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Abstract In this paper it is shown how a 2-DoF inherently force balanced spatial deployable Butterfly Linkage is found consisting of four entangled similar Bennett linkages moving synchronously and with the common center of mass in the central joint. This linkage is derived from the Grand 4R Four-Bar Based Inherently Balanced Linkage Architecture by selecting a planar linkage with four entangled similar 4R four-bar linkages to which the Bennett conditions are applied. The inherent balance conditions are calculated, which are independent of the Bennett angles, and a CAD-model of the linkage is presented.

Key words: Inherent force balance, Static balance, Deployable, Bennett linkage, Similar four-bar

1 Introduction

For deployable linkages, static balance - when gravity does not affect the motion of the linkage - can be of urgent importance, especially when they become large of size with significant mass. This is for instance the case in kinetic architecture where deployable structures are used as movable walls, facades, roofs and for temporary portable housing [2,4]. The use of springs in the design for balance can be very complex, dangerous in case of failure and, since in practice springs lose their stiffness properties quickly over time, not very durable [3]. Using large countermasses for balance however is not desired as well as they cause a high demand on the structural design in terms of strength and space and they also limit the portability significantly for which lightweight is essential.

The contribution of this paper is to show how the inherent balance approach can be applied to synthesize advanced deployable structures with inherent balance,

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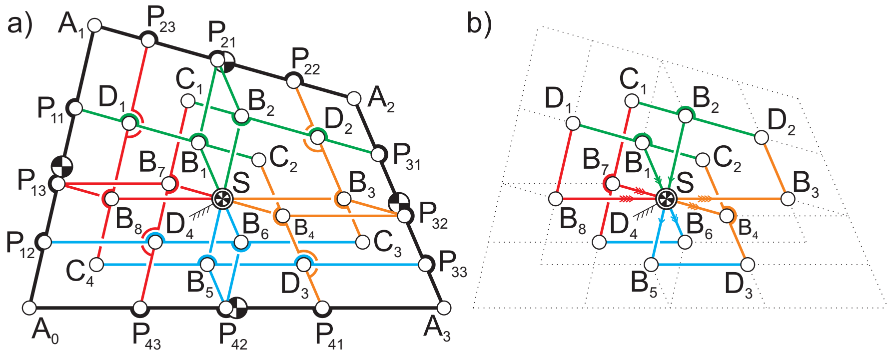

Fig. 1 a) Grand 4R Four-Bar Based Inherently Balanced Linkage Architecture from [8] where S is the common CoM of all the links for 2-DoF motions; b) 2-DoF inherently balanced linkage solution with S as the common CoM and as base pivot, obtained from the Grand Architecture by eliminating all other links.

which means that they are force balanced with solely the links of the linkage, not needing countermasses [6, 7, 9]. This is possible for specific designs of the kinematics. In this paper the synthesis of a 2-degree-of-freedom (2-DoF) inherently force balanced spatial deployable Butterfly Linkage with four entangled similar Bennett linkages moving synchronously is presented. First it is shown how this linkage is obtained from a Grand Inherently Balanced Linkage Architecture, subsequently the inherent balance conditions are derived.

2 Synthesis from the Grand 4R Four-Bar Based Inherently Balanced Linkage Architecture

Figure 1a shows the Grand 4R Four-Bar Based Inherently Balanced Linkage Architecture with solely mass symmetric links - when link centers of mass (CoMs) are on the line through the joints - as presented in [8]. It consists of a 4R four-bar linkage $A_0A_1A_2A_3$ with inside 20 links that are aligned with the four-bar links in a specific way based on principal vectors. The linkage architecture is 26 times overconstrained yet movable with the common CoM of all links stationary in joint S for all motions. This means that when the linkage architecture is pivoted to a base with S as fixed base pivot, the architecture is balanced with 2-DoF motion capability.

Since the architecture is highly overconstrained, a variety of links can be eliminated to obtain a normally constrained linkage solution. The choices for this do not affect the inherent balance capability, all derived solutions can be inherently balanced without the need of countermasses or additional elements. In [8] 32 inherently balanced linkages were presented which were derived from the Grand Architecture

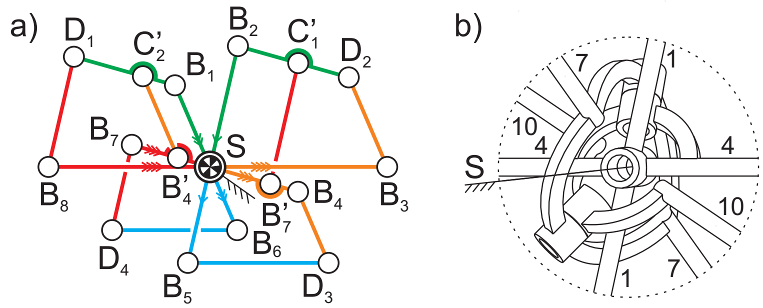

Fig. 2 a) Modification of the linkage solution in Fig. 1b where link segments C_1B_7 and C_2B_4 have become the separated and shifted links $C_1'B_7'$ and $C_2'B_4'$ to obtain a compact and normally constrained design; b) Close-up of the central joint of the spatial Butterfly Linkage in Fig. 3.

when keeping the four links of the four-bar $A_0A_1A_2A_3$. However it is also possible to find solutions without the links of the four-bar $A_0A_1A_2A_3$.

Figure 1b shows a two times overconstrained 2-DoF linkage solution that is derived from the Grand Architecture by eliminating the links of the four-bar $A_0A_1A_2A_3$ and all other internal links. It consists of the 12 links B_5SB_2, $C_1B_2D_2$, D_2B_3, B_3SB_8, B_8D_1, $D_1B_1C_2$, B_1SB_6, B_6D_4, $D_4B_7C_1$, B_7SB_4, $C_2B_4D_3$, and D_3B_5. Within the linkage, the four similar four-bars $SB_2D_2B_3 \sim B_8D_1B_1S \sim D_4B_7SB_6 \sim B_5SB_4D_3$ can be recognized. The four-bars $SB_2D_2B_3$ and $B_5SB_4D_3$ share the common link B_5SB_2, the four-bars $SB_2D_2B_3$ and $B_8D_1B_1S$ share the common link B_8SB_3, the four-bars $B_8D_1B_1S$ and $D_4B_7SB_6$ share the common link B_1SB_6, and the four-bars $D_4B_7SB_6$ and $B_5SB_4D_3$ share the common link B_7SB_4. S is the central joint where these four links have a revolute pair. Parallelograms $C_1B_2SB_7$ and $C_2B_4SB_1$ constrain the four similar four-bars to move synchronously, maintaining similarity.

From the linkage in Fig. 1b it is possible to obtain a normally constrained 2-DoF linkage by removing either joint C_1 or C_2 to keep only one parallelogram. Instead it is also possible to break link $C_1B_7D_4$ into two separate links C_1B_7 and B_7D_4 and to break link $C_2B_4D_3$ into two separate links C_2B_4 and B_4D_3 to obtain a normally constrained linkage. Then it is also possible to shift the links C_1B_7 and C_2B_4 to another location without affecting the kinematics as illustrated in Fig. 2a where they are located, respectively, as $C_1'B_7'$ and $C_2'B_4'$. Practically their function is to keep links B_2D_2 and B_1D_1, respectively, in synchronous motion with link B_7SB_4.

The four similar four-bar linkages can be transformed into four similar Bennett linkages by including equal Bennett conditions [1] for each four-bar. This results in the 2-DoF spatial linkage in Fig. 3 which, because of its four deployable segments, will be referred to as the *Bennett Based Balanced Butterfly Linkage*. It is interesting to find out that the parallel links $C_1'B_7'$ and $C_2'B_4'$ in Fig. 2a are not needed to constrain the Bennetts to move similarly, which is because of the known axial overconstraints of the revolute pairs in Bennett linkages [1].

The Butterfly Linkage is shown for the Bennett conditions that opposite links have equal length ($l_1^a = l_3$, $l_2 = l_4^a$, $l_4^b = l_6$, $l_5 = l_7^a$, $l_7^b = l_9$, $l_8 = l_{10}^a$, $l_{10}^b = l_{12}$, and

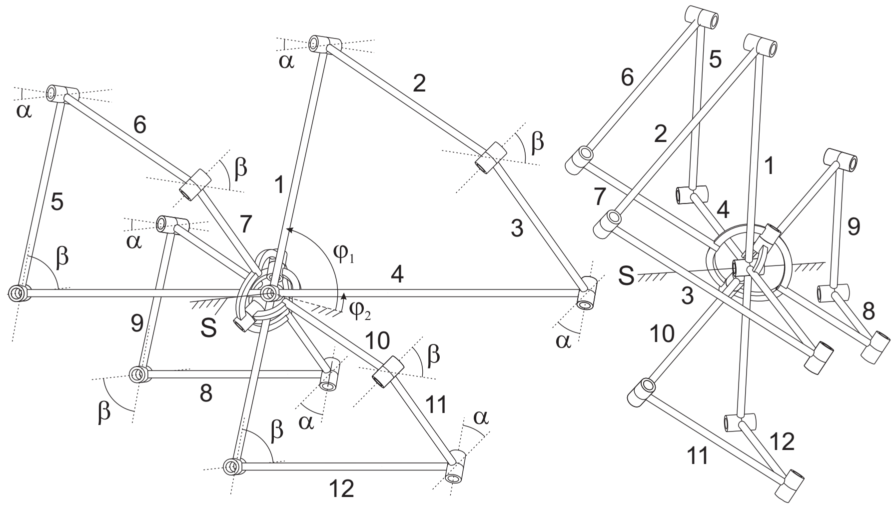

Fig. 3 The spatial deployable *Bennett Based Balanced Butterfly Linkage*, inherently balanced with four similar Bennett linkages entangled, obtained by including the Bennett conditions in the linkage of Fig. 2a. The linkage is normally constrained without the links $C_1'B_7'$ and $C_2'B_4'$ and can move with 2-DoF motions about the central pivot with all four Bennetts moving synchronously (frontview and sideview).

$l_{11} = l_1^b$, with the parameters explained in Fig. 4) and $\sin\alpha/l_1^a = \sin\beta/l_2$ where $\alpha = 40°$ and $l_1^a/l_2 = 7/9$, resulting in $\beta = 55.7°$, which are the same for all four Bennetts for similarity. The relative scaling of the four Bennetts is, in frontview, 1.0 (top right), 0.8 (top left), 0.6 (bottom left), and 0.7 (bottom right) and the angle between link 1 and link 4 is $\varphi_1 - \varphi_2 = 77°$. With respect to the balance solution of a single Bennett linkage with countermasses [5], this solution can be regarded as that the four Bennetts balance one another.

The central joint in S where four links come together with revolute pairs may be challenging to design for the spatial motions. Figure 2b shows a close-up of the design presented here. Link 1 and link 4 rotate with respect to the fixed base with revolute pairs in S, link 7 rotates with respect to link 4 with a revolute pair in S, which is drawn with an offset to the back, and link 10 rotates with respect to link 7 with a revolute pair in S, which is drawn with an offset to the front. Because of the axial overconstraints of the revolute pairs in Bennett linkages it is also possible to replace a few revolute pairs with spherical, cylindrical, or universal joints.

3 Calculation of Parameter Values for Inherent Balance

After deriving the desired linkage solution from the Grand Architecture, the parameter values of the linkage need to be calculated for which joint S is the common

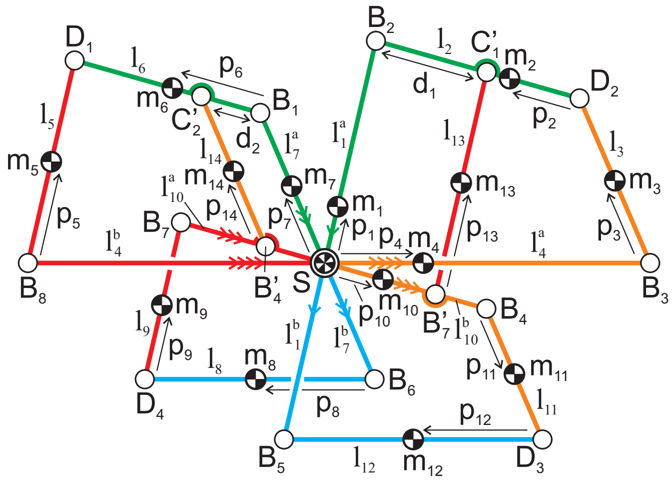

Fig. 4 Mass and link parameters of the linkage in Fig. 2a and of the spatial Butterfly Linkage in Fig. 3, which are equal.

CoM for all motions, i.e. for which the linkage is force balanced about S. Since the linkage solution is based on principal vectors, which are equal for planar and spatial motions, the balance conditions of the planar Butterfly Linkage in Fig. 2a are equal to the balance conditions of the spatial Butterfly Linkage in Fig. 3. In fact, the planar linkage can be seen as the single pose of the spatial mechanism when it becomes flat, i.e. assuming the Bennett angles also movable and becoming $\alpha = \beta = 0°$.

In Fig. 4 the planar Butterfly Linkage of Fig. 2a is shown with the parameters of the links and the link masses. Each link has a length l_i and a mass m_i with the link CoM located at a distance p_i from the indicated joint. Also the parallelogram links 13 and 14 are considered, located at offsets d_1 and d_2 from links 1 and 7, respectively. For the spatial Butterfly Linkage these links can be disregarded for balance by making them massless with $m_{13} = m_{14} = 0$.

There are two groups of inherent balance conditions to be fulfilled, the kinematic balance conditions and the mass balance conditions [8]. The kinematic balance conditions determine the similarity of the four four-bar linkages with:

$$\frac{l_1^a}{l_1^b} = \frac{l_2}{l_{10}^b} = \frac{l_3}{l_{11}} = \frac{l_4^a}{l_{12}}, \qquad \frac{l_1^a}{l_5} = \frac{l_2}{l_6} = \frac{l_3}{l_7^a} = \frac{l_4^a}{l_4^b}, \qquad \frac{l_1^a}{l_9} = \frac{l_2}{l_{10}^a} = \frac{l_3}{l_7^b} = \frac{l_4^a}{l_8} \tag{1}$$

and with $l_{13} = l_1^a$ and $l_{14} = l_7^a$. Since these properties of similarity are completely determined within the kinematics by the design of the links, they turn out to be purely geometric conditions.

The mass balance conditions determine the relations among the link mass values and the locations of the link CoMs and can be derived with a method where the linkage obtains three relative DoFs of which the linear momentum equations are written individually [6, 7].

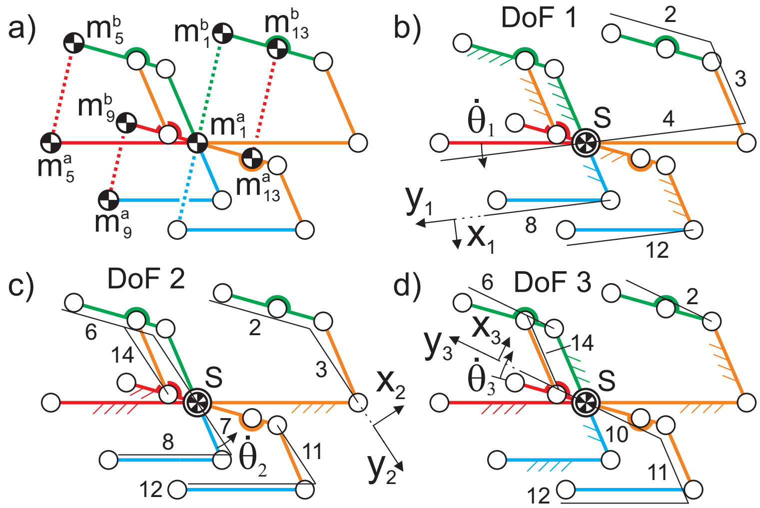

Fig. 5 a) Links 1, 5, 9, and 13 are substituted with equivalent masses to obtain an open-chain model with 3-DoF relative motion; b-c-d) Individual motion of each DoF, the mass balance conditions are derived from the linear momentum equations of each DoF individually.

First the linkage is modeled with open loops to obtain three relative DoFs. This, for instance, is possible as illustrated in Fig. 5a where link 5 and all links in parallel, links 1, 9, and 13, are substituted with equivalent masses. Link 5 (B_8D_1) is substituted with equivalent masses m_5^a and m_5^b in B_8 and D_1, respectively, where mass equivalence is determined by $m_5^a + m_5^b = m_5$ and $m_5^a p_5 = m_5^b(l_5 - p_5)$. Similarly link 9 ($D_4B_7$) is substituted with equivalent masses m_9^a and m_9^b in D_4 and B_7, respectively, where mass equivalence is determined by $m_9^a + m_9^b = m_9$ and $m_9^a p_9 = m_9^b(l_9 - p_9)$. Link 1 ($B_5SB_2$) is substituted with equivalent masses m_1^a and m_1^b in S and B_2, respectively, where mass equivalence is determined by $m_1^a + m_1^b = m_1$ and $m_1^a p_1 = m_1^b(l_1^a - p_1)$. Link 13 ($B_7'C_1'$) is substituted with equivalent masses m_{13}^a and m_{13}^b in B_7' and C_1', respectively, where mass equivalence is determined by $m_{13}^a + m_{13}^b = m_{13}$ and $m_{13}^a p_{13} = m_{13}^b(l_{13} - p_{13})$.

The motions of the three relative DoFs of the opened linkage are illustrated in Figs. 5b-c-d. For DoF 1 in Fig. 5b link 4 is rotating about S and the links parallel to link 4 (8 and 12) rotate similarly along while all other links solely translate (links 2 and 3) or remain fixed. For DoF 2 in Fig. 5c link 7 is rotating about S and the links parallel to link 7 (3, 11, and 14) rotate similarly along while all other links solely translate (links 2, 6, 8, and 12) or remain fixed. For DoF 3 in Fig. 5d link 10 is rotating about S and the links parallel to link 10 (2 and 6) rotate similarly along while all other links solely translate (links 11, 12 and 14) or remain fixed. The linear momentum equations of these three individual motions can be written with respect to the illustrated reference frames as [6]:

$$\frac{L_1}{\dot{\theta}_1} = \begin{bmatrix} -(m_1^b+m_2+m_3+m_{13}^b)l_4^a - m_4p_4 + m_5^a l_4^b + \\ m_8p_8 + m_9^a l_8 + m_{12}p_{12} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2}$$

$$\frac{L_2}{\dot{\theta}_2} = \begin{bmatrix} -(m_1^b+m_2+m_{13}^b)l_3 - m_3p_3 - (m_5^b+m_6)l_7^a - m_7p_7 + \\ (m_8+m_9^a)l_7^b + m_{11}p_{11} + m_{12}l_{11} - m_{14}p_{14} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3}$$

$$\frac{L_3}{\dot{\theta}_3} = \begin{bmatrix} m_1^b l_2 + m_2p_2 + m_5^b l_6 + m_6p_6 + m_9^b l_{10}^a - m_{10}p_{10} - \\ (m_{11}+m_{12})l_{10}^b + m_{13}^b(l_2-d_1) - m_{13}^a d_1 + m_{14}d_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4}$$

with equivalent masses:

$$m_1^b = m_1\frac{p_1}{l_1^a} \qquad m_5^a = m_5(1-\frac{p_5}{l_5}) \qquad m_5^b = m_5\frac{p_5}{l_5} \tag{5}$$
$$m_9^a = m_9(1-\frac{p_9}{l_9}) \qquad m_9^b = m_9\frac{p_9}{l_9} \qquad m_{13}^a = m_{13}(1-\frac{p_{13}}{l_{13}}) \qquad m_{13}^b = m_{13}\frac{p_{13}}{l_{13}}$$

The common CoM of the linkage is in the central joint S for force balance when the linear momentum equations are equal to zero. From the linear momentum equations the three mass balance conditions of the planar and the spatial Butterfly Linkage can be directly found as:

$$-(m_1^b+m_2+m_3+m_{13}^b)l_4^a - m_4p_4 + m_5^a l_4^b + m_8p_8 + m_9^a l_8 + m_{12}p_{12} = 0 \tag{6}$$

$$-(m_1^b+m_2+m_{13}^b)l_3 - m_3p_3 - (m_5^b+m_6)l_7^a - m_7p_7 + (m_8+m_9^a)l_7^b + m_{11}p_{11} + m_{12}l_{11} - m_{14}p_{14} = 0 \tag{7}$$

$$m_1^b l_2 + m_2p_2 + m_5^b l_6 + m_6p_6 + m_9^b l_{10}^a - m_{10}p_{10} - (m_{11}+m_{12})l_{10}^b + m_{13}^b(l_2-d_1) - m_{13}^a d_1 + m_{14}d_2 = 0 \tag{8}$$

For the spatial Butterfly Linkage these three balance conditions can be simplified by including the Bennett conditions on the link lengths $l_1^a = l_3$, $l_2 = l_4^a$, $l_4^b = l_6$, $l_5 = l_7^a$, $l_7^b = l_9$, $l_8 = l_{10}^a$, $l_{10}^b = l_{12}$, and $l_{11} = l_1^b$ and $m_{13} = m_{14} = 0$ to eliminate links 13 and 14, with which the balance conditions become:

$$-(m_1\frac{p_1}{l_1^a} + m_2 + m_3)l_2 - m_4p_4 + m_5(1-\frac{p_5}{l_5})l_6 + m_8p_8 + m_9(1-\frac{p_9}{l_9})l_8 + m_{12}p_{12} = 0 \tag{9}$$

$$-m_1p_1 - m_2l_1^a - m_3p_3 - m_5p_5 - m_6l_5 - m_7p_7 + m_8l_9 + m_9(l_9-p_9) + m_{11}p_{11} + m_{12}l_1^b = 0 \tag{10}$$

$$m_1\frac{p_1}{l_1^a}l_2 + m_2p_2 + m_5\frac{p_5}{l_5}l_6 + m_6p_6 + m_9\frac{p_9}{l_9}l_8 - m_{10}p_{10} - (m_{11}+m_{12})l_{12} = 0 \tag{11}$$

where also the equivalent masses of Eq. 5 have been substituted. The inherent balance conditions have been verified by Matlab simulations.

4 Conclusion

In this paper the synthesis of a 2-DoF inherently force balanced spatial deployable Butterfly Linkage was presented. The linkage consists of four entangled similar Bennett linkages moving synchronously with the common center of mass in the central joint. The linkage was derived from the Grand 4R Four-Bar Based Inherently Balanced Linkage Architecture by eliminating the outer four-bar links and numerous other links to obtain a planar normally constrained linkage with four entangled similar 4R four-bar linkages. By applying the Bennett conditions the planar linkage was transformed into the spatial inherently balanced Butterfly Linkage. The balance conditions of the spatial Butterfly Linkage were derived from the planar linkage with a method based on linear momentum equations. These balance conditions are independent of the Bennett angles. A CAD-model of a realistic design of the spatial linkage was shown.

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