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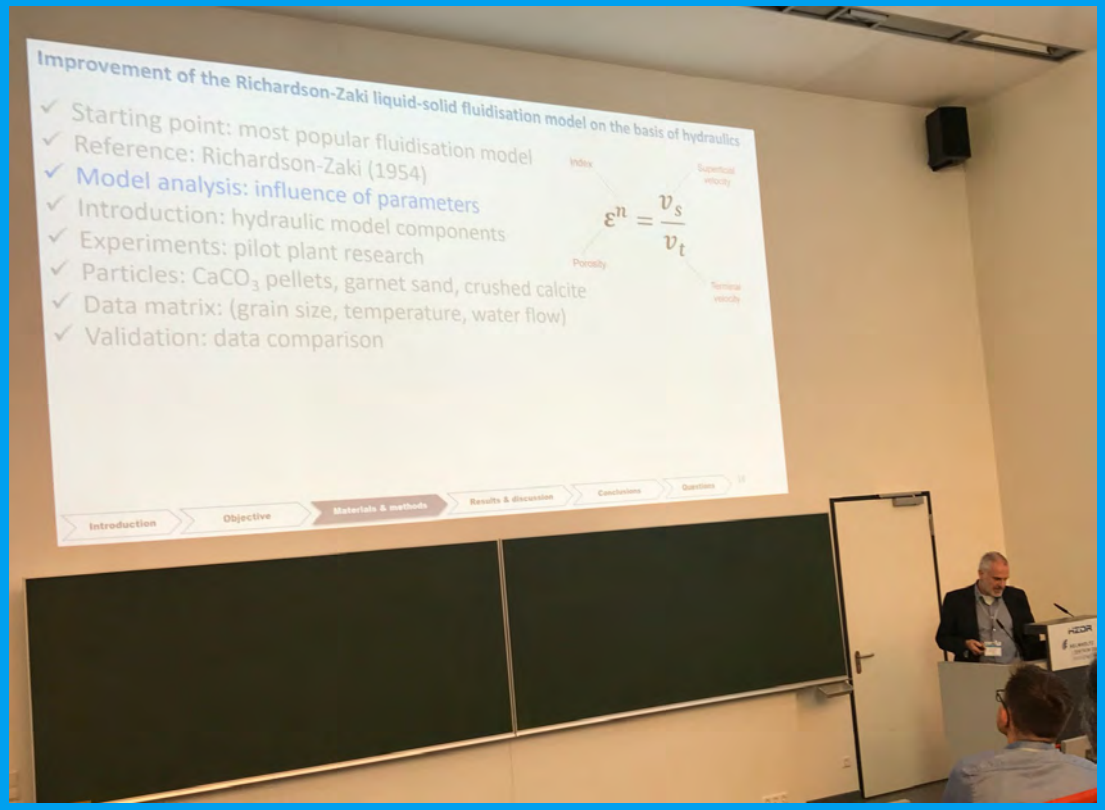
15 November 2018

16th Multiphase Flows conference & shore course – Simulation, Experiment and Application



waternet
waterschap amstel gooi en vecht
gemeente amsterdam

Onno Kramer

15 November 2018

16th Multiphase Flows conference & shore course – Simulation, Experiment and Application

Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

Contents

- Introduction
- Objectives
- Materials and methods
- Results and discussion
- Conclusions
- Questions

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- **Introduction**
  - Objectives
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Hydraulic modelling of liquid-solid fluidisation in drinking water treatment processes

Onno Kramer[1, 2, 3, 4]
Eric Baars[1]
Peter de Moel[3, 5]
Wim van Vugt[2]
Johan Padding[4]
Jan Peter van der Hoek[1, 3]

[1] **Waternet** Drinking Water Department

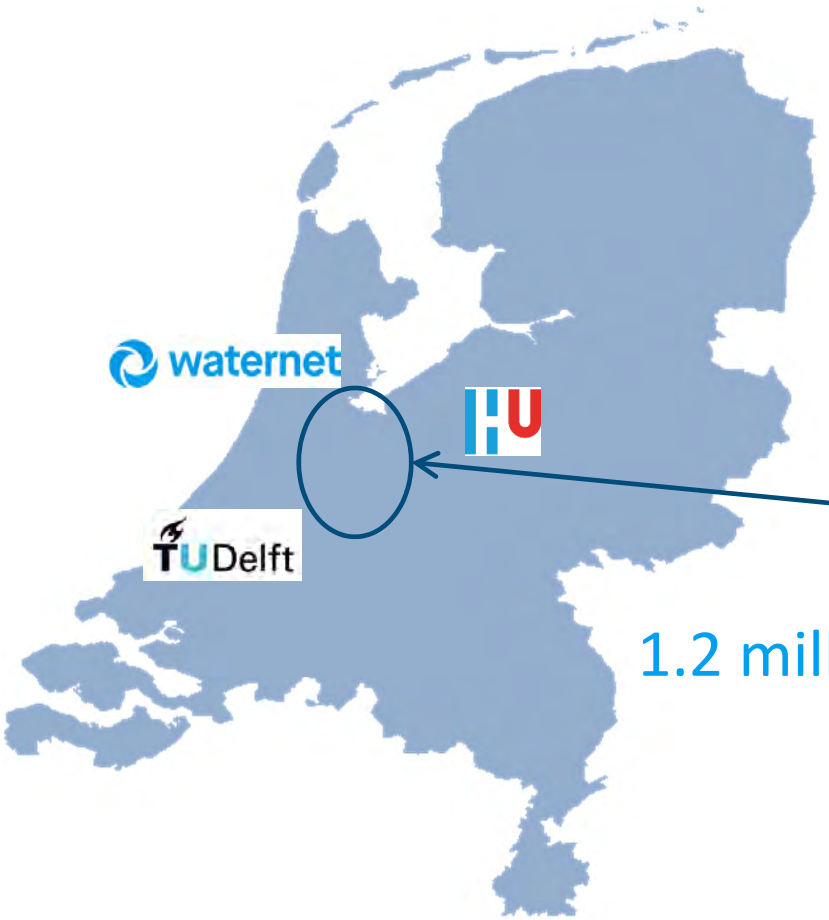
[2] **HU** University of Applied Sciences Utrecht, Institute for Life Science and Chemistry,

[3] **TUD** Delft University of Technology, Faculty of Civil Engineering and Geosciences

[4] **TUD** Delft University of Technology, Faculty of Mechanical, Maritime and Materials Engineering

[5] **Omnisys** Consultancy

Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics



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Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

- ✓ Background: (water cycle)
- ✓ Field: (drinking water treatment processes)
- ✓ System: (multiphase flows)
- ✓ Process: (softening)
- ✓ Fluidisation: (liquid-solid = water-calcite pellets)

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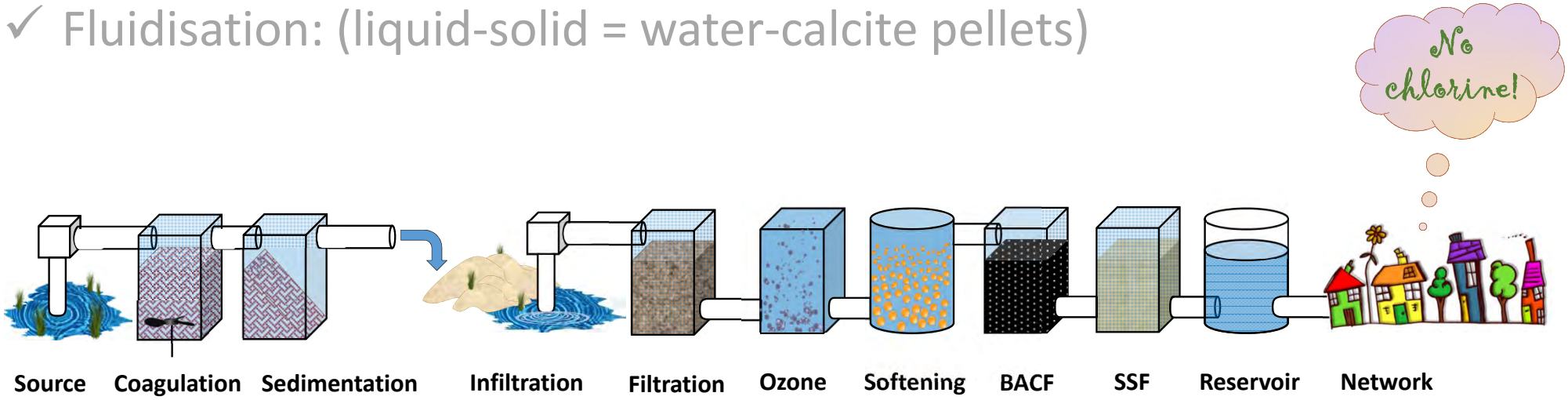

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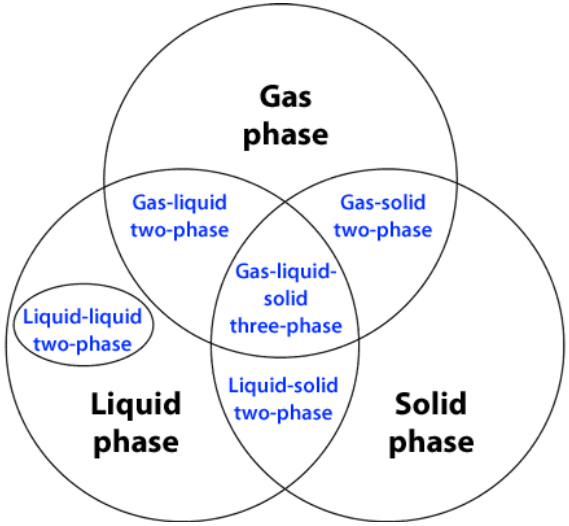

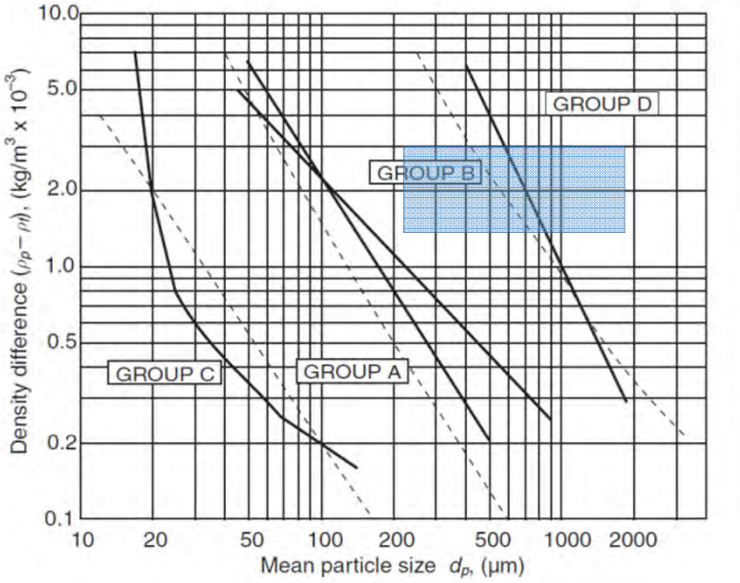

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$OH^- + HCO_3^- \leftrightarrow CO_3^{2-} + H_2O$

$CO_3^{2-} + Ca^{2+} \rightarrow CaCO_3\downarrow$

- Hardness reduction to 1.4 mmol/L
- Reduces solubility of lead (public health) and copper (environment)
- Economic benefits and comfort
    - Reduction of washing powder
    - Increase life time hot water equipment
    - Cleaner laundry, tasteful tea

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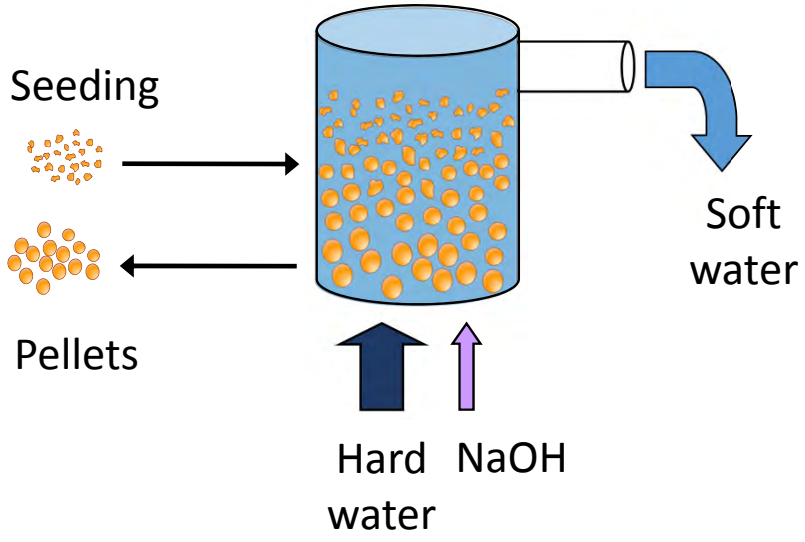

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Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

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  - Increasing sustainability
  - Reducing chemical use
  - Improving water quality
- ✓ Method: improved model based on hydraulics (porosity)
- ✓ Focus: crystallisation on specific surface area

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- ✓ Experiments: pilot plant research
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$$\varepsilon^n = \frac{v_s}{v_t}$$

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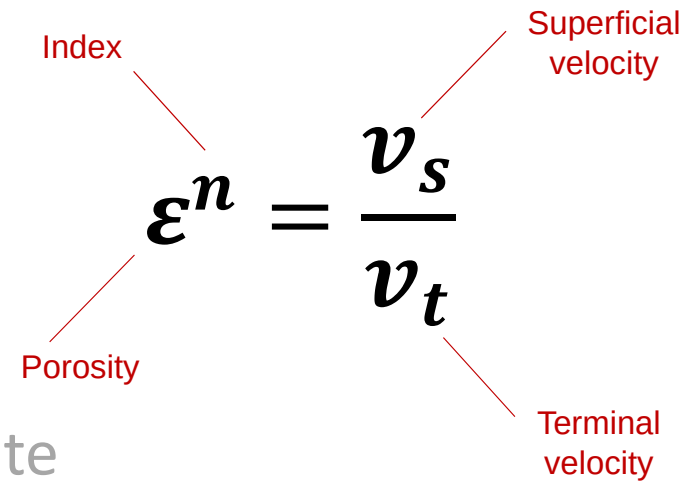

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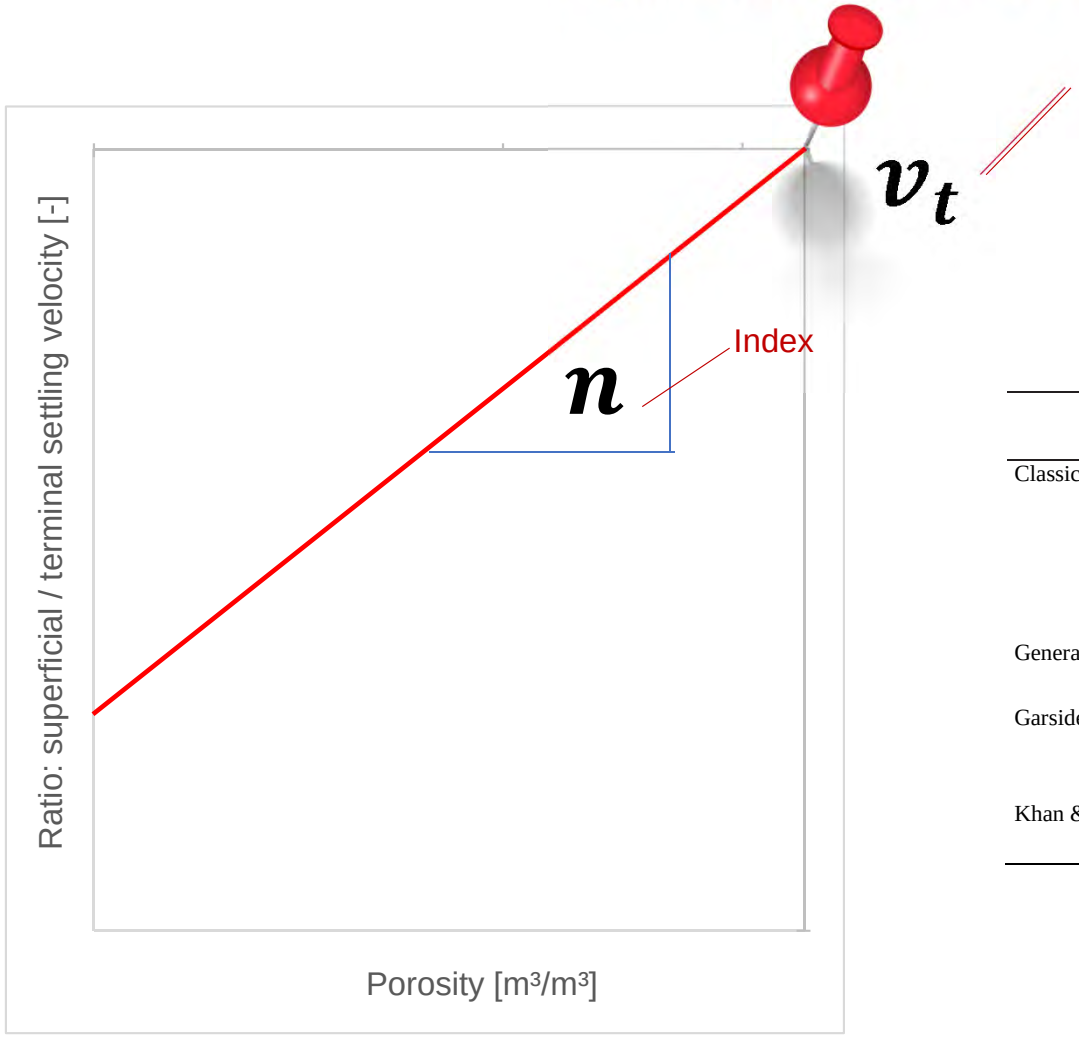
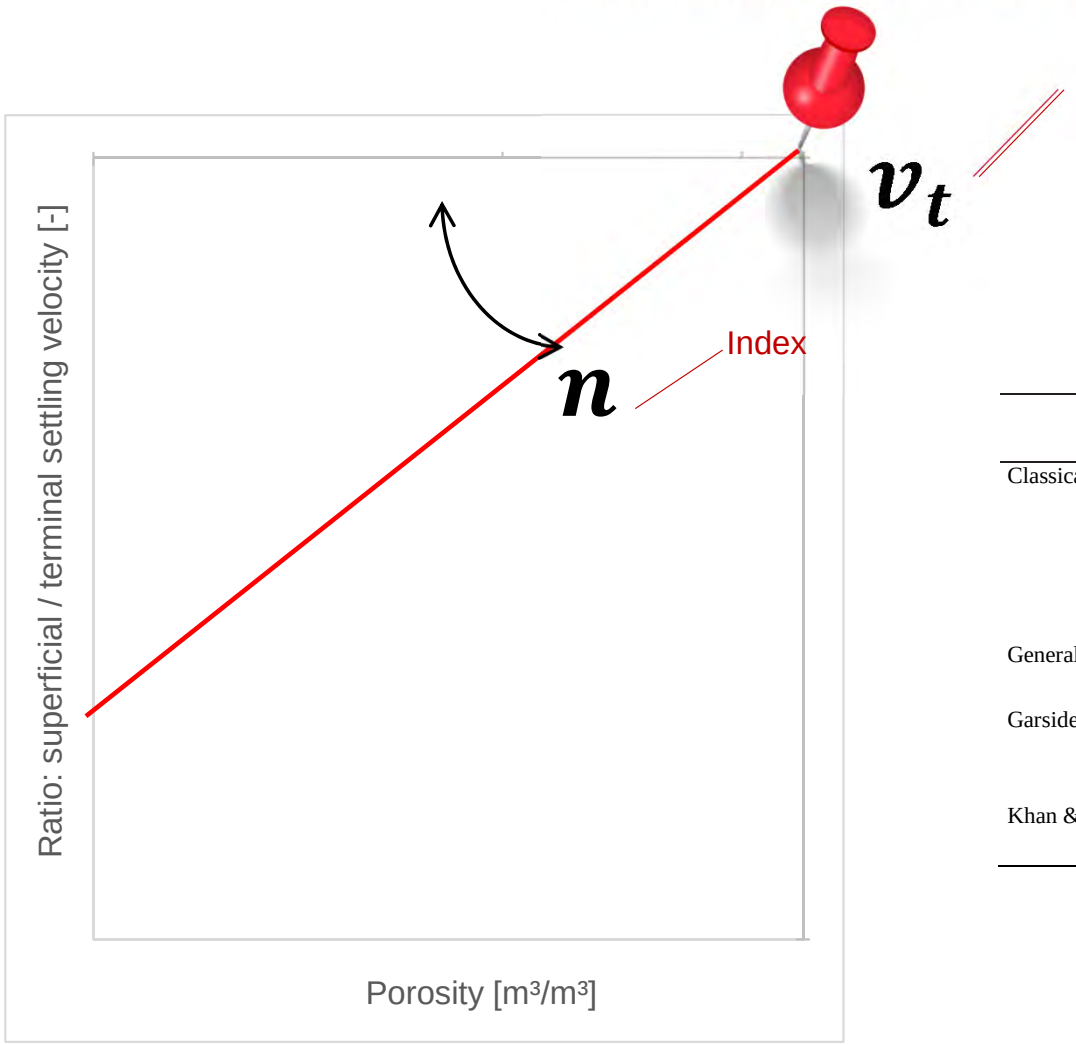


	Formula
Classical Richardson-Zaki equation	$n = \begin{cases} Re_t < 0.2, & n = 4.65 \\ 0.2 \leq Re_t < 1, & n = 4.4\ Re_t^{-0.03} \\ 1 \leq Re_t < 500, & n = 4.4\ Re_t^{-0.1} \\ Re_t \geq 500, & n = 2.4 \end{cases}$
General expression	$n = c_1 {Re_t}^{c_2}$
Garside & Al-Dibouni equation	$\frac{n_L - n}{n - n_T} = \alpha\ {Re_t}^{\beta}$
Khan & Richardson	$\frac{n_L - n}{n - n_T} = \alpha\ Ar^{\beta}$

Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics



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Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

Terminal velocity

v_t

Superficial velocity

v_s

$\frac{v_s}{v_t}$

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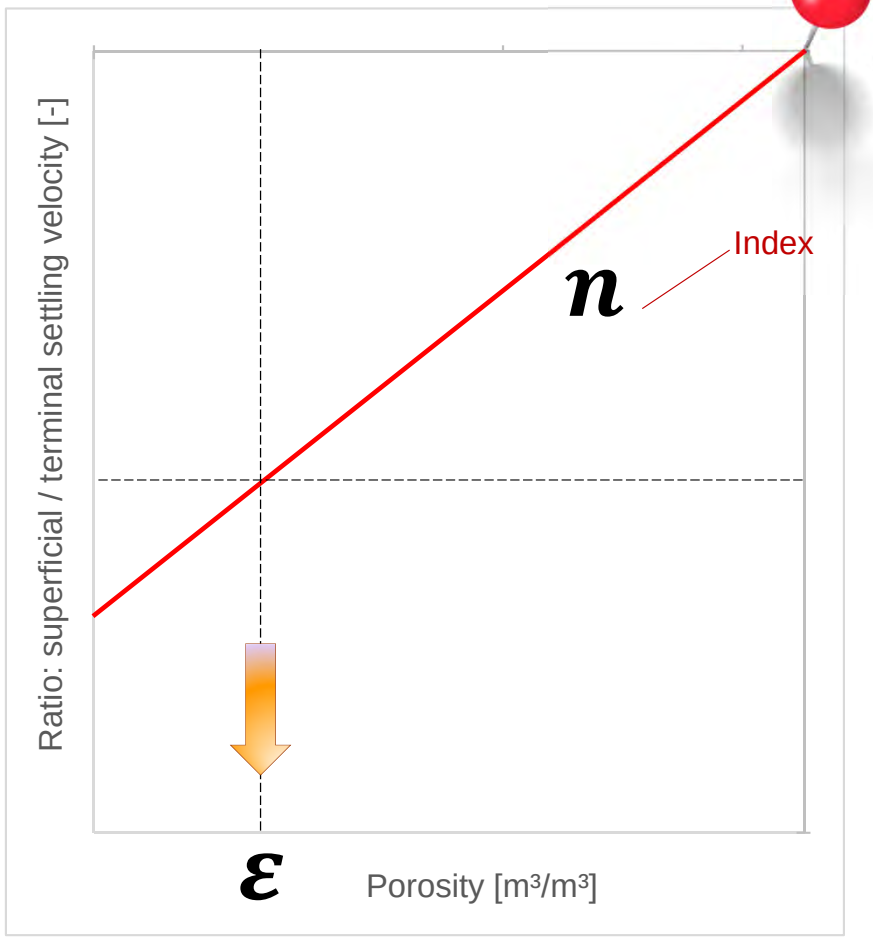
n



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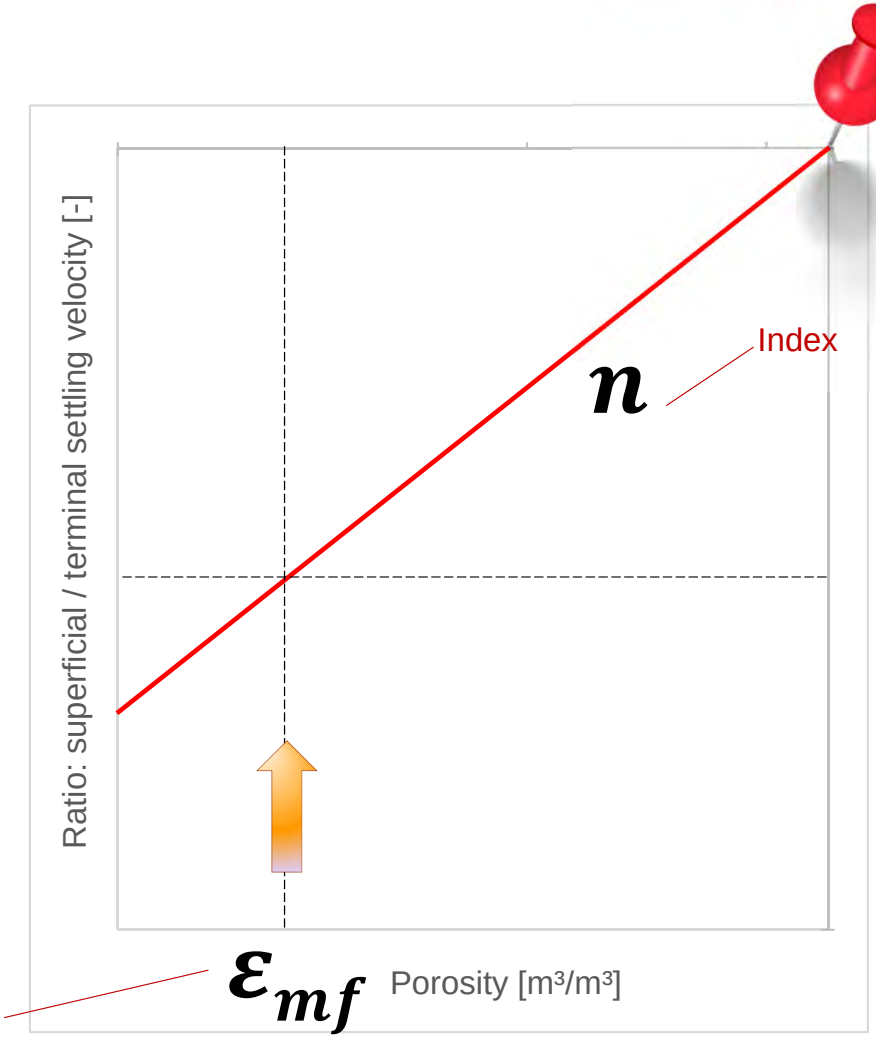

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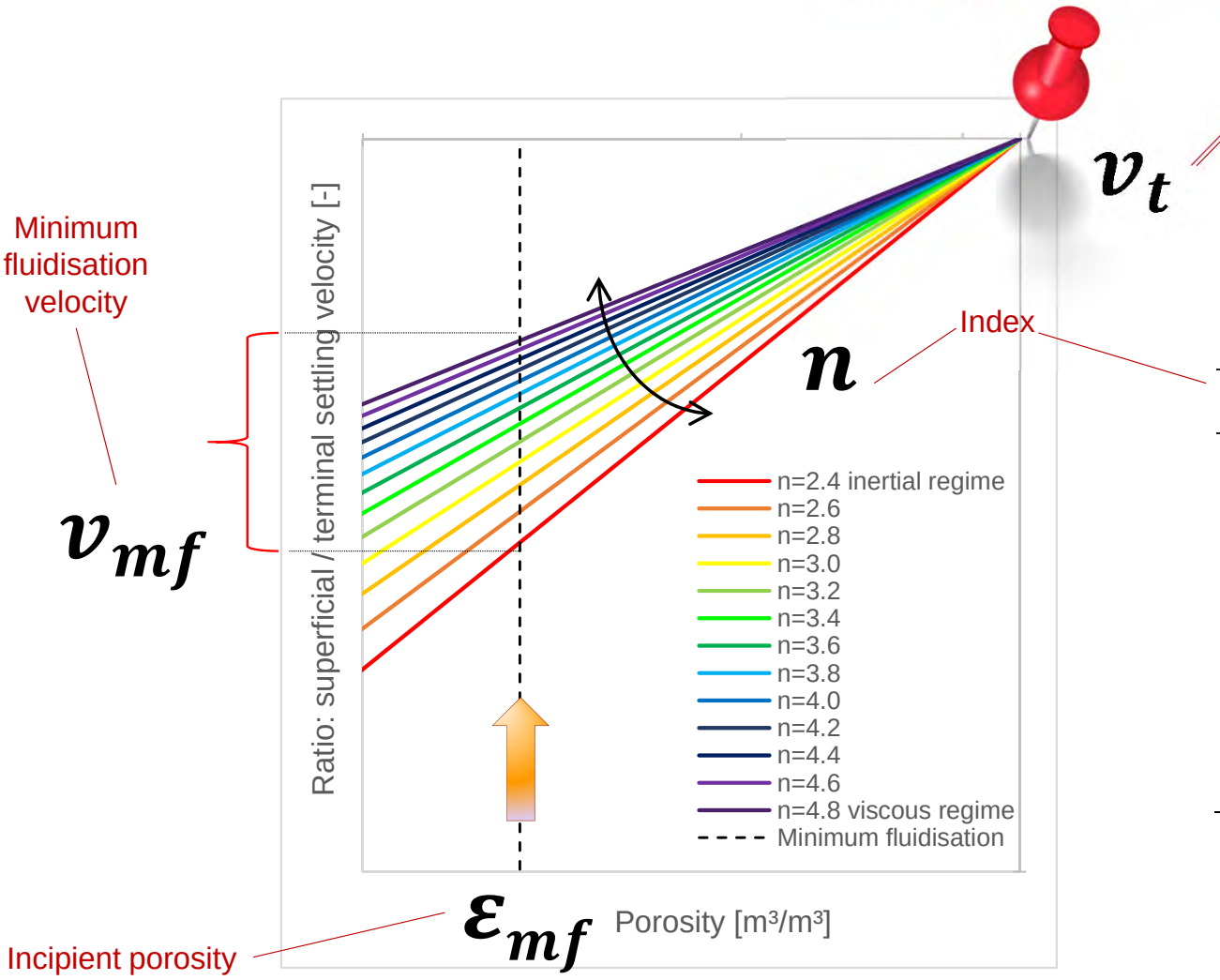

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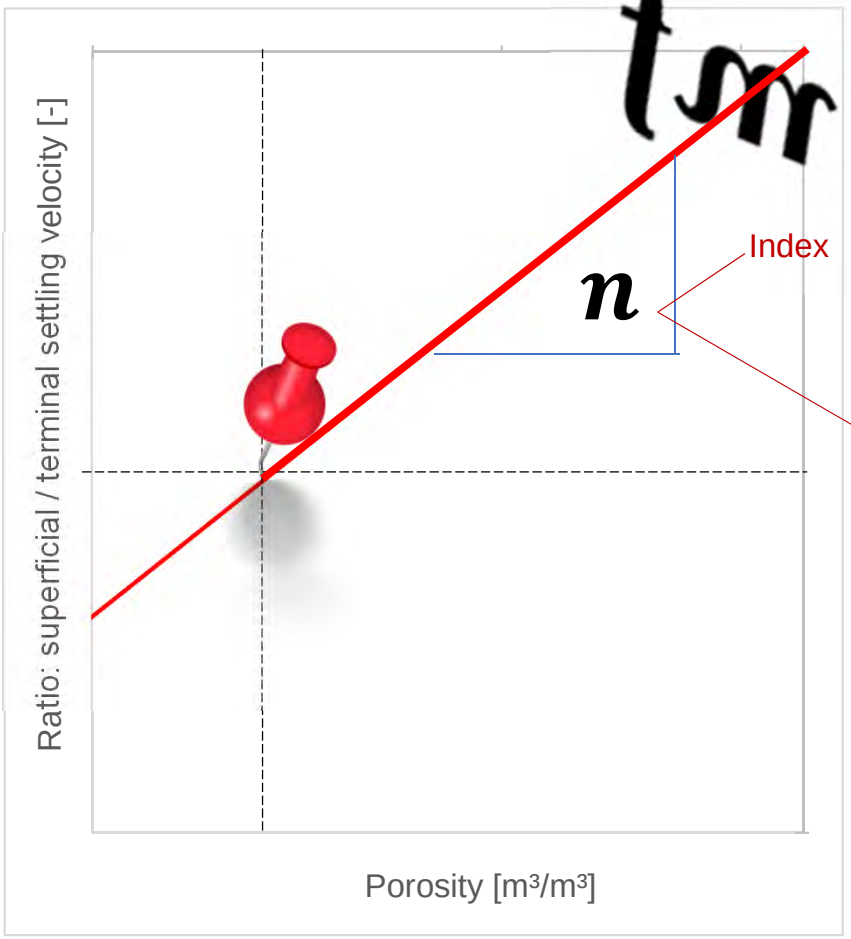
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

Minimum fluidisation velocity

v_{mf}

Carman-Kozeny

$$C_D = \frac{180}{Re_\varepsilon} + \frac{2.87}{Re_\varepsilon{}^{0.1}}$$



Terminal velocity

v_t

Index

Brown-Lawler (improved Schiller-Naumann)

$$C_D = \frac{24}{Re_t}\left(1 + 0.15 Re_t{}^{0.681}\right) + \frac{0.407}{1 + \frac{8710}{Re_t}}$$

Interpolation

$$n = \frac{\log\left(\frac{Re_{\varepsilon,mf}}{Re_t}\left(1 - \varepsilon_{mf}\right)\right)}{\log \varepsilon_{mf}}$$

$$Re_t = \frac{\rho_f d_p v_t}{\eta}$$

$$Re_{\varepsilon,mf} = \frac{\rho_f d_p v_{mf}}{\eta}\frac{1}{1 - \varepsilon_{mf}}$$

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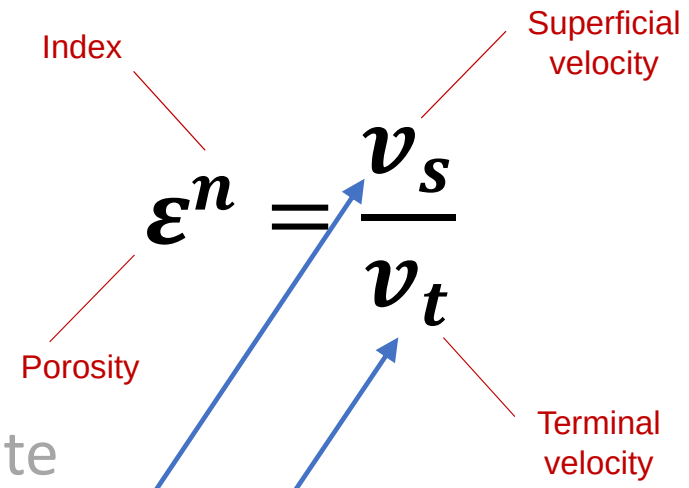

Carman-Kozeny (at minimum fluidisation)

$$C_D = \frac{180}{Re_\varepsilon} + \frac{2.87}{Re_\varepsilon{}^{0.1}}$$

Brown-Lawler (at terminal settling settling)

$$C_D = \frac{24}{Re_t}\left(1 + 0.15Re_t{}^{0.681}\right) + \frac{0.407}{1 + \frac{8710}{Re_t}}$$

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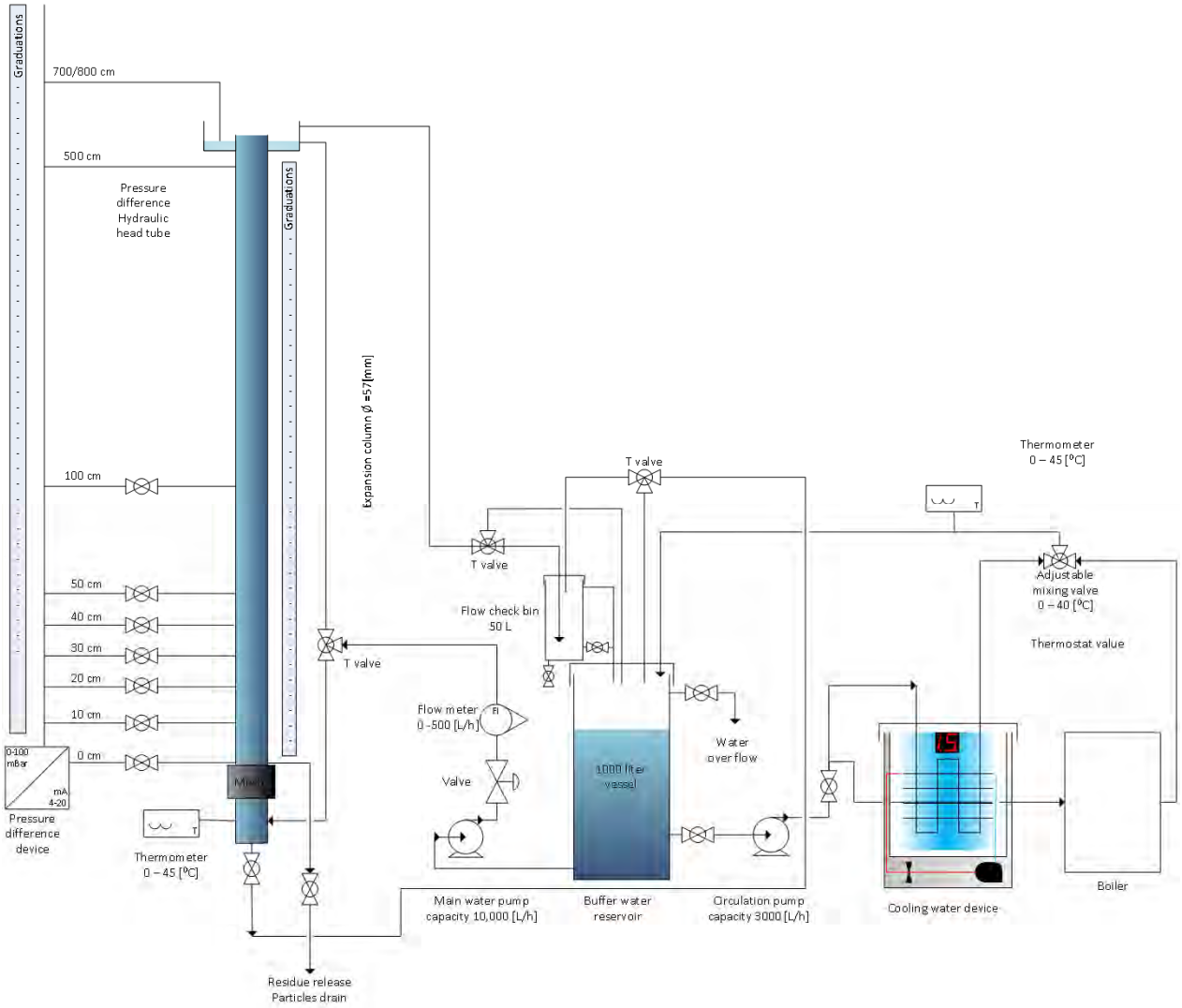

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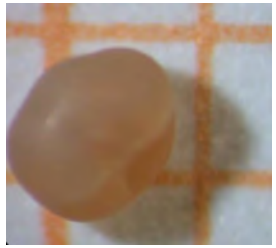

1.4 - 1.7 mm calcite pellets

0.425 - 0.5 crushed calcite

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- ✓ Particles: $CaCO_3$ pellets, garnet sand, crushed calcite
- ✓ Data matrix: (grain size, temperature, water flow)
  - ✓ 10 sieved fractions ($0.4 < d_z < 2.0$ mm)
  - ✓ 4 temperatures (5, 15, 25, 35 °C)
  - ✓ 25 ascending water flows (0-180 m/h)
- ✓ Validation: data comparison

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- ✓ Application: drinking water pellet softening
- ✓ Model accuracy improvement

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$$\frac{4.8 - n}{n - 2.4} = 0.015\, Ar^{0.5}$$

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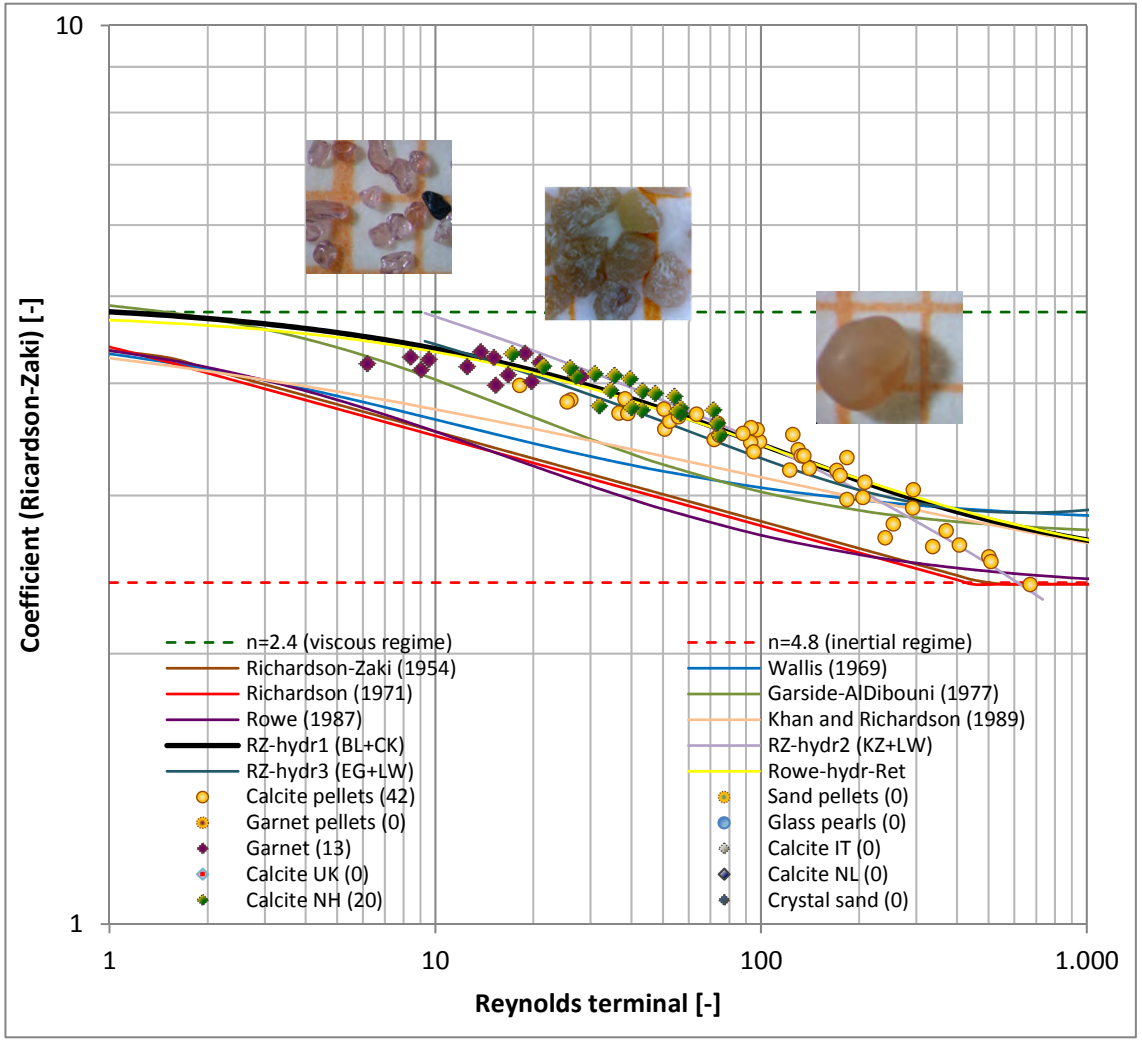
$$\varepsilon^{n} = \frac{v_s}{v_t}$$

$$Re_t = \frac{\rho_f d_p v_t}{\eta}$$



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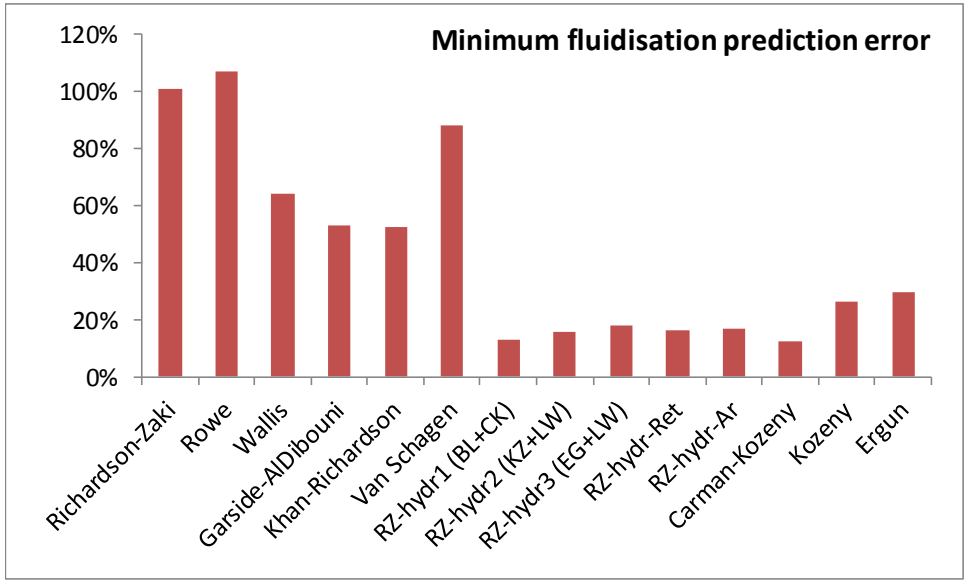

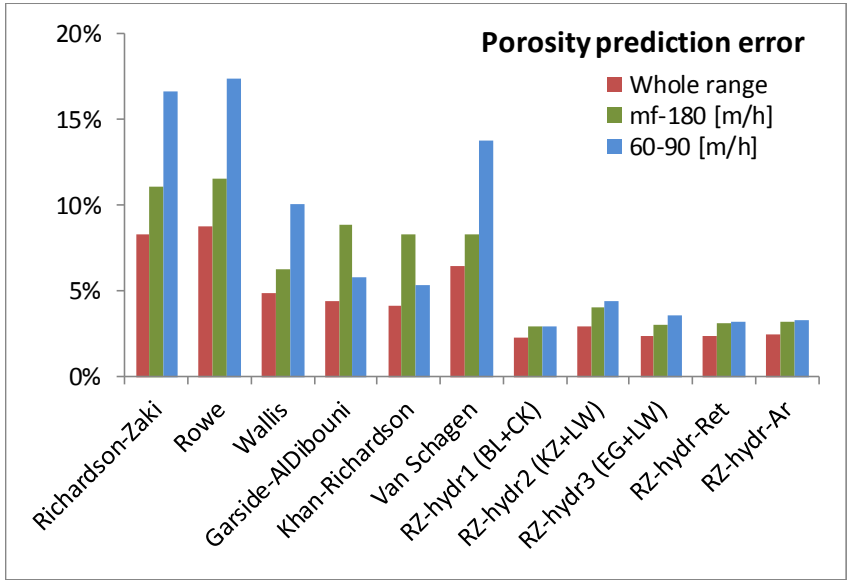

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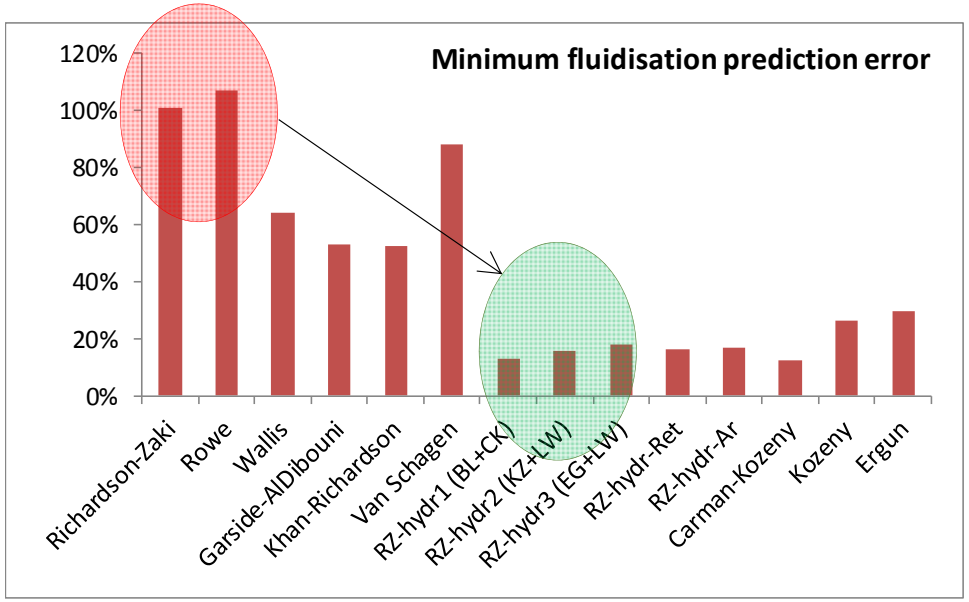
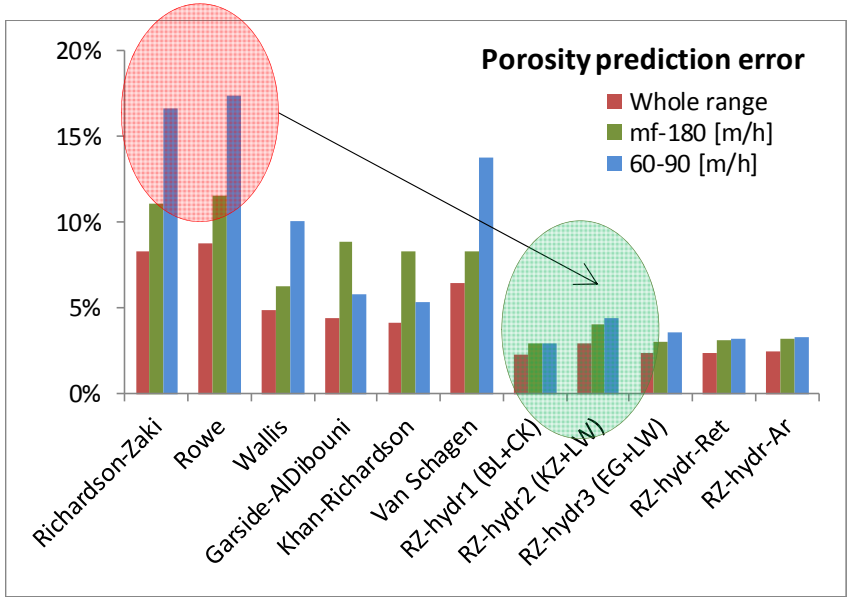
minimum fluidisation >100%→12%



porosity >15%→3%



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- ✓ Porosity can be predicted more accurately
- ✓ Recommendations:
  - Model enhancement (more general)
  - Identification of irregularly shaped particles
  - Implications for specific surface area (Interfacial Area Density)

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Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

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HU University of Applied Sciences Utrecht, Institute for Life Science and Chemistry

Delft University of Technology
Faculty of Civil Engineering and Geosciences, Department Water Management, Section Sanitary Engineering, Research Group Drinking Water
Faculty of Mechanical, Maritime and Materials Engineering, Department Process and Energy, Section Intensified Reaction and Separation Systems

Optional for questions

A.R. Khan, J.F. Richardson, Fluid-particle interactions and flow characteristics of fluidized beds and settling suspensions of spherical particles, Chem. Eng. Commun. 78 (1989) 111–130. doi:10.1080/00986448908940189.

Chem. Eng. Comm. 1989, Vol. 78, pp. 111–130
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FLUID-PARTICLE INTERACTIONS AND FLOW CHARACTERISTICS OF FLUIDIZED BEDS AND SETTLING SUSPENSIONS OF SPHERICAL PARTICLES

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(Received October, 1987; in final form August 4, 1988)

The published correlations for the velocity-voidage relationship observed during fluidization and sedimentation of uniformly sized spherical particles in solid–liquid systems are compared with published experimental results. It is found that the expression suggested by Richardson and Zaki represents the experimental data well over a wide range of values of Galileo number ($10^{-2} < Ga < 10^{10}$) and voidage ($0.4 < \varepsilon < 1$). Methods are given for predicting the constants in the expression as a function of the properties of the system, including wall effects.
KEYWORDS Fluidized beds Fluid–particle interactions Spherical particles.

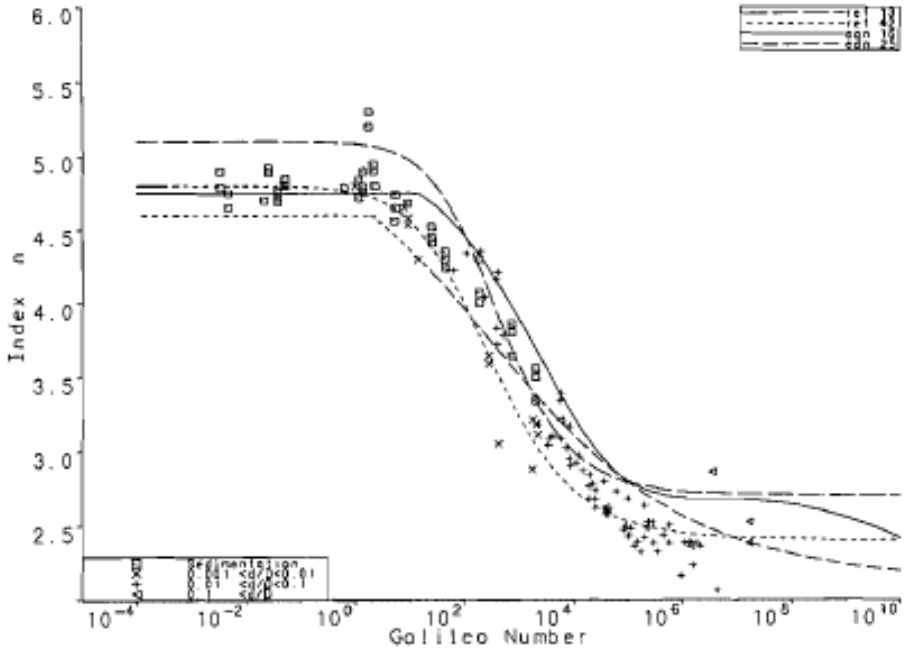

FIGURE 5 Comparison of published correlations with experimental values of index *n*.

J. Garside, M.R. Al-Dibouni, Velocity-voidage relationships for fluidization and sedimentation in solid-liquid systems, Ind. Eng. Chem. Process Des. Dev. 16 (1977) 206–214. doi:10.1021/i260062a008.

Velocity–Voidage Relationships for Fluidization and Sedimentation in Solid–Liquid Systems

John Garside* and Maan R. Al-Dibouni

Department of Chemical Engineering, University College London, London WC1E 7JE

Published experimental results for the velocity–voidage relationship observed during fluidization and sedimentation of uniformly sized spheres in solid–liquid systems are compared and new experimental results are presented. Predictions of various published correlations that are available to describe this relation are compared with these experimental results and the inadequacy of most of the correlations is demonstrated. New correlations are suggested. The most reliable of these is based on a logistic curve and can be represented by the equation $(U_R - A)/(B - U_R) = 0.06\mathrm{Re}_t^{+0.2}$, where $A = \epsilon^{4.14}$ and $B = 0.8\epsilon^{1.28}$ for $\epsilon \leq 0.85$ or $B = \epsilon^{2.65}$ for $\epsilon > 0.85$. $U_R = U_i/U_t$ where U_i is the relative average velocity between particles and fluid, U_t is the terminal velocity of a single particle, Re_t is the particle Reynolds number based on U_t, and ϵ is the bed voidage.

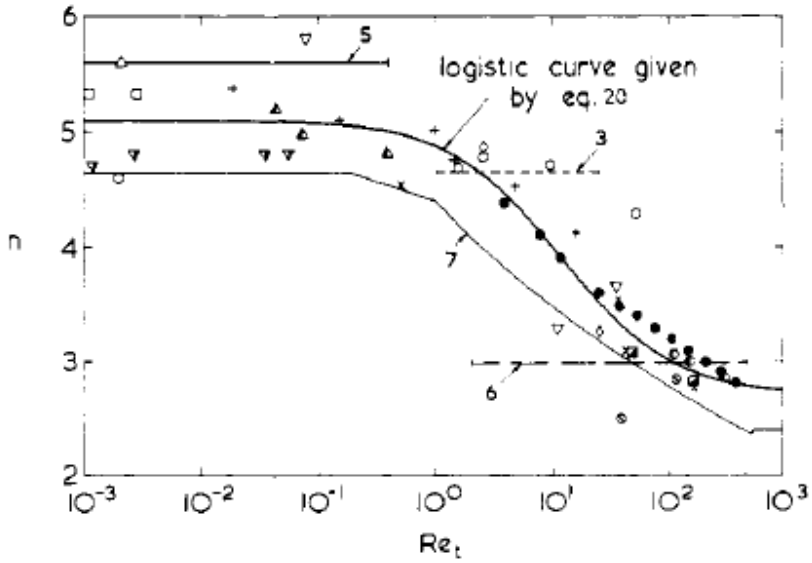

Figure 8. Variation of exponent n with Reynolds number. (See Table I for explanation of symbols; See Table III for key to different curves.)

A.D. Maude, R.L. Whitmore, A generalized theory of sedimentation, Br. J. Appl. Phys. 9 (1958) 477–482. doi:10.1103/RevModPhys.20.35.

A generalized theory of sedimentation

By A. D. Maude, B.Sc., Ph.D., A.Inst.P.,* and R. L. Whitmore, B.Sc., Ph.D., A.Inst.P., Department of Mining and Fuels, University of Nottingham

[*Paper first received* 3 *January, and in final form* 25 *June*, 1958]

A theoretical relationship between the concentration and the sedimentation velocity of non-flocculated suspensions of particles is derived. It is shown that the settling velocity relative to that of a single particle in the suspension is $(1 - c)^{\beta}$ where β is a function of particle shape, size distribution and Reynolds number and c is the volume of solid per unit volume of suspension. The expression is shown to satisfy the experimental results of other workers. An empirical relationship between β and the Reynolds number is suggested.

1. Introduction

When a body falls through a fluid, it accelerates until it reaches a constant terminal velocity. This velocity is determined by the density of the fluid ρ_f, the density of the body ρ_s, the viscosity of the fluid η, the shape and orientation of the body and by some length characterizing the size of the body d. The velocity may also depend, to some extent, upon the size and shape of the containing vessel, but if this is large its influence may be neglected.

The problem becomes more complicated if many bodies are present and the system becomes a sedimenting suspension. When the bodies are more or less evenly dispersed throughout the fluid, their rate of fall is decreased, and it is of considerable practical interest to know the relation between the concentration of bodies and the magnitude of this decrease.

Many theoretical and empirical relations[1–8] have been proposed to solve this problem, but they suffer from various defects, in particular they lack generality. It would be of considerable practical value if an equation could be derived which would cover a wide range of particle shapes, size ranges and rates of fall, and in the following paper an attempt is made to do this.

2. Fall of spheres at low Reynolds number

Consider a mass of equi-velocity particles falling through a pure fluid. If v is the average volume of one particle, and g is the acceleration due to gravity, then the average force on one particle is $F = (\rho_s - \rho_f)gv$. If c is the volume of solid in unit form of suspension, then it may be shown from

* Now at the City of Liverpool College of Technology.

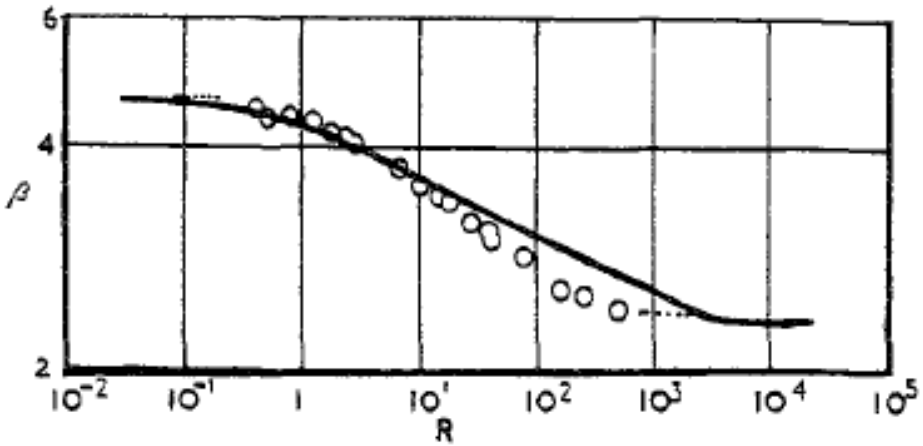

Fig. 2. Variation of the exponent of $(1 - c)$ with Reynolds number

J.F. Richardson, M.A. da S. Jerónimo, Velocity-voidage relations for sedimentation and fluidisation, Chem. Eng. Sci. 34 (1979) 1419–1422. doi:10.1016/0009-2509(79)85167-2.

Chemical Engineering Science Vol 34 pp 1419–1422
Pergamon Press Ltd 1979 Printed in Great Britain

Velocity–voidage relations for sedimentation and fluidisation

(*Received* 26 *February* 1979, *accepted* 30 *April* 1979)

A concentrated suspension of uniform particles settles at a lower rate than one of the particles in isolation in a large expanse of fluid This phenomenon arises from a combination of factors Thus in a suspension there is a significant upflow of displaced fluid, there are changed buoyancy effects and steeper velocity gradients at a given particle velocity relative to the fluid The relation between sedimentation velocity and concentration in a suspension is similar to that between fluidisation velocity and concentration in a liquid solid system, and various empirical relations have been suggested It is now proposed to examine how the constant in one of these relations can be calculated from the slope of the curve of drag coefficient against particle Reynolds number, and to show how calculated and experimental values compare

THE EFFECT OF CONCENTRATION ON THE DRAG FORCE–VELOCITY RELATION FOR A SPHERE

For a single isolated particle settling in a fluid

$$\frac{\pi}{6}d^3(\rho_s-\rho)g=\left(\frac{R'}{\rho u_0^2}\right)\rho u_0^2\frac{\pi}{4}d^2 \qquad (1)$$

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Portugal

from which

$$\left(\frac{R'}{\rho u_0^2}\right)\frac{u_0^2d^2\rho^2}{\mu^2}=\frac{2}{3}\frac{d^3g(\rho_s-\rho)\rho}{\mu^2} \qquad (2)$$

i e

$$\frac{R'}{\rho u_0^2}\,\mathrm{Re}_0^2=\frac{2}{3}\mathrm{Ga} \qquad (3)$$

Suppose that in a suspension of voidage ϵ, the force on a particle at a given relative velocity is increased by some factor $f(\epsilon)$ Then

$$\frac{\pi}{6}d^3(\rho_s-\rho)g=\left(\frac{R'}{\rho u^2}\right)\rho u^2\frac{\pi}{4}d^2f(\epsilon) \qquad (4)$$

where $f(\epsilon)$ takes account of all interparticle effects including the increase in buoyancy force, and $(R'/\rho u^2)$ is still the friction factor for the isolated particle Combining eqns (3) and (4)

$$f(\epsilon)=\frac{2}{3}\frac{\mathrm{Ga}}{(R'/\rho u^2)\,\mathrm{Re}^2} \qquad (5)$$

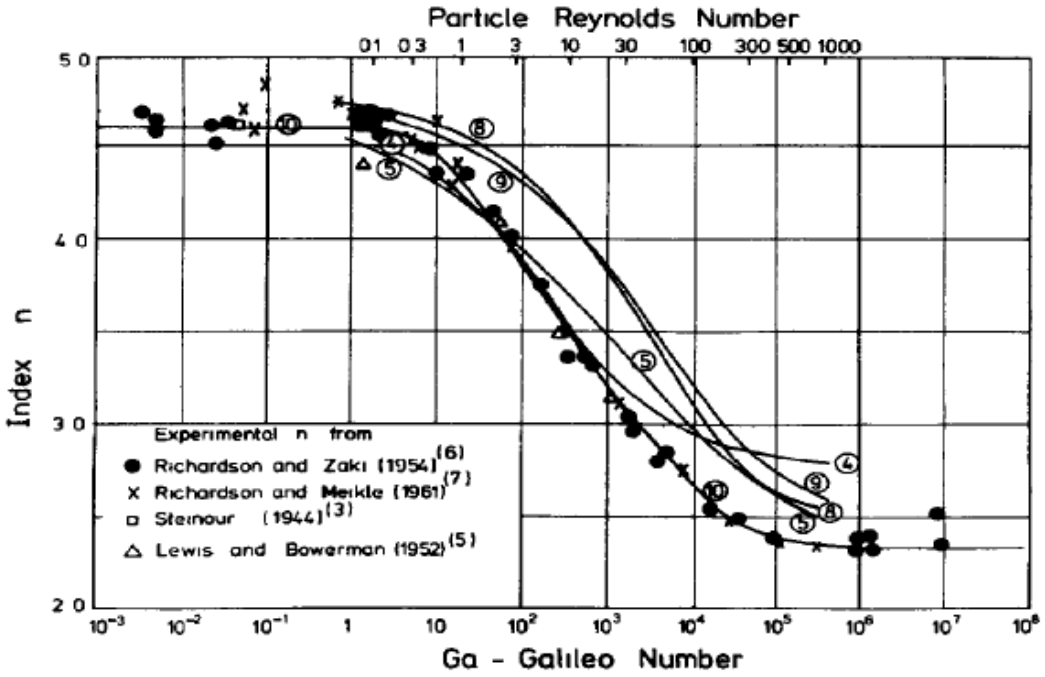

Fig 1 Experimental and calculated values of n as a function of Galileo and particle Reynolds numbers

P.N. Rowe, A convenient empirical equation for estimation of the Richardson-Zaki exponent, Chem. Eng. Sci. 43 (1987) 2795–2796. doi:https://doi.org/10.1016/0009-2509(87)87035-5.

Chemical Engineering Science, Vol. 42, No. 11, pp. 2795–2796, 1987.
Printed in Great Britain.

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A convenient empirical equation for estimation of the Richardson–Zaki exponent

(*Received* 1 *April* 1987; *accepted* 11 *May* 1987)

Wilhelm and Kwauk (1948) were the first to publish studies of the variation of voidage with fluid velocity for fluidized particles and to show that their results using water as the fluid (described as particulate fluidization) were correlated by an equation of the form

$$Re = K\varepsilon^n \quad (1)$$

Richardson and Zaki (1954) showed some of the logic behind this choice which can be conveniently written

$$u = u_t\varepsilon^n \quad (2)$$

and made a systematic experimental study of how the exponent, n (commonly referred to now as the Richardson–Zaki exponent) varies with Re_t. Their results, conveniently presented in Richardson (1971), for cases where particle size is small compared with vessel diameter, were described by four empirical equations covering different Reynolds number ranges. These equations are a little awkward to use particularly in the regions where they overlap and a continuous functions covering all values of Re_t is more useful especially when embodied in a general theory such as that of Foscolo and Gibilaro (1984).

Inspection of their data suggests it can be fitted by a logistic curve which is symmetrical and asymptotes to limiting values

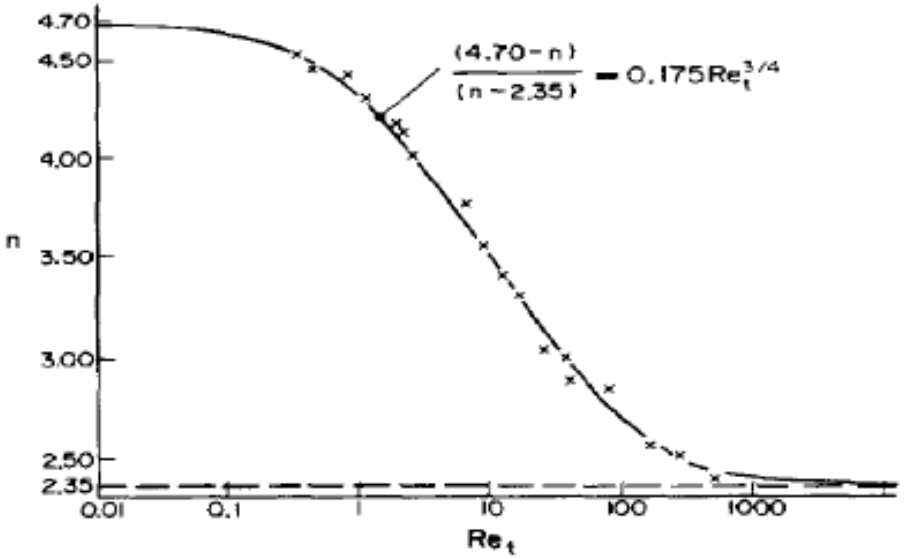
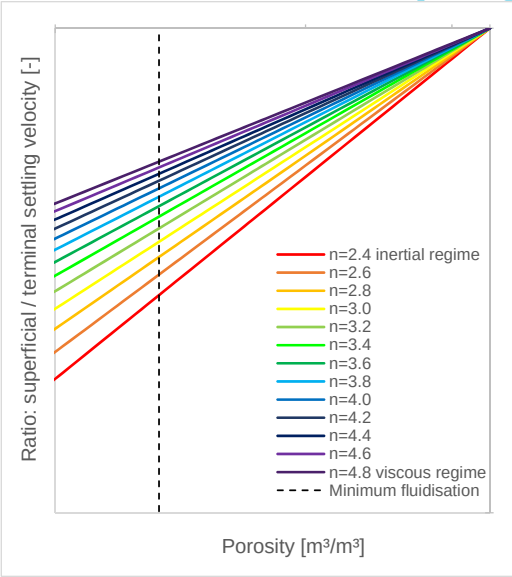


Fig. 1.

Linear relationship log v_s/v_t versus log ε like Richardson-Zaki



J. Fluid Mech. (1972), *vol.* 52, *part* 2, *pp.* 245–268
Printed in Great Britain

245

Sedimentation in a dilute dispersion of spheres

By G. K. BATCHELOR
Department of Applied Mathematics and Theoretical Physics, University of Cambridge

(Received 26 August 1971)

The dispersion considered c
spheres with random position
gravity. The volume fraction
dispersion is statistically hor
that the mean volume flux a
is to determine the mean val
scribed a systematic and rig
ficulty presented by the occ
choice of a quantity **V** whose
same long-range dependence
mean of **U** − **V** can be expres
The result is that, correct to order c, the mean value of **U** is $\mathbf{U}_0(1-6{\cdot}55c)$, where $\mathbf{U}_0$ is the velocity of a single sphere in unbounded fluid. The only assumption made in the calculation is that the centres of spheres in the dispersion take with equal probability all positions such that no two spheres overlap; arguments are

The results (5.3), (5.4) and (5·8), together with (3.14), show that the mean translational velocity of a sphere in a dispersion of identical rigid spheres with (small) volume concentration c is

$$\bar{\mathbf{U}} = \mathbf{U}_0 + c\mathbf{U}_0(-5{\cdot}50 + 0{\cdot}50 - 1{\cdot}55)$$
$$= \mathbf{U}_0(1 - 6{\cdot}55c), \qquad (5.9)$$

ersion over
is difficult, and many observers have sought other kinds of average
icle velocity, such as the speed of fall of the relatively sharp 'top'
to the cloud of particles in a vessel. The available data of different kinds have been examined by Maude & Whitmore (1958) who concluded that the observed mean settling velocity of identical spheres moving with small Reynolds number is best represented, over a wide range of values of c including small values, by a relation of the form

$$\bar{\mathbf{U}} = \mathbf{U}_0(1-c)^{\beta},$$

G. K. Batchelor, 1972, Sedimentation in a dilute dispersion of spheres, Journal of Fluid Mechanics, Volume 52, Issue 2, pp. 245-268, https://doi.org/10.1017/S0022112072001399

A generalized theory of sedimentation

By A. D. Maude, B.Sc., Ph.D., A.Inst.P.,* and R. L. Whitmore, B.Sc., Ph.D., A.Inst.P., Department of Mining and Fuels, University of Nottingham

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sion. The expres

Thus equation (16) becomes

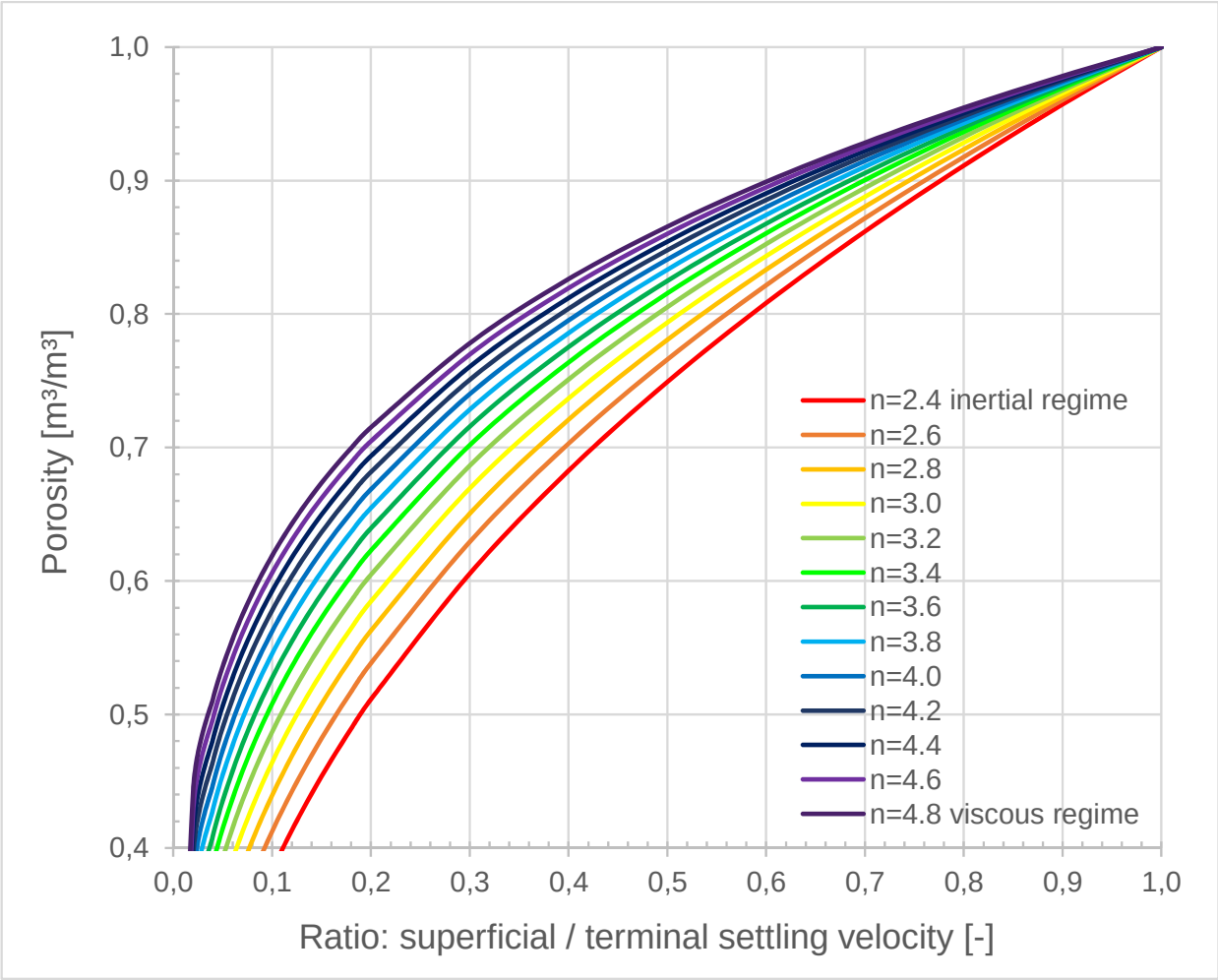
$$\frac{F\rho_i}{\eta^2} \propto \left(\frac{U\rho_i d}{\eta}\right)^m (1-c)^{\alpha}$$

which gives

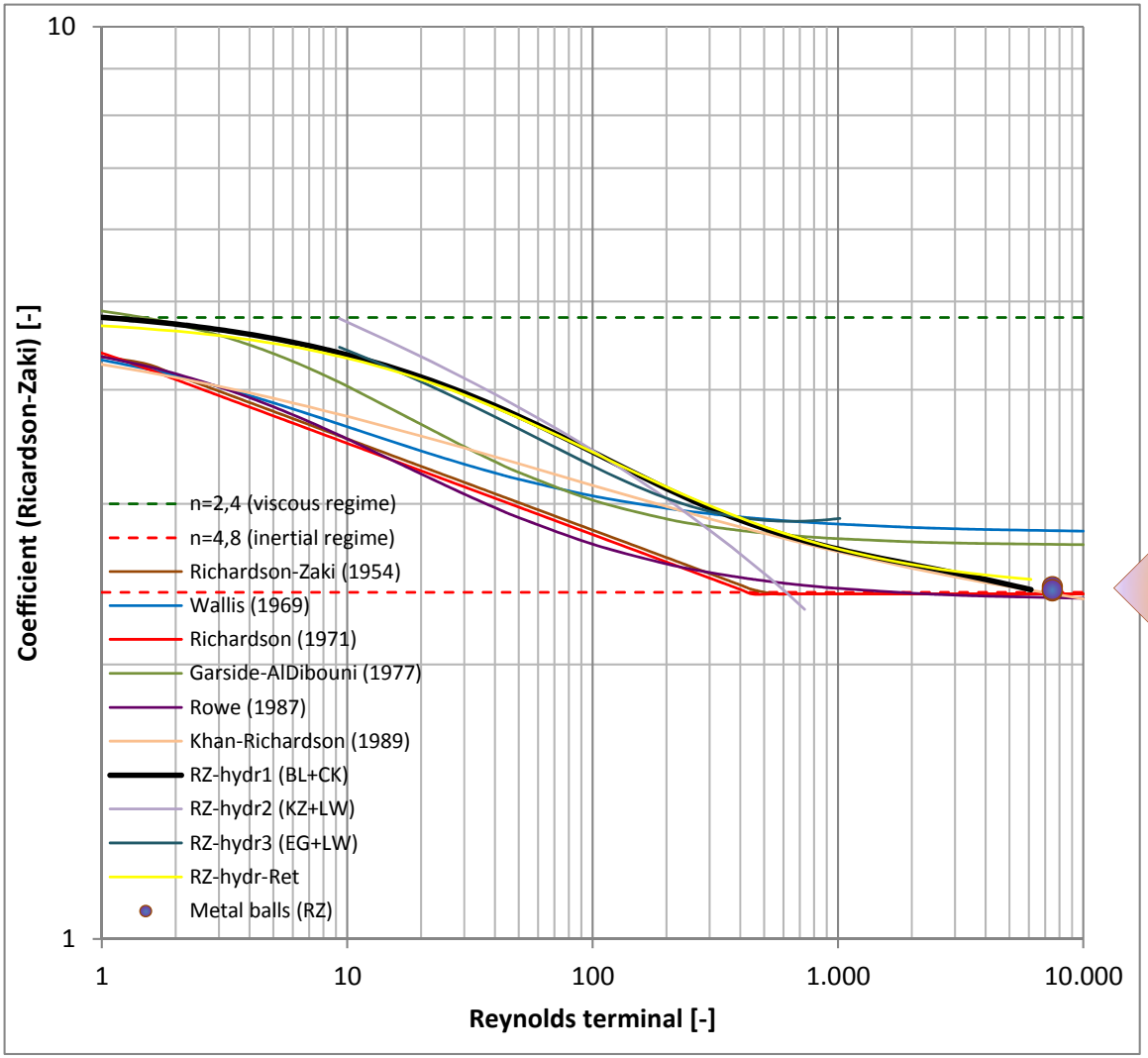
$$U = U_0(1-c)^{\alpha/m}$$

A.D. Maude, R.L. Whitmore, A generalized theory of sedimentation, Br. J. Appl. Phys. 9 (1958) 477–482. doi:10.1103/RevModPhys.20.35.

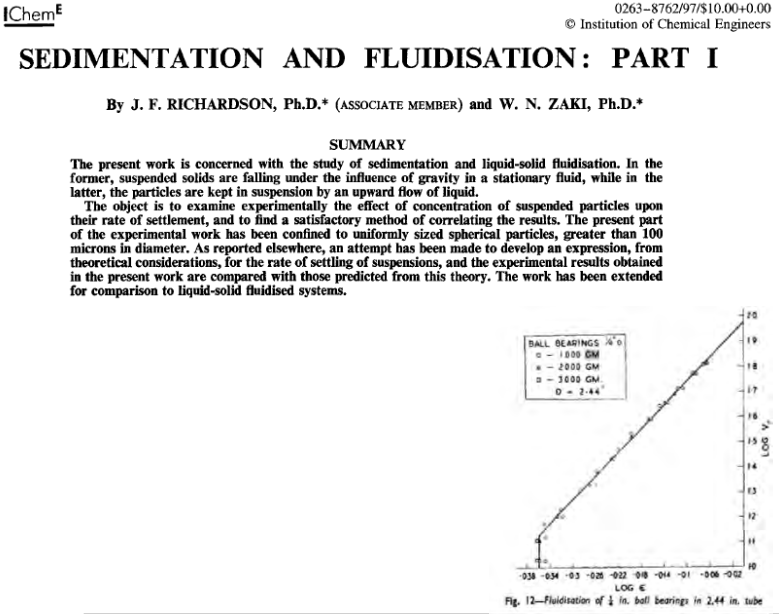
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics



Richardson-Zaki (1954) experimental data



RZ data



IChemE 0263–8762/97/$10.00+0.00 © Institution of Chemical Engineers

SEDIMENTATION AND FLUIDISATION: PART I

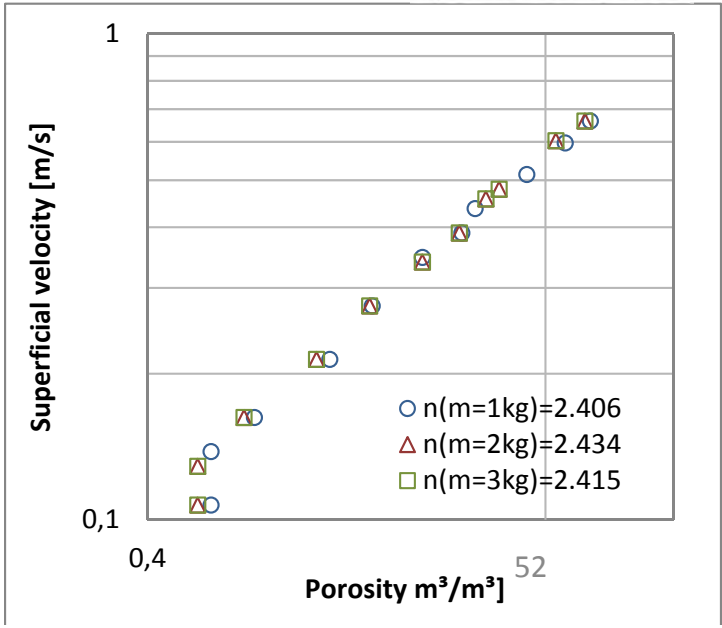
By J. F. RICHARDSON, Ph.D.* (ASSOCIATE MEMBER) and W. N. ZAKI, Ph.D.*

SUMMARY

The present work is concerned with the study of sedimentation and liquid-solid fluidisation. In the former, suspended solids are falling under the influence of gravity in a stationary fluid, while in the latter, the particles are kept in suspension by an upward flow of liquid.

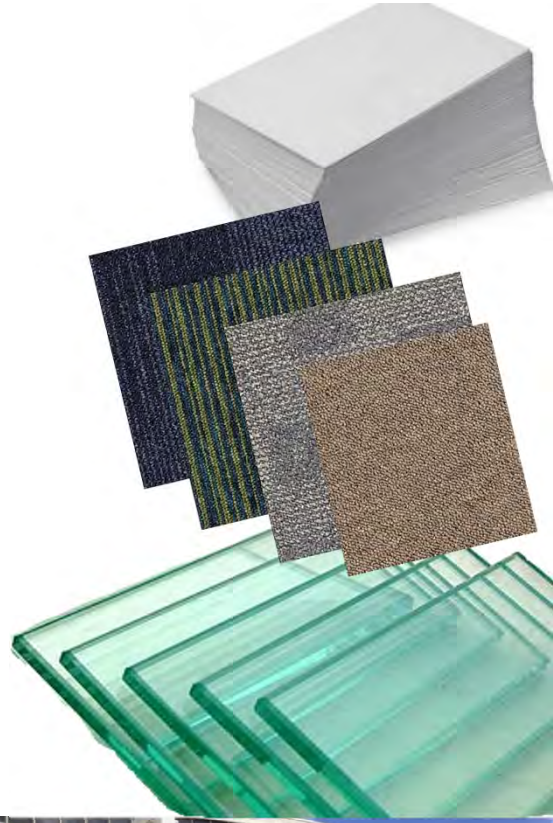
The object is to examine experimentally the effect of concentration of suspended particles upon their rate of settlement, and to find a satisfactory method of correlating the results. The present part of the experimental work has been confined to uniformly sized spherical particles, greater than 100 microns in diameter. As reported elsewhere, an attempt has been made to develop an expression, from theoretical considerations, for the rate of settling of suspensions, and the experimental results obtained in the present work are compared with those predicted from this theory. The work has been extended for comparison to liquid-solid fluidised systems.

Fig. 12—Fluidisation of ¼ in. ball bearings in 2,44 in. tube



Drinking water softening (circular economy)

- ✓ Profit: re-use calcite as a seeding material
  - Cost reduction: 100.000 €/year (0,4%)
  - Sustainability: 40.000 eco-points/year (5%)
  - Valorisation: high market segments: glass/paper/capet…
  - Vision: possibilities introduction of process cycles in industry
  - So much to learn…
    - Legislation
    - Hydraulic
    - LCA calculation



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Circular economy in drinking water treatment: reuse of ground pellets as seeding material in the pellet softening process

M. J. A. Schetters, J. P. van der Hoek, O. J. I. Kramer, L. J. Kors, L. J. Palmen, B. Hofs and H. Koppers

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Overview

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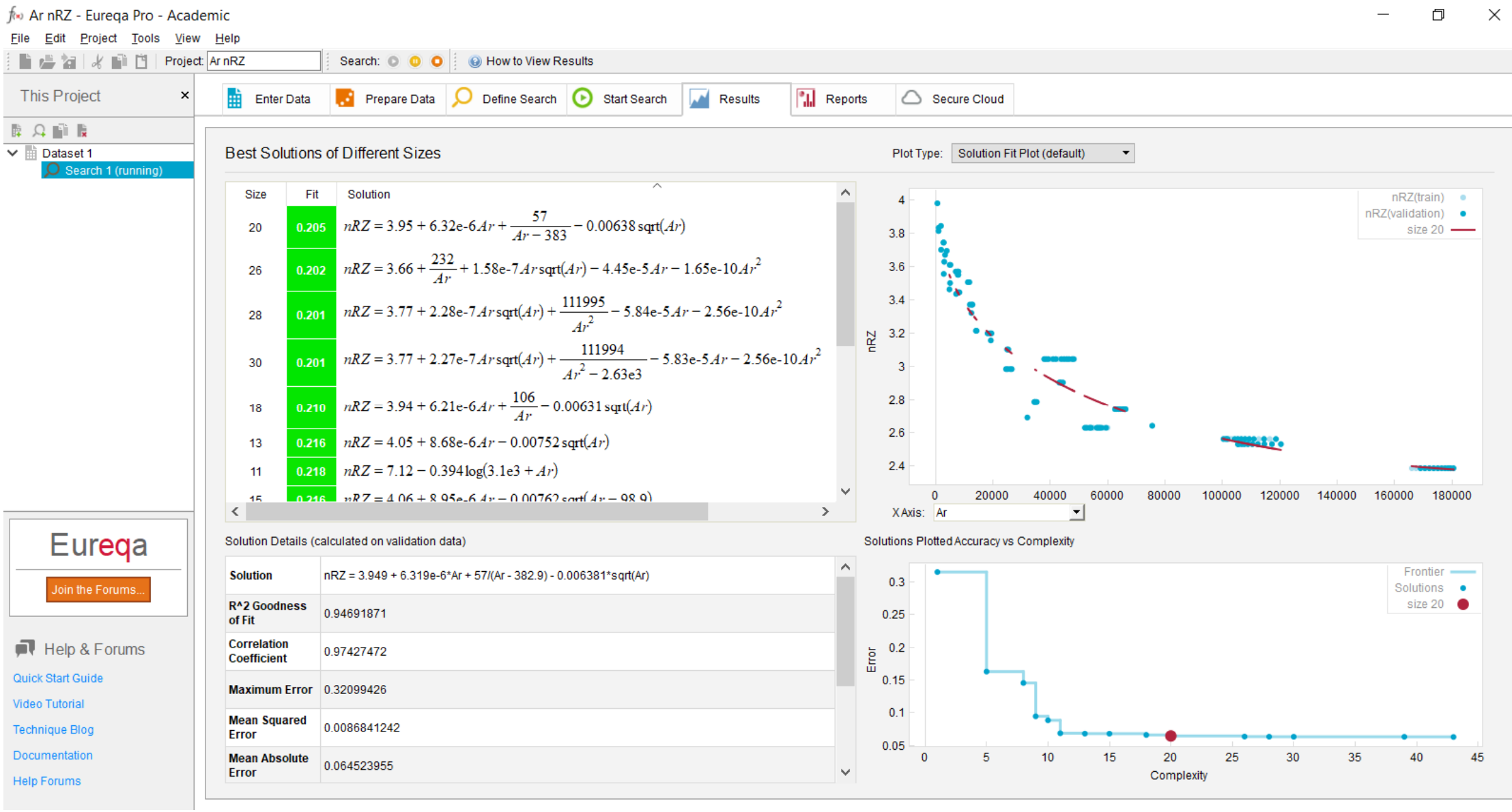
Symbolic regression: Archimedes number → n_{RZ}



Symbolic regression: Reynolds terminal number → n_{RZ}

RZ - Ret n - Eureqa Pro - Academic

File Edit Project Tools View Help

Project: RZ - Ret n | Search: | How to View Results

This Project

Dataset 1

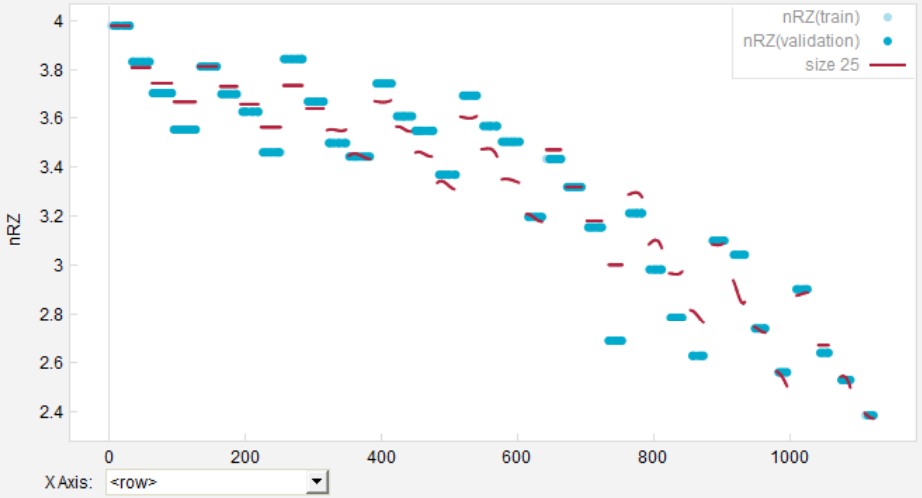
Search 1 (running)

Enter Data | Prepare Data | Define Search | Start Search | Results | Reports | Secure Cloud

Best Solutions of Different Sizes

Plot Type: Solution Fit Plot (default)

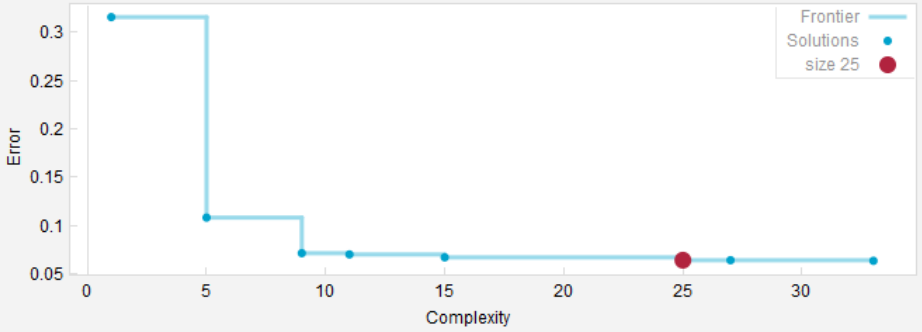
Size	Fit	Solution
25	0.204	$nRZ = 4.02 + 2.22e32\,0.0148^{Ret} + 0.00173\,Ret\log(Ret) - 0.0137\,Ret$
27	0.204	$nRZ = 4.02 + 2.22e32\,0.0148^{Ret} + 0.00173\,Ret\log(0.0443 + Ret) - 0.0137\,Ret$
33	0.202	$nRZ = 3.95 + 2.84e32\,0.0147^{Ret} + 5.94e\text{-}6\,Ret^2 - 0.0055\,Ret - 0.0546\,(0.0024\,Ret)^{4.7}$
15	0.213	$nRZ = 4.05 + 0.00186\,Ret\log(Ret) - 0.0146\,Ret$
11	0.223	$nRZ = 4.11 - 0.0714\,\mathrm{sqrt}(Ret - 11.1)$
9	0.227	$nRZ = 4.19 - 0.0756\,\mathrm{sqrt}(Ret)$
5	0.343	$nRZ = 3.81 - 0.00293\,Ret$
1	1.000	$nRZ = 3.5$



Solution Details (calculated on validation data)

Solution	nRZ = 4.017 + 2.223e32*0.01484^Ret + 0.001733*Ret*log(Ret) - 0.01371*Ret
R^2 Goodness of Fit	0.94951995
Correlation Coefficient	0.97517377
Maximum Error	0.31294104
Mean Squared Error	0.0082585596
Mean Absolute Error	0.064296751

Solutions Plotted Accuracy vs Complexity



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CFD oppertunities

- Interstitial velocity versus terminal settling velocity
- Tortuosity versus ratio terminal and interstitial velocity
- Influence of the geometric representation (shape) on the specific surface area
- Particle interactions and collisions versus drag
- Relevant forces buoyancy, gravity and friction
- Surface roughness impact
- …

Any suggestions are welcome.
Please mail me at: o.j.i.kramer@tudelft.nl