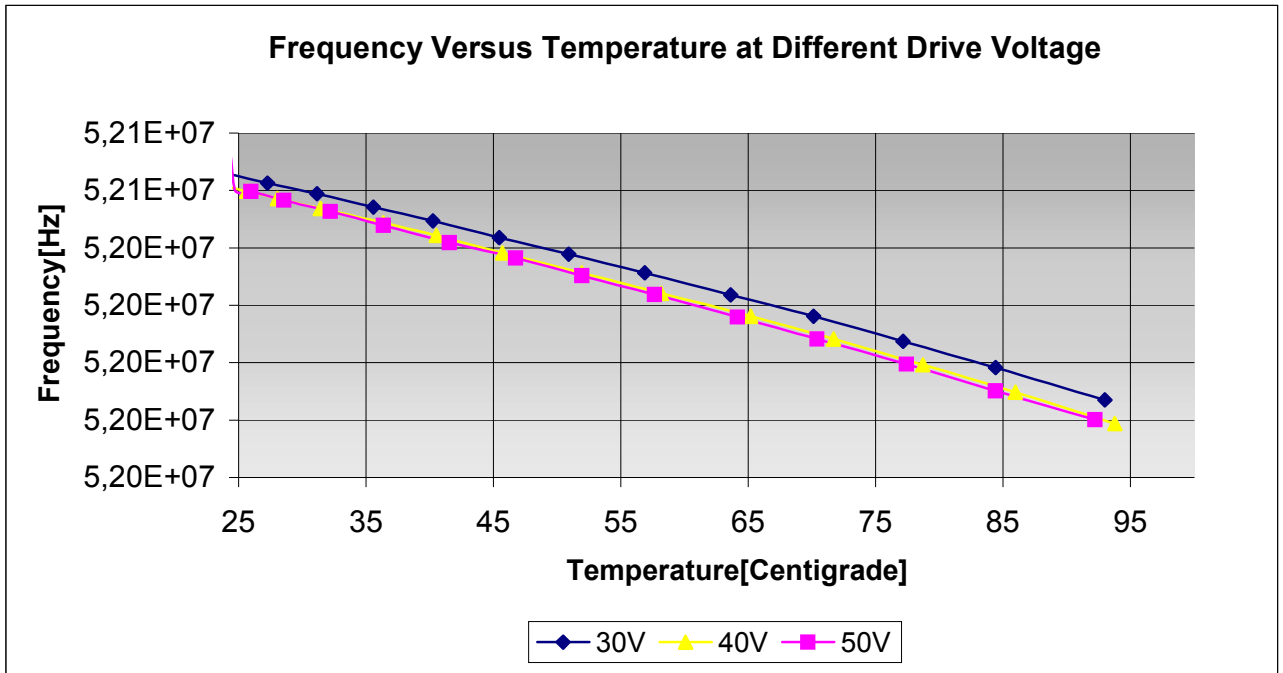


Fig 7.5: Drive-mode frequency versus temperature at different drive voltages

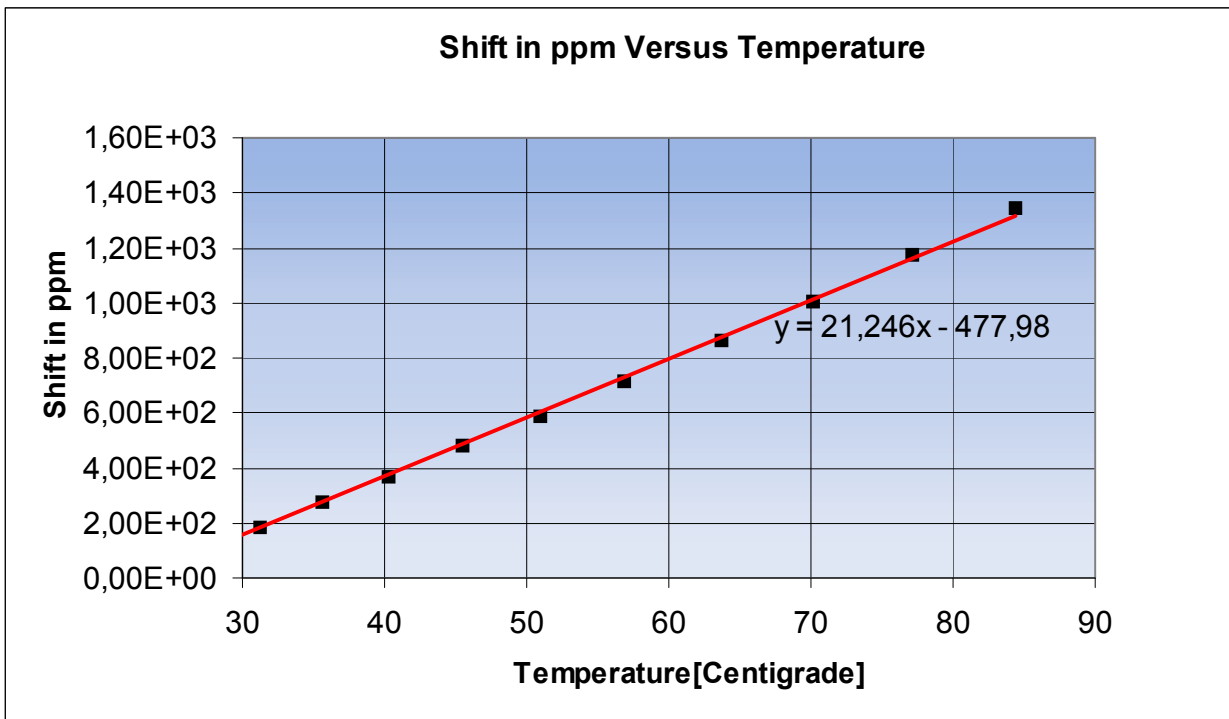


Fig 7.6: Drive mode shift in ppm versus temperature at Vdc=30V.

Furthermore, the measurements showed a frequency shift of 21ppm/ $^{\circ}\text{C}$ as seen in Figure 7.6.

7.3 S-parameter measurements

7.3.1 RESONANCE FREQUENCY TUNING

To derive the relationship between frequency and the drive and sense electrode voltages, we start by writing the total potential energy in the system in figure 6.1:

$$E = \frac{1}{2}k_d x^2 + \frac{1}{2}C V_d^2 + \frac{1}{2}k_s y^2 + \frac{1}{2}C V_s^2 \quad (7.1)$$

Where,

k_d = Spring constant in the drive direction (x-axis)

k_s = Spring constant in the sense direction (y-axis)

V_d = Drive electrode Voltage

V_s = Sense electrode Voltage

x = Drive mode amplitude

y = Sense mode amplitude

During measurements, the device was resonating in lamé-mode. Hence, we can assume $x = y$ and $k_x = k_y$.

$$E = k_d x^2 + \frac{1}{2}C V_d^2 + \frac{1}{2}C V_s^2 \quad (7.2)$$

$$E = k_d x^2 + \frac{1}{2} \frac{\epsilon_o A_{el}}{(gap - x)} V_d^2 + \frac{1}{2} \frac{\epsilon_o A_{el}}{(gap - x)} V_s^2 \quad (7.3)$$

$$E = k_d x^2 + \frac{1}{2} \frac{\epsilon_o A_{el}}{(gap - x)} (V_d^2 + V_s^2) \quad (7.4)$$

Hence the force acting on the electrodes is:

$$F = \frac{\partial E}{\partial x} = 2k_d x + \frac{1}{2} \frac{\epsilon_o A_{el}}{(gap - x)^2} (V_d^2 + V_s^2) \quad (7.5)$$

Furthermore, the effective spring constant of the system is:

$$k_{eff} = \frac{\partial F}{\partial x} = 2k_d + \frac{\epsilon_o A_{el}}{(gap - x)^3} (V_d^2 + V_s^2) \quad (7.6)$$

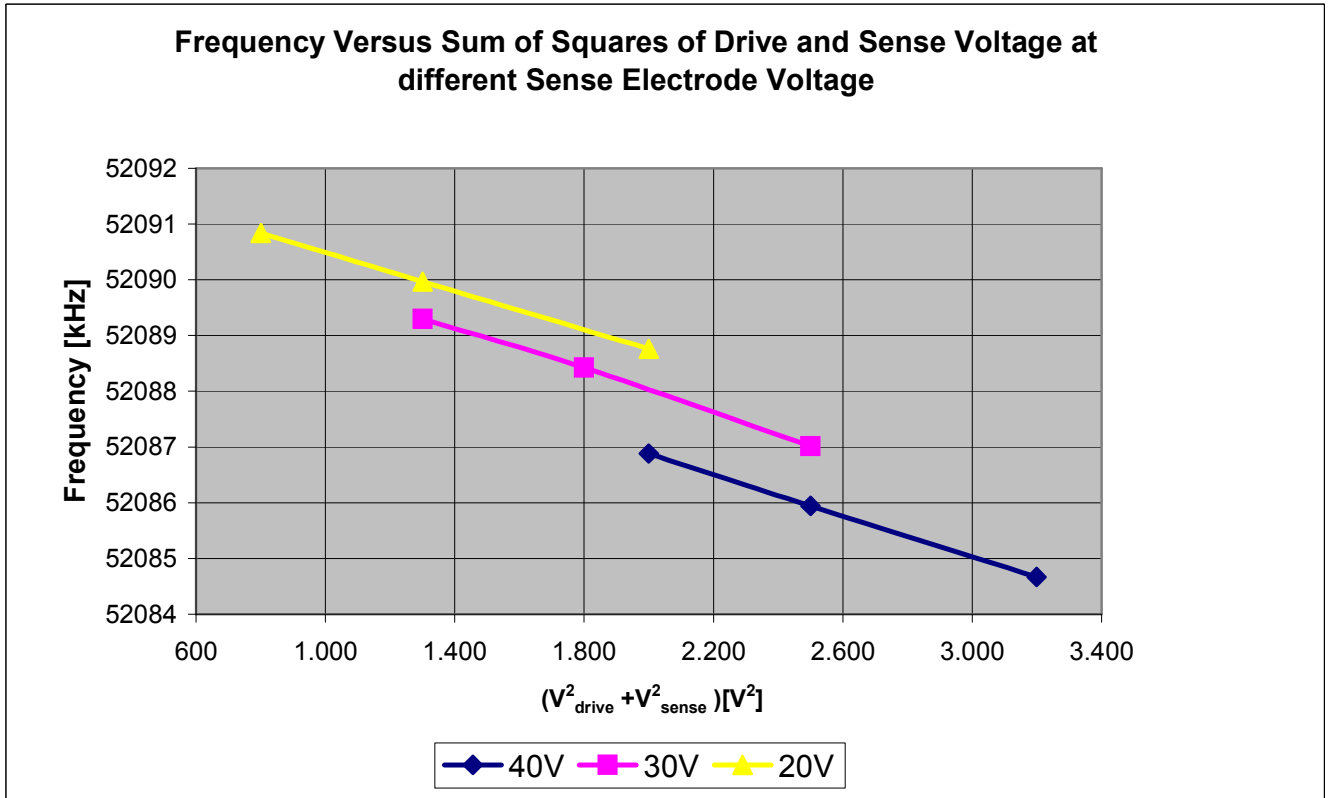


Fig 7.7: Resonant drive mode frequencies versus sum of squares of drive and sense electrode voltage

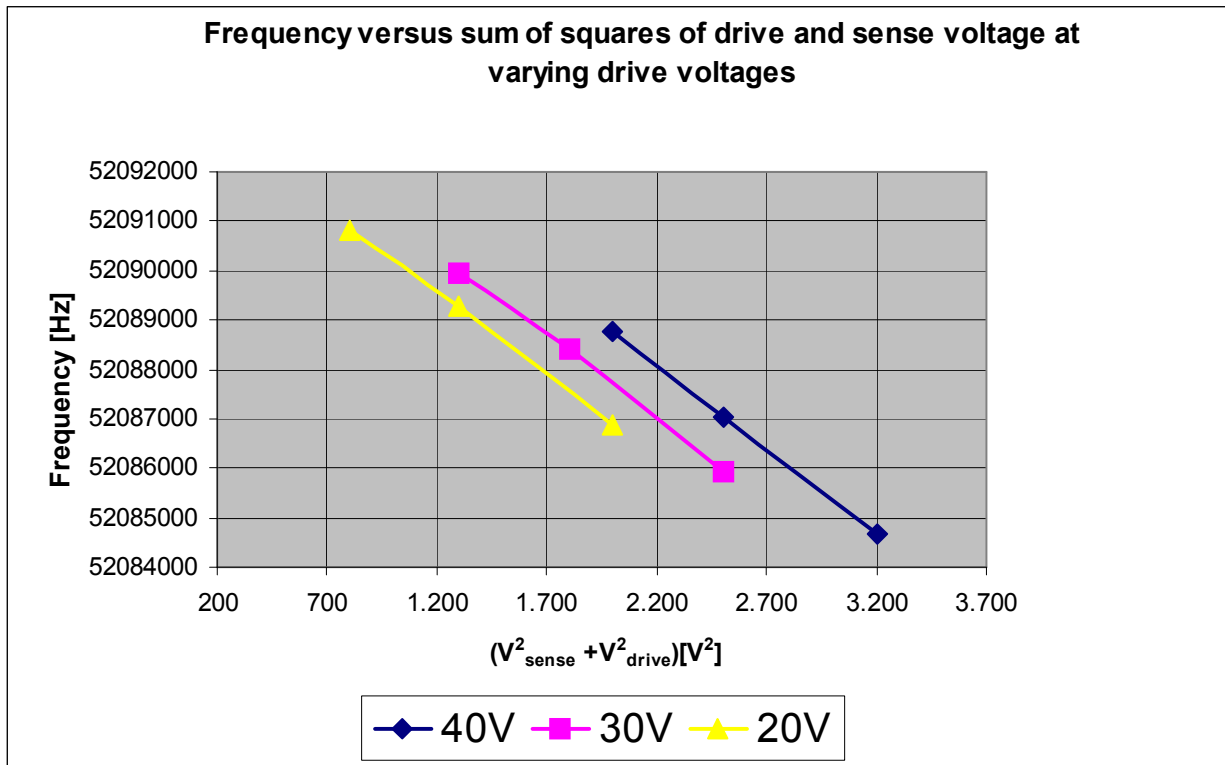


Fig 7.8: Resonant drive mode frequencies frequencies versus sum of squares of drive and sense electrode voltage

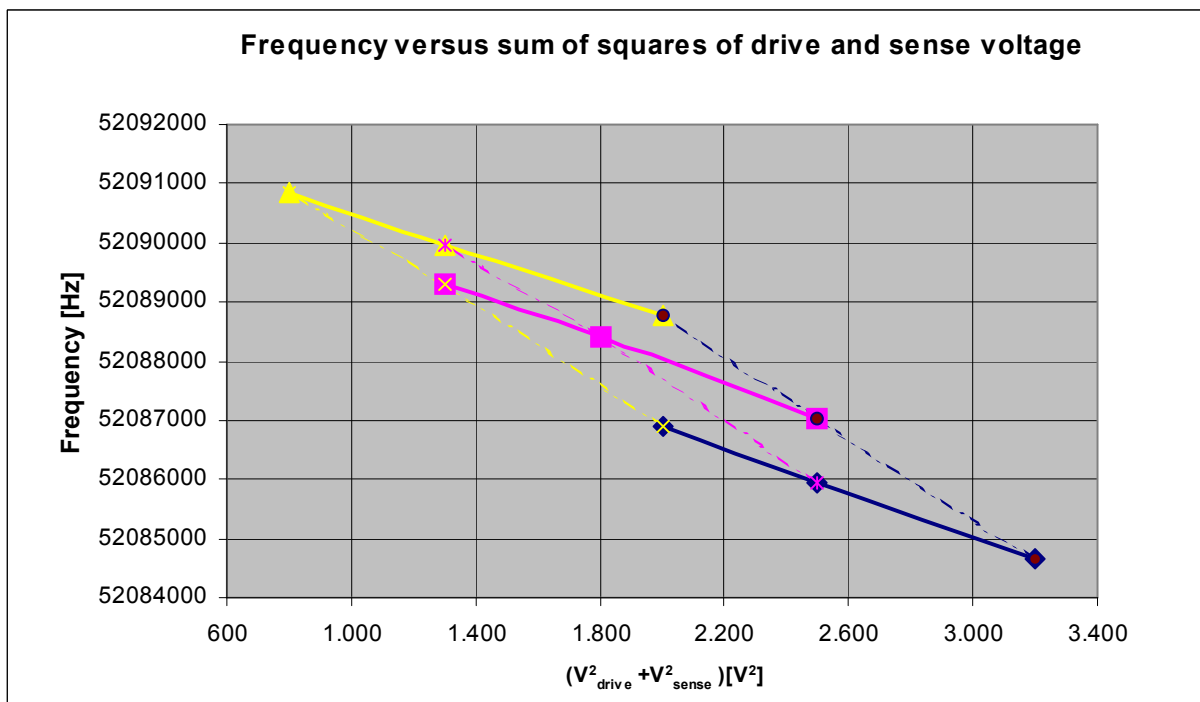


Fig 7.9: Dashed lines are measurements for a fixed drive voltage and varying sense voltage. The solid lines are measurements for a fixed sense voltage and varying drive voltage.

Hence the resonance frequency of the system is given by:

$$f_{res} = \frac{1}{2\pi} \sqrt{\frac{2k_d}{m} + \frac{1}{m} \frac{\epsilon_o A_{el}}{(gap-x)^3} (V_d^2 + V_s^2)} \quad (7.7)$$

The second term in equation 7.7 is a source of nonlinearities in the resonator. However, the nonlinear effect becomes useful in the frequency tuning of the device. Furthermore, decreasing the gap increases the electromechanical coupling and tuning capability.

TABLE 7.3 – Frequency shift measurements for device P1520A at different sense and drive voltage wafer D13.

Sense[V]	Drive[V]	V ² sense+V ² drive[V ²]	Frequency[Hz]
40	40	3.200	52084670,69
	30	2.500	52085944,15
	20	2.000	52086882,49
	10	1.700	52087284,63
30	40	2.500	52087016,54
	30	1.800	52088424,04
	20	1.300	52089295,36
	10	1.000	52089697,5
20	40	2.000	52088759,16
	30	1.300	52089965,6
	20	800	52090836,91
	10	500	52091306,08

7.3.2 Electrical equivalent model

To derive the relationship between Lm, Cm and Rm with the drive and sense electrode voltages, we start from equation 7.4. However, this time we take the alternating actuation voltage into consideration. That means $V_d = V_{d_dc} + V_{ac}e^{j\omega t}$ and $V_s = V_{s_dc} + V_{ac}e^{j\omega t}$

Where,

V_{d_dc} = DC voltage on the drive electrode.

V_{s_dc} = DC voltage on the sense electrode.

V_{ac} = AC actuation voltage on the electrodes.

After substituting the equation above into equation 7.5:

$$F_{el} = \frac{\partial E}{\partial x} = 2k_d x + \frac{1}{2} \frac{\epsilon_o A_{el}}{(gap - x)^2} (V_{d_dc}^2 + 2V_{ac} e^{j\omega t} (V_{d_dc} + V_{s_dc}) + V_{s_dc}^2 + 2V_{ac} e^{2j\omega t}) \quad (7.8)$$

The term V_{ac} is small, hence $2V_{ac} e^{2j\omega t}$ is negligible hence can be ignored. Furthermore the terms $V_{d_dc}^2$ and $V_{s_dc}^2$ are excluded since they are constants and are not alternating. Note that only alternating terms can drive the resonator. Hence equation 7.8 becomes:

$$F_{el} = \frac{\partial E}{\partial x} = 2k_d x + \frac{\epsilon_o A_{el}}{(gap - x)^2} V_{ac} e^{j\omega t} (V_{d_dc} + V_{s_dc}) \quad (7.9)$$

The electromechanical coefficient is given with:

$$\eta = \frac{\epsilon_o A_{el}}{(gap - x)^2} (V_{d_dc} + V_{s_dc}) \quad (7.10)$$

Substitution of equation 7.10 into equation 7.9 results:

$$F_{el} = \frac{\partial E}{\partial x} = 2k_d x + V_{ac} \eta e^{j\omega t} \quad (7.11)$$

The electrostatic force in equation 7.11 is inserted in the second order differential equation for a mechanical resonator:

$$m\ddot{x} + c\dot{x} + kx = 2k_d x + V_{ac} \eta e^{j\omega t} \quad (7.12)$$

If we compare this equation with the general equation of an RLC circuit in equation 6.13, an analogy can be made.

$$L \frac{\partial^2 q}{\partial t^2} + R \frac{\partial q}{\partial t} + \frac{1}{C} q = 2k_d x + V_{ac} \eta e^{j\omega t} \quad (7.13)$$

Thus we can identify the coefficients in both equations and introduce parameters L_m , R_m and C_m that describes the motion parameters of the resonator.

$$R_m = \frac{c}{\eta^2} = \frac{\sqrt{km}}{Q\eta^2} = \frac{\sqrt{km}}{Q} \left[\frac{(gap - x)^2}{\epsilon_o A_{el} (V_{d_dc} + V_{s_dc})} \right]^2 \quad (7.14)$$

$$L_m = \frac{m}{\eta^2} = \frac{m}{\eta^2} = m \left[\frac{(gap - x)^2}{\epsilon_o A_{el} (V_{d_dc} + V_{s_dc})} \right]^2 \quad (7.15)$$

$$C_m = \frac{\eta^2}{k} = \frac{\eta^2}{k} = \frac{1}{k} \left[\frac{\epsilon_o A_{el} (V_{d_dc} + V_{s_dc})}{(gap - x)^2} \right]^2 \quad (7.16)$$

From the analogy above the electrical mode in figure 7.10 was developed. A pstar file with the name 'lamé_mode' was written. The pstar model is used to fit the measured data and extract the electrical parameters of the resonator.

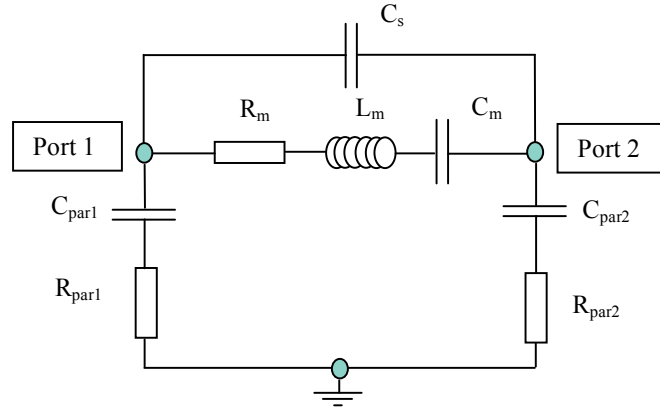


Figure 7.10: Electrical equivalent model for the resonator.

7.3.3 Measurements

Below are the plots obtained from the measurement results. The plots below have been plotted in accordance to the derived equations relating drive and sense voltages with motional parameters R_m , L_m and C_m .

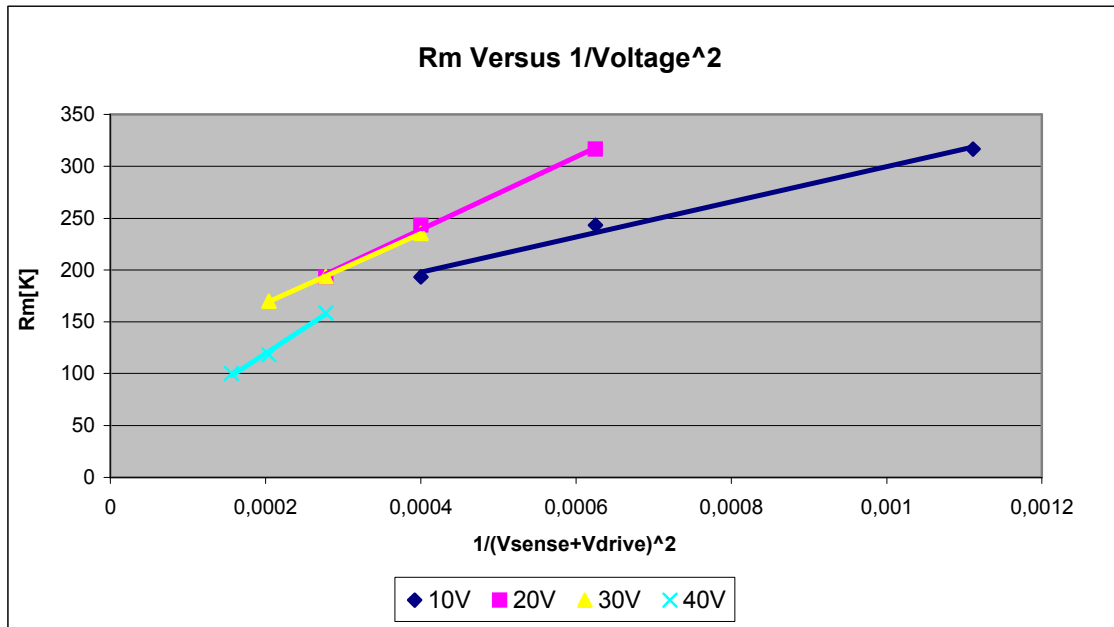


Figure 7.11: R_m versus inverse of the square of the sum of sense and drive electrode voltage

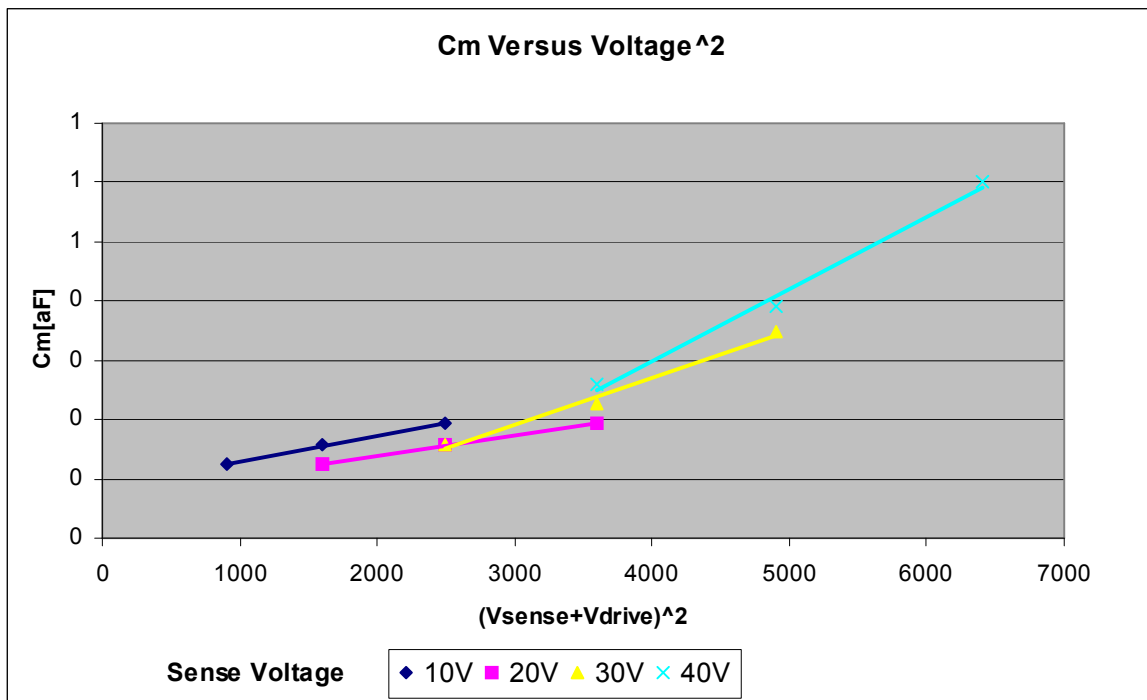


Figure 7.12: Cm Vs Square of the sum of sense and drive electrode voltage

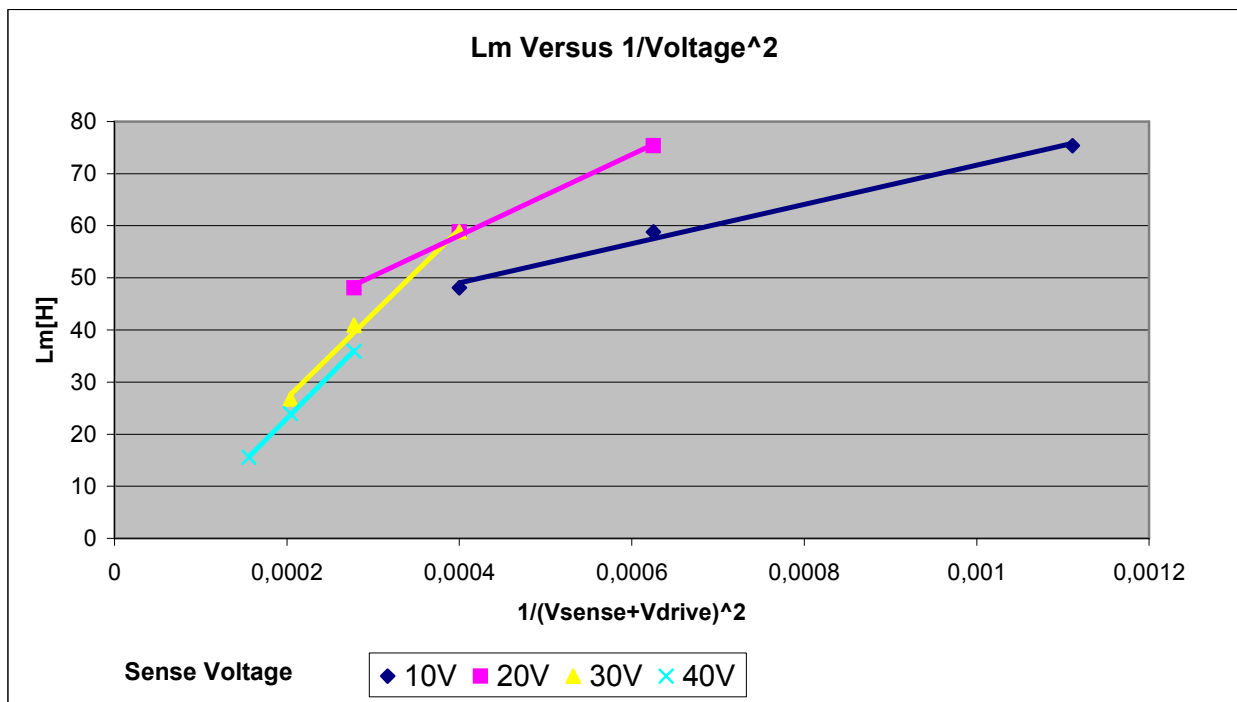


Figure 7.13: Lm Vs Inverse of the square of the sum of sense and drive electrode voltage

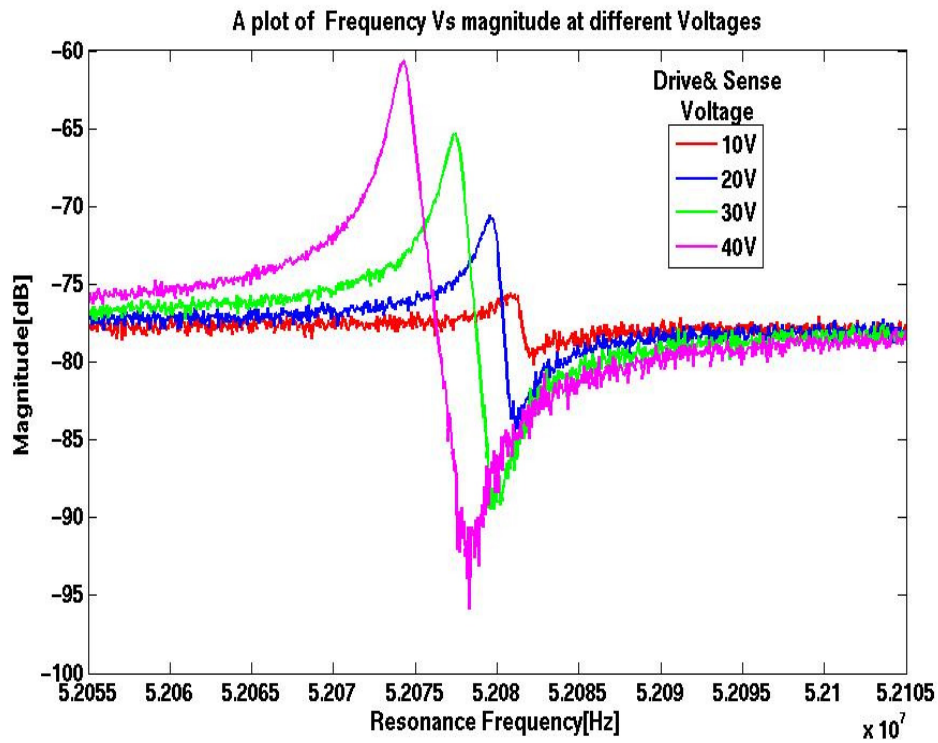


Figure 7.14: A plot of Frequency versus magnitude of the square plate. In the case above both the drive and sense electrodes have the same voltage.

7.3.4 Extracted parameters

Sense Voltage 40[V]	Drive Voltage[V]	Rm[K]	Cm[aF]	Lm[H]
Resonance Frequency[MHz]				
52,111317	10	253,25	0,18	51,78
52,110557	20	158,49	0,2594	35,93
52,1093	30	118,03	0,3897	23,92
52,1074377	40	100,12	0,6	15,54

Sense Voltage 30[V]	Drive Voltage[V]	Rm[K]	Cm[aF]	Lm[H]
Resonance Frequency[MHz]				
52,112594	10	350,36855	0,1101539	84,6068383
52,111968	20	235,50493	0,15848932	58,80520603
52,1103181	30	193,58222	0,2280342	40,87363781
52,108437	40	170	0,34833732	26,75929255

Sense Voltage 20[V]	Drive Voltage[V]	Rm[K]	Cm[aF]	Lm[H]
Resonance Frequency[MHz]				
52,113547	10	600,56436	0,09183	101,4857056
52,112951	20	316,47836	0,1236422	75,3759276
52,111636	30	243,22	0,1585	58,80199288
52,108874	40	193	0,1936422	48,1357055

Sense Voltage 10[V]	Drive Voltage[V]	Rm[K]	Cm[aF]	Lm[H]
Resonance Frequency[MHz]				
52,1140778	10	600,56436	0,09183	101,4836383
52,112951	20	316,47836	0,1236422	75,3759276
52,111636	30	243,22	0,1585	58,80199288
52,108874	40	193	0,1936422	48,1357055

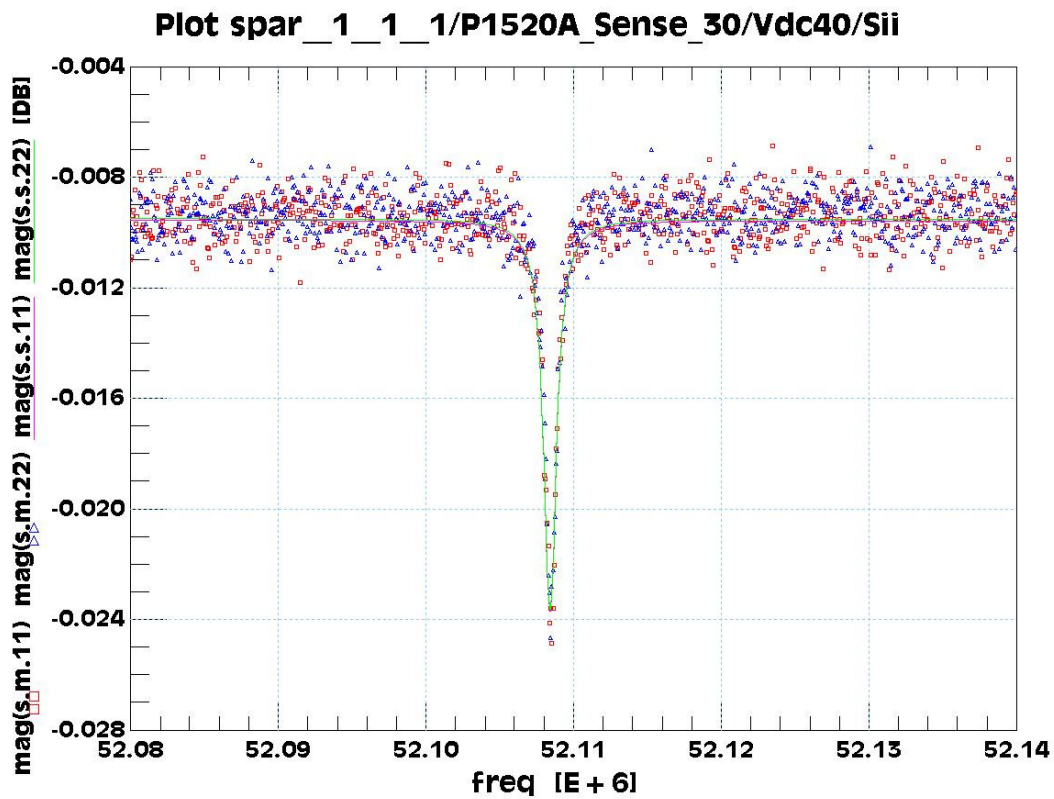


Figure 7.15: Device P1520A. Magnitude of reflection signal (S22) from measurements at Vdrive = 40V and Vsense=30V.

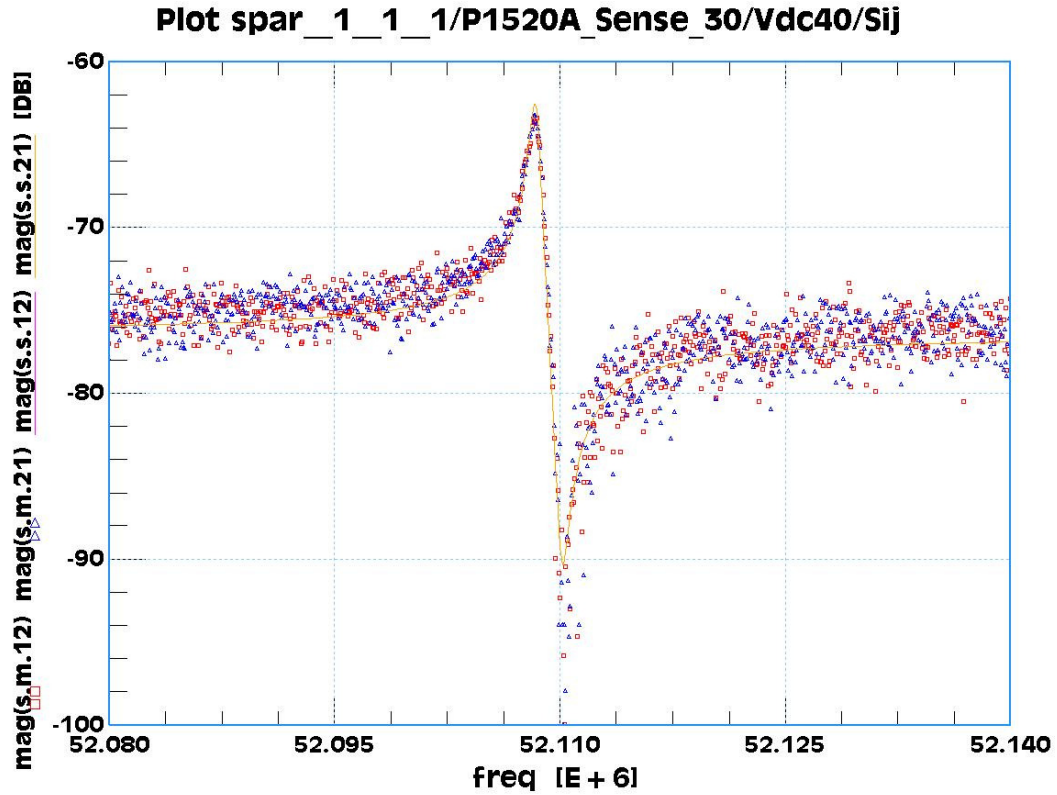


Figure 7.16: Device P1520A. Magnitude of transmission signal (S21) from measurements at $V_{drive} = 40V$ and $V_{sense}=30V$.

7.4 Conclusions

The drive mode frequency varies linearly with temperature as seen in figure 7.5. The measurements were performed at 30V, 40V and 50V. The temperature measurements showed a frequency shift of 21ppm/ $^{\circ}C$ as seen in figure 6.6. These measurements were performed for packaged devices.

Furthermore, to understand the tuning capability of the electrode voltages, measurements were performed for various drive and sense electrode voltages. The measurements were done at sense voltages of 20V, 30V and 40V while the drive voltage was varied from 10V to 0V in each case.

The results were fitted with an electrical model in figure 7.4 in ICCAP. The parameters R_m and L_m varies inversely proportional to $(V_{sense}+V_{drive})^2$ while L_m varies proportional to $(V_{sense}+V_{drive})^2$. The measured parameters are fitted with a straight line according to the expected analytical scaling and derivations done in section 6.2.2.

A manufacturing spread of $\pm 10\text{nm}$ would require a frequency control over a range of 8 kHz. That equals 150ppm drive frequency shift. The measurements in Figure 7.14 show a 9 kHz shift in resonance frequency for 25V shift in electrode voltage. Hence, the tuning factor of the drive mode is $9\text{kHz}/25\text{V} = 360\text{Hz/V}$. That means $9\text{ kHz}/52\text{Mhz}/25\text{V} = 7\text{ppm/V}$.

This tuning range is adequate to enable matching of the drive and sense frequency despite process spread.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

In this work a simulation model of a 52MHz MEMS gyroscope is presented. The FEM model included the Coriolis force. The model enabled the matching of the drive and sense frequency by optimization of the design of the gyroscope. In addition, the model enabled the analysis of the dynamic behavior of the gyroscope.

Device parameter	Value
Drive mode frequency	52.165MHz
Device thickness	1.5um
Capacitive gap	200nm
Device size	63umx63um $\cong$ Bond pad
Bandwidth	1,300Hz
Drive amplitude	0.03nm
DC voltage	15V – 40V
Tuning range	9kHz
Tuning factor	7ppm/V $\equiv$ 360Hz/V
Quality factor	40,000
Theoretical noise floor	$0.00322^0 / s / \sqrt{Hz}$
Sensor sensitivity	1.254aF ⁰ /s
Rate sensitivity	16.38mV
Time resolution	1ms

Table 8.1: Specifications of the MEMS Gyroscope.

The manufacturing process has spread of ± 10 nm. This would require a frequency control over a range of 8 kHz. That equals 150ppm drive frequency shift. The measurements in Figure 7.14 show a 9 kHz shift in resonance frequency for 25V shift in electrode voltage.

Hence, the tuning factor of the drive mode is $9\text{kHz}/25\text{V} = 360\text{Hz/V}$. That means $9\text{kHz}/52\text{Mhz}/30\text{V} = 7\text{ppm/V}$. The proposed design has adequate tuning range to enable matching of the drive and sense frequency despite process spread.

Furthermore a capacitive readout topology was developed. In addition an electrical model was developed and was used to fit the measured data. Electrical parameters of the resonator were extracted using the model.

8.2 Future Work

The design concept was demonstrated by FEM simulations. However, these sense-mode simulation results have not been verified experimentally yet. Thus, the first step of future work will be to implement the capacitive readout topologies. This would enable the experimental evaluation of performance of the fabricated prototype gyroscopes.

Furthermore, future work might focus on developing piezoresistive detection for the gyroscope. Thus far, a method to simulate the piezoresistive effect including Coriolis force is already developed. Refer to Appendix C.5 for more information on it.

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