# LATERAL FORCE AND MOMENT ON SHIPS IN OBLIQUE WAVES

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# Lateral Force and Moment on Ships in Oblique Waves[1]

By Pung Nien Hu[2]

A method for evaluating the exciting force and moment on surface ships as well as on fully submerged bodies in oblique waves is developed, based on the assumptions of long regular waves and slender bodies. The differential equation, together with the boundary conditions, for each component of the velocity potential is studied. Momentum theorems for slender-body sections are derived and applied to the evaluation of stripwise force and moment on bodies in the presence of a free surface. The result is found to be directly related to the added masses of the body sections. Lateral added masses of body sections in the presence of a free surface are investigated in detail and numerical values are presented for Lewis sections.

In the study of the linearized motion of a body in a fluid, an important task is to evaluate the exciting force and moment on the body due to regular waves encountered. Much work has been done in recent years for the case of fully submerged bodies, including the investigations made by Havelock [1],[3] Cummins [2], Korvin-Kroukovsky [3], Korvin-Kroukovsky and Jacobs [4], Kaplan [5] and Kaplan and Hu [6, 7]. Havelock obtained the force and moment on a prolate spheroid from the pressure integration by use of spheroidal harmonics. Cummins treated the case of an arbitrary slender body of revolution and evaluated the force and moment by applying his extension of Lagally's theorem to unsteady flows [8]. Korvin-Kroukovsky and Jacobs evaluated the force and moment of a slender body of revolution by integrating the pressure along the periphery of cross-section, assuming the flow in the plane of cross section to be two-dimensional. This method was employed again by Kaplan, based on a more rational formulation of the problem, and was further extended by Kaplan and Hu to the evaluation of stripwise force and moment on slender bodies of noncircular cross sections, utilizing the technique of conformal transformation.

In the case of surface ships, only the vertical force and pitching moment have so far been treated. Haskind [9] studies the force and moment on a thin ship. Peters and Stoker [10] developed a perturbation technique for evaluating force and moment on ships of Michell type (thin), or planing type (flat), or a combination of the two. Newman [11] further extended the perturbation technique by using three perturbation parameters and investigated higher order terms. All these studies utilized the same Green's function satisfying the appropriate boundary condition on the free surface. The velocity potential was derived from Green's theorem, while the force and moment were found by integration of pressure over the hull surface.

As to the lateral force and moment on ships in waves, Ursell [12] and Levine and Rodemich [13] have treated two-dimensional cases. Haskind [14] has solved the problem of infinite plate and cylinder in oblique waves. However, the treatment of three-dimensional bodies in oblique waves is still lacking. The difficulty in problems of three-dimensional bodies appears to arise from the fact that the integral equation for the velocity potential derived from Green's theorem involves an extremely complicated kernel function which cannot be solved exactly in the present state of mathematics. To avoid this, it is necessary to make further simplifications to or formulate the problem from a different approach.

Since the restoring force and moment on ships in lateral motions are small, motions at low frequencies are of particular interest. In the present paper, consequently, only long regular waves are treated. For this case, it is found that velocity potential attributed to the body-wave interaction can be determined from two-dimensional analysis. In addition to the simplification of the problem thus achieved, the two-dimensional analysis also offers the advantage that it is possible to treat a large class of ship forms such as those represented by two or more parameters [15, 16, 17].

The evaluation of the force and moment from the pressure integration is not always a simple task because the nonlinear terms in the pressure expression must be included even in the case of slender bodies [18, 19]. Although the extended Lagally's theorem [8] enables one to evaluate the force and moment from singularity distributions, the feasibility of applying the theorem to

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[3] Numbers in brackets designate References at end of paper.

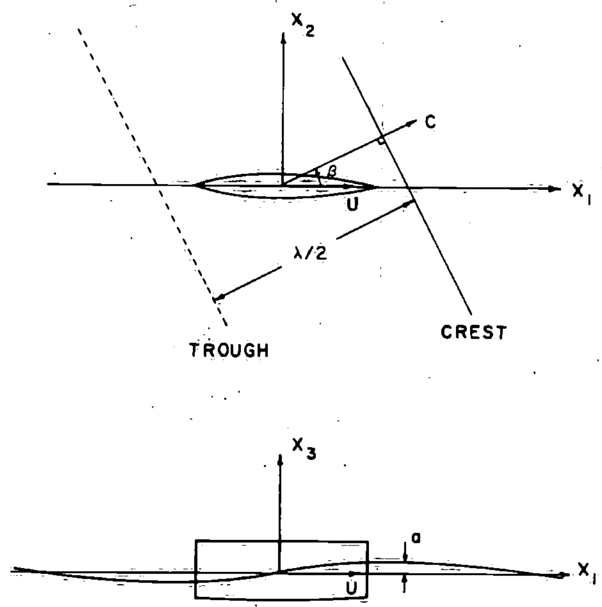


Fig. 1 Co-ordinate system

flows involving bodies on a free surface remains to be examined. In the present paper, the stripwise force and moment are evaluated by use of momentum theorems derived in forms appropriate to slender-body sections. It is interesting to note that results thus obtained are directly related to the added masses of ship sections.

By application of the perturbation method, the lateral added masses of ship sections up to the second order are studied, and the second-order terms of the added masses are found to represent the potential energies on the free surface due to waves generated by the first-order solutions. Numerical values of lateral added masses of Lewis sections are calculated for both first and second-order terms.

## Fundamental Equations

A Cartesian co-ordinate system is chosen with the axes fixed relative to the ship, which is restrained to move only forward at a constant speed $U$ in the $x_1$-direction on the free surface. The $x_3$-axis is positive upward, the $x_2$-axis to the port, and the origin is placed on the free surface in the median plane of ship.

The free surface is disturbed by regular waves propagated with speed $c$ in a direction oblique to the forward motion of the ship at an angle $\beta$ which lies in the range $-\pi/2 \leq \beta \leq \beta/2$ as shown in Fig. 1. The wave-propagation speed $c$ is positive for following seas and negative for head seas.

The perturbation velocity potential $\phi(x_1, x_2, x_3, t)$ of the motion of the fluid satisfies:

## Nomenclature

$A$ = cross-sectional area of ship below free surface
$A_{ij}(i,j = 1,2,3)$ = added mass, defined by equations (38) and (41)
$A_{42}$ = added moment of inertia, defined by equation (42)
$A_{ij}'$, $A_{ij}''$ = first and second-order terms of $A_{ij}$, respectively
$A_{42}'$, $A_{42}''$ = first and second-order terms of $A_{42}$, respectively
$a$ = amplitude of regular waves
$a_1$, $a_3$ = coefficients of Lewis transformation
$b$ = half-beam of ship section
$C$ = submerged surface of ship section
$c$ = wave-propagation speed
$C_{ij}$, $C_{42}$ = added mass coefficients
$C_{ij}'$, $C_{ij}''$, $C_{42}'$, $C_{42}''$ = first and second-order terms of $C_{ij}$ and $C_{42}$, respectively
$\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} - U\dfrac{\partial}{\partial x_1}$ = total time derivative
$F$ = free surface outside ship in plane of cross section
$F_j(j = 2,3)$ = force along $x_j$ – direction
$g$ = gravitational acceleration
$H$ = draft of ship section
$h_2'$, $h_4'$ = wave heights generated by $\phi_2'$ and $\phi_4'$, respectively
$i = (-1)^{1/2}$
$k = \dfrac{2\pi}{\lambda} = \dfrac{g}{c^2}$ wave parameter
$L$ = large surface at infinity
$M_1$ = moment about $x_1$-axis
$m$ = mass of fluid displaced by ship section
$\bar{n}$ = normal pointing into fluid
$n_j = \cos(\bar{n}, x_j)$ = direction cosine ($j$ = 1,2,3)
$n_4 = x_2 n_3 - x_3 n_2$
$p$ = pressure
$r$, $\theta$ = polar co-ordinate in $\zeta_1$ plane
$S$ = surface bounded by $F$, $C$ and $L$ in plane of cross section
$S'$ = surface of control volume
$T = x_1 \cos\beta + (U\cos\beta - c)t$
$t$ = time
$U$ = forward speed of ship
$\bar{U}$ = velocity vector of fluid
$v$, $w$ = components of $\bar{U}$ along $x_2$- and $x_3$-direction, respectively
$v_0 = -ikac \sin\beta$
$w_0 = -kac$
$v^* = v_0 e^{ikT}$ = horizontal component of orbital velocity at $x_2 = x_3 = 0$
$w^* = w_0 e^{ikT}$ = vertical component of orbital velocity at $x_2 = x_3 = 0$
$x_j(j = 1,2,3)$ = Cartesian co-ordinate
$z = x_2 + ix_3$ = complex plane
$\rho$ = fluid density
$\beta$ = angle between $x_1$-axis and normal to crests of regular waves
$\zeta = \xi + i\eta$ = complex plane, on which unit circle is mapped into a flat plate
$\zeta_1 = \xi_1 + i\eta_1 = re^{i\theta}$ = complex plane, on which ship section is mapped into unit circle
$\sigma = \dfrac{A}{2bH}$ = area ratio
$\lambda$ = wave length
$\gamma = \dfrac{H}{b}$
$\phi$ = perturbation velocity potential
$\phi(b)$, $\phi_j(b)$ = abbreviations of $\phi(x_1, b, 0)$ and $\phi_j(b, 0)$, respectively
$\phi_j(j = 1,2,3,4)$ = normalized velocity potential
$\phi_w$ = velocity potential of waves
$\phi_b$ = velocity potential due to body-wave interaction
$\psi$, $\psi_w$ = see equations (8) and (16)
$\tau$ = control volume

(a) Laplace equation

$$\frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2} + \frac{\partial^2 \phi}{\partial x_3^2} = 0 \tag{1}$$

everywhere in the fluid,

(b) The linearized boundary condition[4]

$$\frac{\partial^2 \phi}{\partial t^2} + 2U \frac{\partial^2 \phi}{\partial t\, \partial x_1} + U^2 \frac{\partial^2 \phi}{\partial x_1^2} + g \frac{\partial \phi}{\partial x_3} = 0 \tag{2}$$

on the free surface ($x_3 = 0$), and

(c) The boundary condition

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial x_1} n_1 + \frac{\partial \phi}{\partial x_2} n_2 + \frac{\partial \phi}{\partial x_3} n_3 = -Un_1 \tag{3}$$

on the surface of ship, where $\bar{n}$ is the normal pointing into the fluid and $n_j = \cos(\bar{n}, x_j)$ is the direction cosine.

One may expresss the potential $\phi$ as

$$\phi(x_1, x_2, x_3, t) = U\phi_1(x_1, x_2, x_3) + \phi_w(x_1, x_2, x_3, t) + \phi_b(x_1, x_2, x_3, t), \tag{4}$$

where $\phi_1$ represents the potential due to the forward motion of ship in calm water, $\phi_w$ the potential of waves, and $\phi_b$ the body-wave interaction. Obviously, each potential satisfies the Laplace equation.

The potential $\phi_1$ satisfies the boundary condition

$$U^2 \frac{\partial^2 \phi_1}{\partial x_1^2} + g \frac{\partial \phi_1}{\partial x_3} = 0 \tag{5}$$

on the free surface, and the boundary condition

$$\frac{\partial \phi_1}{\partial n} = -n_1 \tag{6}$$

on the surface of the ship. For thin ships, the potential $\phi_1$ may be determined by the well-known Michell's approximate method. For slender bodies, Cummins [21] has developed a method to obtain $\phi_1$ in terms of the coordinates fixed in space, based on the concept of impulsive flow. In the case that the forward speed of the ship is small, the free surface can be regarded as a rigid wall and the potential $\phi_1$ is identical with that for a double body, consisting of the submerged position of the body and its image over the free surface, moving in an infinite fluid. The standard slender-body theory can then be applied (see, e.g., Reference [22]).

For regular waves of wave length $\lambda$ and applitude $a$, one has

$$\phi_w = ace^{k\{x_3 + i[x_1 \cos\beta + x_2 \sin\beta + (U\cos\beta - c)t]\}}, \tag{7}$$

where

$$k = \frac{2\pi}{\lambda} = \frac{g}{c^2}$$

is the wave parameter, $i = (-1)^{1/2}$ and only the real part of the potential is to be taken. As is suggested by the form of $\phi_w$, one may write

$$\phi_w + \phi_b = \psi(x_2, x_3)e^{ikT}, \tag{8}$$

where

$$T = x_1 \cos\beta + (U\cos\beta - c)t, \tag{9}$$

and the function $\psi$ satisfies the two-dimensional Helmholtz equation

$$\frac{\partial^2 \psi}{\partial x_2^2} + \frac{\partial^2 \psi}{\partial x_3^2} = k^2 \cos^2\beta\,\psi, \tag{10}$$

the boundary condition

$$\frac{\partial \psi}{\partial x_3} = k\psi \tag{11}$$

on the free surface and the boundary condition

$$\frac{\partial \psi}{\partial x_2} n_2 + \frac{\partial \psi}{\partial x_3} n_3 = 0 \tag{12}$$

on the surface of the ship, where $\bar{n}$ is approximated by the normal in the $x_2 - x_3$ plane for slender bodies.

It is extremely difficult to solve equation (10) subjected to the mixed boundary conditions (11) and (12) even for very simple cases. Haskind [14] obtained a solution for an infinitely long plank in terms of series of Mathieu functions. However, for the case of long waves, a great simplification may be achieved.

Assuming that the wave length is adequately large and the linear dimensions of the cross section of ship are small relative to the wave length, the potential of waves may be expanded near the body into a Taylor series around the $x_1$-axis; i.e., $x_2 = x_3 = 0$. Neglecting higher order terms of the wave parameter $k$ beyond the linear, one has

$$\phi_w = ac[1 + k(x_3 + ix_2 \sin\beta)]e^{ikT} \tag{13}$$

or

$$\phi_w = (ac - v_0 x_2 - w_0 x_3)e^{ikT} \tag{14}$$

where

$$v_0 = -ikac \sin\beta,$$

and

$$w_0 = -kac \tag{15}$$

This shows that for long waves, the wave potential near the body behaves like that of a uniform flow in the plane of cross section. One may, consequently, set

$$\psi = \psi_w - v_0\phi_2 - w_0\phi_3, \tag{16}$$

where $\psi_w$ represents the contribution due to waves and

$$\psi_w = \begin{cases} ac\, e^{k(x_3 + ix_2 \sin\beta)} & \text{everywhere,} \\ ac - v_0 x_2 - w_0 x_3 & \text{near the body.} \end{cases} \tag{17}$$

Since the function $\psi$ now satisfies the two-dimensional

[4] Newman [11] has studied the boundary condition on the free surface very rigorously and has shown that equation (2) should be inhomogeneous if the beam-length ratio of the ship is in the same order as the incident wave perturbation parameter. The present homogeneous condition, therefore, implies that the beam-length ratio of the ship is smaller than the incident wave parameter. This is consistent with the slender-body treatment in the present study.

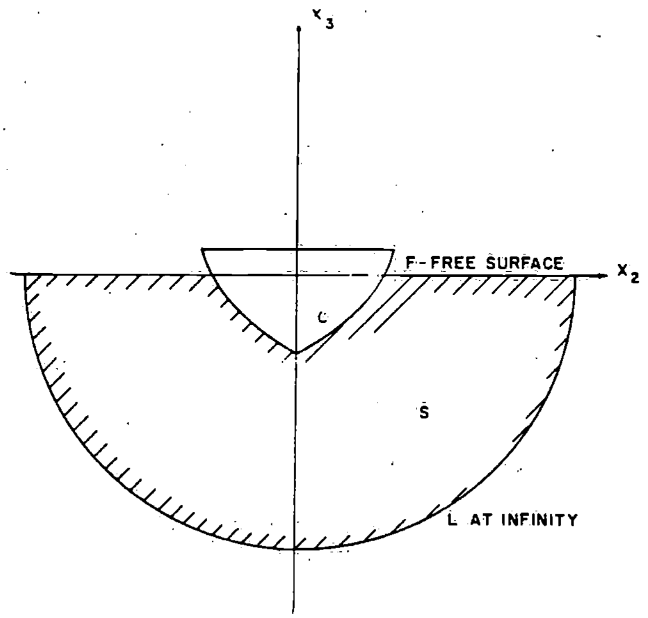


Fig. 2 Elementary control volume (shaded)

Laplace equation up to the linear term of $k$ as seen in equation (10), it follows that the potential $\phi_j$ ($j = 2, 3$) satisfies:

(a) The equation

$$\frac{\partial^2\phi_j}{\partial x_2^2} + \frac{\partial^2\phi_j}{\partial x_3^2} = 0, \tag{18}$$

(b) The boundary condition

$$\frac{\partial\phi_j}{\partial x_3} = k\phi_j \tag{19}$$

on the free surface, and

(c) the boundary condition

$$\frac{\partial\phi_j}{\partial n} = -n_j \tag{20}$$

on the surface of the ship where $\bar{n}$ is approximated by the normal in $x_2 - x_3$-plane. Evidently, equation (18) together with boundary conditions (19) and (20) is identical with the system which describes the vertical and/or horizontal oscillations of two-dimensional bodies on a free surface in calm water. For Lewis sections, Grim [23] has developed a method to evaluate $\phi_j$ by expanding it into an infinite series constructed by functions which satisfy the boundary condition (19) individually.

## Momentum Theorems for Slender-Body Sections

To evaluate the force and moment on a body, one may utilize the simple but powerful momentum theorems, instead of integrating the pressure, usually a laborious procedure for bodies having complicated forms. In the present study, since the flow in the plane of cross section is by no means exactly two-dimensional, these theorems have to be expressed in a form appropriate to the slender-body treatment.

The momentum theorem states that the resultant force on the fluid within a control surface is equal to the rate of change of the momentum of the control volume and the net efflux rate of momentum from the volume. It may be expressed mathematically as

$$-\int_{S'} p\bar{n}\, dS' = -\frac{\partial}{\partial t}\int_\tau \rho\bar{U}d\tau + \int_{S'} (\bar{n}\cdot\bar{U})\rho\bar{U}dS' \tag{21}$$

where $S'$ is the surface of the control volume $\tau$, $\bar{n}$ the normal vector pointing into the volume, and $\bar{U}$ the velocity of the fluid.

Choosing a thin strip as the elementary control volume, bounded by the free surface outside the ship, the submerged surface of the ship, the plane $x_1 = x_1$, the plane $x_1 = x_1 + \Delta x_1$ (both planes are parallel to the $x_2 - x_3$-plane) and a large surface at infinity, one may apply the theorem given by equation (21) to the control volume and obtain

$$-\Delta x_1 \int_{F+C+L} p\bar{n}\, ds - \int_S p\bar{n}\, dS - \int_{S \text{ at } x_1+\Delta x_1} p\bar{n}\, dS$$
$$= -\frac{\partial}{\partial t}\left[\Delta x_1 \int_S \rho\bar{U}dS\right]$$
$$+ \Delta x_1 \int_{F+C+L} (\bar{n}\cdot\bar{U})\rho\bar{U}\, ds + \int_S (\bar{n}\cdot\bar{U})\rho\bar{U}\, dS$$
$$+ \int_{S \text{ at } x_1+\Delta x_1} (\bar{n}\cdot\bar{U})\rho\bar{U}\, dS, \tag{22}$$

where all quantities except those specifically indicated are taken in the plane $x_1 = x_1$, $ds$ is the elementary length along the contour of integration, $dS$ the elementary area, $F$ the free surface outside the ship, $C$ the submerged surface of ship, $L$ the large surface at infinity and $S$ the surface bounded by $F$, $C$ and $L$, Fig. 2.

Assuming that the control volume is moving with the body and the perturbation velocity of fluid is small compared with the free-stream velocity $-U$, one finds that $\bar{n}\cdot\bar{U}$ approximately is equal to $-U$ in the plane $x_1 = x_1$, and is equal to $U$ in the plane $x_1 = x_1 + \Delta x_1$. Thus, by the definition of differentiation, one may reduce equation (22), for the force components in the plane of cross-section to

$$-\int_{C+L} pn_2 ds = -\frac{D}{Dt}\int_S \rho v\, dS + \int_{F+C+L} (\bar{n}\cdot\bar{U})\rho v\, ds \tag{23}$$

and

$$-\int_{C+L} pn_3\, ds = -\frac{D}{Dt}\int_S \rho w\, dS + \int_{F+C+L} (\bar{n},\bar{U})\rho w\, ds \tag{24}$$

where $v$ and $w$ are horizontal and vertical components of $\bar{U}$, respectively, and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - U\frac{\partial}{\partial x_1}, \quad (25)$$

the pressure on the free surface being assumed zero. The first term of the right-hand side in equations (23) and (24) represents the rate of change of fluid momentum, and the second term represents the momentum transfer through the boundary.

Similarly, one may express the moment-of-momentum theorem for the moment about $x_1$-axis, i.e., the roll moment on the strip, as

$$-\int_{C+L} p(x_2n_3 - x_3n_2)ds = -\frac{D}{Dt}\int_S \rho(x_2w - x_3v)dS + \int_{F+C+L} (\bar{n}\cdot\bar{U})\rho(x_2w - x_3v)ds, \quad (26)$$

The first term of the right-hand side in equation (26) represents the rate of change of moment of momentum of the fluid, and the second term represents the moment-of-momentum transfer through the boundary.

In deriving the foregoing expressions for momentum theorems, no assumption other than that of slender body has been made, therefore, one may apply those theorems to a compressible and viscous fluid as well. In the case of a perfect fluid (incompressible and inviscid), one has, for example,

$$\int_S w\,dS = -\int_S \frac{\partial\phi}{\partial x_3}dS = \int_{F+C+L} \phi\, n_3 ds \quad (27)$$

by the use of Gauss's theorem. But

$$\int_F \phi\, n_3 ds = -\int_{-\infty}^{-b} \phi\, dx_2 - \int_b^{\infty} \phi\, dx_2, \quad (28)$$

where $b = b(x_1)$ is the half-beam of ship. It follows that

$$\frac{\partial}{\partial x_1}\int_F \phi\, n_3 ds = \int_F \frac{\partial\phi}{\partial x_1} n_3 ds + \frac{db}{dx_1}[\phi(b) + \phi(-b)], \quad (29)$$

where $\phi(b) = \phi(x_1, b, 0)$.

On the other hand,

$$\frac{\partial}{\partial x_1}\int_L \phi\, n_3 ds = \int_L \frac{\partial\phi}{\partial x_1} n_3 ds. \quad (30)$$

since the large surface $L$ at infinity is independent of $x_1$. Thus

$$\frac{D}{Dt}\int_{F+L} \rho\,\phi\, n_3\, ds = \int_{F+L} \rho\frac{D\phi}{Dt} n_3\, ds - \rho U\frac{db}{dx_1}[\phi(b) + \phi(-b)] \quad (31)$$

Since the linearized pressure equation

$$p = \rho\frac{D\phi}{Dt} \quad (32)$$

may be used on the free surface $F$ and on the surface $L$ at infinity to be consistent with the linearized boundary condition on the free surface, equation (24) reduces to

$$-\int_C p n_3\, ds = -\frac{D}{Dt}\int_C \rho\phi n_3\, ds + \int_C (\bar{n}\cdot\bar{U})\rho\, w\, ds + \rho U\frac{db}{dx_1}[\phi(b) + \phi(-b)] \quad (33)$$

where the line integral along $F$ and $L$ for the momentum transfer has been neglected in accordance with the linearization already made. The other two integrals may, similarly, be reduced to

$$-\int_C p n_2 ds = -\frac{D}{Dt}\int_C \rho\,\phi n_2\, ds + \int_C (\bar{n}\cdot\bar{U})\rho v\, ds \quad (34)$$

and

$$-\int_C p(x_2n_3 - x_3n_2)ds = -\frac{D}{Dt}\int_C \rho\,\phi(x_2n_3 - x_3n_2)ds + \int_C (\bar{n}\cdot\bar{U})\rho(x_2w - x_3v)ds + \rho U b\frac{db}{dx_1}[\phi(b) - \phi(-b)] \quad (35)$$

Obviously, the foregoing theorems, [equations (33) to (35)], in the present form are also applicable to the case in which the body moves at small perturbation lateral velocities in addition to the forward speed.

## Force and Moment on Slender-Body Section

The foregoing momentum theorems, [equations (33) to (35)], may now be applied to the evaluation of force and moment on ships in oblique waves.

The line integral in equation (34) may be written as

$$\int_C \phi n_2\, ds = \int_C [U\phi_1 + (ac - v_0x_2 - w_0x_3 - v_0\phi_2 - w_0\phi_3)e^{ikT}]n_2 ds \quad (36)$$

by the use of equations (4), (8) and (16). On the assumption that the ship is symmetrical about its median plane, it is found that the potentials $\phi_1$ and $\phi_3$ are both even functions of $x_2$, while $\phi_2$ is odd. Consequently, equation (36) may be reduced to

$$\rho\int_C \phi\, n_2 ds = -v_0 e^{ikT}(m + A_{22}), \quad (37)$$

where $m$ is the mass of the displaced fluid, and, using the conventional double index notation for a tensor element, $A_{22}$, the added mass of the ship section along $x_2$-direction due to its motion in that direction, is defined as

$$A_{22} = \rho\int_C \phi_2 n_2 ds, \quad (38)$$

Similarly, one finds that

$$\rho\int_C \phi\, n_3 ds = UA_{31} - w_0e^{ikT}A_{33} - w_0e^{ikT}m, \quad (39)$$

and

$$\rho \int_C \phi(x_2 n_3 - x_3 n_2) ds = v_0 e^{ikT} (m\bar{x}_3 - A_{42} + \tfrac{2}{3}\rho b^3) \quad (40)$$

where

$$A_{3j} = \rho \int_C \phi_j n_3 ds \quad (41)$$

is the added mass of the ship section along the $x_3$-direction due to its motion along $x_j$-direction ($j = 1,3$).

$$A_{42} = \rho \int_C \phi_2 (x_2 n_3 - x_3 n_2) ds \quad (42)$$

is the added moment of inertia of the ship section about $x_1$-axis due to its motions along $x_2$-direction, and

$$\bar{x}_3 = \frac{\rho}{m} \int_C x_2 x_3 n_2 ds \quad (43)$$

is the position of the center of buoyancy which is always negative in the present co-ordinate system.

Substituting the boundary condition (3) into the line integral for momentum transfer in equation (24), one has

$$\int_C (\bar{n} \cdot \bar{U}) \rho v \, ds = -\rho U \int_C \frac{\partial \phi}{\partial x_2} n_1 ds, \quad (44)$$

which may further be reduced to

$$\int_C (\bar{n} \cdot \bar{U}) \rho v \, ds = \rho U v_0 e^{ikT} \int_C \frac{\partial \phi_2}{\partial x_2} n_1 ds - U v_0 e^{ikT} \frac{dm}{dx_1}, \quad (45)$$

where the relation

$$\frac{dm}{dx_1} = -\rho \int_C n_1 ds \quad (46)$$

has been used. Similarly, one finds that

$$\int_C (\bar{n} \cdot \bar{U}) \rho w \, ds = \rho U w_0 e^{ikT} \int_C \frac{\partial \phi_3}{\partial x_3} n_1 ds - U w_0 e^{ikT} \frac{dm}{dx_1} - \rho U^2 \int_C \frac{\partial \phi_1}{\partial x_3} n_1 ds \quad (47)$$

and

$$\int_C (\bar{n} \cdot \bar{U}) \rho (x_2 w - x_3 v) ds = \rho U v_0 e^{ikT} \int_C \left( x_2 \frac{\partial \phi_2}{\partial x_3} - x_3 \frac{\partial \phi_2}{\partial x_2} \right) n_1 ds - U v_0 e^{ikT} \frac{d}{dx_1} (m \bar{x}_3), \quad (48)$$

where

$$\frac{d}{dx_1} (m\bar{x}_3) = -\rho \int_C x_3 n_1 ds \quad (49)$$

Furthermore, equations (4), (8) and (16) yield

$$\phi(b) = U\phi_1(x_1, b, 0) + [ac - v_0 b - v_0 \phi_2(b, 0) - w_0 \phi_3(b, 0)] e^{ikT}$$

and (50)

$$\phi(-b) = U\phi_1(x_1, -b, 0) + [ac + v_0 b - v_0 \phi_2(-b, 0) - w_0 \phi_3(-b, 0)] e^{ikT},$$

which leads to

$$\phi(b) + \phi(-b) = 2U\phi_1(b) + 2[ac - w_0 \phi_3(b)] e^{ikT}$$

and

$$\phi(b) - \phi(-b) = -2v_0 e^{-ikT} [b + \phi_2(b)], \quad (51)$$

where $\phi_j(b)$ is the abbreviation of $\phi_j(b, 0)$.

Combining these results, one obtains finally the force component

$$\frac{dF_2}{dx_1} = \frac{D}{Dt} (A_{22} v^*) + m \frac{Dv^*}{Dt} + \rho U v^* \int_C \frac{\partial \phi_2}{\partial x_2} n_1 ds \quad (52)$$

along the $x_2$-direction, the force component

$$\frac{dF_3}{dx_1} = \frac{D}{Dt} (A_{33} w^* - A_{31} U) + m \frac{Dw^*}{Dt} + 2\rho U \frac{db}{dx_1} [U\phi_1(b) + ac\, e^{ikT} - w^* \phi_3(b)] + \rho U w^* \int_C \frac{\partial \phi_3}{\partial x_3} n_1 ds - \rho U^2 \int_C \frac{\partial \phi_1}{\partial x_3} n_1 ds \quad (53)$$

along the $x_3$-direction, and the roll moment

$$\frac{dM_1}{dx_1} = \frac{D}{Dt} (A_{42} v^*) - \left( m\bar{x}_3 + \frac{2}{3} \rho b^3 \right) \frac{Dv^*}{Dt} - 2\rho U v^* b \frac{db}{dx_1} \phi_2(b) + \rho U v^* \int_C \left( x_2 \frac{\partial \phi_2}{\partial x_3} - x_3 \frac{\partial \phi_2}{\partial x_2} \right) n_1 ds \quad (54)$$

about the $x_1$-axis, where

$$v^* = v_0 e^{ikT}$$

and

$$w^* = w_0 e^{ikT} \quad (55)$$

are the horizontal and vertical orbital velocity components, respectively, of waves at the origin on the $x_2 - x_3$-plane; i.e., on the $x_1$-axis.

The foregoing result is quite general, without any restriction on the form of ship sections. Once potentials $\phi_1$, $\phi_2$, and $\phi_3$ have been found, the stripwise force and moment may be evaluated readily. The resultant force and moment on the entire ship may then be obtained by integration.

The same analysis may also be used for evaluating the force and moment on fully submerged bodies. The result is found to be the same as equations (52) to (54) except that the terms containing $b$ are deleted.

For infinitely long cylinders having uniform cross sections on a free surface, all line integrals of the momentum transfer together with $A_{31}$ and $db/dx_1$ vanish in equations (52) to (54), and the time derivative $D/Dt$ reduces to $\partial/\partial t$. If the body is fully submerged, then the term containing $b$ in equation (54) also vanishes.

It is important to note that the added masses, given by equations (38), (41) and (42), can no longer represent

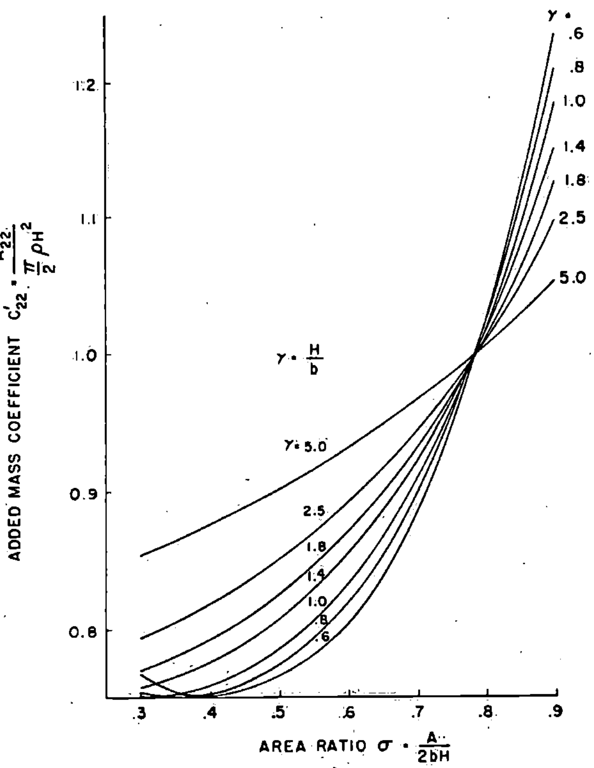


Fig. 3 Added-mass coefficient $C_{22}'$ versus area ratio $\sigma$

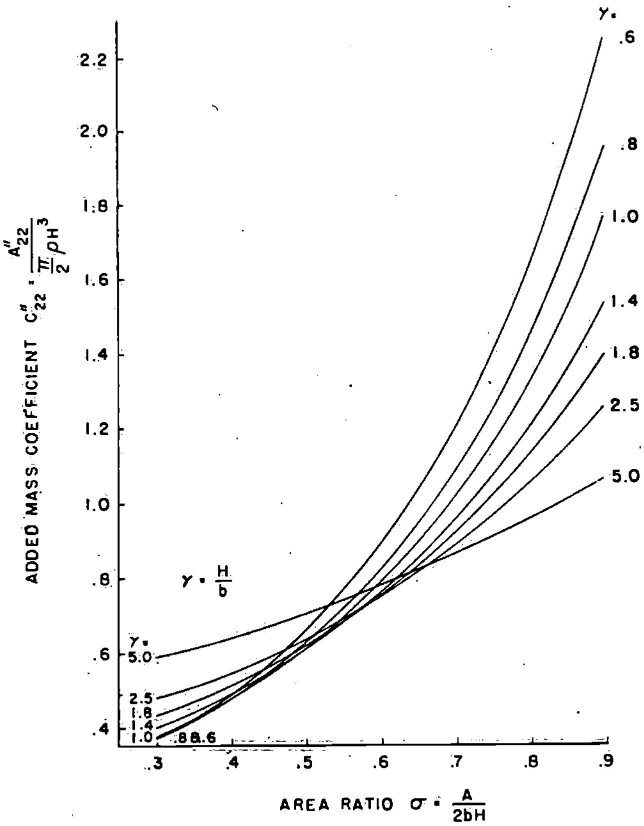


Fig. 4 Added-mass coefficient $C_{22}''$ versus area ratio $\sigma$

the entire kinetic energy of the fluid when a free surface is present since the contour of integration in the expressions of added masses is only a portion of the boundary surrounding the fluid.

## Asymptotic Values of Lateral Added Masses

In the expressions of the stripwise force and moment given in equations (52) to (54), one finds that the lateral added masses $A_{22}$ and $A_{42}$ of a body section, either on a free surface or fully submerged, can be evaluated up to the linear term of the wave parameter $k$ without solving the problem exactly.

In accordance with the assumption of long wave length in the present analysis, one may, utilizing the small perturbation method, write

$$\phi_2 = \phi_2' + k\phi_2'' + k^2\phi_2''' + \ldots\ldots \tag{57}$$

where the number of primes represents the order of approximation. Evidently, the solution of $\phi_2$ of all order satisfies the Laplace equation. As to the boundary condition (19), by substituting equation (57) and equating the same order terms, one finds that

$$\frac{\partial\phi_2'}{\partial x_3} = 0 \quad \text{and} \quad \frac{\partial\phi_2''}{\partial x_3} = \phi_2' \tag{58}$$

on the free surface. Furthermore setting

$$\frac{\partial\phi_2'}{\partial n} = -n_2 \tag{59}$$

on the body section, boundary condition (20) becomes

$$\frac{\partial\phi_2''}{\partial n} = 0 \tag{60}$$

on the surface of the body. Therefore, the first-order solution $\phi_2'$ actually represents the flow generated by a double body, consisting of the submerged portion of body section and its image above the free surface, oscillating laterally in an infinite fluid.

Now, by the use of boundary condition (19), the added mass

$$A_{22} = \rho \int_C \phi_2 n_2 ds \tag{61}$$

may be written as

$$A_{22} = -\rho \int_C \phi_2 \frac{\partial\phi_2}{\partial n} ds, \tag{62}$$

the usual form of added mass. Writing $\phi_2$ in the expansion (57), equation (62) becomes

$$A_{22} = -\rho \int_C \phi_2' \frac{\partial\phi_2'}{\partial n} ds - \rho k \int_C \left(\phi_2' \frac{\partial\phi_2''}{\partial n} + \phi_2'' \frac{\partial\phi_2'}{\partial n}\right) ds \tag{63}$$

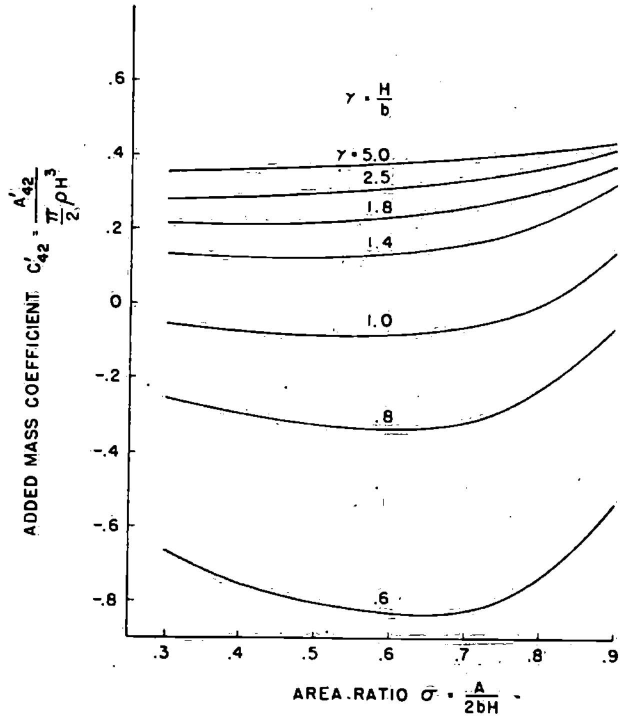


Fig. 5 Added-mass coefficient $C_{42}'$ versus area ratio $\sigma$

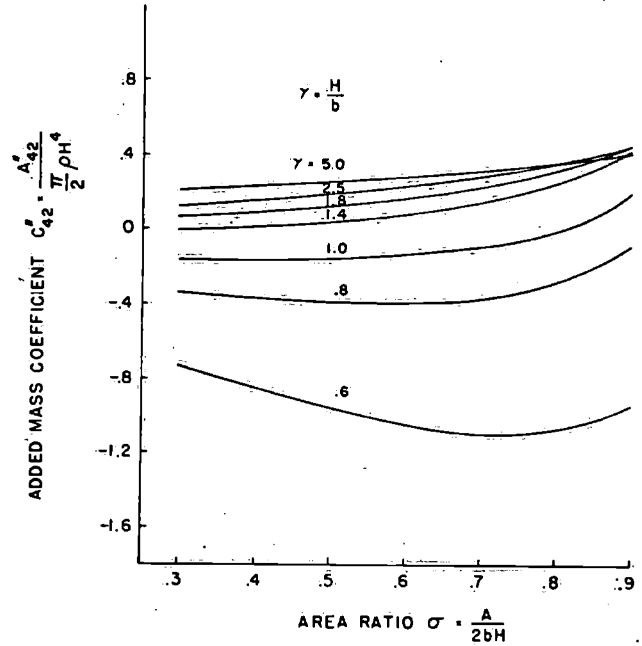


Fig. 6 Added-mass coefficient $C_{42}''$ versus area ratio $\sigma$

But, from Green's formula,

$$\int_{F+C} \left( \phi_2' \frac{\partial \phi_2''}{\partial n} - \phi_2'' \frac{\partial \phi_2'}{\partial n} \right) ds = 0 \qquad (64)$$

Therefore, equation (63) reduces to

$$A_{22} = -\rho \int_C \phi_2' \frac{\partial \phi_2'}{\partial n} ds - 2\rho k \int_{F+C} \phi_2' \frac{\partial \phi_2''}{\partial n} ds + \rho k \int_F \left( \phi_2' \frac{\partial \phi_2''}{\partial n} + \phi'' \frac{\partial \phi_2'}{\partial n} \right) ds \qquad (65)$$

Substitution of boundary conditions (58) to (60) leads to

$$A_{22} = -\rho \int_C \phi_2' \frac{\partial \phi_2'}{\partial n} ds + \rho k \int_F \phi_2'^2 ds, \qquad (66)$$

or

$$A_{22} = A_{22}' + k A_{22}'' \qquad (67)$$

where

$$A_{22}' = -\rho \int_C \phi_2' \frac{\partial \phi_2'}{\partial n} ds \qquad (68)$$

is the first-order solution for the added mass $A_{22}$ which may be taken as half of the added mass of the fully submerged double body consisting of the submerged portion of the body section and its image above the free surface in an infinite fluid, since $\phi_2'$ is an even function of $x_3$ as shown by the boundary condition (58), and

$$A_{22}'' = \rho \int_F \phi_2'^2 ds \qquad (69)$$

is the second-order term of the solution which can be evaluated readily once the first-order solution $\phi_2'$ is found. It is seen that $A_{22}''$ given in the foregoing expression is always positive, which shows that the presence of the free surface, to the linear term of the wave parameter $k$, always tends to augment the added mass. This is due to the fact that the disturbed free surface carries a certain amount of potential energy which can be identified as the second-order term of the added mass. Since the complete expression of the first-order potential is

$$\phi_2' e^{ikT}$$

the wave height $h_2'$ is therefore

$$h_2' = \frac{1}{g} \frac{\partial}{\partial t} (\phi_2' e^{ikT}) = -i \frac{kc}{g} \phi_2' e^{ikT}, \qquad (70)$$

while the potential energy is

$$\tfrac{1}{2} \rho g \int_F h_2'^2 ds = -\tfrac{1}{2} \rho k e^{2ikT} \int_F \phi_2'^2 ds, \qquad (71)$$

and one can easily see that the second-order term $kA_{22}''$ of the added mass given in equation (69) is identical with twice the amplitude of the first-order potential energy carried in the elevated free surface. This parallels the ordinary definition of added mass which represents twice the kinetic energy of the fluid when the velocity is normalized. However, the added mass $A_{22}$ does not represent the entire kinetic energy of the fluid since the line integral in equation (62) excludes the contribution from the free surface, which, as seen in equation (65), gives an additional term

$$\int_F \phi_2'^2 ds$$

With respect to the added mass $A_{42}$, one may introduce a potential $\phi_4$ which satisfies the two-dimensional Laplace equation (18), the boundary condition (19) on the free surface, and the boundary condition

$$\frac{\partial \phi_4}{\partial n} = -n_4 = -(x_2 n_3 - x_3 n_2) \qquad (72)$$

on the ship section. Then the potential $\phi_4$ represents the motion of the fluid for a body section oscillating in roll in the free surface. Again, set

$$\phi_4 = \phi_4' + k\phi_4'' \qquad (73)$$

with the first and second-order solutions $\phi_4'$ and $\phi_4''$ satisfying, respectively, the boundary condition

$$\frac{\partial \phi_4'}{\partial x_3} = 0 \quad \text{and} \quad \frac{\partial \phi_4''}{\partial x_3} = \phi_4' \qquad (74)$$

on the free surface, and the boundary condition

$$\frac{\partial \phi_4'}{\partial n} = -n_4 \quad \text{and} \quad \frac{\partial \phi_4''}{\partial n} = 0 \qquad (75)$$

on the surface of the body, the added mass may now be written as

$$A_{42} = \rho \int_C \phi_2 n_4 ds = -\rho \int_{F+C} \phi_2 \frac{\partial \phi_4}{\partial n} ds + \rho \int_F \phi_2 \frac{\partial \phi_4}{\partial n} ds \qquad (76)$$

which may further be reduced to

$$A_{42} = A_{42}' + k A_{42}'' \qquad (77)$$

where

$$A_{42}' = -\rho \int_C \phi_2' \frac{\partial \phi_4'}{\partial n} ds = \rho \int_C \phi_2' n_4 ds \qquad (78)$$

is the first-order solution and

$$A_{42}'' = \rho \int_F \phi_2' \phi_4' ds \qquad (79)$$

the second-order term. In this case, it can also be shown that $A_{42}''$ represents twice the potential energy

$$\tfrac{1}{2} \rho g \int_F h_2' h_4' ds \qquad (80)$$

on the free surface where $h_2'$ and $h_4'$ are the wave-heights generated by the first order potential $\phi_2'$ and $\phi_4'$, respectively. However, $A_{42}''$ may be either positive or negative, depending on the form of the body section, as illustrated in Fig. 6 for Lewis sections on a free surface.

### Lateral Added Masses of Lewis Sections on a Free Surface

For Lewis sections, i.e., sections which can be mapped into a unit circle in the $\zeta_1 = \xi_1 + i\eta_1$ plane by conformal transformation

$$z = \zeta_1 + \frac{a_1}{\zeta_1} + \frac{a_3}{\zeta_1^3} \qquad (81)$$

where $z = x_2 + ix_3$, $a_1$ and $a_3$ are coefficients depending on the form of section, the first-order solution $\phi_2'$ has been obtained by Landweber and Macagno [15] and Grim [20] as

$$\phi_2' = (1 - a_1) \frac{\cos\theta}{r} - a_3 \frac{\cos 3\theta}{r^3} \qquad (82)$$

where $\zeta_1 = re^{i\theta}$. The added mass $A_{22}'$, which is half the the value for the fully submerged double body, can be shown to be

$$A_{22}' = \frac{\pi}{2} \rho \left[(1 - a_1)^2 + 3a_3^2\right] \qquad (83)$$

Furthermore, one has

$$A_{22}'' = 2\rho \int_1^\infty \left(\phi_2'^2 \frac{\partial x_2}{\partial r}\right)_{\theta=0} dr \qquad (84)$$

from equation (69). Since

$$x_2 = r\cos\theta + \frac{a_1}{r}\cos\theta + \frac{a_3}{r^3}\cos 3\theta \qquad (85)$$

as given in equation (81), it is found, by substituting equations (82) and (85) into equation (84), that

$$A_{22}'' = 2\rho \left[(1 - a_1)^2 \left(1 - \frac{a_1}{3} - \frac{3}{5} a_3\right) + a_3^2 \left(\frac{1}{5} - \frac{a_1}{7} - \frac{a_3}{3}\right) - 2a_3(1 - a_1) \left(\frac{1}{3} - \frac{a_1}{5} - \frac{3}{7} a_3\right)\right]. \qquad (86)$$

On the other hand, equation (78) gives

$$A_{42}' = -\rho \int_C \phi_2'(x_2 dx_2 + x_3 dx_3) = -\rho \int_\pi^{2\pi} \phi_2' \left(x_2 \frac{\partial x_2}{\partial \theta} + x_3 \frac{\partial x_3}{\partial \theta}\right)_{r=1} d\theta \qquad (87)$$

Noting that

$$x_3 = r\sin\theta - \frac{a_1}{r}\sin\theta - \frac{a_3}{r^3}\sin 3\theta \qquad (88)$$

as given in equation (81), one obtains

$$A_{42}' = -8\left[\tfrac{1}{3}a_1(1 - a_1) + \tfrac{1}{15} a_3(4 + 4a_1 - 5a_1^2) - \tfrac{1}{35} a_3^2(20 - 7a_1)\right] \qquad (89)$$

The potential $\phi_4'$ for Lewis sections was obtained by Grim [20] who applied a second conformal transformation

$$\zeta = re^{i\theta} + \frac{e^{-i\theta}}{r} = \xi + i\eta \qquad (90)$$

to map the unit circle in $\zeta_1$-plane into a horizontal flat plate in the $\zeta$-plane and found that

$$\phi_4' = \int_0^\infty f(q) e^{-q\eta} \sin(q\xi) dq \qquad (91)$$

where

$$f(q) = \frac{1}{2\pi} \int_{-2}^{2} [(q_1^2 - 4)(a_1 + a_1a_3 - 4a_3) + (q_1^4 - 16)a_3] \cos (qq_1) dq_1 \quad (92)$$

Interchanging the order of integrations in equation (91), one obtains

$$\phi_4' = \frac{1}{4\pi} \int_{-2}^{2} [(q^2 - 4)(a_1 + a_1a_3 - 4a_3) + (q^4 - 16)a_3] \cdot \left[\frac{q + \xi}{(q + \xi)^2 + \eta^2} - \frac{q - \xi}{(q - \xi)^2 + \eta^2}\right] dq \quad (93)$$

where the subscript of the dummy variable has been deleted.

On the free surface, $\theta = 0$ or $\pi$, therefore $\eta = 0$ and $|\xi| > 2$ as seen from equation (90), and the potential $\phi_4'$ becomes

$$\phi_4' = -\frac{1}{2\pi} \left\{(a_1 + a_1a_3 - 4a_3) \times \left[4\xi - (\xi^2 - 4) \log \left|\frac{2 + \xi}{2 - \xi}\right|\right] + a_3 \left[\frac{16}{3} \xi + 4\xi^3 - (\xi^4 - 16) \log \left|\frac{2 + \xi}{2 - \xi}\right|\right]\right\} \quad (94)$$

Substituting equations (82) and (94) into equation (79), one finds that

$$A_{42}'' = 2\rho \int_1^\infty \left(\phi_2'\phi_4' \frac{\partial x_2}{\partial r}\right)_{\theta=0} dr = -\frac{\rho}{\pi} \int_1^\infty \left(\frac{1 - a_1}{r} - \frac{a_3}{r^3}\right) \left\{(a_1 + a_1a_3 - 4a_3) \left[4\xi - (\xi^2 - 4) \log \left|\frac{2 + \xi}{2 - \xi}\right|\right] + a_3 \left[\frac{16}{3} \xi + 4\xi^3 - (\xi^4 - 16) \log \left|\frac{2 + \xi}{2 - \xi}\right|\right]\right\} \cdot \left(1 - \frac{a_1}{r^2} - \frac{3a_3}{r^4}\right) dr \quad (95)$$

where

$$\xi = r + \frac{1}{r} \quad (96)$$

The foregoing expression can be written, after integration, as

$$A_{42}'' = -\frac{\rho}{\pi} (a_1 + a_1a_3 - 4a_3) \left\{\pi^2(1 - a_1) - \left(8 - \frac{\pi^2}{2}\right) [a_3 + a_1(1 - a_1)] + \tfrac{16}{9} a_3(4a_1 - 3) + \tfrac{56}{15} a_3^2\right\} - \frac{\rho}{\pi} a_3 \left\{5\pi^2(1 - a_1) - 2\left(\tfrac{160}{9} - \pi^2\right) [a_3 + a_1(1 - a_1)] + \left(\tfrac{128}{9} - \frac{\pi^2}{2}\right) a_3 (4a_1 - 3) + \tfrac{10304}{525} a_3^2\right\} \quad (97)$$

The added-mass coefficients, defined as

$$C_{22} = \frac{A_{22}}{\frac{\pi}{2} \rho H^2}, \qquad C_{42} = \frac{A_{42}}{\frac{\pi}{2} \rho H^3} \quad (98)$$

with $H$ as the draft of ship, may then be written as

$$C_{22} = C_{22}' + kHC_{22}'', \qquad C_{42} = C_{42}' + kHC_{42}'' \quad (99)$$

where

$$C_{22}' = \frac{A_{22}'}{\frac{\pi}{2} \rho H^2}, \quad C_{22}'' = \frac{A_{22}''}{\frac{\pi}{2} \rho H^3}, \quad C_{42}' = \frac{A_{42}'}{\frac{\pi}{2} \rho H^3}, \quad C_{42}'' = \frac{A_{42}''}{\frac{\pi}{2} \rho H^4} \quad (100)$$

Introducing the area ratio

$$\sigma = \frac{A}{2bH} \quad (101)$$

where $A$ is the area of the section below the free surface, the first and second-order terms of the added-mass coefficients given by equation (100) have been calculated and plotted versus $\sigma$ in Figs. 3 through 6, taking the ratio

$$\gamma = \frac{H}{b} \quad (102)$$

as parameter. The permissible ranges of $\sigma$ have been evaluated by Landweber and Macagno [15] and are reproduced in Table 1 for convenience.

**Table 1 Permissible Ranges of $\sigma$**

| $\gamma$ | $\sigma$ |
|---|---|
| 0.6 | 0.412–0.93 |
| 0.8 | 0.353–0.942 |
| 1.0 | 0.294–0.957 |
| 1.4 | 0.379–0.937 |
| 1.8 | 0.425–0.925 |
| 2.5 | 0.471–0.914 |
| 5.0 | 0.530–0.898 |

It is obvious that the asymptotic values of lateral added masses of other ship sections such as the two-parameter forms developed by Prohaska [16] and the three-parameter forms of Landweber and Macagno [17] can also be found by the technique utilized in the present study.

## Conclusions

This paper has developed a "strip method" for the evaluation of the exciting force and moment on slender bodies, either on a free surface or fully submerged, in long regular waves. From the foregoing analysis, certain general conclusions can be drawn:

1 It is possible to determine, from two-dimensional analysis, the velocity potential attributable to the body-wave interaction up to the linear term of the wave parameter. The differential equation and the boundary conditions which govern the potential are identical with those which describe the oscillation of two-dimensional bodies in the presence of an otherwise undisturbed free surface.

2 The stripwise force and moment on a body in oblique waves are directly related to the added masses of the body section as well as to the momentum transfer through the boundary which varies from section to section.

3 The added mass of a body section when a free surface is present can no longer represent the entire kinetic energy of the fluid in contrast with the case of bodies in an infinite fluid.

4 The second-order term of the solution for the lateral added mass of a body section in the presence of a free surface represents physically the potential energy on the free surface due to waves generated by the velocity potential of the first order. The latter can be obtained from the oscillation of a double body, consisting of the submerged portion of the body-section and its image above the free surface, in an infinite fluid.

The present study has also shown that the application of momentum theorems to the evaluation of force and moment on bodies in waves offers the advantage of possible treatment of the general problem without solving for the velocity potential explicitly. Furthermore, the physical interpretation of the result thus obtained can readily be recognized.

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