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PREFACE

In November, 1979 the International Geographical Union's Commission on the Coastal Environment met at Newport, R.I., in conjunction with the annual meeting of The Coastal Society. During the two-day IGU-CCE gathering, four paper-presentation sessions were held. One of these was the Per Bruun symposium, convened and chaired by John Fisher.

As far as the participants knew, this was the first gathering ever devoted solely to the subject of the Bruun Rule. We were honored, of course, to have Dr. Per Bruun present.

Over the years, the hypothesis that was named after Per Bruun has grown in recognition, has been incorporated in textbooks, and has motivated a number of field studies. Though we could not have everyone connected with this topic participate in the symposium; the papers presented there, and reprinted in this proceedings volume, represent the highlights of what has been done. In this collection you will find the work of Maury Schwartz and Vladimir Milicic, Roger Dubois, John Fisher, Peter Rosen, Edward Hands, Henry Allison, and Per Bruun. It is also anticipated that a supplement by Pavel Kaplin will be distributed to everyone who receives the proceedings.

We, the editors, believe that this collection represents something of a milepost in the study and application of the phenomenon that has become known as the Bruun Rule. We sincerely hope that reviewing these papers will give you as much pleasure as we have had in bringing them to you.

M.L. Schwartz
J.J. Fisher

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HOLOCENE SEA LEVEL RISE, SHORELINE EROSION AND THE BRUUN RULE-OVERVIEW

by

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Introduction

These papers on the application of the Bruun Rule to shoreline erosion were part of a symposium that I conducted as conference chairman as part of the Atlantic Regional Conference of the International Commission of the Coastal Environment at Newport, Rhode Island, in the fall of 1979. As background information, briefly, the Bruun Rule postulates that erosion of the shoreline is necessary on a rising sea level to maintain a profile of equilibrium if there is sediment supply deficit. At the 23rd International Geographic Union's Congress in Moscow, U.S.S.R., in the summer of 1976, which I attended, application of this concept was suggested, in part, as an explanation of some shoreline erosion. I reported on the symposium in Geotimes (Fisher, 1977a). The following summer, 1977, at the 10th International Quaternary Association's Congress in Birmingham, England, at the session on Quaternary shorelines, I presented (Fisher, 1977b) information on this concept as applied to the Rhode Island and North Carolina coasts of the U.S.A. as it was affected by the Holocene rising sea level.

Shoreline Erosion

The significance of a review of the Bruun Rule as a factor in shoreline erosion can be traced to increased documentation of shoreline erosion. Erosion appears recently to have been especially severe along the barrier islands of the Atlantic and Gulf of Mexico coasts of the United States. The problem is even more significant along those shorelines where there are costly beach front developments. Many man-made shoreline protection efforts may not be able to protect these beaches to justify their high costs. In some cases, these efforts do not realize what natural coastal processes are in effect causing the erosion and thus are only minimal in their protection or sometimes they even interfere with the natural processes.

Soviet Symposium

Shoreline erosion is not only a local or a regional problem, but it is a worldwide problem. Worldwide reports of coastal erosion during the last several decades led to the convening of an international symposium on the Dynamics of Shoreline Erosion during the summer of 1976, in the Soviet Union, as part of the 23rd International Geographic Congress. The chairmen of this symposium on shoreline erosion were V. P. Zenkovich of the Soviet Union and E. C. F. Bird of Australia. Many earlier coastal researchers felt that shoreline erosion was balanced by deposition elsewhere - a sort of coastal "cut" and "fill" - and that the "problem" was that the areas of shoreline deposition were not as well documented as those of erosion and that there was no real problem. Evidence for shoreline erosion not balanced by deposition was pointed out in 27 papers. Meetings, together with field seminars, were held for a full week along the Black Sea coast. About half the papers presented covered erosion of the Soviet coasts, while the remainder were worldwide in coverage (Fisher, 1977a).

All these reports and hundreds of others discussing worldwide coastal erosion were summarized by the Chairman of the Commission on the Dynamics of Shoreline Erosion, E. C. F. Bird of the University of Melbourne. His report (Bird, 1976) confirmed that, during the last century, the world's sandy shorelines have on the average been retreating, even on coasts where Holocene beach ridges indicated previous shoreline advancement. Over a hundred correspondents from 60 countries supplied information on different aspects of coastal erosion. The only coasts where shorelines were not retreating were either where: (a) excess sediment was being supplied by river sources, (b) the land was being elevated due to tectonic uplift or (c) the land was elevating due to isostatic glacial rebound.

Several hypotheses were advanced in the commission's report to explain this dominance of shoreline erosion on a worldwide scale. The first possibility was to suggest the action of destructive coastal activities by man in his overdevelopment of the coastal zone. However, Chairman Bird pointed out that his correspondents indicated that significant shoreline erosion is also taking place on sparsely populated coasts. A second possibility was that of a climatic variation leading to increasing storm activities and increasing shore erosion. This implies, however, that increasing storm activity would have to be worldwide. Another possibility suggested what that beach development took place initially on a worldwide basis some 5,000 years ago, when Holocene marine transgression brought the sea to near its present level. In this development, the sea used the sands of the shelf to construct the present day shoreline. Now, a lack of shelf sand allows coastal processes to erode rather than construct present day beaches. The final possibility, the Bruun Rule, is that the continuing eustatic worldwide sea level rise due to melting glaciers requires sand to be eroded from sandy shorelines and deposited offshore to compensate for the rise. There was no conclusion as to which of the various possibilities might be correct, although Maurice Schwartz (from his early model and field studies) and I (based on my on-going Rhode Island studies, Fisher, 1977c) supported the Bruun Rule. However it also came out from our discussions that P. Kaplin of the Soviet Union felt that possibly V. P. Zenkovitch, also of the Soviet Union, might possibly have priority in the concept embodied in the Bruun Rule. M. L. Schwartz volunteered to research this aspect further.

Bruun Rule Symposium

The possibility of having a symposium devoted to the Bruun Rule at the Atlantic Regional Conference of the IGU's Coastal Environment Commission Convention resulted from the facts that: (1) J. Fisher and M. Schwartz planned to report on their continuing studies of the Bruun Rule, (2) a Commission field trip was planned for the Cape Cod National Seashore to revisit the field sites of the earlier Schwartz studies and (3) coincidently, almost all the studies relating to the Bruun Rule in the United States have been in the eastern United States from the Great Lakes east to Cape Cod and then south to Chesapeake Bay. It was therefore planned to plan for a Bruun Rule symposium separate from the basic regional meeting. In addition, Dr. Per Bruun was also invited and accepted an invitation to present his work at the symposium. The results of that symposium make up this volume.

Bruun Rule Chronology (1960-1970's)

One of the interesting aspects of the Bruun Rule studies has been its development from its earliest concept from observations along the Florida coast by Per Bruun (1962). The studies increased in time frame and scope as shown in the following selected chronology. First were the early laboratory wave tank studies of M. Schwartz (1965), and later he (Schwartz, 1967) conducted shallow water field surveys at two sites over the neap-spring tide period. This tidal change allowed a temporary sea level rise which affected the foreshore beach profile. Next M. El Ashry (1971) suggested that the increased erosion he had noticed on sequential coastal aerial photographs might be best explained by the Bruun Rule concept. Some time later, R. Dubois (1975, 1976) conducted a study similar to M. Schwartz but in Lake Michigan at two sites over a 4 month summer period when the lake level rises. Again the predicted deposition was in the nearshore zone. E. Hands (1976), at the same time, in the same lake at 34 sites extended the study to a 9 year period using the long term (5-15 yr) lake level rise due to climatic (increase ppn) variations. Again, within one year J. Fisher (1977a,b), using aerial photographs from 1939, extended his study along the entire Rhode Island coast at 113 sites over a 35 year period for primarily the Rhode Island and also the North Carolina coasts. This was followed again within a year with a study by P. Rosen (1978) along the Virginia Chesapeake Bay shoreline at 146 beach units. The study extended over a 100 year period using map data. Finally, the next year, the cycle was completed with J. Weggel (1979) presenting a general empirical technique to apply the Bruun Rule to predict long term shore erosion rates using shore profile and sea level rise data. Thus, over a period of about 15 years, the studies went from theoretical concept through model studies and then through a series of studies extending over an increasing period of time (1 month - 100 years) and over a greater scope of shoreline (2 profiles - 146 beach units). The chronology below indicates this development by listing studies that specifically concerned the testing of the Bruun Rule. It does not include many other studies (e.g., D. Swift, 1975) where the Bruun Rule is mentioned in passing as offering a possible explanation of observed changes whether of a short term or long term nature.

Selected Chronology of Bruun Rule Studies

P. Bruun, 1962 - Introduces concept and applied to SE Florida coast
M. Schwartz, 1965 - Laboratory study of Bruun Rule
M. Schwartz, 1967 - Field study, Cape Cod, MA, time frame - neap to spring tide, scope - 2 sites
M. El Ashry, 1971 - Suggests increasing United States shore erosion follows Bruun Rule
R. Dubois, 1975, 1976 - Field study, Lake Michigan, time frame - 4 months, scope - 2 sites
E. Hands, 1976 - Field observation, Lake Michigan, time frame - 9 years, scope - 34 sites
J. Fisher, 1977a,b - Field and aerial photograph observations, Rhode Island and North Carolina, time frame - 30 to 35 years, scope - 113 sites
P. Rosen, 1978 - Field and map observations, Chesapeake Bay, time frame - 100 years, scope - 146 units
J. Weggel, 1979 - General Bruun Rule application technique using profile and sea level change data

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THE BRUUN RULE: A HISTORICAL PERSPECTIVE

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Since this is a fairly subjective history, covering the last 17 years, it will be written informally in the first person singular of the senior author; just as was the oral presentation, delivered at the Bruun Rule Symposium in Newport.

The story starts at Columbia University in 1963, where I was a graduate student. In that year, I built a stream-table as an aid in teaching the introductory geology courses. (Schwartz, 1964a, 1964b). One day, Rhodes Fairbridge handed me a copy of Per Bruun's 1962 paper, Sea level rise as a cause of shore erosion, and asked if I thought the hypothesis could be tested on the stream-table. The concept was largely intuitive and had not been tested in the laboratory or in the field.

Bruun's statement (Fig. 1) was as follows: a) There is a shoreward displacement of the beach profile as the upper beach is eroded; b) The material eroded from the upper beach is equal in volume to the material deposited on the nearshore bottom; and c) The rise of the nearshore bottom as a result of this deposition is equal to the rise in sea level, thus maintaining a constant water depth in that area.

It is important here that two points be made: a) It is not the Bruun Rule that erodes the shore as sea level rises. Rather, as sea level rises and erodes the shore, the Bruun Rule describes the characteristics of that process; and b) The Bruun Rule assumes a profile of equilibrium, where there is either no shore drift at all or shore drift into the unit cell under consideration equals the shore drift out. In other words, the shore is neither receding or prograding. Bruun has described a profile of equilibrium as one that maintains its form, through tidal or seasonal (i.e. storm-calm) fluctuations, for long periods of time.

Testing the hypothesis on the stream-table was a fairly simple matter. Utilizing different wave parameters and varying amounts of sea level change, measurements were made before and after each run to determine the water depth in the nearshore zone. Profile translation and erosion-deposition relationships were observed at the same time. These elementary experiments showed support for Bruun's hypothesis.

In order to continue the laboratory study, but with better equipment, a wave-basin was constructed in another lab at Columbia. With a

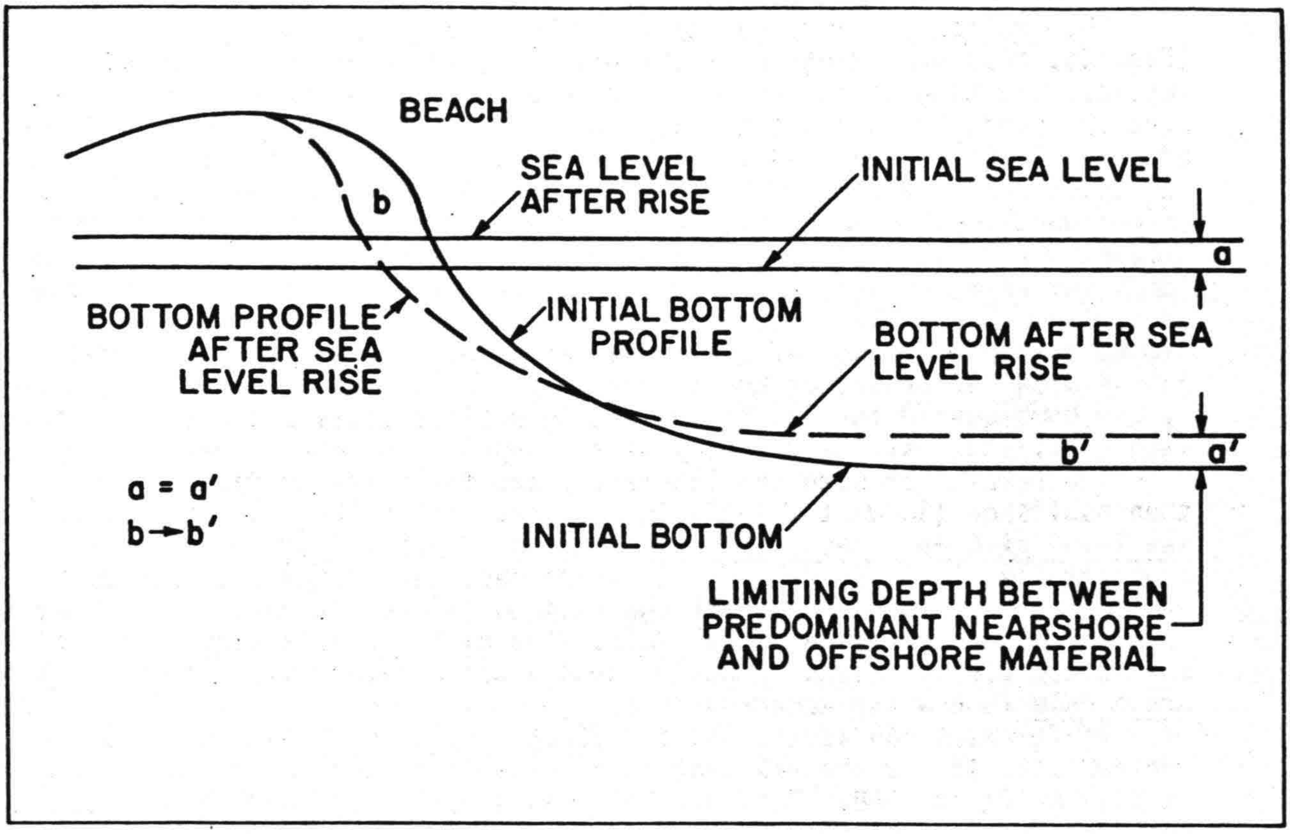

FIGURE 1. Shore erosion following a rise in sea level according to the Bruun Rule (after Schwartz 1967)

better wave generator and its slightly larger size, the wave-basin proved to be more suitable for the investigation at hand. Numerous runs were made utilizing this new equipment.

The results of both laboratory studies were published in a report titled Laboratory study of sea level rise as a cause of shore erosion (Schwartz, 1965) as an affirmation of the Bruun concept.

In conference with Rhodes Fairbridge and Arthur Strahler it was then deemed necessary to study a field model of sea level rise if the investigation was to proceed any further. What was finally settled upon, was the effective rise in sea level between neap and spring high tides. In the summer of 1964, field work based on this approach was carried out on two Cape Cod beaches (Schwartz, 1979).

The two beaches were Nauset Light Beach and Herring Cove Beach (Fig. 2), thus providing, respectively, an open ocean and protected bay regime. Starting points for profile measurements throughout the summer were the protected-beach signs at each beach. The profiles were surveyed by employing a modified version (in this case called the Schwartz One-man Beach-profiler, or S.O.B.) of K.O. Emery's two-profile-stick method, in conjunction with Scuba gear and enough weights to maintain negative buoyancy. The data collected at both sites was then plotted as profiles for a series of neap-spring events. At first a problem appeared to crop up as a result of the vertical and lateral displacement of the profiles caused by the migration of sand waves or humps. However, after consultation with Per Bruun, at an informal meeting in Woods Hole, this anomaly was compensated for and the resulting profile plots proved, like the laboratory studies, the validity of the hypothesis under investigation.

The results of both the laboratory and field investigations were then published (Schwartz, 1967) in a report titled The Bruun theory of sea level rise as a cause of shore erosion. In the last sentence of that article, I proposed "...that the concept henceforth be known as Bruun's Rule." That phrase, though adopted in the literature, has been somewhat corrupted to The Bruun Rule. The difference is very minor and the latter version seems to now be used almost universally, so that The Bruun Rule is now the accepted term.

As far as I can ascertain, the first mention of The Bruun Rule in the geologic literature was that in an article in the Journal of Geology by D.J. Swift in 1968. This was followed closely, in 1969, by Bird's book Coasts, then King's Beaches and Coasts in 1972. Also in 1972, Fisher included the Nauset Light and Herring Cove beach sites, together with a discussion of the early Bruun Rule research, in his guide to the geology of The Cape Cod National Seashore. The rule found its way into the Soviet literature in 1973, with its first mention there in Kaplin's Recent History of the Coasts of the World Ocean. Further testing and refinement of the rule followed in Dubois (1975, 1976, 1977), Hands (1976, 1977), Rosen (1978), and others.

During the summer of 1976 I participated in the International Geographic Union's Commission on the Coastal Environment field symposium along the east coast of the Black Sea. The excursion was led by Professor V.P. Zenkovich and included about 45 delegates from the Soviet and Socialist countries and 16 "foreigners". Pavel Kaplin, with whom I had

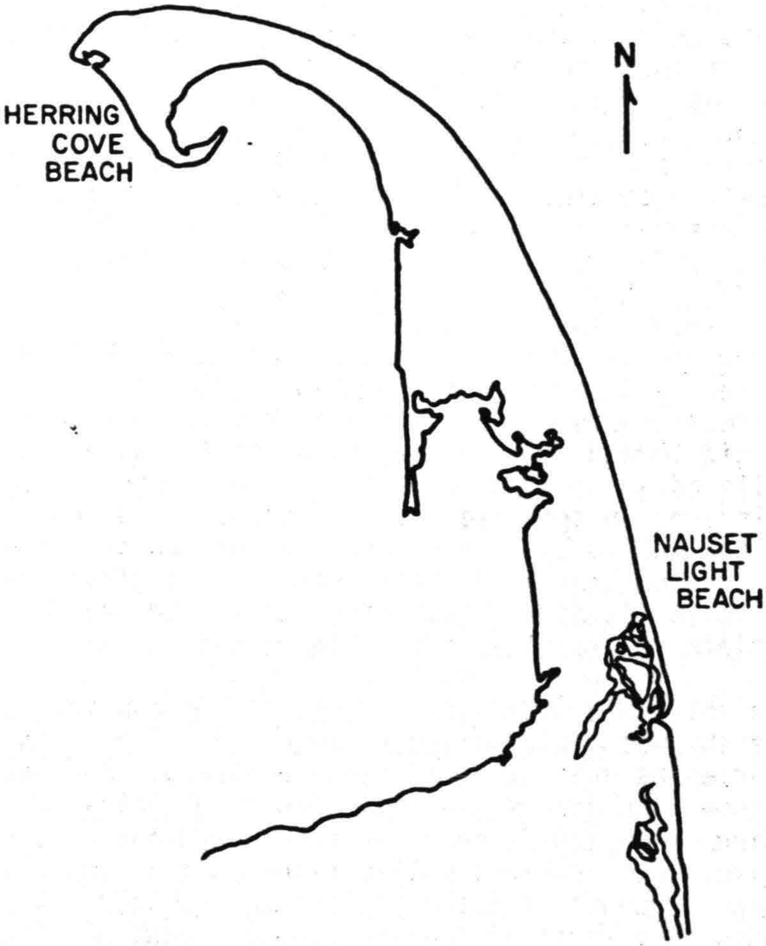

FIGURE 2. Cape Code National Seashore beach location map (after Schwartz 1967).

corresponded but had never met, was among the Soviet participants, and we had many chances to talk together. It was in one of these chats that I thanked him for the courtesy of his having included the Bruun diagram (Fig. 1), and references to both mine and Bruun's work, in his book on coastal morphology. Kaplin laughed and replied "Maury, you do not understand. I disagreed with you." With further conversation, this point was clarified; Kaplin agreed with the conditions of erosion and profile translation as outlined by Bruun, what he disagreed with was that Bruun was the first to propose this hypothesis. It was his contention that Zenkovich and other Soviet workers in the coastal community had delineated such relationships prior to 1962. Not being conversant with the Soviet literature on this subject, there was not much that I could do but ask a few more questions about it and politely let the matter drop.

Upon returning to my office at Western Washington University, I decided to pursue the matter of the Soviet literature on this topic. I, therefore, enlisted the aid of Vladimir Milicic, a specialist in Slavic languages. First we obtained copies of all the Soviet publications cited by Kaplin in his book in connection with the Bruun Rule. These were: Zenkovich (1950, 1957), Ionin (1955), Budanov and Ionin (1956), and Kaplin (1957, 1959). Milicic read these and translated into English any and all portions which mentioned profile translation, erosion-deposition, shore erosion, or sea level rise. We then reviewed these segments several times, to verify the technical meaning of each statement.

From this detailed review, it appears that the Soviet literature, prior to 1962, did indeed contain references to landward migration of the beach profile as an accompaniment to sea level rise. However, in no way could we discern, in the literature that was cited and reviewed, any description of the one-to-one correspondence between eroded and deposited material, or the constancy of water depth at any given nearshore site after the rise in sea level. While point "a" in the Bruun Rule had adequate Soviet claim to priority, claims to points "b" and "c" were unsupported.

Armed with this information, I returned to Moscow in the summer of 1978 (I was, at this time, traveling through the U.S.S.R. on the National Academy of Sciences specialist exchange program). Meeting with Kaplin at his home, the discussion continued amid typical Russian hospitality consisting of copious amounts of food and drink. We each outlined, respectively, the various publications on this topic in the English and Russian languages; tracing ideas from one source to another. After a few hours in pursuit of the one single truth, we finally came to an agreement that the concept in question was the result of "simultaneous, but independent, research converging upon the same conclusions" (Schwartz and Milicic, 1978). That seems to be where the matter rests at this point in time. However, due to its history of development, as outlined in this paper, the concept will probably continue to be known as the Bruun Rule.

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HYPOTHETICAL SHORE PROFILES IN RESPONSE

TO RISING WATER LEVEL

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INTRODUCTION

The purpose of this paper is to present some of my thoughts on how shore profiles should respond to rising water levels. The first part of this paper will focus on the behavior of an equilibrium shore profile in response to a rise in water level; the state of equilibrium will be with respect to wave action and sediment supply. The second part of this paper will deal with the behavior of disequilibrium shore profiles in response to rising water levels; the state of disequilibrium will be with respect to sediment supply and wave action, respectively. Throughout this paper, it is assumed that the physical properties of sediments in each segment of a shore profile remain reasonably constant as the magnitude of coastal processes varies.

The behavior of a shore profile can be conceptualized to range from a state of complete equilibrium to a state of complete disequilibrium with respect to coastal processes. A shore profile adjusts to wave dimensions, sediment supply, and water levels. If the magnitude of each of these primary variables remains constant, then the profile can be viewed as being in a state of complete equilibrium. On the other hand, if the magnitude of all process variables continues to change with the passage of time, then the profile can be viewed as being in a state of complete disequilibrium. A shore profile may also exist in a state between complete equilibrium and complete disequilibrium; for example, a profile may be in equilibrium with one or two of the process variables and in disequilibrium with the rest of the variables.

When a shore profile is in complete equilibrium, the position of the total profile relative to a fixed point on land and the shape of the profile remain constant. If sediments are lost from a shore zone and the shore profile retreats landward while its shape remains constant, then a parallelogram can be used to qualitatively and quantitatively describe the change that has occurred on the shore (Bruun 1962; Coastal Engineering Research Center 1973, p. 4-122; Dubois 1977). Most of the shore models in this paper are constructed from parallelograms and represent a first approximation of the possible behavior of shore profiles in response to rising water levels.

EQUILIBRIUM SHORE PROFILE

For a profile at equilibrium with respect to sediment supply and wave action, the input and output of sediment in the longshore direction is in balance, and the magnitude of wave dimensions is reasonably constant; if some beach erosion occurs because of wave action, it is assumed that those beach sediments deposited in the nearshore will be transported and redeposited on the beach in a relatively short period of time. Given the above assumptions, no net beach erosion nor deposition takes place; the shape and position of the shore profile relatively to a point on land are fixed.

Given a shore profile at equilibrium and a rise in water level, Bruun's Rule (Bruun 1962; Schwartz 1965, 1967) states that beach erosion occurs in order to provide sediments to the shore bottom (Fig. 1) so that the shore bottom (a') can be elevated in proportion to the rise in water level (a). The volume of sediment eroded from the beach (V) is equal to the volume of sediment deposited on the shore bottom (V'). Although the results of field (Schwartz 1967; Dubois 1975, 1976) and laboratory (Schwartz 1965) studies have shown that as water level rises beach erosion occurs and deposition takes place on the shore bottom, the seaward extent of deposition is still in question. Does deposition extend from the base of the foreshore to where waves begin to feel bottom, to the surge or surf base (Dietz 1963), or to the average position where waves break? The results of a study (Dubois 1975, 1976, 1977) conducted along the Wisconsin shore of Lake Michigan have shown that, for a shore profile at a reasonable degree of equilibrium, a rise in lake water level was associated with beach erosion and nearshore deposition; the volume of material eroded from the beach was approximately equal to the volume of material deposited over a distance that extended from the base of the foreshore to the average position where waves broke on the crest of the first longshore bar, a distance of about 18 m. Materials eroded from the beach were not deposited on the second longshore bar which extended parallel to the shoreline.

In my opinion it would appear logical, from two points of view, that deposition on the shore bottom should be confined to the nearshore zone. The first point deals with the general principle of beach erosion as caused by wave height. Simply stated, an increase in wave height is ususally followed by beach erosion; sediments eroded from the beach are deposited in the nearshore where waves break. This basic principle of beach erosion may be applicable to Bruun's Rule. When water level rises, beach erosion occurs because larger waves than usual are permitted to break on the shore; in turn, the sediments eroded from the beach may be deposited in the nearshore zone. An increase in the elevation of the nearshore bottom acts as a negative feedback mechanism which reduces the height of waves that break on the shore. As wave height is reduced to a dimension that existed

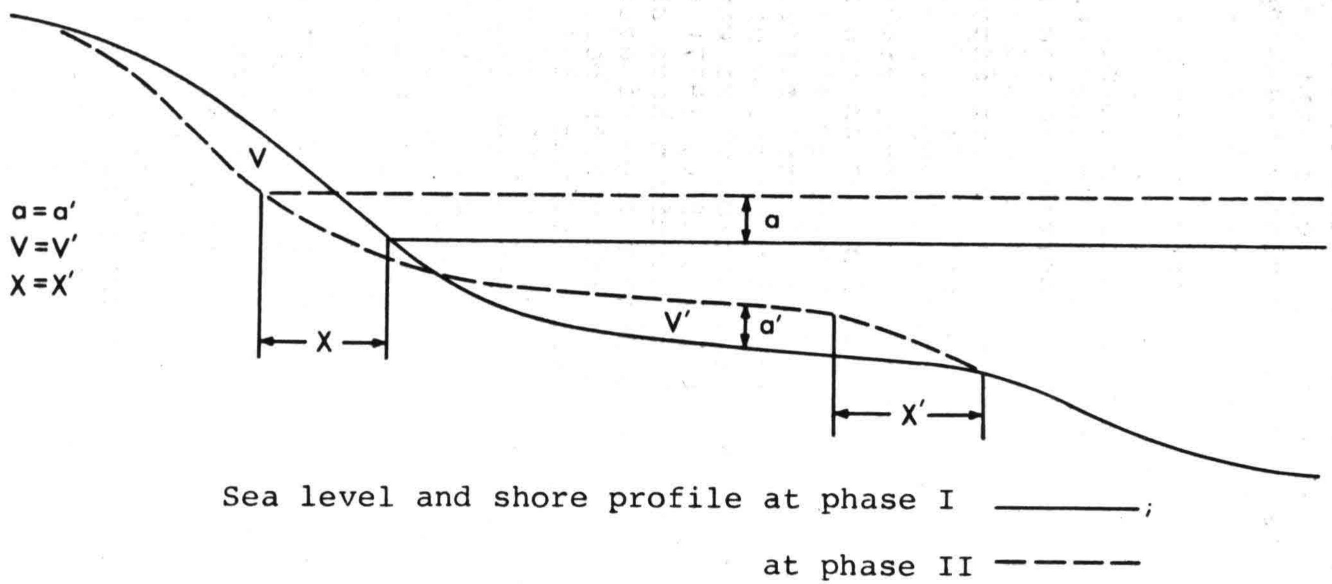

FIGURE 1. The response of a shore profile to a rise in sea level (Modified after Bruun, 1964).

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prior to the rise in water level, the shore profile attains a state of equilibrium. When compared to the former profile, the new profile of equilibrium has the same shape; however, the new profile has moved upward and landward (Fig. 1). Seaward from the average position where waves break, the angle of the shore bottom slope remains constant because the dimensions of incoming waves are constant; the total position of the slope remains fixed, neither advancing nor retreating from shore, because of the balance of sediment input and output in the longshore direction (Dubois 1976).

The second point deals with the concept of minimization of effects. The concept states that once a system is in equilibrium that system tends to adjust to an applied force so as to minimize the disturbance of the system (reviewed and used by Williams 1978). The fact that deposition on the shore bottom may be confined to the nearshore zone can be interpreted as a shore system responding in such a way so as to minimize the expenditure of energy in order to maintain its shape. For example, if deposition were extended from the base of the foreshore to the position where waves begin to feel bottom or to the surge base, then the shore system would be required to expend an additional amount of energy in order to erode more beach sediments and transport them to the offshore zone (Fig. 2). Although the new profile would have a similar shape as the former profile, an unnecessary amount of expended energy would have been used to construct such a profile. Thus, as water level rises, the shape of the shore bottom profile can be maintained, with a minimal amount of disturbance, by confining deposition to the nearshore zone.

There is, however, field evidence that can be used to oppose the idea that in response to rising water levels beach sediments are deposited on the shore bottom close to the beach. Hands (1976) analyzed shore profile and water level data that were collected from 1967 to 1971 along the eastern shore of Lake Michigan near Little Sable Point, Michigan. Four longshore bars existed parallel to the shore of his study area. The crest of the outer bar was about 400 m from shore. The results of his study showed that as water levels rose during the span of 5 years, subaerial shore sediments were eroded, transported lakeward, and deposited over a zone that extended from the beach to about 500 m from shore. Thus, the results of our studies are in disagreement (Dubois 1976; Hands 1976). The discripancy between the two studies may be resolved if one evaluates the total geomorphic processes that could have an influence on the sediment supply entering the nearshore zone.

Coastal bluffs are found along many segments of the shoreline that borders Lake Michigan. Generalizing, the bluffs on the western shore are derived from glacial drift

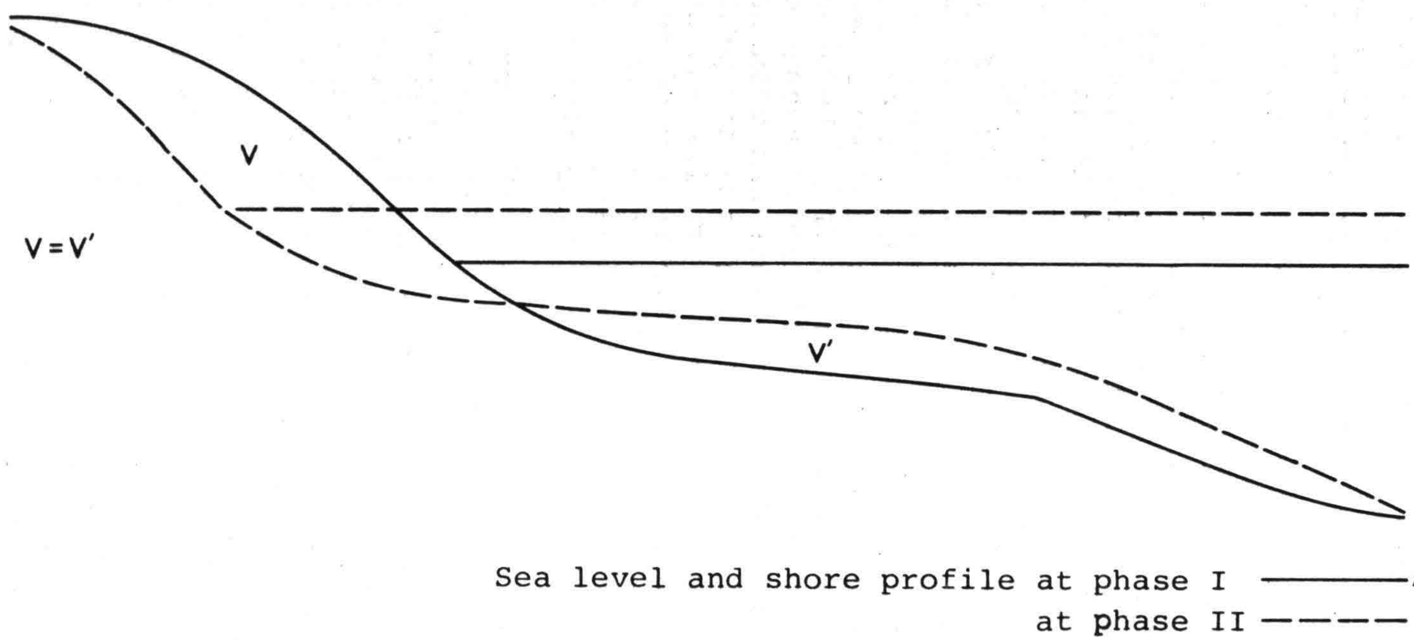


FIGURE 2. In response to a rise in sea level, deposition has extended along the total length of the profile.

(Martin 1965) while on the eastern shore, the bluffs are formed from sand dunes (Hough 1958). During times when lake level is high, the beaches are relatively narrow, and storm waves can attack the base of the bluff; erosion of the base slope could trigger mass wasting especially if the bluff is composed of sands. Thus, it is reasonable to assume that during times of storms mass wasting could deliver sediments directly to the foreshore and nearshore zones. Indeed, even during calm wave conditions, the momentum of the sediments moving down slope could be sufficient to carry these sediments across a narrow beach and into the nearshore zone.

In the summer of 1975, I traveled and noted the evidence of mass wasting along the eastern shore of Lake Michigan. Evidence of mass wasting was prevalent in the vicinity of Van Buren State Park; here, the bluffs derived from sand dunes were approximately 10 to 20 m above lake water level. During calm wave conditions, the width of the backshore which extended from the crest of the foreshore to the base of the bluff varied from about 1 to 7 m. On the sandy slope of the bluffs, down trees and large patches of grass covered sod were observed. Down trees were also found on the narrow beach and in the nearshore. At one time this vegetation existed on top of the bluff. Evidence of mass wasting was also noted at New Buffalo, Michigan where the backshore was narrow (3-5 m) and was flanked landward by a dune scarp with a relief of about 5 to 7 m. Here, homes that had been built on top of the dunes and some distance away from the lake were now positioned at the edge of the scarp because of the continuous erosional activity of mass wasting.

The evidence of mass wasting found along the eastern shore of Lake Michigan suggests that wave action is not the sole process responsible for delivering coastal sediments to the nearshore. In turn, the amount of sediment delivered to the nearshore by mass wasting may be more than that system needs in order to readjust its profile during times of rising water levels. For a shoreline with several longshore bars, one may postulate that sediments not needed by the first longshore bar system could be past on to the second bar system; once the profile of the second bar was adjusted, then the surplus sediment could be past on to the third bar system, and so on. Thus, the results from the data collected during a 5 year period (Hands 1976) may reflect the combined action of waves and currents, mass wasting, and rising water levels.

On the other hand, the material which forms the bluff on the western shore of Lake Michigan is derived from glacial drift and has some clay content (Martin 1965). The lakeward slope of the bluff is vegetated with trees and shrubs, and in turn, is generally more resistant to mass wasting when compared to the bluff slopes of the eastern shore. During my study period (Dubois 1976) which extended from April through July of 1971, no evidence of any significant amount of mass wasting was noted in the vicinity of the study area. Thus, as water level rose during the spring of 1971, wave and current action were the sole processes responsible for delivering sediments to the shore bottom. The waves eroded from the beach only an amount of sediment that was required to readjust the shore bottom profile which extended from the base of the foreshore to the crest of the first longshore bar; little or no sediment was transfered to the second longshore bar system. If the study time had been extended to cover a number of years when lake water level continuously rose, then waves could have attacked the base of the bluff and triggered mass wasting; hence, the results of that study might have been similar to the results reported by Hands (1976).

DISEQUILIBRIUM SHORE PROFILES

When the attempt was first made to try to construct a simple conceptual model of the changes that take place along a shore profile as water level rises, it was convenient to consider the profile as being in a state of equilibrium with respect to all other coastal processes (Bruun 1962). In reality, however, shores may not be in equilibrium as they are subjected to a rise in water level. Disequilibrium of a shore profile, which yields beach erosion, may be caused by a reduction of sediments that feed into the longshore system and/or by an increase in wave height. For the moment, let us assume that wave height is reasonably constant and that disequilibrium of a shore is caused by a loss of sediment in the longshore direction. Given such a condition, the shore bottom profile should migrate landward as the foreshore recedes (Fig. 3). In response to a retrograding beach, evidence of a landward migration of the shore bottom, specifically the second longshore bar system, can be seen in the results of a Lake Michigan study (Dubois 1976). As the beach retrograded in response to rising water levels, little if any beach sediments were deposited in the zone of the second longshore bar; thus, in order to maintain an equilibrium distance from the base of the foreshore to the bar crest, the total bar profile had to advance landward as the beach retrograded.

The shores along many barrier islands and bay mouth barriers may suffer from a net lost of sediment in the longshore direction. Because rivers deposit their sediments in lagoons, sounds, or bays that exist on the landward side of barriers, little or no fluvial sands may enter in the

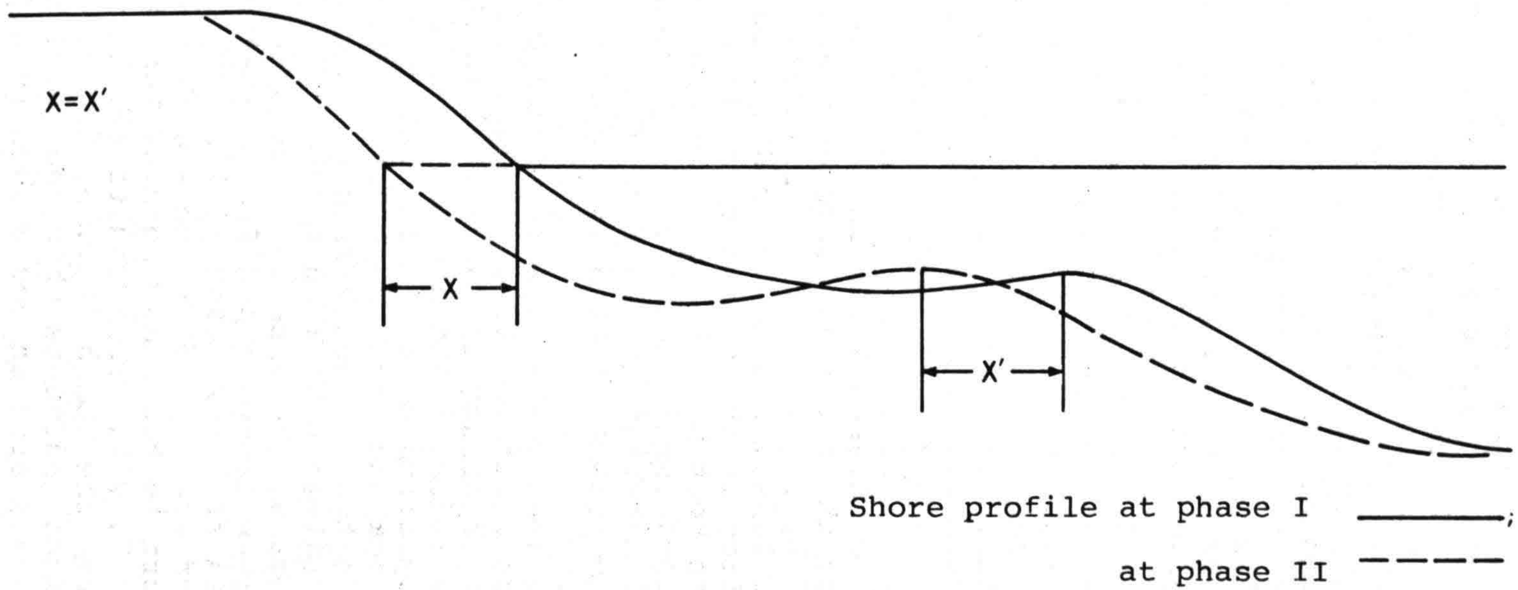
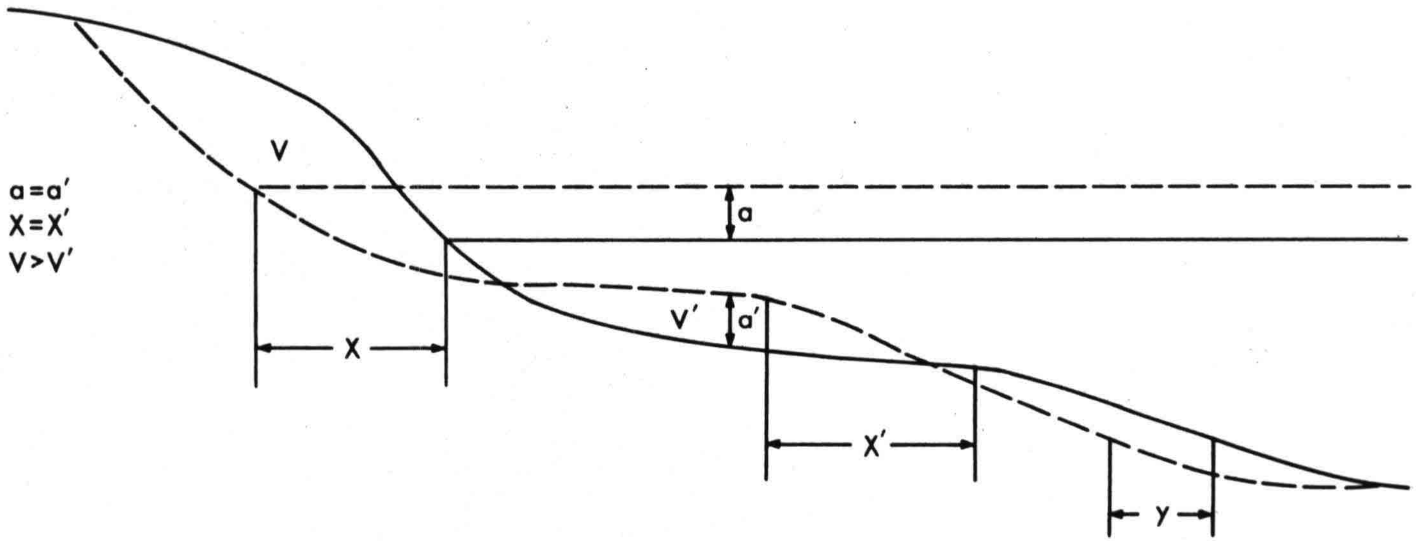


FIGURE 3. The response of a shore profile to a loss of sediment from the beach and nearshore zone (Modified after Coastal Engineering Research Center, 1973, p. 4-44).

longshore drift. In response to the continuous action of waves breaking at an oblique angle to the shore and of longshore currents, some barrier sediments may be eroded from the beach and nearshore zone, transported downdrift, and deposited at the end of the barrier system to form a spit or a cape; these sediments may also be transported through tidal inlets and deposited in the adjacent lagoon (Armon and McCann 1979).

If the arguments that have here been presented are reasonably correct concerning the response of a shore profile to a rise in water level and to a loss of beach and nearshore sediments, then a general model can be constructed of a shore profile that is being simultaneously affected by a rise in water level and by a loss of beach and nearshore sediments. A rise in water level and a loss of shore sediments in the downdrift direction would combine to cause beach erosion (Fig. 4). The position where waves break would advance landward by a distance (X') equal to the horizontal distance of beach retrogradation (X). A portion (Y) of the total distance of beach retrogradation (X) would be attributed to the net loss of sediment in the downdrift direction; the loss of sediment would cause the total shore bottom profile to advance landward (Y). The remaining portion (X-Y) of the total distance of beach retrogradation would be caused by the rise in water level. In response to a rise in water level, sediments eroded from the beach would be deposited in the nearshore so that the elevation of the nearshore bottom would be increased (a') to a value equal to the elevational increase of water level (a). The total volume of material eroded from shore (V) would be greater than the volume of material deposited in the nearshore (V').

Let us now assume that the input and output of sediment in the downdrift direction is in balance and that sediments are eroded from the beach and deposited in the offshore zone. During a storm, sediments are removed from the beach by waves and deposited on the offshore bottom by waves and rip currents (Cook and Gorsline 1972). After the storm, calm waves erode the sediments from the shore bottom and deposit these sediments back on the beach. If the intensity of storms increases, the energy of waves and rip currents will also increase (Shepard and Inman 1950); in turn, the sediments eroded from the beach will be deposited in a greater depth of water. After the storm, calm waves may not be able to return to the beach some of the deep, deposited sediments (Fig. 5, V'). The results should yield a net loss of sediments from the beach (Fig. 5, V). If large storm waves remove beach sediments when sea level is rising, then the resultant beach erosion would be a function of both processes; the shore bottom profile should respond by



Sea level and shore profile at phase I ———;

at phase II - - - -

FIGURE 4. A hypothetical response of a shore profile to a rise in sea level and a loss of beach sediment.

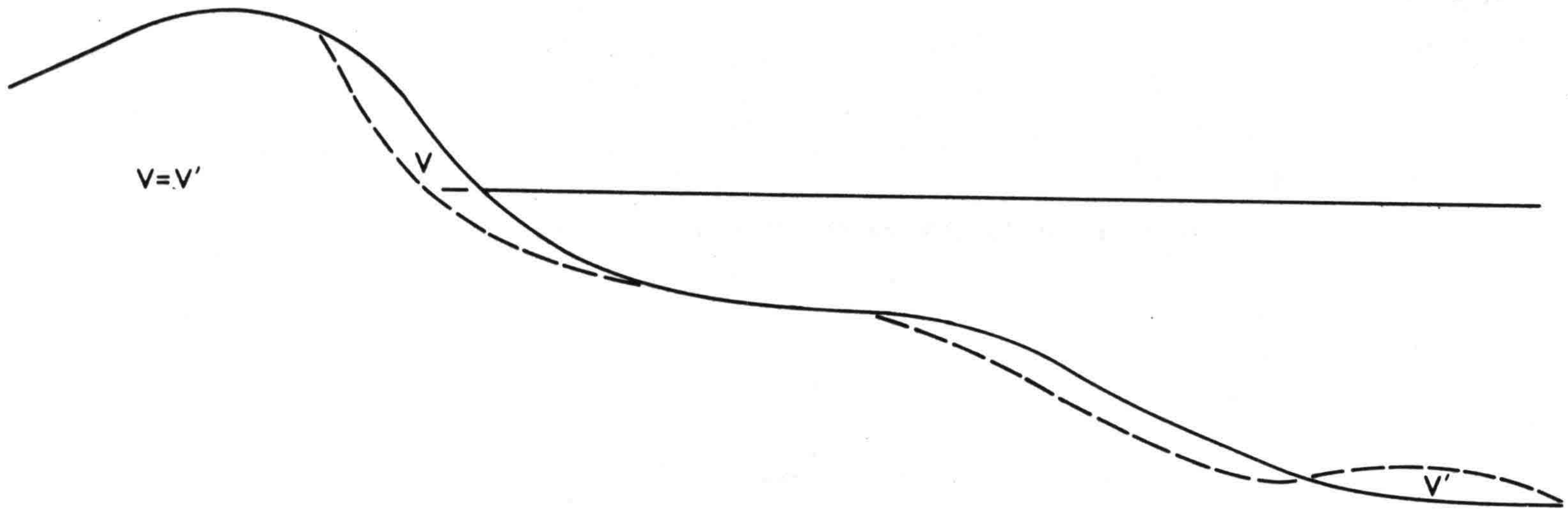
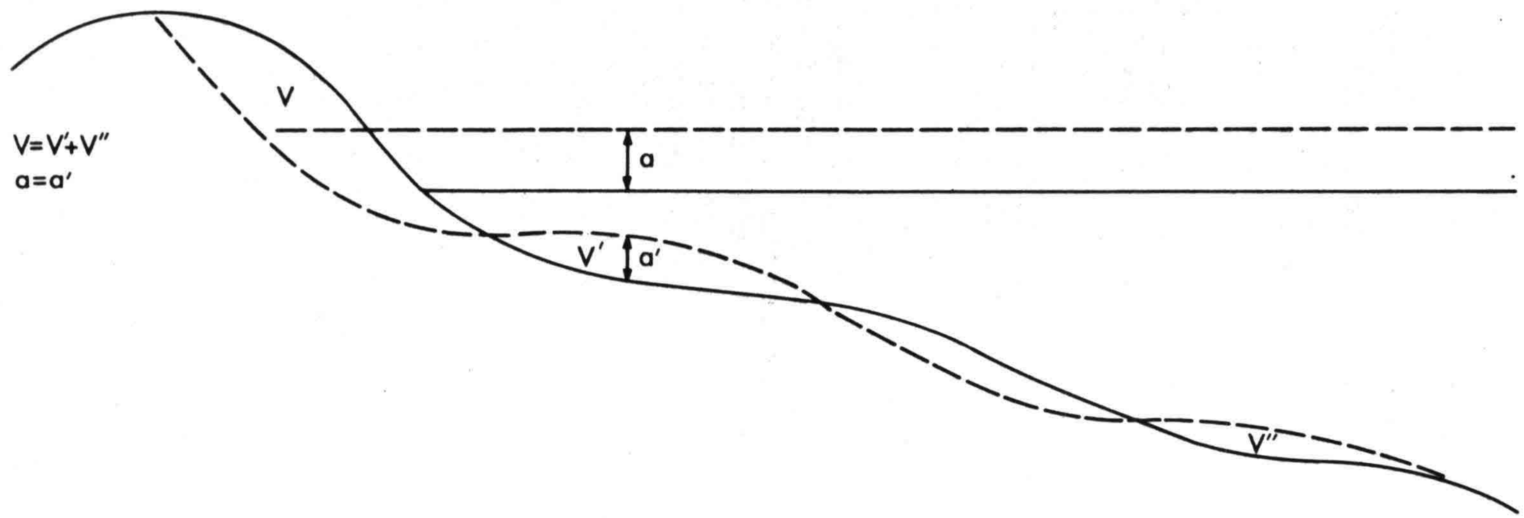


FIGURE 5. The response of a shore profile before (————) and after (– – – – – –) a storm.

moving upward and landward (Fig. 6). The volume of material eroded from the beach (V) should be equal to the volume of material deposited on the shore bottom (V' and V"). Deposition should be confined at both ends of the shore bottom. A rise in water level and wave action would cause beach erosion and subsequent nearshore deposition (V') so that the bottom would be elevated (a') in proportion to the elevational increase of sea level (a). The accumulation of sediments at the seaward end of the shore profile (V") would be caused by the failure of calm waves to return to the beach these sediments that were deposited during storm conditions. If waves were to overtop the foredune, then sediments would be eroded from the beach and shore bottom, and deposited on the landward side of the foredune; the extensiveness of offshore deposition during this event is not fully known. It could be argued that because waves are overtopping the foredune the backwash energy would be reduced which, in turn, would reduce the amount of sediment transported to the offshore zone.

For the Mid-Atlantic region, there is evidence to suggest that storm wave regimes have changed during the last three decades. Hayden (1975) noted that the frequency and duration of storm waves and the length of the winter storm wave season have all increased. In addition, Hicks (1972) reported that a relative rise in sea level is occurring in this region. Thus, many of the barrier beaches of the Mid-Atlantic coast may be in complete disequilibrium as they try to adjust to a change of storm wave regimes, a lack of fluvial sediments , and a rise in water level.

The process of storm waves overtopping the foredune is fairly common along barrier systems (Dillon 1970; Pierce 1970; Dolan 1973; Bartberger 1976; Leatherman 1979; Fisher and Simpson 1979). Indeed, the combination of rising water levels and overwash has been attributed as a cause of the transgressive behavior of barrier systems (Hoyt 1967). To some people, there may seem to be a paradox between the processes that sustain a transgressive barrier and Bruun's Rule. Transgression of a barrier system is sustained by sediments being eroded from the beach and shore bottom and being deposited on the landward side of a barrier; Bruun's Rule is sustained by sediments being eroded from the beach and being deposited on the shore bottom. The shore bottom becomes an area of conflict; one concept suggests that erosion must occur on the bottom while the other concept suggests that deposition must take place. This conflict of ideas has led Dillon (1970) to write that Bruun's Rule was not applicable to the barrier systems along the coast of Rhode Island because erosion occurred on the shore bottom. The paradox of ideas may be resolved if deposition of



Sea level and shore profile at phase I ———;

at phase II – – – –

FIGURE 6. Hypothetical response of a shore profile to a rise in water level and to offshore deposition.

sediments eroded from the beach in response to rising water level is confined to the shore bottom that extends from the base of the foreshore to the average position where waves break, and if erosion of the shore bottom in response to overwash is confined seaward from the average position where waves break to the surge base (Fig. 7). Phase I in Figure 7 represents an equilibrium shore profile during calm wave conditions; Phase II represents an equilibrium shore profile after a period of rising water level and overwash. The elevation of the nearshore bottom has increased (a') in proportion to the elevational increase of water level (a). In response to overwash, sediments are lost from the beach and offshore zone. The barrier sediments that are lost from the beach and deposited in the nearshore are resupplied from the offshore zone; the offshore sediments that are deposited on the barrier also help to increase the elevation of the foredune in response to rising water level.

There is some evidence in support of the concepts that are presented in Figure 7. Evidence of beach erosion and nearshore deposition in response to rising water level has been presented earlier in this paper (Dubois 1976). There is evidence to suggest that sediments are eroded from the offshore and deposited on barriers (Shepard 1962; Giles and Pilkey 1965; Pierce 1969). Kraft and others (1973) reported that off the coast of the Delmarva Peninsula the shore bottom slope which extended from the beach to the -30 ft contour was relatively steep; from -30 to -60 ft, the bottom was fairly flat. Kraft and others (1973) suggested that the shore bottom with the relatively steep slope was the zone where submarine erosion occurred at their study site.

Swift (1975, Fig.4, lower A) has also constructed a shore model that combines the concepts of a transgressive barrier system and Bruun's Rule. He likewise was faced with the problem of diagraming where erosion and deposition occurred on the shore bottom. Judging from the arrows in Figure 4A (Swift 1975), the concept of foreshore erosion and shore bottom deposition in response to rising water level appears to be missing. His model has submarine erosion occurring in the offshore zone; some of the sediments eroded from this zone are transported seaward and deposited on the inner shelf floor while others are transported landward and deposited on the backbarrier.

SUMMARY AND CONCLUSION

The purpose of this paper was to offer some hypothetical models on the behavior of shore profiles in response to a rise in water level. For an equilibrium profile, a rise in water level coupled with wave action causes erosion on the

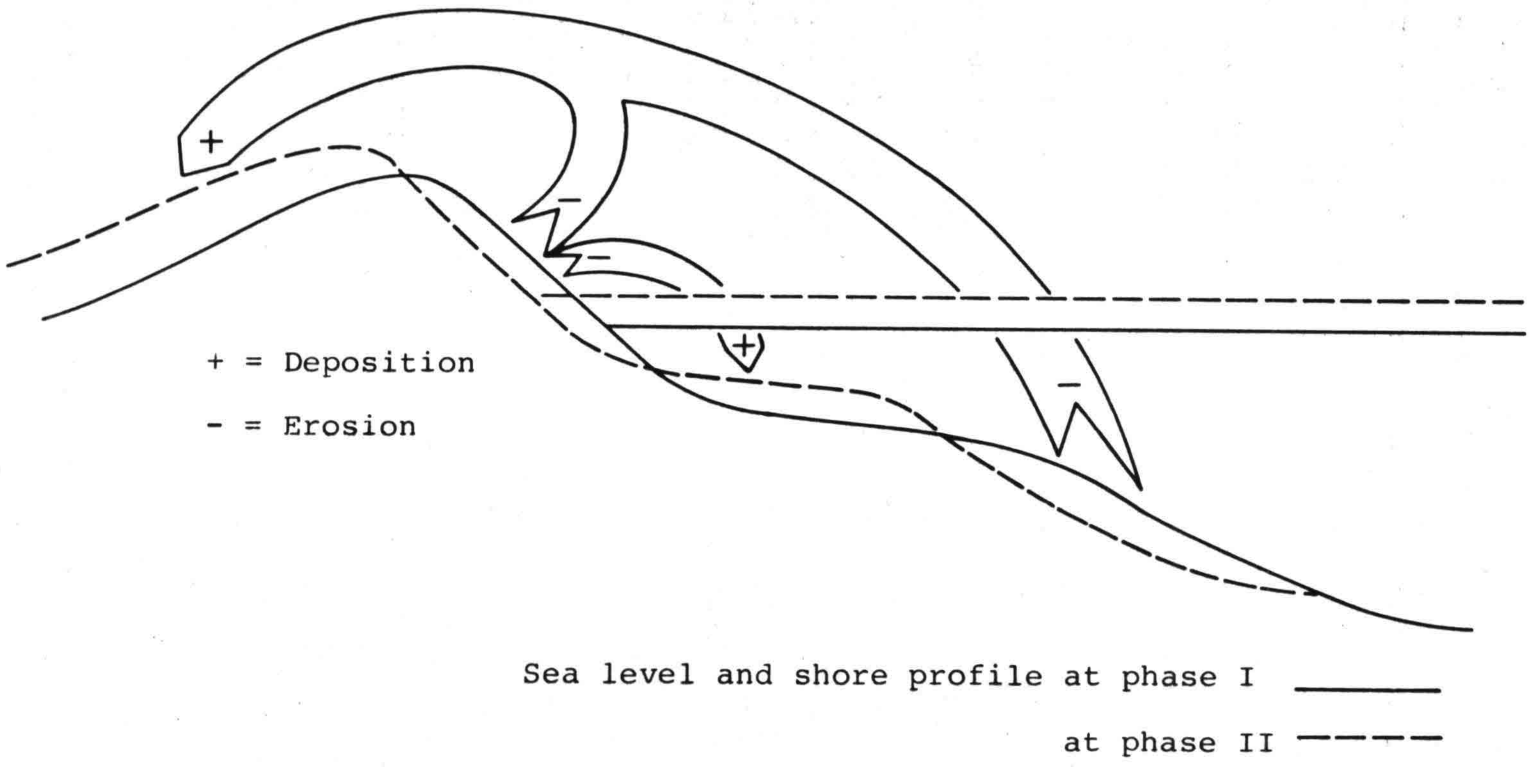


FIGURE 7. Hypothetical response of a shore profile to a rise in water level and to overwash. Offshore deposition in response to storms is not considered.

beach and deposition on the shore bottom. The seaward extent of deposition is still in question; some evidence suggests that deposition may be confined to the nearshore zone. If deposition is confined to the nearshore zone, then the shore profile that extends from the average position where waves break to the zone of the effective swash action should move upward and landward as water level rises. Further, in response to rising water levels, a shore profile that is losing sediment in the downdrift, landward, and/or seaward directions should also move upward and landward. In this case, however, the total shore profile from the zone of the surge base to the zone of swash action should move upward and landward. Deposition on the shore bottom should be confined to the nearshore zone while erosion should occur in the offshore zone that ranges from about the position of breaking waves to the surge base.

In conclusion, I believe that our comprehension of the behavior of an equilibrium or a disequilibrium (erosional type) profile in response to rising water levels is still limited; we have the results of only a few experiments to guide us. It is clear, therefore, that more controlled experiments will have to be conduced before we can have a better understanding of the complex behavior of some erosional beaches.

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SHORELINE EROSION, RHODE ISLAND AND NORTH CAROLINA COASTS - TEST OF BRUUN RULE

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Abstract

Analysis of Rhode Island shoreline retreat, measured on aerial photographs from 1939 to 1975 together with sea level rise rates allows a test of the Bruun Rule. This rule suggests that as sea level rises, sediment eroded from the shore is deposited offshore equal to this sea level rise. Submergence by a sea level rise of 0.3cm/yr accounts for only 15% of the average shoreline retreat of 0.2 m/yr. Overwash accounts for 26%, while inlet deposition accounts for 35% of this retreat. The remaining 24% of the eroded sediment is deposited offshore between the breaker zone and wave base limit. A similar sedimentation situation exists along the higher energy North Carolina coast with erosion averaging 2.0 m/yr. These are the first studies of the Bruun Rule on barrier island coastlines.

Acknowledgements

Funds for this study were provided by the University of Rhode Island Sea Grant 506-010, Project R/E-9, Principal Investigator, John J. Fisher, for main study "Photogrammetric Remote Sensing Inventory of Long Term Rhode Island Shoreline Changes, Part I - Southern Barrier Coasts". The photogrammetric measurements were made by Donald R. Regan (shoreline and duneline changes) and Elizabeth J. Simpson (overwash and tidal delta changes), and Stephen C. Gautie conducted the beach slope surveys.

Introduction

The Per Bruun theory for shore erosion (Bruun, 1962) suggests that as sea level rises increasing sediment must be deposited offshore equal to the amount of this sea level rise to maintain "a profile of equilibrium." If this sediment is not available from elsewhere, it will be eroded from the adjacent beach face (Fig. 1). Validity of this theory was indicated by Schwartz (1967) based on wave tank model studies and a short-term (one month high-low tidal cycle) field study. He suggested that this theory therefore now be called the "Bruun Rule." El Ashry (1971) thought that the Per Bruun theory might account for the increasing erosion he had observed earlier on aerial photographs of certain United States shorelines.

A Sea Grant funded study to produce an inventory of long term shoreline changes of the Rhode Island barrier beach coast to provide management information also provided information on which to test this theory. Rates of shoreline change, primarily erosion, were determined from measurements of sequential aerial photography taken over a 36 year period. Detailed sea level rise curves for the same coast over the same time period were used to determine if there was the proposed balance between shoreline erosion and sea level rise as indicated by this theory.

Most short-term tests of the Bruun theory (on the order of one or two years or at the most 10 years) have the opportunity to measure the following three parameters:

1. the amount of shoreline erosion
2. the amount of offshore deposition
3. sea level rise.

The test then determines whether the amount of eroded sediment equals the amount of sediment deposited, this amount being in excess of that occurring simply due to shoreline retreat due to submergence by sea level rise. In addition, the amount eroded and deposited must also equal that predicted by sea level rise. Of course, sediment gains and losses due to other sources (e.g., river input, longshore transport in and out of system, etc.) are determined, quantified and incorporated into this sediment budget.

However, for this long-term (36 year) analysis of the Bruun theory along the Rhode Island coast, only the following two parameters could be determined:

1. the amount of shoreline erosion
2. sea level rise.

No series of bathymetric charts exist over this period of time which are sufficiently detailed to show the expected offshore deposition. Thus, this test is whether the amount of measured shoreline erosion equals that predicted by the measured sea level rise, both over the 36 year period.

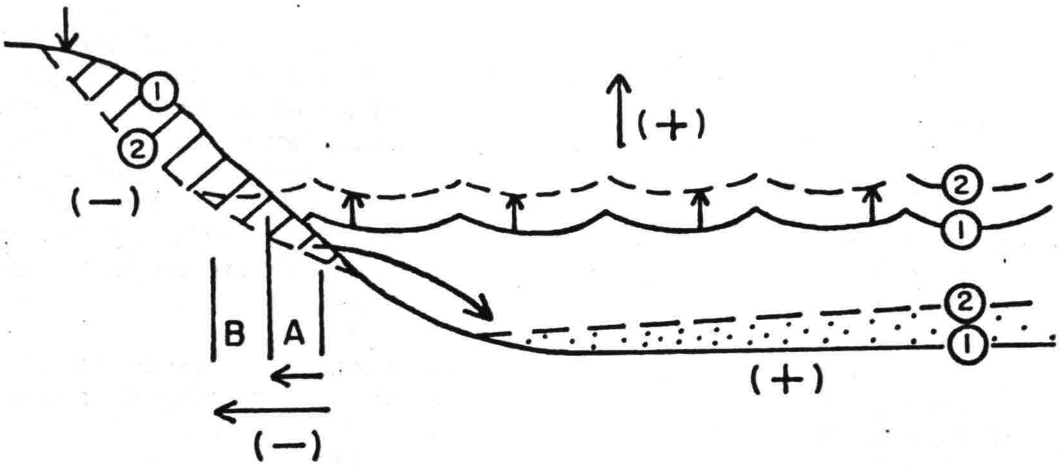


Fig. 1 - Relationship of sea level rise to shore erosion (after Bruun 1962). Sea level rise (1 to 2) requires erosion (B) in excess of retreat (A) due to submergence with deposition offshore equal to sea level rise (1 to 2) to maintain profile of equilibrium.

Rhode Island Coast - Special Features

The Rhode Island southern shoreline, 30 km in length, is primarily a barrier beach complex, developed from a mainland consisting of primarily a glacial outwash plain. It has been submerged by recent sea level rise. Headlands (locally called "points") composed of till and outwash plain deposits separate a series of lagoon-like bays (locally called "ponds") that are drowned glacial outwash channels. Interconnecting baymouth barriers (locally called "barrier beaches") with several inlets make up the major shoreform of the coastline (Fig. 2).

This Rhode Island barrier beach shoreline is an especially favorable coast for a test of the Bruun theory for the following reasons:

1. The coastline study area has natural sedimentological boundaries - Narragansett Bay on the east and Little Narragansett Bay on the west.
2. There are no sediment sinks out of the area due to longshore transport.
3. There is no sediment transport into the coast by longshore transport.
4. There is no additional sediment source from within the system since the lagoon streams are very short and relict from glacial outwash channels.
5. There is no additional sediment source from outside the system - Narragansett Bay is over 30 m deep, while Little Narragansett Bay is drained by meandering streams with little or no sediment load.
6. Detailed sea level rise curves are available from nearby Newport Harbor, at the mouth of Narragansett Bay.
7. Suitable coastal vertical aerial photographs are available from 1939 to 1975, a 36 year period, for photogrammetric shoreline change mapping.

Photogrammetric Analysis of Shoreline Changes

To determine the rate of shoreline changes, aerial photographs covering the entire Rhode Island coast were mapped photogrammetrically for the period covered by aerial photography from 1939 (oldest) to 1975 (most recent). Amounts of areal changes of all shoreline, duneline, washover and inlet deltas were measured using a standard grid point-count technique on stable mylar overlays, rectified as to scale and ground controls with a Bausch and Lomb Zoom Transfer Scope (Fig. 3). Linear measurements were made with a direct reading micro-rule to a precision of 0.001 inch.

The Bausch and Lomb Zoom Transfer Scope is an optical, anamorphic copy system that allows ratio comparisons of 1:14 and was initially designed to permit overlays of larger scaled satellite imagery on small scaled topographic maps. Early photogrammetric shoreline

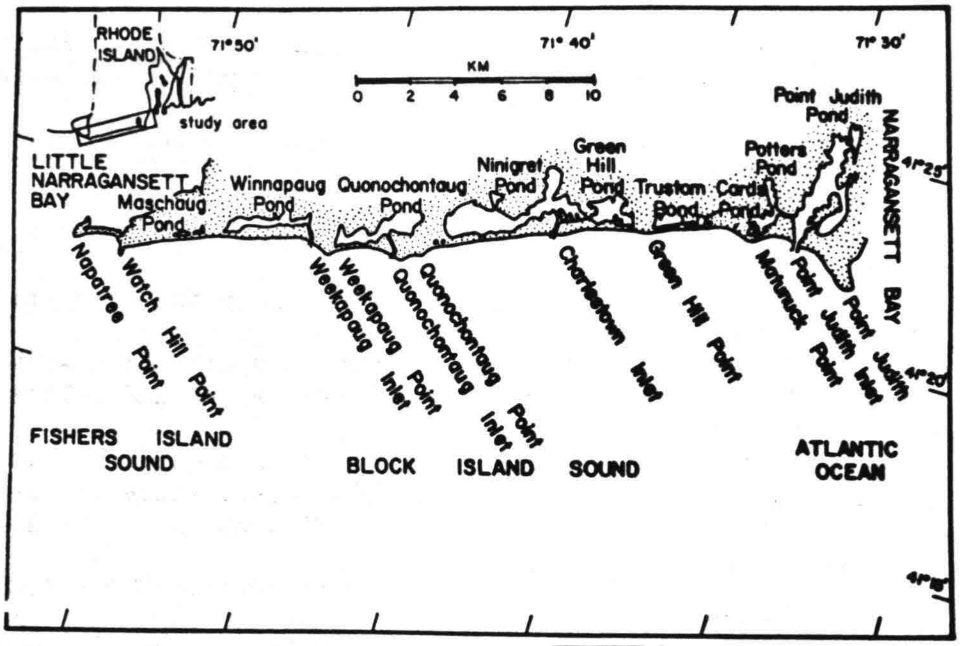


Fig. 2 - Rhode Island barrier beach coast developed on an embayed glacial outwash plain under glacio-eustatic sea level rise. Barrier beaches formed as bay-mouth barriers between till headlands ("points").

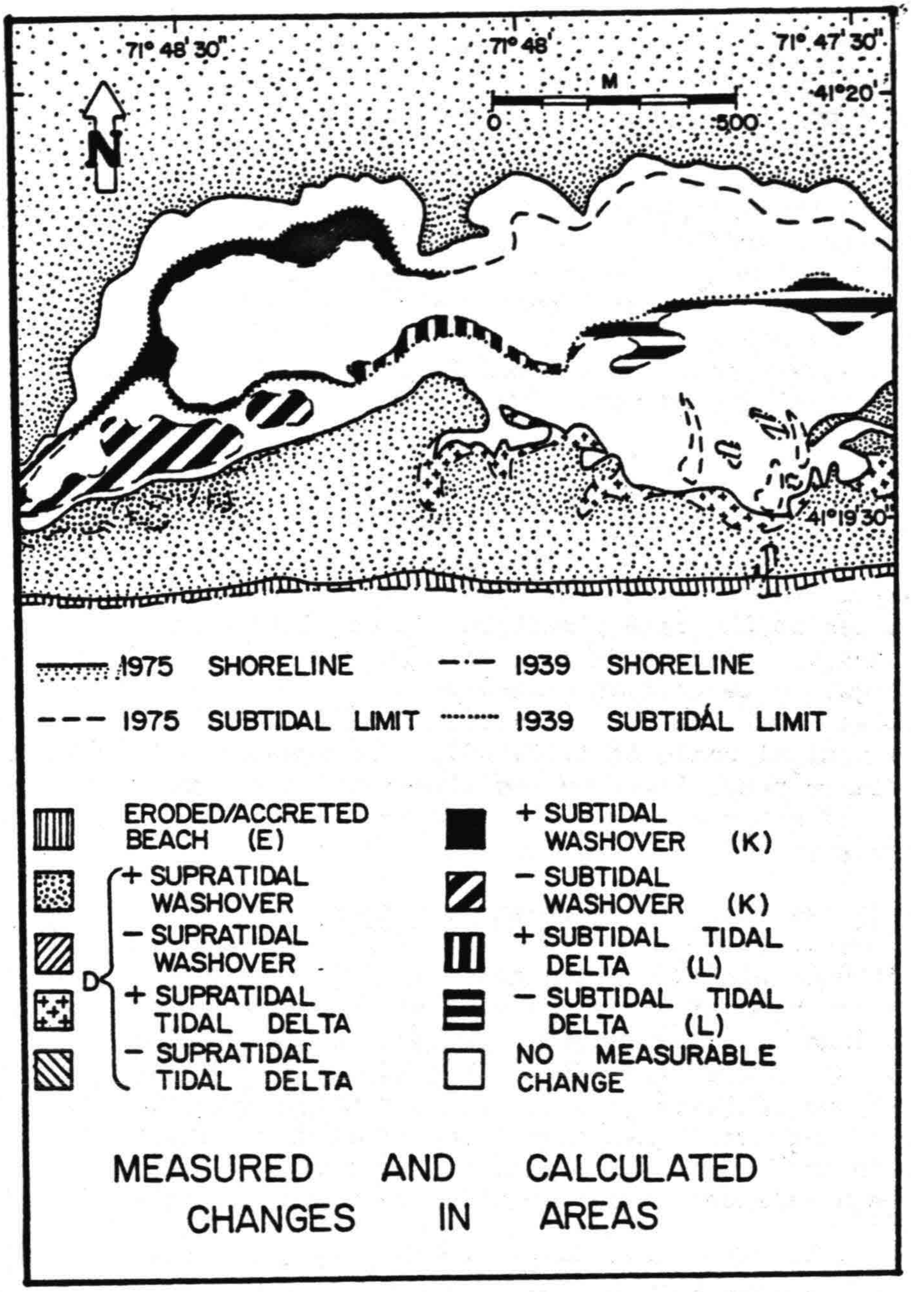


Fig. 3 - Photogrammetrically mapped coastline changes from 1939-1975 aerial photographs including beach, tidal deltas and washovers.

change studies (e.g., Stafford 1972, 1971) required rectified enlargements of each aerial photograph. Direct measurements or overlays on unrectified aerial photographs may produce errors due to scale differences (sometimes as great as 5 percent), camera tilts, radial displacement, print paper shrinkage, and relief displacement. A photographically rectified print can reduce these errors, but it is expensive. The anamorphic feature of the Zoom Transfer Scope allows on-line displacement of the photographic image in both the x and y axis and thus can optically rectify each overlay image to stable ground control points. In addition, the continuous enlargement system allows each aerial photograph to be optically enlarged to exactly the scale of ground control survey points. Several ground control survey points were used for each aerial photograph. Comparison of aerial and ground control measurements indicated an accuracy of 3 m.

Quantitative analysis requires that the order of accuracy of photogrammetric measurements be related to the true field values. To reduce micro-rule errors, scale and cartographic variability, and operator variability, a ground-truth survey of linear distances and areas was made. Linear distances were measured in the field at several localities at the same elevation. These linear ground measurements were used to calculate the ground areas of rectangular fields and buildings, to be used as ground-truth values for both linear distances and areas that were then measured on a 1972 photograph (073-72 series, with a nominal scale of 1:12,000). The quantitative amounts of error or variance resulting from the linear and areal measurements of the objects of known areas (determined from the field measurements) average 2.1%.

Rhode Island Shoreline Retreat as Submergence

It was initially anticipated that our photogrammetric shoreline change inventory from 1939 - 1975 would show that the headlands have an erosional trend while the barrier beaches would show accretional trends in a manner similar to the classical model of a shoreline of submergence of Davis (1896). In this model, headland areas are the focus of refracted wave energy and erode with accretion taking place in the adjacent baymouth barriers or bayhead beaches. There is thus a balance between erosion and accretion in this model.

However, the total amount of shoreline erosion (Fig. 4) shows that the barrier beaches erode as much or greater than the adjacent headlands. The only areas of advance have been in the immediate up-drift areas of jetties and groins. The headlands eroded during this 36 year period at a rate of about 0.2-0.6 meters per year. The annual erosion rate of the Charlestown - Green Hill barrier (stations 60 to 80) was 0.5-1.0 m/yr, while the Misquamicut barrier eroded at 0.5-1.5 m/yr. In general, the entire Rhode Island barrier and headland coast is erosional at an average annual rate of 0.2 m/yr.

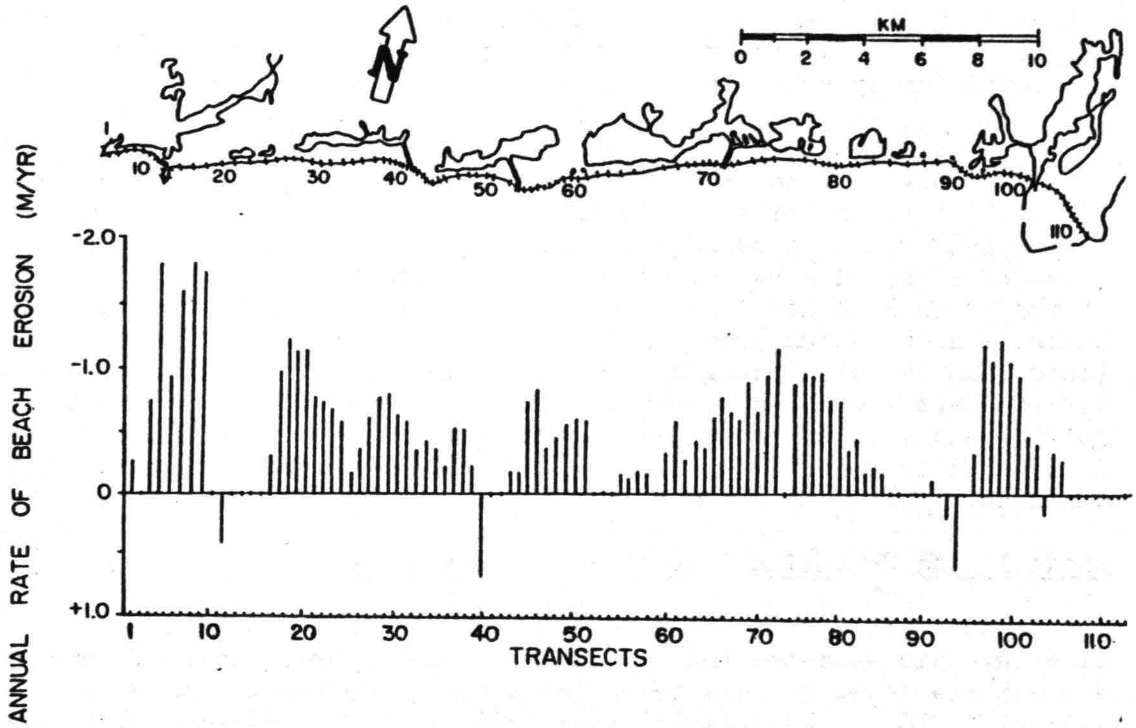


Fig. 4 - Annual rate of Rhode Island beach erosion (m/yr) based on photogrammetric mapping for 1939-1975 period. Coast completely erosional except where up-drift from jettied inlets. Average rate of erosion is 0.2 m/yr.

However, this average coastal erosion of 0.2 meters/year on this coast may simply be "retreat" due to submergence under sea level rise and not actual erosion. The sea level rise curve of Hicks and Crosby (1974) shows an average rise of 0.3 centimeters per year at Newport, Rhode Island, over the past forty years (Fig. 5). The vertical component of the measured horizontal retreat of 0.7 meters/year on the average beach slope of five degrees would be 2.0 centimeters/year. A submergence of 0.3 centimeters/year due to sea level rise therefore accounts for only fifteen percent of the vertical component of the measured yearly retreat. Thus, eighty-five percent of the shoreline retreat must be due to erosion since only fifteen percent can be accounted for by submergence due to sea level rise.

This sand, eroded from the high tide shoreline, probably has not gone extensively into dune building on the barrier island. A similar photogrammetric mapping of the frontal barrier duneline for the period 1939 - 1972 was also conducted. This study shows that the duneline (seaward edge) also was almost completely in retreat (Fig. 6). Only at the Green Hill headland (stations 78 to 83) have the frontal dunes advanced at an annual rate of 2-3 m/yr. Similarly, duneline advances (less than 1 m/yr) occurring at other areas (stations 35-40 and 110-113) are where beaches have also advanced in the updrift sides of jetties and groins. In general, the pattern of shoreline retreat is duplicated in regional pattern by the retreat of the frontal barrier duneline.

Rhode Island Shoreline Retreat - Subsurface Record

A subsurface study of Ninegret and Green Hill Ponds found stratigraphic evidence that the barrier beach was probably formed at a lower sea level and has moved landward as the sea transgressed (Dillon, 1970). The term "barrier roll-over" was coined to describe this barrier action. Dillon further noted that "the small size of this barrier places its base at a shallow depth (less than 5 meters), resulting in erosion of the entire seaward side by storm waves and also permitting considerable transport of sand across the barrier to the lagoon side." He observed that lack of sand supply seems to be the dominant factor allowing landward migration (erosion of the seaward beach face) of the barrier and that no significant amounts of sand are contributed from the land because the only rivers entering the ocean in the Charlestown - Green Hill vicinity flow into effective sediment traps (Narragansett Bay and Long Island Sound). He also suggested that little sand is supplied from offshore because offshore sampling indicated that these sediments ranged from coarse sand to gravel in size. The landward movement of this barrier as the sea transgressed as described by Dillon, is a process occurring over thousands of years. On the other hand, while this Rhode Island photogrammetric study covers a time period of only thirty-seven years, a short time with respect to sea level rise, photogrammetric measurements still indicate that barrier retreat landward, or erosion,

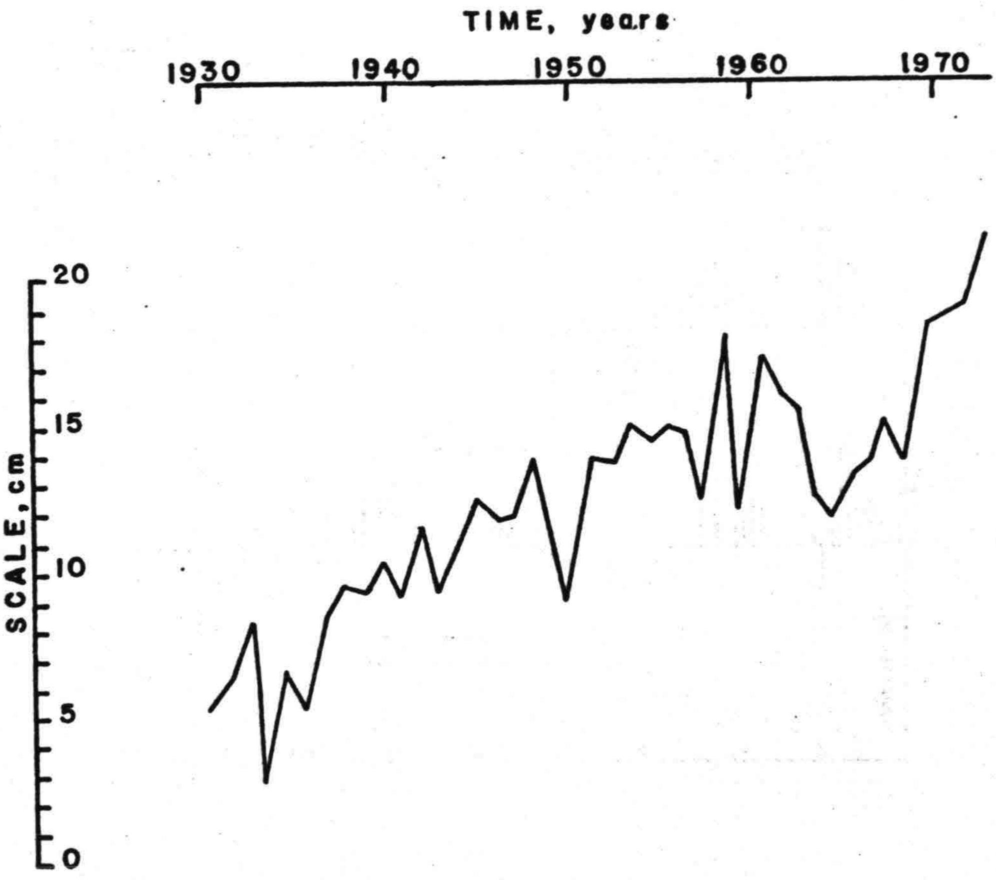


Fig. 5 - Sea level rise curve for Rhode Island coast based on tidal data from Newport, R.I. (after Hicks and Crosby, 1974). Average value for 1930-1972 period is 3 mm/yr.

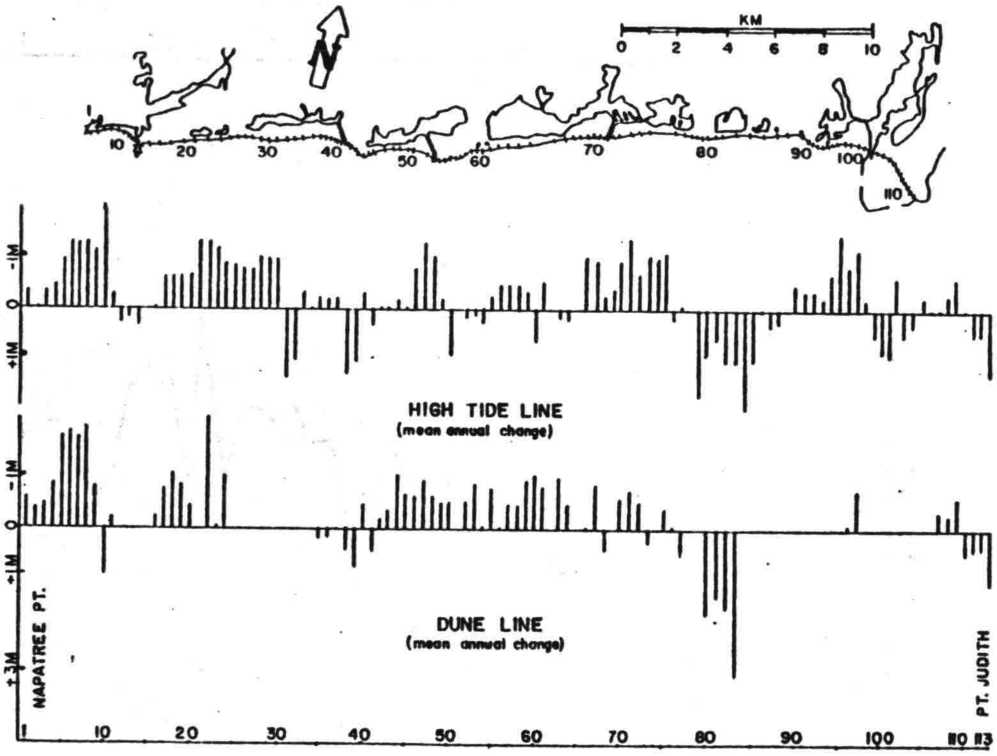


Fig. 6 - Relationship of high tide shoreline erosion to dune line erosion (1939-1972) showing that while shoreline is eroding, the sand is not deposited on dunes since they are also eroding in a similar pattern.

has occurred.

Barrier Islands - Bruun Rule Modifications

Modifications to the Bruun Rule must be considered when applying it to barrier island coastlines. Previous field tests of the Bruun Rule (e.g., Schwartz, 1967, and Dubois, 1976) have taken place on mainland beaches and it appears that for barrier island beaches several other factors have to be considered. Most barrier islands, including these Rhode Island barrier beaches, are now thought to form under a rising sea level as a feature of a shoreline of submergence (Fisher, 1968). Under these conditions, the erosion and accretion sediment budget of a barrier island must take into account certain features of a transgressive barrier island coast. Sediment lost from shoreline erosion will not exclusively move offshore as predicted by the Bruun Rule, assuming that any loss or gain by longshore transport is accounted for in the budget. Transgressive barrier islands exhibit extensive flood tidal deltas at inlets as well as storm washovers (Pierce, 1970). These depositional features are potential significant reservoirs of sediment in barrier island systems and also are important sediment sinks in the overall littoral sediment budget of a shoreline (Fig. 7). These various "back-barrier" sediment sinks were first identified in the field to develop "ground-truth keys". These field units were then mapped on aerial photographs of 1939 and 1975 respectively and the areal change of the subtidal and supratidal washover and inlet deposits along the entire coast were measured.

Backbarrier sediment sink inventory:

The relationships between the areal changes in washover and tidal delta deposits and the amount of eroded beach (Regan and Fisher, 1977) over the whole study period for the entire south shore are as follows:

(1) Total area of eroded beach:	-608,558 m^2
(2) A, total area of subtidal washover accretion:	+267,953 m^2
B, total area of subtidal tidal delta accretion:	+862,322 m^2
C, total area of subtidal washover and tidal delta loss:	- 76,147 m^2
Total (2) subtidal washover and tidal delta accretion:	+786,442 m^2
(3) A, total area of supratidal washover accretion:	+522,792 m^2
B, total area of supratidal tidal delta accretion:	+188,238 m^2
Total (3) supratidal washover and tidal delta accretion:	+711,030 m^2
Total (2) + (3), subtidal and supratidal washover and tidal delta accretion:	+1,497,472 m^2

Subtidal washover accretion (+267,953 m^2) is only about one third (0.31) as effective as subtidal tidal delta accretion (+862,322 m^2) in the lagoons, whereas supratidal tidal delta accretion (+188,238 m^2) is only about one third (0.36) as effective as supratidal washover accretion

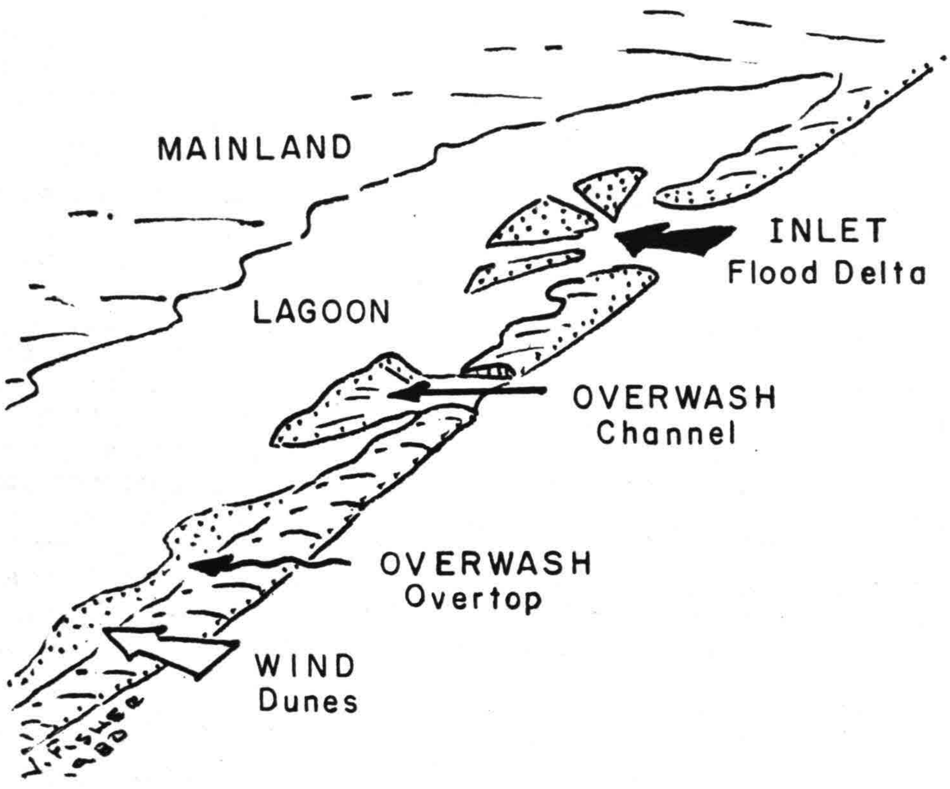


Fig. 7 - Sediment budget modifications for Bruun Rule Analysis on a barrier island shoreline, includes above "back barrier" sediment sinks.

(+522,792 m^2) (Fig. 8). Subtidal and supratidal washover accretion (+790,745 m^2) is three-quarters (0.75) as effective in transporting sediment landward as subtidal and supratidal tidal delta accretion (+1,050,560 m^2). It is to be expected that tidal delta sedimentation is more effective in transporting sediment into the lagoon and onto the backbarrier than are overwash processes, because tidal delta sedimentation processes are steady and relatively continuous, whereas overwash is catastrophic (i.e., discontinuous and erratic).

Using the calculated value for the annual rate of washover accretion along the Rhode Island south shore to approximate the rates of accretion for washover fans as well as for tidal deltas (in the absence of any rates of tidal delta sedimentation) and using the Coastal Engineering Research Center's estimate of the volume of beach lost (0.76 m^3) per areal loss of beach (0.09 m^2) to derive a value of 8.44 m^3 sediment loss per 1 m^2 areal units of beach erosion, the following values for the whole south shore over the entire study period can be computed:

Washover accretion: $0.05\ m/yr \times 36\ yr \times (267{,}953+522{,}792-\frac{76{,}147}{2}) = 1{,}354{,}809\ m^3$

Tidal delta accretion: $0.05\ m/yr \times 36\ yr \times (862{,}322+188{,}238-\frac{76{,}147}{2}) = 1{,}822{,}476\ m^3$

Beach erosion: $8.44\ m^3/m^2 \times -608{,}558 = -5{,}138{,}934\ m^3$

Washover: $\frac{1{,}354{,}809}{-5{,}138{,}934} \times 100 = 26\%$

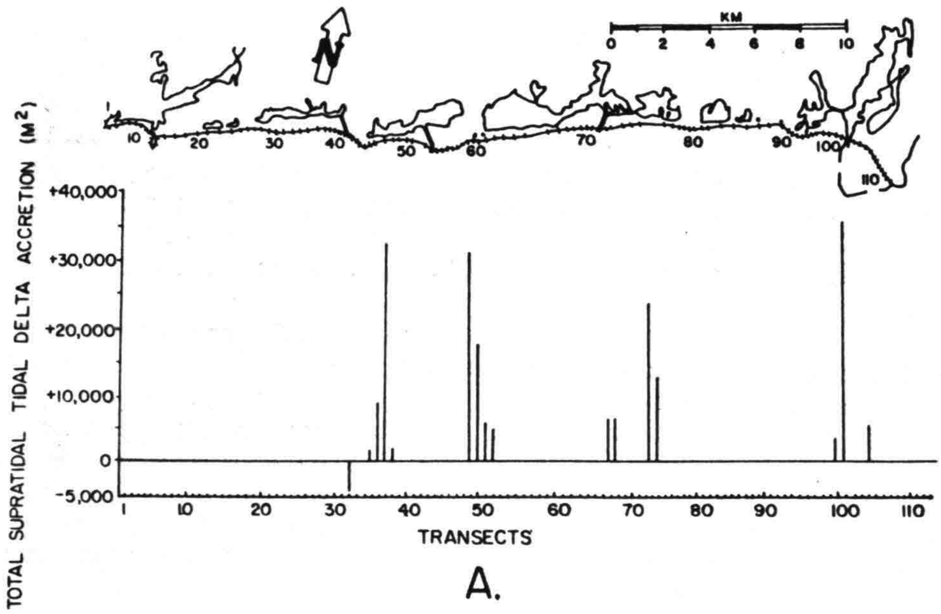
Tidal delta: $\frac{1{,}882{,}476}{-5{,}138{,}934} \times 100 = 35\%$

Washover and tidal delta: $\frac{3{,}177{,}285}{-5{,}138{,}934} \times 100 = 62\%$

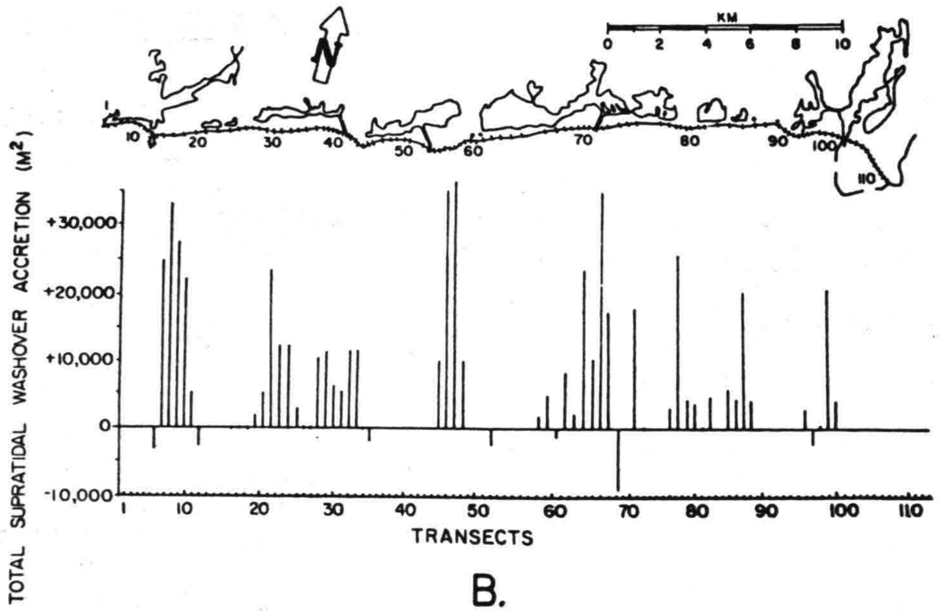
According to these calculations, washover accretion accounts for 26% of the volume of beach eroded, and tidal delta accretion accounts for 35% of the volume of beach eroded (Fisher and Simpson, 1979). Washover and flood tidal delta sedimentation over the short term represent a loss of sediment to the littoral sediment budget. Over the long term, however, this sediment becomes incorporated into the barrier beach system as the barriers erode and migrate landward. These calculations from the photogrammetric analysis of the areal backbarrier shoreline changes indicate that washover and tidal delta sedimentation may be responsible for 62% of the sediment that is eroded from the beaches, and 38% of the sediment being eroded from the beaches is available to be transported offshore in application of the Bruun Rule.

Bruun Rule Offshore Sediment Sink

As a regional long-term test of the Bruun Rule for the Rhode Island barrier coast, the input to the sediment budget includes the amount lost to overwash and inlet sediment sinks. In addition, sea



A.



B.

Fig. 8 - Relationship of supratidal tidal delta accretion (A) and supratidal washover accretion (B) for 1939-1976 period. This accretion represents 62% of eroded beach material.

level rise during this time is also known. However, offshore deposition could not be measured over the 36 years. The test is then to determine whether the amount of eroded beach sediment could be contained in the potential offshore sediment "sink" as limited by the sea level rise.

In determining the volume of this offshore potential sediment sink, in addition to sea level rise, two other dynamic sedimentation parameters must be determined. The first is the point offshore to which this eroded sediment is deposited. A logical conclusion is to assume that, since deposition is ultimately by wave action, effective regional wave base might be considered as the limit of seaward deposition. It is realized that this deposition would not be uniform in this zone but would be less offshore and more nearshore. This sediment deposition prism, however, can be considered as a prism of uniform thickness relative to this sea level rise parameter. A second and equally important parameter is the "fulcrum" point (in profile) location where the shoreface erosion gives over to deposition. Thus the three parameters which govern the potential offshore dimensions of the sediment sink as indicated by the Bruun Rule are as follows:

1. amount of sea level rise
2. limit of offshore deposition (depth)
3. location of fulcrum (erosion vs. deposition).

Rhode Island Bruun Rule Budget

The sediment budget inventory of a shore in testing the Bruun Rule depends in part on placement of the fulcrum point between the erosive and depositional zones. First, if we assume that this fulcrum (F_0 of Fig. 9) is at sea level and that what is eroded above sea level is then deposited offshore below sea level, then problems will develop. Bruun pointed out that a sea level rise relative to an adjacent shoreline would require an offshore profile adjustment (deposition) in order to continuously maintain the same depth of water as sea level rises. This deposition in the nearshore zone would be the vertical equivalent of the amount of sea level rise. Using the sea level rise of 3 mm/yr for the Rhode Island coast (from Hicks and Crosby, 1974) and deposition by wave action to an effective wave base of about nine meters (thirty feet) depth which averages one kilometer distance from shore, the potential sediment deposition per unit length of shoreline, as required by the Bruun model, can be calculated (Fig. 9). The cross-sectional profile area of sediment that could be deposited offshore to maintain a constant water depth as sea level rises is about 3.0 meters^2/year per unit length of shoreline. For the Rhode Island shoreline, with an average beach slope of five degrees and 0.2 meters/year of horizontal erosion, the cross-sectional profile area of beach loss per unit length of shoreline is only 0.002 meters^2/year. Of this beach loss, about 60% is redeposited as "back barrier" sedimentation. Therefore, the volume of the potential offshore sediment "sink" is about 600 times

RHODE ISLAND (1939-1975)

LENGTH 30 KM

TYPE SHORELINE	BAY MOUTH BARRIERS
TYPE SEDIMENT	SAND
AVER. BEACH SLOPE	5°
AVER. SHORE EROSION	0.2 M/YR
AVER. SEA LEVEL RISE	.003 M/YR
ANNUAL LOSS	.002 M^2/YR
WAVE BASE	9 M
WAVE BASE DIST.	1000 M (1 KM)
SEDIMENT "SINK"	3.0 M^2/YR
"UNBALANCE"	1:1500 (LAND/SEA)
BALANCE: SURF	0.5 M (OFFSHORE)
BALANCE: BREAKER	15.0 M (OFFSHORE)
OVERWASH-INLETS	62 % ANNUAL LOSS

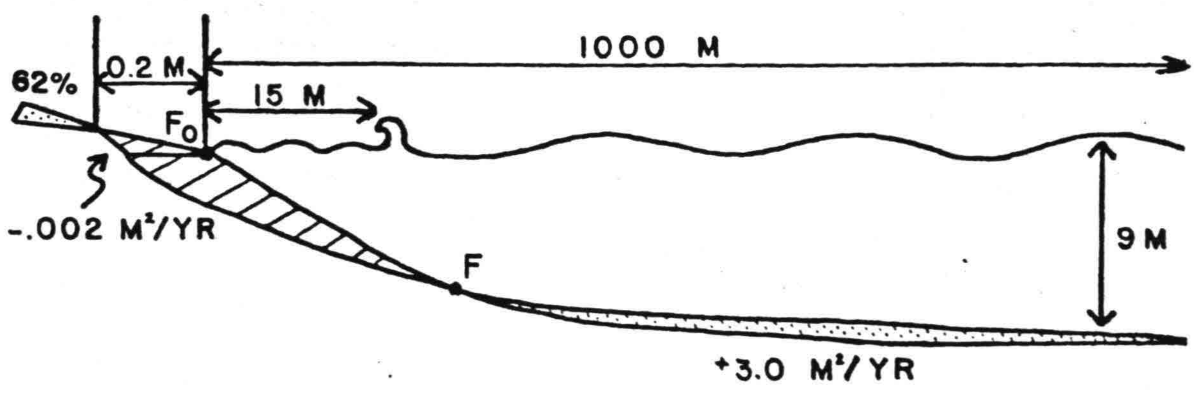


Fig. 9 - Sediment budget inventory of Rhode Island coast for Bruun Rule analysis.

greater (1500 times if no back barrier loss) than the actual eroded material if the fulcrum (F_o) is assumed to be at sea level. Criteria for locating this fulcrum point is important in determining whether the sediment budget will balance as required by the Bruun Rule. It can be seen that, using sea level as the fulcrum (F_o), a large unbalance results; however, this unbalance can be changed to balance by movement of the fulcrum offshore.

The original Bruun diagram does indicate that this fulcrum point is, however, offshore and at depth. Bruun (1962) suggests that for the southeast coast of Florida this "intersection point of the old and new profiles" will be about 135 m offshore and at a depth of 2 m (new profile) or 235 m offshore and at a depth of 3 m (old profile) upon a sea level rise of 1 m. Over 167 years, this rise will produce 100 m of retreat. These values are based on a mathematical formula modified by beach slope for an equilibrium profile. For the Rhode Island coast, with a measured sea level rise of 3 mm/yr, application of the values from the Bruun model gives the following Rhode Island fulcrum and retreat values.

Relationship of Fulcrum to Sea Level Rise and Erosion

	Florida - Southeast (after Bruun, 1962)	Rhode Island - Barriers (Fisher, this paper)
Sea Level Rise:	6 mm/yr	3 mm/yr
Fulcrum-offshore:	135 m	65 m *
Fulcrum-depth:	2 m	1 m *
Shoreline Retreat:	0.6 m/yr	0.3 m/yr *

*calculated after Bruun model

These Rhode Island values above were calculated by ratio from the Bruun, 1962, model for Florida. The actual measured value for Rhode Island shoreline retreat (erosion) was found to be 0.2 m/yr over a 37 year period and close to the calculated 0.3 m/yr value.

The physical factors that control the location of the fulcrum point should be found in the wave dynamics of the system. While it may be expressed above as a factor of sea level rise and shore profile geometry, it must be related to an actual dynamic process in the shoreface sedimentation system. For the short-term Cape Cod study, Schwartz (1967) found that the location of this fulcrum point was within the surf zone (personal communication, 1976). For the Rhode Island coast, this surf zone 5 m offshore does not "balance" the volume of erosion with deposition and thus is not the fulcrum for this long-term study. The erosion/deposition budget can be balanced for the Rhode Island coast with a fulcrum at a distance of 20 m offshore. This fulcrum distance appears to correlate with the breaker zone. Thus it appears that this "inflection point" of the old and new profiles (Bruun, 1962) which is a "fulcrum point" of the erosion and deposition sedimentation units (Fisher, this paper) is probably also the location of the breaker zone.

That this "inflection/fulcrum" point is related to wave dynamics is not unusual since the pre-existing geometry of the offshore profile of equilibrium has to relate to wave dynamics. The subsequent profile developed under sea level rise would also need to be in equilibrium with the existing wave dynamics. Wave breaker zones, however, are not independent of the offshore topography. Combinations of wave parameters and offshore slope determine exactly where offshore the breaker zone will develop. Inherent in this process is a feedback mechanism whereby long-term offshore erosion or deposition can change the offshore slope and thus shift this breaker zone landward or seaward respectively.

North Carolina Bruun Rule Budget

A similar Bruun Rule sediment budget inventory was also developed for the North Carolina coast using the techniques developed for the Rhode Island coast. Fortunately, shoreline changes for a similar time period for that coast had been completed previously. The North Carolina shoreline coast, some 500 km in length, is a barrier island chain shoreline which includes the well-known "Outer Banks" barrier islands as well as several prominent cuspate capes, including Cape Hatteras. Although a classical barrier island shoreline, recent morphological (Fisher, 1967) and subsurface evidence (Pierce and Colquhoun, 1970) indicate that rather than developing on a shoreline of emergence as formerly thought (Johnson, 1919), these barrier islands have developed on a shoreline of submergence primarily by spit growth (Fisher, 1968) although some (Hoyt, 1967) have suggested engulfment of coastal dunes, but again during a sea level rise. Thus the above evidence of short-term (1940-1970) erosion along this coast due to recent sea level rise is compatable with this long-term Holocene geological development on a shoreline of glacio-eustatic submergence.

Using similar photogrammetric techniques for the North Carolina coast to those used for the Rhode Island coast (Langfelder, Stafford and Amein, 1968; Wahls, H.E., 1973), I calculate from their studies that the average annual rate of erosion for the North Carolina coast during the period 1940-1970 was 2.0 m/yr. That this shoreline loss rate is 10 times the Rhode Island coast rate is not surprising. Sea level rise records (Hicks and Crosby, 1974) in this coastal area (nearest Federal tide gauges north at Portsmouth, Virginia, and south at Charleston, South Carolina) indicate a calculated rise averaging 0.4 cm per year during the same 30 years, just slightly more than the Rhode Island Coast. Development of the sediment erosion inventory (Fig. 10) indicates that along the North Carolina coast, for the measured rate of shoreline erosion of 2.0 m/yr, on an assumed average

NORTH CAROLINA (1940-1975)

LENGTH	500 KM
TYPE SHORELINE	BARRIER ISLAND
TYPE SEDIMENT	SAND
AVER. BEACH SLOPE	6°
AVER. SHORE EROSION	2.0 M/YR
AVER. SEA LEVEL RISE	.004 M/YR
ANNUAL LOSS	.02 M^2/YR
WAVE BASE	15 M
WAVE BASE DIST.	10,000 M (10 KM)
SEDIMENT "SINK"	40 M^2/YR
"UNBALANCE"	1:2000 (LAND/SEA)
BALANCE: SURF	5.0 M (OFFSHORE)
BALANCE: BREAKER	20.0 M (OFFSHORE)
OVERWASH-INLETS	? 60 %

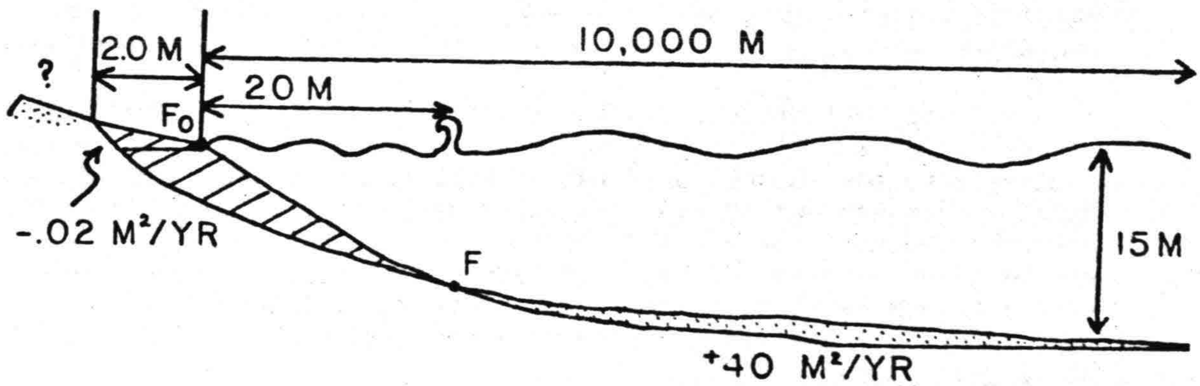


Fig. 10 - Sediment budget inventory of North Carolina coast for Bruun Rule analysis.

beach of 6° slope (estimated), there will be a loss of about .02 m^2/yr per unit length of shoreline. For the measured average rate of sea level rise of 0.4 cm/yr, there should be deposition offshore to a depth of perhaps 10 m (estimated effective wave base) or to a measured distance offshore of about 10 km along this coast. This would give a potential offshore deposition sink of 40 m^2/yr per unit length of shoreline. Submergence of the coast by sea level rise would account for perhaps 20% of the measured shoreline retreat. There is no measured information on the volume of washover and inlet sedimentation for this time period in the studies by Langfelder et al., 1968, but it can be assumed that the values of 60% for the Rhode Island coast is of the correct order of magnitude. Thus, 80% of the eroded beach material can be accounted for by submergence and back barrier sedimentation. The remaining 20% of eroded beach sediments is the amount that is available for offshore deposition. Again, the fulcrum point between the erosion and deposition does not balance the sediment budget if it is assumed to be in the surf zone (5.0 m offshore) but does balance if it is in the breaker zone (20.0 m offshore).

Conclusion

1. The Rhode Island barrier beach shoreline was chosen for a long-term (37 year) sediment budget test of the Bruun Rule. This coast allowed a regional study, with natural boundaries that allow no input or loss of sediment and no river input of sediment.

2. Photogrammetric mapping of aerial photographs from 1939 to 1975 showed that the entire coast, both headlands and barrier beaches, was retreating at an average rate of 0.2 m/yr, measured at the high tide shoreline datum.

3. This shoreline retreat is not due completely to simple submergence. Long-term Rhode Island tidal records indicate that sea level rise during this period average 0.3 cm/yr. Only 15% of the shoreline retreat can be attributed to this amount of sea level rise.

4. Sediment eroded from the beach has not gone to form dunes, since photogrammetric mapping indicates that the duneline has retreated similarly to the shoreline retreat. Photogrammetric mapping of back barrier overwash and inlet tidal delta sedimentation indicates that overwash accounts for 26% and inlet deposition accounts for 35% of the eroded beach material. Thus, for barrier islands, the Bruun Rule must consider back barrier sediment sinks (overwash, inlets, etc), especially as these are features of a transgressing barrier on a rising sea level.

5. Sediment budget for this coast indicates the following losses: (a) submergence - 15%, (b) washover - 26% and (c) inlet deposit - 35%. This accounts for a total of 76% of the eroded beach material. The remaining 24% may therefore be deposited offshore as proposed by the

Bruun Rule.

6. Sediment budget inventory analysis shows that location of the point between beach erosion and offshore deposition (inflection/ fulcrum point) is located in the wave breaker zone, while the offshore limit of deposition is controlled by wave base depth. A similar inventory analysis of the North Carolina coast shows a similar pattern except that all parameters including the scale of erosion (2.0 m/yr vs. 0.2 m/yr) are greater due to the higher energy shoreline.

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AN APPLICATION OF THE BRUUN RULE IN THE CHESAPEAKE BAY

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INTRODUCTION

This application of the Bruun Rule (Bruun, 1962) was part of a regional study of the erodability of the Virginia Chesapeake Bay Shoreline (Rosen, 1978). The study area consists of 350 km of estuarine shoreline in the southern half of the Chesapeake Bay (Fig. 1). Although fetch restrictions result in a low to moderate energy wave climate (Rosen, 1976), there is an extremely high and variable erosion rate ($\bar{x}$ = 0.94 m/yr, Fig. 2). The Bruun Rule was utilized to determine how much of this erosion can be accounted for by the rise in sea level.

METHODS

The Bruun Rule is an onshore-offshore sediment budget. In order to test the role of sea level rise in the model, several assumptions are necessary, including: 1) longshore transport is in equilibrium, and, 2) short-term processes (wind, waves, tides, groundwater effects) are constant through the test period. To quantitatively apply the model either at a single profile or short time period would require adding factors to account for these variations of short-term processes. To circumvent these effects, the model was applied over a long time period (~100 years) and over a large region, so short-term variations would be averaged out.

The data base that exists for the Virginia Chesapeake Bay shoreline is well suited to an application of the Bruun Rule because 1) accurate records of long-term shoreline changes are available, 2) the limit of offshore transport of sediment can be defined by both sedimentological and geomorphic evidence, 3) accurate data exists of rates of sea level rise from two independent sources, and 4) the morphology of the shoreline is clearly defined (Rosen, 1979).

Relative sea level rise is the result of two components: eustatic sea level rise and crustal movements. Since the eustatic rate is assumed to be constant worldwide at about 1.2 mm/yr (Walcott, 1975), variations in rates of sea level rise are a function of crustal movements. The crustal movement component can be estimated by comparison of successive ground surveys tied to a stable datum. Figure 3 shows the regional variations in subsidence rates along the shoreline of the Virginia Chesapeake Bay derived from resurveys of benchmarks (Holdahl and Morrison, 1974). Tide guage-derived estimates incorporate both components of relative sea level rise. By adding the eustatic rate of 1.2 mm/yr to to rate of crustal movements, a comparison can be made. Six of ten tide guage-derived rates (Table 1) fall within one standard deviation of the survey-derived rates. As the recording period on the tide guages is as little as 17 years, this comparison is considered favorable.

The model was applied using the following techniques:

1) The calculations were performed on each of 146 process-constant reaches in the study area, and averaged over the various regions using weighted means.

2) Calculations for beach shorelines were separated from marsh shorelines, as the model loses physical meaning on marsh shorelines.

3) The limit of the nearshore zone was defined at the 3.6 m depth contour. A regional break-in-slope occurs at this depth, which probably represents the limit of the flood plain of the ancestral Susquehanna River. Ryan (1953) showed that sand deposition is confined to the nearshore terraces landward of this slope break.

4) The total vertical distance cut by erosion of the shore is 3.6 m + measured beach elevation + measured bluff height at each reach.

5) The local rate of relative sea level rise was computed for each reach using the subsidence data of Holdahl and Morrison (1974) and adding 1.2 mm/yr for eustatic sea level rise.

DISCUSSION AND CONCLUSIONS

Bruun's model suggests that an area with higher bluffs will erode slower than an area with lower bluffs. When observationally applied to the Virginia Chesapeake Bay, the areas having the highest bluffs on their respective sides of the Bay (Northampton County and Potomac River to Rappahannock River) each have the narrowest nearshore terraces (Fig. 4). In addition, the areas with lowest bluff heights, Accomack County and Mobjack Bay to York County, have the widest nearshore terraces. Since the seaward margin of the nearshore terraces has been proposed as a time-constant shoreline location (Rosen, 1976), these regional trends agree with the Bruun Rule.

Table 2 shows the results of calculations for all beach reaches in the Virginia Chesapeake Bay. The calculated mean shore retreat rate was 0.98 m/yr, which is within 3% of the measured rate of 0.94 m/yr. Some degree of compensating errors is recognized, as the eastern shore prediction was 58% higher than measured and the western shore 7% below measured. However, if the data is segmented by counties, the error increases with increasing amounts of marsh shoreline in the area and improved on dominantly beach shorelines.

It appears that the concurrance of the field data with estimates from the Bruun Rule demonstrate that the relative sea level rise can account for the high erosion rates along the Virginia Chesapeake Bay shoreline. However, sea level rise itself does not cause the translation of sediment associated with shore erosion. The action of short-term processes (waves, tide, surge, groundwater effects) are the agents effecting this erosion in the presence of a rise in sea level. Sea level "...plays only a permissive role in coastal erosion, not a causitive one." (Davis et al, 1973).

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TABLE I

Estimated rates of relative sea level rise from tidegauge data*

Location	Record length (years)	Trend (mm/yr)	Standard error	Variability
Gloucester Point, Va.	17	2.09	1.130	22.85
Old Point Comfort, Va.	15	5.43	1.280	26.01
Kiptopeke, Va.	22	3.93	0.877	26.08
Richmond, Va.	25	−0.46	2.120	76.46
Portsmouth, Va.	38	3.93	0.396	26.87
Hampton Roads, Va.	47	4.68	0.323	29.95
Solomons Island, Md.	34	3.86	0.430	24.87
Washington, D.C.	43	3.40	0.430	33.20
Baltimore, Md.	71	3.42	0.152	26.35
Annapolis, Md.	44	4.26	0.287	24.36

*Tidegauge data was reduced using the technique of Hicks and Crosby (1974).

TABLE II

Application of the Bruun model to the Virginia Chesapeake Bay all beach environments

Location	Measured erosion (weighted means) (m/yr)	Predicted erosion by Bruun model (weighted means) (m/yr)	Error (%)	Number of Reaches
Virginia Chesapeake Bay	0.94	0.98	+ 3	146
Eastern Shore	0.87	1.38	+ 58	57
Western Shore	1.03	0.95	− 7	84
Counties				
Northampton	0.88	0.76	− 13	39
Accomack	0.85	2.77	+224	18
Northumberland	1.08	0.63	− 41	35
Lancaster	1.49	0.76	− 48	12
Middlesex	0.74	0.76	− 2	6
Mathews	1.13	0.88	− 22	12
Gloucester	0.57	1.06	+ 85	3
York	0.64	1.92	+199	10
Hampton	0.97	0.35	− 64	6
Norfolk/Virginia Beach	0.76	0.24	− 68	5

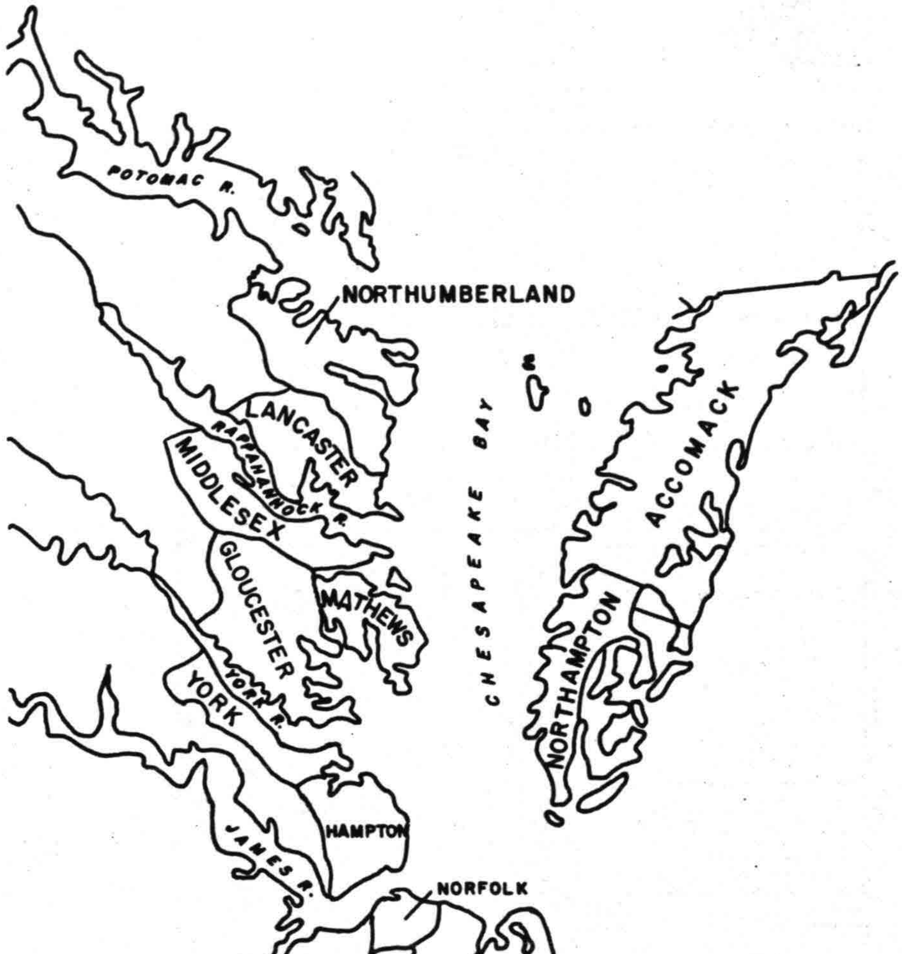


Figure 1. Map of the Virginia Chesapeake Bay showing counties and major rivers.

FIGURE 2

VIRGINIA CHESAPEAKE BAY SHORELINE EROSION-ALL REACHES

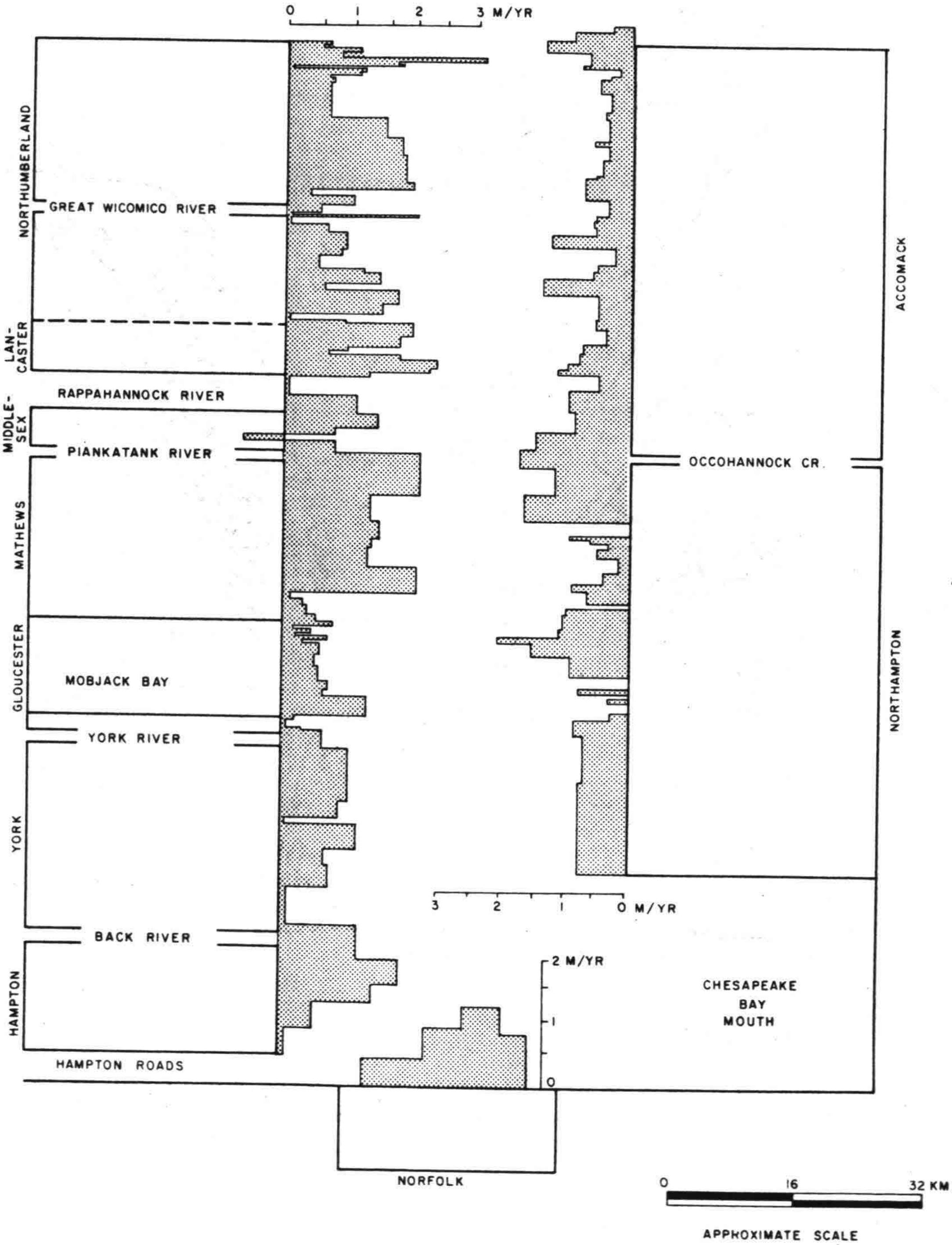
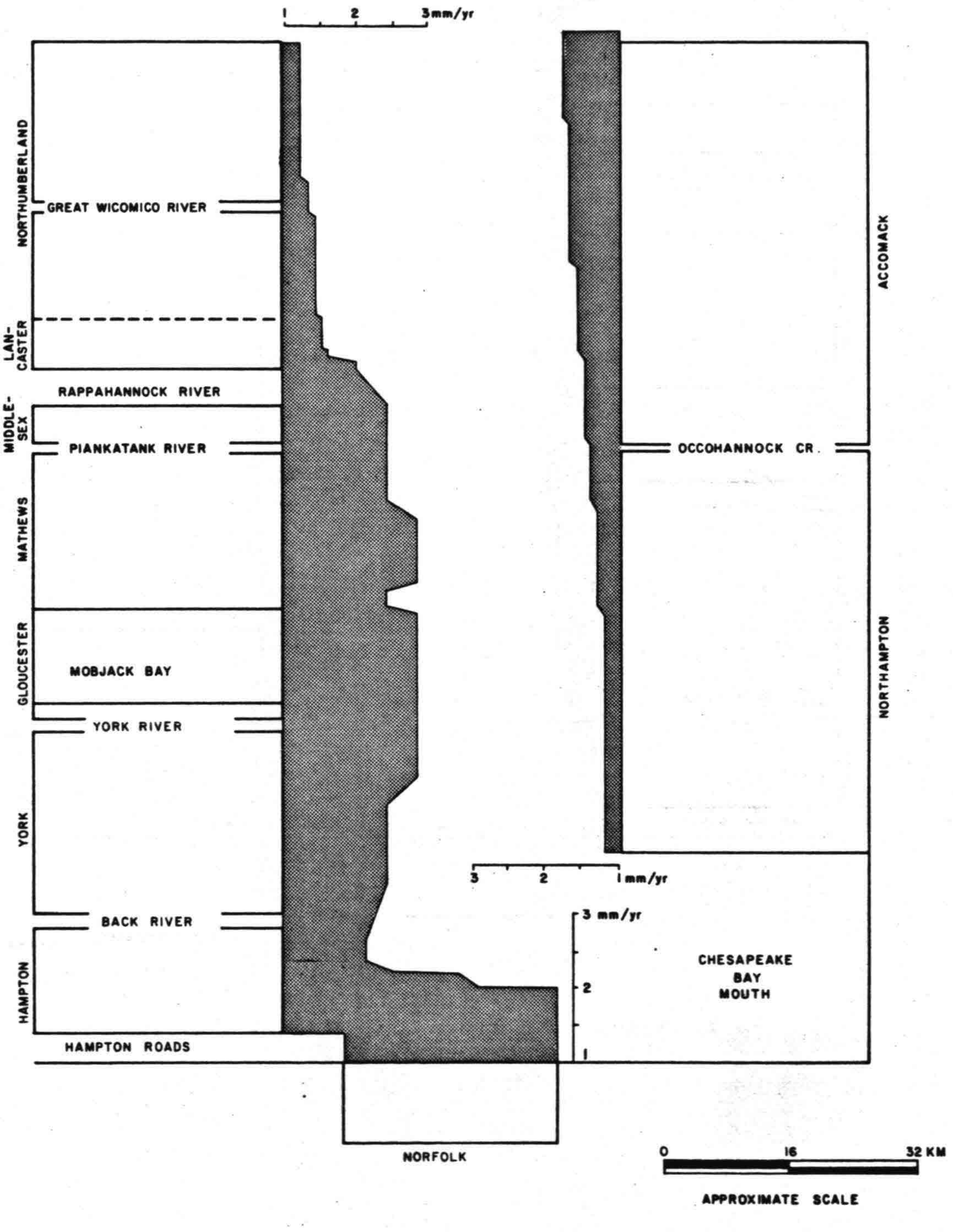


FIGURE 3

VIRGINIA CHESAPEAKE BAY
LOCAL SUBSIDENCE ALONG SHORELINE



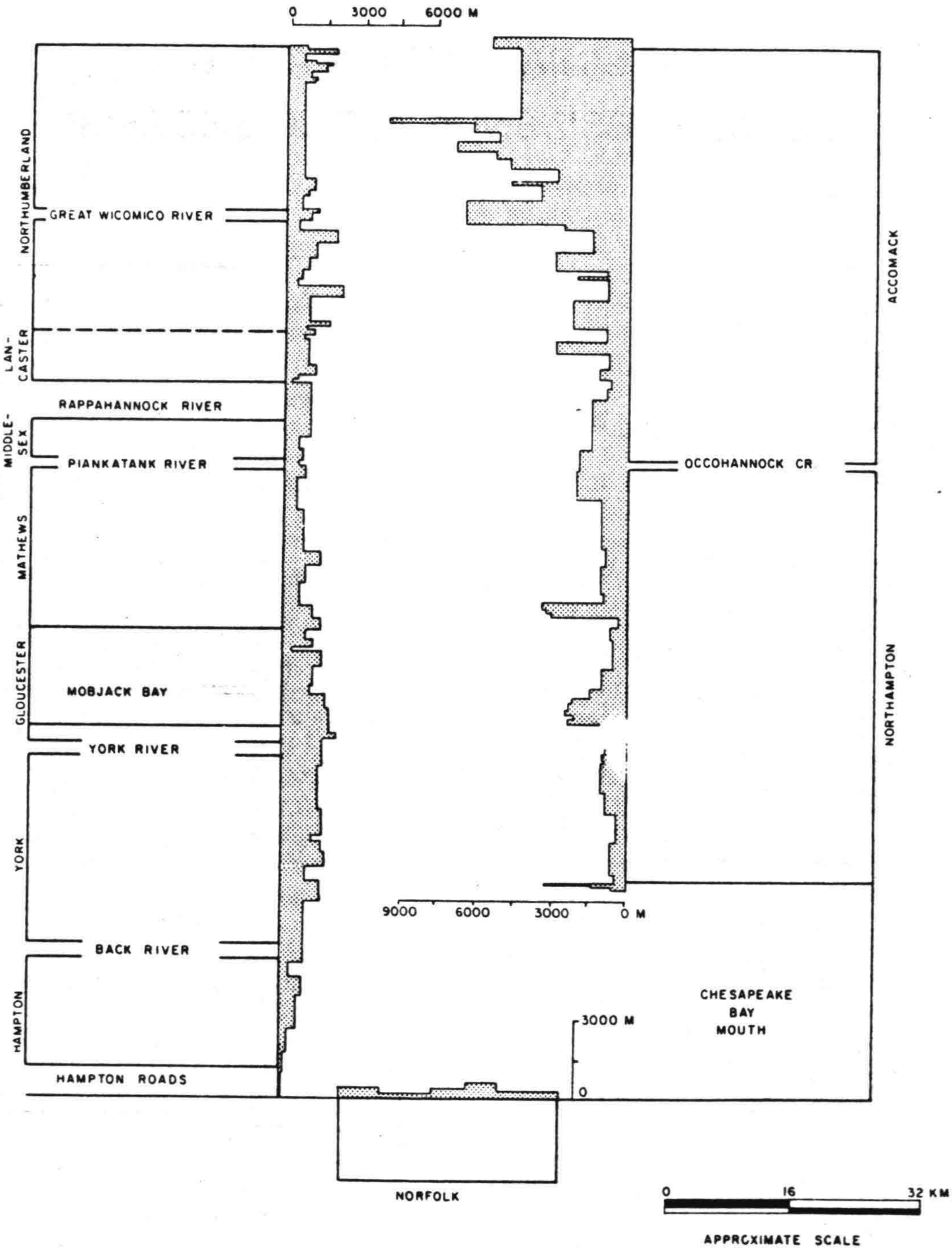


Figure 4. Nearshore terrace widths along the Virginia Chesapeake Bay shoreline.

BRUUN'S CONCEPT APPLIED TO THE GREAT LAKES

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MODEL. As described by Bruun (1962) a rise in sea level tends to shift the equilibrium shore profile upward and landward. Erosion prevails landward of a null point, and the shoreline retreats (Fig. 1). Conceptually, this erosion supplies material to the outer portion of the responding profile and the entire profile is thus raised above the initial equilibrium profile through a distance equal to the change in water level. The landward distance through which the profile moves to regain equilibrium can be calculated if the width of the entire responding profile, and the proportion of eroded material which would be stable in the outer zone, are known. This concept is straightforward, but application is difficult. The assumption of an equilibrium profile may be unrealistic. The width of the responding profile, or equivalently the closure depth (Fig. 1), is an unknown variable, dependent on wave climate. Field verification of profile response has been scanty. While coastal submergence is widespread, it is also usually so slow that few measurements span a long enough time to accurately reveal profile adjustment. Furthermore, the transition between the elevated equilibrium profile and the stable seabed below closure depth was never shown in previous diagrams of ideal profile response. Some undue skepticism about the model's validity may have arisen because of the apparent inadequacy of the model for explaining this transition (Fig. 1).

FIELD CONFIRMATION. The Great Lakes are subject to sustained periods of relatively rapid changes in water level as a result of long term climatic fluctuations. During periods of increasing lake levels, which last from 5 to 15 years, average shore erosion rate increases 3 to 6 times the historic (100+ year average) rate. Response of Lake Michigan shore and offshore profiles to increased water levels has been documented over a 9-year period. The closure depth is near 11 m (Fig. 2A). A system of 4 to 5 longshore bars dominate the active profile. The bars respond relatively rapidly, migrating to maintain a constant depth beneath the rising lake surface (Fig. 2B). The inner profile recedes steadily, but is relatively sluggish compared to bar mobility (Fig. 2C). There is a timelag of several years between lake level stabilization and complete profile readjustment. Though the mean lake level peaked in 1973, recession continued unabated at most stations until 1976 at which time the beach began to accrete (Fig. 2D).

FURTHER IMPLICATIONS. Due to the absence of any active supply of fluvial sediment and the closed nature of the 50 km shoreline under the study, sediment budget calculations could be readily applied to document sediment balance. The ratio of shore retreat to submergence over the 9-year study period was roughly 1 to 70. For each unit increase in elevation, the profile moved 70 units landward leaving a smooth 1:70 slope behind. Thus a smooth bottom marked the transition between the raised equilibrium profile and inactive bottom (Fig. 3). A concave trailing edge would have suggested a secular increase in profile retreat per unit of submergence. In areas with similar geology, geomorphology, and wave exposure roughly similar responses may be expected. In areas having broader active profiles, lower backshores, offshore or longshore sediment sinks, and where the eroding backshore contains a large percentage of material which could be unstable as a nearshore deposit, the ratio of profile retreat to submergence should be larger. The closure depth can be estimated from wave climate data as roughly twice the height of waves with a 5 year return period. A recommendation that shore retreat be evaluated in terms of a sediment balance model similar to Bruun's (Hands, in prep.) should lead to rigorous testing of these concepts on the Great Lakes.

ACKNOWLEDGEMENT. This work was carried out under the coastal engineering research program of the U.S. Army Corps of Engineers. Permission to publish is appreciated.

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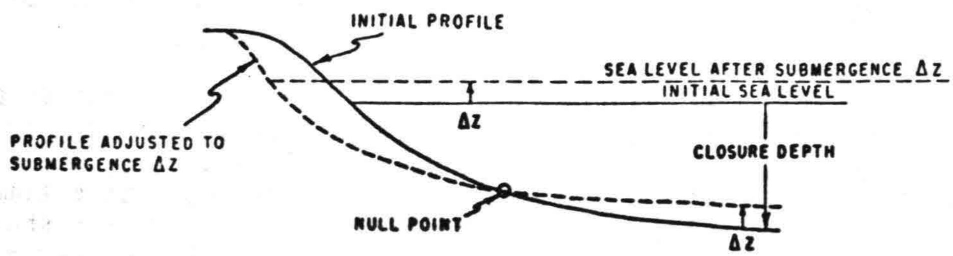

Figure 1. Concept of Profile Adjustment to Increased Sea Level (after Bruun, 1962).

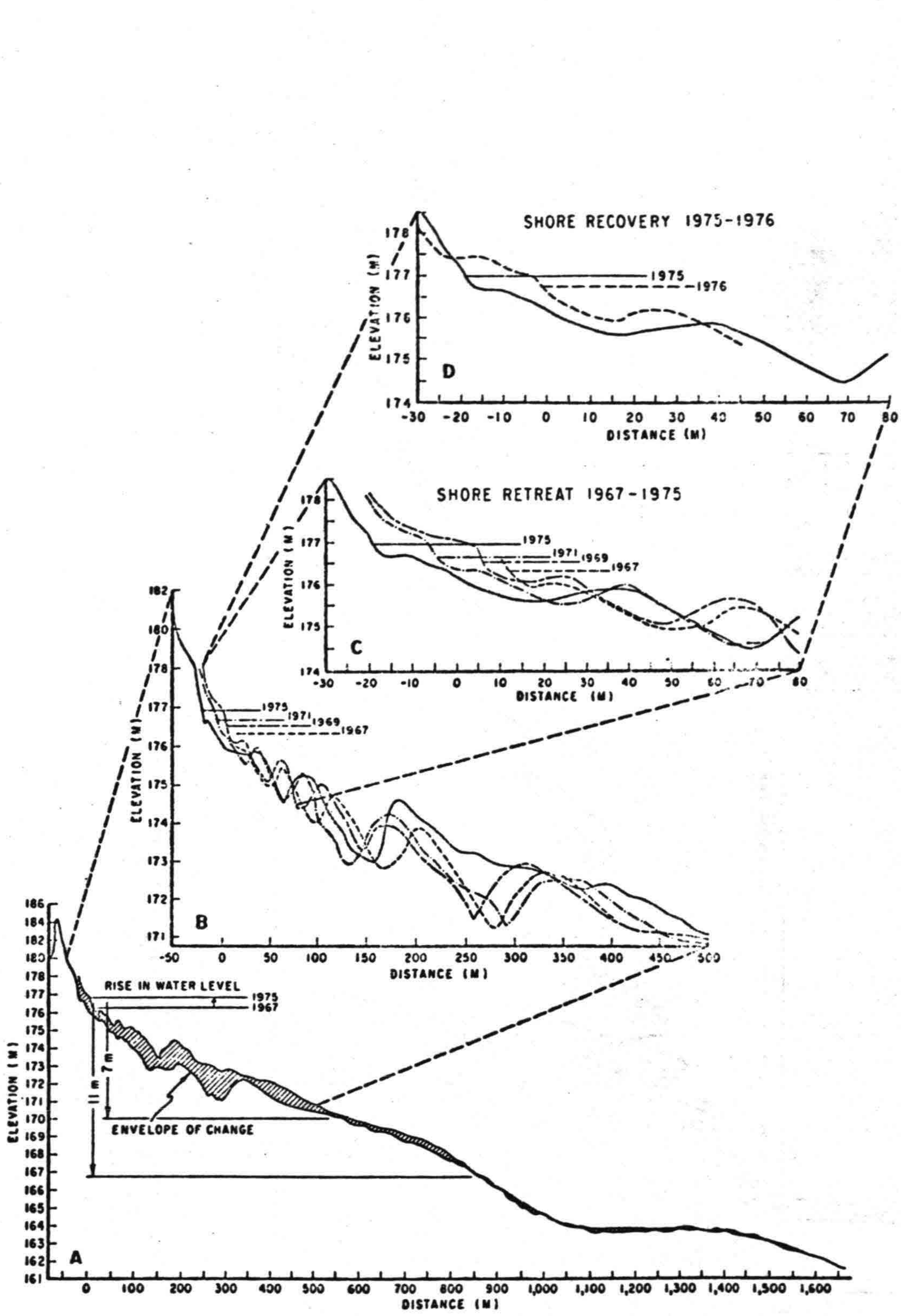

Figure 2. Measurements of actual profile adjustments over a 9-year period of increasing, then stable mean lake levels. The example above comes from one of 34 sites monitored on Lake Michigan.

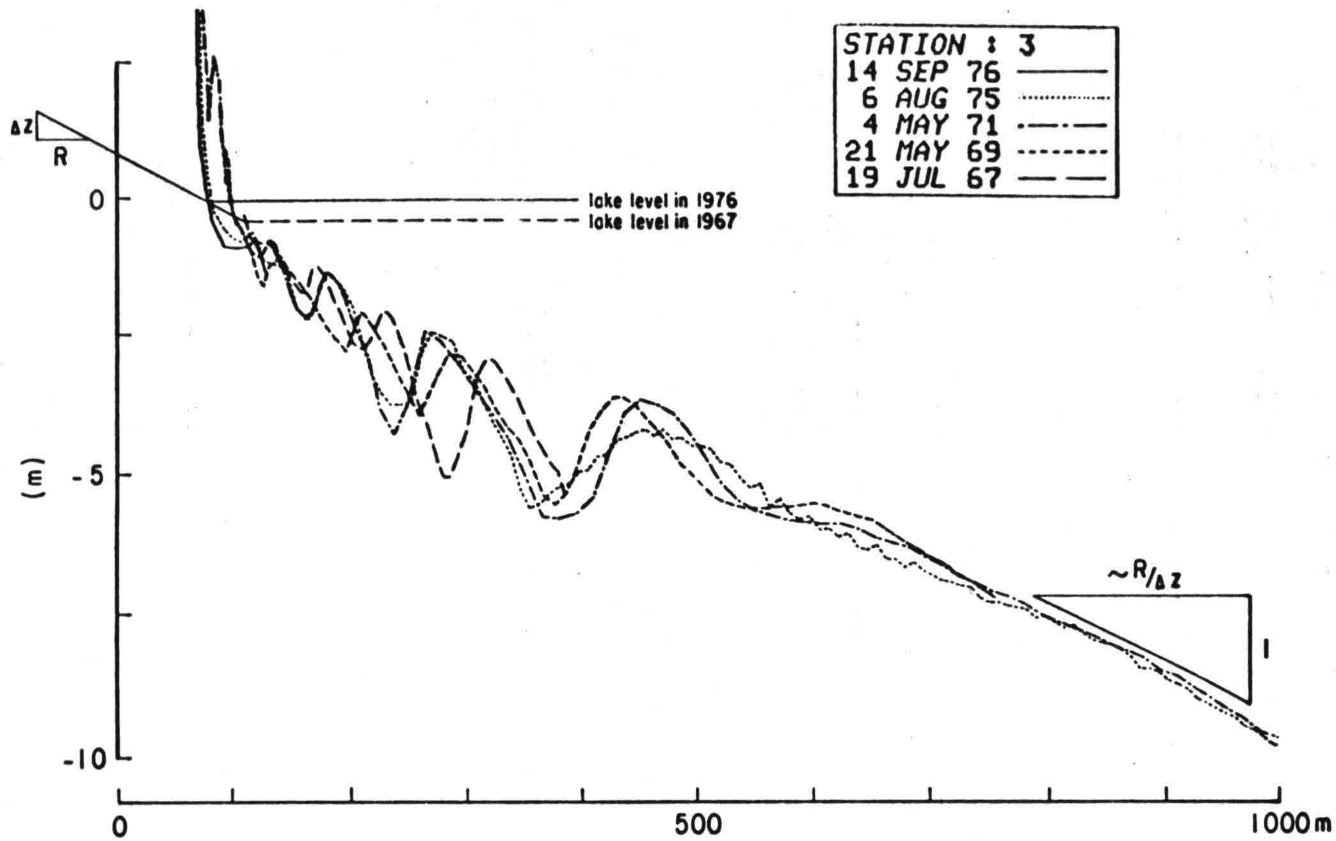

Figure 3. As the profile retreats R units in response to an increase, Δz, in lake level it will tend to leave a "trailing edge" offshore with a slope of Δz in R. The trailing edge merges gently with a similarly sloped area on the outer part of the responding profile. Thus the boundary between the responding and static portions of the profile need not be marked by any abrupt change in slope, as illustrated above with actual profiles from another of the Lake Michigan profile sites.

ENIGMA OF THE BRUUN'S FORMULA IN SHORE EROSION

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1. Introduction

Sound experimental support for the Bruun's theory of shore erosion due to sea-level rise (Bruun, 1962); Schwartz, 1965, 1967; Dubois, 1975, 1976, 1977; Rosen, 1978b) makes it necessary to have a rigorous analysis of the theory, because a significant controversy exists between the different interpretations of the theory as such, and the variables involved.

The theory as formulated by M. Schwartz (1967), comprises the following:

1. Change in sea-level causes translation of the transverse beach profile whilst retaining its original shape.

2. A rise in sea-level causes shoreward displacement of the beach profile.

3. The material eroded from the upper beach is equal in volume to the material deposited on the nearshore bottom.

4. The rise of the nearshore bottom is equal to the rise in sea-level.

5. The relationship between sea-level rise a and shoreward displacement s of the beach profile is given by the formula (Bruun, 1962):

$$a = \frac{hs}{l} \qquad (1)$$

where: l is the length of the transverse profile,
h is the profile height, being the sum of sea-depth at the distance l from the shore and the shore elevation above the sea level (Fig. 1).

It was the formula (1) which obviously caused dissatisfaction amongst some of the most active proponents of the Bruun's theory. Thus, Dubois (1977) quotes the following formula (in the notations of Fig. 1):

$$a = \frac{ds}{c} \qquad (1a)$$

while Rosen (1978[b]) cites the Bruun's formula as:

$$a = \frac{hs}{c} \tag{1b}$$

Both these formulae, although being claimed to be the Bruun's formulae, are obviously different from the original Bruun's formula (1). Presence of the distance c in both (1a) and (1b) is seemingly quite understandable, because if one wants to calculate the volume of eroded sediment and equate it to the volume of deposited sediment, according to the point (3) of the above formulation of the Bruun's theory, then the distance c must be involved (Fig. 1).

However, to know the distance c one must know the position of the point z of intersection of the old and new profiles. As position of this point z depends on the initial shape of the profile, it follows inevitably, the profile shape is needed to be known for use of (1a) and (1b). This contradicts completely to the Bruun's theory formulated above, for which the original shape of the beach profile is not required.

In fact, Dubois (1977) went still further in this direction, claiming that the angle of seaward slope of the nearshore bar should be involved in the Bruun's theory. The angle is, of course, just one of the parameters of the bottom profile which, consequently, is claimed by Dubois to be required for use of the Bruun's theory. This controversy is of major importance to the application of the Bruun's Rule, the main question being: does the Bruun's theory depend on the profile shape or not? If it does, it makes the application of the theory very difficult, because in each case one must know the beach profile to calculate the shore recession due to sea-level rise. On the other hand, if the Bruun's theory is invariant on the beach profile, this makes it a universal principle of shore erosion.

It is shown in this paper that the last statement is nearly correct. Namely, in §2 we demonstrate, how the Bruun's formula might seemingly have been derived from the above statements 1-4. However, in §3 it is proved that it is, in reality, another formula, which follows from the statements; the Bruun's formula being a zero-order approximation to the exact formula derived. Accuracy of the Bruun's formula is assessed. Effect of shoreward and seaward slopes of the profiles is considered.

2. Heuristic Derivation of the Bruun's Formula

Taking into account the p.(1-4) we may formulate the problem as follows.

A certain function $f(x)$, describing the profile shape, is lifted up to a value a, equal to the rise in sea-level and shifted in the positive x-direction (shorewards) to a value s.

A relationship must be found between a and s which provides equal volumes of erosion and deposition (Fig. 1).

Let us consider the effects of the profile shift and lift separately. Assume initially the monotonous function $f(x)$ which is shifted onshore (in the positive x-direction) to the value s (Fig. 2a). The shifted function may be written as $f(x-s)$.

The area labelled "erosion" is the difference between the areas under the curve $f(x) + AB$ and the curve $f(x-s)$. (Here "under" means between the curve and the x-axis).

These areas A_1 and A_2 are given correspondingly by the integrals:

$$A_1 = \int_0^l f(x)dx + hs \tag{2a}$$

$$A_2 = \int_s^{l+s} f(x-s)dx = \int_0^l f(x)dx \tag{2b}$$

The equality in (2b) can be proved by a change of variable. Substracting (2b) from (2a) results in the eroded area Δ_E being equal to the area of the rectangle ABCD:

$$\Delta_E = A_1 - A_2 = hs, \tag{3}$$

where h is the maximum profile height (at the point $x=l$) and s is the profile shift.

Let us note now, that the assumption that the function $f(x)$ is monotonous, was not used in the derivation of (3). The Fig. 2b shows the case, when the function $f(x)$ is not monotonous; in fact it has a hump, which may imitate an offshore bar. The derivation given by equations (2) and (3) holds also for this case, but the area of the rectangle ABCD now represents the difference between erosion and deposition areas (Fig. 2b).

Hence, we may conclude that due to the profile shift in the onshore direction there is an erosion or at least a net balance between erosion and deposition which, according to equation (3), is independent of the initial shape of the profile $f(x)$.

Consider now the effect of lift of the profile by the value $a(s)$ which, according to the Bruun's Rule (p. 4) equals the sea-level rise. From the Figs. 3a and 3b it is seen that independently of the profile shape, only deposition may occur and the area Δ_D of deposition is given by:

$$\Delta_D = \int_0^l [f(x) + a(s)]dx - \int_0^l f(x)dx = la(s). \tag{4}$$

According to the Bruun's Rule (p. 3), the volume of erosion must be equal to the volume of accretion. Hence, from (3) and (4) we obtain:

$$hs = la(s), \tag{5}$$

It follows from (5) that

$$a(s) = \frac{hs}{l}; \tag{5a}$$

coinciding with the formula (1).

It is worth noting that neither the shape of the profile $f(x)$, nor the point of intersection of the new and old profile, or the position or seaward angle slope of the offshore bar (Fig. 2b) were used or needed for derivation of the formulae (5) and (5a).

The aim of the above "derivation" was to demonstrate how one can arrive to the Bruuns' formula (1) in principle, starting from the statements (1-4).

However, the "derivation" is faulty. We show below, that the formula entirely different from (1) follows in fact from the statements (1-4).

3. *Rigorous derivation of the relationship between sea-level rise and shore recession.*

The following derivation is based on and only on the statements (1-4).

Let us consider the integral I, which represents the difference between areas of erosion and deposition when the arbitrary function $f(x)$, with the domain of definition $-s<x<l+s$ is shifted by the value s and lifted by the value $a(s)$:

$$I = \int_0^{l+s} \{f(x) - [f(x-s) + a(s)]dx = I_1 - I_2 - I_3, \tag{6}$$

where $$I_1 = \int_0^{l+s} f(x)dx; \quad I_2 = \int_0^{l+s} f(x-s)dx; \quad I_3 = \int_0^{l+s} a(s)dx.$$

By a change of variables the integrals may be rewritten as:

$$I_1 = \int_0^{l+s} f(x)dx = \int_0^{l} f(x)dx + \int_l^{l+s} f(x)dx = \int_0^{l} f(x)dx + \int_0^{s} f(x+l)dx \tag{7a}$$

$$I_2 = \int_0^{l+s} f(x-s)dx = \int_{-s}^{l} f(x)dx = \int_0^{l} f(x)dx + \int_{-s}^{0} f(x)dx = \tag{7b}$$

$$= \int_0^{l} f(x)dx + \int_0^{s} f(-x)dx.$$

$$I_3 = \int_0^{l+s} a(s)dx = (l+s)a(s). \tag{7c}$$

Combining the expressions 7a, b and c according to (6) it is seen, that the integrals $\int_0^{l} f(x)dx$, being the effect of a beach profile shape between the points 0 and l, cancel each other and:

$$I = \int_0^{s} [f(x+l) - f(-x)]dx - (l+s)a(s). \tag{8}$$

According to the Bruun's theory (p. 3), the value I should equal zero and the final rigorous expression for the relationship $a(s)$ follows:

$$a(s) = \frac{\int_0^{s} [f(x+l) - f(-x)]dx}{l+s} \tag{9}$$

It is seen from (9) that $a(s)$ is independent on the profile shape between the points 0 and l. It nevertheless depends on the behaviour of the function $f(x)$ at the intervals $-s<x<0$ and $l<x<l+s$ just outside of the interval $[0,l]$.

It is clear now why the heuristic derivation of the Bruun's formula in the previous paragraph was incorrect: because behaviour of the function $f(x)$ beyond the interval $[0,l]$ was neglected.

There exist classes of profiles, for which the exact expression (9) can be greatly simplified.

A) Let us consider one class of profiles, which are arbitrary between the points $[0,l]$ and horizontal beyond these points:

$$\left.\begin{aligned} f(x+l) &\equiv f(l) = h \\ f(-x) &\equiv f(0) = 0 \end{aligned}\right\} \text{ for } 0 \le x \le s. \tag{10}$$

(Notice that the assumption $f(0) = 0$ does not cause any loss of generality, because the coordinate system can always be adjusted to satisfy this condition).

Under the condition (10) the integral in (9) can be easily calculated, being in fact equal to hs and the simple formula is obtained:

$$a(s) = \frac{hs}{l+s} \tag{11}$$

One can see, that the formula (11) reduces to the Bruun's formula (1) when the shore recession s is small, comparing to the profile length l.

B) Let us consider another class of profiles, which again are <u>arbitrary</u> between the points $[0,l]$, but have extensions beyond the terminal points as linear functions:

$$\begin{aligned} f(-x) &= \alpha x \quad &&\text{- seaward} \\ f(x+l) &= h + \beta x \quad &&\text{- shoreward.} \end{aligned} \tag{12}$$

The parameter α is, in fact, the seaward slope of the profile, while β is the shoreward slope.

Substitution of (12) into the rigorous formula (9) gives:

$$a(s) = \frac{1}{l+s} \int_0^s [h + \beta \quad + \alpha x] dx =$$

$$= \frac{1}{l+s} \quad [hs + (\beta + \alpha) \frac{s^2}{2}] \tag{13}$$

It is seen, that the seaward slope α is accompanied in the formula (13) by the corresponding shoreward slope β and <u>both</u> slopes play equal role, hence Dubois' (1977) point of view of exclusive role of seaward slope is incorrect.

It is seen further, that for small s the second term in (13), containing s^2, can be neglected in comparison with the term hs and then (13) reduces to (11), from which, in turn, the original Bruun's formula (1) follows for small ratio $s/l \ll 1$.

We give below a numerical example to compare the relative accuracies of formulae (1), (9) and (11).

4. *Accuracy of the Bruun's Formula.*

Consider the profile which was described (P. Bruun, 1962) by the analytical expression:

$$y^{3/2} = px. \tag{14}$$

In our notations this should be rewritten as

$$f(x) = (px)^{2/3}, \tag{14a}$$

where p = 0.04 and the profile length l = 2000 m. For the beach profile recession s = 100 m as given by Bruun, we calculate the corresponding values of $a(s)$, using the Bruun's formula (1), formula (11) and, for the comparison, the exact value by use of the equation (9). Taking the value $h = f(l) = (0.04 \cdot 2000)^{2/3} = 18.56$ m, we obtain:

a) From the Bruun's formula (1):

$$a_1 = \frac{hs}{l} = \frac{18.56 \cdot 100}{2000} = 0.928 \text{ m} \tag{15}$$

b) From the formula (11):

$$a_2 = \frac{hs}{l+s} = \frac{18.56 \cdot 100}{2000+100} = 0.883 \text{ m} \tag{16}$$

c) From the exact formula (9), using the eq. (14a):

$$a_3 = \frac{\int_0^s \{[p(x+l)]^{2/3} - [p(-x)]^{2/3}\}dx}{l+s} = \frac{F}{l+s} \tag{17}$$

Denoting the integral in the numerator in (17) as F, one can write:

$$F = p^{2/3} \int_0^s [(x+l)^{2/3} - (-x)^{2/3}]dx =$$

$$= p^{2/3}[\frac{3}{5}(x+l)^{5/3}|_o^s - \frac{3}{5}(x)^{5/3}|_o^s] = \quad (18)$$

$$= p^{2/3}\,\frac{3}{5}[(s+l)^{5/3} - l^{5/3} - s^{5/3}].$$

Substituting the given values l = 2000 m, s = 100 m, p = 0.04 the numerical value F = 1737 m^2 is obtained. Hence, the exact value of sea-level rise, corresponding to s = 100 m is

$$a_3 = \frac{1737}{2100} = 0.827 \text{ m}. \quad (19)$$

The Bruun's formula (1) gives + 12.2% and the formula (11) + 6.7% error in respect to an exact value a_3. It is obvious, that for any practical purposes these errors can be neglected.

5. *Conclusions*

1. The formulae (1a) and (1b) by Dubois (1977) and Rosen (1978b) are incorrect.

2. The use of seaward slope alone of the offshore profile as in Dubois (1977) is inadequate in principle. The shore-ward slope of the profile in combination with seaward slope as in (13) must be used in the Bruun's theory.

3. The exact formula (9) is derived from the statements (1-4) of the Bruun's Rule. There is no need to know the point of new and old profiles intersection or indeed any detail of the bottom profile except its seaward and shoreward behaviour to use the exact formula (9).

4. For practically most interesting cases of arbitrary bottom profile with known slopes of seaward and shoreward ends the exact formula (13) follows from (9). When the ends of the profile are horizontal, the simple formula (11) can be used.

5. The Bruun's formula (1) does not follow rigorously from the statements 1-4 of his theory. However, it follows from (9) as a particular case for small ratio s/l, where s is the shore recession and l is the profile length. For realistic profiles the Bruun's formula (1) gives high accuracy, in comparing to results given by the exact formula (9).

6. For small ratio $\frac{s}{l} << 1$ there is, consequently, no need to know any detail of bottom profile at all and Bruun's formula (1) correctly reflects this, not containing any parameters, describing the shape of bottom profile. Hence, the value h/l in formula (1), (having nothing to do with the slope of the bottom profile) is, to a zero order approximation, an invariant, valid for any profile shape for calculation of the ratio a/s. It is proposed therefore, that this value h/l be known as Bruun's Invariant.

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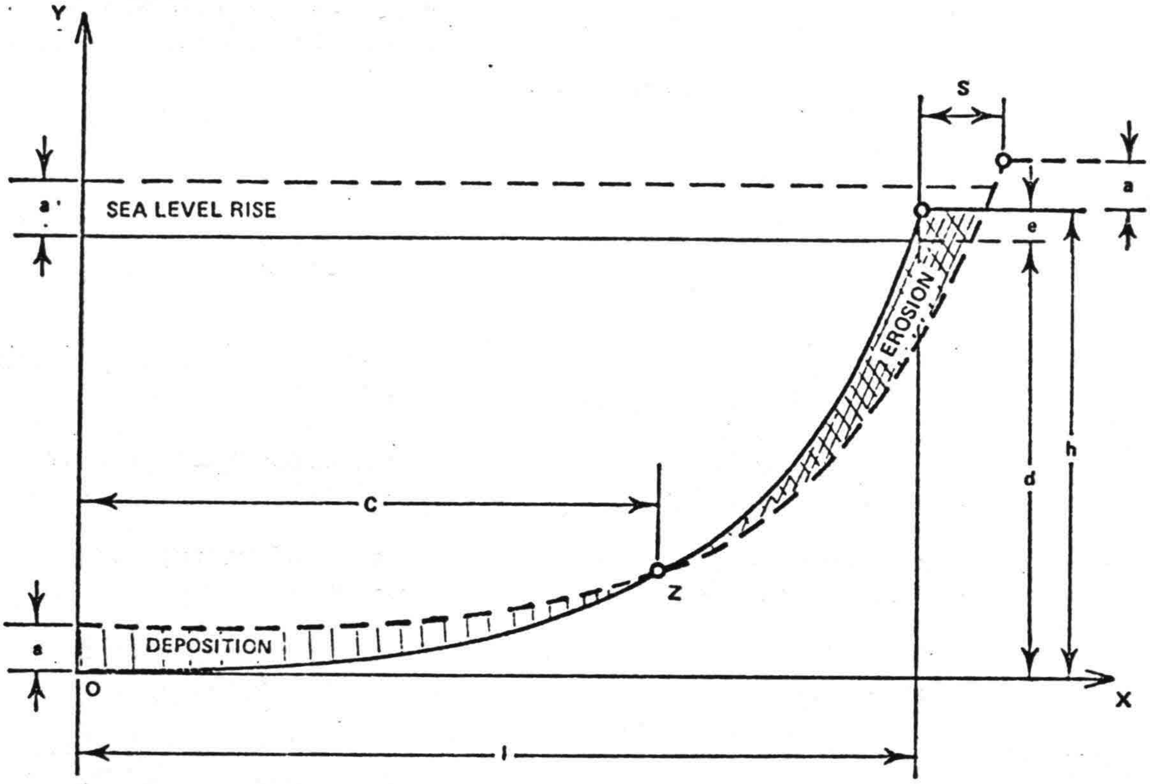

Fig. 1. The Bruun's Effect - translation of the beach profile, resulting in shore erosion and deposition of sediments.

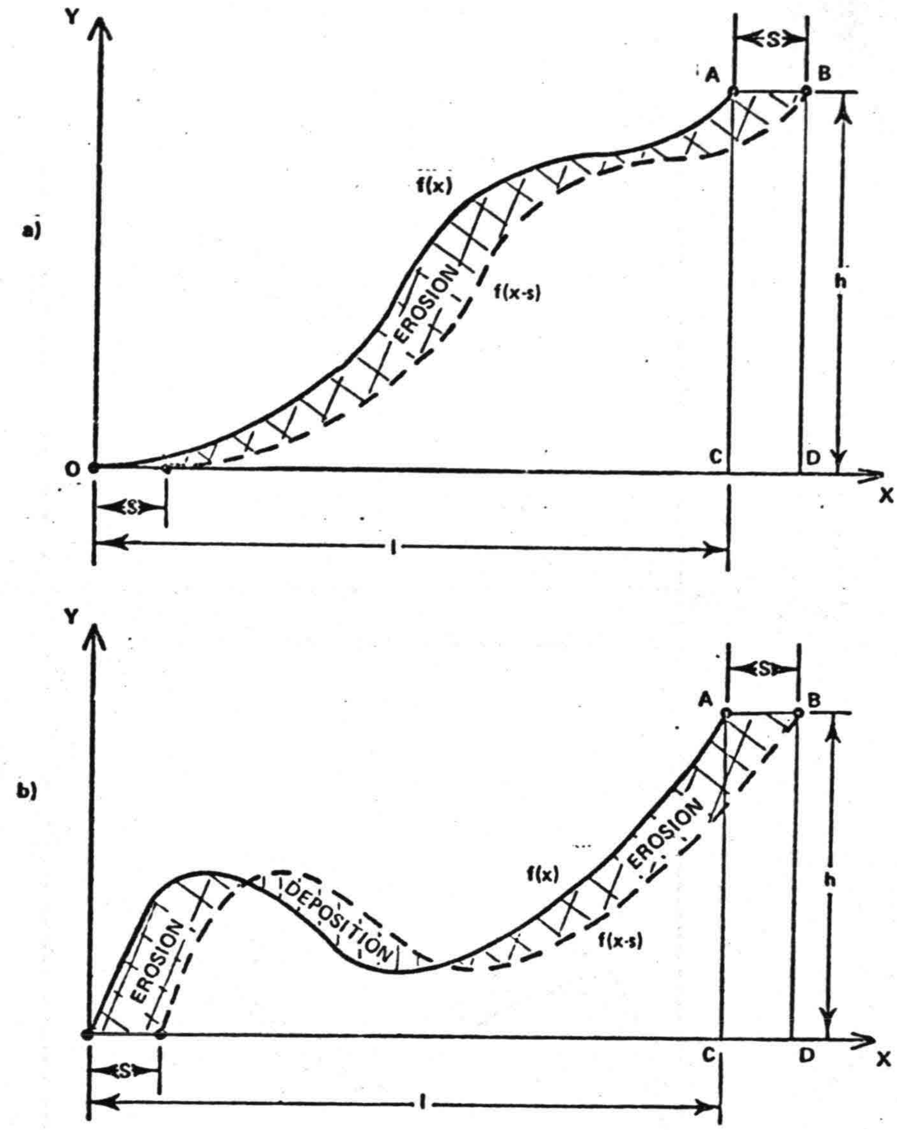

Fig. 2. Profile *f(x)* shifted horizontally in the onshore direction to the value *S*.

a) Pure erosion in the case of monotonous profile. Area of erosion equal the area of the rectangle ABCD.

b) Non-monotonous profile - presence of an off-shore bar. Area of erosion minus area of deposition is still equal to the area of the rectangle ABCD.

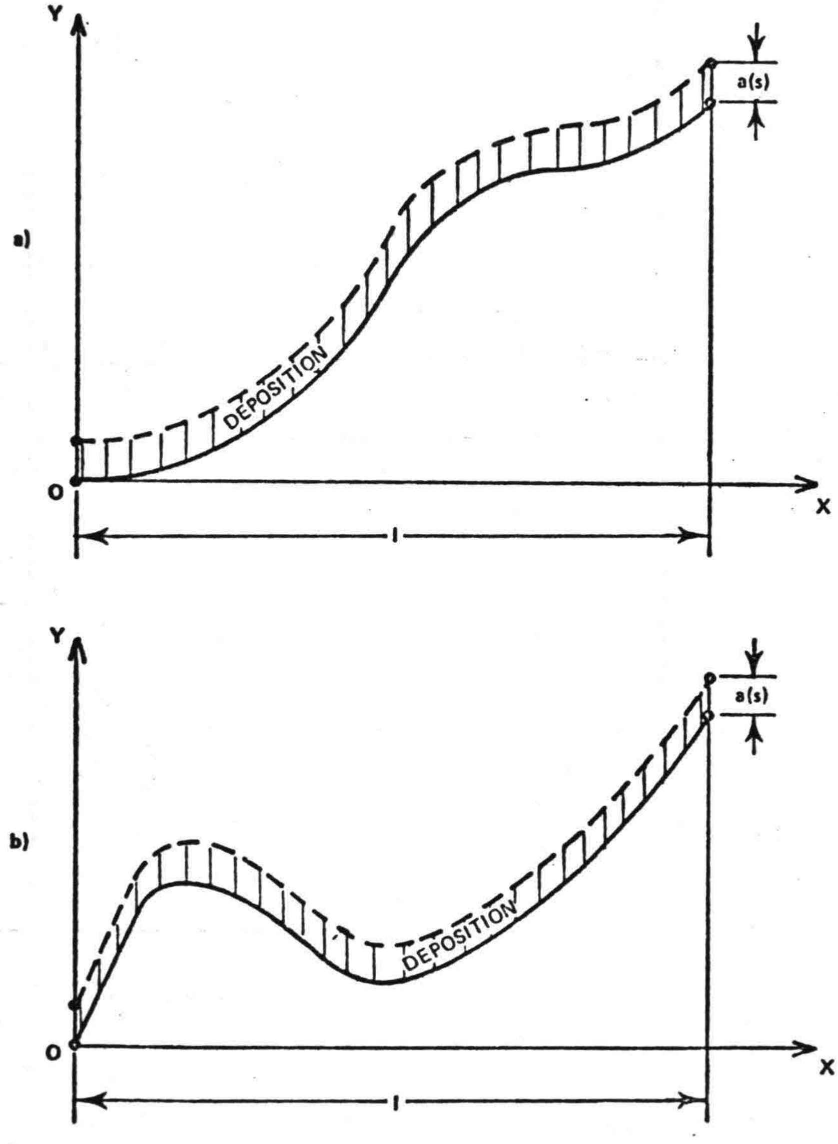

Fig. 3. Profile $f(x)$ is lifted vertically to the value a, which is a function of S. In both cases (a and b) only deposition occurs.

THE "BRUUN RULE"
DISCUSSION ON BOUNDARY CONDITIONS

by P. Bruun

The theory of the influence of sea level rise on erosion is proposed two dimensional but nature is 3-dimensional. This in turn means that one must consider a certain uninterrupted length of shore when the material transport is contained in a "box", xyz, x (length offshore from a defined shoreline point), y (length of box along the shore) and z (depth from a defined water table). The material balance in and out of two remote x-z sections y meters apart is assumed to be zero. If there is no balance between the two quantities this must be considered in the total material balance equations. There are two y-z sections, one located on the beach, the other at a certain water depth which separates the "nearshore drift" from the "offshore drift" in the x-direction. On the beach the effect of wind-drift may then have to be added. It is usually negligible but may in some cases present a non-negligible quantity. The outer y-z boundary is more difficult to define because there is no clear distinction between the limit of exchange between beach and offshore drifts of material. Often the terminology "wave base" for material agitation on the bottom is used. Recent research by the USCE, CERC, reported in TP-9 by Hallermeier in paper presented at this symposium attempt to present rational methods for calculation of limits of active agitation of the sandbottom and long-term erosion rates based on long term rises of sea level.

The wave base defined is closely related to the capacity or limit of the wave action in agitating the bottom material "actively" which in turn also depend upon the time interval considered. A five-year wave base obviously is not the same as a "figty-year-base". To this comes the problem of diffusion of material along the bottom and other boundary layer phenomena. The original theory, however, is based on a quantitative balance between beach

- nearshore and offshore and it will of course be a certain transition area between "nearshore" and "offshore" adjustments to a change in sea level. If the physical forces situation does not change the nearshore area the equilibrium-balance condition apparently will be maintained. But the situation may be different in the offshore area where forces and supplies of materials are very different to those managing the nearshore area. No equilibrium balance situation needs to exist in offshore area. Currents are offshore-originated. Bottom sediments in movement are clay and silt, occasionally fine sand where currents are strong enough to carry the material. Current ripple marks have been found in very great depths including indications of scour due to currents. Depending upon grain size the "base" may extend to shallower or deeper water. This refers to the open sea coast where fine sediments may be carried for distances in suspension before deposition. In defining the area of exchange one therefore has to relate this area to grain sizes and materials of certain characteristics available on the shore. The finest parts of this material may therefore have to be disclosed from the balance condition which refers to a certain depth beyond which fines may still be transported to much deeper waters for deposition. A "devision line" between inshore and offshore bottom areas therefore does not exist in a strict sense of limits. At the most one may be able to define a "division area" of a certain width, which could in turn be explored by comparing onshore and offshore sedimentary characteristics or by tracing of the bottom sediments. Such tracing would then have to be continued over a sufficient long period of time to establish the boundary area with a reasonable accuracy. For the establishment of a qualitative balance criteria it is, however, not absolutely necessary to go into the smallest details in the transversal exchange. Extending the offshore area of exchange until a limit set by long time study of the occurring variances in depth one may only have to consider the losses of a certain very limited quantity of fines - if available - in the beach and nearshore sediments. See also CETA 79-2 (CERC) by Weggel.

The four figures below describe four different situations.

Fig. 1a shows a closed basin e.g. a lake where it is possible to account for all material deposition on the bottom as erosion and river discharges are known. A rise of lake level causing erosion will therefore - with a certain phase delay - be balances by a bottom deposit corresponding to the yield of sediment to the lake.

Fig. 1b has a wide shelf where all erosion material or other material discharged will be deposited on the shelf for which reason it is traceable.

Fig. 1c presents a narrow shelf limited oceanward by a steep slope extending to deep water. In this case it will be necessary to introduce a "loss function" at the outer edge which may be determined by topographic surveys or by tracing.

Fig. 1d is a not unusual case which shows how nearshore depth contours some distance from shore develop a relatively steep slope which indicate a deposit area for sediment "creeping out from shore". This does not necessarily mean that some fines may not escape beyond that limit but on a sand shore this will usually be minor quantities only. The "toe" of the slope among other words indicate the limit of the exchange area.

Usually it will be possible to evaluate the outer limit of the exchange area by more than one method, e.g. using depth topography and results of sedimentological investigation and thereby arrive at a reasonable result useful for practical purposes.

At the Hadera offshore terminal in Israel, profile analyses using the statistical methods described in TM 44 by CERC (Beach Erosion

Board) radioactive tracing and pegs placed in grid systems have been used to determine the offshore longshore versus the transversal drift. On exposed shore there is many indications that the offshore limit may be in the 50 to 70 ft (15 - 21 m) area on a long term basis.

FIG. 1a CLOSED BASIN

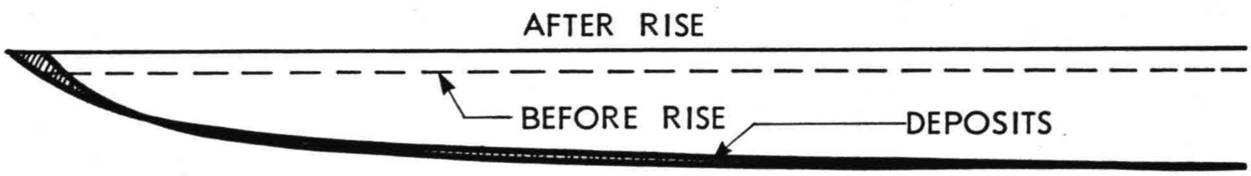

FIG. 1b WIDE SHELF

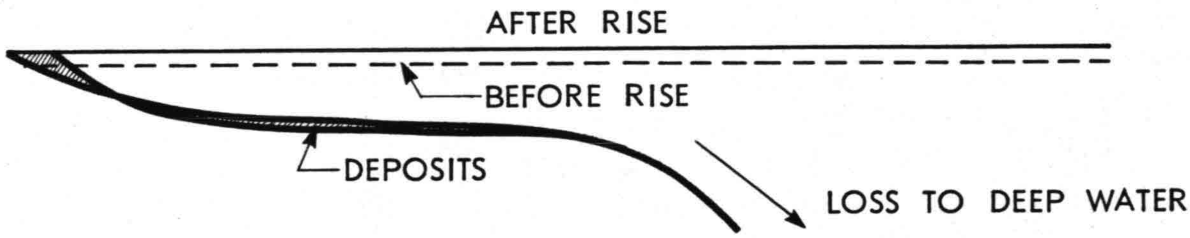

FIG. 1c NARROW SHELF

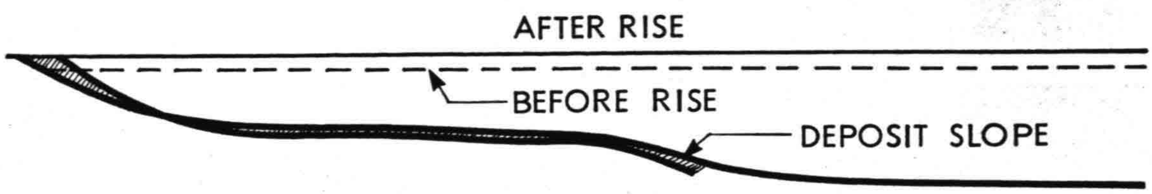

FIG. 1d PROFILE WITH DEPOSIT SLOPE