Design of a passive self-levelling device

T.A. Heldoorn

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T.A. Heldoorn

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Table of Contents

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Thijs A. Heldoorn and Just L. Herder

***Abstract*—In this paper the design of a passive self-levelling device is presented. Self-levelling means that the device adjusts itself to compensate for inclinations of the floor without user intervention. This has the advantage that a single device can be used on floors with various inclinations without losing its functionality. First an analysis is presented on functions (e.g. levelling, measuring and locking) and strategies (possible orders in which these functions take place) that are useful for self-levelling devices. After this a prototype of the designed device is presented and tested on its levelling accuracy for inclinations of 0,5,10,15,20,25 and 30 degrees relative to the horizontal. The test results show that the system for most inclinations can level its platform within a 3 degrees deviation from the values that were expected based on a computer model.**

I. Introduction

ALMOST all devices that are used in everyday life are designed for use on horizontal floors. Wheelchairs, tables, chairs, ladders and many other devices only function correctly if they are used on smooth horizontal floors. However lots of these devices are also used outdoors, were inclined floors are often encountered. If these devices are used on inclined floors this can result in annoying situations, for example a pencil that cannot be placed on a table without rolling of. In other situations, an inclined floor can lead to more serious problems, for example when a wheelchair tumbles over due to an inclination.

In the past several attempts have been made to solve the aforementioned problems. In [1] and [2], mechanisms are presented that can compensate for small irregularities of a floor, or for slight differences in the length of a platform its legs. In both these cases the differences were compensated by using compliant materials that deform under pressure.

Previous work has also been done on trying to remain a horizontal platform in situations with non-horizontal floors. These works mainly focus on systems that can be used on ships to compensate for the rolling and pitching movements caused by the waves. Some of these systems are solely based on a counterweight to keep a platform levelled horizontally [3][4]. Other systems make use of four-bar mechanisms that behave like a pendulum in combination with springs (both leaf springs [5] as well as normal springs [6] are used). Yet another system uses a combination of springs, a four-bar mechanism and a counterweight to keep a horizontal level [7].

Though the previous systems (in combination with one another) can be used to reduce the problems caused by uneven floors, they do not provide full solutions. Some of the systems can only be used for the compensation of small irregularities in the order of millimeters, causing the system to remain instable on floors with larger irregularities. These systems also do not provide a rigid platform that can be actively used. They are sensitive to forces, which cause them to get a deviation from the horizontal position. These systems also tend to be very heavy and are therefore not useful for active outdoor use.

An ideal self-levelling platform is able to compensate for any inclination or irregularity of the floor. It is also rigid after performing the levelling motion, so external forces can be applied without losing the new orientation.

The goals of this paper are:

1) *Present an analysis on function and strategies that benefit the design of passive self-levelling devices.*
2) *Present a design/prototype of a passive self-levelling device for uneven surfaces.*
3) *Present experimental results on the levelling accuracy of the prototype.*

The choice for a passive system was made because passive systems can be widely used. First, the method used to reach these goals is presented. After this the analysis of passive self-levelling devices is made, followed by the results, the discussion and the conclusion.

II. Method

A. Requirements & Assumptions

To be able to properly quantify the assumptions and requirements an application for the final design should be chosen. It was decided to base the quantification on a passive self-levelling chair. It should be noted that the remainder of this work is still based on the design of a self-levelling device rather than a self-levelling chair.

Requirements:

- Passive system: The system should work without active element(s).
- Compensation of inclinations: The device should compensate for inclinations of up to 30 degrees.
- Maximal platform deviation: After levelling the maximal deviation relative to the horizon for any given inclination should be less than 3 degrees.
- Applicable load: The platform must sustain loads up to 100kg.

T.A Heldoorn and J.L. Herder are with the Delft University of Technology, Faculty of Mechanical, Maritime and Materials Engineering, Department of Bio-Mechanical Engineering, Mekelweg 2, 2628 CD Delft, The Netherlands.

- Maximal levelling time: Time to set up the device must be less than 5 seconds.
- Easily transportable: Limited dimensions and light weight (6 kg max, similar to a foldable chair).
- Easy to use: Require minimal user input and minimal thought.

Assumptions:

- System compensates for a single direction of inclination. (Levelling in 2D.)
- System levels once per use: The system does not need to correct for changes after initial levelling.
- No disturbance: The system is not exposed to disturbing forces during levelling.
- Platform legs never slide: The chair does not slide away regardless of the inclination.
- Firm surface: The surface does not deform, thereby adjusting the platforms position.
- System is used correctly: The device is not used in unintended ways.

B. *Problem Analysis*

First a general analysis useful for the design of a self-levelling platform was made, which is presented in the next chapter. The analysis was made based on reasoning and knowledge related to engineering. Different necessities for a self-levelling platform could be identified by thinking about the intended use of this device and the shortcomings of already existing solutions. These necessities resulted in the identification of different functions that the platform should have. Strategies were identified by ordering these functions in different ways. Different ideas for self-levelling mechanisms were thought of using the required functions as a guide. These ideas were analyzed using the functions and strategies, which in some cases led to the identification of new functions and strategies. The functions that were necessary to meet the requirements have been analyzed thoroughly, and their sub-functions are discussed.

C. *Design Method*

The design method started with brainstorming about different ways in which a self-levelling system can work. This did not result in any great solution, but it formed a basis to start with the analysis. By considering the ideas that were generated new strategies and functions for the system could be thought of and were analyzed. During the analysis itself new ideas were also generated, but these were much more focused than the ideas obtained during brainstorming.

All ideas that were generated during and before the analysis were checked on their feasibility first based on solid reasoning, and after that by making simplified computer models. It was also checked to what extent they met the requirements, and how easy they were to fabricate. It was clear that one concept stood out, since it was the only concept that could fulfill its tasks in an easy and straightforward way. To make sure the idea worked as intended a small LEGO®model was made to prove its functionality. After this concept was chosen, variations to this concept were thought off and their obtainability was checked using MatLab and LEGO®. These different concepts were compared. The best concept was chosen from these variations, after which several different shapes for the realization of this concept were considered and analyzed using Ansys. Finally, a SolidWorks model was made after which the prototype was constructed.

D. *Experimental Analysis*

All the experiments will be performed on a surface plate, to ensure that the surface is as horizontal as possible. The device will be inclined by a system attached to the device, which can increase the height of one side of the device. The device will be positioned at angles of 0,5,10,15,20,25 and 30 degrees relative to the horizontal. After the platform has levelled, the platforms angle relative to the horizontal will be measured using a digital angle measurement device (Bosch DWM40L, with an accuracy of 0.1 degree). The values found will be compared to values found using computer simulations to be able to distinguish errors introduced by simplifications (e.g. the assumption of a linear relation) from errors introduced by the system itself (e.g. due to friction).

III. Analysis

Since the system will be passive, the energy required for motion has to come from the device itself. This only happens if the system moves to a place with less potential energy than in its original position. Since there will always be energy dissipation due to friction, it is impossible for the system to have a constant amount of energy in different positions.
If the platform has to rotate a force resulting in a moment is needed. There are many different forces that can result in the rotation of the platform. Considering these different forces they can be separated in two different groups, the unidirectional forces and the directional forces. Unidirectional forces are forces that work in a single direction, regardless of the orientation of the system (gravitational force, Archimedes force). The directional forces on the other hand change their orientation in relation to changes in the orientation of the system (spring force, normal force etc). The following figures make this more clear.

In figure 1 it can be observed that in situation A both the gravitational force and the spring force are working downwards. In situation B the exact same system is placed on an inclined surface, due to which the orientation of the gravitational force relative to the system has changed, while the orientation of the spring force relative to the system has remained unchanged. Since the gravitational force does change its direction relative to the system based on orientation, this force is suitable for identifying the systems

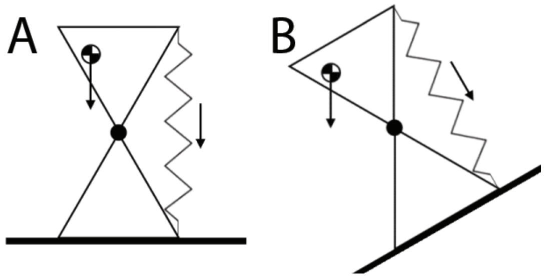

Fig. 1: Spring and gravitational forces relative to the system.

current orientation. Since a self-levelling system needs to be aware of when it has reached the correct position, it is concluded that a unidirectional force is needed for the system to be able to level itself.

A stable platform is always preferred regardless of the surface on which it is placed. Since these surfaces are not known upfront, it is best to make a system that always has a stable position. This can be realized by making a platform with just three contact points with the floor. Since these three contact points always form a plane, the system will always be stable (assuming it does not tilt over).

Considering the different functions that a passive self-levelling platform can possess, four main functions can be identified:
Settling: Placement of the device in a stable position on the floor.
Levelling: The platform its movement toward the horizontal position.
Measuring / Sensor: Reference to identify if the platform is in the horizontal position.
Locking: Locks the platform to prevent movement under the influence of loads.

Given these functions, different orders (strategies) of 'settling' and 'levelling' can be identified:
Level before settling: If the system has to level before it is settled on the floor, it needs upfront knowledge of the curvature and inclination of the surface. Since this information cannot be obtained in a passive way, levelling cannot occur before settling.
Level during settling: In a system that levels during settling the settling part has an influence on the final position of the platform. If the settling is not performed well the system will not be properly leveled. A disadvantage of this method is that the system always needs to start in the same starting position in order to level well. Since there is no manner to predict the angle over which the base has to rotate around its contact point before it is settled, the system can only use information about the movement required relative to a fixed starting position.
Level after settling: Systems that level after settling have reached a stable position on the floor before the platform starts to level. These systems have the advantage that the floor for which compensation is needed is known. Systems that start levelling during settling are included in this group if the settling part has no influence on the final position of the system.

From the three orders of levelling and settling that were discussed, only one option can lead to a truly self-levelling system. Therefore, 'levelling after settling' is the only strategy that will be used in the continuum of this analysis.
Now that an initial strategy has been chosen, the levelling, the locking and the measuring of the platform will be analyzed. Since the system has already settled when levelling starts, no further analysis will be done regarding settling.

A. *Levelling of platform*

Movement of platform:
Rotation: To get a horizontal platform a rotation is needed.
Translation: A translation cannot result in a horizontal platform unless the platform is horizontal from the start.
Rotation and translation: This can result in a horizontal platform because of the rotation, the translation is not necessary.

It was decided to mainly focus on a solely rotating platform, since a rotation is the only necessary type of motion.

Platforms position with minimal potential energy:
In horizontal: If the position with minimal potential energy is in the platforms horizontal position, and assuming there is some sort of damping, the platform can come to rest before any locking action takes place.
Not in horizontal: If the position with the least amount of potential energy is not in the platform's horizontal position, the platform should be locked in the horizontal position during one of its passes through this position.

To decide if a minimal amount of potential energy is needed in the horizontal position, more information is needed about the lock activation. Therefore, both options are considered.

Number of horizontal states:
No horizontal states: The platform never reaches or passes through the horizontal position.
Single horizontal state: The platform reaches or passes through the horizontal position one single time.
Multiple horizontal states: The platform will pass through the horizontal position multiple (n) times.

At least one pass through the horizontal position is necessary to enable the possibility of locking the platform in a horizontal position. If multiple swings are wished for, one needs to keep track of previous passes of the horizontal, meaning an extra reference is needed. Therefore, only a 'single horizontal state' is a viable option.

B. *Locking*

There are several different options that need to be considered for locking of the platform. Locking of the platform is necessary to make sure that loads can be applied without causing deviations.

How to lock:

No movement: The platform can be locked without allowing any movement of the platform.

Allow movement, no return: Small deviations are allowed after which the system does not return to the exact horizontal position.

Allow movement, return: Small deviations are allowed after which the system does return to its initial position.

A disadvantage of allowing slight movements without return is that it can lead to buildup of error. Therefore, 'no movement' and 'allow movement with return' are considered better options.

When to lock:

Before the levelling motion: It is not possible to lock the platform in the correct position before levelling.

During the levelling motion: If the platform is locked during levelling, the platform only needs to pass through the horizontal without a need for minimal potential energy at the horizontal position.

After the levelling motion: Locking after levelling requires the system to have fully come to a rest in the horizontal position.

Locking after the levelling motion is possible, but requires the system to know when the system is at rest and requires minimal potential energy in the horizontal position. Therefore, locking during the levelling motion is the best option.

Locking reference:

Speed: The speed of the system is dependent on its starting position. Therefore there is no unique speed at the horizontal position that can be used as a reference.

Distance / position: The distance between elements cannot be used as a reference for the horizontal position on its own, because the correct reference value is dependent on the orientation of the system.

Angle: The correct reference value is dependent on the orientation of the system, so an angle cannot be used as a reference on its own.

Force: A force can be used as a reference as long as it is a unidirectional force. Directional forces cannot be used as reference.

Moment: Can be used as a reference if the moment is caused by a unidirectional force.

Only a unidirectional force, or a moment caused by a unidirectional force, can be used as a reference value on its own. Therefore these are the only suitable references that can be used for locking.

C. Measuring

To determine the moment at which locking can take place a reference is required. In order to compare the current state of the platform with the reference, a measuring device or sensor is needed. If it is assumed that the sensor measures the position of the platform during its motion, the following variations can be made.

Time of measurement:

Continuous: The sensor measurements can be taken on a continuous basis.

Discrete: Discrete measurements only compare the position of the platform to the reference value at certain instants. Therefore the accuracy of these systems is dependent on the frequency at which the measurements occur.

Continuous systems tend to be more accurate than discrete systems and are therefore preferred. If discrete measurements are used they should occur at a high frequency.

Measurement signal:

Binary: If the measurement signal is binary it is only known if the system is in the horizontal position or if it is not in the horizontal position.

Continuous: A continuous measurement signal knows the current position of the platform relative to the platform.

For self-levelling systems it does not matter whether the sensor is binary or continuous.

For now it has been assumed that levelling happens simultaneously with the measuring of the sensor. But there are also situations where the platform only levels after the sensor has found its correct reference value. In this situation there are some other options that need to be considered.

Activate platform levelling after sensor settling:

Manual: After the sensor has found its proper position relative to the surface platform levelling can be manually activated.

Automatic: After settling of the sensor the platform will automatically start to level.

Even though an automatic transition would obviously be the nicest option, it requires additional elements. Taking the foldable chair as application, most users will not bother to give a small input to the system. It might seem that a manual transition does not comply with the goals. However, due to complexity and robustness of the final design, this is the most logical choice.

When to lock the sensor:

After reaching correct position: After the sensor has reached the correct reference position it can be locked immediately to prevent unwanted motion.

Lock before influence: In this situation the sensor is locked before the platform can influence the sensor position with its levelling motion.

Lock on influence: Here the sensor is locked as soon as it is influenced by the levelling motion of the sensor. Locking should take place fast to reduce deviation of the sensor.

Lock on influence seems nice, but requires a fast lock activation to reduce the influence on the sensor position. Locking the sensor directly after reaching the correct position requires extra elements that notice when the sensor has reached the correct position, which is redundant. Lock before influence is the best option, where e.g. the motion of the platform can be used to lock the sensor, thereby preventing any influence on its position.

IV. Results

A. Prototype results

The prototype is depicted in figure 3 and figure 4, and a schematic overview of the working principle is depicted in figure 5. The device is placed at an inclination with the pendulum in the upward direction. A 1218N/m linear spring running over a pulley simulates a torsional spring and balances the inverted pendulum at an angle that is dependent on the inclination α of the surface. If the platform in figure 3 is lifted, the rack attached to the downside of the platform and the gear fixed to the end of the pendulum will lose contact with each other. The pendulum can now rotate and will find a new equilibrium position. After the new equilibrium position is found the platform is lowered causing the rack and gear to make contact and lock the platform in its new position (figure 4). When lowering the platform a bicycle brake is activated and grabs the pulley. This frictional brake is necessary to prevent pendulum movement at initial contact with the platform.

Fig. 2: The prototype before levelling.

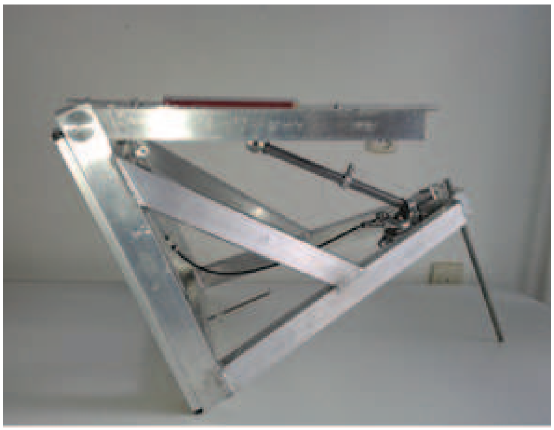

Fig. 3: The prototype after levelling.

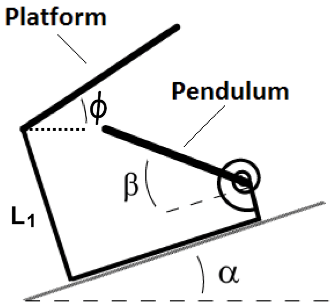

Fig. 4: Schematic overview of the system.

B. The models

1) Deviation from horizontal: The values found during the experiments are compared to the values that are expected based on a computer model. This model is based on an equilibrium of the torques around the point of rotation of the pendulum.

The equation that forms the basis of the model is:

$$\min\{mgl\cos(\beta-\alpha)-(3+kx)r\} \tag{1}$$

Where m is the mass ($0.1976kg$) of the pendulum measured at a length l ($0.315m$) for the horizontal position of the pendulum, g the gravitational constant $9.81m/s^2$, k the spring constant ($1218N/m$) and r the radius of the pulley ($0.024m$). The 3 in the equation 1 denotes the pretension of the spring (in N).
The value of x is given by:

$$x=\frac{90-\beta}{180}\pi r \tag{2}$$

For every $\alpha \in \{0, 0.01, .., 30\}$ there exists a $\beta \in [0, 90]$ that satisfies equation 1. With the values of β known for every α the angle of the platform can be calculated.

2) Angle difference per tooth increment: The angle γ as depicted in figure 2 can be calculated using a rewritten form of the cosine rule:

$$L_d=\sqrt{(p_{3x}-p_{1x})^2+(p_{1y}-p_{3y})^2} \tag{3}$$

$$\gamma=cos\left(\frac{L_p^2-(L_{pf}-w_t i)^2-L_d^2}{-2L_gL_d}\right) \tag{4}$$

Where w_t is the distance between adjacent teeth, which can be calculated using formulas described in [10], and $i \in \{0, 1, .., n\}$ the tooth on the rack used for locking.
Now $\Delta\gamma$ for a tooth increment can be calculated:

$$\Delta\gamma_i=\gamma_i-\gamma_{i+1}. \tag{5}$$

Since γ has a constant relation to ϕ (depicted in figure 5), it can be concluded that $\Delta\phi=\Delta\gamma$.

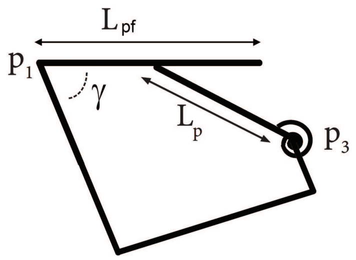

Fig. 5: Overview for the calculation of $\Delta\gamma$.

C. Experimental evaluation

Now the prototype is known the experimental procedure to test the prototype can be explained in more detail. The deviation of the platform will be tested for different angles of the floor. The entire device is placed on a surface plate to ensure an accurate horizontal reference. The inclination of the surface is obtained by adjusting the height of one side of the prototype by means of a screw thread. First the angle between L_1 and the surface plate is determined as a reference value. Since L_1 is not perfectly perpendicular to the horizontal, other inclinations will be determined by comparison with this reference value. The device starts in the position depicted in 3, after which the platform will be raised. After the pendulum has come to a complete rest the platform is lowered, and a manual load is applied to ensure proper contact between the gear and rack. The angle between L_1 and the platform will be measured, which can be used to determine the platforms deviation relative to the horizontal. Since the prototype used a spring that is an estimated linearization of the optimal spring for this design and has an undesirable pretension, the results found in these measurements will be compared to a computer model. This computer model simulates a spring with identical properties as the spring used in the prototype. The comparison between the actual results and the results of the computer model are important to determine if the system behaves as expected.

Another property of the system that will be measured is the difference in angle between L_1 and the platform for every increment in teeth of the rack. This will be done by placing the platform in a horizontal position and shifting the position of the gear relative to the rack one tooth at a time. The angles between L_1 and the platform will be measured for every tooth, after which the difference in angle per tooth skip can be calculated. This increment in angle is used to get an idea of the influence of a tooth skip on the angular deviation of the platform. To test if the results correspond to the expected values, they will be compared with results from a computer model.

D. Experimental results

In graph 6 the average value of the deviation of the platform relative to the horizontal has been plotted together with values according to the computer model. The black bars originating from the measured values represent the 3 degrees error margin. The measured values should stay within a 3 degree range relative to the model.

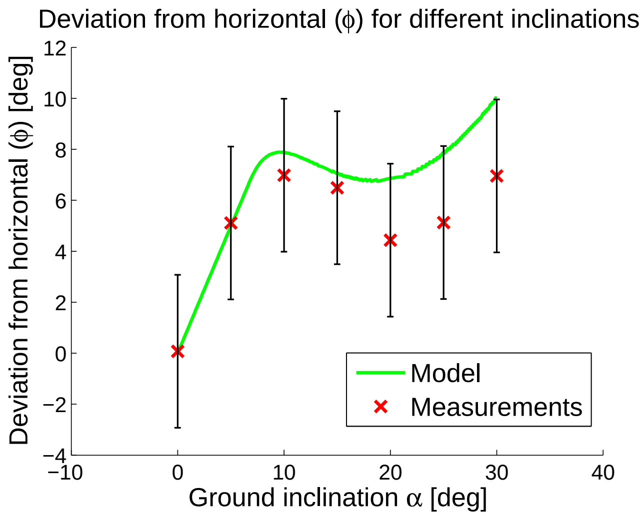

Fig. 6: Deviation from the horizontal for different inclinations of α.

In graph 7 the distributions of the difference between the measured and modeled deviations have been plotted.

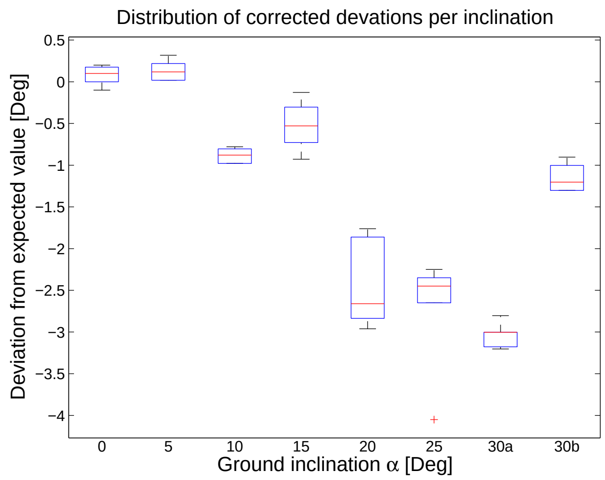

Fig. 7: Distribution of the difference between the measured and modeled values.

In graph 8 the angular deviation per tooth increment is depicted together with the values expected based on a computer model.

V. Discussion

When examining figure 7 it can be noticed that there is a box for $\alpha = 30a$ and for $\alpha = 30b$. Measurement 30a consists of values that where obtained as described in the method. However, it was noticed that the teeth of the rack and gear did not realize full contact by just lowering the platform.

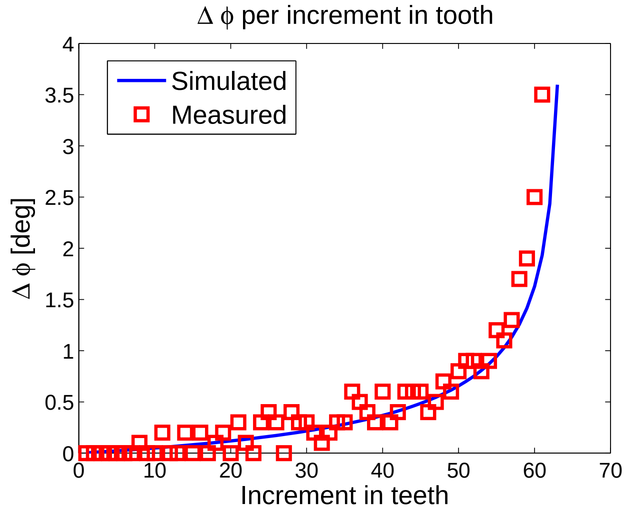

Fig. 8: Angular difference per tooth increment.

Only after a load was applied to the platform full contact was realized. Therefore it was decided to manually adjust the position of the gear relative to the rack by 1 tooth, which resulted in measurement 30b. It can be clearly seen that 30b has a smaller deviation from the modeled value than 30a. Since the rack and gear at first did not make full contact, it is assumed that the preferred position of the pendulum relative to the rack is somewhere in between the teeth corresponding to values 30a and 30b. This also implies that the preferred position of the pendulum would result in a deviation in between the deviations of 30a and 30b. It is also a clear indication that a gear and rack with a higher resolution can result in more accurate levelling for certain positions.

This design requires a spring that has no pretension because this pretension prevents rotation of the pendulum for small inclination angles. Zero-pretension springs can be purchased, but due to time limitations this was not an option (custom fabrication). Initially a pulley system and counter-weight were used to overcome the pretension. However, the friction caused by the pulleys led to inaccurate levelling results that did not show any relation to the models. Therefore it was decided to run the same test without the weight and with pretension of the spring, which was also accounted for in the model. The accurate results for an inclination of 5 degrees is a direct cause of this pretension, because pendulum movement is not possible yet at this angle. Only when the inclination reaches an angle of 6.7 degrees the pendulum starts to move. Therefore it is no surprise to see a close match between the modeled and measured value at 5 degrees. This also means that the real range of inclinations for which the system levels starts at 6.7 degrees.

The spring used in this design is a linear spring with k = 1218N/m. Even though a linear spring can be used to approximate horizontal levelling, calculations have shown that the ideal spring to do this would not be linear. This could have been corrected for by altering the shape of the pulley, not making it perfectly round, as described in [8]. Another option would be to design a non-linear torsional spring as described in [9]. Since this would have been a time consuming process it was decided to use a linear spring and normal pulley that approximated the optimal (torsion) spring for this design.

The final results of the prototype are compared relative to the model and not relative to the actual horizontal. The model describes the ideal behavior of the system for a given spring (continuous locking, no friction etc.). Therefore the comparison between the measured values and the predicted values is a measure of how much deviation is added by the designed system, without adding deviation caused by the sub-optimal spring. The deviation caused by this sub-optimal spring can be removed by using the proper spring for this design, which is the reason why this error should not be added to the levelling error. This justifies using the model as a reference for the measured values instead of the horizontal.

The stiffness of the spring used in this design needed to be checked for deviations from the manufactured value. These measurements were done by loading the spring with various loads and compare the differences in length. Pictures were made of each elongation to enable an accurate estimation of the stiffness. However, figure 9 shows the effect of a slight increase and decrease in spring stiffness. Both the increase and decrease are by less than 1% of the spring stiffness, but it can be seen that they have a clear influence on the levelling accuracy of the system. This shows that small measurement errors of the spring stiffness value can influence the levelling accuracy to the extent that the requirements are met or not.

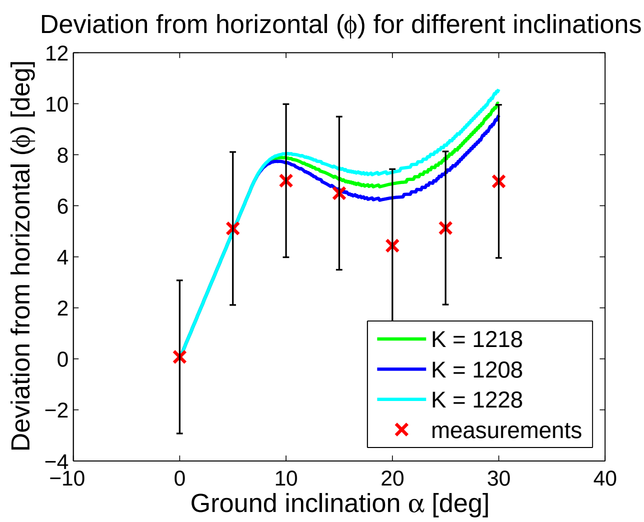

Fig. 9: Deviations of the measured and modeled values for different spring constants.

In the boxplots of fig 7 it can be seen that the boxplots at 15 and especially at 20 degrees have a larger spread than the boxes at other angles. (Except for a single outlier at 25 degrees.) A possible explanation for this is that early tooth skips (occurring at small inclinations) hardly have any influence on the final position of the platform, while later tooth skips (occurring at bigger inclinations) have a big influence on the final position. However, the bigger the deviation caused by a single tooth skip, the less likely it is that this tooth skip occurs. Apparently the combination of the number of possible tooth encounters and deviation per tooth skip causes the biggest deviation in measured value at an inclination α of 20 degrees. In this same figure it can also be observed that there is little spread in the deviations found for the inclination of 0 degrees. Since no pendulum movement has taken place at this inclination, the angle of the platform should always be the same. This means that the distribution observed at $\alpha = 0$ is a measure for the measurement error caused by manual measuring, which is $\pm$0.2 degrees.

It can be observed in figure 8 that the behavior of the measured and modeled difference in angle for every tooth increment matches. Some inaccuracy is caused by the resolution of the angle measurement device and by manual measurement errors. The results give a good indication of the difference in angle if the rack and gear skip a tooth. From this figure it can be deduced that tooth skipping has an increasing influence on the deviation of the platform for increasing inclinations of the floor. The model used to predict these values also showed that a gear and rack with a higher resolution results in less error due to tooth skipping. By doubling the resolution of the gear and rack the maximal deviation $\Delta\phi$ can be reduced to 2.04 degrees.

The systems around 16.5 seconds to level for the maximal inclination of 30 degrees. This is much more than the 5 seconds set in the requirements. It was expected beforehand that the system would take more than 5 seconds to come to a rest, which was reasoned using the formula:

$$\phi(t) = e^{-\omega_0 \zeta t}, \tag{6}$$

which can be used to predict the reduction of the amplitude over time. The damping coefficient of the steel spring was estimated at $\zeta = 0.01$ [11], and the natural frequency ω_0 was estimated around $11 rad/s$. It can be calculated that it takes 53.4 seconds before the system has an amplitude of 0.1 degrees (about 0.28% of its original value). The big difference between the 53.4 and 16.5 seconds is because the calculations assumed spring damping only. It is possible to reduce the settling time by adding extra damping to the system. However, this damping should have minimal influence on the levelling accuracy of the system. Therefore adding viscous damping is preferred over coulomb damping.

VI. Conclusion

The presented analysis proved to be useful for the design of a passive self-levelling device. It also offers an overview of alternative functions and strategies that are useful for designing self-levelling or levelling related devices.

For the first two inclination values of 0 and 5 degrees no difference with the model could be observed. This is due to the fact that the pretension did not allow any movement up to an angle of 6.7 degrees. For inclinations of 10, 15, 20 and 25 degrees the system levels itself within a 3 degrees error margin relative to the model. However, for an angle of 30 degrees the error between the average measured value and the modeled value was 0.045 degrees after subtraction of the 3 degrees error margin. Extra measurements showed that the system will level accurately within the error margin if a gear and rack with a higher resolution is used.

It took the system around 16.5 seconds to level for the maximal inclination of 30 degrees. This is much more than the 5 seconds set in the requirement. The spring and pulley do not provide enough damping on their own, which means that extra elements to increase the damping have to be added.

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Appendix A

Recommendations

Here recommendations for future research are given.

The current system can only level in 2 dimensions. Future research should focus on various ways in which the current system can be implemented in a 3D environment. This is an important and necessary step if this design has to be implemented in a wide variety of real-life situations.

In the analysis two different feasible strategies for settling and levelling were explained. 'Level after settling' was chosen, because it was considered the only real self-levelling option of the two. However, levelling during settling also has interesting properties. Future can be done on the design of such devices, and allows for the experimental comparison of the different strategies.

If the inclination of the underground increases the force in the inverted pendulum also increases. The force in the inverted pendulum can reach huge amounts, especially if the pendulum is in the near horizontal position. These extreme forces limit the maximal applicable force, or cause the system to be heavy in able to sustain these forces. Therefore, it is interesting to find a way that allows the inverted pendulum to reach near horizontal positions, but without the involvement of these huge forces.

The current system was not optimized for forces, stresses and weight. This means savings in weight should be obtainable if stress and force optimizations are performed. Optimization can also be done on the shape if the pulley, thereby allowing for more accurate levelling of the platform.

One of the requirements is that the system should have minimal dimensions. The current system design could allow for a foldable variant that can be folded to use minimal space if it is not utilized. Analysis has to show if this is obtainable without losing the current performance of the system.

The current design can level more accurately if the resolution of the locking mechanism is increased. Ideally a continuous lock that is able to withstand the big forces as set in the requirement is wanted. Tests have to show if such a lock is feasible and if it can level accurately within smaller margins than the set 3 degree.

Appendix B

Proof of principle

For this proof of principle it is assumed that L_1,L_2,L_3,L_4 all have a length of $1m$ and a mass of $1kg$. It is assumed that L_4 is directly connected to L_3. The gravitational constant g is taken as $9.81m/s^2$. A schematic overview of the situation is depicted in figure B-1.

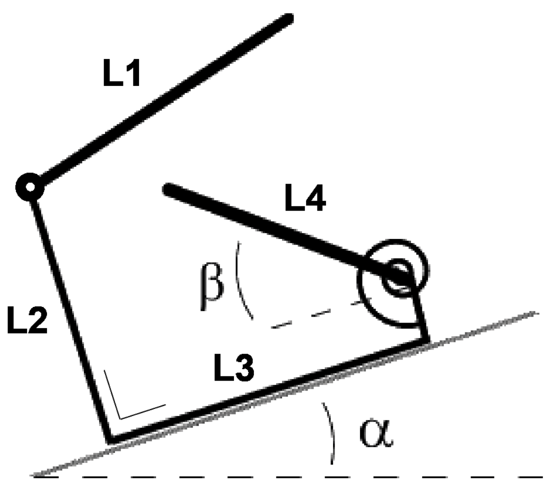

Figure B-1: Schematic overview for the proof of principle.

First the different values for angle β are calculated for which the platform will be horizontal for every inclination of the ground α. This results in the plot of figure B-2, which represents the relation between α and β for which the platform L_1 will be horizontal.

Next the torque required at the base of the inverted pendulum L_3 for which the pendulum is in static equilibrium can be calculated for the various values of α and the corresponding values of β. The relation between the angle β and the torque required for static equilibrium

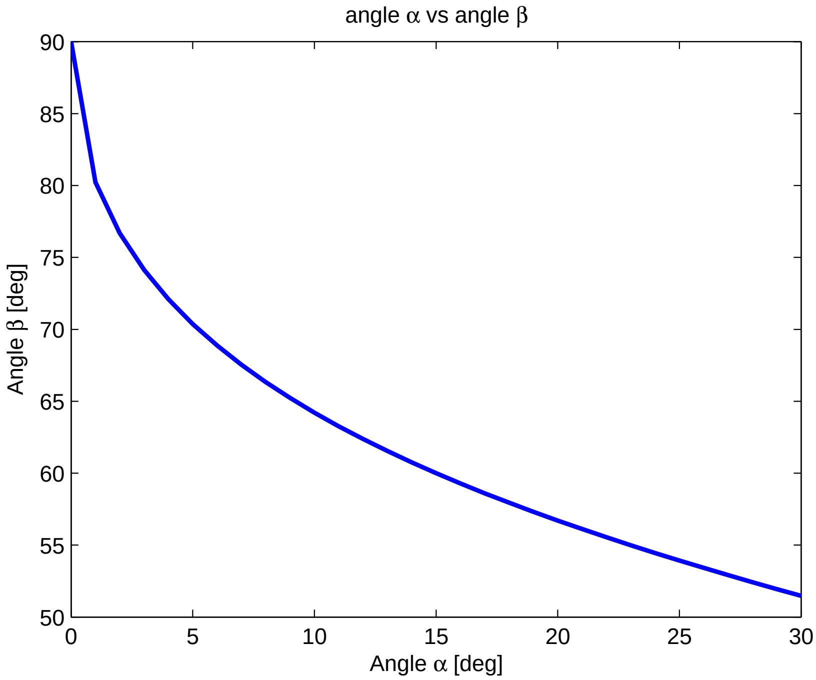

Figure B-2: Angle α versus angle β for which the platform L_1 is horizontal.

can be seen in figure B-3.

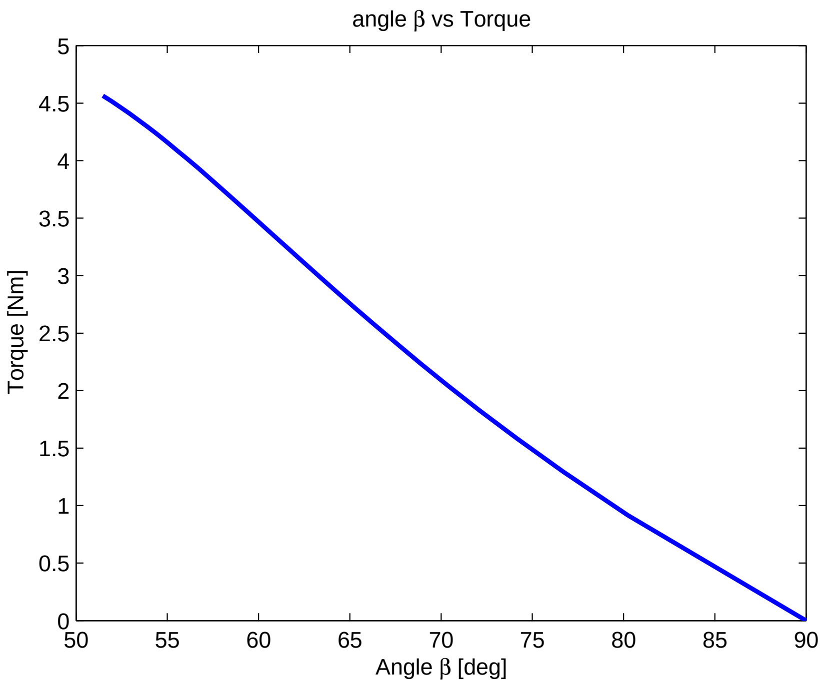

Figure B-3: Angle β vs the torque.

It can be seen that the relationship between angle β and the torque can be approximated to be linear. Assuming a linear torsion spring at the base of the inverted pendulum L_3, the stiffness of this spring has a value of:

$$K = \frac{\Delta x}{\Delta y} = \frac{4.565 - 0}{90 - 51.47} = 0.119[Nm/deg] = 6.79[Nm/rad] \tag{B-1}$$

Now that the value of the torsional spring at the base of the inverted pendulum L_3 is known, the potential energy of this spring for all combinations of α and β can be calculated:

$$E_{spring} = 0.5K\theta^2 \tag{B-2}$$

The gravitational potential energy can be calculated for all combinations of α and β:

$$E_{height} = 0.5mgL_3\sin(\beta - \alpha) \tag{B-3}$$

So the total potential energy of the system can be calculated as follows:

$$E_{tot} = E_{spring} + E_{height} \tag{B-4}$$

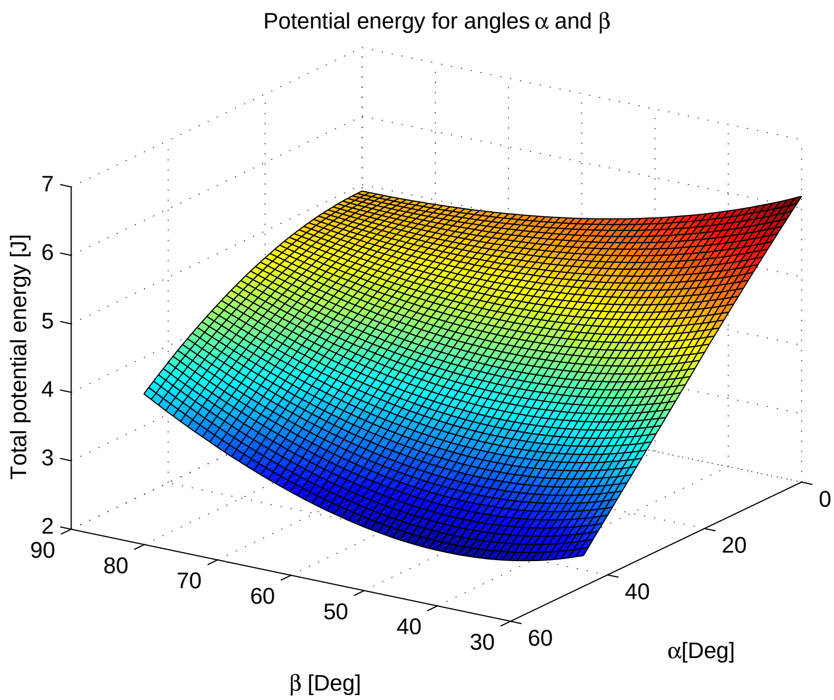

Figure B-4: Plot of the potential energy of L_3.

A 3d-plot of the potential energy as a function of α and β is plotted in figure B-4. With this plot it can be checked if the system behaves as intended. The pendulum will always move to the position with the least amount of potential energy, meaning that the pendulum will find a certain β (which is variable) for any given inclination α. So by finding the β with the lowest amount of potential energy for every α the equilibrium position of the inverted pendulum can be determined for every inclination. The path derived in this way is depicted in figureoptANDder, together with the optimal path of figure B-2. It can be seen that the two graphs have a similar shape, with a maximal difference of about 8.3 degree, which corresponds to a platform deviation of 1.7 degree relative to the horizon. (NOTE: This is not necessarily the position with the largest deviation from the horizon, it is just the position with the largest difference between the optimal and derived path!)Bear in mind that a linear estimation of K can be done more accurately, resulting in less deviation of the platform. This proves that this system can even work with an estimated linearization of the spring.

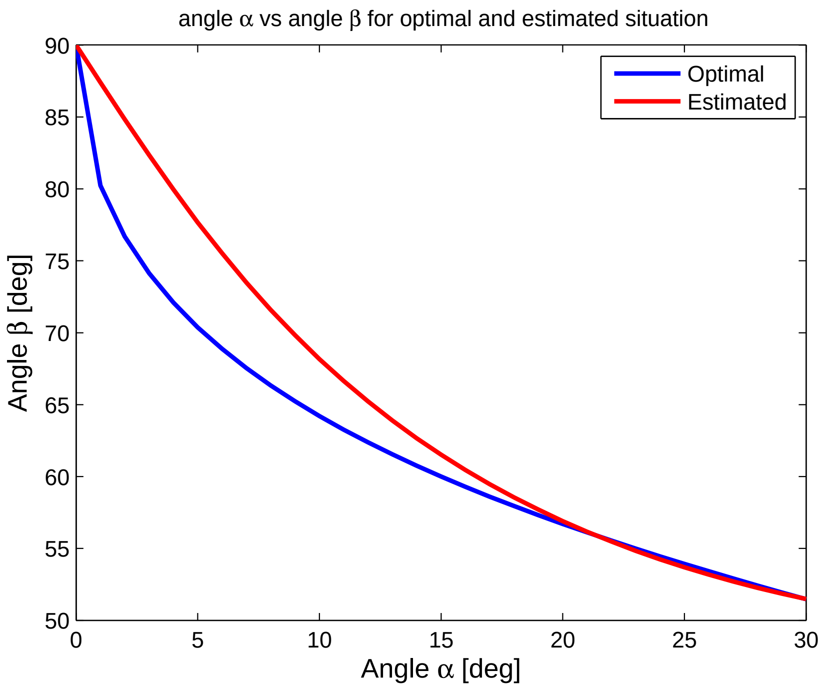

Figure B-5: Plot of the optimal and derived trajectory.

This concept has also been tested with a model made of LEGO®, which is depicted in figure B-6 and B-7.

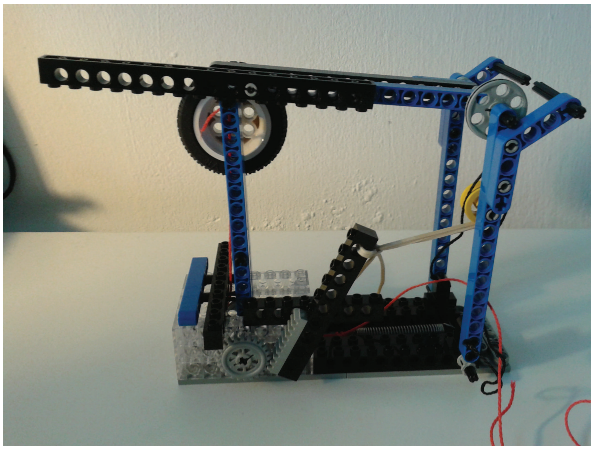

Figure B-6: Picture of the model made in LEGO®.

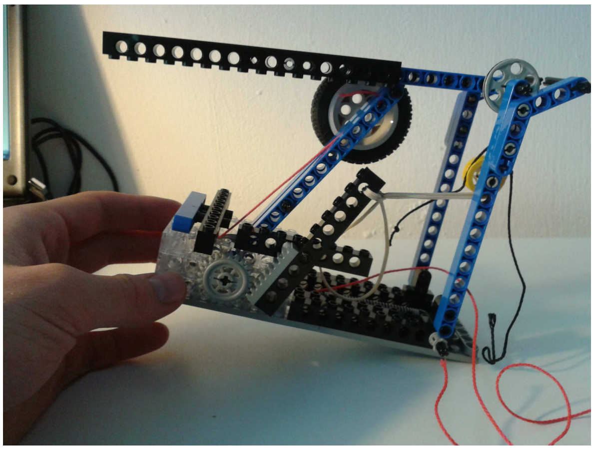

Figure B-7: Picture of the LEGO®model compensating for an inclination.

Appendix C

Influence of friction

In the previous appendix figure B-4 was shown with the minimal potential energy for various combinations of angle α and β. It can be seen that the values for the potential energy lie very close together, sometimes the difference is hardly distinguishable. If the values of the potential energy lie close to each other very little energy loss or addition is needed to change the equilibrium position of the pendulum. Since the device is passive, and since it was assumed that no disturbances occur, energy cannot be added. However, it is very likely that energy will be lost due to friction in the joint. To make sure that the system does not function incorrect due to this joint friction, some dynamic simulations were made using MSC Adams, which is a dynamics simulation software package.

The model that was made is depicted in fig C-1, and is a very simple representation of the concept. The lower pink bar represents the inclined underground and is not able to move. The blue bar is connected to this ground by means of a rotational joint for which the friction coefficients can be altered. A torsional spring is placed in this joint, which has the same values as the spring estimated in the previous appendix, which hass a stiffness $K = 6.79\frac{Nm}{rad}$, and a damping $c = 0\frac{Ns}{m}$. The inverted pendulum has the same properties as during the previous analysis, with a mass $m = 1kg$ and a length $L_3 = 1m$. All simulations were run over 300 seconds, with a total of 30000 steps (100 steps per second). This resulted in graphs like seen in figure C-2.

In table C-1 different values of the friction coefficients of the joint and the resulting angle of the pendulum are presented. It can be seen that the friction coefficients corresponding to wood on wood and steel on steel joint surfaces do not allow any real movement (β starts at 90 degree!). The friction coefficients corresponding to Teflon joint surfaces allow much more movement, and the value for β approximates the value of the MatLab situation much better, with an error of about 1.5 degree. Finally, it can be seen that the value of β corresponding to a situation with a ball bearing placed at the joint makes for a close match with the MatLab value (error of 0.24 degree). It can be concluded that accurate levelling is possible if the

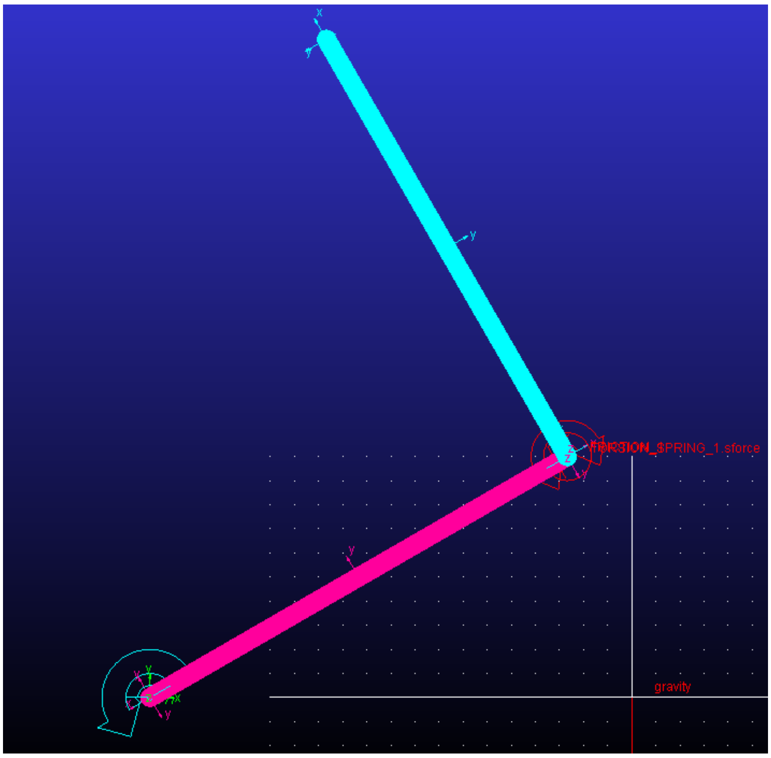

Figure C-1: Model made in MSC Adams.

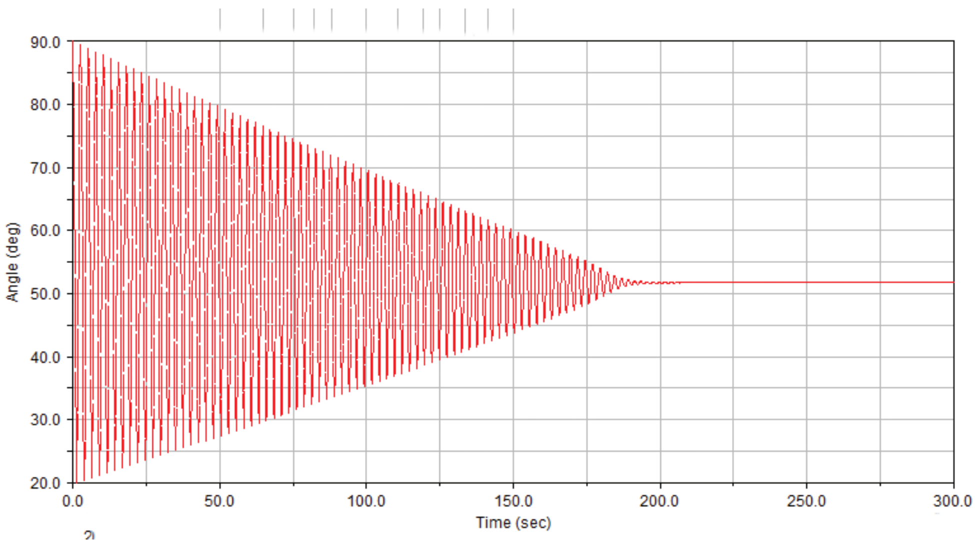

Figure C-2: Plot of angle β versus the time for a joint with a ball bearing

friction in the joint is low enough. Therefore the joint of the pendulum should be equipped with a ball bearing.

Situation	μ_s	μ_d	α [deg]	β [deg]
Matlab model:	-	-	30	51.49
Steel on Steel: [1]	0.6	0.5	30	89.87
Wood on wood:	0.5	0.3	30	89.86
Teflon on Teflon:	0.04	0.04	30	50.01
Ball bearing: [1]	0.0015	0.001	30	51.73

Table C-1: Values of β found for different friction coefficients.

Appendix D

Alternative concepts

For the system presented in appendix B it was assumed that L_1 and L_3 were of equal length. A drawback of this is that for this situation the system can only level in one direction. For a system with equal length of L_1 and L_3, one needs to make sure that the pendulum L_3 is always placed at the higher end of the inclination. If this requirement is not met it is impossible for the platform to be supported in a horizontal position. Several alternative systems that are able to level in two directions are discussed here.

D-1 Alternative 1: Increased pendulum length

The first alternative is the same system as presented in appendix B, only with an increased length of the pendulum L_3. By increasing the length of the pendulum it will become possible to level in two directions. A simple model is depicted in fig D-1. The relationship between α and β for different lengths of L_3 is depicted in fig D-2. It can be observed that the steepnes of the curve is largely dependent on the length of L3. The steeper the curve is, the less sensitive the system is to small errors in angle β, which can be caused by e.g. friction in the system. It can also be observed that the systems levelling behavior is different for the two directions of levelling, causing it to level more accurate in one of the two directions.

D-2 Alternative 2: Slider joint and spring

The second alternative works with a linear slider and a spring, as depicted in figure D-3. If this system is placed on an inclination the force compressing the spring changes, thereby allowing change of the lenght of the spring (and thus a change of L_3). The angle of the slider and spring relative to the base of the system is also of importance if a system that levels in two directions is required. In figure D-4 the relation between the length of L_3 and the ground inclination α are given for different values of ϕ, assuming the platform will always be in a

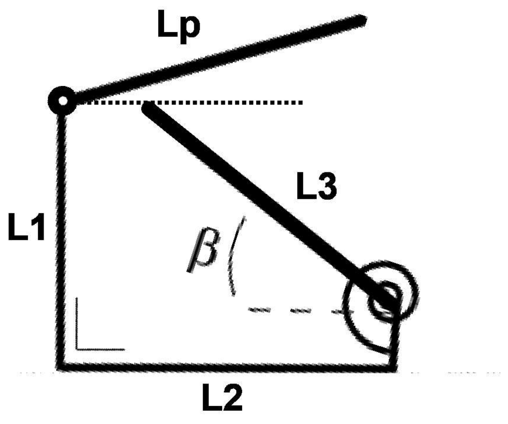

Figure D-1: Schematic overview of the alternative with an increased length of the pendulum.

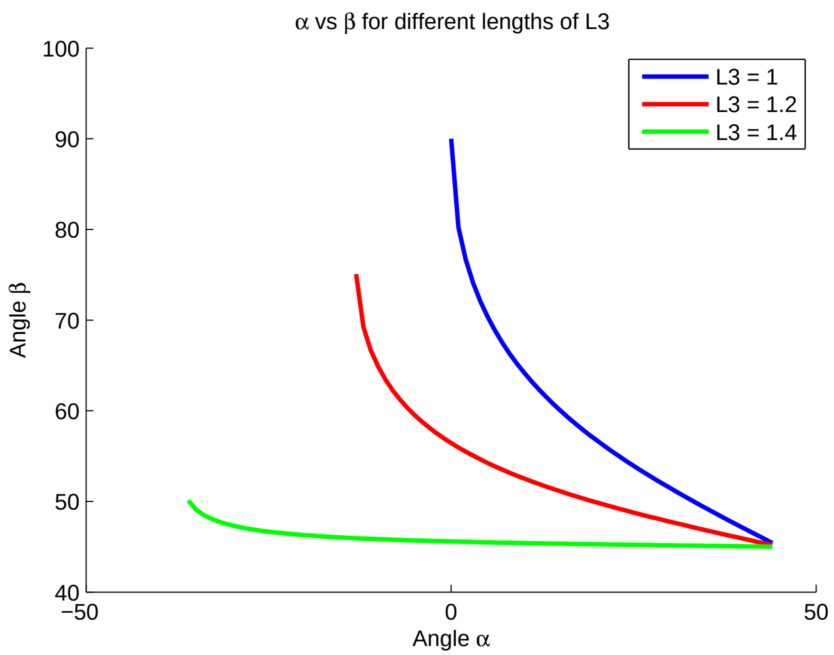

Figure D-2: Relation between α and β for different lengths of L3.

horizontal position. It can be seen that this system with a ϕ of 90 degree cannot level in two directions (the required spring elongation relative to the change in gravitational force is not possible). The system with a ϕ of 45 degree is the only system that can compensate over the full range of α (-30 till 30 degrees), but it can be observed that this requires a very nonlinear spring that can stretch to up to 5 times its initial length, which is hard to realize. Next to this, linear sliders have a high energy loss compared to rotational joints, which makes this system relatively inaccurate.

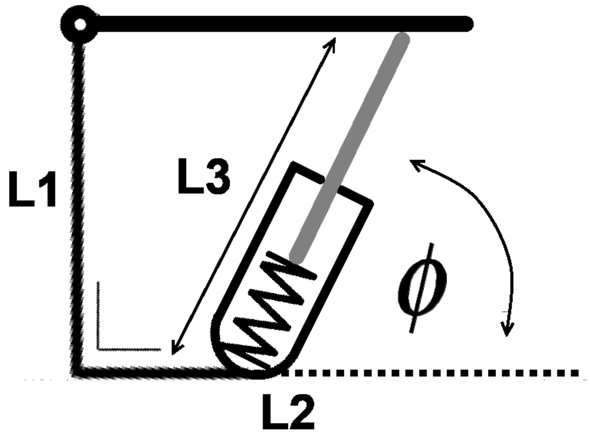

Figure D-3: Schematic overview of the 'slider joint and spring' system.

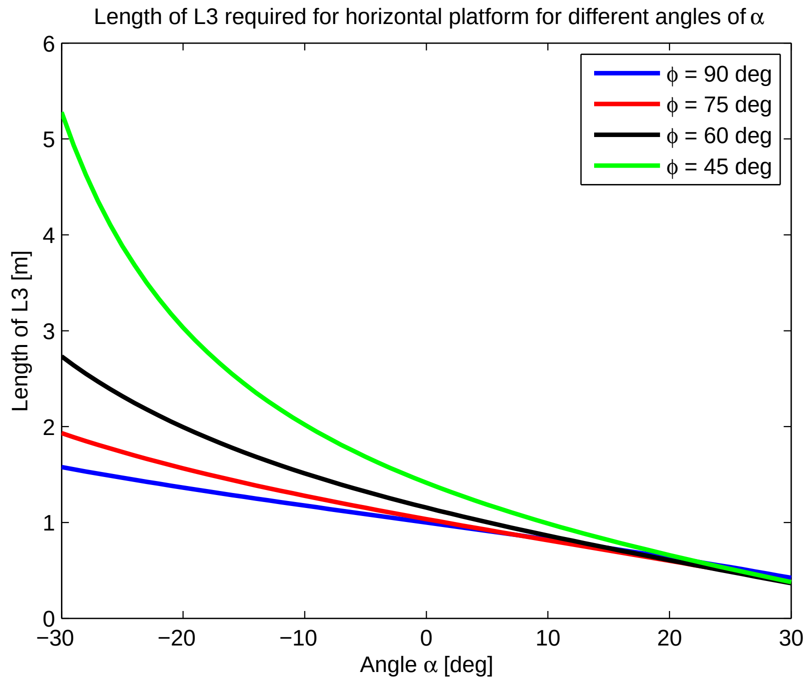

Figure D-4: Relation between α and β for different lengths of L_3

D-3 Alternative 3: Increase pendulum length

The third alternative works with a counter-weight that controls the position of an inverted pendulum L_5. A simplified model of this system can be seen in fig D-5. If proper lengths are chosen, this system should be able to orient pendulum L_5 in such a way that it has the same height as the end of L_1. In figure D-6 a plot is shown of the relation between the inclination α and the angle ϕ for different lengths of L_5. Angle α is also an indication of the angle over which the mass will move due to the shift in gravity. It can be seen that there is a non-linear relation between the movement of the weight and the movement of the pendulum. This would require a non-linear transmission between the weight and pendulum L_5. This transmission will cause loss of energy by itself and because of the additional joints required. This will have a negative influence on the accuracy of the system. Next to that, this system will be heavier than the other alternatives because it requires a counterweight to function properly.

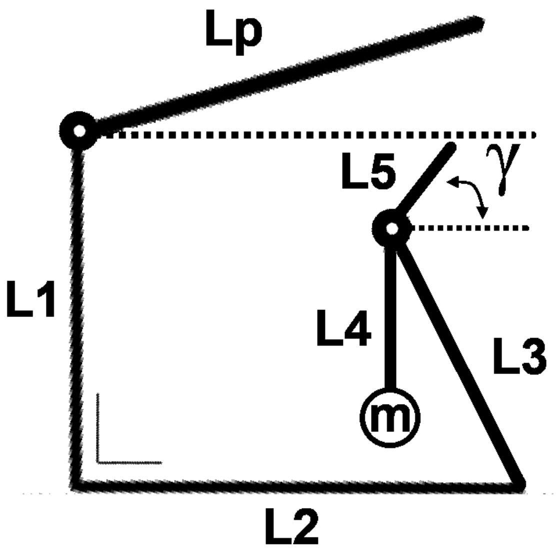

Figure D-5: Schematic overview of the contraweight system.

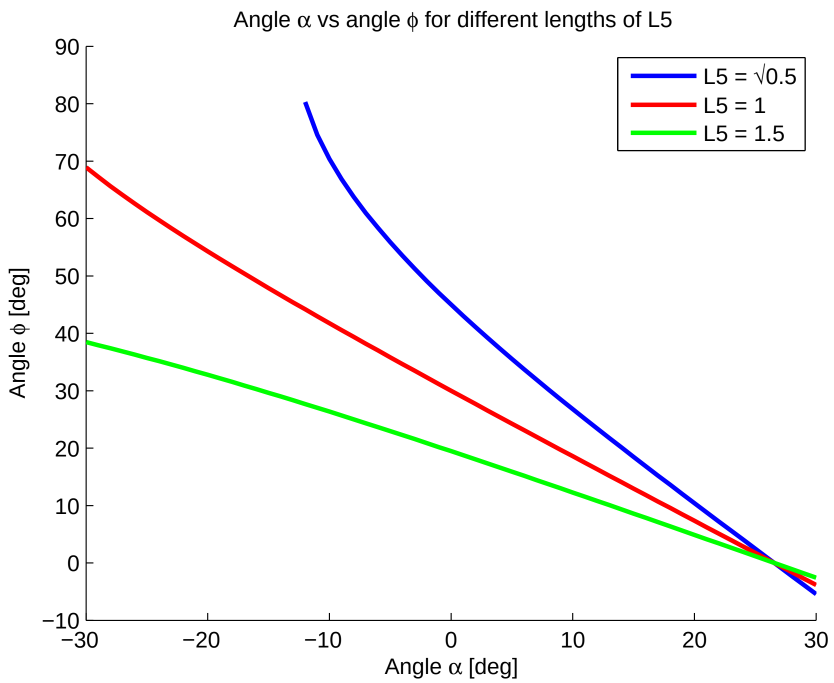

Figure D-6: Relation between α and β for different lengths of L3)

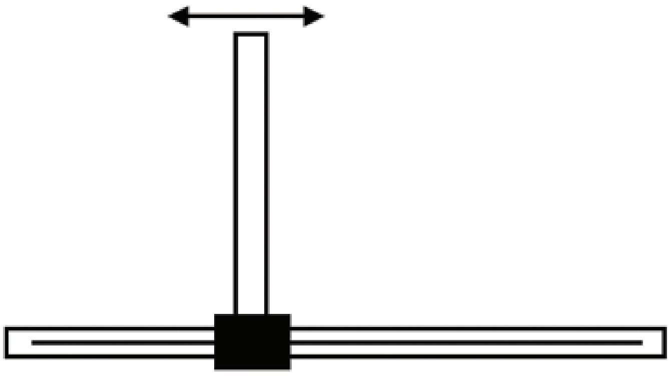

Figure D-7: Example of a movable backrest.

D-4 Alternative backrest

When designing a chair the main disadvantage of only being able to level in one direction is that the backrest can be placed on only one side relative to the inclination. This means you cannot sit facing uphill and downhill on the same inclination. Other than levelling in two directions, a possible solution for this problem might be to make an alternative backrest, that can switch between the two positions. An example of such a backrests can be seen in fig D-7. No research was done regarding alternative backrests.

D-5 Conclusion

When looking at the different alternatives it can be seen that the counterweight system and slider-spring system have some clear disadvantages. The counterweight system needs an extra weight which increases the total mass, and also requires gears which results in extra friction. The linear slider system also suffers from friction and requires a complicated spring. Moreover, these systems do not level equally well in both directions. This is also the case for the 'increased pendulum length' alternative. Even though this variant of the original system can level in two directions, it does not level in both directions equally well and it has a lower sensitivity for small inclinations. Since we want to especially level accurately for small inclinations, the original system was favored.

Appendix E

Choosing the dimensions

Now that the concept has been chosen the dimensions of the device have to be estimated. Proper dimensioning is important because the chosen dimensions will all be used for further calculations. In order to get a good idea of proper dimensions, it is best to look at some already commercialized products. Since the device that is designed has some similarities in intended usage with foldable chairs, these will be used as a reference to estimate proper dimensions. In the table below an overview is given of some foldable chairs and their respective characteristics:

Name:	**Height [cm]:**	**Width [cm]:**	**Depth [cm]:**	**Weight [kg]:**	**Load[kg]:**
Crespo 215-23	44	48	40	3	110
Helinox Chair One	35	52	50	0.85	145
Hypercamp Beach	25	44	44		100
Umefa Dynamic	45	47			
Katsura Forest	39			2.9	100

Table E-1: Overview of commercial foldable chairs and their respective properties.

Based on the findings as shown in the table and personal feeling, the following dimensions were chosen.

Height: 40 centimeters

Width: 45 centimeters

Depth: 45 centimeters

The height is the height at which the platform will be assuming it is in a horizontal position. The width is the distance from the base of the pendulum to the base of L_1. The depth is the width of the platform observed from an occupants perspective. These values will be used as reference values for future calculations.

Appendix F

Choosing a lock

Now the concept has been chosen, there still is need for a locking mechanism. This locking mechanism is needed to keep the inverted pendulum in position after the platform is placed on it. Several different locking mechanisms were considered, some of which are discussed here.

F-1 Differential belt brake

One locking mechanism that could be used is a differential band brake, which is depicted in figure F-1. By applying a force F on the right end of lever a the belt is pulled against the pulley with diameter r. The belt will be stretched in the clockwise direction, thereby increasing the tension T_1, while simultaneously lowering tension T_2. This results in an equilibrium between the torques caused by T_1, T_2 and the torque resulting in the rotation n. The advantage of this system is that only a small force F_i is needed to brake systems that exert large torques due to the self-locking properties of the differential band brake. This system can only brake in one direction and will be freewheeling in the other direction. Disadvantages of this system are that for large torques large tensions T_1 and T_2 are needed to brake the system. This does not only result in large forces on the axis, but this also calls for a strong belt, big radius r, and wide pulley.

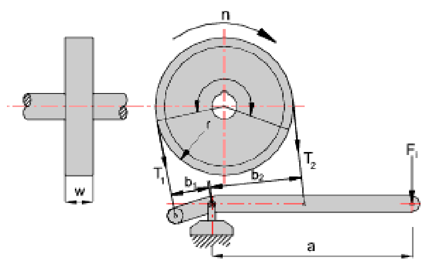

Figure F-1: Differential band brake. (Source: [2])

F-2 Gear on pendulum

One possible solution is based on (part of) a gear that is attached to the inverted pendulum, which can be locked by the downward movement of the platform. A simplified overview of the working principle can be seen in figure F-2. After the pendulum has found its equilibrium position the platform will be lowered, thereby pulling the wire which is attached to the end of the lever. The other side of the lever has a rack attached to it, which will now be pressed against the gear. A spring is placed between the cables to allow all rotations of the platform, regardless of the inclination of the underground.

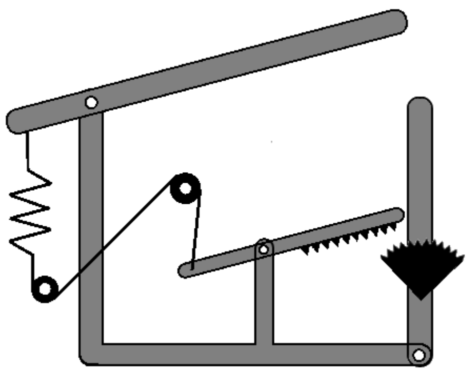

Figure F-2: Lock with a (partial) gear on the pendulum.

F-3 Gear on pendulum & rack on platform

Another possible solution is to attach a gear at the end of the inverted pendulum, and a rack to the bottom of the platform, as depicted in figure F-3. If the platform is lowered after the pendulum has found its correct position the gear and the rack will make contact. Since the gear is fixed it cannot rotate, this causes a singularity. This singularity prevents movement of the pendulum and obstructs any downward motion of the platform. The advantage of this way of locking is that the forces on the teeth are smaller than for the previously discussed lock. This lock is also easier to integrate within the current system, without needing any extra elements; the fixed gear can be placed at the end of the pendulum, while the rack can be placed under the platform.

To make sure that the gear does not deform/fail, some calculations need to be made. According to [3] the bending load on a single tooth of a gear in static situations can be calculated as follows:

$$\theta_b = \frac{F_c}{bmY}. \tag{F-1}$$

Where b is the tooth width in mm, m the module of the gear, Y the Lewis factor and F_c the force on a single tooth. Using the lengths which were presented in the previous appendix, assuming a load of 100kg in the middle of the platform and compensation for an inclination of 30 degrees, it can be calculated that for a gear with a reference diameter of 50mm, a module of 1 and a tooth width of 35mm, the $\theta_b = 380MPa$. According to [4] the allowable bending

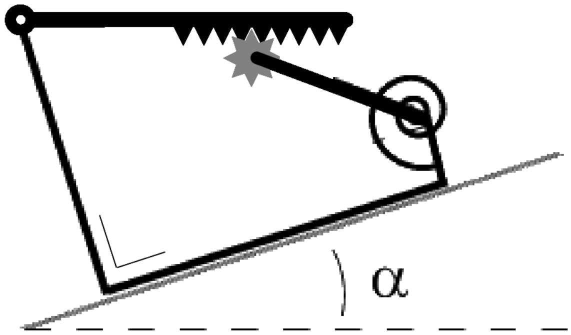

Figure F-3: Locking mechanism with a gear and rack.

strength can be estimated at 1/3 of the ultimate tensile strength of a material. The ultimate tensile strength of Hardened tool steel 45NiCrMo16 (ISO 1.2767) lies around 1500MPa, which means the allowable bending strength lies around 500 MPa. This means that it is possible to implement this system.

Conclusion

Of the locks discussed, the system with the fixed gear and rack is clearly the easiest to implement in the current system. It does not require many additional elements, and it also requires the least amount of space. Also, the load on a single tooth is much smaller for this system than for the other lock using a gear. Therefore, it was decided to implement this lock in the system.

Appendix G

ANSYS Analysis

To make sure the chosen design is able to comply with the requirements stated earlier an analysis using the ANSYS software is made. This analysis has to show if the maximum load specified earlier can be exerted on the platform without resulting in failure of the platform. A schematic overview of the device that will be tested can be seen in figure G-2
For this analysis simplified models of the individual parts of the platform were tested. This was done because it saved a lot of time, while still offering an overview of the behavior of the system under big loads. Keep in mind that this analysis is done just to get an estimate of the forces and loads on the system, and is not intended for optimization purposes.

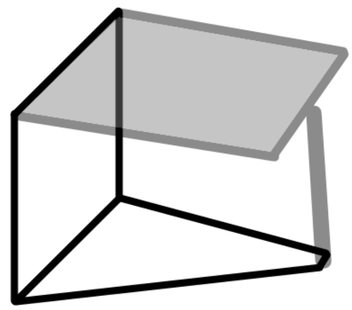

Figure G-1: Schematic overview of the design that was analyzed.

The following settings were used in ANSYS for all elements:

Element type → Beam 188 2 node
E-modulus → 70Gpa
Poison Ratio → 0
Material model → Linear, Elastic, Isotropic
Mesh → 10 elements per bar

The cross section of the 0.4m long pendulumn was modelled as follows:

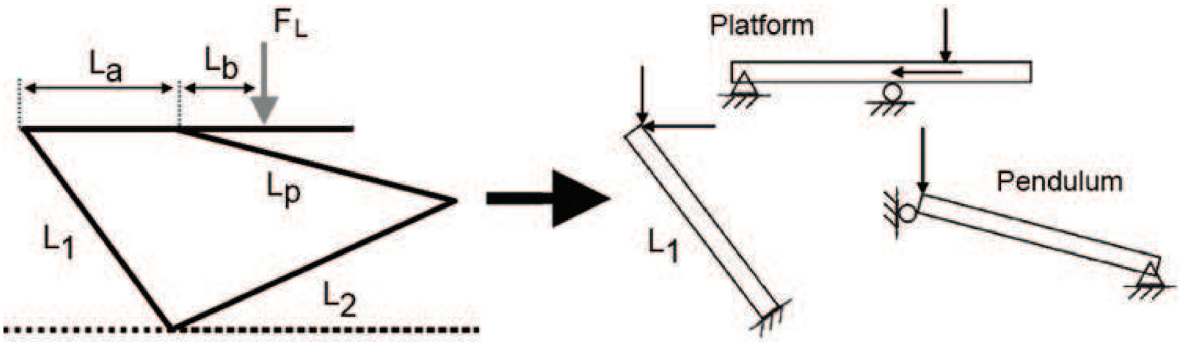

Figure G-2: Simplified model of the system that was analyzed in MatLab. All elements have been simplified as beams with one or multiple supports.

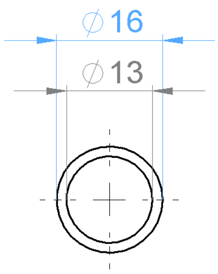

Figure G-3: Cross-section of the pendulum. (Distance in mm)

The bars of the platform and L_1 were modeled as square beams, and were always oriented in such a way that they provided most resistance to the bending moment:

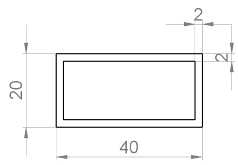

Figure G-4: Cross-section of the square beam.

The maximal allowable load (100kg) is estimated to be applied at 10 centimeters from the right edge of the platform, while the system compensates for an inclination of 30 degrees. This results in the vertical force applied at the end of the inverted pendulum:

$$F = \frac{(L_a + L_b)F_L}{L_a} \left(= 1.646 * 10^3[N]\right) \tag{G-1}$$

When simulating the vertical force on the inverted pendulum in Ansys, this resulted in the deformation as seen in figure G-5, with a maximal deviation of 1.495mm. The maximal value of the bending stress according to Ansys was approximately 0 Pa, while the normal stress throughout the pendulum turned out to be $-79.4MPa$. According to [5], the maximal bending stress for an Aluminium alloy is about $69MPa$, while for steel (1020) the allowable bending stress is about $180MPa$. Therefore it was decided to change the material of the pendulum into steel. According to Ansys the pendulum also exerted a force in the horizontal direction on the platform, with a force of $-5.167kN$.

Figure G-5: Deformation of the inverted pendulum after applying a vertical load of $1.646kN$.

On the platform two different forces are applied. One force is caused by the load of the user, which is set at the right side of the platform. The other force is the horizontal force caused by the pendulum. Since the platform will consist out of two bars and the Ansys model only out a single one, half of these forces is applied. The resulting deformation of the platform can be seen in figure G-6, with a maximal deviation of $0.7mm$. The maximal bending stress is $28.1MPa$ and the normal stress is $11.5MPa$. The resultant forces on the fixture on the left side of the platform are $332.49N$ in the vertical, and $-2583.5N$ in the horizontal direction.

The forces on the fixture on the left side of the platform are the same forces that work on the top of bar L_1. Bar L_1 was originally modeled as a single beam which lead to the bending stresses as seen in figure G-7. It can be seen that the maximal bending stress on the beam has a value of $411MPa$, which is far beyond the maximal bending stress of aluminium, which is about $69MPa$ [5]. Therefore, it was decided to put an extra supporting bar. Trial and error was used until the bending stress had reduced to an acceptable level. The final result of this can be seen in figure G-8 , where the maximal bending stress has been reduced to $21.6MPa$. Since these supporting beams are needed for a properly working system, they are also implemented in the final design.

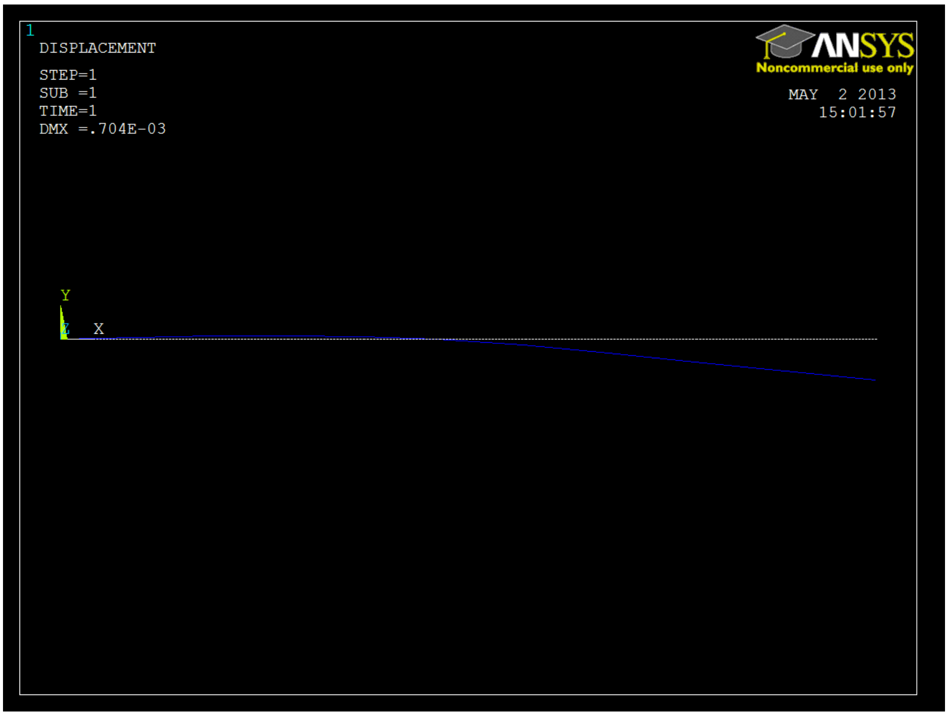

Figure G-6: Deformation of the platform after applying the vertical load of $100kg$ and a horizontal force of $-5.167kN$

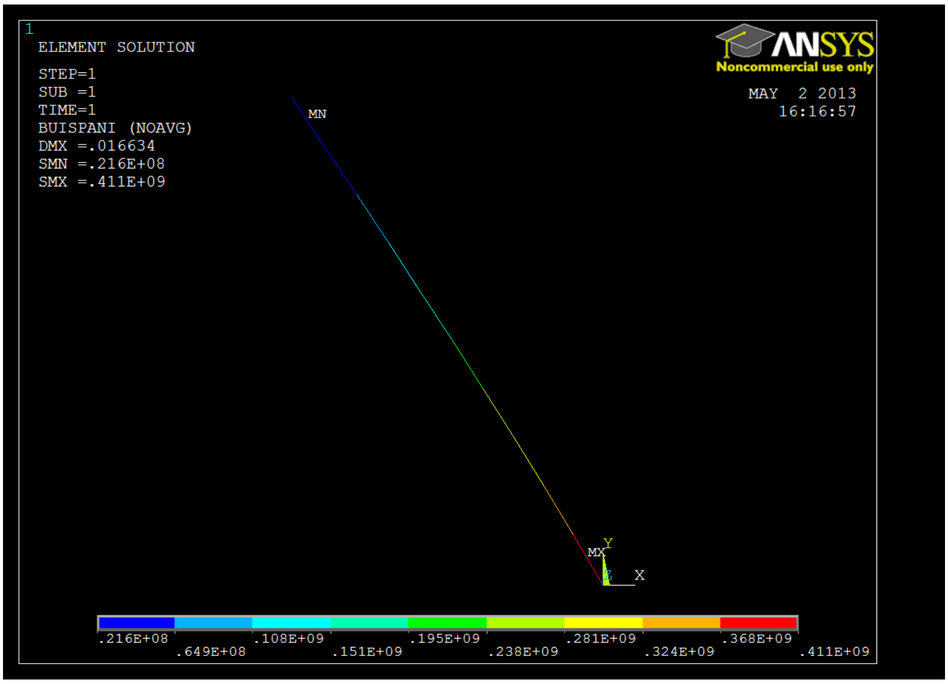

Figure G-7: Bending stresses in L_1 after applying a horizontal and vertical force caused by the loads on the platform.

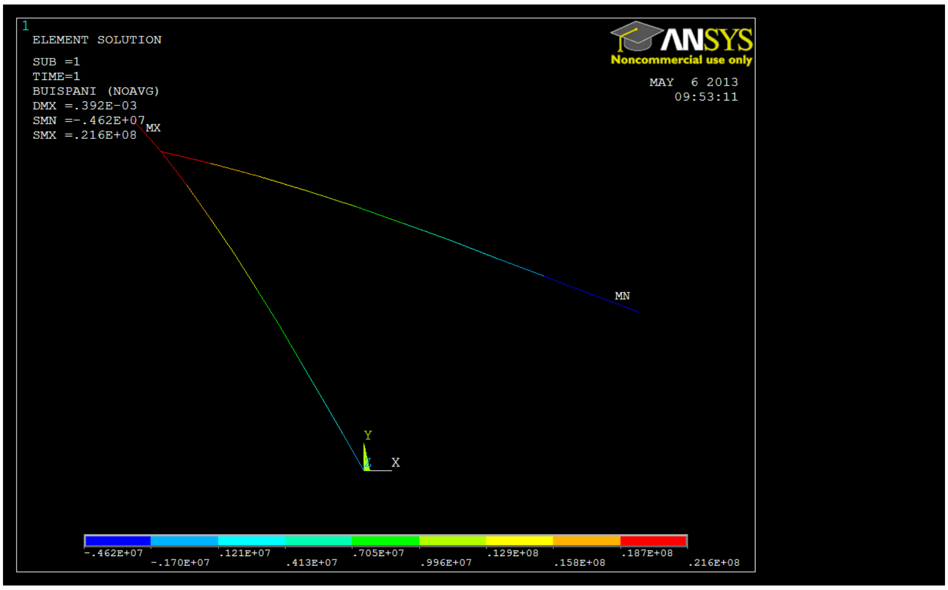

Figure G-8: Bending stresses in L_1 after adding an extra support.

Appendix H

Gear module

Different sizes of the gear and rack will have a different influence on the accuracy of the system. The larger the teeth of a gear and rack, the larger the average deviation from the wanted value will be. In some situations a tooth skip might occur, which can result in relatively large deviations of the platform relative to the horizontal. A computer model was made with which the influence of a single tooth skip on the levelling accuracy of the platform can be tested for different sizes of the teeth. Figure H-1 shows a simplified model based on which the difference in γ for a change in teeth can be described. In this model L_{pf} is the length of the platform, L_g the distance from the joint of the platform to the gear attached to the end of the pendulum, which is denoted by L_p, and w_t is the width of a single tooth.

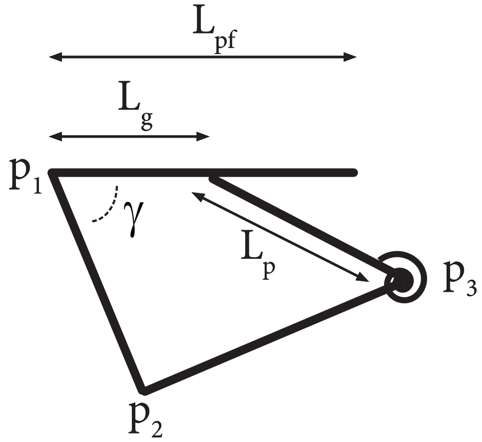

Figure H-1: Simplified overview of system.

The difference in γ for a single tooth increment (from $i-1$ to i) can be calculated as follows:

$$L_d = \sqrt{(p_{3x} - p_{1x})^2 + (p_{1y} - p_{3y})^2} \tag{H-1}$$

$$L_{g1} = L_{pf} - w_t(i-1) \tag{H-2}$$

$$\gamma_1 = cos(\frac{L_p^2 - L_{g1}^2 - L_d^2}{-2L_g L_d}) \tag{H-3}$$

$$L_{g2} = L_{pf} - w_t(i) \tag{H-4}$$

$$\gamma_2 = cos(\frac{L_p^2 - L_{g2}^2 - L_d^2}{-2L_g L_d}) \tag{H-5}$$

$$\Delta\gamma = \gamma_2 - \gamma_1 \tag{H-6}$$

This process was repeated until γ_2 reached, or was bigger than, the value at which the platform can perfectly compensate for an 30 degree inclination of the ground.

The width of a tooth can be approximated if the module (m) or the reference diameter (d) and the number of teeth (z) of a gear are known. If the module is not known it can be calculated as follows:

$$m = \frac{d}{z} \tag{H-7}$$

After which the reference pitch (p, clarified in figure H-2), which is an estimation of the tooth width in mm can be calculated:

$$p = \pi m \tag{H-8}$$

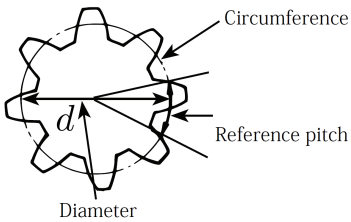

Figure H-2: Reference pitch of a gear. (Source: [6])

The effect of different modules on the difference in γ for a single tooth skip is depicted in figure H-3. From this figure it can be seen that smaller modules decrease the influence of a single tooth skip on the levelling accuracy. However, a smaller module also increases the bending stress on the gears teeth, thereby decreasing the maximal applicable force on the platform.

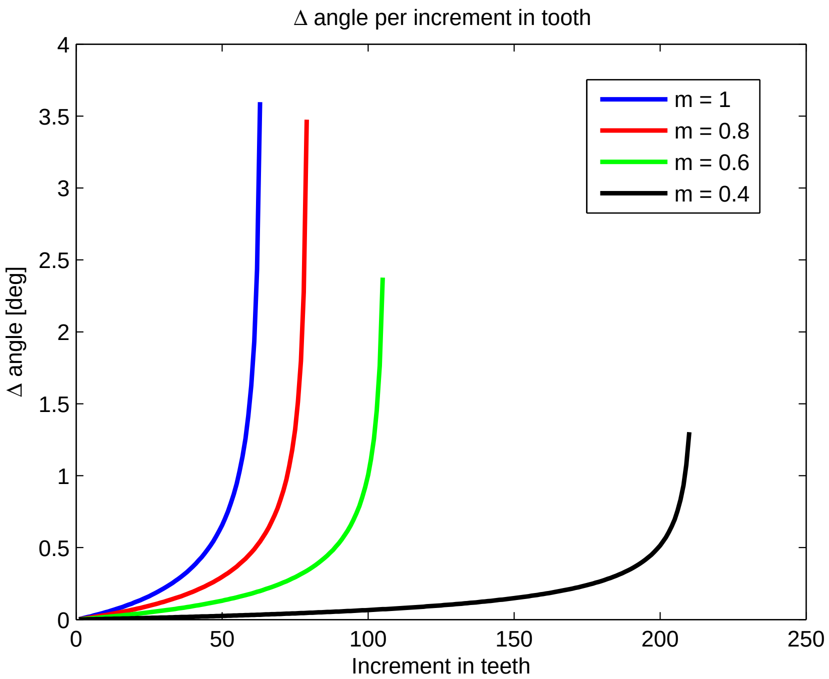

Figure H-3: $\Delta\gamma$ for different modules of the gear and rack.

Appendix I

Dynamic analysis

Since the chosen concepts uses a spring but also requires minimal friction, there is the chance that the time before the pendulum reaches its equilibrium position takes longer than the set maximum of 5 seconds. The following calculations were made to check if the system can reach its equilibrium position within the set time.

Figure I-2 presents a schematic overview of the system that will be analyzed. Here α has a constant value and β (not depicted in the figure) is a constant that represents the angle between the ground and the starting position of the pendulum. It is assumed that the spring also acts as a damper for the system. The torques working around the rotational point of the pulley are:

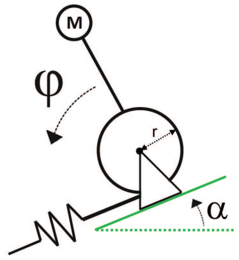

Figure I-1: Schematic overview of the system.

$$T_1 = mgl\cos(\beta - \alpha - \phi) \tag{I-1}$$

$$T_2 = -kxr \tag{I-2}$$

$$T_3 = -c\dot{x}r \tag{I-3}$$

Where the spring elongation x and its derivative $\dot{x}$ are both dependent on ϕ as follows:

$$x = \phi r \tag{I-5}$$

$$\dot{x} = \dot{\phi} r \tag{I-6}$$

It is generally known that:

$$\sum M = I\ddot{\phi} \tag{I-7}$$

Which leads to:

$$I\ddot{\phi} = mgL\cos(\alpha + \beta - \phi) - kxr - c\dot{x}r = mgL\cos(\alpha + \beta - \phi) - \phi kr^2 - \dot{\phi}cr^2 \tag{I-8}$$

This can be linearised $(cos(\phi) = 1))$ and written in the following form:

$$I\ddot{\phi} + \dot{\phi}cr^2 + \phi kr^2 = mgl\cos(\alpha + \beta) \tag{I-9}$$

The left part of equation I-9 must satisfy the form of the second order differential equation

$$\frac{d^2\phi}{dt^2} + 2\zeta\omega_0\frac{d\phi}{dt} + \omega_0^2\phi = T_p \tag{I-10}$$

Where T_p is a prescribed torque. From this it follows that:

$$\omega_0 = \sqrt{\frac{kr^2}{I}} \tag{I-11}$$

The relative damping ζ depends on the material and geometry of the spring. The damping coefficient c of this system can now be determined:

$$c = \frac{2\zeta\omega_0 I}{r^2} = \frac{2\zeta\sqrt{kI}}{r} \tag{I-12}$$

Now that the damping coefficient c is known, the time it takes for the pendulum to come to a rest can be estimated. This was done using MatLab, in which the differential equation solver ODE45 was used. This was done for the non-linearised differential equation I-8, using the estimated properties m = 0.2[kg], l = 0.3[m],g = 9.81[m/s^2], which are estimated based on the SolidWorks model. The spring stiffnes was estimated at k = 1250[N/m], and $\zeta = 0.01$ which is the viscous damping coefficient of a metal in the elastic range according to [7]. This resulted in the oscillatory behaviour seen in figure I-2:

The reduction of the amplitude over time can be found using the following formula:

$$\phi(t) = e^{-\omega_0\zeta t} \tag{I-13}$$

From which it follows that it takes 54 seconds for the amplitude of the system to reduce to 0.1 degrees. (Which is about 0.27% of its original value). According to the ODE simulation

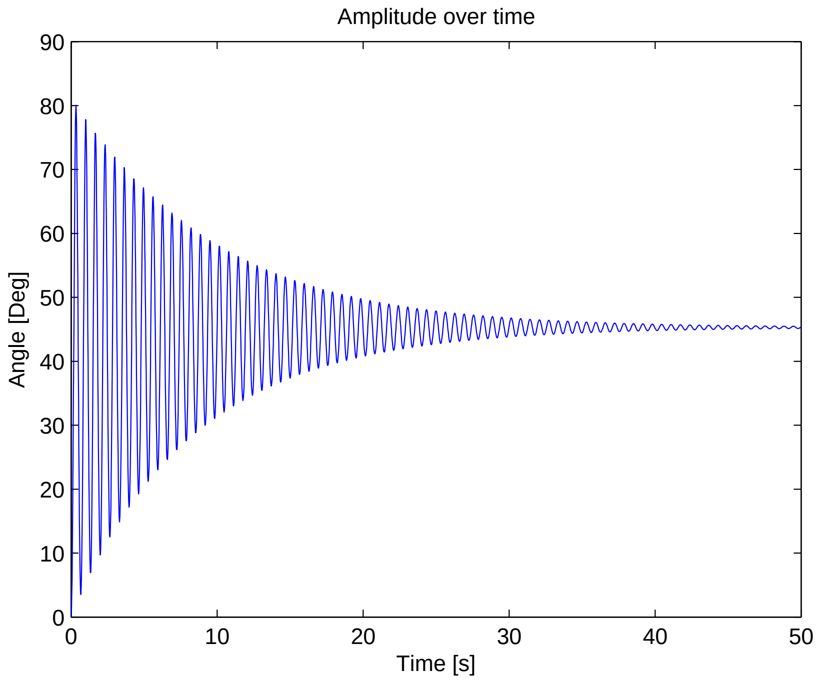

Figure I-2: Oscillatory behaviour of the inverted pendulum.

this takes around 54.94 seconds. These values correspond well.

The requirements stated that it should be able to set up the system in 5 seconds. This means the pendulum has to come to a rest within these 5 seconds. This means that according to this simulation the inverted pendulum will take about 11 times too much time to come to its equilibrium position. In the prototype the ball bearing, cable friction and air resistance will also damp the system, which will reduce the settling time of the pendulum. Extra damping might still be necessary, in which case a viscous damper can be added. Viscous damping only influences the time to reach an equilibrium, without influencing the equilibrium position of the system. Since the settling time is small enough to perform multiple tests within reasonable time, no work is done on adding a damper to the current system.

Appendix J

The prototype

Figure J-1: The self-levelling device before levelling can be seen here. The inclination is simulated by means of a screw thread that is attached to the left side of the platform. If the platform is raised, the lock is released and the pendulum will find an equilibrium position after which the platform can be lowered to a horizontal position.

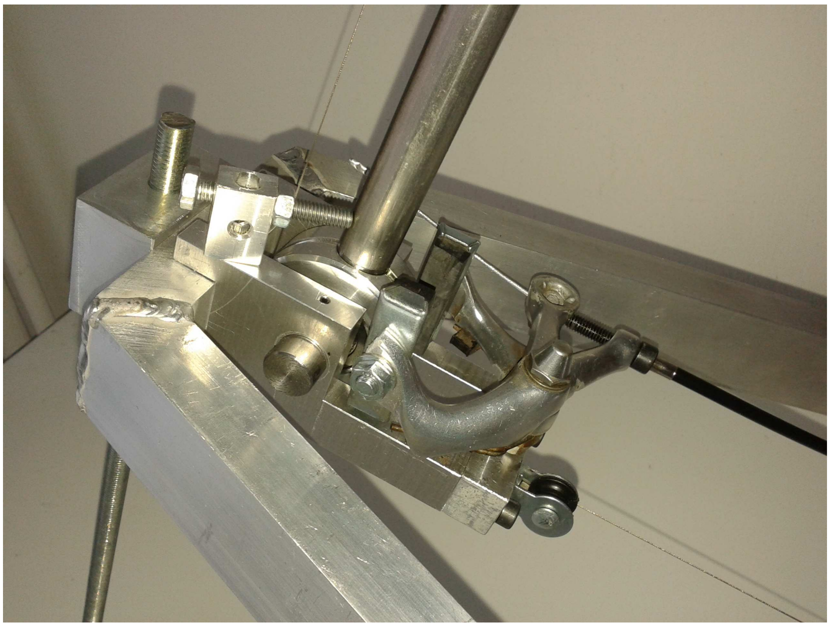

Figure J-2: A close up of the base of the pendulum. The pendulum is attached to a bearing case. The brake, which is a normal bicycle break, is activated when the platform is lowered, after which it grips the sides of the bearing case.

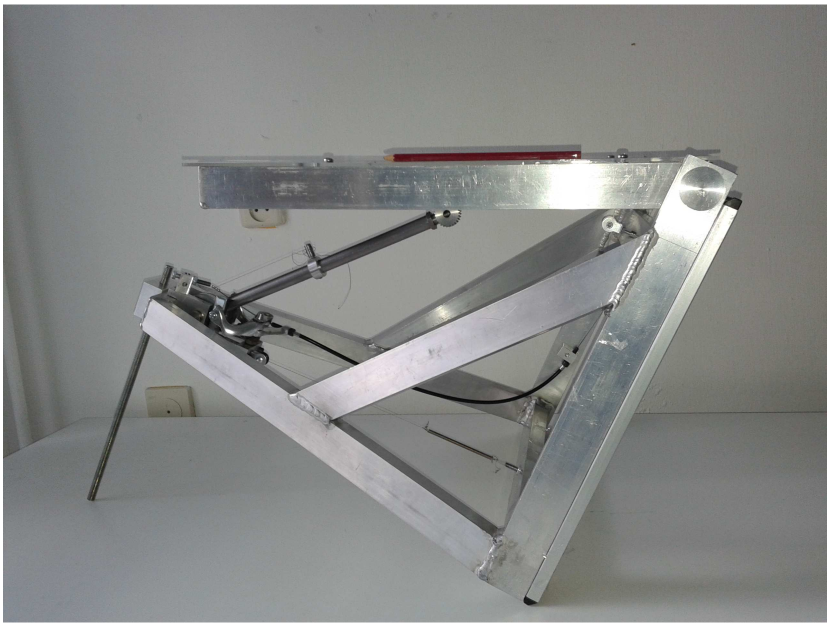

Figure J-3: The self-levelling device after levelling. It can be clearly seen that the pendulum has shifted its position relative to figure J-1. Here the device has a (close to) horizontal platform, while the platform initially had a deviation relative to the horizon.

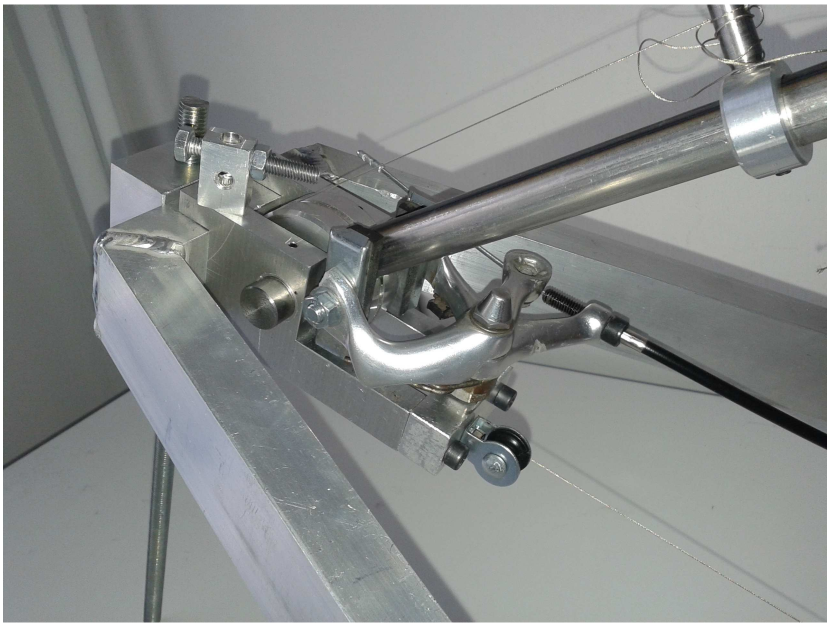

Figure J-4: A close up of the base of the pendulum for its shifted position.

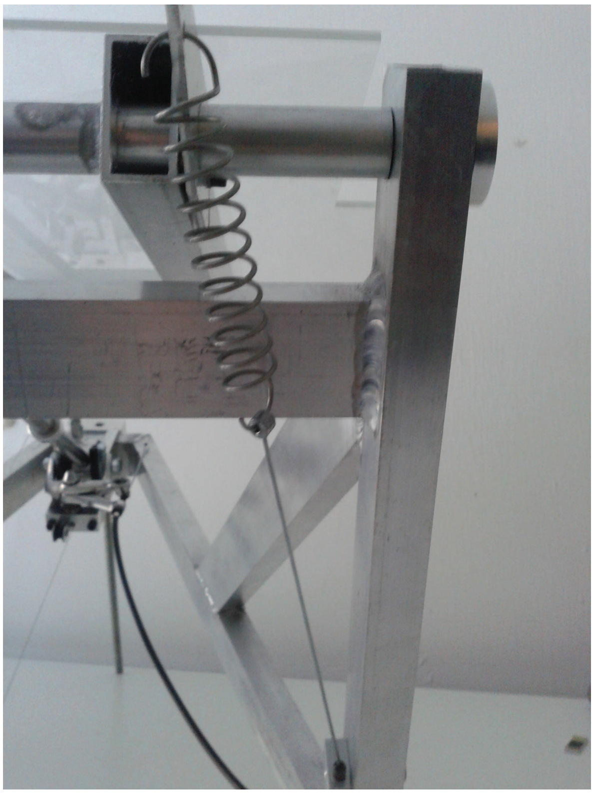

Figure J-5: Here a close-up of the pendulum locking mechanism can be seen. The purpose of this lock is to prevent large movements of the pendulum at the moment of contact with the platform. Small movements can be made, since the gears do not make proper contact for every position of the pendulum. If the platform is moved towards the horizontal the spring is stretched and a tension is applied on the cable. This cable is connected to a regular bicycle brake, that grips on to the bearing case to which the pendulum is attached.

Appendix K

The model

The values found during the experiments were compared to values based on a computer model. Here that model will be presented.

Figure K-1 depicts an overview of the system that is used for making the final model. Figure K-2 is the actual prototype with clarifications.

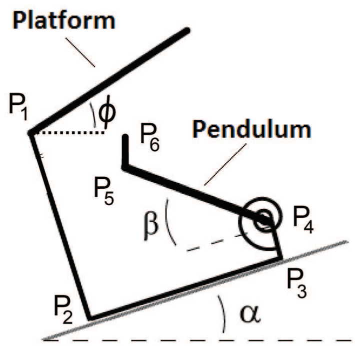

Figure K-1: Schematic overview of the system used for the final model.

First we try to find a minimal value for the sum of the moments.

$$\min\{mgL_4\cos(\beta-\alpha)-(3+kx)r\} \tag{K-1}$$

Where the value of x is given by:

$$x=\frac{90-\beta}{180}\pi r \tag{K-2}$$

For every $\alpha \in \{0, 0.01, .., 30\}$ we look for a $\beta \in [0, 90]$. In equation K-1 m is the mass of the pendulum measured at length L_4 for the horizontal position of the pendulum. g is the gravitational constant, k the spring constant and r the radius of the pulley. The 3 in equation K-1 denotes the pretension of the spring (in N).

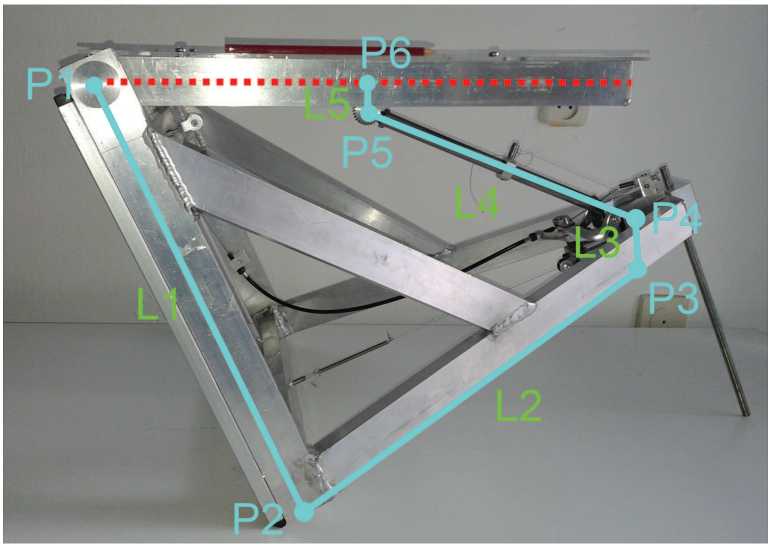

Figure K-2: Schematic overview presented on top of the actual prototype. (Schematic overview in 2D).

This way for every angle of α a corresponding β can be found. Now the positions of P_1 till P_6 can be calculated. First the origin is assumed at P_2.

$$p2x = 0 \tag{K-3}$$

$$p2y = 0 \tag{K-4}$$

From P_2, the positions of P_1, P_2 and P_3 can be calculated for any value of α :

$$P_{1x} = P_{2x} - L_1 \sin(\alpha) \tag{K-5}$$

$$P_{1y} = P_{2y} + L_1 \cos(\alpha) \tag{K-6}$$

$$P_{3x} = P_{2x} + L_2 \cos(\alpha) \tag{K-7}$$

$$P_{3y} = P_{2y} + L_2 \sin(\alpha) \tag{K-8}$$

$$P_{4x} = P_{3x} - L_3 \sin(\alpha) \tag{K-9}$$

$$P_{4y} = P_{3y} + L_3 \cos(\alpha) \tag{K-10}$$

From P_4 the position of P_5 can be calculated using β.

$$P_{5x} = P_{4x} - L_4 \cos(\beta - \alpha) \tag{K-11}$$

$$P_{5y} = P_{4y} + L_4 \sin(\beta - \alpha) \tag{K-12}$$

Now that P_5 is known the position of P_6 can be determined. Since a gear is attached to the end of the pendulum the platform always will make contact with the highest point on this gear. Since the gear is round P_5 was taken as its center position, and L_6 is the distance from the center of the gear to the centerline of the platform (depicted in red dots in figure K-2.

$$P_{6x} = P_{5x} \tag{K-13}$$

$$P_{6y} = P_{6y} + L_5 \tag{K-14}$$

Now that both point P_1 and point P_6 are known the inclination of the platform can be determined according to:

$$\phi = \arctan(\frac{p_{6y} - p_{1y}}{p_{6x} - p_{1x}}). \tag{K-15}$$

The values in table K-1 are used for this model:

L_1	$0.38[m]$
L_2	$0.42[m]$
L_3	$0.03[m]$
L_4	$0.315[m]$
L_5	$0.035[m]$
m	$0.1976[kg]$
r	$0.024[m]$
k	$1218[N/m]$
g	$9.81[m/s^2]$

Table K-1: Values used in the model.

Appendix L

Raw measurement data

Here the raw measurement data is presented. First the angle between L_1 and the platform for various angles of α is given.

α	angle	angle	angle	angle	angle	angle	angle	angle	angle	angle	angle
0	89.9	90.0	89.9	89.9	89.8	89.8	90.0	89.7	90.0	89.8	89.8
5	89.8	89.8	90.0	89.9	89.8	89.8	90.0	90.0	90.1	89.8	90.0
10	86.9	86.7	86.7	86.8	86.7	86.7	86.7	86.9	86.8	86.8	86.9
15	81.7	81.6	81.7	81.2	81.1	80.9	81.3	81.3	81.1	81.3	81.0
20	74.8	74.9	74.8	74.9	73.9	73.8	73.8	73.9	74.1	73.7	74.0
25	70.0	70.4	70.3	70.3	70.4	70.0	70.1	70.3	70.2	68.6	68.6
30a	66.6	66.8	66.6	66.7	66.8	66.9	66.8	66.6	66.8	66.7	67.0
30b	68.7	68.8	68.5	68.6	68.5	68.5	68.6	68.5	68.8	68.8	68.9

Table L-1: Measured angles for the different ground inclinations α.

In table L-2 the measured angles between L_1 and the platform for every increment in tooth is presented.

Tooth #:	1	2	3	4	5	6	7	8	9	10
Angle [deg]:	89.9	89.9	89.9	89.9	89.9	89.9	89.9	89.9	89.8	89.8

Tooth #:	11	12	13	14	15	16	17	18	19	20
Angle [deg]:	89.8	89.6	89.6	89.6	89.4	89.4	89.2	89.2	89.1	88.9

Tooth #:	21	22	23	24	25	26	27	28	29	30
Angle [deg]:	88.9	88.6	88.5	88.5	88.2	87.8	87.5	87.5	87.1	86.8

Tooth #:	31	32	33	34	35	36	37	38	39	40
Angle [deg]:	86.5	86.3	86.2	86.0	85.7	85.4	84.8	84.3	83.9	83.6

Tooth #:	41	42	43	44	45	46	47	48	49	50
Angle [deg]:	83.0	82.7	82.3	81.7	81.1	80.5	80.1	79.6	78.9	78.3

Tooth #:	51	52	53	54	55	56	57	58	59	60
Angle [deg]:	77.5	76.6	75.7	74.9	74.0	72.8	71.7	70.4	68.7	66.8

Tooth #:	61	62
Angle [deg]:	64.3	60.8

Table L-2: Angles for every tooth.

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Designing a passive self-levelling device for uneven grounds; balanced pendulums, variable stiffness & locking mechanisms.

Thijs A. Heldoorn and Just L. Herder

***Abstract*—In this paper an overview is made of separate mechanisms that are useful for the design of a passive self-leveling platform for uneven grounds. These mechanisms are balanced pendulums, variable stiffness mechanisms and locking mechanisms. First a reasoning of why these systems are useful is presented, followed by an overview and a comparison of these mechanisms. The overview shows some basic properties/characteristics of the mechanisms. For the comparison the efficiencies of the stiffness mechanisms are calculated, and the possible combinations between locking mechanisms and balanced pendulums are assessed. The overview and comparison of the mechanisms will provide a basis to see which mechanisms are available and why they are useful in the design a self-leveling platform for uneven grounds. It turned out that it is difficult to compare the different mechanisms, due to unavailability of all relevant data. Two locking mechanisms turned out to be the most widely applicable in locking the various balanced pendulums.**

***Index Terms*—self-levelling, device, balanced pendulums, variable stiffness mechanisms, locking mechanisms.**

I. Introduction

CHAIRS are often used in places without level or horizontal ground. Uneven ground levels cause instability of these chairs. Non-horizontal grounds reduce the sitting comfort and might even cause tilting of these chairs. In the past several attempts have been made to solve both these issues separately. These researches mainly focused on chairs intended for indoor use, where no major fluctuations in ground-level or major inclinations are present. In [1] and [2] , two mechanisms are presented that can compensate for slight height-differences in floor surfaces, or for slight differences in the length of the chairs legs. These differences are compensated by springs or soft materials that are flexible and can deform when under pressure.

Previous work has also been done on remaining a horizontally leveled chair on non horizontal surfaces. These works have mainly focused on chairs that can be used on ships, which need to remain in a horizontal position despite the rolling and pitching motions of the ship caused by the waves. Some of these systems are solely based on a counterweight to keep the chair leveled [3] [4]. Other systems make use of four-bar mechanism that behave like a pendulum in combination with springs (leaf springs [5] as well as normal springs [6]). Yet another system makes use of a combination of springs, four-bar mechanisms and a counterweight [7].

Though the previous systems (in combination with one another) might be able to reduce the earlier mentioned problems, they do not provide a full solution. The systems are only suited for situations where the difference in ground height is small. Next to that these systems, once leveled, do not offer a rigid chair that can be actively used by its occupant. Sudden motions of the chairs occupant or even trying to sit down on the chair can already result in rocking motions of the system. Since most of these systems (especially the ones based on counterweights) are very heavy, they are not useful for an active outdoor use.

In this paper a foundation is made for the design of a new kind of passive self-leveling platform device for uneven grounds. This device should be able to keep a platform leveled horizontally, regardless of the slope and other irregularities of the underground. The choice for a passive system was made because this can be applied in many fields. Moreover, making a passive system was considered a fun challenge.

The platform, once leveled, should be able to withstand forces without losing its current position and orientation. Because no literature on similar devices could be found this paper provides an overview of separate mechanisms that are useful for the design of such a device. The goals of this paper are: 1. ”*Make an analysis of mechanisms that are usefull for the design of a self-levelling platform*”. 2. ”*Create an overview of the mechanisms to aid the design of a self-levelling platform*”. 3. ”*Compare the mechanisms to see which ones are most usefull for the design of a self-levelling platform*”.
First the method used to reach these goals is discussed. After this, the results will be shown, followed by the discussion and conclusion.

II. Method

A. Problem Analysis

The following analysis will provide reasoning of why certain mechanisms are useful for the design of a passive self-leveling platform. During the analysis the following requirements on the system are taken into consideration.

T.A Heldoorn and J.L. Herder are with the Delft University of Technology, Faculty of Mechanical, Maritime and Materials Engineering, Department of Bio-Mechanical Engineering, Mekelweg 2, 2628 CD Delft, The Netherlands.

The complete device should:

- Have a low mass
- Be simple
- Have minimal dimensions

It can be argued that the first requirement is included in the third, but the first argument is stated anyway to make sure no excessive weight are used to balance the system. Next to that the system should be as simple as possible, meaning it consists out of simple elements or mechanisms. The final requirement, minimal dimensions, should ensure that the system is as compact as possible.

A passive self-levelling device should find its level relative to the horizontal irrespective of the underground on which it is placed. To be able to do this the system should be able to move under the influence of forces and have an equilibrium position which is dependent on the orientation of the device. The equilibrium position is the position in which the system will have a (local) minimal amount of potential energy, which for a system without energy storing elements means the center of mass is positioned as low as possible. By adding elements with energy storing capabilities, it is possible to alter the equilibrium position to virtually every position in the systems range. Next to that these energy storing elements can be able to compensate for external forces thereby cancelling them out. Both of these situations describe systems that are balanced under certain conditions. Since a passive self-levelling device requires the alteration of equilibrium position and cancelling of external forces, these kind of systems are useful for the design of a self-levelling chair. This means simple and movable mechanisms with an equilibrium position that can be altered through balancing is needed. The simplest mechanism that fits these requirements is a pendulum, which is the reason why this group of mechanisms is referred to as 'balanced pendulums' from now on. However, the search method involved active search for pendulums, as well as for another simple mechanism with behavior which resembles a pendulum: four-bar mechanisms. Though there are other systems with a minimal of potential energy that can fit the description, the two previously mentioned were considered the most general ones. No active search for other mechanisms than four-bar mechanisms and pendulums was conducted. However, if these mechanisms were found and considered relevant, they were added to the analyses.

As already explained the system should be movable in order to be able to find an equilibrium position, so the system can find its level. However, once leveled, the systems platform has to be able to sustain (external) forces without any deviation from this level. These contradictive demands (movable system non-movable system) can be realized if the forces causing movement of the system are counteracted by other forces once the level position is reached, thereby creating a force equilibrium. These countering forces can be generated easily by securing the system completely, thereby not allowing any movement. All forces applied to the system will now be instantaneously counteracted at the securing points, thereby keeping the system in equilibrium. In the movable situation these securing points should be released, thereby allowing movement under the influence of forces. To solve these points only one kind of mechanism is logical; locking mechanisms.

As previously explained locking mechanisms are a logical choice to allow a system to be movable in certain situations, and not movable in others. In the locked situations no movement of the system is allowed, independent of the size of the force that is applied. This situation is comparable to a very high stiffness, which does not allow displacement despite very large forces. The unlocked situation on the other hand can be compared to a very low (going to zero) stiffness, which allows large displacements with little (zero) resistance. Therefore it can be argued that a locking mechanism is a binary stiffness. This raises the question if there are mechanisms with more than two stiffness values that can be altered? If these mechanisms are available they can be used for multiple things. They can be used for fixing the system once leveled, but these mechanisms can also reduce the time required for the system to reach its equilibrium position by adjusting the stiffness dependent on the deviation from horizontal.

Balanced pendulums can use elasticity (e.g. from springs) to alter a systems position with minimal potential energy. However, the equilibrium position obtained this way is dependent on the angle with the gravitational force and is therefore not constant relative to the horizon. A mechanism that can change its stiffness can be used here to continuously adjust the elasticity force relative to compensate for the changes in the gravitational force. For this reason and the reasons mentioned in the previous paragraph, variable stiffness mechanisms are usefull for the design of a passive self-leveling chair.

The three useful mechanisms for the design of passive self-levelling platforms that will be considered in this paper are:

Balanced pendulums: "A balanced pendulum (four-bar mechanism) consists of a regular pendulum (four-bar mechanism) that can be balanced in other positions than its non-balanced equivalent."

Variable stiffness mechanisms: "A stiffness that can be increased or decreased by altering its internal parameters, thereby allowing alteration of the force-displacement curve."

Locking mechanisms: "A locking mechanism is considered a binary stiffness, with either a (very) low or a very high (infite) stiffness."

B. Search Method

The search for literature is separated into four different parts. In the first part a search was conducted for self-leveling mechanisms, to find out what knowledge and products are already available. In the second to fourth part the three previously mentioned mechanisms (variable stiffness mechanisms, locking mechanisms and balanced pendulums) are subject of the research. The search engine 'Scopus' was used to find literature about these four individual subjects. All searches were limited to only include the 'physical sciences', and within this group the results were limited to only include topics related to 'Engineering'. The keywords used are presented in table I. These keywords have been combined to limit the number of results. Initially the literature got selected based on titles, after which the abstracts, figures and tables were checked to determine the papers relevance for this research. The literature that remained was read, and if the literature was considered to be adequately relevant its references were checked for other relevant literature. The search engine 'Scholar Google' was used to find patents and also to find references that could not be tracked with Scopus.

Mechanism	Search terms
Self levelling	- Self- - levelling, leveling - chair, mechanism, platform, system, uneven ground
Balanced pendulum	- passive - balanc*, weight-compensation - pendulum, four-bar mechanism
Locking mechanism	- Self-, Passive - Locking mechanism
Variable stiffness	- Variable, Adjust*, Chang* - Stiffness, Stiffness mechanism

TABLE I: Overview of search terms that were used.

C. Analysis

The conducted literature research led to the finding of a variety of mechanisms. The mechanisms as found all have different characteristics and properties. Since it is unknown which mechanisms are suited most for implementation in a self-leveling device comparisons need to be made. The results of these comparisons can be useful in a later stage, when precise design requirements are known. First an overview of the selected mechanisms from literature and their properties is presented. The different aspects that will be looked at are explained here.

1) Balanced pendulums: For the comparison of the balanced pendulums first the different balancing methods are compared. The balancing methods can be based on a hellical spring, a leaf spring, a counterweight or on compliance. After this the mechanisms are qualified as systems based on a pendulum, a four-bar mechanism or on an 'other' system. Some of the systems consist of a series of pendula or four-bar mechanisms. A distinction is made between systems that can be placed in series and systems that cannot. A distinction is also made between rotational and translational balancing. (It might be possible for some devices to balance in both these ways.) Some devices once set to balance in a certain position, cannot maintain this balance when the underground (direction of gravity) is tilted. Therefore a distinction is made between systems that can be tilted without the need of adjusting and between systems that need adjustment. Finally, the possibility to create complicated value functions for balancing under various conditions is considered.

2) Variable stiffness mechanism: The variable stiffness mechanisms will be compared on their basic characteristics and their performance. First the working principles on which the mechanisms are based are identified. These working principles can be based on a hellical spring, a leaf spring or compliance. After this the way in which the variable stiffness mechanisms can be loaded is analyzed. The mechanisms can be loaded in bending, compression or rotation. The range over which the stiffness can be varied is also an important factor and thus will be compared as well. Finally, the highest stiffness value that can be obtained by the mechanism is compared.

3) Locking mechanisms: Different ways of locking are distinguished. A lock can prevent for example a rotational or a translational movement. Some locks can prevent one type of movement in two directions (e.g. a negative and positive rotation), while other locks can only prevent one movement (e.g. only a positive rotation). If only one direction of movement can be locked this is noted with a (+). A two-directional movement that can be locked is noted with a $(+/-)$. (The $+$ and $-$ do not give any information about the actual direction of this movement or the related forces.) Some locks provide more freedom than others. For every lock the range of all the allowable rotations is compared. (The translational motion allowed is dependent on a systems dimensions, and therefore not considered.) Finally the failure load of the different locking mechanisms is compared.

D. Comparison of stiffness mechanisms

The stiffness values of the stiffness mechanisms can be immediately compared to one another, though this gives a distorted view. Since all the mechanisms tend to have different dimensions, masses and initial stiffness's it is normal to see large differences in the final stiffness values. Next to that the displacement and working direction of the initial stiffness's can also vary. (Note: The initial stiffness is seen as the working principle of a mechanism. If a spring is connected to a pulley to create a rotational stiffness mechanism, the spring is seen as the 'initial stiffness', while the resulting rotational stiffness is the 'mechanism stiffness'.)

In order to make a fair comparison between the stiffness mechanisms it is decided to look at the potential energies of the stiffness mechanisms relative to the potential energies of the initial stiffness's. The potential energy gives a value for the energy stored, independent of the direction of movement, the elongation or rotation and the stiffness.

The formula used to calculate the potential energy of a rotational stiffness is:

$$U = \frac{1}{2}K\theta^2$$

The formula used to calculate the potential energy of a stiffness/spring is:

$$U = \frac{1}{2}kx^2$$

where K is the rotational stiffness (Nm/rad) , k the stiffness (N/m), θ the angle of rotation (rad) and x the elongation (m).

With these two values the mechanical efficiency of a system can be calculated as follows:

$$\eta = \frac{U_1}{U_2}$$

where U_1 is the 'mechanism stiffness' and U_2 is the 'initial stiffness' as defined above. These efficiencies give a value of how much energy a system can produces relative to the energy initially stored in the system. This can be used to determine how effective a mechanism can produce a stiffness relative to a given initial stiffness.

Note: The values that will be used for these calculations have to come, or must be derivable from practical experimental data. This is to make sure that the efficiencies as presented in this paper are attainable.

E. Combining locking mechanisms and balanced pendulums

Balanced pendulums are interesting because of their ability to balance pendulums in non-vertical positions. However, if external forces are applied to these pendulums, they can lose this balance. If on the other hand the pendulums could be locked in their (non-horizontal) equilibrium position, forces can be applied without influencing the pendulums position. Because of this the possible integration of locking mechanisms and balanced pendulums is analyzed. This is done by looking at the working principles of the locks and pendulums and see if these can be combined. If it is considered likely that a combination between a lock and a pendulum will work, this combination gets rated a $+$. If a combination of the two is possible, but minor alterations need to be made to one or both of the mechanisms, the combination gets rated a $\bullet$. If the combination is considered unfeasibly or requires major alterations to one or both of the mechanisms, it gets rated a $-$.

Some assumptions are made before the combinations are reviewed. It is assumed that the pendulum can have multiple equilibrium positions or a variable equilibrium position relative to its base. Therefore an optimal lock should be able to support all equilibrium positions. Another assumption is that the actuation that might be required for some of the locks is provided in a mechanical way. Details about this actuation are not considered, a rough idea about how this actuation might be realized is considered enough.

Note that in some cases data has been estimated based on figures, graphs, charts or other data. This was only done if these values could not be obtained from text.

III. Results

A. Overview of the mechanisms

Balanced pendulums: In this paper several kinds of balanced pendulums are analyzed, of which an overview is made in table III . It can be noticed that all mechanisms except for one rely on a spring for balancing, while the remaining mechanism of Hirose [9] works with a counterweight. Hirose's mechanism is however the only system that does not necessarily require a level horizontal in order to work properly. All systems found are pendulums or four-bar mechanisms. The system of Riele [8], does at first glance not look like a pendulum. However, the part that Riele described as a link can also be regarded an inverted pendulum that extends on both side of its revolute joint. All systems except for one perform rotational balancing, meaning that they balance a system that tends to rotate. Only the system of Cole [11] balances a translational motion. Endo's mechanism [12] is the only system which can describe complicated value functions. This can be done by changing the shape of the non-circular pulley used to balance the system. Finally it can also be observed that all systems that have the possibility of adding multiple links balance a rotation.

Adjustable stiffness mechanism: Table IV gives an overview of the variable stiffness mechanisms that were found. The working principles (initial stiffness's) behind these mechanisms vary; systems based on hellical springs, leaf springs and on other principles were found. The stiffness's of the complete mechanisms also differed. The mechanisms that were found can be loaded in torsion, compression or bending.

The value of the stiffness can be varied for every system, but for some it can be varied more than for others. Where Kilic [15] can obtain a maximum increase of 3 times its lowest stiffness value, Myung [17] can reach an increase of up to 50 times. The maximum stiffness that can be reached by the mechanisms also differs a lot. The mechanism proposed by Gonzalez [14] can reach a stiffness of up to 16.000 N/m, while the system of Myung [17] can reach up to $38.56N/m$. And while Hayashi [16] reaches a rotational stiffness of $0.248Nm/deg$, Kilic [15] reaches a much lower maximum stiffness of $0.0071Nm/deg$. The final point in the comparison is the possibility of implementing a value function for the stiffness. Only the system of Kilic [15] has this possibility, which can be achieved by changing the shape of wrapping cams. This system is comparable to the way in which Endo [12] constructs value functions for balanced pendulums.

Locking mechanism: An overview of the locking mechanisms can be observed in table V. The system of Qiao [21] is the only system that locks a translational movement. This is done by pressing a wedge between two surfaces, thereby increasing the friction. The other systems all block one or multiple rotations. This is done by creating a singularity ([23]), wedging of a clutch ball ([24]), or by friction ([22]). The freedom of movement before locking also varies. The translational freedom of movement can be very large, and is independent of the lock. The rotational movement can also be large ($> 360deg$), which means that a system can fully rotate multiple times before locking takes place. The lock of Chu

[24] is able to do this. The locking mechanisms of Peerdeman [22] can move over an estimated maximum range of $360°$, but due the structure of the mechanism it cannot rotate multiple times. The system of Oort [23] allows a rotation of maximum $180°$, while the system of Kern [20] allows a rotation of $20°$ per subsequent collar in all directions.

B. Comparison of stiffness mechanisms

The maximal efficiencies of the several stiffness mechanisms were calculated and are presented in table II. Not all mechanism are presented in this table for reasons mentioned in the discussion. It can be noted that Uemura [18] and Kilic [15] clearly have higher efficiencies than the mechanism presented by Hayashi et all [16]. The efficiencies of both Uemura and Kilic are in the same order of value, at an efficiency of about 50%. Both these systems use springs to create a rotational stiffness.

Authors:	Efficiency:
Hayashi et al. (2004)	0.20
Uemura et al. (2010)	0.44
Kilic et al. (2012)	0.51

TABLE II: Efficiencies of the stiffness mechanisms.

C. Combining locking mechanisms and balanced pendulums

VI presents a table of combinations of balanced pendulums and locking mechanisms. An elaboration on these combinations will be presented here.

The locking mechanism as presented by Kern [20] consists out of 7 sets of mating collars that can all be locked individually. If all 7 collars are locked the system behaves like a stiff rod. One unlocked collar has a rotational freedom of $20°$ in all 3 directions. This system can be used to lock rotating pendulums or rotation four-bar mechanisms, though this calls for a slider joint attached to the pendulum, to account for the changes in the locking mechanisms effective length. Multiple locking positions are possible in which a downward movement of the pendulum cannot be realized. Upward motions on the other hand are still possible in those positions. Sideward movements are possible with this mechanism, though the system cannot be locked in these positions rendering this system useless for balancing in 3D.

The locking mechanism of Qiao [21] is able to block translations in one direction. This can be used to block the system of Cole [11], which contains a platform that can translate vertically. This locking mechanism is not likely to be applicable in other balanced pendulums, because these do not have any translations that can be blocked.

The locking mechanism of Peerdeman [22] is made in such a way that it can be easily implemented to lock a single rotation. Because of this the system can be used to lock the system of Riele [8], by locking the relative rotation of the triangles. It can also be applied on the system of Hirose [9], Nathan [10] and Endo [12] by blocking the joint (or joints) around which the pendulum or four-bar mechanism rotates. The system of Cole [11] can also be locked by locking one or two of the sidebars of the four-bar mechanism. If these sidebars cannot rotate, the platform supported by them cannot translate vertically. A combination between Peerdeman [22] and Morita [13] however is questionable. Since Morita's system has a pendulum that is balanced in 3D, with 3 rotational joints located at the same position (ball joint). It is impossible to lock all these 3 DOF with a lock that can only lock one rotation. If multiple locks of the same sort are used, or if the joints are placed in series, it is possible to use this locking mechanism for Morita as well.

The system of Oort [23] can block one rotation by creating a singularity. This singularity however, can only be achieved in one particular position. This means that this locking mechanism can only lock the pendulum in one position, while other locking mechanisms can lock the pendulum in multiple positions. Because of the way the lock works the pendulums can also not rotate beyond the position in which locking takes place (blocked by the singularity). All the systems with one or more rotational joints can be locked by this mechanism, but it places serious limitations on the pendulums. For this reason these combinations are graded a $\bullet$. This mechanism in combination with Morita [13] gets a $-$, because Morita uses a ball joint which is difficult to lock with this system.

The system of Chu [24], consisting out of a driving and a driven plate within a housing can be used to lock a single rotation. The driving plate can be connected to an existing joint, and the housing to the pendulum or four-bar mechanism. Now an external force applied to the driven plate (which copies the motion of the driving plate relative to the housing) in opposite direction to the driving force can lock this system. This locking is facilitated by wedging balls between the driving and driven plate. The external force can be provided by hand, but might also be provided by other locking mechanism like the system of Peerdeman [22]. As mentioned this system can be used to lock systems with one or more rotational joints. Morita [13] contains a ball joint, which therefore cannot be easily locked. If alterations are made to Morita which replace the ball joint by three single rotational joints this combination would be possible.

IV. Discussion

In this part the results will be discussed. First it should be noted that not all literature related to the three different mechanisms has been reviewed due to the scope of this research. Especially in the case of balancing mechanisms a lot more literature can be found.

Not all balancing mechanisms that were found were actually balanced pendulums or balanced four-bar mechanisms. The mechanism of Cole [11], though originating from the analysis of four-bar mechanisms, does not conform to the idea of a balanced four-bar mechanism. Also the system of Riele (though it can be argued that it is a pendulum, and is also considered a pendulum in table III) does not have to be considered a typical example of a pendulum. Since these system were found with the search methods as described

Paper	Hellical Spring	Leaf Spring	Counterweight	Pendulum	Four-bar mechanism	Other	Rot. balancing	Trans. balancing	Requires horizontal	Value functions	Multiple links
Riele et al. (2001)	•			•			•		YES	NO	NO
Hirose et al. (2003)			•		•		•		NO	NO	YES
Nathan (1985)	•			•	•		•		YES	NO	YES
Cole et al. (2008)	•					•		•	YES	NO	NO
Endo et al. (2010)	•			•			•		YES	YES	YES
Morita et al. (2003)	•			•			•		YES	NO	YES

TABLE III: Comparison of the balanced pendulums.

Paper	Hellical Spring	Leaf Spring	Other	Torsion	Compressing	Bending	Increase (x)	Value function	Max. stiffness
Gonzalez et al. (2011)		•			•		15	NO	$16000 N/m$
Kilic et al. (2012)	•			•			3	YES	$0.0071 Nm/deg$
Hayashi et al. (2004)		•		•			4	NO	$0.248 Nm/deg$
Myung HUH et al. (2012)			•			•	50	NO	$38.56 N/m$
Uemura et al. (2010)	•			•			21	NO	$0.044 Nm/deg$
Anubi et al. (2010)	•			•				NO	

TABLE IV: Comparison of the adjustable stiffness mechanisms.

Paper	Rotations before lock	Translations before lock	Blockable rotations	Blockable translations	Rotational freedom 1	Rotational freedom 2	Rotational freedom 3	Failure load
Kern et al. (2009)	3	0	3 (+/-)	0	$20°$	$20°$	$20°$	$980 N$
Qiao et al. (2011)	0	1	0	1 (+)	-	-	-	$15.2 N$
Peerdeman et al. (2012) Gear:	1	0	1 (+)	0	$360°$	-	-	$2 Nm$
Peerdeman et al. (2012) Pall:	1	0	1 (+)	0	$360°$	-	-	$1 Nm$
Oort et al. (2011)	1	0	1 (+/-)	0	$180°$	-	-	-
Chu et al. (2008)	2	0	2 (+/-)	0	inf	inf	-	-

TABLE V: Comparison of the locking mechanisms.

Balanced pendulum: / Locking mech.:	Kern et al. (2009)	Qiao et al. (2011)	Peerdeman. (2012)	Oort et al. (2011)	Chu et al. (2008)
Riele et al. (2001)	−	−	+	•	+
Hirose et al. (2003)	•	−	+	•	+
Nathan (1985)	•	−	+	•	+
Cole et al. (2008)	−	+	+	•	+
Endo et al. (2010)	•	−	+	•	+
Morita et al. (2003)	−	−	•	−	•

TABLE VI: Combinations of balanced pendulums and locking mechanisms.

earlier, and since both systems were considered relevant and worth consideration, they were added to this research.

All articles that were reviewed were written with different intentions; some wanted to present a new method or design, while in other cases the mechanism could be a component of a bigger system. The system of Oort [23] for example, is intended as a locking mechanism designed for the energy efficient locking of a knee belonging to a walking robot. Because of these different intentions, not all authors presented the same data in their work. For example the failure load of the locking mechanisms is only mentioned in half of the cases. This increased the difficulty to make fair comparisons between the different mechanisms. In some cases it was possible to estimate data values based on other data, figures or graphs. Though this was done as accurately as possible, it leaves room for slight errors. However, these errors are considered small enough to not significantly influence the general results.

In order to compare the various stiffness mechanisms their efficiencies were presented in table II. It can be noticed that not all stiffness mechanisms were present in this table. In the case of Myung [17] and Gonzalez [14] this was due to lack of easily accessible data, which mend that extensive calculations had to be performed requiring numerical computing. The work of Anubi [19] is also not mentioned in the table because his work did not include any practical model, due to which no data was available. It should also be noted (again) that the values used for calculating the efficiencies were taken (or derived) from practical experimental results. This means that in some cases it is possible to achieve higher efficiencies than mentioned in table II, for cases were higher values have not been tested.

In table VI the system of Chu [24] was compared to other locking mechanisms. It must be noted that it is estimated that this system still moves over an estimated 5-10 degrees after the locking is activated. This can be overcome by starting the locking before the final locking position is reached, but it adds to the complexity of the system. In the results it is also mentioned that Chu's system can possible be locked by using one of the other locking mechanisms. This seems very redundant, and in this situation it would be easier to directly apply the second locking mechanism to the pendulum.

It was argued that Kerns [20] mechanism was able to lock several balanced pendulums. However, since Kerns system consists out of 7 different independent locks, this means that 7 individual actions are needed to be able to independently control all the individual locks. This seems rather complicated compared to the other mechanisms. As kern argued however, it is possible to decrease (or increase) the number of collars (and thus locks) in his system. This makes the system easier to control, but has the drawback of reducing the range over which the system can move.

In the comparison of Morita's system [13] with the locking mechanisms, it is argued that in some cases slight alterations need to be made to Morita to overcome possible problems. It is not tested if these alterations are actually possible without decreasing the performance of the system. From the analysis it can be seen that Morita's system is the only balanced pendulum that does not have a good possible combination with a locking mechanism. This is because it is hard to lock a ball joint with locks that are intended for locking single rotations.

V. Conclusion

In this paper an overview is made of several mechanisms. Within each category several different mechanisms have been found and analyzed.

Most of the balanced pendulums are based on springs. This suggests the use of springs is beneficial compared to using other systems as counterweights, leaf springs or compliance. On the other hand all these spring based systems require a horizontal underground in order to work as intended. Since the final goal is to create a level platform on uneven undergrounds this does not match well. Additional mechanisms or a new kind of balanced pendulum are required if the system has to be based on springs. Hirose's system [9] is based on a counterweight and able to handle uneven undergrounds, but the use of a counterweight has the disadvantage of an increased mass.

A variety of stiffness mechanisms could be found. Increases (or decreases) in stiffness of 50 times are achievable if the proper mechanism is selected. In the comparison on efficiency it turned out that the two systems based on a normal spring have a high efficiency compared to the system based on a leaf spring.

A variety of locking mechanisms was found. Because all mechanisms possess some unique properties it is difficult to compare them. Some of the locking systems that were found can offer good possible solutions, but require increased strength in order to deal with large forces.

From table VI it becomes clear that it is unlikely that Morita's 3D balancing system [13] can be properly locked. All the 1D balanced pendulums on the other hand have at least one way in which it is likely that they can be locked correctly. Morita's 3D balancer would ideally require a lock that can effectively lock a ball joint, but this kind of lock has not been found during this research.

The systems of Peerdeman [22] and Chu [24] are the two locks that fit with every balanced pendulum. Chu's system however requires more attention during locking because of play. Therefore it seems that Peerdeman's locking mechanism is the best for locking a balanced pendulum.

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