A STUDY ON THE MECHANISM OF WOOL FELTING

PROEFSCHRIFT
TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN DE TECHNISCHE WETENSCHAP AAN DE TECHNISCHE HOGESCHOOL TE DELFT, OP GEZAG VAN DE RECTOR MAGNIFICUS DR O. BOTTEMA, HOOGLERAAR IN DE AFDELING DER ALGEMENE WETENSCHAPPEN, VOOR EEN COMMISSIE UIT DE SENAAT TE VERDEDIGEN OP WOENSDAG 1 JUNI 1955, DES NAMIDDAGS TE 4 UUR

DOOR

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1. INTRODUCTION

1.1. WOOL AS A TEXTILE FIBRE

Wool was probably the first fibre used by man on clothing. It was worn first in the form of a skin or pelt, later the fibres were matted or felted into a fabric. The next step in its development was the formation of the fibres into yarns, from which fabrics were constructed by weaving or knitting.

The importance of wool as a textile fibre is shown by the fact that it was not only one of the first fibres used but that wool still belongs to the most suitable materials for taylored garments and many household fabrics. Though during the last few decennia several synthetic fibres have been developed and produced, wool will probably maintain its unique position, since it possesses a combination of certain highly desired properties, which is not found in other textile fibres. Some of the special features of wool will be mentioned.

Firstly a wool fabric shows a high amount of wrinkle resistance and stability of shape which is due to the excellent recovery from deformations of the fibre.

Secondly wool possesses an outstanding thermal insulating power. This is partly due to the natural curliness of the fibre which causes a rather bulky cloth with much air in it. Furthermore, a very important contribution to the thermal behaviour is formed by the great water absorbing power of wool, together with its large heat of wetting giving rise to a protection against sudden changes in temperature and relative humidity.

Thirdly the feltability has to be mentioned. A slight degree of felting converts a woven fabric into a thicker and utterly more compact structure which can be applied in garments and blankets. When felting is continued, wool felt is obtained which is of considerable importance for several industrial purposes.

Beside these and other advantages wool shows some serious defects.

Wool is subject to biological attack; the fibre is readily eaten by moth larves and some other insects, a fact which causes large economic losses.

A second defect of wool is its shrinkage, this being an unwanted decrease of the dimensions of a fabric during its use. Now two different kinds of shrinkage must be sharply distinguished, namely the shortening of the fibres themselves and the migration of the fibres with respect to each other.

The first form of shrinkage is called relaxation shrinkage and is shown by nearly all kinds of textile fibres. Generally it results from the previous treatment, for instance in the following way: when a cloth is wetted and put under a certain small tension it gets an elongation which recovers in most cases when the tension disappears. When, however, the cloth is dried without having the possibility of retracting, it retains its deformation. In practice the finishing of the textile fabrics often gives rise to the occurrence of this phenomenon. When, during washing, wetting takes place again, the quasi permanent deformations recover, all fibres are shortened, and the shrinkage is a fact.

The felting shrinkage, on the contrary, is only met in assemblies of animal fibres. In felting the length of the individual fibres is not affected, but they migrate with respect to each other. During this migration a fibre which lies originally stretched in the yarn of which the fabric is composed, undergoes displacements and attains a looped and curled position. In this way its contribution to the length of the yarn decreases, which is accompanied by shrinkage of the cloth. The yarn and the fabric, however, increase in thickness.

The special feature of animal hair fibres by which they and only they show felting power, is their scaliness which gives rise to a difference in friction in the two directions along the fibre. Thus irreversible displacements occur, whereas for fibres without scales the total displacement under the influence of forces directed at random is zero.

This felting phenomenon which, as has been pointed out, possesses its good and its bad aspects, will be the subject of the present study.

1.2. PROPERTIES OF THE WOOL FIBRE

1.21. *Geometry and structure*

The dimensions of the wool fibre show a very large variety. They depend largely upon the geographical origin and the kind of sheep from which the wool is obtained. Fibres of 3 cm in length are found as well as those of 30 cm; the diameters of high quality wool and very coarse kinds may amount to 10 and 60 μm respectively.

Generally the fibres are crimped: they possess a number of curls and waves which are most pronounced in short and thin fibres.

Essentially a wool fibre consists of two components, the cortex and the scale layer. A third component, the medulla, is generally present in animal hairs but not always in wool fibres. In fig. 1 a schematic representation of a longitudinal cross-section is given from which it can be seen that the cortex consists of spindle-shaped elements, and the outer layer of overlapping scales with the edges pointing in the direction of the tip of the fibre. Recent investigations have shown that the scales are covered with a very thin (0.01 μm) and very resistant membrane, the epicuticle; a similar membrane, the subcuticle seems to be present between the scales and the cortex.

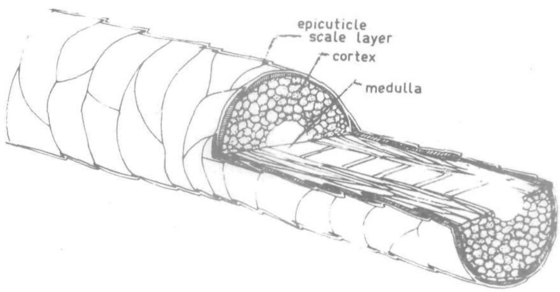

Fig. 1. Structure of a wool fibre

1.22. *The directional frictional effect (d.f.e.)*

As mentioned above (section 1.1) the scaliness of the wool fibre is responsible for the felting phenomenon. Due to this scaliness the fibre exhibits a higher friction when it moves with its tip forward than in the reverse direction. Though the present study only deals with this effect in its relation to the fibre migration, some of the theories which have been established to account for the d.f.e. will be summarized.

Essentially two different opinions have been presented. The first of these starts from the geometry of the scales, the second from a molecular constellation.

The "ratchet theory" in its simplest form which supposes the interlocking of scales, has encountered several difficulties on being examined experimentally. Martin [26] showed that a d.f.e. still existed when a wool fibre was rubbed against a polished glass surface though no interlocking or ploughing action of the scales was to be expected in this case. Viewing these and other objections Rudall [36] proposed a model with flexible scale tips; this model is also able to explain the decrease of d.f.e. after various chemical anti-shrink processes, in terms of softening or even a removal of the scale tips. Makinson [23] performed a theoretical investigation of this model and found simple relations between the frictional coefficients and the angle of friction, the average angle of contact of the surfaces and the shearing force required to separate two asperities in contact. Lincoln [17] applied a purely geometrical treatment on this problem, assuming that the real areas of contact are formed by elastic deformation of the scale material. In this way he was able to predict the variation of friction with load, swelling, fibre diameter, scale distribution and roughness of the second surface.

A different explanation was proposed by Martin [27] who assumed the d.f.e. to originate not from the microscopic structure but from an asymmetric molecular field at the surfaces of the scales. Some experimental evidence appeared to be present for this theory. However, Thomson and Speakman [45] covered some wool fibres with a very thin (0.03 μm) layer of silver or gold and showed experimentally that, though the scales were entirely coated, the fibres still showed a d.f.e. Martin's theory is, therefore, generally rejected.

1.23. *The stress-strain diagram*

A first impression of the elastic properties of the wool fibre can be obtained from the stress-strain diagram. In fig. 2 diagrams are reproduced for a wool fibre in dry and in wet condition.

The first part of the diagrams (OA) is characterized by a slow increase of the stress. This is due to the crimp of the fibre, the removal of which requires a small force. When the fibre is stretched completely a region follows in which Hooke's law is valid (AB). The slope of this straight line is the Young's modulus E of the fibre. At B an apparent yield point can be noticed; the slope of the curve decreases sharply. When the deformation is continued, the slope of the diagram increases somewhat just be-

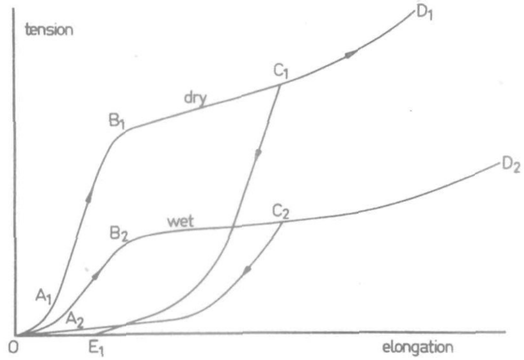

Fig. 2. Stress strain diagrams for a wool fibre in dry and in wet condition

fore the fibre breaks at D. When, however, at C the strain direction is reversed the fibre is allowed to recover. The first part of the recovery curve runs parallel to the part AB of the loading curve, but afterwards its slope decreases. In wet condition the fibre recovers immediately to its original length. The dry fibre, however, retains a deformation OE_1 which recovers only slowly and not always completely, dependent on the total strain applied.

Several parameters of the diagram appear to depend on the rate of elongation; especially the position of B, the apparent yield point, is rather time sensitive.

Between the diagrams of the fibre in wet and in dry condition the principal differences are found in the slope of the straight part AB and in the recovery. These differences can be interpreted by the following simplifying reasoning.

The molecular chains possess several points of interaction between each other. The most important of these linkages are the hydrogen bonds, the salt linkages and the disulfide bonds. These interactions together contribute to the total stiffness of the fibre in dry condition. When, however, the fibre is swollen in water, the salt linkages and the hydrogen bonds are broken or weakened, whereas the disulfide bonds remain unaffected.

During deformation of the dry fibre the salt and the hydrogen bonds are ruptured and are reformed on other spots; the disulphide bonds, however, remain unbroken. The retraction of the fibre during recovery, is partially prevented by these newly formed interactions. In the wet fibre, however, only the disulphide bonds are present, which cause a complete retraction.

These facts account for the smaller stiffness of the wet fibre and for its better recovery power.

1.3. THEORIES OF FELTING

The earliest theories of felting date from the beginning of the 19th century. The scale structure of the wool fibre not yet being discovered, the felting was mainly attributed to the tendency of wool fibres to curl and thus to become entangled.

The first investigator who took notice of the influence of the scaliness on felting was Bowman (1885) [8]. He introduced the interlocking scale theory in which the fibres are supposed to lock into each other by means of their scales and to form in this way an irreversible structure. This theory met the criticism that the scale geometry is not regular enough to enable such interlocking over a number of consecutive scales and that microscopically this phenomenon is never observed. The theory was, therefore, modified in this respect that only few points of interlocking will occur on each fibre. Though the interlocking scale theory has generally been rejected during the last few decennia, some investigators are inclined to believe that it still comprises much of the truth.

Ditzel (1891) [12] was the first who drew the attention to the role played by the root ends of the fibre. His experiments on wool locks, arranged in two ways, viz. with root ends and with tip ends facing each other, clearly showed the unidirectional nature of the fibre movements towards the root end.

These ideas were taken up in 1944 by Martin [27] who prepared some locks of wool by removing the scales from either the root ends or the tip ends. Shrinkage experiments on knitted fabrics made from these fibres also showed the importance of the root end in felting.

In order to explain the mechanics of the fibre migration, Arnold [1] supposed the wool fibres to move by a series of alternate elongations and contractions, somewhat in the same way as a worm crawls. Under the influence of external action the fibres are preferably extended in the direction of their root ends. When released, the fibres recover from this elongation and contract again in the root direction. A repetition of extension and recovery cycles may cause a considerable displacement of the fibre.

Shorter [38] assumed a different migration mechanism. He emphasized the occurrence of fibre entanglements. In a complete entanglement no fibre movement is possible at all; in a partial entanglement the fibre can only move in the root direction. Shorter showed that these spots give rise to a tightening up of the network and to the formation of loops in the fibres when randomly directed forces act on the fabric under suitable conditions. This mechanism requires a low flexural rigidity of the single fibres in order to enable loop formation.

Martin [27] paid special attention to the compression which he considered necessary for the occurrence of felting. During washing or milling the fabric is subjected to many deformations, most of which have a compression component in one direction or another. The recovery from the compression to which the fabric is subjected is hindered by the fibres which migrate into the openings.

Though each of the ideas summarized above assumes a different mechanism of felting, it remains quite well possible that in more than only one way fibre migration takes place and that several mechanisms cooperate together. Which of these predominates may depend on the local fibre constellations and on the nature of the mechanical agitation applied.

1.4. FACTORS WHICH INFLUENCE FELTING

1.41. *Classification*

From the results of the research of many investigators it has been established that felting depends upon a large number of factors. In attempting to arrive at a systematic classification of these factors, it appears that they can be divided into two principal classes, *viz.* those which are inherent in the material and those arising from the external conditions under which the felting occurs.

Consequently the following scheme can be made:

- felting
  - properties of material
    - structure
      - fabric structure
      - yarn structure
    - fibre properties
      - friction
      - elasticity
      - geometry
  - external conditions
    - time
    - mechanical action
    - medium

This scheme is, however, not entirely satisfactory since several factors mentioned are not independent of each other. The various single fibre properties are, for instance, often mutually correlated. The medium may influence the felting through the fibre friction or fibre elasticity as well as through the forces which the liquid acts on the fabric. Consequently, a sharp distinguishment is often difficult.

The most important conclusions from the literature about the effect of the various factors will be briefly summarized.

1.42. *Structure*

Johnson [16] investigated the influence of weave structure of wool fabrics on felting. He found that the tighter the weave and the greater the concentration of threads in the fabric, the less the fabric will shrink. Bogaty and co-workers [5] performed a similar investigation and came to analogous results namely that the compactness of the cloth is the most important factor.

This result is understandable since, when the structure is made tighter, the fibres are packed together more closely thus being hindered in their migration.

An exception is sometimes met in very loosely knitted fabrics which during washing extend rather than shrink. Presumably, in this case the distances between the yarns in the knitted structure are too great to be overcome by the migrating fibres so that no tightening action can take place.

1.43. *Fibre properties*

The influence of fibre dimensions on felting was investigated by various workers by comparing the shrinkage of identical types of fibre assemblies composed of different wool fibres.

As to the relation between felting and fibre diameter, contradictory results are reported. Some investigators found a higher felting power for finer fibres [41]; others, however, reported the same for the coarser ones [43], whereas a third group [27][39] did not find any correlation between these two quantities. Probably this discrepancy is brought forth by other factors which cannot be kept constant, *e.g.* frictional properties and yarn structure; furthermore the difference between the felting methods used may give rise to contradictory results.

The influence of fibre length on felting is studied by Speakman *et al.* [42] and by Sookne *et al.* [39], who both ascertained a higher amount of felting for longer fibres.

Since the directional frictional effect is considered to be the most important factor in felting, many investigations have been performed on the relation between felting and the frictional coefficients of the wool fibre. In most cases a comparison is made between unmodified wool and wool which has undergone an anti-shrink treatment. The majority of these treatments modify the frictional coefficients. Sometimes the lower of the two, μ_1 (the "with-scale" coefficient), is raised; sometimes the higher one, μ_2 (the "anti-scale" coefficient), is lowered; sometimes both are raised. Since the difference of the two coefficients is important in felting, the tendency exists to reduce this difference, but it has appeared that an equal increase of both μ_1 and μ_2 is also effective in reducing the shrinkage.

Several combinations of μ_1 and μ_2 have been proposed as parameters which are significant for felting, namely

$\frac{\mu_2-\mu_1}{\mu_1}$ [41], $\frac{\mu_2-\mu_1}{\mu_2+\mu_1}$ [31], $\mu_2-\mu_1$ [6)22)40], $\frac{1}{\mu_1}-\frac{1}{\mu_2}$ [19], and $\frac{1}{\mu_1+\mu_2}$ [20].

Most of these combinations are purely empirical and are not related to the mechanism of felting. Lindberg [19], however, introduced the expression $(1/\mu_1 - 1/\mu_2)$ on a theoretical basis. His reasoning was as follows: Consider an idealized system of fibres in which each fibre can move into two opposite directions and is subjected to a certain normal pressure N. A force acts on a fibre alternatively in both directions during short periods and transfers to the fibre energy quants of an average value E. The displacements of the fibre in both directions are l_1 and l_2, respectively. Thus

$$E = N\mu_1 l_1 = N\mu_2 l_2 \qquad (1)$$

and the resulting movement which is a measure for the shrinkage is

$$l_1 - l_2 = \frac{E}{N}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right) . \qquad (2)$$

This theory is, however, essentially wrong, because of the assumption that the transferred energy is equal in both directions. When, for instance, the forces are too small to cause a movement in the anti-scale direction, no energy will be transferred at all.

Recently, Lindberg [20] put forward another theory in which the volume shrinkage is related to the fibre friction. Now he starts from the expression

$$Q = \lambda\overline{W}\frac{\mu_{2k} + \mu_{1k}}{2} \qquad (3)$$

in which Q is the energy transferred to the material, λ is the total length of all the fibre displacements, $\overline{W}$ is the average normal pressure between the fibres, and μ_{2k} and μ_{1k} are the coefficients of kinetic friction.

Equation (3) is written in the form

$$\frac{d\lambda}{dQ} = \frac{1}{\overline{W}} \cdot \frac{2}{\mu_{2k} + \mu_{1k}} . \tag{4}$$

For the volume decrease Lindberg assumes the equation

$$\frac{dV}{dt} = - cV . \tag{5}$$

In order to relate eq. (4) to eq. (5), the following proportionalities are postulated:

$$dQ \; (:) \; dt \tag{6}$$

$$dV \; (:) \; d\lambda \tag{7}$$

and

$$\frac{1}{\overline{W}} \; (:) \; V . \tag{8}$$

Substitution leads to the expression

$$c = c_1 \frac{2}{\mu_{1k} + \mu_{2k}} \tag{9}$$

which relates the friction to the rate of shrinkage. Some experiments seem to confirm this relationship. Nearly all the assumptions made are, however, very disputable. If, according to eq. (7), λ denotes the total *resulting* fibre migration, the expression for Q in eq. (3) cannot be valid since Q, the total amount of transferred energy, is used for displacements in both directions. Furthermore, it is not probable that the load on the fibres should be inversely proportional to the volume (eq. (8)). The most serious error is the assumption of eq. (6), because the transferred energy depends on the distances over which the exerted forces are able to move the fibres. When felting is completed the fibres have ceased to migrate and the work per unit time performed by the externally acting forces, has become zero.

Various other investigators have tried to correlate friction and felting. None of the functions of μ_1 and μ_2 mentioned above, can, however, be favoured as being significant for felting since the spreading of the test results is, in general, high. Sookne and co-workers [40] established an empirical relationship between the shrinkage of wool tops and $\mu_2 - \mu_1$. They, however, admit that the application of one of the other combinations of the coefficients of friction gives rise to an equally good correlation.

The results obtained by different investigators are often contradictory. This may be due to differences in methods of felting, in external circumstances or in the materials used.

The influence of elastic properties has not obtained so much attention as the friction, because the directional frictional effect is more spectacularly involved in the felting process than the elasticity.

Speakman, Stott and Chang [42] were the first to point out the importance of easy extensibility and recovery power of the fibre. They derived this statement from their measurements of shrinkage and elastic properties at various temperatures and pH-values of the medium.

Menkart and Speakman [29] treated wool with mercuric acetate and with benzoquinone. These agents enlarge the fibre stiffness by cross-linking the molecular chains. Both treatments reduced the shrinkage though the fibre friction was not affected.

Bogaty, Sookne and Harris [4] studied the relation between felting and elasticity by using a number of different salt solutions as media. They established a correlation between felting and work for extension as well as between felting and resilience *). The existence of these two correlations which are based on the same series of experiments suggests that the work for extension and the resilience are mutually related. It is, therefore, not possible to ascertain which of the two parameters is the one important in felting.

1.44. *Time of felting*

In nearly all the felting experiments the mechanical agitation is applied during a fixed period after which the shrinkage is measured. Some investigators studied the shrinkage as a function of time, and found, in general, a decrease of the rate of shrinkage with increasing time.

Creely en Le Compte [11] expressed the shrinkage of yarns in the form

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*) The resilience is the ratio of work recovered to that required for 20% extension.

$$\frac{dl}{dt} = -cl \qquad (11)$$

in which l is the yarn length. The solution of this equation is

$$l = l_o e^{-ct} . \qquad (12)$$

Since, according to (12), l approaches to zero, eq. (11) can only be valid for not too high values of shrinkage.

Johnson [16] derived from his shrinkage tests on wool fabrics, for the area shrinkage, a, the equation

$$\frac{da}{dt} = k\ (x - a) \qquad (13)$$

which leads to $$a = x\ (1 - e^{-kt}) . \qquad (14)$$

This formula is a better representation of the real shrinkage behaviour than eq. (12), since it allows the shrinkage to proceed to any limit x between 0 and 1.

1.45. *Mechanical action*

The intensity of the applied mechanical agitation is a factor which has, in general, been overlooked. Shrinkage tests are in most cases performed in a commercial or a laboratory washing machine which runs at a constant speed. Some investigators have noted the importance of the effect of the mechanical action on shrinkage, which usually led to special efforts being made to keep the mechanical conditions as constant as possible [11][33][37]. Carter and Grieve [9], in their experiments with the so-called wash weel, paid more attention to the forces involved, by examining the effect of the quantity of water, of the rate of shaking, and of the size of the balls used for applying impulses to the cloth. They, however, did not establish a systematic relationship between shrinkage and mechanical action, presumably because the method they used, was rather complicated.

1.46. *Medium*

It is a well-known fact that wool felts only in moist or in wet condition, which demonstrates clearly the importance of the medium. In section 1.41 it has already been mentioned that the effect of the medium can largely be reduced to the influence of other factors, fibre friction and fibre elasticity being probably the most important ones in this respect. For a fibre in dry condition the frictional difference and the extensibility are both much smaller than for the fibre swollen in water.

Several investigators studied the shrinkage as a function of temperature and pH of the medium or they performed their experiments in various solutions. In so far as the results of these investigations fall within the scope of this chapter, they have already been discussed in section 1.43.

It may be possible that also the viscosity of the medium influences felting since the hydrodynamic forces which act during shaking may depend on the viscosity. Preston [34]) observed a decrease of felting with increasing viscosity. In his experiments, however, the thickened liquid was agitated to form a foam from top to bottom before the fabrics were inserted. This makes the interpretation of the observed phenomenon rather difficult.

1.5. SCOPE OF THE PRESENT STUDY

In the foregoing sections it has appeared that a great number of factors may influence felting. The aim of the present investigation is to separate the effects of several of these factors and, in particular, to study quantitatively the role played by the forces which are responsible for the fibre migration.

Besides, it will become clear that knowledge about the influence of the forces leads in a natural way to a better understanding of felting in relation to the single fibre properties.

2. MEASUREMENT OF SHRINKAGE

2.1. DISCUSSION OF THE METHOD

Most investigators determine the shrinkage capacity of wool by shaking a piece of fabric with a liquid in a commercial washing machine or in a laboratory shaking equipment.

Sometimes extra mechanical means are used, *e.g.* steel or rubber balls. After a certain period of shaking the shrinkage in area or in one of the dimensions is measured. Instead of a fabric, other workers use a yarn [10][30] or a top *), prepared in a special manner [3].

In the present investigation it was decided to use a knitting yarn as the material to be tested. This choice was made on the basis of the following considerations:

a. The experiments on yarns need less material than tests on fabrics or tops require. This is of particular importance when a chemical treatment on laboratory scale has to be applied in order to modify the single fibre properties.
b. Felting tests are performed more simply on yarns than on fabrics or tops in that a smaller testing machine and a smaller amount of liquid can be used.
c. The measurement of yarn shrinkage only consists of a simple determination of the length of the yarn; fabric shrinkage in area or in length is more difficult to perform reproducibly.

Though in practice felting takes place in cloth, there are no reasons to suppose that the mechanism of fibre migration in yarns will differ essentially from that in more complicated structures.

The material being chosen, the problem remains how to bring about the felting of the yarns. In principle each kind of mechanical agitation will serve the purpose, but a suitable method has to meet the following requirements: The reproducibility should be as high as possible; the forces applied to the fibres in the yarn have to be adjustable within wide limits; preferably the magnitude of these forces should be known.

*) A top is the product of one of the first stages of the spinning process; it is twistless and contains a large number of fibres in the cross-section.

Concerning this last requirement it was considered that the simplest form of mechanical action in which the forces are known is a periodical longitudinal stretching. Though shrinkage of the yarns as a result of a repeated stretching and releasing is not very probable, nevertheless some experiments in this direction were made. A number of yarns were suspended in water, each of them being stretched with a moderate weight which was periodically lifted. Sometimes a small amount of shrinkage was observed, especially when the weights were lifted higher so that the yarn became more slack. If an unbalanced twist in the yarn gave rise to crinkling during the unloading period, the shrinkage appeared to be greater. From these observations it becomes clear that not the repeated stretching itself is the important factor but rather the movements of the yarn during release, and that the shrinkage is especially favoured by a periodic torsion.

After this first attempt a periodic twisting and untwisting of the yarn was tried. The shrinkage obtained in this way, however, was rather small. Besides, the magnitude of the mechanical action could not be regulated within wide limits, since, at a higher twist, very soon uncontrolled crinkling of the yarns occurred.

A third attempt consisted of a mechanical rubbing of the yarns between two flat surfaces under adjustable normal pressure. In this case a considerable amount of felting took place indeed. The reproducibility was, however, very poor due to the uncontrollable position of the yarn between the surfaces. An improvement of this method could not be realized.

Finally the yarn was inserted into a tube partially filled with liquid and the tube was shaken. The shrinkage which occurred was satisfactory but not quite well reproducible. When, however, the yarn was attached to both extremes of the tube, its more homogeneous distribution gave rise to a better reproducibility. The mechanical agitation could be varied considerably by changing the rate of shaking, though the actual magnitude of the forces was unknown. Despite of this disadvantage it was decided to use this method for the determination of the felting power of yarns.

2.2. DESCRIPTION OF THE APPARATUS

The shaking movement of the tubes is derived from a rotation in order to avoid large inertial forces. Fig. 3 and fig. 4 re-

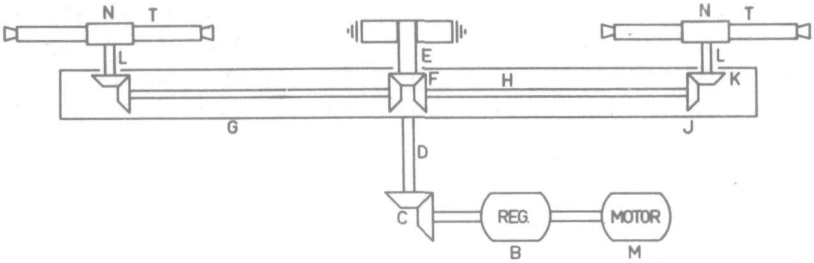

Fig. 3. Scheme of the apparatus for yarn shrinkage

present a schematic view and a photograph of the arrangement. A rotating arm, G, is driven by a synchronous motor, M, via a shaft, D, a transmission, C, and a continuously variable speed control, B. The arm is provided with two clamping blocks, N, each of which can hold 8 tubes, and which are attached to the shafts L. By means of a stationary bevel gear wheel, E, and the transmission elements F, H, J and K, the shafts L are made to rotate in a direction opposite to that of the main shaft D. Consequently, the movement of the blocks, N, with the tubes, T, constitutes only a translation, so that during the rotation of the arm each tube remains in a fixed direction.

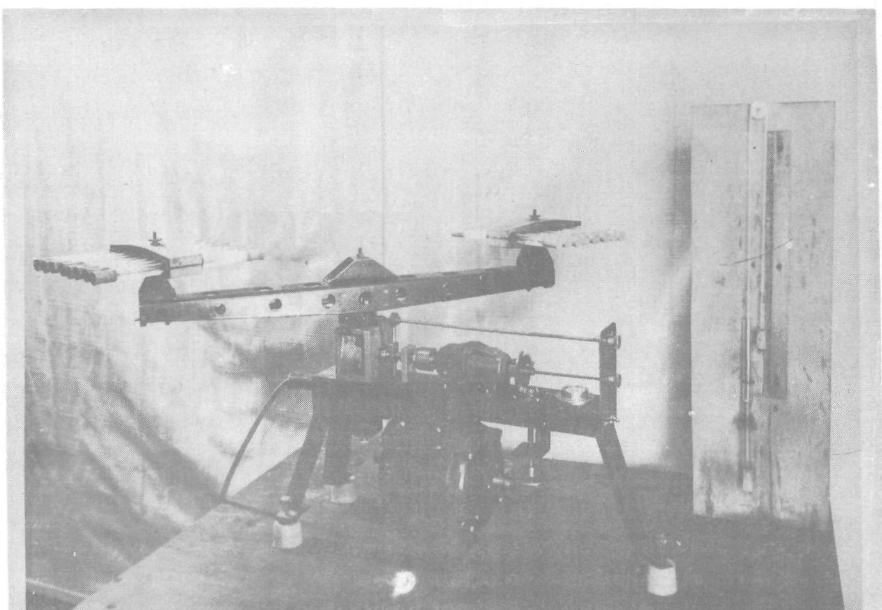

Fig. 4. General view of the apparatus for yarn shrinkage

As a result of this movement the liquid in the tubes streams alternatively from one side to the other, thus exerting hydrodynamic forces on the fibres in the yarn. These forces can be varied in two ways. With the aid of the control, B, the rotation speed of the arm can be regulated between 20 and 300 r.p.m. In addition, the blocks N can be placed at three different distances from the shaft D. The diameter, *A*, of the circle described can thus be adjusted to 60, 45 and 30 cm, respectively. These two features make it possible to investigate the extent to which the shrinkage is influenced by the applied forces. In the majority of the experiments only the speed control is used and the diameter *A* is adjusted to its largest value, 60 cm.

When starting the rotation, the definite speed required can only be attained very slowly owing to the high moment of inertia of the arm with the tubes. Particularly in the case of high speeds when the time intervals after which the yarn length is measured, are very short (10 s for instance), this circumstance may give rise to incorrect results. It is therefore, desirable to prevent the shaking of the liquid in the tubes during the speeding up of the arm. For this reason, the bevel gear wheel E has been designed so that it can be disengaged. By doing this, the shafts H and L, as well as the tubes, will be stationary with respect to the arm G and no shaking movement will take place. The rotation speed of the apparatus can then be slowly raised to the required value; when this speed is reached the bevel gear wheel E is engaged by means of a clamping device and, after the required shaking period has elapsed, it is once more disengaged.

A revolution counter is put into operation when E is engaged, thus registering the number of revolutions during which shaking takes place.

2.3. PERFORMANCE OF THE MEASUREMENTS

The yarn used throughout this investigation is an untreated three ply knitting yarn indicated as E_o. It is composed of fibres with an average length of 7.10 cm and an average diameter of 23 μm.

The metric count of the fibres, Nm, is 1850, which means that the mass of 1850 meters of the fibre is 1 gram. The yarn consists of three threads. Each of these contains on the average 115 fibres in the cross-section and has a metric count Nm 16.1. The single thread contains a twist of 1.9 turns per cm; in the composite yarn the three threads are twined round each other with 0.8 turns per cm in a direction opposite to the single thread twist.

The shrinkage test is performed as follows: First a length of yarn of about 110 cm is cut off and is soaked for 10 min in the liquid in which the felting will take place. By doing so the relaxation shrinkage is eliminated. Then the yarn is attached to the rubber stoppers after its length has been adjusted to 100.0 cm, measured under the tension exerted by one of the stoppers. For this purpose the tube is clamped in a vertical position while one of the stoppers seals off the lower end; the yarn is led over a pulley and the second stopper indicates its length on a graduated scale. The tube is then half filled with the shaking liquid, the yarn is inserted into it, and the tube is closed and placed into one of the clamping blocks of the apparatus. After a certain period of shaking the yarn length is measured again, whereupon the yarn is returned to the tube and the shaking is resumed. In this manner the shrinkage is determined as a function of time. The way of measuring the yarn length provides the possibility to keep the liquid in the tubes during the measurement and to measure the length under a constant tension.

Some of the results of the shrinkage determinations are reproduced in table I. This table reports measurements made at a speed of 100 r.p.m. and a diameter of movement of 60 cm, on the untreated standard sample E_o. In this experiment eight yarns were shrunk simultaneously. The columns s_1 to s_8 show the shrinkage of each of the yarns as a function of time, expressed as a percentage; $\bar{s}$ denotes the mean shrinkage and σ the standard deviation of this mean. The standard deviation is small enough to consider the experiments as being satisfactorily reproducible. When the yarns are treated with anti-shrink agents, sometimes larger variances are found, frequently due to non-uniform treatment.

In fig. 5 the average value of the shrinkage, $\bar{s}$, is plotted as a function of shaking time. In the next section this curve will be discussed.

2.4. PARAMETERS OF SHRINKAGE-TIME BEHAVIOUR

For an efficient comparison of the shrinkage under different circumstances it is desirable to express the shrinkage dependence

Table I
Shrinkage of eight yarns as a function of time

Time (min)	Number of completed revolutions N	s_1 (%)	s_2 (%)	s_3 (%)	s_4 (%)	s_5 (%)	s_6 (%)	s_7 (%)	s_8 (%)	$\bar{s}$ (%)	σ (%)	$65.5(1-e^{-0.235t})$	$\frac{\bar{s}}{N}$
0	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
1	100	13.8	12.4	14.6	15.8	14.9	14.0	12.8	13.7	14.0	1.2	13.3	0.140
2	200	25.3	22.5	25.1	26.5	26.7	24.1	23.1	24.8	24.8	1.4	24.6	0.124
4	400	40.5	38.7	37.5	40.5	40.6	37.8	39.2	40.0	39.4	1.1	39.9	0.099
6	600	51.2	47.6	48.0	49.8	49.8	47.8	48.0	50.2	49.0	1.4	49.5	0.082
8	800	56.5	55.2	54.3	56.3	55.4	53.9	55.1	55.4	55.3	0.9	55.5	0.069
10	1000	61.0	58.7	57.9	59.9	59.8	57.9	58.3	58.8	59.0	1.2	59.2	0.059
12	1200	62.9	61.4	61.4	62.0	61.1	60.5	60.1	61.3	61.3	0.9	61.7	0.051
14	1400	64.3	63.0	62.3	64.0	63.3	62.5	62.9	63.0	63.2	0.7	63.2	
18	1800	65.9	64.8	65.0	65.5	65.3	64.4	64.6	65.2	65.1	0.5	64.7	
22	2200	67.6	66.4	66.0	66.5	66.7	66.2	65.4	67.4	66.5	0.7	65.2	
26	2600	67.7	66.3	66.0	67.2	67.0	66.3	65.7	67.4	66.7	0.7	65.4	
34	3400	68.0	66.3	67.8	67.8	67.6	66.8	67.1	67.9	67.4	0.7	65.5	

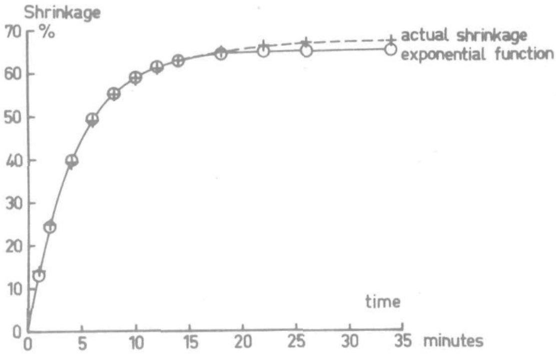

Fig. 5. Shrinkage, s, as a function of time, compared with exponential function

on time in a small number of parameters. The most obvious way in this respect is to represent the shrinkage-time curve as a whole by means of a mathematical expression. The shape of the curve suggests the validity of an exponential function of the form

$$s = a(1 - e^{-bt}) \ . \tag{1}$$

It appears that for the first half of the time interval the best fit is obtained with

$$s = 65.5\ (1 - e^{-0.235t}). \tag{2}$$

For longer times, however, the experimental curve deviates from this exponential one (table I and fig. 5). Presumably the addition of a second function of this kind with much smaller values of a and b would be able to account for these deviations; the determination of the four parameters then needed is, however, rather complicated and time consuming.

Moreover, even in the case of a single exponential function, the parameter b is not of any direct practical significance, though a can be interpreted simply as the limit of the shrinkage for infinitely increasing time of shaking. It is therefore better to express the course of the shrinkage with the aid of the quantities

$$s_\infty = \lim_{t\to\infty} s(t) \tag{3}$$

which is the shrinkage limit, and

$$\dot{s}_o = \lim_{t \to o} \frac{ds}{dt} \tag{4}$$

which is the initial rate of shrinkage. When equation (1) is valid, it is clear that $s_\infty = a$ and $\dot{s}_o = ab$. The use of these parameters has also the advantage that $\dot{s}_o$ can be determined without measuring the shrinkage up to the limit.

The use of $\dot{s}_o$ is, however, not yet entirely satisfactory. An increase in the rate of shaking not only causes the force to increase but also the number of times this force transmits an impulse to the fibres in the yarn. For this reason it is preferable to express the initial shrinkage not per unit of time, but rather per revolution of the apparatus. At the same time the experiments will then correspond more logically with those in which the amplitude of the shaking movement is varied, since this causes a change in the forces but not in the number of times they act on the yarn. The initial shrinkage per turn equals $\dot{s}_o/n$, if n represents the number of revolution per minute. This quantity, which will be denoted by S, can also be defined as

$$S = \lim_{N \to 0} \frac{ds}{dN} \tag{5}$$

in which $N = nt$, the total number of completed revolutions.

The two parameters s_∞ and S can be determined graphically in a fairly simple manner. The shrinkage limit, s_∞, can be estimated from the graph, if necessary with the aid of a logarithmic time scale. The quantity S can be approximated by plotting $\log(\Delta s/\Delta N)$, being the logarithm of the ratio between the shrinkage increase during a certain interval and the number of rotations during this interval, against N. By extrapolating the curve thus obtained towards $N = 0$, S can be estimated. The same result is, however, reached when s/N is taken instead of $\Delta s/\Delta N$, since

$$\lim_{N \to 0} \frac{s}{N} = \lim_{N \to 0} \frac{\Delta s}{\Delta N} = \left(\frac{ds}{dN}\right)_{N=0} . \tag{6}$$

The quantity s/N is calculated more easily and shows a smaller spreading than $\Delta s/\Delta N$. It is therefore used in most cases. Fig. 6 shows how from the plot of $\log(s/N)$ against N, an approximation of S is obtained. From the data recorded in fig. 5 and fig. 6 it appears that $s_\infty = 69\%$ and $S = 0.158\%$/turn. These values roughly represent the shrinkage as a function of time.

In some cases it is found that the slope of the shrinkage-time

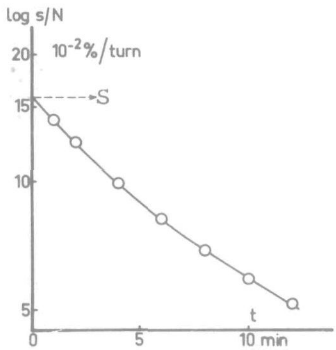

Fig. 6. Determination of S from the data of table I

curve firstly increases, then reaches a maximum, after which it decreases in the normal way. It is obvious that in this case the maximum slope is more significant for the shrinkage behaviour than the initial one. In this case the ratio s/N also shows a maximum which has, however, not the same values as $(ds/dN)_{max}$. When this phenomenon occurs, for S is taken the maximum reached by $\Delta s/\Delta N$ as a function of N.

2.5. INFLUENCE OF THE QUANTITY OF LIQUID

To ascertain the extent to which the shrinkage is affected by the quantity of water present in the tubes, the shrinkage of the standard sample, E_o, was measured with liquid volumes, f, of 1/8, 2/8, 3/8, 4/8, 5/8, 6/8 and 7/8 of the total volume of the tube at a speed of 157 r.p.m. With each different liquid volume, the shrinkage of four yarns was measured; S was derived from the average results. The values of S with reference to the quantity of water are given in table II and fig. 7. It was found, that the effect is very slight at about $f = ½$, which is generally used, so that measurements of S are not affected by small variations in the quantity of liquid. In all these experiments the limit s_∞ remained unchanged at about 70%.

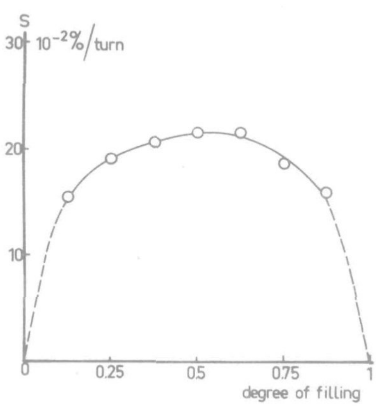

Fig. 7. Dependence of S on quantity of liquid in the tubes

Table II
Influence of the quantity of liquid

f	S (10^{-2}%/turn)
1/8	15.3
2/8	19.0
3/8	20.5
4/8	21.4
5/8	21.4
6/8	18.4
7/8	15.8

3. INFLUENCE OF FORCES

3.1. EFFECT OF SHAKING RATE

The standard sample, E_o, was shaken at various rates and the diameter A adjusted to 60 cm. Most measurements were carried out on four or eight yarns at a time. From the average shrinkage-time behaviours the parameters S and s_∞ were determined in the way described in the preceding chapter. The values found for these parameters as a function of the rate of shaking are recorded in table III and fig. 8.

Table III
Effect of shaking rate on the shrinkage parameters for yarn E_o

n (r.p.m.)	S (10^{-2}%/turn)	s_∞ (%)
25	0.0	2
30	0.8	37
40	2.4	69
56	6.7	69
75	10.1	70
100	15.4	69
128	18.5	71
156	19.0	70

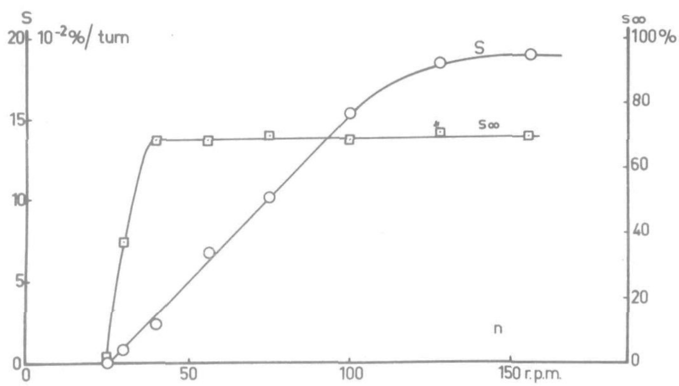

Fig. 8. Initial shrinkage per turn, S, and shrinkage limit, s_∞, as a function of rotational speed, n

It appears from fig. 8 that below a certain rate of shaking no shrinkage occurs at all. The magnitude of this threshold, n_1, is for the experiment under discussion 25 r.p.m. When $n < n_1$, S and s_∞ are both zero. For speeds exceeding the threshold, S appears to be proportional to $(n - n_1)$ up to n = 100 r.p.m. Though the rate of shrinkage is strongly force dependent, the limit of shrinkage is constant at approximately 70%, except in the immediate neighbourhood of the threshold, where the curve for s_∞ takes a very steep course. Apart from this restriction, the constancy of s_∞ means that when shrinkage occurs, the yarn always shrinks down to 30% of its original length, independent of the applied forces.

3.2. EFFECT OF SHAKING AMPLITUDE

The same experiments as described in the preceding section were carried out again with yarn E_o, but this time also other amplitudes, viz. A = 45 cm and A = 30 cm, and higher rates of shaking were applied. In these experiments in most cases only S was determined, the measuring of the limit s_∞ being particularly time consuming.

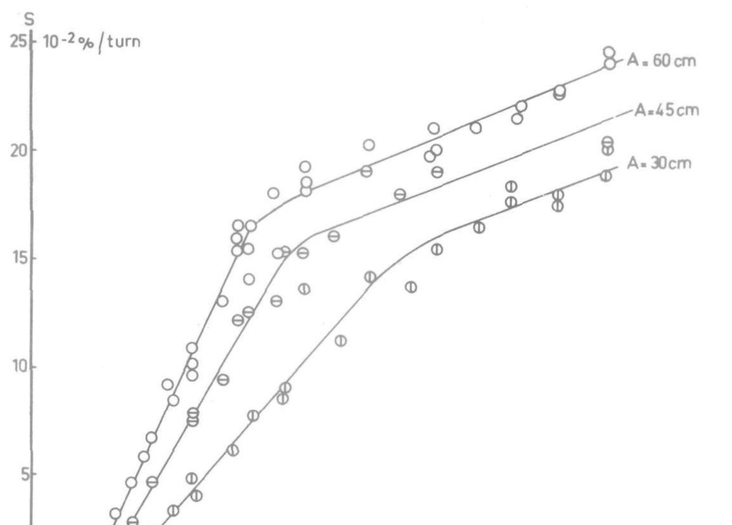

Fig. 9. Dependence of S on n for three amplitudes, A

Table IV
Effect of rotational speed, n (r.p.m.), and amplitude, A (cm), on the initial rate of shrinkage, S (10^{-2}%/turn)

A = 60				A = 45		A = 30	
n	S	n	S	n	S	n	S
29	0.8	101	14.0	47	2.8	56	2.25
30	0.8	101.5	16.5	56	4.65	66	3.3
30	1.6	113	18.0	75	7.45	74.5	4.85
39	3.2	115	15.2	75	7.8	77	4.0
40	2.35	127	19.2	89	9.4	94	6.1
46	4.6	128	18.1	96	12.1	103	7.7
52	5.8	128	18.5	101	12.5	117	8.5
52	5.8	156	19.0	114	13.0	118	9.0
55.5	6.7	157	20.4	117.5	15.3	127	13.6
63	9.2	186	19.7	127	15.2	144	11.2
65.5	8.4	187	21.0	141	16.0	158	14.1
74.5	10.1	189	20.0	156	19.0	176	13.7
74.5	10.8	207	21.0	172	18.0	189	15.4
75	9.6	226	21.4	189	19.0	209	16.4
88.5	13.0	228	22.0	269	20.3	224	17.3
95	15.9	246	22.5			224	18.3
95.5	16.5	246	24.5			246	17.9
95.5	16.5	246	22.7			246	17.4
100.5	15.4	260	24.0			269	20.0
						269	18.8

The values of S at different values of A as a function of n are recorded in table IV and fig. 9. The three curves obtained are strongly resemblant and they give the impression of differing only as regards the scale value for n. When S is plotted against log n, it is found that the three curves thus obtained run approximately parallel. Thus a multiplication factor for n can be found for each value of A, so that the curves merge into each other. It appears that this factor can best be approximated by $A^{0.82}$. Fig. 10 shows the result of this transformation. The curves for A = 45 cm and A = 30 cm have both been superimposed on that of A = 60 cm by taking $n(A/60\text{ cm})^{0.82}$ as the horizontal scale value. It follows, therefore, that the shrinkage per turn depends solely upon this combination of the quantities n and A.

This result will prove to be of value in the following sec-

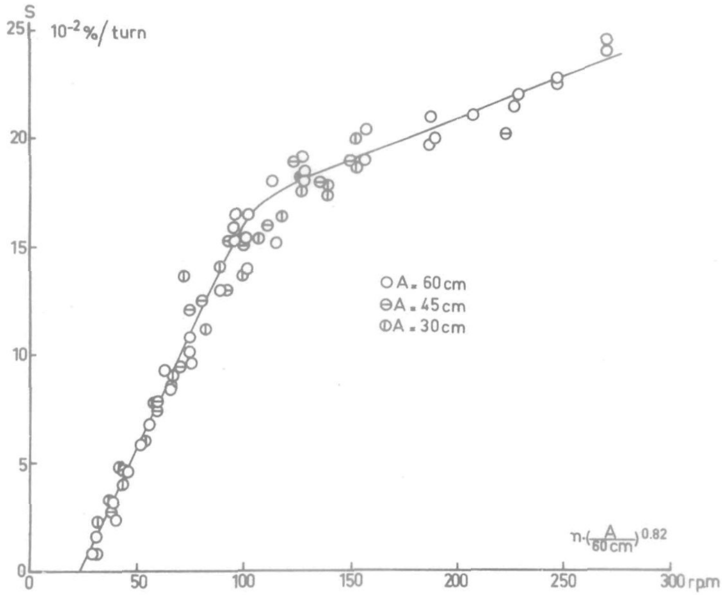

Fig. 10. S as a function of $n(A/60\text{ cm})^{0.82}$

tion; it also indicates that the study of felting as a function of the force can be performed by using one and the same amplitude and by varying only the speed of rotation, application of different amplitudes giving no extra information. Consequently the experiments described in the following chapters are all performed with A adjusted to 60 cm.

3.3. RELATION BETWEEN FORCES, SHAKING RATE AND AMPLITUDE

The actual magnitude of the forces exerted by the liquid on the fibres during shaking is not known, since this magnitude is dependent upon the relative velocity between the liquid and the fibres in the yarn. Even if the forces exerted on the yarn itself could be measured, which is already a rather complicated experiment, then even no knowledge would be present about the forces which cause the fibres to migrate with respect to each other.

About the way in which these forces depend on the rate of shaking a very rough estimation can be made in the following way: When there is no yarn in the tube and the viscosity of the liquid is disregarded, the maximum velocity of the liquid occurring is

$$2\pi(n/60)(A/2) \qquad (\text{cm/s})$$

which is proportional to nA. This velocity is, however, modified by the presence of the yarn, the modification being brought about, among other causes, by the fraction of the cross-section occupied by the yarn, by the movement of parts of the yarn with respect to the tube, and by capillary forces acting between glass, liquid, yarn and air. About the nature of the flow little can be said, since the flow orifices are of very different dimensions. It is probable that with velocities which are not unduly high, laminar flow conditions will occur, especially when the liquid flows in between the fibres. In that case, the force on the fibres is proportional to the velocity of the liquid. If, in a first approximation, the velocity is assumed to be proportional to nA, it follows that the force also obeys this law.

If it is also assumed that the shrinkage per turn depends solely upon the force acting on the single fibres, reasonable agreement is obtained with the results found in the preceding section, namely that S is a function of $nA^{0.82}$.

Although the absolute magnitude of the force on the fibres remains unknown, it is possible in this way to furnish some approximate information concerning its dependence upon number of revolutions and amplitude. This will especially prove to be useful in section 3.5 in which the influence of the force on felting is discussed more in detail and in which the hypothesis is used that the scale for n can be considered as a linear force scale.

3.4. EFFECT OF YARN TWIST

The object of the preceding experiments was to ascertain the effect of the externally exerted forces on the shrinkage. It will be interesting to investigate, however, how the shrinkage changes when the inter-fibre forces are varied. This variation may be achieved by using yarns twisted to different measures. By inserting extra twist in the yarn the fibres are more closely packed, which gives rise to a higher pressure on each other and, consequently, to a higher frictional force. At the same time other features of the structure are altered, as, for instance, the average angle under which the fibres lie in the yarn and the dimensions of the openings through which the fibres can migrate.

In order to obtain a series of yarns with different amounts of twist, the following method was used: The original 3-ply yarn, E_o, was separated into its components by untwining. A 250 cm length of the single yarn was untwisted and afterwards retwisted by giving it a certain number of turns. The number of turns applied, *m*, was successively 200, 400, 600, 800, 1000, 1200 and 1600. From this twisted single yarn a balanced 2-ply yarn was made by doubling the yarn and letting it tordate freely until equilibrium between twist and twine was reached. The 2-ply structures thus obtained were used as initial material for a series of shrinkage experiments at different speeds. The shrinkage was proceeded till an estimation of the limit could be made so that the parameters S and s_∞ were both determined. The dependence of

Table V
Influence of yarn twist, *m*, on shrinkage parameters, S (10^{-2}%/turn) and s_∞ (%)

	m = 200		m = 400		m = 600		m = 800	
n	S	s_∞	S	s_∞	S	s_∞	S	s_∞
19	0.55							
29	2.7		1.3		0.75		0.23	
40	6.9	63.5	3.6	62	1.8	55.5	0.82	39
52	9.2	66.5	5.0	65.5	2.5	63.5	1.25	61
76	11.5	67.5	6.0	66.5	3.5	65	1.9	63.5
102	13.8	67.5	8.3	67.5	5.0	66	2.9	64.5
128	16.4	67.5	10.3	67.5	6.2	67	3.25	65
158	17.8	67	12.1	67.5	6.8	66.5	4.0	65
189	18.6	67.5	12.6	67.5	6.7	67	4.9	65

	m = 1000		m = 1200		m = 1400		m = 1600	
n	S	s_∞	S	s_∞	S	s_∞	S	s_∞
19								
29	0.11							
40	0.32	33	0.18	23	0.05	12	0.025	4
52	0.60	58	0.28	44	0.14	33	0.042	21
76	1.38	61	0.77	59	0.42	55.5	0.22	47
102	1.71	63	1.15	60	0.66	57	0.41	53.5
128	2.4	63	1.22	59	0.80	56.5	0.59	53.5
158	2.7	63.5	1.8	59.5	1.15	58	0.81	54
189	3.4	63.5	2.1	61	1.33	58	0.85	54

these parameters on speed and twist are recorded in table V and in the figures 11 and 12. Each curve represents the average be-behaviour of 4 yarns.

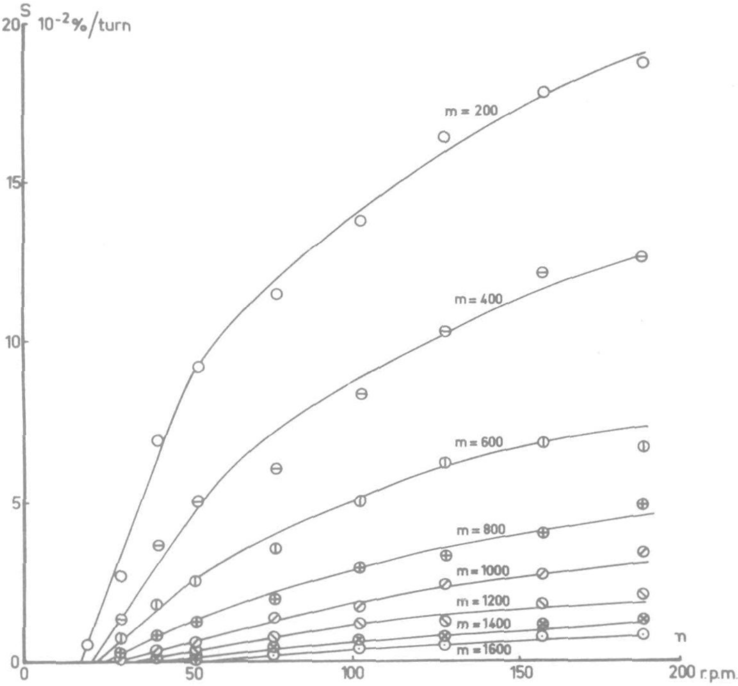

Fig. 11. *S versus n* for different twist parameters, *m*

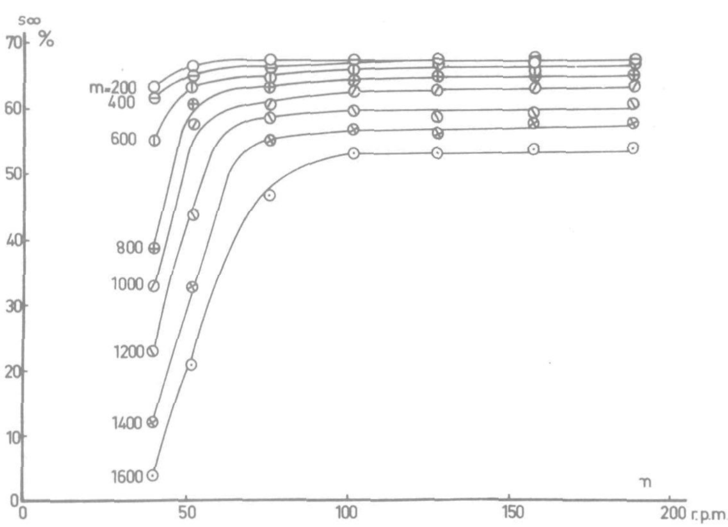

Fig. 12. s_∞ *versus n* for different twist parameters, *m*

It is evident from the results shown in fig. 12 that the upper limit of the values of s_∞ drops somewhat as the twist increases, while, moreover, the threshold moves to higher values of n. This shift of the threshold can also be seen in the course of S (fig. 11), where, in addition to a very pronounced change in the slopes of the $S-n$ plot, the intersection of the curves with the n axis moves to higher values of n when the twist increases.

In order to ascertain the nature of these scale changes, each of the two graphs has been plotted two-side logarithmically. In both cases the curves for various values of m turned out to be of the same shape; they can, by shifting the two axes, be made to merge into each other. For fig. 11 and fig. 12 the multiplication factors in n appear to be the same and can, with sufficient accuracy, be represented by the following function of m:

$$h(m) = 0.89^{m/200} = e^{-0.1165 \cdot m/200} . \qquad (1)$$

The scale factors for S and s_∞ are not easily expressed mathematically; these factors are denoted by $f(m)$ and $g(m)$ respectively and are recorded numerically in table VI. The course of the three functions $f(m)$, $g(m)$ and $h(m)$ is reproduced in fig. 13.

Table VI
Scale multiplication factors for S, s_∞ and n

m	$f(m)$	$g(m)$	$h(m)$
200	1.56	1.00	0.89
400	2.33	1.00	0.79
600	3.83	1.01	0.70
800	5.90	1.04	0.63
1000	8.55	1.06	0.56
1200	11.7	1.11	0.50
1400	16.7	1.17	0.44
1600	23.1	1.25	0.39

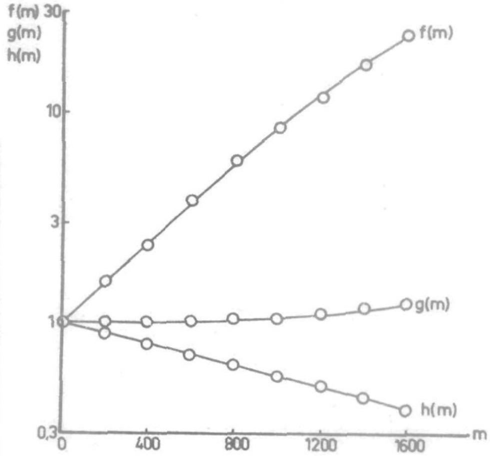

Fig. 13. The transformation functions $f(m)$, $g(m)$ and $h(m)$

The result of the performance of these transformations is shown in fig. 14 and fig. 15. In these figures $f(m) \cdot S$ and $g(m) \cdot s_\infty$ are plotted, both against $h(m) \cdot n$. All the curves have thus been reduced to $m = 0$. Though the spreading of the points in fig. 14

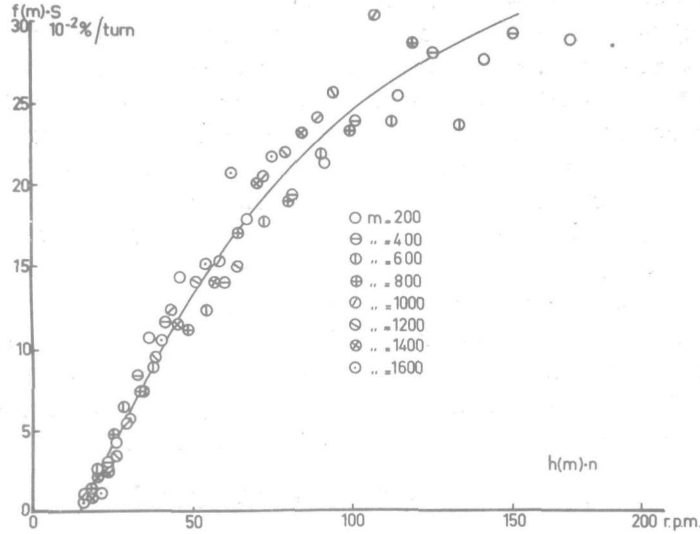

Fig. 14. Combination of the data of figure 11 by means of the functions $f(m)$ and $h(m)$

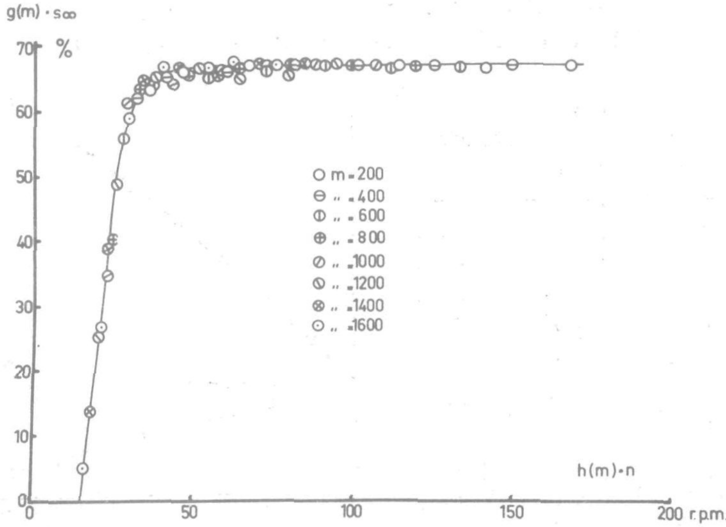

Fig. 15. Combination of the data of figure 12 by means of the functions $g(m)$ and $h(m)$

is rather high, no systematic deviations of the individual curves from the generalized one occur.

If the form of the functions of fig. 14 and fig. 15 is represented by $P(n)$ and $Q(n)$ respectively, the results may be combined mathematically into

$$S \cdot f(m) = P[n \cdot h(m)] \quad (2)$$

and $$s_{\infty} \cdot g(m) = Q[n \cdot h(m)] \ . \quad (3)$$

For the first straight part of the curve of fig. 14, the expression (2) becomes

$$S = c \frac{h(m)}{f(m)} \left[n - \frac{n_1}{h(m)}\right] \quad (4)$$

in which c and n_1 are constants.

3.5. DISCUSSION OF THE RESULTS

When discussing the influence of the forces on the felting, on the basis of the experimental results described in this chapter, the $S - n$ characteristic (for instance that of fig. 10) will be used as a starting point, the other results coming into consideration when needed during the discussion of the special features of this graph.

In order to treat the data in a logical order, the following properties of the $S - n$ curve will be discussed consecutively:

a. the threshold value of n

b. the first straight part of the $S - n$ curve

c. the further course of S with n.

a. The existence of the threshold value gives rise to the following observations: It appears that a certain minimum force is needed to make the fibres migrate. This minimum force will probably be related to the friction between the fibres, since in order to displace a fibre in the yarn with respect to the surrounding fibres, it is necessary to overcome frictional forces. These forces are dependent on the yarn structure and on the frictional properties of the individual fibre. Since the wool fibre has different frictional coefficients in the two directions along the fibre, the movements in the "with scale" direction will invariably be greater than those in the other direction, provided that these movements actually occur. When the frictional force in the direction of the scales, R_1 (which is smaller than R_2, the anti-scale friction), is not exceeded by the force externally applied to the fibre, no movement will take place. Consequently, it may be assumed that the force exerted on the fibres by the flowing liquid at the rotational speed n_1, equals R_1.

When R_1 increases, either as a result of a modification of the frictional coefficient μ_1 or by an increase in the normal pressure between the fibres, caused by a change in the yarn structure, the threshold n_1 will shift to a higher value of n. The first case will be discussed later on; the second case occurs in the experiments with increased twist coefficient m. From the existence of the scale transformation function $h(m)$ (eq. 1), the conclusion may be drawn that this function denotes the relationship between the twist and the rotational speed required to overcome the friction between the fibres. If the speed is supposed to be proportional to the hydrodynamic force, and the friction is proportional to the fibre pressure according to Amontons' law $R = \mu N$, then the function $h(m)$ may be regarded as a first approximation of the rate of increase of the fibre pressure with the twist. This may be an interesting result for the very complicated mechanics of composite twisted yarns, but it will not be considered here further.

The frictional force between the fibres is, however, not equal for each fibre. The frictional coefficients, μ, as well as the normal pressures exhibit a certain spreading. For the yarns under discussion this spreading manifests itself mainly in the course of s_∞ with n. Here a transition region in n exists, in which s_∞ rises from zero up to its ultimate value. The absolute width of this region increases with increasing twist (fig. 12), the relative width remaining constant. This fact is mathematically expressed by the existence of the transformation function $h(m)$ for n, enabling the coincidence of the s_∞ curves, except for a slight difference in level. This indicates that with increasing twist the relative deviations in R_1 remain the same, which may be interpreted in terms of a spreading in μ_1 as well as in N.

b. The second point to be considered is the first straight part of the $S - n$ curve. In this region the external force F exceeds R_1 so that the fibres are able to move, though only in their root direction as long as $R_1 < F < R_2$. When a fibre moves, the resulting force acting on it is $F\text{-}R_1$, the difference between static and kinetic friction being disregarded. If the assumption made in section 3.3 is used, concerning the proportionality between the force F and the shaking speed n, then it is evident from the measurements that the initial shrinkage per turn is proportional to the resulting force acting on the individual fibres. This shrinkage is brought about by small displacements of the fibres.

As originally the fibres lie nearly parallel, the initial shrinkage per turn may be assumed to be proportional to the displacement of an individual fibre, l_1. It follows, therefore, that this displacement per turn is proportional to the resulting force by which it is brought about,

$$l_1 = \alpha (F - R_1) \tag{5}$$

The constant α is proportional to the slope of the straight part of the $S - n$ characteristic. The factors determining this constant are still to be investigated. One of them is no doubt the structure of the yarn; it is obvious that the density of packing and the geometrical position of the fibres will influence the ease of fibre migration in more ways than only via the friction. From the data obtained on twisted yarns it appears that α varies with the degree of twist according to the function $h(m)/f(m)$, differing for the highest and for the lowest used values of m by a factor 34.

Beside this very important influence of the structure, single fibre properties will probably also affect the slope. The conclusion that the displacement of a fibre is proportional to the resulting force, suggests the occurrence of fibre deformations during felting, so that the elastic properties of the fibres may influence the rate of felting. This suggestion is in agreement with the results reported by Speakman and co-workers [29)42)] and by Bogaty and co-workers [4)], mentioned in section 1.42. The deformation may consist of bending or elongation of the fibres, according to Martin's [27)] or Arnold's [1)] theories respectively (see section 1.3).

c. After the first part of the $S - n$ characteristic of fig. 10 the curve bends off rather suddenly and becomes less steep. This phenomenon may be interpreted as follows:

When the force, F, is large enough to overcome the frictional force in the anti-scale direction, so when $F > R_2 > R_1$, then movement of the fibre in both directions is possible. If the magnitude of the movement against the scales is also proportional to the resulting force and if the same proportionality constant, α, as the one introduced in eq. (5) is valid, then the resulting displacement, being the difference between the with-scale and the anti-scale displacements, is

$$l = l_1 - l_2 = \alpha(F - R_1) - \alpha(F - R_2) = \alpha(R_2 - R_1) \tag{6}$$

which is independent of the force. The shrinkage is again supposed to be proportional to the resulting fibre displacement so that according to this reasoning the $S - n$ curve should run horizontally for speeds higher than the value of n corresponding to R_2. To some extent this is evident from fig. 10. It is true that S does not become independent of n, but at $n > 100$ r.p.m. the course is considerably less steep than at lower values of n.

This representation is, of course, only approximate, since the treatment is simplified in several respects.

Firstly, the difference between static and kinetic friction is disregarded. When this difference is taken into account, the idealized picture of the $S - n$ curve derived in the foregoing is modified somewhat and becomes (fig. 16):

$$S = 0 \qquad \text{for } F \leqq R_{1s}$$

$$S \; (:) \; (F - R_{1k}) \qquad \text{for } R_{1s} < F \leqq R_{2s}$$

$$S \; (:) \; (R_{2k} - R_{1k}) \qquad \text{for } R_{2s} < F \; .$$

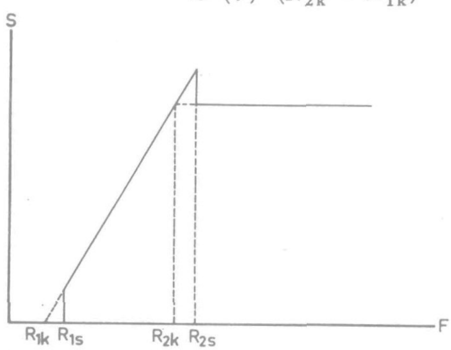

Fig. 16. Idealized dependence of initial shrinkage per turn, S, on the applied force, F

Furthermore the kinetic friction has been supposed to be independent of the velocity of movement; in general, however, a small increase in friction with increasing speed of sliding occurs so that fig. 16 may become still more complicated.

Secondly, the reasoning is based on the assumption that the frictional forces are equal for each fibre. However, a considerable spreading may often exist, which has already been discussed when dealing with the $s_\infty - n$ curves for yarns of different twist. This spreading may also influence the curve under discussion and may give rise to a rounding off of the sharp transitions in the diagram. In particular at $F = R_2$ the curve will not sharply bend off to become horizontal but will remain slowly rising, since the spreading in μ_2 is generally high, as will appear from the measurements reported in chapter 6.

Thirdly, fibres lying at the surface of the yarn will probably be subjected to greater hydrodynamic forces than those in the centre; furthermore the pressure between the fibres, and therefore also the frictional forces, may depend too on the position of the fibres in the yarn. Since the structure of the yarns is rather loose, these effects will presumably not give rise to serious complications.

Fourthly, it has throughout been suggested that the fibres will move as a whole during felting. However, it is not probable that a fibre which has several points of contact with its neighbours will be able to move over its whole length through the yarn. It is easier to imagine that movements of parts of the fibre occur. Investigations of the structure of felted yarns, reported in chapter 8, confirm this supposition. The considerations on fibre movement in felting remain equally well valid when they are applied to a part of a fibre instead of to a whole fibre.

The discussion of the influence of the forces on felting, presented in this section, has given rise to some presumptions about the role played by frictional and elastic properties of the single fibres. Confirmation of these ideas can only be obtained by investigating the shrinkage of yarns in which these fibre properties are systematically varied. The methods used to measure the friction and the elasticity of single fibres will be discussed in the next few chapters.

4 MEASUREMENT OF FRICTION

4.1. DISCUSSION OF THE METHOD

Several methods have been followed to determine the frictional properties of wool fibres; they will be summarized briefly.

The first measurements of wool fibre friction were performed by Speakman and Stott [41]. They arranged fifty fibres with all their tips pointing in the same direction, and mounted these fibres, as nearly parallel as possible, into a kind of miniature violin bow. This violin bow was placed upon an inclined plane, the elevation of which could be increased till the fibre assembly started to glide at an angle of elevation, θ, for which tg $\theta = \mu$. By placing the violin bow in two directions, μ_1 and μ_2 could be measured. This method was used in a modified form by Bohm [6] who applied it in air as well as in liquids and who covered the lower surface also with fibres. The way of mounting the fibres, however, is rather tedious and, furthermore, the method is not suitable to determine kinetic coefficients of friction.

Afterwards, Speakman, Chamberlain and Menkart [44] designed the so-called lepidometer which was simpler to operate and was believed to give results correlating better with the fibre behaviour during felting. A fibre is placed with its root end downwards and is pressed between two soft surfaces which are made to rub against each other in a direction perpendicular to the fibre axis. The fibre starts to travel downwards until the tension it develops, stops its movement. This tension is supposed to be a measure for the feltability. Makinson [24] performed a theoretical analysis of this method and arrived at the conclusion that the tension in the fibre is proportional to a function of the frictional coefficients lying between $(\mu_{2s} - \mu_{1s})$ and $(\mu_{2s} - \mu_{1k})$. The value of the results obtained with the lepidometer therefore depends on the question whether this combination of frictional coefficients has something to do with the mechanism of felting.

Other investigators determined the friction of wool fibres by sliding a single fibre over a cylinder in a direction perpendicular to the cylinder axis (fig. 17). If at the moment of slip the tensions in both ends of the fibre are P_1 and P_2, respectively, and if θ is the angle of wrap, it can easily be proved that the coefficient of static friction is given by

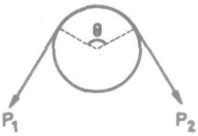

Fig. 17. Principle of friction measurement with a fibre sliding over a cylinder

$$\frac{P_1}{P_2} = e^{\mu_s \theta} \quad \text{or} \quad \mu_s = \frac{1}{\theta} \ln \frac{P_1}{P_2} \qquad (1)$$

Lipson and Howard [21] made use of a horn surface; Martin and Mittelmann [28] did the same, but in some of their experiments they covered the cylinder with a layer of wool fibres; Frishman, Smith and Harris [13] applied a cover of wool felt.

Mercer [31] introduced an adaptation of the apparatus which Bowden and Leben [7] used to measure friction of metals. Mercer brought about stick-slip movements by carrying forward a single fibre whilst it was pressed against a cylindrical piece of horn, and registered the forces which acted on this body. The movement is characterized by a discontinuous sliding, being caused by the difference between static and kinetic friction and by the elasticity of the fibre. This method thus allows the determination of both static and kinetic coefficients of friction.

Graleń and Olofsson [15] applied the same principle but made the fibre slip against a second fibre which could be placed at different angles with the moving one.

Lindberg and Graleń [18] designed a method in which two fibres are twisted round each other n times under a small angle, β (fig. 18). The two lower ends, C and D, of the fibres are attached to balance arms, the upper ends, A and B, each to the lever of a torsion balance. First the four forces are adjusted to the same value, P_1, then the balance at C is arrested and the tension at B is increased until, at a certain tension P_2, the right-hand fibre starts sliding. The authors showed that the static coefficient of friction is given by

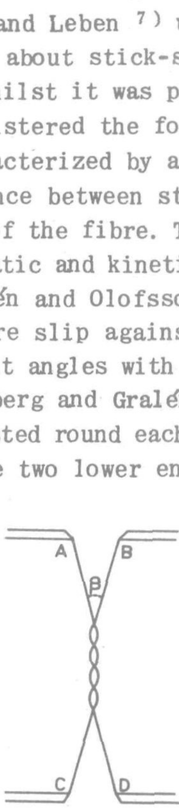

Fig. 18. Principle of friction measurement between two twisted fibres

$$\mu_s = \frac{\ln P_2/P_1}{\pi n \beta}. \qquad (2)$$

In order to determine the kinetic coefficient of friction, the torsion balance at B is adjusted at a value P_2' which is somewhat lower than P_2, and the balance at D is given a small impulse. If the value of P_2' is too low according to the kinetic friction, no slip will occur and a higher value is tried. After a few trials μ_k can be ascertained.

Later on, Lindberg [20] presented a modification of this method, in which a stick-slip movement is brought about. The tension in the non-moving fibre is measured and recorded by means of the strain gauge load cell of an Instron Tensile Tester. From the record both coefficients of friction can be calculated.

For the present investigation it is desirable to choose a method by which fibre-fibre friction can be measured, since this will obviously give the best correlation with felting. Methods which require the preparation of fibre assemblies are too laborious so that only the measurement of friction between two single fibres comes into consideration. The stick-slip method, applied to single fibres, used by Gralén and Olofsson [15], has the disadvantage that the area of contact is very small, probably only one or two scales. This may give rise to a considerable dispersion in the results, especially when the fibre slides in the anti-scale direction [18].

The fibre twist method appears to be the most suitable one, though the measurement of μ_k is not satisfying. The modification with the strain gauge does not show this disadvantage but it requires a rather expensive electronic equipment. However, it appears to be possible to simplify the fibre twist method considerably. In order to determine μ, according to equation (2), Lindberg and Gralén fixed the values for P_1, n and β, and varied the other force till its right value P_2 was reached. When, however, P_1 and P_2 are fixed and the number of turns is varied till slip occurs, the method becomes much simpler; the tensions in the fibres remain constant, so that no balances or strain gauges are needed. Besides, it will be shown that this modification enables a simple determination of the kinetic coefficient of friction too.

It was, therefore, decided to measure the friction of wool fibres according to this principle. The apparatus will be described in the following section.

4.2. THE APPARATUS FOR MEASURING FIBRE FRICTION

The essential parts of the apparatus are shown schematically

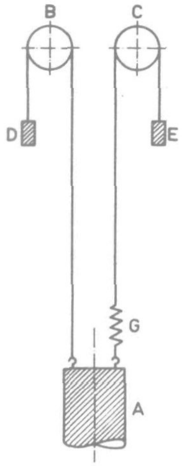

Fig. 19. Essential parts of the apparatus for fibre friction

in fig. 19. A vertical shaft, A, contains two small hooks to each of which a fibre can be attached. One of the fibres is mounted to the shaft via a steel spring, G; the other one is fixed directly. The fibres are led over the pulleys B and C; they are kept under tension by the weights D and E, which must be large enough to remove the curliness of the fibres entirely.

The measurement of friction is performed as follows: By rotating the shaft the fibres are twisted round each other, the initial tensions being both P_1. When a certain twist is reached the weight at E is increased from P_1 to P_2 but the twist is chosen so high that the fibres do not start sliding during this manipulation. Then the twist is decreased until slip occurs. When this is the case the rotation is stopped immediately and the number of turns is read on a revolution counter. This counter reads zero when the fibres are just in contact with each other, *i.e.* when the shaft is turned over 180^o from the position indicated in fig. 19. The static coefficient of friction can be calculated from the forces P_1 and P_2, the number of turns n, and the angle between the fibres, β, according to equation (2).

During the movement of the right-hand fibre the spring, G, is being elongated so that the difference between the forces acting on the fibre decreases. After a certain displacement the fibre comes to rest again as a result of the equilibrium between the driving force and the kinetic friction. In principle it is possible to derive the kinetic coefficient of friction from the fibre displacement and the stiffness of the spring. This is, however, rather tedious, and therefore a different method is followed. After the first slip the number of turns is diminished again until a second slip occurs at n', the corresponding angle being β'. Now eq. (2) can again be applied, so that the elongation of the spring can be eliminated, which results is the relation

$$\frac{\mu_k}{\mu_s} = \tfrac{1}{2}\left(1 + \frac{n'\beta'}{n\,\beta}\right) \qquad (3)$$

as will be shown in section 4.4.

This method is only applicable when $\mu_k < \mu_s$ as is most often the case. When, however, $\mu_k \geqq \mu_s$, no distinct slip occurs; at a certain n the fibre starts sliding very slowly over a very short distance; during further untwisting it creeps further again. In this case only μ_s can be measured.

If the left-hand fibre is suspended with its root end downwards and the right-hand one in the opposite direction, then the with-scale frictional coefficient, μ_1, is measured in the way described above. In order to determine μ_2, a similar procedure is followed; now the initial tensions are both P_2, and after twisting the tension in the upper end of the right-hand fibre is decreased to P_1. The same formulae (2) and (3) remain valid.

If the fibres are both suspended with their root ends upwards (or downwards), both ways of slipping will yield the same coefficient of friction namely a value between μ_1 and μ_2. This value is, however, not of any practical significance since only in this exceptional position of the fibres with respect to each other, the friction is independent on the direction of sliding, whereas in any other position a difference between with-scale and anti-scale friction occurs.

Fig. 20 represents a general view of the complete apparatus, some parts of which will now be described in more detail.

The pulleys, B and C, are mounted in jewel bearings to keep their friction as small as possible.

The spring, F, is made of steel wire of 0.1 mm diameter. It is useful to keep the total length of the spring small enough to prevent it touching the other fibre when the fibres are twisted. At the same time the stiffness must preferably be small enough to enable a well observable displacement of the fibre. These contradictory requirements can be met by applying a spring manufactured with a certain pre-stress. Its characteristic is shown in fig. 21.

The shaft is driven by a small asynchronous motor via a retarding transmission in the ratio 1 : 100. A revolution counter is directly coupled to the motor so that hundredths of revolutions can be read.

Since it is useful to perform the initial twisting fast and the untwisting slow, an electrical circuit is designed which allows the motor to run considerably slower than its ordinary speed of 2800 r.p.m. The circuit diagram is represented in fig. 22. When the slow-fast switch is in the indicated position, the motor will run under normal conditions. When this switch is clos-

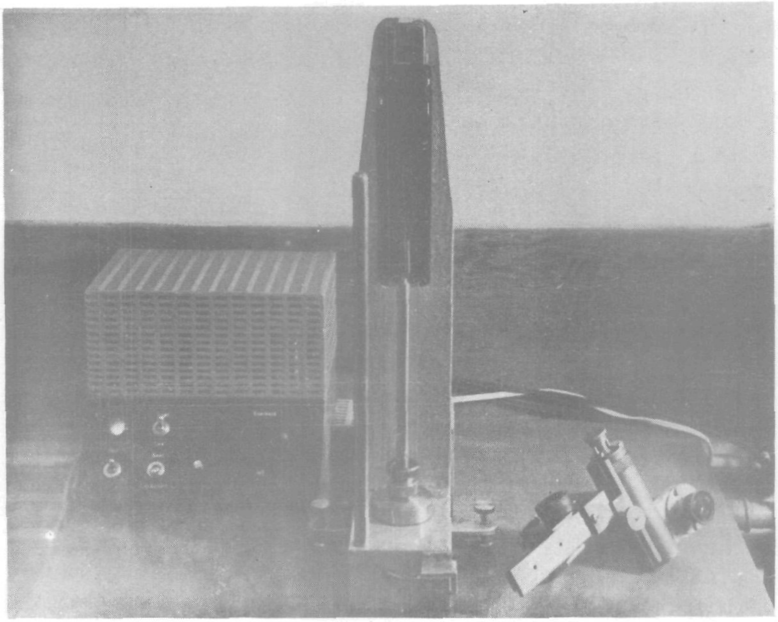

Fig. 20. General view of the apparatus for fibre friction

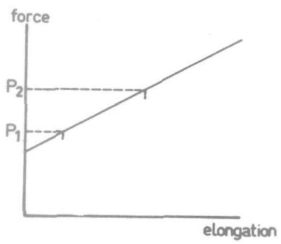

Fig. 21. Characteristic of the steel spring

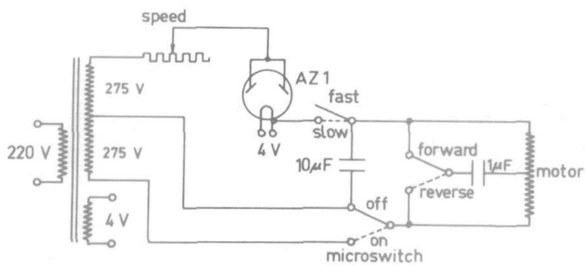

Fig. 22. Motor circuit diagram

ed, a direct current component is superimposed on the alternating current, which retards the speed of the motor considerably. By means of a variable resistance in the d.c. circuit the speed can be regulated between 60 and 400 r.p.m. When the microswitch is pressed, the motor starts running; when it is released, the motor receives only a d.c. tension and is braked quickly. The direction of rotation is reversed by switching the 1 μF capacitor which is parallel to one of the two motor windings, to the other one.

In order to enable measurements in liquids, the apparatus is provided with a cylindrical glass vessel which can slide up and down along the vertical shaft, and which rotates together with the shaft. In the upper position it can be filled with a liquid which surrounds the twisted parts of the fibres. When it is lowered (dotted position), the vessel is emptied automatically by means of a small hole in the shaft (fig. 23).

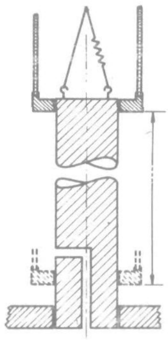

Fig. 23. Detail of the apparatus for fibre friction

The weight attached to the sliding fibre is observed by means of a low-power microscope, mounted in a frame which is movable in vertical and in horizontal direction. The frame is provided with a calibrated scale enabling the measurement of the twist length.

In general, wool fibres are too short to cover the distance from the shaft over the pulley to the weight. Therefore, most often auxiliary nylon monofilaments are used in order to obtain the desired length. The wool fibres are glued to these filaments with collodion in such a way that only the wool fibres will become twisted. Before the fibres are inserted into the apparatus, the direction of their scales is ascertained by microscopical examination.

4.3. DERIVATION OF THE EXPRESSION FOR μ_s

The expression (2), relating μ to the forces P_2 and P_1 and to the twist parameters n and β was derived by Lindberg and Gralén under the following simplified conditions:

a. the fibres are exactly cylindrical
b. the fibres have the same diameter
c. the four forces are initially equal
d. the angle β is constant along the twist length.

It follows from these conditions that both fibres lie in an ideal helix under an angle $\beta/2$ with the axis, whilst the line of contact between the fibres coincides with the axis. It is clear that in practice the first two conditions will seldom be fulfilled by wool fibres, so that the problem will be considered in a more general form.

If the forces acting on the right-hand fibre are F_1 and F_3 and those on the other fibre F_2 and F_4, and if the angles between the directions of these forces and the twist axis are γ_1, γ_3, γ_2 and γ_4, respectively (fig. 24), then the conditions for equilibrium are

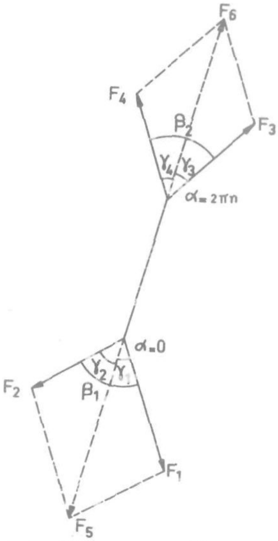

Fig. 24. Forces acting on the twisted part of the fibres

$$F_1 \cos \gamma_1 + F_2 \cos \gamma_2 =$$

$$= F_3 \cos \gamma_3 + F_4 \cos \gamma_4 \quad (4)$$

and $$\frac{F_1}{\sin\gamma_2} = \frac{F_2}{\sin\gamma_1} = \frac{F_5}{\sin(\gamma_1+\gamma_2)};$$

$$\frac{F_3}{\sin\gamma_4} = \frac{F_4}{\sin\gamma_3} = \frac{F_6}{\sin(\gamma_3+\gamma_4)} \quad (5)$$

in which F_5 and F_6 are the resulting forces of F_1 and F_2 and of F_3 and F_4, respectively.

The angles γ are all smaller than 3^o so that the approximations $\cos\gamma = 1$ and $\sin\gamma = \gamma$ may be safely used.

Now the equations (4) and (5) become

$$F_1 + F_2 = F_3 + F_4 = F \qquad \text{(say)} \qquad (6)$$

and $$\frac{F_1}{\gamma_2} = \frac{F_2}{\gamma_1} = \frac{F}{\beta_1}, \qquad \frac{F_3}{\gamma_4} = \frac{F_4}{\gamma_3} = \frac{F}{\beta_2} \qquad (7)$$

in which β_1 and β_2 are the angles between the two fibres at the extremities of the twisted part.

Fig. 25 represents schematically two fibres of radii r_1 and r_2 twisted round each other. The axis of each of the fibres follows an ideal helix when the diameters are constant along the fibre; when this is not the case, every small part of the curve can be closely approximated by a helix. A part of the helix formed by one of the fibres is represented in fig. 26. The part dz is tordated over an angle $d\alpha$; its angle with the axis is ϵ_1. The distances between the axes of the fibres and the axis of twist are a_1 and a_2, respectively. Now dz may be written as

$$dz = \frac{a_1 d\alpha}{\epsilon_1} \qquad (8)$$

because BC = $a_1 d\alpha$ and dz = AC = BC/sin ϵ_1.

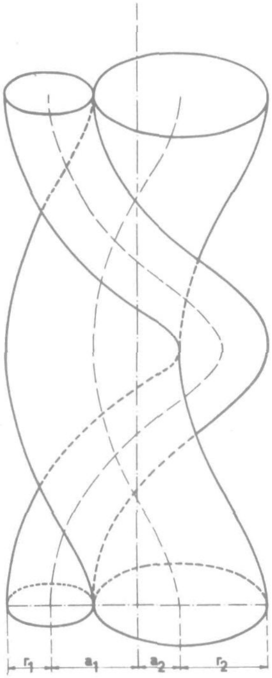

Fig. 25. Schematig representation of twisted fibres

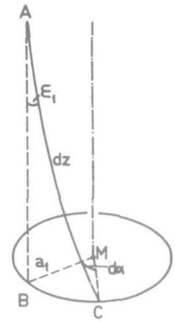

Fig. 26. A small part, dz, of the helix formed by one of the fibres

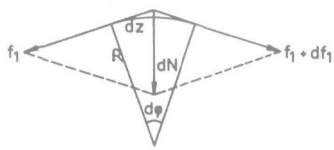

Fig. 27. Forces acting on the part dz of one of the fibres

In fig. 27 the element dz is approximated by a circular curve with radius R. The forces acting upon this element are f_1 and $f_1 + df_1$, the resulting force of which is dN; the angle at the centre of curvature of the element is $d\varphi$, so that $dz = Rd\varphi$. The radius of curvature, R, equals $a_1/\sin^2\epsilon_1 = a_1/\epsilon_1{}^2$, according to a well-known geometrical theorem; hence

$$dz = \frac{a_1}{\epsilon_1^2}\, d\varphi \; . \tag{9}$$

Combination of (8) and (9) yields

$$d\varphi = \epsilon_1 \; d\alpha \; . \tag{10}$$

The normal force between the two fibres is

$$dN = f_1 \; d\varphi = f_1 \; \epsilon_1 \; d\alpha \; . \tag{11}$$

For the second fibre similar considerations are valid; dN and $d\alpha$ have both the same value, so $dN = f_1 \epsilon_1 d\alpha = f_2 \epsilon_2 d\alpha$. If df_1 is the force difference necessary to make the fibres slide then $\mu dN = df_1$, or $\mu f_1 \epsilon_1 d\alpha = df_1$, or

$$\frac{df_1}{f_1} = \mu\epsilon_1 \, d\alpha \, . \tag{12}$$

This equation must be integrated from $\alpha = o$ and $f_1 = F_1$ to $\alpha = 2\pi n$ and $f_1 = F_3$:

$$\int_{F_1}^{F_3} \frac{df_1}{f_1} = \mu \int_{o}^{2\pi n} \epsilon_1 \, d\alpha \, . \tag{13}$$

To express ϵ_1 in known quantities, eq. (7) is applied to the fragment of twist under discussion with a length dz:

$$\frac{f_1}{\epsilon_2} = \frac{f_2}{\epsilon_1} = \frac{f_1 + f_2}{\epsilon_2 + \epsilon_1} = \frac{F}{\beta} \quad \text{or} \quad \epsilon_1 = \beta \frac{f_2}{F} \, . \tag{14}$$

Both f_2 and β vary along the twist length, namely f_2 from F_2 to F_4 and β from β_1 to β_2. In a first approximation these quantities may be supposed to depend linearly on α:

$$f_2 = F_2 + \frac{\alpha}{2\pi n}(F_4 - F_2)$$

and

$$\beta = \beta_1 + \frac{\alpha}{2\pi n}(\beta_2 - \beta_1) \; ,$$

so that, according to eq. (14):

$$\epsilon_1 = \frac{1}{F}\left[\beta_1 + \frac{\alpha}{2\pi n}(\beta_2 - \beta_1)\right]\left[F_2 + \frac{\alpha}{2\pi n}(F_4 - F_2)\right] \, . \tag{15}$$

When eq. (15) is applied to the right-hand side of eq. (13), one obtains

$$\int_{o}^{2\pi n} \epsilon_1 d\alpha = \frac{2\pi n}{F}\left[\beta_1 \left(F_2 + \frac{F_4 - F_2}{2}\right) + \left(\frac{\beta_2 - \beta_1}{2}\right)\left(F_2 + \frac{F_4 - F_2}{1.5}\right)\right] . \tag{16}$$

Since $(\beta_2 - \beta_1)/2 << \beta_1$ and $(F_4 - F_2)/1.5 << F_2$, equation (16) may be approximated by

$$\int_0^{2\pi n} \epsilon_1 d\alpha = \frac{2\pi n}{F}\left(\beta_1 + \frac{\beta_2 - \beta_1}{2}\right)\left(F_2 + \frac{F_4 - F_2}{2}\right) =$$

$$= \frac{2\pi n}{F}\left(\frac{\beta_1 + \beta_2}{2}\right)\left(\frac{F_2 + F_4}{2}\right) =$$

$$= \pi n \bar{\beta} \, . \, \frac{F_2 + F_4}{F} \tag{17}$$

in which $\bar{\beta}$ is the average of β_1 and β_2. Equation (13) therefore becomes

$$\ln \frac{F_3}{F_1} = \mu \pi n \bar{\beta} \frac{F_2 + F_4}{F}$$

or

$$\mu = \frac{\ln F_3/F_1}{\pi n \bar{\beta}} \, . \, \frac{F_1 + F_2}{F_2 + F_4} \, . \tag{18}$$

When $F_1 = F_4$, the second factor is unity, hence

$$\mu = \frac{\ln F_3/F_1}{\pi n \bar{\beta}} \, . \tag{19}$$

When originally $F_1 = F_2 = F_3 = F_4 = P_1$, and when F_3 is increased to P_2, $F_1 + F_2$ must also increase with the difference $P_2 - P_1$. If the fibre has a much higher stiffness than the spring, the tension in the rigidly fixed fibre, F_2, becomes P_2, so that $F_1 = F_4$ remains valid. The expression for μ_s thus becomes

$$\mu_s = \frac{\ln P_2/P_1}{\pi n \bar{\beta}}$$

which corresponds to eq. (2). If $P_2 < P_1$, μ_s has the negative sign which indicates that slip in the reverse direction occurs.

The normal pressure between the fibres can be calculated as follows: At each spot of the fibres $dN = df/\mu_s$ when slip begins. The total normal force, N_{tot}, is given by

$$\int dN = \frac{1}{\mu_s} \int df$$

or

$$N_{tot} = \frac{F_3 - F_1}{\mu_s} = \frac{P_2 - P_1}{\mu_s} \, .$$

The normal force per unit length is

$$\frac{N_{tot}}{z} = N_1 = \frac{P_2 - P_1}{\mu_s z} . \tag{20}$$

4.4. DERIVATION OF THE EXPRESSION FOR μ_k

When the fibres have been untwisted to the value of n at which they begin to slide, the tension in the spring, F_1, is initially equal to P_1. The static frictional force, R_s, equals $P_2 - P_1$.

During slip the kinetic frictional force, R_k, takes the place of R_s, while at the same time the tension of the spring, F_1, increases because the spring is being elongated. Now the moving fibre is subjected to the forces R_k, P_2 and F_1; if the inertia of the moving parts during sliding is represented by a mass, m, the equation of motion reads

$$m\frac{d^2x}{dt^2} - P_2 + F_1 + R_k = 0. \tag{22}$$

The displacement x is measured along the fibre starting from the original position of the point where the fibre is attached to the spring. This equation is valid if the friction in the pulley is small enough to be neglected and if the kinetic friction is independent of the velocity.

The spring tension F_1 can be written as $F_1 = P_1 + cx$, in which c is the stiffness of the spring, so that eq. (22) becomes

$$m\frac{d^2x}{dt^2} + cx + (R_k + P_1 - P_2) = 0 \tag{23}$$

with the solution:

$$x = \frac{P_2 - P_1 - R_k}{c} + b \sin(\omega t + \delta) \tag{24}$$

in which $\omega = (c/m)^{1/2}$. The initial conditions are $x = o$ and $dx/dt = o$ for $t = o$, so that $\delta = \pi/2$ and $b = -(1/c)(P_2 - P_1 - R_k)$. Equation (24) therefore may be written as

$$x = \frac{P_2 - P_1 - R_k}{c}(1 - \cos\omega t) = X(1 - \cos\omega t) \tag{25}$$

in which $X = (1/c)(P_2 - P_1 - R_k)$.

This expression is only valid for the first half period, since at $\omega t = \pi$, $dx/dt = 0$, and the reversal of the movement is prevented by the friction. From (25) it appears that the total displacement of the fibre is $2X$; when $x = X$, the forces R_k, P_2 and F_1 are in equilibrium because at that point $cx = P_2 - P_1 - R_k$ or $P_2 = R_k + F_1$. At $x = X$ and $x = 2X$ the tension of the spring, F_1, reaches the values $P_1 + cX$ and $P_1 + 2cX$, respectively. Now equation (18) can be applied to express the kinetic coefficient of friction, μ_k, in the forces which occur at $x = X$, namely $F_3 = P_2$, $F_4 = P_1$, $F_1 = P_1 + cX$ and $F_2 = P_2 - cX$:

$$\mu_k = \frac{\ln P_2/(P_1 + cX)}{\pi n \bar{\beta}} \cdot \frac{P_1 + P_2}{P_1 + P_2 - cX} \tag{26}$$

When the twist is decreased once more until a second slip occurs at n', μ_s can be expressed in n', the corresponding average angle $\bar{\beta}'$ and the forces at $x = 2X$:

$$\mu_s = \frac{\ln P_2/(P_1 + 2cX)}{\pi n' \bar{\beta}'} \cdot \frac{P_1 + P_2}{P_1 + P_2 - 2cX} \tag{27}$$

From the equations (26), (27), and the original equation

$$\mu_s = \frac{\ln P_2/P_1}{\pi n \bar{\beta}}, \tag{28}$$

X can be eliminated in order to obtain a relation between μ_s, μ_k, $n'\bar{\beta}'$ and $n\bar{\beta}$. For this purpose the following substitutions are used: $P_1/P_2 = \alpha$, $\pi n \bar{\beta} = \varphi$, $cX/P_1 = a$, so that the equations (28), (26) and (27) are transformed into

$$\mu_s = -\frac{\ln \alpha}{\varphi} \tag{29}$$

$$\mu_k = -\frac{\ln \alpha(1 + a)}{\varphi} \cdot \frac{1 + 1/\alpha}{1 + 1/\alpha - a} \tag{30}$$

and

$$\mu_s = -\frac{\ln \alpha(1 + 2a)}{\varphi'} \cdot \frac{1 + 1/\alpha}{1 + 1/\alpha - 2a}. \tag{31}$$

Division of eq. (31) by eq. (29) yields

$$\frac{\varphi'}{\varphi} = \frac{\ln \alpha(1 + 2a)}{\ln \alpha} \cdot \frac{1 + 1/\alpha}{1 + 1/\alpha - 2a} \tag{32}$$

and, from dividing eq. (30) by eq. (29) one obtains

$$\frac{\mu_k}{\mu_s} = \frac{\ln \alpha(1 + a)}{\ln \alpha} \cdot \frac{1 + 1/\alpha}{1 + 1/\alpha - a}. \tag{33}$$

The elimination of a from the equations (32) and (33) appears to be a very complicated procedure; therefore, the value for $\alpha = P_1/P_2$ always used, viz. $\alpha = 2/3$, is substituted.

$$\frac{\varphi'}{\varphi} = \frac{\ln\,(2/3)(1+2a)}{\ln\,(2/3)} \cdot \frac{5}{5-4a} \approx 1 - 4a \tag{34}$$

$$\frac{\mu_k}{\mu_s} = \frac{\ln\,(2/3)(1+a)}{\ln\,(2/3)} \cdot \frac{5}{5-2a} \approx 1 - 2a \tag{35}$$

The approximations by means of simple linear functions of a appear to be surprisingly good; the maximum error made is smaller than 0.003.

From (34) and (35) it follows that

$$\frac{\mu_k}{\mu_s} = \frac{1 + \varphi'/\varphi}{2} = \frac{1 + (n'\bar{\beta}'/n\bar{\beta})}{2} . \tag{36}$$

It may be remarked that, according to the equations (34) and (35), μ_k must be greater than 0.5 μ_s; if this is not the case, no second slip point will occur, and μ_k cannot be measured. Fortunately, in practice μ_k/μ_s is always greater than 0.5.

4.5. PERFORMANCE OF THE CALCULATION OF FRICTIONAL COEFFICIENTS

From the measurement of n, n', $\bar{\beta}$ and $\bar{\beta}'$, the coefficients μ_s and μ_k can be calculated according to the equations (28) and (36). The angles $\bar{\beta}$ are difficult to measure directly. It is obvious that when $\beta_1 = \beta_2 = \bar{\beta}$, $\bar{\beta}$ is determined by the dimensions of the apparatus and the length of the twist. In this case $\bar{\beta} = 2d/(l-z)$ if d denotes the distance between the fibres in untwisted condition, l their length from the shaft to the pulley and z the length of the twisted part. It can be shown that, when $\beta_1 \neq \beta_2$, under the circumstances prevailing in practice only a very small error is made when the same formula is applied. The expression for μ_s becomes then

$$\mu_s = \frac{\ln P_2/P_1}{\pi n} \cdot \frac{l - z}{2d} .$$

Substitution of $P_2/P_1 = 1.5$, $d = 0.67$ cm and $l = 19.2$ cm yields

$$\mu_s = \frac{0.0963}{n}\,(19.2 - z) . \tag{37}$$

For the calculation of μ_k, according to eq. (36), it is convenient to introduce a parameter μ' defined as

$$\mu' = \frac{\ln P_2/P_1}{\pi n' \bar{\beta}'} = \frac{0.0963}{n'}(19.2 - z') ,$$

which transforms eq. (36) into

$$\mu_k = \frac{\mu_s}{2}\ (1 + \frac{\mu_s}{\mu'}) . \qquad (38)$$

In order to simplify the computation of data obtained from large series of routine measurements, nomograms have been designed enabling a rapid determination of the coefficients of friction. The equations of the curves in the nomograms are as follows:

For μ_s and μ' (eq. (37)):

$$x = 0,\ y = 40 \log 51.91/(19.2 - z)$$
$$x = 10,\ y = 20 \log(5/2\mu)$$
$$x = 20,\ y = 40 \log (n/2)$$

For μ_k (eq. (38)):

$$x = 0,\ y = 20 - 3/\mu'$$
$$x = 20,\ y = 18\mu_k$$
$$x = 20/(1 + 3\mu_s^2),\ y = (9\mu_s + 60\mu_s^2)/(1 + 3\mu_s^2)$$

Fig. 28 and fig. 29 represent these nomograms.

On a pair of fibres from the standard sample, E_o, in water, the following points of slip have, for instance, been observed in the with-scale direction: $n = 10.97$ and $n' = 10.18$; in the opposite direction $n = 3.09$ and $n' = 2.74$.

The twist length has been measured on several pairs of fibres at various values for n. It appears that for this sample z can be approximated by the expression $z = 0.20\ n$ cm. The error in z made by this approximation may amount to 5%, but according to eq. (37) the resulting error in μ_s is only about ½% so that this relation between z and n may be safely used.

Consequently the values of z are $z = 2.19$, $z' = 2.04$ and $z = 0.62$, $z' = 0.55$ cm, respectively. By application of the first nomogram (fig. 28) the following values are obtained: $\mu_s = 0.149$, $\mu' = 0.162$ and $\mu_s = 0.579$, $\mu' = 0.655$. The values of μ_k are, according to the second nomogram (fig. 29), 0.143 and 0.545, re-

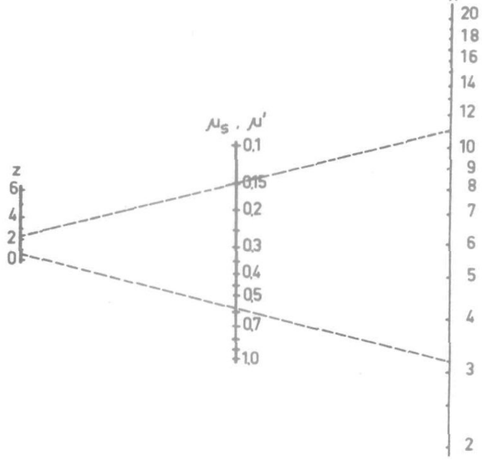

Fig. 28. Nomogram for the determination of μ_s and μ'

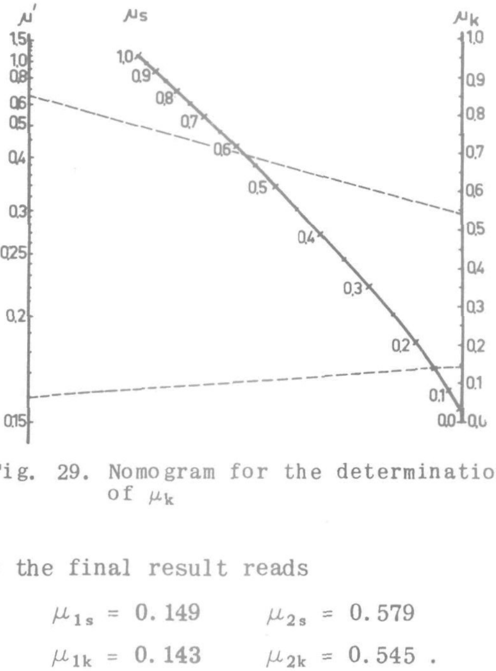

Fig. 29. Nomogram for the determination of μ_k

spectively. So the final result reads

$$\mu_{1s} = 0.149 \qquad \mu_{2s} = 0.579$$

$$\mu_{1k} = 0.143 \qquad \mu_{2k} = 0.545 \, .$$

4.6. PRELIMINARY RESULTS

The coefficients of friction of various fibres were measured in air-dry condition. Of each kind of fibres four or five different pairs were taken, on each of which the measurement was repeated five times. In order to give an impression of the reproducibility, the results obtained on viscose rayon filaments are represented in fig. 30; on each pair of vertical lines in this graph the observed coefficients of static and kinetic friction are indicated, respectively, together with their mean values; the two horizontal dotted lines show the average of all fibres together, $\bar{\mu}_s$ and $\bar{\mu}_k$.

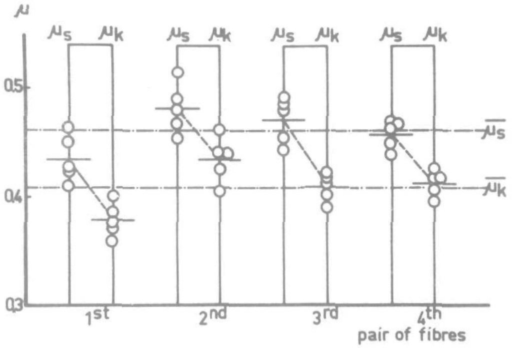

Fig. 30. Observed coefficients of friction on four pairs of viscose rayon filaments

The results obtained in this way on wool, viscose rayon, dacron, orlon, two kinds of nylon and teflon are collected in table VII; the normal pressure between the fibres ranged from 300 to 600 dyn/cm.

Table VII
Coefficients of friction of various fibres in air-dry condition

fibre	μ_s	μ_k
wool (anti-scale)	0.278	0.250
wool (with-scale)	0.141	0.137
viscose rayon	0.461	0.408
dacron	0.519	0.491
orlon	0.419	0.334
nylon (19 μm)	0.611	0.502
nylon (43 μm)	0.646	0.593
teflon	0.209	0.184

5. MEASUREMENT OF YOUNG'S MODULUS

5.1. PRINCIPLE OF MEASUREMENT

The Young's modulus, E, of a fibre can be measured in various ways. Generally the methods are divided into static and vibrational (or dynamic) measurements, in the first of which single deformations are involved, in the second periodic repetitions of deformations. Static experiments are, for instance, measurements of the stress-strain diagram, creep and relaxation tests. Vibrational experiments can be distinguished into those with either forced or free vibrations or into measurements with or without an extra mass.

Various types of deformation can be applied to a fibre, the most frequently used ones being elongation, bending and torsion. A torsion test yields the shear modulus, G, which is not simply related to E, due to the anisotropy of fibres. From a bending test, in the first instance E is obtained, though for fibres with inhomogeneous cross-section the results of tests in bending and in elongation may differ considerably.

The present investigation requires a method in which the conditions as to type of deformation are as closely as possible related to the circumstances during felting. Both elongation and bending have been mentioned in the literature, as being important in the mechanism of felting, namely by Arnold and Shorter, respectively (see section 1.3). Viewing the observations on the structure of felted yarns, reported in chapter 8, it seems that a low bending stiffness favours felting.

Unfortunately it is not possible to measure the bending stiffness of wool fibres in a simple way. A static measurement can practically not be performed on a single fibre. Transverse vibrations of short pieces of a fibre can easily be brought about in air; the stiffness must, however, be measured in the liquid in which felting takes place, and the damping of this kind of vibrations in a liquid is much higher than the critical one. Hence, the deformations must be applied in longitudinal direction.

The method most frequently used in studying longitudinal elastic or viscoelastic properties of fibres is the determination of a stress-strain diagram. By this method not only the modulus E is measured but also other characteristic properties as mentioned in section 1.23. The following objection may, however, be raised against the use of the stress-strain diagram in the present investigation. Most often the influence of elasticity on felting is studied by means of a variation of the medium without modification of the material itself. Since as a rule the spreading in thickness of the fibres is high, it is most convenient to measure the modulus E of one and the same fibre in these different media. With the determination of a stress-strain diagram considerable deformations of the fibre occur and it has been shown [35] that if the strain has amounted to 20% one has to allow the fibre 24 hours for recovery before the behaviour reproduces. In this way the total duration of the measurements on one and the same fibre is as many days as there are media used. Furthermore, the effect of time of soaking on the elasticity cannot be investigated.

It is, therefore, better to apply a method in which the deformations are very small, which is the case with a longitudinal vibrational measurement. The most obvious method in this respect is the determination of the resonance frequency of a system formed by a fibre and an extra mass. When the fibre is suspended vertically and is loaded by a small weight, several difficulties arise, however, the chief one being the occurrence of oscillating movements in other directions. An improvement is obtained when the fibre is stretched between two fixed points with the mass attached in its centre, though still uncontrollable transverse vibrations take place. Hence the mass is substituted by a small bar, to one of the extremes of which the fibre is attached, and which can rotate about its centre in the plane formed by fibre and bar (fig. 31). This principle enables a rapid determination of the stiffness of a fibre. In order to ascertain the modulus E, the area of cross-section must also be known. The most convenient way to measure this quantity is to apply the principle of the so-called vibroscope, introduced by Gonsalves [14]; by determining the transverse resonance frequency in air, the mass per unit length can be calculated. Though for comparative measurements of E on one and the same fibre in different media, knowledge of the cross-sectional area is not necessary, it is desirable to provide the equipment with the pos-

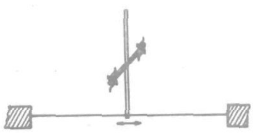

Fig. 31. Principle of fibre stiffness measurement

sibility to determine the thickness of a fibre under investigation rapidly.

5.2. DESCRIPTION OF THE METHOD

On a rigid frame the following three parts of the apparatus are mounted (fig. 32):

A is a grammophone cutting head which acts as an exciter and which is fed by an audio frequency oscillator. The pen B is set into vibration in the way indicated in figure 33; as a result of its shape it will excite the fibre which is attached to it into longitudinal as well as into transverse vibration.

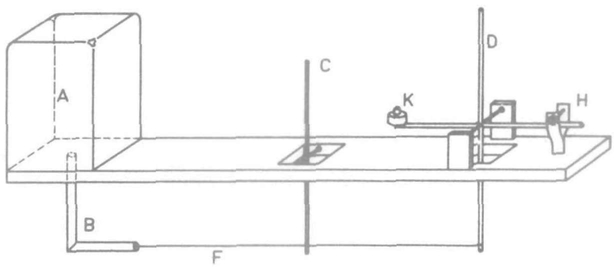

Fig. 32. Schematic representation of the apparatus for measuring the Young's modulus of fibres

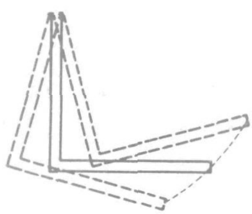

Fig. 33. Way of vibration of the exciter pen

The pen C is pivoted in its centre; when the fibre is fixed to it, it acts as an additional mass to the fibre, as pointed out in the previous section.

The system D is cross-shaped and can rotate in its own plane by virtue of a pivoting in its centre. A weight, K, can be screwed to one of the horizontal arms and serves to put the fibre which is fixed to the lower end of the cross, under a certain tension. The balance system obtained in this way can be arrested by means of the screw H. The position of the top of the vertical arm can be read on a scale, engraved into a mirror.

The whole fibre can be immersed in a liquid. In order to avoid corrosion, the parts of the apparatus which are immersed together with the fibre, are made of stainless steel. The vibrating parts can be observed by means of a low-power microscope.

The apparatus is used in the following way:
First the fibre under investigation is glued to the lower end of D with the balance unarrested, then it is fixed to B in such a way that the balance, D, is in its zero position so that the length and the tension of the fibre are known. Now the transverse resonance frequency is determined by adjusting the oscillator frequency to the value at which the amplitude of transverse vibration shows a maximum. When this measurement is performed, the fibre is glued to the pen C, the balance is arrested and the oscillator frequency is varied till the amplitude of vibration of C is maximal; now the fibre vibrates longitudinally. This measurement can be repeated after immersion of the fibre into a liquid; if necessary the frequency can be studied as a function of the soaking time, since each measurement takes only a few seconds.

The Young's modulus can be calculated from the measured frequencies and the constants of the apparatus; for this purpose the following symbols will be introduced:

- l = the total length of the fibre which is divided into two equal parts by the pen C.
- m = the mass representing the inertia of the pen C.
- P = the tension in the fibre exerted by the screw K.
- S = the area of cross-section of the fibre.
- ρ = the specific mass of the fibre.
- ν_t = the frequency of transverse resonance.
- ν_l = the frequency of longitudinal resonance.
- E = the Young's modulus of the fibre.

When the fibre is brought into transverse vibration, the well-known formula for the resonance frequency of a string is valid:

$$\nu_t = \frac{1}{2l}\sqrt{\frac{P}{S\rho}} \quad \text{or} \quad S = \frac{P}{4l^2\nu_t^2\rho} . \tag{1}$$

The frequency of the longitudinal vibration is

$$\nu_l = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{2}$$

in which k, the total stiffness of the two parts of the fibre, is given by

$$k = 2E\,\frac{S}{l/2} = 4E\,\frac{S}{l} . \tag{3}$$

Substitution of (3) into (2) yields

$$\nu_l = \frac{1}{2\pi}\sqrt{\frac{4ES}{lm}} \qquad \text{or} \qquad E = \frac{\nu_l^2\pi^2 lm}{S}, \tag{4}$$

which, together with (1), becomes

$$E = \frac{4\pi^2\nu_l^2\nu_t^2 l^3 m\rho}{P} . \tag{5}$$

5.3. PRELIMINARY MEASUREMENTS

Since the fibres under investigation are put under a certain tension it is desirable to ascertain to what extent the result of the stiffness measurement is influenced by the magnitude of this tension. Therefore, on several fibres the longitudinal resonance frequency was determined as a function of the applied stress. In order to control the fibre tension, the balance must be kept unarrested during these measurements, which gives rise to the following complications.

Firstly, the fibre gets an extra elongation when the stress is increased, so that the moduli are no more simply proportional to the square of the resonance frequency. However, a correction can easily be performed in the following way. A relative elongation, $\epsilon = \Delta l/l$, diminishes the area of cross-section in the ratio $1/(1 + \epsilon)$ and enlarges the length in the ratio $(1 + \epsilon)$. If l and S represent the dimensions of the unstrained fibre, one can write (see eq. (3))

$$E = k\,\frac{l}{4S}(1 + \epsilon)^2 . \tag{6}$$

Combination of this equation with eq. (2), yields

$$E = \pi^2\, m\, \frac{l}{S}\, \nu_l^2\, (1 + \epsilon)^2 . \tag{7}$$

A second complication arises from the fact that the fibre is not rigidly fixed at D but is coupled to a second mass, formed by the inertia of the balance. Thus the system represented in fig. 34 is formed, consisting of two springs, k_1 and k_2, and two masses, m_1 and m_2. The resonance frequencies of this system are given by [25])

$$2\omega_{1,2}^2 = \frac{k_2}{m_2} + \frac{k_1 + k_2}{m_1} \pm \left[\left(\frac{k_2}{m_2} + \frac{k_1 + k_2}{m_2}\right)^2 - 4\,\frac{k_1\,k_2}{m_1\,m_2}\right]^{1/2} . \qquad (8)$$

Now $k_1 = k_2 = 2k$, $m_1 = m$ and $m_2 = pm$, say, so that eq. (8) becomes

$$\omega_{1,2}^2 = \frac{2k}{m}\left[1 + \frac{1}{2p} \pm \sqrt{1 + \frac{1}{4p^2}}\right] . \qquad (9)$$

Since $p >> 1$, eq. (9) can be approximated by

$$\omega_1{}^2 = \frac{2k}{m}\left(\frac{1}{2p} - \frac{1}{8p^2}\right) ; \qquad \omega_2{}^2 = \frac{2k}{m}\left(2 + \frac{1}{2p}\right) . \qquad (10)$$

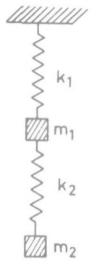

Fig. 34. System formed by two springs and two masses

The ratio of the frequencies does not depend on the stiffness of the fibre; from a few determinations of both frequencies it appears that $\omega_2/\omega_1 = 12$ approximately, when $P = 1/2$ gram is used; for a greater weight the ratio becomes still higher. From this observation it can be calculated easily that $p = 35$, approximately, so that the error in ω which is made when the simple relation $\omega_2{}^2 = 4\,k/m$ is used, is only 0.3%, being within the accuracy of the oscillator frequency.

Thirdly, when the fibre is elongated, the pen C will come into a skew position so that its effective mass is altered. This mass, m, equals I_p/a^2 in which I_p is the moment of inertia of the system and a is the distance in normal direction between the fibre and the axis of rotation of the pen. Now a varies with the angle of the pen, α, according to $a = a_o \cos\alpha$ so that

$$m = \frac{m_o}{\cos^2\alpha} = \frac{m_o}{1 - \sin^2\alpha} = \frac{m_o}{1 - (\Delta l/2a_o)^2} . \qquad (11)$$

When the corrections expressed in eq. (7) and eq. (11) are combined, the expression for E becomes

$$E = \pi^2\, m_o\, \frac{l}{S}\, \nu_l^2 \left[1 + \frac{\Delta l}{l}\right]^2 \Big/ \left[1 - \left(\frac{\Delta l}{2a_o}\right)^2\right] . \qquad (12)$$

Generally Δl is measured by means of the deflection, δ, of the top end of the balance, which enlarges the fibre elongation in the ratio 1.21. The numerical values of l and a_o are 7.7 cm and 1.8 cm respectively, so that

$$E = \frac{1}{c} \nu_l^2 \left[1 + \frac{\delta}{9.3}\right]^2 \Big/ \left[1 - \left(\frac{\delta}{4.35}\right)^2\right] \tag{13}$$

in which c depends only on the initial thickness of the fibre.

The results of the measurements performed on a wool fibre from the sample E_o in water are recorded in table VIII. At each tension, P, ν_l and δ have been measured; cE is calculated according to eq. (12). Though several fibres were investigated in this way, only the results obtained on one of them are represented; the other ones showed a similar behaviour.

Table VIII
Modulus of a wool fibre in water as a function of fibre tension

fibre tension P (grams)	frequency ν_l (c.p.s.)	pen deflection δ (cm)	modulus cE (arbitrary units)
0.25	165	0.00	2.73
0.50	205	0.02	4.21
0.75	209	0.06	4.46
1.00	210	0.10	4.48
1.25	209	0.14	4.50
1.50	203	0.25	4.42
2.00	182	0.80	4.04

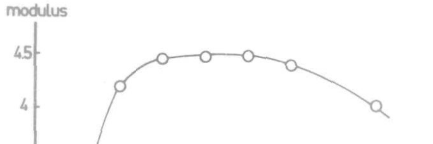

Fig. 35. Young's modulus of wool fibre as a function of the stress, P

Fig. 35 represents cE as a function of the load, P. It appears that the modulus is small at small loads, due to the curliness of the fibre which has not yet entirely been removed by the tension. In the region from $P = 0.5$ g up to $P = 1.5$ g, E is reasonably constant; at still higher load E decreases. This decrease is accompanied by a considerable increase in the elongation; obviously the apparent yield value is exceeded (see fig. 2).

As a consequence of these results the normal stiffness measurements of wet fibres are generally performed at a tension of

P = 1 gram; the thus obtained values may be compared safely. It is true that some relaxation of the tension may occur but the region in which E appears to be constant is large enough to prevent an undesired decrease of the modulus. Furthermore, incomplete removal of the curliness can be observed easily by visual examination of the fibre.

Though determination of the absolute magnitude of the modulus, E, is seldom needed, it is desirable to know the constants in eq. (5) numerically. In order to measure the magnitude of m, the apparatus is turned in such a way that the fibre is suspended vertically. The tension in the fibre is not applied by means of the balance but by a known weight. After the measurements of the frequencies ν_1 and ν_2, p can be calculated from ν_1/ν_2, according to eq. (9). For a weight of 0.589 g, p appears to be 12.7 so that m = 0.0465 g. In order to check this value, m has also been determined by adding a known extra mass M to the pen at the spot where the fibre is attached to it. From the ratio of the resonance frequencies without and with the additional mass, ν and ν' respectively, m can be calculated according to

$$\frac{\nu}{\nu'} = \sqrt{\frac{m + M}{m}}$$

Three different values of M were applied, namely 0.0197, 0.0424 and 0.0805 g; the four observed frequencies were 216, 182, 155 and 131 c.p.s. The resulting values for m are 0.0480, 0.0447 and 0.0468 g; the average happens to be identical to the value found in the other way, namely 0.0465 g.

If m and l are substituted numerically into equation (5), one obtains

$$E = 833\ \nu_l^2\ \nu_t^2\ \rho/P\ . \tag{14}$$

With measurements on wool, the tension P is generally 1 gram or 981 dyne; the specific mass of wool, ρ, is 1.31 g/cm^3, so that in this case

$$E = 1.11\ \nu_l^2\ \nu_t^2\ \ \text{dyn/cm}^2\ . \tag{15}$$

A few measurements were performed on several kinds of fibres in air-dry condition; table IX represents the results. E was calculated according to equation (14).

Table IX
Results of measurements on various fibres

fibre	ν_l (c.p.s.)	ν_t (c.p.s.)	P (grams)	ρ (g/cm^3)	E (10^{10} dyn/cm^2)
nylon (19 μm)	220	760	0.5	1.14	5.4
	222	760	0.5	1.14	5.5
nylon (43 μm)	461	500	1.0	1.14	5.1
	473	470	1.0	1.14	4.8
orlon	276	940	0.5	1.17	13.4
	308	840	0.5	1.17	13.3
dacron	341	760	0.5	1.38	15.7
	271	1010	0.5	1.38	17.5
viscose rayon	370	830	1.0	1.52	12.2
	360	850	1.0	1.52	12.1
kuralon	250	620	0.5	1.31	5.7
	250	640	0.5	1.31	5.7
wool	269	880	1.0	1.32	6.0
	220	1100	1.0	1.32	6.5

6. FELTING AND FRICTION

6.1. MATERIALS USED

In order to investigate the influence of fibre friction on felting it is necessary to have the disposal of yarns in which the friction properties of the fibres have been modified. These modifications can be brought about by chemical treatments, especially by those which are known to reduce the shrinkage of wool, since most of these treatments affect the fibre friction. Chlorination of wool is one of the processes often applied; the nature of its action depends on the circumstances under which chlorine is brought into contact with the fibres, but in most cases the surface of the fibres is modified. Mercuric acetate also reduces felting, though it is of no economic importance in this respect. According to Barr and Speakman [2] this agent affects mainly the stiffness of the single fibres; sometimes, however, the friction is also altered.

For the present investigation, parts of the standard sample were treated in several ways *). The samples thus obtained are listed below.

E_o untreated
E_1 treated by chlorination at pH 3.5; 4% active chlorine
E_2 treated by chlorination at pH 8.5; 4% active chlorine
E_3 treated with 0.1 M mercuric acetate in 0.1 N acetic acid
E_4 degreased by extraction
E_5 degreased by extraction; chlorinated at pH 8.5; 4% active chlorine
E_6 degreased by extraction; chlorinated at pH 3.5; 4% active chlorine
E_7 degreased by extraction; treated with 0.1 M mercuric acetate in 0.1 N acetic acid.
E_8 treated by chlorination at pH 5.5; 4% active chlorine
E_9 treated by chlorination at pH 3.5; 8% active chlorine
E_{10} treated by chlorination at pH 3.5; 8% active chlorine
E_{11} treated by chlorination at pH 7.5; 4% active chlorine
E_{12} treated by chlorination at pH 7.5; 8% active chlorine
E_{13} treated with 0.1 M mercuric acetate in 0.1 N acetic acid.

*) The author is indebted to Mrs Ir C.J. Schooneveldt-v.d.Kloes for the performance of the chemical treatments.

Though, according to this scheme, some treatments are identical, small differences in the circumstances during the treatment (temperature and time of action) often give rise to deviation of the results, so that the yarns mentioned all behave differently. A more detailed description of these circumstances will not be given because the aim of the chemical treatments has not been to study the nature of the action of the various agents on wool, but only to obtain a series of yarns with diverging frictional properties.

6.2. FELTING OF TREATED YARNS

On the yarns mentioned in the previous section shrinkage tests at various rotational speeds were performed. Each time four yarns of the same sample were tested simultaneously; the initial rate of shrinkage was determined from the average behaviour. The results obtained in this way are recorded in table X.

As has been pointed out in section 3.5, two critical values of n may be expected namely the intersection of the $S-n$ plot with the n axis (n_1) and the value of n at which the curve bends off (n_2). The determination of these values is demonstrated in fig. 36 for the yarns E_o, E_1 and E_5. In order not to overcrowd the graph, the other results have been omitted. It appears that in some cases the straight part of the curve is very small so that extrapolation of this part towards the n-axis is rather difficult. The spreading of the points causes an uncertainty in the determination of both n_1 and n_2. Nevertheless for each yarn these two quantities have been estimated; the results have been collected in the lower part of table X.

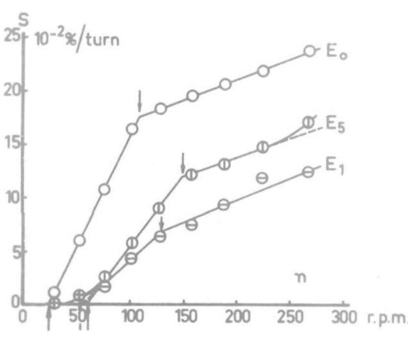

Fig. 36. Determination of characteristic values of rotational speed, n_1 and n_2, from the $S-n$ plot for three different yarns

The limit of shrinkage has only been determined incidentally; it appears that for most yarns the limit at high speeds lies between 60% and 70%, but the region of n in which the limit rises

Table X

Initial rate of shrinkage, S (10^{-2}%/turn), of treated yarns as a function of rotational speed, n (r.p.m.)

n	E_o	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}
19										0.3	0.6	0.1	0.2	0.0
30	1.2	0.1	0.2	0.3	0.3	0.0	0.1	0.1	0.4	0.8	0.9	0.4	0.7	1.0
40										1.0	1.0	1.0	1.0	0.7
53	6.0	0.8	2.4	1.5	2.0	0.8	1.1	1.2	3.2	2.1	1.7	2.3	1.6	1.7
63										3.1	3.3	3.3	1.9	2.3
77	10.8	1.9	5.5	4.0	4.6	2.4	2.6	3.2	6.1	4.7	3.7	4.2	2.9	3.9
90										4.0	4.7	3.7	2.6	4.9
102	16.4	4.3	8.6	7.2	7.1	5.7	5.4	6.8	9.5	5.4	4.3	4.9	2.9	6.1
117										6.7	4.7	4.3	2.7	6.6
128	18.3	6.4	9.9	10.0	9.6	9.0	7.5	9.8	10.9	6.8	5.9	2.6	2.3	6.7
143										6.7	4.9	2.4	2.4	7.6
158	19.5	7.5	12.4	9.8	10.4	12.1	8.2	9.3	13.2	6.1	6.7	3.7	2.9	8.1
175										7.0	5.7	7.0	2.1	
189	20.6	9.4	13.7	10.6	12.0	13.1	11.1	9.8	12.4	6.3	7.4	6.3	3.7	8.6
209										6.3	8.6	4.6	3.6	
225	21.8	11.8	13.1	11.7	13.8	14.7	10.2	11.7	14.2	8.1	6.3	6.4	3.1	8.6
247												4.9	2.4	
267	23.7	12.4	14.8	13.0	17.5	17.0	12.3	12.7	15.3			3.7		8.9
critical values of n														
n_1	24	55	35	45	35	60	53	54	28	35	37	29	23	38
n_2	105	130	100	120	115	150	125	120	105	95	90	78	74	105

from zero to its maximum value is, in general, much greater than for the untreated yarn.

6.3. FRICTION OF FIBRES FROM THE TREATED YARNS

From the fourteen yarns under investigation single fibres were taken, the friction of which was measured in destilled water with the aid of the apparatus described in chapter 4.

On one and the same pair of fibres five or more measurements in both directions were performed; of the untreated sample ten pairs of fibres were tested and of each of the other samples five pairs. The average values of the coefficients of friction of each pair of fibres are collected in table XI, in which the final averages of the samples are also recorded.

It appears that the coefficients of friction in the with-scale direction show, in general, a smaller spreading than those in the anti-scale direction. For some of the yarns, however, the deviations in μ_1 are considerably greater than for the untreated sample, which is obviously due to an inhomogeneous distribution of the chemical attack on the fibres in the yarn.

6.4. RELATION BETWEEN FELTING AND FRICTION

On the basis of the experimental results reported in 6.2 and 6.3 the hypotheses put forward in 3.5 can be tested. In that section it was supposed that the coefficients of friction determine the values of n at which the $S - n$ curve intersects the n-axis and at which this curve bends off. According to fig. 16, extrapolation of the first straight part of the curve determines the value n_1 which is related to μ_{1k}; the intersection of the two idealized straight parts takes place at n_2 which is determined by μ_{2k}.

In order to investigate the validity of this hypothesis, the values found for n_1, n_2, μ_{1k} and μ_{2k} for each yarn have been combined in table XII and in fig. 37.

It appears from fig. 37 that a reasonable good correlation exists between the values for μ_k and for n. A closer fit of the experimental points to a single curve can hardly be expected if one considers firstly the spreading in the experimentally found coefficients of friction and secondly the often rather arbitrary

Table XI

Coefficients of friction of fibres from treated yarns

	μ_{1s}	μ_{1k}	μ_{2s}	μ_{2k}		μ_{1s}	μ_{1k}	μ_{2s}	μ_{2k}		μ_{1s}	μ_{1k}	μ_{2s}	μ_{2k}
E_o	.16	.15	.71	.58	E_4	.19	.19	.54	.47	E_9	.24	.21	.72	.69
	.17	.16	.78	.65		.43	.28	.88	.72		.17	.17	.54	.50
	.16	.15	.59	.55		.27	.21	.70	.54		.25	.25	.67	.65
	.17	.17	.48	.43		.22	.20	.78	.64		.22	.22	.81	.76
	.16	.15	.64	.56		.34	.24	.72	.59		.22	.21	.57	.52
	.17	.16	.52	.48		**.29**	**.22**	**.72**	**.59**		**.22**	**.21**	**.66**	**.62**
	.15	.15	.66	.58										
	.17	.17	.58	.52	E_5	.61	.53	.87	.82	E_{10}	.26	.24	.55	.50
	.22	.21	.82	.63		.55	.50	.93	.88		.26	.23	.60	.57
	.15	.15	.66	.58		.52	.43	.83	.76		.21	.19	.57	.53
	.17	**.16**	**.64**	**.55**		.57	.50	.95	.89		.30	.28	.50	.45
						.52	.51	.90	.83		.39	.33	.61	.55
E_1	.33	.28	.62	.57		**.55**	**.49**	**.90**	**.84**		**.28**	**.25**	**.57**	**.52**
	.40	.35	.68	.64										
	.38	.32	.72	.69	E_6	.34	.32	.69	.62	E_{11}	.21	.19	.48	.45
	.29	.27	.69	.65		.34	.32	.73	.62		.45	.36	.69	.60
	.27	.24	.72	.65		.34	.32	.75	.61		.28	.23	.59	.55
	.33	**.29**	**.69**	**.64**		.37	.34	.81	.71		.28	.23	.60	.54
						.38	.35	.83	.73		.25	.22	.61	.58
E_2	.19	.19	.52	.52		**.35**	**.33**	**.76**	**.66**		**.29**	**.25**	**.59**	**.54**
	.24	.20	.70	.61										
	.14	.14	.45	.45	E_7	.49	.35	.82	.61	E_{12}	.23	.19	.39	.36
	.18	.18	.48	.48		.52	.35	.87	.70		.21	.16	.41	.38
	.19	.19	.52	.52		.52	.30	.90	.66		.35	.29	.55	.49
	.19	**.18**	**.53**	**.52**		.38	.26	.87	.64		.22	.19	.46	.43
						.50	.33	.90	.66		.17	.14	.45	.42
E_3	.26	.22	.66	.58		**.48**	**.32**	**.87**	**.65**		**.24**	**.19**	**.45**	**.41**
	.27	.25	.72	.62										
	.27	.25	.61	.49	E_8	.19	.18	.67	.57	E_{13}	.29	.26	.64	.54
	.27	.25	.75	.61		.17	.17	.62	.57		.19	.17	.45	.43
	.25	.24	.68	.61		.16	.16	.50	.45		.22	.21	.52	.46
	.26	**.24**	**.68**	**.58**		.16	.16	.52	.48		.27	.24	.58	.55
						.17	.17	.61	.60		.26	.24	.53	.49
						.17	**.17**	**.58**	**.53**		**.25**	**.22**	**.54**	**.49**

Table XII

Characteristic values of n and coefficients of kinetic friction

	μ_{1k}	n_1	μ_{2k}	n_2
E_o	.16	24	.55	105
E_1	.29	55	.64	130
E_2	.18	35	.52	100
E_3	.24	45	.58	120
E_4	.22	35	.59	115
E_5	.49	60	.84	150
E_6	.33	53	.66	125
E_7	.32	54	.65	120
E_8	.17	28	.53	105
E_9	.21	35	.62	95
E_{10}	.25	37	.52	90
E_{11}	.25	29	.54	78
E_{12}	.19	23	.41	74
E_{13}	.22	38	.49	105

way of determining n_1 and n_2 from the $S - n$ curve which is drawn through spreading points.

When the coefficients of correlation are calculated, the values of $r_1 = 0.86$ and $r_2 = 0.85$ are found for the correlations between μ_{1k} and n_1 and between μ_{2k} and n_2 respectively; for the combined results $r = 0.96$ is obtained.

The straight line with the best fit to the measuring points of fig. 37 has the equation

$$n = 200(\mu - 0.04);$$

this line, however, does not pass through the origin so that it cannot be generally valid. An equally good fit is obtained by the curve with the equation

$$n = 205 \cdot \mu^{1.13}$$

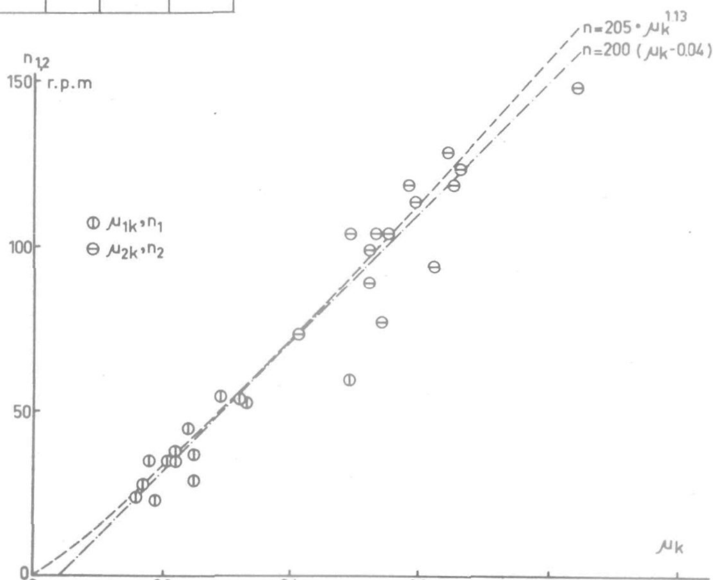

Fig. 37. Correlation between characteristic values of rotational speed n_1 and n_2, and coefficients of kinetic friction μ_{1k} and μ_{2k}, for fourteen yarns

which is considered to be a better representation of the true relation between μ and n.

The deviation from exact proportionality is too small to justify a thorough discussion of the question whether this deviation is due to non-proportionality between the hydrodynamic force and the rate of shaking, or to other causes.

It may be concluded that the results of the experiments with the treated yarns confirm the hypothesis about the influence of the coefficients of friction on felting.

7. FELTING AND ELASTICITY

7.1. INVESTIGATION ON TREATED YARNS

In section 3.5 the discussion of the shape of the $S - n$ curve led to the supposition that the slope of the initial part of this curve is related to the fibre stiffness. Since this slope can easily be determined from the shrinkage data of the treated yarns, reported in section 6.3, it seems obvious to try out this hypothesis by measuring the stiffness of fibres from these yarns. These measurements have been performed in the way described in chapter 5. Of each sample ten fibres were taken, on each of which the two frequencies ν_t and ν_l were determined, the latter being measured in destilled water. The Young's modulus of the wet fibre, E_w, was calculated for each fibre according to eq. (14) of chapter 5; the mean for each sample is recorded in table XIII together with the slope of the $S - n$ curve, dS/dn. Fig. 38 represents a plot of dS/dn against $1/E_w$.

Table XIII
Slope of the S-n curve and Young's modulus of fibres, for treated yarns

Sample	E_o	E_1	E_2	E_3	E_4	E_5	E_6
dS/dn	0.210	0.088	0.128	0.125	0.107	0.133	0.110
E_w (10^{10}dyn/cm^2)	2.65	2.76	2.93	2.87	2.72	2.85	2.50
$1/E_w$	0.387	0.363	0.307	0.348	0.367	0.351	0.400
Sample	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}
dS/dn	0.138	0.126	0.109	0.126	0.095	0.052	0.095
E_w (10^{10}dyn/cm^2)	2.92	2.57	2.52	2.88	2.54	2.47	2.95
$1/E_w$	0.343	0.389	0.396	0.347	0.392	0.404	0.339

From fig. 38 it appears that no correlation exists at all. The highest situated point in the graph represents the untreated yarn E_o; for this yarn dS/dn is much greater than for the treated yarns, which cannot be caused by a difference in fibre stiffness.

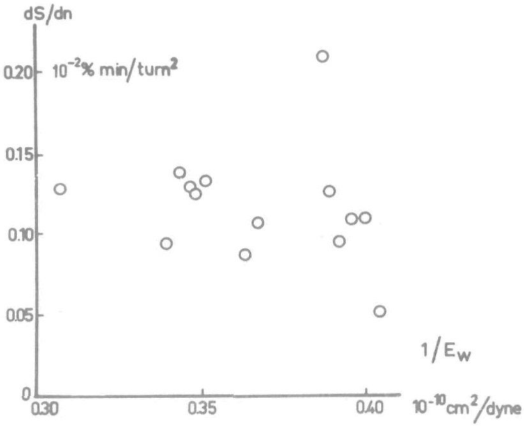

Fig. 38. Slope of the initial part of the $S - n$ curve *versus* fibre compliance for fourteen yarns

Obviously this discrepancy is induced by other factors; from the measurements on yarns with various degree of twist, reported in section 3.4, the yarn structure appeared to affect the slope of the $S - n$ curve considerably, so that probably small structural changes are brought about in the yarns by the application of the treatments. If these changes have a greater effect on dS/dn than the fibre stiffness, the lack of correlation in fig. 38 can be understood. Fortunately the results obtained on twisted yarns also indicate that a small difference in the twist has a much greater influence on the slope than on the characteristic values of n, so that the conclusions of chapter 6 remain unaffected. In order to investigate this effect more closely, the yarn E_0 has been made to undergo a blanco treatment in water. The results of shrinkage tests performed on this sample are given in table XIV which shows that n_1 and n_2 remain practically unaffected, whereas the slope dS/dn is considerably decreased by the blanco treatment.

Table XIV
Influence of blanco treatment on shrinkage parameters

Sample	n_1	n_2	dS/dn
untreated	23	103	0.145
after blanco treatment	23	105	0.111

It appears that two circumstances prevent the drawing of conclusions from the data obtained on the treated yarns; firstly the differences between the Young's moduli of the fibres from different samples are too small; secondly the chemical treatments cause structural changes which affect the slope more than the fibre properties do.

It may be remarked that the slope, dS/dn, for the untreated sample recorded in table XIV differs considerably from the one in table XIII. This fact indicates that small structural changes may also occur during storing of the yarns, since the measurements on treated yarns were performed two years before comparison with a blanco treatment took place.

7.2. FIBRE ELASTICITY IN SOME AQUEOUS SOLUTIONS

Since it is a well-known fact that the stiffness of wool fibres depends on the medium in which it is measured, the use of aqueous solutions offers a simple possibility to study the influence of fibre elasticity on felting.

Several inorganic salts increase the stiffness; the most effective ones in this respect appear to be calcium chloride and magnesium chloride. Formic acid and urea, on the other hand, give rise to a lower stiffness compared to that in water. Therefore the four agents mentioned were applied in various concentrations in order to verify the suggested relation.

A first impression of the influence of these agents on the stiffness is obtained *via* the following procedure: A fibre is mounted into the stiffness meter, it is surrounded with water and its longitudinal resonance frequency is determined. Then the water is replaced by one of the solutions and the measurement is repeated. In some cases the frequency is not constant and must be determined as a function of time. After a certain period, 30 minutes for instance, the solution is removed, the fibre is rinsed with water and is then placed into the following solution, *etc.*

The results obtained in this way on one of the fibres in calcium chloride and formic acid solutions are represented in fig. 39; the observed longitudinal resonance frequency, which is proportional to the square root of the fibre stiffness, is plotted against log time. It appears from this graph that the stiffness of the fibre in $CaCl_2$ solutions is initially much higher than in water but decreases afterwards. This decrease is the more rapid as

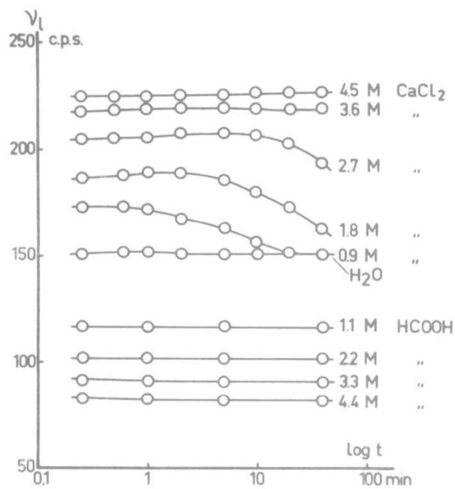

Fig. 39. Longitudinal resonance frequency as a function of time observed on one and the same wool fibre in several media

the concentration is smaller; for the smallest concentration used, namely 0.9 M, the stiffness is after 30 minutes equal to that in water. Magnesium chloride shows essentially the same effect; in formic acid and in urea solutions the stiffness is independent of time.

When the stiffness of the fibre in relation to that in water is calculated by determining the ratio of the squared frequencies, a serious error may be made. Though it has been shown in section 5.3 that with measurements in water a fibre tension of 1 gram gives rise to reliable results, the situation may be different when other media are applied. In order to investigate this question, the longitudinal frequency was determined in some solutions at various fibre tensions in the same way as described in section 5.3. The results obtained in 3.6 M $CaCl_2$, in 2.2 M formic acid and in water are recorded in fig. 40.

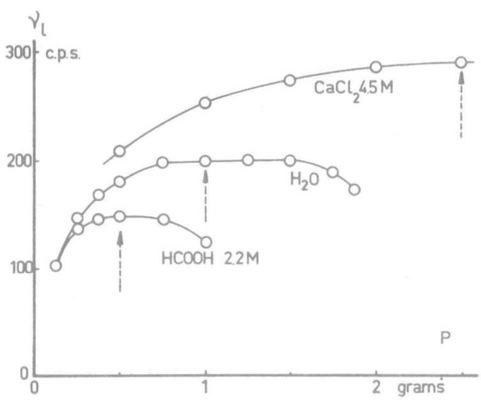

Fig. 40. Longitudinal resonance frequency as a function of fibre tension in three different media

It appears that the tension at which the maximum frequency occurs, is smaller in formic acid and greater in calcium chloride, so that, in general, the measured frequencies in these agents will be too small when the tension is kept constant. This indicates that the stress necessary to remove the curliness and the yield stress both decrease with decreasing stiffness. Therefore, for each medium the more complicated procedure of measuring the frequency as a function of the tension was followed, and from the maximum frequency the fibre stiffness was calculated. If fibre deformations occurred, the correction of eq. (13) in chapter 5 had to be applied.

In this way three fibres were investigated in each series of solutions. The results of these measurements are collected in table XV; this table contains the values of E_{water}/E, which denotes the extensibility or the reciprocal of the stiffness, relative to that in water.

Table XV
Ratio of fibre extensibility in several solutions to that in water

Agent	Fibre 1	Fibre 2	Fibre 3	Mean
water	1.00	1.00	1.00	1.00
HCOOH 0.4M	1.23	1.23	1.22	1.23
1.1M	1.44	1.46	1.47	1.46
2.2M	1.95	1.86	1.86	1.89
3.3M	2.40	2.40	2.34	2.38
4.4M	2.71	2.60	2.66	2.66
urea 5M	1.23	1.24	1.22	1.23
10M	1.32	1.38	1.37	1.36
$CaCl_2$ 0.9M	0.83	0.79	0.81	0.81
1.8M	0.66	0.66	0.66	0.66
2.7M	0.53	0.57	0.56	0.55
3.6M	0.52	0.50	0.51	0.51
4.5M	0.46	0.47	0.46	0.47
$MgCl_2$ 2.5M	0.78	0.78	0.78	0.78
5M	0.59	0.60	0.58	0.59

The difference between the values obtained for E_w/E between the three fibres are very small though the fibres differed considerably in thickness; therefore the data obtained on this small number of fibres seem sufficiently representative for the elastic behaviour of the wool in the applied solutions.

7.2. SHRINKAGE EXPERIMENTS IN SOLUTIONS

Shrinkage tests were performed on the untreated yarn E_o, in the solutions mentioned in the previous section. Due to the decrease of fibre stiffness in dependence of time, as indicated in fig. 39, it was necessary to reduce the testing time in the weaker salt solutions to a minimum. Therefore, only the shrinkage during 100 revolutions of the apparatus was determined, so that the whole measurement could be performed in a few minutes after the immersion of the yarn into the liquid. For the sake of uniformity, not only the tests in the lower concentrations of $CaCl_2$ and $MgCl_2$, but all the experiments were performed in this way. From a few complete measurements of the maximum rate of shrinkage, S, (section 2.4) it appeared that the errors made with the simplified test were small enough to justify this method; an increase of $\Delta s/\Delta N$ with N was never observed.

Since only the slope of the first straight part of the $S - n$ curve must be known, the shrinkage was only determined at the lower speeds. At each speed in each solution 2 or 4 yarns were shrunk simultaneously; the average results are recorded in table XVI, together with the graphically determined slopes dS/dn. Fig. 41 represents only part of the results, namely the shrinkage in some of the calcium chloride and formic acid solutions and in water.

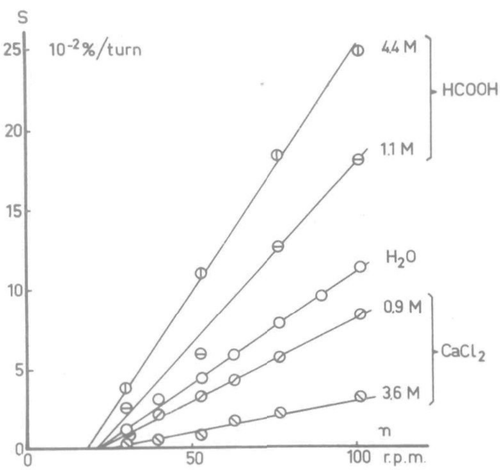

Fig. 41. Initial shrinkage per turn, S, as a function of rate of shaking, n, in various solutions

Table XVI
Shrinkage, S (10^{-2}%/turn), in various solutions as a function of n

agent \ n (r.p.m.)	30	40	53	63	77	90	102	dS/dn
water	1.2	3.1	4.3	5.8	7.7	9.5	11.3	0.139
HCOOH 0.4M	1.2		6.0		12.5		15.4	0.192
1.1M	2.6		5.9		12.6		18.0	0.224
2.2M	2.4		8.1		14.5		23.7	0.257
3.3M	3.8		9.9		16.9		22.6	0.278
4.4M	3.8		11.0		18.3		24.8	0.302
urea 5M	3.0		7.2		11.0		14.8	0.173
10M	2.5		7.5		11.3		16.8	0.194
$CaCl_2$ 0.9M	1.0	2.1	3.2	4.2	5.6		8.3	0.099
1.8M	0.1	0.7	1.1	1.7	2.9		4.3	0.066
2.7M	0.2	0.6	1.0	1.4	2.3		3.5	0.050
3.6M	0.2	0.5	0.8	1.7	2.1		3.2	0.044
4.5M	0.3	0.6	1.3	1.6	2.2		3.3	0.042
$MgCl_2$ 2.5M	0.5		3.3		6.2		8.6	0.115
5M	0.0		1.6		2.2		3.6	0.058
sucrose 0.3M	1.8		5.2		8.3		11.2	0.137
0.6M	2.1		4.8		8.8		12.0	0.144
1.2M	0.8		3.1		6.7		10.1	0.140

In the next section the relation between shrinkage and fibre extensibility will be discussed; however, the results of the shrinkage experiments may also be influenced by the viscosity and the density of the solutions. To investigate the effect of these properties of the liquid, some shrinkage tests were performed in sucrose solutions of three different concentrations. The results of these experiments can be read from table XVI; it appears that no significant differences in dS/dn occur. The relative fibre extensibilities in the 0.3M, 0.6M and 1.2M solutions are found to be 1.02, 1.00 and 0.93 respectively. Table XVII shows the viscosities and the densities of the various liquids.

Table XVII
Viscosity and density of liquids used

solution		viscosity (cP)	density (g/cm^3)
water		1.0	1.00
formic acid	2.2M	1.1	1.03
	4.4M	1.1	1.05
urea	5M	1.3	–
	10M	1.9	–
$CaCl_2$	0.9M	1.3	1.08
	2.7M	2.1	1.22
	4.5M	4.2	1.36
$MgCl_2$	2.5M	2.7	1.21
	5M	6.1	1.43
sucrose	0.3M	1.3	1.04
	0.6M	2.0	1.08
	1.2M	6.1	1.18

The viscosity of the 1.2M sucrose solution exceeds that of most of the other applied liquids; nevertheless it does not influence the quantity dS/dn. The fibre extensibility is practically unaffected too. Consequently, the effect of the viscosity on dS/dn appears to be negligible. As to the density the same conclusion may be drawn; though some of the concentrated salt solutions have a higher density than the 1.2M sucrose solution, the influence of density may probably also be ignored.

7.4. RELATION BETWEEN FELTING AND ELASTICITY

For each applied solution the values of the slope of the $S - n$ curve and of the relative fibre extensibility are now known and these values, shown in the last columns of table XV and table XVI, are plotted against each other in fig. 42.

In this graph a straight line has been drawn through the origin and the point representing the measurements in water. This straight line is a first approximation of the real empirical relationship. The data obtained by means of formic acid form an oscillating curve about this line; those of $CaCl_2$ deviate systematically; the two points representing tests in urea fit the

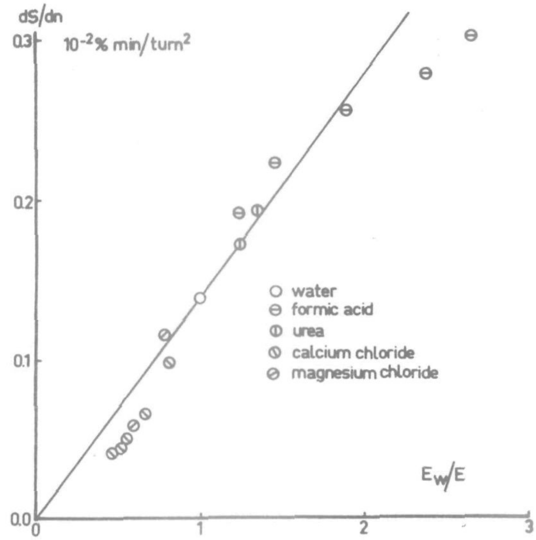

Fig. 42. Relation between slope dS/dn and relative fibre compliance E_w/E

straight line closely, and the data obtained in $MgCl_2$ are too few to show a systematic behaviour if any.

In the idealized picture of the mechanism of felting which is used throughout this study, a proportionality between dS/dn and $1/E$ may be expected on the basis of the following reasoning. In section 3.5 it was supposed that the fibre displacement per stroke is proportional to the resulting force which acts on the fibre. This supposition was based on the experimentally found straight part of the $S - n$ curve and on the hypothesis that the force is proportional to the rotational speed.

If the displacements are accompanied by fibre deformations the slope must be related to the fibre stiffness. In the ideal case, when the displacements are proportional to the deformations, the shrinkage per turn will be proportional to the force and inversely proportional to the fibre stiffness; then the slope of the $S - n$ curve will be proportional to the compliance, $1/E$.

These considerations are essentially confirmed by the results reported in this chapter; though the fit of the experimental points to the straight line through the origin is not ideal, nevertheless this line represents the average relationship. The deviations which occur cannot be interpreted without performing a deeper study; several factors may play a part in complicating the real behaviour.

8. STRUCTURE AND MECHANICAL PROPERTIES OF FELTED YARNS

8.1. INVESTIGATION OF STRUCTURE

When the shrinkage of a yarn is continued up to the final limit, a yarn is obtained which has little in common with the original one. Its length has decreased by a factor 3 or 4 and it has become much thicker and rougher. The surface structure has become very irregular compared to the original yarn, as is shown in fig. 43 which represents a photograph of a small part of each yarn. This photograph suggests that the fibres lose their parallel position during felting, which was already assumed in the discussion of the mechanism of felting. Examination of the surface structure, however, gives only a very rough impression of the position of the fibres in the yarn so that a method was sought enabling a more detailed study of this position.

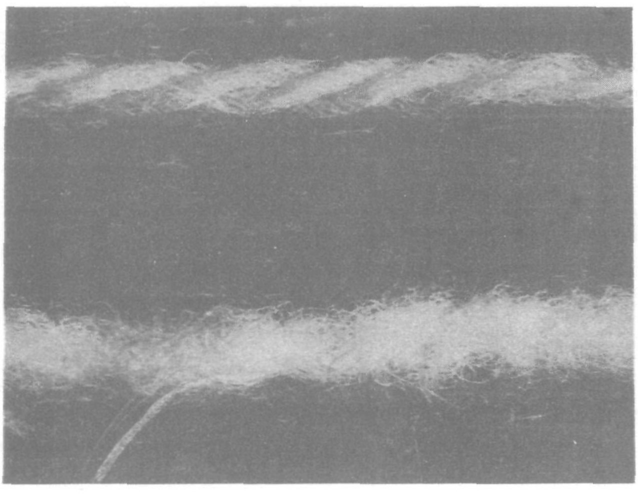

Fig. 43. Photograph of original and felted yarn (magnification 6 ×)

A very attractive method is the immersion of yarns with a small percentage of coloured fibres into a liquid with the same refractive index as the other fibres; then only the dark fibres are visible and their position can be studied easily. Fortunately this kind of knitting yarn is commercially available (the so-called Jeager-wool). The liquid used is a mixture of butylstearate and tricresyl-phosphate with a refractive index n = 1.549. The first investigations on these yarns were performed as follows: Some of the yarns were shrunk 10%, other ones 20%, etc. up to 70%. All these yarns were immersed in the liquid and photographed. Some of the results are shown in fig. 44, namely the yarns with 0%, 30% and 70% shrinkage. These photographs give a reasonable impression of the structure of felted yarns; however, they do not show how the position of one and the same fibre alters during felting.

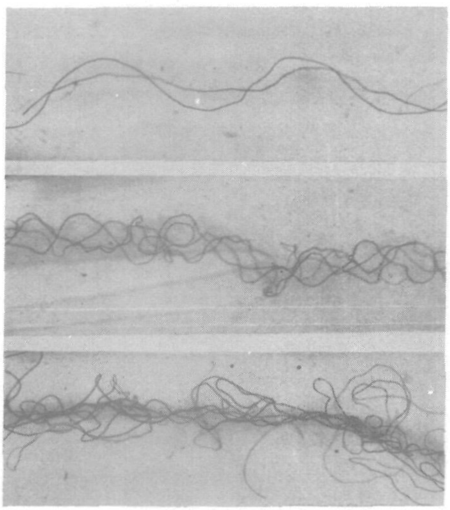

Fig. 44. Structure of yarns with 0%, 30% and 70% shrinkage (magnification 3.5 ×)

Therefore, the method was refined in the following way: Four pieces of yarn were selected, each possessing an isolated coloured fibre which was easily recognizable. These pieces were alternatively shrunk during a certain period and photographed in the liquid mentioned. Between these manipulations the yarns were thoroughly cleaned with alcohol and ether, and dried. In spite of the careful selection, the tracing on the photographs of the fibres under investigation appeared to be rather difficult; with two of the four yarns it was even impossible. Due to this difficulty the photographs themselves are not reproduced here, but the position of the fibres has been copied carefully. The results are shown in fig. 45. It must be remarked that the yarns were cut off in such a way that the fibre under investigation was located at equal distances from the ends of the yarn; during felting it appeared to remain in the middle. Consequently, the conclusion may be drawn that the fibre does not move as a whole during felting but only its parts. From fig. 45 it becomes clear that in the fibre more and more loops are formed, which reduce its effective length and which presumably entangle it thoroughly with its neighbours.

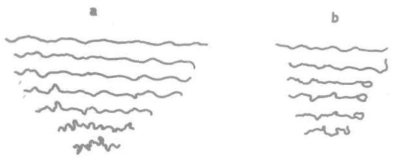

Fig. 45. Positional changes of two single fibres during yarn felting (magnification ¼ x)

Table XVIII
Yarn shortening $\Delta L/L$ and effective fibre shortening $\Delta l/l$ during felting for two fibres from different yarns

fibre a		fibre b	
$\Delta l/l$	$\Delta L/L$	$\Delta l/l$	$\Delta L/L$
0.00	0.00	0.00	0.00
0.11	0.12	0.13	0.11
0.13	0.25	0.25	0.19
0.22	0.36	0.32	0.32
0.42	0.48	0.40	0.40
0.63	0.60	0.60	0.55
0.74	0.72		

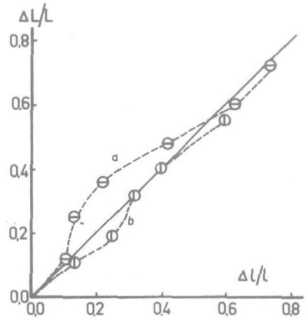

Fig. 46. Relation between yarn shrinkage and effective fibre shortening

The effective length, l, of the fibres can be measured, and can be compared with the total yarn length, L. Table XVIII and fig. 46 represent the relative shortenings, $\Delta l/l$ and $\Delta L/L$, for both series of experiments; the agreement between the two quantities is rather good.

8.2. MECHANICAL PROPERTIES OF FELTED YARNS

A number of yarns were felted to various degrees, their amount of shrinkage ranging from 0% to 70%. Of these yarns stress-strain diagrams were produced with the aid of the Cambridge Textile Extensometer. In order to eliminate the differences in thickness, the force was divided by the mass per unit length of the yarns which is proportional to the effective cross-sectional area. Some of the diagrams obtained are represented in fig. 47.

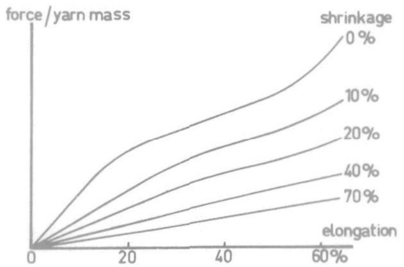

Fig. 47. Load-elongation diagrams of felted yarns

From the diagrams the Young's modulus, E, and the breaking stress, σ_B, were determined. These quantities, expressed in relative units, are collected in table XIX and plotted as a function of shrinkage in fig. 48.

Table XIX
Relative magnitude of Young's modulus and breaking stress for felted yarns

shrinkage (%)	E/E_o	σ_B/σ_{B_o}
0	1.00	1.00
10	0.63	0.81
20	0.26	0.54
30	0.33	0.48
40	0.18	0.36
50	0.18	0.31
60	0.13	0.27
70	0.13	0.24

The decrease of both E and σ_B can easily be understood qualitatively. When the orientation of the fibres is disturbed, the stress in the yarn, which was originally carried by all the fibres together, is distributed over a smaller number of fibres, which results in a higher elongation and thus in a smaller Young's modulus. For the same reason the breaking strength decreases; when the stress of the yarn is still low, few fibres are subject-

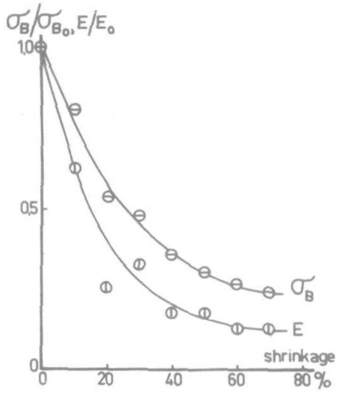

Fig. 48. Decrease of Young's modulus, E, and breaking strength, σ_B, of yarns during felting

ed to a relatively high stress and will break; then other fibres or fibre segments become orientated into the stress direction, and the same occurs. This breaking process cannot be seen from the diagrams obtained with the Cambridge Textile Extensometer since this instrument registers only increasing forces. A few experiments have, however, been performed on an electronic tensile tester which does not show this defect *); the stress-strain diagrams obtained on yarns with 0% and 50% shrinkage are reproduced in fig. 49, which demonstrates clearly the phenomenon described above. In the original yarn all fibres break simultaneously, the felted yarn shows breaking in subsequent stages.

Furthermore it appears from fig. 48 that the desorientation is happening the most rapidly when the shrinkage is still slight. In the neighbourhood of the shrinkage limit, E and σ_B have become approximately constant; presumably the orientation of the fibre segments has practically disappeared, though further decrease of the effective length of the whole fibre may still occur.

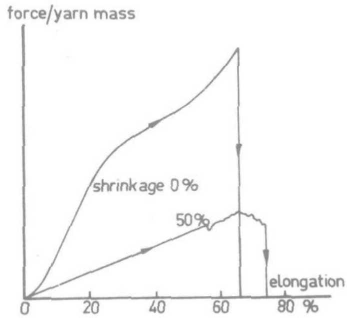

Fig. 49. Load-elongation diagrams of yarns with 0% and 50% shrinkage, the later showing rupture in subsequent stages

8.3. DISCUSSION

The observations reported in 8.1 and 8.2 give rise to the following considerations.

It has already been noted that the fibres in the yarn do not move as a whole during felting. The effective fibre length de-

*) The author is indebted to Ir H. van Lingen for the performance of this measurement.

creases by the formation of bends and loops. During these movements, the fibre segments which are going to form a loop have to overcome the frictional resistances of their contacts with other fibres; the resulting force determines the magnitude of the displacement. As has been shown in 3.5 and 7.4, the displacements are proportional to the resulting force and to the fibre extensibility. It is not possible to ascertain the nature of the deformations which are responsible for felting; on the one hand it may be imagined that the amount of bending determines the irreversible displacement; on the other hand the possibility exists that, apart from the bending, felting goes accompanied by a series of alternate elongations and contractions, and that the magnitude of these deformations is significant for the rate of shrinkage.

During felting the fibres become more and more chaotically situated; in this respect a faint analogy with rubbermolecules exists. Just as rubbermolecules tend to a state of minimal order which they reach *via* the heat motion of the molecular segments, the wool fibres which lie originally in a stretched position, try to reach a chaotical state when subjected to forces directed at random. An important difference is, however, the irreversibility of the fibre displacements; the same directional frictional effect which allows the fibres to reach a state of disorder, prevents them from coming back again and causes the formation of a more and more tightened and fixed network.

The tightening up of the felted structure is presumably one of the factors which determine the limit of shrinkage or, more generally, the shrinkage as a function of time. Without the formation of entanglements the limit ought to be higher, for fig. 45 shows clearly that complete disorder of the fibres is not yet reached. The shrinkage limit was found to be approximately independent of the applied force, so the entanglements are completely fixed and prevent locally further fibre movements.

S U M M A R Y

When wool felts, the individual wool fibres undergo irreversible displacements with respect to each other and arrive more and more in a state of disorder and entanglement. This phenomenon occurs when the wool is subjected to mechanical agitation; the displacements are irreversible as a result of the scale structure at the fibre surface (chapter 1).

In order to study the mechanism of felting, a method and apparatus have been designed for determining the relation between the felting shrinkage of wool yarns and the magnitude of the applied mechanical action when shaken up in water. Preliminary experiments gave satisfactorily reproducible results and showed that the most practical parameters by which the shrinkage of any sample can be identified are the limiting shrinkage and the initial shrinkage per revolution of the shaking apparatus (chapter 2).

The applied force was varied by changing the shaking rate and amplitude (chapter 3). The results show that shrinkage does not occur below a certain value of the applied force. From that value the rate of shrinkage increases linearly with increasing force up to a second critical force level at which the rate of shrinkage becomes more or less independent of the force (fig. 10). Measurements with twisted yarns show that the first limit is raised by increasing the inter fibre forces (fig. 11 and fig. 12).

From these results the suggestion arises that the two critical values of the force are determined by the friction of the fibres in the direction with the scales and the direction against the scales, respectively, and that the rate of increase of initial shrinkage with the applied force is related to the stiffness of the fibres. These hypotheses were tested by investigating yarns in which the single fibre properties were varied systematically.

The variations in frictional properties were brought about by chemical treatments and were measured with the aid of an apparatus described in chapter 4. The correlation between the coefficients of friction and the critical values of the rate of shaking is reported in chapter 6 (fig. 37).

The elastic properties were modified by the application of several aqueous solutions in which the measurements on untreated

yarns were performed. The fibre extensibility was measured with the aid of longitudinal forced vibrations; the method is described in chapter 5. Chapter 7 reports the results of this investigation; it appears that the rate of increase of the shrinkage with the force is approximately proportional to the fibre extensibility (fig. 42).

On the basis of these observations the following picture of the mechanism of felting seems justified: When the forces are too small to overcome the fibre friction in the scale direction, no fibre movements occur at all. With higher forces, movements in one direction are possible; they give rise to irreversible displacements, the magnitude of which is proportional to the resulting force and to the compliance of the fibres, and which are also strongly dependent on the yarn structure. When the applied force exceeds the friction in the direction against the scales as well, the displacements are partly reversible and the shrinkage becomes virtually independent of the force.

Finally the structure and the mechanical properties of felted yarns were investigated (chapter 8); it is shown that the fibres do not move as a whole but rather in parts, and that the orientation of fibre segments is completely disturbed by the felting process (fig. 45).

SAMENVATTING

Bij het vervilten van wol ondergaan de individuele vezels onder invloed van uitwendige krachten kleine verplaatsingen ten opzichte van elkaar. Hierbij beweegt iedere vezel zich tengevolge van de geschubde oppervlaktestructuur bij voorkeur in die richting, waarin de wrijving het geringst is. De verplaatsingen worden daardoor gedeeltelijk irreversibel en geven aanleiding tot het ontstaan van een meer en meer chaotische ligging der vezels in het garen waaruit het weefsel of breisel is samengesteld. Dit gaat gepaard met krimp.

Verscheidene theorieën over het mechanisme van het verviltingsproces zijn reeds gelanceerd, waarbij al dan niet gepoogd werd de krimp te correleren met de eigenschappen van de enkele vezels. Geen van deze is echter bevredigend, hoofdzakelijk aangezien de bewegingen van de vezels tot dusver niet quantitatief in verband gebracht werden met de toegepaste krachten (hoofdstuk 1).

In dit proefschrift wordt een onderzoek beschreven naar het bewegingsmechanisme der individuele vezels onder invloed van krachten, daarop uitgeoefend door een schuddende vloeistof. Het onderzoek werd verricht aan wollen breigarens aangezien de experimenten hiermede het eenvoudigst zijn, terwijl verwacht mag worden, dat het gedrag der vezels in garens niet essentieel zal verschillen met dat in meer samengestelde structuren.

Een toestel werd ontworpen waarmee de neiging tot vervilten van garens gemeten kan worden door deze garens te schudden met een vloeistof; de schudsterkte kan binnen wijde grenzen gevarieerd worden. De reproduceerbaarheid der metingen bleek voldoende te zijn; bovendien bleek de lengtekrimp onder bepaalde omstandigheden het meest practisch weergegeven te kunnen worden door twee parameters, namelijk de aanvankelijke krimp per schudperiode en de limiet waartoe de krimp nadert bij lange schudtijden (hoofdstuk 2).

De op de vezels uitgeoefende kracht werd gevarieerd via de schudsnelheid en de amplitude (hoofdstuk 3). De resultaten wijzen uit, dat geen krimp optreedt beneden een bepaalde drempelwaarde van de kracht. Vanaf deze waarde neemt de krimp per periode practisch lineair toe bij toenemende kracht tot deze een tweede critische waarde bereikt waarboven de krimp veel minder sterk toe-neemt (fig. 10). De krimplimiet is voor krachten groter dan de drempelwaarde practisch constant (fig. 8). Metingen verricht aan garens met verschillende twist tonen aan, dat bij sterkere twist de krachtdrempel hogere waarden aanneemt (fig. 11 and 12).

Uit voorlopige beschouwingen over het verviltingsmechanisme blijkt, dat de wijze waarop de krimp van de kracht afhangt, bij gegeven garenstructuur vermoedelijk in hoofdzaak bepaald wordt door de wrijvings- en elastische eigenschappen van de enkele vezels. Deze veronderstelling is getoetst door metingen te verrichten aan garens, waarin deze vezeleigenschappen op bekende wijze gevarieerd zijn. De veranderingen in de wrijving zijn teweeggebracht door middel van een aantal chemische behandelingen; de vezelstijfheid is systematisch gevarieerd door waterige oplossingen van diverse stoffen toe te passen.

Teneinde de bewuste vezeleigenschappen op verantwoorde wijze te kunnen meten is een apparaat ontwikkeld, waarmee de wrijving tussen twee vezels bepaald kan worden (hoofdstuk 4) en een waarmee een snelle meting van de elasticiteitsmodulus mogelijk is (hoofdstuk 5).

Het onderzoek naar het verband tussen wrijving en krimp wijst uit, dat er een goede correlatie bestaat tussen de critische waarden van de schudsnelheid enerzijds en de kinetische wrijvingscoëfficiënten anderzijds (hoofdstuk 6; fig. 37). Uit de experimenten die in diverse milieu's verricht werden blijkt de snelheid van toename van de krimp per slag met de schudsnelheid, bij benadering omgekeerd evenredig te zijn met de elasticiteitsmodulus der vezels (hoofdstuk 7; fig. 42).

Op grond van deze waarnemingen schijnt het volgende beeld van het verviltingsmechanisme gerechtvaardigd: Als de uitgeoefende krachten te klein zijn om de vezelwrijving in elk van de beide richtingen te overwinnen, vindt er in het geheel geen vezelbeweging plaats. Bij grotere krachten zijn bewegingen in één richting mogelijk; deze veroorzaken irreversibele verplaatsingen, waarvan de grootte evenredig is met de resulterende kracht op de vezel en omgekeerd evenredig met de elasticiteitsmodulus, terwijl deze verplaatsingen bovendien sterk van de garenstructuur afhangen. Wanneer de kracht zo groot wordt, dat hij ook de wrijving tegen de schubben in kan overwinnen, worden de verplaatsingen gedeeltelijk reversibel, waardoor de krimp per slag in eerste benadering onafhankelijk van de kracht wordt (fig. 16).

Tenslotte worden in hoofdstuk 8 enkele onderzoekingen naar de structuur en de eigenschappen van vervilte garens beschreven; het blijkt o.a. dat de vezels niet als geheel doch in gedeelten verplaatsingen ondergaan en dat de oriëntatie van de vezelsegmenten geheel te niet gaat tijdens het verviltingsproces (fig. 45).

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STELLINGEN

1. Bij het bepalen van de neiging tot vervilten van wol wordt over het algemeen te weinig rekening gehouden met de invloed van de grootte der toegepaste krachten.

2. De beschouwingen van Lindberg over het mechanisme van het vervilten van wol zijn fundamenteel onjuist.
J. Lindberg, Text. Research J. **18**, 470 (1948), **23**, 225 (1953)

3. De theorie van Van den Abeele betreffende de ideale gelijkmatigheid van garens, is aanvechtbaar.
A.M. van den Abeele, I.W.T.O. Techn. Comm. Proc. **2**, 48 (1948)

4. Bij het vermoeidheidsonderzoek van vezels dienen de periodieke spanningen niet in de lengterichting der vezel doch loodrecht daarop te worden aangebracht.

5. Het o.a. door Meredith waargenomen verschijnsel, dat bij sommige vezels de krachtrelaxatiecurven voor verschillende deformaties elkaar snijden, hangt ten nauwste samen met de systematische afwijkingen van lineair gedrag bij kruip.
R. Meredith, J. Text. Inst. **45**, T438 (1954)

6. De veelvuldig toegepaste interpretatie van het visco-elastisch gedrag van polymeren met behulp van de viscositeitstheorie van Eyring, is in de meeste gevallen ongeoorloofd.
Zie b.v. A. Tobolsky en H. Eyring, J. Chem. Phys. **11**, 125 (1943)
G. Halsey, J. Appl. Phys. **18**, 1072 (1947).

7. De door Mack ingevoerde modelvoorstelling van niet-Newtonse plastische vloei is aanvechtbaar.
C. Mack, J. Appl. Phys. **17**, 1086 (1946)

8. Aan de uitdrukking "kinetische wrijving" dient de voorkeur gegeven te worden boven de dikwijls gebruikte term "dynamische wrijving".

9. De door Prins, Schenk en Dumoré waargenomen temperatuurverschillen in een electrisch verhitte draad moeten waarschijnlijk aan onregelmatig verdeelde convectiestromen worden toegeschreven.
J.A. Prins, J. Schenk en J.M. Dumoré, Appl. Sci. Res. **A3**, 272 (1952)

10. Het is wenselijk, dat bij het hoger onderwijs in de natuurkunde een grotere plaats wordt ingeruimd aan de trillingsverschijnselen in materie.

11. De waarde van vele der in zwang zijnde slijtage-apparaten moet in twijfel getrokken worden op grond van het feit, dat zij noch een maat voor de practijkslijtage, noch physische informatie verschaffen.

12. Het verdient aanbeveling om bij het elementair muziekonderwijs meer aandacht aan solfège te besteden dan thans gebruikelijk is.