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Averting Undetectable Attacks on Over-Actuated Systems with Sparse Control

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1 Introduction

Over-actuated systems, defined as having more inputs than outputs, arise in a multitude of applications, with the benefit of providing redundancy and increasing control performance. However, this property makes these systems susceptible to undetectable actuator attacks. This research focuses on a method to protect these over-actuated systems against these attacks.

2 Undetectable Cyber-Attacks

We consider the following LTI system:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k, \quad \mathbf{y}_k = \mathbf{C}\mathbf{x}_k, \quad (1)$$

where $\mathbf{x}_k \in \mathbb{R}^n$, $\mathbf{u}_k \in \mathbb{R}^m$, $\mathbf{y}_k \in \mathbb{R}^p$, and we assume that $(\mathbf{A}, \mathbf{B})$ is controllable, $(\mathbf{A}, \mathbf{C})$ is observable, $\text{rank}(\mathbf{B}) = m$, $\text{rank}(\mathbf{C}) = p$ and $m > p$. The plant (1) is controlled over a network. We assume that this network has been compromised by an attacker that has gained access to the input channel, and can alter the input signal:

$$\mathbf{u}_k \longrightarrow \tilde{\mathbf{u}}_k = \mathbf{u}_k + \mathbf{a}_k,$$

where $\mathbf{a}_k \in \mathbb{R}^m$. The adversary has full knowledge of $\mathbf{A}, \mathbf{B}, \mathbf{C}$. The goal of such an adversary is to diverge the state trajectory from the nominal trajectory as much as possible while staying undetected. In this work we are specifically interested in the following subset of attack signals.

Definition 1 (Undetectable Attacks [1]). *The attack $\tilde{\mathbf{u}}_k$ is undetectable if $y(\mathbf{x}_0, \mathbf{a}, k) = 0$, where $y(\mathbf{x}_0, \mathbf{u}, k)$ denotes the output at time k driven by inputs $\mathbf{u}$ with initial condition $\mathbf{x}_0$.*

Note that knowledge of $\mathbf{x}_0$ reduces the class of undetectable attacks. In this work, we consider the more general case where $\mathbf{x}_0$ is not known.

Theorem 1 (Existence of Undetectable Attacks [1]). *An undetectable attack $\mathbf{a}$ exists iff there exist $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{g} \in \mathbb{R}^m$ and $z \in \mathbb{C}$ such that*

$$\begin{bmatrix} z\mathbf{I}_n - \mathbf{A} & -\mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{g} \end{bmatrix} = \mathbf{0}. \quad (2)$$

Moreover, in the case that $m > p$, an undetectable attack that is able to drive the state unbounded is guaranteed to exist.

Note that condition (2) in Theorem 1 is equivalent to the existence of (invariant) zeros in system (1).

3 Removal of Undetectable Attacks

Motivated by the observation that changing the input matrix $\mathbf{B}$ impacts the existence of undetectable attacks, we employ *s-sparse control* [2]. In this framework, every time step only a subset of the actuators is utilized. That is, we constrain $\|\mathbf{u}_k\|_0 \leq s$ for some sparsity level s , where $\|\cdot\|_0$ counts the number of non-zero elements.

Theorem 2 (*s*-Sparse Controllability [2]). *The system (1) remains controllable, under the constraint $\|\mathbf{u}_k\|_0 \leq s$ if and only if $s \geq n - \text{rank}(\mathbf{A})$.*

Furthermore, we are guaranteed to be able to find a *periodic* schedule $S \in \{1, \dots, m\}^{\mathbb{N}}$, $S_{k+h} = S_k$ that dictates which actuators to utilize at a given time-step, enforcing the sparsity condition while guaranteeing controllability. By enforcing this schedule at both control and actuator side, we are able to fundamentally alter the input matrix $\mathbf{B}$ at every time-step: $\mathbf{B} \rightarrow \mathbf{B}_k$. The matrices $\mathbf{B}_k$ consist of the columns $\mathbf{B}^j$ of $\mathbf{B}$: $\mathbf{B}_k := [\mathbf{B}^{j_1} \dots \mathbf{B}^{j_s}]$, where $S_k = \{j_1, \dots, j_s\} \subset \{1, \dots, m\}$. Hence, system (1) can be regarded as an *h*-periodic linear system. For a given schedule, we are able to check the existence of undetectable attack by checking a condition analogous to (2) for periodic systems. However, finding such a schedule S is inherently combinatorial in nature. As such we are exploring other formulations and conditions that are computationally tractable. Furthermore, we demonstrate the principle by an example that admits a suitable schedule that removes undetectable attacks.

References

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